WRITING IN MATHEMATICS ENHANCED BY TECHNOLOGY

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Abstract

Research on writing in mathematics has shown that this skill is most beneficial when it is directed at the learning aspect. Educators agree that writing benefits in the promotion of understanding in an inquiry based environment. This skill however necessitates proficiency on the part of the students that may not have been developed at their earlier learning stages. But the burden placed on teachers and learners to master this skill is rewarded by the mathematical learning in such an environment. This paper explores the effect of writing on improving student understanding of some topics in differential equations and investigates how the students' writing skills developed throughout the semester.

Introduction

In recent history, mathematical representations have been recognized as symbolic, verbal, graphical, and numeric. The 2000 NCTM Standards called for using multiple representations within a problem-solving situation. Since then, many instructors began redesigning their classes in such a way that the algebraic, visual, and numeric aspects are all emphasized simultaneously. This came to be known as *the Rule of Three*. This call was paralleled with attempts to integrate the skill of *writing* across the curriculum. According to Hallet, Gleason, et al. (1998), students need to learn "to reason with the intuitive ideas and explain their reasoning in plain English" (p. iv). Indeed, the skill of writing in mathematics assists students to learn more since it necessitates the combination of the numeric, symbolic expressions, and graphical representations. Thus mathematical writing can be considered as one single representation (Freitag, 1997). As with the various approaches to learning mathematics, students need to be trained to write mathematically (Shibli, 1992). It is clear that students appreciate more the writing aspect of mathematics if they observe their own teachers utilizing such a teaching strategy in the classroom. Needless to say that skill of writing can be further promoted through assignments and exams (Thompson, Austin, and Beckmann, 2002).

The experiment, object of this paper, is about improving student understanding of some key concepts in an introductory differential equations class by introducing a writing component to an already non-traditional teaching environment. The study was conducted in spring of 2010 at the Lebanese American University (LAU) in Beirut, Lebanon. The textbook used was Elementary Differential Equations, 9th edition, by William Boyce, Richard DiPirma (2009). The book was chosen because it served the purpose of a reformed curriculum. Indeed its authors combine the quantitative and qualitative approaches for solving odes and often analyze outcomes in writing. Thus, and during the course of the semester, I, as the course instructor, emphasized the role of writing to communicate the solution behavior of differential equations. In fact, I have been experimenting with various other instruction methods in this course with an eye on promoting a visual representation of concepts (see for instance Habre, 2000 and Habre, 2003). This has become possible and more learner-friendly because of the availability of dynamical software programs, primarily the software program *ODE Architect* (1999). The program has been instrumental in my reform attempts since it can easily be employed to add a dynamical dimension for understanding various concepts in differential equations such as the solutions to a differential equation and also the solution curves of a linear system of first-order differential equations; this can be done through an investigation of slope fields in the former case, phase portraits and time series in the latter case (Figure 1). In this paper, I present some preliminary results on how student's understanding of these two concepts improved through some writing exercises, and I also report on the improvement of the writing skill throughout the semester.

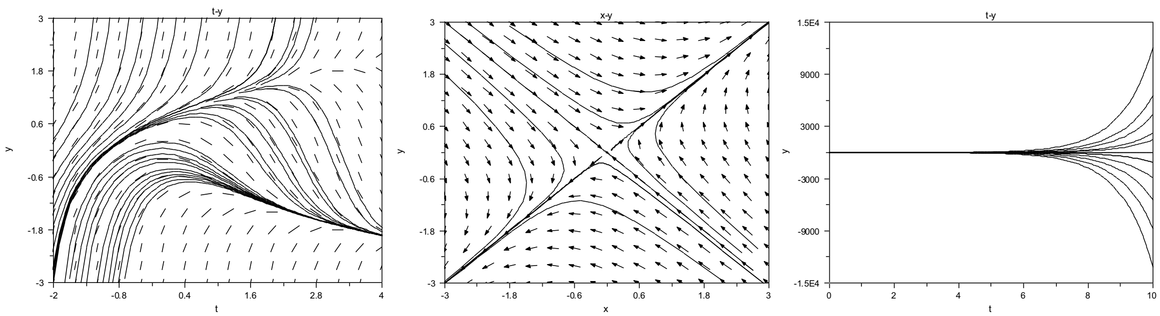

Figure1. A slope field, a phase portrait, and time series

Results and Conclusions

The class, object of my investigation, consisted initially of forty three students (43). By the end of the semester, only two had dropped and another three failed. As a matter of fact, the students of this class may be considered academically strong with 72% earning a final grade above 70, 25% a final grade above 80 and 17% earning a final grade above 90.

To achieve the learning objectives of this course, strategies and special lesson plans were specifically developed in order to highlight the visual aspect the meaning of a solution to a differential equation and the solution curves for systems of first order differential equations (odes). Various indicators were used to assess the success of the experiment. On one hand, exams included writing exercise and on the other hand the same exercises were assigned as take-home projects and guidance was given as to my expectations. On Exam 1, students were asked in one problem to discuss in writing the shape of a solution satisfying an initial value problem; in another problem, they were asked to match differential equations with direction fields and to justify their choice in writing; in yet a different problem, students were asked to write a short paragraph explaining the concept of an ordinary differential equation to an audience with a minimal mathematical background. On Exam 2, a writing problem required the discussion of the motion of a door while opening/shutting based on graphs of solutions to odes. On the Final Exam, a writing assignment required that students justify the shape of time series for a solution curve of a linear system of differential equations, as well as a matching problem similar to what was given on the first exam. Copies of the relevant exam problems were collected and grouped according to one basic criterion: Improvement in the student's writing skills. When categorizing their work on Exam 1, writing skills were assessed as unsatisfactory if students barely wrote any explanatory statement, and thus the work they presented was purely analytical or quantitative. Twenty nine (29) students belonged to this category. Following Exam 2 and the Final Exam, the classification was revisited and 10 out of the 29 showed improvement in their writing skills.

The teaching of Math in Lebanon is still very traditional be it at the school or college level. Students are exposed only rarely to a reformed approach. This applies to the teaching environment at LAU as well. Only in few cases, and upon personal initiatives by the instructor, are non conventional teaching approaches implemented. It is only natural therefore to expect a large number of students with weak writing skills in mathematics. In the case of this study, initial findings show that 67% (29 students out of forty three) were found to have unsatisfactory writing skills. And even those who were categorized otherwise, not all their work was satisfactory. Following the first exam, the writing exercises on the exam were given again as an optional take home assignment but this time my expectations as an instructor were made clearer (see Appendix). For instance in the first writing question, students were asked in particular to discuss the existence and uniqueness of solutions, their increase, decrease, their concavity, their long-term and asymptotic behavior. Such details showed to be important since they served as a scoring rubric for the learner. It is a well-known fact that scoring rubrics when shared with students provide a guide for the teachers' expectations. Unfortunately, only few people returned this assignment; however discussing it in class and the continuous emphasis of the skill by the instructor must have contributed to some enhancement of the writing skills on Exam 2 and on the Final Exam. As an indicator for this improvement, I present in particular the work of one student, Ali. Initially this student was categorized as having weak writing skills. His work on question 1 of Exam 1 (discussing in writing the solution of an initial value problem) and on question 2 of the same exam (matching odes with direction fields) was purely analytical (See Figures 2 and 3 below). Even though his answers might have been mathematically correct, however Ali neither described in question 1 the solution curve verbally nor discussed the uniqueness of the solution. In question 2, his answer was slightly better (e.g. *as x approaches 0 and 3, slopes get less steep*), however it did not rise to my expectations.

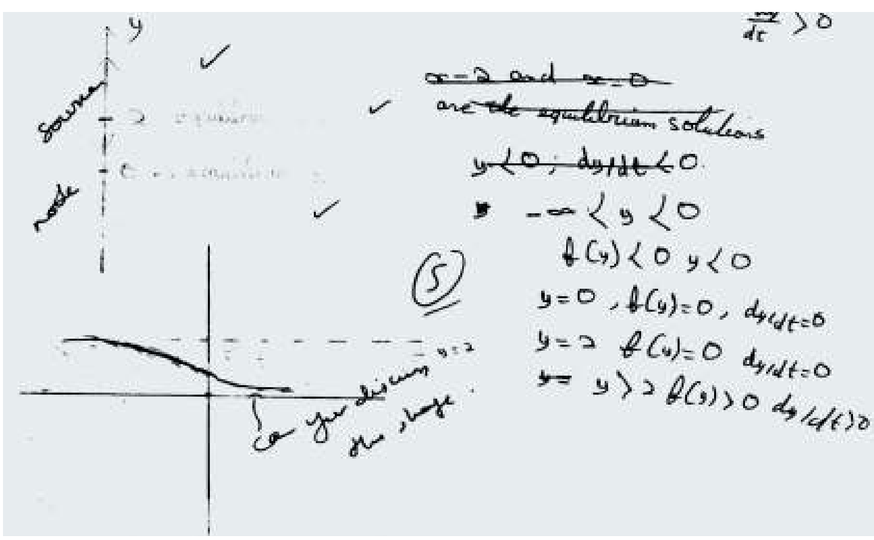

Figure 2: Answer to writing question 1-Exam 1(Ali)

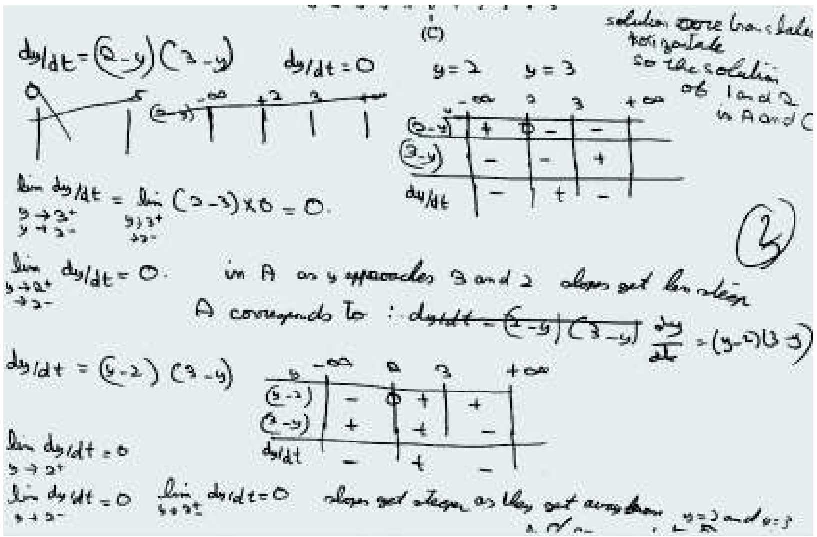

Figure 3: Answer to writing question 2 – Exam1 (Ali)

The answer of Ali to question 3 of Exam 1 (explaining the concept of an ordinary differential equation to laymen) was however better than others whose answers were purely quantitative, something that could be completely alien to an audience with minimal mathematical background. A statement like "*it relates a function to its rate of change*" is an indication of Ali's attempt to explain the real meaning of a differential equation. Such a stamen did not even exist in many of the other exam papers and students described an ode by describing say how to solve it in the case it is separable or linear.

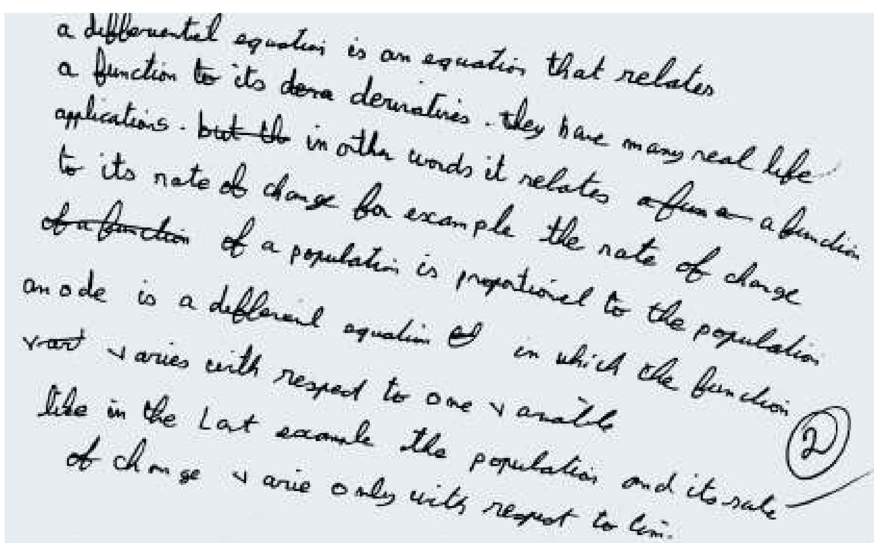
a differential equation is an equation that relates
a function to its ~~dera~~ derivatives . they have many real life
applications . ~~but th~~ in other words it relates ~~a fun a~~ a function
to its rate of change for example the rate of change
~~of a function~~ of a population is proportional to the population
an ode is a different equation ~~of~~ in which the function
~~var~~ varies with respect to one variable
like in the last example the population and its rate
of change varie only with respect to time.

Figure 3: Answer to writing question 3 – Exam 1 (Ali)

On the second and final exams however, Ali showed improvement. I present his work on the second exam in Figure 4 below.

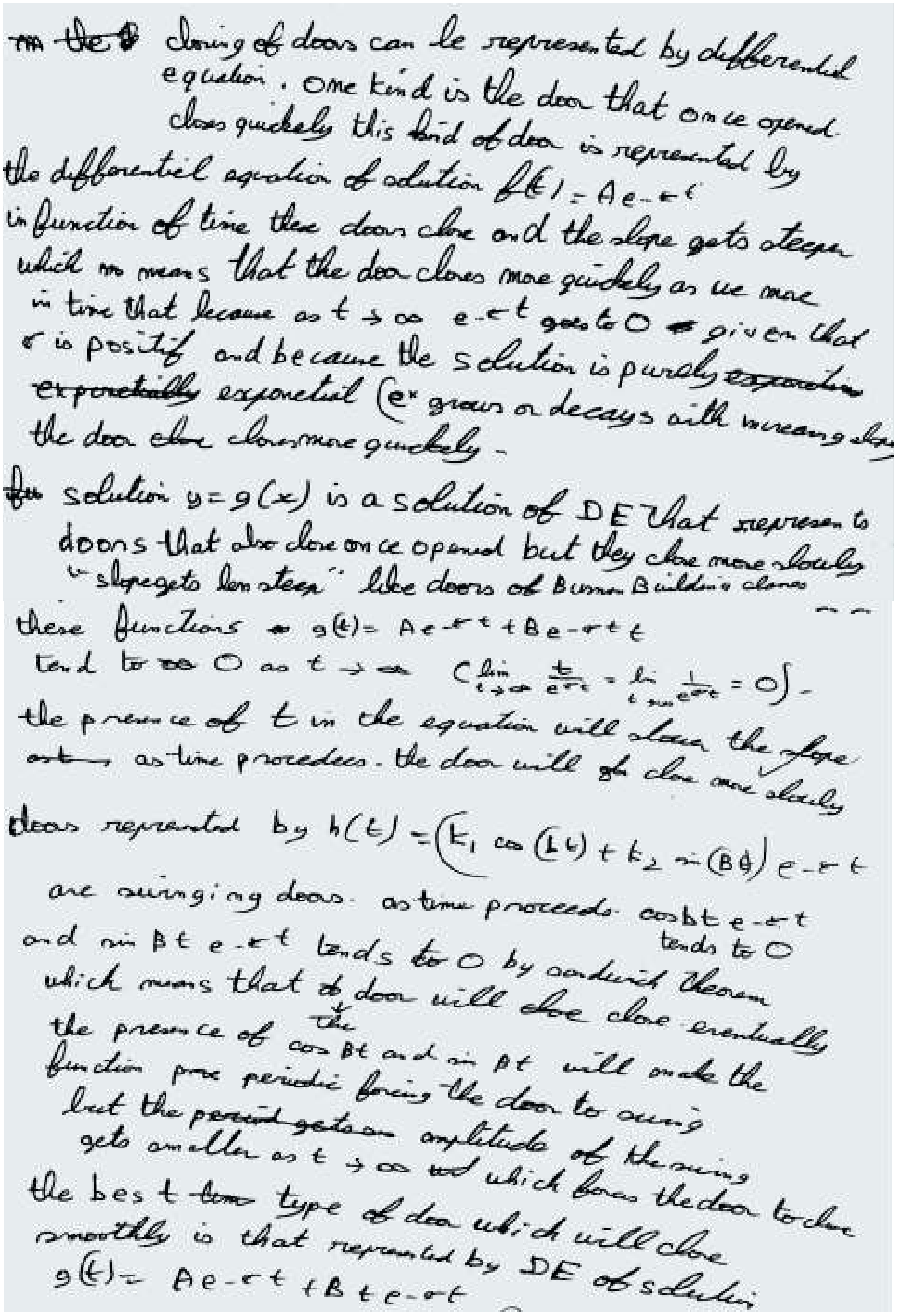
closing of doors can be represented by differential equation. One kind is the door that once opened closes quickely this kind of door is represented by
the differentiel equation of solution f(t) = Ae-rt
in function of time these doors close and the slope gets steeper which means that the door closes more quickely as we move in time that because as t → ∞ e-rt goes to 0 given that r is positif and because the solution is purely exponetial (e^x grows or decays with increasing slope) the door closes more quickely.
solution y = g(x) is a solution of DE that represents doors that also close once opened but they close more slowly "slope gets less steep" like doors of Busman Building closes
these functions g(t) = Ae-rt + Be-rt t
tend to 0 as t → ∞ (lim t→∞ t/e^rt = lim t→∞ 1/e^rt = 0).
the presence of t in the equation will slow the slope as time procedees. the door will close more slowly
doors represented by h(t) = (k1 cos(βt) + k2 sin(βt)) e-rt
are swinging doors. as time proceeds cos βt e-rt and sin βt e-rt tends to 0 by sandwich theorem
which means that the door will close eventually
the presence of cos βt and sin βt will make the function periodic forcing the door to swing
but the amplitude of the swing gets smaller as t → ∞ which forces the door to close
the best type of door which will close smoothly is that represented by DE of solution
g(t) = Ae-rt + Bte-rt

Figure 4: Answer to writing question on Exam 2 (Ali)

As for the general class performance, 10 additional students were classified as having acquired satisfactory writing skills so that almost 50% of the entire class ended up in this category. The improvement observed on the second and final exams such as Ali's is an evidence of the importance of conjoining the various mathematical modes of learning. In Ali's case for instance, not only his writing skills improved but also his "essay" on the writing question of exam 2 shows a deep mathematical understanding of the problem.

In conclusion, writing skills can be improved if clearly emphasized by the instructor as an important mean for communicating mathematical ideas and for intertwining these ideas. Better results may be achieved if the teachers' expectations are clearly spelled out in the writing assignments. In addition the inclusion of a writing component whenever appropriate may lead sometimes to a better understanding of concepts. Thus even though students initially view the idea of writing in mathematics very alien to them and may sometimes show resistance to it, yet with time many of them reap its benefits through learning more, learning better, and acquiring new skills.

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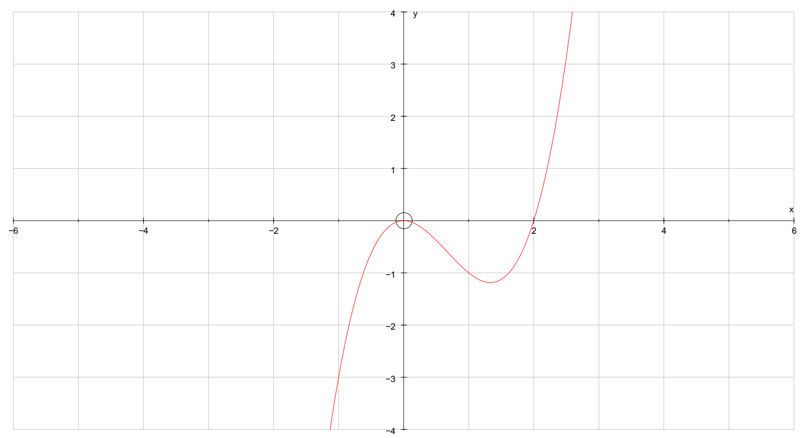
Appendix

Writing Question 1 on Exam 1

Consider the differential equation $\frac{dy}{dt} = f(y)$, *where the graph of f(y) takes the shape:*



Draw the phase line of this autonomous equation; identify its equilibrium solutions; classify them and then discuss the shape of the solution satisfying the initial condition $y(0) = 1$.

Writing Question 1 – Take Home Assignment

In addition to the question above, the following guidelines were provided to the students:

While discussing the shape of the solution satisfying the initial condition, emphasize the following (not necessarily in the order given below):

1. *Existence of the Solution*
2. *The increase and decrease of the solution*
3. *The concavity of the solution*
4. *The long-term behavior of the solution:*
    a. *In particular the asymptotic behavior*
    b. *The uniqueness of the solution*