Developing and Piloting a teaching/learning unit on Similar Triangles using DGS

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By
Nisreen Mohammad Chayya

Under the Direction of
Dr. Iman Osta

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Project approval Form

Student Name: Nisreen Chayya
I.D. #: 200600490

Project Title: Developing and Piloting a teaching/learning unit on Similar Triangles using DGS

Program: MA in Education

Department: Education

School: School of Arts and Sciences

Approved by:

Project Advisor: Dr. Iman Osta, PhD, Associate Professor.

Member: Dr. May Hamdan, PhD, Associate Professor.

Date: 15 February, 2010
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Abstract

Technology has recently become a vital factor in the teaching of mathematics. Yet, a key issue remains how technology could be ultimately used to support students in reaching a better understanding of Mathematics and to design meaningful learning experiences. In Geometry, forms of software known as Dynamic Geometry Software (DGS) are shown to be useful in realizing such a purpose. In an attempt to examine the possible effects of DGS on a learning situation, a learning/teaching sequence on Similar Triangles was developed then piloted and clinically analyzed to investigate its effect on a pair of 8th grade Lebanese students' learning and problem solving strategies. The aim was to assess the effectiveness of DGS on students' learning of the topic (9th grade level in the Lebanese curriculum) and the transfer of knowledge and competencies into a computer-free environment. An interview with a 9th grade math teacher was conducted to investigate the difficulties faced in teaching and learning Similar Triangles. The teaching/learning sequence was implemented over a period of 5 sessions each lasting for 1-2 hrs, including solving problems in a clinical interview setting. The sessions were videotaped and transcripts were generated. Data analyzed were collected from the curriculum text, textbook, transcript of the teacher's interview, videotapes and transcripts of the teaching/learning sequence and conducted clinical interviews, students' computer files and solutions to the activities in worksheets and diagnostic and summative tests. The comparison of the diagnostic and summative tests showed that meaningful learning took place and knowledge was gained. The analysis of the clinical interviews highlighted the fact that DGS helps in reinforcing a wide variety of skills and evolving mathematical problem strategies, and that those skills would be transferred into a computer-free environment. The study brought forth that properly designed tasks based on constructivist approach and supported by DGS can help in reaching better learning of mathematics and acquisition of essential mathematical skills at an earlier level than expected in the curriculum.
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CHAPTER I

Introduction

Providing students with a meaningful experience in mathematics seems to be a demanding task. An important body of research (Mariotti, 2000, Jones, 2000, Gerretson, 2004, Ward, 2005) has reported that even after relatively long teaching, students fail to fulfil the requirements of deductive reasoning and proof or to develop efficient problem solving skills. Students find it hard to construct proofs and tend to base their conviction about properties or relationships on empirical data or evidence. Therefore, curriculum developers’ and teachers’ challenge is to design a rigorous experience, based on problem solving approach, where students are prompted to develop a genuine cognitive urge for conviction whereby proof offers them the satisfaction of understanding why their reasoning is true (Vincent, 2005).

Mathematical proof has two roles. One has to do with mathematics in general and the other is strictly related to the understanding of mathematics by students. A proof in mathematics is a kind of discourse meant both for validating the truth of a statement and for convincing others with the validity of an assertion (Laborde, 2000). On the other hand, the process of proving guides students to conceive mathematics as a result of human endeavour and as a logically constructed discipline rather then an odd body of knowledge made up from unrelated theorems, postulates, axioms and rules (Vincent, 2005). Hence, mathematical problem solving, which requires proving as a need, ought to become a central activity in students’ learning of mathematics.

Working in a technological environment has been found to enhance the process of mathematical problem solving and knowledge construction (Battista, 2002, Jones, 2000, Mariotti, 2000, Hanna, 2000). An essential goal in instruction is to provide students with a rich environment whereby they get the chance to demonstrate
their own ways of thinking, elaborate their ideas, and develop unique strategies to
deal with mathematical problems (Ward, 2005). These ideas and strategies need to be
challenged and illustrated and the use of technology arises to be a convenient tool to
meet such goals.

Technological tools play a considerable role in the emergence of deductive
reasoning and related processes such as dynamic visualization of concepts (Rivera,
2005). Computers equipped with specialized software are able to facilitate
visualization of abstract concepts and create a powerful medium that extends beyond
students' physical capabilities (Gerretson, 2004). The role of visual representations, or
dynamic visualization in the case of technology, is not restricted to providing
evidence for mathematical assumptions but rather extends to explaining and justifying
them. "Visual imagery", as defined by Garderen (2006), is used in establishing the
meaning of a mathematical problem, channelling problem-solving approaches and
influencing cognitive constructions. Hence, visual representations are considered to
be "heuristic accompanies to proofs" (Garderen, 2006, Hanna, 2000).

In geometry, types of software known as DGS, Dynamic Geometry Software,
are proving to be significant in promoting deep understanding and reasoning about
figures, shapes and various geometric concepts (Battista, 2002).

The development of impressive advanced technologies, such as Cabri
Géomètre (hereafter referred to as Cabri) with its drag-mode and dynamic capabilities
have made possible the visualization of several instances of geometric configurations,
allowing students to easily investigate and verify the truth of various conjectures.
Cabri has the potential to provide students with direct experience with geometrical
theory and hence bridge the gap between what seems to be a separation in the eyes of
the student between the geometric construction and deduction (Jones, 2000, Garderen,
DGS affects the actions that are possible during problem solving and the feedback that is reflected back to the student on those actions. A solution to a problem involving construction is valid if and only if it can't be ruined upon dragging one of the elements of the figure. And for the figure not to be "messed up" it has to be constructed according to geometrical theory (Jones, 2000). Therefore, Cabri prompts students to develop deductive explanations and provide a base to generate ideas of proof and proving as it encourages both exploration and deductive reasoning (Jones, 2000).

Therefore, DGS creates a world of mathematics far beyond the limits of conventional teaching and classical tools of classroom interaction.

**Purpose**

In an attempt to examine the possible effects of DGS on a learning situation, this study aims at the following:

- Developing a teaching/learning didactical sequence on *Similar Triangles* to be taught to 8th graders using Cabri, one year ahead its regular inclusion in the curriculum (9th grade level).

- Piloting the developed teaching/learning sequence to assess its effectiveness in students’ learning of the topic, earlier then expected in the curriculum.

- Investigating the evolution of students’ geometric problem solving strategies and their proving capabilities in a situation where DGS is integrated.
Research questions

The study targets the following research questions:

- Are 8th grade students able to learn Similar Triangles (9th grade according to the curriculum) if they are provided with a learning experience based on DGS?
- What features of DGS are helpful in realizing such a purpose?
- How do students' geometrical problem solving strategies pertaining to the topic Similar Triangles evolve while learning using DGS?
- Is mathematical knowledge gained in Cabri-based situations transferred to a computer-free setting?

Rationale and Significance

The Lebanese curriculum, (ECRD, 1997), represented by its introduction (aims) and objectives, ought to be an encouraging prospective to the future of Mathematics in Lebanon. It perceives Mathematics as a field of exploration, critical thinking, discovering, conjecturing and above all a science that is not directed towards elite only rather "accessible by a larger public". Therefore, a reform in teaching should take place. It stresses learning based on the individual construction of knowledge, the use of representations and modelling that is to be supported by computers and technological tools and emphasizing mathematical reasoning. Along its Basic Education chapter – intermediate level, the curriculum recommends that the concept of Similar Triangles should be introduced at the ninth grade level (age 14-15 years).

The study includes two basic sections: The pilot study and the problem solving setting (clinical interviews). The students are guided in a way that they would be, on
their own, coming up with the Thales theorem, the definition of similar triangles and the postulates relative to similar triangles. Students are given the opportunity to observe, describe, analyze, interpret, reflect, deduce, conjecture and hypothesize. The researcher’s role was limited towards providing hints when students reach an end in analyzing and solving, resolve misunderstandings and name students’ finding. For example, the relationship existing between two triangles where the corresponding angles are congruent and corresponding sides are proportional is called *Similarity*. Therefore, students are prompted to explore, make use of and extend their scientific and mathematical skills and above all, assisted with technology, individually generate the sought concepts and relationships on their own.

This study attempts to show that such a concept (along with all the required skills) could be successfully integrated in the 8th grade program if it is supported with DGS. It is expected that the study would significantly contribute to the field of math education and more specifically geometry teaching. The design of the experiment and the kind of instructional sequence produced provide teachers with a meaningful experience to be applied in math classrooms. Students would be actively constructing their knowledge, generating a sense of ownership of the properties and theorems. Such a situation would develop a positive attitude, in particular the low achievers’, towards mathematics and generate deep and meaningful understanding of the concept away from rote learning.
CHAPETR II

Literature Review

During the recent years, mathematics education researchers have shaped the fields of interest; Rather then seeking means to encourage the acquisition of proving skills, they aimed to investigate the student’s understanding of mathematical proof and search for effective ways to help them reach a better understanding (Gutierrez & Marrades, 2000). The closest resolution ought to be in the kind of teaching – learning approaches adopted by teachers in the math classrooms, whereby constructivism and technology are emphasized (Arsac et al., 1992, Jones, 2000, Laborde, 2000). In Geometry, forms of software, useful in providing meaningful experiences with deductive and geometric reasoning, are known as Dynamic Geometry Software (DGS) (Laborde, 2000).

Theoretical Framework

It is reported that the basic premise about learning underlying teaching situations is constructivism. The learner is considered to be the basic constructor of his/her own knowledge through efficient classroom interaction where the student practices less listening and more reasoning (Hartter et al., 2006). The teacher’s role is more of a facilitator of learning in a setting offering multiple opportunities for students to actively participate, emphasizing autonomy and meaningful learning.

Such approaches of teaching and learning would take place in an environment that promotes questioning (open-ended or closed-ended), wondering, demonstrating, exploring and testing to elicit students’ thinking. Those environments are known to be “thinking classrooms” (Loertscher, 2006, p. 1)

The term “scaffolding” has been introduced almost 20 years ago, to indicate the kind of interaction between an adult and a child, with Wood, Burner and Ross
(Wood & Wood, 1996, p. 5). The notation was originated to describe the kind of assistance an adult can offer to guide the learner to achieve a certain chore that she/he can’t do on his/her own. Scaffolding is close in meaning to Vygotsky’s concept of Zone of Proximal Development (ZPD), referring to the mismatch between the kinds of performance the student can show on his/her own versus under the guidance of specific tutor (Wood & Wood, 1996). However, the question remains what kind of help or guidance is to be provided. Tutoring could take place either through an adult or peers that could be described as follows:

- Tutor’s role is filling the gap between the requirements of the new task and the learner’s existing background knowledge.

- The tutor provides a plan to help the learner’s problem solving actions through granting activity – contextualized instructions.

- Learning takes place best when the student actively enrols in the process of learning and problem solving. The participation is optimized when it is properly guided.

Scaffolding, by peer or adult, should progressively lead to the “hand-over of task responsibility from tutor to learner” (Wood & Wood, 1996, p. 6). Tutorial activity contributes to the acquisition of task-related competence without which a novice learner might lose direction into the core of the task.

*Problem Solving Approach*

Research reflects the appreciation of the interactive roles of subject matter, teachers, students and technology in designing classrooms that promote understanding of mathematics and specifically geometry and space (Chazan & Lehrer, 1998, Jones, 2000, Vincent, 2005).
Vincent (2005) believes that promoting proof as a basic process to construct mathematical knowledge and understanding will not motivate students unless they feel that they are actively taking part in a meaningful mathematical discovery activity. Hence, the goal is to create a mathematical task that provokes the need to convict and proving provides the satisfaction of demonstrating the truth of their conjectures (2005).

However, designing a meaningful task ought to be difficult. It is reported that even after a long period of teaching, students yet fail to foresee the difference between forms of mathematical reasoning. This may be referred back to the fact that proving and reasoning requires the integration of several skills each of which is not easy to acquire (Jones, 2000).

Garderen (2006) argues that all mathematical tasks entail spatial reasoning and hence visualization ought to be considered to be an important factor in mathematics problem solving, even when geometry is not manifested. This is due to the basic concept underlying “visualization” as the “mental scheme depicting visual or spatial information” (Garderen, 2006, p. 1).

Despite the complexity of the tasks requiring deduction, analysis and reasoning, students can learn to argue mathematically once they are provided with efficient problem solving settings. It is the form of student – teacher interactions throughout problem solving that helps in achieving such a goal (Jones, 2000, Vincent, 2005).

Hence, students should be offered the opportunity to think, argue, analyze, convict, conjecture, work in groups, discuss solutions with peers, and hypothesize then test.
Garderen (2006) claims that “visualization is considered to be a powerful problem representation process for solving problems” (p. 1). The use of visual images can help in generating the actual meaning of the problem, bringing forward the relevant problem-solving strategies, hence affecting the problem solving and the cognitive analyses.

DGS

“Students can learn mathematics more deeply with the appropriate use of technology (NCTM, 2000, p. 25)”. About a decade ago, educators agreed on the fact that there is no longer need to discuss whether technology is appropriate in schools – education (Clements and Sarama, 2005).

In geometry, tools that proved to be beneficial in the curriculum ought to be related to the deductive methods are computer software packages known as Dynamic Geometry Software (DGS), offering a direct experience with geometrical theory and hence bridging the gap between construction and deduction (Jones, 2000). Laborde (2000) argues that with its “exploration powers and checking facilities, DGS offers an interplay between conjectures and checks and between certainty and uncertainty” (p. 154). Throughout task competence, the processes carried out by the students are due to mutual interaction between the user and the system, providing various possibilities of actions and responses, where the user acts and the system reacts back offering a direct feedback (Jones, 2000, Guitierrez & Marrades, 2000, Laborde, 2000).

Reasoning and specifically geometric reasoning (and proofs) tends to be an essential part of mathematics. It is claimed that such software is capable of changing both the environment and the methods in which mathematical ideas are explored (Rivera, 2005, Marrades & Gutierrez, 2000, Hanna, 2000). Because DGS offers non-traditional ways of learning through freely and efficiently experimenting, the way
justification, reasoning and proof are utilized started taking various forms. Some justifications ought to be empirical, portrayed through heavy dependence on Cabri figures and constructions to illustrate the truth and validity of statements and validity. Hence, examples are used to verify the correctness of conjectures. Others tend to be deductive, through “decontextualization and relying on abstract and symbolic justification without referring back to examples” (Marrades & Gutierrez, 2000, p. 133).

DGS helps in creating efficient learning environments where students are capable of experimenting, observing the sustainability or the lack of mathematical properties, state and verify conjectures in an easier set up than paper-and-pencil. Christou et. al (2004), believes that DGS offers non-traditional ways of generating deep understanding and meaningful learning that is hard to be found in traditional math classrooms. According to Mariotti (2000), DGS, specifically Cabri, works according to the logic of Euclidean geometry enabling an essential connection between geometric construction under Cabri and geometrical theorems and axioms. The main advantage lies in allowing the user to construct complex figures and perform several transformations on figures. Hence, the user gains access to various examples each time a transformation is applied which could hardly be done through other tools (Marrades & Guiterrez, 2000).

Jones (2000) distinguished drawing from figures; “drawing refers to the material entity while the figure refers to the theoretical object” (p. 58). With its dragging mode abilities, DGS highlights a specific method to validate solutions to construction problems; “a solution is valid if it could not be messed up under dragging” (Jones, 2000, p. 58). The dragging test, applying object movements on a figure, would result in preserving all the existing properties and relations between the
components (Marrades & Gutierrez, 2000, Hoyles & Jones, 1998). This would be a wise tool to assess the validity of construction and the truth of statements (Rivera, 2005). Hoyles and Jones (1998) argue that Cabri, through the dragging test, prompts students to pose and test conjectures through emphasizing the theorems and properties linking the various objects of the figure. And once the figure passes under the test, Mariotti (2000) illustrates that a justification is required that eventually has to be based on geometrical theorems. Hence, through DGS and with the drag mode, figures are no more static, rather they are dynamic and students could explore multiple possibilities, generate conjectures then test them to either prove or disprove them. The process of proving or refuting could take place either through theoretical (or visual) justification or via counter examples (Rivera, 2005).

DGS software is equipped with commands that reflect definitions, postulates and theorems that are represented in Euclidean Geometry. Therefore, through its built-in facilities and while constructing objects, DGS illustrates for students the logical system lying under Euclidean Geometry (Rivera, 2005).

Several research papers investigated the impact of the use of DGS on students' mathematical understanding (Hanna, 2000, Rivera, 2005). “Mathematical learning with DGS engages students and encourages them to construct and experience what it means to produce knowledge” (Rivera, 2005, p. 135).

However, the influence is extended towards affecting the approaches in which students construct and process knowledge (Rivera, 2005). It prompts the development of various kinds and levels of mathematical skills and competencies such as pattern recognition, verification, explanation, justification and proof (2005).
DGS and Proofs

Research highlighted the fact that students fail to conceive the need for deductive proof or to distinguish between various forms of mathematical reasoning such as explanation, argument, verification and proof. The reasons may be summarized as: 1) emphasizing verification and neglecting exploration and explanation, 2) proving requires utilizing several skills each is not easy to acquire, and 3) learning how to prove requires students to undergo a critical shift from perceiving math as a computational system to a field of interrelated structures and axioms (Jones, 2000).

According to Laborde (2000), proof is considered to be “a specific kind of discourse meant both for validating the truth of the statement and for convincing others of the validity of this assertion” (p. 155). But what is the role of proof? The role of proof could be be seen in different contexts: mathematics, classroom and curriculum.

Role of proof in mathematics education: Mathematicians see proofs not as “syntactic derivations (sequence of sentences, each of which is an axiom or the immediate consequence of preceding sentences by applying the rules of inference)” (Hanna, 2000, p. 7), but rather, they are conceptual. The approaches towards proving are secondary. Hence, the function of proving extends beyond justifying. It does not only illustrate if it is true but why it is true (2000).

Role of proof in mathematics classroom: Vincent (2005) indicates that the “proof is not merely to support conviction, nor to respond to distrustful nature of self-doubts; proof serves to provide explanation” (p. 94).

The basic role of proof in the math classroom is to provoke and deepen mathematical understanding. The teacher’s job is to inspire the excitement and
enjoyment of exploration activities (Hanna, 2000, Vincent, 2005). Proof leads students to perceive mathematics as a result of human accomplishment and as a logically constructed structure. It brings students' satisfaction on the reasons behind the truth of their conjectures. Conjectures and accompanied justifications arising throughout the deductive argument play a major role in the proper sequencing of ideas and construction of valid proofs (Jones, 2000, Vincent, 2005).

*Role of proof in curriculum:* Highlighting the essential relation between proofs and understanding, curricula makers emphasize the need for students to justify and explain their reasoning (Vincent, 2005). NCTM (2000) recommends that proof and geometric reasoning to be part of the mathematics curriculum from early school years through 12th grade level. Hanna (2000) asserts that proofs are essential in the curriculum as they do bring "clarity and reliability barely matched by other sciences which in turn highlights the unique contribution of mathematics to the culture of sciences" (p. 11). Yet, proofs still do receive special value in the mathematical practices and curricula (2000).

It is reported that one of the effective ways to teach proofs is through DGS. DGS helps in developing sound mathematical reasoning, generate valid proofs of various propositions and improve the global understanding of mathematics (Hanna, 2000). Proof assumes the multiple functions of verifying the truth of conjectures, meaningful understanding of geometric relations and an explanation role; DGS enables students to generate deductive explanations, providing a basis for generating ideas of proofs and proving (Jones, 2000, Vincent, 2005). It affects the interpretation that students give to mathematical objects, enabling them to learn how to articulate explanations and verification of theorems and properties (Hanna, 2000).
DGS triggers and exposes the actions available while problem solving. A task that is to be solved through dynamic geometry requires strategies and arguments that are different from those in the classical paper-and-pencil context (Jones, 2000). It introduces direct feedback and special criteria to test the validity of the construction (Jones, 2000, Laborde, 2000, Vincent, 2000). It is the “robustness of the figure under the drag test” (Jones, 2000, p. 58) that shows whether the construction that took place is actually based on geometrical theory. The outer appearance seems to be less in importance in this case as the figure would change in shape, without changing properties, using the drag facility, hence focusing efforts on exploring the relevant existing mathematical relationships (Vincent, 2005). DGS, and specifically Cabri, presents a “sequential organization of actions necessary to produce a figure and introduces explicit order of construction” (Jones, 2000, p. 59) which for some users ought to be less in importance. A hierarchy of reliance is generated, where each construction depends on earlier steps.
CHAPTER III

Method

This is a case study, in which a learning/teaching sequence is developed to integrate the use of DGS, then piloted and clinically analyzed to investigate its effect on a pair of students' learning and problem solving strategies.

Participants

Two Lebanese students, a boy and a girl 8<sup>th</sup> graders, enrolled in a Lebanese private school, participated in the pilot study. The students are 14 years old, with the boy being a high achiever in Mathematics (average above 80), and the girl average to low achiever (average 60). The students are computer-literate as the use of computers is integrated in their school’s program as early as the third grade. Both students use computer on a daily basis with an average of 2-3 hours per day. During the intervention, both students were working on one laptop with Cabri. However, before the intervention students had no prior knowledge or skills regarding the use of Cabri or any other dynamic geometry tools.

Procedure

The study is conducted based on the following procedures:

- Review of the Lebanese math curriculum (ECRD, 1997) (Appendix A) as pertains to Similar Triangles in terms of prerequisites, objectives and content.

- Investigation of difficulties faced by students learning Similar Triangles. To achieve this purpose, an interview with a 9<sup>th</sup> grade Lebanese math teacher was conducted over a period of 60 minutes, during which she reflected on three basic parts: problems encountering the students in learning this specific topic and hence
hindering their improvement in geometry and problems that would come across teachers while teaching Similar Triangles and finally how technology can play an essential role in overcoming the above mentioned difficulties. A transcript of the interview was generated for analysis (Appendix B).

- Listing of concepts and skills involved in Similar Triangles and their prerequisites. For this purpose, the Lebanese math curriculum and the 9th grade math book issued by the ECRD are used (Appendix C).

- Development of the lessons on Similar Triangles (integrating activities using Cabri).

- Administering, to the two participants, a diagnostic test (Appendix D) for prerequisites and base-line problem solving abilities. The results were reviewed, and accordingly the gaps in background, prerequisites and problem solving approach, were highlighted.

- Implementing the learning/teaching sequence in a tutoring setting (two students). The sequence was carried out over a period of 5 sessions, lasting for one to two hours each, as follows:

  *Session 1: Covering Prerequisites and Cabri exploration activities*
  *Sessions 2 and 3: Teaching/learning sequence on similar triangles*
  *Sessions 4 and 5: Clinical interviews*

The first session was devoted to cover the prerequisites (concepts related to congruency of triangles and proportionality) and familiarizing students with the use of Cabri (Appendix E).

Throughout the second and third session, two worksheets (Appendix F and Appendix G) were distributed. The activity sheets contained
exercises that were constantly provoking students to come up with the basic properties and rules of similar triangles. The aim was to
guide students towards constructing their own knowledge, and the researcher's role is meant to provide hints and correct misconceptions whenever they occur. During the implementation, the researcher
prompted students to constantly save their work under a new file name. These files helped follow up on and analyze their construction.
Moreover, the sessions were videotaped for later analysis.
The fourth and fifth session represented conducting the clinical interviews. Two problem solving activities (consisting of three and four sub-activities respectively) (Appendix H) were covered in a clinical interview setting.
The students were working together on one laptop. By the end of each exercise one solution sheet was submitted by the students.
- Administering a paper-based summative test to investigate the transfer of understanding and problem solving abilities to a computer-
free environment.

Data Collection and Analysis

Data is collected and analyzed using the following texts and documents:
- Curriculum text and textbook, specifically the section on Similar Triangles (Appendix C).
- Transcript of the interview with 9th grade math teacher (Appendix B).
- Videotapes and transcripts of the teaching/learning sequence.
- Videotapes and transcripts of the conducted clinical interviews.
- Observation notes recorded during clinical interviews.
- Students' computer files.
- Students' notes taken throughout the sessions and their solutions to the activities in the worksheets.
- Diagnostic and summative tests (Appendix D and Appendix I).

A qualitative/clinical analysis is conducted to exploit the above data towards the purpose of this project. For each set of data, themes that are related to the research questions are selected according to which the data are classified, compared and synthesized.
CHAPTER IV

The Unit

The following chapter will cover four main areas: Similar Triangles as it appears in the Lebanese curriculum, teacher's interview, development of the unit and implementation.

*Similar Triangles in the Lebanese Curriculum*

In the Lebanese curriculum (ECRD, 1997), *Similar Triangles* is introduced throughout the middle of the academic year of Grade 9. Few geometry lessons precede *Similar Triangles* that are *Tangents and Circles, Inscribed Quadrilaterals* and *Thales' Theorem*.

It is to be noted that in the textbook (Appendix C) the term "corresponding" is used to refer to homologous angles and sides in similar triangles. However, in Geometry, the term ‘corresponding’ (Figure 1) is used to represent the position of angles in lines cut by a transversal.

![Diagram of corresponding angles](image)

*Figure 1: Corresponding angles*

Therefore, to preserve consistency with the book, the term ‘corresponding’ will be used in this section to indicate homologous angle or sides in similar triangles.

The lesson *Similar Triangles* (Chapter 9) occurs right after *Thales' Theorem* (Chapter 8). The book recommends that *Thales' Theorem* take place over a period of 7 hours whereas *Similar Triangles* 13 hours.
**Thales’ Theorem:** In this lesson students would cover:

1) Definition of Thales’ Theorem

In \( \Delta ABC \), if (BC) and (B’C’) are parallel (Figure 2), then \( \frac{AB'}{AB} = \frac{AC'}{AC} = \frac{B'C'}{BC} \)

![Figure 2](image)

**Figure 2:** Thales’ Theorem in triangles

2) Construct the proportionality table:

**Table 1**

<table>
<thead>
<tr>
<th>( \triangle ABC )</th>
<th>( \triangle AB'C' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>AB'</td>
</tr>
<tr>
<td>AC</td>
<td>AC'</td>
</tr>
<tr>
<td>BC</td>
<td>B'C'</td>
</tr>
</tbody>
</table>

3) Generate the consequences following Thales’ Theorem

If (BC) is parallel to (B’C’) and (B’’C’’) (Figure 3) then \( \frac{BB'}{AB} = \frac{CC'}{AC} \) and

\[
\frac{BB'}{BB''} = \frac{CC'}{CC''}
\]
Figure 3: Consequences of Thales’ Theorem in triangles

4) Illustrate the concept of reduction and expansion of triangles

ABC is a triangle, M is a point of the side [AB] distinct from A and B, and N is a point of [AC] distinct from A and C (Figure 3) such that

\[ \frac{AM}{AB} = \frac{AN}{AC} = \frac{MN}{BC} = k, \] where \( k < 1 \) and \( k \) is a real number.

\[ \implies \text{The triangle AMN is a reduction of the triangle ABC of ratio } k \]

\[ \implies \text{The triangle ABC is an enlargement of the triangle AMN of ratio } \frac{1}{k}. \]

Figure 4: \( \Delta ABC \) is an enlargement of \( \Delta AMN \)

5) Converse of Thales’ Theorem

In triangle ABC, if \( \frac{AB'}{AB} = \frac{AC'}{AC} \) (Figure 5), then (B’C’) is parallel to (BC)
Figure 5: Converse of Thales’ Theorem in triangles

6) Thales’ Theorem: Generalized

Knowing that (d) and (d’) are two intersecting lines at A, B and B’ are two points of (d) distinct from A and C and C’ two points of (d’) distinct from A (Figure 6).

If the lines (BC) and (B’C’) are parallel, then \[ \frac{AB'}{AB} = \frac{AC'}{AC} = \frac{B'C'}{BC} \].

Figure 6: Generalization of Thales’ Theorem

7) Fourth proportional

Given a segment [AB], and two numbers m and n. Point B’ should be placed on (AB) such that \[ \frac{AB'}{AB} = \frac{m}{n} \]. AB’ is the fourth proportional.

Throughout the pilot study, the sections relative to Reduction – Expansion, Thales’ Theorem – Generalized and Fourth proportional were not covered.
Similar Triangles: This lesson covers the following topics:

1) Definition of Similar Triangles

Two triangles are said to be similar if one of these two conditions are fulfilled:

- The sides of one have lengths respectively proportional to the lengths of the corresponding sides of the other.
- The corresponding angles have the same measure.

Whenever two triangles are similar, the ratio of the lengths of the corresponding sides is called the ratio of similitude.

2) Properties (Conditions of similarity)

- If two angles of one triangle are respectively equal to two angles of another triangle, then these two triangles are similar.
- Two triangles having an angle in one equal to an angle in the other one and the two sides of each angle respectively proportional, are similar.
- If the sides of one triangle are respectively proportional to the sides of another triangle, then these two triangles are similar.

3) Ratio of Similitude, Length and Area

4) Metric relations in a right triangle

5) Center of gravity in a triangle: In a triangle, the medians intersect at the same point G called the center of gravity of the triangle. G divides the medians in the ratio $\frac{2}{3}$ from the vertices of the triangle.

Concepts 3 and 4 (Ratio of Similitude, Length, Area and Metric relations in right triangle) were not explained due to shortage in time.

The properties of similar triangles, covered in the pilot study, were introduced as postulates Angle Side Angle (ASA), Side Angle Side (SAS) and Side Side Side (SSS).
Difficulties in teaching and learning of Similar Triangles

Before the study was actually launched, there was a need to investigate the main difficulties encountered by the teacher and the student while teaching/learning the concept of similar triangles in class. Accordingly, an interview with a 9th grade math teacher (Appendix B) was carried out.

The teacher is Lebanese with three years of experience in teaching Mathematics. She has no experience with integration of technology into the teaching of Math. The teacher had not use Cabri or any other DGS means before but she’s aware of their use and features. She reported that similar triangles is one of the most interesting concepts in Math as it is very close to reality and students can easily relate it to their everyday life experience (shadows, blue prints, models, floor plans, etc…). However, students fail to perceive its importance due to certain difficulties they face while learning.

It is known that proportionality is a major area of difficulty in similar triangles, for students, having equality in mind from Congruent Triangles, find it hard to comprehend proportionality and accordingly use it in a meaningful way. They might solve a direct application on proportionality correctly, but they would struggle in spotting proportional sides in a given figure. Eventually, the concept of proportionality seems to be hindering students’ high achievement in similar triangles.

The teacher revealed as well that the major area of concern in Similar Triangles is Proofs. Proofs form an essential requirement in the Lebanese curriculum, book and general mathematics education. However, students seem to be struggling when they are asked to solve a problem involving proof. The most common approach that they tend to use is the Angle Angle postulate as it is free from proportionality and they disregard the SAS or SSS postulates. The teacher asserted that problem solving
with proofs seems to be difficult for students as well as for teachers to explain. She believes that the process of explaining a proof seems to be a difficult task as the students fail to put together all the data from the given problem, from the figure and from knowledge.

However, the figure ought to be a critical tool in *Similar Triangles*. The teacher pointed to the fact that if a student is able to “make more” out of the figure then he/she should be able to tackle proofs in an easier way. Eventually, what is needed is a mean to manipulate or play with the figure. Students would then be able to move points, alter measures, derive hypotheses and test their conclusions. The figure is no more a fixed drawing but dynamic. This tool can be technology.

Development of the Unit

The Lebanese Mathematics curriculum (Appendix A) considers Mathematics as a “fertile field for the development of critical thinking, for the formation of the habit of scientific honesty, for objectivity and for rigor precision” (ECRD, 1997, p. 288).

Therefore, it emphasizes mathematical arguments and reasoning, problem-solving, scientific approach, mathematical communication and valuing mathematics as an art. It assigns five periods of Math per week to 8th and 9th graders along a sequence of 150 periods per year.

Within the Intermediate Level – Geometry, *Thales' Theorem* and *Similar Triangles* are embraced under the 9th grade level. They belong to the section of Plane Figures, together with *cyclic quadrilaterals*. Twenty hours of teaching are allocated to the three topics, all of them included in one section of the textbook.

The Lebanese curriculum (Appendix A), the teacher’s interview (Appendix B) and the book (Lessons in Appendix C) shaped the structure of the unit in terms of the
diagnostic test (Appendix D), the prerequisites session, the teaching/learning sequence and kind of activities introduced (Appendices F, G and H respectively) as well as the summative test (Appendix I).

Before the teaching part took place, there was an urge to test students’ acquisition of the prerequisites. Accordingly a diagnostic test (Appendix D) took place.

The results showed that the prerequisites needed to be revisited.

Eventually, a 90 minute-session was reserved to cover the prerequisites and get students acquainted with the use of Cabri (Appendix E). The prerequisites were limited into the concepts of proportionality, congruent triangles and locus.

**Proportionality**

This section covered ratios of the form \(x/y\), ratios in simplest form and definition of proportions. The main properties of proportions are revised:

\[
\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{d}{b} = \frac{c}{a}, \quad \frac{a+b}{b} = \frac{c+d}{d}, \quad \frac{a-b}{b} = \frac{c-d}{d}
\]

The students practiced on proportionality through direct application.

**Congruent Triangles**

This section covered the main definition of two congruent triangles. Accordingly three main postulates are covered:

i. SSS postulate

ii. ASA postulate

iii. SAS postulate

A direct application on Congruent Triangles followed the instruction.

**Locus**

The main definition and specific cases of Locus were revisited.

Within this session the students were introduced into Cabri. They explored its main features, the function of each option and constructed basic geometric figures.
Implementation

By the time the study was piloted, it was clear that participants had a solid understanding of *Congruent Triangles* and *Proportionality*, though it varied relative to each student's level in Math.

Working on *Similar Triangles*, the preference regarding the teaching approach was towards the Social Constructivism (as reported in literature review). Knowing that “learners respond not to external stimuli but to their interpretation of those stimuli”, students tend to gain more whenever they get the chance to construct their own knowledge (Vygotski, 1978). Hence, the student-researcher interaction was shaped in a way that students would be developing the definition, properties and postulates relative to *Similar Triangles* on their own. Accordingly, four activity sheets were generated covering the entire lesson.

*Worksheet I (Appendix F)*

The aim of the following activity is to generate and apply Thales’ Theorem. Throughout the activity students were asked to construct the following figure (Figure 7):

![Diagram](image)

*Figure 7: Worksheet I*

To avoid difficulties caused by construction, the students were guided through the construction process. Then they were prompted to measure the angles, measure the sides, move the points and notice what is happening upon each alteration.
Furthermore, they were asked to calculate several proportions, as $\frac{AB}{AB'}$, $\frac{AC}{AC'}$ and $\frac{BC}{B'C'}$, to spot the kind of relation existing between the segments.

Eventually, the students would come up with the relation, state its main characteristics and the researcher would interfere to name the above relation as Thales’ Theorem. On the board, the findings would be recorded: Definition and the formal stating of Thales’ Theorem.

During the exploration, the researcher’s role was limited to providing help whenever the students were facing difficulties in terms of Cabri.

Later on, throughout the session, the students would be asked to apply the theorem, generate its converse and finally generalize their findings.

By the end of the session, students would have developed the theorem on their own, and it would be easy to be applied as they have taken an essential role in the construction of their own knowledge regarding Thales’ Theorem.

*Worksheet II (Appendix G)*

This sheet specifically contained the core of the lesson *Similar Triangles* in terms of:

i. Definition of Similar Triangles

ii. Ratio of Similitude

iii. Properties (Conditions of Similarity):

   a. Angle Angle Postulate (AA)
   
   b. Side Angle Side Postulate (SAS)
   
   c. Side Side Side Postulate (SSS)

Having in mind the Constructivist approach, the purpose was that the students would develop the above conjectures on their own.

Note that the main aim behind activities I, II and III was to start from specific cases then reach a general definition of similar triangles, which was to be done by
elimination. By the end of activities I and II, the conclusion would be recorded on the board. Eventually, the following results were drawn:

Similar Triangles:

- Have common vertex → Two corresponding sides in each triangle would be respectively overlapping
- Have at least one of the corresponding sides parallel
- Corresponding sides are proportional
- Corresponding angles are congruent

Activity I: To familiarize students with the concept of Similarity in an easy way, the plan was to start from a common example which is equilateral triangles. Accordingly, they would construct two equilateral triangles and compare their results with congruent triangles.

Students would infer that though angles are congruent, sides are not equal, thus there should be another relation close to Congruency but not Congruency that exists between the two constructed triangles. I would interfere to name this relation as Similarity or Similar Triangles. Prompted by students’ answers, the findings would be written on the board.

By the end of this activity, students would have grasped the basic feature of Similarity that is congruent angles among two given triangles.

Activity II: The purpose of the following activity is to determine the other condition to obtain Similarity which is proportionality among homologous sides.

The students have to the construct the following figure (Figure 8):
Figure 8: Worksheet II, activity II

They would measure all the angles of the two triangles and the sides $AB$, $AM$, $AC$, $AN$, $BC$ and $MN$. The first conclusion they would come up with is that angles are congruent but triangles, rather than congruent, are similar based on the results of Activity Sheet I. They would be prompted to calculate, tabulate and animate $AM/AB$, $AN/AC$ and $MN/BC$. Therefore, they would notice that upon dragging any of the points, the sides will remain proportional. Hence, another condition to Similar Triangles is proportionality among homologous sides.

Activity III: A direct application to Similar Triangles is provided. As students proceeded in solving, they realized that they are obtaining similar triangles but not necessarily satisfying all of the above listed conditions. Hence, they started eliminating the non-sufficient conditions to obtain similarity which is the first and the second.

By the end of the activity, a generalization is provoked trying to show students that the case they had is very specific and results tend to occur even if triangles are not overlapping or parallelism doesn’t exist. They then succeeded to provide a formal definition of Similar Triangles. The definition was recorded, by one of the students, on the board.
Activities IV, V and VI: The aim is to generate the three postulates of similarity (AA, SAS and SSS). Students would be given pre-constructed, under Cabri, figures with specific measurements then prove that their exist similarity among given figures. They would prove their result, and put together the given they used in an attempt to name the method that had actually worked. Eventually, they would have generated the three main postulates.

Activity VII: The purpose is to show students that the above postulates represent sufficient conditions to obtain similarity. So, students were given a case where they had to construct, under Cabri, two triangles with two of their sides are respectively proportional but angles are non-congruent angles. The students would notice that similarity doesn’t exist in such a case and the postulates are the only proper cases.

Worksheet III and IV (Appendices II and I)

Those two activity sheets represent the problem solving part of the pilot study, a clinical interview during which a “talk-aloud problem solving” setting took place (Malloy & Jones, 1998).

The problems ranged from easy to difficult, through which several mathematical concepts are involved such as perpendicular lines, fixed/free points, perpendicular bisectors, heights, circle and all its components, angles facing diameters, inscribed angles, center of gravity and tracing Locus in a circle or triangle. Students are prompted to look for similar triangles, verify their results through proofs, determine length of sides, verify equations (such as HB = 2OH), determine measure of sides in terms of fixed values (such as calculate in terms of R the lengths OH, HM, MA and MB) and evaluate areas of triangles.

The problems are not considered to be “routine”, they are rather complex. To solve them, students need to integrate several concepts in Math and put together
relative data from the given, the construction process, postulates, theorems and properties. Furthermore, they need to look for patterns, guess and check, draw a separate diagram, generate hypotheses and test the validity of those hypotheses using dragging mode and other options from Cabri (such as Trace On/Off). Various strategies can be used, some are direct, others need long stages of development; hence it is up to the student to perceive and choose the easiest method to be utilized. The proofs, inferences and deductions need an organized plan to be constructed.

Eventually, inductive and deductive reasoning are needed.

Summative Test (Appendix J)

The purpose of the summative test was to investigate students’ acquisition of knowledge regarding Thales’ Theorems and Similar Triangles. The results would be used to support the research question on the capability of 8th graders to learn similar triangles once they are supported with DGS.

However, the summative test was paper-and-pencil based in order to assess the transfer of knowledge and strategies into a computer-free environment.

The test consisted of four problems embracing the concepts that were addressed throughout the lessons. The level of difficulty ranged from direct application to critical thinking.

Problem I: This is a direct application on Similar Triangles, whereby students had to construct any similar triangles. They had to verify their answer and determine the ratio of similitude.

Problems II, III and V: The purpose of this problem is to assess students’ understanding of the newly learned concept when similar triangles appear in a circle (Figure 9) and a triangle (Figures 10 and 11):
Figure 9: The case of similar triangles in a circle (Problem II)

Figure 10: The case of similar triangles in a triangle (Problem III)

Figure 11: The case of similar triangles in a circle (Problem V)

The students have to prove that existing triangles are similar and generate certain deductions from the diagram. They would use properties of angles in a circle (inscribed angles and angles facing diameter), Thales’ Theorem, Mid-Line Theorem and deducting reasoning to provide a correct solution.

Problem IV: In this problem, the student was prompted to construct a diagram based on given data. Then, the student had to use deductive reasoning using the figure and
Pythagorean Theorem to prove segments are perpendicular and determine measure of sides.

*Problem VI:* The final problem encompassed critical thinking and high levels of analysis. The objective was to assess the ceiling of students’ mathematical thinking and deductive reasoning skills.

Given the following figure (Figure 12), students had to use several inferences, look

![Diagram](image)

*Figure 12: Problem VI*

for patterns, use similarity in generating deductions about segments, determine nature of triangles and find locus.

Several skills are integrated in these problem solving situations and students needed to organize the various given data so that they are not lost with the several underlying concepts in the figure.
CHAPTER V

The Pilot Study

The following chapter will focus on analyzing students' work. The following will be covered: Comparing diagnostic test and summative tests, analyzing the conducted clinical interviews, students' work on Cabri, the features of Cabri that seem to be helping in achieving the main purpose and the evolution of students' problem solving strategies.

Note that diagnostic and summative tests could be found in Appendices D and J, whereas worksheets I, II, III and IV could be found in Appendices F, G, H and I respectively.

Global Comparison of Diagnostic and Summative Test Scores

The conceptual scoring criteria represented in the following table (Table 2) was used as a scoring rubric of the diagnostic test (Appendix D) and summative test (Appendix J) to assess students' success in solving the required problems. The criteria was based on the National Assessment of Educational Progress (NAEP) scoring criteria and problem solving evaluation guidelines set by Malloy and Jones (1998).

Table 2

<table>
<thead>
<tr>
<th>Scoring Criteria</th>
<th>Conceptual Scoring Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 points</td>
<td>There was no response or the work was completely incorrect or irrelevant</td>
</tr>
<tr>
<td>1 point</td>
<td>The response demonstrated minimal understanding of the problem, but the approach would not have led to a correct solution</td>
</tr>
<tr>
<td>2 points</td>
<td>The response demonstrated some conceptual understanding of the solution, but the solution was not developed</td>
</tr>
<tr>
<td>3 points</td>
<td>The response demonstrated a thorough conceptual understanding, but the solution was incorrect</td>
</tr>
<tr>
<td>4 points</td>
<td>The response demonstrated clear understanding and was correct</td>
</tr>
</tbody>
</table>

(Malloy & Jones, 1998, p. 149)
A sample case would be given as an example of each scoring criterion:

\[ \frac{a+b}{h} - \frac{c+d}{d} \]

**Figure 13:** A sample of a solution where the score given is 0 points (blank)

2) Show that the four points M, N, H and K are the vertices of an isosceles trapezoid.

[Diagram with points M, N, H, and K labeled]

**Figure 14:** A sample of a solution where the score given is 1 point

1) \[ \frac{a}{c} = \frac{b}{d} \]

[Diagram with a cross mark]

**Figure 15:** A sample of a solution where the score given is 2 points
Figure 16: A sample of a solution where the score given is 3 points

\[
\frac{x + 2}{6} = \frac{6}{x - 2}
\]

\[
(x + 2) (\frac{x + 2}{2}) = 36
\]

\[
x^2 + 2x + \frac{3x + 4}{4} = 36
\]

\[
x^2 + \frac{4x + 4}{4} = 32
\]

\[
x^2 + x = \frac{32}{4}
\]

\[
x^2 + x = 8
\]

Figure 17: A sample of a solution where the score given is 4 points

\[
\frac{x + 3}{4} \times \frac{9}{2}
\]

\[
(x + 3)(x) = 36
\]

\[
2x + 6 = 36
\]

\[
x = 30
\]

\[
x = \frac{30}{2}
\]

\[
x = 15
\]

Table 3

<table>
<thead>
<tr>
<th></th>
<th>Problem I</th>
<th>Problem II</th>
<th>Problem III</th>
<th>Problem IV</th>
<th>Problem V</th>
<th>Problem VI</th>
<th>Total Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>15</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>33</td>
</tr>
<tr>
<td>Student 2</td>
<td>14</td>
<td>10</td>
<td>12</td>
<td>4</td>
<td>9</td>
<td>6</td>
<td>55</td>
</tr>
<tr>
<td>Maximum</td>
<td>16</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>8</td>
<td>72</td>
</tr>
</tbody>
</table>

Possible Score

The maximum possible score of each problem is the number of parts in the problem multiplied by 4
Table 4

**Summative Test Results**

<table>
<thead>
<tr>
<th></th>
<th>Problem I</th>
<th>Problem II</th>
<th>Problem III</th>
<th>Problem IV</th>
<th>Problem V</th>
<th>Total Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>12</td>
<td>11</td>
<td>15</td>
<td>10</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>Student 2</td>
<td>11</td>
<td>12</td>
<td>23</td>
<td>25</td>
<td>79</td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>12</td>
<td>12</td>
<td>24</td>
<td>27</td>
<td>83</td>
<td></td>
</tr>
</tbody>
</table>

Possible Score

The maximum possible score of each problem is the number of parts in the problem multiplied by 4

Table 5

**Summary of Results**

<table>
<thead>
<tr>
<th></th>
<th>Diagnostic Test Score (out of 100)</th>
<th>Summative Test Score (out of 100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>33/72 = 46% (Around)</td>
<td>55/83 = 66%</td>
</tr>
<tr>
<td>Student 2</td>
<td>55/72 = 76%</td>
<td>79/83 = 95%</td>
</tr>
<tr>
<td>Average Student Score</td>
<td>41.5/72 = 58% (Around)</td>
<td>67/83 = 81% (Around)</td>
</tr>
</tbody>
</table>

The scores in Table 5 show that student 1 scored 46% and 66% on the diagnostic and summative tests, whereas student 2 scored 76% and 95% respectively. Therefore, the performance of student 1 improved from 46% to 66% and student 2 from 76% to 95%. These scores imply that meaningful learning of Similar Triangles took place and DGS was helpful in achieving this purpose.

The results reflected by the scores are supported by the following analysis which presents a qualitative comparison of the students’ performance on the diagnostic test versus the summative test.

Students’ performance skills/competencies on the diagnostic test are categorized into:

- Direct application of proportional relationships
- Proofs including direct application on congruent triangles

- Proofs including proportionality

- Proofs including basic analysis and geometric reasoning

- Deductions about lengths of sides

- Construction

- Locus

_Direct application of proportional relationships_

Both students (Figures 18 and 19) were able to apply the basic properties of proportionality through cross multiplication to find the value of unknowns.

\[
3) \quad \frac{16}{4x} = \frac{4x}{9}
\]

\[
\left(\frac{4x}{16}\right)^2 = 144
\]

\[
x^2 = \frac{144}{16}
\]

\[
x = \sqrt{\frac{144}{16}}
\]

*Figure 18: Student 1, work on proportionality, diagnostic test, problem 1*
Figure 19: Student 2 work on proportionality, diagnostic test, problem I

Proofs including direct application on Congruent Triangles

Both students were able to develop proofs involving congruent triangles. However, the proofs provided by student 1 focused on transcribing information from the figure and stating correct reasons (Figure 20). But he failed to back up the final congruency result with a valid reason (SAS, SSS and ASA postulate).

![Problem II:]

Write a congruence relation between each two triangles. Then, prove your answer.

1) Relation: \( GI = LK \)

\( GH = JL \)

\( HGI = LK \)

\( \text{Proof: } i/s \text{ given} \)

Figure 20: Student 1, work on direct proofs, diagnostic test, problem II

Student 2 provided elaborate and organized proofs. Each statement was justified with a valid reason and each time, a certain congruency postulate was being used and stated to explain the final result (Figure 21).
Figure 21: Student 2, work on direct proofs, diagnostic test, problem II

Proofs including proportionality

Given that \( \frac{a}{b} = \frac{c}{d} \), students were asked to develop several further related proportions as \( \frac{a}{c} = \frac{b}{d} \) and \( \frac{a+b}{b} = \frac{c+d}{d} \). The easiest approach was to start from \( ad = bc \).

Student 1 generated the relation \( ad = bc \) (Figure 22), but failed to use it properly to explore the other required proportionalities. Hence, she scratched it out and didn’t reach any other proportional form.

Figure 22: Student 1, work on proofs including proportionality, diagnostic test, problem III
On the contrary, Student 2 cross multiplied the above ratios and showed that each proportion could end up with \( ad = bc \) and deduced that the statement is true (Figure 23).

![Problem III: Given that \( \frac{a}{b} = \frac{c}{d} \), show that each proportion is true:]

1) \( \frac{a}{c} = \frac{b}{d} \)
2) \( \frac{a}{b} \times \frac{1}{d} \)
3) \( a \frac{1}{d} = b \frac{1}{c} \)

\[ \text{Then: } \text{They are equal. Proportion is true.} \]

*Figure 23: Student 2, work on proofs including proportionality, diagnostic test, problem III*

*Proofs including basic analysis and geometric reasoning*

Problems IV, V and VI included basic analysis and proofs regarding circles and all its components (radii, diameters and perpendicular bisectors), properties (angles facing diameters measure 90°, two perpendicular diameters cut the circle into four equal quadrants, etc...) and definitions (the length of a diameter equals double the length of the radius), midpoints, isosceles and right triangles and isosceles trapezoids.

Student 1 was aware of all the given data and the implication of each. But she was unable to undergo the proper linking between the various given data. Hence, she failed to provide a valid proof to any of the proof – problem solving problems (Figure 24). In problems V and VI she barely attempted to solve any and a condition of ‘giving up’ took place.
Figure 24: Student 1, work on proofs with basic analysis, diagnostic test, problem IV

Student 2 attempted to solve all the problems requiring proofs. He organized all his given data, developed the necessary deductions out of the given to be used and placed them in a proper order to make his proof clear. He organized and supported his arguments with valid reasons.

For instance, upon solving problem IV in the diagnostics test (Figure 25):
Figure 25: Student 2, work on proofs with basic analysis, diagnostic test, problem IV

To prove that the quadrilateral CMUE is a square, student 2 managed to prove that it is a parallelogram having two pairs of opposite sides parallel. Furthermore, the diagonals (diameters CU and EM) are given perpendicular and are congruent (diameters are equal in length); hence the quadrilateral is a square (Figure 25).

Deductions about lengths of sides

Each problem encompassed a part where the student has to undergo several steps of deductions to develop the length of certain sides. The approach depends on making use of the preceding parts in each problem.

Student 1 failed to construct the required proofs in problems V and VI hence the deductions about length of sides could not be done (Figure 26).
Figure 26: Student 1, work on deducing values of length of sides, Diagnostic Test, Problem V

Student 2 exerted significant amount of effort trying to use all of the given data, proved statements and background knowledge about concepts such as perpendicular bisectors and right angle triangles.

For instance, in problem V (Figure 27), to calculate the length of $OI$, the student noticed that $OI$ is a median relative to hypotenuse $AB$. Then, $OI$ measures half $AB$ and equals $\frac{2a}{2} = a$.

Figure 27: Student 2, work on deducing values of length of sides, Diagnostic Test, Problem V
Construction

Both students showed adequate construction skills (Figures 28 and 29).

Figure 28: Student 1, construction sample, Diagnostic Test, Problem IV

Locus

The locus appeared in problem V. Both students showed weakness in solving the locus problems (Figures 30 and 31).
Problem V:
Given a right angle \( \angle x y \). Let \( A \) be a variable point on \( [OX] \) and \( B \) be a variable point on \( [OY] \) such that \( AB = 2a \) = constant.
Let \( I \) be the midpoint of \( [AB] \).
1) Construct the figure.

3) Determine the locus of the point \( I \).
   Attention: \( I \) is always inside the angle \( \angle x y \).

Figure 30: Student 1, solution on locus, diagnostic test, problem V

Problem V:
Given a right angle \( \angle x y \). Let \( A \) be a variable point on \( [OX] \) and \( B \) be a variable point on \( [OY] \) such that \( AB = 2a \) = constant.
Let \( I \) be the midpoint of \( [AB] \).
1) Construct the figure.

3) Determine the locus of the point \( I \).
   Attention: \( I \) is always inside the angle \( \angle x y \).

Figure 31: Student 2, solution on locus, diagnostic Test, problem V

Students' performance skills/competencies in the summative test are categorized into:

- Geometric construction and basic knowledge application

- Rich geometric reasoning, analysis and proofs including similarity and other geometric concepts
- Integrating proportionality and similarity to generate relations and information about sides

- Locus

**Geometric construction and basic knowledge application**

Both students managed to directly apply the main definition of similar triangles through drawing two triangles that are similar. They succeeded in backing up their drawings with valid reasons (AA postulate) and generated correct ratio of similitude.

However, student 1 considered a specific case where triangles have a common vertex and parallel bases; accordingly she reasoned that they are similar because corresponding angles are congruent (Figure 32). Hence AA postulate was adopted (though it was named AAA on the test).

![Problem 1: Construct any two similar triangles.](image)

![Figure 32: Student 1 drawing of similar triangles, Summative test, Problem 1](image)
Student 2 drew two triangles each with angles measuring $60^\circ$ and $70^\circ$, and he claimed that they are similar because of the two pairs of congruent angles (Figure 33). So, the AA postulate was used as well.

Figure 33: Student 2 drawing of similar triangles, summative test, problem I

The construction that the students did along with the illustration and the ratio of similitude showed that the concept of similarity was well understood and could be easily applied.

Rich geometric reasoning, analysis and proofs including similarity and other geometric concepts

Throughout the summative test, students were asked to prove similar triangles (Problems II and III), isosceles triangles (Problem IV), parallel lines (Problem IV) and perpendicular lines (Problem IV). Furthermore, they had to undergo wise analysis to determine the nature of triangles and measure of angles (Problems IV and V).

Student 1 was able to cover most of the proof-problems with considerable accuracy relative to the sequence of ideas and the quality of provided reasons (Figure 34). Her analysis was based on efficient deductions through using the concepts of parallel lines (to find congruent angles), converse of Pythagorean Theorem (to prove
that lines are perpendicular), Mid-line segment Theorem (to find congruent angles through parallel lines) and circle (congruent radii and right angles to deduce right isosceles triangle).

**Figure 34:** Student 1, geometric reasoning and analysis sample, summative test, problem IV

Similarly, student 2 showed efficient proving skills (Figure 35) and a wise use

**Figure 35:** Student 2, geometric reasoning and analysis sample, summative test, problem IV
of various geometric definitions and concepts.

*Integrating proportionality and similarity to generate relations and information about sides*

Throughout problem solving tasks, students were asked to generate relations among sides and find the measures of various sides of triangles. In both cases, they were supposed to use proportionality (without being stated or given as a hint). Both students noticed that they should use similar triangles and specifically ratio of similitude and eventually they developed correct solutions.

A wise use of the concept of similarity was clear in the solutions that the students generated. They were able to relate proportionality to similar triangles and above all wisely decide when to use the fact that matching sides are proportional in such triangles.

*Locus*

Both students failed to develop sufficient analysis regarding the Locus. This difficulty may be due to the fact that the prerequisites session did not cover the topic and techniques of Locus need to be reviewed in terms of cases and consequences. In addition, at the level of 8th grade students would not have had a considerable amount of teaching input and practice on proofs especially Locus. That’s why the students showed weakness in solving problems requiring locus of points.

As a summary, comparing the diagnostic vs. summative test analysis, the following could be implied:

- The concept of similar triangles was properly acquired and applied by both students

- High-quality and efficient geometric reasoning, analysis and proof skills were developed by both students and to a greater extent by student 2. The
improvement is significantly noticeable especially in the proving and reasoning capabilities.

- Mathematical knowledge gained was successfully transferred into a computer-free environment.

Analysis of Clinical Interviews

The aim behind this analysis is to report the impact of Cabri on the students' problem solving skills, hence, the kind of strategies adopted and the final solutions generated by the students through solving will be thoroughly analyzed.

After the introduction of the lesson Similar Triangles and applying directly on it, the students were asked to solve two exercises (Worksheets III and IV). Worksheets III and IV consisted of three and four problems respectively. Students were solving, arguing and discussing their solutions out loud in a clinical interview setting.

The transcript of the sessions' tapes showed that the students resorted to the following strategies while solving:

- Verification

_Defined as the process of testing a conjecture to determine its truth_

- Guess and Check/Trial and error

_Considered to be an approach in problem solving that takes place through examining and trying several possibilities._

- Smart Guessing

_It is the kind of guessing that is made based on some reasoning._

- Analysis

_It is the process of breaking down a certain idea into smaller pieces to gain better understanding of it._
- Pragmatic Justification (based on the results from Cabri – figures)

It is the justification that is based on certain actions or examples that is not related to conceptual explanation.

- Conceptual Justification

Described to be the kind of explanation that is characterized by depending on basic concepts and theory.

- Visual Reasoning

It is the reasoning that is based on perceiving relationships in the drawing

- Drawing conjectures and testing them

Conjectures are considered to be propositions or hypotheses that are plausible but not proved.

- Geometric reasoning

The cognitive process of looking for reasons, conclusions, deductions and relations throughout geometric problem solving.

Verification

Verification was used as a mean to confirm or refute an assumption that has been generated by the students. For instance in worksheet III (Figures 36 and 37):

Open the following activity where:
\[ \angle A = 60^\circ, \ OA = 2\text{ cm}, \ OB = 9 \text{ cm}, \ OC = 6\text{ cm} \text{ and } OD = 3\text{ cm} \]

1) Find two similar triangles. Verify your answer
2) Show that these triangles are similar.
3) Find the ratio of similitude.

Figure 36: Worksheet III, part 1
Figure 37: The given figure in worksheet III, part 1

When the students were asked to find two similar triangles (OAD and OBC) without explaining the reasons, they spotted two triangles but knew that it is not enough to just state them (Figure 37). There has to be a way that could actually support their choice of triangles without going into formal proofs; and verification was the way out. Eventually, the students determined the matching sides, measured them and then calculated the proportions. The students reasoned that if the selected triangles are similar then the ratio of matching sides should be equal. However, upon calculating the ratios, the result was that they weren't equal. Hence, three possibilities could take place: The measurements were wrong, the choice of matching sides was wrong or the triangles are not similar.

In the following excerpts from the clinical interview transcripts, R and S shall stand for researcher and student respectively.

- **R:** What are you doing?
- **S:** Checking the sides measurements
- **R:** What do you get?
- **S:** They are not correct
- **R:** Why?
- **S:** OC/OD is not equal to OA/OD.
- **R:** Why should they be equal?
- **S:** Because those sides are matching so the ratios should be equal

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However, they failed to verify that those two triangles were similar. As a result, there was an urge for a different method.

_Trial and Error/Guess and check_

Along the same exercise (Worksheet III, part 1) (Figures 36 and 37) and after facing problems in checking measures of the sides, the students switched their attention towards angles (Figure 38). Hence, they agreed that they need a different approach then sides. They measured the angles, and found out that there is a common vertex angle and one pair of matching congruent angles existing in the two triangles (Figures 37 and 38). Hence, the triangles are similar (AA postulate).

![Diagram of congruent angles](image)

*Figure 38: Congruent angles in the figure of worksheet III, part 1*

Accordingly the students have drawn two different but related conclusions:

- If the pre-supposed triangles are similar then the matching sides should be proportional.

- Since sides are hard to spot, an easier approach should be angles.

So, they first searched for the triangles that might appear to them as similar, and then measured, using Cabri, the matching angles. If Cabri showed that each pair has the exact measurement (even if the points are dragged measures stay the same),
then the triangles are similar. Eventually, the students depended on trial and error, using smart guessing as they had a solid understanding of the concept of similarity that led them throughout the search for congruent angles.

- R: Based on what do you suppose that OC and OD are matching sides?
  (No answer, so the other student is being asked)
- R: Do you agree that those two sides are matching?
- S: No they are not
  (The student knows that the sides are not matching but fails to explain why)
- R: So, what would you do in this case?
- S: Check for angles

............... 
- R: So, what are the similar triangles?
- S: OAD and OCB
- R: Why?
- S: Because they have two congruent angles

Pragmatic Justification

After the students found the similar triangles, it was not enough to base their justification solely on Cabri, they had to explain it in a more mathematically sound way (Figure 37):

- R: So, what are the similar triangles?
- S: OAD and OCB.
- R: Why?
- S: Because they have two congruent angles.
- R: What are they?
- S: \( \angle O = \angle O = 60^\circ \) and \( \angle ODA = \angle OBC = 40.9^\circ \)
- R: What method did you use?
- S: AA postulate (Angle – Angle postulate)
The students' conclusions were based on actions they have taken through Cabri and not solely simple reasoning. Hence, they attempted to develop pragmatic justification to explain their results.

*Smart Guessing*

In order to prove that the above triangles are similar (Figure 37) The students had to figure out what are the matching sides. Since the figure was not clear, it was not easy on them to spot the sides.

- **R:** Can you come up with the matching sides?
- **S:** OC and OD
- **R:** Why is it just because they are overlapping? *(They found no other way but using the figure.)*

Finally, and while students were writing down the solution on the notebook:

- **S:** $\angle ODA \equiv \angle OCB \Rightarrow OC$ and $OA$ are matching
- **R:** What is the other pair?
- **S:** $OB$ and $OD$
- **R:** How did you find it out that those are matching?
- **S:** $OB$ is a part of this triangle and $OD$ is part of this triangle.
- **R:** So, randomly speaking?
- **R:** Yes

The students found no other way to spot the sides but guessing. However, it was smart guessing because they were aware of all the pairs of congruent angles that are in the triangles and their characteristics.

*Conceptual justification*

While solving worksheet IV, part 2, the students were supposed to find two similar triangles then prove similarity. After the students succeeded in finding the similar triangles ($\triangle ABM$ and $\triangle MBH$) (Figures 39 and 40), the way they have chosen
to explain their finding was through providing justification based on the concept of angle facing diameter, similar triangles and the similarity postulates (referring to AA postulate).

(C) is a circle of fixed diameter [AB] of center O and radius R.
H is a point of [OB] and (d) is the perpendicular line to (AB) that passes through H.
Designate by M one of the intersection points of (d) and of (C) and by N the point diametrically to M on (C).

Part 2 will be saved under as Q2 and each attempt will be saved under a different title (Q2.1 - Q2.2 - Q2.3 etc... )

2) a- Find two similar triangles. Verify your answer
   b- Can you prove Similarity in this case?

Figure 39: Worksheet IV, part 2

Figure 40: The corresponding figure to worksheet IV, part 2

- R: How can you explain your result? (that $\Delta ABM = \Delta MBH$)
- S: $\angle B$ is a common angle $\Rightarrow \angle ABM \equiv \angle HBM$
- R: $m\angle B = 90^\circ$ (angle facing diameter) and $m\angle BHM = 90^\circ$ (definition of perpendicular lines)
- S: Triangles are congruent by AA postulate
Visual Reasoning

The visual reasoning was used throughout solving all problems by strictly relying on the figure. However, this was clearly shown when the students were asked about the Locus. They had to find the set of points of $G$ as $H$ moves (Figure 41):

(C) is a circle of fixed diameter $[AB]$ of center $O$ and radius $R$. 
$H$ is a point of $[OB]$ and $(d)$ is the perpendicular line to $(AB)$ that passes through $H$.
Designate by $M$ one of the intersection points of $(d)$ and of $(C)$ and by $N$ the point diametrically to $M$ on $(C)$.

Part $3$ will be saved under as $Q3$ and each attempt will be saved under a different title ($Q3.1$ – $Q3.2$ – $Q3.3$ etc…)

3) Construct $G$ the center of gravity of triangle $BAM$.
   a) Move $H$ along $BC$. What is happening to $G$?
   b) Using Trace On/Off, find the set of points of $G$ when $H$ describes the segment $[AB]$.
   c) Can you explain your result?

Figure 41: Worksheet IV, part 3

Therefore, what students did, was refer to the Trace On/Off option offered by Cabri (Figure 42).

- $R$: So, what is the set of points of $G$?
- $S$: Upon moving $H$ on diameter $AB$, $G$ is moving along a semicircle centered at $O$ and radius $OG$.
- $R$: How did you find it?
- $S$: Using of Trace On/Off Cabri

Figure 42: Tracing locus through Cabri in Worksheet IV, part 3
After determining the set of points, the students had to justify their answer. Accordingly, they looked at the figure and noticed that when H moves, OG remains fixed. So, the question is why OG stays fixed. Taking a look again back to the figure, they noticed that the hypotenuse of triangle AMB is a diameter which is fixed. At this point of analysis, there was a shift into another strategy (See the following paragraph).

*Analysis based on breaking down data into pieces and reflecting on each*

Along the same problem where students were supposed to find the set of points G in Worksheet IV part 3 (Figure 41), the result obtained by animating the figure showed that as the locus of point G to be a semi-circle (Figure 42), but the tough part was still to determine the reason behind this. The students tried moving backwards by questioning themselves “when are we supposed to get set of points as a circle?” However, they failed to answer this question. Therefore, they found no other way but going back to the drawing to check what they were missing. So, they moved point H again, and noticed what was going on with the other parts of the circle. They came out with the following:

- **S**: The diameter is fixed
- **R**: AB is fixed and the length of MG does not change
- **S**: Only MG?
- **R**: No, MG and OG
- **S**: Why does the length of OG remains fixed? (No answer)

Finding no answer on why the length of OG remains the same, the students decided to go back to the original given, shown in their following dialogue:

- **M moves on the circle and O is the center**
- **But G is the center of gravity**
- **But we know that the length of radius does not change**
The length of OM will not change

- So, no matter where M moves, it will always divide the segment into 3 parts: MG consist of 2/3 of OM and OG as 1/3 of OM and OM is fixed

OG remains constant

A thorough analysis has taken place, and the key was that G is the center of gravity. Having it in the figure, the students disregarded the features of center of gravity. However, when they got back to the given and drew conclusions from each piece, they remembered the major role of the center of gravity.

Geometric reasoning

Geometric reasoning was common throughout all problem-solving in the clinical interviews. The students were constantly looking for angles and sides, relying on basic definitions of circles, center of gravity, similar triangles, fixed segments and perpendicular/parallel lines. They were trying to put together the given, the characteristics drawn out form the figure and their geometric background from properties, definitions and postulates to solve the problems.

However, special reasoning skills were obvious. For instance in the same problem Worksheet IV, part 3 (Figures 41 and 43) the students were supposed to construct B such that HB = 2 OH:

- R: Construct H such that HB = 2 OH
- R: So, what do we mean by HB = 2 OH?
- S: It means OB we have to cut it into three equal parts then plot B.

The students noticed that HB = 2 OH means that there’s a segment OB that has to be divided into three equal units; the first unit represents OH and the second two represent HB and hence the length of HB is double that of OH.
Within the same problem, the students were asked first to construct B such that HB = 2OH, then calculate the length of HM. Accordingly, the students used their proposition as shown by their following dialogue:

- *We want to divide segment OB into three equal parts*
- *So HB will take 2 parts of the three and the third will be for OH*
- *But the problem is how to draw this.*

So the strategy was to measure segment OB, which turned out to be equal to 2.78, and then divide it by 3, so that the measure of each part would be 0.93.

Because of the fact that there are three equal parts, the students decided to construct a circle center B and of radius 0.93 (Figure 43). Then, the point of intersection of the circle with diameter AB will form the center of another circle that has to be constructed of the same radius 0.93 (Figure 43).

![Figure 43: Students' construction of figure in worksheet IV, part 4](image)

- *S: OH will constitute the first part and HB constitutes the other two parts.*
- *R: Calculate HM.*
- *S: To calculate HM: we have perpendicular lines ➔ Right away use Pythagoras (Conceptual Understanding)*
Students' Work on Cabri

Throughout the clinical interviews and the pilot study, the main technological tool that was used is Cabri. The students showed heavy reliance on Cabri, whether in the kind of strategies they have followed (validation and verification through Cabri, etc…) or the kind of analysis that took place (visual reasoning including dragging, calculating the measures of sides and angles, or constructing).

Upon analyzing the sessions’ videos, the features of Cabri that seemed to help the students achieve their purpose were:

- *Measurement Transfer*
- Calculator, Tabulation and Animation
- Animation
- Dragging points
- Exact measurement of angles and sides
- Constructing heights
- Hide/Show options

*Measurement Transfer*

*Measurement Transfer* was basically used in construction. The main aim behind *Measurement Transfer* was exploration and providing help in terms of exact measurements. When students had to construct segments of precise length and angles with accurate measurements, this option seemed to be a valuable tool to rely on. It was an essential element in the process of learning of *Similar Triangles* because, based on those measurements students were coming up with the definition and
properties of similar triangles and deriving the methods that would prove that any two given triangles are similar.

For instance in worksheet III, part 1, the students were given the following problem (Figure 36). They manipulated the figure, moving points and checking the measurements as follows:

- R: How can I make sure that the measurements are correct?
- S: We used Measurement Transfer
- R: If we didn't choose to use measurement transfer, what would we do?
- S: Measure the sides.
- R: Other than the sides?
- S: Drag the points and check

The above conversation implies that students fully understood the main role of Measurement Transfer. They developed a new way to check on their measurements by dragging one of the points and checking if the figure got messed up or not.

Therefore, Measurement Transfer seemed to be essential in simplifying the process of construction, obtaining exact measurements, and encouraging exploration activities.

Calculator, Tabulation and Animation

The built-in calculator had two main roles:

- Tabulating for the purpose of animating
- Calculating ratios

Tabulating for the purpose of animating: One of the very important features of Cabri
is animation. By grabbing couple of points, the figure and its parts will change shape in such a way that all the properties are preserved. Hence, the student can form a “mental image” of what is actually going on (Hanna, 2000). However, as a supplement to this activity, the student can establish a table with the main elements of the figure (with length and measure of angles) and upon animation the system would tabulate all the new values (Figures 45 and 46). Hence, deductions and conclusions could be generated.

As a sample, in worksheet I, activity I (Figure 44), the problem was as follows:

2)  
1- Measure all the angles.
4- Measure all the sides. Move aside and rename each as follows: If AB measures 3 cm, then move the 3 cm and rename it as AB: 3 cm
5- Calculate, Tabulate and animate AB/AB’, AC/AC’ and BC/B’C’
6- What do you notice?

3) Give a precise statement of your conclusion.

Figure 44: Worksheet I, activity I

The students’ work on Cabri (Figures 45 and 46) resulted in:

Figure 45: The figure in activity 1, worksheet I
<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>AB*</th>
<th>AB/AB*</th>
<th>AC</th>
<th>AC*</th>
<th>AC/AC*</th>
<th>BC</th>
<th>B'C</th>
<th>BC/B'C*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.01</td>
<td>3.44</td>
<td>2.04</td>
<td>12.25</td>
<td>6.02</td>
<td>2.04</td>
<td>12.60</td>
<td>6.18</td>
<td>2.04</td>
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<tr>
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<td>6.92</td>
<td>3.40</td>
<td>2.04</td>
<td>12.32</td>
<td>6.05</td>
<td>2.04</td>
<td>12.53</td>
<td>6.15</td>
<td>2.04</td>
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<td>6.77</td>
<td>3.32</td>
<td>2.04</td>
<td>12.46</td>
<td>6.11</td>
<td>2.04</td>
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<td>6.09</td>
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<td>12.35</td>
<td>6.06</td>
<td>2.04</td>
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<td>6.55</td>
<td>3.22</td>
<td>2.04</td>
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<td>12.22</td>
<td>6.00</td>
<td>2.04</td>
</tr>
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<td>6.49</td>
<td>3.19</td>
<td>2.04</td>
<td>12.73</td>
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<td>12.16</td>
<td>5.97</td>
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<td>12.80</td>
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<td>6.21</td>
<td>3.05</td>
<td>2.04</td>
<td>13.07</td>
<td>6.42</td>
<td>2.04</td>
<td>11.85</td>
<td>5.82</td>
<td>2.04</td>
</tr>
</tbody>
</table>

Figure 46: The tabulated results of animation in activity 1, worksheet 1

After deeply observing the animation process, the students started discussing their results, (the values of the ratios) and agreed on their final result to write down:

- **R:** What do you notice?
- **S:** (Pinpointing into the screen) The ratios $AB/AB'$, $AC/AC'$ etc... always stayed the same and constant.
- **S:** Right, even though we have changed the measure of the angles that we have.
- **R:** So, can you give me now a precise statement of your conclusion?
- **S:** In any two triangles if they have two respective parallel sides, then the ratio of the length of each side to its parallel is constant.
- **R:** More precise one in terms of $AB'$, $AC'$ etc... regarding this figure in particular. (To make the formulation of the conclusion easier, but no reply occurred)

When they failed to find the solution, the researcher asked them to go back to the table and see what is remarkable about it (One of the students was calling the other and pinpointing to the number 2.04 on the screen).

- **S:** 2.04, the ratio is common
- **R:** So, what do you conclude?
- **S:** The ratios, pointing to the screen with a hand gesture that showed confidence, are equal.
- **R:** What ratios?
- **S:** $AB/AB'$, $AC/AC'$ and $BC/B'C'$ are equal
- \[ R: \text{Conclusion: IF } (B'C') \parallel \text{to} (BC) \text{ then the ratios } \frac{AB}{AB'} = \frac{AC}{AC'} = \frac{BC}{B'C'} \]

Later on the students changed this specific conclusion into a general statement and the researcher named it as Thales' Theorem.

The same process took place while generating the definition and properties of similar triangles.

So, the basic role of the tabulation – animation was providing students with means to discover and explore existing relations between the several components of the figure. The students, and on their own, were able to derive the theorem. And once they were able to come up with it, they felt they owned it.

**Calculating ratios:** Students were constantly using the calculator to determine the values of ratios. They used those ratios for two purposes:

- Verify that triangles are similar

- Determine ratio of similitude

**Verify that the given triangles are similar:** In worksheet II, activity III (Figure 47), the triangles were given (Figures 47 and 48) and the students were supposed to look for similar triangles:

**Activity III:**
Within the file open a folder <PartII-3> where all the work will be saved.

Open the folder Activity III

1) Are the triangles Similar?
2) Is there at least one pair of opposite sides parallel?

**Figure 47: Worksheet II, activity III**
Figure 48: Figures of worksheet II, activity III

- R: Are those triangles similar?
- S: No they are not similar
- R: Are the ratios of the matching sides equal?
- S: No

They could not figure out the answer without referring back to a calculator, so the researcher asked them that it might be helpful to use the built-in calculator:

- R: Using the calculator, what did you get?
- S: The ratio of the first two matching sides is 0.5 \((AB/NM)\)
- S: \(AC/\, NP = 0.5\) and \(BC/\, MP = 0.5\)
- S: The ratios are equal \(\Rightarrow\) triangles are similar

The students relied on the calculator to verify their result. They had in mind that the triangles were not similar, but upon calculating the ratios, the students refuted their own suggestion and derived the correct one that the triangles are indeed similar.

Determine the ratio of similitude: As a direct application to the use of the built-in calculator, the students depended on it to determine the ratio of similitude whenever it was asked for.
Dragging points

Cabri offers the students a special facility known as dragging. By dragging, the figure changes shape. If it gets “messed up”, then the construction was wrong because it was not based on geometrical theories, otherwise it would have considered to be passing the “drag test” (Jones, 2000). However, the role of dragging was not limited towards the purpose of only validation but rather it took place along three levels:

- Checking the validity of construction

- Noticing the existing relations in the figure and given (Discovery)

- Testing and verifying conjectures

Checking the validity of construction: Throughout the pilot study, most of the difficult figures were pre-constructed through Cabri and given. Therefore, the students did not have a hand in the process of certain constructions and hence were not confident about their correctness. They wanted to check the construction and the simplest method to do this was through dragging.

Unlike the clinical interview activities, the students were responsible for constructing all the given data. Before starting the solving process, several points were dragged for them to test the “robustness” of their figures (Jones, 2000). If the result was positive they would go ahead and solve, if not they would keep on trying until the figure takes its proper geometric shape.

For instance, in worksheet III, part 1 (Figure 36) the following dialogue took place:

- R: How can I make sure that those values are true, and not that I got them by any chance?

- S: I move (drag) one of the points.
Noticing the existing relations in the figure and given: In worksheet I, activity I (Figure 49), the students were supposed to determine the kind of relation existing between the points and in between angles and lines. As a result the following was recorded:

**Activity I:**
Within the file open a folder and name it <Q1>

Consider the following figure:

![Diagram of triangle with points A, B, C, B', and C']

1. Measure all the angles.
2. Move the points A, B and C randomly. What do you notice? What is happening to B' and C'?
3. Move the points B' and C' randomly. What do you notice?

**Figure 49:** Worksheet I, activity I

- **R:** As you drag angle B, what is happening to the other angles?

- **S:** The measurement stays the same and measures of angles are equal:
  \[ m \angle AB'C' \equiv m \angle ABC \text{ and } m \angle AC'B' \equiv m \angle ACB \]
Figure 50: Measurements of angles and sides in the figure of worksheet I, activity I

Figure 51: Consequences of dragging the figure in worksheet I, activity I

Hence, students noticed that no matter how they change the length of the sides of similar triangles (Figures 50 and 51), the congruent angles will always remain congruent.

The same procedure took place when the students deduced that no matter how they change the length of the sides, matching sides will always remain proportional (Figures 50 and 51):

- **R**: What is happening to $B'$ and $C'$?
- **S**: $BC$ and $B'C'$ are always parallel
- **R:** What about angles B and C, are they changing in measure when you drag B and C

- **S:** Yes definitely they are changing

Hence, students noticed that dragging the points did not change the kind of relation that exists between BC and B’C’ (Figures 50 and 51):

- **R:** Can you drag B’ and C’

- **S:** B’ has moved \( \Rightarrow \) lines stay parallel and C’ does not move

- **R:** Why did B’ move? Why didn’t C’ move?

- **S:** Because the parallel is drawn through B’ and C’ is the point of the intersection of the parallel line with segment AC.

- **S:** Conclusion: \( \angle C’ \) can’t be dragged. Upon moving B’, \( \angle B \) is always congruent to \( \angle B’ \) and \( m \angle B \) doesn’t change so the measure of \( \angle B’ \) stays the same. C’ moves only when B’ moves, \( \angle C \) is congruent to \( \angle C’ \) and \( m \angle C \) doesn’t change so the measure of \( \angle C’ \) stays the same.

Thus, students concluded if the parallel line is taken though B’ then the point of intersection of this line with AC, point C’, is fixed and can’t be dragged (Figure 51).

Doing the above analysis, the students matched B with B’ and C with C’ and reasoned why the measurement of the angels remain the same even if they are dragged.

**Testing and verifying Conjectures:** Throughout problem solving tasks, students were coming up with conjectures (in the cases of similarity between triangles and parallel lines) and to test their conjectures they referred back to dragging. Upon dragging one of the points the students noticed that measurement of angles change and not equal so the so-assumed to be matching angles are not congruent. Similarly, in the cases of parallel lines, when one of the points is being dragged no more parallelism occurs. Hence, both hypotheses ought to be wrong.

**Exact measurement of angles and sides**

Cabri offers the user an exact measurement of sides and angles. Thus, the
first attempt students took to find any two similar triangles was to measure the angles and the lengths of sides (Figure 52).

![Diagram of triangle with measurements]

**Figure 52**: Measurements and labeling of angles in the figure in worksheet III, part 3

The fact that any measurement could be edited (Figure 52) makes the use of such elements easy and trivial especially when it is dragged to the table where calculations take place.

**Direct information on relations among lines**

Relations among lines on Cabri ought to be deceptive. Lines/segment may seem to be parallel or perpendicular, but students can’t use this piece of information before validating their assumptions (unless it is provided in the given or proved). But Cabri has the potential to indicate to the kind of relation existing in between specified lines. For instance, in case of parallelism, within a click the checking would take place and the outcome would be displayed on the figure as *parallel* or *not parallel* (Figure 53).

This particular feature helped in determining the characteristics of similar triangles. Starting from a specific case and moving into a general one and while students where exploring similar triangles, the first contact they had with such
triangles is a case where bases of triangles are parallel, common vertex, three pairs of congruent angles and proportional sides. To show that not all of those conditions are sufficient, several activities (worksheet II, activities I, II, III and IV) were introduced so that students can eliminate the unnecessary conditions one after the other. In worksheet II, activity III (Figure 47), students failed to solve properly because the common vertex option was eliminated and they had in mind that lines were not parallel as follows:

- **R:** Are those sides parallel?
- **S:** No
- **R:** Why don’t you ask Cabri whether they are parallel or not?

![Diagram](image)

*Figure 53:* The ‘Parallel’ label generated by Cabri in the figure of worksheet I, activity III

- **S:** Yes they are parallel.
- **R:** Great, let’s move to the next: Do they have a common vertex?
- **S:** No they don’t

*Hide/Show options*

This option helps in the process of construction. Rather then obtaining messy figures,
the un-needed elements, that were essential for drawing, could be hidden so that the figure is neat (Figure 54). However, the hidden parts with a click can re-appear.

Figure 53: The hidden lines in the figure of worksheet III, part 1

Trace On/Off

The concept of Locus is one of the difficult topics that students encounter in Geometry. Cabri simplifies this by tracing the locus and showing the students what is the set of the resulting points (Figure 55). And what remains for the students to do is to provide the correct justification behind such results as in worksheet IV, part 3 (Appendix I):
Figure 55: Tracing locus in the figure of worksheet IV, part 4

Using Trace On/Off, students noticed that upon moving H on diameter AB, G is moving along a semicircle centered at O and radius OG (Figure 55). So students went back to the figure and started moving H again to see what they were missing:

- S: The diameter is fixed
- S: So, AB is fixed and the length of MG stays the same as well.
- R: Only MG?
- ..........
CHAPTER VI

Discussion of Results

The results of diagnostic test, pilot study (including clinical interviews) and summative test, about the ability of 8th grade students to learn Similar Triangles (9th grade according to the curriculum), once they are provided with a learning experience based on DGS, and the evolution of geometrical problem solving strategies pertaining to the topic Similar Triangles, lead to the following discussion and corresponding recommendations.

On Learning Similar Triangles

The students participating in this study acquired meaningful learning of the topic Similar Triangles. Students were able to solve problems involving similarity, ranging from direct application to problems requiring deep analysis and reasoning. They built knowledge that was effectively used in proofs and deductive reasoning.

On the effect of DGS

The main tool that was utilized throughout the study was Cabri. Students were able to make more use of the constructed figure through applying various manipulations and observing the behaviour of the components of the figure accordingly. They described, explained and justified their observations. The figure is no more static, rather dynamic and visual reasoning is the dominant element in terms of proving. Those results are supported by Hanna (2000), who contends that though exploring and proving are two separate actions yet they are complementary. DGS can help students perceive mathematical concepts as a system of related statements that are validated by a proof. In addition, Sinclair (2003) highlighted the fact that DGS is capable of leading students to notice geometric details, explore relationships and develop reasoning skills that are all related to geometric proof.
On the development of geometric skills and competencies

Throughout the clinical interviews, and with the help of Cabri, the students demonstrated the development of rich problem solving skills and strategies such as validation, verification, guess and check/trial and error, pragmatic justification (based on the results from Cabri – figures), analysis, conceptual justification, visual reasoning, drawing and testing conjectures, proofs and various geometric reasoning strategies. Those skills are basic and essential in Geometry especially when it pertains to proofs. Jones (2000) emphasized this idea that DGS guides students to generate precise statements regarding properties and relationships and to carry out correct deductions, both basic in the construction of proofs. This idea was demonstrated in the work of the two participants.

On the transfer of knowledge into computer-free environment

The administered summative test was paper-and-pencil based, and yet students managed to show high level of performance; implying that knowledge and problem solving skills and strategies were successfully transferred into a computer free environment. Hence, DGS tends to be a great aid to the learning of mathematics and the acquisition of efficient problem solving skills even in an environment where technology doesn’t exist. The results of this study are also consistent with Marrades and Gutierrez (2000) findings, whereby DGS affects the students’ conception of mathematical proof and on top of everything it enhances their methods of justification.

On Constructivism and Pair Work

Constructivism and pair work formed a basic approach towards learning the new concept of similarity in the study. Students were working on activities that would prompt them to pose and test conjectures, and then convert those conjectures to
formal definitions, properties and postulates. The role of the researcher was limited to leading and guiding this learning experience, providing hints and emphasizing ideas when necessary. Hence, students would be, on their own, developing the newly learned concept, generating a sense of ownership to the concept and making it is easy to be applied.

**Recommendations on Teaching of Mathematics**

Thoughtfully designed activity sheets, the constructivist learning approach and use of DGS, form an effective learning experience to be introduced by the math teachers in their classes. Teachers can use DGS in classroom activities to ensure meaningful knowledge and the proper acquisition of problem solving skills. Those results tend to be aligned with Jones (2000) and Chazan & Lehrer (1998), indicating that aside with DGS use, carefully designed tasks, sensitive teacher input, and a classroom environment that stimulates conjecturing and explaining, students can make progress in developing efficient mathematical reasoning and proving skills.

In addition, this study showed that Grade 8 students were able to learn a concept that the Lebanese curriculum assigns on the 9th grade level program. Hence, once supported with DGS and properly designed activities, topics could be introduced at a level earlier than expected in the curriculum.

**Recommendations on Curriculum**

The aim of this study is not to generalize the results, but rather to address curricula. More specifically it targets the Lebanese curriculum where technology needs to be an important objective. The actual integration of technology in mathematics classrooms helps not only in fulfilling the main vision statement of the curriculum but also in introducing lessons at an earlier level in the curriculum.
Due to the significant impact of technology in the better learning and understanding of mathematics, special efforts should be put forth to make sure it is properly integrated in the schools' math classrooms.

Limitations of the Study

This study is an exploratory investigation to the impact of integration of DGS on the learning of mathematics and problem solving skills. Two 8th grade Lebanese students, studying the Lebanese curriculum, one high achiever and the other low achiever have participated.

It is hoped that this study came out with significant findings. Yet, it is important to draw attention to the fact that those results cannot be generalized and limitations occur as follows:

The relatively small number of participants (two students), the short period of teaching/learning duration (5 sessions each 1-2 hours long), and the lack of wide variety of math achievement levels among the participants (weak students were not among the participants) make it impossible to generalize the results of the study. On the other hand, the low number of students making the student-to-teacher ratio 2-to-1 may have positively affected the extent of learning.

In addition, special concern should be given to the kind of problems introduced in the diagnostic and summative tests. The tests were not constructed based on similar competencies or skills. Rather each was used for a different purpose: Diagnostic test to check on prerequisites and summative test to assess actual learning of the concept under discussion, the effect of using DGS on acquisition of skills and the transfer of knowledge into computer free environment. Hence, comparison of scores, solely, to derive conclusions about findings would not be valid.
Perspectives for Further Research

The significance of this study is the focus on lower secondary school students. Most of the research reported results about upper secondary school students, where they would have received considerable teaching input with learning plane geometry, where as few tackled lower secondary level students those having limited experience with geometry and proofs specifically (Jones, 2000).

Therefore, this study highlights the needs to carry out further research on the effect of DGS on the learning of mathematics of students in lower secondary. In addition, it calls for studies targeting the effect of DGS on learning concepts other than similar triangles and investigating the kind of skills and strategies that are strongly reinforced by this technology.

Finally, this study could be replicated using a larger sample, integrating several schools and with participants of various levels in math whereby other concepts in Geometry would be further investigated.
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1. INTRODUCTION

Mathematics constitute an activity of the mind which takes the dimensions of a big human adventure. It is a fertile field for the development of critical thinking, for the formation of the habit of scientific honesty, for objectivity, for rigor and for precision. It offers to students the necessary knowledge for the social life and efficient means to understand and explore the real world whatever the domain is: physical, chemical, biological, astronomical, social, psychological, computer, etc.

The flashing advancement in science and technology has deeply marked modern society. We speak today of the era of "information"; like we spoke, a quarter of a century ago, of the industrial era. Now, everybody agrees on the fact that this development could not have been accomplished but by the mathematical tool whose use has allowed to substitute the qualitative description of reality by its quantification and its operational modeling. Today, more than ever, Mathematics proves to be an ineluctable necessity to the life of societies and to their development. This science can no longer remain the property of a specialized elite, but many of its results and means must be acquired by a more considerable number of citizens.

This extension of Mathematics to all the reality, and the increasing demand for its learning have, without doubt, modified the spirit and the use. The reform of its teaching is to be operated in three axes: a new formulation of the objectives, a remodeling of contents and a suitable choice of methods.

1. Formulation of objectives: The fundamental objectives concerning the mental activities and the formation of mathematical reasoning, continue to figure, the stress is mainly on the individual construction of Mathematics; it no longer consists of teaching already made Mathematics but of making it by oneself. Starting with real-life situations in which the learner raises questions, lays down problems, formulates hypotheses and verifies them, the very spirit of science is implanted and rooted.

Our intention is also to form the students to the communication: reading a mathematical text, understanding it, interpreting it, using symbols, graphs, tables etc..., writing a demonstration, explaining a situation, etc... remain essential objectives of the teaching.

2. Remodeling contents: The subjects are not judged according to their theoretical interest but according to their practical interest. They must be accessible to all the students and respond to their need of formation and to their cultural development. Every theoretical overuse was abolished, every virtuosity in the accomplishment of the tasks was omitted. This allowed a significant reduction in the programs which aim to form "well made heads". The introduction to the calculator and the possibility of using the computer are two technological novelties which will have benefits on the formation. Other subjects which deal with the treatment of information, such as Statistics, allow the new generations to adapt better to socio-economic problems.

3. Method of teaching: The teaching of Mathematics must be organized in such a way as to demythicize it and make it accessible to a larger public. The recommended method consists of starting from real-life situations, lived or familiar, to show that there is no divorce between Mathematics and everyday life. This practice of Mathematics will lead students to the intelligence of conceptual models whose effectiveness will be understood by the transfer of successful teachings.
That was the context in which this new program has been prepared. Our essential aim is to form a citizen capable of critical thinking and intellectual autonomy.

II. GENERAL OBJECTIVES

The present curriculum, through the acquisition of adequate mathematical knowledge, aims to achieve the following general objectives.

1. Training in the construction of arguments and evaluating them, developing critical thinking, and emphasizing MATHMATICAL REASONING. These are the major goals of this curriculum. Toward this end, student will be given the chance to observe, analyse, abstract, doubt, foresee, conjecture, generalize, synthesize, interpret and demonstrate.

2. SOLVING MATHMATICAL PROBLEMS is perhaps the most significant activity in the teaching of mathematics. On the one hand, every new mathematical knowledge must start from a real-life problem. On the other hand, students must learn to use various strategies to tackle difficulties in solving a problem. Toward this end, he must be able to serialize, classify, quantify, discover mathematical methods, manipulate simulation techniques, construct and use algorithms, take decisions, verify, apply, measure, use ad hoc techniques and manipulate information.

3. Modern society has a greater need for highly qualified workers and researchers in all areas. The Mathematics curriculum responds to these demands by offering the student an opportunity of practicing the scientific approach, developing the scientific spirit, improving skills in research, establishing relations between mathematics and the surrounding reality in all its dimensions and valuing the role of Mathematics in technological, economical and cultural development.

4. Our intention is to train the student to COMMUNICATE MATHMATICALLY. To achieve this, he must learn to encode and decode messages, formulate, express information orally, in writing and/or with the help of mathematical tools.

5. Aside from being a utilitarian science, Mathematics is also an art. The curriculum gives the student a chance to VALUE Mathematics by helping him to acquire confidence in mathematical methods, to appreciate precision, rigor, order and harmony of mathematical theories, to develop intuition, imagination and creativity, to find pleasure in intellectual activities and persevere at work.
| Class | First | Second | Third | Intermediate Level | First | Second | Third | Life Sciences | General Studies | Social Studies | Economics | Humanities | Science | Literature | English | Math | Sixth | Fifth | Fourth | Third | Second | First | First | Second | Third | Fourth | Fifth | Sixth |
|-------|-------|--------|-------|-------------------|-------|--------|-------|-------------|-------------|--------------|-----------|-----------|---------|-----------|--------|------|-------|-------|--------|-------|--------|-------|-------|--------|-------|--------|-------|
| 150   | 300   | 2      | 6     | 4                 | 5     | 5      | 5     | 5           | 5           | 5            | 5         | 5         | 5       | 5        | 5      | 5    | 5     | 5     | 5      | 5     | 5      | 5     | 5     | 5      | 5     | 5      | 5     | 5     |

Table of Distribution of Periods Per Week/Year
### Table 1: Mathematics Curricula

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<tr>
<th>Operation</th>
<th>Seventh Year</th>
<th>Eighth Year</th>
<th>Ninth Year</th>
</tr>
</thead>
</table>
| 2. OPERATIONS | 1. Subtraction and multiplication of integers.  
2. Powers of a positive number having positive integer exponent.  
3. Common factor.  
Factorization. (30 h) | 1. Powers of a positive number having positive integer exponent.  
2. Powers of a negative integer exponent of 10. (5 h) | 1. Rationalizing the denominator of a numerical fraction.  
2. Calculation on real numbers. (10 h) |
| 3. PROPORTIONALITY | Directly proportional magnitudes. (10 h) | Inversely proportional magnitudes. (5 h) | Linear functions and proportionality. (5 h) |
| 4. ALGEBRAIC EXPRESSIONS | Calculation on algebraic expressions. (15 h) | 1. Remarkable identities.  
2. Literal expressions in fractional form. (20 h) | 1. Algebraic expressions having radicals.  
2. Polynomial in one variable. (10 h) |
| 5. EQUATIONS AND INEQUALITIES | Equations reduced to \( ax = b \). (10 h) | 1. Equations of the form:  
\( (ax + b)(cx + d) = 0 \).  
2. First degree equations and inequalities in one unknown. (15 h) | 1. Equations of the form:  
\( ax + b = 0 \).  
\( cx + d = 0 \).  
2. Systems of equations of the first degree in two unknowns.  
3. Systems of inequalities of the first degree in one unknown. (40 h) |

### Table 2: Geometry Curricula

<table>
<thead>
<tr>
<th>Location</th>
<th>Seventh Year</th>
<th>Eighth Year</th>
<th>Ninth Year</th>
</tr>
</thead>
</table>
| 1. LOCATION | 1. Geometric loci and constructions.  
2. Orthogonal system and coordinates of a point in a plane. (10 h) | 1. Relative positions of two circles.  
2. Geometric loci and constructions.  
3. Coordinates of the midpoint of a segment. (15 h) | 1. Tangents and circles.  
2. Geometric loci and constructions.  
3. Graphic representation of a straight line.  
4. Analytical properties of two parallel and of two orthogonal straight lines.  
5. Length of a segment in an orthonormal system.  
6. Solving graphically a system of linear equations in two unknowns. (35 h) |
| 2. SOLID GEOMETRY | Plane representation of a cube and a rectangular prism. (5 h) | 1. Plane representation of a cylinder, a pyramid, a cone and a sphere.  
2. Relative positions of straight lines and of planes. (10 h) | 1. Intersection of a straight line and a common solid.  
2. Intersection of a plane and a common solid. (5 h) |
| 3. PLANE FIGURES | 1. Cases of congruent triangles.  
2. Angles formed by two parallel straight lines cut by a transversal.  
3. Characteristic properties of the perpendicular bisector of a segment.  
2. Theorem of midpoints in a triangle, in a trapezoid.  
3. Characteristic properties of a parallelogram.  
2. Thales’ theorem.  
3. Similar triangles. (20 h) |
<table>
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<tbody>
<tr>
<td></td>
<td>(5 h)</td>
<td>(5 h)</td>
<td>(5 h)</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>5. TRIGONOMETRY</th>
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<tr>
<td></td>
<td>Sine, cosine and tangent of an acute angle in a right triangle.</td>
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<tr>
<td></td>
<td>(5 h)</td>
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**STATISTICS**

<table>
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<th>Eighth Year</th>
<th>Ninth Year</th>
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<td>2. Representation of data: bar graph, frequency polygon.</td>
<td>(5 h)</td>
<td>2. Representation of data: circular diagram, cumulative frequency polygon.</td>
<td>2. Mean and weighted mean.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(10 h)</td>
<td>(10 h)</td>
</tr>
</tbody>
</table>

**A - Elementary Level**

**FIRST CYCLE**

**1. OBJECTIVES**

The Mathematics curriculum must, in the following domains, make the student able to:

**A. MATHEMATICAL REASONING**

1. Recognize tendencies or relations in sequences of simple facts.
2. Justify an answer.

**B. PROBLEM SOLVING**

1. Take initiatives.
2. Use appropriate mathematical techniques in solving concrete problems of daily life.
3. Use ad-hoc means to find a result.

**C. COMMUNICATION**

1. Use pictorial or symbolic representations.
2. Express himself correctly, both orally and/or in writing.
3. Ask and answer questions.

**D. SPACIAL**

1. Find directions with the help of a map.
2. Recognize solid figures and plane figures.

**E. NUMERICAL**

1. Recognize natural integers, use Indo-Arabic numeration.
2. Recognize the four arithmetic operations.
3. Master the computational techniques of addition and subtraction.
4. Get training in the computational techniques of multiplication and division.
5. Apply relations among numbers in well-thought out calculations.
6. Use simple fractions to indicate parts of a whole.

**F. MEASUREMENT**

1. Measure length, mass and duration.
2. Tell time.
Teacher's Interview Transcript

Q: For how long have you been teaching?

For 3 years

Q: What are the classes that you have taught?

9th and 10th grade level

Q: Are you aware of DGS? Have you had any direct experience with it?

I've heard about it, as it is close to Geometer's Sketchpad. It is used to construct figures, where the student tends to move the figure that they have in a way that its shape is not destroyed.

Q: Have you ever taught Similar Triangles?

Yes definitely, throughout the curriculum of 9th grade

Q: How do you see it?

As a teacher, explaining it is easier than Congruent Triangles. I can easily give typical examples that might work as the shadow of one and himself. So for students, they feel it is closer to reality and easily understood.

Q: Where do students struggle in Similar Triangles?

Usually in developing proofs
Q: Why in proofs?

Because usually students in proofs tend to use properties and theorems from previous grade levels hence it is not an easy job. For them, it is hard to put together all those given data and the ones that can be easily generated from the figure to come up with a proper proof. A student might not know where to start from, what piece of data he/she needs to use and which he/she should be disregarded.

However, the most important reason is the drawing that they have. Students just see a drawing that is fixed, they see perpendicular lines but they are not sure whether they are perpendicular or not, they see parallel lines but they are not sure whether they are actually parallel, hence they can’t visualize.

Q: What about students’ part, are they able to choose right away what theorem to use from the figure?

Definitely not, that’s why most of the time they tend to use the AA theorem because it’s easier on them. It only requires angles that should be congruent, there’s no proportionality and no combination of sides with angles.

Q: Why do you think this happens?

Because they are still confused between proportionality and equality. Equality is easy but proportionality is hard to see in the figure as you need to make relations in terms of sizes and measures. Usually, students struggle in fractions in general, so how would it be the case if we are using fractions and proportions in triangles and sometimes in circles. It’s not easy to do that.
Q: So based on your experience, how could this be remedied?

They need to explore figures. They need to move points, check and see if lines are parallel, what different relations exist between various parts of the figure. Therefore, there need to be a way where they can check for all those features without actually making their task even more difficult. Thus, technology is the solution.

Q: Do you think DGS can help in learning Similar Triangles?

Sure. Because once the kids told me, how would we ever know that those sides are proportional, you are a teacher they are easy on you, for us it’s not. We can’t just speculate. That’s why they need this tool that allows them if they ever come up with a certain hypothesis or a conclusion, they can easily test and check whether it is valid or not.

Q: How do you think you can improve the students’ problem solving skills when it comes to proofs and Similar Triangles?

There are two basic ways: The first is to improve the students’ knowledge and understanding of proportionality. The second is that they should have a feasible tool that allows them to vary the figure that they have constructed. However, they can both be accomplished through technology.

Q: How?

With technology, speaking about sides and proportionality, if the student is able to see that no matter how you changed the measure of sides, the ratio of measure of sides will always be constant. Hence, when we speak about sides and their measures, sides
need not to be equal anymore rather they are proportional. With practice such an understanding about proportionality will be solid enough that students can easily use it whenever they see it on the figure.

The student should be able to vary measurements of sides, angles, play with the figure, not just look at the figure and think what would the best applied approach to solve the problem be! Hence, the weak and strong students in math would have a big chance to improve their problem solving skills and abilities.

Q: For teachers, in what means technology may be helpful?

For teachers it makes teaching Geometry an easier job. Rather than keep on holding a ruler and a compass and eventually generating figures that are not precise, with technology you can just print out worksheets for the students to guide them and show the work on white board screen. You can eliminate parts of the figure, vary sizes and measures of triangles, and ruin the figure sometimes to show the students the results of such actions.
APPENDIX C

Math Lessons: Thales' Theorem & Similar Triangles
Thales of Milet (625 B.C. - 547 B.C.) was one of the Seven Sages of Antiquity. He was a Greek scientist, astronomer, philosopher and mathematician.

He became famous when he made his astonishing prediction of the eclipse of the sun for a specific day of the year 585 B.C.; in fact, this eclipse occurred!

He lived in the Ionian city of Miletus, the famous harbor of Asia Minor.

It is told that he calculated the height of the pyramid of Cheops by comparing the height of his shadow to that of a stick planted in the soil.

Thales' theorem has been known and used since Antiquity; it helps to calculate inaccessible lengths.

However, Thales is not the inventor of this theorem, but he had established some principles concerning angles in a triangle.

At the end of 19th century in France, the theorem of the proportionality of lengths was called Thales' theorem.

It may be also called "Euclid's Theorem": since Euclid himself proved this theorem in The Elements in the 3rd century B.C.

"The most important is being great without appearing."

Romain Roland

Objectives

1. Know and use Thales' theorem concerning triangles, and its converse.
2. Construct the fourth proportional.
3. Expand and reduce a figure following a given ratio.

Plan of the chapter

I. Preliminary activities

II. Course
1. Thales' theorem
2. Reduction - expansion
3. Converse of Thales' theorem
4. Thales' theorem: generalized
5. Fourth proportional

III. Exercises and problems

IV. Test
Activity 1

\[ABC\] is a right triangle at \(B\) of sides \(AB = 12\) cm and \(BC = 16\) cm.

Designate by \(B', B_1\) and \(B_2\) the points on the side \([AB]\) such that

\[AB' = B'B_1 = B_1B_2 = 3\] cm. The parallels to \((BC)\) drawn respectively from \(B', B_1\) and \(B_2\) cut \((AC)\) at \(C', C_1\), and \(C_2\).

Is point \(B_1\) the midpoint of segment \([AB]\)? Calculate \(B_1C_1\).

Is segment \([B_2C_2]\) the midsegment of the trapezoid \(RCC_1B_1\)?

Calculate \(B_2C_2\).

Calculate \(B'C'\) and \(AC'\).

Verify that \(AC' = C'C_1 = C_1C_2 = C_2C\).

Compare \(\frac{AB'}{AB}, \frac{AC'}{AC}\), and \(\frac{B'C'}{BC}\).
Activity 2

Using the square pattern, find the ratios $\frac{AB'}{AB}$ and $\frac{AC'}{AC}$.

Do we have $\frac{AB'}{AB} = \frac{AC'}{AC}$? Are the lines $(BC)$ and $(B'C')$ parallel?

The line $(B_1C_1)$ is parallel to $(BC)$. Verify that $\frac{AB_1}{AB} = \frac{1}{4}$. Find $\frac{AC_1}{AC}$.

Knowing the lengths of the sides of triangle $AB_1C_1$, by what number should they be multiplied respectively to obtain the lengths of the sides of triangle $ABC$?

Knowing the lengths of the sides of triangle $ABC$, can we say that triangle $AB_1C_1$ is expanded in the ratio $k$?

To reduce triangle $ABC$ in order to have triangle $AB'C'$, by what number $k'$ should the lengths of the sides of triangle $ABC$ be multiplied?
Thales' theorem

ABC is a triangle.

If (BC) and (B'C') are parallel, then

\[
\frac{AB'}{AB} = \frac{AC'}{AC} = \frac{B'C'}{BC}
\]

The adjacent table is a table of proportionality.

<table>
<thead>
<tr>
<th></th>
<th>AB'</th>
<th>AC'</th>
<th>B'C'</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>AC</td>
<td>BC</td>
<td></td>
</tr>
</tbody>
</table>

We also say that if a line \((d)\) is parallel to one side of a triangle, it determines proportional segments on the other sides.

Application 1

ABC is a triangle such that

\(AB = 4\, \text{cm}, AC = 5\, \text{cm},\) and

\(BC = 6\, \text{cm}.\)

B' is a point of \([AB]\) such that

\(AB' = 3\, \text{cm}.\)

The parallel to \((BC)\) drawn through \(B'\) cuts \([AC]\) at \(C'.\)

Calculate \(\frac{AC'}{AC}, \frac{AC'}{AC}, \frac{B'C'}{BC},\) and \(B'C'.\)
Application 2

In the adjacent figure, $ABC$ is a triangle, $(B'C')$ is parallel to $(BC)$ such that $B'C' = 2 \text{ cm}$ and $\frac{AB'}{AB} = \frac{1}{3}$.

Calculate $\frac{AC'}{AC}$ and $BC$.

Consequences

$ABC$ is a triangle.

$B'$ and $B_1$ are two points of $(AB)$ distinct from $A$ and $B$, $C'$ and $C_1$ are two points of $(AC)$ distinct from $A$ and $C$.

If $(BC)$ is parallel to $(B'C')$ and $(B_1C_1)$ then

$$\frac{BB'}{AB} = \frac{CC'}{AC} \quad \text{and} \quad \frac{BB'}{AC} = \frac{CC'}{BB_1}$$

Following Thales’ theorem, we have:

$$\frac{AB'}{AB} = \frac{AC'}{AC}.$$

We also have $1 - \frac{AB'}{AB} = 1 - \frac{AC'}{AC}$ which gives:

$$\frac{AB - AB'}{AB} = \frac{AC - AC'}{AC}, \quad \text{so} \quad \frac{BB'}{AB} = \frac{CC'}{AC} \quad (1)$$

we also have:

$$\frac{BB_1}{AB} = \frac{CC_1}{AC} \quad (2)$$

Both relations $(1)$ and $(2)$ give:

$$\frac{BB'}{BB_1} = \frac{CC'}{CC_1}.$$
Reduction - Expansion

$ABC$ is a triangle with the point $D$ on the side $[AB]$ distinct from $A$ and $B$ and a point $N$ on $[AC]$ distinct from $A$ and $C$ such that:

$$\frac{AM}{AB} = \frac{AN}{AC} = \frac{MN}{BC} = k \text{ where } k \text{ is a real number.}$$

Since $AM < AB$, then $k$ is a strictly positive real number less than 1, $0 < k < 1$.

We also say that:

The triangle $AMN$ is a reduction of the triangle $ABC$ of ratio $k$.

The triangle $ABC$ is an enlargement of the triangle $AMN$ of ratio $\frac{1}{k}$ because:

$$\frac{AB}{AM} = \frac{AC}{AN} = \frac{BC}{MN} = \frac{1}{k} \text{ and } \frac{1}{k} > 1.$$  

**Example**

In the adjacent figure,

$$\frac{AM}{AB} = \frac{AN}{AC} = \frac{MN}{BC} = \frac{1}{3}.$$  

Triangle $AMN$ is a reduction of triangle $ABC$ of ratio $\frac{1}{3}$.

Triangle $ABC$ is an enlargement of triangle $AMN$ of ratio 3.
Converse of Thales' theorem

Let \(ABC\) be any triangle.

\(B'\) is a point of \([AB]\) distinct from \(A\) and \(B\) and \(C'\) is a point of \([AC]\) distinct from \(A\) and \(C\).

If \(\frac{AB'}{AB} = \frac{AC'}{AC}\), then \((B'C')\) is parallel to \((BC)\).

We also say that if the points \(B'\) and \(C'\) determine proportional segments on two sides of a triangle, then \((B'C')\) is parallel to the third side.

Remark

To apply Thales' theorem and its converse, the three collinear points \(A, B', B, C\) on one hand and the three collinear points \(A, C', C\) on the other hand should be in the same order. Otherwise, the property is not true.

In the adjacent figure, \((B'C')\) is parallel to \((BC)\), therefore

\[
\frac{AB'}{AB} = \frac{AC'}{AC}.
\]

\(B_1\) is the symmetry of \(B'\) with respect to \(A\), that is to say that \(A, B_1, B'\) are collinear and \(AB_1 = AB'\);

in this case, we have \(\frac{AB_1}{AB} = \frac{AC'}{AC}\),

but \((B_1C')\) is not parallel to \((BC)\)
because \(A, B_1, B, C\), and \(A, C', C\) do not have the same order.

Application 3

\(ABC\) is an isosceles triangle of vertex \(A\).

\(B'\) is a point of \([AB]\) and \(C'\) is a point of \([AC]\) such that

\[
AB' = 1.5 \text{ cm}, B'C' = 2 \text{ cm and } \frac{AB'}{AB} = \frac{AC'}{AC} = \frac{1}{3}.
\]

Calculate \(AC'\), \(BC\), and \(AB\).
Thales' theorem: Generalized

Let \( d \) and \( d' \) are two intersecting lines at \( A \), \( B \) and \( B' \) are two points of \( d \) distinct from \( A \) and \( C \) and \( C' \) two points of \( d' \) distinct from \( A \).

If the lines \((BC)\) and \((B'C')\) are parallel, then
\[
\frac{AB}{AC} = \frac{AC'}{BC}.
\]

Two cases arise.

First case

Point \( A \) is not a point of the segments \([BB']\) and \([CC']\).

In this case, this is similar to paragraph 1. Therefore the property is true.

Second case

Point \( A \) is a common point of both segments \([BB']\) and \([CC']\).

\( B_1 \) and \( C_1 \) are two points placed respectively on lines \((d)\) and \((d')\) such that point \( A \) is the midpoint of segments \([B_1B']\) and \([C_1C']\).

Then we have \( AB_1 = AB' \) and \( AC_1 = AC' \).

The quadrilateral \( B_1C_1B'C' \) is a parallelogram because the diagonals \([B'B_1]\) and \([C'C_1]\) intersect at their midpoint \( A \). Therefore \((B'C')\) and \((B_1C_1)\) are parallel.
By applying Thales' theorem to the triangle $ABC$, we obtain:

$$\frac{AB_1}{AB} = \frac{AC_1}{AC} = \frac{B_1C_1}{BC}.$$ Since $AB_1 = AB'$, $AC_1 = AC'$ and $B_1C_1 = B'C'$,

therefore

$$\frac{AB'}{AB} = \frac{AC'}{AC} = \frac{B'C'}{BC}.$$

Application 4

$BCB'C'$ is a trapezoid such that its diagonal $[BB']$ is perpendicular to the bases $[BC]$ and $[B'C']$.

$A$ is the intersection point of the diagonals.

Suppose that $AB = 2$ cm, $BC = 1.5$ cm, and $AB' = 5$ cm.

1) Calculate $B'C'$.

2) Calculate $AC$ and deduce $AC'$.

5. Fourth proportional

Given a segment $[AB]$, $m$ and $n$ are two numbers. We should place a point $B'$ on $(AB)$ such that

$$\frac{AB'}{AB} = \frac{m}{n} \quad (1)$$

$AB'$ is the fourth proportional in the equality (1).

Two cases arise:

First case

$B'$ is a point of segment $[AB]$.
Construction

We suppose that \( \frac{AB'}{AB} = \frac{2}{3} \), for example.

We draw a semi-line \([Ax]\). With an opening of the compass, we mark on \([Ax]\) and from \(A\), three consecutive segments having the same length \(\ell\).

On \([Ax]\), we place the point \(C'\) and \(C\) such that \(AC' = 2\ell\) and \(AC = 3\ell\).

We then draw the parallel to \((BC)\) through \(C'\) which cuts \([AB]\) at \(B'\).

The lines \((B'C')\) and \((BC)\) being parallel, therefore following Thales' theorem we obtain:

\[
\frac{AB'}{AB} = \frac{AC'}{AC} = \frac{2}{3}.
\]

Second case

\(B'\) is a point exterior to \([AB]\).

Construction

We suppose that \(\frac{AB'}{AB} = \frac{2}{3}\), for example.

We draw a line \((x'Ax)\).

With an opening of the compass, we mark the semi-line \([Ax]\) and from \(A\), three consecutive segments having the same length \(\ell\).

On \([Ax]\), we place the point \(C\) such that \(AC = 3\ell\) and on \([Ax']\), we place the point \(C'\) such that \(AC' = 2\ell\).

We then draw the parallel to \((BC)\) through \(C'\), and which cuts \((AB)\) at \(B'\).

The lines \((B'C')\) and \((BC)\) being parallel, therefore following Thales' theorem we obtain:

\[
\frac{AB'}{AB} = \frac{AC'}{AC} = \frac{2}{3}.
\]
Most of the results of geometry studied till the first year secondary were already known by the Greeks in the 3rd century B.C. These principles appear in the famous book Elements written by Euclid, in which similar triangles that are the topic of this chapter are discussed.

The work of Euclid reached our era because of mathematicians such as Theon of Alexandria or Proclus who commented and criticized the Elements about seven centuries after Euclid.

The daughter of Theon of Alexandria, Hypatia (370-415) was famous for being a philosopher and a mathematician; however, her commentaries about the work of Diophante and the work of Appolonius were lost. The governor of Alexandria, Orestes, used to consult her about public affairs.

Hypatia died in 415, she was assassinated by a group that was hostile concerning the Greek pagan science.

«To know more about the rose, some use geometry and others use the butterfly.»

Paul Claudel

Objective

Understand and apply the conditions of the similarity of two triangles.

Plan of the chapter

I Preliminary activities

II Course

1- Definition
2- Properties
3- Summary table
4- Ratio of similarity
5- Metric relations in a right triangle
6- Center of gravity in a triangle

III Exercises and problems

IV Test
Activity 1

$A'B'C'$ and $AB_1C_1$ are two congruent triangles where

$A'B' = AB_1$ ;

$A'C' = AC_1$ and

$B'C' = B_1C_1$.

On the semi-lines $[AB_1)$ and $[AC_1)$, place respectively the points $B$ and $C$ so that $(B_1C_1)$ and $(BC)$ are parallel.

Notice that $k = \frac{AB_1}{AB}$.

1°) Compare the angles $\angle ABC$ and $\angle A'B'C'$.

2°) Compare the angles $\angle ACB$ and $\angle A'C'B'$.

3°) Calculate $\frac{A'B'}{AB}$, $\frac{A'C'}{AC}$ and $\frac{B'C'}{BC}$ in terms of $k$.

Activity 2

$ABC$ is a right triangle at $A$ and $H$ is the orthogonal projection of $A$ on $(BC)$.

Suppose that $BC = a$, $CA = b$, $AB = c$, $AH = h$ and $BH = x$.

1°) Do we have $a^2 = h^2 + c^2$?

$h^2 = c^2 - x^2$ ? $bc = ha$ ?

2°) Can you deduce from the equality $b^2c^2 = h^2a^2$ that $AB^2 = BH \cdot BC$ ?

3°) Is the equality $AC^2 = BC \cdot CH$ true ?

4°) Using the equalities $AB^2 \cdot AC^2 = BC^2 \cdot BH \cdot CH$ and $AB \cdot AC = AH \cdot BC$, verify that $AH^2 = HB \cdot HC$. 

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Definition

Two triangles are said to be similar if these two conditions are fulfilled:

- The sides of one have lengths respectively proportional to the lengths of the corresponding sides of the other.
- The corresponding angles have the same measure.

Therefore, triangle $A'B'C'$ is similar to triangle $ABC$ if:

\[
\frac{B'C'}{B'C} = \frac{A'B'}{A'B} = \frac{A'C'}{A'C} = \frac{B'C'}{B'C} = \frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC}.
\]

When two triangles are similar, the angles of these triangles, having the same measure, are called corresponding angles, their vertices are called corresponding vertices and the sides of these triangles, whose lengths are in the same ratio, are called corresponding sides.

$A$ and $A'$ as well as $B$ and $B'$ and also $C$ and $C'$ are two corresponding vertices.

The sides $[AB]$ and $[A'B']$ as well as $[AC]$ and $[A'C']$ and also $[BC]$ and $[B'C']$ are two corresponding sides.

The angles $\angle BAC$ and $\angle B'A'C'$ as well as $\angle ABC$ and $\angle A'B'C'$ and also $\angle ACB$ and $A'C'B'$ are two corresponding angles.

Whenever two triangles are similar, the ratio of the lengths of the corresponding sides is called ratio of similarity.

Therefore, \[\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC} = k, k \text{ is called ratio of similitude}.
\]

Remark

Two equal triangles are similar and the ratio of similitude is equal to 1.
Properties (Conditions of similitude)

1°)
If two angles of one triangle are respectively equal to two angles of another triangle, then these two triangles are similar.

\[ \triangle ABC \text{ and } \triangle A'B'C' \text{ are two triangles such that:} \]
\[ \hat{B}'A'C' = \hat{BAC} \text{ and} \]
\[ \hat{A'B'C'} = \hat{ABC}. \]

On the semi-line \([AB]\), place the point \(B_1\) such that \(AB_1 = A'B'\) and from \(B_1\) draw the parallel to \((BC)\) that cuts \((AC)\) in \(C_1\).

We then have \(\hat{AB_1C_1} = \hat{ABC}\).

Since \(\hat{A'B'C'} = \hat{ABC}\), then \(\hat{AB_1C_1} = \hat{A'B'C'}\).

The two triangles \(\triangle AB_1C_1\) and \(\triangle A'B'C'\) are congruent, therefore,
\[ A'B' = AB_1, A'C' = AC_1 \text{ and } B'C' = B_1C_1. \]

Following Thales' Theorem in the triangle \(\triangle ABC\), we have:
\[ \frac{AB_1}{AB} = \frac{AC_1}{AC} = \frac{B_1C_1}{BC}, \text{ then } \frac{\hat{A'B'}}{\hat{AB}} = \frac{\hat{A'C'}}{\hat{AC}} = \frac{\hat{B'C'}}{\hat{BC}}. \]

Since \(\hat{B'A'C'} = \hat{BAC}\) and \(\hat{A'B'C'} = \hat{ABC}\), then \(\hat{A'C'B'} = \hat{ACB}\).

The two triangles \(\triangle ABC\) and \(\triangle A'B'C'\) are then similar.

Remarks

• In two right triangles, if an acute angle in one is equal to an acute angle in the other one, then these two triangles are similar.

• If \(\triangle ABC\) is a triangle similar to the triangle \(\triangle A'B'C'\) and \(\triangle A'B'C'\) is similar to the triangle \(\triangle A''B''C''\), then \(\triangle ABC\) and \(\triangle A''B''C''\) are similar.
Application 1

- In the adjacent figure, the two angles \( \widehat{MON} \) and \( \widehat{MAO} \) are equal.

Show that the two triangles \( MON \) and \( MAO \) are similar.

- In the adjacent figure, \( TRES \) is a parallelogram. The angles \( \widehat{SIT} \) and \( \widehat{GRE} \) are two right angles.

Show that the two triangles \( SIT \) and \( GRE \) are similar.

2°)

Two triangles, having an angle in one equal to an angle in the other one and the two sides of the angles respectively proportional, are similar.

\( ABC \) and \( A'B'C' \) are two triangles such that \( \widehat{BAC} = \widehat{B'A'C'} \) and \( \frac{A'B'}{AB} = \frac{A'C'}{AC} \).

On the semi-lines \([AB]\) and \([AC]\), place respectively the points \( B_1 \) and \( C_1 \) such that \( AB_1 = A'B' \) and \( AC_1 = A'C' \).

The two triangles \( A'B'C' \) and \( AB_1C_1 \) are congruent, therefore \( B_1C_1 = B'C' \).

We have \( \frac{A'B'}{AB} = \frac{A'C'}{AC} \). Since \( A'B' = AB_1 \) and \( A'C' = AC_1 \), then \( \frac{AB_1}{AB} = \frac{AC_1}{AC} \).
Following are two similar triangles. Let $AB'C'$ be parallel to $(BC)$. Therefore, \[
\frac{AB}{AC} = \frac{B'C'}{BC} \text{ and the angles of the triangle } \triangle ABC \text{ are respectively equal to the angles of the triangle } \triangle A'B'C'.
\]

This proves that the two triangles $\triangle ABC$ and $\triangle A'B'C'$ are similar.

**Application 2**

The diagonals $[JO]$ and $[LT]$ of the quadrilateral $ILOT$ intersect at $P$.

Given $TP = 5$, $OP = 10$, $LP = 4$ and $IP = 8$.

Show that the two triangles $\triangle PLJ$ and $\triangle POT$ are similar.

3°) **If the sides of one triangle are respectively proportional to the sides of another triangle, then these two triangles are similar.**

$\triangle ABC$ and $\triangle A'B'C'$ are two triangles such that \[
\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC}
\]

On the two semilines $(AB)$ and $(AC)$, place respectively the points $B_1$ and $C_1$ such that $AB_1 = A'B'$ and $AC_1 = A'C'$.
Since \( \frac{A'E'}{AB} = \frac{A'C'}{AC} \), then \( \frac{AB_1}{AB} = \frac{AC_1}{AC} \). Following the converse of Thales' theorem, \((B_1C_1)\) is parallel to \((BC)\).

Since \( \frac{AB_1}{AB} = \frac{AC_1}{AC} = \frac{B_1C_1}{BC} = \frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC} \), then \( B_1C_1 = B'C' \).

So triangles \(A'B'C'\) and \(AB_1C_1\) are congruent.

Since \(ABC\) and \(AB_1C_1\) are similar, then \(ABC\) and \(A'B'C'\) are similar.

**Application 3**

\(ABCD\) is a trapezoid of bases \([BC]\) and \([AD]\) such that \(AB = 3\) cm, \(BC = 4\) cm, \(AC = 5\) cm, \(AD = 6.25\) cm and \(CD = 3.75\) cm.

Verify that the two triangles \(ABC\) and \(ACD\) are similar.

**Summary table**

<table>
<thead>
<tr>
<th>Similar triangles</th>
<th>(A'B'C')</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corresponding vertices</td>
<td>(A')</td>
<td>(B')</td>
<td>(C')</td>
<td></td>
</tr>
<tr>
<td>Corresponding angles</td>
<td>(\hat{A}' = \hat{A})</td>
<td>(\hat{B}' = \hat{B})</td>
<td>(\hat{C}' = \hat{C})</td>
<td></td>
</tr>
<tr>
<td>Corresponding sides</td>
<td>([A'B'])</td>
<td>([A'C'])</td>
<td>([B'C'])</td>
<td></td>
</tr>
<tr>
<td>[AB]</td>
<td>[AC]</td>
<td>[BC]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportionality</td>
<td>(\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cases of similitude</td>
<td>(\hat{A} = \hat{A})</td>
<td>(\hat{A}' = \hat{A})</td>
<td>(\hat{B}' = \hat{B})</td>
<td></td>
</tr>
<tr>
<td>(\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Similar Triangles** 123
Ratio of similarity - Length - Area

$A'B'C'$ and $ABC$ are two similar triangles of ratio

$$k = \frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC}.$$  

1°) Reduction - Enlargement

$A'B' = kAB', A'C' = kAC$ and $B'C' = kBC$.

As a result, the similarity multiplies the distances by the same strictly positive real number $k$.

If $k < 1$, triangle $A'B'C'$ is a reduction of triangle $ABC$ by a ratio $k$.

If $k > 1$, triangle $A'B'C'$ is an enlargement of triangle $ABC$ by a ratio $k$.

2°) Area

$[A'H']$ and $[AH]$ are the respective heights drawn from the corresponding vertices of triangles $A'B'C'$ and $ABC$.

Triangles $A'B'H'$ and $ABH$ are similar since both are right triangles having an equal acute angle $\hat{B} = \hat{B}$.

Therefore

$$\frac{A'B'}{AB} = \frac{A'H'}{AH} = \frac{B'H'}{BH}.$$  

Hence

$$\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC} = \frac{A'H'}{AH} = k \text{ and } A'H' = kAH.$$  

$\mathcal{A}$ is the area of triangle $ABC$ and $\mathcal{A}'$ is the area of triangle $A'B'C'$.

$$\mathcal{A} = \frac{BC \times AH}{2} \text{ and } \mathcal{A}' = \frac{B'C' \times A'H'}{2} = kBC \times kAH = k^2 \frac{BC \times AH}{2}.$$  

So $\mathcal{A}' = k^2 \mathcal{A}$.
As a result, the similarity multiplies the areas by $k^2$.

We also prove that if $[A'T']$ and $[AI]$ are the medians drawn from the corresponding vertices $A'$ and $A$, then $A'T' = kAI$.

**Application 4**

JUS is an equilateral triangle having a side of $3$ cm and a height $[UR]$.

CRU is also an equilateral triangle having a height $[CE]$.

1°) Calculate $RU$ and the area $\mathcal{A}$ of the triangle JUS.

2°) Deduce $CE$ and the area $\mathcal{A}'$ of the triangle CRU.

5. **Metric relations in a right triangle**

ABC is a right triangle at $A$ and $[AH]$ is its height.

The right triangles $ABH$ and $CBA$ are similar since they have a common acute angle $B$. Therefore:

\[
\frac{AB}{BC} = \frac{AH}{AC} = \frac{BH}{AB}.
\]

This gives:

\[
AH \cdot BC = AB \cdot AC \quad \text{and} \quad AB^2 = BH \cdot BC.
\]

Also, the right triangles $ACH$ and $BCA$ are similar since they have a common acute angle $C$. Therefore, \[
\frac{AC}{BC} = \frac{AH}{AB} = \frac{CH}{AC}.
\] This gives: $AC^2 = CH \cdot CB$.

The right triangles $ABH$ and $CAH$ are similar since they have an equal acute angle: $\widehat{ABH} = \widehat{HAC}$. Therefore \[
\frac{AB}{AC} = \frac{AH}{CH} = \frac{BH}{AH}.
\] This gives: $AH^2 = BH \cdot CH$.
The following relations are called the algebraic relations in a right triangle.

\[ AH \cdot BC = AB \cdot AC \]
\[ AH^2 = BH \cdot CH \]
\[ AB^2 = BC \cdot BH \]
\[ AC^2 = CB \cdot CH \]

**Remark**

A triangle, in which one of these relations is verified, is a right triangle.

**Application 5**

*JAD* is a right triangle at J and H is the foot of the height drawn from J.

Given \( AH = \frac{9}{5} \) cm and \( DH = \frac{16}{5} \) cm.

Calculate \( JA \), \( JD \), and \( JH \).

*KIF* is a triangle.

H is the foot of the height drawn from K such that \( IH = 1 \) cm, \( FH = 4 \) cm and \( KH = 2 \) cm.

Show that the triangle *KIF* is right and calculate its sides.
Center of gravity in a triangle

$ABC$ is a triangle, $I$ and $J$ are the respective midpoints of the sides $[BC]$ and $[AC]$.

The medians $[AI]$ and $[BJ]$ intersect at $G$.

Following the theorem of the midpoints, $(IJ)$ is a line parallel to $(AB)$ and $IJ = \frac{1}{2} AB$.

The two triangles $GJL$ and $GAB$ are similar because $\hat{IGJ} = \hat{AGB}$ (vertically opposite angles) and $\hat{GJL} = \hat{GAB}$ (alternate-interior angles).

The ratio of similitude is:

$$\frac{IJ}{AB} = \frac{GL}{GA} = \frac{GJ}{GB}.$$

Since $IJ = \frac{1}{2} AB$, then $GI = \frac{1}{2} GA$ and $GJ = \frac{1}{2} GB$.

So: $AG = \frac{2}{3} AI$ and $BG = \frac{2}{3} BJ$.

We prove in the same way that the medians $[CK]$ and $[AI]$ intersect at point $G$, and that $CG = \frac{2}{3} CK$.

In a triangle, the medians then intersect at the same point $G$ called center of gravity of the triangle.

$G$ divides the medians in the ratio $\frac{2}{3}$ from the vertices of the triangle.
APPENDIX D

Diagnostic Test
Diagnostic Test

Session №: __________________________
Date: __________________________

Student №: _______

Final Score: _______

Notes:
- All the problems will be solved on this sheet.
- The back of each page could be used as scratch unless extra space is needed for your final solution. In this case please write down ‘SOLUTION’ on the top of the page.
- No scratches or corrector is allowed to be used, mark an X next to idea and go on.
**Problem 1:**
Find the value of $x$ in each case:

1) $\frac{x}{2} = \frac{3}{9}$

2) $\frac{x + 3}{4} = \frac{9}{2}$

3) $\frac{16}{4x} = \frac{4x}{9}$

4) $\frac{x + 2}{6} = \frac{6}{x + 2}$

Score:_________
Problem II:
Write a congruence relation between each two triangles. Then, prove your answer.

1) Relation:

![Image of triangle GHJ]

Proof:

---

2) Relation:

![Image of triangle MNOS]

Proof:

---

3) Relation:

![Image of parallelogram TXYZ]

Proof:

---

Score: ________
Problem III:

Given that $\frac{a}{b} = \frac{c}{d}$, show that each proportion is true:

1) $\frac{a}{c} = \frac{b}{d}$

3) $\frac{b}{a} = \frac{d}{c}$

4) $\frac{a + b}{b} = \frac{c + d}{d}$

Score: ________
Problem IV:
Draw a circle (C) center I and of radius r, and two perpendicular diameters [CU] and [EM].
The perpendicular bisector of [CM] cuts the arc $CM$ at O and the arc $EU$ at T.
The perpendicular bisector of [CE] cuts the arc $CE$ at R and the arc $MU$ at P.

1) Construct the figure

2) What is the nature of the quadrilateral CMUE? Calculate its sides in terms of R.

3) Show that O is the midpoint of $CM$ and that $OC = OM$
Problem V:
Given a right angle $x\hat{O}y$. Let $A$ be a variable point on $[Ox]$ and $B$ be a variable point on $[Oy]$ such that $AB = 2a = \text{constant}$.
Let $I$ be the midpoint of $[AB]$.
1) Construct the figure.

2) Calculate $OI$ in terms of $a$.

3) Determine the locus of the point $I$.
   Attention: $I$ is always inside the angle $x\hat{O}y$
Problem VI:
ABC is a triangle such that $AB > AC$. M, N and K are the midpoints of the sides [AB], [BC] and [AC] respectively.
Let $H$ be the foot of the height issued from $A$.
1) Draw the figure

2) Show that the four points M, N, H and K are the vertices of an isosceles trapezoid.

Score: ____________
APPENDIX E

Cabri Exploration Activity
1) Construct any triangle, label it, measure the sides than measure the angles
2) Drag point A, measure the sides and angles again
3) Find the area of the triangle
4) Find the perimeter of the triangle
5) Construct a circle
6) Construct a segment measuring 4 cm
7) Construct a circle of radius 5 cm
8) Construct two parallel lines
9) Construct two parallel segments
10) Construct any triangle than tabulate and animate the area of the triangle
11) Construct an isosceles triangle
12) Construct two perpendicular segments
13) Construct any triangle:
    i. Construct all the medians
    ii. What do you notice?
    iii. What do we call this point? State its property.
14) Construct $\triangle ABC$ such that $M$ and $N$ are the midpoints of $AB$ and $AC$ respectively.
    i. What relation exists between $MN$ and $BC$?
    ii. Compare $MN$ and $BC$
    iii. Calculate, tabulate and animate:

    \[
    \begin{array}{ccc}
    \text{MN} & \text{BC} & \text{MN/BC} \\
    \hline
    \end{array}
    \]

    iv. What do you notice?
    v. State the postulate.
Worksheet I

Date: _______________________________________

Session №: __________________________________

Duration: ____________________________________

Note: Open a file <Similar Triangles> on the desktop, and within the file open three folders:
- Part I
- Part II
- Part III
- Part IV
Open a folder <Part I>, where all the work will be saved

Activity I:
Within the file open a folder and name it <Q1>

Consider the following figure:

Using Cabri:
1)  
   1- Consider a point and label it as A.
   2- From A, and using triangle construct a triangle and label the rest of the points as B and C.
   3- Through point on an object, plot the point B'.
   4- Through parallel lines, construct a line parallel to BC and passing through B'.
   5- Through point of intersection, plot the point of intersection between the line and the segment AC and label it as C'.
   6- Hide the line
   7- Construct the segment from B' to C'.

2)  
   1- Measure all the angles.
   2- Move the points A, B and C randomly. What do you notice? What is happening to B' and C'?
   3- Move the points B' and C' randomly. What do you notice?
   4- Measure all the sides. Move aside and rename each as follows: If AB measures 3cm,
      then move the 3cm and rename it as AB: 3m
   5- Calculate, Tabulate and animate AB/AB', AC/AC' and BC/B'C'
   6- What do you notice?

3) Give a precise statement of your conclusion.

4) What do we call this theorem?
Application:
Within the file open a folder and name it <app>

Using Cabri, and going through the same procedure as above, construct the following figure:

1- Measure all the distances, then move aside and rename each as above.
2- Measure all the angles.
3- Move the points A, B'', and C''. What do you notice?
4- Try moving B' and C'. What do you notice?
5- Try moving B and C. What do you notice?
6- Calculate, Animate and Tabulate BB'/AB and CC'/AC.
7- What do you notice?
8- Calculate, Animate and Tabulate BB''/BB'' and CC'/CC''.
9- What do you notice?

Using paper and pencil prove that:

1) \[
\frac{BB'}{AB} = \frac{CC''}{AC}
\]

2) \[
\frac{BB''}{BB''} = \frac{CC'}{CC''}
\]
Activity II:
Within the file open a folder and name it <Q2>

Try producing a converse for Thales’ theorem. (Each attempt save it as Part1_2.1 – 2.2., 2.3.)
APPENDIX G

Worksheet II
Worksheet II

Date: ______________________________

Session №: _______________________

Duration: _________________________

All the work will be saved in the file <PartII>

Activity I:
Within the file open a folder <PartII-I> where all the work will be saved.

1) Construct any two equilateral triangles ABC and MNP:
   1- Consider a point A then draw segment AB.
   2- Construct a circle of center A and radius AB.
   3- Construct a circle of center B and radius BA.
   4- Mark the point of intersection as C.
   5- Hide the circles.
   6- Verify that the triangle is equilateral (check sides and angles)
   7- Draw another equilateral triangle.

2) Are those triangles congruent? Why?
Activity II:
Within the file open another folder <PartII-2> where all the work will be saved.

Construct the above figure:
1)  
   1- Draw any triangle and label the points as A, M and N.  
   2- Plot the point B, through *point on an object.*  
   3- From B draw the segment BC parallel to MN.

2)  
   1- Measure all the angles. What do you notice?  
   2- What are the corresponding sides in this case?  
   3- Move the points M and N. What do you notice?  
   4- Move the point C. what do you notice?  
   5- Move the point B what do you notice?  
   6- What property has been applied?

3)  
   1- Measure the sides AB, AM, AC, AN, BC and MN.  
   2- Move the measurements aside and rename each by its label. (e.g. AB: 3 cm)

4)  
   1- Are those two triangles congruent?  
   2- Calculate, tabulate and animate the following: AM/AB – AN/AC – MN/BC  
   3- What do you notice?

5)  
   What do you call such triangles?

6)  
   1- What are the key elements to obtain this relation in this particular case?  
   2- Is it the only case?
Activity III:  
Within the file open a folder < PartII-3> where all the work will be saved.

Open the folder Activity III  
1) Are the triangles Similar?  
2) Is there at least one pair of opposite sides parallel?

Open the folder Activity III - 2  
1) What happened to the triangle MNP?  
2) Are the triangles still similar?  
3) What do you conclude?

Consider the cardboard triangles in the file you have.  
Are you able to demonstrate what have been stated using cardboards?

Can you give a formal definition for Similar Triangles?

Construct any two similar triangles. Verify your answer.
Activity IV:
Within the file open a folder < PartII-4> where all the work will be saved.

Open the file Activity IV

Given: $AB \parallel EF$ and $\hat{A} \equiv \hat{E}$

Using Cabri:
1) Verify that $\triangle ABC \approx \triangle DEF$
2) What are the corresponding sides
3) State the ratio of Similitude.
4) Away from Cabri, can you prove the above result?
5) What particular piece of information did you use? (Sides – Angles – etc…)

Can you give a precise description of the method used?

Can you give a precise definition of the postulate used?
Activity V:
Within the file open a folder <PartII-5> where all the work will be saved.

Open the file Activity V

Given: AM = 4 cm and AN = 3 cm
      AB = 6 cm and AC = 8 cm

Using Cabri:
   1) Verify that $\triangle ABC \cong \triangle AMN$
   2) What are the corresponding sides
   3) State the ratio of Similitude.
   4) Away from Cabri, can you prove the above result?
   5) What particular piece of information did you use? (Sides – Angles – Sides & Angles, etc…)

Can you give a precise description of the method used?

Can you give a precise definition of the postulate used?
Activity VI:
*Within the file open a folder < PartII-6> where all the work will be saved.*

Open the folder Activity VI

**Given:** AB = 10 am, BC = 18 cm and AC = 16 cm

MN = 5 cm, NP = 9 cm and MP = 8 cm

**Using Cabri:**
1) Verify that $\triangle ABC \approx \triangle MNP$
2) What are the corresponding sides
3) State the ratio of Similitude.
4) Away from Cabri, can you prove the above result?
5) What particular piece of information did you use? (Sides – Angles – Sides & Angles, etc...)

Can you give a precise description of the method used?

Can you give a precise definition of the postulate used?
Activity VII:
Within the file open a folder < PartII-7> where all the work will be saved.

Construct $\triangle XYZ$ having two of its sides measuring 3 cm and 5 cm
Construct $\triangle RST$ having two of its sides measuring 6 cm and 10 cm

1) What relation does exist between the four sides mentioned above?
2) Are those triangles similar

Is having two proportional sides enough to guarantee similarity among triangles?
Worksheet III

Date: _______________________

Session №: ______________________

Duration: ______________________
All the work will be saved in the file <PartIII>

Open the following activity where:
\( xOy = 60^\circ, \ OA = 2 \ cm, \ OB = 9 \ cm, \ OC = 6 \ cm \) and \( OD = 3 \ cm \)

Part 1 will be saved as Q1 and each attempt will be saved under a different title (Q1.1 - Q1.2 - Q1.3 etc...)

1)
1- Find two similar triangles. Verify your answer
2- Show that those triangles are similar.
3- Find the ratio of similitude.

Part 1 will be saved as Q2

2) Draw the altitude \([CH]\) in triangle COB.
   1- Calculate CH. Verify your answer
   2- Calculate OH. Verify your answer.

Part 1 will be saved as Q3 and each attempt will be saved under a different title (Q3.1 - Q3.2 - Q3.3 etc...)

3) Let \( M \) be the point of intersection of the lines (AD) and (CB).
   1- Find another two similar triangles. Verify your answer
   2- Prove your answer
APPENDIX I

Worksheet IV
Worksheet IV

Date: ________________________________

Session №: __________________________

Duration: ____________________________
All the work will be saved in the file `<Part IV>`

(C) is a circle of fixed diameter [AB] of center O and radius R.
H is a point of [OB] and (d) is the perpendicular line to (AB) that passes through H.
Designate by M one of the intersection points of (d) and of (C) and by N the point diametrically to M on (C).

**Part 1 will be saved under as Q1**

1) Construct the figure through using the following tools:
   - **Fix/Free** to fix diameter [AB].
   - **Perpendicular Line** to draw (d).
   - **Line** to connect M and O. The point of intersection of the line with the circle is N. Label it.
   - **Segment** to mark the segment [MN].

**Part 2 will be saved under as Q2 and each attempt will be saved under a different title (Q2.1 - Q2.2 - Q2.3 etc...)**

2) a- Find two similar triangles. Verify your answer
   b- Can you prove Similarity in this case?
   c- Deduce that $MA \times MB = MH \times MN$
3) Construct $G$ the center of gravity of triangle BAM.
   a) Move $H$ along $BC$. What is happening to $G$?
   b) Using Trace On/Off, find the set of points of $G$ when $H$ describes
      the segment $[AB]$.
   c) Can you explain your result?

4) Suppose that $H$ is fixed and that $HB = 2OH$
   a) Can you reconstruct $H$?
   a) Calculate in terms of $R$ the lengths $OH$, $HM$, $MA$ and $MB$.
   b) Deduce the area of triangle $NAM$
   c) Verify your answer.
   d) Calculate $GH$.  

Part 3 will be saved under as Q3.1 - Q3.2 - Q3.3 etc...

Part 4 will be saved under as Q4.1 - Q4.2 - Q4.3 etc...
APPENDIX J

Summative Test
Summative Test

Date: __________________________________________

Session No.: __________________________________

Duration: _____________________________________

Name: _________________________________________

Student No.: ________________________________

Note: The back of each page could be used as scratch, and in case extra space is needed for solving, at the bottom write ‘Continue →’ and on the back of the page write ‘SOLUTION’
Problem 1:
Construct any two similar triangles.

1) Verify your answer

2) Write the ratio of similitude
Problem II:

1) Show that triangles TEN and SEB are similar

2) Deduce that $SE \times EN = TE \times EB$
Problem III:
ABC is any triangle, [AM] and [BN] are medians. G is the point of intersection of the medians in triangle ABC.

1) Show that triangles CGA and NGM are similar.

2) Write the ratio of similitude.

3) Suppose AC = 12 cm and MG = 5 cm.
Write all the measure of sides that could be determined.
Problem IV:
Draw a segment $[AC]$ with $AC = 15$ cm. Place the points O and F on this segment such that $AO = OF = 3$ cm.

On the perpendicular at O to $(AC)$, place the point B such that $OB = 6$ cm.

Construct the figure:

1) Prove that $AB = 3\sqrt{5}$ cm and that $BC = 6\sqrt{5}$ cm.

2) Prove that $(AB)$ and $(BC)$ are perpendicular.
3) Draw the circle (T) of diameter [CF] that cuts (BC) again in H. Prove that (AB) and (FH) are parallel.

4) Calculate CF then CH.

5) Prove that triangle BAF isosceles
Problem V:
(C ) is a circle of center O and of radius R.
[AB] is a diameter of (C ) and M is a point on the semi-line [OA) such that OM = 4 R and \( O TM = 90^\circ \)

1) Calculate MA, MT, MH and AH.

2) Calculate TH.

3) Verify that \( HO \times HM = HA \times HB \)

4) Calculate TA and TB.
Problem VI:
In the following figure, given a semi-circle of center O and diameter [AB]. The perpendicular to [AB] at O cuts the circle in C. M is any point of arc $B\hat{C}$ and the bisector of angle $C\hat{OM}$ cuts the semi-circle in E. D is the point of intersection of [AM] and [OC], and by I the point of intersection of [AM] and [OE].

1) Show that triangles AMB and AOD are similar.

2) Deduce that the product $AM \times AD$ is constant.

3) What is the measure of angle $C\hat{MA}$?
4) What is the nature of triangle CIM? Justify your answer.

5) Show that (CI) and (AM) are perpendicular.

6) Determine the locus of point I as M describes the arc $BC$.

7) Let F be the midpoint of [CM]. Show that triangles CIA and OFM are similar.
School of Business
Byblos Campus

Graduate Research Topics in Business (BUS 898)
Dr. Elias Raad
Spring 2009

Are Islamic Banks More Cost, Revenue and Profit Efficient Than Commercial Banks: An Empirical Comparative Study from the Middle East.

Amine Abi Aad
June, 30 2009
LEBANESE AMERICAN UNIVERSITY

Project Approval Form (Annex IV)

Student Name: Amine Abi Aad
I.D. #: 199605510

Project Title: Are Islamic Banks More Cost, Revenue, and Profit Efficient than Commercial Banks:
An Empirical Comparative Study from the Middle East

Program: MBA

Division/Dept: Business Studies

School: Business

Approved by:

Project Advisor-name, Ph.D. (Advisor)
Elias A. Raad
Associate Professor of Finance

Member One-name, Ph.D.
Sebouh Aintablian
Assistant Professor of Finance

Date: June 30, 2009
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Signature: [Signature]  Date: July 22, 2009
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Abstract

This project compares the efficiencies of commercial and Islamic banks in nine Arab countries in the Middle East. We find that Islamic banks are more efficient in terms of cost, revenue and profit than commercial Banks. This result is thoroughly depicted in small Islamic banks when compared to small commercial banks but does not hold when we compare big Islamic to big commercial banks. In Bahrain, Jordan, UAE, and Yemen commercial banks are more efficient than Islamic banks, but the results of Qatar are similar to those of the cross sectional data. We also find that after controlling for size, the data indicate that big banks are more cost and profit efficient than small banks, which is typically the case of commercial banks but not of Islamic banks. Through a semi-log regression analysis we find that the Average Return on Assets (AROA) of commercial banks is significantly affected by six independent variables while the AROA of Islamic banks is only significantly affected by three independent variables from the same pool of independent variables that we consider for both types of banks. Out of the three independent variables that are common for both types of banks, only off balance sheet lead to opposite result: it is negatively significantly related to AROA of commercial banks, and it is positively significantly related to AROA of Islamic banks. The other two variables, loan loss provisions and cost to income ratio are both negatively significantly related to AROA of commercial and Islamic banks.
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I. Introduction:

The Islamic banking industry has made a decent entry into the realm of the huge commercial banking industry in the Arab world. Although it began to emerge in the mid of the 1970’s, it was not until the last decade, that Islamic banking became the source of attraction for socially responsible investors and creditors, and a source of competition for the commercial banks. To hedge against their new competitors, several large commercial banks opened Islamic windows\(^1\). Side by side, the fully-fledged Islamic banks and the Islamic windows of commercial banks grew rapidly.

Being operational along the well established commercial banking sector, the Islamic banking sector has to secure its own share of the financial market to survive. Still in its introductory stage, the Islamic banking industry has a huge growth potential, thus making it an attractive field to be analyzed. In this project, we study the efficiency of Islamic banks and compare it to that of commercial banks, in nine Arab countries: Bahrain, Jordan, Kuwait, Lebanon, Qatar, Saudi Arabia, Syria, United Arab Emirates, and Yemen.

The rest of the project proceeds as follows: section II overviews Islamic banking; section III discusses related literature review; section IV describes the collection of data; section V explains the methodology used in the project; section VI analyses the results, and section VII concludes the project.

---

\(^1\) Islamic Windows: under this structure a commercial banks offer simultaneously commercial and Islamic financial services.
II. Overview of Islamic Banking:

When asked about Islamic banking, most commercial bankers would relate it to the Islamic religion and to the ban of interest without ever questioning its roots, and benefits. Indeed, the stereotype image of the Islamic banking would lead them to say that it is like the commercial banking but in different dress. As we shall explain in this section, Islamic banking is based on Islamic ethical principles which differ from the commercial principles and lead to a different type of banking institutions.

Islamic banks are financial institutions that base their objectives and operations on *Qur'anic* principles. As such, they only provide the commercial services that comply with Islamic religious laws. For instance, Islamic banks provide services for their customers interest-free or with no *riba* (Arabic word for interest as stated in the *Quran*). The ban of *riba* makes the Islamic banking system differ fundamentally from the commercial banking system.

Technically, *riba* means the addition, however small it might be, to the amount of the principle of a loan over time, and the size of the loan. The prohibition of interest or *riba* makes us wonder about the mechanism that runs Islamic institutions i.e. how do they operate. The answer would simply be on profit and loss sharing (PLS). As such "an Islamic bank does not charge interest but rather participates in the yields resulting from the use of funds" (Kettell, 2008).

While commercial banks borrow from depositors and pay them interest on one side, then lend to borrowers and charge them interest on the other side, Islamic banks enter in
partnership with their depositors and borrowers by sharing the profits and losses with them according to predetermined ratios. Hence, Islamic banks act as managers of resources to be invested in productive manners.

The six principles that drive the Islamic banking system are:

1. "Predetermined payments are prohibited": Islam only allows qard al Hassan (meaning good loan) whereby the lender does not charge interest or any additional amount over and above the main amount of the loan. According to one Islamic scholar: "the prohibition applies to an advantage or benefits that the lender might secure out of the qard (loan) such as riding the borrower’s mule, eating at his table, or even taking advantage of the shade of his wall."

2. "Profit and loss sharing": the creditor and the borrower shall share the risks and the rewards that would arise out of the business venture for which the money was lent. This is unlike commercial banking where the borrower is held accountable to repay his loan with an increment regardless of the outcome of the business venture for which the money was lent.

3. "Making money out of money is not acceptable": In Islam, money does not have an intrinsic value. Indeed it is used only as a medium of exchange or a way of defining value of other things. As such money should not earn money by staying idle or deposited in banks. Moreover, money would only become capital if it is to be invested in business along with human efforts and risks.

4. "Uncertainty is prohibited": Any Islamic transaction or operation should be free from gharar i.e. it should be free from uncertainty, risk and speculation. All the
parties involved in an Islamic transaction should have clear and perfect knowledge of all the details pertaining to the transaction. Moreover, no guaranteed profit could be predetermined. In addition, the prohibition of *gharar* would protect the weak from mistreatment.

5. "*Only Sharia'a approved contracts are acceptable*": Any project that contradicts the moral value of Islam should not be financed. For instance, Islamic banks cannot finance a wine factory, a casino, a night club, or any activity that is known to be harmful to society.

6. "*Sanctity of contracts*": responsible Muslims have a moral obligation to conduct business and commerce according to the requirements and *Sharia'a* rules of their religion.

Since the emergence of Islamic banking in the mid seventies, the commercial sector viewed it as a marginal financial service targeting an exotic niche mainly focusing on household investors. But the start of the new millennium brought along a new dimension for the Islamic banking sector. Fueled by the massive increase in oil prices, and the ever increasing world of Muslim population that exceeds one billion, the Islamic banking industry mushroomed rapidly. Nowadays the size of Islamic banking is estimated to range from: "$500 billion to $750 Billion" (Kettell, 2008). Not only Islamic banking has attracted the huge savings of the Gulf Cooperation Council (GCC) countries, but also it has increased the appetite of the largest commercial banks to open Islamic windows. These banks include but are not limited to the following names: HSBC, Lloyds Bank, Deutsche Bank, Citigroup, Barclays
Bank Plc, UBS A.G, BNP Paribas, Bloom bank, Al-Ahli Bank... Hand in hand, governing bodies of Islamic banks entered the picture. In 1991, the Accounting and Auditing Organization for Islamic Financial Institutions (AAOIFI) was inaugurated in Bahrain to set accounting, auditing, ethics and governance standards under the umbrella of Sharia'a. Subsequently, in 2002, the international Islamic Financial Market (IIFM), the Liquidity Management Center (LMC) and the Islamic Financial Services Board (IFSB) were created.

A. Islamic Bank Balance Sheet Analysis

In this section we discuss the Islamic banks’ balance sheet composition in order to better understand their sources and uses of funds. In their article:’” Balance Sheet Analysis: Islamic vs. Commercial” Hennie & Zamir (2009) decompose the balance sheets of commercial as well as Islamic banks based on functionality as shown in tables 1 and 2:

Table 1: Stylized balance sheet of a commercial bank-based on functionality

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans and advances to customers</td>
<td>Customers’ deposits</td>
</tr>
<tr>
<td>Cash and cash balances with other banks</td>
<td>Due to banks and other financial institutions</td>
</tr>
<tr>
<td>Investments in associates, subsidiaries and joint ventures</td>
<td>Other liabilities</td>
</tr>
<tr>
<td>Financial assets held for trading</td>
<td>Sundry creditors</td>
</tr>
<tr>
<td>Cash and cash balances with the central bank</td>
<td>Equity and reserves</td>
</tr>
</tbody>
</table>
Table 2: Stylized balance sheet of an Islamic bank-based on functionality

<table>
<thead>
<tr>
<th>Application of funding</th>
<th>Sources of funding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash balances</td>
<td>Demand deposits (amnah)</td>
</tr>
<tr>
<td>Financing assets (murabaha, salam,</td>
<td>Investment accounts (mudarabah)</td>
</tr>
<tr>
<td>ijara, istisna)</td>
<td></td>
</tr>
<tr>
<td>Investment assets (mudarabah,</td>
<td>Special investment accounts</td>
</tr>
<tr>
<td>musharakah)</td>
<td>(mudarabah, musharakah)</td>
</tr>
<tr>
<td>Fee-based services (ju’ala, kafala,</td>
<td>Reserves</td>
</tr>
<tr>
<td>and so forth)</td>
<td></td>
</tr>
<tr>
<td>Non-banking assets (property)</td>
<td>Equity capital</td>
</tr>
</tbody>
</table>

Definition of some of the above stylized balance sheets terms:\n
- Other liabilities: they are divided into other current liabilities and other long term liabilities. Other current liabilities is “a balance sheet entry used by companies to group together current liabilities that are not assigned to common liabilities such as debt obligations or accounts payable”. Other long term liabilities, is “a balance sheet item which includes obligations that do not currently require interest payments. This would include items such as remaining leases, future employee benefits and deferred taxes.”

- Sundry creditors or Sundry income is an “external income that results from factors outside of a firm’s control. Examples of sundry income include gains from foreign exchange, royalty income, or even income from the sale of various investments.”

---

3 www.investopedia.com/terms website is used as a reference for some definitions.
• Reserves are "capital reserves held by a bank or financial institution in excess of what is required by regulators, creditors or internal controls. For commercial banks, excess reserves are measured against standard reserve requirement amounts set by central banking authorities. These required reserve ratios set the minimum liquid deposits (such as cash) that must be in reserve at a bank; more is considered excess. Banks that carry excess reserves have an extra measure of safety in the event of sudden loan losses or cash withdrawals by customers. Reserves need to be in liquid forms of capital such as cash in a vault, which does not create income. Banks will therefore try to minimize their excess reserves by lending the maximum allowable amount to borrowers" with deficits in their reserves.

• Fee-based services (ju’ala, ka’ala, and so forth): Ju’ala: is a contract for performing a given task against a prescribed fee in a given period. Kafalah: it is a pledge given to the creditor that the debtor will pay the debt or fine. (Kettell, 2008).

• Non-banking assets (property): Due to the profit and loss sharing mechanism the Islamic bank enters into real partnership with its customers. As such the Islamic bank would literally own part or the whole of the underlying asset.

• Demand deposits (amanah) are the deposits held in trust. It entails the absence of liability for loss except in the breach of duty. (Kettell, 2008).

According to Hennie & Zamir (2009) the balance sheets of Islamic banks differ from the balance sheets of commercial banks in the following aspects:
1. "The foremost feature of an Islamic bank is the 'pass-through' nature of the balance sheet". Meaning that the mismatch of asset-liability exposure of commercial banks is removed because the return on deposits of Islamic banks’ clients are directly related to the return on assets of the bank as per the PLS mechanism (explained in previous section).

2. "The nature of assets of two institutions is different". Unlike commercial banks, the bulk of Islamic banks’ assets are concentrated on asset-based investments, thus incurring high credit risk that are backed by tangible assets. As such, Islamic banks do not have leverage credit creation.

3. "The assets of Islamic banks contain financing assets where tangible goods and commodities are purchased and sold to the customers". As such the financing and its underlying asset are coupled together thus increasing the exposure to risks of Islamic banks.

4. The prohibition on interest, forbids Islamic banks to issue debts to finance assets. The lack of leverage in Islamic banks makes them less risky during financial crisis.

B. Islamic Financing Methods

"Allah has permitted trade and forbidden usury" this is the basic rule that governs all types of Islamic finance and banking. In Islam, trade is not only permitted but also encouraged if and only if a mutual consensus exists between the seller and the buyer. Any transaction that is free from Riba and Gharar and that does not deal with Haram products
and services could be subject of trade. The lack of true knowledge of the underlying rules of Islamic trade has led to the formation of few worldwide acceptable Islamic financial instruments detailed below.

1. *Murabaha* (Cost –plus sales): It is a kind of sale where the buyer knows exactly the cost incurred by the seller and the buyer is willing to pay a specified profit margin or premium over the cost to the seller. A crucial condition of this kind of sale is that the seller (Islamic bank) should own the item (to be sold) at the time the buyer agrees to conclude the sale.

2. *Bay’ bi-thaman ‘ajil* (credit sales): it is only permitted if and only if the price was increased for deferment i.e. the price was raised before the deferment (as in the case of *murabaha*) not by the deferment (where it would be considered *riba*).

3. ‘*Ijarah* (leasing): it is the sale of the *usufruct* of the object for a certain period of time. The main condition for the ‘*ijarah* to be valid is that the leasing agency must own the object of lease throughout the period or duration of the lease. An extension of the ‘*ijarah* contract that would end by transferring the ownership of the object of the lease to the lessee (by paying a pre-determined residual value at the end of the contract) is also available and called: ‘*ijarah wa ‘iqtina*’.

4. *Musharaka* and *Mudaraba* (partnerships): are the two allowed partnerships in Islamic banking. One is silent partnership (*mudaraba*) and the other is full partnership (*musharaka*). Another form of the later called *musharaka*
mutanaqisah (diminishing partnership) could replace a commercial mortgage of commercial banks.

5. *Salam* and *'Istisna'* (Islamic forwards): in the exception of these two contracts that were tailored to finance certain types of businesses, forward sale of non-existing objects is forbidden in Islam since it would be considered *gharar*. *Salam* is used to produce agricultural products. The main conditions of *Salam* are: price paid in full when the contract is signed and the delivery due date clearly specified. As for *'istisna'* , it is mainly used in production or construction. Unlike Salam, the price in *'istisna'* is paid on installments as the work progress.
III. Literature Review

In this section, we conduct a literature review related to our study. First we review the literature on the efficiency of Islamic banking. Second, we discuss the efficiency of commercial banks. Finally, we review studies on the comparison of efficiency of commercial and Islamic banks.

A. Islamic Banking

Given the fact that the Islamic banking industry is relatively new, few academic articles are published in this area. Another important factor that leads to the scarcity of research regarding Islamic banking is the lack of availability of data. In this section, we review some of the published research in this field. It is good to note that none of the studies came out with a finite result, but all added in a way or another to the literature of understanding and evaluating Islamic banking.

In his paper “Assessing the Performance of Islamic Banks: Some Evidence from the Middle East” Abdel-Hammed M.(2000) argues that globalization has a great effect on the efficiency of Islamic banks because it puts the later in fierce competition with the much larger, well-developed commercial banks. The author studies how the efficiency of Islamic banks is related to the relationship between profitability and banks’ characteristics. He used total liabilities to total assets ratio as a proxy of the high risk that Islamic banks incur because of the nature of their core work that is governed by the PLS mechanism.
Abdel-Hammed M. also finds that capital and loan ratios affect positively Islamic banks profitability, which means that the performance of Islamic banks is positively related to the adequate capital ratios and loan portfolios. He also reports that promoting Islamic banks profits should be related to customers, short term funding, non-interest earning assets and overhead. Finally, he notes that reserves, that are a major requirement in Islamic banking, hurt the later in two ways: first they do not yield any return and second they reduce the amount of funds available for investment.

In his study "Can Islamic Banking Survive? A Micro-evolutionary Perspective" Amine El-Gamal (1997) argues that although all Muslim countries concede that Islam prohibits interest, not a single country runs its financial institutions without resorting to interest. The fact is that no one knows how to run it without interest especially when political pressure mounts. He adds that a survey conducted on depositors of Islamic banks by Abdel-Kader (1995) reveals that the majority of depositors participate in interest paying commercial as well as Islamic banks. Indeed Abdel-Kader also notes that the transactions of Islamic banks are not purely according to Sharia’a. He explains:

- In Islamic banks, whereas demand deposits do not pay interest, the savings deposits that remain with the banks for a period greater than 12 months are rewarded with a “gift” that has high correlation with market interest rates.

- In murabaha contracts, the entrepreneur to avoid the banks audit would typically pay the later a threshold rate regardless of the actual profit or loss. This threshold rate is typically equivalent to an interest rate.
The article "Islamic Banking Comparative Analysis" of The Arab Bank Review, notes that the Islamic banking industry, because of the profit and loss sharing (PLS) mechanism, is actually more favorable to economies’ needs than its commercial counterparts since it not only shares profits but losses as well; thus encouraging profitable projects. The article also indicates that since Islamic banks prefer to lend money for short term periods, they are more readily available for small entrepreneurs. Islamic banks have to compete with the commercial banks to attract customers. Zaher & M.K.(2001) call for the standardization of Islamic products to encourage additional growth by offsetting the different Shariah's rulings and interpretations at the country or individual banks levels. The article notes as well that many Muslim countries have few Islamic banks, which predicts huge growth possibilities.

In her study:"Market Structure and Competitive Conditions: A comparative analysis of Islamic and commercial banking” Turk-Ariss (2009) selects thirteen countries in which Islamic as well as commercial banks coexisted during the period extending from 2000 to 2006 and performs comparative analysis on their structures and competitive conditions. Ariss assumes that Islamic financial services have a distinct global market than those offered by commercial banks. Moreover, she notes that their balance sheets are dominated by credit-based financing mainly murabaha; all the other Islamic finance instruments constituted on average less than ten percent of the assets.

Ariss also indicates that univariate statistics provides a broad evidence of the differences in the portfolios and returns of the Islamic and commercial banks. Islamic banks allocate a greater share of their assets to financing/loans than commercial banks, thus incurring a higher credit risk. But on the other hand, Islamic banks have higher capitalization levels
that lower their financial risk; this is translated into higher ROAs. She concludes that asset composition and return of Islamic and commercial banks operating in the same country differ significantly.

As such, she argues that Islamic banking has a competitive advantage or a higher degree of market power in some segments where they could have a higher profitability levels.

In another study: “On the Rise of Islamic Banking among Giant Commercial Banks: a Performance and Efficiency Analysis”, Ariss indicates that commercial banks on average waste twice as much resources as Islamic banks. The reason is that higher efficiency is directly correlated to higher profitability that is the result of better capitalization and lower concentration levels. She adds that Islamic customers would feel more confident if they can deal with Islamic banks that offer products according to Sharia’a rules and regulations but at the same time adhere to international standards of performance and services.

**B. Commercial Banking**

In their paper titled “The Performance of Banks in Post-War Lebanon” Peters, Raad, & Sinkey, (2004) give a clear insight on the performance of the Lebanese commercial banks. They argued that, although the Lebanese banks have improved their performances in terms of profitability and safeness (less risky) during the post civil war period especially during the years 1993 through 2000, they remained much less efficient when compared to the control group of several banks from five Middle Eastern countries. Several univariate and multivariate analysis led them to infer that improvement in the
banking industry in Lebanon was due to the following four factors: "political (cessation of war), economic (lower inflation), regulatory (BIS capital requirements), and managerial (greater recognition of the importance of the capital adequacy and risk management)". The authors recommend that Banque du Liban (the Lebanese central bank) should lower the interest on its T-bills (less than the market interest) so that the Lebanese banks would be encouraged to give loans to the private sector, thus driving the sluggish Lebanese economy to flourish and grow again.

In their paper titled: "Efficiency Performance of Commercial Banks in Lebanon", Djoundourian & Raad (2008) use the stochastic frontier production function to evaluate the key performance indicators of the Lebanese commercial banking sector post the civil war that ended in 1990. The authors observe that the size of the commercial banking sector significantly grew if measured by its assets, but this growth is not accompanied by a growth in profits. The authors add that the Lebanese commercial banking sector is more stable during the period of the study (1993 through 2002) due to its significant growth in equity capital. According to the authors, the Lebanese commercial banks during the period of the study, did not play their traditional role of financing the private sector initiative, instead they heavily invested in the high yield Lebanese treasury bills. Djoundourian & Raad’s findings also indicate that the Lebanese commercial banks level of inefficiency in producing loans and in the cost of producing loans are increased with the increases in the number of branches, number of employees per branch and the ratio of staff to operating expenses. They conclude that banks have become more efficient over the period of the study.
In his paper titled: "Performance of banks on countries of the Gulf Cooperation Council" Ramanathan (2005) studies the performance of a sample of fifty five banks from six Gulf Cooperation Council. Using the data envelopment analysis (DEA) method, the author finds that only fifteen out of the fifty five banks display efficiency under the assumption of constant return to scale (CRS), with at least one efficient bank in each of the GCC countries that are considered. Using the variable return to scale (VRS) the number of efficient banks rises to twenty seven and a minimum number of efficient banks in each country rise to two. Using the MPI approach, the results show that Bahrain, Kuwait, Saudi Arabia and UAE progress in terms of MPI from 2000 to 2004 although there has been no change in the average productivity of the region.

In their paper titled: "Bank Size, Specification and Efficiency: the Netherlands" Bos, JWP, Kool, & CJM (1998) investigate the profit and cost efficiencies in the Dutch banking industry during 1992-1998, a period that is characterized by a lack of major institutional change. In their analysis, the authors use stochastic frontier cost and profit models. Their results show that both cost and profit frontiers are deterministic. Their results also indicate that: profit efficiency varies more than cost efficiency when comparing bank to bank. Although many banks have relatively high cost efficiency, the profit efficiency of many banks is relatively low which is in line with previous literature that shows that output side inefficiencies appear to be of a larger magnitude than inefficiencies of the input side. The authors also find that there is no clear correlation between profit efficiency and cost efficiency. They add that the average performance, which is measured by profit and cost efficiencies of the banking industry in Netherland remains stable during the period of the study (1992-1998) ; thus suggesting that previous
consolidation and regulations had little impact on industry efficiency. The authors conclude that whereas all banks appear to perform similarly in terms of cost efficiency, large general banks and specialized banks outperformed small general banks in terms of profit efficiency for two main reasons: large general banks benefit from sheer size and market power, while specialized banks benefit from the niche market in which they operate.

In his paper: "Operating performance of banks among Asian economies: An international and time series comparison" Kwan (2003) compares the observed operating costs of banks across seven Asian economies. He finds that bank's per unit operating costs varied significantly across the Asian countries and over time after controlling for loan quality, liquidity, capitalization and output mix. After further analysis of the labor cost and the cost of physical capital, the two components of operating costs, Kwan notes that the country ranking per unit labor cost and the country ranking per unit physical capital cost are highly correlated; thus inferring that banks incurring high labor cost would also incur high capital cost. Kwan adds that if per unit operating cost is used as a measure of efficiency, the results would suggest that there exist systematic differences in bank operating efficiency across the studied countries. However, per unit operating cost is not related to the degree of openness of the banking sector.

Kwan also finds that banks’ operating costs in the studied Asian countries were declining in the period from 1992 to 1997, indicating that banks were on average lowering their operating costs thus improving their operating performances. Post 1997, per unit operating costs started to build up indicating that banks incurred additional costs to deal with loan problems and output declination that resulted due to the Asian financial crisis.
Kwan also indicates that the declining labor cost share shows that banks were adjusting their labor input but not their physical capital input when demands were falling. He detects significant differences in labor cost shares across the Asian countries, suggesting that each country has a different bank production functions. More importantly, he identifies that the variations in labor cost shares are significantly positively related to each country financial services wage rate, which provides evidence that banks use relatively more labor because their country’s banking labor force is more productive not because it is cheaper.

In their paper titled: “Foreign banks entry, deregulation and bank efficiency: Lessons from the Australian experience” Sturm & Williams (2004) find that technical inefficiency in Australian banks was dominated by scale inefficiency. They add that during and after deregulation the biggest four banks in Australia, used size as a barrier to entry via mergers before the entry of the foreign banks. Moreover, domestic banks increased their spending upon branch networks post deregulation. The authors also note that the main source of the gain in Australian banks’ productivity in post –deregulation was the technological change rather than technical efficiency. This efficient technological change was introduced by foreign banks immediately post-deregulation. In addition, the authors report that foreign banks were superior, to domestic banks, on scale efficiency which resulted in increased overall banks’ efficiency. They also suggest that foreign banks that did not convert to branch status were the most efficient of the foreign banks. Add to that, the authors indicate that an important source of competitive improvements in banking system productivity would be the diversity in the types of banks participating in
it. Finally, the authors argue that even though foreign banks were more efficient than domestic banks, they were not more profitable.

C. Comparison of Islamic Banking versus Commercial Banking

Bader, Shamsher, & Mohamad. Hassan (2008) study on: "Cost, Revenue, and Profit Efficiency of Conventional versus Islamic Banks: Financial Ratios Approach" is the most directly related to our project. In this study the authors investigate the cost, revenue and profit efficiency to compare the efficiency of commercial and Islamic banks. Not only they analyze the efficiencies of commercial and Islamic banks, but also they go further to distinguish cost, revenue, and profit efficiency differences among big versus small, old versus new, and banks based on their regions. They use the following six financial ratios as proxies for cost, revenue and profit: cost to income ratio, non-interest expense ratio, net interest margin, Other Operating Income, AROA (Average return on assets) and AROE (average return on equity).

The authors find that there are no significant differences between the efficiency scores of commercial and Islamic banks, which indicate that Islamic banks are not impediment to efficiency when compared to the well established commercial banks. However, they add that there are substantial room for improvement in cost minimization and revenue and profit maximization in both types of banks.

On the bank size effect, the authors report no significant difference in the mean scores between big and small banks. They argue that the size of banks does not affect their cost, revenue and profit efficiency. As for bank’s age, data indicate that old banks are better in minimizing costs and are able to generate slightly better profits than new banks. On the
other hand, the new banks pay significantly higher interest rate to their clients and have slightly higher revenue efficiency. Finally, on the location difference, the authors note that are no significant.
IV. Data Collection

Data are collected from BankScope database for banks from nine Arab countries (Bahrain, Jordan, Kuwait, Lebanon, Qatar, Saudi Arabia, Syria, United Arab Emirates, and Yemen) that host commercial and Islamic banks in their financial systems. To be included in the sample, banks must have complete data from 2003 to 2007.

Lebanon, which is a major player in the Middle Eastern financial market, was considered in the study even though the Islamic banks operating in it did not pass the second criterion. Islamic banks in Syria did not pass the second screening as well. We kept Lebanon and Syria in the study since we believe that the Islamic banking sectors in both countries will survive and grow by gaining even a small share in the market in the coming few years, and because both types of banks are readily available to investors at the time of the study. This project evaluates a cross-country data compiled from the balance sheets of 83 commercial banks and 20 Islamic banks located in the Middle East and distributed as shown in table 3:

<table>
<thead>
<tr>
<th>Country</th>
<th>Commercial banks</th>
<th>Islamic banks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bahrain</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>Jordan</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>Kuwait</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Lebanon</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>Qatar</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Saudi Arabia</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>Syria</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>United Arab Emirates (UAE)</td>
<td>15</td>
<td>4</td>
</tr>
<tr>
<td>Yemen</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>83</strong></td>
<td><strong>20</strong></td>
</tr>
</tbody>
</table>
The list of names of all commercial and Islamic banks that are included in this project is available in appendix 1.

All the data, initially provided in the currency of their respective countries, are converted into US dollars using the average of five conversion ratios\(^3\) in each year to adjust for inflation.

Table 4 reports all the variables used in this project for commercial and Islamic banks

**Table 4: Variables used in the study collected from BankScope**

<table>
<thead>
<tr>
<th></th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Total Assets</td>
</tr>
<tr>
<td>2</td>
<td>Total Earning Assets</td>
</tr>
<tr>
<td>3</td>
<td>Equity</td>
</tr>
<tr>
<td>4</td>
<td>Off Balance Sheet</td>
</tr>
<tr>
<td>5</td>
<td>Loan Loss Reserves</td>
</tr>
<tr>
<td>6</td>
<td>Loan Loss Provisions</td>
</tr>
<tr>
<td>7</td>
<td>Net Income</td>
</tr>
<tr>
<td>8</td>
<td>Loan Loss Reserves/ Gross Loan</td>
</tr>
<tr>
<td>9</td>
<td>Equity/Total Assets</td>
</tr>
<tr>
<td>10</td>
<td>Net Interest Margin</td>
</tr>
<tr>
<td>11</td>
<td>Net Interest Revenue/Average Assets</td>
</tr>
<tr>
<td>12</td>
<td>AROA</td>
</tr>
<tr>
<td>13</td>
<td>AROE</td>
</tr>
<tr>
<td>14</td>
<td>Cost To Income Ratio</td>
</tr>
<tr>
<td>15</td>
<td>Net Loans/Total Assets</td>
</tr>
</tbody>
</table>

\(^3\) Taken at these points in time: January 1, March 1, June 1, September 1, and December 1
V. Methodology

We assume that commercial as well as Islamic banks are keen to minimize costs and maximize both revenues and profits. We examine the differences in mean cost, revenue and profit of commercial and Islamic banks. Our main objective is to test the following null hypothesis:

1. The efficiency, measured by the mean cost, mean revenue and mean profit, of commercial banks is significantly better than that of their Islamic counterparts.

2. The efficiency, measured by the mean cost, mean revenue and mean profit, of big commercial and Islamic banks is significantly better than that of small commercial and Islamic banks

3. The efficiency, measured by the mean cost, mean revenue and mean profit, of big commercial banks is significantly better than that of small commercial banks.

4. The efficiency, measured by the mean cost, mean revenue and mean profit, of big Islamic banks is significantly better than that of small Islamic banks.

5. The efficiency, measured by the mean cost, mean revenue and mean profit, of big commercial banks is significantly better than that of big Islamic banks.

6. The efficiency, measured by the mean cost, mean revenue and mean profit, of small commercial banks is significantly better than that of small Islamic banks.

To test for the above six null hypotheses, we use paired sample t-test in SPSS 17.0 to compare the means of the different samples. Traditionally, financial ratios analysis has been used to evaluate the performance of banks. Although it has been criticized, ratios
analysis is still widely used as a simple approach to evaluate banks’ financial performances. In our analysis, we use four critical financial ratios as proxies for cost, revenue and profit as detailed in the table 5.

Table 5: Financial ratios used as proxies for cost, revenue and profit

<table>
<thead>
<tr>
<th>Definitions of financial ratios (used as proxies)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cost efficiency ratio</strong></td>
</tr>
<tr>
<td>Cost To Income Ratio (CTIR)</td>
</tr>
<tr>
<td><strong>Revenue efficiency ratio</strong></td>
</tr>
<tr>
<td>Net Interest Margin (NIM)</td>
</tr>
<tr>
<td><strong>Profit efficiency ratios</strong></td>
</tr>
<tr>
<td>Average Return on Assets (AROA)</td>
</tr>
<tr>
<td>Average Return on Equity (AROE)</td>
</tr>
</tbody>
</table>

On the other hand, and in order to investigate what contributes to AROA of commercial and Islamic banks we would run the following two semi-log regressions on each type:

- \( \text{AROA} = \alpha + \beta_1 \ln(\text{Total Assets}) + \beta_2 \ln(\text{off balance sheet}) + \beta_3 \ln(\text{loan loss reserve}) + \beta_4 \ln(\text{Cost to income ratio}) + \beta_5 \ln(\text{equity to total assets}) + \beta_6 \ln(\text{net loans to total assets}) + \varepsilon_i^4 \)

\( ^4 \varepsilon_i \) is an error term
- AROA = α + β1 ln(Total Assets) + β2 ln(off balance sheet) + β3 ln(loan loss reserve) + β4 (Cost to income ratio) + β5 (equity to total assets) + β6 (net loans to total assets) + B7 ln (loan loss provisions) + εi

Table 6 displays the definition of some of the independent variables.

Table 6: Definition of some independent variables

<table>
<thead>
<tr>
<th>Description</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Off-Balance Sheet financing</td>
<td>A form of financing in which large capital expenditures are kept off of a bank’s balance sheet through various classification methods. Banks will often use off-balance-sheet financing to keep their debt to equity (D/E) and leverage ratios low, especially if the inclusion of a large expenditure would break negative debt covenants.</td>
</tr>
<tr>
<td>Loan Loss Reserve</td>
<td>Is an estimate of future loan losses, a valuation reserve. When loans are charged off, they deplete the loan loss reserve.</td>
</tr>
<tr>
<td>Loan Loss Provision</td>
<td>In anticipation of some borrowers’ defaulting, banks set aside part of their earnings through an expense account called loan loss provisions. The provision is an inflow that augments the stock or level of the reserve.</td>
</tr>
</tbody>
</table>

5 The definition of off-balance sheet is taken from www.investopedia.com/terms, the definitions of Loan Loss Reserve and Provision are taken from the book Commercial Bank Financial Management by Dr. Joseph Sinkey.
VI. Empirical Results

Table 7 displays the means and standard deviation obtained from the descriptive statistics using SPSS 17.0, for all commercial and Islamic banks during the period 2003 to 2007. Figures 1 to 4 show the trend variations, for the same period, in the cost to income ratio, net interest margin, AROA and AROE of commercial and Islamic banks.

Table 7: Mean comparison of all commercial and Islamic banks for the years 2003-2007

<table>
<thead>
<tr>
<th></th>
<th>Mean comparison</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>μ</td>
<td>σ</td>
<td>N</td>
<td>μ</td>
<td>σ</td>
</tr>
<tr>
<td>Commercial Cost To</td>
<td></td>
<td>47.58</td>
<td>18.20</td>
<td>83</td>
<td>47.40</td>
<td>20.35</td>
</tr>
<tr>
<td>Income Ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Islamic Cost To</td>
<td></td>
<td>52.26</td>
<td>17.63</td>
<td>19</td>
<td>47.76</td>
<td>16.55</td>
</tr>
<tr>
<td>Income Ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commercial Net</td>
<td></td>
<td>2.93</td>
<td>1.20</td>
<td>83</td>
<td>2.83</td>
<td>1.11</td>
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<td>3.30</td>
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<td>19</td>
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</tr>
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<td>Commercial AROE</td>
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<td>23.61</td>
<td>83</td>
<td>18.23</td>
<td>15.16</td>
</tr>
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<td>Islamic AROE</td>
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<td>11.31</td>
<td>8.76</td>
<td>19</td>
<td>13.44</td>
<td>10.22</td>
</tr>
</tbody>
</table>

μ sample mean
σ sample standard deviation
N sample size
Figure 1: Commercial versus Islamic banks Cost to Income Ratio

Figure 2: Commercial versus Islamic Banks Net Interest Margin
Figure 3: Commercial versus Islamic Banks AROA

Figure 4: Commercial versus Islamic Banks AROE
A. Overall Efficiency: Commercial versus Islamic Banks

The results of the paired sample t-test, shown in table 8, indicate that our sample of Islamic banks (IBs) is significantly more cost efficient than our sample of commercial banks (CBs). Moreover, IBs are more efficient on the revenue side also. On the profit efficiency, we can conclude that IBs are more profitable even though there is no significant difference between Islamic AROE and commercial AROE but there exist a significant difference in their respective AROA’s. The main reason why AROA and AROE do not exhibit in the same manner (one is significantly different and the other is not) is due to the equity multiplier (EM= total equity capital/total assets) which is a measure of a bank leverage. In this sample, the EM mean of commercial banks is 7.56, while the EM mean of Islamic bank is 3.53. The huge difference in EM mean of commercial banks and the EM mean of Islamic banks is due to the nature of the work of the Islamic banks which is explained in previous sections. As a result, we can reject the first null hypothesis and claim that our sample of Islamic banks is more efficient than our sample of commercial banks.
Table 8: Descriptive statistics: cost, revenue and profit efficiency of all banks, commercial and Islamic

<table>
<thead>
<tr>
<th>Group</th>
<th>Statistics</th>
<th>Cost Efficiency Ratio</th>
<th>Revenue Efficiency Ratio</th>
<th>Profit Efficiency Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CTIR</td>
<td>NIM</td>
<td>ROAA</td>
</tr>
<tr>
<td>All Banks</td>
<td>N</td>
<td>508.00</td>
<td>511.00</td>
<td>513.00</td>
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<td>Minimum</td>
<td>0.00</td>
<td>-6.80</td>
<td>-7.23</td>
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<td>Maximum</td>
<td>323.28</td>
<td>64.00</td>
<td>53.09</td>
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<td>Mean</td>
<td>46.31</td>
<td>3.58</td>
<td>2.82</td>
</tr>
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<td></td>
<td>Std. Deviation</td>
<td>24.76</td>
<td>4.07</td>
<td>4.45</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Commercial banks</td>
<td>N</td>
<td>415.00</td>
<td>415.00</td>
<td>415.00</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>0.00</td>
<td>-1.06</td>
<td>-3.65</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>323.28</td>
<td>6.18</td>
<td>30.18</td>
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<td>Mean</td>
<td>46.74</td>
<td>2.99</td>
<td>2.27</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>26.12</td>
<td>1.10</td>
<td>2.80</td>
</tr>
<tr>
<td>Islamic banks</td>
<td>N</td>
<td>93.00</td>
<td>96.00</td>
<td>98.00</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>9.57</td>
<td>-6.80</td>
<td>-7.23</td>
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<tr>
<td></td>
<td>Maximum</td>
<td>100.91</td>
<td>64.00</td>
<td>53.09</td>
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<tr>
<td></td>
<td>Mean</td>
<td>44.38</td>
<td>6.13</td>
<td>5.13</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>17.43</td>
<td>8.69</td>
<td>8.03</td>
</tr>
<tr>
<td>T-test Sig (2-tailed)</td>
<td>After adjusting for sample size</td>
<td>0.05*</td>
<td>0.00*</td>
<td>0.00*</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

* the mean difference is significant at the 5% level

It should be noted that the paired sample t-test takes the lowest number of observations when comparing two sets of variables. As such, and since we have 100 Islamic observations and 415 commercial ones we replicated the Islamic observations four times thus maximizing the number of observations for the paired t-test without changing the means of Islamic data but slightly alternating the commercial data by keeping out only 15 observations. The results are shown in table 9. The Islamic means do not change but the observations increase from 100 to 384 which leads to higher number of degrees of freedom and thus a better Paired t-test results. The results of the paired t-test when using the adjusted sample size are more accurate, because the coefficients that were significant (when using the un-adjusted sample size) remained significant but at a lower level of
significance and some independent variables that were insignificant at a level just above the 5% became significant when using the adjusted sample size.

Table 9: Paired sample statistics after adjusting for sample size

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>N</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
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</thead>
<tbody>
<tr>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Net Interest Margin</td>
<td>2.97</td>
<td>384</td>
<td>1.08</td>
<td>0.055</td>
</tr>
<tr>
<td>NIM Islamic</td>
<td>6.13</td>
<td>384</td>
<td>8.66</td>
<td>0.44</td>
</tr>
<tr>
<td>Pair 2</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AROA</td>
<td>2.25</td>
<td>392</td>
<td>2.86</td>
<td>0.14</td>
</tr>
<tr>
<td>AROA Islamic</td>
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<td>392</td>
<td>8</td>
<td>0.40</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>AROE</td>
<td>17.06</td>
<td>392</td>
<td>15.6</td>
<td>0.79</td>
</tr>
<tr>
<td>AROE Islamic</td>
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<td>392</td>
<td>14.89</td>
<td>0.75</td>
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<tr>
<td>Pair 4</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Cost To Income Ratio</td>
<td>47.67</td>
<td>372</td>
<td>27</td>
<td>1.4</td>
</tr>
<tr>
<td>CTIR Islamic</td>
<td>44.38</td>
<td>372</td>
<td>17.35</td>
<td>0.9</td>
</tr>
</tbody>
</table>

B. Efficiency of Big Banks versus Small Banks in Commercial and Islamic Banks

In this section, we divide each of our samples of commercial and Islamic banks into two sub-samples: big and small according to total assets. All banks of both types that have total assets greater than two billions US dollars were classified as big and the rest as small. We end up with 239 observations for big commercial banks, 35 observations for big Islamic banks, 176 observations for small commercial banks and 65 observations for small Islamic banks.
The results of the paired t-test shown in table 10 prove that there exist significant differences in the cost, revenue, and profit efficiencies between big commercial and Islamic banks and small commercial and Islamic banks. Indeed, big banks are by far better cost efficient (CTIR mean big= 39.70< CTIR mean small= 53.88). Even though on the revenue efficiency, big banks slightly lack behind small banks, on the profit efficiency, big banks outperform the small banks as there exists a significant difference in their respective AROE’s (Big mean AROE=18.99>Small mean AROE= 15.49). As such, we fail to reject the second null hypothesis and conclude that big commercial and Islamic banks are significantly more efficient on the cost and profit level than small commercial and Islamic banks.

Table 10: Descriptive statistics: cost, revenue and profit efficiency of commercial and Islamic big and small banks

<table>
<thead>
<tr>
<th>Group</th>
<th>Statistics</th>
<th>Cost Efficiency Ratio</th>
<th>Revenue Efficiency Ratio</th>
<th>Profit Efficiency Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CTIR</td>
<td>NIM</td>
<td>ROAA</td>
</tr>
<tr>
<td>Commercial and Islamic Big</td>
<td>N</td>
<td>273.00</td>
<td>273.00</td>
<td>274.00</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>0.00</td>
<td>0.00</td>
<td>-3.65</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>323.28</td>
<td>31.00</td>
<td>30.18</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>39.70</td>
<td>3.09</td>
<td>2.58</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>23.15</td>
<td>2.08</td>
<td>2.58</td>
</tr>
<tr>
<td>Commercial and Islamic Small</td>
<td>N</td>
<td>235.00</td>
<td>238.00</td>
<td>239.00</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>0.00</td>
<td>-6.80</td>
<td>-7.23</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>177.45</td>
<td>64.00</td>
<td>53.09</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>53.98</td>
<td>4.15</td>
<td>3.09</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>24.40</td>
<td>5.48</td>
<td>5.90</td>
</tr>
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<td>T-test Sig (2-tailed)</td>
<td>After adjusting for sample size</td>
<td>0.00*</td>
<td>0.00*</td>
<td>0.07</td>
</tr>
</tbody>
</table>

* the mean difference is significant at the 5% level
To investigate these results further, we test two additional hypotheses (3 and 4) to check if the same results hold within each type of banks (Islamic and commercial).

Table 11: Descriptive statistics: cost, revenue and profit efficiency of commercial big and small banks

<table>
<thead>
<tr>
<th>Group</th>
<th>Statistics</th>
<th>Cost Efficiency Ratio</th>
<th>Revenue Efficiency Ratio</th>
<th>Profit Efficiency Ratios</th>
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<tr>
<td></td>
<td></td>
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<td>NIM</td>
<td>ROAA</td>
</tr>
<tr>
<td>Commercial</td>
<td>N</td>
<td>239.00</td>
<td>239.00</td>
<td>239.00</td>
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<tr>
<td>Big</td>
<td>Minimum</td>
<td>0.00</td>
<td>0.00</td>
<td>-3.65</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>323.28</td>
<td>6.18</td>
<td>30.18</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>39.92</td>
<td>2.85</td>
<td>2.42</td>
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<tr>
<td></td>
<td>Sd. Deviation</td>
<td>24.34</td>
<td>0.90</td>
<td>2.42</td>
</tr>
<tr>
<td>Commercial</td>
<td>N</td>
<td>176.00</td>
<td>176.00</td>
<td>176.00</td>
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<tr>
<td>Small</td>
<td>Minimum</td>
<td>0.00</td>
<td>-1.06</td>
<td>-2.10</td>
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<tr>
<td></td>
<td>Maximum</td>
<td>177.45</td>
<td>6.18</td>
<td>28.49</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>55.99</td>
<td>3.18</td>
<td>2.07</td>
</tr>
<tr>
<td></td>
<td>Sd. Deviation</td>
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<td>1.30</td>
<td>3.24</td>
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<tr>
<td>T-test Sig</td>
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<td>0.00*</td>
<td>0.42</td>
<td>0.00*</td>
</tr>
<tr>
<td>(2-tailed)</td>
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<td>for sample</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>size</td>
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</table>

* the mean difference is significant at the 5% level

Table 12: Descriptive statistics: cost, revenue and profit efficiency of Islamic big and small banks

<table>
<thead>
<tr>
<th>Group</th>
<th>Statistics</th>
<th>Cost Efficiency Ratio</th>
<th>Revenue Efficiency Ratio</th>
<th>Profit Efficiency Ratios</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>CTIR</td>
<td>NIM</td>
<td>ROAA</td>
</tr>
<tr>
<td>Islamic</td>
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<td>34.00</td>
<td>35.00</td>
</tr>
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<td>Big</td>
<td>Minimum</td>
<td>15.70</td>
<td>1.03</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>62.69</td>
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<td>38.16</td>
<td>4.75</td>
<td>3.66</td>
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<tr>
<td></td>
<td>Sd. Deviation</td>
<td>11.95</td>
<td>5.16</td>
<td>3.33</td>
</tr>
<tr>
<td>Islamic</td>
<td>N</td>
<td>59.00</td>
<td>62.00</td>
<td>63.00</td>
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<td>Small</td>
<td>Minimum</td>
<td>9.57</td>
<td>-6.80</td>
<td>-7.23</td>
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<td></td>
<td>Maximum</td>
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<td>47.97</td>
<td>6.89</td>
<td>5.95</td>
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<td>Sd. Deviation</td>
<td>19.10</td>
<td>10.09</td>
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<td>T-test Sig</td>
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</tr>
<tr>
<td>(2-tailed)</td>
<td>After</td>
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</tr>
<tr>
<td></td>
<td>size</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* the mean difference is significant at the 5% level
Table 11 reports the results of comparison of big and small commercial banks. The results lead exactly to the same interpretation as in the case of comparing all big versus all small commercial and Islamic banks. As such, we fail to reject the third null hypothesis. We conclude that big commercial banks are significantly more efficient than small commercial banks. Results in table 12 lead to the rejection of the fourth null hypothesis, thus we can conclude that big Islamic banks are not significantly more efficient than small Islamic banks, since the small Islamic banks show more revenue and profit efficiencies. Need to mention that in this group, the profitability efficiency is based on AROA instead of AROE that was used for the overall group and for the commercial group.

In addition, in all three groups based on size differences (detailed above) although there exist a significant difference in the revenue efficiency, the differences in the mean of Net Interest Margins are small and are offset by either the differences in means of the Cost to Income Ratios or the means of AROA’s or AROE’s, that is why we conclude overall efficiency.

Finally, we check the effect of size on our first hypothesis (commercial banks are more efficient than Islamic banks) by comparing big commercial to big Islamic banks and small commercial to small Islamic banks. Based on the results reported in table 13, we can conclude that big Islamic banks are significantly more revenue and profit (if measured by AROA) efficient than big commercial banks but they are not necessarily more cost efficient since there does not exist a significant difference between the means of CTIR commercial and CITR Islamic at the 5% level of significance. As such, we cannot reject the fifth null hypothesis and stick with the claim that commercial banks
efficiency is better than Islamic banks efficiency as measured by cost, revenue and profit efficiencies.

Table 13: Descriptive statistics: cost, revenue and profit efficiency of commercial big and Islamic big banks

<table>
<thead>
<tr>
<th>Group</th>
<th>Statistics</th>
<th>Cost Efficiency Ratio</th>
<th>Revenue Efficiency Ratio</th>
<th>Profit Efficiency Ratios</th>
</tr>
</thead>
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<tr>
<td></td>
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<td>CTIR</td>
<td>NIM</td>
<td>ROAA</td>
</tr>
<tr>
<td>Commercial Big</td>
<td>N</td>
<td>239.00</td>
<td>239.00</td>
<td>239.00</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
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<td>0.00</td>
<td>-3.65</td>
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<td></td>
<td>Maximum</td>
<td>323.28</td>
<td>6.18</td>
<td>30.18</td>
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<td>Mean</td>
<td>39.92</td>
<td>2.85</td>
<td>2.42</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>24.34</td>
<td>0.90</td>
<td>2.42</td>
</tr>
<tr>
<td>Islamic Big</td>
<td>N</td>
<td>34.00</td>
<td>34.00</td>
<td>35.00</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>15.70</td>
<td>1.03</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>62.69</td>
<td>31.00</td>
<td>18.33</td>
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<td>Mean</td>
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<tr>
<td></td>
<td>Std. Deviation</td>
<td>11.95</td>
<td>5.16</td>
<td>3.33</td>
</tr>
<tr>
<td>T-test Sig (2-tailed)</td>
<td>After adjusting for sample size</td>
<td>0.33</td>
<td>0.00*</td>
<td>0.00*</td>
</tr>
</tbody>
</table>

* the mean difference is significant at the 5% level

The results of table 14 proves that small Islamic banks are indeed more cost, revenue and profit (measured by AROA) efficient than small commercial banks. As a result, we can reject the sixth null hypothesis.
Table 14: Descriptive statistics: cost, revenue and profit efficiency of commercial small and Islamic small banks

<table>
<thead>
<tr>
<th>Group</th>
<th>Statistics</th>
<th>Cost Efficiency Ratio</th>
<th>Revenue Efficiency Ratio</th>
<th>Profit Efficiency Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CTIR</td>
<td>NM</td>
<td>ROAA</td>
</tr>
<tr>
<td>Commercial</td>
<td>N</td>
<td>176.00</td>
<td>176.00</td>
<td>176.00</td>
</tr>
<tr>
<td>Small</td>
<td>Minimum</td>
<td>0.00</td>
<td>-1.06</td>
<td>-2.10</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>177.45</td>
<td>6.18</td>
<td>28.49</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>55.99</td>
<td>3.18</td>
<td>2.07</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>25.67</td>
<td>1.30</td>
<td>3.24</td>
</tr>
<tr>
<td>Islamic</td>
<td>N</td>
<td>59.00</td>
<td>62.00</td>
<td>63.00</td>
</tr>
<tr>
<td>Small</td>
<td>Minimum</td>
<td>9.57</td>
<td>-6.80</td>
<td>-7.23</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>100.91</td>
<td>64.00</td>
<td>53.09</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>47.97</td>
<td>6.89</td>
<td>5.95</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>19.10</td>
<td>10.09</td>
<td>9.64</td>
</tr>
<tr>
<td>T-test Sig</td>
<td>After</td>
<td>0.00*</td>
<td>0.00*</td>
<td>0.00*</td>
</tr>
<tr>
<td>(2-tailed)</td>
<td>adjusting</td>
<td>for sample size</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* the mean difference is significant at the 5% level

C. Efficiency by Country: Commercial versus Islamic Banks

To further investigate the first null hypothesis (The efficiency, measured by the mean cost, mean revenue and mean profit, of commercial banks is significantly better than that of their Islamic counterparts), we perform the test in five Arab countries: Bahrain, Jordan, Qatar, UAE and Yemen because of the availability of data.
a) Bahrain case:

The results of the paired sample t-test shown in table 15 reveal that Islamic banks are only more revenue efficient than commercial banks in Bahrain. As such, we fail to reject the first null hypothesis and conclude that the efficiency, measured by the mean cost, mean revenue and mean profit, of commercial banks in Bahrain is significantly better than that of their Islamic counterparts.

Table 15: Descriptive statistics: cost, revenue and profit efficiency of commercial and Islamic banks in Bahrain

<table>
<thead>
<tr>
<th>Group</th>
<th>Statistics</th>
<th>Cost Efficiency Ratio</th>
<th>Revenue Efficiency Ratio</th>
<th>Profit Efficiency Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CTIR</td>
<td>NIM</td>
<td>ROAA</td>
</tr>
<tr>
<td>Bahrain</td>
<td>N</td>
<td>45.00</td>
<td>45.00</td>
<td>45.00</td>
</tr>
<tr>
<td>Commercial</td>
<td>Minimum</td>
<td>0.00</td>
<td>-1.06</td>
<td>-3.65</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>323.28</td>
<td>4.07</td>
<td>30.18</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>54.26</td>
<td>1.87</td>
<td>4.40</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>46.17</td>
<td>1.16</td>
<td>6.84</td>
</tr>
<tr>
<td>Bahrain</td>
<td>N</td>
<td>32.00</td>
<td>33.00</td>
<td>35.00</td>
</tr>
<tr>
<td>Islamic</td>
<td>Minimum</td>
<td>9.57</td>
<td>-6.80</td>
<td>-7.23</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>94.89</td>
<td>64.00</td>
<td>53.09</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>48.40</td>
<td>10.61</td>
<td>6.89</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>18.38</td>
<td>13.19</td>
<td>9.81</td>
</tr>
<tr>
<td>T-test Sig (2-tailed)</td>
<td>After adjusting for sample size</td>
<td>0.49</td>
<td>0.00*</td>
<td>0.38</td>
</tr>
</tbody>
</table>

* the mean difference is significant at the 5% level
b) Jordan case:

Based on the paired samples t-test results shown in table 16, we infer that Islamic banks in Jordan are less revenue and profit efficient than commercial banks. However, they are not significantly less cost efficient, as such we fail to reject the first null hypothesis and conclude that the efficiency, measured by the mean cost, mean revenue and mean profit, of commercial banks in Jordan is significantly better than that of their Islamic counterparts.

Table 16: Descriptive statistics: cost, revenue and profit efficiency of commercial and Islamic banks in Jordan

<table>
<thead>
<tr>
<th>Group</th>
<th>Cost Efficiency Ratio</th>
<th>Revenue Efficiency Ratio</th>
<th>Profit Efficiency Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CTIR</td>
<td>NIM</td>
<td>ROAA</td>
</tr>
<tr>
<td>Jordan</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commercial</td>
<td>N</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>22.45</td>
<td>1.80</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>105.32</td>
<td>5.19</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>48.72</td>
<td>3.48</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>14.92</td>
<td>0.79</td>
</tr>
<tr>
<td>Jordan</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Islamic</td>
<td>N</td>
<td>10.00</td>
<td>10.00</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>30.51</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>75.48</td>
<td>5.14</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>47.59</td>
<td>2.38</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>15.76</td>
<td>1.41</td>
</tr>
<tr>
<td>T-test Sig</td>
<td>After adjusting for</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2-tailed)</td>
<td>sample size</td>
<td>0.67</td>
<td>0.00*</td>
</tr>
</tbody>
</table>

* the mean difference is significant at the 5% level
c) Qatar Case:
The results of the paired sample t-test shown in table 17 reveal that Islamic banks are more cost, revenue, and profit efficient than commercial banks in Qatar. As such, we reject the first null hypothesis and conclude that the efficiency, measured by the mean cost, mean revenue and mean profit, of Islamic banks in Qatar is significantly better than that of their commercial counterparts.

Table 17: Descriptive statistics: cost, revenue and profit efficiency of commercial and Islamic banks in Qatar

<table>
<thead>
<tr>
<th>Group</th>
<th>Statistics</th>
<th>Cost Efficiency Ratio</th>
<th>Revenue Efficiency Ratio</th>
<th>Profit Efficiency Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CTIR</td>
<td>NIM</td>
<td>ROAA</td>
</tr>
<tr>
<td>Qatar</td>
<td>N</td>
<td>30.00</td>
<td>30.00</td>
<td>30.00</td>
</tr>
<tr>
<td>Commercial</td>
<td>Minimum</td>
<td>22.72</td>
<td>2.07</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>128.68</td>
<td>4.72</td>
<td>6.04</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>38.37</td>
<td>3.23</td>
<td>2.94</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>18.88</td>
<td>0.58</td>
<td>1.04</td>
</tr>
<tr>
<td>Qatar</td>
<td>N</td>
<td>10.00</td>
<td>10.00</td>
<td>10.00</td>
</tr>
<tr>
<td>Islamic</td>
<td>Minimum</td>
<td>14.30</td>
<td>3.01</td>
<td>1.85</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>40.72</td>
<td>9.72</td>
<td>8.43</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>25.39</td>
<td>5.52</td>
<td>5.22</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>10.07</td>
<td>1.79</td>
<td>2.41</td>
</tr>
<tr>
<td>T-test Sig</td>
<td>After</td>
<td>0.00*</td>
<td>0.00*</td>
<td>0.00*</td>
</tr>
<tr>
<td>(2-tailed)</td>
<td>adjusting for sample size</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* the mean difference is significant at the 5% level
d) UAE Case:
The results of the paired sample t-test shown in table 18 reveal that Islamic banks are significantly less cost efficient than commercial banks in UAE. At the revenue and profit levels, there is no significant difference. As such we fail to reject the first null hypothesis and conclude that the efficiency, measured by the mean cost, mean revenue and mean profit, of commercial banks in UAE is significantly better than that of their Islamic counterparts.

Table 18: Descriptive statistics: cost, revenue and profit efficiency of commercial and Islamic banks in UAE

<table>
<thead>
<tr>
<th>Group</th>
<th>Statistics</th>
<th>Cost Efficiency Ratio</th>
<th>Revenue Efficiency Ratio</th>
<th>Profit Efficiency Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>UAE</td>
<td>N</td>
<td>75.00</td>
<td>75.00</td>
<td>75.00</td>
</tr>
<tr>
<td>Commercial</td>
<td>Minimum</td>
<td>9.77</td>
<td>1.59</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>96.40</td>
<td>6.18</td>
<td>13.15</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>32.87</td>
<td>3.35</td>
<td>3.23</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>11.61</td>
<td>1.10</td>
<td>1.66</td>
</tr>
<tr>
<td>UAE</td>
<td>N</td>
<td>19.00</td>
<td>19.00</td>
<td>19.00</td>
</tr>
<tr>
<td>Islamic</td>
<td>Minimum</td>
<td>20.49</td>
<td>1.03</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>62.69</td>
<td>5.36</td>
<td>35.10</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>40.72</td>
<td>2.95</td>
<td>4.70</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>9.95</td>
<td>1.04</td>
<td>7.54</td>
</tr>
<tr>
<td>T-test Sig</td>
<td>After</td>
<td>0.00*</td>
<td>0.33</td>
<td>0.14</td>
</tr>
<tr>
<td>(2-tailed)</td>
<td>adjusting for sample size</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The mean difference is significant at the 5% level
e) Yemen Case:
The results of the paired sample t-test shown in table 19 reveal that Islamic banks are significantly less cost, revenue and profit efficient than commercial banks in Yemen. As such, we fail to reject the first null hypothesis and conclude that the efficiency, measured by the mean cost, mean revenue and mean profit, of commercial banks in Yemen is significantly better than that of their Islamic counterparts.

Table 19: Descriptive statistics: cost, revenue and profit efficiency of commercial and Islamic banks in Yemen

<table>
<thead>
<tr>
<th>Group</th>
<th>Cost Efficiency Ratio</th>
<th>Revenue Efficiency Ratio</th>
<th>Profit Efficiency Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Statistics</td>
<td>CTIR</td>
<td>NIM</td>
</tr>
<tr>
<td>Yemen Commercial</td>
<td>N</td>
<td>15.00</td>
<td>15.00</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>19.07</td>
<td>3.16</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>55.71</td>
<td>6.18</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>35.22</td>
<td>4.19</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>13.22</td>
<td>0.85</td>
</tr>
<tr>
<td>Yemen Islamic</td>
<td>N</td>
<td>15.00</td>
<td>14.00</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>35.79</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>100.91</td>
<td>4.11</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>56.48</td>
<td>2.49</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>17.56</td>
<td>1.05</td>
</tr>
<tr>
<td>T-test Sig (2-tailed)</td>
<td>After adjusting for sample size</td>
<td>0.00*</td>
<td>0.00*</td>
</tr>
</tbody>
</table>

* the mean difference is significant at the 5% level

Out of the five countries that we consider, the only results that led to the same conclusion as that of the overall cross sectional data are that of Qatar. This discrepancy in the results, between country level and regional level, is probably due to the underperformance of some commercial banks, for instance the Lebanese commercial banks when compared to the better established and more efficient GULF commercial banks. Indeed table 20 shows the results of the paired t-test of all commercial banks without Lebanon versus all Islamic
banks. While Islamic banks are significantly more revenue and profit efficient than commercial banks without Lebanon, they are significantly less cost efficient. As such, we fail to reject the first null hypothesis and conclude that the efficiency, measured by the mean cost, mean revenue and mean profit, of all commercial banks without Lebanon is significantly better than that of all Islamic banks. When commercial banks of Lebanon are kept out, the efficiency of all commercial banks becomes better than that of all Islamic banks. The efficiency of Lebanese commercial banks versus the GCC commercial banks remains an issue to be further investigated.

Table 20: Descriptive statistics: cost, revenue and profit efficiency of all commercial without Lebanon and all Islamic banks

<table>
<thead>
<tr>
<th>Group</th>
<th>Statistics</th>
<th>Cost Efficiency Ratio</th>
<th>Revenue Efficiency Ratio</th>
<th>Profit Efficiency Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CTIR</td>
<td>NIM</td>
<td>ROAA</td>
</tr>
<tr>
<td>Commercial Banks</td>
<td>N</td>
<td>293.00</td>
<td>293.00</td>
<td>293.00</td>
</tr>
<tr>
<td>without Lebanon</td>
<td>Minimum</td>
<td>0.00</td>
<td>-1.06</td>
<td>-3.65</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>323.28</td>
<td>6.18</td>
<td>30.18</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>39.06</td>
<td>3.11</td>
<td>2.96</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>23.73</td>
<td>1.10</td>
<td>3.06</td>
</tr>
<tr>
<td>Islamic Banks</td>
<td>N</td>
<td>93.00</td>
<td>96.00</td>
<td>98.00</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>9.57</td>
<td>-6.80</td>
<td>-7.23</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>100.91</td>
<td>64.00</td>
<td>53.09</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>44.38</td>
<td>6.13</td>
<td>5.13</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>17.43</td>
<td>8.69</td>
<td>8.03</td>
</tr>
<tr>
<td>T-test Sig (2-tailed)</td>
<td>After adjusting for sample size</td>
<td>0.00*</td>
<td>0.00*</td>
<td>0.00*</td>
</tr>
</tbody>
</table>

* the mean difference is significant at the 5% level
D. Semi-log Regression Results:
The following semi-log regressions depict the significant effects of total assets, off-balance sheet, loan loss reserves, cost to income ratio, equity to total assets, and net loans to total assets on the Average Return on Assets (AROA) of commercial and Islamic banks.

a) All commercial banks semi-log regressions:
   1. \[ \text{AROA} = \alpha + \beta_1 \ln(\text{Total Assets}) + \beta_2 \ln(\text{off-balance sheet}) + \beta_3 \ln(\text{loan loss reserve}) + \beta_4 (\text{Cost to income ratio}) + \beta_5 (\text{equity to total assets}) + \beta_6 (\text{net loans to total assets}) + \epsilon_i \]

Table 21: All commercial banks semi-log regression 1 results

<table>
<thead>
<tr>
<th>Model</th>
<th>R</th>
<th>R Square</th>
<th>Adjusted R Square</th>
<th>Std. Error of the Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.745*</td>
<td>.555</td>
<td>.547</td>
<td>1.060412</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>509.477</td>
<td>6</td>
<td>84.913</td>
<td>75.513</td>
<td>.000*</td>
</tr>
<tr>
<td>Residual</td>
<td>409.309</td>
<td>364</td>
<td>1.124</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>918.786</td>
<td>370</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>Unstandardized Coefficients</th>
<th>Standardized Coefficients</th>
<th>t</th>
<th>Sig.</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>Std. Error</td>
<td>Beta</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Constant)</td>
<td>1.002</td>
<td>.436</td>
<td>.2302</td>
<td>.022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LnTotalAssets</td>
<td>.176</td>
<td>.086</td>
<td>.170</td>
<td>2.034</td>
<td>.043</td>
<td>.006</td>
</tr>
<tr>
<td>LnOffBalanceSheet</td>
<td>-.110</td>
<td>.065</td>
<td>-.150</td>
<td>-1.710</td>
<td>.088</td>
<td></td>
</tr>
<tr>
<td>LnLoanLossReserve</td>
<td>-.030</td>
<td>.072</td>
<td>-.027</td>
<td>-.421</td>
<td>.674</td>
<td></td>
</tr>
<tr>
<td>CostToIncomeRatio</td>
<td>-.030</td>
<td>.003</td>
<td>-.457</td>
<td>-11.507</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>EquityOverTotalAssets</td>
<td>.042</td>
<td>.005</td>
<td>.277</td>
<td>7.599</td>
<td>.000</td>
<td>.031</td>
</tr>
<tr>
<td>NetLoansOverTotalAssets</td>
<td>.031</td>
<td>.004</td>
<td>.361</td>
<td>7.108</td>
<td>.000</td>
<td>.023</td>
</tr>
</tbody>
</table>

\* Semi Log regression: a regression in which some of the variables in the model are in logarithmic form.
The overall model is significant at the 5% confidence level (F-test) and 55.5% (R Square) of the variation in the AROA of commercial banks is explained by the semi-log regression equation. However, only Off Balance Sheet, and Loan Loss Reserves are not significantly, at the 5% confidence level, related to AROA. Total assets, equity over total assets and net loans over total assets are all positively significantly related to AROA at the 5% confidence level. On the other hand, Cost to income ratio is negatively significantly related to AROA at the 5% confidence level.

2. AROA = \( \alpha + \beta_1 \ln(\text{Total Assets}) + \beta_2 \ln(\text{off balance sheet}) + \beta_3 \ln(\text{loan loss reserve}) + \beta_4 (\text{Cost to income ratio}) + \beta_5 (\text{equity to total assets}) + \beta_6 (\text{net loans to total assets}) + \beta_7 \ln (\text{loan loss provisions}) + \xi \)

Table 22: All commercial banks semi-log regression 2 results

<table>
<thead>
<tr>
<th>Model Summary</th>
<th>R</th>
<th>R Squared Adjusted R Squared Std. Error of the Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.771†</td>
<td>.594</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ANOVA²</th>
<th>Model</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Regression</td>
<td>482.087</td>
<td>7</td>
<td>68.870</td>
<td>62.266</td>
<td>.000*</td>
</tr>
<tr>
<td></td>
<td>Residual</td>
<td>329.605</td>
<td>298</td>
<td>1.106</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>811.692</td>
<td>305</td>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Coefficients²</th>
<th>Model</th>
<th>Unstandardized Coefficients</th>
<th>Standardized Coefficients</th>
<th>t</th>
<th>Sig.</th>
<th>95.0% Confidence Interval for B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lower Bound</td>
</tr>
<tr>
<td>(Constant)</td>
<td>1</td>
<td>2.220</td>
<td>.545</td>
<td>4.077</td>
<td>.000</td>
<td>1.148</td>
</tr>
<tr>
<td>LnTotalAssets</td>
<td>2</td>
<td>.262</td>
<td>.099</td>
<td>.242</td>
<td>2.642</td>
<td>.009</td>
</tr>
<tr>
<td>LnOffBalanceSheet</td>
<td>3</td>
<td>-.214</td>
<td>.074</td>
<td>-.269</td>
<td>-2.868</td>
<td>.004</td>
</tr>
<tr>
<td>LnLoanLossReserve</td>
<td>4</td>
<td>.009</td>
<td>.100</td>
<td>.008</td>
<td>.093</td>
<td>.926</td>
</tr>
<tr>
<td>CostToIncomeRatio</td>
<td>5</td>
<td>-.055</td>
<td>.005</td>
<td>-.572</td>
<td>-11.798</td>
<td>.000</td>
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<tr>
<td>EquityOverTotalAssets</td>
<td>6</td>
<td>.037</td>
<td>.006</td>
<td>.230</td>
<td>5.823</td>
<td>.000</td>
</tr>
<tr>
<td>NetLoansOverTotalAssets</td>
<td>7</td>
<td>.030</td>
<td>.005</td>
<td>.340</td>
<td>6.302</td>
<td>.000</td>
</tr>
<tr>
<td>LnLoanLossProvisions</td>
<td>8</td>
<td>-.130</td>
<td>.065</td>
<td>-.125</td>
<td>-1.987</td>
<td>.048</td>
</tr>
</tbody>
</table>

44
The overall model is significant at the 5% confidence level (F-test) and 59.4% (R Square) of the variation in the AROA of commercial banks is explained by the semi-log regression equation. However, only loan loss reserve is not significantly, at the 5% confidence level, related to AROA. Total assets, equity over total assets, and net loans over total assets are all positively significantly related to AROA of commercial banks at the 5% confidence level. On the other hand, off balance sheet, cost to income ratio, and loan loss provisions are all negatively significantly related to AROA of commercial banks. Need to mention that the second regression (including Loan Loss Provisions) is a better predictor of the variation of commercial banks AROA since its R Square (0.594) is higher than that of regression one (without Loan Loss Provisions).
b) All Islamic banks semi-log regressions:

1. AROA = \( \alpha + \beta_1 \ln(\text{Total Assets}) + \beta_2 \ln(\text{off balance sheet}) + \beta_3 \ln(\text{loan loss reserve}) + \beta_4 (\text{Cost to income ratio}) + \beta_5 (\text{equity to total assets}) + \beta_6 (\text{net loans to total assets}) + \epsilon_i \)

Table 23: All Islamic banks semi-log regression 1 results

<table>
<thead>
<tr>
<th>Model</th>
<th>R Square</th>
<th>Adjusted R Square</th>
<th>Std. Error of Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.583*</td>
<td>.340</td>
<td>.259</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>91.089</td>
<td>6</td>
<td>15.181</td>
<td>4.201</td>
<td>.002*</td>
</tr>
<tr>
<td>Residual</td>
<td>177.057</td>
<td>49</td>
<td>3.613</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>268.146</td>
<td>55</td>
<td></td>
<td></td>
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<table>
<thead>
<tr>
<th>Model</th>
<th>Unstandardized Coefficients</th>
<th>Standardized Coefficients</th>
<th>t</th>
<th>Sig.</th>
<th>95.0% Confidence Interval for B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unstandardized</td>
<td>Standardized</td>
<td></td>
<td></td>
<td>Lower Bound</td>
</tr>
<tr>
<td></td>
<td>Coefficients</td>
<td>Coefficients</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>(Constant)</td>
<td>8.831</td>
<td>2.747</td>
<td>.002</td>
<td>3.311</td>
</tr>
<tr>
<td></td>
<td>lnTotalAssets</td>
<td>-8.03</td>
<td>.473</td>
<td>.096</td>
<td>-1.754</td>
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<tr>
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<td>lnOffBalanceSheet</td>
<td>5.03</td>
<td>.356</td>
<td>.165</td>
<td>-2.14</td>
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<tr>
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<td>lnLoanLossReserves</td>
<td>3.16</td>
<td>.306</td>
<td>.307</td>
<td>-2.99</td>
</tr>
<tr>
<td></td>
<td>CostToIncomeRatio</td>
<td>-0.96</td>
<td>.020</td>
<td>.000</td>
<td>-1.26</td>
</tr>
<tr>
<td></td>
<td>EquityOverTotalAssets</td>
<td>0.17</td>
<td>.013</td>
<td>.210</td>
<td>-.010</td>
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<tr>
<td></td>
<td>NetLoansOverTotalAssets</td>
<td>0.10</td>
<td>.019</td>
<td>.580</td>
<td>-0.048</td>
</tr>
</tbody>
</table>

The overall model is significant at the 5% confidence level (F-test) and 34% (R Square) of the variation in the AROA of Islamic banks is explained by the semi-log regression equation. However, only Cost to Income ratio is negatively significantly, at the 5% confidence level, related to AROA.
2. AROA = α + β1 ln(Total Assets) + β2 ln(off-balance sheet) + β3 ln(loan loss reserve) + β4 (Cost to income ratio) + β5 (equity to total assets) + β6 (net loans to total assets) + B7 ln (loan loss provisions) + εi

Table 24: All Islamic banks semi-log regression 2 results

<table>
<thead>
<tr>
<th>Model</th>
<th>R</th>
<th>R Square</th>
<th>Adjusted R Square</th>
<th>Std. Error of the Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.820⁰</td>
<td>.673</td>
<td>.606</td>
<td>1.30931</td>
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</table>

ANOVA

<table>
<thead>
<tr>
<th>Model</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>120.059</td>
<td>7</td>
<td>17.151</td>
<td>10.005</td>
<td>.000⁰</td>
</tr>
<tr>
<td>Residual</td>
<td>58.286</td>
<td>34</td>
<td>1.714</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>178.345</td>
<td>41</td>
<td></td>
<td></td>
<td></td>
</tr>
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</table>

Coefficients

<table>
<thead>
<tr>
<th>Model</th>
<th>Unstandardized Coefficients</th>
<th>Standardized Coefficients</th>
<th>t</th>
<th>Sig.</th>
<th>95.0% Confidence Interval for B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>Std. Error</td>
<td>Beta</td>
<td></td>
<td>Lower Bound</td>
</tr>
<tr>
<td>1</td>
<td>(Constant)</td>
<td>5.436</td>
<td>2.558</td>
<td>2.125</td>
<td>.041</td>
</tr>
<tr>
<td></td>
<td>LnTotalAssets</td>
<td>-.017</td>
<td>.433</td>
<td>-0.11</td>
<td>.039</td>
</tr>
<tr>
<td></td>
<td>LnOffBalanceSheet</td>
<td>.682</td>
<td>.316</td>
<td>.340</td>
<td>2.162</td>
</tr>
<tr>
<td></td>
<td>LnLoanLossReserves</td>
<td>-.130</td>
<td>.278</td>
<td>-1.16</td>
<td>-.468</td>
</tr>
<tr>
<td></td>
<td>CostToIncomeRatio</td>
<td>-.115</td>
<td>.017</td>
<td>-0.86</td>
<td>-6.314</td>
</tr>
<tr>
<td></td>
<td>EquityOverTotalAssets</td>
<td>.008</td>
<td>.011</td>
<td>.085</td>
<td>7.01</td>
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<tr>
<td></td>
<td>NetLoansOverTotalAssets</td>
<td>-.022</td>
<td>.018</td>
<td>-.173</td>
<td>-1.202</td>
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<tr>
<td></td>
<td>LnLoanLossProvisions</td>
<td>-.335</td>
<td>.160</td>
<td>-.287</td>
<td>-.2101</td>
</tr>
</tbody>
</table>

The overall model is significant at the 5% confidence level (F-test) and 67.3% (R Square) of the variation in the AROA of Islamic banks is explained by the semi-log regression equation. However, only off-balance sheet is positively significantly related to AROA at the 5% significance level, and cost to income ratio and loan loss provisions are negatively
significantly related to AROA of Islamic banks at the 5% confidence level. Need to mention that the second regression (including Loan Loss Provisions) is a better predictor of the variation of Islamic banks AROA since its R Square (0.673) is higher than that of regression one (without Loan Loss Provisions).

In commercial and Islamic banks cases, the second regression (including Loan Loss Provisions) is a better predictor (having higher R-Squares) of the variation in AROA. Table 25 compares the independent variables that are significantly related to AROA of commercial and Islamic banks based on regression 2 of both types.

Table 25: Comparison of the independent variables that are significantly related to AROA of commercial and Islamic banks based on regression 2.

<table>
<thead>
<tr>
<th></th>
<th>Commercial AROA</th>
<th>Islamic AROA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Assets</td>
<td>Positively</td>
<td>Positively</td>
</tr>
<tr>
<td></td>
<td>Significantly related</td>
<td>Significantly related</td>
</tr>
<tr>
<td>Off Balance Sheet</td>
<td>Negatively</td>
<td>Positively</td>
</tr>
<tr>
<td></td>
<td>related</td>
<td>Significantly related</td>
</tr>
<tr>
<td>Loan Loss Reserves</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loan Loss Provisions</td>
<td>Negatively</td>
<td>Negatively</td>
</tr>
<tr>
<td></td>
<td>related</td>
<td>Significantly related</td>
</tr>
<tr>
<td>Cost to Income Ratio</td>
<td>Negatively</td>
<td>Negatively</td>
</tr>
<tr>
<td></td>
<td>related</td>
<td>Significantly related</td>
</tr>
<tr>
<td>Equity over Total Assets</td>
<td>Positively</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Significantly related</td>
<td></td>
</tr>
<tr>
<td>Net Loan over Total Assets</td>
<td>Positively</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Significantly related</td>
<td></td>
</tr>
</tbody>
</table>
VII. Conclusion

In this project, we compare the efficiency of commercial and Islamic banks in nine Arab countries. We include non-Muslim countries for two reasons: first because non-Muslim countries are hosting Islamic banking activities through well developed Islamic banks or through the Islamic windows of big commercial banks; and second because Islamic banking is not restricted to Muslim, indeed it is open to any socially responsible investors or depositor.

We find out that our sample of 20 Islamic banks is significantly more efficient than our sample of 83 commercial banks when comparing their relative mean cost, mean revenue and mean profit efficiencies. This finding is held constant when we compare the efficiency of small Islamic and small commercial banks, but it fails when we compare big Islamic and big commercial banks because we cannot significantly claim that there exists a difference in the means of the Cost to Income Ratios of big Islamic and big commercial banks. This finding also fails on the country level, where we compare performance of commercial and Islamic banks in Bahrain, Jordan, Qatar, UAE and Yemen. The only results that are similar to the overall cross sectional data are that of Qatar.

We also find that size does matter in both commercial and Islamic banking industries. First we compare big commercial and big Islamic banks (grouped together) to small commercial and small Islamic banks (grouped together) and find that the big banks outperformed the small banks in cost, and profit but failed to do so in revenue efficiency. A closer look at the cost, revenue and profit figures led us to conclude that big banks are
more efficient, but this result does not hold in both banking systems. Indeed, while we find that big commercial banks are significantly more efficient than small commercial banks, we fail to claim that big Islamic banks are significantly more efficient than small Islamic banks.

Through a semi-log regression analysis we find that the AROA of commercial banks is significantly affected by six independent variables while the AROA of Islamic banks is only significantly affected by three independent variables from the same pool of independent variables that we consider for both types of banks. The three common independent variables that affect both types of banks are: off balance sheet, loan loss provisions and cost to income ratio. While loan loss provisions and cost to income ratio negatively significantly affect the AROA of both types of banks, off balance sheet affects negatively significantly the AROA of commercial banks and positively significantly the AROA of Islamic banks.

These results are representative of our samples and could not be generalized to cover the overall performance of commercial and Islamic banks. Further research could use the same or different methodologies and or proxies for cost, revenue and profit efficiencies to test the same hypotheses in different regions.
References


Appendix 1: List of Commercial and Islamic Banks by Country (used in the study)

<table>
<thead>
<tr>
<th>Bank name</th>
<th>Country</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahli United Bank BSC</td>
<td>BAHRAIN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Arab Banking Corporation BSC</td>
<td>BAHRAIN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Bahrain Financing Company B.S.C.</td>
<td>BAHRAIN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Bahraini Saudi Bank (The) BSC</td>
<td>BAHRAIN</td>
<td>Commercial</td>
</tr>
<tr>
<td>BBK B.S.C.</td>
<td>BAHRAIN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Gulf International Bank BSC</td>
<td>BAHRAIN</td>
<td>Commercial</td>
</tr>
<tr>
<td>International Banking Corporation BSC</td>
<td>BAHRAIN</td>
<td>Commercial</td>
</tr>
<tr>
<td>National Bank of Bahrain</td>
<td>BAHRAIN</td>
<td>Commercial</td>
</tr>
<tr>
<td>TAIB Bank B.S.C.</td>
<td>BAHRAIN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Arab Bank Group</td>
<td>JORDAN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Arab Bank Plc</td>
<td>JORDAN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Arab Banking Corporation (Jordan)</td>
<td>JORDAN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Bank of Jordan Plc</td>
<td>JORDAN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Cairo Amman Bank</td>
<td>JORDAN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Capital Bank of Jordan</td>
<td>JORDAN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Housing Bank for Trade &amp; Finance (The)</td>
<td>JORDAN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Jordan Ahli Bank Plc</td>
<td>JORDAN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Jordan Commercial Bank</td>
<td>JORDAN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Jordan Kuwait Bank</td>
<td>JORDAN</td>
<td>Commercial</td>
</tr>
<tr>
<td>Bank of Kuwait &amp; The Middle East (The)</td>
<td>KUWAIT</td>
<td>Commercial</td>
</tr>
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<td>Burgan Bank SAK</td>
<td>KUWAIT</td>
<td>Commercial</td>
</tr>
<tr>
<td>Commercial Bank of Kuwait SAK (The)</td>
<td>KUWAIT</td>
<td>Commercial</td>
</tr>
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<td>Gulf Bank KSC (The)</td>
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<td>Commercial</td>
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<td>KUWAIT</td>
<td>Commercial</td>
</tr>
<tr>
<td>Audi Saradar Private Bank SAL</td>
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<td>B.L.C. Bank S.A.L</td>
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<td>Commercial</td>
</tr>
<tr>
<td>Bank Audi SAL - Audi Saradar Group</td>
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<td>Commercial</td>
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<tr>
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<td>Commercial</td>
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<td>Bank of Kuwait &amp; The Arab World SAL</td>
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<td>Commercial</td>
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<td>Bankmed, sal</td>
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</tr>
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</tr>
<tr>
<td>Banque de l'Industrie et du Travail SAL</td>
<td>LEBANON</td>
<td>Commercial</td>
</tr>
<tr>
<td>Banque Libano-Francaise</td>
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<td>Banque Misr Liban</td>
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</tr>
<tr>
<td>Banque Pharaon &amp; Chiha SAL</td>
<td>LEBANON</td>
<td>Commercial</td>
</tr>
<tr>
<td>Bank Name</td>
<td>Country</td>
<td>Type</td>
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<tr>
<td>-----------------------------------------------</td>
<td>--------------</td>
<td>---------------</td>
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<tr>
<td>BBAC sal</td>
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<tr>
<td>BLOM Bank s.a.l.</td>
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<td>Byblos Bank S.A.L.</td>
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<td>Crédit Libanais S.A.L.</td>
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<td>First National Bank SAL</td>
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</tr>
<tr>
<td>Fransabank sal</td>
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</tr>
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<td>IBL Bank sal</td>
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<td>Lebanese Swiss Bank SAL (The)</td>
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<td>Commercial</td>
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<td>Lebanon &amp; Gulf Bank S.A.L.</td>
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<td>Commercial</td>
</tr>
<tr>
<td>MEAB SAL</td>
<td>LEBANON</td>
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</tr>
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<td>Near East Commercial Bank SAL</td>
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<td>North Africa Commercial Bank SAL</td>
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<td>Commercial</td>
</tr>
<tr>
<td>Ahli Bank QSC</td>
<td>QATAR</td>
<td>Commercial</td>
</tr>
<tr>
<td>Commercial Bank of Qatar (The) QSC</td>
<td>QATAR</td>
<td>Commercial</td>
</tr>
<tr>
<td>Doha Bank</td>
<td>QATAR</td>
<td>Commercial</td>
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<td>International Bank of Qatar Q.S.C.</td>
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<td>Commercial</td>
</tr>
<tr>
<td>Qatar Development Bank Q.S.C.C.</td>
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<td>Bank Al-Jazira</td>
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<td>Banque Saudi Fransi</td>
<td>SAUDI ARABIA</td>
<td>Commercial</td>
</tr>
<tr>
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