FSO Cooperative All-Active and Selective Relaying Schemes with Backup RF Antennas

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***Abstract*—In this paper, we consider the problem of parallel relaying decode-and-forward (DF) cooperative free space optical (FSO) networks with backup radio frequency (RF) links. We analyze and compare the all-active and selective relaying schemes for 2-relay and 3-relay systems where these schemes can be implemented in the absence and presence of channel state information (CSI), respectively. The analysis is carried out in the case of fully connected systems where each relay is connected to all other relays. We compare the performance of both schemes using an exact outage probability analysis for gamma-gamma FSO fading channels and Rician/Rayleigh RF fading channels. We then apply the cut-set method to derive an upper-bound on the asymptotic performance for the sake of quantifying the achievable diversity orders and coding gains.**

Keywords—FSO; RF; cooperation; relaying; all-active; selective; outage probability; diversity order; gamma-gamma; Rician; Rayleigh.

I. INTRODUCTION

When transmitting a packet from a source S to a destination D, several transmission methods can be used. Free Space Optics (FSO) constitutes one of these methods that have been widely investigated for the sake of achieving high data rates. User-cooperation has also been advised for FSO systems where the all-active and selective relaying techniques were implemented [1]-[3]. All-active relaying is defined when all relays participate in the cooperation effort by simultaneously forwarding decoded or amplified versions of the received information symbols to the destination node. This approach is characterized by a remarked simplicity since it can be implemented without the need of acquiring any form of channel state information (CSI) [1]. Selective relaying was introduced as an alternative to all-active relaying where only one relay is selected from all available relays based on the state of the FSO network. The selective-relaying is superior to all-active relaying at the expense of an increased complexity since we need to have CSI for the sake of selecting the relay with the best end-to-end link [2], [3].

In addition to adopting the conventional two-phase source-relay and relay-destination techniques, exploiting inter-relay links was found to be beneficial either through one-way links or two-way links [4]. The added value of inter-relay links was investigated in the case where only adjacent relays were connected [4]. This model was further extended in [5] to the case of all-to-all connections where each relay is connected to all other relays through the backup radio frequency (RF) links. In fact, RF links backup the FSO links in commercially available FSO transceivers and all-to-all connections can be established owing to the broadcast nature of RF transmissions [6]. More precisely, the FSO links provide high data rates, but become practically unavailable under severe weather conditions like fog or snow. This makes FSO systems with RF backup links beneficial as they take advantage from the high data rate of the FSO link and the high reliability of the RF links [7], [8].

The contributions of the paper are as follows. We extend the adjacent-connectivity model in [4] to the complete-connectivity model for systems with 2 and 3 relays. While this latter model was investigated in [5] under the context of all-active relaying with any number of relays, this work considers the scenario of selective relaying as well. We compare the all-active and selective schemes both analytically and numerically and we prove that they achieve the same diversity order. In this context, the advantage of selective relaying resides in a coding gain.

II. SYSTEM MODEL

We consider an intensity modulation with direct detection (IM/DD) FSO system with a source node S and a destination node D. Parallel relaying is considered with N neighboring relays denoted by $R_1,R_2,\ldots,R_N$ such that the S-D, S-R and R-D links are of FSO nature. In addition, the system establishes RF communications between the relays using the backup RF links. An example of a cooperative FSO network with RF inter-connected relays is provided in Fig. 1 for 3 relays.

The FSO links are modeled as gamma-gamma distributed with a probability density function (pdf) given by [4]:

$$f(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{(\alpha+\beta)/2-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta I}\right) \quad (1)$$

where I is the irradiance fluctuations along the FSO link caused by atmospheric turbulence, $\Gamma(.)$ is the Gamma function and $K_c(.)$ is the modified Bessel function of the second kind of order c. The parameters α and β are given by:

$$\alpha(d) = \left[\exp\left(0.49\sigma_R^2(d)/\left(1+1.11\sigma_R^{12/5}(d)\right)^{7/6}\right)-1\right]^{-1} \quad (2)$$

$$\beta(d) = \left[\exp\left(0.51\sigma_R^2(d)/\left(1+0.69\sigma_R^{12/5}(d)\right)^{5/6}\right)-1\right]^{-1} \quad (3)$$

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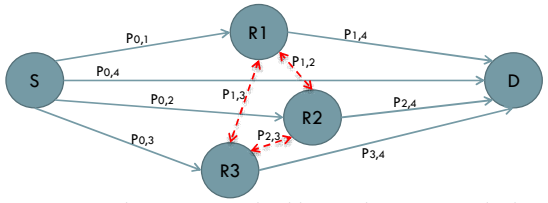

Fig. 1. A cooperative FSO network with 3 RF inter-connected relays. Solid lines and dashed lines correspond to FSO and RF links, respectively.

where $\sigma_R^2(d)$ is the Rytov variance related to the link distance d by:

$$\sigma_R^2(d) = 1.23 C_n^2 k^{7/6} d^{11/6} \quad (4)$$

The outage probability will be calculated in terms of the optical power margin that is defined as follows:

$$P_M = \sqrt{\frac{\gamma_{0,N+1}}{\gamma_{th}}} \quad (5)$$

where $\gamma_{0,N+1}$ is the average electrical SNR along the S-D link and γ_{th} is the threshold SNR below which an outage event occurs. In other words, the FSO link will be in outage when the signal-to-noise ratio (SNR) falls below the threshold level γ_{th} implying that the outage probability of the FSO link can be determined as:

$$P_{ij}^{(N_{tot})} \triangleq \Pr(\gamma_{i,j} < \gamma_{th}) = \Pr\left(I_{i,j} < \frac{N_{tot}}{G_{i,j} P_M}\right) \quad (6)$$

where N_{tot} denotes the number of activated links with $N_{tot} = 2N+1$ for all-active relaying where the optical power is evenly split among the $2N+1$ available links. For selective relaying, N_{tot} will be either 1 or 2 depending on whether the one-hop direct S-D link or an indirect two-hop S-R-D link is selected. Finally, the gain factor $G_{i,j}$ can be calculated as follows:

$$G_{i,j} = e^{-\sigma(d_{i,j} - d_T)} \quad (7)$$

where σ is the attenuation coefficient and d_T is the distance of the S-D link. In (7), $d_{i,j}$ denotes the distance between R_i and R_j (where S and D are denoted by R_0 and R_{N+1}, respectively).

Now for the RF R-R links, we use either the Rician model when the Line-of-Sight (LOS) is available or the Rayleigh fading model when the LOS is absent. The resulting outage probabilities for the RF links are given by [9]:

For Rayleigh fading:

$$P_{ij} = 1 - \exp\left(-\frac{\Omega_{th}}{\Omega_{i,j}}\right) \quad (8)$$

For Rician fading:

$$P_{ij} = 1 - Q_1\left(\sqrt{2K_{i,j}}, \sqrt{\frac{2(K_{i,j}+1)\Omega_{th}}{\Omega_{i,j}}}\right) \quad (9)$$

Where $\Omega_{th} = 2^{2R} - 1$ is the threshold SNR where R is number of bits transmitted per channel use, $\Omega_{i,j}$ is the average SNR along the RF link connecting R_i with R_j, Q_1 stands for the first-order Marcum Q-function, and $K_{i,j}$ is the Rician factor of the power in the LOS component compared to the power in the non-LOS components.

III. Cooperation Strategies

A. All-Active Relaying

In all-active relaying, no knowledge of the channel state is assumed implying that all links are activated in the process. The cooperation strategy is divided into four phases:

- S-D phase: The packet is sent from the source to the destination along the direct S-D FSO link.
- S-R phase: In case the S-D link fails in delivering the packet, the packet is resent from the source to all relays using the S-R FSO links.
- R-R phase: The packet received by any of the relays is broadcasted to all other relays using the R-R RF links.
- R-D phase: All the relays that successfully received the packet forward this packet along the R-D FSO links to the destination D.

B. Selective Relaying

In selective relaying, the channel state information (CSI) is available and thus the strongest path is selected for transmission. The candidate paths that can be selected for carrying the information from S to D are as follows:

- S-D, S-R_1-D, S-R_2-D, S-R_1-R_2-D, and S-R_2-R_1-D for two relays.
- S-D, S-R_1-D, S-R_2-D, S-R_3-D, S-R_1-R_2-D, S-R_1-R_3-D, S-R_2-R_1-D, S-R_2-R_3-D, S-R_3-R_1-D, S-R_3-R_2-D, S-R_1-R_2-R_3-D, S-R_1-R_3-R_2-D, S-R_2-R_1-R_3-D, S-R_2-R_3-R_1-D, S-R_3-R_1-R_2-D and S-R_3-R_2-R_1-D for three relays.

In order to simplify the outage analysis, conditioning on the R-R links will be carried out first. Thus, based on the state of the R-R links we will have smaller number of paths to investigate. In fact, our system will look as multiple S-R-D paths where new virtual relays are constructed combining all connected relay. For example, with 2 relays, if the R_1-R_2 link is active, then $R_{1\text{-}2}$ is a new virtual relay combining relays R_1 and R_2; the new system has only 1 path S-$R_{1\text{-}2}$-D where the strongest path from S-R_1 and S-R_2 is chosen to represent S-$R_{1\text{-}2}$ and the strongest path between R_1-D and R_2-D is chosen to represent $R_{1\text{-}2}$-D. Now with R_1-R_2 in outage, we have 2 separate paths to investigate: S-R_1-D and S-R_2-D. It is important to note that the S-D link is always taken into consideration in the paths' strength analysis. The same strategy is applied for any number of relays as will be highlighted in detail in the outage probability analysis section.

One of the main gains in the selective scheme is power distribution. Instead of splitting the power among all the FSO links, the power will be split only on either 1 link (when S-D is the strongest) or on 2 links (S-R_i and R_j-D where i can be either equal or different from j).

Fig. 2. FSO Network Reduction when Combining Relays.

IV. Outage Probability Analysis for 2 Relays

A. Outage Probability

In this section we study the outage probability of both all-active and selective schemes. We compare the outage probabilities of these schemes with each other and then we benchmark with the non-inter-relay scheme. The exact results are derived using conditional probability approach by conditioning on the inter-relay links.

1) All-Active Relaying

In what follows we separate the S-D link from our calculations. In fact, as this link will be checked in the first phase if it delivers the transmitted packet, thus the determined outage probability can be multiplied with P_{03} to get the final outage probability. In what follows, $P_{i,j}$ denotes the outage probability along the (*i-j*)-th link where the corresponding expression is given in (6) for the FSO links and (8)-(9) for the RF links.

By conditioning on the inter relay link, the outage probability can be expressed as:

$$P_{out} = P_{12}P_{out1} + (1- P_{12})P_{out2} \quad (10)$$

where P_{out1} is determined when the system reduces to the parallel connection between 2 subsystems S-R_1-D and S-R_2-D:

$$P_{out1} = (P_{01} + P_{13} - P_{01}P_{13})(P_{02} + P_{23} - P_{02}P_{23}) \quad (11)$$

and P_{out2} is determined when the system reduces to series connection between 2 parallel subsystems as follows:

$$P_{out2} = P_{01}P_{02} + P_{13}P_{23} - P_{01}P_{02}P_{13}P_{23} \quad (12)$$

After expanding and simplification, P_{out} can be written as:

$$\begin{aligned} P_{out,a} = {} & P_{01}P_{02} + P_{13}P_{23} + P_{01}P_{12}P_{23} + P_{02}P_{12}P_{13} - \\ & P_{01}P_{02}P_{12}P_{23} - P_{02}P_{12}P_{13}P_{23} - P_{01}P_{02}P_{12}P_{13} - \\ & P_{01}P_{12}P_{13}P_{23} - P_{01}P_{02}P_{13}P_{23} + 2P_{01}P_{02}P_{12}P_{13}P_{23} \end{aligned} \quad (13)$$

2) Selective Relaying

In the selective scheme, we define the strength of the link as α. On the other hand, a path of *n* serial links will be in outage when at least one of these links is in outage, usually the weakest link; thus, the strength of this path is defined as the minimum strength among these *n* links. For *n* parallel links, the strength is calculated as the maximum among these *n* links.

In what follows, the S-D link is treated separately as it is always parallel to the remaining subsystem. This means that the total strength of the system will be the maximum of the strength of the S-D link along with the whole remaining system. Thus, we can simply multiply the final result by the outage probability of the S-D link to get the total final answer.

For a system of 2 relays, we will condition on the R-R link. If this link is not in outage, the system will look as in Fig. 2.

Thus, the strength of this system is calculated as:

$$\alpha = \min\{\max\{\alpha_{01},\alpha_{02}\}, \max\{\alpha_{13},\alpha_{23}\}\} \quad (14)$$

The system will be in outage when its strength is below a threshold value X:

$$\begin{aligned} P_{out1} &= \Pr(\alpha<X) = 1-\Pr(\alpha>X) \\ &= 1-\Pr(\min\{\max\{\alpha_{01},\alpha_{02}\}, \max\{\alpha_{13},\alpha_{23}\}\}>X) \\ &= 1-[\Pr(\max\{\alpha_{01},\alpha_{02}\}>X)\Pr(\max\{\alpha_{13},\alpha_{23}\}>X)] \\ &= 1-[(1-\Pr(\max\{\alpha_{01},\alpha_{02}\}<X))(1-\Pr(\max\{\alpha_{13},\alpha_{23}\}<X))] \\ &= 1-[(1-(\Pr(\alpha_{01}<X)\Pr(\alpha_{02}<X)))(1-(\Pr(\alpha_{13}<X)\Pr(\alpha_{23}<X)))] \\ &= 1-[(1-P_{01}P_{02})(1-P_{13}P_{23})] \end{aligned}$$

$$P_{out1} = P_{01}P_{02} + P_{13}P_{23} - P_{01}P_{02}P_{13}P_{23} \quad (15)$$

If the R-R link is in outage, the system simplifies to 2 parallel paths S-R_1-D and S-R_2-D. Thus, the strength is calculated as:

$$\alpha = \max\{\min\{\alpha_{01},\alpha_{13}\}, \min\{\alpha_{02},\alpha_{23}\}\} \quad (16)$$

Similarly, the system will be in outage when its strength is below a threshold value X:

$$\begin{aligned} P_{out2} &= \Pr(\alpha<X) \\ &= \Pr(\max\{\min\{\alpha_{01},\alpha_{13}\}, \min\{\alpha_{02},\alpha_{23}\}\}<X) \\ &= \Pr(\min\{\alpha_{01},\alpha_{13}\}<X)\Pr(\min\{\alpha_{02},\alpha_{23}\}<X)] \\ &= (1-\Pr(\min\{\alpha_{01},\alpha_{13}\}>X))(1-\Pr(\min\{\alpha_{02},\alpha_{23}\}>X)) \\ &= (1-(\Pr(\alpha_{01}>X)\Pr(\alpha_{13}>X)))(1-(\Pr(\alpha_{02}>X)\Pr(\alpha_{23}>X))) \\ &= (1-(1-\Pr(\alpha_{01}<X))(1-\Pr(\alpha_{13}<X))) \\ &\qquad (1-(1-\Pr(\alpha_{02}<X))(1-\Pr(\alpha_{23}>X))) \\ &= (1-(1-P_{01})(1-P_{13}))(1-(1-P_{02})(1-P_{23})) \end{aligned}$$

$$\begin{aligned} P_{out2} = {} & P_{01}P_{02} + P_{01}P_{23} - P_{01}P_{02}P_{23} + P_{13}P_{02} + P_{13}P_{23} - \\ & P_{13}P_{02}P_{23} - P_{01}P_{02}P_{13} - P_{01}P_{13}P_{23} + P_{01}P_{02}P_{13}P_{23} \end{aligned} \quad (17)$$

Consequently, the outage probability P_{out} can be determined as follows:

$$P_{out} = P_{12}P_{out2} + (1- P_{12})P_{out1} \quad (18)$$

After expanding and simplifying:

$$\begin{aligned} P_{out,s} = {} & P_{01}P_{02} + P_{13}P_{23} + P_{01}P_{12}P_{23} + P_{02}P_{12}P_{13} - \\ & P_{01}P_{02}P_{12}P_{23} - P_{02}P_{12}P_{13}P_{23} - P_{01}P_{02}P_{12}P_{13} - \\ & P_{01}P_{12}P_{13}P_{23} - P_{01}P_{02}P_{13}P_{23} + 2P_{01}P_{02}P_{12}P_{13}P_{23} \end{aligned} \quad (19)$$

3) Comparision between All-Active and Selective Schemes

By comparing the outage probability expressions of the all-active and selective schemes, we realize that the expressions derived in (13) and (19) are exactly the same. However, the main difference between them lies in the exact calculation of the probability terms i.e. when going back to the main formula for the outage probability of a single link in (6) and replacing the corresponding power factor value. In fact, with all-active relaying, the power will be distributed among $2N+1$ FSO links where N is the number of relays in the system. On the other hand, with selective relaying, the optical power will be distributed either on 1 FSO link (the direct S-D link) or 2 FSO links (S-R_i and R_j-D) depending on the strength of the S-D link. This means that the FSO links in the selective scheme receive higher power from that in all-active scheme, resulting in lower outage probabilities for the selected links. Also, the RF R-R links are identical in both schemes thus they don't affect the comparison. Therefore, the total outage probability for the Selective scheme will be less than that of All-Active scheme. We will validate this numerically in the numerical results section.

4) Comparision with Non-Inter-Relay scheme

The non-inter-relay scheme can be treated as a special case of the inter-relay schemes where the inter-relay link is always in outage (i.e. $P_{1,2}$=1 in (10) and (18)). This means that the outage probability of the non-inter-relay scheme will be:

$$P_{out,n} = P_{01}P_{02} + P_{13}P_{23} + P_{01}P_{23} + P_{13}P_{02} - P_{01}P_{02}P_{23} - P_{13}P_{02}P_{23} - P_{01}P_{02}P_{13} - P_{01}P_{13}P_{23} + P_{01}P_{02}P_{13}P_{23} \quad (20)$$

Comparing $P_{out,n}$ in (20) with $P_{out,a}$ and $P_{out,s}$ in (13) and (19) as expressions, respectively, we deduce that:

$$P_{out,s}=P_{out,a}=P_{12}P_{out,n}+(1-P_{12})(P_{01}P_{02}+P_{13}P_{23}-P_{01}P_{02}P_{13}P_{23}) \quad (21)$$

Where $P_{01}P_{02} + P_{13}P_{23} - P_{01}P_{02}P_{13}P_{23}$ is much smaller than $P_{out,n}$,and P_{12} is a relatively small number for high signal-to-noise ratios. Thus the inter-connected schemes are superior to the non-inter-relay scheme, and due to the power distribution gain, the selective scheme is superior among all of them.

B. Diversity Order

A good measure of the system performance is the diversity gain. However, it is hard to extract the diversity order from the exact outage probability expressions. Consequently, we will apply the minimum cut set method for deriving approximate outage probability expressions (upper-bounds) and deduce the diversity gain.

1) All-Active Relaying

Using the minimum cut set method, and by including the direct S-D link, the outage probability in (13) becomes:

$$P_{out} = P_{03}(P_{01}P_{02} + P_{13}P_{23} + P_{01}P_{12}P_{23} + P_{02}P_{12}P_{13}) \quad (22)$$

Thus, the diversity order is given by:

$$d_A = \beta_{0,3} + \min\{\beta_{0,1}+\beta_{0,2}, \beta_{1,3}+\beta_{2,3}, \beta_{0,1}+\beta_{1,2}+\beta_{2,3}, \beta_{0,2}+\beta_{1,2}+\beta_{1,3}\} \quad (23)$$

where $\beta_{i,j}$ is defined in (3) for the FSO links ($\beta_{i,j}$<1 for long distances) while $\beta_{i,j}$=1 for Rayleigh-faded RF links and $\beta_{i,j}$>1 for Rician-faded RF links. Since the smallest terms in (23) follow from the FSO links, the diversity gain can be calculated as follows:

$$d_A = \beta_{0,3} + \min\{\beta_{0,1}+\beta_{0,2}, \beta_{1,3}+\beta_{2,3}\} \quad (24)$$

2) Selective Relaying

Similarly, the outage probability expression of the selective relaying scheme was proved to be exactly the same as the all-active scheme but with different calculation values due to different power distribution. This results in the same diversity order:

$$d_S = d_A = \beta_{0,3} + \min\{\beta_{0,1}+\beta_{0,2}, \beta_{1,3}+\beta_{2,3}\} \quad (25)$$

As a conclusion, (25) highlights that the all-active and selective schemes achieve the same diversity order.

3) Comparison with Non-Inter-Relay schemes

For the non-inter-relay scheme, we will have a special case of the system as shown before in (20) where the minimum cut set method, along with including the direct S-D link outage probability P_{03}, can be applied to have the following approximate outage probability expression:

$$P_{out,n} = P_{03}(P_{01}P_{02} + P_{13}P_{23} + P_{01}P_{23} + P_{13}P_{02}) \quad (26)$$

This results in the following diversity order:

$$d_N = \beta_{0,3} + \min\{\beta_{0,1}+\beta_{0,2}, \beta_{1,3}+\beta_{2,3}, \beta_{0,1}+\beta_{2,3}, \beta_{0,2}+\beta_{1,3}\} \quad (27)$$

In fact, min{x,y} is always greater than or equal min{x,y,z}; this implies that:

$$\min\{\beta_{0,1}+\beta_{0,2}, \beta_{1,3}+\beta_{2,3}\} \geq \min\{\beta_{0,1}+\beta_{0,2}, \beta_{1,3}+\beta_{2,3}, \beta_{0,1}+\beta_{2,3}, \beta_{0,2}+\beta_{1,3}\} \quad (28)$$

and, consequently, $d_S = d_A \geq d_N$ which proves the advantage of cooperation among the relays.

V. Outage Probability Analysis for 3 Relays

A. Outage Probability

Similar to the case of 2 relays, the outage probabilities of the all-active and selective schemes can be derived using the conditional probability approach by conditioning on the inter-relay links.

1) All-Active Relaying

In what follows we separate the S-D link from our calculations, as this link will be checked in the first phase if it delivers the transmitted packet, thus the determined outage probability can be multiplied with P_{04} to get the final outage probability.

By conditioning on the inter relay links, the following four scenarios might arise:

- All 3 inter-relay links are active.
- Only 1 inter-relay link is in outage (3 cases).
- 2 inter-relay links are in outage (3 cases).
- All 3 inter-relay links are in outage.

In the first two scenarios, the system will have 1 common virtual relay $R_{1\text{-}2\text{-}3}$ as explained in section III.B resulting in:

$$P_{out1} = P_{01}P_{02}P_{03} + P_{14}P_{24}P_{34} - P_{01}P_{02}P_{03}P_{14}P_{24}P_{34} \quad (29)$$

This outage probability will be weighed by either one of the four following probabilities (corresponding to the first two scenarios): $(1-P_{12})(1-P_{23})(1-P_{13})$, $P_{12}(1-P_{23})(1-P_{13})$, $(1-P_{12})P_{23}(1-P_{13})$, and $(1-P_{12})(1-P_{23})P_{13}$. Consequently:

$$P_{out1,2} = [(1-P_{12})(1-P_{23})(1-P_{13}) + P_{12}(1-P_{23})(1-P_{13}) + (1-P_{12})P_{23}(1-P_{13}) + (1-P_{12})(1-P_{23})P_{13}]P_{out1} \quad (30)$$

For the third scenario, we have three similar cases where 2 relays are combined together and 1 relay is left alone and the 2 subsystems are treated as parallel systems. This results in the following outage probability expression:

$$P_{out3} = (1-P_{12})P_{23}P_{13}(1-(1-P_{01}P_{02})(1-P_{14}P_{24}))(1-(1-P_{03})(1-P_{34})) + P_{12}(1-P_{23})P_{13}(1-(1-P_{02}P_{03})(1-P_{24}P_{34}))(1-(1-P_{01})(1-P_{14})) + P_{12}P_{23}(1-P_{13})(1-(1-P_{01}P_{03})(1-P_{14}P_{34}))(1-(1-P_{02})(1-P_{24})) \quad (31)$$

For the last scenario, all inter-links are in outage thus we have the same system as in the non-inter-relay scheme which results in 3 parallel paths: S-R_1-D, S-R_2-D, and S-R_3-D with outage probability:

$$P_{out4} = P_{12}P_{23}P_{13}[(1-(1-P_{01})(1-P_{14}))(1-(1-P_{02})(1-P_{24}))(1-(1-P_{03})(1-P_{34}))] \quad (32)$$

From (30)-(32), the system outage probability can be calculated from:

$$P_{out,a} = P_{out1,2} + P_{out3} + P_{out4} \quad (33)$$

2) Selective Relaying

Similar to the case of 2 relays, we use the strength α of the links in the outage analysis for the selective scheme.

Again, in all what follows, the S-D link is treated separately as it is always parallel to the remaining subsystem. For the remaining system, we will also condition on the inter-relay links which results in the following four scenarios in a manner that is completely analogous to the all-active scheme:

- All 3 inter-relay links are active.
- Only 1 inter-relay link is in outage (3 cases).
- 2 inter-relay links are in outage (3 cases).
- All 3 inter-relay links are in outage.

For the first 2 scenarios, the strength of the system is calculated as:

$$\alpha = \min\{\max\{\alpha_{01},\alpha_{02},\alpha_{03}\}, \max\{\alpha_{14},\alpha_{24},\alpha_{34}\}\} \quad (34)$$

The system will be in outage when its strength is below a threshold value X:

$$\begin{aligned} P_{out1} &= \Pr(\alpha<X) = 1-\Pr(\alpha>X) \\ &= 1-\Pr(\min\{\max\{\alpha_{01},\alpha_{02},\alpha_{03}\}, \max\{\alpha_{14},\alpha_{24},\alpha_{34}\}\}>X) \\ &= 1-\Pr(\max\{\alpha_{01},\alpha_{02},\alpha_{03}\}>X)\Pr(\max\{\alpha_{14},\alpha_{24},\alpha_{34}\}>X) \\ &= 1-((1-P_{01}P_{02}P_{03})(1-P_{14}P_{24}P_{34})) \end{aligned}$$

$$P_{out1} = P_{01}P_{02}P_{03} + P_{14}P_{24}P_{34} - P_{01}P_{02}P_{03}P_{14}P_{24}P_{34} \quad (35)$$

Weighing by the four inter-relay probabilities corresponding to the first two scenarios, the corresponding outage probability can be written as:

$$P_{out1,2} = [(1-P_{12})(1-P_{23})(1-P_{13}) + P_{12}(1-P_{23})(1-P_{13}) + (1-P_{12})P_{23}(1-P_{13}) + (1-P_{12})(1-P_{23})P_{13}]P_{out1} \quad (36)$$

For the third scenario, we have 3 similar cases where 2 relays are combined together while 1 relay is left alone where the 2 subsystems are treated as parallel systems. This results in the following strength expression:

$$\alpha = max\{min\{max\{\alpha_{0i},\alpha_{0j}\}, max\{\alpha_{i4},\alpha_{j4}\}\}, min\{\alpha_{0z},\alpha_{z4}\}\} \quad (37)$$

where i and j are the combined relays and z is the left alone one. This will result in an outage probability expression corresponding to a threshold X as follows:

$$\begin{aligned} P_{out3'} &= \Pr(\alpha<X) \\ &= \Pr(\max\{\min\{\max\{\alpha_{0i},\alpha_{0j}\},\max\{\alpha_{i4},\alpha_{j4}\}\}, \min\{\alpha_{0z},\alpha_{z4}\}\}<X) \\ &= \Pr(\min\{\max\{\alpha_{0i},\alpha_{0j}\},\max\{\alpha_{i4},\alpha_{j4}\}\}<X) \Pr(\min\{\alpha_{0z},\alpha_{z4}\}\}<X) \\ &= (1-\Pr(\min\{\max\{\alpha_{0i},\alpha_{0j}\}, \max\{\alpha_{i4},\alpha_{j4}\}\}>X) (1-\Pr(\min\{\alpha_{0z},\alpha_{z4}\}\}>X)) \\ &= (1-(\Pr(\max\{\alpha_{0i},\alpha_{0j}\}>X)\Pr(\max\{\alpha_{i4},\alpha_{j4}\}\}>X)) (1-\Pr(\alpha_{0z}>X)\Pr(\alpha_{z4}>X)) \end{aligned}$$

$$P_{out3'} = (1-(1-P_{0i}P_{0j})(1-P_{i4}P_{j4}))(1-(1-P_{0z})(1-P_{z4})) \quad (38)$$

Substituting i, j, and z by their values and including the R-R possible combinations and corresponding outage probabilities for the 3 predefined cases, the resulting outage probability is:

$$\begin{aligned} P_{out3} = {} & (1-P_{12})P_{23}P_{13}(1-(1-P_{01}P_{02})(1-P_{14}P_{24}))(1-(1-P_{03})(1-P_{34})) \\ & + P_{12}(1-P_{23})P_{13}(1-(1-P_{02}P_{03})(1-P_{24}P_{34}))(1-(1-P_{01})(1-P_{14})) \\ & + P_{12}P_{23}(1-P_{13})(1-(1-P_{01}P_{03})(1-P_{14}P_{34}))(1-(1-P_{02})(1-P_{24})) \end{aligned} \quad (39)$$

For the last scenario, all inter-links are in outage thus we have the 3 parallel paths: S-R_1-D, S-R_2-D, and S-R_3-D with total strength of the system corresponding to the maximum between all of the 3 paths as follows:

$$\alpha = \max\{\min\{\alpha_{01},\alpha_{14}\}, \min\{\alpha_{02},\alpha_{24}\}, \min\{\alpha_{03},\alpha_{34}\}\} \quad (40)$$

The outage probability for this case is calculated as follows:

$$\begin{aligned} P_{out4'} &= \Pr(\max\{\min\{\alpha_{01},\alpha_{14}\}, \min\{\alpha_{02},\alpha_{24}\}, \min\{\alpha_{03},\alpha_{34}\}\}<X) \\ &= \Pr(\min\{\alpha_{01},\alpha_{14}\}<X)\Pr(\min\{\alpha_{02},\alpha_{24}\}<X) \Pr(\min\{\alpha_{03},\alpha_{34}\}\}<X) \\ &= (1-\Pr(\min\{\alpha_{01},\alpha_{14}\}>X))(1-\Pr(\min\{\alpha_{02},\alpha_{24}\}>X)) (1-\Pr(\min\{\alpha_{03},\alpha_{34}\}\}>X) \end{aligned}$$

$$P_{out4'} = (1-(1-P_{01})(1-P_{14})(1-(1-P_{02})(1-P_{24}))(1-(1-P_{03})(1-P_{34})) \quad (41)$$

Therefore, the resultant outage probability for this scenario is:

$$P_{out4} = P_{12}P_{23}P_{13}P_{out4'} \quad (42)$$

Therefore, the outage probability will be the sum of all cases:

$$P_{out,s} = P_{out1,2} + P_{out3} + P_{out4} \quad (43)$$

3) Comparision between All-Active and Selective Schemes

Similar to the 2 relays case, both outage probability expressions resulting from each of the all-active and selective schemes have exactly the same terms. However, the main difference between them lies in the real calculation of the probability terms i.e. when going back to the main formula for the outage probability of a single link and replacing the power factor value. Thus, selective scheme shows superiority again.

4) Comparision with Non-Inter-Relay schemes

Similar to the case with 2 relays, the non-inter-relay scheme represents special case of the whole system where the inter-links do not exist and thus yielding the highest outage probability result among all cases. This means that the outage probability of the non-inter-relay scheme will be:

$$P_{out,n}=(1-(1-P_{01})(1-P_{14}))(1-(1-P_{02})(1-P_{24}))(1-(1-P_{03})(1-P_{34})) \quad (44)$$

Where $P_{out,n}$ can be shown to be much larger than $P_{out,s}$ and $P_{out,a}$. Thus, the inter-connected schemes are superior to the non-inter-relay scheme, and due to the power distribution gain, the selective scheme is superior among all of them.

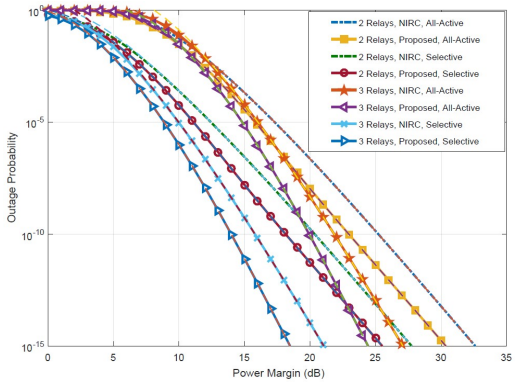

Fig. 3. Comparision between the propsed system and non-inter-relay systems for all-active and selective relaying schemes with 2 and 3 relays. Lines with markers correspond to exact outage probability while associtaed dotted lines correspond to approximate outage probability from minimum cut set method.

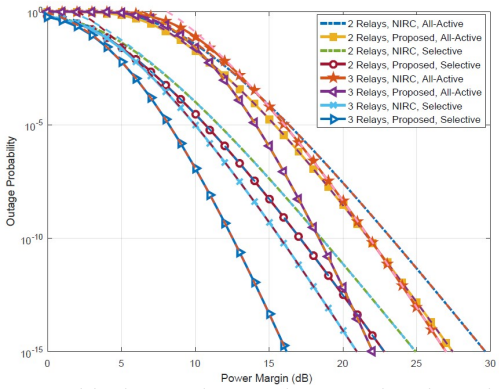

Fig. 4. Comparision between the propsed system and non-inter-relay systems for all-active and selective relaying schemes with 2 and 3 relays. Lines with markers correspond to exact outage probability while associtaed dotted lines correspond to approximate outage probability from minimum cut set method.

B. *Diversity Order*

1) All-Active Relaying

Similar to the case of 2 relays, the diversity order of all-active scheme with 3 relays is given by:

$$d_A = \beta_{0,4} + \min\{\beta_{0,1}+\beta_{0,2}+\beta_{0,3}, \beta_{1,4}+\beta_{2,4}+\beta_{3,4}\} \quad (45)$$

2) Selective Relaying

Similar to all-active scheme and the reasoning in 2 relays:

$$d_S = d_A = \beta_{0,3} + \min\{\beta_{0,1}+\beta_{0,2}, \beta_{1,3}+\beta_{2,3}\} \quad (46)$$

3) Comparision between All-Active and Selective Schemes

$$d_S = d_A \quad (47)$$

4) Comparision with Non-Inter-Relay schemes

Similar to the case of 2 relays, the diversity order for non-inter scheme is given by:

$$\begin{aligned} d_N = \beta_{0,4} + min\{&\beta_{0,1}+\beta_{0,2}+\beta_{0,3}, \beta_{1,4}+\beta_{2,4}+\beta_{3,4}, \\ &\beta_{0,1}+\beta_{2,4}+\beta_{3,4}, \beta_{1,4}+\beta_{0,2}+\beta_{3,4}, \beta_{1,4}+\beta_{2,4}+\beta_{0,3}, \\ &\beta_{0,1}+\beta_{0,2}+\beta_{3,4}, \beta_{0,1}+\beta_{2,4}+\beta_{0,3}, \beta_{1,4}+\beta_{0,2}+\beta_{0,3}\} \end{aligned} \quad (48)$$

In fact, $\min\{x,y\}$ is always greater than or equal $\min\{x,y,z,..\}$. As a conclusion, $d_S = d_A \geq d_N$ which proves the advantage of cooperation among the different relays.

VI. Numerical Results

The refractive index structure constant and the attenuation constant are set to $C_n^2 = 1\text{x}10^{-14}$ $m^{-2/3}$ and $\sigma = 0.44$ dB/km, respectively. We also set $\lambda = 1550$ nm and the distance between S and D to 3 km in all network setups. Results are shown for 2-relay and 3-relay networks where the distances of the FSO links are provided in TABLE I ($d_{S,3}$ and $d_{3,D}$ are limited to the scenario of 3 relays). Regarding the RF links, we consider Rayleigh fading in Fig. 3 with $\Omega_{i,j} = 20$ dB and we consider Rician fading in Fig. 4 with $K_{i,j} = 10$. The results validate the equal diversity order between the selective and all-active schemes as predicted by (25) and (36). In fact, the corresponding outage probability curves are practically parallel to each other (for large power margins) whether with two relays or three relays.

TABLE I. FSO Links' Distances (in km)

	$d_{S,1}$	$d_{1,D}$	$d_{S,2}$	$d_{2,D}$	$d_{S,3}$	$d_{3,D}$
Fig. 3 (Rayleigh R-R links)	1.5	2.5	2.5	1.5	1	2
Fig. 4 (Rician R-R links)	2.25	1.75	1	2	2.75	1.25

On the other hand, lower outage probabilities are achieved by the selective scheme due to the enhanced power gain as explained in sections IV.A.3 and V.A.3. Results in Fig. 3 and Fig. 4 also highlight the performance gains with respect to the non-inter-relay scheme. The gain harvested from increasing the number of relays is also evident in all scenarios. However, a system with 2 relays operating in the selective mode can achieve better performance than a system with 3 relays without any inter-relay cooperation.

VII. Conclusion

The cooperative FSO network with RF backup inter-relays can be exploited with both all-active and selective relaying. The results are encouraging as the system overall performance is enhanced in both cases over non-inter relay schemes. Selective schemes showed superiority over all-active schemes.

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