Performance Analysis of FSO Cooperative Communications with RF Inter-Connected Relays in the Absence and Presence of Channel State Information

By

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ABSTRACT

Free Space Optics (FSO) communication systems have been heavily studied in the literature during the recent decade. Compared to the widely used and traditional Radio Frequency (RF) communication systems, FSO systems are believed to achieve much higher data rates and better performance in terms of feasibility and practicality to compile with the worldwide increased speed need and uprising fast technologies. However, as the case for any new technology, various limitations and challenges such as bad weather conditions, atmospheric turbulence, scintillation, and building sways, face FSO technology. As a result, continuous research is being conducted in the field to overcome these challenges and enhance the system reliability and performance. Among the explored solutions, deploying cooperative schemes in the context of the FSO communication systems was proven to add another degree of freedom for the existent systems. More specifically, diversity methods and inter-relay cooperative schemes are the leading between the discussed techniques as they suggest exploiting the existing relays in the system without any extra needed hardware or resources. Parallel relaying without inter-relay cooperation (NIRC), unidirectional inter-relay cooperation (IRC1), and bidirectional inter-relay cooperation (IRC2) are three main categories that have been studied recently which exploit the existence of adjacent relays in the system to cooperate in the communication process. NIRC schemes use two types of FSO links in the communication system: source to relay $S - R$ and relay to destination $R - D$ links, while IRC schemes benefit additionally from the existence of inter-relay $R - R$ links connecting each pair of adjacent relays. However, none of the done work in the field tackles the possibility of talking advantage of another potential inter-relay links other than those connecting adjacent pairs. Therefore, the work in this thesis proposes a novel method which deploys a full inter-relay connectivity scheme labeled as All-to-All relaying scheme. Conversely, some limitations face the proposed scheme such as the
difficulty of ensuring clear line of sight (LOS) between all the relays in the system which is required for deploying the FSO links. Moreover, the RF links still offer higher reliability in bad weather conditions and at the same time are of broadcasting nature which is more practical for the proposed All-to-All scheme. Moreover, RF links are used as backup for the FSO links of practical FSO systems which exist in the market today. Therefore, we propose in this thesis a novel mixed RF/FSO All-to-All connected system which combines the advantages of both FSO and RF technologies. The proposed system ensures the high data rates empowered by the FSO links deployed for $S-R$, $R-D$, and $S-D$ links. Moreover it maintains high reliability by activating the $R-R$ RF links whenever needed in the communication process. In fact, the proposed system is encouraging as it does not require any extra hardware or resources.

In this thesis, we describe the proposed system components, functionalities, requirements, strategies, advantages and the gains carried with it compared to existing systems and relaying schemes. We also present two protocols that the system can work with: (1) All-Active relaying where all the FSO links are activated to participate in the cooperation strategy and (2) Selective relaying where only the strongest path is selected for the communication process. Selective relaying is showed to achieve higher gains compared with All-Active but with the cost of prior knowledge of channel state information (CSI). Therefore, a detailed comparison and analysis are presented in this thesis at two levels: (1) The proposed system vs existing systems and (2) All-Active relaying vs Selective relaying. The conducted analysis is carried out in terms of outage probability analysis and diversity order analysis where the first studies the effects of implementing the proposed system under the different relaying protocols in lowering the outage probability of the system thus increasing its reliability. In addition, the diversity order analysis presents another performance measurement of the system behavior under different relaying schemes and techniques. The results show superiority of the proposed mixed RF/FSO All-to-All scheme over other existing relaying schemes such as NIRC, IRC1, and IRC2, in addition to the superiority of the Selective relaying protocol over the All-Active relaying protocol. The results are studied theoretical in three aspects: exact, approximate, and simulated analysis, and supported by experimental modeling with numerical results which validate the theoretical analysis.

The flow of work in this thesis is given as follows: Chapter one presents a review of the com-
communication technologies with special highlights on the FSO generalities and the implemented diversity methods. Then, chapter two explains the mathematical tools and concepts such as conditional probability approach, minimum cut set method, and Monte Carlo simulation that are used throughout the whole thesis. Moreover, the system model along with all used channel models and corresponding equations are presented in chapter three. The core of the thesis is presented in chapters four and five where a detailed outage probability analysis and diversity order analysis are presented, respectively. The presented analysis in both chapters covers both special cases and general cases for any number of relays under both relaying protocols: All-Active and Selective. A proper comparison is conducted between the relevant schemes under different network setups and correspondingly valid conclusions are derived theoretically and then verified numerically in chapter six. Finally, the presented work is summarized and potential future work is suggested at the end of this thesis in chapter seven.

Keywords: Free Space Optics, FSO, Radio Frequency, RF, communication, cooperation, cooperative schemes, diversity, relaying, parallel relaying, inter relay, serial relaying, outage, probability, NIRC, IRC1, IRC2, All-to-All, All-Active relaying, Selective relaying, CSI, diversity order, Gamma-Gamma, Rician, Rayleigh, minimum cut set, Monte Carlo, atmospheric turbulence, scintillation.
# Table of Contents

1 Introduction ........................................ 1
   1.1 Communication Techniques ......................... 1
      1.1.1 RF Communication ............................ 2
         1.1.1.1 Description ............................ 2
         1.1.1.2 Channel Models ......................... 3
            1.1.1.2.1 Rayleigh Distribution ............... 3
            1.1.1.2.2 Rician Distribution ................. 3
            1.1.1.2.3 Nakagami-m Distribution ............ 3
         1.1.1.3 Advantages and Limitations ................. 4
      1.1.2 FSO Generalities ............................. 4
         1.1.2.1 Historical Perspective .................... 4
         1.1.2.2 FSO Definition ........................... 6
         1.1.2.3 FSO Applications ......................... 8
         1.1.2.4 Advantages ............................... 9
            1.1.2.4.1 High Data Rate ...................... 10
            1.1.2.4.2 License Free ......................... 10
            1.1.2.4.3 Low Cost ............................ 10
            1.1.2.4.4 Huge Bandwidth and Capacity .......... 10
            1.1.2.4.5 Noise and Interference Resistance .... 11
            1.1.2.4.6 Security ............................ 11
         1.1.2.5 Limitations ............................... 11
            1.1.2.5.1 LOS Immobility and Sensitivity ....... 11
            1.1.2.5.2 Atmosphere and Weather Conditions .... 11
      1.1.3 FSO Channel ................................. 12
   1.2 FSO Channel ....................................... 12
1.1.3.1 Weather .................................................. 12
  1.1.3.1.1 Fog ................................................. 12
  1.1.3.1.2 Rain ................................................. 13
  1.1.3.1.3 Snow ................................................. 13
1.1.3.2 Building Sway ......................................... 13
1.1.3.3 Atmospheric Turbulence ............................... 14
  1.1.3.3.1 Beam Wandering .................................... 14
  1.1.3.3.2 Scintillation ....................................... 14
1.1.3.4 Channel Models ....................................... 14
  1.1.3.4.1 Lognormal Distribution ............................ 16
  1.1.3.4.2 Exponential Distribution ........................... 16
  1.1.3.4.3 Rayleigh Distribution .............................. 17
  1.1.3.4.4 Gamma-Gamma Distribution ........................ 18
1.1.4 Diversity Methods ...................................... 21
  1.1.4.1 MIMO ................................................. 21
  1.1.4.2 Relaying .............................................. 21
    1.1.4.2.1 Serial Relaying .................................. 22
      1.1.4.2.1.1 Advantages and Disadvantages .............. 23
    1.1.4.2.2 Parallel Relaying ................................. 23
      1.1.4.2.2.1 Inter Relay Cooperation (IRC) ............. 24
      1.1.4.2.2.2 Unidirectional IRC ........................... 25
      1.1.4.2.2.3 Bidirectional IRC ............................. 26
      1.1.4.2.2.4 Limitations motivating the All-to-All inter-
        relay communications ................................. 28
    1.1.4.2.3 Relaying Protocols ............................... 29
      1.1.4.2.3.1 All-Active Relaying ......................... 29
      1.1.4.2.3.2 Selective Relaying ........................... 30
1.1.5 Mixed RF/FSO System .................................. 31
  1.1.5.1 Hybrid RF/FSO Systems - Backup RF Links .......... 32
  1.1.5.2 Dual-Hop Mixed RF/FSO Systems ..................... 32
  1.1.5.3 Novel Mixed RF/FSO System .......................... 33
2 Mathematical Tools

2.1 Conditional Probability Approach .................................................... 35
2.2 Monte-Carlo Simulation ................................................................. 38
   2.2.1 Definition ................................................................................. 38
   2.2.2 Advantages ............................................................................. 38
   2.2.3 Applications .......................................................................... 39
   2.2.4 Relation to Our Work ............................................................. 39
2.3 Cut-Set Method ............................................................................. 39
   2.3.1 Definition in Reliability ......................................................... 39
   2.3.2 Algorithms and General Examples ......................................... 40
   2.3.3 Application to Our Work ......................................................... 46

3 System Model and Cooperation Strategies .................................. 48

3.1 System Description and Relaying Protocols .............................. 48
3.2 FSO Links .................................................................................... 51
3.3 RF Links ...................................................................................... 53
3.4 Rate of Transmission .................................................................. 53
3.5 All-Active vs Selective Relaying .................................................. 55
   3.5.1 Affected Equations ................................................................. 55
   3.5.2 New Parameters and Expressions ........................................... 55

4 Outage Probability Analysis ......................................................... 57

4.1 All-Active .................................................................................... 57
   4.1.1 NIRC ..................................................................................... 58
      4.1.1.1 Exact Analysis ................................................................. 58
         4.1.1.1.1 Special Cases ............................................................. 58
         4.1.1.2 General Case ............................................................... 65
      4.1.1.2 Approximate Analysis ..................................................... 67
         4.1.1.2.1 Application to Special Cases .................................... 67
         4.1.1.2.2 Concluding a General Case .................................... 69
      4.1.1.3 Simulation Analysis ....................................................... 71
   4.1.2 IRC1 ...................................................................................... 72
      4.1.2.1 Exact Analysis ................................................................. 72
4.3 All-Active vs Selective ................................................. 117
  4.3.1 Special Cases .................................................. 117
    4.3.1.1 NIRC .................................................. 117
    4.3.1.2 All-to-All ........................................... 120
  4.3.2 General Cases .................................................. 121
    4.3.2.1 NIRC .................................................. 121
    4.3.2.2 All-to-All ........................................... 122
  4.3.3 Identical Expressions - Different Power Distribution ......... 124

5 Diversity Order Analysis .............................................. 125
  5.1 Concept and Derivation ......................................... 125
  5.2 Simplification and Comparison Standards ....................... 127
  5.3 Application to different Relaying Schemes ...................... 128
    5.3.1 NIRC .................................................. 128
      5.3.1.1 Special Cases .................................. 128
      5.3.1.2 General Case .................................... 129
    5.3.2 IRC1 & IRC2 ............................................ 130
    5.3.3 All-to-All Relaying .................................... 131
      5.3.3.1 Special Cases .................................. 131
        5.3.3.1.1 Exact Expressions ......................... 131
        5.3.3.1.2 Approximate Expressions .................. 131
      5.3.3.2 General Case - Approximate Expression ......... 132
  5.4 Comparison and Analysis ........................................ 133

6 Numerical Results .................................................... 137
  6.1 Comparing Relaying Schemes while Varying Relays Positions ... 138
    6.1.1 2 Relays ................................................. 138
    6.1.2 3 Relays ................................................. 141
  6.2 Effect of Increasing the Number of Relays ...................... 143
  6.3 All-Active vs Selective ........................................ 148
  6.4 Rician vs Rayleigh fading for RF links ......................... 152

7 Conclusion and Future Work ......................................... 155
Appendices 171

A  Simulation Analysis Algorithm - All-Active All-to-All Relaying Scheme 172

B  Simulation Analysis Algorithm - Selective NIRC Relaying Scheme 175

C  Simulation Analysis Algorithm - Selective All-to-All Relaying Scheme 177
List of Tables

5.1 Comparing Diversity Order Expressions for different relaying schemes with $N = 2$ relays . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 134

5.2 Comparing Diversity Order Expressions for different relaying schemes with $N = 3$ relays . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 134
List of Figures

1.1 RF Communication System [1] ........................................... 2
1.2 FSO Communication System [2] ...................................... 6
1.4 FSO Transceiver Examples ............................................. 7
1.5 FSO Weather Challenges [4] .......................................... 12
1.6 Atmospheric Turbulence: Scintillation and Beam Wandering [5] 15
1.7 Lognormal Distribution PDF for a 2-km FSO link .................. 17
1.8 Negative Exponential Distribution PDF for a unity mean irradiance 18
1.9 Gamma-Gamma parameters variation with distance ................ 19
1.10 Gamma-Gamma PDF versus Irradiance for various distances .... 20
1.11 m x n MIMO FSO Channel [6] ................................. 22
1.12 FSO system with Serial Relaying [7] .............................. 23
1.13 FSO system with Parallel Relaying [7] ............................ 24
1.14 FSO system with Inter Relay Cooperation [8] .................. 25
1.15 FSO system with IRC1 ............................................. 25
1.16 FSO system with Links Failure Example ........................ 26
1.17 IRC1 saving FSO system with Links Failure ...................... 26
1.18 FSO system with IRC2 ............................................. 27
1.19 FSO system with IRC1 System Failure Example ............... 27
1.20 IRC2 saving FSO system with IRC1 Failure ...................... 28
1.21 FSO system with IRC2 System Failure Example ............... 28
1.22 All-to-All saving FSO system with IRC2 Failure ............... 29
1.23 All Active Relaying Protocol S – R phase ....................... 29
1.24 All Active Relaying Protocol R – D phase ....................... 30
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.9</td>
<td>NIRC with 3 Relays - Case 3</td>
<td>63</td>
</tr>
<tr>
<td>4.10</td>
<td>NIRC with 3 Relays - Case 4</td>
<td>63</td>
</tr>
<tr>
<td>4.11</td>
<td>NIRC with 3 Relays - Case 5</td>
<td>63</td>
</tr>
<tr>
<td>4.12</td>
<td>NIRC with 3 Relays - Case 6</td>
<td>64</td>
</tr>
<tr>
<td>4.13</td>
<td>NIRC with 3 Relays - Case 7</td>
<td>64</td>
</tr>
<tr>
<td>4.14</td>
<td>NIRC with 3 Relays - Case 8</td>
<td>65</td>
</tr>
<tr>
<td>4.15</td>
<td>NIRC with N Relays</td>
<td>66</td>
</tr>
<tr>
<td>4.16</td>
<td>NIRC with 2 Relays - Reliability System</td>
<td>68</td>
</tr>
<tr>
<td>4.17</td>
<td>Incidence Matrix of NIRC system with 2 Relays</td>
<td>68</td>
</tr>
<tr>
<td>4.18</td>
<td>NIRC with 3 Relays - Reliability System</td>
<td>70</td>
</tr>
<tr>
<td>4.19</td>
<td>Incidence Matrix of NIRC system with 3 Relays</td>
<td>70</td>
</tr>
<tr>
<td>4.20</td>
<td>IRC1 with 2 Relays</td>
<td>73</td>
</tr>
<tr>
<td>4.21</td>
<td>IRC1 with 2 Relays - Case 1</td>
<td>73</td>
</tr>
<tr>
<td>4.22</td>
<td>IRC1 with 2 Relays - Case 2</td>
<td>73</td>
</tr>
<tr>
<td>4.23</td>
<td>IRC1 with 2 Relays - Case 3</td>
<td>74</td>
</tr>
<tr>
<td>4.24</td>
<td>IRC1 with 2 Relays - Case 4</td>
<td>74</td>
</tr>
<tr>
<td>4.25</td>
<td>IRC1 with 3 Relays</td>
<td>75</td>
</tr>
<tr>
<td>4.26</td>
<td>IRC1 with 3 Relays - Case 1</td>
<td>76</td>
</tr>
<tr>
<td>4.27</td>
<td>IRC1 with 3 Relays - Case 2</td>
<td>76</td>
</tr>
<tr>
<td>4.28</td>
<td>IRC1 with 3 Relays - Case 3</td>
<td>76</td>
</tr>
<tr>
<td>4.29</td>
<td>IRC1 with 3 Relays - Case 4</td>
<td>76</td>
</tr>
<tr>
<td>4.30</td>
<td>IRC1 with 3 Relays - Case 5</td>
<td>77</td>
</tr>
<tr>
<td>4.31</td>
<td>IRC1 with 3 Relays - Case 6</td>
<td>78</td>
</tr>
<tr>
<td>4.32</td>
<td>IRC1 with 3 Relays - Case 7</td>
<td>78</td>
</tr>
<tr>
<td>4.33</td>
<td>IRC1 with 3 Relays - Case 8</td>
<td>78</td>
</tr>
<tr>
<td>4.34</td>
<td>IRC1 scheme with 2 Relays - Reliability System</td>
<td>80</td>
</tr>
<tr>
<td>4.35</td>
<td>Incidence Matrix of IRC1 system with 2 Relays</td>
<td>80</td>
</tr>
<tr>
<td>4.36</td>
<td>IRC1 scheme with 3 Relays - Reliability System</td>
<td>82</td>
</tr>
<tr>
<td>4.37</td>
<td>Incidence Matrix of IRC1 system with 3 Relays</td>
<td>82</td>
</tr>
<tr>
<td>4.38</td>
<td>IRC2 with 2 Relays</td>
<td>84</td>
</tr>
<tr>
<td>4.39</td>
<td>IRC2 with 2 Relays - Case 1</td>
<td>84</td>
</tr>
<tr>
<td>4.40</td>
<td>IRC2 with 2 Relays - Case 2</td>
<td>84</td>
</tr>
</tbody>
</table>
6.3 Comparison between the four relaying schemes for $N = 2$ relays, Case 3 . . . . 140
6.4 Comparison between the four relaying schemes for $N = 2$ relays, Case 4 . . . . 141
6.5 Comparison between the four relaying schemes for $N = 3$ relays, Case 1 . . . . 143
6.6 Comparison between the four relaying schemes for $N = 3$ relays, Case 2 . . . . 144
6.7 Comparison between the four relaying schemes for $N = 3$ relays, Case 3 . . . . 144
6.8 Comparison between the four relaying schemes for $N = 3$ relays, Case 4 . . . . 145
6.9 Performance of NIRC scheme under increasing number of relays . . . . . . . . . 146
6.10 Performance of IRC1 scheme under increasing number of relays . . . . . . . . . 146
6.11 Performance of IRC2 scheme under increasing number of relays . . . . . . . . . 147
6.12 Performance of All-to-All scheme under increasing number of relays . . . . . . . 147
6.13 All-Active vs Selective for NIRC and All-to-All schemes with $N = 2$ relays . . . . 149
6.14 All-Active vs Selective for NIRC and All-to-All schemes with $N = 3$ relays . . . . 149
6.15 All-Active vs Selective for NIRC scheme with 2 and 3 relays . . . . . . . . . . 150
6.16 All-Active vs Selective for All-to-All scheme with 2 and 3 relays . . . . . . . . 150
6.17 All-Active with 2, 3, 4, and 5 relays vs Selective with 2 and 3 relays for All-to-
All scheme . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 151
6.18 Rayleigh vs Rician for All-to-All scheme with 2 relays . . . . . . . . . . . . . . 153
6.19 Rayleigh vs Rician for All-to-All scheme with 4 relays . . . . . . . . . . . . . . 153
6.20 Rayleigh vs Rician for All-to-All scheme with 2 relays - zoomed view . . . . . 154
6.21 Rayleigh vs Rician for All-to-All scheme with 4 relays - extended view . . . . . 154
Chapter 1

Introduction

1.1 Communication Techniques

The first thing that a child learns is to communicate with the surroundings. Humans by nature tend to communicate with each other, and for that purpose, wide communication techniques have been implemented throughout ages.

“I won’t compare ants and people, but ants give us a useful model of how single members of a community can become so organized that they end up resembling, in effect, one big collective brain. Our own exploding population and communication technology are leading us that way.” — Lewis Thomas

Distance and time have been always two main concerns for communicating people, thus increasing the demand for long distance yet fast communication techniques. For achieving this purpose, various technologies have been developed in this area. Many ancient and simple techniques inspired the revolution in communication field such as communication by fire during disasters, smoke signals, acoustic signals, etc... In the 19th century, telegraphy raised using consecutive electrical on/off pulses to transmit data across distance. The same century witnessed the invention of telephone, radio, and computer exploiting electrical communication techniques. Extending these discoveries and inventions, Radio Frequency (RF) is one of the conventional and still currently used communication technologies.
1.1.1 RF Communication

1.1.1.1 Description

The technical term RF stands for Radio Frequency. RF technology is a wireless technology based on electromagnetic signals for transmitting data; this technology has been in use for more than 100 years and occupies wide range of applications [9]. Technically, when talking about radio Frequency, we expect a range of frequencies related to the term; it is from 3 KHz to 300 GHz. In fact, the major chunk of the RF range was from 1 kHz to 1 GHz and was used mainly for radio in addition to military applications [10]. Later on, higher desired data rates and less noise requirement led to extending the frequency range. The field drew huge attention and investments leading to better data security, more efficient transmission, and even more compact and practical transmitters and receivers [11, 12].

Technically, RF transmitters, which are usually installed in buildings unless a mobile user is acting as a transmitter, code and broadcast electromagnetic signals traveling through air to the desired destination area. In their turn, the desired users who have their own receiver antennas, which can be mobile or fixed in a building, receive and decode the transmitted message [13]. Figure 1.1 represents a simplex RF communication system composed of transmitters (TX), transcievers (TRX), and recievers (RX).
1.1.1.2 Channel Models

When talking about RF technology, we consider multipath fading and shadowing represented in lognormal based channel fading models. RF channel models can be represented as any of the Rayleigh, Rician, or Nakagami fading models \[14\]. In what follows, we present each of these channel fading models.

1.1.1.2.1 Rayleigh Distribution

Rayleigh Fading channel model has been heavily linked with RF links \[15–17\]. Rayleigh fading is associated to RF links when there is no direct LOS between the transmitter and the receiver. Its probability density function (PDF) is given by \[17\]:

\[
f(\gamma) = \frac{1}{\bar{\gamma}} \exp \left( -\frac{\gamma}{\bar{\gamma}} \right) \tag{1.1}
\]

Where \(\bar{\gamma}\) stands for the average signal to noise ratio (SNR) of the RF link.

1.1.1.2.2 Rician Distribution

Rician Fading can also be used to model the RF link channel when a direct LOS between the transmitter and the receiver exists \[18, 19\]. Its PDF is given by \[18\]:

\[
f(r) = \frac{2(K + 1)r}{\Omega} \exp \left( -K - \frac{(K + 1)r^2}{\Omega} \right) I_0 \left( 2\sqrt{\frac{K(K + 1)}{\Omega}} r \right) \tag{1.2}
\]

Where \(\Omega\) is the scale parameter defined as average SNR of the RF link, \(I_n(\cdot)\) is the \(n\)th order modified Bessel function of the first kind, and \(K\) defined as the ratio of LOS power component to non-LOS power component \[18\].

1.1.1.2.3 Nakagami-m Distribution

Nakagami-m fading channel model which is related to the gamma function, is another distribution to model the RF links \[14, 20\]. Its PDF is given by \[20\]:

\[
f(x) = \frac{2m^m x^{2m-1}}{\Omega^m \Gamma(m)} \exp \left( -\frac{mx^2}{\Omega} \right) \tag{1.3}
\]

Where \(m\) is the shape parameter of the Nakagami fading model ranging from 0.5 and above.
\( \Omega \) is the spread control parameter, and \( \Gamma(\cdot) \) is the gamma function [20].

1.1.1.3 Advantages and Limitations

For a technology to stay in use for more than 100 years, during the era of technology revolution, it needs really practical and effective strength points regardless of all the new rising technologies. In fact, RF technology has proved to be practical and essential in the domain till today. The main reason lies in its high resistance for any weather condition permitting large range coverage while maintaining capacity and bandwidth requirements. In addition, RF signals can cover long distances and complex paths due to their broadcast nature [21].

Regardless of the fact that RF technology is still considered among the best communication technologies in the world till today, this technology has certain limitations. Licensing, Space, and power are some of these limitations. RF systems need licensing to work in most of the cases. Moreover, they occupy large space for the systems to be installed. Also, with data rates increase, even larger space is required with huge power requirement for their equipment [22, 23]. Even more, due to the broadcasting nature of RF technology, data security is in addition a limitation to the technology [13]. Moreover, due to the continuous data increase and thus higher capacity and bandwidth demands, RF technology has been suffering from weakness in this point. In fact, the large wavelength nature of RF leads to short frequency ranges and small bandwidths leading to a limitation in the capacity of RF systems [21, 24]. All the previously mentioned limitations raised the need for new technologies to follow with the world needs and technology revolution. The next sections discuss these new technologies and how they cover the gaps that are caused by RF. More specifically, we will tackle the optical domain by highlighting optical fibers and Free Space Optics (FSO) technologies and comparing them to Radio Frequency technology and how they can improve the weakness points that were presented with RF technology.

1.1.2 FSO Generalities

1.1.2.1 Historical Perspective

FSO by definition means Free Space Optics; it is in fact a wireless communication technology in the Optical Field domain. To start, we present a historical view of the Optical Communication Field.
Looking into the dictionary, optics can be defined as “scientific study of sight and the behavior of light, or the properties of transmission and deflection of other forms of radiation”. This means that optics is directly related to light, thus when we talk about optical communication we go back historically to the simple and ancient means of communication such as using the sunlight as a mean of communication during wars by ancient Greek and Romans (reflecting the sunlight using their shields to send specific signals) [25].

In recent decades, the optical domain has got huge attention and evolved rapidly. Laser and Light Emitting Diodes (LEDs) are considered among optical wireless communication technologies and they were used and tested in the mid and late 20th century but failed to achieve the desired goals due to laser beams’ large divergence and low resistivity to weather conditions [26, 27]. In the same era, optical fiber links arise with low loss over long distances which encouraged the investment in the field, but what are optical fiber links?

Optical Fiber communication is a technology that transmits information from a source to a destination using optical fiber links. These fibers are made of glass, they are thin and long as well as immune against electromagnetic interference noise and long distances loss. These characteristics of fiber give advantage for optical fiber links over any other metal links in communication field. In fact, fiber links were not initially used for optical communication purposes. Instead, they were first used to look into inaccessible areas in human body by transmitting light and directing it into the desired areas. This can explain the concept of optical fiber communication where a beam of light carrying information from the source traverses an end to end path through fiber links which are practical and efficient for this mission [28].

Optical Fiber communication field proved to be practical and efficient even for long distance communication, but the optical fiber links have some limitations. They are at the end physical links meaning that to transmit huge data they need special installment equipment, enough space, careful cabling procedure due to the fiber glass fragility nature, resulting in high money and time costs for the optical fiber links. Instead, what if we can use fiber optics technology without the actual physical fiber links? [29]

Here came the idea of transforming the cable-based optical fiber technology into a cable-free one. In other words, the goal was to transmit optical information wirelessly without optical fiber links. This technology is known as Free Space Optics (FSO).
1.1.2.2 FSO Definition

FSO is the wireless version of Fiber Optics; it is optical technology for transmitting data through atmosphere without the need for optical physical links [21]. FSO uses optical carriers such as visible, Infrared (IR) or even ultraviolet (UV) beams for the transmission of the message in free space [30]. This free space medium can be classified as outer space, vacuum, and mostly air or atmosphere in addition to any relevant medium [31].

Assume we have a source $S$, a destination $D$ and a message that is needed to be delivered from $S$ to $D$. The message will be first initiated by $S$ as waveforms, modulated onto an optical carrier and then transmitted through a line of sight (LOS) toward $D$. Upon arrival, the optical message is collected and then transformed using a photodetector into an electrical current that the receiver will use to regenerate the initiated message by $S$ [30]. The whole process is represented in Figure 1.2.

So first, the optical source such as laser or LED at the transmitter level produces data bits which will be subjected to encoding and then modulation resulting in a laser beam. This laser beam is to be amplified using an optical amplifier, collected again and transferred through beam forming optics, before being ready for transmission through the channel medium [30].

In the same context, after the message propagates through the channel medium, it reaches the receiver side. At the receiver side, the delivered information in the form of an optical beam is subjected to filtering, collection and focusing before being detected by the photodetector. The detected electrical signal is then further filtered against noise before being demodulated into the recovered message [30].

FSO products are usually bidirectional, meaning they carry a transmitter and a receiver at the same time. To have a clear LOS, transceivers are usually placed at very high places such as roof tops of buildings as shown in Figure 1.3 [32]. Some examples of these FSO transceivers are shown in Figure 1.4.
Figure 1.3: Placement of FSO Transceivers on High Buildings [3]

(a) FSO 10Gbps Transceiver [33]  (b) FSO 1.25Gbps Transceiver [34]

Figure 1.4: FSO Transceiver Examples
1.1.2.3 FSO Applications

FSO networks are drawing more attention and investments in the wireless communication field. The applications of these FSO networks are very wide and powerful ranging from basic applications at home to widest applications at global scale [35]. Due to the many advantages that FSO systems proposed over Optical fiber systems and RF systems, FSO systems have made it widely into the global market in various applications [36]. Among these applications, we state the following [29, 37–43]:

- **Wireless access**: As FSO doesn’t require license, wireless service providers use FSO instead of RF for wireless communication in outdoor activities.

- **Backup Links**: For environment of optical fiber links, FSO can be used as a backup when the primary transmission through the fiber link fails.

- **Last mile access**: FSO links are suggested to be the solution for last mile access issue. The issue lies in providing cables for last mile users which result in very high cost to implement these fiber cables. Instead, FSO is applied to form a high speed yet low cost solution in the last mile.

- **Service Acceleration**: For users who are waiting optical fiber to be implemented in their environment, FSO can provide alternative service during that time as it is very fast to be deployed.

- **Point-to-Point Links**: FSO links are used for direct communication between two points such as two buildings, two towers, two stations, or even two ships.

- **Point-to-Multipoint**: FSO links are also used for multipoint connections such as connecting an aircraft or satellite to earth.

- **WAN Access**: Wide Area Network (WAN) is another field for FSO applications where high speed data services are required by mobile users.

- **SAN Formation**: With increasing storage requirements and technology advancements, Storage Area Network (SAN) has been widely and highly demanded for highly available yet fast storage access. These requirements are supported by FSO links making them potential for SAN formation.
• Campus Connectivity: It allows schools, universities, and big organizations to benefit from the very high speed of FSO links connecting the campus buildings. This corporates in solving the issue of overwhelmed network connections due to huge and wide network traffic on campus.

• Metropolitan Extensions: FSO links are beneficial in metro-network when an extension for the existing fiber rings of the metro-network is needed. FSO links are favorable for this type of applications as they don’t require a lot of time for deployment and the needed infrastructure for newly added FSO networks is simple.

• High Quality Video Monitoring: FSO links are being strongly applied instead of conventional wireless technologies for high quality video streaming in various applications where video monitoring is required such as public safety and commercial applications.

• Cellular Systems Backhaul: Instead of the traditional links that reside in between the core stations and the mobile edge centers, FSO links can be used as better alternative as they provide higher bandwidth for the increasing number of mobile phone services.

• Live Broadcasting: For live broadcasting of important events such as big ceremonies or sports popular events, FSO links can provide high speed and quality transmission links between the cameras recording the event and the broadcasting vehicles. One relevant example was during the World Cup in 2010, when BBC TV station used FSO links between their temporary studios in South Africa for the high quality broadcasting mission.

• Military Applications: FSO links comply with security and safety requirements, and they also can be invisible. These characteristics along with fast deployment capability favor FSO links for military applications and use.

• Disaster Recovery: FSO links are used for disaster recovery or as redundant links. They are used to avoid loss of data or unavailability of the system in disasters and emergency cases. One famous application is after September 11 events where FSO links were deployed urgently and quickly to recover from the loss of landlines in the area.

1.1.2.4 Advantages

As described earlier, FSO links are believed to have advantages against each of optical fiber links and RF links. In fact, FSO links combine the benefits of the two technologies; they
provide the speed of optical fiber links but wirelessly just like with RF links.

1.1.2.4.1 High Data Rate

FSO links can provide very high data rate speed which is definitely higher than the speed of RF links [31]. FSO transceivers are even found in the market with data rate speed up to Gbps [44]. This data rate is even comparable to the data rate offered by optical fiber links [45].

1.1.2.4.2 License Free

Contrary to RF, FSO is a license-free technology which can be deployed in a straightforward manner [45]. Saying that FSO is a license-free technology means that its optical spectrum is license-free and thus the corresponding optical channels can be accessed by the users without any need for license or permission [46]. This characteristic of FSO technology leads to unlimited spectrum without any required licensing cost thus leading to lower initial cost of the FSO system [46, 47].

1.1.2.4.3 Low Cost

As mentioned earlier, due to the license-free trait of FSO systems, they are characterized by low initial cost. In addition, compared to RF technology, FSO systems have lower cost due to the lower cost and power consumption of optical components [48]. In addition, FSO links offer lower cost compared to optical fiber links due to the absence of the high cost of manufactured fiber-glass material which is present in optical fiber links in addition to the installation cost (digging of fibers) [49].

1.1.2.4.4 Huge Bandwidth and Capacity

FSO systems are characterized by the advantage of their huge bandwidth [50]. In fact, this huge bandwidth comes from the fact that FSO systems have thousand times smaller wavelengths than RF systems and thus resulting in thousand times larger frequency, and in its turn larger frequency implies larger bandwidth [21]. In addition, capacity is directly proportional to bandwidth. Knowing that FSO systems have hundreds of thousands larger bandwidth than RF systems, the fact that FSO systems provide larger capacity than RF systems is clarified [51].
1.1.2.4.5 Noise and Interference Resistance

FSO links satisfy high immunity against external noise sources that RF links suffer from. In fact, FSO links are not subjected to radio frequency interference; and the transmitted information by FSO links are not disturbed by neither electromagnetic interference nor radio-magnetic interference [45, 50]. Even when optical sources are reused with same source specifications in the same building, they don’t cause significant mutual interference as their signals cannot breach walls [52].

1.1.2.4.6 Security

FSO systems are considered secure systems as they can’t be intercepted. This come from the fact that FSO links transmit narrow and highly directive optical beams that attackers cannot breach unless if they disrupt the signal which will easily reveal their identity. This security property of FSO links is known as Low Probability of Intercept/Low Probability of detection (LPI/LPD) [21, 45, 52].

1.1.2.5 Limitations

Even though of all the advantages that FSO technology proposes, FSO technology still faces some limitations that challenge its application and deployment in the practice [31, 35].

1.1.2.5.1 LOS Immobility and Sensitivity

Due to the high directivity and narrow beams of FSO links, the fixed line of sight that these links need is considered as a limitation. The presence of any physical obstacles in this LOS will affect the FSO link. In addition, very precise alignment of the beams is required between the transmitter and the receiver so that the communication can occur without loss of information and optical power [31, 35].

1.1.2.5.2 Atmosphere and Weather Conditions

Atmosphere is considered the medium for FSO transmission meaning it plays a main role in determining the success of the FSO communication. Thus, any changes in the weather conditions are considered to affect the FSO system in most cases negatively. There are various
aspects of this limitation classified as Scintillation, Absorption, Atmospheric turbulence, Scattering, and typical bad weather conditions such as Fog, Rain, Haze, Smoke, Sandstorms, and Snow. All these aspects impact the system negatively by affecting the FSO transmitted signal such as affecting the signal’s amplitude and strength and optical beam’s power density and intensity [53–57]. Figure 1.5 presents some of the weather challenges that FSO links might face. All of these aspects will be discussed further and tackled deeper in section 1.1.3.

1.1.3 FSO Channel

In this section, the FSO channel is described against weather conditions and possible challenges that might affect the performance of the FSO link. Later on, some practical channel models will be introduced and explained.

1.1.3.1 Weather

As mentioned in section 1.1.2.5.2, weather conditions play a major role in affecting the FSO link performance. Among these weather conditions, we specify fog, rain and snow which cause huge attenuation of the FSO signal leading to limiting the achievable range by the FSO link and lowering system reliability [4]. In the following sections, we define each of these aspects and how it affects the FSO channel.

1.1.3.1.1 Fog

Fog is considered as one of the top weather conditions degrading the FSO channel performance. It causes absorption, scattering and reflection of the transmitted optical beam thus leading to the need of increasing the transmitted power to reach longer distances [31]. Thick
layers of fog can cause dramatic attenuation values exceeding even those obtained with RF under rain. These values can reach 300 dB per Km thus limiting the transmission range to 100 meters only [4].

1.1.3.1.2 Rain

Rain drops lead to an attenuation in the FSO transmitted signal. The attenuation loss with FSO channel under rain ranges from 1 dB/km with light rain up to 10 dB/km with heavy rain [53, 58]. Compared to the attenuation values achieved by thick fog (300 dB/Km and above), rain is believed to have minor effect on FSO channel where fog is the main contributor to the loss [59].

1.1.3.1.3 Snow

Snow particles are classified into two categories: wet snow and dry snow. The difference between both categories is related to the density of the particles which plays a role in the attenuation process. More specifically, wet snow is denser than dry snow thus wet particles result in higher attenuation factor. Moreover, compared to fog and rain, snow sets in the middle causing attenuation loss values higher than those with rain and lower than those with fog. These values can even reach high levels during heavy snow (30-350 dB/km). This attenuation is caused by increasing density of snow particles and ice formation on receiver window, which block the path of the optical beam [53, 59].

1.1.3.2 Building Sway

Among the factors that affect the performance of the FSO channel, we present Building Sway phenomenon. Building Sway is described as the little movement of the building under external circumstances. In FSO communication systems, transceivers are usually placed on high buildings to ensure clear LOS [32]. Due to strong wind heaps, thermal expansion of building edges, and small earthquakes, these high buildings sway [60–63]. These sways can disturb the transmission process as the high placed transceiver will transmit a vibrated optical beam toward the receiver [62]. These vibrations might cause misalignment in the LOS between the transmitter and the receiver in addition to the fading of the received signal [63]. As a result, probability of error will increase which requires increasing the power of the transmitted sig-
nal in addition to widening the beam divergence angle at the transmitter in order to ensure successful LOS [62, 63].

1.1.3.3 Atmospheric Turbulence

Even under clear weather conditions and within stable environments, without rain, fog, snow, strong wind, earthquakes, and any other similar situation, FSO channel can suffer from Atmospheric Turbulence [30, 53, 59]. This phenomenon is caused by the variations of atmosphere’s temperature and pressure resulting in formed turbulent cells which will interfere with the transmitted optical beam across the LOS [59]. Two main aspects of Atmospheric Turbulence are described in the next paragraphs: Beam Wandering and Scintillation. The effects of these aspects are visualized in Figure 1.6.

1.1.3.3.1 Beam Wandering

This aspect of atmospheric turbulence is experienced when the turbulent cells are larger than the size of the optical beam. In this case, the transmitted optical beam will be deflected by these large turbulent cells away from its propagation path. The result will be the loss of the desired propagation of the transmitted optical beam across the LOS; the result is FSO link failure [59, 64–66].

1.1.3.3.2 Scintillation

Scintillation is one of the major aspects that affect the FSO channel. This phenomena is caused by fluctuations in the temperature and the pressure of the atmosphere due to solar heating and wind, varying the air refractive index along the propagation path [30]. If the size of the resulting turbulent cells is comparable to the size of the optical beam, then these cells will cause random fluctuations of the received signal in its phase and amplitude [30, 59]. As a result, the FSO channel will suffer from channel fading specially across long distances resulting in major degradation of the system performance [30, 53, 59].

1.1.3.4 Channel Models

Due to channel fading, attenuation and path loss, an aggregated channel model for the FSO channel should be considered and described in any FSO system. These aggregated channel
models take into consideration the fading resulting from atmospheric turbulence in addition to a path loss accompanied by the distance that the FSO transmitted signal propagates; these FSO channels could be modeled as lognormal, exponential, Rayleigh, or Gamma-Gamma channel models [4, 7, 30, 67–82].

In what follows, we consider the following definition for modeling the FSO link with distance $d$ by taking into consideration the atmospheric effects [7, 67, 76, 77]:

$$h = L(d)\tilde{h}$$  \hspace{1cm} (1.4)$$

Where $\tilde{h}$ corresponds to the turbulence-induced fading term, and $L(d)$ represents the path loss along the FSO channel with distance $d$ and can be defined as [76, 77]:

$$L(d) = \frac{D_T^2}{(D_T + \theta_T d)^2} e^{-\sigma d}$$  \hspace{1cm} (1.5)$$

Where $D_T$ and $D_R$ correspond to the aperture diameters of the transmitter and receiver, respectively. Also, $\theta_T$ represents the divergence angle of the optical beam, and $\sigma$ is the attenuation coefficient which is dependent on weather conditions.

In this system, the instantaneous signal-to-noise ratio (SNR) can be represented as [67, 77]

$$\gamma = \bar{\gamma}|\tilde{h}|^2$$  \hspace{1cm} (1.6)$$

Where $\bar{\gamma}$ is the average SNR of the FSO link, and $|\tilde{h}|$ is the amplitude of the turbulence
fading term as shown in equation (1.4).

1.1.3.4.1 Lognormal Distribution

Lognormal Distribution is the most widely adopted and adequate channel modeling for FSO systems with weak atmospheric turbulence fading [7, 30, 67, 74, 76]. In what follows we define $X$ to follow a normal distribution with mean $\mu_X$ and variance $\sigma_X^2$ where $|\tilde{h}|^2$ symbolized as irradiance $I$ in what follows is said to be equal to $e^X$ and thus $I^2 = |\tilde{h}|^2 = e^{2X}$. Moreover, to preserve the average power, the fading amplitude is normalized so that $E[|\tilde{h}|^2] = 1$, and thus $\mu_X = -\sigma_X^2$ [67]. This implies that the probability density function (PDF) of the lognormal distribution can be written as [7]:

$$f(I) = \frac{1}{I\sqrt{2\pi\sigma_X^2}} \exp\left(-\frac{(\ln(I) - \mu_X)^2}{2\sigma_X^2}\right) \tag{1.7}$$

With $\sigma_X^2 = \frac{\sigma_R^2}{4}$ where $\sigma_R^2$ is the Rytov variance defined as [8, 73]:

$$\sigma_R^2(d) = 1.23 C_n^2 (k)^{7/6} d^{11/6} \tag{1.8}$$

Where $C_n^2$ is the refractive index constant, $k$ is the wave number, and $d$ is the FSO link distance.

Another representation with respect to scintillation index $\sigma_I$ would be $\sigma_X^2 = \ln(1 + \sigma_I^2)$ [72].

In figure 1.7, the PDF of the lognormal distribution of an FSO link with length $d = 2$ km is plotted.

The corresponding cumulative density function (CDF) is given by [67]:

$$F(I) = 1 - Q\left(\frac{\ln(I) + 2\sigma_X^2}{2\sigma_X}\right) \tag{1.9}$$

Where $Q(\cdot)$ is the Gaussian-Q function.

1.1.3.4.2 Exponential Distribution

The Negative Exponential distribution is used for channel modeling of FSO links suffering from strong atmospheric turbulence fading conditions and more specifically operating in saturation regime; it is valid for long distances in the order of several kilometers [74, 81, 82].
The pdf of the negative exponential distribution is defined as follows [78–80]:

\[ f(I) = \frac{1}{I_0} \exp\left(\frac{-I}{I_0}\right) \]  
(1.10)

Where \(I_0 = E[I]\) is the mean irradiance, and the scintillation index which is equal to \(\frac{E[I^2]}{\langle E[I]\rangle^2} - 1\) is an additional important parameter for describing turbulence strength [79, 80].

The PDF of the negative exponential distribution is plotted in figure 1.8 where the mean irradiance is set to 1.

1.1.3.4.3 Rayleigh Distribution

For cases with severe atmospheric turbulence, the Rayleigh distribution can be used [30, 83, 84]. In fact, in the case where Rayleigh distribution is applied to the path gain of the FSO link, the irradiance of this FSO link will follow a negative exponential distribution that was described in section 1.1.3.4.2 [83].
In this case, the pdf of the path gain $a (a > 0)$ provided by the Rayleigh distribution is given by\cite{75, 83, 84}:

$$f_a(a) = 2ae^{-a^2}$$  \hspace{1cm} (1.11)$$

With a normalized mean path intensity $E[A^2] = 1$ and the scintillation index equals to 1.

1.1.3.4.4 Gamma-Gamma Distribution

Contrary to previously described distributions such as lognormal distribution and exponential distribution which are adequate for specific atmospheric turbulence conditions, the Gamma-Gamma distribution can be used with any turbulence conditions (weak, moderate, or strong); it is widely accepted and used for FSO links’ channel modeling \cite{4, 8, 30, 59, 67, 77}. Its pdf is given by:

$$f(I) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} \int^{\frac{\alpha\beta}{2}}_0 K_{\alpha-\beta}\left(2\sqrt{\alpha\beta I}\right)$$  \hspace{1cm} (1.12)$$

Where $\Gamma(\cdot)$ is the Gamma function, $K_{\cdot}(\cdot)$ is the modified Bessel function of the second kind

Figure 1.8: Negative Exponential Distribution PDF for a unity mean irradiance
Figure 1.9: Gamma-Gamma parameters variation with distance

of order c, and \( \alpha \) and \( \beta \) representing the terms related to the effective atmospheric conditions which are dependent on the Rytov variance as described in section 1.1.3.4.1. They are defined as [8, 67]:

\[
\alpha(d) = \left[ \exp(0.49\sigma_R^2(d)/(1 + 1.11\sigma_R^{12/5}(d))^{7/6}) - 1 \right]^{-1}
\] (1.13)

\[
\beta(d) = \left[ \exp(0.51\sigma_R^2(d)/(1 + 0.69\sigma_R^{12/5}(d))^{5/6}) - 1 \right]^{-1}
\] (1.14)

Where \( \sigma_R^2(d) \) was defined in equation (1.8).

As we can see in equations (1.13) and (1.14), the values of \( \alpha \) and \( \beta \) are dependent on the Rytov variance. Moreover, as equation (1.8) indicates, the Rytov variance \( \sigma_R^2 \) is dependent on the distance \( d \); therefore both \( \alpha \) and \( \beta \) will vary with the distance too. This can be illustrated in figure 1.9 where each of these turbulence and scintillation parameters \( \alpha \) and \( \beta \), in addition to Rytov variance \( \sigma_R^2 \), are plotted against variation of distance \( d \) in km.

In what follows, the following notation will be used:

\[
\alpha_{i,j}(d) \triangleq \alpha(d_{i,j}); \beta_{i,j}(d) \triangleq \beta(d_{i,j})
\] (1.15)

Where \( \alpha_{i,j} \) and \( \beta_{i,j} \) are the link parameters with \( i \) and \( j \) representing the two nodes con-
connected by the FSO link. In addition, $d_{i,j}$ is the distance of this FSO link connecting $i$ and $j$.

The pdf of the FSO link represented by $f(I)$ will be inheritably affected by the FSO link distance $d$ through the distance-dependent parameters $\alpha$ and $\beta$. This is validated in figure 1.10 where an increase in the distance of the FSO link will shift the pdf of the channel to the left. This indicates that the density of the function is shifted to smaller values of irradiance $I$. 

Figure 1.10: Gamma-Gamma PDF versus Irradiance for various distances
1.1.4 Diversity Methods

Due to atmospheric turbulence effects on the FSO channel, especially with long distances, the reliability of the FSO system can be affected. This issue comes from the fact that each FSO link has a single point of failure and no redundancy. Therefore, diversity methods were introduced in FSO systems in order to improve their performance. Diversity methods can be Single-Input-Multiple-Output (SIMO), Multiple-Input-Single-Output (MISO), the combination of both Multiple-Input-Multiple-Out (MIMO), or relay-assisted techniques [30, 59, 76, 77, 84–87]. The most used diversity methods in the FSO literature are MIMO and relay-assisted communication techniques which are categorized as local diversity and cooperative diversity, respectively [59]. These diversity methods constitute fading mitigation techniques in which they reduce the turbulence effects on the systems, enhance the performance, and increase the reliability of the FSO channels [59, 85, 86]. In what follows, we describe these methods in details.

1.1.4.1 MIMO

MIMO FSO systems constitute a localized diversity method where multiple apertures are placed at both the transmitter and receiver sides [30, 59, 85, 86]. MIMO FSO systems are used to reduce the atmospheric turbulence fading which affects the FSO channels [30, 86]. An example of MIMO system channel is presented in figure 1.11 where we have $m$ input transmitters and $n$ output receivers forming $m$ by $n$ subchannels. To achieve the goal, MIMO FSO systems deploy repetition coding (RC) or transmit laser selection (TLS) [7, 86, 88, 89]. With RC, we transmit the same information redundantly in parallel from all the input apertures, indicating that there is no need for prior knowledge of Channel State Information (CSI). On the other side, TLS is based on selecting the laser link which results in best performance, thus we need a feedback from the receiver to the transmitter in addition to its CSI [86].

1.1.4.2 Relaying

Another way to apply diversity in the communication system is by using a relay-assisted approach. This approach has been studied heavily in the literature of FSO as in [7, 8, 75–77, 90–96]. The work in [7] contributes to the implementation of relay-assisted diversity methods in FSO systems inspired from the similar previous work applied to RF systems. Applying
relay-assisted approach means that the message transmitted from the source will be delivered to the adjacent relays which will assist in transmitting the message to the destination. For this approach, two configurations were discussed: serial relaying and parallel relaying. Serial configuration is described as a multi-hop transmission where the message is forwarded from a relay to the next relay in a serial way [97]. On the other side, the parallel configuration implements the concept of cooperative diversity which can be explained as cooperation between the adjacent relays for the sake of transmitting a message from a certain source to a specific destination. Moreover, with relaying, we consider two forwarding strategies: Amplify and Forward (AF) and Decode and Forward (DF) where both will be discussed with relay-assisted configurations. These configurations will be detailed in the following sections.

1.1.4.2.1 Serial Relaying

The main idea behind serial relaying relies on the fact that long distance FSO links suffer from high performance degradations due to strong fading. Dividing the long link into shorter parts or chunks can thus improve the system performance. The application of Serial Relaying or multi-hop transmission into FSO systems as shown in [7] was inspired from its application in RF systems. However, with RF systems, the results show no real mitigation against fading effects due to their broadcasting nature as shown in [98]. Contrary to RF systems, FSO systems can exploit the benefit of employing multi-hops as discussed in [7] since FSO links were proved to be distance dependent which is a major difference from RF systems. The presented work in [7] considers both path-loss as the case with RF systems and fading effects as in the work
done with FSO systems in [99–101]. The serial configuration is shown in figure 1.12 where a source (S) starts transmitting the message into an adjacent relay (R1). In its turn, R1 forwards the message into the next relay (R2) and the procedure continues on till we reach the final destination (D). More specifically, an intensity modulated signal is first transmitted by S to R1 before having two possible forwarding protocols to be applied: AF and DF. With AF, the received message by the relay is amplified and directly forwarded to the next relay. However with DF, the received message is decoded by R1 and then encoded again before forwarding it to the next relay. In both cases, the procedure is applied to the next relays until the message reaches the desired destination.

1.1.4.2.1 Advantages and Disadvantages

The work described in [7] classifies serial relaying as an advantageous option for maintaining the signal strength when the transmitter power is limited. Therefore, serial relaying helps in achieving lower fading, higher reliability, transmission for longer distances, better system performance, and conservation of the signal strength.

At the same time, serial relaying suffers from a single point of failure, where the outage of one hop leads to the failure of the whole system. Hence, the number of relays in a serial system should be studied precisely to lower the effects of this disadvantage. Another solution to consider is multiple redundant paths, which introduces us to the parallel relaying scheme.

1.1.4.2.2 Parallel Relaying

Parallel relaying or cooperative diversity has been studied heavily in FSO literature as in [7, 75, 91–96]. While RF systems are categorized by their broadcasting nature, FSO systems can implement another approach but similar in functionality. The vital work in [7] proposes the placement of multiple transmitter apertures like in MIMO however these multiple transmitter apertures will be aligned with multiple adjacent relays instead of multiple receiver apertures. The FSO system can use cooperation among the relays as shown in figure 1.13 where the source
(S) transmits the message using its multi apertures in the direction of all adjacent relays. These relays working in parallel, will forward the message into the destination (D) after applying either AF or DF on the received message by the relays.

Moreover, it is important to mention the conclusion of the work explained in [86] which compares the performance of FSO systems between MIMO and cooperative diversity. The results show the superiority of MIMO over adding new relays to an existing FSO system. This is justified by avoiding the unnecessary additional cost for upgrading the existing system with newly added resources and infrastructure since the same number of transmitter and receiver apertures with MIMO was proven to achieve higher diversity gain. However, the advantage of the cooperative diversity approach was examined with exploiting existing nodes in the system to work as assistant relays without the need for additional new resources and infrastructure.

### 1.1.4.2.2.1 Inter Relay Cooperation (IRC)

The work mentioned in section 1.1.4.2.2 examines the benefits of using adjacent relays to cooperate in forwarding the message to the destination by exploiting two types of links: Source to Relay links and Relay to destination links. However, it does not discuss the potential of inter relay links in participating in the cooperation strategy. Inter Relay Cooperation (IRC) was first discussed in [8] where the cooperation strategy is similar to the parallel relaying scheme but with benefiting from the existence of inter-relay links. Therefore, FSO systems employing IRC consist of three phases as described in [102]: Source to Relay, Relay to Relay, and Relay to Destination communications. These communications are studied with DF mechanism in [8, 102] where unidirectional and bidirectional communication schemes between the inter FSO links are described. The presented work shows potential benefits due to the fact that these inter-relay links exist and there
is no need for additional hardware and infrastructure to apply the IRC scheme. Figure 1.14 shows an FSO system with inter connected relays enabling the deployment of IRC cooperative diversity scheme for the system.

1.1.4.2.2 Unidirectional IRC

The work presented in [102] introduces unidirectional IRC schemes, named as IRC1, to deploy a one-way communication between Relay ($R_i$) and its adjacent Relay ($R_j$) in which $R_i$ can only forward the message to $R_j$, but the opposite way is not allowed. The process is better described in figure 1.15 where three relays are presented in the FSO system, connected in a unidirectional way from relay $R_1$ to relay $R_2$, and from relay $R_2$ to relay $R_3$.

The system will work as follows: first, the message is transmitted from source $S$ to the
three adjacent relays $R_1$, $R_2$, and $R_3$ while transmitting it at the same time to the destination $D$. The next phase is only activated if the transmission on the direct link between $S$ and $D$ fails. In this case, the relays that were able to receive the message successfully will forward it to their adjacent relays using the inter relay links.

For the FSO system provided in figure 1.15, $R_1$ will forward the message to $R_2$, and in its turn $R_2$ will forward the message to $R_3$. At a final stage, all the relays that have successfully decoded the message will forward it to the final destination $D$. This mechanism is very effective when taking the example provided in 1.16 where the following links are in outage: $S - D$, $S - R_2$, $R_1 - D$, and $R_3 - D$. Without IRC, the system will definitely experience a failure since all the parallel paths $S - D$, $S - R_1 - D$, $S - R_2 - D$, and $S - R_3 - D$ have at least one failed link in them, thus the message cannot be delivered. However the existence of inter relay links enabled the message to follow the highlighted path shown in figure 1.17 i.e. $S - R_1 - R_2 - D$.

1.1.4.2.2.3 Bidirectional IRC  
Similar to IRC1, bidirectional IRC named as IRC2, corresponds to the two-way communication between the relays using the existing FSO transceivers. This adds more available paths for the transmitted message to propagate along in order to reach
the targeted destination. As described in [8], this diversity method increases the number of degrees of freedom in the FSO system as the system can fight against worse fading effects, and can handle more link failures. Figure 1.18 shows an FSO system with IRC2.

The transmission with IRC2 goes similar to the case with IRC1 with the only difference in the second phase. Instead of forwarding the message in one way from $R_i$ to $R_j$, the information is exchanged simultaneously in both directions: from $R_i$ to $R_j$ and from $R_j$ to $R_i$.

The gain from this approach can be illustrated in figure 1.19 where the FSO system with IRC1 fails to deliver the message through any of the existing paths to the destination. However the use of IRC2 as shown in figure 1.20 provides the highlighted path to deliver the message through the path $S − R_3 − R_2 − D$ exploiting the existing bidirectional link between $R_2$ and $R_3$.
1.1.4.2.2.4 Limitations motivating the All-to-All inter-relay communications  The main limitation in the existing work is that only adjacent pairs of FSO links are connected leaving some relays unconnected. For example, in the provided FSO system in figure 1.18, relays $R_1$ and $R_3$ are not connected. The scenario demonstrated in figure 1.21 shows the importance of connecting all the relays together where the system fails with IRC2. However, connecting all the relays together would have ensured a path for the message from $S$ to $D$ ($S - R_3 - R_1 - D$) as shown in figure 1.22; this motivates our All-to-All relaying scheme.

However, due to the non-broadcasting nature of FSO links, new apertures should be installed at each relay and ensuring a clear LOS between the connected relays would be needed; thus connecting all the relays together with large number of relays would be a challenge and might be unfeasible sometimes. This inspired also the potential of deploying RF links between the inter relays as will be discussed later in section 1.1.5.

Figure 1.20: IRC2 saving FSO system with IRC1 Failure

Figure 1.21: FSO system with IRC2 System Failure Example
1.1.4.2.3 Relaying Protocols

The work presented in [8, 75–77, 83, 86, 90–96, 103–105] discusses two types of relaying protocols: All-Active Relaying and Selective Relaying. These protocols are further explained in what follows.

1.1.4.2.3.1 All-Active Relaying  As explained in [86, 91–94, 103–105], All-Active relaying scheme activates all the available FSO links in the transmission process. In other words, the message will be transmitted from the source to all adjacent relays using all the $S - R$ FSO links in the system as shown in figure 1.23. Depending on whether the system deploys inter-relay cooperation or not, the relays which successfully received the transmitted message will forward it to their neighbors or not, respectively. At a final stage, as shown in figure 1.24, all the relays will forward the message to the desired destination activating all the available $R - D$ FSO links regardless of the current channel state information (CSI).
The strength of the All-Active relaying protocol relies in the fact that no CSI is needed, thus no more complexity is added to the system [86]. Hence, the All-Active relaying scheme is simple and can induce more paths for the system to mitigate fading and path-loss without the need for CSI.

From another perspective, a real limitation to the All-Active relaying scheme is the transmission power. When activating all the available FSO links, the power has to be split among all of them. Since the dedicated power for the FSO link directly affects its performance as will be shown in chapter 4, the system reliability will be affected by this limitation. A better approach would be to select certain links in the cooperation process that will ensure the delivery of the transmitted message; this protocol is named Selective Relaying.

1.1.4.2.3 Selective Relaying

The work presented in [75, 77, 86] defines selective relaying as choosing the strongest path for transmission. The analysis of the path strength requires a prior knowledge of the CSI of each of $S-R$, $R-D$, and if applicable $R-R$ links. Based on the system state, the strongest path is chosen for transmission. For example, an FSO system with parallel relaying and without inter-relay cooperation, named as NIRC, will have two phases for the message transmission under selective relaying protocol. First, all the available paths are studied, even the direct link between $S$ and $D$, and the strongest path is determined. Second, the strongest path is chosen for transmission and the power will be split among the links constituting this path. It is important to note that the strength of a path is highly depen-
dent on the strength of all the links forming this path. A system with IRC is more complicated, as all the possible paths should be considered, where a system with 3 relays deploying IRC2 as shown in figure 1.18 will have 10 available paths: $S - D$, $S - R_1 - D$, $S - R_2 - D$, $S - R_3 - D$, $S - R_1 - R_2 - D$, $S - R_1 - R_3 - D$, $S - R_2 - R_1 - D$, $S - R_2 - R_3 - D$, $S - R_3 - R_2 - D$, and $S - R_3 - R_1 - D$.

The main advantage of this protocol is saving unneeded available resources and optimizing the use of the needed ones. In other words, if the strongest path fails then all the other paths should fail, so why do we need to activate all of them together? At the same time, this will enable us to maximize the power allocated for the selected links instead of splitting the total power on higher number of links. Therefore, selective relaying protocol is proved to be superior to All-Active relaying protocol [86].

Even though of its presented advantages and vital performance, the selective relaying scheme requires the knowledge of the CSI to determine the strength of the available paths. This required knowledge of the channel state under varying weather conditions, requires specific infrastructure and resources which is the cost of the enhanced performance of the system. In particular, pilot training symbols need to be exchanged between the different communicating nodes for the sake of estimating the underlying channels. Moreover, as the number of relays increases, the number of inter-relay links presented with IRC2 approach will increase as multiple of 2, and thus the number of available paths in the system will increase exponentially. This will dramatically increase the complexity of the system for determining the strength of all the available paths.

1.1.5 Mixed RF/FSO System

Mixed and Hybrid RF/FSO Systems have been widely studied in the literature [106–115] where RF links serve as backup for FSO links. The work in [106–111] proposes the benefits of hybrid RF/FSO systems as the FSO links provide a very high speed, but yet suffer from the severe weather conditions. On the other hand, RF links achieve a lower speed than FSO link, but they are more reliable in such weather conditions. Thus the proposed work exploited the advantages of both types of links to achieve better system performance.
1.1.5.1 Hybrid RF/FSO Systems - Backup RF Links

In this context, two types of hybrid systems are described: hard switching and soft switching [108, 110]. As described in [108], hard switching corresponds to the case where the RF link stays inactive until the FSO link fails to meet the required threshold, then the system switches to the RF link. However, [110] defines another type of switching: soft switching, in which both FSO and RF links are activated in parallel to each other and where channel encoding and decoding is implemented for the coordination between the two links. Figures 1.25 and 1.26 illustrate the described difference between hard switching and soft switching.

1.1.5.2 Dual-Hop Mixed RF/FSO Systems

Moreover, another type of mixed RF/FSO systems is described in [112–115]: dual-hop mixed RF/FSO system. As demonstrated in figure 1.27, the information is sent from the source (S) to the relay (R) using a high speed FSO link, then R broadcasts the information to the multi end users at the destination (D) using the RF links. The opposite way of communication is also valid with this system (multiple access scheme). In addition, this system can be deployed with single relay AF scheme as in [113], multiple relays AF scheme as in [114], or even single relay DF scheme as in [115].

![Hybrid RF/FSO System with Hard Switching](image)
1.1.5.3 Novel Mixed RF/FSO System

Inspired from the existing mixed RF/FSO systems, and motivated by addressing the presented limitations for full FSO cooperative systems, our work proposes a novel method for FSO cooperative communications. The presented work takes advantage of existing RF backup antennas at the relays for better relay cooperation when needed. The proposed system as shown in figure 1.28 is described as a mixed RF/FSO system with direct source todestination ($S − D$), source to relay ($S − R$), and relay to destination ($R − D$) links of FSO nature, while inter-relay ($R − R$) links being of RF nature. These RF links serving as a backup for the FSO direct link, will be activated when the $S − D$ FSO link fails. They will participate in sharing the received message by at least one of the relays to form All-to-All cooperative scheme among them. Thus the source ($S$) will try to send the packet through the direct $S − D$ FSO link, if the transmission fails, the system sends the packet to the existing relays ($R_1$, $R_2$, and $R_3$) through the $S − R$ FSO
links. The next phase will be sharing the packet between the relays through the $R - R$ RF links, after which the relays that successfully received the packet forward it to the destination ($D$) through the $R - D$ FSO links.
Chapter 2

Mathematical Tools

In this chapter, we present the mathematical tools, methods, and approaches that will be used heavily throughout the next chapters.

2.1 Conditional Probability Approach

Conditional Probability by definition is the probability of occurrence of a certain event \( B \) given that another event \( A \) has already taken place in the past [116]. From this definition, the conditional probability of occurrence of event \( B \) given the occurrence of event \( A \) is represented as \( P(B/A) \). Moreover, the probability of the occurrence of two events \( A \) and \( B \) together is represented by \( P(A \cap B) \). The mathematical formula of the conditional probability is given by [117]:

\[
P(B/A) = \frac{P(A \cap B)}{P(A)}
\]  

(2.1)

Another way to describe the conditional probability formula derived from equation (2.1) would be:

\[
P(A \cap B) = P(B/A)P(A)
\]  

(2.2)

In this context, a probability tree diagram is presented in figure 2.1 where we follow the tree to get a certain probability.

In our work, we apply the conditional probability approach on the studied communication
systems for calculating their probability of failure. We take the simple example provided in figure 2.2 where we have a source $S$, a relay $R$, and a destination $D$. The probability of failure for each link is represented by $P_{i,j}$ where $i$ and $j$ are the connected nodes. We condition first on the direct link $S \rightarrow D$:

$$
Pr(\text{failure of the system}) = P_{S,D}Pr(\text{failure of the system}/ S \rightarrow D \text{ link failed})
+ P_{S,D}Pr(\text{failure of the system}/ S \rightarrow D \text{ link is operational})
$$

In fact, if $S \rightarrow D$ link is operational, the system will not fail, thus $Pr(\text{failure of the system}/ S \rightarrow D \text{ link is operational}) = 0$. This induces that:

$$
Pr(\text{failure of the system}) = P_{S,D}Pr(\text{failure of the system}/ S \rightarrow D \text{ link failed})
$$

Moreover, if $S \rightarrow D$ link fails, the system will look as shown in figure 2.3. If we condition on the $S \rightarrow R$ link now, we get:

$$
Pr(\text{failure of the system}) = P_{S,R}Pr(\text{failure of the system}/ S \rightarrow D \text{ and } S \rightarrow R \text{ links failed})
+ P_{S,R}Pr(\text{failure of the system}/ S \rightarrow D \text{ link failed and } S \rightarrow R \text{ link is operational})
$$
In the first case, it is impossible for the message to reach $D$, thus the system fails meaning $\Pr(\text{failure of the system/ } S - D \text{ and } S - R \text{ links failed}) = 1$. While in the second case, the system has the message delivered to $R$ and the only questioned part is the $R - D$ link as shown in figure 2.4 which indicates the failure of the link will lead to the failure of the system. This will result in the following expression:
Pr(failure of the system) = P_{S,D}[P_{S,R} + P_{S,R} P_{R,D}] \\
= P_{S,D}[P_{S,R} + (1 - P_{S,R}) P_{R,D}]

This approach is exact where no approximations are made to derive the failure probability. In communication systems, the approach is helpful in studying the reliability of the system and what scenarios would lead to the failure of the system. However, with big and complex systems, the involved expressions can be very complicated. In our work, we developed a MATLAB code that automatically derives the exact reliability or probability of failure for any given system. This code takes as an input the connections in the system between all the nodes represented in a connection matrix and returns as an output the final symbolic expression corresponding to the reliability or probability of failure of the system (depending on the requirements). This code was applied, throughout our work, for the presented communication systems. More specifically, we used the code in determining the exact Outage Probability expressions using the conditional probability approach in Chapter 4.

2.2 Monte-Carlo Simulation

2.2.1 Definition

As explained in [118], Monte Carlo simulation is a computerized method for generating random variables with a very large number of trials in order to simulate the behavior of the current system. These random variables could be either natural, where the statistics are collected from a real-life system model such as traffic monitoring systems, or artificial, where they contribute to solving deterministic problems such as the reliability of a given network system.

2.2.2 Advantages

Monte Carlo Simulation is considered to be an easy and efficient technique, strengthened by its randomness, provided with an insight into the systems, while being supported with theoretical justification in the related mathematical researches [118, 119]. This can reduce the complexity of the system, allowing general models to be studied and implemented in easy manner when other approaches face complexity in that. Moreover, an insight to the numerical values and
random variables can be explored thus providing more realistic information about the system behavior.

2.2.3 Applications

The applications of Monte Carlo Simulation are wide as it has been an important technique for many system models [118, 120–124]. These applications can be categorized into Sampling, Estimation, and Optimization problems. The areas of applications for these categories can fit into Industrial Engineering and Operations Research, Physical Processes and Structures, Random Graphs and Combinatorial Structures, Economics and Finance, and Computational Statistics [122–126].

2.2.4 Relation to Our Work

In our work, we are most concerned with Sampling category along with Industrial Engineering and Operations Research area. The reasons behind that are in our need to model communication systems with outage probability analysis in a similar way to studying reliability systems which fit under the Industrial Engineering and Operations Research application area. Moreover, with sampling we can get the values for all the required random variables and probability distributions in the studied communication systems by running the Monte Carlo simulation technique for very large number of trials. For this purpose, we developed multiple MATLAB code files dedicated for each communication system that we studied in the presented work.

2.3 Cut-Set Method

2.3.1 Definition in Reliability

The Cut-Set Method is one the most famous algorithms for evaluating the reliability of complex systems [127, 128]. In this context, a cut set is a “set of the system components which, when failed, causes failure of the system” [127]. In the same context, a minimal cut set can be defined as “a set of system components which, when failed, causes failure of the system but when any one component of the set has not failed, does not cause the system failure” [128].
2.3.2 Algorithms and General Examples

For better explanation, we provide a simple general example with the system shown in figure 2.5. The components $A$, $B$, $C$, $D$, and $E$ are the reliability components which describe the reliability of the link connecting two nodes. For example, component $A$ represents the reliability of the link connecting Node 1 and Node 2. We note that all the paths go in a unidirectional way from the left to the right except the link between nodes 2 and 3 which is bidirectional from 2 to 3 and from 3 to 2.

In the provided example, we can get all the cut sets visually by cutting all the possible paths from the start point to the end point. This can be illustrated in figure 2.6 where the first cut set is determined to be $\{A,B\}$. The second cut set is shown in figure 2.7 to be $\{C,D\}$. Third and fourth cut sets are given by figure 2.8 to be $\{A,E,D\}$ and $\{B,E,C\}$, respectively. These cut sets are in fact minimal as all the involved components for each cut set are essential for the failure of the system. However, more cut sets could have been deduced from the system which are not minimal such as: $\{A,B,C\}$, $\{A,B,D\}$, $\{A,B,E\}$, $\{A,C,D\}$, $\{B,C,D\}$, and $\{C,D,E\}$.

The next important step after determining the minimal cut sets is the derivation of the unreliability expression for the whole system. In fact, the failure of any minimal cut set will lead to a system failure, and for each minimal cut set, all the involved components should fail together. This can be explained as a serial connection of all the minimal cut sets where the components of each minimal cut set are treated as parallel components. This can be further illustrated in figure 2.9 where $C_1$, $C_2$, $C_3$, and $C_4$ correspond to the minimal cut sets that we derived for the given system in figure 2.5.
Figure 2.6: Applying Cut-Set Method - Step 1

Figure 2.7: Applying Cut-Set Method - Step 2

Figure 2.8: Applying Cut-Set Method - Steps 3 & 4
Figure 2.9: Visualizing the System in terms of Minimal Cut Sets

Now the final unreliability ($Q_S$) expression of the system can be derived as the union of unreliability expressions of all the minimal cut sets:

$$Q_S = \Pr(\text{failure of } C_1 \cup \text{failure of } C_2 \cup \text{failure of } C_3 \cup \text{failure of } C_4)$$

$$Q_S = \Pr(\text{failure of } C_1) + \Pr(\text{failure of } C_2) + \Pr(\text{failure of } C_3) + \Pr(\text{failure of } C_4)$$

$$- \Pr(\text{failure of } C_1 \cap \text{failure of } C_2) - \Pr(\text{failure of } C_1 \cap \text{failure of } C_3) - \Pr(\text{failure of } C_1 \cap \text{failure of } C_4)$$

$$+ \Pr(\text{failure of } C_1 \cap \text{failure of } C_2 \cap \text{failure of } C_3) + \Pr(\text{failure of } C_2 \cap \text{failure of } C_3 \cap \text{failure of } C_4)$$

$$- \Pr(\text{failure of } C_1 \cap \text{failure of } C_2 \cap \text{failure of } C_3 \cap \text{failure of } C_4)$$

$$Q_S = Q_A Q_B + Q_C Q_D + Q_A Q_B Q_D Q_E + Q_B Q_C Q_E$$

$$- Q_A Q_B Q_C Q_D - Q_A Q_B Q_D Q_E - Q_A Q_C Q_D Q_E - Q_B Q_C Q_D Q_E - Q_A Q_B Q_C Q_D Q_E$$

$$+ Q_A Q_B Q_C Q_D Q_E + Q_A Q_B Q_C Q_D Q_E + Q_A Q_B Q_C Q_D Q_E + Q_A Q_B Q_C Q_D Q_E - Q_A Q_B Q_C Q_D Q_E$$

Where $Q_X$ is the unreliability of the system component $X$

As we can notice, the involved expressions are complicated and long. However, unreliability values in reliability systems are usually very low thus the dominant terms in the expression are only the first four terms. This is usually referred to in the literature as an upper bound where only the terms directly related to the minimal cut sets are added together to get the unreliability.
of the system. This results in the following general expression for the minimal cut set method:

\[ Q_S \approx \Pr(\text{failure of } C_1) + \Pr(\text{failure of } C_2) + \cdots + \Pr(\text{failure of } C_n) = \sum_{i=1}^{n} \Pr(\text{failure of } C_i) \quad (2.3) \]

The visualization procedure is effective with simple systems; however it can be challenging to trace all the possible minimal cut sets in bigger and more complex systems. Therefore, we need a systematic way that can be applied to any system. The algorithm is described by [129]:

1. Determine all the minimal paths, where a minimal path is a “path between the input and output such that no node or intersection between branches is traversed more than once”.
2. Build the incidence matrix which corresponds to the components in each path.
3. A cut set of first order exists if all the values in its corresponding column in the incidence matrix are greater than zero.
4. In a second run, add the elements of all possible combinations of two columns; if the result of the addition is greater than zero for all the values in the added columns, then the corresponding components form a cut set of second order. It is necessary to eliminate any cut sets that has a subset of it repeated from the first order cut sets in order to determine the minimal cut sets.
5. Repeat step (4) with three columns and by eliminating all cut sets which contain repeated cut sets from all previous orders (first order and second order for now).
6. Keep repeating the process with increased number of columns and by eliminating all cut sets containing subsets from previous orders, until the highest order of the minimal cut sets is reached.

For illustration purposes, we take the previous example provided in figure 2.5, and apply this algorithm as follows:

1. The minimal paths in the system are: AC, BD, AED, and BEC.
2. The incidence matrix is provided in figure 2.10.
3. No non-zero columns exist, therefore no first order cut sets are available.
4. By adding combinations of two columns, the second order cut sets are: AB and CD. However there are no first order cut sets to check against for the elimination step, thus AB and CD are minimal cut sets.

5. By adding combinations of three columns, the third order cut sets are: ABC, ABD, ABE, ACD, ADE, BCD, BCE, and CDE. After eliminating all the sets containing either AB or CD (from second order cut sets), we obtain the third order minimal cut sets: ADE and BCE.

6. With fourth and fifth order, all the cut sets are already containing cut sets from the second and third order, thus no new minimal cut sets are added.

However, when applying the previous procedures determining the minimal paths was done visually, but it could have been done also systematically. The algorithm presented by [128] for this purpose is described as follows:

1. Build the connection matrix of the system which defines the transmission of flow between input and output (Use 0 for no connection, 1 for connection to itself, and the component name for the rest connections).

2. Multiply the Matrix by itself until there are no more changes in the resultant matrix. However, we have to note the following special addition and multiplication rules:

- \( A + A \equiv A \)
- \( A + 1 \equiv 1 \)
\[
A \times 1 \equiv A \\
A \times A \equiv A \\
A + 0 \equiv A \\
A \times 0 \equiv 0
\]

3. The resultant Matrix gives the minimal paths between any two nodes (This is helpful in determining all the minimal paths from the source node to the destination node).

![Connection Matrix for Reliability System](image1)

![Resultant Matrix for Minimal Path Deduction](image2)

The same example provided in figure 2.5 is studied for the illustration of this algorithm:

1. The connection matrix for this system is represented in figure 2.11.

2. The resultant stable matrix after several multiplications by itself is given in figure 2.12.
3. The minimal paths from node 1 to node 4 are given by the intersection of first row and fourth column of the resultant matrix which was presented in figure 2.12 as:

\[ AC + BD + BEC + AED \]

Hence, this expression validates the results that were derived earlier.

2.3.3 Application to Our Work

The Cut-Set method is a very effective approximation method for reliability systems and can be systemized. Moreover, in our work, we study the outage probability of communication systems presented as a network of connected nodes through communication links. Therefore, the communication systems that we study in this work can be represented by reliability systems where the communication links are the system components. Moreover, the source, the relays, and the destination in the communication system are represented by the nodes in the reliability system. Hence, the presented work for the cut-set method can be directly applied to the studied communication systems in our work. This can be further explained in figure 2.13 where a communication system is transformed into a reliability system easily. This is done by translating the links \( S - R_1 \), \( S - R_2 \), \( R_1 - D \), \( R_2 - D \), and \( R_1 - R_2 \) into components \( A \), \( B \), \( C \), \( D \), and \( E \), respectively. Moreover, \( S \), \( R_1 \), \( R_2 \), and \( D \) are translated into Nodes 1, 2, 3, and 4 respectively.

In our work, we developed MATLAB codes for each of the described algorithms for computerizing the process and applying it to any communication system with any connection matrix allowing us to determine the minimal paths and apply the minimal cut set method systematically to derive the outage probabilities accordingly.
Figure 2.13: Transforming Communication System into Reliability System
Chapter 3

System Model and Cooperation Strategies

3.1 System Description and Relaying Protocols

As introduced by section 1.1.5.3 and described in our work in [130, 131], we propose a novel mixed RF/FSO cooperative system. In this system, the goal is to establish a communication from a source $S$ to a destination $D$. To establish this communication, a direct FSO link connecting $S$ and $D$ is deployed. Moreover, the system benefits from the existence of $N$ adjacent relays, labelled as $R_1, R_2, \ldots, R_N$, in the cooperation strategy. These relays assist in the communication between $S$ and $D$ when the direct $S-D$ link fails. The corresponding deployed links between the source and the relays, named as $S-R$ links, are of FSO nature. Similarly, the $R-D$ links, deployed between the relays and the destination, are of FSO nature. Moreover, the system implements an inter relay cooperation benefiting from the existence of RF backup links between the relays as discussed in section 1.1.5.1. These $R-R$ links which are of RF nature provide a full connectivity between the relays due to their broadcasting nature. The resultant cooperation strategy is described to be an All-to-All relaying scheme where all the relays are connected with each other by the RF broadcasting links. An example of the proposed system is given in figure 3.1 with $N = 3$.

It is important to mention that these relays and links are not added to the system for the purpose of assisting the communication between $S$ and $D$, but they already exist for their own communication purposes. In fact, the described system takes advantage from the existence of these relays and links to implement the cooperation strategy only when the direct $S-D$ link
 fails. This approach is promising since we can exploit an additional \( \binom{N}{2} \) links without the need to add any extra infrastructure or hardware to the system.

For each of the All-Active and Selective relaying protocols, we describe the phases that the system undergoes during the communication process between S and D.

For All-Active Relaying protocol, as discussed in section 1.1.4.2.3.1 no CSI is needed since all the links will be activated during the cooperation process. Therefore, the proposed scheme can be divided into four phases

1. **S – D phase:** In this phase, the packet is transmitted using the direct S – D FSO link from S to D. In order to proceed to the next phases, an acknowledgment signal is required to inform the transmitter whether the packet was successfully received or not.

2. **S – R phase:** This phase and the next phases are initiated only if the packet fails to be delivered in the S – D phase. In this case, all the S – R FSO links are activated to forward the message from S to all adjacent relays.

3. **R – R phase:** Benefiting from the existence of RF broadcasting links between the relays, the packet is broadcasted along the R – R RF links from the relays that succeeded to receive and decode the packet into all the other relays. It is important to note that the relays are given different RF frequency bands for the purpose of avoiding any interference.

4. **R – D phase:** Now, combining all the relays that successfully received and decoded the packet either from the S – R phase or from the R – R phase, we proceed to forward the packet using all these relays through the available R – D FSO links to the desired destination D.
For the Selective Relaying protocol, as discussed in section 1.1.4.2.3.2, CSI is needed for determining the strongest path which will be selected for the packet transmission. It is important to note that with our mixed RF/FSO system, the RF links are not considered when determining the possible paths due to their broadcasting nature which means it is useless to apply the selective protocol on them. Instead we first study the state of the RF links and how they are connected, and then investigate the available paths in the form of $S - R - D$ where $R$ may be one specific relay, combination of two or more relays, or may not exist. This strategy can tackle the limitation presented earlier in section 1.1.4.2.3.2. In what follows, we describe these phases in more detail:

1. $R - R$ status phase: In this phase we study the current status of the $R - R$ connections. In other words, any set of relays connected together successfully can be combined in a one big relay meaning the successful reach for any one of these relays will lead to the successful reach for the rest of the relays in this set. In this phase, test packets can be sent throughout the RF links for determining the status of these links.

2. Rebuilding the System phase: As described in the $R - R$ status phase, some relays can be combined together before proceeding with determining the available paths for transmission. Therefore, in this phase we rebuild the system virtually where any links coming into or out of the set of combined relays are set to be parallel to each other as can be shown in figure 3.2 where $R_1$ and $R_2$ are combined into $R_{1,2}$.

3. Determining all Possible Paths: At this phase, all the possible paths in the system should be determined. For example, in the provided system in figure 3.2, we have the following possible paths: $S - D$, $S - R_{1,2} - D$, and $S - R_3 - D$.

4. Path Strength Analysis: In this phase the strongest path should be selected for packet transmission.
transmission. Moreover, whenever encountering parallel links, the strongest link should be chosen. For example, in the provided system in figure 3.2, if we choose to transmit through \( S - R_{1,2} - D \) path, we should choose the strongest link between the parallel links connecting \( S \) and \( R_{1,2} \). We have to note here that \( R_{1,2} \) is a virtual relay, thus physically we still have relays \( R_1 \) and \( R_2 \) and we should select one of the two FSO links: \( S - R_i \) or \( S - R_j - D \) where \( i \) can be equal to or different from \( j \).

### 3.2 FSO Links

In this work we adopt the gamma-gamma distribution given in section 1.1.3.4.4 for modeling the FSO links under atmospheric fading. The corresponding pdf of the irradiance \( I_{i,j} \) of the FSO link connecting nodes \( i \) and \( j \) is given by:

\[
f_{I_{i,j}}(I) = \frac{2(\alpha_{i,j}\beta_{i,j})^{\alpha_{i,j}+\beta_{i,j}}}{\Gamma(\alpha_{i,j})\Gamma(\beta_{i,j})} I^{\alpha_{i,j}-1} K_{\alpha_{i,j}-\beta_{i,j}} \left( 2\sqrt{\frac{\alpha_{i,j}\beta_{i,j}}{I}} \right)
\]

(3.1)

With the link parameters \( \alpha_{i,j} \) and \( \beta_{i,j} \) defined as in section 1.1.3.4.4 in terms of the Rytov variance \( \sigma_{R_{i,j}}^2(d_{i,j}) \) as follows:

\[
\alpha_{i,j}(d_{i,j}) = [\exp(0.49\sigma_{R_{i,j}}^2(d_{i,j})/(1 + 1.11\sigma_{R_{i,j}}^{12/5}(d_{i,j})^{7/6}) - 1)]^{-1}
\]

(3.2)

\[
\beta_{i,j}(d_{i,j}) = [\exp(0.51\sigma_{R_{i,j}}^2(d_{i,j})/(1 + 0.69\sigma_{R_{i,j}}^{12/5}(d_{i,j})^{3/6}) - 1)]^{-1}
\]

(3.3)

With the Rytov variance \( \sigma_{R_{i,j}}^2(d_{i,j}) \) defined in section 1.1.3.4.1 as:

\[
\sigma_{R_{i,j}}^2(d_{i,j}) = 1.23C_n^2(k)^{7/6}d_{i,j}^{11/6}
\]

(3.4)

Moreover, the FSO communications in the system are considered to be based on Binary Pulse Position Modulation (BPPM) with Intensity-Modulation and Direct-Detection (IM/DD). As in [8], the electrical signal that is delivered to a node \( j \) from node \( i \) along FSO link \( R_i - R_j \) is given by:

\[
\gamma_{i,j} = \frac{R^2T_b^2G_{i,j}^2L_{i,j}^2P_i^2}{N_{tot}^2N_0}
\]

(3.5)

Where \( R \) is the responsivity of the photodetector and \( T_b \) is the bit duration. \( P_i \) corresponds
to the total transmitted optical power, $\frac{N_0}{2}$ represents the noise variance (AWGN noise), and $I_{i,j}$ is the irradiance from equation (3.1). Here the value of $N_{\text{tot}}$ corresponds to the total number of activated FSO links which is $2N +1$ ($N$ being the number of relays in the system) for All-Active relaying protocol, and either 1 (in the case of selecting $S-D$ link) or 2 (in the case of selecting any $S-R-D$ path) for the Selective Relaying protocol. Moreover, the gain factor $G_{i,j}$ is given by:

$$G_{i,j} = e^{-\sigma(d_{i,j} - d_T)}$$

(3.6)

Where $d_T$ correspond to the length of the direct $S-D$ link, $\sigma$ is the attenuation coefficient, and $d_{i,j}$ in all the equations of this section represents the distance of the FSO link connecting nodes $i$ and $j$.

Based on the previous equations, the FSO link connecting node $i$ to node $j$ will be in outage if the SNR of the received signal at node $j$ is below the desired threshold value annotated by $\gamma_{th}$. Therefore, the FSO link outage probability can be described by [8]:

$$p_{(N_{\text{tot}})}^{(i,j)} \triangleq \Pr (\gamma_{i,j} < \gamma_{th}) = \Pr \left( I_{i,j} < \frac{N_{\text{tot}}}{G_{i,j}P_M} \right)$$

(3.7)

Where $P_M$ is the optical power margin defined in terms of the average electrical SNR along the S-D link denoted by $\gamma_{0,N+1}$ and in terms of $\gamma_{th}$ as shown by [130]:

$$P_M = \sqrt{\frac{\gamma_{0,N+1}}{\gamma_{th}}}$$

(3.8)

Finally, it is important to mention that the variable $N_{\text{tot}}$ which appears in the equations (3.5) and (3.7) of this section, represents the number of activated links as described in the start of this section. This directly contributes to the power splitting factor as the total power in the system $P_t$ should be divided between the activated FSO links. Here appears the main difference between the All-Active relaying protocol and the Selective relaying protocol where in the later the number of activated FSO links is lower. In fact, the number of activated links with All-Active relaying is $2N +1$ where $N$ denotes the number of the relays in the system. However, for selective relaying, the number of activated FSO links is at most 2. This is explained as follows, when the strongest path in the system is the direct $S-D$ link, then the number of activated FSO links is only 1 i.e. $N_{\text{tot}} = 1$. Otherwise, the strongest path will be composite of two FSO links: $S-R_i$ and $R_j-D$ where $i$ and $j$ could be different depending on the current
status of the system, but whatever the values of \( i \) and \( j \) are, the path will have only two FSO links activated. In this case, the number of activated FSO links will be limited to 2 regardless of the number of the relays in the system i.e. \( N_{\text{tot}} = 2 \).

### 3.3 RF Links

For the inter-connected relays through \( R - R \) RF links, we apply either the Rayleigh or the Rician fading channel models that were described in sections 1.1.1.2.1 and 1.1.1.2.2. The Rayleigh distribution corresponds to the absence of a direct LOS between \( R_i \) and \( R_j \) whereas the Rician distribution is associated with the presence of a direct LOS between the relays. For both distributions, we are interested in the outage probability of the RF link \( R_i - R_j \).

In the case of Rayleigh Fading, the outage probability is given by [130]:

\[
p_{i,j} = 1 - \exp \left( -\frac{\Omega_{th}}{\Omega_{i,j}} \right)
\]

(3.9)

Where \( \Omega_{th} \) denotes the threshold SNR for the RF links and is given by \( \Omega_{th} = 2^{2R} - 1 \) with \( R \) being the number of transmitted bits per channel use. In addition, \( \Omega_{i,j} \) represents the average SNR for the \( R_i - R_j \) RF link.

On the other hand, the outage probability for the Rician Fading is given by [19]:

\[
p_{i,j} = 1 - Q_1 \left( \sqrt{\frac{2K_{i,j}}{\Omega_{i,j}}} \right) \sqrt{\frac{2(K_{i,j} + 1)\Omega_{th}}{\Omega_{i,j}}} \]

(3.10)

Where \( Q_1(\cdot, \cdot) \) represents the Marcum Q function of first order. Moreover, \( K_{i,j} \) stands for the power ratio between those of LOS component and of non-LOS component; it is described as the Rician factor.

The outage probability expressions of the RF links along with the ones described in the previous section for the FSO links will be critical for the next chapter in which an outage probability analysis is done. However a main concern to address before proceeding to chapter 4 is the difference in transmission rates between RF and FSO systems.

### 3.4 Rate of Transmission

Among the main advantages of the mixed RF/FSO system is maintaining high data rate from FSO links while benefiting from the high reliability of the RF links in bad weather conditions.
that hugely degrade the performance of FSO links. However the difference in the data rates between FSO and RF links raises two questions: Is the average data rate of the system affected? And will the system encounter any dropped packets due to mismatched data rates?

Answering the first question is straightforward, as the average data rate of the system will not be affected. This comes from two facts: First, we will not be activating the $R - R$ RF links unless the direct $S - D$ FSO link fails. Second, we will continue transmitting the packets through the direct FSO link in parallel with the process of transmission through the RF links.

For the second question, we are concerned with the separation time between two consecutive failures i.e. two consecutive uses of the $R - R$ RF links. In this context, we assume that while the RF links are in use, the system proceeds to transmit packets through the FSO link, therefore we need to assure that there are no two failures of the direct FSO link while the RF links are in use. In this context, we can witness on average that two failures of the direct FSO link are separated by $1/P_e$ where $P_e$ is the outage probability of the FSO link with typical values below $10^{-6}$ as shown in figure 3.3. Moreover, we discussed that the RF links have lower data rates by a ratio ranging from 10 to 100; we denote this ratio by $M$. The condition needed for this system to avoid dropping packets is encountering $1/P_e > M$. In fact, only the packets that can not be transmitted along the $S - D$ link will be relayed. Therefore, at $P_e = 10^{-6}$, the RF
links will be activated $10^{-6}$ of the time and no problem will arrive if $\frac{\text{Rate of FSO}}{\text{Rate of RF}} < 10^6$ which is the case since this ratio ranges from 10 to 100 [106, 110, 132]. Now if we assume that the system could still encounter such a situation, even if with a very low probability, we can suggest the solutions proposed by [133] such that a small buffer for the data can be used for such rare cases.

### 3.5 All-Active vs Selective Relaying

As described in sections 1.1.4.2.3.1, 1.1.4.2.3.2, and 3.1, there are two relaying protocols to be considered with the described system: All-Active relaying scheme and Selective relaying scheme. There are major differences to consider between the two protocols such as the system phases and cooperation strategy which were discussed in section 3.1. In addition, we discuss in the next sections the affected equations and introducing new parameters and expressions for strength analysis.

#### 3.5.1 Affected Equations

Both relaying protocols correspond to FSO links which are modeled with a gamma-gamma distribution as presented in section 3.2. Therefore, all the presented equations hold for both protocols, but the only difference is the number of activated links $N_{\text{tot}}$ which is presented in equations (3.5) and (3.7). This variable was discussed in section 3.2 to vary with the number of relays $N$ when deploying the All-Active relaying scheme as a function of $2N + 1$, while sticking to 1 or 2 in the case of Selective Relaying scheme. This directly affects the outage probability equation (3.7) leading to lower value for the expression $\frac{N_{\text{tot}}}{\text{Pr}(I < X)}$. Mathematically, $\text{Pr}(I < X)$ is smaller than $\text{Pr}(I < Y)$ when $X$ is smaller than $Y$. Therefore the outage probability for the FSO links will decrease as the value of $N_{\text{tot}}$ decreases, which is a favorable behavior for our system increasing the reliability of the individual FSO links.

#### 3.5.2 New Parameters and Expressions

As described in section 3.1, the fourth step in the Selective relaying cooperation strategy is: Path Strength Analysis, where a number of possible paths for the packet transmission are available. These paths, serving the packet transmission from the source to the destination of the system, are studied with respect to their strength against bad atmospheric conditions. In this
context, a new parameter is introduced to the system describing the strength of the FSO link $R_i - R_j$, namely $\alpha_{i,j}$. This parameter is essential in determining the strongest path and in calculating the outage probability of the system later in chapter 4. For determining the strongest path in the system, we recall two types of paths: parallel and serial.

The strength of parallel links is the strength of the strongest path among them. This comes from the concept of parallelism where redundant links are available and only one link is to be chosen which is supposed to be the strongest link. If the strongest link fails, then logically the remaining links will fail too. This results in the following definition for the example given in figure 3.4:

\[
\text{Strength of Parallel System} = \text{Strength of the Strongest Link} = \max\{\alpha_{0,1}, \alpha_{0,2}, \ldots, \alpha_{0,N}\}
\]

(3.11)

On the other side, the strength of serial links is determined by the weakest link which is the first probable to fail thus leading the failure to the whole path. This concept is defined as a single point of failure where the failure of one link leads directly to the failure of the whole path; this link usually is the weakest link along the path. Such a system is described in figure 3.5, and its strength expression is given by:

\[
\text{Strength of Serial System} = \text{Strength of the Weakest Link} = \min\{\alpha_{0,1}, \alpha_{1,2}, \ldots, \alpha_{N-1,N}\}
\]

(3.12)
Chapter 4

Outage Probability Analysis

In this chapter, we perform an outage probability analysis for the described systems under the various presented relaying schemes and protocols. We start by presenting the analysis for All-Active relaying protocol that was presented in sections 1.1.4.2.3.1 and 3.1 applied for FSO systems with: No Inter Relay Cooperation (NIRC), Unidirectional Inter Relay Cooperation (IRC1), Bidirectional Inter Relay Cooperation (IRC2), and the proposed All-to-All scheme which applies more practically to mixed RF/FSO systems as discussed in sections 1.1.5.3 and 3.1. We then perform the same outage probability analysis but for the Selective Relaying protocol that was discussed in sections 1.1.4.2.3.2 and 3.1. For each presented scenario, we discuss the special cases for small number of relays in the system, and conclude a general case with any number of relays where applicable. Moreover, we use in our analysis the three presented mathematical methods in chapter 2: Exact Analysis (with Conditional Probability Approach), Approximation Analysis (powered by the Minimum Cut Set Method), and Simulation Analysis (using Monte Carlo simulation). Even more, for the exact analysis using the conditional probability approach, we present two ways of the analysis by conditioning on the $S - R$ links followed by conditioning on the $R - R$ links if applicable (such as with IRC2 and All-to-All schemes). As for the outage probability terms, we reference the presented equations in section 3.2 for the FSO links, and in section 3.3 for the RF links.

4.1 All-Active

For the sake of clarity, in what follows, we remove the direct $S - D$ FSO link from our analysis, and we multiply the final expressions by the term $p_{0,N+1}$ which corresponds for the outage
probability of the direct $S - D$ link.

### 4.1.1 NIRC

#### 4.1.1.1 Exact Analysis

#### 4.1.1.1.1 Special Cases

In this section, we consider two special cases relevant to NIRC systems with 2, and 3 relays, respectively.

For $N = 2$ Relays, we consider the system presented in figure 4.1. By conditioning on the $S - R$ links, we have 4 cases to consider for the provided system:

1. Both links $S - R_1$ and $S - R_2$ are in outage, with probability $p_{0,1}p_{0,2}$

2. Link $S - R_1$ is in outage but $S - R_2$ is active, with probability $p_{0,1}(1 - p_{0,2})$

3. Link $S - R_2$ is in outage but $S - R_1$ is active, with probability $(1 - p_{0,1})p_{0,2}$

4. Both links $S - R_1$ and $S - R_2$ are active, with probability $(1 - p_{0,1})(1 - p_{0,2})$

For the first case, as shown in figure 4.2, the system automatically falls in outage which results in the outage probability expression $p_{0,1}p_{0,2}$ for this case.

For the second case presented in figure 4.3, the system has only one way to reach the destination when the packet reaches $R_2$. This can only happen through the link $R_2 - D$, which results in the following expression for this case: $p_{0,1}(1 - p_{0,2})p_{2,3}$. 
Figure 4.2: NIRC with 2 Relays - Case 1

Figure 4.3: NIRC with 2 Relays - Case 2
For the third case, as we can see in figure 4.4, the system has only one way to reach the destination from the delivered packet to $R_1$ which is through the link $R_1 - D$, thus the resultant expression for this case is: $(1 - p_{0,1})p_{0,2}p_{1,3}$.

However for the fourth case, the two relays are reached successfully as shown in figure 4.5. This indicates that there are two ways to reach the destination either through $R_1 - D$ link or $R_2 - D$ link, resulting in the following expression for this case: $(1 - p_{0,1})(1 - p_{0,2})p_{1,3}p_{2,3}$.

The resultant final outage expression for the NIRC scheme with 2 relays, after multiplying with the $S - D$ outage probability term $p_{0,3}$, denoted by $P_{out,NIRC}^{(2)}$ is:

$$P_{out,NIRC}^{(2)} = p_{0,3}[p_{0,1}p_{0,2} + p_{0,1}(1 - p_{0,2})p_{2,3} + (1 - p_{0,1})p_{0,2}p_{1,3} + (1 - p_{0,1})(1 - p_{0,2})p_{1,3}p_{2,3}]$$

(4.1)
Now for $N = 3$ Relays, we consider the system presented in figure 4.6. By conditioning on the $S - R$ links, we have 8 cases to consider here for the presented system:

1. All 3 links $S - R_1$, $S - R_2$, and $S - R_3$ are in outage, with probability $p_{0,1}p_{0,2}p_{0,3}$

2. Only $S - R_1$ is active while both $S - R_2$ and $S - R_3$ are in outage, with probability $(1 - p_{0,1})p_{0,2}p_{0,3}$

3. Only $S - R_2$ is active while both $S - R_1$ and $S - R_3$ are in outage, with probability $p_{0,1}(1 - p_{0,2})p_{0,3}$

4. Only $S - R_3$ is active while both $S - R_1$ and $S - R_2$ are in outage, with probability $p_{0,1}p_{0,2}(1 - p_{0,3})$

5. Both $S - R_1$ and $S - R_2$ are active while $S - R_3$ is in outage, with probability $(1 - p_{0,1})(1 - p_{0,2})p_{0,3}$

6. Both $S - R_1$ and $S - R_3$ are active while $S - R_2$ is in outage, with probability $(1 - p_{0,1})p_{0,2}(1 - p_{0,3})$

7. Both $S - R_2$ and $S - R_3$ are active while $S - R_1$ is in outage, with probability $p_{0,1}(1 - p_{0,2})(1 - p_{0,3})$

8. All 3 links $S - R_1$, $S - R_2$, and $S - R_3$ are active, with probability $(1 - p_{0,1})(1 - p_{0,2})(1 - p_{0,3})$

For the first case, as shown in figure 4.7, the system automatically fails which would result in outage expression $p_{0,1}p_{0,2}p_{0,3}$ for this case.
For the second case presented in figure 4.8, the system has only one way to reach the destination after reaching $R_1$. This can only happen through the link $R_1 - D$, which results in the following outage expression for this case: $(1 - p_{0,1})p_{0,2}p_{0,3}p_{1,4}$. Similarly, for the third and fourth cases presented in figures 4.9 and 4.10, the outage expressions are $p_{0,1}(1 - p_{0,2})p_{0,3}p_{2,4}$ and $p_{0,1}p_{0,2}(1 - p_{0,3})p_{3,4}$, respectively.

In the fifth case which is presented in figure 4.11, two relays successfully receive the packet through the active links. Therefore, there are two ways to reach $D$, through $R_1 - D$ or $R_2 - D$ links which should be in outage for the whole system to be in outage. This results in the following outage expression for this case: $(1 - p_{0,1})(1 - p_{0,2})p_{0,3}p_{1,4}p_{2,4}$. Similarly, for the sixth and seventh cases presented in figures 4.12 and 4.13, the outage expressions are $(1 - p_{0,1})p_{0,2}(1 - p_{0,3})p_{1,4}p_{3,4}$ and $p_{0,1}(1 - p_{0,2})(1 - p_{0,3})p_{2,4}p_{3,4}$, respectively.

Finally, in the last case presented in figure 4.14, all the $S - R$ links are active leading to the successful delivery of the packet to the 3 relays $R_1$, $R_2$, and $R_3$. For this system to be in outage
Figure 4.9: NIRC with 3 Relays - Case 3

Figure 4.10: NIRC with 3 Relays - Case 4

Figure 4.11: NIRC with 3 Relays - Case 5
Figure 4.12: NIRC with 3 Relays - Case 6

Figure 4.13: NIRC with 3 Relays - Case 7
all the subsequent $R - D$ links should be in outage i.e. $R_1 - D$, $R_2 - D$, and $R_3 - D$ implying
the following outage expression: $(1 - p_{0,1})(1 - p_{0,2})(1 - p_{0,3})p_{1,4}p_{2,4}p_{3,4}$.

Combining all these cases together, we get the following final outage expression for the
NIRC scheme with 3 relays ($P_{\text{out,NIRC}}$) after multiplying with the $S - D$ outage probability term $p_{0,4}$:

$$P_{\text{out,NIRC}} = p_{0,4}[p_{0,1}p_{0,2}p_{0,3} + (1 - p_{0,1})p_{0,2}p_{0,3}p_{1,4} + p_{0,1}(1 - p_{0,2})p_{0,3}p_{2,4}$$
$$+ p_{0,1}p_{0,2}(1 - p_{0,3})p_{3,4} + (1 - p_{0,1})(1 - p_{0,2})p_{0,3}p_{1,4}p_{2,4} + (1 - p_{0,1})p_{0,2}(1 - p_{0,3})p_{1,4}p_{3,4}$$
$$+ p_{0,1}(1 - p_{0,2})(1 - p_{0,3})p_{2,4}p_{3,4} + (1 - p_{0,1})(1 - p_{0,2})(1 - p_{0,3})p_{1,4}p_{2,4}p_{3,4}]$$

(4.2)

4.1.1.2 General Case

Before proceeding with deducing a general expression for the outage probability with the
NIRC scheme, we present another way for calculating the outage probabilities. This method
is based on the parallel and serial subsystems concept where for parallel paths, all the paths
should be in outage for the communication to fail; this implies that the outage probability of
these paths will be the multiplication of the outage probabilities of each individual path. This
can be described as $\prod_{i=1}^{n} P_i$ where $P_i$ is the outage probability of the i-th path. However, for serial
paths or links, it is enough for one link to be in outage for the whole communication to fail;
this can be described as the unreliability of the system given by $1 - \prod_{i=1}^{n} (1 - P_i)$ where $(1 - P_i)$
is the reliability of the i-th link.

In this context, NIRC systems are composed of $N$ parallel paths, where each path is formed
by two serial links. All these paths are in parallel with the $S - D$ link as shown in figure 4.15
For the whole system to fail, then all these paths should fail. In fact, the outage probability of the first path is given by: \( P_1 = (1 - (1 - p_{0,1})(1 - p_{1,N+1})) \). Similarly, for the second path, \( P_2 = (1 - (1 - p_{0,2})(1 - p_{2,N+1})) \). Therefore, the outage probability of the general \( i \)-th link \( S - R_i - D \) is given by: \( P_i = (1 - (1 - p_{0,i})(1 - p_{i,N+1})) \). As a special case, the direct path from \( S \) to \( D \) has an outage probability \( P_0 = p_{0,N+1} \). Now, since all these paths are parallel to each other, then the outage probability of the whole system can be given as:

\[
P^{(N)}_{\text{out, NIRC}} = \prod_{i=0}^{N} P_i = P_0^{N+1} \prod_{i=1}^{N} (1 - (1 - p_{0,i})(1 - p_{i,N+1}))
\]  

(4.3)
4.1.1.2 Approximate Analysis

Using the Minimum Cut Set method that was presented in chapter 2, we can approximate the final outage probability expressions by an upper bound by transforming any given communication system network into a reliability system as shown in section 2.3.3.

4.1.1.2.1 Application to Special Cases

First, we apply the minimum cut set method for the case of NIRC with 2 relays after transforming the communication system into a reliability system as shown in figure 4.16. In the given figure components A, B, C, and D represent the FSO links $S - R_1$, $S - R_2$, $R_1 - D$, and $R_2 - D$ respectively.

1. The minimal paths in the system are: AC, and BD.

2. The incidence matrix is provided in figure 4.17.

3. No non-zero columns exist, therefore no first order cut sets are available.

4. By adding combinations of two columns, the second order cut sets are: AB, AD, BC and CD. However there are no first order sets to check against for the elimination step, thus AB, AD, BC and CD are minimal cut sets.

5. By adding combinations of three columns, the third order cut sets are: ABC, ABD, ACD, and BCD. All these cut sets contain either AB, AD, BC or CD (from second order cut sets), thus no more minimal cut sets are available.

6. The minimal cut sets in this system are: AB, AD, BC and CD.

The resultant upper bound outage expression for NIRC with 2 relays, after replacing the components with the relevant outage probability terms and considering the direct term $p_{0,3}$, is:

$$P_{U,B,NIRC}^{(2)} = p_{0,3}[p_{0,1}p_{0,2} + p_{0,1}p_{2,3} + p_{0,2}p_{1,3} + p_{1,3}p_{2,3}]$$

Then we apply the minimum cut set method for the case of NIRC with 3 relays after the transformation of the communication system into the presented reliability system in figure 4.18 where components A, B, C, D, E, and F correspond to the FSO links $S - R_1$, $S - R_2$, $S - R_3$, $R_1 - D$, $R_2 - D$, and $R_3 - D$ respectively.
Figure 4.16: NIRC with 2 Relays - Reliability System

Figure 4.17: Incidence Matrix of NIRC system with 2 Relays

<table>
<thead>
<tr>
<th>Path</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>BD</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
1. The minimal paths in the system are: AD, BE, and CF.

2. The incidence matrix is provided in figure 4.19.

3. No non-zero columns exist, therefore no first order cut sets are available.

4. By adding combinations of two columns, we can’t find any combination resulting in a non-zero column, therefore no second order cut sets are available.

5. By adding combinations of three columns, the third order cut sets are: ABC, ABF, ACE, AEF, BCD, BDF, CDE, and DEF which are minimal since no previous order cut sets are available.

6. No more minimal cut sets are available.

7. The minimal cut sets in this system are: ABC, ABF, ACE, AEF, BCD, BDF, CDE, and DEF.

The resultant upper bound outage expression for NIRC with 3 relays, after replacing the components with the relevant outage probability terms and considering the direct term $p_{0,4}$, is:

$$P_{U.B.,NIRC}^{(3)} = p_{0,4} \left[ p_{0,1}p_{0,2}p_{0,3} + p_{0,1}p_{0,2}p_{3,4} + p_{0,1}p_{0,3}p_{2,4} + p_{0,1}p_{2,4}p_{3,4} 
+ p_{0,2}p_{0,3}p_{1,4} + p_{0,2}p_{1,4}p_{3,4} + p_{0,3}p_{1,4}p_{2,4} + p_{1,4}p_{2,4}p_{3,4} \right] \quad (4.5)$$

Similarly, we can prove that for $N = 4$ relays, the upper-bound takes the following expression:

$$P_{U.B.,NIRC}^{(4)} = p_{0,5} \left[ p_{0,1}p_{0,2}p_{0,3}p_{0,4} + p_{0,1}p_{0,2}p_{0,3}p_{4,5} + p_{0,1}p_{0,2}p_{3,5}p_{0,4} + p_{0,1}p_{0,3}p_{2,4}p_{4,5} + p_{0,1}p_{2,5}p_{0,3}p_{0,4} 
+ p_{0,1}p_{2,5}p_{0,3}p_{4,5} + p_{0,1}p_{2,5}p_{3,5}p_{4,5} + p_{1,5}p_{0,2}p_{0,3}p_{4,5} + p_{1,5}p_{0,2}p_{0,3}p_{4,5} + p_{1,5}p_{0,2}p_{3,5}p_{0,4} 
+ p_{1,5}p_{0,2}p_{3,5}p_{4,5} + p_{1,5}p_{2,5}p_{0,3}p_{4,5} + p_{1,5}p_{2,5}p_{0,3}p_{4,5} + p_{1,5}p_{2,5}p_{3,5}p_{4,5} + p_{1,5}p_{2,5}p_{3,5}p_{4,5} \right] \quad (4.6)$$

4.1.1.2 Concluding a General Case

As we can notice from applying the minimum cut set method on special cases, the upper bound equations follow a certain pattern. This pattern comes from the fact that NIRC system with $N$ relays is composed of $N$ parallel $S - R - D$ paths. Therefore, we always have in the minimal cut sets one link of each path in outage, either the $S - R$ link or the $R - D$ link while
Figure 4.18: NIRC with 3 Relays - Reliability System

Figure 4.19: Incidence Matrix of NIRC system with 3 Relays

<table>
<thead>
<tr>
<th>Path</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BE</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>CF</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
taking into consideration all possible combinations. To conclude the general formula, we define a set \( L_{n,i} \) which have \( n \) \( R-D \) links in outage and \( i \) goes from 1 to \( \binom{N}{n} \) representing all possible subsets of the \( N \) links with \( n \) failures. Complementary to \( L_{n,i} \), we have the set \( \overline{L}_{n,i} \) representing the other links which are active. As a result, the upper bound general case equation can be written as (while taking into consideration the direct \( S-D \) link with outage term \( p_{0,N+1} \)):

\[
P^{(N)}_{U.B.,NIRC} = p_{0,N+1} \left[ \sum_{n=0}^{N} \sum_{i=1}^{\binom{N}{n}} \left( \prod_{j \in L_{n,i}} p_{j,N+1} \right) \left( \prod_{j' \in \overline{L}_{n,i}} p_{0,j'} \right) \right]
\]  

(4.7)

Where \( p_{j,N+1} \) terms correspond to the \( R-D \) links that are in outage, and \( p_{0,j'} \) terms correspond to the \( S-R \) links that are opposite to the active \( R-D \) links.

4.1.1.3 Simulation Analysis

As discussed earlier in section 2.2.4, the Monte Carlo simulation is used to provide simulation of the real behavior of the studied systems. The results can confirm the validity of the presented theoretical exact and approximate results. For this purpose, the computerized sampling method is implemented through a MATLAB code that can be run for any NIRC system with any number of relays for large number of trials. The main algorithm behind this simulation goes as follows:

1. First, we need to assign random binary values (0 for outage and 1 for active) for all the presented links in the system based on their predefined individual outage probability values.

2. Second, based on the binary value of the direct \( S-D \) link, we have to check if the sent information can be delivered directly or not. This means, if the binary value is 1, then there is no outage in the system and we can stop the analysis of this trial here and we go to the sixth step. Else, a binary value of 0 means that this link is in outage and thus we need to check the other links for a possible outage in the system by going to the third step.

3. Third, we check the binary values of all the \( S-R \) links. For any active link with binary value 1, we add the corresponding relay index into a new set of successfully reached relays. This means that each relay in this set is able to receive the sent information by the source \( S \). If this set is empty, then we go immediately to the fifth step meaning the system is in outage. Else, we proceed with the fourth step.
4. Fourth, we check all the $R - D$ links that are associated with the successfully reached relays. If all of these links are with a binary value of 0 meaning they are in outage, then the system is in outage and we go to the fifth step. Else, if any link among the checked $R - D$ links is active, then the system does not experience an outage and therefore we proceed to the sixth step.

5. Now the system is in outage so we increment the counter of outages during the simulation. We then check if we reached the desired number of trials: if yes, we go to the final step; else, we increment the counter of trials and go back to the first step.

6. Here, the system is not in outage, so we check if we reached the desired number of trials. If yes, we go to the final step. Else, we increment the counter of trials and go back to the first step.

7. Finally, we divide the counter of outages by the number of trials to obtain the simulated outage probability value of the system.

4.1.2 IRC1

4.1.2.1 Exact Analysis

4.1.2.1.1 Special Cases

In a similar manner to the NIRC scheme, we consider two special cases in this section relevant to IRC1 systems with 2 and 3 relays, respectively.

For $N = 2$ Relays, we consider the system presented in figure 4.20. By conditioning on the $S - R$ links, we have 4 cases to consider for the provided system:

1. Both links $S - R_1$ and $S - R_2$ are in outage as shown in figure 4.21, with probability $p_{0,1}p_{0,2}$. In this scenario the system automatically fails. The resultant outage expression remains $p_{0,1}p_{0,2}$.

2. Link $S - R_1$ is in outage but $S - R_2$ is active as shown in figure 4.22, with probability $p_{0,1}(1 - p_{0,2})$. This case is similar to that of NIRC since only $R_2$ receives the packet, resulting in an outage expression: $p_{0,1}(1 - p_{0,2})p_{2,3}$.

3. Link $S - R_2$ is in outage but $S - R_1$ is active as shown in figure 4.23, with probability $(1 - p_{0,1})p_{0,2}$. In this case the packet has two possible paths from $R_1$: $R_1 - D$ or $R_1 -$
Therefore the consequent outage expression is: \((1 - p_{0,1})p_{0,2}p_{1,3}p_{1,2,3}\) with 
\[p_{1,2,3} = (1 - (1 - p_{1,2})(1 - p_{2,3}))\] (given by serial links property that was explained in section 4.1.1.1.2).

4. Both links \(S - R_1\) and \(S - R_2\) are active as shown in figure 4.24, with probability \((1 - p_{0,1})(1 - p_{0,2})\). This case is the same as with NIRC requiring the links \(R_1 - D\) and \(R_2 - D\) to be in outage regardless of the state of \(R_1 - R_2\) link. This implies an outage
expression: \((1 - p_{0,1})(1 - p_{0,2})p_{1,3}p_{2,3}\).

The resultant final outage expression for the IRC1 scheme with 2 relays, after multiplying with the \(S - D\) outage probability term \(p_{0,3}\), is:

\[
P_{\text{out,IRC1}}^{(2)} = p_{0,3}[p_{0,1}p_{0,2} + p_{0,1}(1 - p_{0,2})p_{2,3} + (1 - p_{0,1})p_{0,2}p_{1,3}(1 - (1 - p_{1,2})(1 - p_{2,3}))]
+ (1 - p_{0,1})(1 - p_{0,2})p_{1,3}p_{2,3} \tag{4.8}
\]
Similarly, for $N = 3$ Relays, we consider the system presented in figure 4.25. By conditioning on the $S - R$ links, we have also 8 cases to consider here for the presented system:

1. All 3 links $S - R_1$, $S - R_2$, and $S - R_3$ are in outage as shown in figure 4.26, where the system automatically fails and the resultant outage probability expression is: $p_{0,1}p_{0,2}p_{0,3}$.

2. Only $S - R_1$ link is active while both $S - R_2$ and $S - R_3$ are in outage as shown in figure 4.27, with probability $(1 - p_{0,1})p_{0,2}p_{0,3}$. In this case we have 3 available paths for the packet starting from $R_1$ through either $R_1 - D$, $R_1 - R_2 - D$, or $R_1 - R_2 - R_3 - D$. So we surely need $R_1 - D$ to be in outage with the term $p_{1,4}$ but we also condition further on the common link between the other 2 paths which is $R_1 - R_2$. Now if $R_1 - R_2$ is in outage, then the 2 paths will be in outage. However if $R_1 - R_2$ is active, then we are left with 2 independent paths to deliver the packet: $R_2 - D$ and $R_2 - R_3 - D$. The resultant outage expression for all these scenarios is: $(1 - p_{0,1})p_{0,2}p_{0,3}p_{1,4}(p_{1,2} + (1 - p_{1,2})p_{2,4}p_{2,3,4})$ where $p_{2,3,4} = (1 - (1 - p_{2,3})(1 - p_{3,4}))$ (given by serial links property that was explained in section 4.1.1.1.2).

3. Only $S - R_2$ link is active while both $S - R_1$ and $S - R_3$ are in outage as shown in figure 4.28, with probability $p_{0,1}(1 - p_{0,2})p_{0,3}$. This scenario results in delivering the message to $R_2$ where we have 2 available paths to reach $D$ through either $R_2 - D$ or $R_2 - R_3 - D$. This results in the following outage expression: $p_{0,1}(1 - p_{0,2})p_{0,3}p_{2,4}p_{2,3,4}$.

4. Only $S - R_3$ link is active while both $S - R_1$ and $S - R_2$ are in outage, as shown in figure 4.29 with probability $p_{0,1}p_{0,2}(1 - p_{0,3})$. This scenario is the same as the one with NIRC, so the resultant outage expression is: $p_{0,1}p_{0,2}(1 - p_{0,3})p_{3,4}$.
Figure 4.26: IRC1 with 3 Relays - Case 1

Figure 4.27: IRC1 with 3 Relays - Case 2

Figure 4.28: IRC1 with 3 Relays - Case 3

Figure 4.29: IRC1 with 3 Relays - Case 4
5. Both $S - R_1$ and $S - R_2$ links are active while $S - R_3$ is in outage as shown in figure 4.30, with probability $(1 - p_{0,1})(1 - p_{0,2})p_{0,3}$. Similarly to the NIRC scheme, two relays successfully receive the packet through the active links implying there are two links that can directly reach $D$ and should be in outage: $R_1 - D$ and $R_2 - D$. However, with IRC1, one additional path is added regardless of the state of $R_1 - R_2$ link which is $R_2 - R_3 - D$. Therefore, the resultant outage expression for this case is given by: $(1 - p_{0,1})(1 - p_{0,2})p_{0,3}p_{1,4}p_{2,4}p_{2,3,4}$.

6. Both $S - R_1$ and $S - R_3$ links are active while $S - R_2$ is in outage as shown in figure 4.31, with probability $(1 - p_{0,1})p_{0,2}(1 - p_{0,3})$. In this scenario, 3 paths should be in outage: $R_1 - D$, $R_1 - R_2 - D$, and $R_3 - D$. The correspondent outage expression for this case is: $(1 - p_{0,1})p_{0,2}(1 - p_{0,3})p_{1,4}p_{3,4}p_{1,2,4}$ where $p_{1,2,4} = (1 - (1 - p_{1,2})(1 - p_{2,4}))$.

7. Both $S - R_2$ and $S - R_3$ links are active while $S - R_1$ is in outage as shown in figure 4.32, with probability $p_{0,1}(1 - p_{0,2})(1 - p_{0,3})$. In this scenario, we have to care about only 2 links regardless of the state of $R_2 - R_3$ link; these links are: $R_2 - D$ and $R_3 - D$. The resultant outage expression is: $p_{0,1}(1 - p_{0,2})(1 - p_{0,3})p_{2,4}p_{3,4}$.

8. All 3 links $S - R_1$, $S - R_2$, and $S - R_3$ are active, as shown in figure 4.33, with probability $(1 - p_{0,1})(1 - p_{0,2})(1 - p_{0,3})$. As with NIRC, for this system to be in outage, all the opposite $R - D$ links should be in outage i.e. $R_1 - D$, $R_2 - D$, and $R_3 - D$ implying the following outage expression: $(1 - p_{0,1})(1 - p_{0,2})(1 - p_{0,3})p_{1,4}p_{2,4}p_{3,4}$.
Combining all these cases together, we get the following final outage expression for the IRC1 scheme with 3 relays ($P_{\text{out},\text{IRC1}}^{(3)}$) after multiplying with the $S – D$ outage probability term
\( p_{0.4} \):

\[
P_{out,IRC1}^{(3)} = p_{0.4} p_{0.1} p_{0.2} p_{0.3} + (1 - p_{0.1}) p_{0.2} p_{0.3} p_{1.4} (p_{1.2} + (1 - p_{1.2}) p_{2.4} p_{2.3,4}) + p_{0.1} (1 - p_{0.2}) p_{0.3} p_{2.4}^2 p_{2,3,4} + p_{0.1} p_{0.2} (1 - p_{0.3}) p_{3.4} + (1 - p_{0.1}) (1 - p_{0.2}) p_{0.3} p_{1.4} p_{2.4} p_{2,3,4} + (1 - p_{0.1}) p_{0.2} (1 - p_{0.3})
\]

\[
p_{1.4} p_{3.4} p_{1,2,4} + p_{0.1} (1 - p_{0.2}) (1 - p_{0.3}) p_{2.4} p_{3,4} + (1 - p_{0.1}) (1 - p_{0.2}) (1 - p_{0.3}) p_{1.4} p_{2.4} p_{3,4}^4
\]

(4.9)

Where the probability terms labelled as \( p_{i,j,z} \) are defined as the outage probability of a serial path as:

\[
p_{i,j,z} = (1 - (1 - p_{i,j})(1 - p_{j,z}))
\]

(4.10)

4.1.2.2 Approximate Analysis

4.1.2.2.1 Application to Special Cases

Here, we apply the previously described minimum cut set method for the unidirectional IRC FSO networks for the case of 2 and 3 relays. For the case of 2 relays, the transformed system is given in figure 4.34 where components A, B, C, D, and E represent the FSO links \( S - R_1 \), \( S - R_2 \), \( R_1 - D \), \( R_2 - D \), and \( R_1 - R_2 \), respectively.

1. The minimal paths in the system are: AC, BD, and AED.
2. The incidence matrix is provided in figure 4.35.
3. No non-zero columns exist, therefore no first order cut sets are available.
4. By adding combinations of two columns, the second order cut sets are: AB, AD, and CD. Since there is no first order sets, thus AB, AD and CD are minimal cut sets.
5. By adding combinations of three columns, the third order cut sets are: ABC, ABD, ABE, ACD, ADE, BCD, BCE, and CDE. However each of ABC, ABD, ABE, ACD, ADE, BCD, and CDE contains one of AB, AD or CD (from second order cut sets), thus they are eliminated. So we are left with BCE as third order minimal cut set.
6. No more minimal cut sets are available.
7. The minimal cut sets in this system are: AB, AD, CD, and BCE.
Figure 4.34: IRC1 scheme with 2 Relays - Reliability System

Figure 4.35: Incidence Matrix of IRC1 system with 2 Relays

<table>
<thead>
<tr>
<th>Path</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BD</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<tr>
<td>AED</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
The resultant upper bound outage expression for IRC1 with 2 relays, after replacing the components with the relevant outage probability terms and considering the direct term $p_{0,3}$, is:

$$P_{U.B..IRC1}^{(2)} = p_{0,3}[p_{0,1}p_{0,2} + p_{0,1}p_{0,2} + p_{0,2}p_{1,2}p_{1,3} + p_{1,3}p_{2,3}]$$

(4.11)

Similarly, with the case of 3 relays, the transformed system is shown in figure 4.36 where components A, B, C, D, E, F, G, and H represent the FSO links $S - R_1$, $S - R_2$, $S - R_3$, $R_1 - D$, $R_2 - D$, $R_3 - D$, $R_1 - R_2$, and $R_2 - R_3$, respectively.

1. The minimal paths in the system are: AD, BE, CF, AGE, BHF, and AGHF.
2. The incidence matrix is provided in figure 4.37.
3. No non-zero columns exist, therefore no first order cut sets are available.
4. By adding combinations of two columns, no cut sets are discovered.
5. By adding combinations of three columns, the third order cut sets are: ABC, ABF, AEF, and DEF. However there are no first order or second order sets to check against for the elimination step, thus ABC, ABF, AEF, and DEF are minimal cut sets.
6. Similarly, the fourth order minimal cut sets are: ACEH, BDFG, and CDEH.
7. In the same manner, the only fifth order minimal cut set is: BCDGH.
8. No more minimal cut sets are available.
9. The minimal cut sets in this system are: ABC, ABF, AEF, DEF, ACEH, BDFG, CDEH, and BCDGH.

The resulting upper bound outage expression for IRC1 with 3 relays, after replacing the components with their outage probability terms and considering the direct term $p_{0,4}$, is:

$$P_{U.B..IRC1}^{(3)} = p_{0,4}[p_{0,1}p_{0,2}p_{0,3} + p_{0,1}p_{0,2}p_{0,3} + p_{0,2}p_{1,2}p_{1,3}p_{2,3} + p_{0,1}p_{0,3}p_{2,4}p_{2,3} + p_{0,2}p_{1,4}p_{3,4}p_{1,2} + p_{0,2}p_{0,3}p_{1,4}p_{1,2}p_{2,3} + p_{0,3}p_{1,4}p_{2,4}p_{2,3} + p_{1,4}p_{2,4}p_{3,4}]$$

(4.12)
Figure 4.36: IRC1 scheme with 3 Relays - Reliability System

Figure 4.37: Incidence Matrix of IRC1 system with 3 Relays

<table>
<thead>
<tr>
<th>Path</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
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<tbody>
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<td>1</td>
<td>1</td>
<td>1</td>
</tr>
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</table>
4.1.3 IRC2

4.1.3.1 Exact Analysis

With IRC2, we introduce a new way of analysis by conditioning on the inter relay $R - R$ links. This will simplify the calculations since now we have $2^{N-1}$ cases to consider instead of $2^N$ cases where $N$ is the number of relays and $N - 1$ correspond to the number of the inter-relay links in the system. Moreover, this will enable us to benefit from the previous results of the NIRC system and to use parallel and serial relaying concepts that were introduced in section 4.1.1.1.2.

4.1.3.1.1 Special Cases

In this section, we consider two special cases relevant to IRC2 systems with 2 and 3 relays, respectively.

For $N = 2$ Relays, we consider the system presented in figure 4.38. By conditioning on the single $R - R$ link, we have 2 cases to consider for the provided system:

1. $R_1 - R_2$ link is in outage, as shown in figure 4.39 with probability $p_{1,2}$. The resulting system is exactly like the NIRC system whose outage probability expression was given by equation (4.1) (without the $p_{0,3}$ term). So for this case, the corresponding outage expression is given by: $p_{1,2}(p_{0,1}p_{0,2} + p_{0,1}(1 - p_{0,2})p_{2,3} + (1 - p_{0,1})p_{0,2}p_{1,3} + (1 - p_{0,1})(1 - p_{0,2})p_{1,3}p_{2,3})$.

2. $R_1 - R_2$ link is active, with probability $(1 - p_{1,2})$. The resultant system will have the 2 relays combined into 1 relay as can be seen in figure 4.40. This system has 2 paths from $S$ to $D$ with 2 parallel $S - R$ links followed by 2 parallel $R - D$ links. Following from the serial and parallel properties, the resulting outage expression for the system is given by: $(1 - p_{1,2})(1 - (1 - p_{0,1}p_{0,2})(1 - p_{1,3}p_{2,3}))$. 
Figure 4.38: IRC2 with 2 Relays

Figure 4.39: IRC2 with 2 Relays - Case 1

Figure 4.40: IRC2 with 2 Relays - Case 2
The resulting outage expression for the IRC2 scheme with 2 relays, after taking into account $p_{0,3}$ term, is given by:

$$P_{\text{out,IRC2}}^{(2)} = p_{0,3}[p_{1,2}(p_{0,1}p_{0,2} + p_{0,1}(1 - p_{0,2})p_{2,3} + (1 - p_{0,1})p_{0,2}p_{1,3}$$

$$+ (1 - p_{0,1})(1 - p_{0,2})p_{1,3}p_{2,3} + (1 - p_{1,2})(1 - (1 - p_{0,1}p_{0,2})(1 - p_{1,3}p_{2,3}))] \quad (4.13)$$

Similarly for $N = 3$ Relays, we consider the system presented in figure 4.41. By conditioning on the $R - R$ links, we have 4 cases to consider:

1. Both inter links $R_1 - R_2$ and $R_2 - R_3$ are in outage as shown in figure 4.42, with probability $p_{1,2}p_{2,3}$. This results in the same system as with NIRC with 3 relays whose outage expression was expressed in equation (4.2). However for simpler representation, we use the general case form following serial and parallel connections that was presented in section 4.1.1.1.2. The induced outage expression for this case is given by:

$$p_{1,2}p_{2,3}(1 - (1 - p_{0,1})(1 - p_{1,4}))(1 - (1 - p_{0,2})(1 - p_{2,4}))(1 - (1 - p_{0,3})(1 - p_{3,4})).$$

2. $R_1 - R_2$ link is active while $R_2 - R_3$ is in outage, with probability $(1 - p_{1,2})p_{2,3}$. In this scenario, the system simplifies into serial and parallel connections with a combined relay for $R_1$ and $R_2$ represented as $R_{1,2}$ as shown in figure 4.43. In this case the outage expression can be written as: $(1 - p_{1,2})p_{2,3}(1 - (1 - p_{0,1}p_{0,2})(1 - p_{1,4}p_{2,4}))(1 - (1 - p_{0,3})(1 - p_{3,4})).$

3. Alternatively, $R_2 - R_3$ link is active while $R_1 - R_2$ is in outage, with probability $p_{1,2}(1 - p_{2,3})$. In this case, the system simplifies into serial and parallel connections with a combined relay for $R_2$ and $R_3$ represented as $R_{2,3}$ as shown in figure 4.44. The resultant outage expression can be written as: $(1 - p_{2,3})p_{1,2}(1 - (1 - p_{0,2}p_{0,3})(1 - p_{2,4}p_{3,4}))(1 - (1 - p_{0,1})(1 - p_{1,4})).$

4. Both $R_1 - R_2$ and $R_2 - R_3$ links are active, with probability $(1 - p_{1,2})(1 - p_{2,3})$. The system simplifies into one big relay combining all the relays and paths as shown in figure 4.45. The resultant outage expression for this case is: $(1 - p_{1,2})(1 - p_{2,3})(1 - (1 - p_{0,1}p_{0,2}p_{0,3})(1 - p_{1,4}p_{2,4}p_{3,4})).$

Combining all these cases together, we get the following outage probability expression for the IRC2 scheme with 3 relays ($P_{\text{out,IRC2}}^{(3)}$) after multiplying by the $S - D$ outage probability.
Figure 4.41: IRC2 with 3 Relays

Figure 4.42: IRC2 with 3 Relays - Case 1

Figure 4.43: IRC2 with 3 Relays - Case 2
term $p_{0,4}$:

$$P^{(3)}_{out,IRC2} = p_{0,4}[p_{1,2}p_{2,3}(1-(1-p_{0,1})(1-p_{1,4}))(1-(1-p_{0,2})(1-p_{2,4}))(1-(1-p_{0,3})(1-p_{3,4}))$$
$$+ (1-p_{1,2})p_{2,3}(1-(1-p_{0,1}p_{0,2})(1-p_{1,4}p_{2,4}))(1-(1-p_{0,3})(1-p_{3,4}))$$
$$+ (1-p_{2,3})p_{1,2}(1-(1-p_{0,2}p_{0,3})(1-p_{2,4}p_{3,4}))(1-(1-p_{0,1})(1-p_{1,4}))$$
$$+ (1-p_{1,2})(1-p_{2,3})(1-(1-p_{0,1}p_{0,2}p_{0,3})(1-p_{1,4}p_{2,4}p_{3,4})]$$

(4.14)

4.1.3.2 Approximate Analysis

4.1.3.2.1 Application to Special Cases

First, we apply the minimum cut set method for the case of 2 relays with a transformed reliability system shown in figure 4.46 where A, B, C, D, and E stand for the $S - R_1$, $S - R_2$, $R_1 - D$, $R_2 - D$, and $R_1 - R_2$ links, respectively.

1. The minimal paths in the system are: AC, BD, AED, and BEC.

2. The incidence matrix is provided in figure 4.47.

3. No non-zero columns exist, therefore no first order cut sets are available.
Figure 4.46: IRC2 scheme with 2 Relays - Reliability System

Figure 4.47: Incidence Matrix of IRC2 system with 2 Relays
4. By adding combinations of two columns, the second order cut sets are: AB and CD. However there are no first order sets to check against for the elimination step, thus AB and CD are minimal cut sets.

5. By adding combinations of three columns, the third order cut sets are: ABC, ABD, ABE, ACD, ADE, BCD, BCE, and CDE. However each of ABC, ABD, ABE, ACD, BCD, and CDE contains either AB or CD (from second order cut sets), thus they are eliminated. So we are left with ADE and BCE as third order minimal cut sets.

6. No more minimal cut sets are available.

7. The minimal cut sets in this system are: AB, CD, ADE, and BCE.

The resulting upper bound outage expression for IRC2 scheme with 2 relays, after replacing the components with the relevant outage probability terms and considering the direct term $p_{0,3}$, is:

$$P_{U,B,IRC2}^{(2)} = p_{0,3} \left[ p_{0,1} p_{0,2} + p_{0,1} p_{1,2} p_{2,3} + p_{0,2} p_{1,2} p_{1,3} + p_{1,3} p_{2,3} \right]$$  \hspace{1cm} (4.15)

Then, we apply the minimum cut set method for the case of 3 relays. The transformation of the communication system into a reliability system is given by figure 4.48. In the given figure, the components A, B, C, D, E, F, G, and H represent the FSO links $S - R_1$, $S - R_2$, $S - R_3$, $R_1 - D$, $R_2 - D$, $R_3 - D$, $R_1 - R_2$, and $R_2 - R_3$, respectively.

1. The minimal paths in the system are: AD, BE, CF, AGE, BGD, BHF, CHE, AGHF, and CHGD.

2. The incidence matrix is provided in figure 4.49.

3. No non-zero columns exist, therefore no first order cut sets are available.

4. By adding combinations of two columns, no second order cut sets are discovered.

5. By adding combinations of three columns, the third order minimal cut sets are: ABC and DEF.

6. Similarly, fourth order minimal cut sets are: ABFH, AEFG, BCDG, and CDEH.

7. Moreover, the fifth order minimal cut sets are: ACEGH and BDFGH.

8. No more minimal cut sets are available.
Figure 4.48: IRC2 scheme with 3 Relays - Reliability System

Figure 4.49: Incidence Matrix of IRC2 system with 3 Relays

<table>
<thead>
<tr>
<th>Path</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
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<tr>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CF</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>AGE</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>BGD</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>BHF</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>CHE</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>AGHF</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>CHGD</td>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
9. The minimal cut sets in this system are therefore: ABC, DEF, ABFH, AEFG, BCDG, CDEH, ACEGH, and BDFGH.

The resulting upper bound outage expression for IRC2 scheme with 3 relays, after replacing the components with the corresponding outage probability terms and considering the direct term \( p_{0,4} \), is:

\[
P_{\text{out}}^{\text{IRC2}} = p_{0,4}[p_{0,1}p_{1,2}p_{0,3} + p_{0,1}p_{0,2}p_{3,4}p_{2,3} + p_{0,1}p_{0,3}p_{2,4}p_{1,2}p_{2,3} + p_{0,1}p_{2,4}p_{3,4}p_{1,2} + p_{0,2}p_{0,3}p_{1,4}p_{1,2} + p_{0,2}p_{1,4}p_{3,4}p_{1,3}p_{2,3} + p_{0,3}p_{1,4}p_{2,4}p_{2,3} + p_{1,4}p_{2,4}p_{3,4}] \tag{4.16}
\]

4.1.4 All-to-All Relaying

4.1.4.1 Exact Analysis

4.1.4.1.1 Special Cases

With \( N = 2 \) Relays, the system with All-to-All relaying scheme is the same as with IRC2. This comes from the fact that there is only two relays that can be connected with one inter relay link thus there is no difference between the 2 schemes. The resulting outage probability for the system is the same as described in equation (4.13):

\[
P_{\text{out,ALL}}^{(2)} = p_{0,3}[p_{1,2}(p_{0,1}p_{0,2} + p_{0,1}(1 - p_{0,2}))p_{2,3} + (1 - p_{0,1})p_{0,2}p_{1,3} + (1 - p_{0,1})(1 - p_{0,2}p_{1,3}p_{2,3}) + (1 - p_{1,2})(1 - (1 - p_{0,1}p_{0,2})(1 - p_{1,3}p_{2,3}))] \tag{4.17}
\]

For \( N = 3 \) Relays, we consider the system presented in figure 4.50. By conditioning on the \( R - R \) links, we have 4 scenarios to consider for the presented system:

1. All inter-relay links \( R_1 - R_2, R_1 - R_3, \) and \( R_2 - R_3 \) are in outage as shown in figure 4.51, with probability \( p_{1,2}p_{1,3}p_{2,3} \). This results in the same system as with NIRC with 3 relays.

The resultant outage expression for this case is given by: \( p_{1,2}p_{1,3}p_{2,3}(1 - (1 - p_{0,1})(1 - p_{1,4}))(1 - (1 - p_{0,2})(1 - p_{2,4}))(1 - (1 - p_{0,3})(1 - p_{3,4})). \)

2. One inter relay link is active while the other two links are in outage. This divides the system into 2 parallel paths: one path containing a big relay \( R_1, \) combining the 2 connected relays \( R_i \) and \( R_j \) through the active link \( R_i - R_j \), and the other path containing the unconnected relay \( R_k \). The 3 possible combinations for this category are pre-
sented in figures 4.52, 4.53, and 4.54. The corresponding outage expression is given by:

\[
(1 - p_{1,2})p_{1,3}p_{2,3}(1 - (1 - p_{0,1}p_{0,2})(1 - p_{1,4}p_{2,4}))(1 - (1 - p_{0,3})(1 - p_{3,4})) + p_{1,2}(1 - p_{1,3})p_{2,3}(1 - (1 - p_{0,1}p_{0,3})(1 - p_{1,4}p_{3,4}))(1 - (1 - p_{0,2})(1 - p_{2,4})) + p_{1,2}p_{1,3}(1 - p_{2,3})(1 - (1 - p_{0,2}p_{0,3})(1 - p_{2,4}p_{3,4}))(1 - (1 - p_{0,1})(1 - p_{1,4})).
\]

3. Two inter relay links are active while only one is in outage. In such scenario, the 3 relays will be connected together through the two active links. We can therefore combine these cases with the final category. The corresponding outage terms for the combinations of this category are:

\[
(1 - p_{1,2})(1 - p_{1,3})p_{2,3} + (1 - p_{1,2})p_{1,3}(1 - p_{2,3}) + p_{1,2}(1 - p_{1,3})(1 - p_{2,3}).
\]

4. All the inter links are active, thus the 3 relays are combined together with a one big virtual relay as shown in figure 4.55; this is given with a probability term \((1 - p_{1,2})(1 - p_{1,3})(1 - p_{2,3})\). Moreover, for this category along with the previous one combined together, we get the following outage expression:

\[
[(1 - p_{1,2})(1 - p_{1,3})p_{2,3} + (1 - p_{1,2})p_{1,3}(1 - p_{2,3}) + p_{1,2}(1 - p_{1,3})(1 - p_{2,3})] + (1 - p_{1,2})(1 - p_{1,3})(1 - p_{2,3})(1 - p_{0,1}p_{0,2}p_{0,3})(1 - p_{1,4}p_{2,4}p_{3,4}).
\]

Combining all these scenarios together, we get the following final outage expression for the All-to-All scheme with 3 relays \(P_{\text{out,ALL}}^{(3)}\) after multiplying with the \(S - D\) outage probability term \(p_{0,4}\):

\[
P_{\text{out,ALL}}^{(3)} = p_{0,4}[p_{1,2}p_{1,3}p_{2,3}(1 - (1 - p_{0,1})(1 - p_{1,4}))(1 - (1 - p_{0,2})(1 - p_{2,4}))(1 - (1 - p_{0,3})(1 - p_{3,4}))
+ (1 - p_{1,2})p_{1,3}p_{2,3}(1 - (1 - p_{0,1}p_{0,2})(1 - p_{1,4}p_{2,4}))(1 - (1 - p_{0,3})(1 - p_{3,4}))
+ p_{1,2}(1 - p_{1,3})p_{2,3}(1 - (1 - p_{0,1}p_{0,3})(1 - p_{1,4}p_{3,4}))(1 - (1 - p_{0,2})(1 - p_{2,4})))
+ p_{1,2}p_{1,3}(1 - p_{2,3})(1 - (1 - p_{0,2}p_{0,3})(1 - p_{2,4}p_{3,4}))(1 - (1 - p_{0,1})(1 - p_{1,4}))
+ ((1 - p_{1,2})(1 - p_{1,3})p_{2,3} + (1 - p_{1,2})p_{1,3}(1 - p_{2,3}) + p_{1,2}(1 - p_{1,3})(1 - p_{2,3})
+ (1 - p_{1,2})(1 - p_{1,3})(1 - p_{2,3})(1 - (1 - p_{0,1}p_{0,2}p_{0,3})(1 - p_{1,4}p_{2,4}p_{3,4}))]
\]

(4.18)
Figure 4.50: All-to-All with 3 Relays

Figure 4.51: All-to-All with 3 Relays - Case 1

Figure 4.52: All-to-All with 3 Relays - Case 2

Figure 4.53: All-to-All with 3 Relays - Case 3
4.1.4.1.2 General Case

As shown in the provided special cases in section 4.1.4.1.1, conditioning on the RF links can divide the system into parallel subsystems. Each of these subsystems is composed of a source $S$, a destination $D$, and an intermediate virtual relay $R_{i,j,...,n}$ where $i,j,...,n$ are the indices of the system relays forming this virtual relay; these indices range from 1 (being a single relay) to $N$ (combining all the relays together) depending on the inter-links conditions. The virtual relay in practice is a combination of multiple relays that are successfully connected with each other through the RF links. So we define a set $S_R$ containing all the combined relays for each subsystem. This enables us to determine the outage probability for each subsystem, having $i$ elements out of $N$ relays in the set $S_R$; this means having $i$ parallel links going from $S$ to the virtual relay and similarly $i$ parallel links going from the virtual relay to $D$, as follows:

$$P_{out,S_R} = 1 - \left( 1 - \prod_{i \in S_R} P_{0,i} \right) \left( 1 - \prod_{i \in S_R} P_{i,N+1} \right)$$  \hspace{1cm} (4.19)

Therefore we can determine the outage probability of the whole system under these conditions as a multiplication of the outage probabilities of each subsystem since all subsystems are
parallel to each other. This results in the following expression:

\[ P_{i_{\text{out}},S}^j = \prod_{x=1}^{n} P_{x_{\text{out}},S}^x \]  

(4.20)

Where \( n \) is the number of parallel subsystems and can vary from 1 to \( N \) depending on the conditions of the RF links. Therefore the final outage expression of the All-to-All system can be expressed as a summation of the whole \( 2^{(N(N-1)/2)} \) cases multiplied by their corresponding weights. These \( 2^{(N(N-1)/2)} \) cases come from the various combinations of the \( (N(N-1)/2) \) inter links where each link can be either active or in outage. However the main challenge is to partition the initial system into parallel subsystems based on the conditions of the RF links. In other words, we need first to study what are the possible combinations of the parallel subsystems. To do so, we define a vector of all the available \( (N(N-1)/2) \) inter \( R_n - R_{n'} \) links corresponding to their state if in outage (0) or active (1), as follows:

\[ C = [c_{1,2}, c_{1,3}, \ldots, c_{1,N}, c_{2,3}, c_{2,4}, \ldots, c_{2,N}, \ldots, c_{N-1,N}] \]  

(4.21)

Now to determine the connected relays, we discover the \( c_{n,n'} \) values and if any active path \( (k_1, k_2, \ldots, k_z) \) from \( n \) to \( n' \) exists through other links such that \( c_{n,k_1} = c_{k_1,k_2} = \cdots = c_{k_z,n'} = 1 \). Based on the state of inter-connected relays, parallel subsystems are determined such that all elements of each set of relays are not connected with any element of the other sets. This will result in a number of parallel subsystems varying from 1 (when more than \( \frac{(N-1)(N-2)}{2} \) inter-relay links are active) to \( N \) (when all \( \frac{N(N-1)}{2} \) links are in outage). For each case, a weight is defined in terms of the vector \( C \) translating it as follows:

\[ p(C) = [p(c_{1,2}), p(c_{1,3}), \ldots, p(c_{1,N}), p(c_{2,3}), p(c_{2,4}), \ldots, p(c_{2,N}), \ldots, p(c_{N-1,N})] \]  

(4.22)

Where \( p(c_{n,n'}) \) is determined based on the binary value of \( c_{n,n'} \) where a 0 value of \( c_{n,n'} \) means an outage value \( p_{n,n'} \) for \( p(c_{n,n'}) \), and a 1 value of \( c_{n,n'} \) means no outage value \( 1 - p_{n,n'} \) for \( p(c_{n,n'}) \). Finally, the final outage probability of the general case for an All-to-All system can be expressed as:

\[ P_{\text{out,ALL}}^N = \sum_{i=1}^{2^{(N(N-1)/2)}} p(C_i) P_{i_{\text{out}},S}^i \]  

(4.23)

Where \( P_{i_{\text{out}},S}^i \) was given in equation (4.20) for each system composed of \( n \) parallel subsystems.
For example, for $N = 7$ relays, we consider a vector $C = [0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1]$. This means that $c_{1,6} = c_{1,7} = c_{2,3} = c_{2,4} = c_{4,5} = 1$ resulting in two partitions $S^1_R = \{1, 6, 7\}$ and $S^2_R = \{2, 3, 4, 5\}$. These partitions are determined as follows: $c_{1,6} = 1$ implies that the link $R_1 - R_6$ is not in outage, and thus $c_{1,6} = c_{1,7} = 1$ implies that relays $R_6$ and $R_7$ are connected through $R_1$ even though the link $R_6 - R_7$ is in outage ($c_{6,7} = 0$). Similarly, $c_{2,3} = c_{2,4} = c_{4,5} = 1$ implies that the relays $R_2, R_3, R_4$, and $R_5$ are connected together. As a result, for the given vector $C$, the outage expression can be expressed as: $p(C)P^1_{out,S_R}P^2_{out,S_R}$ where $P_{out,S_R}(C) = p_1p_{1,3}p_{1,4}p_{1,5}(1 - p_{1,6})(1 - p_{1,7})(1 - p_{2,3})(1 - p_{2,4})p_{2,5}p_{2,6}p_{2,7}p_{3,4}p_{3,5}p_{3,6}p_{3,7}(1 - p_{4,5})p_{4,6}p_{4,7}p_{5,6}p_{5,7}p_{6,7}$. $P^1_{out,S_R} = 1 - (1 - p_{0,1}p_{0,6}p_{0,7})(1 - p_{1,8}p_{6,8}p_{7,8})$, and $P^2_{out,S_R} = 1 - (1 - p_{0,2}p_{0,3}p_{0,4}p_{0,5})(1 - p_{2,8}p_{3,8}p_{4,8}p_{5,8})$.

4.1.4.2 Approximate Analysis

One again, by using the Minimum Cut Set method that was presented in chapter 2, we can approximate the final outage probability expressions by an upper bound. This is done by transforming the All-to-All mixed system network into a reliability system as shown in section 2.3.3.

4.1.4.2.1 Application to Special Cases

We apply the minimum cut set method for the case of 3 relays. The transformation of the communication system into a reliability system is given by figure 4.56. In the given figure components A, B, C, D, E, and F represent the FSO links $S - R_1$, $S - R_2$, $S - R_3$, $R_1 - D$, $R_2 - D$ and $R_3 - D$ respectively while the components G, H, and I represent the $R_1 - R_2$, $R_2 - R_3$, and $R_1 - R_3$ RF links.

1. The minimal paths in the system are: AD, BE, CF, AGE, AIF, BGD, BHF, CHE, CID, AGHF, AIHE, BGIF, BHID, CHGD, and CIGE.

2. The incidence matrix is provided in figure 4.57.

3. No non-zero columns exist, therefore no first order cut sets are available.

4. By adding combinations of two columns, no cut sets are discovered.

5. By adding combinations of three columns, the third order cut sets are: ABC and DEF.

However there are no first order nor second order sets to check against for the elimination
step, thus ABC and DEF are minimal cut sets.

6. Similarly, all fourth order cut sets contain subsets from third order minimal cut sets, therefore no minimal fourth cut sets

7. Moreover, Fifth order minimal cut sets are: ABFHI, ACEGH, AEFGI, BCDGI, BDFGH, and CDEHI.

8. No more minimal cut sets are available.

9. The minimal cut sets in this system are therefore: ABC, DEF, ABFHI, ACEGH, AEFGI, BCDGI, BDFGH, and CDEHI.

The resultant upper bound outage expression for All-to-All with 3 relays, after replacing the components with the corresponding outage probability terms and considering the direct
Figure 4.57: Incidence Matrix of All-to-All system with 3 Relays

\[
P_{U.B,ALL}^{(3)} = P_{0,4} \left[ P_{0,1} P_{0,2} P_{0,3} + P_{0,1} P_{0,2} P_{3,4} P_{2,3} P_{1,3} + P_{0,1} P_{0,3} P_{2,4} P_{1,2} P_{2,3} + P_{0,1} P_{2,4} P_{3,4} P_{1,2} P_{1,3} + P_{0,2} P_{3,4} P_{1,2} P_{1,3} + P_{0,2} P_{1,4} P_{3,4} P_{1,2} P_{2,3} + P_{0,3} P_{1,4} P_{2,4} P_{2,3} P_{1,3} + P_{1,4} P_{2,4} P_{3,4} \right]
\]

(4.24)
In a brief manner, the reliability system for the case of 4 relays is provided in figure 4.58 where components A, B, C, D, E, F, G, and H represent the FSO links corresponding to $S-R_1$, $S-R_2$, $S-R_3$, $S-R_4$, $R_1-D$, $R_2-D$, $R_3-D$, and $R_4-D$, respectively. In the same manner, components I, J, K, L, M, and N represent the inter RF links corresponding to $R_1-R_2$, $R_2-R_3$, $R_3-R_4$, $R_1-R_3$, $R_2-R_4$, and $R_1-R_4$. For this system, the minimum cut sets are determined in the same manner as described in detail with the case of 3 relays, where the final result is composed of $2^4 = 16$ minimal cut sets which are: ABCD, EFGH, ABCHKM, ABGDJL, AFCDJM, AFGHILN, EBCDILN, EBGHIJM, EFCHJKL, EFGDKMN, ABGHJLMN, AFCHJKN, AFGDIKLM, EBCHIKLM, EBGDIJKN, and EFCDJLMN.

4.1.4.2.2 Concluding a General Form

Moreover, as previously stated in section 2.3, we implemented a MATLAB code that can apply the described Minimum cut set method algorithm for any number of relays. We ran the
code further for 5 and 6 relays and analyzed the resulting pattern. In fact, any given system can be divided into 3 types of links: $S - R$, $R - D$, and $R - R$ links. The closest components to the destination are the $R - D$ links, so referring to our hereby presented results and to our results in the previous section, we present a deduced general form for any number of relays. We start looking into all possible combinations of $R - D$ links which results in an equal number of minimal cut sets to the number of possible combinations. The general form expression, inspired by that of NIRC in equation (4.7) while taking into consideration all possible paths resulting from $R - R$ links and leading to $2^N$ minimal cut sets, is described as follows:

- First, a set $L_{n,i}$ containing $n R - D$ links is in outage where the number of elements $n$ in this set varies from 1 to $N$, and by taking all the possible combinations, $i$ varies from 1 to $(N^n)$. The corresponding outage probability term is described as: $\prod_{j \in L_{n,i}} p_{j,N+1}$

- On the opposite way, the set $L_{n,i}$ represents the active $R - D$ links which automatically implies that the complement $S - R$ links should be in outage, otherwise a direct delivery through the $S - R_i - D$ path will happen. These probability terms are expressed as: $\prod_{j' \in L_{n,i}} p_{0,j'}$

- So since by design all the $S - R_i - D$ paths experience an outage in one hop (either $S - R_i$ or $R_i - D$), the only links still to be checked are the $R - R$ links that ensure a path from $S$ to $D$. In this manner, we look into the relays that successfully received the message from $S$ through the active $S - R$ links which are opposed by failed $R - D$ links in set $L_{n,i}$. So if these relays are prevented from reaching the complementary set of relays belonging to links in $L_{n,i}$, the system will be in outage. This contributes to the $R - R$ links as: $\prod_{j \in L_{n,i}, j' \in L_{n,i}} p_{j,j'}$

The resulting general formula for the upper bound expression of All-to-All scheme with any number of relays, while taking into consideration the direct term, is given by:

$$
P_{U.B., ALL}^{(N)} = p_{0,N+1} \left[ \sum_{n=0}^{N} \sum_{i=1}^{(N^n)} \left( \prod_{j \in L_{n,i}} p_{j,N+1} \right) \left( \prod_{j' \in L_{n,i}} p_{0,j'} \right) \left( \prod_{j \in L_{n,i}, j' \in L_{n,i}} p_{j,j'} \right) \right]^{(4.25)}$$

Where $p_{j,N+1}$ terms correspond to the $R - D$ links that are in outage, $p_{0,j'}$ terms correspond to the $S - R$ links that are on the opposite side of the active $R - D$ links, and $p_{j,j'}$ terms represent...
the $R-R$ links that should be in outage to prevent the message from being delivered to relays having active $R-D$ links.

4.1.4.3 Simulation Analysis

In a similar manner to the case of NIRC schemes that was discussed in section 4.1.1.3, the Monte Carlo simulation was implemented for the All-to-All relaying scheme. The only difference is in the existence of $R-R$ inter links that need to be checked when determining the final set of successfully reached relays. The algorithm that was implemented to derive the outage probability is provided in Appendix A.

4.2 Selective Relaying

4.2.1 Mathematical Concept Introduction

As described in sections 1.1.4.2.3.2, 3.1, and 3.5, we can perform an outage probability analysis for the selective relaying scheme using the system strength analysis. We start by recalling some definitions that will be used heavily in this section.

4.2.1.1 Strength of a Link

As described in section 3.5.2, the strength of an FSO link will be denoted by $\alpha_{i,j}$ where $i$ and $j$ are the nodes connected through this link.

4.2.1.2 Strength of a Path

As in equations (3.11) and (4.27), the strength of a path will be dependent on whether it is composed of parallel links, serial links or even combination of both. For parallel and serial links, we recall the following equations:

\[
\text{Strength of N Parallel Links} = \text{Strength of the Strongest Link} = \max\{\alpha_{0,1}, \alpha_{0,2}, \cdots, \alpha_{0,N}\}
\]

\[
(4.26)
\]

\[
\text{Strength of N Serial Links} = \text{Strength of the Weakest Link} = \min\{\alpha_{0,1}, \alpha_{1,2}, \cdots, \alpha_{N-1,N}\}
\]

\[
(4.27)
\]

However, when witnessing a combination of both, we have to divide the path into sub-parts where serial and parallel links apply.
4.2.1.3 Strength of a Network

The strength of a communication network is defined by the strength of its strongest path when \( n \) multiple parallel paths exist. This can be done in 2 steps:

1. Determining the strength of each complete path from the source to the destination of the system network, following serial and parallel links properties.

2. Applying the following equation on the obtained strength expressions for each path:

\[
\alpha_s = \text{Strength of System} = \text{Strength of the Strongest Path} = \max\{\alpha_1, \alpha_2, \ldots, \alpha_n\}
\] (4.28)

Where \( \alpha_1, \alpha_2, \ldots, \alpha_n \) are the strength expressions relative for each complete path.

4.2.1.4 Outage Probability in terms of Strength

The outage probability of a link connecting nodes \( i \) and \( j \) is given in terms of its strength value as follows:

\[
P_{i,j} = \Pr(\alpha_{i,j} < X)
\] (4.29)

Where \( P_{i,j} \) is the outage probability of the link. In addition, \( X \) is a predefined threshold which is SNR-related.

In a similar manner, the system will be in outage when its strength value falls below the predefined threshold \( X \). This can be described as:

\[
P_{\text{out,system}} = \Pr(\alpha_s < X)
\] (4.30)

4.2.2 NIRC

4.2.2.1 Exact Analysis

4.2.2.1.1 Special Cases

In what follows, the \( S - D \) link is treated separately as it is always parallel to the remaining subsystems. This means that the total strength of the system will be the maximum between the strength of the \( S - D \) link and the strength of the whole remaining system. Thus, we can simply multiply the final result by the outage probability of the \( S - D \) link to get the final answer.

For \( N = 2 \) relays, as we can see in figure 4.59, there are 2 available paths from \( S \) to \( D \):
(1) $S - R_1 - D$ and (2) $S - R_2 - D$. The strength of $S - R_1 - D$ path is given by: $\alpha_1 = \min\{\alpha_{0,1}, \alpha_{1,3}\}$. Similarly, the strength of $S - R_2 - D$ path is given by: $\alpha_2 = \min\{\alpha_{0,2}, \alpha_{2,3}\}$. Therefore, the strength of NIRC system with 2 relays is given by:

$$\alpha_{s,NIRC}^{(2)} = \max\{\alpha_1, \alpha_2\} = \max\{\min\{\alpha_{0,1}, \alpha_{1,3}\}, \min\{\alpha_{0,2}, \alpha_{2,3}\}\}$$

(4.31)

![NIRC scheme with 2 Relays - Selective Relaying](image)

Figure 4.59: NIRC scheme with 2 Relays - Selective Relaying

This implies that the outage probability is given by:

$$\frac{1}{P_{out,NIRC}^{(2)}} = \Pr(\alpha_{s,NIRC}^{(2)} < X)$$

$$= \Pr(\max\{\min\{\alpha_{0,1}, \alpha_{1,3}\}, \min\{\alpha_{0,2}, \alpha_{2,3}\}\} < X)$$

$$= \Pr(\min\{\alpha_{0,1}, \alpha_{1,3}\} < X)\Pr(\min\{\alpha_{0,2}, \alpha_{2,3}\} < X)$$

$$= (1 - \Pr(\min\{\alpha_{0,1}, \alpha_{1,3}\} > X))(1 - \Pr(\min\{\alpha_{0,2}, \alpha_{2,3}\} > X))$$

$$= (1 - \Pr(\alpha_{0,1} > X)\Pr(\alpha_{1,3} > X))(1 - \Pr(\alpha_{0,2} > X)\Pr(\alpha_{2,3} > X))$$

$$= (1 - (1 - \Pr(\alpha_{0,1} < X))(1 - \Pr(\alpha_{1,3} < X)))$$

(4.32)

$$= (1 - (1 - \Pr(\alpha_{0,1} < X))(1 - \Pr(\alpha_{1,3} < X)))$$

Similarly to the case of 2 relays, the NIRC scheme with 3 relays will have 3 available paths to be studied as we can see in figure 4.60. These paths are: (1) $S - R_1 - D$, (2) $S - R_2 - D$, and (3) $S - R_3 - D$. The strength of each of these paths is given by: $\alpha_1 = \min\{\alpha_{0,1}, \alpha_{1,4}\}$, $\alpha_2 = \min\{\alpha_{0,2}, \alpha_{2,4}\}$, and $\alpha_3 = \min\{\alpha_{0,3}, \alpha_{3,4}\}$. Therefore, the strength of NIRC system with
3 relays is given by:

\[
\alpha_{s,NIRC}^{(3)} = \max \{\alpha_1, \alpha_2, \alpha_3\} = \max \{\min \{\alpha_{0,1}, \alpha_{1,4}\}, \min \{\alpha_{0,2}, \alpha_{2,4}\}, \min \{\alpha_{0,3}, \alpha_{3,4}\}\} \tag{4.33}
\]

This implies that the outage probability is given by:

\[
\frac{1}{P_{0,4}} P_{out,NIRCs}^{(3)} = \Pr(\alpha_{s,NIRC}^{(3)} < X)
\]

\[
= \Pr(\max \{\min \{\alpha_{0,1}, \alpha_{1,4}\}, \min \{\alpha_{0,2}, \alpha_{2,4}\}, \min \{\alpha_{0,3}, \alpha_{3,4}\}\} < X)
\]

\[
= \Pr(\min \{\alpha_{0,1}, \alpha_{1,4}\} < X)\Pr(\min \{\alpha_{0,2}, \alpha_{2,4}\} < X)\Pr(\min \{\alpha_{0,3}, \alpha_{3,4}\} < X)
\]

\[
= (1 - \Pr(\min \{\alpha_{0,1}, \alpha_{1,4}\} > X))(1 - \Pr(\min \{\alpha_{0,2}, \alpha_{2,4}\} > X))
\]

\[
(1 - \Pr(\min \{\alpha_{0,3}, \alpha_{3,4}\} > X))
\]

\[
= (1 - \Pr(\alpha_{0,1} > X)\Pr(\alpha_{1,4} > X))(1 - \Pr(\alpha_{0,2} > X)\Pr(\alpha_{2,4} > X))
\]

\[
(1 - \Pr(\alpha_{0,3} > X)\Pr(\alpha_{3,4} > X))
\]

\[
= (1 - (1 - \Pr(\alpha_{0,1} < X))(1 - \Pr(\alpha_{1,4} < X)))(1 - (1 - \Pr(\alpha_{0,2} < X))
\]

\[
(1 - \Pr(\alpha_{2,4} < X)))(1 - (1 - \Pr(\alpha_{0,3} < X))(1 - \Pr(\alpha_{3,4} < X)))
\]

\[
\frac{1}{P_{0,4}} P_{out,NIRCs}^{(3)} = (1 - (1 - p_{0,1})(1 - p_{1,4}))(1 - (1 - p_{0,2})(1 - p_{2,4}))(1 - (1 - p_{0,3})(1 - p_{3,4}))
\]

\[
(4.34)
\]
4.2.1.2 General Case

As we can observe from the special cases, with NIRC we have \( N + 1 \) parallel paths (\( N \) \( S-R-D \) paths for the \( N \) relays in addition to one direct \( S-D \) path). Following from the parallel and series properties, the strength of this system can be determined as the maximum value among the strength values of all the parallel paths; this can be described as follows:

\[
\alpha_{s,NIRC}^{(N)} = \max\{\alpha_1, \alpha_2, \cdots, \alpha_N, \alpha_{N+1}\} \tag{4.35}
\]

Where each of \( \alpha_1, \alpha_2, \cdots, \) and \( \alpha_N \) corresponds to the strength of the \( S-R_1-D, S-R_2-D, \cdots, \) and \( S-R_N-D \) paths, respectively. Exceptionally, \( \alpha_{N+1} \) represents the strength of the direct \( S-D \) link with \( \alpha_{0,N+1} \). Since each path consists of 2 serial links, then the strength of each path is determined as the minimum value among the strength values of the 2 serial links:

\[
\alpha_i = \min\{\alpha_{0,i}, \alpha_{i,N+1}\}; i = 1, \cdots, N \tag{4.36}
\]

This implies the following expression for the strength expression of the NIRC system with \( N \) relays:

\[
\alpha_{s,NIRC}^{(N)} = \max\{\min\{\alpha_{0,1}, \alpha_{1,N+1}\}, \min\{\alpha_{0,2}, \alpha_{2,N+1}\}, \cdots, \min\{\alpha_{0,N}, \alpha_{N,N+1}\}, \alpha_{0,N+1}\} \tag{4.37}
\]

Using the strength expression of the NIRC system with \( N \) relays, we can obtain the outage expression as follows:
This also verifies the relation obtained in equation (4.3) for All-Active relaying protocol:

\[
P_{out,NIRCs}^{(N)} = \Pr(\alpha_{s,NIRC}^{(N)} < X)
\]
\[
= \Pr(\max\{\min\{\alpha_{0,1}, \alpha_{1,N+1}\}, \min\{\alpha_{0,2}, \alpha_{2,N+1}\}, \ldots, \min\{\alpha_{0,N}, \alpha_{N,N+1}\}, \alpha_{0,N+1}\} < X)
\]
\[
= \Pr(\min\{\alpha_{0,1}, \alpha_{1,N+1}\} < X)\Pr(\min\{\alpha_{0,2}, \alpha_{2,N+1}\} < X)\cdots
\]
\[
\Pr(\min\{\alpha_{0,N}, \alpha_{N,N+1}\} < X)\Pr(\{\alpha_{0,N+1}\} < X)
\]
\[
= (1 - \Pr(\min\{\alpha_{0,1}, \alpha_{1,N+1}\} > X))(1 - \Pr(\min\{\alpha_{0,2}, \alpha_{2,N+1}\} > X))\cdots
\]
\[
(1 - \Pr(\min\{\alpha_{0,N}, \alpha_{N,N+1}\} > X))p_{0,N+1}
\]
\[
= p_{0,N+1}(1 - \Pr(\alpha_{0,1} > X)\Pr(\alpha_{1,N+1} > X))(1 - \Pr(\alpha_{0,2} > X)\Pr(\alpha_{2,N+1} > X))\cdots
\]
\[
(1 - \Pr(\alpha_{0,N} > X)\Pr(\alpha_{N,N+1} > X))
\]
\[
= p_{0,N+1}(1 - (1 - \Pr(\alpha_{0,1} < X))(1 - \Pr(\alpha_{1,N+1} < X)))(1 - (1 - \Pr(\alpha_{0,2} < X))
\]
\[
(1 - \Pr(\alpha_{2,N+1} < X)))\cdots(1 - (1 - \Pr(\alpha_{0,N} < X))(1 - \Pr(\alpha_{N,N+1} < X)))
\]
\[
P_{out,NIRCs}^{(N)} = p_{0,N+1}[(1 - (1 - p_{0,1})(1 - p_{1,N+1}))(1 - (1 - p_{0,2})(1 - p_{2,N+1}))\cdots
\]
\[
(1 - (1 - p_{0,N})(1 - p_{N,N+1}))]
\]
\[
(4.38)
\]

This also verifies the relation obtained in equation (4.3) for All-Active relaying protocol:

\[
P_{out,NIRCs}^{(N)} = p_{0,N+1}\prod_{i=1}^{N}(1 - (1 - p_{0,i})(1 - p_{i,N+1}))
\]
\[
(4.39)
\]

This implies that, with NIRC, the outage probability expressions are the same. Only the values of the constituent probabilities \(p_{i,j}\) will vary (they are smaller for the selective scheme) following from the power splitting procedure.

### 4.2.2.2 Simulation Analysis

Since selective relaying schemes are based on strength analysis strategy, the application of Monte Carlo simulation will work on generating random values for the strength of each link in the system. This step will be essential in determining the outage probability of the system by the algorithm described in Appendix B.
4.2.3 All-to-All Mixed RF/FSO Selective Relaying Scheme

With All-to-All selective relaying, we implement a mixed RF/FSO system as described in sections 1.1.5.3 and 3.1. In this case we follow the algorithm described in section 3.1 for applying the selective relaying scheme, where we first condition on the RF inter links before choosing the strongest path. Following the described strategy, we condition on \( R - R \) links with the corresponding outage probability terms for each case. Then we apply the strength analysis in order to get the outage probability of the described system.

4.2.3.1 Exact Analysis

4.2.3.1.1 Special Cases

In what follows, the \( S - D \) link is treated separately as it is always parallel to the remaining subsystems. This means that the total strength of the system will be the maximum between the strength of the \( S - D \) link and the strength of the whole remaining system. Thus, we can simply multiply the obtained outage probability of the simplified system by the outage probability of the \( S - D \) link to get the total system outage probability.

First, for an All-to-All selective scheme with 2 relays given by figure 4.61, we have 1 RF link to condition on: \( R_1 - R_2 \) with outage probability term \( p_{1,2} \). So there are 2 cases to consider here:

1. If \( R_1 - R_2 \) is in outage, then the system would look like an NIRC system with 2 relays as we can see in figure 4.62. The strength analysis in this case is the exact one that was done with NIRC in section 4.2.2.1.1 where the resultant outage probability \( P_{out,NIRCs}^{(2)} \) was described as in equation (4.32). This gives the outage probability for this case as:
   \[
   p_{1,2}P_{out,NIRCs}^{(2)} = p_{1,2}(1 - (1 - p_{0,1})(1 - p_{1,3}))(1 - (1 - p_{0,2})(1 - p_{2,3}))
   \]

2. Else \( R_1 - R_2 \) will be active and the system will reduce into a combined relay \( R_{1,2} \) with one single path \( S - R_{1,2} - D \) as shown in figure 4.62. However along this path, we have 2 parallel links from \( S \) to \( R_{1,2} \) followed by 2 parallel links from \( R_{1,2} \) to \( D \). Following serial and parallel concepts for path strength analysis given by section 4.2.1.2, the strength of this path is given by:
\[ \alpha_{s_2,\text{ALL}}^{(2)} = \min\{\max\{\alpha_{0,1}, \alpha_{0,2}\}, \max\{\alpha_{1,3}, \alpha_{2,3}\}\}. \] (4.40)

Figure 4.61: All-to-All scheme with 2 Relays - Selective

Figure 4.62: All-to-All scheme with 2 Relays - Selective - Case 1

Figure 4.63: All-to-All scheme with 2 Relays - Selective - Case 2
This implies that the outage probability for this case is given by:

\[
P_{\text{out,ALLs}}^{(2)} = \Pr(\alpha_{s2,ALL}^{(2)} < X) = 1 - \Pr(\alpha_{s2,ALL}^{(2)} > X)
\]

\[
= 1 - \Pr(\min\{\alpha_{0,1}, \alpha_{0,2}\}, \max\{\alpha_{1,3}, \alpha_{2,3}\}) > X
\]

\[
= 1 - \Pr(\max\{\alpha_{0,1}, \alpha_{0,2}\} > X)\Pr(\max\{\alpha_{1,3}, \alpha_{2,3}\} > X)
\]

\[
= 1 - [(1 - \Pr(\max\{\alpha_{0,1}, \alpha_{0,2}\} < X))(1 - \Pr(\max\{\alpha_{1,3}, \alpha_{2,3}\} < X))]
\]

\[
= 1 - [(1 - \Pr(\alpha_{0,1} < X)\Pr(\alpha_{0,2} < X))(1 - \Pr(\alpha_{1,3} < X)\Pr(\alpha_{2,3} < X))]
\]

\[
P_{\text{out,ALLs}}^{(2)} = 1 - [(1 - p_{0,1}p_{0,2})(1 - p_{1,3}p_{2,3})]
\]

(4.41)

Therefore, the outage probability of the All-to-All selective mixed system with 2 relays can be expressed as:

\[
P_{\text{out,ALLs}}^{(2)} = p_{1,2}P_{\text{out,NIRCs}}^{(2)} + (1 - p_{1,2})P_{\text{out,ALLs}}^{(2)}
\]

\[
= p_{1,2}(1 - (1 - p_{0,1})(1 - p_{1,3}))(1 - (1 - p_{0,2})(1 - p_{2,3}))
\]

\[
+ (1 - p_{1,2})(1 - ((1 - p_{0,1}p_{0,2})(1 - p_{1,3}p_{2,3})))
\]

(4.42)

Now with \(N = 3\) relays, as presented in figure 4.64, we have 3 RF links to condition on: \(R_1 - R_2, R_1 - R_3,\) and \(R_2 - R_3\). By conditioning on these RF links, we have 4 categories to consider:

1. All inter links \(R_1 - R_2, R_1 - R_3,\) and \(R_2 - R_3\) are in outage as shown in figure 4.65, with probability \(p_{1,2}p_{1,3}p_{2,3}\). This results in the same system as with NIRC. The resulting outage expression for this case is given by:

\[
P_{\text{out,ALLs}}^{(3)} = p_{1,2}P_{1,3}P_{2,3}p_{\text{out,NIRCs}}^{(3)}
\]

\[
= p_{1,2}p_{1,3}p_{2,3}(1 - (1 - p_{0,1})(1 - p_{1,4}))
\]

\[
(1 - (1 - p_{0,2})(1 - p_{2,4}))(1 - (1 - p_{0,3})(1 - p_{3,4}))
\]

(4.43)
2. One inter relay link is active while the other two links are in outage. This divides the system into 2 parallel paths: one path contains a big relay $R_{i,j}$ combining the 2 connected relays ($R_i$ and $R_j$) through the active link $R_i - R_j$, and the other path containing the alone relay $R_z$. The 3 possible combinations for this category are presented in figures 4.66, 4.67, and 4.68 with outage terms $(1 - p_{1.2})p_{1.3}p_{2.3}$, $p_{1.2}(1 - p_{1.3})p_{2.3}$, and $p_{1.2}p_{1.3}(1 - p_{2.3})$, respectively. For this scenario, the 2 subsystems are in parallel resulting in the following strength expression:

$$\alpha_{s2,ALL}^{(3)} = \max\{\min\{\max\{0,0\}, \max\{\alpha_{i,4}, \alpha_{j,4}\}\}, \min\{\alpha_{0,z}, \alpha_{z,4}\}\} \quad (4.44)$$

Where $i$ and $j$ are the combined relays and $z$ is the left alone one.
This will result in an outage probability expression corresponding to the threshold $X$ as follows:
\[
P_{\text{out,ALLs}2}^{(3)} = \Pr(\alpha_{2,\text{ALL}}^{(3)} < X)
\]

\[
= \Pr(\max \{ \min \{ \max \{ \alpha_{0,i}, \alpha_{0,j} \}, \max \{ \alpha_{j,4}, \alpha_{j,4} \} \}, \min \{ \alpha_{0,z}, \alpha_{z,4} \} \} < X)
\]

\[
= \Pr(\min \{ \max \{ \alpha_{0,i}, \alpha_{0,j} \}, \max \{ \alpha_{j,4}, \alpha_{j,4} \} \} < X) \Pr(\min \{ \alpha_{0,z}, \alpha_{z,4} \} < X)
\]

\[
= (1 - \Pr(\min \{ \max \{ \alpha_{0,i}, \alpha_{0,j} \}, \max \{ \alpha_{j,4}, \alpha_{j,4} \} \} > X))
\]

\[
= (1 - (1 - (1 - \Pr(\max \{ \alpha_{0,i}, \alpha_{0,j} \} < X))(1 - \Pr(\max \{ \alpha_{j,4}, \alpha_{j,4} \} < X))))
\]

\[
= (1 - (1 - (1 - \Pr(\alpha_{0,i} < X) \Pr(\alpha_{0,j} < X))(1 - \Pr(\alpha_{j,4} > X))))
\]

\[
P_{\text{out,ALLs}2}^{(3)} = (1 - (1 - p_{0,i} p_{0,j}))(1 - p_{1,4} p_{j,4}))(1 - (1 - p_{0,z})(1 - p_{z,4}))
\]

\[
(4.45)
\]

Now by substituting \(i\), \(j\), and \(z\) by their values with respect to the presented \(R - R\) possible cases under this scenario, the resultant outage probability becomes:

\[
P_{\text{out,ALLs}2}^{(3)} = (1 - p_{1,2})p_{1,3}p_{2,3}(1 - (1 - p_{0,1} p_{0,2})(1 - p_{1,4} p_{2,4}))(1 - (1 - p_{0,3})(1 - p_{3,4}))
\]

\[
+ p_{1,2}(1 - p_{1,3})p_{2,3}(1 - (1 - p_{0,1} p_{0,3})(1 - p_{1,4} p_{3,4}))(1 - (1 - p_{0,2})(1 - p_{2,4}))
\]

\[
+ p_{1,2}p_{1,3}(1 - p_{2,3})(1 - (1 - p_{0,2} p_{0,3})(1 - p_{2,4} p_{3,4}))(1 - (1 - p_{0,1})(1 - p_{1,4}))
\]

\[
(4.46)
\]

3. Two inter relay links are active while only one is in outage. In such scenario, the 3 relays will be connected together through the two active links. We can therefore combine these cases with the final category. The corresponding outage terms for the possible combinations of this category are: \((1 - p_{1,2})(1 - p_{1,3})p_{2,3} + (1 - p_{1,2})p_{1,3}(1 - p_{2,3}) + p_{1,2}(1 - p_{1,3})(1 - p_{2,3})\).

4. All the inter links are active, thus the 3 relays are combined together with a one big virtual relay as can be shown in figure 4.69, this is given with a probability term \((1 - p_{1,2})(1 - p_{1,3})(1 - p_{2,3})\). Moreover, for this category and the previous one combined we get the following outage expression: \(((1 - p_{1,2})(1 - p_{1,3})p_{2,3} + (1 - p_{1,2})p_{1,3}(1 - p_{2,3}) +\)
\[ p_{1.2}(1 - p_{1.3})(1 - p_{2.3}) + (1 - p_{1.2})b(1 - p_{1.3})(1 - p_{2.3}) \] where \( P^{(3)}_{\text{out,ALLs34'}} \) will be determined based on path strength analysis. In these cases, we have one path consisted of 2 parts: \( S - R_{1.2.3} \) and \( R_{1.2.3} - D \), where on both sides we have 3 parallel links. This implies the following path strength:

\[
\alpha^{(3)}_{34,ALL} = \min \left\{ \max \{ \alpha_{0.1}, \alpha_{0.2}, \alpha_{0.3} \}, \max \{ \alpha_{1.4}, \alpha_{2.4}, \alpha_{3.4} \} \right\}
\]  

(4.47)

This gives the following outage expression:

\[
P^{(3)}_{\text{out,ALLs34'}} = \Pr(\alpha^{(3)}_{2,ALL} < X)
\]

\[
= 1 - \Pr(\min \{ \max \{ \alpha_{0.1}, \alpha_{0.2}, \alpha_{0.3} \}, \max \{ \alpha_{1.4}, \alpha_{2.4}, \alpha_{3.4} \} \} > X)
\]

\[
= 1 - \left( \Pr(\max \{ \alpha_{0.1}, \alpha_{0.2}, \alpha_{0.3} \} > X) \Pr(\max \{ \alpha_{1.4}, \alpha_{2.4}, \alpha_{3.4} \} > X) \right)
\]

\[
= 1 - \left( (1 - \Pr(\alpha_{0.1} < X)\Pr(\alpha_{0.2} < X)\Pr(\alpha_{0.3} < X))
\right)
\]

\[
(1 - \Pr(\alpha_{1.4} < X)\Pr(\alpha_{2.4} < X)\Pr(\alpha_{3.4} < X))
\]

(4.48)

This implies that the outage expression for these 2 categories can be written as:

\[
P^{(3)}_{\text{out,ALLs34}} = \left[ (1 - p_{1.2})(1 - p_{1.3})p_{2.3} + (1 - p_{1.2})p_{1.3}(1 - p_{2.3})
\right.
\]

\[
+ p_{1.2}(1 - p_{1.3})(1 - p_{2.3}) + (1 - p_{1.2})(1 - p_{1.3})(1 - p_{2.3}) \right] P^{(3)}_{\text{out,ALLs34'}}
\]

\[
= \left[ (1 - p_{1.2})(1 - p_{1.3})p_{2.3} + (1 - p_{1.2})p_{1.3}(1 - p_{2.3}) + p_{1.2}(1 - p_{1.3})(1 - p_{2.3})
\right.
\]

\[
+ (1 - p_{1.2})(1 - p_{1.3})(1 - p_{2.3}) \right] \left[ (1 - (1 - p_{0.1}p_{0.2}p_{0.3})(1 - p_{1.4}p_{2.4}p_{3.4})) \right]
\]

(4.49)

![Figure 4.69: All-to-All scheme with 3 Relays - Selective - Cases 5,6,7,8](image)

Combining all these categories together, we get the following outage expression for the All-to-All Selective scheme with 3 relays \( P^{(3)}_{\text{out,ALLs}} \) after multiplying with the \( S - D \) outage.
probability term \( p_{0,4} \):

\[
P^{(3)}_{\text{out},\text{ALLs}} = p_{0,4}[p^{(3)}_{\text{out},\text{ALLs}1} + p^{(3)}_{\text{out},\text{ALLs}2} + p^{(3)}_{\text{out},\text{ALLs}34}]
\]

\[
= p_{0,4}[p_{1,2}p_{1,3}p_{2,3}(1 - (1 - p_{0,1})(1 - p_{1,4}))(1 - (1 - p_{0,2})(1 - p_{2,4}))(1 - (1 - p_{0,3})(1 - p_{3,4})) + (1 - p_{1,2})p_{1,3}p_{2,3}(1 - (1 - p_{0,1}p_{0,2})(1 - p_{1,4}p_{2,4}))(1 - (1 - p_{0,3})(1 - p_{3,4})) + p_{1,2}(1 - p_{1,3})p_{2,3}(1 - (1 - p_{0,1}p_{0,3})(1 - p_{1,4}p_{3,4}))(1 - (1 - p_{0,2})(1 - p_{2,4})) + p_{1,2}p_{1,3}(1 - p_{2,3})(1 - (1 - p_{0,2}p_{0,3})(1 - p_{2,4}p_{3,4}))(1 - (1 - p_{0,1})(1 - p_{1,4})) + ((1 - p_{1,2})(1 - p_{1,3})p_{2,3} + (1 - p_{1,2})p_{1,3}(1 - p_{2,3}) + p_{1,2}(1 - p_{1,3})(1 - p_{2,3}) + (1 - p_{1,2})(1 - p_{1,3})(1 - p_{2,3})))(1 - ((1 - p_{0,1}p_{0,2}p_{0,3})(1 - p_{1,4}p_{2,4}p_{3,4})))]
\]

\[(4.50)\]

### 4.2.3.1.2 General Case

Following from the presented special cases, and by applying the basic concepts for determining the outage probability of All-to-All connected system in the presence of selective schemes, a general case can be determined as follows:

- First, we need to consider all the possible states of the \( R - R \) links. In fact, for \( N \) fully connected relays, there will be \( x = \frac{N(N-1)}{2} \) inter links. As a result there will be \( 2^x = 2^{\frac{N(N-1)}{2}} \) cases to consider.

  - The outage probability of the whole system is calculated as the summation of the outage probability of each case:

\[
P^{(N)}_{\text{out},\text{ALLs}} = \sum_{i=1}^{x} p(C_i)p^i_{\text{out},S}
\]

\[(4.51)\]

Where \( p^i_{\text{out},S} \) corresponds to the outage probability corresponding to the \( i^{th} \) case. In addition, \( p(C_i) \) was described in section 4.1.4.2.2 to be the probability of having vector \( C_i \) which corresponds to the states of all the inter relay links as described in equation (4.21):

\[
C = [c_{1,2}, c_{1,3}, \ldots, c_{1,N}, c_{2,3}, c_{2,4}, \ldots, c_{2,N}, \ldots, c_{N-1,N}]
\]

\[(4.52)\]

Where each \( c_{i,j} \) value corresponds to the state of the \( R_i - R_j \) link (0 for outage and 1 for active).
In order to determine the outage probability $P_{out,S}$ of each case of the system, we need to determine the strength expression corresponding to that case:

- Based on the current state of these inter-relay links, connected relays can be combined into virtual big relays if a path exists between any relay of the set to all other relays as was further discussed in section 4.1.4.2.2, and as a result the system can be divided virtually into $y$ parallel subsystems where $y$ is a positive integer that varies between 1 and $N$.

- For each subsystem, a strength expression $\alpha_i$ will be expressed as the minimum strength value among the two serial $S-R$ and $R-D$ hops following from equation (4.27). In addition, each hop will have a strength value represented as the maximum among the parallel links (if we have a combined relay) connecting the virtual relay with source or destination following from equation (4.26).

- The final System Strength expression is expressed as the maximum strength value among the $y$ parallel subsystems as:

$$\alpha_{s,xi} = \max_{i=1,...,y} \{\alpha_i\}$$  \hspace{1cm} (4.53)

- Consequently, the corresponding outage probability of each case will be given as shown in equation (4.30) in terms of threshold $X$:

$$P_{out,S}^i = Pr(\alpha_{s,xi} < X)$$  \hspace{1cm} (4.54)

- Finally, as expressed above in equation (4.51), the final outage probability expression will be the summation of all the cases multiplied by the probability of obtaining the $R-R$ case combination:

$$P_{out,ALLs}^{(N)} = 2^{\binom{N}{2} - 1} \sum_{i=1}^{\binom{N}{2}} p(C_i)P_{out,S}^i$$  \hspace{1cm} (4.55)

- In addition, it is important to note some facts that can simplify the above described procedure:

  - There will be $\binom{x}{n}$ different combinations for each considered case with $n$ inter links in outage out of the total $xR-R$ links.
– If \( n < \frac{x}{2} \) \( \Rightarrow \) all relays will be connected together and combined in one big virtual relay for the strength analysis process.

– If \( n = x \) \( \Rightarrow \) all relays are separated from each other leading to \( N \) parallel subsystems to be considered in the strength analysis step.

4.2.3.2 Simulation Analysis

In this section, Monte Carlo simulation is applied to the All-to-All Selective relaying scheme. The main difference from its application with NIRC Selective schemes is in the presence of inter-relay links which need to be studied at first stage as they can lead into different network setups. The detailed algorithm for applying this simulation is provided in Appendix C.
4.3 All-Active vs Selective

In this section, we present a mathematical comparison between the different obtained results for All-Active and Selective Schemes. We compare the outage probability expressions obtained by the NIRC and All-to-All relaying schemes for all cases (special and general) in terms of both All-Active and Selective relaying strategies. We then analyze the compared results and conclude for this chapter.

4.3.1 Special Cases

4.3.1.1 NIRC

We start with $N = 2$ relays:

As shown in equation (4.1), the outage probability for the All-Active scheme is given by:

$$P_{out,NIRC}^{(2)} = p_{0,3}[p_{0,1}p_{0,2} + p_{0,1}(1-p_{0,2})p_{2,3} + (1-p_{0,1})p_{0,2}p_{1,3} + (1-p_{0,1})(1-p_{0,2})p_{1,3}p_{2,3}]$$

(4.56)

Now, if we expand, simplify, and reorganize the equation, it can be written as:

$$P_{out,NIRC}^{(2)} = p_{0,3}[p_{0,1}p_{0,2} + p_{0,1}(1-p_{0,2})p_{2,3} + (1-p_{0,1})p_{0,2}p_{1,3} + (1-p_{0,1})(1-p_{0,2})p_{1,3}p_{2,3}]$$

$$= p_{0,3}[p_{0,1}p_{0,2} + p_{0,1}p_{2,3} - p_{0,1}p_{0,2}p_{2,3} + p_{0,2}p_{1,3} - p_{0,1}p_{0,2}p_{1,3} +$$

$$+ (1-p_{0,1} - p_{0,1}p_{0,2})p_{1,3}p_{2,3}]$$

$$= p_{0,3}[p_{0,1}p_{0,2} + p_{0,1}p_{2,3} - p_{0,1}p_{0,2}p_{2,3} + p_{0,2}p_{1,3} - p_{0,1}p_{0,2}p_{1,3} +$$

$$+ p_{1,3}p_{2,3} - p_{0,1}p_{1,3}p_{2,3} - p_{0,2}p_{1,3}p_{2,3} + p_{0,1}p_{0,2}p_{1,3}p_{2,3}]$$

(4.57)

On the other hand, equation (4.32) shows the outage probability for the NIRC system under selective relaying scheme (without taking into account the direct $S-D$ link):

$$P_{out,NIRC_s}^{(2)} = (1 - (1-p_{0,1})(1-p_{1,3}))(1 - (1-p_{0,2})(1-p_{2,3}))$$

(4.58)

So if we count for the direct $S-D$ link and apply the mathematical needed steps to rewrite
the equation in an expanded form, the equation becomes as follows:

\[
P_{\text{out,NIRCs}}^{(2)} = p_{0,3}[(1 - (1 - p_{0,1})(1 - p_{1,3}))(1 - (1 - p_{0,2})(1 - p_{2,3}))]
\]

\[
= p_{0,3}[(1 - (1 - p_{0,1} - p_{1,3} + p_{0,1}p_{1,3}))(1 - (1 - p_{0,2} - p_{2,3} + p_{0,2}p_{2,3}))]
\]

\[
= p_{0,3}[(p_{0,1} + p_{1,3} - p_{0,1}p_{1,3})(p_{0,2} + p_{2,3} - p_{0,2}p_{2,3})]
\]

\[
= p_{0,3}[p_{0,1}p_{0,2} + p_{0,1}p_{2,3} - p_{0,1}p_{0,2}p_{2,3} + p_{0,2}p_{1,3} - p_{0,1}p_{0,2}p_{1,3}
\]

\[
+ p_{1,3}p_{2,3} - p_{0,1}p_{1,3}p_{2,3} - p_{0,2}p_{1,3}p_{2,3} + p_{0,1}p_{0,2}p_{1,3}p_{2,3}]
\]

\[
(4.59)
\]

Now by comparing the obtained expressions in equations (4.57) and (4.59), we can clearly see that they are fully identical and built up from the same individual outage probability terms. However a critical question here is: Are these individual outage probability terms equal in both All-Active and Selective relaying schemes? This question is answered in a detailed manner in section 4.3.3.

In a similar manner, we deploy the mathematical comparison for \(N = 3\) relays as follows:

For All-active scheme, the outage probability expression was determined in equation (4.2) as:

\[
P_{\text{out,NIRC}}^{(3)} = p_{0,4}[p_{0,1}p_{0,2}p_{0,3} + (1 - p_{0,1})p_{0,2}p_{0,3}p_{1,4} + p_{0,1}(1 - p_{0,2})p_{0,3}p_{2,4}
\]

\[
+ p_{0,1}p_{0,2}(1 - p_{0,3})p_{3,4} + (1 - p_{0,1})(1 - p_{0,2})p_{0,3}p_{1,4}p_{2,4} + (1 - p_{0,1})p_{0,2}(1 - p_{0,3})p_{1,4}p_{3,4}
\]

\[
+ p_{0,1}(1 - p_{0,2})(1 - p_{0,3})p_{2,4}p_{3,4} + (1 - p_{0,1})(1 - p_{0,2})(1 - p_{0,3})p_{1,4}p_{2,4}p_{3,4}]
\]

\[
(4.60)
\]
By expanding and reorganizing, the equation becomes:

\[
P_{\text{out},\text{NIRC}}^{(3)} = p_{0.4}[(1 - (1 - p_{0.1}))(1 - (1 - p_{0.2})(1 - p_{2.4}))(1 - (1 - p_{0.3})(1 - p_{3.4}))]
\]

Similarly, the expanded outage probability expression for the Selective scheme can be derived as:

\[
P_{\text{out},\text{NIRC}}^{(3)} = p_{0.4}[(1 - (1 - p_{0.1}))(1 - (1 - p_{0.2}))(1 - (1 - p_{0.3})(1 - p_{3.4}))]
\]

As a result, we can observe that the 27 probability subexpressions that are forming each outage probability expression are fully identical. However as discussed with the case of 2 relays, we need to check if the individual outage probability terms are equal or not; this will be discussed
The outage probability of All-Active scheme was determined in equation (4.17) to be:

\[ P_{\text{out}, \text{ALL}}^{(2)} = p_{0,3} |p_{1,2}(p_{0,1}p_{0,2} + p_{0,1}(1 - p_{0,2})p_{2,3} + (1 - p_{0,1})p_{0,2}p_{1,3}) + (1 - p_{0,1})(1 - p_{0,2})p_{1,3}p_{2,3}) + (1 - p_{1,2})(1 - (1 - p_{0,1}p_{0,2})(1 - p_{1,3}p_{2,3})) | \]  

(4.63)

Applying expand, simplify, factorize, and reorganize operations, the outage expression can be written as follows:

\[ P_{\text{out}, \text{ALL}}^{(2)} = p_{0,3}|p_{1,2}(p_{0,1}p_{0,2} + p_{0,1}(1 - p_{0,2})p_{2,3} + (1 - p_{0,1})p_{0,2}p_{1,3}) + (1 - p_{0,1})(1 - p_{0,2})p_{1,3}p_{2,3}) + (1 - p_{1,2})(1 - (1 - p_{0,1}p_{0,2})(1 - p_{1,3}p_{2,3})) | \]

(4.64)

In a similar manner, the outage probability expression for the selective scheme can be written as:

\[ P_{\text{out}, \text{ALLs}}^{(2)} = p_{0,3}|p_{1,2}(1 - (1 - p_{0,1})(1 - p_{1,3}))(1 - (1 - p_{0,2})(1 - p_{2,3})) + (1 - p_{1,2})(1 - ((1 - p_{0,1}p_{0,2})(1 - p_{1,3}p_{2,3}))) | \]

(4.65)
Therefore, both outage probability expressions are shown to be identical.

Similarly, for the case of 3 relays:

The outage probability expressions for the All-Active and Selective relaying schemes are totally compatible and identical as can be seen in equations (4.18) and (4.50):

$$P_{\text{out},3,\text{ALL}} = P_{\text{out},3,\text{ALLs}} = p_0,4[p_{1,2}p_{1,3}p_{2,3}(1 - (1 - p_{0,1})(1 - p_{1,4}))(1 - (1 - p_{0,2})(1 - p_{2,4}))(1 - (1 - p_{0,3})(1 - p_{3,4}))
+ (1 - p_{1,2})p_{1,3}p_{2,3}(1 - (1 - p_{0,1}p_{0,2})(1 - p_{1,4}p_{2,4}))(1 - (1 - p_{0,3})(1 - p_{3,4}))
+ p_{1,2}(1 - p_{1,3})p_{2,3}(1 - (1 - p_{0,1}p_{0,3})(1 - p_{1,4}p_{3,4}))(1 - (1 - p_{0,2})(1 - p_{2,4})))
+ p_{1,2}p_{1,3}(1 - p_{2,3})(1 - (1 - p_{0,2}p_{0,3})(1 - p_{2,4}p_{3,4}))(1 - (1 - p_{0,1})(1 - p_{1,4}))
+ ((1 - p_{1,2})(1 - p_{1,3})p_{2,3} + (1 - p_{1,2})p_{1,3}(1 - p_{2,3}) + p_{1,2}(1 - p_{1,3})(1 - p_{2,3})
+ (1 - p_{1,2})(1 - p_{1,3})(1 - p_{2,3}))((1 - (1 - p_{0,1}p_{0,2}p_{0,3})(1 - p_{1,4}p_{2,4}p_{3,4}))))$$

However, as explained earlier with the NIRC scheme comparison in section 4.3.1.1, the identical expressions does not necessary mean equal values for the individual probability terms which will be shown in section 4.3.3 to be untrue due to different power distributions between the two analyzed relaying schemes.

### 4.3.2 General Cases

#### 4.3.2.1 NIRC

The general case equations for the outage probabilities of All-Active and Selective relaying schemes were given in equations (4.3) and (4.39). As can be seen below, both equations achieve the same formula:

$$P_{\text{out},N,\text{NIRC}} = P_{\text{out},N,\text{NIRCs}} = p_{0,N+1} \prod_{i=1}^{N}(1 - (1 - p_{0,i})(1 - p_{i,N+1}))$$ (4.67)
4.3.2.2 All-to-All

To compare the general case between all-active and selective for All-to-All connected system, we present the following analysis:

As shown in equations (4.23) and (4.55), both All-Active and Selective relaying schemes achieve the same general case form given by:

\[ P_{\text{out,ALL}}^N = P_{\text{out,ALLs}}^N = \frac{2}{N(N-1)} \sum_{i=1}^{\lfloor y/2 \rfloor} p(C_i) P_{\text{out,S}}^i \]  \hspace{1cm} (4.68)

Where \( p(C_i) \) was proved to be the same for both schemes, but \( P_{\text{out,S}}^i \) is still questionable as each scheme has its own definition of it:

For All-Active, equation (4.20) defines \( P_{\text{out,S}}^i \) as follows:

\[ P_{\text{out,S}}^i = \prod_{x=1}^n P_{\text{out,S}}^{i_x} \]  \hspace{1cm} (4.69)

With \( n \) being the number of parallel subsystems and \( P_{\text{out,S}}^{i_x} \) given by equation (4.19):

\[ P_{\text{out,S}}^{i_x} = 1 - \left( 1 - \prod_{i \in S_R} P_{0,i} \right) \left( 1 - \prod_{i \in S_R} P_{i,N+1} \right) \] \hspace{1cm} (4.70)

With \( S_R \) being the set of combined relays forming the corresponding parallel subsystem.

On the other side, the selective scheme defines the equation of \( P_{\text{out,S}}^i \) in terms of strength expressions as in equation (4.54):

\[ P_{\text{out,S}}^i = \Pr(\alpha_{s,xi} < X) \] \hspace{1cm} (4.71)

Where \( \alpha_{s,xi} \) was defined in equation (4.53) as \( \max \{ \alpha_i \} \) with \( y \) being the number of parallel subsystems presented in the system. Moreover we recall that the strength of \( y \) parallel paths is given as: \( \max \{ \alpha_1, \alpha_2, \ldots, \alpha_y \} \). As a result, \( P_{\text{out,S}}^i \) can be written as:

\[ P_{\text{out,S}}^i = \Pr(\alpha_{s,xi} < X) = \Pr(\max \{ \alpha_1, \alpha_2, \ldots, \alpha_y \} < X) = \Pr(\alpha_1 < X) \Pr(\alpha_2 < X) \cdots \Pr(\alpha_y < X) \] \hspace{1cm} (4.72)

Moreover, each parallel subsystem is composed of 2 serial hops \( S - R \) and \( R - D \), therefore the equation becomes as follows:
\[ P_{out,S} = \Pr(\min\{\alpha_{1s}, \alpha_{1d}\} < X) \Pr(\min\{\alpha_{2s}, \alpha_{2d}\} < X) \cdots \Pr(\min\{\alpha_{yS}, \alpha_{yD}\} < X) \]
\[ = (1 - \Pr(\min\{\alpha_{1s}, \alpha_{1d}\} > X)) (1 - \Pr(\min\{\alpha_{2s}, \alpha_{2d}\} > X)) \cdots (1 - \Pr(\min\{\alpha_{yS}, \alpha_{yD}\} > X)) \]
\[ = (1 - (\Pr(\alpha_{1s} > X)\Pr(\alpha_{1d} > X))) (1 - (\Pr(\alpha_{2s} > X)\Pr(\alpha_{2d} > X))) \cdots (1 - (\Pr(\alpha_{yS} > X)\Pr(\alpha_{yD} > X))) \]
\[ (4.73) \]

Where the values of \( \alpha_{js} \) and \( \alpha_{jd} \), with \( j \) being any integer between 1 and \( y \) representing the index of the combined relay, depend on the set of combined relays forming each parallel subsystem. We define this set to be \( S_{R,j} \). As a result, \( \alpha_{j,S} \) can be written as \( \max_{\alpha_{z}} \{\alpha_{0,z}\} \) where the values of \( z \) represent the parallel links connecting \( S \) with combined relay index \( j \) (same as the index of the parallel subsystem). Similarly, \( \alpha_{j,D} = \max_{\alpha_{z}} \{\alpha_{c,N+1}\} \) with \( N \) being the number of relays in the system and \( N + 1 \) indicating the index of the destination \( D \).

Consequently, \( P_{out,S} \) can be written as:

\[ P_{out,S}^{i} = \left(1 - \left(\prod_{\alpha_{z} \in S_{R,1}} \Pr(\alpha_{0,z} < X) \right) \left(1 - \left(\prod_{\alpha_{z} \in S_{R,1}} \Pr(\alpha_{c,N+1} < X) \right) \right) \right) \]
\[ \left(1 - \left(\prod_{\alpha_{z} \in S_{R,2}} \Pr(\alpha_{0,z} < X) \right) \left(1 - \left(\prod_{\alpha_{z} \in S_{R,2}} \Pr(\alpha_{c,N+1} < X) \right) \right) \right) \cdots \]
\[ \left(1 - \left(\prod_{\alpha_{z} \in S_{R,y}} \Pr(\alpha_{0,z} < X) \right) \left(1 - \left(\prod_{\alpha_{z} \in S_{R,y}} \Pr(\alpha_{c,N+1} < X) \right) \right) \right) \]
\[ = \left(1 - \left(\prod_{\alpha_{z} \in S_{R,1}} \Pr(\alpha_{0,z} < X) \right) \left(1 - \left(\prod_{\alpha_{z} \in S_{R,1}} \Pr(\alpha_{c,N+1} < X) \right) \right) \right) \]
\[ \left(1 - \left(\prod_{\alpha_{z} \in S_{R,2}} \Pr(\alpha_{0,z} < X) \right) \left(1 - \left(\prod_{\alpha_{z} \in S_{R,2}} \Pr(\alpha_{c,N+1} < X) \right) \right) \right) \cdots \]
\[ \left(1 - \left(\prod_{\alpha_{z} \in S_{R,y}} \Pr(\alpha_{0,z} < X) \right) \left(1 - \left(\prod_{\alpha_{z} \in S_{R,y}} \Pr(\alpha_{c,N+1} < X) \right) \right) \right) \]
\[ = \left(1 - \left(\prod_{\alpha_{z} \in S_{R,1}} p_{0,z} \right) \left(1 - \left(\prod_{\alpha_{z} \in S_{R,1}} p_{c,N+1} \right) \right) \right) \left(1 - \left(\prod_{\alpha_{z} \in S_{R,2}} p_{0,z} \right) \right) \cdots \]
\[ \left(1 - \left(\prod_{\alpha_{z} \in S_{R,y}} p_{0,z} \right) \right) \left(1 - \left(\prod_{\alpha_{z} \in S_{R,y}} p_{c,N+1} \right) \right) \right) \]
\[ (4.74) \]

This equation is shown to be the product of the outage probabilities of each subsystem in a similar manner to what was discussed with All-Active scheme in equation (4.20) with change
of variables but yet same values ($y$ instead of $n$). Moreover, each subsystem outage probability was proven to follow the given formula in equation (4.19) for All-active scheme but again with change of variables ($z$ instead of $i$).

As a result, the final equation validates that both schemes achieve identical outage probability expressions for the general case of All-to-All connected scheme, but they are not really equal due to the induced power gain for the selective scheme as will be shown in the next section in details.

### 4.3.3 Identical Expressions - Different Power Distribution

As was shown in sections 4.3.1 and 4.3.2, both All-Active and Selective schemes have identical outage probability expressions with the exact formula. However the final numerical values were not proven to be equal which is to be discussed in this section.

To start, we discuss the difference between both schemes setups. There is one main difference: All FSO links are activated with the All-Active scheme while only two or a single FSO link is activated in the Selective scheme. This is the real gain of the Selective scheme: power gain, as we can divide the available power in the system into 1 or 2 FSO links instead of $2N + 1$ FSO links as with All-Active scheme where $N$ is the number of relays in the system. So how does this difference in power distribution affect the outage probability expressions?

In fact, each FSO link individual outage probability is directly affected with the given power for the FSO link. This can be shown clearly in equation (3.7) where the outage probability of the FSO link is affected by the power margin of the system $P_M$ and the number of activated links $N_{tot}$; this relation implies that as $\left( \frac{N_{tot}}{P_M} \right)$ increases, the individual link outage probability increases. As a result, as the given power to this FSO link $\left( \frac{P_M}{N_{tot}} \right)$ increases, its outage probability decreases which is the case with the Selective relaying schemes.

Consequently, the Selective relaying scheme achieves lower system outage probability values compared to the All-active relaying scheme, but for sure this comes at the cost of prior knowledge of the channel state information (CSI) as was discussed earlier in sections 1.1.4.2.3.2 and 3.1.
Chapter 5

Diversity Order Analysis

In the previous chapter, we presented an outage probability analysis to study the reliability and performance of the proposed relaying system and to compare it with other existing systems. All three aspects of the presented analysis: exact, approximation, and simulation verify the same theoretical assumptions. However, with increased number of relays, the exact analysis becomes harder to be done and the final expressions appear to be complex and complicated for comparison. Instead, the approximate expressions make it much easier for the comparison and analysis, as we will see also in the diversity order analysis. In fact, the diversity order analysis is another measurement of the system performance that captures the spatial diversity of the system. In this chapter, we present the concept of diversity order and how it is derived, we provide simplification and comparison standards, and we apply the presented analysis strategy on the different relaying schemes and systems considered in this work.

5.1 Concept and Derivation

The diversity order of the system is another measurement of how reliable the system is. It can be derived from the outage probability expression of the system. The diversity order is defined as the slope of the outage probability expressions for large SNRs. A large slope for the outage probability means higher diversity order and thus better performance. Mathematically, the diversity order of a link can be defined as:

\[
d_{\text{link}} = \lim_{{\text{SNR} \to \infty}} \frac{\log(p_{\text{link}}(\text{SNR}))}{\log(\text{SNR})}
\]  

(5.1)
Where $p_{\text{link}}(\text{SNR})$ is the outage probability term of the link.

Starting from the approximation of the channel model pdf, the asymptotic expression for the pdf equation can be expressed as [94]:

$$f_{I_{i,j}}(I) \approx a_{i,j} I^{\beta_{i,j}-1}$$  \hspace{1cm} (5.2)

Where $a_{i,j}$ is given by:

$$a_{i,j} = \frac{(\alpha_{i,j} \beta_{i,j})^{\beta_{i,j}} \Gamma(\alpha_{i,j}) \Gamma(\beta_{i,j})}{\Gamma(\alpha_{i,j} \beta_{i,j}) \Gamma(\beta_{i,j})}$$  \hspace{1cm} (5.3)

As a result, the asymptotic outage probability expression can be approximated by the following expression for large values of $P_M$ [86]:

$$p_{(\text{out})}^{(N_{\text{tot}})} \approx a_{i,j} \left( \frac{G_{i,j} P_M}{N_{\text{tot}}} \right)^{-\beta_{i,j}}$$  \hspace{1cm} (5.4)

Where $a_{i,j}$ was described earlier in equation (5.3), $G_{i,j}$ is the gain factor that was defined in equation (3.6), $P_M$ being the power margin, $N_{\text{tot}}$ corresponds to the total number of activated FSO links, and $\beta_{i,j}$ is the gamma parameter that was defined in equation (3.3).

By observing equation (5.4) for large SNR values, we can observe that the outage probability reaches an asymptotic expression of $P_M^{-\beta_{i,j}}$. This expression will be used in determining the diversity order expressions which follow the slope of the outage expressions. By applying the logarithmic function on both sides, we can conclude that the diversity order is equal to $\beta_{i,j}$ parameters.

In what follows, we introduce two basic observations for deriving the diversity order of any system:

First, if the outage probability of the system is expressed as the product of individual outage probabilities of the links, as for example $P_{\text{out}} = p_1 p_2$, then the diversity order will be the summation of the $\beta$ parameters for these parallel links as follows:

$$d_{\text{parallel}}^{(2)} = \beta_1 + \beta_2$$  \hspace{1cm} (5.5)

Where $d_{\text{parallel}}^{(2)}$ is the diversity order of 2 parallel links. In the same manner, the diversity order of $n$ parallel links can be described as:
\[ d_{\text{parallel}}^{(n)} = \sum_{i=1}^{n} \beta_i \] (5.6)

On the other side, if the approximate outage probability of the system is expressed as the sum of individual outage probabilities of \( n \) serial links, then the diversity order of the system will be corresponding to the minimum \( \beta \) term among them as follows:

\[ d_{\text{series}}^{(n)} = \min \{ \beta_1, \beta_2, \cdots, \beta_n \} \] (5.7)

In reality, the outage expressions of complex systems will have mixed sum and product terms, therefore we have to use both definitions where needed to derive the diversity order of the applicable systems, as we will see in section 5.3. Before proceeding to the application of diversity order concept on the different relaying schemes in section 5.3, we present in the following section some simplification and comparison standards that will be used throughout our analysis.

### 5.2 Simplification and Comparison Standards

As mentioned earlier in the beginning of this chapter, using exact outage probability expressions will make it difficult for us to derive the diversity order expressions. This can be seen when looking on the general case for All to All scheme that was presented in section 4.1.4.2.2, where we need to determine all the possible combinations for the number of links in outage and the corresponding system connections accordingly, before being able to get very long outage probability expressions to be used for deriving diversity order expressions. Instead, using the upper bound expressions derived from the minimum cut set method can simplify the process of deriving the diversity order expressions where we always have only \( 2^N \) terms (in addition to the direct term) in the outage probability expressions with \( N \) being the number of relays in the system.

On the other hand, determining the diversity expressions without involving them in performance analysis and comparing the different schemes accordingly would be useless. Therefore, we introduce some comparison mechanisms that will be used for determining the diversity gain added by each system:

- \( \beta \) is dependent on distance \( \Rightarrow \) Distance and diversity order relationship exists
– As we can observe in equation (3.3), the $\beta_{i,j}$ term is related to the distance $d_{i,j}$ of the link. Even more, we have showed in figure 1.9 that as the distance of the link increases, the value of its $\beta$ parameter decreases. Moreover, the diversity order is directly related to the $\beta$ parameter, and therefore the higher the distance of the link is, the lower diversity order the link can achieve.

- We report some rules for obtaining the minimum term in the analyzed diversity order expressions:
  
  - $\min\{x, y, z\} = \min\{\min\{x, y\}, z\} = \min\{x, \min\{y, z\}\} = \min\{\min\{x, z\}, y\}$
  
  - $\min\{x + y, x + z\} = x + \min\{y, z\}$
  
  - $\min\{x, y\} \geq \min\{x, y, z\}$

5.3 Application to different Relaying Schemes

It is important to note at first that both All-Active and Selective relaying schemes achieve same diversity order expressions since it was concluded in section 4.3.3 that both schemes achieve identical outage probability expressions with the only difference being in the individual outage values of each link due to the conceptual different power distribution between both schemes. This is validated by the fact that diversity order expressions are dependent on the $\beta$ values of these links which are in their turn independent from the power distribution, therefore the deduced diversity expressions in term of the $\beta$ parameters are fully identical between both schemes.

5.3.1 NIRC

5.3.1.1 Special Cases

We start with the case of 2 relays:

Looking back into equation (4.4), where the approximate outage probability of NIRC system with 2 relays is given by $p_{0.3}[p_{0.1}p_{0.2} + p_{0.1}p_{2.3} + p_{0.2}p_{1.3} + p_{1.3}p_{2.3}]$, we can derive the diversity order expression by changing each multiplication into addition, and each addition into a minimum as follows:

$$d_{NIRC}^{(2)} = \beta_{0.3} + \min\{\beta_{0.1} + \beta_{0.2}, \beta_{0.1} + \beta_{2.3}, \beta_{0.2} + \beta_{1.3}, \beta_{1.3} + \beta_{2.3}\}$$  (5.8)
Similarly for the case of 3 relays:

\[ d_{NIRC}^{(3)} = \min \{ \beta_{0,1} + \beta_{0,2} + \beta_{0,3}, \beta_{0,1} + \beta_{0,2} + \beta_{3,4}, \beta_{0,1} + \beta_{2,4} + \beta_{0,3}, \beta_{0,1} + \beta_{2,4} + \beta_{3,4} \} \]

\[ \beta_{1,4} + \beta_{0,2} + \beta_{0,3}, \beta_{1,4} + \beta_{0,2} + \beta_{3,4}, \beta_{1,4} + \beta_{0,3}, \beta_{1,4} + \beta_{2,4} + \beta_{3,4} \} \]

\[ \text{(5.9)} \]

5.3.1.2 General Case

In order to conclude the diversity order expression for the general case of NIRC scheme, we refer to its general outage probability expression that was presented in equation (4.7). By applying the concept of changing the product terms in the outage probability expression into summation terms in the diversity order expression, and similarly the summation terms into minimum ones, the diversity expression can be given as:

\[ d_{NIRC}^{(N)} = \min_{n=0}^{N} \left\{ \min_{i=1}^{\binom{N}{i}} \left[ \left( \sum_{j \in L_{n,i}} \beta_{j,N+1} \right) + \left( \sum_{j' \in L_{n,i}} \beta_{0,j'} \right) \right] \right\} \]

\[ \text{(5.10)} \]

For example if \( N = 2 \), the equation resolves into:

\[ d_{NIRC}^{(2)} = \beta_{0,3} + \min_{n=0}^{2} \left\{ \min_{i=1}^{\binom{2}{i}} \left[ \left( \sum_{j \in L_{0,i}} \beta_{j,3} \right) + \left( \sum_{j' \in L_{0,i}} \beta_{0,j'} \right) \right] \right\} \]

\[ = \beta_{0,3} + \min \left\{ \min_{i=1}^{\binom{2}{i}} \left[ \left( \sum_{j \in L_{0,i}} \beta_{j,3} \right) + \left( \sum_{j' \in L_{0,i}} \beta_{0,j'} \right) \right], \min \left[ \left( \sum_{j \in L_{1,1}} \beta_{j,3} \right) + \left( \sum_{j' \in L_{1,1}} \beta_{0,j'} \right) \right], \min \left[ \left( \sum_{j \in L_{1,2}} \beta_{j,3} \right) + \left( \sum_{j' \in L_{1,2}} \beta_{0,j'} \right) \right] \right\} \]

\[ \text{(5.11)} \]

By analyzing the above general case in equation (5.10), we discover \( 2^N \) diversity terms from which we determine the minimum. Each diversity term in its turn is composed of the sum-
mation of $N$ $\beta$ terms selected from the $N$ paths connecting the source and the destination (depending on the different combinations of possible links’ failures and states). In other words, the final diversity expression will contain the $\beta$ term corresponding to the direct $S - D$ link in addition to the minimum of all $2^N$ possible combinations of the $\beta$ parameters of the $N$ paths. This conclusion will be helpful in the comparison step that will be discussed in section 5.4.

### 5.3.2 IRC1 & IRC2

As with NIRC, the diversity orders can be determined from the outage probability expressions of the IRC1 scheme as follows:

For 2 relays:

$$d^{(2)}_{IRC1} = \beta_{0,3} + \min\{\beta_{0,1} + \beta_{0,2}, \beta_{0,1} + \beta_{2,3}, \beta_{0,2} + \beta_{1,2} + \beta_{1,3}, \beta_{1,3} + \beta_{2,3}\} \quad (5.12)$$

For 3 relays:

$$d^{(3)}_{IRC1} = \beta_{0,4} + \min\{\beta_{0,1} + \beta_{0,2} + \beta_{0,3}, \beta_{0,1} + \beta_{0,2} + \beta_{3,4}, \beta_{0,1} + \beta_{2,4} + \beta_{0,3} + \beta_{2,3}, \beta_{0,1} + \beta_{2,4} + \beta_{3,4}, \beta_{1,4} + \beta_{0,2} + \beta_{1,2}, \beta_{1,4} + \beta_{0,2} + \beta_{3,4} + \beta_{1,2}, \beta_{1,4} + \beta_{2,4} + \beta_{0,3} + \beta_{2,3}, \beta_{1,4} + \beta_{2,4} + \beta_{3,4}\} \quad (5.13)$$

For IRC2 with 2 relays:

$$d^{(2)}_{IRC2} = \beta_{0,3} + \min\{\beta_{0,1} + \beta_{0,2}, \beta_{0,1} + \beta_{1,2} + \beta_{2,3}, \beta_{0,2} + \beta_{1,2} + \beta_{1,3}, \beta_{1,3} + \beta_{2,3}\} \quad (5.14)$$

For IRC2 with 3 relays:

$$d^{(3)}_{IRC2} = \beta_{0,4} + \min\{\beta_{0,1} + \beta_{0,2} + \beta_{0,3}, \beta_{0,1} + \beta_{0,2} + \beta_{3,4} + \beta_{2,3}, \beta_{0,1} + \beta_{2,4} + \beta_{3,4} + \beta_{1,2}, \beta_{0,1} + \beta_{2,4} + \beta_{0,3} + \beta_{1,2} + \beta_{2,3}, \beta_{1,4} + \beta_{0,2} + \beta_{0,3} + \beta_{1,2}, \beta_{1,4} + \beta_{0,2} + \beta_{3,4} + \beta_{1,2} + \beta_{2,3}, \beta_{1,4} + \beta_{2,4} + \beta_{0,3} + \beta_{2,3}, \beta_{1,4} + \beta_{2,4} + \beta_{3,4}\} \quad (5.15)$$
5.3.3 All-to-All Relaying

5.3.3.1 Special Cases

5.3.3.1.1 Exact Expressions

In what follows, we assume at a first step that all links presented in the diversity order analysis are of FSO nature.

First, with the case of 2 relays:

\[ d_{\text{AlltoAll}}^{(2)} = \beta_{0.3} + \min\{\beta_{0.1} + \beta_{0.2}, \beta_{0.1} + \beta_{1.2} + \beta_{2.3}, \beta_{0.2} + \beta_{1.2} + \beta_{1.3}, \beta_{1.3} + \beta_{2.3}\} \] (5.16)

Similarly for the case of 3 relays:

\[ d_{\text{AlltoAll}}^{(3)} = \beta_{0.4} + \min\{\beta_{0.1} + \beta_{0.2} + \beta_{0.3}, \beta_{0.1} + \beta_{0.2} + \beta_{3.4} + \beta_{2.3} + \beta_{1.3}, \beta_{0.1} + \beta_{2.4} + \beta_{0.3} + \beta_{1.2} + \beta_{2.3} + \beta_{1.3}, \beta_{0.1} + \beta_{2.4} + \beta_{1.2} + \beta_{1.3} + \beta_{1.4}, \beta_{0.1} + \beta_{2.4} + \beta_{0.3} + \beta_{2.3} + \beta_{1.3} + \beta_{1.4} + \beta_{2.4} + \beta_{3.4}\} \] (5.17)

However with RF links, these \( \beta \) parameters (which are smaller than 1 with long FSO links) have different range of values (equal to 1 with Rayleigh-faded RF links and greater than 1 with Rician-faded RF links), so we can exploit this observation to further simplify the diversity order expressions.

5.3.3.1.2 Approximate Expressions

First with \( N = 2 \) relays, the 4 diversity terms (excluding the direct \( S - D \) term) presented in equation (5.16) are: \( \beta_{0.1} + \beta_{0.2}, \beta_{0.1} + \beta_{1.2} + \beta_{2.3}, \beta_{0.2} + \beta_{1.2} + \beta_{1.3}, \) and \( \beta_{1.3} + \beta_{2.3}, \) where \( \beta_{0.1} + \beta_{0.2} \) and \( \beta_{1.3} + \beta_{2.3} \) correspond to pure FSO links. Comparing \( \beta_{0.1} + \beta_{1.2} + \beta_{2.3} \) with \( \beta_{0.1} + \beta_{0.2}, \) we can find a common term \( \beta_{0.1} \) between them, thus we are left with comparing \( \beta_{1.2} + \beta_{2.3} \) with \( \beta_{0.2} \) where \( \beta_{0.2} \) and \( \beta_{2.3} \) correspond to FSO links i.e. their values are smaller than 1, and \( \beta_{1.2} \) corresponds to RF link i.e. \( \beta_{1.2} \geq 1 \). As a result \( \beta_{1.2} + \beta_{2.3} \geq 1 \) and \( \beta_{0.2} < 1, \) hence \( \beta_{1.2} + \beta_{2.3} > \beta_{0.2} \) and \( \beta_{0.1} + \beta_{1.2} + \beta_{2.3} > \beta_{0.1} + \beta_{0.2} \) which means we can eliminate \( \beta_{0.1} + \beta_{1.2} + \beta_{2.3} \) from the diversity order expression. In a similar manner, \( \beta_{0.2} + \beta_{1.2} + \beta_{1.3} \) is compared to \( \beta_{1.3} + \beta_{2.3} \) with \( \beta_{1.3} \) being simplified from the comparison and with \( \beta_{0.2} + \beta_{1.2} \geq 1 \)
and $\beta_{1,3} < 1$, i.e. $\beta_{0,2} + \beta_{1,2} > \beta_{1,3}$ and $\beta_{0,2} + \beta_{1,2} + \beta_{1,3} > \beta_{1,3} + \beta_{2,3}$ which means we can eliminate $\beta_{0,2} + \beta_{1,2} + \beta_{1,3}$ too from the diversity order expression. The final given approximate diversity expression for the case of 2 relays is:

$$d^{(2)}_{\text{AlltoAll,approx}} = \beta_{0,3} + \min\{\beta_{0,1} + \beta_{0,2}, \beta_{1,3} + \beta_{2,3}\}$$

(5.18)

Similarly for the case of 3 relays, the minimization in equation (5.17) involves 8 terms among which we have 2 pure FSO subexpressions consisting of the $\beta$ parameters of 3 FSO links each: $\beta_{0,1} + \beta_{0,2} + \beta_{0,3}$ and $\beta_{1,4} + \beta_{2,4} + \beta_{3,4}$, while the rest contains 5 $\beta$ terms (3 FSO $\beta$ terms and 2 RF ones). Moreover, the $\beta$ parameters corresponding to the FSO part of the mixed subexpressions can be divided as either: 2 $S - R$ terms and 1 $R - D$ term, or 1 $S - R$ term and 2 $R - D$ terms. This division helps in the comparison part since the 2 pure FSO subexpressions are either composed of 3 $S - R$ terms or 3 $R - D$ terms, therefore we can find each time 2 terms common between the compared expressions to be simplified. This leaves us with (1 FSO term) to be compared with (1 FSO term and 2 RF terms). However the $\beta$ parameters of the RF terms were mentioned to be greater than or equal to one while those of FSO terms are less than one, hence all the subexpressions involving RF terms will be always greater than the pure FSO subexpressions that we are comparing with, so all of them can be eliminated. As a result, the final given approximate diversity expression for this case with 3 relays is:

$$d^{(3)}_{\text{AlltoAll,approx}} = \beta_{0,4} + \min\{\beta_{0,1} + \beta_{0,2} + \beta_{0,3}, \beta_{1,4} + \beta_{2,4} + \beta_{3,4}\}$$

(5.19)

5.3.3.2 General Case - Approximate Expression

The above approximation can be generalized with any number of relays. Examining the upper bound for the general case presented in equation (4.25), we can divide the final expression into 2 parts: one part with 2 pure FSO subexpressions involving either the $N S - R$ links or the $N R - D$ ones, and another part containing $2^N - 2$ mixed subexpressions. By analyzing the mixed subexpressions part, we can see that they will always contain $N$ FSO terms divided between $S - R$ and $R - D$ links, therefore based on which type of links are dominant in the expression, we compare it with its relevant pure FSO subexpression accordingly. In the worst case, the terms will be divided equally between $S - R$ and $R - D$ links; in that case half of the $N$ FSO terms can be simplified during the comparison. In the other section of these expressions and
by looking on the part corresponding to the RF terms coming from \( \prod_{j \in I_n, f \in T_n} p_{j,f} \), we can conclude that the minimum number of involved RF terms in each subexpression will be \( N - 1 \). Consequently, the diversity order value of each mixed subexpression will be \( d_{\text{mixed}} \geq N - 1 \) compared to \( d_{\text{pure}} < \frac{N}{2} \) for the remaining part of pure FSO subexpressions after simplification. This implies that \( d_{\text{pure}} \) will never be greater than \( d_{\text{mixed}} \) for any number of relays, thus we can eliminate all the corresponding mixed subexpressions from the diversity order expression. As a result, we can write the final approximate diversity order expression for any number of relays as:

\[
d_{\text{AlltoAll,approx}}^{(N)} = \beta_{0,N+1} + \min\left\{ \sum_{i=1}^{N} \beta_{0,i}, \sum_{i=1}^{N} \beta_{i,N+1} \right\}
\] (5.20)

### 5.4 Comparison and Analysis

In this section, we use the previously presented simplifications to compare the diversity order expressions between the presented relaying schemes under different possible network setups (based on relays positions and links distances). We apply the mentioned comparison analysis for special cases as shown in table 5.1 and table 5.2 for 2 relays and 3 relays, respectively. We then generalize our analysis using the approximate diversity expression for All-to-All mixed RF-FSO scheme presented in equation (5.20).

We start with the comparison tables for special cases \( N = 2 \) relays and \( N = 3 \) relays. It is important to mention that the terms \( \beta_{0,3} \) and \( \beta_{0,4} \) corresponding for 2 relays and 3 relays respectively, are removed from the comparison tables as they are common between all relaying schemes.
<table>
<thead>
<tr>
<th>Network Setup</th>
<th>Relaying Scheme</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{0,1} &lt; d_{1,3}$, $d_{0,2} &lt; d_{2,3}$</td>
<td>$\beta_{1,3} + \beta_{2,3}$</td>
<td>$d_{NIRC} = d_{IRC1} = d_{IRC2} = d_{All-to-All}$</td>
</tr>
<tr>
<td>$d_{0,1} &gt; d_{1,3}$, $d_{0,2} &gt; d_{2,3}$</td>
<td>$\beta_{0,1} + \beta_{0,2}$</td>
<td>$d_{NIRC} = d_{IRC1} = d_{IRC2} = d_{All-to-All}$</td>
</tr>
<tr>
<td>$d_{0,1} &lt; d_{1,3}$, $d_{0,2} &gt; d_{2,3}$</td>
<td>$\beta_{1,3} + \beta_{2,3}$, $\beta_{1,3} + \beta_{2,3}$, $\beta_{1,3} + \beta_{2,3}$</td>
<td>$d_{NIRC} &lt; d_{IRC1} = d_{IRC2} = d_{All-to-All}$</td>
</tr>
<tr>
<td>$d_{0,1} &gt; d_{1,3}$, $d_{0,2} &lt; d_{2,3}$</td>
<td>$\beta_{1,3} + \beta_{0,2}$, $\beta_{1,3} + \beta_{0,2}$, $\beta_{1,3} + \beta_{0,2}$</td>
<td>$d_{NIRC} = d_{IRC1} = d_{IRC2} = d_{All-to-All}$</td>
</tr>
</tbody>
</table>

Table 5.2: Comparing Diversity Order Expressions for different relaying schemes with $N = 3$ relays

<table>
<thead>
<tr>
<th>Network Setup</th>
<th>Relaying Scheme</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{0,1} &lt; d_{1,4}$, $d_{0,2} &lt; d_{2,4}$, $d_{0,3} &lt; d_{3,4}$</td>
<td>$\beta_{1,4} + \beta_{2,4} + \beta_{3,4}$</td>
<td>$d_{NIRC} = d_{IRC1} = d_{IRC2} = d_{All-to-All}$</td>
</tr>
<tr>
<td>$d_{0,1} &gt; d_{1,4}$, $d_{0,2} &gt; d_{2,4}$, $d_{0,3} &gt; d_{3,4}$</td>
<td>$\beta_{0,1} + \beta_{0,2} + \beta_{0,3}$</td>
<td>$d_{NIRC} = d_{IRC1} = d_{IRC2} = d_{All-to-All}$</td>
</tr>
<tr>
<td>$d_{0,1} &lt; d_{1,4}$, $d_{0,2} &lt; d_{2,4}$, $d_{0,3} &gt; d_{3,4}$</td>
<td>$\beta_{1,4} + \beta_{2,4} + \beta_{0,3}$</td>
<td>$d_{NIRC} &lt; d_{IRC1} = d_{IRC2} = d_{All-to-All}$</td>
</tr>
</tbody>
</table>
\[
\begin{array}{|c|c|c|c|c|}
\hline
& d_{0.1} < d_{1.4}, & d_{0.2} > d_{2.4}, & d_{0.3} < d_{3.4} & \\
\hline
\beta_{1.4} + \beta_{0.2} + \beta_{3.4}, & \min(\beta_{0.1} + \beta_{0.2} + \beta_{3.4}, & \beta_{1.4} + \beta_{1.2} + \beta_{0.2} + \beta_{3.4}, & \min(\beta_{0.1} + \beta_{0.2} + \beta_{0.3}, & \min(\beta_{0.1} + \beta_{0.2} + \beta_{0.3}, \\
& \beta_{0.1} + \beta_{0.2} + \beta_{3.4}, & \beta_{1.4} + \beta_{0.2} + \beta_{3.4}, & \beta_{0.1} + \beta_{0.2} + \beta_{3.4}, & \beta_{0.1} + \beta_{0.2} + \beta_{3.4}, \\
& \beta_{0.1} + \beta_{0.2} + \beta_{3.4}, & \beta_{1.4} + \beta_{0.2} + \beta_{3.4}, & \beta_{0.1} + \beta_{0.2} + \beta_{3.4}, & \beta_{0.1} + \beta_{0.2} + \beta_{3.4}, \\
& \beta_{1.4} + \beta_{0.2} + \beta_{3.4}) & \beta_{1.4} + \beta_{0.2} + \beta_{3.4}) & \beta_{1.4} + \beta_{0.2} + \beta_{3.4}) & \beta_{1.4} + \beta_{0.2} + \beta_{3.4}) \\
\hline
\end{array}
\]

As a result, the All-to-All relaying scheme shows superiority in the diversity order expressions for both \( N = 2 \) relays and \( N = 3 \) relays systems.

In addition, following from the general case equations presented for NIRC and All-to-All relaying schemes in equations (5.10) and (5.20), we can observe that the presented subexpressions
in the All-to-All scheme diversity order expression are always subset of the presented subexpressions in the NIRC scheme. Therefore, a mathematical relation can be derived between the two schemes using the mathematical rule \((\min\{x,y\} \geq \min\{x,y,z\})\) as follows:

\[
d^{(N)}_{\text{AlltoAll,approx}} = \beta_{0,N+1} + \min\{x,y\} \geq \beta_{0,N+1} + \min\{x,y,z_1, z_2, \cdots, z_{2^N-2}\}
\]

\[
\Rightarrow d^{(N)}_{\text{AlltoAll,approx}} \geq d^{(N)}_{\text{NIRC}}
\]

(5.21)

Where \(x\) and \(y\) are common subexpressions between the approximate All-to-All diversity expression and the NIRC one, and \(\{z_1, z_2, \cdots, z_{2^N-2}\}\) correspond to the remaining \(2^N-2\) subexpressions from the total \(2^N\) subexpressions presented in the NIRC general case diversity order expression.

As a conclusion, this proves the superiority of the All-to-All mixed RF-FSO relaying scheme for any number of relays.
Chapter 6

Numerical Results

In this chapter, we present numerical results for the theoretical conclusions that were determined earlier in chapter 4 for the outage probability analysis and in chapter 5 for the diversity order analysis. We start by simulating the proposed mixed RF-FSO model that was introduced in section 1.1.5 and further described in chapter 3. The proposed system is compared with other existing systems in the cooperative literature such as NIRC, IRC1, and IRC2 which were compared theoretically with the proposed scheme throughout the previous chapters. We compare both the outage probabilities and diversity order slopes between the different relaying schemes to confirm the gains delivered by the proposed scheme. We then highlight the superiority of Selective scheme over All-Active scheme and we plot the effect of increasing the number of relays in the system in each scheme and interpret the results. We also consider different channel fading models such as Gamma-Gamma, Rayleigh, and Rician distributions.

Throughout this chapter, the refractive index structure constant and the attenuation constant which were described in equations (3.4) and (3.6) are set to $C_n^2 = 1 \times 10^{-14} \text{m}^{-2/3}$ and $\sigma = 0.44$ dB/km, respectively. We also set $\lambda = 1550$ nm and the distance between S and D to be equal to 3 km in all network setups. The results show the variation of the outage probability of each system versus the system power margin in decibels (dB) varying from 0 dB to 50 dB. In all presented plots, both exact and approximate (using minimum cut set method) outage probabilities are plotted where the approximate outage probability is always presented with dotted lines.
6.1 Comparing Relaying Schemes while Varying Relays Positions

In this whole section, gamma-gamma channel model fading is used with all FSO links, while Rician fading with $\Omega_{i,j} = 20$ dB and $K_{i,j} = 10$ is applied for the RF links. In addition, All-Active relaying is considered with $N_{tot}$, being the number of activated FSO links, equal to $2N + 1$ for NIRC and mixed RF/FSO All-to-All schemes and equal to $3N$ with IRC1 and IRC2 where $N$ is the number of relays in the system.

6.1.1 2 Relays

We consider the four possible scenarios as presented in table 5.1: $d_{0,1} < d_{1,3}$ and $d_{0,2} < d_{2,3}$, $d_{0,1} > d_{1,3}$ and $d_{0,2} > d_{2,3}$, $d_{0,1} < d_{1,3}$ and $d_{0,2} > d_{2,3}$, and $d_{0,1} < d_{1,3}$ and $d_{0,2} > d_{2,3}$.

Case 1:

The link distances are chosen as: $d_{0,1} = 1.5$ km, $d_{1,3} = 2.5$ km, $d_{0,2} = 1$ km, and $d_{2,3} = 2.5$ km. The results are shown in figure 6.1 where the parallel curves for large $P_M$ values validate the expected equal diversity order between the 4 schemes as stated in table 5.1 for the first case. However, the performance of NIRC and All-to-All schemes appear to be better than IRC1 and IRC2 schemes due to the lower number of activated links, therefore the system power is divided into less number of FSO links. This indicates lower individual link outage probabilities leading to an overall gain in the system outage probability for both NIRC and All-to-All scheme. Here, we have to mention that in concept, both IRC2 and All-to-All relaying schemes appear to be the same system and with the same outage probability expressions as stated in equations (4.13) and (4.17). However, the main difference is in the nature of the inter link connecting the 2 relays in the system (FSO nature for IRC2 while RF nature for All-to-All); this is the reason for the better performance under the All-to-All scheme.

Case 2:

In this case, the link distances are chosen as: $d_{0,1} = 2.5$ km, $d_{1,3} = 1$ km, $d_{0,2} = 2.5$ km, and $d_{2,3} = 1.5$ km. Similarly to the first case, the results which are shown in figure 6.2 validate the expected equal diversity order between the 4 schemes as stated in table 5.1 for the second case with better performance of NIRC and All-to-All schemes over IRC1 and IRC2 due to the power gain as explained in the first case. Case 3:
Figure 6.1: Comparison between the four relaying schemes for $N = 2$ relays, Case 1

Figure 6.2: Comparison between the four relaying schemes for $N = 2$ relays, Case 2
The link distances are given by: $d_{0,1} = 1.5$ km, $d_{1,3} = 2.5$ km, $d_{0,2} = 2.5$ km, and $d_{2,3} = 1.5$ km. In this case, as shown in figure 6.3, the NIRC scheme has the lowest diversity order while the remaining three relaying schemes share equal diversity order values; this validates the result shown in table 5.1 for the third case. Moreover, the system outage probability values validate the superiority of the mixed All-to-All scheme due to the power gain as mentioned earlier in cases 1 and 2.

Case 4:

$d_{0,1} = 2.5$ km, $d_{1,3} = 1.5$ km, $d_{0,2} = 1.5$ km, and $d_{2,3} = 2.5$ km. The presented results in figure 6.4 confirm the equal diversity order between NIRC and IRC1 schemes with a lower shift for NIRC scheme due to lower number of FSO links activated as explained in the first case. Moreover, the results show an equal diversity order between IRC2 and All-to-All relaying schemes with superiority too for the All-to-All scheme due to power gain, where both schemes achieve higher diversity order than NIRC and IRC1 schemes. These results validate the result in table 5.1 for the fourth case which states that $d_{NIRC} = d_{IRC1} < d_{IRC2} = d_{All-to-All}$. 

![Figure 6.3: Comparison between the four relaying schemes for N = 2 relays, Case 3](image-url)
6.1.2 3 Relays

In this section we show the performance of the four relaying schemes under 3 relays. We consider for this purpose 4 out of the total 8 cases presented in table 5.2. The selected cases show all different possible outcomes: $d_{NIRC} = d_{IRC1} = d_{IRC2} = d_{All-to-All}$ (Cases 1 & 2 in the table), $d_{NIRC} < d_{IRC1} = d_{IRC2} \leq d_{All-to-All}$ (Cases 3 & 6), $d_{NIRC} < d_{IRC1} \leq d_{IRC2} \leq d_{All-to-All}$ (Cases 4 & 7), and $d_{NIRC} = d_{IRC1} < d_{IRC2} \leq d_{All-to-All}$ (Cases 5 & 8). As a result, the four network setups to be considered in this section are as follows:

Case 1: $d_{0,1} < d_{1,4}$, $d_{0,2} < d_{2,4}$, and $d_{0,3} < d_{3,4}$ with $d_{0,1} = 1.5$ km, $d_{1,4} = 2.5$ km, $d_{0,2} = 1.25$ km, $d_{2,4} = 2.75$ km, $d_{0,3} = 1.75$ km, and $d_{3,4} = 2.25$ km.

Case 2: $d_{0,1} < d_{1,4}$, $d_{0,2} < d_{2,4}$, and $d_{0,3} > d_{3,4}$ with $d_{0,1} = 1.5$ km, $d_{1,4} = 2.5$ km, $d_{0,2} = 1.25$ km, $d_{2,4} = 2.75$ km, $d_{0,3} = 2.75$ km, and $d_{3,4} = 1.25$ km.

Case 3: $d_{0,1} < d_{1,4}$, $d_{0,2} > d_{2,4}$, and $d_{0,3} < d_{3,4}$ with $d_{0,1} = 1.5$ km, $d_{1,4} = 2.5$ km, $d_{0,2} = 2.75$ km, $d_{2,4} = 1.25$ km, $d_{0,3} = 1.75$ km, and $d_{3,4} = 2.75$ km.

Case 4: $d_{0,1} > d_{1,4}$, $d_{0,2} < d_{2,4}$, and $d_{0,3} < d_{3,4}$ with $d_{0,1} = 2.75$ km, $d_{1,4} = 1.25$ km, $d_{0,2} = 1.5$ km, $d_{2,4} = 2.5$ km, $d_{0,3} = 1.75$ km, and $d_{3,4} = 2.75$ km.

Figure 6.4: Comparison between the four relaying schemes for $N = 2$ relays, Case 4
Case 1:

The results shown in figure 6.5 confirm the similar results for the case with 2 relays where the parallel curves for large power margin values validate the expected equal diversity order between the 4 schemes as stated in table 5.2 for the first two cases. Also, the performance of NIRC and All-to-All schemes show lower outage probability values than IRC1 and IRC2 schemes due to the power gain as explained earlier with 2 relays.

Case 2:

In this case, the results that are shown in figure 6.6 validate that the diversity order of the IRC1 and IRC2 schemes are equal to each other and at the same time greater than the NIRC scheme. Moreover, the mixed All-to-All scheme is shown to be superior to all three schemes validating the expected greater or equal diversity order than IRC schemes ($d_{\text{NIRC}} = 6.655$, $d_{\text{IRC1}} = d_{\text{IRC2}} = 8.834$, and $d_{\text{All-to-All}} = 8.96$; it is a little bit greater in this case) that was stated in table 5.2 for cases 3 and 6.

Case 3:

In this case, as shown in figure 6.7, the NIRC scheme has the lowest diversity order ($d_{\text{NIRC}} = 6.66$), after which comes the IRC1 scheme ($d_{\text{IRC1}} = 7.97$), followed by the IRC2 scheme ($d_{\text{IRC2}} = 8.96$), and finally the mixed All-to-All scheme ($d_{\text{All-to-All}} = 8.97$ which is almost equal to the diversity order of the IRC2 scheme but better performance due to FSO power gain with 3 RF inter-relay links instead of 2 FSO ones). These results confirm the expected diversity order relationship that was presented in table 5.2 for cases 4 and 7.

Case 4:

Finally, the plotted curves in figure 6.8 show equal diversity order values between NIRC and IRC1 (NIRC performs better due to power gain), with IRC2 being superior to both of them. In addition, mixed RF-FSO All-to-All relaying scheme is proven to have the highest diversity order and the lowest outage probability values as implied theoretically in table 5.2 for cases 5 and 8.

It is important to highlight that all results presented in figures 6.1 → 6.21 show that the ap-
proximate outage probability values are extremely close to the exact values for the values of $P_M$ exceeding 10 dB.

### 6.2 Effect of Increasing the Number of Relays

In this section, we simulate the system behavior under the effect of increasing the number of relays presented in the system. Therefore, we plot the outage probability for each of the four relaying schemes with $N = 2, 3, 4, \text{ and } 5$ relays. In a similar manner to the previous section, all the plotted curves in this section correspond to systems with gamma-gamma channel model fading for the FSO links and Rician fading with $\Omega_{i,j} = 20 \text{ dB and } K_{i,j} = 10$ for the RF links. Also, All-Active relaying is considered with $N_{tot}$ being the number of activated FSO links is equal to $2N + 1$ with NIRC and mixed All-to-All schemes and equal to $3N$ with IRC1 and IRC2. In addition, the following network setups are considered for the different number of relays with each relaying scheme:

$N = 2$: $d_{0,1} = 1.5 \text{ km, } d_{1,3} = 2.5 \text{ km, } d_{0,2} = 2.5 \text{ km, and } d_{2,3} = 1.5 \text{ km}$

$N = 3$: $d_{0,1} = 1.5 \text{ km, } d_{1,4} = 2.5 \text{ km, } d_{0,2} = 2.5 \text{ km, } d_{2,4} = 1.5 \text{ km, } d_{0,3} = 1.75 \text{ km, and } d_{3,4} = 2.75 \text{ km}$
Figure 6.6: Comparison between the four relaying schemes for $N = 3$ relays, Case 2

Figure 6.7: Comparison between the four relaying schemes for $N = 3$ relays, Case 3
Figure 6.8: Comparison between the four relaying schemes for N = 3 relays, Case 4

\[ N = 4: d_{0,1} = 1.5 \text{ km}, d_{1,5} = 2.5 \text{ km}, d_{0,2} = 2.5 \text{ km}, d_{2,5} = 1.5 \text{ km}, d_{0,3} = 1.75 \text{ km}, d_{3,5} = 2.75 \text{ km}, d_{0,4} = 2.8 \text{ km}, \text{ and } d_{4,5} = 2 \text{ km} \]

\[ N = 5: d_{0,1} = 1.5 \text{ km}, d_{1,6} = 2.5 \text{ km}, d_{0,2} = 2.5 \text{ km}, d_{2,6} = 1.5 \text{ km}, d_{0,3} = 1.75 \text{ km}, d_{3,6} = 2.75 \text{ km}, d_{0,4} = 2.8 \text{ km}, d_{4,6} = 1.7 \text{ km}, d_{0,5} = 2.2 \text{ km}, \text{ and } d_{5,6} = 2.9 \text{ km} \]

As a result, the impact of increasing the number of relays is reported in figures 6.9, 6.10, 6.11, and 6.12 for NIRC, IRC1, IRC2, and mixed All-to-All relaying schemes, respectively. The effect is positive for large values of \( P_M \) with all relaying schemes as increasing the number of relays in the system leads to a decrease in the outage probability of the system and thus improving the system reliability. However, the effect of adding new relays becomes less significant with higher number of relays in the system. On the other hand, some performance losses can be observed for small values of \( P_M \). This follows from the fact of splitting the power among a larger number of FSO links when \( N \) increases where \( N_{tot} \) is an increasing function of \( N \).
Figure 6.9: Performance of NIRC scheme under increasing number of relays

Figure 6.10: Performance of IRC1 scheme under increasing number of relays
Figure 6.11: Performance of IRC2 scheme under increasing number of relays

Figure 6.12: Performance of All-to-All scheme under increasing number of relays
6.3 All-Active vs Selective

In this section we simulate the system performance under All-Active and Selective techniques for both NIRC and mixed All-to-All relaying schemes. We compare the system performance between both schemes under each technique to highlight the advantages that the Selective technique presents versus the All-Active technique as explained in section 4.3. We also compare both relaying schemes to present the system performance gain that the mixed All-to-All scheme presents even compared to the NIRC Selective scheme. For this purpose, gamma-gamma channel modeling is considered for the FSO links and Rician fading with $\Omega_{i,j} = 20$ dB and $K_{i,j} = 10$ for the RF links. It is important to note that $N_{\text{tot}}$ is equal to $2N + 1$ for All-Active technique while for Selective it is taken equal to either 1 when the direct $S-D$ link is activated or equal to 2 otherwise. For all plotted figures in this section, same network setup is used as in section 6.2.

We start by comparing NIRC with mixed All-to-All relaying schemes under All-Active and Selective techniques for system with 2 and 3 relays as shown in figures 6.13 and 6.14. It can be validated from the plotted figures that selective and all-active techniques result in parallel outage probability curves i.e. equal diversity orders due to comparable outage variations but with a shift due to the induced power gain by Selective relaying as explained theoretically in section 4.3.3. Moreover, we can see that the mixed All-to-All relaying scheme performs better than the NIRC scheme in both techniques; the performance gain is in the order of 3 dB at an outage probability of $10^{-10}$.

We then extend this comparison into an arguable question: Is it better to increase the number of relays or implement Selective technique for a given network setup? As an answer for this question, we combine both cases $N = 2$ relays and $N = 3$ relays for each relaying scheme and we compare the All-Active and Selective relaying techniques accordingly. As can be shown in figures 6.15 and 6.16, Selective relaying technique achieves lower outage probabilities than increasing the number of relays from 2 to 3 with All-Active Relaying.
Figure 6.13: All-Active vs Selective for NIRC and All-to-All schemes with $N = 2$ relays

Figure 6.14: All-Active vs Selective for NIRC and All-to-All schemes with $N = 3$ relays
Figure 6.15: All-Active vs Selective for NIRC scheme with 2 and 3 relays

Figure 6.16: All-Active vs Selective for All-to-All scheme with 2 and 3 relays
Figure 6.17: All-Active with 2, 3, 4, and 5 relays vs Selective with 2 and 3 relays for All-to-All scheme

Even more, it can be shown in figure 6.17 that a mixed All-to-All relaying scheme with 2 relays under Selective relaying technique achieves lower outage probability values compared to systems with 3 and 4 relays (even with 5 relays for $P_M < 22$ dB) under All-Active technique. This gain increases further with 3 relays under Selective relaying as can be shown clearly in the plotted figure. This result highlights the huge gain that the Selective relaying technique introduces rather than changing the system network setup by adding more relays. But we have to mention that this gain comes with the cost of a prior knowledge of CSI as was explained in section 1.1.4.2.3.2.
6.4 Rician vs Rayleigh fading for RF links

In the previous sections, we always considered Rician fading for the RF links. However, we mentioned earlier in sections 1.1.1.2.1 and 1.1.1.2.2 that Rayleigh and Rician distributions are the most commonly linked to RF links. Moreover, as explained in the second chapter section 3.3, the difference between both models is in the presence or absence of a direct LOS between the relevant relays for Rician and Rayleigh models, respectively. Therefore, we present a comparison applied to the system behavior under each channel model fading where we vary the parameter $\Omega_{i,j}$ (between 10, 20, and 30 dB) representing the average SNR for the RF link as presented in equations (3.9) and (3.10) for the outage probability of Rayleigh and Rician RF modeled links, respectively. Moreover, we vary the power ratio $K_{i,j}$ (between 3 and 10) presented in equation (3.10) corresponding to Rician channel model outage probability. In order to calculate the threshold SNR value for the channel $\Omega_{th}$ given by $\Omega_{th} = 2^{2R} - 1$, we set the transmitted bits per channel use to $R = 1$. We also apply this comparison for 2 relays (figure 6.18) and 4 relays (figure 6.19) with the same network setup that was presented in the two sections 6.2 and 6.3.

As we can explore in both figures 6.18 and 6.19, both channel models result in approximately equal system outage probability values for equal average link SNR with slightly lower outage values for the Rician channel model specially when $k = 10$. Moreover, by having deeper look into a zoomed in view of the system outage probability with 2 relays as in figure 6.20, we can see that the system under Rician fading with $k = 10$ and average SNR of 20 dB leads to lower outage values than under Rayleigh fading with 30 dB or even Rician fading with 30 dB but $k = 3$. This explains our choice of 20 dB and $k = 10$ for all the RF links in the previous sections. This result is also highlighted in the extended view of the system outage probability with 4 relays as shown in figure 6.21 but with a smaller impact on the system performance as the number of relays increases.
Figure 6.18: Rayleigh vs Rician for All-to-All scheme with 2 relays

Figure 6.19: Rayleigh vs Rician for All-to-All scheme with 4 relays
Figure 6.20: Rayleigh vs Rician for All-to-All scheme with 2 relays - zoomed view

Figure 6.21: Rayleigh vs Rician for All-to-All scheme with 4 relays - extended view
Chapter 7

Conclusion and Future Work

In this thesis, we proposed, modeled, and tested a novel method for cooperative communication systems. This system was shown to profit from the advantages of both FSO and RF links in order to increase the reliability of the communication process without adding complex hardware and costly operations to the existent communication network. The system was inspired by inter-relay cooperative (IRC) schemes but by adding an additional degree of freedom through connecting the inter relays in a fully connected way (All-to-All). Moreover, in order to overcome the limitations that hinder the existing FSO IRC systems from being connected in an All-to-All way, the proposed system exploits the existence of backup RF links between the relays which have broadcasting nature making the All-to-All relaying scheme feasible without any added complexity or cost.

In order to highlight the gains introduced by our proposed system, the system was studied and simulated as per outage probability and diversity order analysis under different possible network setups. For this purpose, we compared the proposed system with well-known cooperative FSO schemes in the literature such as parallel relaying without any inter relay cooperation (NIRC), unidirectional inter relay cooperation (IRC1), and bidirectional inter relay cooperation (IRC2). The systems were studied for the special cases with small number of relays and then generalized for any number of relays $N$. The outage probability analysis was supported by exact results through conditional probability approach, approximate expressions determined by the minimum cut set method, and simulated results obtained from the Monte Carlo simulation method which all together revealed approximately equal results. The results showed the superiority of the proposed All-to-All scheme in all different network setups by achieving the lowest
outage probability and highest diversity order values.

In addition, the analysis was extended to the Selective relaying technique in addition to the existing All-Active one. The comparison between both techniques showed superiority of the Selective technique due to the power gain that was introduced by activating 1 or 2 links instead of $2N + 1$ links as the case is with All-Active technique. However, the achieved that which was categorized as a power gain was achieved by dividing the power equally between the activated links, but it could have been taken a step further by determining the optimal power distribution that can achieve highest gains to the system. In fact, changing the power distribution can lead sometimes to selecting another path for the message transmission which could be more reliable.

Therefore, a future work will target the power allocation problems in order to reach a general mathematical equation for optimizing the power distribution in the system to select the most reliable path. On the other hand, it is important to mention that the gains introduced by the Selective scheme come with the cost of prior knowledge of the Channel State Information which should be taken into consideration too when engineering the desired communication system. In fact, it is highly needed to establish a common baseline and cost metrics to compare the gain introduced with the cost required. Thus, another next step would be studying the cost of establishing this technique in terms of CSI and how it can be compared with the induced gains to determine the best scenario in terms of both reliability and cost to proceed with.

Finally, this thesis proved the importance of implementing the proposed mixed RF-FSO All-to-All relaying scheme in the existing communication systems. The system which was validated to be effective, feasible, and much more reliable can be very beneficial for further studying and applications in the communications field.


Appendices
Appendix A

Simulation Analysis Algorithm -
All-Active All-to-All Relaying Scheme

This appendix describes the algorithm that was implemented to determine the outage probabil-
ity with Monte-Carlo simulation under the All-Active All-to-All Relaying scheme.

A main challenge here is that a relay can be reached through different ways from other
relays. For example if we have $R_1 - R_2$ and $R_2 - R_3$ as active links but only $R_1$ was successfully
reached from the source $S$, then we have to update the successfully reached relays to include
all of $R_1$, $R_2$, and $R_3$. This will require updating the list after checking the link $R_1 - R_2$ and
rechecking all the links going out from the new elements in the updated list (such as $R_2$). Each
time we have new element in the list we will need to check all the inter-connected links going
out from this added relay to update the list accordingly until no more relays can be added into
the successfully reached set. This analysis was added after the third step that was described in
the NIRC Monte Carlo simulation section 4.1.1.3. As a result, the updated algorithm looks as
follows:

1. First, we assign the random binary values (0 for outage and 1 for active) for all the
presented links in the system based on their predefined individual outage probability
values.

2. Second, we check the binary value of the direct $S - D$ link: if it is 1, then there is no
outage in the system and we can stop the analysis of this trial here and we go to the
seventh step; else, we go to the third step to analyze any possible outage of the system
from the status of the remaining links in the system.

3. Third, we check the outage of the $S-R$ links where each active link means that the corresponding relay is successfully reached, thus we add its index into the set of successfully reached relays. If this set is empty at the end of checking all the $S-R$ links, then the system is in outage and we go to the sixth step. Else, we proceed with the fourth step to check the connectivity of the $R-R$ links.

4. Fourth, we check the $R-R$ links to update the set of successfully reached relays. This can be done as follows:
   
   (a) Check all the outgoing links from each relay in the successfully reached set. For each active link reaching a relay that does not exist in the current set, update the set with the newly reached relay accordingly.

   (b) In the next round, check all the outgoing links from the newly added relays in the set for any possible new relays to be reached. Update the set accordingly, and repeat this step until no more new reached relays are found.

   (c) At the end of this step, an updated set with all successfully reached relays is obtained to be used in the fifth step.

5. Fifth, we check all the $R-D$ links that are associated with the successfully reached relays. If any of these links is active, then there is no outage in the system and therefore we proceed to the seventh step. However, if all these links are in outage, then the destination $D$ cannot be reached and the system is in outage consequently; therefore we move to the sixth step.

6. Reaching this step means the system is in outage, so we increment the counter of outages and we check if we reached the desired number of trials. If yes, we go to the final step; else, we increment the counter of trials and go back to the first step for a new random trial.

7. Alternatively, reaching this step means the system is not in outage, so we directly check if we reached the desired number of trials without incrementing the outages counter. Again, if yes, we go to the final step; else, we increment the counter of trials and go back to the first step.
8. Finally, we divide the counter of outages by the number of trials to obtain the simulated outage probability value for the studied system.
Appendix B

Simulation Analysis Algorithm -
Selective NIRC Relaying Scheme

This appendix describes the algorithm that was implemented to determine the outage probability with Monte-Carlo simulation under the Selective NIRC Relaying scheme.

1. First, we need to generate random strength values for all the presented links in the system: 1 $S\rightarrow D$ link, $N S\rightarrow R$ links, and $N R\rightarrow D$ links. However the strength value of each link can be expressed in terms of gamma-gamma random variable as follows:

$$\text{strength} \triangleq Gxh$$  \hspace{1cm} (B.1)

Where $h$ is the gamma-gamma random variable and $G$ is the static gain factor that was introduced in equation (3.6). Therefore, we need to generate the gamma-gamma random variables for the FSO links (This can be done using the “gamrnd” function in MATLAB) and multiply them by their static gain values.

2. Second, we get the minimum strength value along each path between the $S\rightarrow R$ and $R\rightarrow D$ hops. As a result we have $N$ minimum strength values in addition to the strength value of the direct $S\rightarrow D$ link.

3. Third, we divide all the $N$ values by 2 to account for the power distribution (we assume power is divided fairly between activated links, no power optimization is determined here).
4. Fourth, we determine the maximum value among all the obtained strength values ($N$ divided strength values for $S - R - D$ paths and one direct full strength value for the $S - D$ link).

5. Fifth, we compare the final strength value with the system threshold ($\frac{1}{P_M}$ where $P_M$ is the system power margin). If the strength value falls below this threshold, then the system is in outage and we increment the number of outages. Otherwise, the system is not in outage.

6. Finally, we repeat the described procedure until we reach the desired number of trials so that we divide the obtained outage counter by the number of trials to get the simulated system outage probability.
Appendix C

Simulation Analysis Algorithm -
Selective All-to-All Relaying Scheme

This appendix describes the algorithm that was implemented to determine the outage probability with Monte-Carlo simulation under the Selective All-to-All Relaying scheme.

1. Since the $R - R$ links are of RF nature, the states of these links are studied prior to applying the selective relaying scheme on the FSO links. This can be done using random binary values generated for these RF links based on their individual outage probabilities. The randomly generated binary values determine the states of the $R - R$ links.

2. In a next phase, the inter-connectivity between the inter-relays is studied to determine the combined relays and thus the simplified network setup (this was done using a separate MATLAB code that takes as an input the active $R - R$ links and returns sets of the combined relays together). The simplified network setup will be composed of $y$ parallel subsystems ($y$ is a positive integer which varies between 1 and $N$) where each subsystem is composed of $S - R$ and $R - D$ hops. In their turn, the $S - R$ and $R - D$ hops contain $x$ parallel links (where $x$ is the number of combined relays in the subsystem).

3. Based on the system simplified network setup, and using the concepts of strength analysis that was presented in the beginning of this chapter in sections 4.2.1.2 and 4.2.1.3, along with the presented concepts in applying Monte Carlo Simulation for NIRC Selective schemes as in section 4.2.2.2 (generating gamma-gamma random variables for FSO links, multiplying them by their static gain values, and dividing them by the power factor
which is 2 for all the links except the direct $S - D$ link), the strength value of the system is determined.

4. Finally, the obtained strength value of the system is compared with the system threshold to determine if an outage is experienced or not. As a result, a counter associated with the number of experienced outages during the system trials is either incremented or not. The final result is obtained after repeating all the described steps until reaching the desired number of trials by dividing the resultant counter of outages by the number of trials; this is the simulated outage probability of the system.