Exploring the Effect of Using Mathematical Group Discussions on Seventh Graders’ Problem Solving Abilities in Geometry

by

FATEN HUMEIDAN

B.S., Math, Lebanese University, 1992

Thesis submitted in partial fulfillment of the requirements for the Degree of Master of Education/ Emphasis Math Education

Department of Education
LEBANESE AMERICAN UNIVERSITY

October 2009
Thesis approval Form

Student Name:  Faten Hemeidan

I.D. #: 200202758

Thesis Title: Exploring the Effect of Using Mathematical Group Discussions on Seventh Graders' Problem Solving Abilities in Geometry

Program: MA in Education, Emphasis Math Education

Department: Education

School: School of Arts and Sciences

Approved by:

Thesis Advisor: Dr. Iman Osta, Associate Professor

Committee Member: Dr. Rima Bahous, Associate Professor

Committee Member: Dr. Ahmad Quein, Associate Professor

Date: October 5, 2009
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ACKNOWLEDGMENTS

I would like to express my gratitude to the committee members, Dr. I. Osta for her constant support, encouragement, and guidance throughout my thesis work; Dr. Oueini and Dr. Bahous for their help and for being committee members for this thesis.

I would like to dedicate this to my caring family especially my mom for her support, and encouragement during my work; and to my loving husband for his patience, understanding, support and encouragement. Thanks for helping me through this way. In addition, I would like to dedicate this work to my beloved son Ram.
ABSTRACT

Problem solving is a major standard in most modern mathematics curricula. Problem solving should not be taught as an isolated part of the mathematics program. The purpose of the study is to explore the types of questions used by the teacher in a math class and to investigate the effect of using mathematical group discussions on seventh graders' problem solving abilities in geometry. The study is a qualitative research analyzing the processes and the effect of an intervention in an intermediate-level class (grades 7-8). This study was implemented in a school located in Beirut, Lebanon. The participants are the students of grade 7 class and their math teacher. The class consists of seven females and ten males. The age of the students ranges between 12 and 13 years. Students have different levels of academic achievement in mathematics. The study extended over two academic years. During the second year the same students were in grade 8. Data were collected through: 1) class observations in grade 7 and grade 8 to investigate the teacher's types of questions and method of teaching in a problem solving situation prior to intervention, 2) interview with the teacher to obtain data about his perception of using mathematical group discussions and his belief about the possible positive effects of mathematical discussions on students' problem solving abilities, 3) clinical focus group interviews with the students in grade 7 to explore their initial problem solving abilities, 4) class observations in grade 8 to investigate students' mathematical discussions during intervention, and 5) clinical focus group interviews with the students in grade 8 to investigate students' final state of problem solving abilities. Results of the study showed that teacher's questions are mostly probing questions to assess students' mathematical knowledge. They also showed many effects of using mathematical group discussions on the interactions among students in class and on students' problem solving abilities. Interactions among students increased and shifted to higher levels of thinking and argumentation, metacognitive processes increased, and students' problem solving abilities improved.
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INTRODUCTION

Discussions of mathematical concepts provide opportunities for students to reason and articulate their mathematical thinking, refute other students' ideas, prove their conjectures to one another, and discuss their problem solving strategies using mathematical language. Language is considered to be so important in learning that researchers name it "the key for cognitive development" (Berliner & Gage, 1998, p. 109). The teacher should choose "worthwhile mathematical tasks" to motivate students to communicate mathematically (Lee, 2006, p.77; NCTM, 1991, p.25) and to provide students with chances to "solidify and extend what they know" (NCTM, 2000, p.52). The math teacher thus has an important role that allows getting benefits from mathematical discussions. The teacher should orchestrate discourse by posing questions that elicit responses to reflect students' thinking (NCTM, 1991; NCTM, 2000).

Many teachers in traditional classrooms apply group discussions in problem solving sessions just to ask students about their correct answers. Apparently, they don't tackle wrong solutions and discuss it with their students. As teachers, we should give time for students to solve problems and have the chance to discuss and share their solutions with their peers. Also, we need to engage our students, whether high or low achievers, in solving tasks that need extended effort (Bushman, 2003; NCTM, 1989). As a result, problem solving should be an integral part of the instructional program from lower grades to upper grades (NCTM, 2000). Additionally, when teachers trust in students' problem solving abilities, then the students trust themselves and develop self-confidence to become successful problem solvers (Bushman, 2003).
Purpose and Research Questions

The purpose of the study is to explore the types of questions used by the teacher in a math class and to investigate the effect of using mathematical group discussions on seventh graders' problem solving abilities in geometry. Small-group work is very essential in talking and learning mathematics. For this reason, in this study, students worked in small groups. The analysis will investigate the type of questions the teacher used. Moreover, the study investigates the cognitive processes that are enhanced by mathematical discussions, the type of interactions among students, the strategies used to formulate arguments and to convince others, and whether mathematical group discussions enhance problem solving abilities.

The study intends to tackle the following questions:

1. What types of questions does the teacher mostly use in interaction with students in a math class?

2. What are the problem-solving cognitive processes that are enhanced by the use of mathematical discussions?

3. What types of interactions do students use during mathematical discussions?

4. What strategies do students use to formulate arguments and to convince each other?

5. Do mathematical group discussions enhance problem solving abilities?

Rationale and Significance of the Study

Problem solving is a major standard in most modern mathematics curricula. Many math teachers emphasize in their teaching fluency in computation and manipulation of symbols leaving no room for problem solving. With the latest
technologies such as the calculators and computers, teachers can shift the importance from performing calculations to understanding of concepts and finding mathematical relationships. NCTM (2000) describes problem solving as a "hallmark of mathematical activity and a major means of developing mathematical knowledge" (p.116).

Problem solving situations require students to draw on their prior knowledge and to connect mathematical concepts, procedures, reasoning, and communication skills to solve a problem. Many researchers report that using group discussions encourages students to become active learners and critical thinkers (Tanner & Casados, 1998). However, many teachers use classroom discussions just to hear right or wrong answers; they do not ask their students to justify their answers. Moreover, discussions in traditional classrooms usually follow a sequence in which the teacher initiates a question, the students reply, and then the teacher evaluates the answers; this is known as IRE sequence (Mehan, 1979; as cited in Sherin, 2002) Teachers can start discussions by asking open-ended questions to stimulate students' mathematical thinking. Solutions of open-ended questions provide insight into students' mathematical ideas, cognitive processes, and problem-solving strategies and approaches they use.

The Lebanese curriculum emphasizes the fact that communication plays an important role in teaching mathematics. Communication is considered as one of the seven mathematical objectives at the intermediate level. Under "Communication", it is stated that students should be able to read and communicate using the mathematical language, as well as to present their work orally or in writing "with particular care to writing a proof" (Center for Educational Research and Development (CERD), 1997, p.302). Also, the Lebanese curriculum stresses the important role that problem solving
plays in teaching mathematics. It is stated that students should be able to "analyze a situation and deduce the relevant elements", and "to choose a strategy to find the solution" (p.302). Students articulate their mathematical thinking using their language in lower grades. Later, they use the formal mathematical language in the higher grades. Many students face difficulties expressing their mathematical thinking using mathematical language. Teachers can help students overcome this difficulty by building "discourse communities" (Sherin, Louis, & Mendez, 2000, p. 190) from lower grades that promote mathematical communication, and enhance students' problem solving abilities.

Hence, this study will explore the effect of using mathematical group discussions on seventh graders' problem solving abilities in geometry.

**Definition of Terms**

This study assumes the following explanations of terms:

1. Cognitive Processes: The cognitive processes in this study are categorized as reading the problem, understanding the problem, analyzing, exploring, planning for the solution, implementing, verification (Artzt & Armour-Thomas, 1992).

2. Mathematical Group Discussions: Students' use of mathematical vocabulary and mathematical symbols in interaction situations, to articulate knowledge about mathematical concepts, to communicate mathematically, and to explain mathematical ideas to their peers or to the teacher.
3. Peer to peer discussions: Students explain mathematical ideas to other students, or try to convince other students about certain relationships between mathematical objects or the suitability of a strategy for solving a problem.

4. Teacher to student discussions. The teacher may initiate the discussion by asking challenging mathematical questions or assigning a mathematical task in a situation of communication that leads the students into a discussion, or she/he may start the lesson with a challenging activity that motivates students to begin the discussion. The teacher can play the role of moderator by asking questions that help students proceed in their discussions.

5. Problem Solving Abilities: Students demonstrate problem solving ability in mathematics when they:

   recognize and formulate problems; determine the consistency of data; use strategies, data, models; generate, extend, and modify procedures; use reasoning in new settings; and judge the reasonableness and correctness of solutions. Problem-solving situations require students to connect all of their mathematical knowledge of concepts, procedures, reasoning, and communication skills to solve a problem (National Center for Education Statistics, 2005).

The following chapter is dedicated to identifying the review of literature: reasoning and proof, teachers and proof, communication and group work, role of
teachers in classroom discussions, difficulties in developing a discourse community, communication and language, and problem solving and metacognition.
REVIEW OF LITERATURE

Chapter two reviews the literature about reasoning and proof. The second part tackles the teachers’ way in approaching proof. The third part highlights the relation between group work and communication and the advantages of organizing students in small groups. The fourth part focuses on the role of the teacher in classroom discussions followed by a discussion about the difficulties that teachers face when developing a discourse community. The sixth part is devoted to the demonstration of the importance of communication and language. The last part is concerned about the significant effect of metacognition on problem solving abilities.

Reasoning and Proof

Mathematical reasoning and proof are considered to be the essence of mathematics. NCTM (2000) acknowledges their importance by considering them as one of the five process standards in mathematics. It states that “mathematical proof is a formal way of expressing particular kinds of reasoning and justifications” (NCTM, 2000, p. 56). However, many research studies have shown that students find the study of proof difficult (Chazan, 1993; Healey & Hoyles, 2000; Hoyles, 1997) because it requires the coordination of many competencies such as identifying assumptions and organizing logical arguments (Hoyles, 1997). Also, they have difficulty in recognizing what a proof is (Chazan, 1993).

Hoyles (1997) found that students: 1) prefer to use an empirical argument and not a deductive one, 2) assume that the deductive proof is evidenced in the diagrams or examples in the text, and 3) have difficulty in identifying premises and following a logical argument from premises to conclusion.
To overcome this difficulty, reasoning and proof can't be taught in isolation from other topics or in a single unit. NCTM (2000) recommends that reasoning and proof should be taught as a natural part of the curriculum and should be integrated in students' mathematical experience from lower grades to upper grades and not as a separate activity (Hanna, 1990; Schoenfeld, 1994). In addition, reasoning mathematically should be developed through consistent use in many contexts. Students can learn to "make, refine, and test conjectures" in elementary grades (NCTM, 2000, p.57). Then, they use "inductive and deductive reasoning to formulate mathematical arguments" in middle grades (NCTM, 2000, p.262).

However, Hanna (1990) differentiates between two kinds of proof: 1) proof that proves or shows "that a theorem is true", and 2) proof that explains or shows "why a theorem is true" (p.9). She recommends that both formal and explanatory proofs can be embedded in the curriculum. However, to shift to the use of explanatory proofs, educators should stress that mathematical understanding is the goal. Hanna (2000) asserts that rigorous proof is secondary to understanding, and the key role of proof in the classroom is to promote mathematical understanding. Proofs are "essential to understanding mathematics" (NCTM, 2000, p.56). As a result, students should be taught formal proof in order to tell whether a result is true or not. However, they should as well have the experience to view and use proofs that "convey understanding" (Hanna, 2000, p.37).

Simpson (1995, as cited in Hanna 2000) distinguishes between "proof through logic" and "proof through reasoning". The former emphasizes the formal proof, whereas the latter involves investigations. He believes that "proof through logic" is
“alien” to students since it has no connection with their “existing mental structures”, and it can be mastered by a minority of students. However, “proof through reasoning” appeals to the learner because it represents heuristic argument; hence, it is accessible to the majority of students. Moreover, he expresses the view that deductive reasoning needs no longer to be taught. He focuses on reasoning and justification and believes that heuristic techniques are useful in developing skills even in justification (NCTM, 1989).

Sfard (2000) describes proof as a part of a discourse practice, with a distinction between discourse with oneself and discourse with others. That is to say, the distinction is between trying to produce a proof and trying to communicate a proof to someone else.

Another distinction has been made between “an essentially public and an essentially private” aspects of proof (Raman, 2003, p. 320). Proof involves both public and private arguments. Raman (2003) stated that a private argument is an “argument which engenders understanding,” whereas a public argument is an “argument with sufficient rigor for a particular mathematical community” (p.320). Students regard the public and private aspects of mathematics differently. Moreover, students’ ability to generate proofs is limited for two reasons: lack of knowledge and insufficient epistemology. They don’t see the important connection between their “privately held idea and what they expect to produce as a formal, public proof” (Raman, 2003, p. 321). On the contrary, Lawson and Chinnappan (2000) believed that the difficulty that students face with geometry is due to their inability to access the geometric available knowledge during memory search. They identified three aspects for failure to access
available knowledge: 1) lack of persistence with the solution, 2) ineffective use of cues, and 3) lack of connected knowledge relevant to the problem.

Proof and proving have many functions such as: “verification, explanation, systematization, discovery, communication, construction, exploration, incorporation” (Hanna, 2000, p.8). Thus, one of the important functions of proof is communication. Students should explain their arguments to the other students or to a teacher and convince themselves of their truth (Hoyles, 1997; NCTM, 2000).

**Teachers and Proof**

As most students have difficulty in producing proofs, research indicates that high school teachers have difficulty in understanding the nature of proof (Knuth, 2002). So, it is important for teachers to understand the nature of proof in order to teach students about it (NCTM, 2000).

According to Knuth (2002), teachers’ previous experiences as students affect their conception of proof. As a result, the teachers did not view that promotion of understanding as a role of proof. However, teachers should find effective ways of using proof as a “vehicle to promote mathematical understanding” (Hanna, 1995, p.42). They play an important role in developing and choosing tasks that are suitable for students and require reasoning to check mathematical relationships (NCTM, 2000). It is the teacher’s role to create opportunities for students to do mathematical proofs (Herbst, 2002). The challenge remains to create situations that “scaffold a coherent and connected conception of proof” and motivate students “to prove in all its functions” which is not necessarily from induction to deduction (Hoyles, 1997, p.15). Moreover, Malloy (1999) revealed that teachers can use students’ own reasoning skills and then
extend students' mathematical reasoning using classroom instruction. So, teachers can begin the discussion using students' intuition and then use inductive and deductive reasoning to reach conclusions.

The teacher needs to challenge students by choosing interesting, complex, and "worthwhile mathematical tasks" that provide a context for mathematical reasoning (Artzt & Femia, 1999, p.124). Also, teachers need to pose appropriate questions that enable students to use their own reasoning approaches to solve the problem (Lampert, 1990; Malloy, 1999; Posamentier & Jaye, 2006).

*Communication and Group Work*

Communication plays an important role in the teaching of mathematics. Its importance is evidenced through its recognition by NCTM as one of the five process standards in mathematics: Problem Solving, Reasoning and Proof, Communication, Connections, and Representations (NCTM, 2000). Communication plays an essential role in clarifying and developing students' understanding. Students who are encouraged in their "speaking, writing, reading and listening in mathematics classes reap dual benefits; they communicate to learn mathematics, and they learn to communicate mathematically" (NCTM, 2000, p. 60). Communication also enhances students' mathematical reasoning (Pugalee, 2001).

According to Sfard (2001), communication should not be viewed as a mere aid to thinking but as "almost tantamount to the thinking itself" (p.13). Communication and participation in class activities are related; participation requires communication with the other participants (Lee, 2006). Thus, “Communicating is thinking and discourse is a strong motivator in the learning process” (Lee, 2006, p.95). Sfard (2001), on the other
hand, attributed to communication and language an ultimate importance by defining the learning of mathematics as an “initiation to mathematical discourse” (p.28).

Small-group work is very useful in mathematics classroom. The small-group strategy offers a productive environment where students can communicate with each other about mathematics (Artzt, 1996, Winsor, 2007) as they discuss and develop problem-solving strategies (Curcio & Artzt, 1998). Organizing students in small groups has many advantages: 1) it enables all students to actively participate in a lesson, 2) it reduces requests for help from teacher, and 3) it enables students to help each other if they are teamed with students who can provide help (Stacey & Gooding, 1998). Moreover, when group work strategy is applied, it increases students’ use of mathematical terms in English and students’ understanding of mathematics (Winsor, 2007). Also, it provides a window into students’ mathematical reasoning, and promotes spontaneous verbalization (Artzt & Armour-Thomas, 1992; Artzt & Femia, 1999), and it helps the teacher to check the mathematics students are generating (Thom & Pirie, 2002).

Results of Stacey’s and Gooding’s (1998) study showed that members who learn effectively in groups interact more with one another and with the task. In addition, one of the most useful functions of group work in mathematics seemed to be the reduction of misconceptions. The analysis of the results showed that the effective groups have at least two improvers (students who improved their scores on the posttest), and their members had more turns in talking and interacted more than did those in the ineffective groups. Also, teachers should play an important role not only in correcting their students’ misconceptions, but also in helping them to learn from procedures that
resulted in the correction of their misconceptions (Ben-Hur, 2006). As a result, students who learn from their mistakes become the most skillful problem solver (Buschman, 2003).

Lee (2006) states that in whole-class discussions, each student receives a variety of inputs from their peers, but there is not enough time for their own involvement. However, in organized small-group work, all students learn to talk and communicate about their mathematical ideas. They have to think aloud when they work in groups, trying out ideas and receiving feedback from their peers. They have to give reasons and defend their solutions (Abele, 1998). In addition, the discourse that takes place reveals students’ understanding of mathematics. More specifically, it reveals students’ behaviors about problem-solving (Artzt, 1996).

However, not all students work so well in small groups. The effectiveness of the small group strategy is dependent on two factors: the task chosen and the personalities of the group members. In their research, Curcio and Artzt (1998) chose the assigned task in a way that it made it interesting to students because it was related to their everyday experiences, and at the same time it was challenging (Pandisco, 2001). The teacher formed the groups based on her knowledge of students’ personalities, their preferences of the group members and their different abilities.

Vygotsky (1986) asserted that when students participate in such a social setting (group work), the behaviors that occur within the group will be transferred by the students when they work individually. In the same line, Artzt (1996) asserted that the communication students encounter in their cooperative learning becomes the communication they use when they work alone. Thus, students can gain much when
they work in effective problem-solving groups (Curcio & Artzi, 1998). However, Sfard (2001) stated that the benefits of students' collaborative problem solving can't be taken for granted. If students' interactions are to improve learning, then the communicative skills should be taught.

In small group work, depending on the situation, the teacher plays the role of a judge between right and wrong solutions suggested by the students, or the role of a silent observer leaving room for the students to do the discussions and find solutions (Abele, 1998). Also, the teacher can play the role of a manager of the discussion (Lampert, 1990). However, one of the difficult aspects of small-group discussions is teacher intervention. The teacher should decide when to intervene to redirect students' efforts in the correct direction (Brenner, 1998; Civil, 1998; Goos & Galbraith, 2002).

Role of Teachers in Classroom Discussions

It is not an easy task for teachers to promote mathematical communication in the classroom. Most of them lack the personal experience with such environments, as they have learned mathematics in traditional classrooms (Silver & Smith, 1996). However, if given the opportunity, students can communicate mathematically (NCTM, 1989; Vace, 1994). The teacher needs to create a classroom environment of mutual trust and respect in which students can critique mathematical thinking without personally criticizing their peers (Ball & Freil, 1991; Lee, 2006; Silver & Smith, 1996). According to Vace (1994), teachers should support mathematical communication by encouraging student participation in discussions, reducing teacher control and evaluation, directing students to communicate with each other, and giving students a chance to express their ideas in writing, and allowing them to introduce their strategies (Ackles, Fuson & Sherin, 2004).
Mathematical Discussions

The teacher will play the role of a facilitator who encourages students to critically analyze the answers to the questions being posed.

As a consequence, students' role will change. According to Lee (2006), students should recognize that their role has changed from being passive recipients to active participants in their learning process. They should participate in the discourse, suggesting their ideas and opinions. Also, they should be responsible for their own learning, thinking and talking about what and how they are learning. They will take the responsibility for sharing their results, explaining and justifying their solutions. Also, students should listen to others to learn from them and benefit from their ideas (Ackels et al., 2004; Hiebert, Carpenter, Fennema, & Fuson, 1996).

Sherin, Louis, and Mendez (2000) stress the fact that students should not only talk about mathematical ideas and strategies, but they should also build on other students' ideas. The authors identified three kinds of building: 1) agreeing or disagreeing, 2) providing new evidence, and 3) creating new conjectures from students' ideas. These methods of building help students to be engaged in a “discourse community”, an environment in which students discuss mathematical concepts with other students, defend their own ideas and build on the other students' ideas. Their ideas will be the center of the discussions, and this will motivate them to think deeply about mathematical concepts. In this same line, Kazemi (1998) observed that student achievement in problem solving as well as in conceptual understanding increased when teachers helped students to build on their thinking.

According to Tanner and Casados (1998), the teacher may notice students' misconceptions just by listening to their discussions. Furthermore, teachers can gain
insight into students' thinking, simply, by asking them to explain the strategy used (Ball & Friel, 1991; Buschman, 2003). In this way, the teacher can create opportunities to encourage students to justify and explore whether a certain solution is correct or not (Artzt & Femia, 1999; Kazemi, 1998; Lee, 2006). In addition, this will help students articulate their mathematical thinking.

Researchers report that many students are non-participants in classroom discussions (Tanner & Casados, 1998; Vace, 1994). According to Lee (2006), some students are reluctant to share their ideas and thinking (Jansen, 2006) or to contribute to the discussions because they are concerned about not making mistakes or giving a wrong answer. However, teachers can overcome this problem by: 1) asking students questions that do not need factual information, and 2) having students restate what the previous student has said (Vace, 1994). In their study, Tanner and Casados (1998) describe the effect of using the “Socratic Discussion Method” on students’ attitude toward reading mathematics, and students’ participation in the math class. The participants were 17 high school students. The researchers used the “Socratic Method” seven times, videotaped two sessions for student evaluation, and gave two questionnaires to the students to check their attitude toward the “Socratic Method”. Students were engaged in discussions, either by listening to one another, taking notes, or speaking to others about a mathematical concept. Every student had the chance to participate in the debriefing session to reflect on the work done. Results showed that students enjoyed the discussions and learned from them. They even started to initiate questions to begin a discussion. Using the “Socratic Discussion Method” helped students improve their attitudes toward reading mathematics, and increased their
participation in the math class. According to Tanner and Casados (1998), using this technique encouraged students not only to participate in the math discussions, but also to "become active learners and critical thinkers" (p.349).

Also, Fonzi and Smith (1998) assigned essays about mathematical experience for students to read. Results showed that students were engaged in the negotiation and generation of ideas, and communicated not only the end product of their thinking, but also the way they used to get their product. Also, in this study, telling and showing provided students a chance to share what they know with the rest of the class; students read a variety of texts related to mathematics and approached them in a "generative" way; students elaborated, clarified, and revised their ideas in response to students’ and teacher’s questions which allowed them to go beyond just getting information from text (Siegel, Borasi, Fonzi, Sanridge, & Smith, 1996). As a result, reading is one form of communication that can find a place in mathematics instruction to engage students actively in meaningful learning (Siegel et al., 1996).

Loska (1998) applied the "neo-socratic method" where students’ responsibility was to explore the topic without instructive help from the teacher. They have to think whether they understand what is being said, and they should agree or disagree about what is said by other students. The teacher’s role was to ensure a safe environment that allows students to express their thoughts and perceptions (Ben-Hur, 2006; Pandisco, 2001). Results showed that even weak and quiet students were able to participate in the discussion. Moreover, students’ thinking process went deeper and deeper; they were able to find connections among different topics and gained more insight.
Writing is another form of communication. Writing is not only used as a way to communicate, but also as a tool for thinking (Fonzi & Smith, 1998; Sierpinska, 1998). Winsor (2007), created an approach for teaching students mathematics called Mathematics as a Second Language (MSL). The main components are vocabulary activities, journals, group work, and projects. Results showed that students benefited from journal writing (Thompson & Rubenstein, 2000) because writing about mathematics required them to decide what they understand and what they do not understand. Moreover, students became more expert in communicating mathematically. Writing, unlike spoken language, needs special training. Spoken language can be learned and taught in unsystematic and unplanned ways (Sierpinska, 1998). Writing, however, is a barrier for students. Many students find writing their mathematical ideas very difficult and overwhelming; teachers should support their students to produce any written mathematical communication (Lee, 2006). Students’ writing can provide a source for teachers to assess students’ learning and thinking about mathematics. As a consequence, teachers should implement various writing activities in their instruction (Pugalee, 2001). However, Lee (2006) supports more thinking and less writing. More thinking will include more talking; talking will help the students to understand the concept under discussion and to use and control it. According to Lee (2006), when the students have thought, talked, and explained their ideas, they will be ready to write them down.

Difficulties in Developing a Discourse Community

In creating and maintaining a “mathematical discourse community”, the teacher may face many difficulties and dilemmas. Sherin (2002) conducted a study to describe
the pedagogical tensions which a teacher faced in order to find a balance between using students' ideas as the center of the classroom discussions, and ensuring that mathematical concepts are discussed in a deep and meaningful way. This study was a part of FCTL project, which stands for "Fostering a Community of Teachers as Learners". The teacher met with the researcher to discuss how to implement Community Of Learners (COL) units. The participants were the eighth-grade students. A total of 68 lessons were identified through the school year to investigate classroom discourse. The teacher needed to work hard throughout the school year to maintain a balance between "process" and "content" in classroom discussions. At the beginning of the year, the goal of the teacher was to create a structure for discussions. So, the teacher focused on "process" rather than on "content". Once the rules were known and practiced, the teacher shifted his emphasis to "content". The teacher sometimes achieved a balance between "process" and "content" by: 1) idea generation through eliciting ideas from students, 2) comparison and evaluation whereby students agree or disagree about an idea that had been suggested, and 3) filtering, which involves narrowing ideas raised by students and exploring few ideas in details. However, the balance was not shown throughout the year. It is the teacher who decided when to emphasize process over content or vice versa.

According to Sherin (2002), two factors influence teachers' decisions in managing classroom discourse. First, teachers' beliefs about mathematical discourse affect the way discussions are orchestrated in class. Second, teachers need to develop new understandings of mathematical content and pedagogy (Silver & Smith, 1996). Educational reforms may as well affect teachers' abilities of conducting mathematics
discourse, because they are usually based on curricula changes only, while the communication patterns are ignored (Cestari, p.164).

Moreover, Silver and Smith (1996) stress the fact that using “worthwhile mathematical tasks” alone does not ensure students’ discussions on certain important mathematical ideas. Teachers should help students link their explanations to mathematical reasons (Kazemi, 1998). Silver and Smith (1996) and Vace (1994) mentioned other barriers that challenge teachers in classroom discussions. Some students may feel uncomfortable to share their ideas; others don’t listen to their peers. In addition, some students cannot articulate their mathematical thinking using the mathematical language (Tanner & Casados, 1998; Vace, 1994), and they worry that peers may make fun of them (Lee, 2006). Also, students may ask questions that do not address mathematical issues (Silver & Smith, 1996).

To build a discourse community requires teachers to change their traditional practices. Teachers should increase their subject matter knowledge as well as their pedagogical content knowledge (Ackles et al., 2004).

In conclusion, mathematics teachers face many difficulties when establishing a discourse community in their classes. On one hand, students should learn a specific content. On the other hand, students’ ideas are supposed to guide discussions. Literature provides some guidelines that may help the teacher solve this dilemma by eliciting and listening to students’ ideas, encouraging them to elaborate their thinking and to evaluate and compare their ideas. The teacher can focus the discussion on a few ideas that are mathematically significant. Finally, teachers can use productive classroom discourse to promote students’ problem solving abilities.
Communication and Language

Language is an important component of mathematics instruction. It is regarded as a "window to the child's conceptual world" (Kress, 1994; as cited in Monaghan, 2000, p.192). According to Thompson and Rubenstin (2000), language has at least three roles: 1) it is a means of communication, 2) students build understanding as they express ideas through language, and 3) teachers assess students' understanding by listening to their oral communication and by reading their mathematical writings. As a result, students should master the language of mathematics if they are to read, understand, and discuss mathematical ideas (Thompson & Rubenstin, 2000). However, the difficulties caused by the language used to express mathematical ideas affect students' "conceptualization" of mathematical notions (Lee, 2006).

Ben-Hur (2006) and Lee (2006) assert that successful communication in a classroom is important to learning. When students argue about a mathematical statement, they justify their answers by using language and symbols (Lampert, 1990; Lee, 2006). Sfard (2001) described language as a symbolic tool that will be used in communication, either "inter-personal or self-oriented" (p.28). Also, communication is the transmission of thoughts mediated by language (Sierpinska, 1998). However, language issues in mathematics can be a barrier to communication (Ben-Hur, 2006; Lee, 2006; Winsor, 2007), to learning (Pimm, 1987), and to reasoning about mathematics (Ben-Hur, 2006). But the teacher plays an important role in reducing those barriers (Ben-Hur, 2006; Lee, 2006). The teacher should explicitly discuss with students the way language is used in order to express mathematical ideas (Lee, 2006), and should help them to build the formal language that replaces their "metaphoric, informal
expressions” (Ben-Hur, 2006, p. 67). Consequently, students will learn to use language to express their ideas effectively (Lee, 2006).

Vygotsky (1962 as cited in Lee, 2006) discussed how children learn to have a place in society through the mediation of “more competent peers using tools and signs, many linguistic, that are part of the social world” (p.86). Therefore, children develop “higher mental functions” (Vygotsky, 1981, p.162 as cited in Lee, 2006, p.86) through interactions with their society that take place, not exclusively, in verbalized language.

Mathematics has a specific way of using language and its own way of expressing ideas, which is named the “mathematics register” (Pimm, 1987). The mathematics register is a way of using symbols, vocabulary, and grammatical structures that results in expressing mathematical ideas; it is a way of using language to express concepts and to present and discuss arguments (Lee, 2006). Also, mathematics is a language that is both oral and written and that can be formal or informal (Usiskin, 1996).

According to Ben-Hur (2006), the formal verbal language of mathematics is important for a minimum of two reasons: 1) most of the verbal terms- for example angle, product, quotient, and many others- function as “conceptual packages”. These terms do not represent specific objects, but they are “abstract categories that conceptually organize mathematics” (p.67), and 2) language is a significant tool of dialogue about particular mathematical concepts. As a result, students should know how to read mathematics. Failing to do so, students will not be able to register the mathematics because the oral language is essential for memory. Students will have difficulty internalizing the meaning when the teacher reads the sentence (Usiskin,
1996). Therefore, for students to be engaged in mathematical discourse, they should be able to engage with the rules of mathematical language and teachers must help them to do this (Lee, 2006).

**Problem Solving and Metacognition**

NCTM describes problem solving as a “hallmark of mathematical activity and a major means of developing mathematical knowledge” (NCTM, 2000, p. 116). It is important to choose problems that help students understand the mathematical concept (NCTM, 2000; Thom & Pirie, 2002) and leave them with a “mathematical residue” (Thom & Pirie, 2002, p.26). Also, it is important to choose problems that can be solved using different problem-solving strategies. Moreover, sharing these strategies with their peers provides students with opportunities to assess the strengths and limitations of these methods (NCTM, 2000). However, Thom and Pirie (2002) assert that the teacher plays an essential role in creating an environment that encourages students to be active participants in their discovery of mathematical problem solving. Then, students become able to pose problems for themselves (NCTM, 2000; Thom & Pirie, 2002) through the use of metacognitive skills (Davis, 1992 as cited in Thom & Pirie, 2002) such as reflecting on their work and monitoring their solutions (NCTM, 2000).

Polya (1957) identified four phases to solve problems: understand the problem, devise a plan, carry out the plan, and look back. Polya’s four-step process of problem solving describes cognitive and metacognitive behaviors (Ben-Hur, 2006). Then, Schoenfeld (1983 as cited in Artzt & Armour-Thomas, 1992) described mathematical problem solving in five episodes: read, analyze, explore, plan/ implement, and verify. However, Artzt and Armour-Thomas (1992) made some changes to Schoenfeld’s
framework. They made the framework to include eight problem-solving episodes which are: read, understand, analyze, explore, plan, implement, verify, watch and listen. These behaviors are categorized into cognitive or metacognitive behaviors. Behaviors of reading was categorized as cognitive, episodes of understanding, analyzing, and planning were categorized as metacognitive, where as behaviors of exploring, implementing, and verifying were sometimes categorized as cognitive and sometimes as metacognitive. (See Table adopted from the study done by Artzt and Armour-Thomas, 1992)

Research about cognitive processes has focused on the “knowledge, monitoring, evaluation, and overseeing” that students use during any problem-solving activity (Artzt and Armour-Thomas, 1992, p.139). The term used for these cognitive processes is metacognition which is defined as “one’s knowledge concerning one’s own cognitive processes or anything related to them” (Favell, 1976 as cited in Schoenfeld, 1992, p.347). In order to differentiate between cognitive and metacognitive behaviors, Artzt and Armour-Thomas (1992) describe them as follows: “Cognition is involved in doing, whereas metacognition is involved in choosing and planning what to do and monitoring and regulating what is being done”(p.141). In other words, metacognitive behaviors are demonstrated by statements made about the problem or about the problem-solving process, while cognitive behaviors are demonstrated by verbal and nonverbal actions that specify actual processing of information (Artzt & Armour-Thomas, 1992).
<table>
<thead>
<tr>
<th>Episode</th>
<th>Predominant Cognitive Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Read</td>
<td>Cognitive</td>
</tr>
<tr>
<td>Understand</td>
<td>Metacognitive</td>
</tr>
<tr>
<td>Analyze</td>
<td>Metacognitive</td>
</tr>
<tr>
<td>Explore</td>
<td>Cognitive and metacognitive</td>
</tr>
<tr>
<td>Plan</td>
<td>Metacognitive</td>
</tr>
<tr>
<td>Implement</td>
<td>Cognitive and metacognitive</td>
</tr>
<tr>
<td>Verify</td>
<td>Cognitive and metacognitive</td>
</tr>
<tr>
<td>Watch and listen*</td>
<td>Level not assigned.</td>
</tr>
</tbody>
</table>

* Level not assigned.


Analysis of results of the study done by Artzt and Armour-Thomas (1992) showed that a continuous interplay between the cognitive and metacognitive processes is necessary for successful problem solving to occur. These behaviors did not occur in a linear way which reflects the thoughts and behaviors of expert problem solvers working alone (Curcio & Artzt, 1998). Students should understand that the use of these behaviors does not necessarily happen in a linear way (Artzt, 1996; Artzt & Femia, 1999; Ben-Hur, 2006). Students usually become frustrated when they can't solve problems using a direct, sequential approach, because they tend to think that problem solving and mathematical reasoning occur in a linear and direct fashion (Artzt & Femia, 1999). Also, students' difficulty in problem solving may be that students can't monitor or

Expert problem solvers learn how to organize their knowledge, create meaningful patterns in problem solving, implement different strategies, and use self-monitoring skills to guarantee effective performance (Ben-Hur, 2006; Chinnappan, 1998; NCTM, 2000; Montague & Applegate, 1993). Chinnappan (1998) asserts that students who structure their prior geometrical knowledge into schemas may develop an understanding of how and when to use that knowledge effectively during problem solving. Also, in their study Artzt and Armour-Thomas (1992) showed that the group who failed to solve the problem was the group who got the lowest percentage of episodes at the metacognitive level and the highest percentage at the cognitive level. In addition, Goos and Galbraith (2002) documented that students’ poor metacognitive decisions contributed to their failure in solving problems (Schoenfeld, 1992, Pugalee, 2001).

The research presented in this report adopts the framework of Artzt and Armour-Thomas (1992) for protocol analysis in order to examine the interactions between the cognitive and metacognitive processes observed in the students’ problem-solving behaviors working in small groups on two geometry problems.
METHOD

This study is a qualitative research analyzing the processes and the effect of an intervention in an intermediate-level class (grades 7-8). Unstructured interviews, class observations, and focus group interviews are used.

Participants

The setting of the study is a grade 7 class in a school located in Beirut, Lebanon. The participants are the students in this class and their math teacher. The study extended over two academic years. During the second year the same students were in grade 8. The school is one of three private schools that are affiliated to a foundation and located in a low income neighborhood. The school follows the Lebanese National Curriculum Program that prepares students at the intermediate level (grades 7 through 9) for the national Brevet Diploma. The class consists of seven females and ten males. The age of the students ranges between 12 and 13 years. Students have different levels of academic achievement in mathematics: low, average, high and gifted. Almost all participants completed their elementary education in the same school, and they learnt math in English which is not their native language. Moreover, the students come from low socioeconomic status families. The math teacher of the grade 7 class considered in the research has volunteered to engage in this study, according to an agreement with the researcher who is a math teacher of another grade 7 class, in the same school.

The participant math teacher is a male in his late thirties, has a Bachelor of Science (B.S) in mathematics and has been teaching in the school for two years. In addition, he has been teaching math for grade 12 in another school for 10 years. Hereafier, the teacher is referred as Mr. X.
Procedures

This study is an action research whereby the researcher agreed with the class teacher about the techniques and procedures to follow during the research. Techniques that were used in the study consist of class observations, an unstructured interview with the math teacher Mr. X, and clinical focus group interviews with the students. A research sequence was prepared and implemented in cooperation with Mr. X. The research consists of the following procedures: 1) The researcher observed grade 7 class for one session and grade 8 class for one session (around 30 minutes each) that are devoted for problem solving, prior to any intervention, 2) the researcher conducted an interview with Mr. X to investigate his perception of using mathematical group discussions, and the frequency according to which he uses them, 3) the researcher used clinical focus group interviews during which students solve geometry problems, 4) a year after, when students were promoted to grade 8, the researcher discussed with Mr. X the way to promote students' interactions and class discussions by applying mathematical group discussions and agreed on a unit to be taught according to those guidelines, 5) the researcher observed the same students in grade 8 during three math sessions- one of which is a group work session, and 6) the researcher again conducted clinical focus group interviews to check students' final state of their problem solving abilities. Following are details about each of the above mentioned procedures.

First Sequence of Class Observations in Grade 7 and Grade 8

The researcher observed grade 7 for one session prior to intervention to investigate the teacher's method of teaching in a problem solving situation. Also, she observed grade 8 for one session. While observing the two classes, the researcher sat in
the back of the classroom to take notes about eventual mathematical discussions, whether peer to peer or teacher to student discussions. An observation chart was used (Appendix A). The two sessions were audio-taped for further analysis. The objective is to check the types of questions the teacher and students ask, and to investigate the roles that the teacher assigns to students in a problem solving session.

Interview with the Teacher

The researcher conducted an interview with the teacher to obtain data about his perception of using mathematical group discussions, whether he uses group work and discussions, the frequency of using group discussions, and his belief about the possible positive effects of mathematical discussions on students' problem solving abilities (Appendix B for sample of the questions asked). Accordingly, the researcher discussed with the teacher the method of teaching that would be applied during the implementation phase, for example, proceeding by questioning, using mathematical language by students as well as teacher, applying group discussion strategy in a problem solving situation, and providing opportunities for students to reason, to articulate their mathematical thinking, to refute other students' ideas, or to prove their conjectures to one another (Appendix D)

First Clinical Focus Group Interviews

The main procedure of the study was the focus group interviews. The first focus group interview was applied prior to any intervention from the researcher. According to Krueger (1994), implementing this technique: 1) places students in natural situations to capture the dynamic nature of group discussions in which students are able to talk freely and share insights; they "are influencing and influenced by others"(p.19), 2) produces data that offer insight into the "attitudes", "perceptions" and "opinions of participants"
possesses flexibility that helps by probing to explore unexpected issues, and places significance on understanding the thought processes (Krueger, 1994). The researcher used the focus group interviews method with students in a problem-solving situation where students solved two geometry problems (Appendix C). The interview started by asking open-ended questions to reveal what is on the interviewees' mind (Krueger, 1994). During the interview, students were reminded to articulate their thoughts, give explanations of the procedure applied to solve the problem using mathematical language, and to provide more information when necessary with the help of a moderator (Litoselliti, 2003). The researcher did not demand participants to reach consensus. However, she observed, listened and later analyzed discussions that took place in the focus groups to understand the thought processes and initial problem solving strategies and approaches that are applied during discussions. In addition, the researcher developed understanding about the way using mathematical group discussions helps students in problem solving situations.

Heterogeneous groups were formed according to gender and academic achievement. This was done with the help of Mr. X.

After obtaining the school's approval, the interviews took place during school days. The classrooms were prepared ahead of time to seat students around tables with their names written on their seats. The researcher conducted three focus group interviews; one group (group 2) consisted of five students, and two groups consisted of six students each. Group 1 included three girls and three boys. Group 2 included two girls and three boys, and group 3 included two girls and four boys.
In each group, a writer and an organizer were assigned. The researcher, with the help of Mr. X, decided on which students to play these roles. The writer was chosen from average students to be involved in the discussion, whereas the organizer was chosen from students who have leadership characteristics. His/her task was to organize the turn-talking during the focus group discussions. Moreover, the researcher and Mr. X agreed about the distribution of students in the groups according to their achievement level. Group 1 included three students with high level of achievement, two students with low level of achievement, and one student with average level of achievement. Group 2 included one student with high level of achievement, one student with low level of achievement, and three students with average level of achievement. Group 3 included three students with high level of achievement, one student with low level of achievement, and two students with average level of achievement.

The focus group interview consisted of two problems. Students were given 70 to 100 minutes for to solve the problems. The problems were chosen according to the following criteria: 1) they were consistent with the content of the lesson that students were learning at that time, 2) they lent themselves to different methods for solving, 3) the solution process reflected students' problem solving abilities and 4) solving them required higher order thinking level.

The researcher, with the help of a mathematics teacher as a moderator (other than Mr. X), conducted the interviews. The moderator is a female in her late thirties and has been teaching mathematics for 10 years. The researcher met with the moderator prior to the focus group discussions and agreed on the way the session will proceed and which questions to be addressed to students during their solving the problems, for
example, to prompt them to explain the way they reached their answer, or ask them if they agree or disagree about the solution found. The moderator helped in organizing the discussions, ensuring that all students were participating in finding the solution, and reminding students to talk aloud and verbalize their thoughts. She helped in probing questions that don’t lead or guide students’ answers.

Students had their pens, geometry sets, and their copybooks to write the solution of the problems (no erasers were allowed). The researcher notified students that they should discuss the problem and explain their solution method, using correct mathematical vocabulary. Also, the researcher informed students that questions such as "is our answer correct?" or "can we use this method?" will not be answered. In each group, one student had the role of an organizer and one student had the role of the writer. The other students in the group should ask the organizer for permission to talk and participate. Students should have the chance to articulate their mathematical thinking, to defend their reasoning, and to prove their conjectures to one another. They were encouraged to discuss the procedures and agree on a strategy or a solution. The writer wrote the final answer and submitted it to the researcher. The researcher took observation notes during the interviews, and the moderator helped in monitoring discussions.

Class Observations in Grade 8

The researcher discussed with Mr. X the guidelines for promoting students' interactions and class discussions (Appendix D). They agreed that those guidelines be applied for 2 months of instruction (November and December). The researcher observed the class during three sessions- one of which is a group work session- within that period
in a problem solving situation to check students' mathematical discussions. While observing the class, the researcher sat in the back of the classroom to take notes about eventual mathematical discussions, whether peer to peer or teacher to student discussions. The observation chart (Appendix A) was used. The two sessions were audio-taped for further analysis. The objective is to check the type of questions the teacher and students ask, and to investigate the roles that the teacher attributes to students in a problem solving session.

*Second Clinical Focus Group Interviews*

After the implementation phase, the researcher conducted clinical focus group interviews to investigate students' final state of problem solving abilities in a problem solving situation where students solved two geometry problems (Appendix E). The same criteria and procedures used in the first clinical focus group interviews were applied.

*Instruments Used*

The researcher played the role of a non-participant observer. Class observations were audio taped. During these observations, the researcher recorded any "communication about mathematics" and "communication in mathematics" (Brenner, 1994) in an observation log (Appendix A). Communication about mathematics includes students' explanation about processes they used and giving reasons for procedures (Brenner, 1994). Communication in mathematics entails the use of math vocabulary and symbols. Also, the researcher audio-taped the focus group interviews and the open-ended interview with the teacher.
Data Analysis Methods

The researcher transcribed the audiotapes related to class observation before and during intervention. The analysis adopted the following classification of teacher’s questions, according to four types: probing, leading, prompting, or redirecting. Also, verbal flow charts were designed to investigate the level of interaction in class whether teacher- to- student or student- to- student.

Moreover, the six audiotapes of clinical focus groups were transcribed and analyzed according to the seven problem-solving episodes which are read, understand (meaning trying to understand), analyze, explore, plan, implement, verify. Also, the audiotapes were transcribed and analyzed according to the types of interaction among students. The following categories were adopted: asking to repeat, repeating, explaining by giving evidence from exercise, checking understanding, asking questions about the proof, deduction, analysis, stating reasons, asking to read, reading, calling for help, asking to draw, explaining how to draw, checking agreement, agreeing, disagreeing, why questions, correcting others’ mistakes, seeking suggestions, suggesting a plan, requiring a proof for a suggested property, suggesting to find another way, reminding about what is required to prove, no answer, and making conclusion.

Also, these audiotapes were analyzed to check the strategies used by students to formulate arguments and to convince others. The following six categories were noted: explaining a method or solution, requiring a proof, refuting a reason or a proof, defending a proof, use of representations, and planning what to do. Finally, the researcher analyzed the data to find whether or not the use of the mathematical group
discussions enhances students' problem solving abilities in Geometry. The use of different methods and tools ensures the triangulation of the data.
RESULTS

In the review of literature concerning problem solving and metacognition, a framework was identified and adopted to analyze the relation of metacognitive and cognitive behaviors with respect to problem solving. This chapter is devoted to represent the results found. It consists of the following:

Class Observations According to Teacher’s Questions

The researcher observed grade seven for one session (25 minutes) and grade eight for one session (33 minutes) prior to the intervention phase. The aim was to investigate the teacher's method of teaching in a problem solving situation. The two sessions were audiotaped. The researcher transcribed the two audio tapes and analyzed them according to kind of questions being asked. Also, the researcher observed grade eight for three sessions 51 minutes, 41 minutes, 25 minutes -one of which is a group work session- during the intervention phase. The third session will not be included in the analysis.

The analysis adopted the following classification of teacher’s questions, according to four types: probing, leading, prompting, or redirecting.

- Probing questions are a “teaching/assessment strategy that provides insight into the mental processes a student is using by engaging him or her in conversation about the subject” (http://images.rbs.org/assessment/questioning.shtml). In the analysis, two types of probing questions will be separately considered: 1) probing to assess students’ mathematical knowledge (Probing M), and 2) probing to have insight into students’ thinking (Probing T).
Leading questions try to “lead the respondent to teacher’s way of thinking”.
(http://www.mindtools.com/pages/article/newTMC_88.html)

Prompting questions are “a process by which a teacher supports a student by giving hints that clarify the student’s response or to point the student toward appropriate strategies to use in solving a problem”.
(http://images.rbs.org/assessment/questioning.shtml)

Redirecting questions occur “when an instructor turns a student-initiated question or comment back to the student or to the class. This provides students with further opportunities to develop thinking and communication skills”.
(http://teaching.berkeley.edu/compendium/suggestions/file57.html)

The expressions between parentheses are the abbreviations as used in tables 1 and 2.

The teacher varied his questioning techniques. Beside the above four types of questions, the teacher addressed also to students questions about whether they understood the problem/idea such as: 1) Did you understand? 2) Do you have any question? 3) Clear till now? and 4) Did you know how? As these questions don’t affect students’ thinking, they will not be considered in the analysis.

Following are examples of each category of questions

1) Probing Questions to Assess Students’ Mathematical Knowledge:
How do we draw the parallel?
Why you think they are equal?
How do we draw the translate of M by vector BM? Tell us the steps.
So, by proving which 2 triangles?
First of all, look at the figure. Are they parallel?
You want to say D is the symmetric of B with respect to....?

Converse because we have a midpoint and a...?

Because they are opposite angles in a...?

Why here can we divide by two?

We prove it a parallelogram with one right angle or directly we say a quadrilateral with?

2) Probing Questions to have Insight into Students’ Thinking:

What are you going to do now?

You want to say D is the symmetric of B with respect to....?

How can you know that M is the midpoint of AE?

By parallelogram of center what?

Still you are proving 4 equal sides?

Any parallelogram?

Which one fits here?

What can you say now?

Because?

What property you base your proof on?

3) Leading Questions:

Now to prove an isosceles triangle, what do we have to search for?

Who can find 2 equal angles in triangle AMP?

What do you think here in this problem we are going to find?

When are they congruent?

Do we have two midpoints?

So, what do we have now?
Since we have BO and MF ....?

Who can find or who can prove it by a right angle?

Is E on the circle? Is it inscribed?

Where is the angle facing diameter?

Do we have alternate interior now?

4) Prompting Questions:

First you have to draw ...?

What is the required?

First of all, look at the figure. Are they parallel?

Can we get a pair of alternate interior angles or...?

Or, first who can find a pair of alternate interior angles or corresponding angles?

DE as a vector is equal to AB or what?

When we say vector AB is equal to vector DE that means they have the same ....?

MCN is alternating with what?

Every 2 consecutive angles in what?

5) Redirecting Questions

Student: DAM and PMA are alternate interior angles.

Teacher: DAM and?

Student: Is the angle PAM is equal to CAB?

Teacher: PAM?

Student: MD is equals to AM.

Teacher: MD?

Student: And I prove that MF is parallel to. Mid-segment theorem.
Teacher: Here we used the mid-segment theorem?

Student: Prove that MBGE is a rectangle.

Teacher: Prove what is a rectangle?

Student: We can say that AF is parallel to CH since they both intersect at the same line, by property of 2 straight lines intersecting at same straight line are parallel.

Teacher: 2 straight lines they should be intersecting on the same straight line?

*Types of Questions Used by the Teacher*

Table 1 represents the type of questions the teacher used before intervention, their number of occurrences, and their percentages.

<table>
<thead>
<tr>
<th></th>
<th>Number of occurrences</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probing M</td>
<td>24</td>
<td>28</td>
</tr>
<tr>
<td>Probing T</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>Leading</td>
<td>16</td>
<td>19</td>
</tr>
<tr>
<td>Prompting</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>Redirecting</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>85</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

Teacher's types of questions, noted in Table 1, ranged from a low of 11% for redirecting questions to a high of 28% for probing questions that assessed students' mathematical knowledge; whereas the prompting and leading questions got almost the same percentage (18%-19%).

Table 2 represents the type of questions the teacher used during intervention, their number of occurrences, and their percentage.
Table 2

<table>
<thead>
<tr>
<th>Types of questions during two observations during intervention</th>
<th>Number of occurrences</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probing M</td>
<td>59</td>
<td>35</td>
</tr>
<tr>
<td>Probing T</td>
<td>28</td>
<td>16</td>
</tr>
<tr>
<td>Leading</td>
<td>34</td>
<td>20</td>
</tr>
<tr>
<td>Prompting</td>
<td>24</td>
<td>14</td>
</tr>
<tr>
<td>Redirecting</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>Total</td>
<td>170</td>
<td>100</td>
</tr>
</tbody>
</table>

Analyzing table 2 demonstrated that the types of questions ranged from a low of 14% for prompting questions to a high of 35% for probing questions that assessed students' mathematical knowledge; whereas the redirecting and probing questions to have insight into students' thinking got almost the same percentage (15% - 16%).

As shown in table 1 and table 2, the number of questions (170) in the two observations during intervention is double than the number of questions (85) in the two observations before intervention. However, the time in the set of observations done during intervention is not double than the time in the set of observations done before intervention. The average of questions asked by the teacher in the set of observations done during intervention is 1.8 questions per minute and 1.4 questions per minute for the questions asked in the set of observations done before intervention; which means around 29% increase in the number of questions, for the same period of time. This is an evidence that the teacher asked more questions to encourage students to become active learners. It also shows that he implemented the guidelines agreed upon with the researcher, especially “Vary the questioning techniques”, “Probe incorrect or incomplete responses” and “Keep the focus of students on reasoning rather than only getting the correct answers” (Appendix D).
Moreover, the highest percentages were for probing questions that assessed students' mathematical knowledge. It is clear that the teacher made sure that students knew the mathematical properties before and during solving the problem. He also uses frequently this type of questions to remind students constantly about important mathematical properties. Moreover, the teacher used probing questions to have insight into students' thinking in both sets of observations before and during the intervention phase. The teacher did not accept only right answers, but he engaged students in conversation about the problem by asking "why" questions. Additionally, the teacher used leading questions in both set of observations before and during the intervention phase, and their percentages (19% - 20%) were almost the same. The teacher led the students to solve the problem by asking questions that helped them analyze and solve it.

Although the teacher used redirecting questions (11%) in the two observations done prior the intervention phase, it is clear that he used more redirecting questions (15%) in the two observations done during the intervention phase. He provided students with more opportunities to develop thinking and communication skills. Also, he encouraged student to student interaction by promoting discussions. Additionally, the teacher used less prompting questions in the two observations done during the intervention phase. This may be due to the fact that the teacher encouraged students to share their ideas.

We can attribute this change in the number of questions asked before and during intervention and the fact that the redirecting questions increased in the two observations done during the intervention to the implementation of the guidelines that the researcher
and the teacher discussed for promoting students' interaction and class discussions (Appendix D).

**Class Observations According to Level of Interactions among Students**

The researcher, also, transcribed the audiotapes of the class observations conducted before and during intervention to investigate the level of interaction in class whether teacher- to- student or student- to- student. For this purpose, the flow charts were designed.

**Flow Charts First Observation Before Intervention**

Flow charts were designed to investigate the level of interaction in class. The researcher took four to five samples almost every seven to 10 minutes-depending on the duration of the session. Each sample represents the interaction for two minutes.

Following are the situations represented by each flow chart:

- In sample 1.1.1, a student (Mohamad H) drew the figure on the board, the teacher asked him questions while he was drawing the figure. The teacher applied the IRE sequence (Mehan, 1979 as cited in Sherin, 2002): the teacher *Initiates* a question, the student *Replies*, and then the teacher *Evaluates* the answers. The student did not know the answer, another student (Mazen) answered (Appendix F).

- In sample 1.1.2, many students participated in answering questions asked by the teacher (Appendix F).

- In sample 1.1.3, one student (Aya) stated the proof orally. Then, the teacher asked general questions about mathematical properties. In this sample, one
student (Mohamad H) asked the teacher a question, and the teacher did not answer directly, he redirected him a leading question. The student answered wrong, then the teacher explained to him the solution (Appendix).

- In sample 1.1.4, the same student (Aya) who stated the proof orally in sample three wrote the proof on the board. The student did a mistake and the teacher corrected for her (Appendix F).

No discussion among students was recorded. All interactions took place between the teacher and the individual students.

Flow Charts Second Observation Before Intervention

The researcher analyzed the five samples and noted the following:

- In sample 1.2.1, the teacher asked many students, not one, to solve the exercise. When a student did not know the answer, he asked another student to solve it (Appendix F).

- In sample 1.2.2, a student (Ziad) asked a question, the teacher did not answer directly. He redirected the question to the student. The student answered, then the teacher elaborated. Also, in this sample, another student (Mostapha) asked a question. The teacher did not answer, but asked him some leading questions so that the student can answer. Then the teacher explained. Another student (Baraa) asked a question showing that he was confused. Also, the teacher asked him some leading questions so that the student can answer. Then, the teacher elaborated (Appendix F).

- In sample 1.2.3, the teacher asked general questions and students participated in answering these questions. Then, he chose a student (Ziad) to state the proof.
The teacher asked him leading questions to help him state the proof correctly.
The student did not state the reason correctly, the teacher asked another student
(Mahmoud B) to state it (Appendix F).

- In sample 1.2.4, the teacher asked general questions about mathematical
  properties. The students participated and answered. A student (Ziad) asked a
  question, the teacher answered him and asked another student (Mahmoud) to
  state the proof. A student (Mohamad T) made a comment, and the teacher asked
  the student (Mahmoud) to continue (Appendix F).

- In sample 1.2.5, the teacher asked general questions about mathematical
  properties. The students participated and answered. The teacher asked a student
  a question, the student answered incorrectly. The teacher corrected him. He
  asked him a leading question to help him recognize his mistake. Then the
  teacher asked students general questions about the proof. At the end the teacher
  made the conclusion, not the students (Appendix F).

Flow Charts First Observation During Intervention

- In sample 2.1.1, the teacher asked a student (Aya) to read the exercise. Then, he
  asked another student (Inas) to draw the figure on the board. He asked her
  questions about how to draw the figure and the student answered (Appendix F).

- In sample 2.1.2, the teacher asked a general question. Then, a student (Nour)
  asked a question and she answered it. The teacher elaborated and then asked her
  another question that she answered. Again, the teacher elaborated the idea and
  asked students to continue the proof. Another student (Mohamad A) asked the
  teacher a question “Can we can we show them parallel by alternate interior
angles?"; the teacher redirected the question and explained the idea. The student did not know how to continue. The teacher did not answer, and asked another student (Rayan) to state it who answered correctly (Appendix F).

- In sample 2.1.3, the teacher asked a student (Sara R) a question. The student gave a wrong answer, and the teacher corrected the mistake. Then, the teacher asked another student (Nizar) to continue. When the student finished explanation, the teacher asked the previous student (Sara R) to repeat the explanation to make sure that she understood. Another student (Mohamad A) asked a question, the teacher did not answer and redirected the question to another student (Aya) who answered the question, and explained to him (Appendix F).

- In sample 2.1.4, the teacher asked a student (Sandra) a question. The student gave a wrong answer, the teacher did not correct her, but he asked her some prompting questions that helped her to continue the proof. Again, the student made a mistake, the teacher asked another student (Sara R) to correct her (Appendix F).

- In sample 2.1.5, the teacher asked a student (Ali) a question to check understanding. Then, the student (Ali) asked the teacher a question to check if he can solve the exercise in another method, and the teacher accepted his solution. The student made a mistake, and the teacher did not correct him. He asked him to make the correction on his own and he knew the answer. At the end, the teacher asked the students general questions and they knew the answers.
A student (Ghina) made a wrong comment; the teacher corrected her by asking leading questions that helped her to answer correctly (Appendix F).

**Flow Charts Second Observation During Intervention**

- In sample 2.2.1, the teacher asked a general question about the exercise. A student (Sherine) answered. Then the teacher continued to ask general questions, and the students answered. The teacher asked the same student (Sherine) to state the proof. One student (Bilal) said that he did not understand the proof. The teacher asked a general question to check who can explain for him. A student (Sandra) offered her explanation to him (Appendix F).

- In sample 2.2.2, the teacher asked another student (Nour) to continue the proof. The student did a mistake, the teacher corrected her. Then, the student (Nour) continued the proof. The teacher asked the students leading questions about the proof (Appendix F).

- In sample 2.2.3, the teacher asked a general question to check understanding of the proof. A student (Aya) asked the teacher a question; the teacher did not answer and redirected the question to the class for which the students answered. A student (Nizar) stated the proof. Another student (Rouba) asked the student (Nizar) to explain to her. She understood the proof, and suggested another way to solve the exercise. Another student (Abed Latif) also suggested another solution to solve the exercise. He did a mistake, and a student (Nizar) asked him about it. The teacher made a comment, and another student (Ali) did the correction of the mistake (Appendix F).
In sample 2.2.4, a student (Aya) asked the teacher a question about whether she can use a certain method then she answered directly her question by presenting an explanation of her thinking. Then, the teacher redirected the question to the class, and a discussion was launched. The teacher asked a student (Shirine) to state the proof; she did a mistake. The teacher did not make the correction, but asked another student (Sandra) to do it. The teacher asked general questions, and students participated in answering. Also, the teacher elaborated the ideas discussed. The same student (Shirine) who did the previous proof suggested another solution (Appendix F).

In sample 2.2.5, the teacher asked a student (Inas) to repeat her suggestion of proof. The teacher asked her to explain how she did the proof. Also, another student (Nizar) asked her to explain her way. She explained her method, but it was wrong. Again, the teacher asked her to explain. She did not know, so another student (Shirine) answered and stated the proof, and the teacher elaborated. Then, the teacher asked a general question about the conclusion, a student (Sally) answered. At the end, the teacher asked general questions to state the different properties they used in solving the exercise. The students were motivated, and most of them participated in answering (Appendix F).

Analyzing the observations done before and during intervention yielded the following:

In the first observation before intervention, the teacher applied mostly the IRE sequence. However, there were few instances when a student corrected another student, or when the teacher asked either leading or prompting questions to a student so that he/she would find the answer or the proof.
The second observation before intervention showed more teacher-student interaction. The teacher gave more chances to students to participate in answering questions. Moreover, when a student did not know the proof or did a mistake, he did not make the correction. The teacher either asked other students to solve it or asked the same student some leading or prompting questions to help him/her in stating the proof or correcting his/her mistake. Then, he elaborated the idea mentioned. In addition, there were many instances when a student asked the teacher a question either to suggest another solution or to ask about the proof. The teacher sometimes answered or redirected the question to the students.

In the two observations done during intervention, the most obvious change was that students interacted with each other. When a student did not understand a certain statement in the proof, the student would ask the classmate presenting it to explain. This was not found in the two observations before intervention—where all the questions were addressed to the teacher. Also, there were many instances when students asked the teacher either to suggest a new method to solve the exercise—the teacher accepted the different methods— or to inquire about the proof—the teacher either redirected the question and asked other students to explain to him/her, or he asked the student some leading or prompting questions that can help him/her to know the answer. Moreover, when the teacher asked a question and the student did not know the answer, he would ask another student to state the answer and made sure to ask again the student who did not know to repeat, inorder to make sure that he/she understood. In these two observations, more students participated in solving the exercises.
Also, the researcher analyzed the verbal flow charts done before and during the intervention to check students' interaction. It was clear that in the two observations done before the intervention, the number of questions—whether prompting or leading—asked to a student were less in the observations done before than during the intervention. Also, not many students participated to solve an exercise or to answer a question. The students were rather slow in expressing their mathematical ideas in English because it is not their native language. Moreover, there was no student-to-student interaction either in solving an exercise or in discussing a certain idea or proof. It was mainly teacher-to-student interaction. When the teacher asked a question, the student answered. The teacher was the one who did the evaluation to check whether the answer was correct or not.

The most remarkable change in the observations done during the intervention was student-to-student interaction. The flow charts show clearly arrows going across the class between students. Students were motivated to participate and ask each other questions either to evaluate other classmate's solution or to ask about a certain idea if they did not understand it.

As a result, many solutions to the same question appeared. Their correctness and validity was evaluated by the students themselves and their classmates rather than by the teacher. There were instances of different solutions suggested by the same student. We can attribute this change to the fact that the researcher and the teacher agreed on some guidelines for promoting students' interaction and class discussions to be implemented during the intervention phase such as: encouraging students to ask and answer questions, involving students in correcting other students' answers, requiring
students to justify and explain each solution that was presented. Obviously, the implementation of these guidelines helped the students to be better communicators. Another obvious change was that the teacher asked many general questions—mostly leading questions—to the class either to launch a discussion or to make sure that most of the students understood the proof. This allowed students to participate more in the discussions launched.

Teacher's Interview

The researcher conducted an interview with the teacher to obtain data about his perception of using mathematical group discussions, whether he uses group work and class discussions, the frequency of using discussions, and his belief about the possible positive effects of mathematical discussions on students' problem solving abilities.

The analysis of the interview with the teacher yielded the following results:

*Difficulties Teachers may Face in Classroom Discussions*

Mr. X believes that the teacher will face many difficulties in classroom discussions. First, many students “don’t follow the rules” to discuss mathematical concepts. Second, they ask questions not related to the lesson, and then the teacher may be lead into a discussion which has no relation with the lesson. Third, the teacher may find during discussions some students who don’t have an idea about the lesson. In this case according to Mr. X, the teacher has to “do the reteaching in a special way” not by repeating the lesson. Or, the teacher can choose a student “that I can depend on” to help students who do not understand. Fourth, the teacher sometimes faces discipline problems.
Use of Group Discussions

Mr. X has been applying group work since the beginning of the school year. He uses group discussions whenever he wants to introduce a new lesson during an activity, or when there is a new concept to be taught. Mr. X usually divides the students into four or five heterogeneous groups that include low- and high-achieving students. In each group, there is a “writer” and a “representative”. These groups will be changed once or twice during the year depending on whether the group “is working well”. He also uses group discussions when solving homework. Students would discuss the results in their groups, and then one student from each group would state the solution. The teacher will not intervene in the discussion, unless students don’t agree on a solution. Mr. X believes that group discussions are important. Students should follow class rules such as asking for permission before talking to ensure that they are listening to each other and paying attention in order to participate in the discussion. Also, students will compete to do their best. When students work in groups, they will not give up directly. They will do their best to reach the solution.

Students’ Problem Solving Abilities

The teacher identified problem solving abilities as reading the problem, understanding the objective and what is required. Mr. X believes that group discussions may enhance students’ problem solving abilities. According to him, this is due to the fact that students will read the problem, and try to find an answer by identifying the goal they want to reach. However, not all students reach a solution; it depends on which type of problems they are dealing with whether it is one step or multi-step problem. According to Mr. X, the low achieving students are usually depressed when they know
that the session is about problem solving. He stated that they will not try to solve the exercise unless they are told that it is an easy problem.

Some students face difficulties in problem solving sessions because: 1) they don’t divide the problem into smaller parts. “They look at the problem as a whole, they don’t break it into steps”, 2) others don’t read the problem “twice or three times” in order to understand the problem very well. The teacher stated that some students usually face difficulties when solving problems that need higher level of analysis. However, working in groups, students can benefit from their friends’ ideas by following the same strategy. Also, the teacher can guide the students when solving problems by giving hints either during classroom discussions or group work.

As a conclusion, Mr. X believes that group discussions may enhance students’ problem solving abilities by reading the problem, and trying to find an answer by identifying the goal they want to reach. According to him, students will compete to do their best. Mr. X uses group discussions in his class, and this will help students to listen to each other and pay attention in order to participate in classroom discussions. Through classroom discussions, the teacher can check whether students understand a certain topic or not. In this case, Mr. X will do reteaching, or ask other student to explain the topic. However, the teacher should take into consideration the difficulties that arise when implementing discussions in the classroom such as asking questions that are not related to the lesson, and sometimes discipline problems.
Cognitive Processes Used by the Students During Focus Group Interviews

The researcher analyzed the two sets of focus group interviews before and during intervention to investigate the cognitive processes used by the students during mathematical discussions. The framework developed by Artzt and Armour-Thomas (1992) is adopted for protocol analysis in order to examine the interactions between the cognitive and metacognitive processes observed in the students' problem-solving behaviors working in small groups on two geometry problems. The framework includes eight problem-solving episodes which are: read, understand (meaning trying to understand), analyze, explore, plan, implement, verify, watch and listen. The verbal behaviors (hereafter referred to as behaviors) are categorized into cognitive and metacognitive behaviors. The episode “watch and listen” was not considered for the analysis of the behaviors of the present study's participants.

In grade seven before the intervention, the researcher conducted focus group interviews for three sessions: 1 hour and 30 minutes, 1 hour, and 45 minutes respectively. A year after during the intervention, the researcher conducted focus group interviews with the same students in grade eight for three sessions: 1 hour, 1 hour, and 55 minutes respectively. The six sessions were audiotaped. The researcher transcribed the six audiotapes and analyzed them according to the eight problem-solving episodes. Behaviors of reading were categorized as cognitive, episodes of understanding (trying to understand), analyzing, and planning were categorized as metacognitive, whereas behaviors of exploring, implementing, and verifying were sometimes categorized as
cognitive and sometimes as metacognitive, according to the indicators mentioned in the framework (Appendix G).

**Metacognitive and Cognitive Behaviors**

Table 3 shows the global number and percentage (out of the total number) of behaviors coded as metacognitive, and cognitive. The break-down of these behaviors according to their nature is presented in Table 3.

**Table 3**

*Number (and percentage out of the total number) of metacognitive, and cognitive behaviors per group*

<table>
<thead>
<tr>
<th>Behavior category</th>
<th>FII (BI)</th>
<th>FIII (BI)</th>
<th>Total B</th>
<th>FI (DI)</th>
<th>FII (DI)</th>
<th>FIII (DI)</th>
<th>Total D</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metacognitive</td>
<td>32 (58)</td>
<td>23 (53)</td>
<td>83 (53)</td>
<td>47 (55)</td>
<td>33 (54)</td>
<td>35 (59)</td>
<td>115 (56)</td>
<td>198</td>
</tr>
<tr>
<td>Cognitive</td>
<td>23 (42)</td>
<td>20 (47)</td>
<td>72 (47)</td>
<td>39 (45)</td>
<td>28 (46)</td>
<td>24 (41)</td>
<td>91 (44)</td>
<td>163</td>
</tr>
<tr>
<td>Total</td>
<td>55 (100)</td>
<td>43 (100)</td>
<td>155 (100)</td>
<td>86 (100)</td>
<td>61 (100)</td>
<td>59 (100)</td>
<td>206 (100)</td>
<td>361</td>
</tr>
</tbody>
</table>

*Note.* Group FII (BI): Focus group I before intervention.
Group FII (BI): Focus group II before intervention.
Group FII (BI): Focus group III before intervention.
Group FI (DI): Focus group I during intervention.
Group FII (DI): Focus group II during intervention.
Group FIII (DI): Focus group III during intervention.
Total B: Total number of behaviors before intervention.
Total D: Total number of behaviors during intervention.

The first line of numbers for each category (no parentheses) expresses the number of behaviors in that category for the group, while the numbers between parentheses expresses the percentage (out of the total number) of each category for each group.

The total number of behaviors increased by 51 behaviors during intervention.

The average of behaviors in the set of focus groups before intervention is 0.8 behaviors
per minute, and 1.2 behaviors per minute for the behaviors done in the set of focus groups during intervention, which means 50% increase in the number of behaviors, for the same period of time. Also, the number of metacognitive and cognitive behaviors increased. The average of metacognitive (cognitive) behaviors in the set of focus groups before intervention is 0.4 (0.4) behaviors per minute, and 0.6 (0.5) behaviors per minute for the metacognitive (cognitive) behaviors done in the set of focus groups during intervention, which means 50% (25%) increase in the number of metacognitive (cognitive) behaviors, for the same period of time. This is an evidence that the students participated and interacted more in the discussions to solve the problems and they regulated and monitored more their work in the set of focus groups during intervention that helped them to reach the solution. This was clearly reflected in focus groups II and III during intervention which had shown an increase of 10% and 11% in the percentage of metacognitive behaviors.

The metacognitive behaviors before intervention ranged from a low of 49% to a high of 58%; whereas the cognitive behaviors ranged from a low of 42% to a high of 51%. Of 155 behaviors coded, 53% were metacognitive, and 47% were cognitive.

However, the metacognitive behaviors during intervention ranged from a low of 52% to a high of 59%; whereas the cognitive behaviors ranged from a low of 41% to a high of 48%. The metacognitive behaviors were increased as compared to cognitive ones (56% versus 44%). In all the focus groups during intervention, the metacognitive behaviors compared to the cognitive ones is more.
Problem Solving Episodes and Cognitive Levels

Table 4 lists the number and percentage of metacognitive, and cognitive behaviors coded by category for each group before and during intervention.

Table 4

Number (and percentage out of total number of behaviors per group) of cognitive, and metacognitive behaviors by problem solving category

<table>
<thead>
<tr>
<th>Behavior category</th>
<th>FI (BI)</th>
<th>FII (BI)</th>
<th>FIII (BI)</th>
<th>Total B</th>
<th>FI (DI)</th>
<th>FII (DI)</th>
<th>FIII (DI)</th>
<th>Total D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metacognitive</td>
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<td></td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>Understand</td>
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<td>32</td>
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<td>(trying to understand)</td>
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<td>13</td>
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<td>Analyze</td>
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<td>20</td>
<td>12</td>
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<td>13</td>
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<td>(13)</td>
<td>(14)</td>
<td>(8)</td>
<td>(22)</td>
<td>(15)</td>
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<td>7</td>
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<td>10</td>
<td>9</td>
<td>29</td>
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<td>(planning)</td>
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<td>(9)</td>
<td>(5)</td>
<td>(12)</td>
<td>(16)</td>
<td>(15)</td>
<td>(14)</td>
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<tr>
<td>Plan</td>
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<td>8</td>
<td>6</td>
<td>19</td>
<td>6</td>
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<td>4</td>
<td>14</td>
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<tr>
<td>(planning)</td>
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<td>(14)</td>
<td>(14)</td>
<td>(12)</td>
<td>(7)</td>
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<td>Implement</td>
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<td>3</td>
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<td>9</td>
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<td>(planning)</td>
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<td>(2)</td>
<td>(7)</td>
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<td>Verify</td>
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<td>2</td>
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<td>5</td>
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<tr>
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<td>(0)</td>
<td>(1)</td>
<td>(2)</td>
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<td>Cognitive</td>
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<td>Read</td>
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<tr>
<td>(planning)</td>
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<td>(10)</td>
<td>(8)</td>
<td>(9)</td>
<td>(12)</td>
<td>(10)</td>
</tr>
<tr>
<td>Explore</td>
<td>12</td>
<td>20</td>
<td>6</td>
<td>38</td>
<td>18</td>
<td>11</td>
<td>9</td>
<td>38</td>
</tr>
<tr>
<td>(planning)</td>
<td>(22)</td>
<td>(35)</td>
<td>(14)</td>
<td>(25)</td>
<td>(21)</td>
<td>(19)</td>
<td>(14)</td>
<td>(19)</td>
</tr>
<tr>
<td>Implement</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>13</td>
<td>6</td>
<td>3</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>(planning)</td>
<td>(10)</td>
<td>(7)</td>
<td>(7)</td>
<td>(8)</td>
<td>(7)</td>
<td>(5)</td>
<td>(7)</td>
<td>(6)</td>
</tr>
<tr>
<td>Verify</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>8</td>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>(planning)</td>
<td>(5)</td>
<td>(0)</td>
<td>(5)</td>
<td>(3)</td>
<td>(9)</td>
<td>(13)</td>
<td>(7)</td>
<td>(10)</td>
</tr>
<tr>
<td>Total</td>
<td>55</td>
<td>57</td>
<td>43</td>
<td>155</td>
<td>86</td>
<td>61</td>
<td>59</td>
<td>206</td>
</tr>
</tbody>
</table>

As shown in table 3, the percentage of metacognitive behaviors is 53% before intervention. The highest percentage of metacognitive behaviors was in the category “understanding” (trying to understand) 39% - of all metacognitive behaviors, followed
by the category “analyzing” that got 16% - of all metacognitive behaviors. Focus group III (BI) had the highest percentage (23%) in the category “understanding” (trying to understand), but it got the lowest percentage (7%) in the category “analyzing”. Whereas focus group I (BI) had the highest percentage (20%) in the category “analyzing” and focus group II (BI) got the lowest percentage (17%) in the category “understanding” (trying to understand).

Also, as noted in table 3, the percentage of metacognitive behaviors is 56% during intervention. The highest percentage of metacognitive behaviors was in the category “analyzing” 26% - of all metacognitive behaviors, followed by the category “exploring” that got 25%. Focus group III (DI) got the highest percentage (13%) in the category “analyzing”, but it got the lowest percentage (9%) in the category “exploring”. Whereas focus group I and II (DI) got the same highest percentage (10%) in the category “exploring”, and focus group II (DI) got the lowest percentage (5%) in the category “analyzing”.

The 50% increase in the metacognitive behaviors during intervention is clearly shown in the category “exploring” that increased more than the double; whereas the category “analyzing” increased by 15%. However, the category “understanding” (trying to understand) got a decrease of 38%. This may be due to the fact that students understood the conditions of the problem better that helped them not to repeat many times these conditions to themselves or to other students.

The percentage of cognitive behaviors, as noted in table 3, was 47% before intervention. The highest percentage of cognitive behaviors was in the category “exploring” 53% - of all cognitive behaviors, followed by the category “reading” that
got 22% - of all cognitive behaviors. Focus group II (BI) got the highest percentage (35%) in the category “exploring”, and focus group III (BI) got the lowest percentage (14%); whereas focus group III (BI) got the highest percentage (21%) in the category “reading”, and focus group I (BI) got the lowest percentage (4%) in the category “reading”.

Furthermore, the percentage of cognitive behaviors, as noted in table 3, was 44% during intervention. The highest percentage of cognitive behaviors was in the category “exploring” 42% - of all cognitive behaviors, followed by the category “reading” and “verification” that got 22% - of all cognitive behaviors. Focus group I (DI) got the highest percentage (21%) in the category “exploring”, but got the lowest percentage (8%) in the category “reading”. Whereas focus group II (DI) got the highest percentage (13%) in the category “verification”, and focus group III (DI) got the lowest percentage (7%) in the same category.

Also, there was an increase of 25% in the cognitive behaviors during intervention, which was clearly shown in the category of “verification” that increased from 3% to 10%. It is evident that the student made sure that the solution done was correct, and was better able to judge and evaluate their work.

Analyzing tables 3 and 4 gave an evidence that a change had happened between the sets of focus group interviews done before and during intervention. The “exploring” category (cognitive and metacognitive) changed from 30% of the total behaviors before intervention to 33%, during intervention (adding the percentages of cognitive and metacognitive). Also, the “verifying” category (cognitive and metacognitive) showed a change from 4% before intervention to 12% during intervention, but the
“implementing” category did not show any change- 10% before and during intervention. However, the behaviors in the two categories “exploring” and “implementing” were redistributed more at the metacognitive level during intervention, but the behaviors in the “verifying” category were redistributed in the cognitive level. This is an evidence that the students participated in the solutions by making decisions whether to continue solving or stop and look for another way. They checked, built on, or revised the previous decisions. At the end, they made judgments and evaluated the process.

This change can be attributed to the interactive atmosphere set and maintained in the classroom, which encouraged students to ask each other questions, discuss ideas with their peers, suggest and evaluate different solutions or arguments to explain their thinking to their peers or to convince them that a solution is valid.

Discussion of Results

Both metacognitive and cognitive behaviors are important during problem solving. The metacognitive behaviors help students to monitor and regulate their work, and give an idea how they may serve to enhance the problem solving process. However, this can’t happen without students who are able to implement these metacognitive behaviors. A balance between metacognitive and cognitive processes is necessary for successful problem solving to occur.

However, the increase of metacognitive processes as compared to cognitive ones is an indicator of improvement in problem solving abilities.
**Focus Group Interviews According to Students’ Interactions**

The researcher conducted focus group interviews with the students before and during the intervention phase. During the focus group interviews, the researcher made it clear to the students that questions such as: “Is this solution correct?”, “can I do this?”, or “can we use this method?” are not allowed. In grade seven before the intervention, the researcher conducted focus group interviews for three sessions: 1 hour and 30 minutes, 1 hour, and 45 minutes respectively. A year after during the intervention, the researcher conducted focus group interviews with the same students in grade eight for three sessions: 1 hour, 1 hour, and 55 minutes respectively. The six sessions were audiotaped. The researcher transcribed the six audiotapes and analyzed them according to the types of interaction among students.

The analysis adopted the following classification of students’ interactions according to the following categories: asking to repeat, repeating, explaining by giving evidence from exercise, checking understanding, asking questions about the proof, deduction, analysis, stating reasons, asking to read, reading, calling for help, asking to draw, explaining how to draw, checking agreement, agreeing, disagreeing, why questions, correcting others’ mistakes, seeking suggestions, suggesting a plan, requiring a proof for a suggested property, suggesting to find another way, reminding about what is required to prove, no answer, and making conclusion.

*Types of Interactions Among Students Before and During Intervention*

Table 5 summarizes the results of the types of interaction among students before and during intervention.
Table 5  
*Types of interactions during three focus group interviews before and during intervention*

<table>
<thead>
<tr>
<th>Types of interactions</th>
<th>Before Intervention</th>
<th>During Intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of Occurrences</td>
<td>Percentage</td>
</tr>
<tr>
<td>Asking to repeat</td>
<td>51</td>
<td>8</td>
</tr>
<tr>
<td>Repeating</td>
<td>104</td>
<td>15</td>
</tr>
<tr>
<td>Explaining by giving evidence from exercise</td>
<td>10</td>
<td>1.5</td>
</tr>
<tr>
<td>Checking understanding</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>Asking questions about the proof</td>
<td>56</td>
<td>8</td>
</tr>
<tr>
<td>Deduction</td>
<td>103</td>
<td>15</td>
</tr>
<tr>
<td>Analysis</td>
<td>10</td>
<td>1.5</td>
</tr>
<tr>
<td>Stating reasons</td>
<td>48</td>
<td>7</td>
</tr>
<tr>
<td>Asking to Read</td>
<td>10</td>
<td>1.5</td>
</tr>
<tr>
<td>Reading</td>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td>Calling for help</td>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>Asking to draw</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Explaining how to draw</td>
<td>23</td>
<td>3</td>
</tr>
<tr>
<td>Checking agreement</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>Agreeing</td>
<td>42</td>
<td>6</td>
</tr>
<tr>
<td>Disagreeing</td>
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<td>5</td>
</tr>
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<tr>
<td>Correcting others’ mistakes</td>
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<td>4</td>
</tr>
<tr>
<td>Seeking suggestions</td>
<td>20</td>
<td>3</td>
</tr>
<tr>
<td>Suggesting a plan</td>
<td>32</td>
<td>5</td>
</tr>
<tr>
<td>Requiring a proof for a suggested property</td>
<td>10</td>
<td>1.5</td>
</tr>
<tr>
<td>Suggesting to find another way</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Reminding about what is required to prove</td>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td>No answer</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>Making conclusion</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>675</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

The percentages before intervention, noted in table 5, demonstrate that the highest was 15% for the categories “Deduction” and “Repeating” followed by the categories “Asking questions about the proof”, “Asking to repeat” and “Stating reasons”.
that got 8%, 8%, and 7% respectively. This means that the students used mathematical properties when they wanted to solve a problem, answer a question asked by their group members, or when a student called for help. Also, the students asked each other questions about a certain concept that they did not know or to inquire about a proof. Also, it is clear that the students asked each other to repeat whenever they had doubts about any idea.

The category “Checking agreement” - as the other categories “Asking to draw”, “Checking understanding”, “Why questions”, and “No answer” - got the lowest percentage (1%). It means that students didn’t accept all solutions. They agreed or disagreed about a certain idea or solution, and argued if they knew that the solution was wrong and submitted evidence to justify their way. Moreover, they recommended alternative solutions by suggesting other plans.

Also, analyzing table 5 for types of interaction during intervention reveals that the category “Deduction” and “Stating reasons” got the highest percentages, 16% and 14% respectively, followed by the categories “Asking questions about the proof” and “Agreeing” that got 11% and 9%; whereas the categories “Checking understanding”, “Analysis”, “Asking to draw”, “Why questions”, and “Reminding about the required to prove” got 1%, the same lowest percentage.

As shown in table 5, there was a slight increase between the number of interactions among students before and during intervention. The average of interactions done by the students in the set of focus groups before intervention is 3.5 interactions per minute, and 3.9 interactions per minute for the interactions done in the set of focus groups during intervention, which means around 11% increase in the number of
interactions, for the same period of time. This is an evidence that students were more comfortable talking to each other and exchanging ideas.

Change in some of the categories was insignificant such as for “Deduction” that shows an increase of 7%. The other categories “Checking understanding”, “Asking to draw”, “Why questions”, “Correcting others’ mistakes”, and “Suggesting to find another way” did not show any change. In both sets of focus group interviews, the category “Why questions” got a low percentage (1%). Although one of the guidelines, that the researcher and the teacher agreed upon to implement during the intervention, was to encourage students to include “why” questions in their discussions; this was not revealed in the focus group interviews. This may be due to the fact that more time should be given during the intervention phase to make sure that students will adopt this strategy more in their discussions.

However, other categories show significant change. The category “Asking questions about the proof” shows an increase of 37.5% (8% to 11%) and “stating reasons” has doubled (7% to 14%); whereas the other two categories “Asking to repeat” and “Repeating” shows a decrease of 50% (8% to 4%) and 60% (15% to 6%) respectively. This means that the students were more engaged in analyzing and thinking either to ask questions about proof or to defend their ideas by stating reasons rather than asking questions to repeat the reasons or the proof.

Also the categories “Agreeing” and “Disagreeing” shows an increase of 50% (6% to 9%) and 20% (5% to 6%) respectively. This shows that the students were more engaged in thinking to decide whether to agree or disagree about a certain idea or
solution. Moreover, students interacted more with each other and submitted reasons to convince their peers whenever they disagreed about a certain idea or solution.

In the focus group interviews during intervention, the students asked each other questions. This helped students to become active participants in the discussions launched. Although there are times when not all students participated, the other students urged them to participate by asking them questions such as: Can you repeat what she had said? What do you think? Do you have any idea? Even though, when a student did not understand a certain topic and his/her partner explained it to him/her, he/she checked understanding by sometimes asking him/her to repeat the solution. Moreover, the students checked if their partners agreed or disagreed not only by asking questions such as: Do you agree? But also questions such as: Why don't you agree? Also, they asked each other questions about proof not in a direct way such as: Why don't we prove that these 2 triangles QAM and NCP congruent? Why can't we think about it?

The change in the results is an evidence that the teacher implemented the guidelines that were agreed upon with the researcher during the intervention phase. This was clear in the way students participated in the discussions held. They were attentive listeners and contributed to the solutions presented either by agreeing or disagreeing. Moreover, the focus of students was on reasoning rather than only getting the correct answers. If they had any doubts about a certain solution, they didn’t hesitate to ask about its validity.

Discussion of Results

From the results shown, the way students interacted with each other had changed. They were listening more to each other, and being attentive to correct the
mistakes done by their peers. Not only they were motivated to participate in the discussions held, but also they made sure that other students are participating. They submitted reasons for their work, and used the deduction method to write their proofs. The students did not accept that their peers did not state reasons for their statements. They either helped them to state the reason or they themselves stated.

*Strategies Used by Students to Formulate Arguments and to Convince Others*

The researcher conducted two sets of focus group interviews before and during intervention. Not only the researcher analyzed these interviews according to types of interaction among students, but also to the strategies they used to formulate arguments and to convince each other.

*Focus Group Interviews before Intervention*

The students used different strategies in the three focus group interviews done before intervention to formulate arguments and to convince others. A qualitative analysis is done and the following categories were identified:

*Explaining a Method or Solution*

Some students explained to others about whether a certain method can be used or not either by providing evidence from the given, or by stating mathematical properties or reasons that are relevant to the problem (see Figure 1 and 2).

*Requiring a Proof*

Some students did not accept a property without proof especially when a student stated that it was clear from the figure. They asked and required the student to state the reason or the proof. For example:
Mazen: Can you repeat again?
Farah: LB is parallel to MO. LB is parallel to MO.
Mohamad: LB is parallel to MO?
Rayan: They didn't say it in the given.
Mazen: We don't have any proof that is parallel. There is no proof and now if you have any proof Farah please can you say to us so will be more accurate?
Farah: No, but it shows that it is parallel. I don't have a specific proof but it shows that LB is parallel to OM.
Mazen: You can't know from only seeing in eyes we should have proofs.

Mohamad: I think it the best way because we have all the given all the given and the proofs.
Mazen: Can you help her Mohamad?
Mohamad: Eh, eh
Mazen: I said that it is it is important to get that the two triangles are congruent. You can't get that angle MON is equals to angle ALM without congruent triangles. Do you have any suggestions?

\[ \text{Solution} \]

1. Proof: \((\triangle LMB) \sim (\triangle ONC)\) or =
2. \(\hat{A} = \hat{M} = \hat{O}\) (corresponding angles)
3. \(\hat{L} = \hat{M} = \hat{O}\) (alternate interior angles)
4. By substitution, \(\triangle MON \cong \triangle ALM\)
5. \(\hat{L} = \hat{O}\) (given)
6. Therefore, the two triangles are congruent by ASA.

Figure 1. Using explanation of a method by stating reasons to solve problem two part one.
Sally: Now through the given we can prove that MON is equals to ALM.
Mohamad H: In the proof we should write to prove in the 2 triangles MON and ALM we have MO is equals to MO because it is a common side.
Sally: This is one side.
Moderator: Rouba do you agree on this statement?
Zuheir: And.
Moderator: Wait a little bit Zuheir.
Rouba: Here they said that prove that MON MON is equals to ALM it is not a common between the 2 triangles.

*Figure 2.* Using explanation of a method by providing evidence from the given to solve problem two part one.

**Refuting a Reason or a Proof**

Moreover, students held a discussion many times in order to convince another student that his/ her reason or proof is wrong, and they did the correction. (See figure 3)

An example for refuting a proof:

*Mazen: Here in number in number two we said it but it is more more it is the nearest in this answer. It is the nearest for the number two. If we need we need only to write number one to be more to be better. It is number one better.
Moderator: Do you agree Aya with him on what he just said?
Aya: No.
Moderator: Why?
Aya: The solution that we just write it should before number two not number one. Because number one should have another solution.
Sarah: Since we have equal enough angles and sides. In the 2 triangles in the 2 triangles LMA. Rani can you continue?
Rani: Can I see the paper? (no answer)
Sherin: We have we have that ON is parallel equals to LM it is given. Can you continue please Rafic?
Rafic: We have OM is equals to OM common side.
Sarah: OM is equals to what?
Rani: OM is equals to OM common side. No, it is wrong. it is wrong.
Sherin: We have we are talking about LMA triangle and OMN triangle and not ONM and OML triangle. We should prove OMN triangle and LMA are congruent. We proved that ON is equals to LM.
Sarah: We did not prove, it is given.
Sherin: It is given sorry, and we should know. Obeida can you continue?

Figure 3. Using refutation of a reason in solving problem two part one.

Defending a Proof

Although a student may suggest a strategy to solve the problem, he/she had to clarify the conditions of this strategy to the students in order to convince them of its validity. (see Figure 4)

Use of Representations

Moreover, students used representations to convince other students about the position of two angles such as the “zed shape” for alternate interior angles. Moreover,
they used notations on the drawing because this will help them to identify the properties of the figure (see Figure 5). Also, they redrew a wrong figure (see Figure 6)

For example:

*Sherin*: We have here OSN OS is parallel to NI. OSN is equals to SNE. We name it E. *Sarah*: How are they equal?
*Sherin*: Alternate interior angles.
*Sarah*: Ah.
*Rani*: They form the letter zed.

**Planning What to Do**

Students sometimes formulated arguments by planning what to do. Even when a student did not agree on a certain reason or procedure, he/she explained his/her idea by providing evidence and reasons (see Figure 7).
Sherin: How?
Sarah: We knew that $NN$ ONI ONI is equals to OSN. They are alternate interior angles, so we can find alternate interior angles with C.
Sherin: We can we have here vertically opposite angles. Yes, we have SNG is equal to ONI it is vertically opposite angles and SN...
Sarah: How can vertically opposite angles?
Obeida: May be it is bisector to CNO because SO is parallel to NI and SO is making perpendicular bisector to CSA and also CNO also should be making bisector to CNO.
Sherin: We have here OSN OS is parallel to NI. OSN is equals to SNE. We name it E.
Sarah: How are they equal?
Sherin: Alternate interior angles.
Sarah: Ah.
Rani: They form the letter zed.
Sherin: And.
Sarah: Here vertically opposite angles SNE is equals to INC.
Moderator: Do you agree on this Ranii? Rani do you agree on this?
Rani: Yes.
Moderator: Can you rephrase what she had just said?
Rani: She said that we have alternate interior angles that is ENS is equals to NSO and we have that.
Sherin: If SNE, SNE is vertically opposite with INC.
Rani: So please write them so we can see them.
Obeida: Because SNE vertically opposite with CNI.
Sherin: Take turns raise your hands, Sarah?
Sarah: ENG, since SNE is equals to OSN, and O is equal to S by substitution we can say that.

*Figure 4.* Showing the way he/she defended the proof in solving problem one part two.
Aya: Rayan should put the marks on the figure so that we should know just by looking that the 2 lines are parallel.
Nizar: Yes, Mazen.
Mazen: We should put. We should mark the lines and to know better what is the nature of the triangles, and to make better thinking of it

Figure 5. Showing the way how students used notations in problem one part one.

Omar: You have to redraw it, you have to redraw it again so that it would be any triangle not isosceles. Zuheir: Now I draw any triangle and I should put the bisector. Rouba: Check if it is not right. Sally: You should make it bigger the triangle so it will be the drawing clear.

Figure 6. Showing the way how students redrew the figure in solving problem one part one.
Rouba: Prove that the triangles MAL and MOL congruent.
Zuheir: First, we should find sides or angles that are equal
and we found LM LM is equals to LM.
Moderator: Could we listen please to Zuheir? Zuheir could you repeat
what you just said?
Zuheir: To prove that they are congruent, we should find
three we should find angle and sides that are equal and
now we found LM is common side
between the two.
Bilal: And we have ALM and OML
we prove it before and it
is two alternate angles.
Sally: But this what you said the angle
MON.
Bilal: ALM is equals to OML we
proved it before and it is 2
alternate interior angles.
Sally: Yes, now I understood it.
Moderator: Ok, any more
suggestions?
Sally: We found one side and one
angle.
Zuheir: We still have either an angle
or either a side.
Bilal: And we have also OLM is equals to
LMA by two alternate
Interior angles.
Rouba: I don't agree with you. I don't agree
with you.
Because AML is not alternate with. It
doesn't.
Bilal: OLM.
Mohamad H: Is OL and MA equal parallel?
Bilal: No, it is not parallel.
Zuheir: They are parallel.
Rouba: You mean LO and MN but this
angles should be LM and one
parallel to it.
Sally: We need a line parallel to LM. We
don't agree on what he said.
Sally: Can we read the given?
Rouba: From the beginning?
Sally: Yes.
Rouba: LMNO is a quadrilateral such
that LM is equals to
ON and MN is equals to OL. LM is
parallel to ON and OL
is parallel to MN. The parallel to MO at L
cuts ON at B and
MN at A.
Mohamad H: Now we discovered that
OL is parallel to MA
given so now we can say that OLM
alternate interior equal
to LMA alternate interior angles.

Figure 7. Showing the way how students planned, agreed or disagreed by providing reasons in solving problem two part three.

Focus Group Interviews During Intervention

Also, the students used different strategies in the three focus group interviews done during intervention to formulate arguments and to convince others. They used almost the same strategies as the ones that were used by students in the three focus groups done before intervention.

Explaining a Method or Solution

Mohamad: I have another solution. If we take the triangle MBN and QPD we will find that they are congruent here they are. DQ is equal to BN and PD is equal to MB parts of equal lines are equal.
Nizar: Yes.
Mohamad: Yes and also we have the angle NBM is opposite angle with PDQ that's why.

Figure 8. Using explanation of a method by stating reasons to solve problem two part one.
Requiring a Proof

For example:

Sally: Can you repeat?
Omar: MQ and PN are going to be parallel because ABCD is a parallelogram making AD is parallel to BC.
Zuheir: But that's not enough to make to prove that this would be parallelogram. We should find more hints to prove it.
Sally: Omar you said MQ is going to be parallel to NP. But I said that may be QP will be parallel and equal at the same time to MN since M is a point on AB and P is a point on DC and N is a point on BC and Q is a point on AD so given that N, MN is equal to QP and now we just proved it parallel so one one pair of equal and parallel sides is going to be a parallelogram

Refuting a Reason or a Proof

(see Figure 9)

Defending a Proof

(see Figure 10)

Use of Representations

The students used and corrected the notations on the drawing because this will help them to identify the properties of the figure (see Figure 11). Also, they redrew a wrong figure (see Figure 12).

Planning What to Do

(see Figure 13)
Shirine: $KA$ is parallel to $BC$ we proved it opposite sides in a parallelogram.
Sarah: Can you slow down please?
Shirine: $KA$ parallel to $BC$ proved opposite sides in a parallelogram.
And $AH$ is parallel to $BC$ also opposite sides in a parallelogram.
Moderator: What do you think we can conclude Obeda?
Obeda: We can conclude that also $BC$ is parallel to $KH$ because parts of parallels are parallel.
Shirine: It's wrong. We say that from a point outside the line we can draw only one parallel to it.

**Figure 9.** Using refutation of a reason in solving problem two part one.
Zuheir: And now we should prove that ACBK and ABCH are two parallelograms. So first we should join the points.
Sally: The quadrilateral.
Mohamad: Can you repeat?
Zuheir: We should join the points to prove that ACBK and ABCH are parallelograms.
Sally: Do you agree on the figure?
Students: Yes.
Sally: Now we are going to start proving ACBK first is a parallelogram ACBK. Does anyone have hint to start proving it?
Bilal: I can prove CBKA by diagonals. We prove that they bisect and KC that KM equal to MC and AM equal to MB.

Figure 10. Showing the way students defended their proof in solving problem two part one.

Mohamad: Nizar here here should put the correct signs. She had made that AD is parallel to CA. Here Nizar: Ya. You meant here ok these are equal to each other. We have to prove it We have the given tha this is a parallelogram
Mohamad: Yes.
Nizar: And we take like these 2 sides are equal and parallel and these 2 sides are equal and parallel and the opposite angles are also equal in a parallelogram.

Figure 11. Showing the way how students used notations in problem one.
Rouba: Sally you draw the figure wrong.
Sally: We are going to draw it again. Can you draw it?
Rouba: We now drawing ABCD is a parallelogram.
Sally: We are redrawing the figure again.
Moderator: Bilal could you tell us what is Rouba doing?
Bilal: We are redrawing the figure because the first figure was wrong so we need to draw it again to be more clear.
Sally: Rouba take any measure of AM and on BC on BC is the point N.
Rouba: I am drawing P on CD on segment CD.
Sally: But they should be equal.

Figure 12. Showing the way how students redrew the figure in solving problem one.

Nizar: I have a suggestion. Why don't we prove that these 2 triangles QAM and NCP congruent?
Aya: No, but if we can do it we have that PN and QM are equal. But we need another something else to prove to complete that it is parallelogram.
Nizar: Yes, Mazen.
Mazen: If Nizar want this triangles to prove them congruent we can get the other 2 triangles and prove them also congruent.
Nizar: Ok

Figure 13. Showing the way how students planned in solving problem one.
However, the researcher identified a new category when analyzing the three focus group interviews during intervention; which is:

**Suggesting New Properties for the Proof**

A student, in focus group III, tried to solve exercise one (Appendix E) by suggesting a new property.

*Omar:* Since $M$, $N$, $P$, $Q$ the points $M$, $N$, $P$, $Q$ are equidistant relatively from $A$ and $B$ and $C$ and $D$ which are the endpoints of the segments in parallelogram $ABCD$ then I suggest that may be if these four points are connected together with segments they will form a parallelogram.

Although some students agreed about this property, others did not agree. A student convinced others by stating that they did not know such a property “From before, from before we didn't know a property like this...... But I don't think that this is a property”. Other student convinced to start with a new proof “I think it is a wrong property. I think we should find another way to prove it as a parallelogram. Any suggestions?” Students recognized the mistake and were convinced to find another way to prove a parallelogram. A student suggested proving triangles congruent, and the other students agreed and found the solution.

Another strategy for argumentation was:

**Asking a Complicated Question**

Students of group I were motivated and this provoked a student to generate a question that was not mentioned in the exercises “Now these parallelograms KACB and ABCH are equal? Are they congruent?” At the beginning, students refused to answer it and one student said “it's very complicated”. However, the student convinced them to
think about it by encouraging them to solve it even it is a complicated question. A student figured out the solution, and convinced others by giving evidence from the given “Is AB is side AB equal AC? No, it's not. Since it is in the given any triangle. The diagonals are here, and AB is a diagonal in ACBK and AC is a diagonal in AHBC that's why they are not equal parallelograms”

It was clear that the students used different strategies to convince other students in both sets of focus group interviews done before and during intervention. However, in the set of focus group interviews done during intervention, students were motivated and eager to participate in the discussions held, and to provide reasons and evidences to convince each other about a certain reason or proof. This was also reflected in the analysis done to the types of interactions among students. The number of questions asked about the proof during intervention increased by 37.5% (see Table 5). It means that if students were not convinced about a certain property, they asked about it. Also, another remarkable difference was that the percentages for the categories “Agreeing” and “Disagreeing” had shown an increase of 50% and 22% respectively. This shows that students analyzed either the reasons or the solutions mentioned before being convinced about or not.

This can be attributed to the fact that the researcher and the teacher agreed about guidelines for promoting students' interaction and class discussions such as: involving students in “inquiry” and “conjecture”, encouraging students to share their ideas, allowing students to work in pairs or groups, and keeping the focus of students on reasoning.
Problem Solving Abilities

The researcher analyzed the two sets of focus group interviews before and during intervention to investigate whether mathematical group discussions enhance problem solving abilities. According to National Center for Education Statistics (2005), students demonstrate problem solving ability in mathematics when they:

recognize and formulate problems; determine the consistency of data; use strategies, data, models; generate, extend, and modify procedures; use reasoning in new settings; and judge the reasonableness and correctness of solutions. Problem-solving situations require students to connect all of their mathematical knowledge of concepts, procedures, reasoning, and communication skills to solve a problem.

The researcher transcribed the audiotapes and analyzed the two sets of focus group interviews to check whether students demonstrate problem solving ability in mathematics according to: recognizing and formulating problems; using strategies; generating, extending, and modifying procedures; and judging the reasonableness and correctness of solutions.

Analysis of the focus group interviews before and during intervention yielded the following:

Focus Group Interviews Before Intervention

The students in the three focus group interviews before intervention used deductive reasoning to write proofs. However, the students sometimes recognized and formulated problems, and judged the reasonableness and correctness of solutions.
In focus group I, a student suggested a way to solve exercise two part one (Appendix C) by using equilateral triangles “And OL is equals to MN it is is an equilateral triangle LO is equals to MN first because it is given second it is an equilateral triangle. It means all sides are equal”. The students did not recognize the mistake, but they suggested changing the procedure by using corresponding and alternate interior angles.

Rayan: We have that $ALM$ is equals to $LMO$ alternate interior angles. $ALM$ is equals to $OML$.  
Mohamad: Can you repeat? 
Rayan: $ALM$ is equals to $OML$ alternate interior angles................. 
Mohamad: We can say that this $ALM$ $ALM$ is equals to $AML$ so they are corresponding corresponding angles.

After a discussion, the students could not find the solution. So, another student recognized and formulated the problem by suggesting to use the strategy of proving triangles congruent.

Mohamad: I have a suggestion that we can prove that this triangle congruent these 2 triangles are congruent and we can easily find the angles we can do it? 
Mohamad: I think it the best way because we have all the given all the given and the proofs. 
Mazen: Can you help her Mohamad? 
Mohamad: Eh, eh 
Mazen: I said that it is it is important to get that the two triangles are congruent. You can't get that angle $MON$ is equals to angle $ALM$ without congruent triangles. Do you have any suggestions?

Solution

\[
\begin{align*}
AFL & = MON \text{ (corresponding angles)} \\
LFA & = MON \text{ (alternate interior angles)} \\
\text{by substitution} & \quad HON = ALM \\
LFA & = ON \text{ (given)} \\
\text{Therefore the two triangles are congruent by ASA.}
\end{align*}
\]

Figure 14. Recognizing and formulating the problem by using congruent triangles.
A student judged the reasonableness of the strategy by saying: “I think it is the best way because we have all the given and the proofs”. Nevertheless, a student recognized that they were solving part two of exercise two instead of one, and she mentioned that part one should have another solution. But the students were not convinced, and insisted to use the strategy of congruent triangles.

Nizar: In the 2 triangles and LMA and ONM we have ...
Mohamad: Aya can you help Nizar?
Aya: In the next question, they told us to prove that triangle MON triangle MON is congruent to triangle MLA. So it doesn’t mean that in the first question we should prove that. We can find another way to prove.
Nizar: Here it is easier for us to prove that the angles MLA and MON are equal.

In focus group II before intervention, a student formulated the problem for exercise one part two (Appendix C) by suggesting to find two equal angles in order to prove that the line is a bisector “we should find INC is equal to NOS to get that to know that NI is the bisector of angle ONC”. However, another student suggested to use the strategy of proving triangles congruent. The students participated to find the solution, but a student judged the reasonableness and correctness of solution by disagreeing because “we don’t have enough equal sides and angles”.

The students agreed about the incorrectness of the strategy and a student suggested to use vertically opposite angles and stated the solution which was wrong (see Figure 15); no one noticed the mistake done.
Sherin: We can we have here vertically opposite angles. Yes, we have SNG is equal to ONI it is vertically opposite angles and SN...
Sarah: How can vertically opposite angles?
Obeida: May be it is bisector to CNO because SO is parallel to NI and SO is making perpendicular bisector to CSA and also CNO to CNO.
Sherin: We have here OSN OS is parallel to NI. OSN is equals to SNE. We name it E.

Figure 15. Suggesting a wrong method in solving problem one part two.

Another student changed the procedure and suggested to use another method also by using vertically opposite angles. He stated the proof correctly, and the students agreed to use this method because “The second method is the best is best because it is easier” [sic].
Obeida: But there are second way. INO we prove that INO
Moderator: Write please Sarah second way.
Obeida: We proved, we proved in the last last exercise that INO is equals to NOS alternate interior angles, and we found that angle ENS is equals to NOS by substitution, so ENS and we also prove that ENS.
Sarah: ENS is.
Obeida: ENS is vertically opposite with CNI.
Sherin: Obeida continue.
Obeida: And CNI so CNI is equals to ENS and ENS is equal to NSO, and NOS.
Sherin: Wait so she can write.
Obeida: And NOS is equals to INO so by substitution INO is equals to CNI.
Moderator: Do you agree on what he had just said?
Students: Yes.
Obeida: So we found now that NI is the bisector of ONC.

Figure 16. Suggesting a new way to solve problem one part two.

Moreover, a student recognized and formulated a problem by suggesting to use the strategy of proving triangles congruent to solve problem two part one (Appendix C). The student judged the reasonableness of the strategy by stating “Because we don’t have enough equal sides and angles”. However, the students tried to solve the exercise, but could not find the solution. Another student suggested to use the substitution method in order to solve the exercise (see Figure 17). He solved correctly and the other students agreed about the solution.
Sherin: Put we have a side.
Obeida: We should put that OL is equals to MN although it is outside the triangle.
Rani: Why?
Obeida: Because may be we get it by substitution of something.
Moderator: Do you agree on this Rani do you agree on what Obeida had said?
Rani: Yes, but what is the proof?

Sarah: We have AML and MNO are corresponding angles, they are equal and we have also we found one side and one angle. We should now find a side or an angle. Can you help?
Obeida: Yes, NOM is equals to LMO alternate interior angles, and OMN is equals to MNO also alternate interior angles. so by substitution we can get that MNA is equals to NOM

Figure 17. Suggesting to use substitution method to solve problem two part one.

In focus group III before intervention, a student solved exercise one part one proving a triangle isosceles - by specifying that "we should find two equal angles in this figure". Another student stated that we can find two equal sides in order to prove a triangle isosceles. However, the students proved by finding two equal angles. As in focus group I, a student recognized and formulated the problem to solve exercise two part by suggesting to use the strategy of proving triangles congruent. Also, the students
did the same mistake as the students in focus group I, and they were convinced that they solved part one and two at the same time.

Zuheir: In exercise one by proving that MON is equal to ALM we proved that the two triangles that are congruent. So, by this we solved the second exercise.

Bilal: We did number two instead of one and by this we proved number one at the same time.

**Focus Group Interviews During Intervention**

Similar analysis was done for the three focus groups during intervention. Also, the students in the three focus group interviews during intervention used deductive reasoning to write proofs. However, in the three focus groups during intervention, the students were more active and held rich discussions in which they showed more problem solving abilities than before intervention, mostly recognizing and formulating problems, as well as judging the reasonableness and correctness of solutions.

Although the students formulated the problem in focus group I to solve exercise one (Appendix E), but they expressed some of their strategies in a different way. For example: “Why don’t we prove that these 2 triangles QAM and NCP are congruent?” He formulated the problem b asking a question to suggest his method. Moreover, another student extended the procedure by stating that “If Nizar want this triangles to prove them congruent we can get the other 2 triangles and prove them also congruent” [sic]. He formulated the problem by building on his friend’s idea.
Mohamad: I think that if we said that M, MA is equal PC. Yes it is equal PC.
Nizar: I have a suggestion suggestion. Why don't we prove that these 2 triangles QAM and NCP congruent?
Aya: No, but if we can do it we have that PN and QM are equal. But we need another something else to prove to complete that it is parallelogram
Nizar: Yes, Mazen.
Mazen: If Nizar want this triangles to prove them congruent we can get the other 2 triangles and prove them also congruent.
Nizar: Ok.

Figure 18. Suggesting to use congruent triangles and another student built on his idea.

Another student solved exercise two part two (Appendix E) without formulating the problem. However, another student tried to formulate the problem and said that “Can we prove that angle CAH and KAC are adjacent, so K, A, H are collinear”. The student who solved the exercise disagreed with him, and explained his way in a detailed manner to convince his peer.

When the student solved exercise two part three, one student generated a question not found in the exercise and urged the students to think about. The students found the solution, agreed about it and judged its reasonableness by specifying evidence from the previous proof done.

Nizar: Why can't we think about it? Yes Mazen.
Mazen: We can't say that these two parallelograms are congruent. We can say that their opposite sides are equal. Here we have KA is equal to BC and AH is
equal to BC. Therefore, we have two parallelograms and line and a segment of parallelogram is equal to both. Did you understand?
Mazen: It is a good question. What is the conclusion?
Mazen: KA is equal to BC and AH is equal to BC, and BC is a segment of is a segment of the parallelograms for both lines. Nizar: Common segment.
Mazen: Common.
Mohamad: I have to say something. Is AB is side AB equal AC? No, it’s not. Since it is in the given any triangle. The diagonals are here, and AB is a diagonal in A, ACBK and AC is a diagonal in AHBC that’s why they are not equal parallelograms.
Nizar: Thank you Mohamad. You cleared it to me very much.
Moreover, students showed in this focus group more attention to others’ mistakes and challenged their peers’ thinking. For example:
Mohamad: I have another solution. If we take the triangle MBN and QPD we will find that they are congruent here they are. DQ is equal to BN and PD is equal to MB parts of equal lines are equal.

Rayan: He said that PD is equal to MB he said parts of equal lines are equal but parts of parallel lines are parallel.
Mohamad: Equal they are equal.
Nizar: Here here Mohamad you are saying these lines these parts of the lines are equal. Oh they are equal by....
Mohamad: By parallelogram.

Another example for students’ correcting others’ mistakes:
Aya: Since ABCD is a parallelogram then the opposite sides should be equal. AB is equal to DC and since AM is equal to. No, and since BN is equal to DQ given then A,
BM is equal to DP remaining part from equal segments.

Aya: We have equal opposite angles. These angles QDP is equal to angle MBN because ABCD is a parallelogram AB is equal to DC opposite sides in a parallelogram and AM is equal to PC given then BM is equal to DP remaining parts from equal segments.
Mohamad: Yes, Rayan can you please denote this?
Aya: So this the parallelogram the quadrilateral QPNM is a parallelogram because we proved that AQ is equal to...
Mohamad: Aya you made a mistake. We just proved that only QP and MN equal to each other. Now we have to prove that QM and PN equal to each other.

In focus group II, a student formulated the problem to solve exercise one by saying: “I think that we should use congruency. We should prove that AQM triangle is congruent to PCN triangle”. However, when students were discussing the solution, one
student stated the conclusion without checking the steps needed to prove congruent triangles. Another student noticed the mistake and did not correct it, but explained it by asking leading questions such as: “What is the side? We didn't have the third the second side. Can you tell us? AM is equal to PC and….”. (see Figure 19).

Also, a student formulated the problem to solve exercise two part one by stating that “Yes, we can prove it by the diagonals they intersect each other”. Also, in this part the students were attentive to correct the reason of “diagonals intersect” and made it clear for the students that diagonals should bisect for a quadrilateral to be a parallelogram (see Figure 20).

In focus group III, the students started the discussion in order to solve exercise one without formulating the problem. However, a student was not convinced and suggested that there was not enough data to prove the quadrilateral a parallelogram. Also, he proposed that students should find more hints to prove it. Another student formulated the problem by specifying that: “May be since the points M and N and P and Q are respectively equidistant from A and B and C and D which are points the endpoints of the segments of the parallelogram ABCD. May be if connected together with the segments they will also form a parallelogram?” [sic]. Although, some students agreed about the strategy by declaring that it may be a new property, other students disagreed and one student insisted that “I think it is a wrong property. I think we should find another way to prove it as a parallelogram. Any suggestions?”

![Diagram of a parallelogram with labeled points A, B, C, D, M, N, P, Q, and H.](image-url)
Another student formulated the problem to solve exercise one by suggesting that “I think we can prove QP and MN by congruent triangle..... so to get that MN equal QP” Students agreed about the strategy, and started the solution. A student continued and formulated the problem by saying “and that is not enough to for to be for the 2 triangles to be congruent. We need a side or an angle”. Also, another student extended the procedure to continue the solution and stated that “we still need to prove that MQ is equal to NP and we can use the same way to find out that the way we used to find out that MN is equal to PQ which is proving that AMQ is equal is congruent to the triangle CNP” Students were more confident in their work and knew how to solve the exercise in an organized way (see Figure 21).

Also, a student formulated the problem to solve exercise two part one by stating that “I can prove CBKA by diagonals. We prove that they bisect”. Students made sure that other students understood the strategy. If not, they explained it again. Another student extended the procedure by saying that “And we can prove that AHCB is a parallelogram by the same way that we proved the other parallelogram”. Students made analogy with the previous proof.
Shirine: I think that we should use congruency. We should prove that AQM triangle is congruent to PCN Triangle.  
Sarah: Can you please repeat?  
Shirine: Prove that AQM triangle is congruent to PCN triangle. 

Shirine: DQ is a side outside the congruent the triangles that you want to prove that they are congruent. We can say QAM equal to NCP opposite angles in a parallelogram are equal. It is an angle. Now we can say DQ is equal to BN it is given and AQ equal CN proved and also given then by substitution AQ.  
Sarah: Can you repeat please?  
Shirine: DQ equal to BN given.  
AQ, AM. Obeida can you continue what I was saying?  
Obeida: I don't know.  
Shirine: Can you help us Rani?  
Rani: Therefore, AQM and PCN are congruent by SAS.  
Sarah: What is the side? We didn't have the third the second side. Can you tell us? AM is equal to PC and....

Solution

1. In the two triangles AQM and PCN we have:  
   \[ AM = PC \text{ (given)} \]
   \[ QAM = NCP \text{ (opposite angles in a parallelogram)} \]

2. DQ = BN \text{ (given)}

3. DA - DQ = QA and BC - BN

Therefore the two triangles are congruent by SAS

Figure 19. Discussing the solution and correcting student's mistake in solving problem one
Obeida: Yes, we can prove it by the diagonals they intersect each other since AM.
Sarah: BM.
Obeida: BM is equal to AM it is given. I is the midpoint and IC is equal to.. 
Sarah: K 
Obeida: KM by symmetry.
Shirine: Obeida you said something wrong in this. Can you correct it for him Rani?
Rani: We can say that KM is also equal to MC since we get it.
Sarah: By symmetry.
Rani: Yes from the given.
Shirine: Obeida told us that they intersect they bisect each other.
Rani: They bisect each other.

\[ \text{Solution} \]

1) \( KM = NC \) (by symmetry)
\[ BN = NA \] (M is the midpoint of AB)
Therefore, \( ABCD \) is a parallelogram

(diagonals bisect each other at their midpt)

\[ BN = NH \] (by symmetry)
\[ AH = NC \] (N is the midpoint of BC)
Therefore, \( ABCH \) is a parallelogram

(diagonals bisect each other at their midpt)

**Figure 20.** Suggesting to use diagonals bisect each other to solve problem two part one.
Rouba: Do you think it is correct? 
Bilal: I think we can prove $QP$ and $MN$ by congruent triangles $QDP$. We can prove it parallel to $MBN$ so to get that $MN$ equal $QP$. 
Mohamad: Yes that's correct. Can we start proving the 2 triangles congruent? 

Zuheir: We proved we proved up till now that the angles $QD$ $QDP$ and $NBM$ are equal and side $QD$ and $BN$ are equal and that is not enough to for to be for the 2 triangles to be congruent. We need a side or an angle so so we found the side $DP$ equal $MB$ because it is proved it is given up. So now we can can prove them that they are congruent. 

Omar: We can conclude that $MN$ is equal to $QP$, we still need to prove that $MQ$ is equal to $NP$ and we can use the same way to find out that the way we used to find out that $MN$ is equal to $PQ$ which is proving that $AMQ$ is equal is congruent to the triangle $CNP$. 

*Figure 21.* Suggesting to use congruent triangles and discussing it in an organized way.

The students in the two sets of focus group before and during intervention demonstrated problem solving abilities by recognizing and formulating problems; using strategies; generating, extending, and modifying procedures; and judging the reasonableness and correctness of solutions. However, the students in the focus groups during intervention recognized and formulated more problems. Also, the students represented the strategies in an organized way that helped other students to understand the solution. They made sure to judge the reasonableness and correctness of solutions
either by stating reasons needed or giving evidence from exercise. Moreover, the students extended the procedures mentioned to use it in other parts.

The change may be attributed to the implementation of the guidelines that the teacher and the researcher agreed upon during the intervention phase. The students got used to discuss ideas with their peers by asking each other questions, suggesting ideas and evaluating different solutions to check its reasonableness and correctness.

The researcher elaborates on the findings in the next chapter Discussion and Conclusions.
Discussion and Conclusions

The purpose of the research was to explore the types of questions used by the teacher in a math class and to investigate the effect of using mathematical group discussions on seventh graders' problem solving abilities in geometry. The researcher collected data by administering unstructured interview, class observations, and focus group interviews. The observations and the interviews were audiotaped. The researcher analyzed the audiotapes to investigate: 1) the types of questions the teacher mostly used in his interaction with students in the math class, 2) the problem-solving cognitive processes that were enhanced by the use of mathematical discussions, 3) the types of interactions students used during mathematical discussions, 4) the strategies that the students used to formulate arguments and to convince each other, and 5) whether mathematical group discussions enhanced problem solving abilities. The analysis yielded the following results:

Types of Questions

The researcher classified the teacher’s questions into four categories: probing leading, prompting, or redirecting. The probing questions were also separated into: probing to assess students’ mathematical knowledge (Probing M), and probing to have insight into students’ thinking (Probing T). The teacher varied his questioning techniques and this helped him to uncover what students are thinking about. The number of questions increased by 29% in the set of observations conducted during intervention. The teacher asked more questions to encourage students to become active participants in their learning process. He used mostly probing questions, which
increased by 25%, to assess students' mathematical knowledge. Also, there was increase by around 36% in the redirecting questions. The teacher provided students with opportunities to think about their thinking and to become better communicators.

*Problem-Solving Cognitive Processes*

Problem solving cognitive episodes are: read, understand, analyze, explore, plan, implement, and verify. The behaviors were categorized as cognitive and metacognitive according to the indicators mentioned in the framework (Appendix G). The number of total behaviors increased by 50% during intervention; whereas the total number of metacognitive (cognitive) behaviors increased by 50% (25%) during intervention.

Moreover, it was evidenced that the percentage of the behaviors in the category “exploring” (cognitive and metacognitive) changed from 30% of the total behaviors to 33%; whereas the “implementing” category showed no change (10%). The behaviors in the two categories “exploring” and “implementing” were redistributed more at the metacognitive level during intervention. Although, the “verifying” category showed a change from 4% before intervention to 12% during intervention, but the behaviors were redistributed at the cognitive level during intervention. Students were actively engaged in the discussions held. They listened to, monitored, and evaluated the different ideas presented in order to find the solution of the problem.

*Types of Interactions*

Students' interactions were classified according to: asking to repeat, repeating, explaining by giving evidence from exercise, checking understanding, asking questions about the proof, deduction, analysis, stating reasons, asking to read, reading, calling for help, asking to draw, explaining how to draw, checking agreement, agreeing,
disagreeing, why questions, correcting others' mistakes, seeking suggestions, suggesting a plan, requiring a proof for a suggested property, suggesting to find another way, reminding about what is required to prove, no answer, and making conclusion.

The number of interactions in the set of focus groups during intervention increased 11%. Students interacted with others to find the solution of the problem.

The category "Asking questions about the proof" showed an increase of 37.5% and "stating reasons" doubled during intervention; whereas the categories "Asking to repeat" and "Repeating" showed a decrease of 50% and 60% respectively. Students were actively engaged in asking their peers questions about the proof. Also, they submitted reasons for their arguments to make them valid and to defend their solutions rather than asking their peers to repeat the solutions or proofs being stated.

The two categories, also, "Agreeing" and "Disagreeing" showed an increase of 50% and 20% respectively. The students agreed or disagreed about the solution, and they clarified their statements. Also, students explained their reasoning to each other.

**Strategies that the Students Used to Formulate Arguments and to Convince Each Other**

Students formulated their own ideas and arguments in order to convince others about their solutions. The researcher identified six categories in the analysis done for the set of focus groups before intervention: explaining a method or solution, requiring a proof, refuting a reason or a proof, defending a proof, use of representations, and planning what to do.

The students explained to their peers whether a certain method can be used by providing evidence from the given or stating mathematical properties or reasons that
supported their idea. Also, students did not accept any idea or method without proof. Not only they refuted any proof or reason that was not correct, but also they submitted reasons to convince others. Moreover, students defended their solutions by explaining their reasonableness to others. Students, also, planned a solution and submitted reasons to convince others, and agreed or disagreed by providing evidence. In addition, students used representations such as coding the figure or sometimes redrawing the figure to help others understand certain properties.

The same categories were identified in the set of focus groups during intervention. However, there were two new categories identified: suggesting new properties for the proof, and asking complicated questions.

**Problem Solving Abilities**

The problem solving abilities listed in the NAEP framework were adopted: recognize and formulate problems; use strategies; generate, extend, and modify procedures; and judge the reasonableness and correctness of solutions. The students in both sets of focus groups before and during intervention used the deductive reasoning to write their proofs. Also they revealed problem solving abilities. They recognized and formulated problems, modified procedures, and judged the reasonableness of solutions. However, in the set of focus groups during intervention, the students were more active and showed more problem solving abilities than before intervention, mostly recognizing and formulating problems, as well as judging the reasonableness and correctness of solutions. Moreover, the researcher analyzed the observations done before and during intervention and designed flow charts to investigate the level of interaction in class.
Both the roles of the teacher and student changed. In the observations during intervention, the teacher was not the only one who evaluated the students’ work. The other students were active listeners, and corrected the mistakes for their peers. Also, the teacher asked more questions to the students to encourage them to be a part of the mathematical discourse; especially he redirected questions to the students to answer. The interactions changed from being teacher-to-student interactions to student-to-student interactions.

As mentioned before, the teacher and the researcher agreed upon guidelines for promoting students’ interaction and class discussions to be implemented during the intervention phase. It is noteworthy that the implementation of these guidelines led to many changes in the teacher’s role as well as students’ role. As shown from the class observations done before and during intervention, it was noticed that with time the students started to address questions to each other, not only to the teacher. The teacher is not any more the only center and dispenser of knowledge or thinking. The questioning techniques shifted from teacher as questioner to students and teacher as questioners. These findings are compatible with those of Ackles, Fuson, and Sherin (2004) in their study.

Definitely, the teacher played an important role in encouraging the students to ask questions to each other by redirecting questions to the class. Also, he helped them analyze the problem by asking leading questions so that they are able to solve it. Hence, as Lampert (1990) and Malloy (1999) recommended, teachers need to pose appropriate questions that enable students to use their own reasoning approaches to solve the problem. Moreover, students were able to explain and defend their mathematical ideas
to their peers. They presented many solutions to the same problem, and the teacher accepted their strategies. Also, they corrected the mistakes done by their peers because they listened to each other. They were active participants in their learning process. As Lee (2006) mentioned, students should participate in the discourse, suggesting their ideas and opinions. Also, they should be responsible for their own learning, thinking and talking about what and how they are learning. They will take the responsibility for sharing their results, explaining and justifying their solutions. As evidenced, the teacher was trying to build a “math-talk learning community”. As Ackles et al. (2004) described it, the class is a community in which students and teacher use meaningful discourse to develop a learning environment so that all students participate.

As for the focus group interviews done before and during intervention, many changes were noticed. The focus group interview provides a setting where students work in groups and verbalize their thoughts orally and aloud which provides a window into students’ cognitive processes and problem solving abilities.

Students were active participants in these focus groups especially during intervention. Even the low-achieving students were participating, suggesting ideas, and asking questions. The interactions among students increased and this was remarkably reflected in the two categories “Agreeing” and “Disagreeing” during intervention. This supports the recommendations by Sherin, Louis, and Mendez (2000) which stress the fact that students should not only talk about mathematical ideas and strategies, but they should also build on other students’ ideas. The authors identified three kinds of building, one of which is agreeing or disagreeing. Students were able to defend their own ideas and build on the other students’ ideas by providing evidences and submitting
reasons. In these focus groups, students’ ideas were the center of the discussions, and this motivated them to think deeply about mathematical concepts. Not only the students generated the ideas needed for the proof, but also a student generated a new question and convinced the other students to think about and try to solve it. As Lampert (1990) stated in order to specify what the students know about mathematics, they should generate a strategy and argue about its legitimacy. Students were able to decide whether their classmates’ solutions were legitimate to be used.

Artzt and Femia (1999) mentioned that mathematical reasoning is an integral part of problem solving. This is evidenced in the analysis done before and especially during intervention. The students were engaged in mathematical reasoning that helped them to understand the problem, analyze it, and plan which method to apply. Nevertheless, some members of the group may become frustrated when they can’t solve problems using a direct approach, because they tend to think that problem solving and mathematical reasoning occur in a linear and direct way. The teacher mentioned in the interview that some students are frustrated when it comes to a problem solving session. He attributed this to the fact that students don’t divide the problem into smaller parts and they don’t try to solve it if it can’t be solved in a direct way.

Engaging students in mathematical reasoning is not the only reason that helped students to recognize a solution for the problem. Metacognitive processes – regulation and monitoring of one’s work- play an important role in helping students realize the problem solution, and to implement the solution. Artzt and Armour-Thomas (1992) suggested that metacognitive behaviors may be the mechanism that facilitates problem solving.
The students in both sets of focus groups demonstrated both metacognitive and cognitive behaviors. However, metacognitive behaviors compared to cognitive ones increased in all focus groups during intervention. This implies that the groups worked effectively listening to, monitoring, and evaluating the different ideas presented to complete the problem. As Artzt and Armour-Thomas (1992) suggested, that a continuous “interplay” between the cognitive and metacognitive processes is essential for successful problem solving to happen. The increase of metacognitive behaviors during intervention is an indicator of improvement in problem solving abilities.

Results show that the behaviors of “exploring” (metacognitive) during intervention increased (5% to 15%), which is compatible with the findings of the study done by Artzt and Armour-Thomas (1992). It seems that the “exploration” initiates the “analysis”, which then initiates more “exploration” and “analysis”. As noted, the problem solving behaviors did not occur in a linear way which reflects the behaviors of expert problem solvers, which match the findings in the study done by Artzt and Armour-Thomas (1992).

Small-group work is very essential in talking and learning mathematics. In this study, students worked in small groups. The students were more motivated to participate in the focus groups done during intervention. They interacted more with one another, interacted more with the task, and used more cognitive strategies, which is compatible with the findings of Webb (1991, as cited in Stacey & Gooding, 1998). Moreover, the students had the chance for more spontaneous verbalization, and discussion of their ideas with their peers. This is an indicator that the teacher
implemented the guidelines that were agreed upon, and that students were used to work in small groups.

A note to be made is that the researcher observed grade eight for one session that was devoted for group work. The teacher formed five groups in a heterogeneous way. He assigned 3 roles: writer, organizer, representative. The researcher observed one group. The following was noted:

Students were listening to each other, asking each other whenever they did not understand a certain concept. The teacher moved around and checked their work. He helped them by asking leading or prompting questions. He stressed communication especially the use of mathematical language. When the time allotted was over, the teacher asked each representative to submit the solution. Moreover, the students were involved in correcting other students' mistakes. The teacher asked the students "why" questions, and varied the questioning techniques depending on the exercise. He required students to justify and explain their solutions.

The metacognitive behaviors increased compared to cognitive ones which is an indicator of improvement in problem solving abilities. Students in the focus groups during intervention demonstrated more problem solving abilities than before intervention, mostly recognizing and formulating problems, as well as judging the reasonableness and correctness of solutions. The results of this study conform to the findings emphasized by NCTM (1989), which support the use of small-group strategy because students' mathematical problem-solving abilities will be improved.

It is obvious that the teacher mostly implemented all the guidelines that the researcher and he agreed upon to promote students' interactions and class discussions in
the set of observations during intervention. This was reflected on students’ interactions; they asked each other more questions about the proof, and stated reasons to validate their work. Moreover, they agreed or disagreed by providing reasons. They used more metacognitive behaviors during intervention especially to conduct exploration about the strategy they had to apply. They regulated and monitored their work, and this helped them to reach the solution. In addition, the students’ problem-solving abilities improved. All of the above-mentioned factors lead to the conclusion that using mathematical group discussions enhanced students’ problem-solving abilities in geometry. As Lee (2006) mentioned, “Communicating is thinking and discourse is a strong motivator in the learning process” (p. 95). Students learned to talk and communicate about their mathematical ideas as they discussed and developed problem-solving strategies. Hence, they had the opportunity to reflect on their own understanding and reasoning about mathematics.
Recommendations

The results of this study will help teachers gain insight on how the use of mathematical group discussions can improve students' problem solving abilities in Geometry. The teacher's role will be expanded to include facilitating classroom discourse in which students are able to explain and justify their mathematical reasoning, and their problem solving strategies. The teacher will ask "why", "what-if" and "how" questions to make sure that students discuss, understand, and propose alternative approaches to solving problems. In addition, teachers will develop and maintain classroom communities in which discourse is emphasized and students' ideas are valued.

Creating and maintaining an environment where mathematical group discussions take place to enhance students' problem solving abilities, as well as whole class discussions, is a challenging task for teachers. However, teachers may find this study a catalyst for them to implement mathematical group discussions in order to improve students' mathematical thinking and their problem solving abilities. Finally, learning with understanding is essential to help students overcome obstacles in the future.

Limitations of the Study

The researcher is aware that the study has some limitations. It needs to be replicated on a bigger sample, which would make it more generalizable. The sample is a convenience sample, which poses problems on non-representativity of that category of students. Also, the time allotted for the study is short. The number of group work sessions were only seven and ranged from 10 to 15 minutes. In addition, the teacher's personality and perceptions play a role in the outcome of this study. Finally, the way
students were grouped may have affected the social interaction among them. Changing the groups may affect the results of this study.

Perspectives for Further Research

This study showed that using mathematical group discussions enhances students' problem solving abilities in Geometry. Further research is needed to investigate the findings of this study when using a bigger sample of teachers and students. Moreover, studies on cognitive and metacognitive behaviors should focus on finding the balance between them for successful problem solving to happen. In addition, more studies are needed to investigate how the communicative skills can be taught to students, especially in proofs.

The researcher did not notice any gender differences in the discussions among students. However, studies are needed to check whether changing the structure of groups will reflect any gender differences in communication. Finally, studies need to examine the suitable structure of groups needed to ensure that problem solving abilities will be enhanced.
REFERENCES


Sfard, A. (2001). There is more to discourse than meets the ears: looking at thinking as communicating to learn more about mathematical learning. *Educational Studies in Mathematics, 46*(1), 13-57.


APPENDICES
APPENDIX A

CHART USED WHILE OBSERVING THE CLASSROOM.

<table>
<thead>
<tr>
<th>Communication in mathematics</th>
<th>Communication about mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use of mathematical language by students</td>
<td>Use of mathematical language by teacher</td>
</tr>
<tr>
<td></td>
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APPENDIX B

QUESTIONS ASKED TO TEACHER

1) Do you use questioning while you teach math?

2) Do students ask you questions?

3) How often do you use group discussion?

4) When you use group discussion, how do you ensure that classroom discussions will help students in learning specific mathematical content?

5) How can you create and maintain an environment where discussions of mathematical concepts take place?

6) What are the difficulties that are faced by the teacher in classroom discussions?

7) Do you think the use of mathematical group discussions enhance students' problem solving abilities? How?

8) What do you think about your students' problem solving abilities?

9) What are your students' major difficulties when solving problems?
APPENDIX C

PROBLEMS OF FIRST FOCUS GROUP INTERVIEW BEFORE INTERVENTION

I) SAC is any triangle. The bisector of \( \triangle ASC \) cuts \([AC]\) at \( O \). The parallel drawn from \( O \) to \((SA)\) cuts \([SC]\) at \( N \).

1) Prove that triangle \( SON \) is isosceles.

2) The parallel drawn from \( N \) to \((SO)\) cuts \((AC)\) at \( I \). Prove that \( [NI] \) is the bisector of angle \( O\hat{N}C \).

II) \( LMNO \) is a quadrilateral such that \( LM=ON \) and \( MN=OL \).

\( (LM) \) is parallel to \((ON)\) and \((OL)\) is parallel to \((MN)\). The parallel to \((MO)\) at \( L \) cuts \((ON)\) at \( B \) and \((MN)\) at \( A \).

1) Prove that \( MON = ALM \).

2) Prove that the triangles \( MON \) and \( MAL \) are congruent.

3) Prove that the triangles \( MAL \) and \( MOL \) are congruent.
APPENDIX D
GUIDELINES FOR PROMOTING STUDENTS’ INTERACTIONS AND CLASS DISCUSSIONS

The teacher should:

- Encourage students to ask and answer questions.
- Encourage students to include “why” questions in their discussions.
- Involve students in correcting other students’ answers.
- Involve students in “inquiry” and “conjecture”.
- Listen to students’ answers, and give them time before providing feedback.
- Provide positive feedback for answers whether they are right or wrong.
- “Revoice” students’ contributions to clarify ideas, introduce new terms, or direct the discussion.
- Vary the questioning techniques.
- Select and use “worthwhile” mathematical tasks that provide students with opportunities to interpret and justify.
- Encourage students to share their ideas.
- Require students to justify and explain each solution that is presented.
- Probe incorrect or incomplete responses.
- Allow students to work in pairs or groups; this will give them a chance to share their ideas.
- Keep the focus of students on reasoning rather than only getting the correct answers.

These guidelines were developed based on the literature, especially Posamentier & Jaye (2006) and NCTM (2000).
APPENDIX E

PROBLEMS OF SECOND FOCUS GROUP INTERVIEW DURING INTERVENTION

I) Let ABCD be a parallelogram. Locate the points M, N, P, and Q respectively on [AB], [BC], [CD], and [AD] such that AM=BN=CP=DQ.

Prove that the quadrilateral MNPQ is a parallelogram.

II) Let ABC be any triangle. M is the midpoint of [AB] and N is the midpoint of [AC].

Consider K the symmetric of C with respect to M and H the symmetric of B with respect to N.

1) Prove that ACBK and ABCH are two parallelograms.

2) Deduce that points K, A, and H are collinear.

3) Deduce that A is the midpoint of [KH].
APPENDIX F
FLOW CHARTS BEFORE AND DURING INTERVENTION

Legend
Answer from teacher to student: ↘ Q

Answer from teacher to student: ↘ A

Question from student to teacher: → Q

Answer from student to teacher: → A

General Questions by teacher: → Q

Explanation by teacher: → A

Answer by all or group of students: ↓ A

Sample i,j,k represents the following:
i=1, sample before intervention.
i=2, sample during intervention.
j=1 first observation.
j=2 second observation.
k= number of sample.
Mohamad H: Q13 to A16, A16 to A17
Mazen: A16
Verbal Flow Chart First Observation Before Intervention
Sample 1.1.2

Rouba: Q23 to A24, and Q29 to A29; Rayan: A25 to A27; Mazen: Q28 to A28; Rouba: Q29 to A29; Sara: Q30 to A33.
Verbal Flow Chart First Observation Before Intervention
Sample 1.1.4

Aya: A44 to A46
Verbal Flow Chart Second Observation Before Intervention
Sample 1.2.1

Bilal: Q12 to A13; Dana: A14; Mahmoud B: Q15 to A17
Verbal Flow Chart Second Observation Before Intervention
Sample 1.2.2

Ziad: Q26 to A27; Baraa: Q28 to A31; Mostapha: Q32 to A35
Verbal Flow Chart First Observation During Intervention
Sample 2.1. 1

Aya: Q11 to A11; Inas: A12 to A15
Verbal Flow Chart First Observation During Intervention
Sample 2.1.2

Nour: Q37 to Q39; Mohamad A: Q40 to Q43; Rayan: A43 to A44.
Verbal Flow Chart First Observation During Intervention
Sample 2.1.3

Sara R: Q57 to Q59 and Q61 to Q63; Nizar: A59 to A60; Mohamad A: Q64 to Q66, and Q68 to A68; Aya: Q66 to A67
Verbal Flow Chart First Observation During Intervention
Sample 2.1.4

Sandra: Q81 to A88; Sara R: A89 to A91
Verbal Flow Chart First Observation During Intervention
Sample 2.1.5

Ali: Q121 to A123; Ghina: A126 to A129
Mathematical Discussions

Verbal Flow Chart Second Observation During Intervention
Sample2.2. 1

Sherine: A11 to Q12, A13 and Q15 to A15; Bilal: A16 to A17; Sandra: A18.
Verbal Flow Chart Second Observation During Intervention
Sample 2.2. 2

Teacher

A36

Rouha

Bilal

Farah

Obeida

A38

Rayan

Rani

A38

Ghina

Abdel Latif

A38

Mohamad A

Mohamad Y

Omar

Nour

Sara B

Rafic

Mazen

Sally

A39

Inas

Nizar

Bilal

Mohamad H

Dina

Ali

Zulair

Sara R

Sandra

Nour: A30 to Q32, Q34 to Q35, and Q37 to A38; Rouha: A36; Mohamad A: A39.
Verbal Flow Chart Second Observation During Intervention

Sample 2.2.2

Teacher

Nour: A30 to Q32, Q34 to Q35, and Q37 to A38; Rouba: A36; Mohamad A: A39.
Aya: Q65 to A65; Sherine: A67, and Q72 to Q74; Sandra: A69.
APPENDIX G
COGNITIVE - METACOGNITIVE EPISODES, DESCRIPTIONS, INDICATORS, AND CODES

Episode 1: Reading the problem (cognitive)

Description: The student reads the problem silently or aloud to the group.

Indicators: The student is observed as:

- Reading the problem.
- Listening to someone else read the problem.

Code:  
a) Initial reading (R1)
b) Rereading (R2)
c) Partial reading (PR)

Episode 2: Understanding the problem (metacognitive)

Description: The student considers domain-specific knowledge that is related to the problem.

Indicators: The student may exhibit any of the following behaviors:

- Restates the problem in his/ her own words.
- Clarifies the conditions of the problem to himself/ herself or others.
- Represents the problem in a different form.

Code:  U

Episode 3: Analyzing the problem (metacognitive)

Description: The student decomposes the problem and checks the relation between the givens and goals of the problem.

Indicators: The student is engaging in an attempt to:

- Simplify the problem.
- Reformulate the problem.
Code: A

Episode 4: Planning (metacognitive)

Description: The student selects a strategy and decides on a solution path to solve the problem. Also, he/she evaluates the status of the problem solution and make decisions for change if needed.

Indicators: The student explains the approach he/she aims to use to solve the problem by:

- Listing the steps to be taken.
- Listing the strategy to be applied.

Code: P

Episode 5: Exploring (cognitive and metacognitive)

Description (cognitive): The student tries to use a trial-and-error strategy in order to find a solution of the problem.

Indicators: The student engages in a:

- Variety of ways in an attempt to find a solution.

Code: Ec

Description (metacognitive): The student monitors his/her work or others’ attempts and decides whether to continue working through the operations or stop.

Indicators: The student may exhibit any of the following behaviors:

- Draws away from the problem to ask himself or herself or someone else what has been done during the exploration.
- Gives suggestions to other students about what to try next in the exploration.
- Evaluates the status of the exploration.

Code: Em
Episode 6: Implementing (cognitive and metacognitive)

Description (cognitive): The student executes a strategy that develops from his/her understanding, analysis and planning decisions. The student transforms the givens into the goals of the problem in a systematic way.

Indicators: The student appears to be engaged in a well-structured and rational way to execute the strategy.

Code: Ic

Description (metacognitive): As in the exploring phase of problem solving, the student regulates and monitors his/her work or others' attempts. However, in this phase, the student checks, builds on, or revises the previous decisions.

Indicators: The student draws away from the problem to see what has been done or where it is leading.

Code: Im

Episode 7: Verifying (cognitive and metacognitive)

Description (cognitive): The student judges and evaluates the outcome.

Indicators: The student makes sure that the solution done is correct.

Code: Vc

Description (metacognitive): The student evaluates the solution of the problem by deciding whether the results revealed adequate problem understanding, analysis, planning, and implementation. If the student finds inconsistency in the solution, he/she makes a new decision to help correct the false cognitive/metacognitive processing that led to the incorrect solution.

Indicators: The student reviews his/her work in many ways by:
Mathematical Discussions

- Checking the solution to see whether it makes sense.
- Explaining to a groupmate how the solution was obtained.

Code: Vm

These episodes need not to occur in the order listed. They may occur many times, and may be avoided.

The framework is adopted from the study done by Artzt and Armour-Thomas (1992)