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The effects of using Scratch software on students' learning of geometric problem solving

By

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To my loving wife, Hafiza, for her
devotion, support, and inspiration.

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The effects of using Scratch software on students' learning of geometric problem solving

Bassam Mohsen El-Hajj Ali

ABSTRACT

Researchers contend that one of the supreme purposes of mathematics education is to promote students' capabilities to solve an expanded scope of non-routine problems. Accordingly, mathematics education reformers continuously investigate for numerous approaches that augment the student's capability to be a highly effectual problem solver. The current study examines the potency of using Scratch software as a tool to reinforce students' problem-solving skills that form a factor out of four factors of the study. The study also strives for disposing the effect of utilizing such software on their knowledge, conceptual understanding, and cognitive skills, which form the other three factors of the study. This study is performed by using a mixed-method research design that advances systematic integrating quantitative and qualitative data collection and analysis. The study comprises of two groups of grade-6 students, a control group and an experimental group. The control group was exposed to the usual models of teaching, while the teaching model used in the experimental group integrated a set of Scratch-based learning materials that were designed according to the constructionist discipline's viewpoint of Scratch. Regarding this matter, two four-lesson unit plans were designed for the two groups. A pretest and posttest were conducted to both groups to resolve the scale of advancement succeeding the study. Further, clinical interviews were administered with a selected sample of three students of different levels of achievement from each group. The pretest and posttest executed a quantitative quasi-experimental analysis, while, to a lesser extent, the clinical interviews carried out a quantitative analysis. All the analyses were assigned to examine the four factors of the study. The study showed that students who were experienced the Scratch-based activities appeared to exhibit more progress than students who were exposed to the usual methods of teaching. The study concludes with recommendations for improving the quality of problem solving skills with the use of basic programming languages like Scratch for future studies, with a larger target sample and more time dedicated for the accomplishment.

Keywords: Scratch, Problem Solving, Knowledge, Conceptual Understanding, Cognitive Skills

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Chapter One

Introduction

1.1 Recent use of technology at schools

Today's learners were born in the age of the technology. In recent years, computers have become available in schools. Integrating technology with education means enriching the students' technical skills along with their learning opportunities. With this growth, comes the opportunity for regular lessons to become more significant through using the accessible technology in order to improve student learning and to settle the proficiencies for the 21st-century information and communication technology skills.

Although technology in the classroom is no longer new and has been used extensively before, it can now enable new approaches such as blended learning and flipped classroom using online contents. Blended learning amalgamates online digital media with traditional classroom approaches. Flipped classroom is the reversed instructional strategy of the traditional learning environment by supplying online content prior to class discussion. Furthermore, with nowadays technology, learning is assessed using computerized testing. E-assessment is the result of using information technology in several forms of assessment that are becoming more interactive using more sophisticated systems and moving beyond multiple-choice questions. These experiences are reshaping the classroom experience for students and helping to develop their abilities in many ways.

1.2 Historical background of using technology in learning

Calculators, computers, and software have been used in math classes for decades. Desktop computers and graphing calculators were first used in the 1980s. Famous mathematics software, such as Maple, MATLAB, Mathematica, The Geometer's Sketchpad, and Cabri were released in the 1980s. Web-based programs came on board in the twentieth century. Portable electronic devices such as tablets and smartphones emerged in the last decade of the 21^{st} century. Hence, Greenwald and Thomley (2013) observe that these historical activities show that mathematics is a living discipline. Recently, technology is growing at an accelerated pace all around the world and in the future. For example, the Economist Intelligence Unit (2008) examines the role of technology in crystalizing the future of higher education and finds that technology will continue to have a substantial impact on higher education that is responding to globalization and online learning.

Higher education starts experiencing a shift from classroom to distance learning (Akbar, 2016). Currently, technological simulation is growing promptly in many different fields and throughout the world (Gaba & Raemer, 2007). Simulations have been used in many areas of higher education. For example, simulation technology is now regularly integrated into the curriculum of medical higher education (Ahalt & Fecho, 2015). Simulation-based medical education is used most vigorously and effectively to achieve learning objectives along with other educational methods (McGaghie, Issenberg, Petrusa, & Scalese, 2010).

Moreover, virtual reality is growing expeditiously. The importance of using the virtual reality is growing rapidly in the learning framework (Neves & Duarte, 2016). From their research, Mihalca and Miclea (2007) foresee that educational technology is shifting from learning machines to technologies of virtual reality.

However, the positive social change will not occur before the next two decades because of economic, educational, and cultural barriers (Facer, 2011).

1.3 Programming in education

One of the current movements in the educational outlook is the introduction of computer programming in the classroom to develop problem solving in students. Researchers affirm that programming guides to problem solving by improving computational thinking (Lye & Koh, 2014). Wing (2014) defines computational thinking as the set of “thought processes involved in formulating problems and their solutions for the purpose of representing solutions in a form that can be effectively carried out by an information-processing agent” (p.1). Passey (2016) describes that set of processes as being involved fundamentally with problem solving processes. He considers computational thinking as the core of learning. Computational methods and models encourage learners to solve problems and design systems (Wing, 2006). Magana, Marepalli, and Clark (2011) call for creating educational experiences and instructional methods to make a combination of computational thinking and engineering as tools to help students understand and solve realistic problems.

1.4 STEM and problem solving

STEM refers to science, technology, engineering, and mathematics. Jurdak (2016) describes STEM as providing opportunities for real-world problem solving in mathematics. He explains that these opportunities are implanted in mathematics curricula and included in the interaction between mathematics and other STEM disciplines. In STEM education, problem solving is a centripetal object. Sanders

(2009) considers that problem-based learning situates mathematics in the context of technological design and problem solving. Moreover, Capraro, Capraro, and Morgan (2013) deliberate that problem-based learning is a basis of STEM project-based learning.

1.5 Problem solving in today's curriculum

Problem solving is an indispensable skill needed by today's learners. Since problem-solving skill is an important factor that researchers emphasize nowadays, there is a vigorous movement in education to integrate it as a fundamental component of the curriculum. Mathematical problem solving represents tasks that have the prospect to afford thoughtful challenges to build up students' mathematical understanding and development (National Council of Teachers of Mathematics, 1980). Problem-solving abilities are an important part of mathematics development that connects it to life. Supporting mathematical development can become part of the everyday routine (Clements & Sarama, 2005). Pólya (1957) identifies four basic steps of problem solving: understand the problem, devise a plan, carry out the plan, and look back.

When referring the term "problem solving" to mathematics, Cai and Lester (2010) argue that the tasks themselves have the potential to afford intellectual challenges to students in order to let them understand mathematics and develop mathematically.

1.6 Scratch software's environment

One of the software tools that could help enhance problem-solving abilities in mathematics is Scratch. Scratch is a visual programming language software that is obtainable for Windows, Mac OS, and Linux. It is used around the world by students from schools, clubs, or home. It enables them to learn coding to create animations and interactive games through graphical drag-and-drop scripts.

The prominent merit of Scratch is that the user uses a computer mouse for dragging and composing objects instead of typing commands through a computer keyboard. Kalelioğlu and Gülbahar (2014) adopt the idea that since programming the procedures is dependent on constructing blocks of commands instead of writing them, Scratch could be more helpful to students.

The most noticeable feature of Scratch is that one can simply create complex computer programs by snapping together visual programming blocks, which is similar to putting LEGO pieces or jigsaw puzzles together (Lee, 2011).

The interface for the Scratch environment (Fig. 1) divides the screen into several panes: on the left is the Block Palette, in the middle is the Script Area, and on the right, there are the Stage and Sprite List. Figure 1 presents a typical Scratch screen with its different parts.

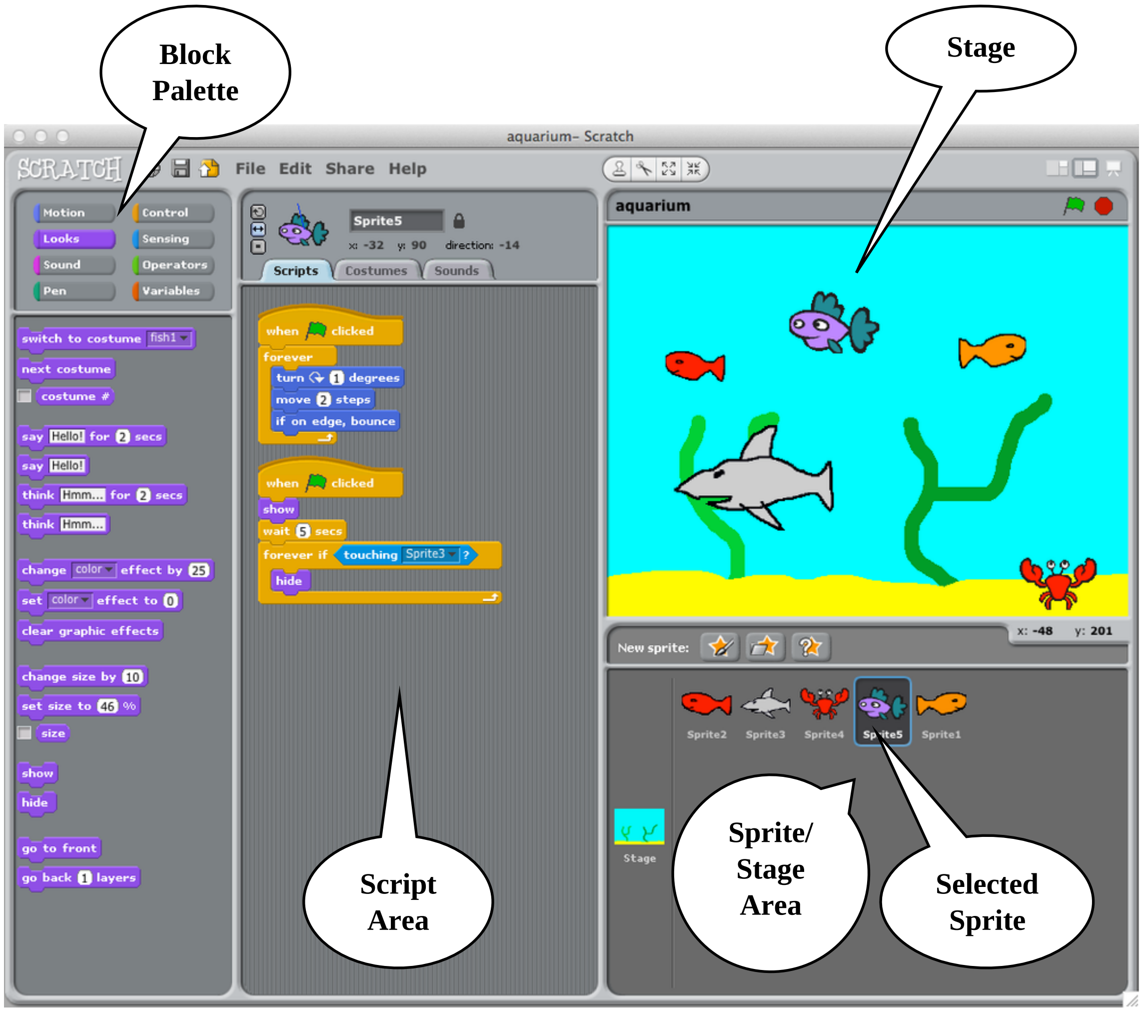

Fig. 1 *Typical Scratch screen with its different parts*

A *sprite* is a two-dimensional graphic image that performs actions. It is found in the *Sprites Library*, uploaded from a file, or created by the user. It can move, sing, dance, and change its appearance. Each sprite can encompass its own set of images, sounds, variables, and scripts. Figure 2 presents a Sprite. This one is the cat sprite that appears by default until the user changes it.

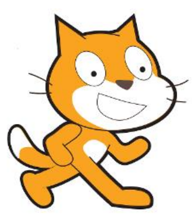

Fig. 2 *The cat sprite that appears by default until the user changes it.*

A *Stage* (Fig. 3) is the backdrop of all programs. It can display different background images with appropriate sounds. When a sprite is created, it will be placed in a random location on the Stage, usually around the center. The stage has two buttons: *Green Flag* that starts the project and *Stop Sign* that stops running all scripts.

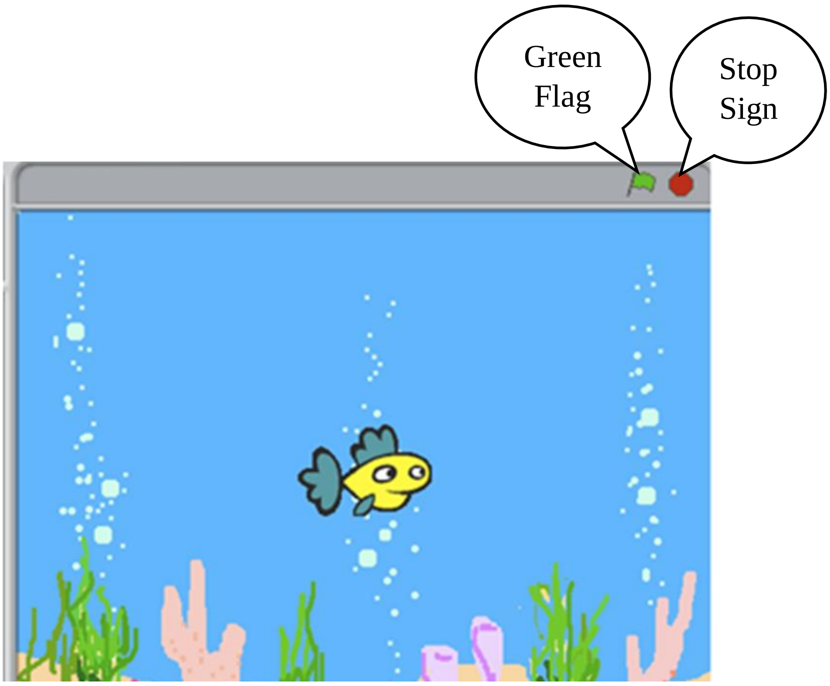

Fig. 3 *Stage with its two main buttons and a fish sprite*

The *block palette* (Fig. 4) offers eight categories of color-coded blocks so that one chooses the appropriate ones and assembles them to construct a script. The eight block categories are: *Motion*, *Looks*, *Sound*, *Pen*, *Control*, *Sensing*, *Operators*, and *Variables*.

Fig. 4 *Block palette*

The *Motion* blocks deal with the movement of sprites. The *Looks* blocks are related to the appearance of sprites and the stage. The *Sound* blocks are related to playing several sounds. Pen blocks draw a line correspondingly with a sprite's location. They can be turned on and off. The *Control* blocks run the project in the desired way. The *Sensing* blocks detect conditions. The *Operators* blocks deal with mathematical functions. The *Variables* blocks store values.

A *Script* (Fig. 5) is a set of blocks that are created by dragging blocks from the palette to make stacks of blocks that all interlock with one another.

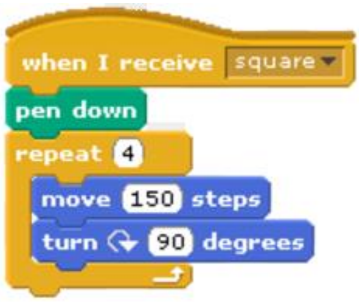

Fig. 5 *A script that is a collection or stack of blocks*

Multiple stacks of blocks make a code. A script can be associated with either the Sprite or Stage area to govern the Sprite or Stage's behavior, such as motion, appearance, and sound.

A *script area* (Fig. 6) is where scripts can be assembled. Blocks are dragged onto the script area to make programs. These then can be combined with other blocks to form scripts, and eventually a project.

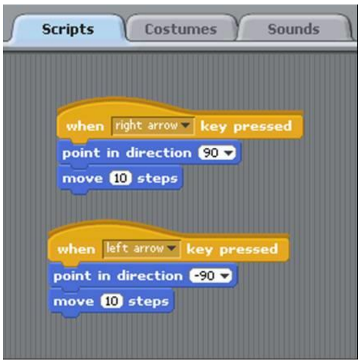

Fig. 6 *Script Area where blocks from the Block Palette are dragged*

1.7 Mathematical skills in Scratch

Many studies and research have been published for educational effects of Scratch programming on various mathematical skills. The developers of this software claim that it enables learners to think creatively, reason systematically, and work collaboratively (n.d.). Lifelong Kindergarten Group at the MIT Media Lab in Cambridge, Massachusetts (n.d.), who manufactured Scratch, argues that the software "supports critical thinking and problem solving through the creative process when students create, enhance, or troubleshoot their projects" (para. 2). Ornelas-Marques and Marques (2012) point that, when students use a programming language like Scratch, they soon face problems that they immediately have to solve in order to advance the project. The authors add that students learn how to properly use a modern basic tool when they use programming languages like Scratch.

1.8 Scratch and constructionism

Scratch designers claim that students can learn in a constructionist approach, which supports their essential skills such as problem solving (n.d.). The constructionist approach is based on the constructivist theory that expresses the idea that one's mind actively constructs mental structures and operations rather than passively acquiring them (Riegler, 2012). Constructionism is an epistemological pedagogical paradigm first established by the mathematician Seymour Papert (Kynigos, 2012) from 1985 and 1990 when he was working with MIT media Lab. The theory was firstly elaborated in combination with the digital media and mathematics. In his book: Mindstorms: Children, Computers, and Powerful Ideas that was published in 1980, Papert offers his vision of education for the future. In his book, he predicted that technology would play an essential role in education as children would face new possibilities for learning, thinking, and developing their emotions and cognitions (Papert, 1980). In the early 1980s, Papert and his team developed the Logo programming language and then tens of thousands of elementary classrooms throughout the United States started using the software (Chakraborty, Graebner, & Stocky, 1999). This project tells us how technology can provide new ways to learn and teach mathematics.

Papert considers that both constructivism and constructionism share building knowledge structures in learning, but constructionism happens when the learner is engaged in constructing a public entity with a full intention (Papert, 1991). Although constructionism advocates student-centered learning, sharing ideas with the community is important in constructionism. Constructionists provide chances for learners to plan and construct shareable artifacts (Boyer, 2010). Constructionism is established on the idea that learners can learn more efficiently when they make significant artifacts and share their ideas and products with others (Chung, 2012). Constructionism attributes equal importance to the individual learner and to the role of social contribution (Peppler & Kafai, 2007).

1.9 Research Purpose

The research purpose of this study is to investigate the effect of using Scratch software on grade-6 students' geometric problem-solving skills.

1.10 Research Questions

This study adopts the following two research questions:

1. How does Scratch programming improve grade-6 students' problem-solving skills?
2. How does Scratch strengthen students' geometric knowledge, conceptual understanding, and cognitive skills?

In order to answer the above two mentioned research questions, a mixed-method design is adopted. It uses quantitative and qualitative data collection and analysis methods. For the quantitative method, the quasi-experimental research design was used in which an intervention, through using Scratch software, is administered to only one of two groups of six graders, in which one group is the experimental, and the other is the control group. A written pretest is to be conducted to both groups to determine the starting level of each group. For the qualitative part, clinical interviews will be administered with a sample of three students of different levels of achievement from each group. This qualitative study is performed for a deeper analysis. One problem will be given to the six students before the intervention and another problem will be given to them after the intervention. At the end of the study, a written posttest is also to be used to analyze the students' problem-solving skills and determine the achievement levels of both groups. The coming Chapter 2 discusses in more details the theoretical background and framework on which the current study is based. Furthermore, a description of the method is used while conducting this study in details in Chapter 3. An in-depth analysis of both the quantitative and qualitative data is offered in Chapter 4. Finally, an inclusive conclusion is discussed in Chapter 5.

Chapter Two

Literature Review and Historical Framework

2.1 Introducing the current chapter

Recent researchers' expectations emphasize on aligning teaching mathematics to students and using problem-solving skills since problem solving has a significant affective factor. Affective aspects of problem solving relate to the influence of emotions during problem solving. The affective domain is seen as an important factor leading to successfulness in mathematics (Knotra, 2001). Positive affective factors arise when a student has a sense of confidence related to his or her problem solving ability, has expectations of success, and has the capabilities to face the difficulties (Grattoni, 2007). Researchers focus on teaching mathematical topics through problem-solving contexts and dynamic environments. However, they appeal to build a bridge between vision and practice.

The current study focuses on the uses of dynamic software, specifically Scratch, as a tool for empowering teaching mathematical problem solving. The current chapter provides a general view about the use of authentic problem solving. It offers a wide spectrum about the definition of a problem, explains the real-world problem solving comparing between authentic and non-authentic and routine and non-routine types of problem solving, and explores a link between problem solving and mathematical proficiency, technology, and information visualization. Also, the review introduces Scratch as a tool for enhancing mathematical thinking, game construction, and programming.

2.2 What is a problem?

Authentic problem solving demands a problem that is non routine. This problem is unfamiliar and challenging in such a way that it is just out of reach of the student's knowledge and skills. The solver will not spontaneously find the problem's solution and should use his/her prior knowledge and skills. Further, an authentic problem in not provided by only the teacher nor by the school tasks. In this perspective, students have to be given the opportunity to engage in real-life problem solving activities that occur in everyday situations. In addition, students have to be productive as to generate problems rather than receive problems to solve. Problem solving is the process of working on a problem to reach a solution for it. Problem solving involves disabling the obstacles that prevent achieving the undetermined goal. Martinez (1998) defines problem solving as "the process of moving toward a goal when the path to that goal is uncertain" (p. 605).

Nevertheless, researchers give various definitions for the term problem. Resnick and Glaser (1976) define a problem as being something that someone has no experience to solve. Mella (2012) defines a problem as any element that impedes a process aimed at achieving an objective of change or at restoring the optimal situation. Posamentier and Krulik (2009) define a problem as a situation that confronts the learner, that requires resolution, and for which the path to the answer is not immediately known. Heppner and Krauskopf (1987) define a problem as a situation in which one may respond in connection with his/her internal or external demands.

Researchers agree that there should be a goal to be achieved in a problem; however, they differ in providing definitions to a problem, as related to that goal. To begin with, Bingham (1958) defines a problem as an interruption that blocks one's control to achieve an anticipated goal. Mayer and Hegarty (1996) define a problem as the transition between a "given state" and a "goal state" when one does not immediately know how to achieve the latter. Chi and Glaser (1985) define a problem as a situation in which someone is trying to reach a goal with a must to find a means to get there. In addition, Kilpatrick (1985) defines a problem "as a situation in which a goal is to be attained and a direct route to the goal is blocked." (p. 2). Finally, Holden, Rivera-Rodriguez, Faye, Scanlon, and Karsh (2013) define a problem as "any occurrence or state that makes goal accomplishment impossible, difficult, or unsatisfying in light of standards for timely and effective performance." (p. 284).

2.3 Real-world problem solving

Problem solving is an important aspect of mathematics (Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016). Over the years, problem solving has been developed as one of the major processes at all levels of school mathematics. Furthermore, problem solving has been a fundamental part of mathematics all through history. Approvingly, Jonassen (2010) reflects that problem solving provides the most essential learning outcomes in most perspectives. Problem solving is the most important of all education goals especially when increasing its higher cognitive ability (Tsai, 2010). Problem solving is not only important in schools, but also in universities. Problem solving is the most useful skill a student can gross when leaving higher education (Rowlett, 2011). In mathematics, researchers consider problem solving as the main element of such discipline. For example, Aydoğdu and Ayaz, (2008) highlight that problem solving is an important component of mathematics education. In addition, problem solving is the ultimate goal in a mathematics (O'Hara, 2015). Others go far when they maintain that problem solving is the heart of mathematics, such as Halmos (1980), Ben-Hur (2006), and Nool (2012).

In fact, professional organizations acknowledge the importance of problem solving. For example, the National Council of Supervisors of Mathematics (1977) indicates in its position papers that solving problems is the major purpose of studying mathematics. Similarly, in its Agenda for Action, the National Council of Teachers of Mathematics (1980) points out that problem solving should be the focus of school mathematics.

Educators discuss the importance of real-world problem solving. Blum and Niss (1991) identify two kinds of mathematical problems: 1) the applied mathematical problems in which the situation and questions belong to the real world, and 2) the pure mathematical problems, which are embedded exclusively in mathematics. In the real world, a problem may exemplify a complex situation whose solution does not have a precise and clear meaning (Christakos, 2011). Problems are usually complex, and solving them needs an expanded dimension of knowledge and experience (Hoffmann, n.d.). Similarly, according to Blum and Niss (1991), a problem is a situation that has certain open questions where a solver has no instant methods, procedures, or algorithms sufficient to solve those questions. However, lack of any experience to solve complex real problems makes it impossible for students to solve real problems with high level of difficulty (Maasz & O’Donoghue, 2011). Jurdak (2016) points out that problem solving in the real world is accredited to situations which necessitate decision making. In assessing such decision making, one needs to know what available options problem solvers have in their hands (Schoenfeld, 1992). The skill of decision making is not characterized only by development and socialization, but also as a life-time process (Srimadevi &

Saraladevi, 2016). Jurdak (2006) identifies situated problem solving as it relates to a situation where the student is involved in solving real-world problems. He considers that one remarkable feature of situated problem solving is that it simulates real-world problem solving as a meaningful mathematical activity. Such activity starts with phenomena that are real to students (Freudenthal, 1983). The mathematical activity of learning real-world problem solving is viewed as a process in which the students are motivated to get involved in actions (Jurdak, 2016). Typically, building a bridge between real-world problem solving and math-problem solving may awake students' interests to capacitate their motivation (Boaler, 1994).

2.4 Authentic and non-authentic problems

Real-world problems are authentic situations that deal with real data and model phenomena. They are probable to have multiple solutions. In addition, while working on real-world problems, finding solutions for the problems is not just required but also showing reasoning for the solution while communicating mathematically (karakoç, 2012). In contrast, non-real-world problems include only mathematical notations, formulas, and phrases rather than statements about reality. Solutions are deduced from data. Researchers differentiate real and non-real-world problem solving. However, they contradict each other in prejudicing in favor of opposite approaches. For example, Cohen, Whitmeyer, and Funk (1960) suggest that problem solving performance improves only on real-world problems. On the other hand, Rickards (1975) suggest that creative training for students is expected to be beneficial only on non-real-world problems. Further, using real-world problem solving improves mathematical literacy (Bokar, 2013), spatial thinking (Gauvain, 1993), and the physical and the psychological advantage (Willis, 1996) of the problem solver in everyday meaningful contexts. On the other hand, non-real-world problems help develop formal reasoning abilities (Toom, 2010). Substantially, Matney (2004) submitted real-world and non-real-world mathematics classroom activities to his students. He found that his students flavored the non-real-world activities because they did not find the real-world problems beneficial, worthwhile, or purposive. That is why a real-world problem requires a high level of interdisciplinarity (Goyette, 2016).

In the current research, triangles were the main part of the unit that the research participants learned. Triangles are used to make triangular structures. For example, old Egyptians built triangular-shaped pyramids. Triangulation is used to measure distances. In particular, right-angled triangles are used to solve real-world distance problems. Some vocational jobs such as carpentering require a right-angled triangle to take measurements. Notably, a common real-life two-dimensional example of a right-angled triangle would be a ladder inclined against a wall.

2.5 Routine and non-routine problem solving

Researchers differentiate routine and non-routine problem solving. In the routine type, the problem solver understands the given and the goal stated and has almost a ready way to move from one to the other. In the non-routine type, the problem and the procedure are neither known nor familiar. For example, Pólya (1957) considers that the routine type “can be solved either by substituting special data into a formerly solved general problem, or by following step by step, without any trace of originality, some well-worn conspicuous example.” (p. 171). To him, the non-routine problem encourages creativity and originality. Additionally, English and

Sriraman (2010) describe the routine type as the problems that require an application of a standard computational procedure. They explain that the non-routine type involves getting from a given to a goal when the path is not evident.

2.6 Problem solving and mathematical proficiency

Milgram (2007) describes that mathematics comprises three key factors: precision, stages, and problem solving. He explains precision as definitions of terms, operations, and properties of operations, which all to be precisely used. He explains a stage as a category that has objects and an appropriate structure to do mathematics. For example, integers build a stage, patterns build another stage, and geometry plays out on another stage. Each stage has its own tools. To Milgram (2007), a problem is "where every term is precisely understood in the context of a single stage." (p. 8). He considers that the mindfulness of these three factors and the ways in which they interact are the essential parts of mathematical proficiency. Improving proficiency in mathematics is an essential goal for all students (Monica, 2003). Mathematical proficiency has four aspects according to Schoenfeld (2007):

1) knowledge base as students should be able to use their mathematical knowledge.

2) strategies that students may use in problem-solving.

3) metacognition. He explains this as "taking one's thinking as an object of inquiry." (p. 8). He gives an example that while getting involved in problem solving, one might reflect on progress and act accordingly. That is what as he describes *monitoring* and *self-regulation*.

and 4) beliefs and dispositions.

The National Research Council (NRC) assigned a committee under the supervision of Kilpatrick, Swafford, and Findell (2001) to conduct a research about teaching for mathematical proficiency. They published a report that addressed the latest educational, psychological, and neurological research (Civil & Turner, 2014). The committee discussed how teachers and curricula should change to improve mathematics learning (Suh, 2007). The committee outlined five strands that together guide effective mathematics learning. The five strands identify mathematical proficiency and describe how students develop this proficiency. The five strands are: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. These strands are not separate but basically connected with each other (Bass, 2003).

The committee considers that students with *conceptual understanding* deal with more than isolated facts or methods because they can organize their knowledge in a “coherent whole” that enables them to connect ideas with each other. The committee explains that a substantial indicator that shows that a student possesses conceptual understanding is when he/she can represent mathematical situations in different ways.

The committee also considers that students with *procedural fluency* know when and how to use the procedures appropriately, flexibly, accurately, and efficiently. Moreover, students with *strategic competence* can formulate and solve mathematical problems. The students also need to represent the problem situations. This requires building a mental image of the problem’s important components, understanding the situation including its main features, and generating its mathematical representation through ignoring its irrelevant features.

In addition, the committee speculates that students with *adaptive reasoning* are capable of thinking rationally about relationships among the situations and concepts, and capable of justifying the conclusions. In this case, students know that their answers are right when they follow the assumptions logically and know that their answers are wrong when they are certain that they are invalid by reasoning.

Finally, the committee declares that students with *productive disposition* tend to see sense in mathematics, to recognize it as both useful and valuable, to believe that firm struggle in learning mathematics has a profit, and to see oneself as an effective learner of mathematics.

2.7 Mathematical proficiency and technology

Various instructional strategies help students develop their mathematical proficiency (Gray, 2014). For example, paying attention to students' discussions of their processes and findings can help develop their mathematical proficiency (Kastberg & Frye, 2013).

Lumpkin, Achen, and Dodd (2015) conducted a research project to examine the effect of using of technology-based instructional strategies on student perceptions of their learning. They found that students engage positively in learning when visual and interactive instructional strategies are incorporated in their classes. Examples of these strategies are PowerPoint presentations, online blogs, response systems, video clips, and interactive review games that helped students in their self-assessment and revision for their exams.

Riccomini, Smith, Hughes, and Fries (2015) discuss the impact of mathematical vocabulary in classroom learning on specific evidence-supported instructional strategies and proficiency. They arrive at a conclusion that when students use mathematical vocabulary properly, they will be able to explain, justify, and communicate mathematically on the road to develop their mathematical proficiency. They call for using technology applications to improve instruction of such mathematical vocabulary by providing visual and auditory catalysts and interactive simulations.

Styers (2012) conducted a research based on math instructional strategies and student assessment. She shows that such strategies can support math learning and can approvingly improve student math outcomes. Additionally, she indicates that incorporating technology and using dynamic content for assessment would help as a positive assistance for students when they review automated generated test items. These items let students overlook and use different questions of the same content.

2.8 Technology and mathematical problem solving

Computer technology is prominently shaping the way we perceive mathematical problem solving in education due to the multi-representational and dynamical nature of digital tools.

Carreira (2015) points out that recent study about problem solving take into account the solver's use of technological tools. For example, Barrera-Mora and Reyes-Rodríguez (2013) discuss how students' use of technological tools in problem solving activities can help them develop their mathematical competences. They declare that the use of technological tools in learning activities encourages students to pay attention on the structural aspects of problem solving.

In their study, Santos, Agüero, Borbón, and Páez (2003) described what students showed when they were asked to use technological tools when they were solving routine problems from different perspectives. The researchers noticed that, with the use of technology, students' ideas were enhanced when they searched for different ways to solve the problems. Those problems became as a powerful tool for students to reflect on their mathematical thinking.

Furthermore, Kuzle (2012) conducted a study to investigate how teachers work in a dynamic geometry environment and how such experiences influence mathematical activities incorporating non-routine problems. Her results drove her to encourage the teachers to be more attentive to the pedagogical and cognitive inferences of technology and to consider that technology is an influential teaching tool that may effect on mathematical problem solving, teaching, and learning. She recommends that teachers should have experience in technology problem solving before getting into service.

In addition, Chang (2013) conducted a study to investigate the effects of online creative problem-solving activities on student technological creativity. She found that the students who used the online creative problem-solving activities showed superior results compared to students who did not use them especially in the analytic thinking domain rather than the synthetic thinking domain. Those students who involved in creative problem solving used metaphor and analogy to produce new ideas and most of them reached solutions by discussing, interacting, and regulating others' thoughts.

In their study, Hurme and Jarvela (2005) find that working with technology allows students to use their mathematical knowledge and stimulates them to visualize their thinking.

Santos-Trigo, Reyes-Martínez, and Aguilar-Magallón (2015) show that students working with technology incessantly reflect their reasoning strategies about concepts and problems through investigating mathematical relationships. The authors point out that digital technologies maybe used in all problem-solving processes that include "problem formulation, comprehension, representation, exploration, generalization, and communication of results." (p. 300). Moreover, they specify that mathematical tasks play a key role for students to develop their awareness of technology as a means to grasp and improve problem-solving abilities. For this viewpoint, they provide examples: a) Students look for information online. In this manner, they depend on strategies and methods that help them select, analyze, and summarize the found information. b) Students learn and use a number of ways to work on problems. In this process, they share and discuss ideas within a community where it is important for them to listen to others and discuss partial results. c) Students represent, explore, solve, and communicate their results using a variety of digital tools. Particularly, they construct and analyze dynamic models of problems, so that they are stimulated to discover creative solutions to problems.

Jones, Geraniou, and Tiropanis (2013) consider that computer technology is changing the approaches that young learners are using to solve and communicate mathematically. They explain that networked computer-based tools have progressively provided technologies for students (such as blogs, wikis, social bookmarking, etc.) to collaborate using visual representations of mathematics, both algebraic and geometric. Additionally, the authors clarify that students chat and share files to organize their collaborative problem-solving activities using digital text, drawings, and mathematical symbols.

Furthermore, technology helps in differentiating instruction and individualizing learning. For example, using databases of problems may provide problems that are suitable to students with different levels of performance. In this perspective, if a student is having a difficulty with a specific type of problems, it will be useful to suggest similar problems on a suitable level of difficulty. Further, the authors add that it will be possible to propose extra problems that entail different argumentation patterns.

Working with technology during problem-solving tasks may save time for teachers and students. Li, Leigh, and Wills (1997) argue that as students use computer algebra systems, they can spare time that used to be lost on solving routine problems, sketching useless graphs, and operating algebraic treatment. On the other hand, students can spend more time on understanding concepts and applying them to solve realistic and challenging problems. Tawfik, Tawfik, and Shahrabi (2008) conducted a research and concluded that computer instructional programming model helped students save their time as they performed their analyses and achieved correct answers with high standards of critical thinking and problem solving skills. By reducing time spent on learning computational algorithms, more time will be accessible to spend on developing and refining problem solving abilities (Pomerantz, 1997).

There are several processes involved in problem solving such as: analysis, design, exploration, implementation, and verification (Schoenfeld, 1985). However, the use of technology affects those processes (Hähkiöniemi, Leppäaho, & Francisco, 2012). For example, Healy and Hoyles (2001) describe that the use of technology improves thoughtfulness and understanding of the problem. Moreover, they show how technology can, not only scaffold the solution processes, but also help learners shift from argumentation to logical deduction.

2.9 Information visualization and problem solving

Digital tools support the visual learning of mathematics (Martinovic, Freiman, & Karadag, 2013). Zimmermann and Cunningham (1991) mention that visualization in learning mathematics contributes to the development of depth and meaning to understanding, serves as a credible index to problem solving, and instigates creative discoveries. Information visualization techniques improve and reinforce learning and investigation of mathematical concepts (Sedig, Rowhani, Morey, & Liang, 2003). In addition, information visualization can permit the student to access mathematical knowledge that produces learning based on doing, touching, moving, and seeing the problem solving processes (Chiappini & Bottino, 1999). Furthermore, an effective information visualization allows for a diversity of different problem solving strategies (Mayr, Smuc, Risku, Aigner, Bertone, Lammarsch, & Miksch, 2010).

2.10 Scratch as a tool for enhancing mathematical thinking

Scratch is a free software. It is easily downloadable from its website, but it can also be used online. It is one of the newer visual programming environments for teaching and learning. It is widely used in primary schools in many countries for introductory programming. In May 2007, the research group at the MIT Media Lab launched the Scratch programming language and online community. Since then, studies on Scratch were conducted by educators (Malan & Leitner (2007); Peppler & Kafai, (2007); De Kereki, (2008); Calder (2010); Lee (2011); Kaučič & Asič (2011); Tsai & Chen (2011); Ferrer-Mico, Prats-Fernàndez, & Redo-Sanchez (2012); Ornelas-Marques & Marques (2012); Wilson, Hainey, & Connolly (2013); Calao, Moreno-Le´on, Correa, & Robles (2015); Korkmaz (2016)).

Resnick (2013), the creator of Scratch, argues that learners gain, through using Scratch, important mathematical and computational ideas.

Brown, Mongan, Kusic, Garbine, Fromm, and Fontecchio (2013) conducted a research to assess the problem-solving skills of students who use Scratch. They showed that when students learn using Scratch, there is an improvement in the students' mathematical reasoning and problem-solving abilities. Shin and Park (2014) conducted a study to check the effect of mathematical logic solving with Scratch on problem-solving ability. They found positive effects on divergent thinking, decision making, and planning ability of students' problem-solving capacity. On the other hand, Kalelioğlu and Gülbahar (2014) conducted a study to investigate the effect of Scratch on students' problem-solving skills. They found that "students have to be supported with different activities and applications that require high-order thinking in order to help students develop problem-solving skills." (p.47). Correspondingly, they found improvement in the students' self-confidence in their problem-solving ability. Congruently, Giordano and Maiorana (2014) conducted a study on visual programming languages, such as Scratch, and they found that it improved students' problem-solving skills and reasoning practices.

2.11 Mathematical thinking in Scratch

Calder (2010) examined the ways mathematical thinking developed when students worked with Scratch for the first time they experienced it. He trained them in designing games, which cultivated students' spatial awareness. In his experiment, Calder (2010) found that Scratch facilitated students' understanding of angles and measurement. It also allowed them to make decisions and explore what was appropriate for their particular purpose. For example, errors with programming had a positive effect on students' learning as they used logical thinking to understand the errors and made more trials to accomplish the preferred movement. Calder (2010) noticed that students used logic and reasoning to interpret and assess situations. He arrived at conclusions that students used mathematical thinking in their approach to problem solving. Thus, their relational thinking developed through the problem-solving processes in Scratch as they made links between their input, the actions that appeared on screen, and the effect of specific procedures. Additionally, he assured that Scratch was an effective tool for encouraging communication and collaboration among students who were cooperatively helping each other to solve programming problems. Furthermore, he proved that Scratch was a helpful tool for encouraging the use of critical, meta-cognitive, and reflective skills.

Investigating the influence of coding on mathematical skills, Calao, Moreno-León, Correa, and Robles (2015) conducted a research to test whether the use of coding, using Scratch in math classes, could have a positive effect on students' mathematical skills. They showed that understanding of mathematical knowledge and processes increased in students who received training in Scratch. They also concluded that the development of computational thinking using Scratch allowed students to improve their performance regarding mathematical processes of modeling, reasoning, and problem solving. In parallel, they also found that Scratch assisted the production of motivation environments, as students were highly motivated to improve their skills and overcome the difficulties.

In order to understand whether students with different cognitive styles would make different results on Scratch motion pictures and game works, Tsai and Chen (2011) conducted a study and found that visual-oriented cognitive style students showed a high level in a dimension namely "composition" compared with the verbal-oriented cognitive style students.

Ferrer-Mico, Prats-Fernàndez, and Redo-Sanchez (2012) conducted a study to investigate the effect of Scratch on the self-directed learning capability. They found that the majority of the students were able to increase their knowledge construction ability. Moreover, they concluded that students' understanding and confidence increased when they spent longer time on Scratch compared to other students who were exposed to the same activity during short periods.

Lee (2011) describes how Scratch can be used as a creative medium for gifted students to facilitate their learning. He adds that Scratch helps gifted and talented students express their inspired imagination, "transforming them from passive receptacles of knowledge into active knowledge producers." (p. 30). He points out that Scratch enables gifted students to easily learn the abstract knowledge of computer programming, which possibly will enrich their problem solving and logical thinking skills (Siegle, 2009).

2.12 Game construction in Scratch

Games are an important stage of young people's cognitive development that fosters awareness, higher order mental processes, abstract thinking, and strategic skills. In addition, games encourage social skills, such as collaborative learning. A child learns through playing games with others. Computer games market is growing boomingly. Computer games support critical thinking and problem solving skills (Mitchell & Savill-Smith, 2004).

Scratch is an open source-programming environment. It is used to create games, animations, and interactive stories. Professional users can construct multiplayer Scratch-based games with cloud data. They can also create single-level or multi-level Scratch games and upload them. However, they cannot show the highest scores online with the current version.

Wilson, Hainey, and Connolly (2013) conducted a research to examine the use of game construction by primary students with Scratch in order to show what programming skills they used within the games they created. Most of the students managed to gain some programming concepts and understood the basic ideas of programming.

Korkmaz (2016, a) conducted a research on Scratch-based game activities. He found that those activities had positive effects on students' attitudes towards learning computer programming, self-efficacy beliefs, and levels of academic achievement. In parallel, he concluded that those activities had positive contributions to the students' algorithmic thinking skills. Korkmaz (2016, b) conducted another study to investigate the effect of Scratch-related game activities and other software on academic achievement concerning students' computer programming, problem solving, and logical-mathematical thinking. The results showed a positive contribution to all of those areas.

2.13 Programming in Scratch

As Scratch is a programming language, where its users can code, it provides multithreading environment, which transcends basic algorithms and programming concepts (Kaučič & Asič, 2011). Scratch users can learn significant mathematical and computational ideas (De Kereki, 2008). They also can gain a deeper understanding in computing (Tangney, Oldham, Conneely, Barrett, & Lawlor, 2010).

Malan and Leitner (2007) view Scratch as a gateway to programming languages. They reflect that Scratch allows students to program with a mouse, thus improves first-time programmers' experiences. They argue that the programmatic constructions in Scratch allow students to focus on problems of logic before syntax. As a result, they recommend its use as a tool to help students understand the basic ideas of programming. For this, they deployed Scratch via Harvard Summer School's Computer Science course.

Likewise, Kereki (2008) points out that Scratch has a positive effect on learning programming. She declares that the most important weaknesses that students demonstrate are linked with solving problems and designing programs. In her study, she found that Scratch improved students' programming experiences. Conjointly, she found that Scratch promotes a high level of motivation.

2.14 Connecting the literature review with the current research

This research investigates the effects of using Scratch software on students' learning of geometric problem solving. Specifically, it focuses on real-world problem solving in education. This research shows various real-world problems that are to be solved by the students: 1) the problems that are included in the unit plan that is assigned for the experimental group and will be taught by the teacher, 2) the problems that the six students from both classes will be exposed to during the clinical interviews, and 3) the problems that are included in the last section of each of pretest and the posttest.

As mentioned in the literature review, using problem solving increases mathematical proficiency. In addition, using various instructional strategies that incorporate computer technology may help students develop their mathematical proficiency since computer technology is shaping the way we approach mathematical problem solving. Information visualization techniques improve learning mathematical concepts. Further, information visualization concedes several problem-solving strategies. The experimental group in this research will be exposed to an instructional strategy based on Information visualization techniques in the school lab.

Some researchers investigated the effect of using software on problem solving. Some of those studies tested the use of Scratch in mathematical problem solving. However, the previous research has a gap in showing how problem solving is raised by students because they focused on other phases of learning skills and abilities, such as mathematical thinking or reasoning, attitudes, self-efficacy, academic achievement, self-confidence, and cognitive styles. This research tries to fill the gap by focusing on how students work on real-world geometric problem solving with the use of Scratch.

Chapter Three

Method

3.1 Research Design

This research study adopts a mixed-method design that uses quantitative and qualitative data collection and analysis methods. For the quantitative research, the quasi-experimental design is adopted to evaluate the implications of an intervention, through teaching an experimental group through using Scratch.

Gall, Gall, and Borg (2007) define quasi-experimental design as a study lacking random assignment to groups. This method is employed to control threats to internal validity (Fraenkel, Wallen, & Hyun, 2012).

In this study, a non-probability sampling method is adopted using the Convenience Sampling. The students are chosen according to their convenient availability, accessibility, and proximity. This method is quick, inexpensive, and convenient to the researcher.

Two different groups A and B of grade 6 in a Lebanese school represented two groups: the control and experimental groups. The two groups were taught by the same teacher, using the same math book that is developed by a local company and aligned with the Lebanese curriculum. Both groups had been initiated to coding and programming and both were taught the same Grade-6 units in geometry. For the unit of types of triangles, the control group was exposed to the usual method of teaching, while the experimental group was taught through Scratch-based activities, using Windows-based laptop computers in the school lab.

3.2 Assigning the two groups

To assign the control group and the experimental, the results of the pretest are compared. The lower achieving class is set to be the experimental group while the other class is the control group. The reason for which this decision was made is that, if the group which earned higher pretest scores was set to be the experimental group in the first place, the Scratch input might not be the reason for its better later achievement. In this case, the result is not guaranteed to be the software's effect. However, if the lower achieving class shows improvement in one or more of the skills after the treatment, it would likely be a result of the Scratch input.

The overall average of class A is 18.6 out of 20 with a percentage of correct answers of 51.6% and standard deviation of 5.4. On the other hand, the overall average of class B is 17.8 out of 20 with a percentage of correct answers of 49.4% and standard deviation of 6.6.

As a result, the overall average of class B is smaller than the average of class A. The percentage of correct answers of class B is smaller than the percentage of correct answers of class A. A comparison of the two standard deviations shows that the scores in class B are much more spread out than the scores in class A. The high standard deviation of class B, together with the lower average, shows the presence of a bigger number of low achievers when compared to class A. These results led the researcher to choose class B as the experimental group and class A as the control group.

3.3 Participating school

The participating school is located in the southern suburb of Beirut. In 2015-2016, the number of students was 3,002 and currently (2016-2017) is 3,055. The number of grade-6 students is 87 in four classes, two of which were assigned to English language instruction. Originally, the full grade in Mathematics in grade 6 is 60 and the passing grade is 30. In this perspective, 70% of the students succeeded in Math in 2015-2016, while 75% of them passed in the first semester this current year. Last year, the highest final grade was 58.05 while the lowest grade was 13.75 with an average of 37.87 and standard deviation of 11.38 over the four grade-6 classes. During 2016-2017 first semester, the highest grade was 59 and the lowest grade was 11 with an average of 40.11 and standard deviation of 12.79.

Table 1 shows the numbers above to compare the two academic years in terms of grades, percentage of success, average, and standard deviation regarding mathematics in the participating school.

	Year 2015-2016	**Year 2016-2017 (First Semester)**
Percentage of Success	70%	75%
Highest Grade	58.05	59
Lowest Grade	13.75	11
Average	37.87	40.11
Standard Deviation	11.38	12.79

Table 1 Mathematics achievement of grade 6 in the participating school during the two years 2015-2016 and 2016-2017

3.4 Technology in the participating school

All the classrooms in the participating school are equipped with white boards; however, only the computer labs and a limited number of classrooms are equipped with LCD projectors. Four computer labs are furnished with either 30 laptops or desktop computers.

Computer education is adopted from grade one to grade eleven excluding grade nine. The institution that runs this school developed the first six-year computer books while the other five books were developed by a local Lebanese publisher. The computer science books and CDs showed progress compared with the Lebanese curriculum in regard to the educational approaches, integration with other subject matters shown in projects and some exercises, interactive activities, modern programs used, and the up-to-date software. Students used Microsoft Windows 8.1 and Microsoft Office 2013 along with multimedia presentations and coding in algorithms during one session each week. The computer subject is allotted a grade of 20.

Besides, students from grade six and above technological projects as they program with Python software to build and model Arduino robots with a Raspberry PI. Such extra-curricular activities are practiced in on-campus clubs on a weekly basis for one hour and a half, but go up to eight hours if there are preparations for competitions.

Nevertheless, the level of integration of technology in instruction is low. Some teachers use CDs with interactive applications to increase opportunities for students' engagement. In mathematics, the school uses geometry software in secondary classes. They usually used Cabri before, but currently they shifted to Geogebra.

3.5 Features of the assigned book

The assigned book was divided into chapters. The book chapters neither explicitly offer a general objective nor a big idea. Basically, the chapter provided lesson objectives. Although the objectives were clear, they were neither coherent nor focused. Most of them were centered on the first three lower Bloom’s Taxonomy levels.

Each chapter delivers some activities. These activities come in a form of problems to solve, yet without instructions that lead to the required steps or terms that would pave the way to the lessons. The chapter then shows the core section that comprises of definitions, explanations, examples, and solved problems.

The chapters do not demonstrate a clear-cut framework of concepts and skills to allow true mastery. As to using pictorial exemplifications, the book does not use concrete or pictorial representations, does not provide drawings to connect visual representations to problem solving, and does not use clear visuals that present concepts or model solutions.

Then, come the exercises at the end of the chapter. Although they are quantitatively sufficient, they do not follow an ascending order of difficulty. Solving them depend on the teacher’s demonstration, not the book, and the students need only to imitate the teacher’s method to complete the exercises correctly. Further, the exercises do not allow the students to explore, search, or create, as advised by Silva and Cabral (2015). As a result, they do not foster the development of logical and mathematical reasoning.

The analysis of the math textbook is important because the teacher uses the math textbook as the major resource for instruction. The teacher depends on the math textbook as the primary guide to prepare her lessons and create assessments. If the book focuses on mere procedural knowledge, it negatively influences teaching, as the teacher focuses less on allowing students to discover new knowledge and utilize material in laborious manners and concentrates more on asking questions exactly the way presented in the book. Thus, the textbook analysis is important for the purpose of the study. The unit plan, which is designed to the control group, is based on the textbook. On the other hand, the unit plan, which is assigned for the experimental group, is designed to allow students to engage through real-life applications and develop their cognitive thinking skills using a constructivist approach. For this reason, the Scratch-based activities, which are embedded in this unit plan, are designed to improve their problem solving skills. The study should describe how content schemes in both unit plans may influence problem solving processes and outcomes.

3.6 Participants

The population that is part of this study composed of 51 grade-6 students of a school in the southern suburb of Beirut. This institution is a private school comprising elementary, intermediate, and secondary level classes. The sample is comprised of two classes: 26 students in class A and 25 students in class B.

The two classes consist of male students of lower to middle-income families who come from the same social backgrounds. They are 12 to 13 years old. According to the school records, the two classes have the same level of achievement in average. Both have an average of 41 out of 60 in mathematics. Compared to the last year when the same students of the current two groups were in grade 5, group A students scored 59.4 in Math as the highest grade and 12.1 as the lowest grade with an average of 40.8 out of 60 and standard deviation of 11.9, whereas in this year in grade 6, group A students scored 57.0 in Math as the highest grade and 14.0 as the lowest grade, with the same average of 40.8 out of 60 and standard deviation of 11.7. On the other hand, group B students who were in grade 5, scored 57.4 in Math as the highest grade and 11.0 as the lowest grade, with an average of 43.2 out of 60 and standard deviation of 12.5, whereas in this year in grade 6, group B students scored 57.0 in Math as the highest grade and 13.0 as the lowest grade with an average of 42.0 out of 60 and standard deviation of 13.7. As a result, according to previous year records, the two groups had the same level of achievement in average. Correspondingly, the two groups were similar as per many criteria.

Table 2 summarizes the comparison of achievements in mathematics between groups A and B. Note that the average of students in group B is higher over the two years, even though they have greater diversity because their standard deviation is bigger.

		Section A	Section B
Year 2015-2016	**Highest Grade**	59.4	57.4
	Lowest Grade	12.1	11
	Average	40.8	43.2
	Standard Deviation	11.9	12.5
Year 2016-2017	**Highest Grade**	57	57
	Lowest Grade	14	13
	Average	40.8	42
	Standard Deviation	11.7	13.7

Table 2 Mathematics achievement between sections A and B of grade 6 during the two years 2015-2016 and 2016-2017

The study tracked the students' math grades for two years in a row, to make sure that the two sections are comparable in their achievement levels, in order to control this variable. Other variables are also controlled, as the two sections are taught by the same teacher, in the same school, using the same curriculum.

3.7 Areas of intervention

The students in both groups usually have Math classes for 50 minutes each day. For the intervention, the two groups were taught the same unit about triangles within the same duration. However, two different approaches were used. The first approach involves using direct instruction regard the use of straightforward, unequivocal teaching techniques, usually to teach specific skills. The second approach involves pedagogic methodology using technology. In chapter 4, reasons show why the traditional approach is assigned to group A and the technology-based approach is assigned to group B.

The unit procedures included investigating the angles of a triangle to find the sum of the measures of the angles in triangles, classifying triangles, finding the missing angle measures, and constructing triangles of types: right-angled, isosceles, and equilateral.

Two four-lesson unit plans were developed for the two groups. Each unit plan had a different approach. Each unit plan consisted of four 50-minute lessons over the duration of a week and was implemented in the academic year 2016-2017. The general and specific objectives of the two unit plans were designed for the research, and they were aligned with the curriculum. The first unit plan that aligned with the usual lessons was applied in the control group class (group A). Another Scratch-based unit plan was developed for the experimental group (group B). It consisted of lessons that integrate computing technology into current curricular objectives.

Note that the two groups did not have classes at the same time because they were taught by the same teacher. The advantage of this was to stabilize and control most of the variables. The independent variable was the use of Scratch software for a group in this study. For this reason, it was important that this single variable to be tested in the experimental group. If more than one variable were to be tested, the research would not be able to tell which one was responsible for any differences that would occur.

3.8 Pretest and posttest

In addition to the four periods of instruction, two additional periods were assigned for the pre- and post-tests. A written pretest (Appendix A) and posttest (Appendix B) were given to both groups. The purpose of the pretest was to uncover the students' abilities to solve geometric problems before taking the unit. The purpose of the posttest was to analyze their problem-solving abilities after implementation of the unit. Each test is composed of four sections to cover the research questions. The four sections are: knowledge, concepts, skills, and problem solving. The problem solving section in the pretest has three parts: low, average, advanced.

3.9 Clinical interviews

The qualitative part of the study used clinical interviews before (Appendix C) and after (Appendix D) unit teaching with six selected students, three from each group.

According to Bingham and Moore (1959), the clinical interview is a conversation with a purpose. It is conducted on a one-to-one basis. It involves a face-to-face interaction between a researcher and a participant. It is a form of pedagogical documentation that can give information about student learning. The researcher asks questions and the participant provides verbal answers. The researcher customizes questioning, in a flexible mode, in reaction to what the participant says to uncover deeper insights and offer valuable information concerning participant's thinking" (Ginsburg, 1997). Clinical interviewing is a non-standardized technique because the interviews reveal differently for every interviewee (Ginsburg, 1997).

The purpose of the interviews is to surface the deep understandings of student learning and to capture students' observations and views of the work being performed on the problem before the intervention and the problem after intervention and the type of solutions they develop. The clinical interviews provide an evidence-based information about how a student understands a word-problem or why a student is struggling in mathematics. Through the interviews after the pretest and posttest, data is collected, analyzed, and compared.

The six students were selected to be closely observed and clinically interviewed, in such a way that three students were chosen from each class, based on levels of achievement: low, average, and high. Each two students (one from each class) had the same level of achievement (high, middle, or low).

During the clinical interviews, the data was recorded using videotaping and notes.

Choosing the six students was based on three criteria: 1) consulting their school teacher, 2) extracting their results on the problem solving section in the pretest, and 3) analyzing their current achievement at school.

One problem was assigned to the six students after having the pretest and before starting the unit. Another problem was assigned to those same students after the posttest. The six students' performance on solving the two problems was closely videotaped and observed while working on their written-format problems.

3.10 Ethical considerations for the clinical interviews

The six students' work on solving the two problems was closely videotaped and observed while working on their written-format problems in such a way their faces didn't appear during filming. Filming was focused on the students' hands only while working on their pencil-paper problems. For this, the researcher needed a permission from each of these six students and their parents whom consent forms were sent to them before starting the intervention. A special room was set for the interviews so that each student was alone with the researcher under the supervision of the corresponding schoolteacher and superintend.

3.11 Types of questions used in the clinical interviews

Each student of those six students was asked some verbal questions during his solving procedures with no cues to the right answers or solutions. Examples of the questions that were asked during solving procedure: a. Have you understood the problem? b. If yes, what exactly did you understand? If no, what part was not understood? c. What strategies would you use to work on the solution? Examples of the end-of-unit questions were: a. Explain how your solution strategy was appropriate. b. What tools or instruments did you use that helped you solve the problem? Other questions were asked during students' solving, to uncover their thinking or the reasons for which they chose to perform certain actions, such as: Why did you do this? How did you know? On what basis did you work this out? Why did you stop? What is the problem? Where is the difficulty? In the advanced way of solving, questions were: Why did you choose a new way to perform the task? Was it successful? How do you know that it was right? Why do you use this? What made you choose this way of solution?

All the questions were selected so as not to give the student any clue to the solution, not to direct him to a specific strategy, not to modify his thinking, and not to imply a certain solution.

3.12 Data analysis

In the qualitative analysis techniques, logical deductions were used to decipher the data collected from the clinical interviews. Chapter four, of the current study, explains in details how the clinical interviews were analyzed. In contrast, the quantitative method used statistical processing. The comparisons of the achievement on the pretest and posttest included more specific comparisons related to the four types of learning: knowledge, conceptual understanding, skills, and problem solving.

In addition, T-Test, for independent samples, was used to determine statistical differences between the two groups in the sense of the posttest results with the help of SPSS software.

Chapter Four

Findings

4.1 Purpose and outline of this chapter

The purpose of this study was to investigate the effect of using Scratch software on grade-6 students' geometric problem-solving skills. The current study adopts a mixed-method design that uses quantitative and qualitative data collection and analysis methods.

The sample that participated in the study is composed of students at the elementary level (grade 6), who were attributed in a random manner to one of two groups: experimental group and control group. In the experimental group, a constructivist unit plan (Appendix E), which includes Scratch-based problem-solving activities, was developed. At the same time, a parallel unit plan (Appendix F) was implemented to the control group in the usual teaching approach. However, the same four lessons and basic instructions were handled in both groups. A pretest (Appendix A) and posttest (Appendix B) were conducted in both classes to study the effect of using a Scratch-based problem-solving approach on students' problem-solving skills. The results of these tests were used for quantitative analysis in this chapter. Furthermore, two sessions of clinical interviews were conducted with students from each group. The first session was after the pretest and before the intervention and the second session was after the intervention and posttest. One written-format problem was assigned in each session. A sample of six students (three from each group) were chosen for the clinical interviews. Choosing them was based on their math achievement level at school, their achievement on the problem-solving section in the pretest, and consultation with their math teacher. The six students' performance on solving the two problems was closely videotaped and students were observed while working on their problems. The work of this sample (six students) is used for qualitative analysis.

The current chapter presents the analysis method and data of pretest and posttest, description of the problems used in the pretest and some students' performance on some of those problems, explanation of the way the two groups are assigned and the way the six students are chosen, and the results of analysis of the clinical interviews.

4.2 Analysis method of tests

Tables in Microsoft Excel were created to analyze the results of both pretests and posttests. Since each test is divided into four sections: knowledge, conceptual understanding, skills, and problem solving, a separate table was made for each section. Each row of the tables shows how many points a student (coded A_i with i = 1 to 26 and B_i with i = 1 to 25) received on each item and a total number of points for each section. For each test item based on the class's answers, the table displays the number of correct answers, the percentage of correct answers, and a standard deviation. Also, for each section of the four test sections, the table displays the average number, the percentage of correct answers, and the standard deviation.

4.3 Analyzing the pretest

The *knowledge* section in the pretest has five items. Each item requires writing the right answer in a blank space. Each item weighs two points. Thus, the total weight of the *knowledge* section is ten. Four items ask about a triangle: the names of its sides, the number of its sides and angles, and the sum of its angles. A fifth item asks about the measure of a right angle.

The table G-A-K-1 (Appendix G) shows the specific results of *Knowledge* section of group A. The table G-A-K-2 (Appendix G) shows the cumulative results of the *Knowledge* section of group A.

The *Conceptual Understanding* section in the pretest has two parts: Part I has three items and Part II has only one item. In Part I, the student writes one word in each blank space: always, sometimes, or never. In Part II, the student answers a question that involves comparing two angles to identify the greater one with an explanation. Each item in Part I weighs one point. The rubric of Part II has six criteria. Each criterion weighs one point. Thus, the total weight of the *Conceptual Understanding* section is nine.

The table G-A-C-1 (Appendix G) shows the specific results of *Conceptual Understanding* section of group A. The table G-A-C-2 (Appendix G) shows the cumulative results of the *Conceptual Understanding* section of group A.

The *Skills* section in the pretest has only one item that requires drawing two complementary angles and writing their measures. This item has four criteria. Each criterion weighs one point except the third criterion that weighs two points because it investigates the sum of the angles, which has to be 90 degrees, because it involves adding the two angles correctly and showing the correct sum. Thus, the total weight of the *Skills* section is five.

The table G-A-S-1 (Appendix G) shows the specific results of *Skills* section of group A. The table G-A-S-2 (Appendix G) shows the cumulative results of group A in the *Skills* section.

The *Problem Solving* section in the pretest contains three real-life problems with three difficulty levels (easy, medium, and hard). A table was created to analyze each of the three problems. The three tables show a number of criteria for solving each problem for each student in both groups. The weight of the first problem is two points, the second problem is four points, and the third problem is six points. Thus, the total weight of the *Problem Solving* section is twelve.

As for the specific results of each of the three problems in the *Problem Solving* section for group A, the table G-A-P-1 (Appendix G) is assigned for the first problem, the table G-A-P-2 (Appendix G) for the second problem, and the table G-A-P-3 (Appendix G) for the third problem. The table G-A-P-4 (Appendix G) shows the specific results of the three problems of the *Problem Solving* section for group A.

Finally, the table G-A-P-5 (Appendix G) shows the percentage of correct answers and standard deviation of the three problems of group A in the *Problem Solving* section of the pretest.

Similarly, the table G-B-K-1 (Appendix G) shows the specific results of *Knowledge* section of group B and the table G-B-K-2 (Appendix G) presents the cumulative results of the *Knowledge* section of group B.

The table G-B-C-1 (Appendix G) reveals the specific results of *Conceptual Understanding* section of group B and the table G-B-C-2 (Appendix G) demonstrates the cumulative results of the *Conceptual Understanding* section of group B.

The table G-B-S-1 (Appendix G) highlights the specific results of *Skills* section of group B and the table G-B-S-2 (Appendix G) displays the cumulative results of the same test section and same group.

For the specific results of each problem of the three problems in the *Problem Solving* section of group B, the table G-B-P-1 (Appendix G) is assigned for the first problem, the table G-B-P-2 (Appendix G) for the second problem, and the table G-B-P-3 (Appendix G) for the third problem.

The table G-B-P-4 (Appendix G) shows the specific results of the three problems of *Problem Solving* section for group B.

Finally, the Table G-B-P-5 (Appendix G) shows the percentage of correct answers and standard deviation of the three problems of group B in the *Problem Solving* section of the pretest.

4.4 Assigning the six students for clinical interviews

Three criteria were set to choose three students from each group for the clinical interviews. These students are:

	Low Achiever	Middle Achiever	High Achiever
From Control Group A	A22	A9	A8
From Experimental Group B	B6	B13	B15

Table 3 Specification of the six students who are chosen for the clinical interviews

The first criterion is based on consultation with their schoolteacher. Her suggestions for the student names met the other two criteria.

The second criterion is based on the six students' achievement in mathematics in the current semester at their school. For the high level of achievement, the student A8 has a grade of 57 out of 60 in math this year with a total average of 17.79 out of 20 in all subjects, while the student B15 has the same grade 57 out of 60 with 17.97 out of 20 as a total average in all subjects. For the middle level of achievement, the student A9 has a 45 out of 60 in math and a total average of 14.5 out of 20 in all subjects, while the student B13 has the same grade of 45 out of 60 in math with 15.16 out of 20 as a total average. For the low level of achievement, the student A22 has a grade of 27 out of 60 in math and 11.76 out of 20 in the total average in all subjects, while the student B6 has 29 out of 60 in math and 11.74 in the total average in all subjects. These numbers would be an evidence of the same criteria of choosing those students as a sample of the two groups.

The numbers mentioned in the second criterion are shown in the following table.

	Level of Achievement	Score (out of 60)	Total Average (out of 20)
A8	High	57	17.79
B15		57	17.97
A9	Middle	45	14.5
B13		45	15.6
A22	Low	27	11.76
B6		29	11.74

Table 4 Data shows level of achievement in mathematics, scoring, and total average in all subjects of the three students of each group

The third criterion is based on the scores, out of 36 points, in the *Problem Solving* section in the pretest. For the high score on problem solving, the student A8 earned 25 and student B15 also earned 25. For the middle score on problem solving, the student A9 earned 17 and student B13 also earned 17. For the low score on problem solving, the student A22 received 10 and student B6 received 8.

These numbers are demonstrated in the following table:

	High Score	Middle Score	Low Score
Group A	A8 (25)	A9 (17)	A22 (10)
Group B	B15 (25)	B13 (17)	B6 (8)

Table 5 Data shows the scores of the six students of the two groups in the problem solving section of the pretest

4.5 Describing the problems taken in the pretest

The three problems of the *Problem Solving* section in the pretest are real-life problems provided with illustrating graphics. They are non-routine problems for the students since they are not found in their regular math books. The first problem, which is of a low-level difficulty, describes a dart thrown by a player at a target. The dart and its shadow form an angle that is given. The student has to find the other angle. The second problem, which is of a middle-level difficulty, demonstrates a map with two streets intersecting a main road. The map shows two angles: one is known and the other is unknown. The student has to find the unknown one. The third problem, which is of a high-level difficulty, shows a flying kite forming two angles with the ground. The student has to solve algebraically a synthesized equation with unknowns.

4.6 Describing some students' performance on complementary angles in the pretest

Complementary angles are angles with measures that add up to 90 degrees. In the *skills* section of the pretest (Appendix A), students are asked to draw two complementary angles and write their measures. This task builds on students' previous understanding with decomposing angles and perceiving that angle measures are additive. The task requires students to use their knowledge of complementary angle measurements and skills to draw a pair of angles. Noting some student results, most of the students drew the angles correctly either with or without measure recording. A few students (8 students from group A and 4 students from group B) did not use tools when they drew the angles and some others (1 student from group A and 3 students from group B) did not answer the problem. On the other hand, some students (9 students from group A and 6 students from group B) had misconceptions towards their understanding of the concept "complementary". Although their strategy for locating complementary angles was right, they wrongly understood that complementary means sharing equally. They assigned 45 degrees for each angle. One student from each group (Fig. 7) used the compass to make an angle bisector to make the precise angular partition.

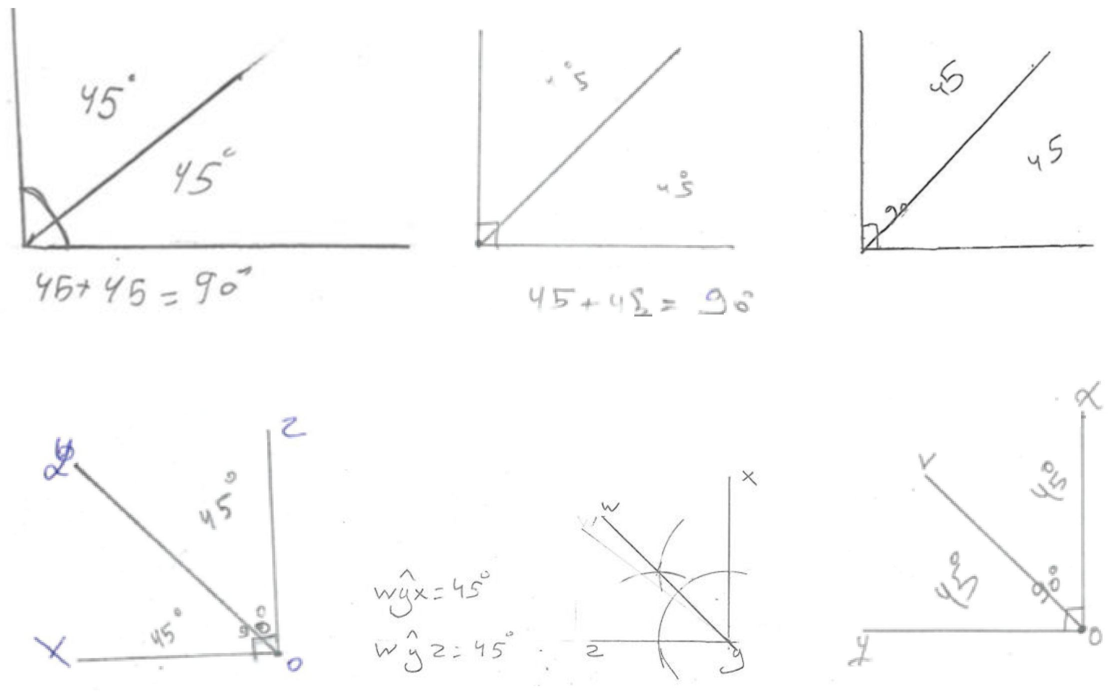

Fig. 7 *Six figures represent examples of students who made precise angular partition (45 degrees for each angle)*

Instructional guidance might be needed to let students develop their conceptual understanding of the complementary angles such that any two angles whose measures add up to 90 degrees are complementary and not necessarily to be both 45 degrees.

4.7 Analyzing the posttest

The *knowledge* section in the posttest (Appendix B) has three items. Each item requires writing the right answer in a blank space. Each item weighs two points. Thus, the total weight of the *knowledge* section is six. The main topic of the three items is about triangles. Each sentence defines a triangle offering its main properties, and the student has to write the type of that triangle to complete each sentence.

The table H-A-K-1 (Appendix H) shows the specific results of the *Knowledge* section of group A, and the table H-A-K-2 (Appendix H) shows the cumulative results of the same *Knowledge* section and same group A.

The *Conceptual Understanding* section in the posttest has three parts: Part I has three items, Part II has two items, and Part III has two items. In Part I, the students write one word in each blank space: always, sometimes, or never. In Part II, the students solve two problems that involve providing short responses with explanations. In Part III, students do the same as in Part II, but they have to show their work using a diagram. Each item in Part I and III weighs two points while each item in Part II weighs either one or two points. The total weight of the *Conceptual Understanding* section is eighteen.

The table H-A-C-1 (Appendix H) shows the specific results of the *Conceptual Understanding* section of group A, and the table H-A-C-2 (Appendix H) shows the cumulative results of the *Conceptual Understanding* section for group A.

The *Skills* section in the posttest has two parts. In each part, the student is asked to draw a triangle with specific features and provide an explanation of the steps of drawing. The total weight of the *Skills* section is ten.

The table H-A-S-1 (Appendix H) shows the specific results of the *Skills* section of group A. The table H-A-S-2 (Appendix H) shows the cumulative results of the same *Skills* section and same group.

The *Problem Solving* section in the posttest includes two problems. The weight of the each problem is six points. Thus, the total weight of the *Problem Solving* section is twelve.

For the specific results of each problem of the two problems in the *Problem Solving* section of group A, table H-A-P-1 (Appendix H) is assigned.

The table H-A-P-2 (Appendix H) shows the cumulative results of the same section and same group.

Similarly, table H-B-P-1 (Appendix H) shows the specific results of the *Knowledge* section of group B, and the table H-B-P-2 (Appendix H) presents the cumulative results of the same section and same group in the posttest.

The table H-B-C-1 (Appendix H) reveals the specific results of the *Conceptual Understanding* section of group B, and the table H-B-C-2 (Appendix H) demonstrates the cumulative results of the same section and same group in the posttest.

The table H-B-S-1 (Appendix H) highlights the specific results of the *Skills* section of group B, and the table H-B-S-2 (Appendix H) expresses the cumulative results of the same section and same group in the posttest.

The table H-B-P-1 (Appendix H) displays the specific results of the *Problem solving* section of group B, and the table H-B-P-2 (Appendix H) conveys the cumulative results of the same section and same group in the posttest.

4.8 Comparing the pretest and posttest

The scores of the tests were transformed into percentage so that each one of the four sections of the tests aligns with each other. As for the pretest, table I-E-A (Appendix I) shows the scores of the pretest of group A, and table I-E-B (Appendix I) reveals the scores of the pretest of group B. As for the posttest, table I-O-A

(Appendix I) represents the scores of the posttest of group A, and table I-O-B (Appendix I) represents the scores of the posttest of group B.

The table 6 shows the total mean score of each ability in each pretest and posttest of the two groups. It also shows the increased or decreased progress shifting from pretest to posttest for each group. The value of this progress is calculated by subtracting the sum of the ability scores of the pretest from the sum of the same ability scores of the posttest and dividing the result by the total number of both scores of this specific ability.

	Knowledge	Conceptual Understanding	Skills	Problem Solving	Total
Mean Score of Pretest A	60.0%	58.5%	66.2%	33.3%	54.5%
Mean Score of Posttest A	85.9%	40.0%	53.1%	53.8%	21.5%
Mean Score of Pretest B	76.8%	38.2%	56.0%	38.0%	52.3%
Mean Score of Posttest B	100.0%	65.1%	64.0%	70.0%	74.8%
From Pretest A to Posttest A	17.8%	-18.9%	-11.0%	23.5%	3.3%
From Pretest B to Posttest B	13.1%	26.0%	6.7%	29.6%	17.7%

Table 6 Data shows the mean scores of the four abilities of the pretest and posttest of both groups A and B

Note that the total mean score of group A in the pretest was 54.5% and group B was 52.3% (table 6). On the other hand, the total mean score of group A in the posttest was 21.5% and group B was 74.8% (table 6).

4.9 Using T-Test

The T-Test is a parametric statistical test used to tell whether there is a significant difference between the means of two samples or not (Fraenkel, Wallen, &

Hyun, 2012). The T-Test analysis is used with two small sample sizes to test the difference between the samples when the variances of two normal distributions are not known. To compare the mean scores of two independent groups, this research conducts a T-Test for independent samples to find the t-value.

An independent-samples T-Test was conducted to compare the score for each factor of the four factors (knowledge, conceptual understanding, cognitive skills, and problem solving) in intervention using Scratch and no using Scratch conditions.

Hypothesis to be tested (H): There is a significant improvement in students' achievement scores after an intervention using Scratch coding in learning types of triangles.

Specific sub-hypotheses:

- (H1): There is a significant improvement in students' knowledge scores after an intervention using Scratch coding in learning types of triangles.
- (H2): There is a significant improvement in students' conceptual understanding scores after an intervention using Scratch coding in learning types of triangles.
- (H3): There is a significant improvement in students' cognitive skills after an intervention using Scratch coding in learning types of triangles.
- (H4): There is a significant improvement in students' problem-solving abilities after an intervention using Scratch coding in learning types of triangles.

Statistically speaking, the significance level of 0.05 pointing to a 5% risk concludes that there is a difference. That is the difference between two groups is interpreted to be statistically significant when $p = 0.05$ or less.

The scores of the experimental group in the geometric knowledge section were higher than the scores of the control group geometric knowledge section before the intervention. This difference was statistically significant (table 7). The students in the experimental group had significantly higher scores than the control group in the *Knowledge* section in the pretest (P ≤0.001).

Pretest Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	fidence Interval of the D	
									Lower	Upper
Knowledge	Equal variances assumed	**0.004**	**.947**	**-3.530**	**49**	**.001**	**-1.680**	**.476**	**-2.636**	**-.724**
	Equal variances not assumed			**-3.546**	**47.362**	**.001**	**-1.680**	**.474**	**-2.633**	**-.727**
Conceptual Understanding	Equal variances assumed	**2.016**	**.162**	**2.360**	**49**	**.022**	**1.829**	**.775**	**.271**	**3.387**
	Equal variances not assumed			**2.352**	**46.906**	**.023**	**1.829**	**.778**	**.264**	**3.394**
Skills	Equal variances assumed	**1.183**	**.282**	**1.405**	**49**	**.166**	**.508**	**.361**	**-.218**	**1.234**
	Equal variances not assumed			**1.408**	**48.842**	**.166**	**.508**	**.361**	**-.217**	**1.232**
Problem Solving	Equal variances assumed	**2.714**	**.106**	**-.911**	**49**	**.367**	**-.560**	**.615**	**-1.796**	**.676**
	Equal variances not assumed			**-.913**	**48.726**	**.366**	**-.560**	**.614**	**-1.793**	**.673**
Total	Equal variances assumed	**0.074**	**.786**	**.063**	**49**	**.950**	**.097**	**1.533**	**-2.985**	**3.179**
	Equal variances not assumed			**.063**	**48.693**	**.950**	**.097**	**1.535**	**-2.988**	**3.182**

Table 7 Comparison between groups A and B in the pretest using T-Test before the intervention

On the other hand, when comparing the scores of the posttest of both groups A and B, using the T-Test, only the *Conceptual Understanding* section showed a significant difference. The results of the posttest (table 8) showed that there was a significant difference in the scores of the *Conceptual Understanding* section of the intervention using Scratch for equal variances assumed (M = -4.53, SD = 0.74) and equal variances not assumed (M = -4.53, SD = 0.75) conditions; t(49) = -6.01, p = 0.00. Furthermore, the overall average score of the posttest (table 8) of the two groups was statistically significant after the intervention using Scratch for equal variances assumed (M = -6.75, SD = 1.98) and equal variances not assumed (M = -6.75, SD = 1.99) conditions; t(44) = -3.4, p = 0.001.

Posttest Independent Samples Test										
		Levene's Test for Equality of		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Knowledge	Equal variances assumed	**16.574**	**.000**	**-1.770**	**49**	**.083**	**-.231**	**.130**	**-.493**	**.031**
	Equal variances not assumed			**-1.806**	**25.000**	**.083**	**-.231**	**.128**	**-.494**	**.032**
Conceptual Understanding	Equal variances assumed	**6.461**	**.014**	**-6.091**	**49**	**.000**	**-4.528**	**.743**	**-6.021**	**-3.034**
	Equal variances not assumed			**-6.026**	**36.874**	**.000**	**-4.528**	**.751**	**-6.050**	**-3.005**
Skills	Equal variances assumed	**.001**	**.970**	**-1.708**	**49**	**.094**	**-1.092**	**.640**	**-2.378**	**.193**
	Equal variances not assumed			**-1.710**	**48.950**	**.094**	**-1.092**	**.639**	**-2.376**	**.191**
Problem Solving	Equal variances assumed	**.303**	**.584**	**-.792**	**49**	**.432**	**-.898**	**1.135**	**-3.179**	**1.382**
	Equal variances not assumed			**-.791**	**48.654**	**.433**	**-.898**	**1.136**	**-3.181**	**1.384**
Total	Equal variances assumed	**2.387**	**.129**	**-3.418**	**49**	**.001**	**-6.749**	**1.975**	**-10.718**	**-2.781**
	Equal variances not assumed			**-3.398**	**44.183**	**.001**	**-6.749**	**1.986**	**-10.752**	**-2.747**

Table 8 Comparison between groups A and B in the pretest using T-Test after the intervention

Note that the indication of the conceptual understanding is so obvious and high. On the other hand, when comparing group statistics between the pretest and posttest, using T-Test, all the four sections are statistically significant to both groups A and B (table 9). In this case, the significance is equal to either 0.001 or less.

		t-test for Equality of Means						
		t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
							Lower	Upper
Knowledge	Pre-test	**3.460**	**100**	**.001**	**.941**	**.272**	**.401**	**1.481**
	Post-test	**3.460**	**56.340**	**.001**	**.941**	**.272**	**.396**	**1.486**
Conceptual Understanding	Pre-test	**-7.951**	**100**	**.000**	**-5.039**	**.634**	**-6.297**	**-3.782**
	Post-test	**-7.951**	**96.726**	**.000**	**-5.039**	**.634**	**-6.297**	**-3.781**
Skills	Pre-test	**-7.456**	**100**	**.000**	**-2.784**	**.373**	**-3.525**	**-2.043**
	Post-test	**-7.456**	**78.529**	**.000**	**-2.784**	**.373**	**-3.528**	**-2.041**
Problem Solving	Pre-test	**-4.086**	**100**	**.000**	**-2.627**	**.643**	**-3.903**	**-1.352**
	Post-test	**-4.086**	**77.127**	**.000**	**-2.627**	**.643**	**-3.908**	**-1.347**
Total	Pre-test	**-7.171**	**100**	**.000**	**-9.510**	**1.326**	**-12.141**	**-6.879**
	Post-test	**-7.171**	**89.363**	**.000**	**-9.510**	**1.326**	**-12.145**	**-6.875**

Table 9 Comparison between pretest and posttest for both groups using T-Test

4.10 Analysis of the clinical interviews before the intervention

The problem before the intervention has three items (Appendix J). It tells that a restaurant sells round and square pizzas. It also gives customers pieces of cartons for the bases of the pizza slices. Customers at each table should cut the pizza into equal slices. The first item states the following: At table 1, three friends order a round pizza. At table 2, five friends order a round pizza. Both pizzas are of the same size. Therefore, the participants are asked to help the waiter identify the type of angles of the carton pieces (acute, obtuse, or right) and their measures. They are required to justify their answers.

The student should consider that the pizza's shape is circular and the full circle is 360°, so if we cut a pizza in 3 pieces, each angle would be 360° ÷ 3 = 120°. Thus, each piece represents an obtuse angle. Also, if we cut the pizza in 5 pieces, we find that each angle measures 72° and represents an acute angle. Students' explanations related to knowledge before the intervention were not accurate. For example, although a correct answer was given to the problem that asked for identifying the type of the angles of the carton pieces of the pizza as the answer was that the type of the angles was obtuse, the explanation was irrelative and was related to the number of the persons who share that pizza. Example:

Student: *The first pizza has to have obtuse-angled slices.*

Interviewer: *Tell how you solved it.*

Student: *It would be logically enough for three friends to share the pizza!*

Other explanations, related to using a tool to measure the angle, show inability to overcome this problem.

Example 1:

Interviewer*: How do you know?*

***Student**: I used my index finger to identify the angle measures.*

Example 2:

***Interviewer**: What are you going to do?*

***Student**: Measure the pizza.*

***Interviewer**: Why are you going to measure it?*

***Student**: In order to make a carton to let it fit inside it.*

***Interviewer**: How are you going to measure it?*

***Student**: With a protractor.*

Although the protractor is a tool that helps students to measure angles, it is not the right tool to be used to solve the problem. The problem needs only using division. It is realized that the students seemed to face a challenge in applying the required knowledge to the problem except for (the high achievers) one student from section A and one student from section B who solved it right. Further, those students lacked the cognitive and process abilities in the understanding of circles and visualizing their properties and the global geometric concepts. Such difficulties are reported in a research conducted by Battista (1999), whereby students failed "to develop an adequate understanding of geometric concepts." (p. 368). Moreover, those students seemed to have some difficulty with measuring angle size, which is a challenge reported in a research conducted by Prescott, Mitchelmore, and White (2002) whereby students faced challenges with the basic concepts of angle measurement.

The second item of the problem before the intervention states the following: At table 3, four friends order a medium round pizza. At table 4, six friends order a large round pizza. The waiter says the sum of the angles of the carton pieces of table

3 is different from the sum of angles of carton pieces of table 4. Do you agree with him? Why or why not?

Regardless of the pizza's size, it is always 360 degrees because the circular shape completes a full rotation; however, some answers given by students were related to the size of the pizza. In this case, the respondents had a misconception. Example:

> ***Student****: The sum of the angles of the carton pieces of each pizza is different from each other. The reason is that there are different sizes of pizzas. One is medium and the other is large.*

Other answers' explanations given by students showed a misconception related to the number of the people at each table. In other words, the number of persons had an effect on the slice angle. Example:

> ***Student****: I agree that the sum of the angles of the carton pieces of table 3 is different from the sum of angles of carton pieces of table 4 because at table 3, there are four people while at table 4, there are six people. So, the sum of the angles is different each time.*

The third item of the problem before the intervention states the following: At table 5, two friends order a square pizza. At table 6, another two friends order a square pizza. The two pizzas are of the same size. Table 5 customers cut from a corner to another corner passing through the center. Table 6 customers cut through the center of the pizza, parallel to its side. Thus, the participants ought to help the waiter identify the forms of the needed carton pieces for each table.

Each pizza slice of table 5 should be a triangle, specifically a right isosceles triangle, and each pizza slice of table 6 represents a rectangle irrespective of the horizontal or vertical direction of the cutting. Thus, each piece of carton should fit its corresponding slice. This problem requires knowing and understanding three terms: center, parallel, and corner. It also requires using visualization and geometric reasoning, developing spatial sense and an ability to apply geometric properties, and forming relationships to solve an everyday-life problem concerning cutting tasks to form congruent pizza slices. Some answers and explanations were acceptable and some showed a poor conceptual understanding.

Table 10 presents students' results in the clinical interviews before the intervention.

Before the Intervention			**Response** (out of 1)	**Proof** (out of 2)	**Sum** (out of 3)
High	Control Group	Part 1	1	2	3
		Part 2	0	0	0
		Part 3	1	1	2
	Experimental Group	Part 1	1	2	3
		Part 2	1	2	3
		Part 3	1	2	3
Middle	Control Group	Part 1	1	0	1
		Part 2	0	0	0
		Part 3	1	2	3
	Experimental Group	Part 1	0	0	0
		Part 2	0	0	0
		Part 3	0	0	0
Low	Control Group	Part 1	1	0	1
		Part 2	0	0	0
		Part 3	1	1	2
	Experimental Group	Part 1	0	0	0
		Part 2	0	0	0
		Part 3	0	0	0

Table 10 Scores of the students' responses in the clinical interviews before the intervention

Before the intervention, the group score (out of 100) of the control group was 44.4% and the group score of the experimental group was 33.3%. These two values are calculated by adding the sums of the table 10 for each group and dividing the result by the total sum of the last column of table 10.

The following table shows how the qualitative method for the individual and group scores is interpreted quantitatively before the intervention (table 11).

Before the Intervention		**Individual Score** (out of 100)	**Group Score** (out of 100)	
High	Control Group	55.6%	Control Group	44.4%
	Experimental Group	100.0%		
Middle	Control Group	44.4%		
	Experimental Group	0.0%	Experimental Group	33.3%
Low	Control Group	33.3%		
	Experimental Group	0.0%		

Table 11 Individual scores of the students' responses in the clinical interviews before the intervention

4.11 Analysis of the clinical interviews after the intervention

The problem after the intervention has four items (Appendix K). It states that at an exhibition, companies are given boards to display their posters. The boards are all of the same height, but of different widths. They come in different colors. The widths of the boards are 3 meters for the red, 4 meters for the blue, and 5 meters for the green. Each company uses three boards only to stick them in a triangular prism model. Each model is built on a wooden base whose sides are traced exactly with the bottom of the boards. Company A model uses boards of colors: red, blue, and green. Company B model uses boards of colors: red, blue, and blue. Company C model uses boards of colors: blue, green, and green. Company D model uses boards of colors: red, red, and red.

The first item states the following: If an angle between two boards of company A model is 90°, what is the type of the base? If another angle between two boards of company A is half the first angle, what is the type of the base? How do you know?

All the six students responded that the answer should be a right triangle, which is correct. However, for the second part of the problem, the control group either said it would remain a right-angled triangle regardless the measure of the other angles as the triangle has 90° or said it would be an isosceles triangle without recognizing the right angle. Evidently, poor conceptual understanding about angles is exhibited.

The second item states the following: Do the bases for companies B and C have the same type? The same size? The same sum of angles? Explain each answer. The correct answers are that both bases are isosceles triangles and have the same sum of angles, which is 180° as long as they are triangles. Although these two triangles have congruent angles, they do not have the same size because the boards are of different lengths. All the interviewed students said that the two bases had the same type. However, explaining the reason was not convincing from some students from both groups. Lack of providing strong proof shows how those students' understanding of the problem situation are not deepened.

The third item states the following: Can the base for company D be a right triangle? If yes, explain why. If not, what is its type? Explain. Definitely, the base represents an equilateral triangle with all three sides of equal length as it matches the borders of the three red boards. All the students responded correctly except the two low achievers from both groups. Both students fell into conceptual mistakes since they could not explore the connection between the sides. Their mistakes showed weak mental thinking and spatial sense.

The fourth item states the following: Tell whether it is always, sometimes, or never that the bases for companies C and D have the same sum of angles. Explain. The answer should be “always” since they are both triangles and the sum of angles for each of them is 180°.

Unfortunately, not all the six students chose the word “sometimes.” Some students chose the word “never.” To them, they perceived that if the two triangles were not congruent, then they had different angle measures and, hence, different sum of angles. Example:

> ***Student:*** *The base C is isosceles and the base D is equilateral. Therefore, they cannot have the same sum of angles.*

Other students chose the word “sometimes.” They wanted to work on geometrical properties such as the measures of the angles because they lacked understanding of mathematical concepts. Example:

> ***Student:*** *Since I do not know the measures of the angles, we might say yes and might say no.*

Those students need to re-identify geometrical properties based on spatial established guidelines as Laborde (2002) describes.

In general, there is deficiency in conceptual understanding related to several forms of knowledge.

Table 12 presents students’ results in the clinical interviews after the intervention.

After the Intervention			Response (out of 1)	Proof (out of 2)	Sum (out of 3)
High	Control Group	Part 1	1	2	3
		Part 2	1	1	2
		Part 3	1	2	3
		Part 4	0	0	0
	Experimental Group	Part 1	1	2	3
		Part 2	1	1	2
		Part 3	1	2	3
		Part 4	1	2	3
Middle	Control Group	Part 1	1	2	3
		Part 2	1	1	2
		Part 3	1	2	3
		Part 4	0	0	0
	Experimental Group	Part 1	1	2	3
		Part 2	1	1	2
		Part 3	1	2	3
		Part 4	1	0	1
Low	Control Group	Part 1	1	2	3
		Part 2	1	0	1
		Part 3	0	0	0
		Part 4	0	0	0
	Experimental Group	Part 1	1	1	2
		Part 2	1	1	2
		Part 3	1	1	2
		Part 4	0	0	0

Table 12 Scores of the students' responses in the clinical interviews after the intervention

After the intervention, the group score (out of 100) of the control group was 55.6% and the group score of the experimental group was 72.2%. These two values are calculated by adding the sums of the table 12 for each group and dividing the result by the total sum of the last column of table 12.

The following table shows how the qualitative method for the individual and group scores is interpreted quantitatively after the intervention (table 13).

After the Intervention		**Individual Score** (out of 100)	**Group Score** (out of 100)	
High	Control Group	Control Group	Control Group	55.6%
	Experimental Group	91.7%		
Middle	Control Group	66.7%		
	Experimental Group	Experimental Group	Experimental Group	72.2%
Low	Control Group	33.3%		
	Experimental Group	50.0%		

Table 13 Individual scores of the students' responses in the clinical interviews after the intervention

4.12 Comparing the clinical interviews' results of the control and experimental groups

Before the intervention, the control group provided responses and proofs regardless of their relative quality to the problem. On the other hand, only the high achiever of the experimental group offered responses and proofs. Further, the other students in his group provided wrong responses and showed weak induction for proving. Hence, the control work earned a group score of 44.4% and the experimental group earned a group score of 33.3% (table 11).

The responses of the students to the problem after the intervention created opposite results. For example, the control group did not work well on the fourth part of the problem while two students from the experimental group worked well on it. Also, the low achiever of the control group worked on two parts of the problem while the low achiever of the experimental group worked on three parts of the problem. Hence, the control work earned a group score of 55.6% and the experimental group earned a group score of 72.2% (table 13). In addition, each individual score of the experimental group is significantly higher than the score of the student of the same level of achievement of the control group (table 12).

Consequently, although the control group's results were higher than those of the experimental group before the intervention, the experimental group's results were higher than those of the control group after the intervention.

Chapter Five

Discussion and Conclusion

The current research aimed to study the effect of using Scratch software with grade-6 students to investigate its effect on their geometric problem-solving skills. This research was conducted in a private school, where real-world activities were implemented in one of two classes using Scratch, with the other class being the control group.

A pretest and a posttest were carried out with both groups in order to check the level of enhancement in the problem solving skills of the control and experimental groups.

Furthermore, clinical interviews before and after a four-day intervention were set for diagnostic purposes before and for qualitative analysis. The interviews were conducted with three students from each group with different levels of achievement. This chapter discusses the results based on the two research questions.

5.1 Discussion about problem solving skills

Margaret (2016) declares that “in order for students to be able to develop effective problem solving skills, they must develop both analytical and creative skills.” (p.1). Researchers differentiate between effective and non-effective problem solving. Dowshen (1980) defines an effective problem-solver as an individual who uses a wide range of “heuristic strategies.” Francis (1990) discusses the characters of an effective problem-solver and determines four distinct areas: strong motivation, positive disposition, powerful self-image, and developed skills. Found and Hughes

(2016) characterize three key elements to effective problem solving: critical thinking, motivation, and knowledge. Besides, knowledge base and transfer of knowledge are the most important elements of problem solving (Carson, 2007).

Lappan and Phillips (1998) established ten criteria for effective problem solving situations in mathematics. The problem must have useful mathematics embedded in it and can be approached in multiple ways. It also requires higher-level thinking, contributes to the solver's conceptual development, creates an opportunity for assessing learning, encourages student engagement and discourse, connects to mathematical ideas, promotes the skillful use of mathematics, allows different decisions to multiple solution strategies, and provides an opportunity to practice important skills.

The current study aims to examine the effect of the Scratch software on improving problem solving of students of elementary school math.

First Research Question:

The first research question is the following: How does Scratch programming improve grade-6 students' problem-solving skills?

The quantitative investigation shows there is no significant difference between the two groups in the pretest (Sig. = 0.366, table 7) and posttest (Sig. = 0.433, table 8). On the other hand, comparing the results in the problem solving section from the pretest to the posttest, there was an increase of 29.6% for the experimental group and 23.5% for the control group (table 6). Nevertheless, there is a significant difference in the problem solving in general when comparing the pretest and posttest (Sig. = 0.000, table 9).

Also, the quantitative study detects a significant increase for the experimental group with an increase of 17.7% which is higher than the control group's increase of

3.3% after the intervention (table 6). Further, the total average of the experimental group in the posttest was 74.8% while it was 52.3% in the pretest (table 6). On the other hand, the total average of the control group in the posttest was 54.5% while it was 21.5% in the pretest (table 6).

Using the qualitative method, problem-solving skills were examined using the clinical interviews. To score the students' work, one point was assigned to a correct answer, one point to the appropriate explanation, and two points to a powerful explanation provided in the clinical interviews.

Comparing the tables 10 and 12, the group score of the control group shifted from 44.4% (table 11) to 55.6% (table 13) while the experimental group shifted from 33.3% (table 11) to 72.2% table 13) with a distinguished progress for the two students (middle and low levels of achievement). It seems that the qualitative thinking skills of experimental group students were more developed in the interviews after the intervention.

Overall, the study reveals a clear improvement for the experimental group in problem solving compared with the control group.

Second Research Question:

The second research question is the following: How does Scratch strengthen students' geometric knowledge, conceptual understanding, and skills?

To answer this question, each ability of the three abilities of the research question is to be put in the spotlight.

5.2 Geometric knowledge in the spotlight

This research seeks to understand the students' learning process of elementary geometric knowledge. Quantitatively Speaking, it was found that there was a significant difference of 0.001 in the knowledge score in the pretest (table 7), but no statistical significant difference in geometric knowledge between the two groups in the posttest was found (table 8). This reveals that knowledge was gained sufficiently for the experimental group compared to the control group before the intervention. On the other hand, the improvement in the *knowledge* section from pretest to posttest was 17.8% for the control group and 13.1% for the experimental group (table 6).

In general, when students from both groups were required to solve the problem before the intervention, they struggled because the problem requires them to investigate and understand the general relationships between cutting pizzas with different shapes into slices and the angles of the carton pieces to find the types of the circular sectors that are represented by the bases. This suggests that the students lack problem-solving abilities.

From a qualitative approach, responses in the interviews showed that there was an ability to progress obtaining knowledge and basic information about triangles in a real-life situation especially for the control group. When constructing triangles, students need to have knowledge about angles. The students' knowledge of angles is essential for the control group to use their tools appropriately and to the experimental group to control the sprite's direction and rotation in Scratch. The students in the experimental group constructed different triangles using the right codes. Hence, it was obvious that this construction with a dynamic software had improved their geometric knowledge and reasoning relations about triangles. This result resembles what Helen (2004) concluded that software users surpassed non-software users even when prior knowledge was not taken into consideration.

Comparing the results of the two groups before the intervention qualitatively, all the students, except for one student B15 (high achiever from the experimental group), got confused when the second part of the problem presented two round pizzas but with different sizes. They could not tell that the sum of angles was the same as the full turn of the circle, 360°. They thought that the size of the pizza would affect the result or thought that the number of the customers for each pizza would change the answer.

The problem after the intervention portrays models that consist of boards with different sizes and colors to display company posters at an exhibition. Again, the fourth part of the problem asks for the sum of angles of the triangular bases of the models. The experimental group (except for the low achiever) got the right answer, which is 180°, while not all the three students of the control group could arrive to the right answer. Although the problem describes models, the control group does not have mental models that can be functioned to solve the problem at hand. Thus, the students had insufficiencies in basic geometric knowledge such as dimensionality. Dimensionality is one of many characteristics of geometric knowledge. It is not a matter of providing names and definitions to students to enable them to construct their geometric knowledge. Memorizing names and definitions does not offer advantage to a utilitarian geometric knowledge. Those students need experience with two- or three-dimensional models to build mental structures of the situations. Researchers suggest that using mental models motivates the students to solve problems in geometry. These mental models accommodate knowledge about concepts, how these concepts connect with each other, and how to employ these concepts (Inquiry-NYC, 1998).

Comparing the results of the control group before and after the intervention qualitatively, the control group failed to gain new knowledge about the sum of angles of a triangle. On the other hand, the experimental group applied their knowledge about the sum of the interior angles of the triangles. It is clear that the experimental group exceeded the control group in the geometric knowledge in such manner.

Table 7 shows that the one group has a higher achievement level in geometric knowledge, but lower achievements in conceptual understanding and problem solving in the pretest. That group was selected as the experimental group, since conceptual understanding and problem solving are the main outcomes that the intervention aims to affect, not the mere geometric, procedural knowledge that is rather based on more memorization and less thinking.

To solve problems, the accentuation should be on implementing activities that help students move progressively, through complex levels of knowledge, from recognizing a shape to using its properties. The activities that are practiced by the students using Scratch allowed for a physical manipulation of figures, and thus the students have developed their knowledge about sum of angles. Such activities promote the students' building of mental models of the situation.

5.3 Geometric conceptual understanding in the spotlight

As mentioned in the literature review, conceptual understanding is one of the five strands of mathematical proficiency. Adding It Up (2001) defines conceptual understanding as “comprehension of mathematical concepts, operations, and relations.” (p.116).

The focus is on gaining conceptual understanding of angles and types of triangles. For the *Conceptual Understanding* section, the statistical data of the pretest results showed that there was no significant difference between the two groups A and B (table 5); however, there was a noticeable and substantial difference in the posttest results between those groups (table 7). It is clear that larger increases in the mean scores were noticed among the experimental group students. Further, the control group demonstrates a widespread lack of understanding of the concept of angle. The control group paid more attention to computational or operational aspects of a given notion rather than to its conceptual aspects. Additionally, the clinical interviews showed that the control group demonstrated weak conceptual understanding of geometric concepts. The control group encountered difficulties in the acquisition of angle concept, which is one of the most important concepts in geometry.

An angle is a figure formed by two rays, called sides, sharing a common endpoint, called vertex. In geometry, the angle arises in real space. Although the concept of angle in trigonometry is different from the concept of angle in geometry, the subject of trigonometry is built on the measurement of angles.

Comparing the results of the two groups from the pretest to posttest quantitatively, the experimental group showed an increase of 26.0% while the control group showed a decrease of 18.9% in the *Conceptual Understanding* section (table 6). Clinical interviews can provide comprehensively more information of conceptual understanding since oral clarifications and graphical representations can be collected, and explanations can be used where appropriate. Comparing the results of the clinical interviews (tables 10 and 12) qualitatively, both groups gained conceptual understanding in general. However, the gain of the experimental group was significantly higher than that of the control group. For example, the control group justified their different answers about identifying the types of angles (acute, obtuse, or right) in the problem before the intervention regardless of the correctness of the answer. Furthermore, they (except for the lower achiever) identified the types of the triangles in the problem after the intervention. On the other hand, the experimental group (except for the higher achiever) could not provide powerful justification in the problem before the intervention, but they all offered sound justification for the problem after the intervention.

Looking at this overview, the control group showed poor conceptual understanding, while the experimental group's gains were significantly higher. That probably means that group B, learning with Scratch software, applied mathematical ideas into new situations, which matches what Sabaté and Raig (2013) indicate that conceptual understanding allows a student to apply and "adapt some acquired mathematical ideas to new situations." (p. 17). Students of the experimental group understood anticlockwise and clockwise turns and generalized their understanding when they knew how the values of degrees matched the direction of the movement because they knew the mathematical principles. They benefited from their understanding when they figured out the value of the third missing angle. When a student "possesses conceptual mathematics knowledge understands the meaning and underlying principles of mathematics principles." (Frederick & Kirsch, 2011, p. 94).

On the other hand, poor conceptual understanding about angles was revealed in the control group because students had difficulties in perceiving the angle concept. Generalizations are formed by "abstraction from experience." (Boas, 1981, p. 727). As a strategy, students use generalization in the conceptual understanding as they solve new similar problems using previous solutions in new situations (Hashemia, Abua, Kashefia, & Rahimib, 2013). The control group has not gone through this process because they did not use methods of mathematical reasoning to make generalizations about the angles. They faced conceptual and rational difficulties when they consided that cutting the pizza into pieces, to form the angles, is related to the number of customers who share it or to the size of each piece of it. They also could not figure out the type of the base of the model because they did not explore the concept of the triangles and their characteristics. As a result, destitute conceptual understanding about angles was revealed in the control group students. If students encounter conceptual or logical difficulties, they require more than just practice.

It is worth to mention that, although problem solving and conceptual understanding are related, the students in the experimental group got statistically significant difference concerning conceptual understanding but not problem solving. This may be interpreted by the fact that problem-solving capability is a more complex construct that includes understanding of all mathematical concepts involved, together with other abilities such as control, strategic thinking, and connections between mathematical operations and concepts. It needs more time and practice to be significantly developed.

5.4 Geometric cognitive skills in the spotlight

Skills are not only necessary for students' development, but also influenced by other capabilities (Sarzosa, 2015). Cognitive skills are mental capabilities needed for learning, thinking, imagining, visualizing, remembering, reading, reasoning, perceiving, conceiving, judging, and paying attention. Non-cognitive skills, defined as personality and stimulus are traits that determine the way learners reflect, sense, and perform (Borghans, Duckworth, Heckman, & Weel, 2008). Eisenhart (1988) assumes that the development of cognitive skills is central to human development.

Statistically speaking, tables 7 and 8 show that there is no significant difference between the two groups concerning the *Skills* section. Sig = 0.282 and F = 1.183 in the pretest (table 7). On the other hand, although Sig = 0.970, F = 0.01 in the posttest (table 8). Since Levene's test shows that the usual F statistic provides significance level where α = 0.01 and 0.05, then the null hypothesis can be said to be rejected. Comparing the results of the groups, the quantitative study showed that the improvement from the pretest to the posttest in the skills section had decreased 11.0% for the control group, but increased 6.7% for the experimental group (table 6).

Nevertheless, the qualitative analysis clearly revealed a number of indications of visualization and reasoning. The teacher said that she noticed that the students in the experimental group formed assumptions about the way of constructing triangles by using compass and ruler because they were taught by using the dynamic software. They also organized their ideas and solutions. Her descriptions were similar to what Cherowitzo (2006) outlines that such students' working on geometric shapes could enrich their reasoning in geometric construction. These records ensure what Duval (1998) has indicated that the geometric thinking involves the cognitive skills of visualization and reasoning. Visualization and reasoning are important mental skills necessary for mathematics (Battista, Wheatley & Talsma, 1989). When working with Scratch, the visual blocks and interactive images, the experimental group students created new triangles as they move the dynamic vertices, and each time they recognize a new shape and explore its relationships, especially the automatic sum of its angles. That is one element of visualization. Visualization is a skill that helps students to recognize shapes, to create new shapes or objects, and to reveal relationships between them (Arcavi, 2003). Arcavi (2003) states that visualization, "as both the product and the process of creation, interpretation and reflection upon pictures and images," (p. 215) provides visibility to math education. It can be claimed that the geometric constructions improved the physical and cognitive mathematical skills about elements of triangles for both groups and especially the experimental group students because their drawings were more effective with the use of the dynamic geometry software. Pea (1985) states that students will improve their cognitive skills to achieve many cognitive processes when they use technology. The experimental group had the capability of recognizing different types of angles. That means they developed their cognitive skills to identify the type of the angle. Scratch took over the cognitive process of constructing and positioning the triangle according to the entered codes as the students focused on how to use the blocks *move* and *turn* from Motion blocks. The students also drew geometric objects (different types of triangles). This would enable them to expand their cognitive processes to develop the required concepts and skills of the four lessons in the intervention of the current study.

5.5 Conclusion

The current study concurs with earlier studies conducted by researchers in the field of promoting geometric problem solving skills with dynamic software. For example, Scratch proved to be a practical tool in constructing triangles, allowing students for using the features of dynamic geometry environments (Förster, 2015). In a study, students had the ability to represent their understanding of the properties of geometric solids in Scratch environment (Boyer, 2010). The process of producing Scratch projects is incorporated with problem solving skills. To solve a problem using Scratch scripts, students need to have logical thinking so that they can develop their problem solving skills. The students are required to evaluate the problem, construct a coding script, and then modify the script to resolve the possible errors. These problem-solving skills are key characteristics of mathematics and essential for everyday life.

The high and middle achieving students in the experimental group reflected more improvement in the posttest compared to the control group especially in constructing triangles. It is concluded that geometric construction activities improved knowledge about forming geometric shapes, geometric understanding, and psychomotor and cognitive skills (Cherowitzo, 2006). This conclusion provides elements of answers to the second research question. This study can arrive to a result that Scratch software is a tool that enhances mathematical processes. It comprises of game construction that improves cognitive development and visual coding that improves basic programming concepts.

5.6 Limitations of the study

This study involved a small sample size. With a small sample, there might be a potential for considerable sampling error. This factor would limit the extent to which the findings and conclusions could be generalized to a bigger target population in which classrooms use Scratch software. The fact that the sample was in only one school could bias the study results. One of the other limitations of the study was the duration of the intervention, which was relatively a short time. The amount of time given to cover the problem-solving abilities was also limited.

5.7 Future research

This study showed that dynamic coding activities benefited students in problems that require geometric knowledge, conceptual understanding, and cognitive skills. Likewise, future studies should use more emphasis on quantitative and qualitative analyses of a larger sample and of a longer duration than this study to gain more substantial information regarding the skill acquisition of problem solving when students are working on Scratch projects.

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Appendix A - Pretest

Pretest on Triangles

Name: ____________________ Date: _____________

Grade 6, section: _____ Time: _____________

KNOWLEDGE

Write the right answers in the blanks.

1. A triangle is made up of line segments called ______.
2. A triangle has ______ sides.
3. A triangle has ______ angles.
4. A right angle is an angle whose measure is exactly ______.
5. The sum of the angles of a triangle is ______.

CONCEPTS

I. Complete the statement using *always*, *sometimes*, or *never*.

6. Vertical angles are _____________ congruent.
7. Intersecting lines are _____________ perpendicular.
8. When a line intersects two parallel lines, corresponding angles are

   _____________ congruent.

II. Solve the following:

9. **Which has a greater measure, an angle complementary to an angle measuring 15° or an angle supplementary to an angle measuring 125°? Explain.**

____

____

____

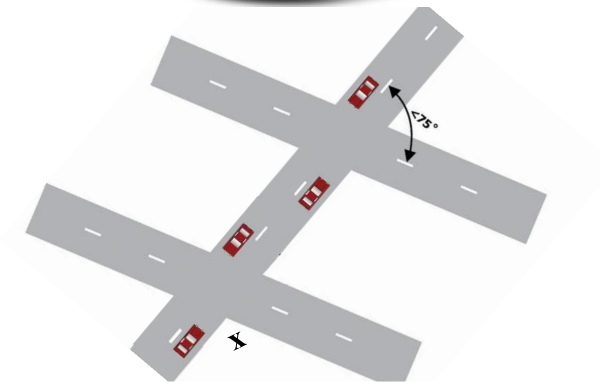
SKILLS

10. Draw two complementary angles and write their measures.

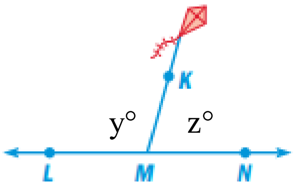
PROBLEM SOLVING

Show your way and explain your reasoning.

11. A player threw a dart at a target. He hit the bull's eye. The dart and its shadow on the dart board formed an angle 70°. What is the value of x?



12. Two streets intersect a main road. The map shows the angle 75° between a street and the road. What is the value of x?



13. The line of a kite is tied to the ground as shown. y = 3x and z = 2x. Find the measures of y and z.

Appendix B - Posttest

Posttest on Triangles

Name: ______________________ Date: _______________

Grade 6, section: ______ Time: _____________

KNOWLEDGE

Write the best word to complete each sentence.

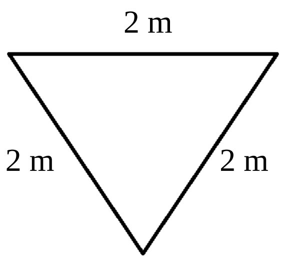
1. A triangle with 3 sides of equal lengths is a(n) ___________________ triangle.

2. A triangle with a right angle is called a a(n) ___________________ triangle.

3. A(n) ___________________ triangle has at least two sides that are the same length.

CONCEPTS

Part I. Classify each triangle.



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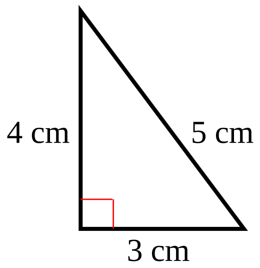

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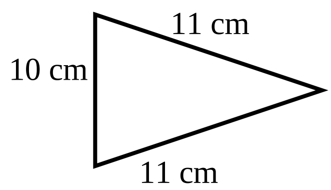


6. ____________

Part II

7. Is an equilateral triangle also an isosceles triangle? Explain.

8. Can a triangle have two right angles? Explain.

Part III

9. Can an isosceles triangle be obtuse?

10. Use a diagram to show.

SKILLS

Part I

11. Draw an isosceles triangle with one angle of measure 70°.

12. Explain the steps of your drawing.

___

___

___

___

Part II

13. Draw a scalene right triangle.

14. Explain the steps of your drawing.

___

___

___

PROBLEM SOLVING

15. A triangular flag contains a right angle and an angle that measures 41°. What is the measure of the third angle of the flag?

____

____

____

16. Find the angles at the base of an isosceles triangle, if the angle opposite to the base has the angle measure of 40°.

____

____

____

Appendix C - Clinical Interviews Before the Intervention

Section A

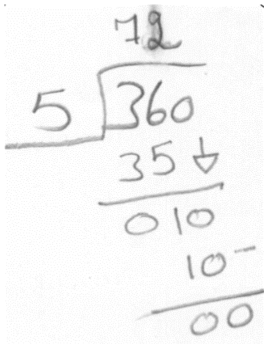
Student A8: Problem 1

Student*: For the first table, we divide 360 by 3 to get 120. For the second table, we divide 360 by 5.*

Interviewer*: Why did you divide 360 by the numbers?*

Student*: Since the pizza is round.*

The student worked on calculations on a piece of paper to solve the second part of the problem.



Student*: We get 72. Therefore, we have obtuse and acute slices.*

Interviewer*: Which table has the obtuse slices and which table has the acute slices?*

Student*: The obtuse slices are for the first table and the acute ones are for the second table.*

Student A8: Problem 2

Interviewer*: Do you agree with the waiter?*

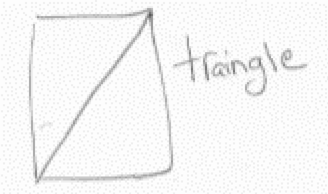
Student*: Yes, I do.*

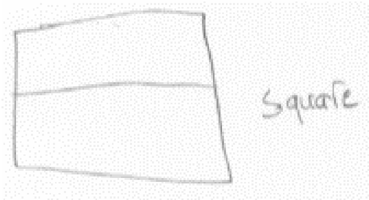
Interviewer*: Why?*

__Student__: Because at table 3, there are four people while at table 4, there are six people.

Student A8: Problem 3

The student used pencil and paper to express his ideas. However, instead of saying *rectangle* for the second part, he said *square*.





Student A9: Problem 1

The student was reading the problem loudly.

Student: *The first pizza has to have obtuse-angled slices.*

Interviewer: *Tell how you solved it.*

Student: *It would be logically enough for three friends to share the pizza!*

__Interviewer__: So, how are you going to solve the second part of the problem?

__Student__: The second pizza has to have acute-angled slices.

__Interviewer__: How do you know?

Student: *Because if they were obtuse, the friends would not take slices of equal sizes.*

__Interviewer__: Describe what they will get.

__Student__: They will get small pieces.

Student A9: Problem 2

__Interviewer__: So, do you agree with the waiter?

__Student__: Yes, I agree with him.

__Interviewer__: What made you take such decision?

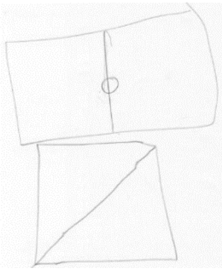
Student: There are different sizes of the pizzas. One is medium and the other is large.

Student A9: Problem 3

The student was silent and thinking.

***Interviewer**: Tell me about what you are thinking of.*

The student drew what he understood. He drew two figures and marked the center of each figure.



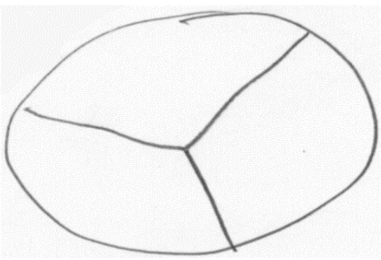
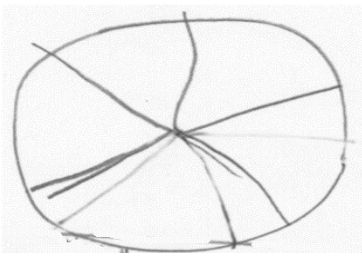
***Student**: Both pizzas have equal slices.*

***Interviewer**: Why did you say that?*

***Student**: Because we divided them from the centers. The cut passed the center in each pizza and divided each one into two parts.*

Student A22: Problem 1

The student drew a circle and divided it into three parts and another circle divided into five parts.

***Student**: The answer is obtuse for the first table and acute for the second table.*

***Interviewer**: How do you know?*

***Student**: I used my index finger to identify the angle measures.*

***Interviewer**: Explain more, please.*

***Student**: If my index finger is along with the extension of the angle is more than 90°, then the angle is obtuse. If it is less than 90°, then it is acute.*

Student A22: Problem 2

***Interviewer**: Do you agree with the waiter?*

***Student**: Yes.*

***Interviewer**: Why yes?*

***Student**: Because…*

***Interviewer**: Because what?*

***Student**: Because this pizza is smaller than the other one. The slice of the first one is similar to a big triangle and the size of the smaller pizza is similar to a small triangle. (He drew the two triangle-shape slices.)*

***Interviewer**: Does the problem ask you about this?*

***Student**: Yes. (He drew a piece of carton for each slice. Each piece is a rectangle shape). So, for sure, they are different.*

***Interviewer**: Different what?*

***Student**: Different size.*

***Interviewer**: How did you know that the problem talk about the sizes?*

***Student**: It is mentioned here that there are medium and large sizes.*

***Interviewer**: What does the difference of sizes lead you to understand?*

***Student**: Wait, please. (He reads again.) Ah, if the problem talks about the sum of angles, the sizes are not needed.*

***Interviewer**: Why did you change your mind?*

Student: *The pieces of cartoon that are under the slices have to be the same size to fit the slices.*

Interviewer: *Even if the slices are of different size?*

Student: *Yes.*

Interviewer: *What is the shape of each piece in this case?*

Student: *Square.*

Interviewer: *Why square?*

Student: *Because each piece has four right angles.*

Interviewer: *If each piece has four right angles, does that imply that it has to be a square?*

Student: *Yes. It would not be a round shape since we have angles.*

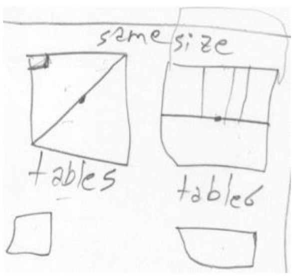
Interviewer: *If there are angles, should it be a square?*

Student: *Yes, because we have right angles where the slices meet by the cut.*

Student A22: Problem 3

Interviewer: *So, what do you say?*

The student drew the two illustrations.



Student: *For the first pizza, the piece should be a square-shaped because there is a right angle at the corner. For the second pizza, the piece should be a rectangle-shaped because if it were a square, it would not fit the slice, or it might be a square, but with a larger size.*

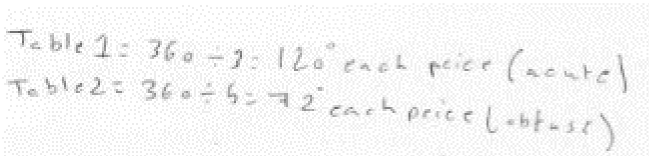
Section B

Student B15: Problem 1

Interviewer: *Explain you way, please.*

Student: *Since the pizza is round, its sum of angles is 360°. I divide 360 over the number of persons at each table. The result tells me the type of each slice.*

The student wrote his responses.



Student B15: Problem 2

Interviewer: *So, what do you think?*

Student: *I am against the waiter.*

Interviewer: *Why not?*

Student: *Because both of them have 360°.*

Interviewer: *Even if they have different sizes?*

Student: *Even so.*

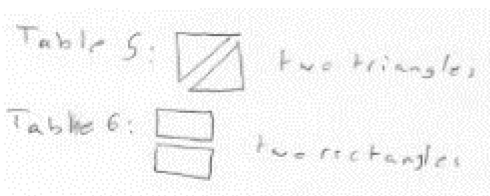
The student wrote his responses.

Student B15: Problem 3

Interviewer**:** *So, what do you say?*

Student: *I have to make the cutting.*

The student represented his way.



Interviewer**:** *And what do you get?*

Student: *For the first table, we will get two triangles. For the second table, we will get two rectangles.*

Student B13: Problem 1

Student: *The problem is about how we cut the pizzas.*

Interviewer*: What are you going to do?*

Student*: Measure the pizza.*

Interviewer*: Why are you going to measure it?*

Student*: In order to make a carton to let it fit inside it.*

Interviewer*: How are you going to measure it?*

Student*: With a protractor.* (After thinking a little bit), *It will be neither obtuse nor acute.*

Interviewer*: Why is that?*

Student*: Because the pizza is 360°.*

Student B13: Problem 2

For the second problem, he offered two contradicted answers.

Student*: The waiter is right and wrong in the same time.*

Interviewer*: You mean you have two answers, right?*

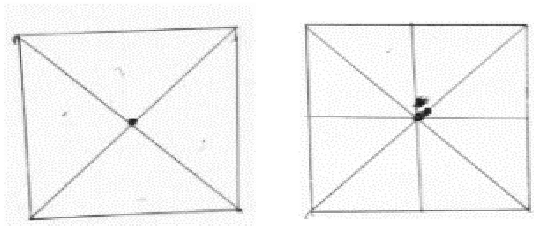
Student*: Yes.*

Interviewer*: How is that? Explain more, please.*

Student*: It could be, because the box sometimes is as the same size of the pizza and sometimes is bigger than it.*

Student B13: Problem 3

For the third problem, he drew the centers and then made the lines that represented the cuts showing four triangles for the first part and eight triangles for the second part.



Student B6: Problem 1

The student was confused a little bit.

Interviewer: *By what can you find the solution: by writing, by drawing, or by thinking? What are you going to do? What is your first step to do?*

Student: *I measure the pizza.*

Interviewer? *Measure what?*

Student: *This one.*

Interviewer: *Measure the graph?*

Student: *Yes, the drawn graph on paper.*

Interviewer: *How are you going to measure it?*

Student: *With a protractor.*

The student could not make the measurement.

Student: *Each piece in each table would have an acute angle.*

Interviewer: *How did you know?*

Student: *I imagined them. It will not work out with different sizes because if we make the cuts, each slice's angle is less than 360°.*

Student B6: Problem 2

Interviewer: *So, do you agree with the waiter or not?*

Student*: I agree with him.*

Interviewer*: Why is that?*

Student*: Because the first pizza's size is medium and the second one's size is large.*

Student B6: Problem 3

In the third problem, He counted on his imagination reaching to an answer for the first part that the pieces are triangles and for the second part the two pieces are large squares (instead of saying two rectangles).

Appendix D - Clinical Interviews After the Intervention

Section A

Student A8: Problem 1

Interviewer: *What is the type of the base?*

Student*: It might be a right triangle or a right isosceles triangle.*

Interviewer*: Why do you have two choices?*

Student*: Because these two types are the only ones that have an angle with 90°. The triangle might have the angles 90°, 30°, and 60°, and another choice is 90°, 45°, and 45°.*

Interviewer*: What about the second part of the problem?*

Student*: It should be one choice: an isosceles right triangle.*

Interviewer*: Why now you have one choice?*

Student*: In the first part, we have only one angle that is 90°, which is why we have two choices. While in the second part, the second angle is identified, and hence the third angle can be concluded. Now we have 90°, 45°, and 45° for sure.*

Interviewer*: How did you know that the second angle is identified?*

Student*: Because they say here, it is half of the first angle 90°. Therefore, it is 45°.*

Interviewer*: And how did you know that measure of the third angle is 45°?*

Student*: I made a calculation.*

Interviewer*: You calculated what exactly?*

Student*: 180 – (90 +45).*

Interviewer*: What do you expect the answer be?*

Student*: 45.*

Student A8: Problem 2

Interviewer*: Are the bases the same type?*

Student*: Yes.*

Interviewer*: What are they?*

Student*: They are both isosceles triangles.*

Interviewer*: How do you know?*

Student*: They both have the same angle sums and same kind of boards: two same boards and one different.*

Interviewer*: What do you mean they have the same angle sums?*

Student*: They both have 180° as a sum of angles.*

Interviewer*: Are the bases the same size?*

The student starts writing the following:

$$B = 3m. + 4m. + 4m. = 11m.$$

$$C = 4m. + 5m. + 5m. = 14m.$$

Interviewer*: Why are you adding?*

Student*: I am trying to find the perimeter of each base.*

Interviewer*: Why are you doing this?*

Student*: To tell if the two triangles are the same size.*

Interviewer*: What do you conclude?*

Student: *No, the triangles are not the same size.*

Interviewer: *How did you know?*

Student: *They have different perimeter. Triangle C is bigger than triangle B.*

Student A8: Problem 3

Interviewer*: What do you think the type of base should be?*

Student*: It should be an equilateral triangle.*

Interviewer*: Can it be a right triangle?*

Student*: No way.*

Interviewer*: Why?*

Student*: Because it has equal sides and equal angles.*

Interviewer*: How do you know that it has equal angles?*

Student*: Because 180 ÷ 3 = 60, therefore each angle measures 60°.*

Student A8: Problem 4

Interviewer: *You have to choose one word: always, sometimes, or never.*

Student: *Never.*

Interviewer: *Why?*

Student: *The two triangles are different.*

Interviewer: *Explain more how different, please.*

Student: *One triangle is isosceles and the other one is equilateral.*

Interviewer: *How did you arrive at this conclusion?*

Student: *From the sizes of the boards. For C, two boards have the same size, therefore the triangle is isosceles. For D, three boards have the same size, therefore the triangle is equilateral.*

Student A9: Problem 1

Interviewer**:** *What is the type of the base?*

Student: *Right-angled triangle*

Interviewer: *Why?*

Student: *Because there is an angle that measures 90°.*

Interviewer: *What about the second part of the problem?*

Student: *It is a right isosceles triangle.*

Interviewer: *How do you know?*

Student: *Because the other angle is half the 90°, which is 45°. We can conclude the third angle.*

Student A9: Problem 2

Interviewer**:** *What do you say about B and C?*

Student: *B is an isosceles and C is an isosceles.*

Interviewer: *What does that imply to?*

Student: *They have the same size.*

Interviewer: *How did you know that?*

Student: *Since they are both isosceles, they have the same size.*

Interviewer: *Do they have the same sum of angles?*

Student: *Yes.*

Interviewer: *How do you know?*

Student: *Since they are both isosceles and each one has two same boards of the same length, they have the same sum of angles.*

Interviewer: *What do you think this sum will be?*

Student: *(After thinking a little bit) I mean, as long they are triangles, they have the same sum of angles.*

Interviewer: *So, you changed your mind. Can you provide an example?*

Student: *one triangle might have the angles 55°, 55°, and 70°.*

Student A9: Problem 3

Interviewer**:** *Can the base D be a right triangle?*

Student: *No way.*

Interviewer: *How come?*

Student: *If it works, it means that the boards would be red, green, and blue to get a right angle.*

Interviewer: *What is the relation between the colors of the boards and the angles?*

Student: *Because since all the boards are red, the base is equilateral, and it is impossible to be a right triangle.*

Student A9: Problem 4

Interviewer**:** *What do you say?*

Student: *Never.*

Interviewer: *Why?*

Student: *The base C is isosceles and the base D is equilateral. Therefore, they cannot have the same sum of angles.*

Interviewer: *You mean it is impossible, right?*

Student: *Yes. However, it is only possible if we changed one color of the boards, for example, we change the red color to blue color.*

Student A22: Problem 1

Interviewer: *Did you understand the problem?*

Student: *Yes. The type of the base is a right triangle.*

Interviewer: *How do you know?*

Student: *Because there is an angle that measures 90°.*

Interviewer: *Is there any other possibility?*

Student: *No, there is not.*

Interviewer: *Let us go to the second part of the problem.*

Student: *It will be remain a right triangle.*

Interviewer: *Why will it remain a right triangle?*

Student: *Since there is a 90°, the base will remain right regardless the measure of the other angles.*

Student A22: Problem 2

Interviewer: *Are the bases B and C the same type?*

Student: *Yes.*

Interviewer: *How do you know?*

Student: *I just guessed it.*

Interviewer: *Are they the same size?*

Student: *Yes.*

Interviewer: *How do you know?*

Student*: Since they are the same type, therefore they are the same size.*

Interviewer*: But you built your result on a guess.*

Student*: (No answer)*

Interviewer*: Do they have the same sum?*

Student*: Yes.*

Interviewer: *How do you know?*

Student*: Since they have the same type and size, therefore they have the same sum.*

Interviewer*: Again, you counted your responses on a guess.*

Student*: (No answer)*

Student A22: Problem 3

Interviewer: *Is there a possibility that this triangle is right?*

Student: *I have to measure them first to answer.*

Interviewer: *Measure what?*

Student: *Measure its angles.*

Interviewer: *What does measuring lead you to?*

Student: *If an angle is 90°, then the triangle is right, but if no angle is 90°, then the triangle is not right.*

Interviewer: *Is there a possibility that this triangle is an equilateral?*

Student: *Yes.*

Interviewer: *How come?*

Student: *Because it has three boards with the same length.*

Student A22: Problem 4

Interviewer: *What word do you choose?*

Student: *"Sometimes"*

Interviewer: *Why?*

Student: *Since I do not know the measures of the angles, we might say yes and might say no.*

Student B15: Problem 1

Interviewer**:** *What is the type of the base?*

Student: *Right-angled triangle*

Interviewer: *How do you know?*

Student: *Because there is an angle that measures 90°.*

Interviewer: *What about the second part of the problem?*

Student*: The second angle is half.*

Interviewer*: Half what?*

Student*: Half the 90°.*

Interviewer*: How much does this equal?*

Student*: 45°.*

Interviewer*: So, what is the type of the base?*

Student*: Isosceles triangle.*

Interviewer*: How do you know?*

Student*: It has two common angles.*

Interviewer*: How do you know that the third angle is common to the second angle?*

Student*: I calculated it. 45 + 45 + 90.*

Student B15: Problem 2

Interviewer**:** *Do the bases B and C have the same type?*

Student: *Yes.*

Interviewer: *Even if the colors of the boards are different?*

Student: *Yes.*

Interviewer: *Why is that?*

Student: *Each base has two similar kind of boards.*

Interviewer: *Do they have same size?*

Student: *No, they have different sizes.*

Interviewer: *How do you know?*

Student: *The colors of the boards are different.*

Interviewer: *Do they have the same sum of angles?*

Student: *No. They have different sum of angles.*

Interviewer: *Why not?*

Student: *Because they have different colors.*

Interviewer: *What is the relationship between the color of the boards and angles?*

Student: (thinking) *they both should have the same sum of angles.*

Interviewer: *Why do you say that?*

Student: *They are both triangles.*

Interviewer: *So, what is the sum of the angles for both base?*

Student: *180°*

Student B15: Problem 3

Interviewer**:** *Do you think the base D is a right triangle?*

Student: *No.*

Interviewer: *Why not?*

Student: *It must be an equilateral.*

Interviewer: *Explain the reason, please.*

Student: *All the three boards of the base have the same color.*

Student B15: Problem 4

Interviewer**:** *So, what do you say?*

Student: *The answer is "always".*

Interviewer: *Explain your answer, please.*

Student: *The two bases are both triangles.*

Student B13: Problem 1

Interviewer**:** *What is the type of the base?*

Student: *Right-angled triangle*

Interviewer: *How do you know?*

Student: *Because there is only one right angle.*

Interviewer: *What would the type of the base later on?*

Student: *Right isosceles*

Interviewer: *Can you explain how did you arrive at this conclusion?*

Student: *We have 90° and the second one is 45°. The third one is definitely 45°.*

Interviewer: *What would the sum be?*

Student: *80°*

Student B13: Problem 2

Interviewer**:** *Do they have the same type?*

Student: *Yes*

Interviewer: *What type is this?*

Student: *isosceles*

Interviewer: *Do they have the same size?*

Student: *No.*

Interviewer: *Why not?*

Student: *Because the length of the green board is bigger than the length of the blue board.*

Interviewer: *Do they have the same sum of angles?*

Student: *Yes.*

Interviewer: *How come?*

Student: *Because each triangle's sum of angles is always 180°.*

Student B13: Problem 3

Interviewer**:** *Can it be a right triangle?*

Student: Never.

Interviewer: *Why not?*

Student: *All the boards are red. The base must be an equilateral. It is impossible to be a right triangle.*

Student B13: Problem 4

Interviewer**:** *What word do you choose?*

Student: *They are both triangles, but ...*

Interviewer: *Explain more, please.*

Student: *The base C is isosceles and the base D is equilateral.*

Student B6: Problem 1

Interviewer**:** *What is your strategy to solve this problem?*

Student: *The base should be a right triangle.*

Interviewer: *Why did you say that?*

Student: *Because there is an angle that measures 90°.*

Interviewer: *How about the second part of the problem?*

Student: *It will remain a right triangle.*

Interviewer: *But there is a new angle mentioned now. The second angle is half the first one.*

Student: *Yes, even so. No matter the measure of the other angles, the triangle remains right as long as it has a 90°.*

Student B6: Problem 2

Interviewer**:** *Do the bases have the same type?*

Student: *Yes they do.*

Interviewer: *Why?*

Student: *Because the bases B and C have number of boards: Three.*

Interviewer: *Do they have the same size?*

Student: *No, they have different sizes.*

Interviewer: *How do you know?*

Student: *Base B has red, blue, and blue boards while base C has blue, green, and green boards. If we add them …*

Interviewer: *Add what?*

Student: *Add their widths. We get 11 meters for B and 14 meters for C.*

Interviewer: *Why did you make addition?*

Student: *Because we have three boards for each base. Each board has its own width.*

Interviewer: *Do they have the same sum of angles?*

Student: *They will have the same sum of angles. They will never be different.*

Interviewer: *Explain your conclusion, please.*

Student: *The triangular model has to become a standstill on the base.*

Student B6: Problem 3

Interviewer**:** *What is the type of the base D?*

Student: *It is a possibility that it is a right triangle.*

Interviewer: *Why is this possible?*

Student: *Because there is a right angle.*

Interviewer: *Is there a possibility that it is an equilateral?*

Student: *If it is not right, then it is possible to be an equilateral.*

Interviewer: *It has to be either right or equilateral. What do you choose?*

Student: *It has to be right since there is a possibility that it has a right angle.*

Student B6: Problem 4

Interviewer**:** *What do you say? Do C and D have the same sum of angles?*

Student: *Never.*

Interviewer: *Why did you say that?*

Student: *The base C has no red-color board and the base D has no blue or green colors. That is why they differ in the sum of the angles.*

Appendix E - Unit Plan (For the Experimental Group)

Subject: Mathematics
Strand: Geometry
Unit Name: Angles and Triangles
Grade Level: 6
Length of Unit: 4 Days

Standard:
Make geometric constructions with specific tools (compass and straightedge or dynamic geometric software.)

Unit Goals: Students will:
- establish facts about angle sums of a triangle.
- build an understanding about the classification of triangles.
- use appropriate tools to construct triangles and explore relationships.

Unit Big Ideas: Students will:
- understand that the sum of the interior angle measures of a triangle is 180 degrees.
- explore properties of triangles to solve real-world problems.
- draw triangles with definite properties.

Mathematical Process Standards:
- Attend to Precision.
- Use appropriate tools strategically.
- Make sense of problems and persevere in solving them.

Essential Questions:
- How can the measure of the angles of triangles be applied to real-world situations?
- How can triangles be classified by the lengths of their sides? By measures of their angles?

Vocabulary:

- **Angle**
  The space between two intersecting lines at a point where they meet. It is usually measured in degrees. In any triangle, the largest angle is opposite to the longest side.
- **Triangle**
  A three-sided plane figure with three angles. The prefix “tri” means three.
- **Acute Triangle**
  A triangle with all three angles are acute (less than 90 degrees).
- **Right Triangle**
  A triangle with one angle is equal to 90 degrees and the other two are acute angles.
- **Obtuse Triangle**
  A triangle with one angle is obtuse (greater than 90 degrees) and the other two are acute angles. Since a triangle’s angles must sum to 180°, no triangle can have more than one obtuse angle.
- **Equiangular Triangle**
  A triangle where all three angles are equal in measure. Since a triangle’s angles must sum to 180°, each angle is equal to 60°.
- **Scalene Triangle**
  A triangle with all three-side lengths are different in measure (unequal sides).
- **Isosceles Triangle**
  A triangle with two-side lengths are the same and one is different. Sometimes it is specified as having at least two sides of equal length.
- **Equilateral Triangle**
  A triangle with all three-side lengths are the same.
- **Interior Angle**
  The angle between two adjacent sides inward of a figure.
- **Exterior Angle**
  The angle between two adjacent sides extended outward of a figure.
- **Vertex**
  A point where sides of angle intersect.
- **Principal Vertex**
  The highest vertex of a figure.
- **Triangle’s Base**
  The lowest side of a triangle.
- **Hypotenuse**
  The longest side of a right-angled triangle. It is the opposite of the right angle.

Unit's Lesson Description

Number	Title	Specific Objectives	Materials	Prerequisite
1	Sums of Angle Measures	Explore the sum of the measures of the angles in triangles.	Scratch software	- Classify angles. - Add adjacent angles. Add non-adjacent angles.
2	Right Triangles	Construct a right triangle.	Scratch software	Classify angles.
3	Isosceles Triangles	Construct an isosceles triangle.	Scratch software	Identify the sides and vertices of a triangle.
4	Equilateral Triangles	Construct an equilateral triangle.	Scratch software	Identify the sides and vertices of a triangle.

Note: The students have already used Scratch and know how to use the needed tools.

Title: Sums of Angle Measures Period: 1

Objective: By the end of this period, students will be able to:
- explore the sum of the measures of the angles in triangles.

Materials: Scratch software
Method: Skill acquisition
Working Form: Whole-class and Individual Work
Prerequisite:
- Classify angles.
- Add adjacent angles.
- Add non-adjacent angles.

Procedures:

A ready-made file is done in Scratch. It is copied on all laptops in the lab.

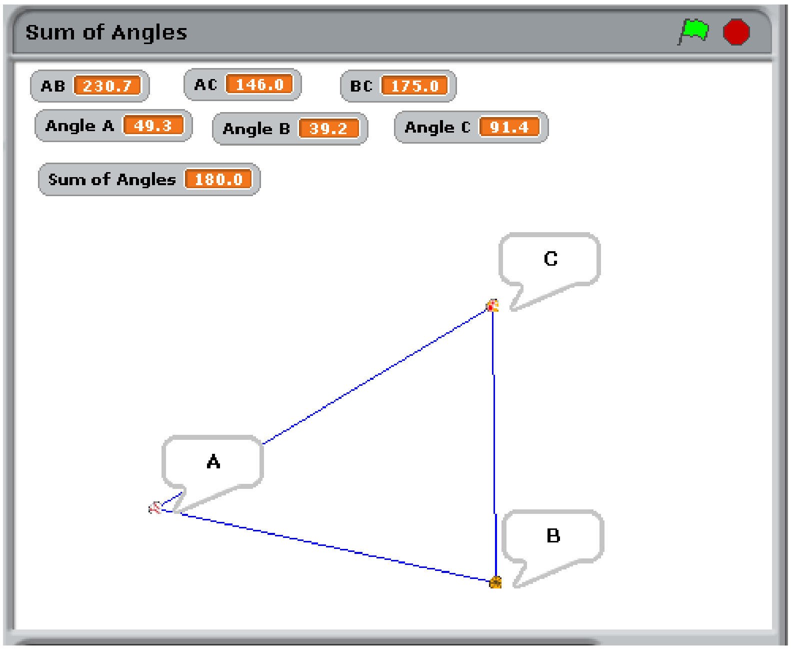
Each student works on a single laptop.

The teacher says:

A plane is traveling from Beirut airport to Cairo airport then to Paris airport. The flight destination forms a triangular shape with three angles. What is the sum of those three angles?

Have the class discuss the question.

Open the file to observe the following:



The class sees a triangle ABC. There are readouts for the side and angle measures and the measure of the three-angle sum shown on the stage.

The students can change the place of each vertex, and click the green flag at the right corner to check the changes each time.

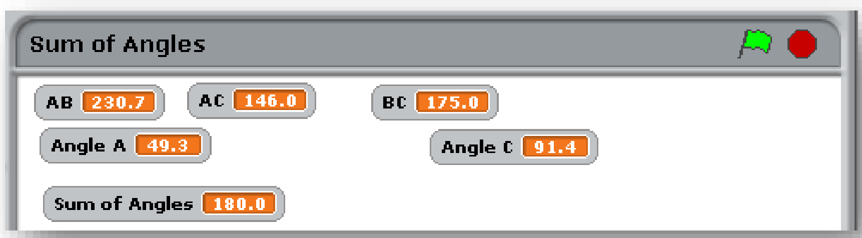
The students can see the length of each side, the measure of each angle, and the sum of the three angles.

They will discover that the sum will still be 180°.

Checkpoint:

The teacher asks the students to move the vertices of the triangle to change the measures of the angles. They then unselect one of the angles (Angle B) (at the check box) from the Variables area.

The readout of the unselected angle will disappear from the stage.



Ask: How can you figure out the missing angle value?

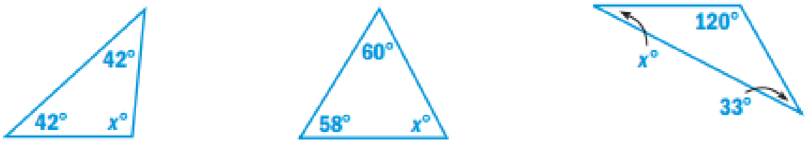
Allow for brain storming. Write the different answers on the board. Then re-select the missing angle in the program to let the class compare their answers with the exact one.

Ask for showing the work and the mathematical representation. In addition, ask for a formal formula to get the answer of a missing angle.

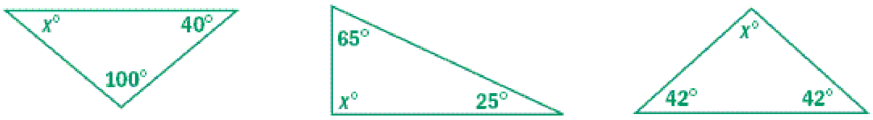
When the class gets the solution, ask the students to select the angle and unselect another one.

Formative Evaluation:

- Find the value value of ***x*** in each triangle.



Assignment Find the value of x.



Title: Right Triangles Period: 2

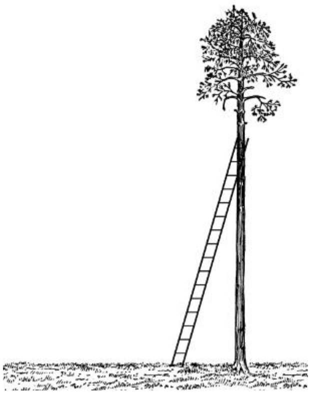
Objective:	By the end of this period, students will be able to: - construct a right triangle.
Materials	Scratch software - Pictures
Method:	Inductive
Working Form:	Whole class discussion and Individual work
Prerequisite:	Classify triangles.
Procedures:	

Look at the furniture in your classroom. Look at a specific place of the chair (pointing to a right angle). Look at the door (at a right angle). Find similar things.

What is common among them?

What measure do you think they have?

Now look at this picture.



Describe the picture. Describe the figure formed.

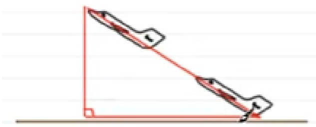
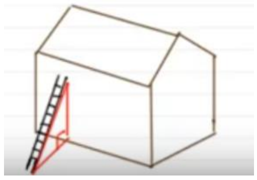
Give us examples from real life about right triangles.

Earnings

Glass ceiling

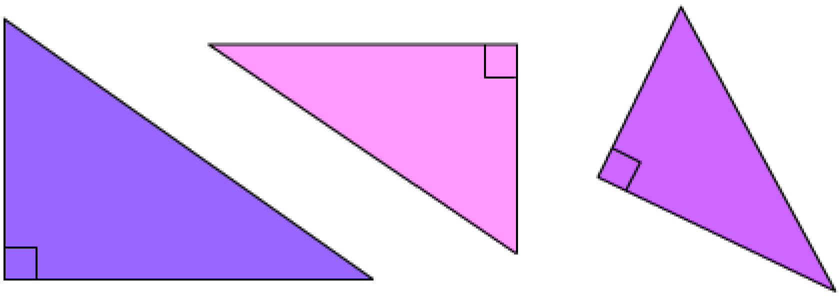
Sandwich

Now, look at these pictures and tell us how right triangles are formed from real-life examples.

Let's go back to the ladder bent on the tree trunk. The tree trunk and the ground are the legs. The ladder is the hypotenuse.

Tell us where the hypotenuse is in each of the following triangles.

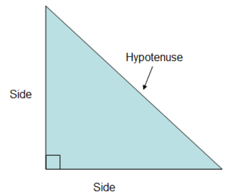


Give us at least two properties about the hypotenuse.

Allow time for the class to think.

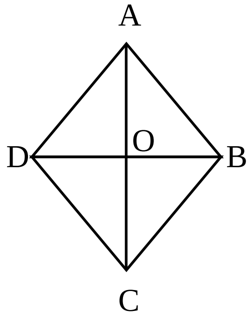
If no appropriate responses are given, give hints or ask questions about the hypotenuse location and length.

(It is opposite the right angle. It is the longest side of a right-angled triangle.)



Checkpoint:

- Name four right triangles with the hypotenuse of each.



Now, let us construct a right triangle using Scratch.

Basic Use

Using the moving and rotating options, you, class, tell us what tracking we would use from the blocks *move* and *turn* from Motion blocks.

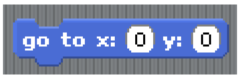
Allow some time to the students to think how to make a triangle with moving and turning. Turning is for making the angles. Remember, turning is always involves the exterior angle.

For making two sides and two angles, let the students think how to use the blocks.

Possible assembling:

If we start from a specific location (vertex), we go back to it in the final step of drawing a triangle.

If we start from the origin, we go back to it. Use the *go to* block from Motion blocks.



Secondary Use

For technical reasons, we will first take the following blocks into consideration:

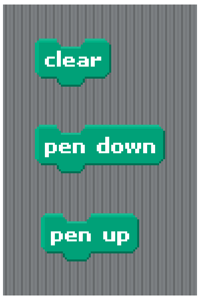
clear, *pen down*, *pen up* from the Pen blocks

hide from the Looks blocks

Clear block for starting again.

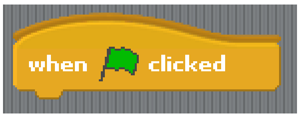
Pen down block for enabling the line visible.

Pen up block disables the line. The line becomes invisible.

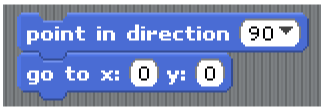


Hide block makes the sprite invisible.

We will use the “when the green flag clicked” from Control blocks to start the project.



We will also use two blocks from the Motion blocks

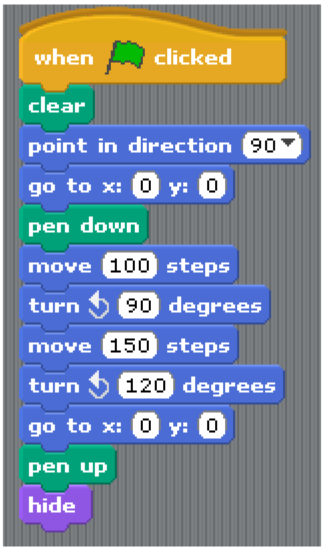


The first block makes the sprite always at a specific direction.

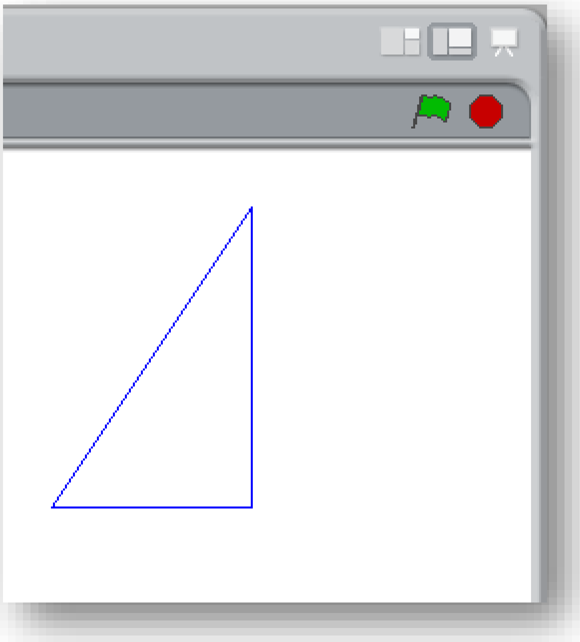
The second one starts the motion from a specific location.

Now, using the moving and rotating options, you, class, tell us what tracking we would use from the blocks *move* and *turn* from Motion blocks.

Possible assembling:



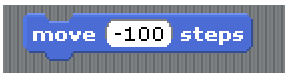
The stage will show the following right triangle:



Have students change the shape of the triangle each time.

They can use the number of steps of motions and the measure of the angles.

They also can use the minus sign as an opposite direction.



Appendix F - Unit Plan (For the Control Group)

Subject: Mathematics
Strand: Geometry
Unit Name: Angles and Triangles
Grade Level: 6
Length of Unit: 4 Days

Standard:
Make geometric constructions with specific tools (compass and straightedge or dynamic geometric software.)

Unit Goals: Students will:
- establish facts about angle sums of a triangle.
- build an understanding about the classification of triangles.
- use appropriate tools to construct triangles and explore relationships.

Unit Big Ideas: Students will:
- understand that the sum of the interior angle measures of a triangle is 180 degrees.
- explore properties of triangles to solve real-world problems.
- draw triangles with definite properties.

Mathematical Process Standards:
- Attend to Precision.
- Use appropriate tools strategically.
- Make sense of problems and persevere in solving them.

Essential Questions:
- How can the measure of the angles of triangles be applied to real-world situations?
- How can triangles be classified by the lengths of their sides? By measures of their angles?

Vocabulary:
- **Angle**
  The opening between two intersecting lines at a point where they meet. It is usually measured in degrees. In any triangle, the largest angle is opposite to the longest side.
- **Triangle**
  A three-sided plane figure with three angles. The prefix "tri" means three.

- **Acute Triangle**
  A triangle with all three angles are acute (less than 90 degrees).
- **Right Triangle**
  A triangle with one angle is equal to 90 degrees and the other two are acute angles.
- **Obtuse Triangle**
  A triangle with one angle is obtuse (greater than 90 degrees) and the other two are acute angles. Since a triangle's angles must sum to 180°, no triangle can have more than one obtuse angle.
- **Equiangular Triangle**
  A triangle where all three angles are equal in measure. Since a triangle's angles must sum to 180°, each angle is equal to 60°.
- **Scalene Triangle**
  A triangle with all three-side lengths are different in measure (unequal sides).
- **Isosceles Triangle**
  A triangle with two-side lengths are the same and one is different. Sometimes it is specified as having at least two sides of equal length.
- **Equilateral Triangle**
  A triangle with all three-side lengths are the same.
- **Protractor**
  An instrument used to measure angles.
- **Compass**
  An instrument used to draw arcs.
- **Interior Angle**
  The angle between two adjacent sides inward of a figure.
- **Exterior Angle**
  The angle between two adjacent sides extended outward of a figure.
- **Vertex**
  A point where sides of angle intersect.
- **Principal Vertex**
  The highest vertex of a figure.
- **Triangle's Base**
  The lowest side of a triangle.
- **Hypotenuse**
  The longest side of a right-angled triangle. It is the opposite of the right angle.

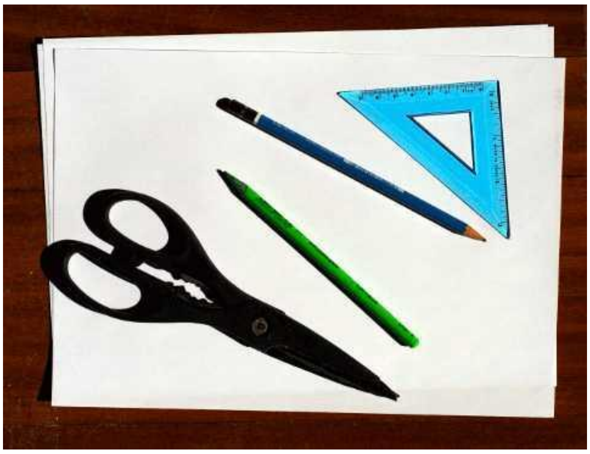
Unit's Lesson Description

Number	Title	Specific Objectives	Materials	Prerequisite
1	Sums of Angle Measures	Explore the sum of the measures of the angles in triangles.	Paper, Pencil, Ruler (Straight Edge), Crayons, Scissors	- Classify angles. - Add adjacent angles. - Add non-adjacent angles.
2	Right Triangles	Construct a right triangle.	Colored Markers, Rulers, Set Square, Compass	Classify angles.
3	Isosceles Triangles	Construct an isosceles triangle.	Ruler, Pen, Compass	Identify the sides and vertices of a triangle.
4	Equilateral Triangles	Construct an equilateral triangle.	Ruler, Pen, Compass, Protractor	Identify the sides and vertices of a triangle.

Title: Sums of Angle Measures Period: 1

Objective: By the end of this period, students will be able to:
- explore the sum of the measures of the angles in triangles.

Materials: Paper, Pencil, Ruler (Straight Edge), Crayons, Scissors

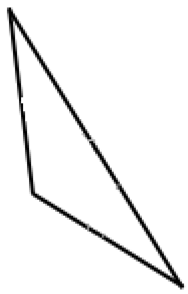


Method: Inductive
Working Form: Group Work
Prerequisite:
- Classify angles.
- Add adjacent angles.
- Add non-adjacent angles.

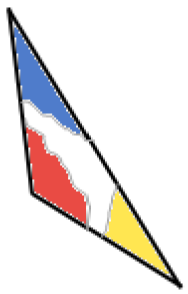
Procedures:

Draw any sort of triangle you like on a sheet of paper. Use the ruler or straight-edge to ensure the sides are straight. Color the sides if you want.

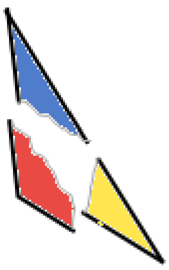
Cut out the triangle.



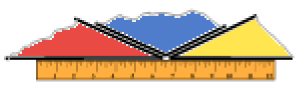
Mark the angles with three different colors.



Cut the corners off the triangle.



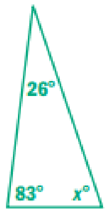
Draw a straight line on a sheet of paper and then assemble the corners on the straight line.



What type of angle do they appear to form? (straight angle).
Based on your work in this activity, what is the measure of the angles in a triangle? (180°)

Checkpoint:

- Find the value of ***x*** in the triangle shown.



$x° + 83° + 26° = 180°$ Sum of angle measures in a triangle is 180°.

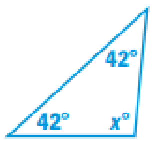
$x + 109 = 180$ Add 83 and 26.

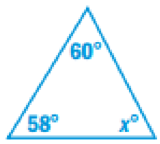
$x = 71$ Subtract 109 from each side.

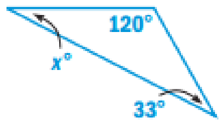
So, what is the value of x? (71°)

Formative Evaluation:

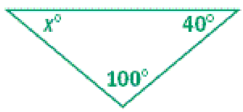
- Find the value value of ***x*** in each triangle.

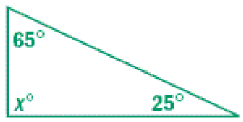


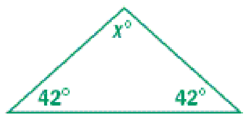




Assignment: Find the value of x.





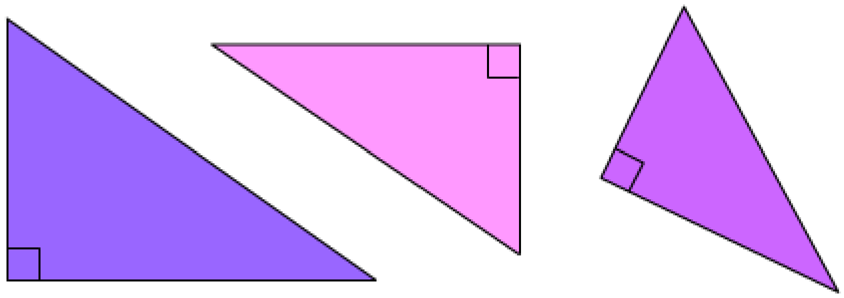


Title: Right Triangles Period: 2

Objective: By the end of this period, students will be able to:
- construct a right triangle.

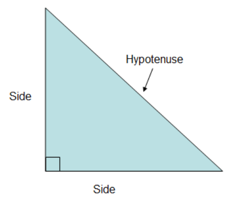
Materials: Colored Markers, Rulers, Set Square, Compass
Method: Inductive
Working Form: Whole class
Prerequisite: Name the sides and angles of a triangle.
Procedures:

Look at these triangles.



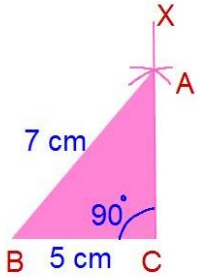
All are right triangles. They are right angled at the angle 90°.

Using Terms: Point to the right angle. (The teacher helps the class point to the right angle.)
Who can tell me which side is opposite to this right angle?
It is called ***hypotenuse***. It is the longest side of a right-angled triangle, opposite the right angle.



How can we draw a right angled triangle at C in which AB = 7 cm and BC = 5 cm? (Answers vary.)

1. Draw BC = 5 cm.
2. Make ∠ BCX = 90°.
3. With B as center and radius 7 cm, cut off BA = 7 cm.
4. Join AB.



Checkpoint:	•	Constrcut a right traingle ABC right at B and with BA = 4 cm and BC = 5 cm..
Evaluation	•	Let the class draw different right triangles based on different measures each time.

Title: Isosceles Triangles   Periods: 3

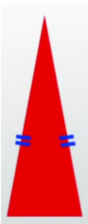
Objectives: By the end of this period, students will be able to:
- construct an isosceles triangle.

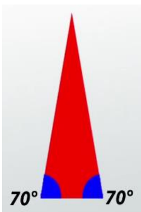
Materials: Ruler, Pen, Compass
Method: Inductive
Working Form: Whole class
Prerequisite: Identify the sides and vertices of a triangle.
Procedures:

Look at this triangle.

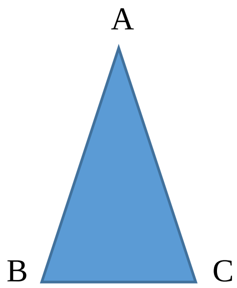
It is the isosceles triangle.

It is with two equal sides and equal angles.





Using Terms: Look at this isosceles triangle.



We will construct an isosceles triangle with given all side lengths.

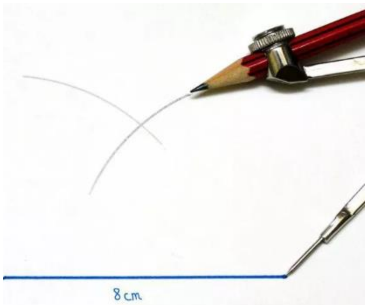
We will start from the base.

Let us say that the base is 8 cm long and each other side is 6 cm long.

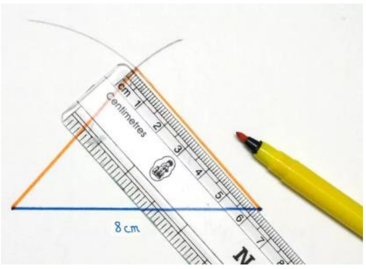
1. To draw the base, use a ruler and a pencil to draw a line 8 cm long.

2. Use the compass. Open it to the width that spans 6 cm on a ruler.

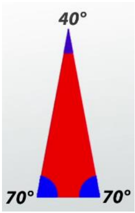
3. To draw an arc above the base, place the tip of the compass on one of the base's vertices. Then sweep the compass above the base. Make another arc from the second vertex. The two arcs cross each other at a point.



4. To draw the sides of the triangle, use a ruler to draw two lines. Each line connects a vertex to the point where the arcs intersect. Now, you have an isosceles triangle.



Evaluation:



- If each of the angles of the base of an isosceles triangle is 70°, what is the value of the third angle at the principal vertex? (40°)
- An isosceles triangle has one angle 70°. Find the measure of the other angles. *Note that students will get two solutions according to the location of the given angle.*
- PAL is an isosceles triangle of vertex P. Its perimeter is 56 cm. If one of the equal sides is 15 cm, find the length of the other two sides.

Using Terms: What would be an ***isosceles right triangle*** be?
A right triangle in which two sides and two angles are equal is called isosceles right triangle.

Checkpoint: Find without measuring, the value of each angle in an isosceles right triangle.

Title: Equilateral Triangles Period: 4

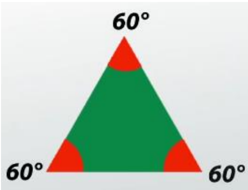
Objective:	By the end of this period, students will be able to: - construct an equilateral triangle.
Materials	Ruler, Pen, Compass, Protractor
Method:	Skill acquisition
Working Form:	Inductive
Prerequisite:	Whole class
Procedures:	

Look at this triangle.

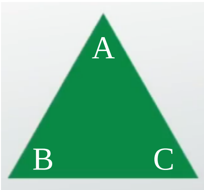
It's the equilateral triangle. It has three equal sides.

It also has three equal angles.

Each angle is 60°.



So according to the following triangle:



AB = AC = BC

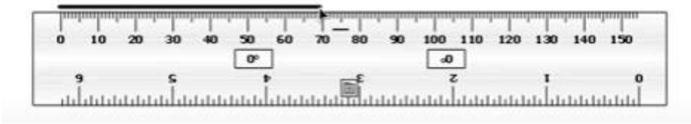
∠ ABC = ∠ ACB = ∠ BAC

Checkpoint:

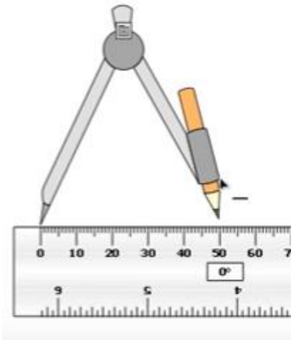
- The perimeter of an equilateral triangle is 48 cm. What is the length of its sides? (16 cm)

How can we draw a triangle whose sides are 7 cm, 5 cm, and 3 cm? (Answers vary.)

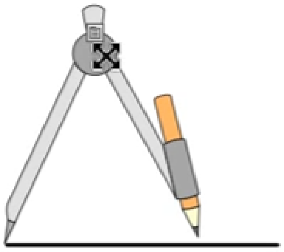
1. Use a ruler to draw a line of length 7 cm.



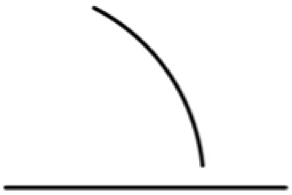
2. Open the compass to a radius of 5 cm.



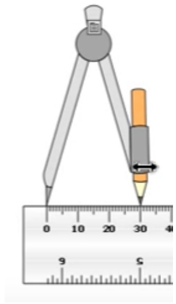
3. Place the compass needle at one of the ends of the drawn line.



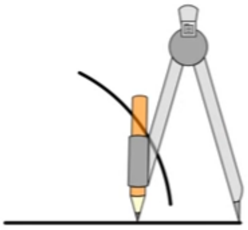
4. Draw an arc above the line.



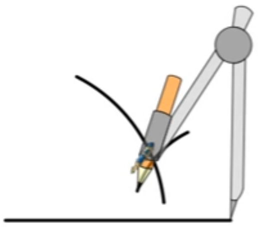
5. Open the compass to a radius of 3 cm.



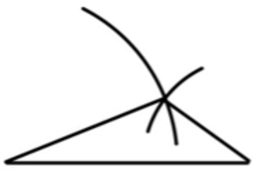
6. Place the compass needle at the second end of the line.



7. Draw a second arc above the line.



8. Join each end of the line to the point where the arcs cross.



Checkpoint:

- Using your protractor and ruler, construct an equilateral triangle with side = 6 cm.

Title: Isosceles Triangles Period: 3

Objective:	By the end of this period, students will be able to: - construct an isosceles triangle.
Materials	Scratch software – Pictures
Method:	Skill acquisition
Working Form:	Whole-class and Individual Work
Prerequisite:	
Procedures:	Name the sides and angles of a triangle.

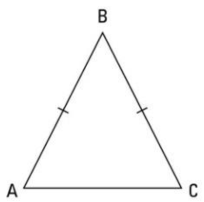
Problem:

Students are preparing for camping. To make a tent, they tie a rope around a tree with a wedge B fixed on the trunk. One student fixes the left side of the rope to the ground with wedge C. The other student fixes the right side to the ground with wedge B.

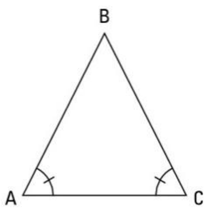
The distance from each ground wedge to the trunk's wedge is the same.

Sketch the figure BAC and tell if there a relationship between the angles A and C.

Allow some time to let the students draw the triangle and tell that it is an isosceles triangle. Have them brainstorm to tell you about the relationship between A and C.



They will guess that they are equal.



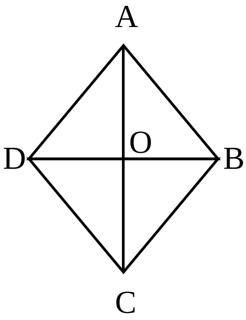
Using Terms:

- If we say that the principal vertex is the highest vertex of the shown triangle, what vertex is the principal vertex in the shown triangle? (B)

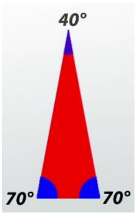
- If we say that the triangle's base is the lowest side of it, what side is the base in the shown triangle? (AC)

Checkpoint:

- Name two isosceles triangles with the base and principal vertex of each.



Evaluation:

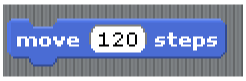


- If each of the angles of the base of an isosceles triangle is 70°, what is the value of the third angle at the principal vertex? (40°)
- An isosceles triangle has one angle 70°. Find the measure of the other angles. *Note that students will get two solutions according to the location of the given angle.*
- PAL is an isosceles triangle of vertex P. Its perimeter is 56 cm. If one of the equal sides is 15 cm, find the length of the other two sides.

Let us draw an isosceles triangle using Scratch.

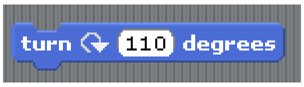
The most important two codes to be used is to **move** (make a side) and **turn** (make an angle). Both codes are from the **Motion** blocks.

The move block requires an input: number of steps; let us say 120.

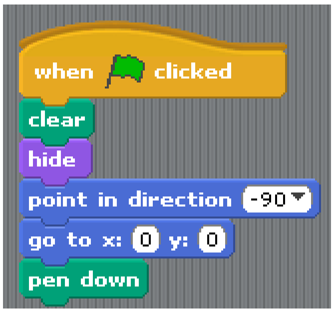


The turn block requires an input: number of degrees; let us say 70°. However, we deal with the exterior angle. Guess, what input shall we use?

We need to enter 110°. Let us make it clockwise.



We first start with the technical blocks:

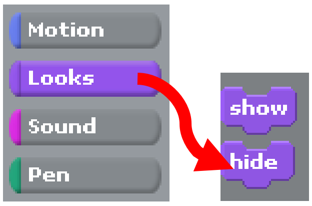


The first one is for starting the project.

Clear block is to start drawing from the beginning.

Hide block is to make the sprite invisible.

We can use "hide" to see the drawn result with nothing else.



Point in direction block is for starting drawing from the same direction.

Note that we can make the script at direction 90° from **Motion** blocks.

Go to x-y axis block is to start from the origin (same location each time).

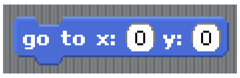
Pen down block for showing the line while drawing.

Now, let us start drawing the triangle.

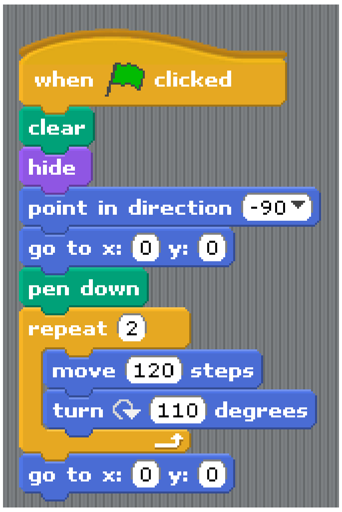
Since we need two congruent sides and two congruent angles, we ca use *repeat* block from *Control* blocks.

Then we insert the two basic blocks inside the repeat block and enter the value 2 as twice.

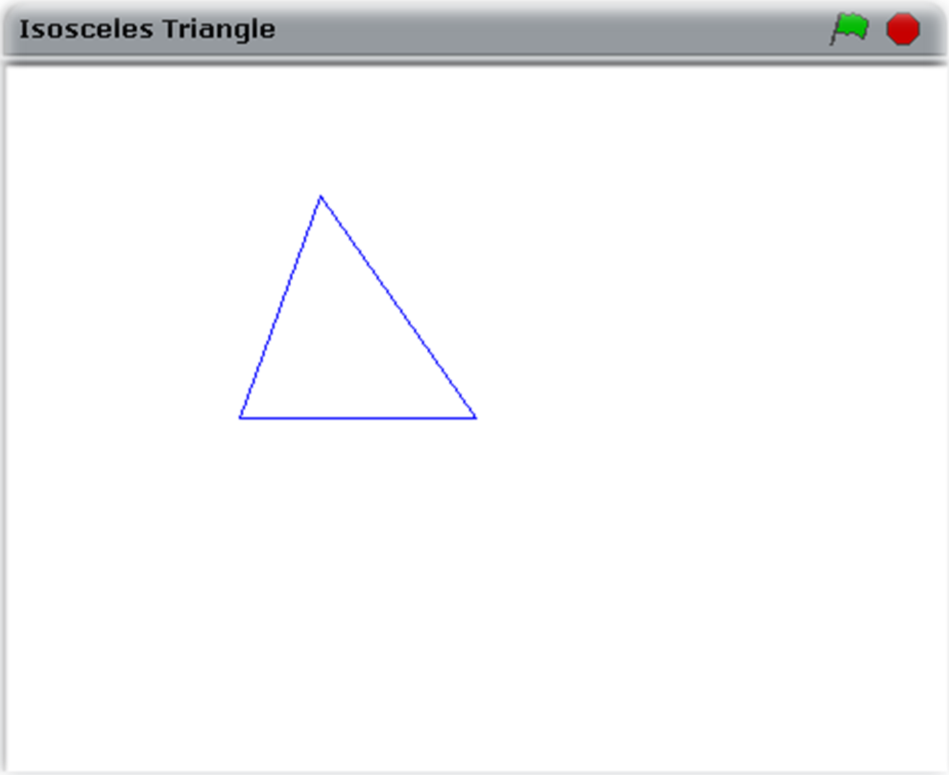
The last step is to let the sprite go back to its original location.



At last, we go to the original vertex.



The output triangle would be:



The students can make changes to measures of lengths and degrees.

Using Terms: What would an ***isosceles right triangle*** be?
A right triangle in which two sides and two angles are equal is called isosceles right triangle.

Checkpoint: Find without measuring, the value of each angle in an isosceles right triangle.

Title: Equilateral Triangles Period: 4

Objective:	By the end of this period, students will be able to: - construct an equilateral triangle.
Materials	Scratch software - Pictures
Method:	Skill acquisition
Working Form:	Inductive
Prerequisite:	Whole class
Procedures:	Name the sides and angles of a triangle.

Look at these pictures:

What is common among them? Focus on the measures of the sides (not their angles.)

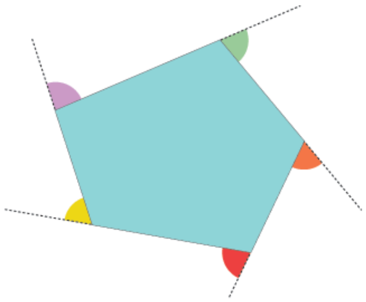
Problem:

A ceiling fan with three blades is decorated with a tape. The tape is stuck on each centered top of each blade. What is the form of the figure made by the tape? What are the measures of the angles that are formed when seen from directly under the fan?

Allow time for students to think about the angle measures.

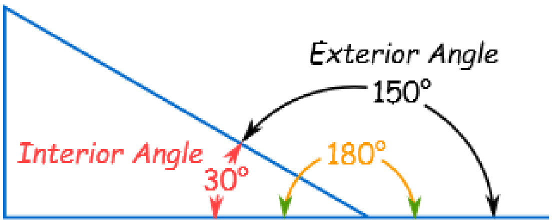
They have to arrive at a solution which is 60°.

Say: The exterior angle is the angle between any side of a shape,
and a line extended from the next side.

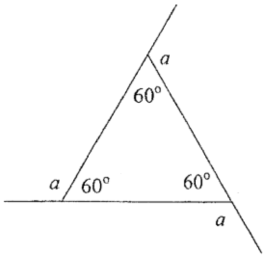


Ask: What is the relation between the interior angle and exterior angle?

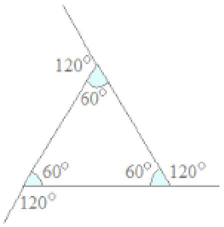
After discussing the answers, show them the following figure.



Ask: based on what you know, what is the exterior angle of each interior angle in the equilateral triangle?

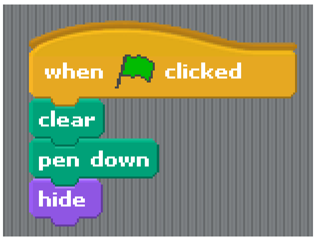


After discussing the answers, show the class the following figure:



Let us now construct an equilateral triangle using Scratch software.

Start with technical blocks.

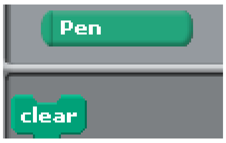


To tell how they come:

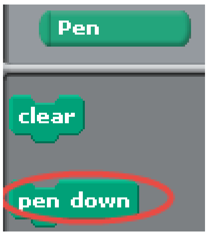
Click on **Control** blocks then choose the block **when clicked** to start the program.

Drag the block and drop on the script area.

Click on **clear** from **Pen** blocks to clear everything and start again.



To let the sprite shows the lines, click on **Pen** blocks then choose **pen down**.

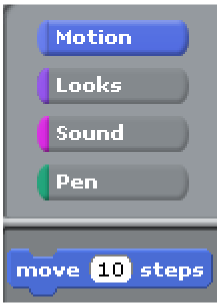


Drag it to the script area.

The hide block is from Looks blocks.

Now comes the triangle construction.

To let the sprite moves, choose from **Motion** blocks the **move steps** block.



Stack it under the previous blocks. Let's make the motion.

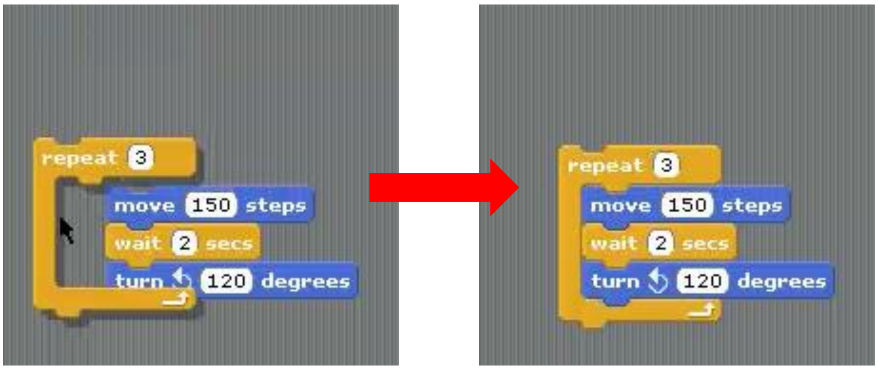
To make a delay in order to see how the triangle is being made, choose the block **wait secs** from the Control blocks. Make it 2 seconds.

Each interior angle in the equilateral triangle is 60°. Here we work with the exterior angle. So, we will make each turn 120°.

Choose **turn degrees** from **Motion** blocks. Let's choose the turning anticlockwise.

In order to repeat this three times, we choose the block **repeat** from **Control** blocks.

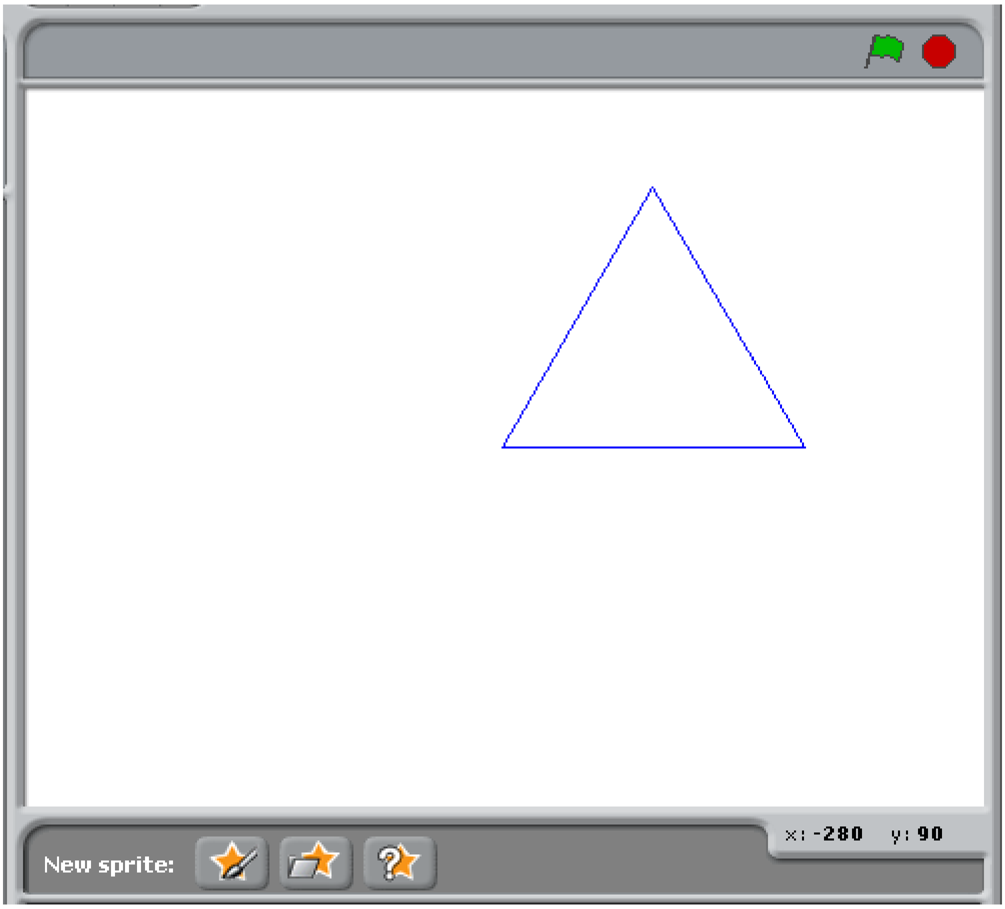
Drag the block to blockade the scripts.



We can add that drawing starts from the origin as to not to let the drawing be away from the sight.

The possible coding would be:

Click on the green flag to run the project.



Evaluation: Have students change the number of steps each time they draw another equilateral triangle.

Appendix G - Statistical Tables of Pretest

	Knowledge					
	Item 1	Item 2	Item 3	Item 4	Item 5	Sum
	2 pts.	2 pts.	2 pts.	2 pts.	2 pts.	10 pts.
A1	0	2	2	2	0	**6**
A2	2	2	2	2	0	**8**
A3	0	2	0	0	0	**2**
A4	0	2	2	2	0	**6**
A5	2	2	2	2	0	**8**
A6	2	2	0	2	0	**6**
A7	0	0	0	0	0	**0**
A8	2	2	2	2	0	**8**
A9	0	2	2	2	0	**6**
A10	0	2	2	2	0	**6**
A11	0	2	2	2	0	**6**
A12	0	2	2	2	0	**6**
A13	2	2	2	2	0	**8**
A14	0	2	2	2	0	**6**
A15	0	2	2	2	0	**6**
A16	0	2	2	2	0	**6**
A17	2	2	2	2	0	**8**
A18	0	2	0	2	0	**4**
A19	0	2	2	2	0	**6**
A20	2	2	0	2	0	**6**
A21	0	2	2	2	0	**6**
A22	0	2	2	0	0	**4**
A23	2	2	2	2	0	**8**
A24	0	2	2	2	0	**6**
A25	2	2	2	2	0	**8**
A26	0	2	2	2	0	**6**

Table G-A-K-1 Specific results of group **A** students in the Knowledge section of the pretest

	Item 1	Item 2	Item 3	Item 4	Item 5	Average
	2 pts.	2 pts.	2 pts.	2 pts.	2 pts.	
Number of correct answers	9	25	21	23	0	
Percentage of correct answers	35%	96%	81%	88%	0%	**60%**
Standard deviation	1.0	0.4	0.8	0.7	0.0	**1.9**

Table G-A-K-2 Cumulative results of group **A** students in the Knowledge section of the pretest

	Conceptual Understanding											
	Part I				Part II							Sum
					Demonstrate an understanding of complementar angles.	Find the complementary angle.	Demonstrate an understanding of supplementary angles.	Find the supplementary angle.	Compare the results.	Write the final answer.	Sum	
	Item 6	Item 7	Item 8	Sum	Item 9							
	1 pt.	1 pt.	1 pt.	3 pts.	1 pt.	1 pt.	1 pt.	1 pt.	1 pt.	1 pt.	6 pts.	9 pts.
A1	1	1	1	**3**	1	1	1	1	0	0	**4**	**7**
A2	1	1	1	**3**	1	1	1	1	1	0	**5**	**8**
A3	1	0	0	**1**	1	0	1	0	0	0	**2**	**3**
A4	1	1	0	**2**	1	1	1	1	1	0	**5**	**7**
A5	1	0	1	**2**	0	0	0	0	0	0	**0**	**2**
A6	0	1	1	**2**	0	0	0	0	1	0	**1**	**3**
A7	0	0	0	**0**	0	0	0	0	1	0	**1**	**1**
A8	1	1	1	**3**	0	1	0	1	1	1	**4**	**7**
A9	0	1	1	**2**	1	1	1	1	1	0	**5**	**7**
A10	1	1	1	**3**	1	1	0	0	1	0	**3**	**6**
A11	1	0	1	**2**	0	0	0	0	0	0	**0**	**2**
A12	1	1	0	**2**	0	0	0	0	0	0	**0**	**2**
A13	1	1	1	**3**	1	1	1	1	0	0	**4**	**7**
A14	0	1	1	**2**	1	1	1	1	1	0	**5**	**7**
A15	1	1	0	**2**	1	1	1	1	0	0	**4**	**6**
A16	0	0	0	**0**	1	1	1	1	0	1	**5**	**5**
A17	1	1	1	**3**	1	1	1	1	1	1	**6**	**9**
A18	0	1	1	**2**	1	0	1	0	1	1	**4**	**6**
A19	0	1	1	**2**	1	1	1	1	1	1	**6**	**8**
A20	1	0	0	**1**	1	0	1	1	0	0	**3**	**4**
A21	1	1	1	**3**	0	0	0	0	0	0	**0**	**3**
A22	1	1	0	**2**	0	0	0	0	0	0	**0**	**2**
A23	1	1	0	**2**	0	0	0	0	0	0	**0**	**2**
A24	1	0	1	**2**	1	1	1	1	1	1	**6**	**8**
A25	1	1	1	**3**	1	1	1	1	1	1	**6**	**9**
A26	0	1	1	**2**	1	1	1	1	0	0	**4**	**6**

Table G-A-C-1 Specific results of group **A** students in the Conceptual Understanding section of the pretest

	Part I				Part II							Total Average
	Item 6	Item 7	Item 8	Average	Item 9						Average	
	1 pt.	1 pt.	1 pt.		1 pt.	1 pt.	1 pt.	1 pt.	1 pt.	1 pt.		
Number of correct answers	18	19	17		17	15	16	15	13	7		
Percentage of correct answers	69%	73%	65%	**69%**	65%	58%	62%	58%	50%	27%	**53%**	**59%**
Standard deviation	0.5	0.5	0.5	**0.8**	0.5	0.5	0.5	0.5	0.5	0.5	**2.2**	**2.5**

Table G-A-C-2 Cumulative results of group **A** students in the Conceptual Understanding section of the pretest

	Skills				
	Accuracy of the figure	Accuracy of the two angles	Sum of the angles is 90 degrees	Showing the right angle symbol	**Sum**
	Item 10				
A1	**1 pt.**	**1 pt.**	**2 pts.**	**1 pt.**	**5 pts.**
A2	1	1	2	1	**5**
A3	1	0	2	0	**3**
A4	1	1	2	1	**5**
A5	1	0	2	1	**4**
A6	1	1	2	0	**4**
A7	1	0	2	0	**3**
A8	1	1	0	0	**2**
A9	1	1	1	1	**4**
A10	1	1	2	0	**4**
A11	1	1	2	1	**5**
A12	1	1	1	1	**4**
A13	1	1	1	0	**3**
A14	1	0	1	0	**2**
A15	1	1	1	1	**4**
A16	1	1	2	1	**5**
A17	1	1	1	0	**3**
A18	1	0	1	0	**2**
A19	1	1	2	0	**4**
A20	1	0	0	0	**1**
A21	1	1	2	0	**4**
A22	1	1	2	0	**4**
A23	1	0	1	0	**2**
A24	0	0	0	0	**0**
A25	1	1	1	1	**4**
A26	1	0	0	0	**1**

Table G-A-S-1 Specific results of group **A** students in the Skills section of the pretest

	Item 10				**Average**
	1 pt.	**1 pt.**	**2 pts.**	**1 pt.**	
Number of correct answers	25	17	22	9	
Percentage of correct answers	96%	65%	85%	35%	**66%**
Standard deviation	0.2	0.5	0.7	0.5	**1.3**

Table G-A-S-2 Specific results of group **A** students in the Skills section of the pretest

	Problem 1		
	Understand the concept of supplementary angle.	Subtract to write the final answer.	Sum
	Item 11		
	1 pt.	1 pt.	2 pts.
A1	1	1	**2**
A2	1	1	**2**
A3	1	1	**2**
A4	1	1	**2**
A5	1	1	**2**
A6	0	0	**0**
A7	0	0	**0**
A8	1	1	**2**
A9	0	0	**0**
A10	1	1	**2**
A11	0	0	**0**
A12	1	1	**2**
A13	1	1	**2**
A14	1	1	**2**
A15	1	1	**2**
A16	1	1	**2**
A17	1	1	**2**
A18	1	0	**1**
A19	1	1	**2**
A20	1	0	**1**
A21	1	1	**2**
A22	0	0	**0**
A23	1	0	**1**
A24	1	1	**2**
A25	1	1	**2**
A26	1	1	**2**

Table G-B-P-1 Specific results of the first problem of group **A** students in the Problem Solving section of the pretest

	Problem 2			
	Understand the concept of angle relations.	Make calculations.	Find the correct answer.	**Sum**
	Item 12			
	2 pts.	**1 pt.**	**1 pt.**	**4 pts.**
A1	1	1	1	**3**
A2	1	1	1	**3**
A3	1	1	1	**3**
A4	1	1	1	**3**
A5	1	1	1	**3**
A6	0	0	0	**0**
A7	0	0	0	**0**
A8	1	1	1	**3**
A9	0	0	0	**0**
A10	1	1	1	**3**
A11	0	0	0	**0**
A12	1	1	1	**3**
A13	1	1	1	**3**
A14	1	1	1	**3**
A15	1	1	1	**3**
A16	1	1	1	**3**
A17	1	1	1	**3**
A18	0	0	0	**0**
A19	2	1	0	**3**
A20	1	1	1	**3**
A21	0	0	0	**0**
A22	1	0	1	**2**
A23	1	0	1	**2**
A24	1	1	1	**3**
A25	0	0	0	**0**
A26	0	0	0	**0**

Table G-B-P-2 Specific results of the second problem of group **A** students in the Problem Solving section of the pretest

	Problem 3						
	Understand the concept of supplementary angle.	Write an algebraic expression.	Write the correct equation.	Find x.	Find y.	Find z.	**Sum**
	Item 13						
	1 pt.	**1 pt.**	**1 pt.**	**1 pt.**	**1 pt.**	**1 pt.**	**6 pts.**
A1	0	0	0	0	0	0	**0**
A2	1	0	0	0	0	0	**1**
A3	0	1	0	0	0	0	**1**
A4	0	0	0	0	0	0	**0**
A5	0	0	0	0	0	0	**0**
A6	0	0	0	0	0	0	**0**
A7	0	0	0	0	0	0	**0**
A8	1	0	0	0	0	0	**1**
A9	0	0	0	0	0	0	**0**
A10	0	0	0	0	0	0	**0**
A11	0	0	0	0	0	0	**0**
A12	1	0	0	1	0	0	**2**
A13	1	0	0	0	0	0	**1**
A14	0	0	0	0	0	0	**0**
A15	0	0	0	0	0	0	**0**
A16	1	0	0	0	0	0	**1**
A17	1	0	0	0	0	0	**1**
A18	0	0	0	0	0	0	**0**
A19	1	0	0	0	0	0	**1**
A20	0	0	0	0	0	0	**0**
A21	0	0	0	0	0	0	**0**
A22	0	0	0	0	0	0	**0**
A23	0	0	0	0	0	0	**0**
A24	0	0	0	0	0	0	**0**
A25	0	0	0	0	0	0	**0**
A26	1	1	1	1	0	0	**4**

Table G-B-P-3 Specific results of the third problem of group **A** students in the Problem Solving section of the pretest

	Problem Solving			
	Problem 1	Problem 2	Problem 3	Sum
	2 pts.	4 pts.	6 pts.	12 pts.
A1	2	3	0	5
A2	2	3	1	6
A3	2	3	1	6
A4	2	3	0	5
A5	2	3	0	5
A6	0	0	0	0
A7	0	0	0	0
A8	2	3	1	6
A9	0	0	0	0
A10	2	3	0	5
A11	0	0	0	0
A12	2	3	2	7
A13	2	3	1	6
A14	2	3	0	5
A15	2	3	0	5
A16	2	3	1	6
A17	2	3	1	6
A18	1	0	0	1
A19	2	3	1	6
A20	1	3	0	4
A21	2	0	0	2
A22	0	2	0	2
A23	1	2	0	3
A24	2	3	0	5
A25	2	0	0	2
A26	2	0	4	6

Table G-B-P-4 Specific results of the three problems of group **A** students in the Problem Solving section of the pretest

	Problem 1	Problem 2	Problem 3	Average
Percentage of correct answers	75%	50%	8%	33%
Standard deviation	0.8	1.4	0.9	2.3

Table G-B-P-5 Percentage of correct answers and standard deviation of the three problems of group **A** students in the Problem Solving section of the pretest

	Knowledge					
	Item 1	Item 2	Item 3	Item 4	Item 5	Sum
	2 pts.	2 pts.	2 pts.	2 pts.	2 pts.	10 pts.
B1	2	2	2	2	0	**8**
B2	0	2	2	2	0	**6**
B3	2	2	2	2	0	**8**
B4	2	2	2	2	2	**10**
B5	0	0	0	2	2	**4**
B6	2	2	2	2	0	**8**
B7	0	2	2	2	0	**6**
B8	0	2	2	2	0	**6**
B9	0	2	2	2	0	**6**
B10	2	2	2	2	2	**10**
B11	2	2	2	2	0	**8**
B12	2	2	2	2	0	**8**
B13	2	2	2	2	0	**8**
B14	2	2	2	2	0	**8**
B15	2	2	2	2	2	**10**
B16	2	2	2	2	0	**8**
B17	2	2	0	2	0	**6**
B18	2	2	0	2	2	**8**
B19	2	2	2	2	0	**8**
B20	2	2	2	2	2	**10**
B21	2	2	2	2	0	**8**
B22	2	2	2	2	0	**8**
B23	2	2	2	2	0	**8**
B24	2	2	2	2	0	**8**
B25	0	2	2	2	0	**6**

Table G-B-K-1 Specific results of group **B** students in the Knowledge section of the pretest

	Item 1	Item 2	Item 3	Item 4	Item 5	Average
	2 pts.	2 pts.	2 pts.	2 pts.	2 pts.	
Number of correct answers	9	25	21	23	0	
Percentage of correct answers	35%	96%	81%	88%	0%	**60%**
Standard deviation	1.0	0.4	0.8	0.7	0.0	**1.9**

Table G-B-K-2 Cumulative results of group B students in the Knowledge section of the pretest

	Conceptual Understanding											
	Part **I**				Part **II**							**Sum**
					Demonstrate an understanding of complementary angles.	Find the complementary angle.	Demonstrate an understanding of supplementary angles.	Find the supplementary angle.	Compare the results.	Write the final answer.	**Sum**	
	Item 6	**Item 7**	**Item 8**	**Sum**	**Item 9**							
	1 pt.	**1 pt.**	**1 pt.**	**3 pts.**	**1 pt.**	**1 pt.**	**1 pt.**	**1 pt.**	**1 pt.**	**1 pt.**	**6 pts.**	**9 pts.**
B1	0	1	0	**1**	1	1	1	1	0	0	**4**	**5**
B2	0	0	0	**0**	0	0	0	0	0	0	**0**	**0**
B3	1	0	0	**1**	1	1	1	1	1	1	**6**	**7**
B4	1	1	0	**2**	1	0	1	0	0	0	**2**	**4**
B5	1	1	0	**2**	1	1	1	1	1	0	**5**	**7**
B6	0	0	0	**0**	0	0	0	0	0	0	**0**	**0**
B7	1	1	0	**2**	0	0	0	0	0	0	**0**	**2**
B8	1	1	0	**2**	0	0	0	0	0	0	**0**	**2**
B9	1	1	0	**2**	0	0	0	0	0	0	**0**	**2**
B10	1	1	0	**2**	1	1	1	1	1	0	**5**	**7**
B11	1	1	0	**2**	0	0	0	0	0	0	**0**	**2**
B12	0	0	0	**0**	0	0	0	0	0	0	**0**	**0**
B13	1	1	0	**2**	1	1	1	1	1	1	**6**	**8**
B14	1	0	0	**1**	0	0	0	0	0	0	**0**	**1**
B15	1	1	0	**2**	1	1	1	1	1	0	**5**	**7**
B16	1	0	1	**2**	1	1	1	1	1	1	**6**	**8**
B17	0	0	0	**0**	0	0	0	0	0	0	**0**	**0**
B18	1	1	0	**2**	0	0	0	0	0	0	**0**	**2**
B19	1	1	0	**2**	1	1	1	1	1	1	**6**	**8**
B20	1	1	0	**2**	0	0	0	0	0	0	**0**	**2**
B21	1	1	0	**2**	0	0	0	0	0	0	**0**	**2**
B22	0	0	0	**0**	0	0	0	0	0	0	**0**	**0**
B23	1	1	0	**2**	1	1	1	1	1	0	**5**	**7**
B24	1	1	0	**2**	0	0	0	0	0	0	**0**	**2**
B25	0	0	1	**1**	0	0	0	0	0	0	**0**	**1**

Table G-B-C-1 Specific results of group **B** students in the Conceptual Understanding section of the pretest

	Part I				**Part II**							**Total Average**
	Item 6	**Item 7**	**Item 8**	**Average**	**Item 9**						**Average**	
	1 pt.	**1 pt.**	**1 pt.**		**1 pt.**	**1 pt.**	**1 pt.**	**1 pt.**	**1 pt.**	**1 pt.**		
Number of correct answers	18	16	2		10	9	10	9	8	4		
Percentage of correct answers	69%	62%	8%	**48%**	38%	35%	38%	35%	31%	15%	**33%**	**38%**
Standard deviation	0.5	0.5	0.3	**0.8**	0.5	0.5	0.5	0.5	0.5	0.4	**2.6**	**3.0**

Table G-B-C-2 Cumulative results of group **B** students in the Conceptual Understanding section of the pretest

	Skills				
	Accuracy of the figure	Accuracy of the two angles	Sum of the angles is 90 degrees	Showing the right angle symbol	**Sum**
	Item 10				
	1 pt.	**1 pt.**	**2 pts.**	**1 pt.**	**5 pts.**
B1	1	1	1	1	**4**
B2	1	1	1	1	**4**
B3	1	1	1	1	**4**
B4	1	1	1	1	**4**
B5	1	1	1	1	**4**
B6	0	0	0	0	**0**
B7	1	1	0	1	**3**
B8	0	0	0	0	**0**
B9	0	0	0	0	**0**
B10	0	1	1	0	**2**
B11	1	1	1	0	**3**
B12	1	1	1	0	**3**
B13	1	1	1	0	**3**
B14	1	0	1	0	**2**
B15	1	1	1	0	**3**
B16	1	0	1	1	**3**
B17	1	1	1	0	**3**
B18	1	1	1	0	**3**
B19	1	1	1	1	**4**
B20	1	1	1	0	**3**
B21	1	1	1	0	**3**
B22	1	0	1	1	**3**
B23	1	1	1	1	**4**
B24	1	0	1	0	**2**
B25	1	1	1	0	**3**

Table G-B-S-1 Specific results of group **B** students in the Skills section of the pretest

	Item 10				**Average**
	1 pt.	**1 pt.**	**2 pts.**	**1 pt.**	
Number of correct answers	21	18	21	10	
Percentage of correct answers	81%	69%	81%	38%	**56%**
Standard deviation	0.4	0.5	0.4	0.5	**1.2**

Table G-B-S-2 Cumulative results of group **B** students in the Skills section of the pretest

	Problem 1		
	Understand the concept of supplementary angle.	Subtract to write the final answer.	**Sum**
	Item 11		
	1 pt.	**1 pt.**	**2 pts.**
B1	1	1	**2**
B2	0	1	**1**
B3	1	1	**2**
B4	1	1	**2**
B5	1	1	**2**
B6	0	0	**0**
B7	1	1	**2**
B8	0	0	**0**
B9	0	0	**0**
B10	0	1	**1**
B11	1	1	**2**
B12	1	1	**2**
B13	0	1	**1**
B14	0	1	**1**
B15	1	1	**2**
B16	1	1	**2**
B17	1	1	**2**
B18	1	1	**2**
B19	0	1	**1**
B20	1	1	**2**
B21	1	1	**2**
B22	1	1	**2**
B23	0	0	**0**
B24	1	1	**2**
B25	0	1	**1**

Table G-B-P-1 Specific results of the first problem of group **B** students in the Problem Solving section of the pretest

	Problem 2			
	Understand the concept of angle relations.	Make calculations.	Find the correct answer.	**Sum**
	Item 12			
	2 pts.	**1 pt.**	**1 pt.**	**4 pts.**
B1	1	1	1	**3**
B2	1	1	0	**2**
B3	1	1	1	**3**
B4	1	1	1	**3**
B5	1	1	1	**3**
B6	0	0	0	**0**
B7	2	1	1	**4**
B8	0	0	0	**0**
B9	0	0	0	**0**
B10	1	1	1	**3**
B11	1	1	1	**3**
B12	1	1	1	**3**
B13	0	0	0	**0**
B14	1	1	1	**3**
B15	1	1	1	**3**
B16	0	0	0	**0**
B17	1	1	1	**3**
B18	2	1	1	**4**
B19	1	1	1	**3**
B20	2	1	1	**4**
B21	1	1	1	**3**
B22	1	1	0	**2**
B23	1	1	1	**3**
B24	1	1	1	**3**
B25	1	1	1	**3**

Table G-B-P-2 Specific results of the second problem of group **B** students in the Problem Solving section of the pretest

	Problem 3						
	Understand the concept of supplementary angle.	Write an algebraic expression.	Write the correct equation.	Find x.	Find y.	Find z.	**Sum**
	Item 13						
	1 pt.	**1 pt.**	**1 pt.**	**1 pt.**	**1 pt.**	**1 pt.**	**6 pts.**
B1	0	0	0	0	0	0	**0**
B2	0	0	0	1	0	0	**1**
B3	0	0	0	0	0	0	**0**
B4	0	0	0	1	0	0	**1**
B5	0	0	0	0	0	0	**0**
B6	0	0	0	0	0	0	**0**
B7	0	0	0	0	0	0	**0**
B8	0	0	0	0	0	0	**0**
B9	0	0	0	0	0	0	**0**
B10	0	0	0	0	0	0	**0**
B11	0	0	0	0	0	0	**0**
B12	0	0	0	0	0	0	**0**
B13	1	1	1	1	1	1	**6**
B14	1	0	0	0	0	0	**1**
B15	0	0	0	0	0	0	**0**
B16	1	0	0	0	0	0	**1**
B17	0	0	0	0	0	0	**0**
B18	0	0	0	0	0	0	**0**
B19	0	0	0	0	0	0	**0**
B20	0	0	0	0	0	0	**0**
B21	0	0	0	0	0	0	**0**
B22	1	0	0	0	0	0	**1**
B23	1	1	1	1	1	1	**6**
B24	0	0	0	0	0	0	**0**
B25	0	0	0	0	0	0	**0**

Table G-B-P-3 Specific results of the third problem of group **B** students in the Problem Solving section of the pretest

	Problem Solving			
	Problem 1	Problem 2	Problem 3	Sum
	2 pts.	4 pts.	6 pts.	12 pts.
B1	2	3	0	5
B2	1	2	1	4
B3	2	3	0	5
B4	2	3	1	6
B5	2	3	0	5
B6	0	0	0	0
B7	2	4	0	6
B8	0	0	0	0
B9	0	0	0	0
B10	1	3	0	4
B11	2	3	0	5
B12	2	3	0	5
B13	1	0	6	7
B14	1	3	1	5
B15	2	3	0	5
B16	2	0	1	3
B17	2	3	0	5
B18	2	4	0	6
B19	1	3	0	4
B20	2	4	0	6
B21	2	3	0	5
B22	2	2	1	5
B23	0	3	6	9
B24	2	3	0	5
B25	1	3	0	4

Table G-B-P-4 Specific results of the three problems of group **B** students in the Problem Solving section of the pretest

	Problem 1	Problem 2	Problem 3	Average
Percentage of correct answers	72%	61%	11%	38%
Standard deviation	0.8	1.3	1.7	2.1

Table G-B-P-5 Percentage of correct answers and standard deviation of the three problems of group **B** students in the Problem Solving section of the pretest

Appendix H - Statistical Tables of Posttest

	Knowledge			
	Item 1	Item 2	Item 3	Sum
A1	2	2	2	**6**
A2	2	2	0	**4**
A3	2	2	2	**6**
A4	2	2	0	**4**
A5	2	2	0	**4**
A6	2	2	0	**4**
A7	2	0	2	**4**
A8	2	2	2	**6**
A9	2	2	0	**4**
A10	2	2	2	**6**
A11	2	2	2	**6**
A12	2	2	2	**6**
A13	2	2	0	**4**
A14	2	2	2	**6**
A15	2	2	2	**6**
A16	2	2	2	**6**
A17	2	2	2	**6**
A18	2	2	2	**6**
A19	2	2	2	**6**
A20	2	2	2	**6**
A21	2	2	2	**6**
A22	2	2	0	**4**
A23	2	2	0	**4**
A24	2	2	0	**4**
A25	2	2	0	**4**
A26	2	2	2	**6**

Table H-A-K-1 Specific results of group **A** students in the Knowledge section of the posttest

	Item 1	Item 2	Item 3	Average
	2 pts.	2 pts.	2 pts.	
Number of correct answers	26	25	16	
Percentage of correct answers	100%	96%	62%	**5.15**
Standard deviation	0.0	0.4	1.0	**1.0**

Table H-A-K-2 Cumulative results of group **A** students in the Knowledge section of the posttest

	Conceptual Understanding													
	Part **I**			**Sum**	Part **II**				**Sum**	Part **III**			**Sum**	**Sum**
					Yes/No Response	Explanation	Yes/No Response	Explanation		Yes/No Response	Draw the obtuse triangle	Demonstrate an understanding of obtuse traingle		
	Item 4	**Item 5**	**Item 6**		**Item 7**		**Item 8**			**Item 9**	**Item 10**			
	2 pts.	**2 pts.**	**2 pts.**	**6 pts.**	**1 pt.**	**2 pts.**	**1 pt.**	**2 pts.**	**6 pts.**	**2 pts.**	**2 pts.**	**2 pts.**	**6 pts.**	**18 pts.**
A1	1	1	1	**3**	0	1	1	2	**4**	1	1	1	**3**	**10**
A2	1	1	1	**3**	0	1	0	0	**1**	0	0	0	**0**	**4**
A3	1	1	1	**3**	0	1	1	1	**3**	1	1	0	**2**	**8**
A4	1	1	1	**3**	0	2	1	1	**4**	1	1	0	**2**	**9**
A5	1	1	1	**3**	0	2	0	0	**2**	0	1	0	**1**	**6**
A6	1	1	1	**3**	0	1	1	1	**3**	1	0	0	**1**	**7**
A7	1	1	1	**3**	1	0	1	2	**4**	0	0	0	**0**	**7**
A8	1	1	1	**3**	1	1	1	1	**4**	0	1	0	**1**	**8**
A9	1	0	1	**2**	0	1	1	0	**2**	1	1	0	**2**	**6**
A10	1	1	1	**3**	1	2	1	2	**6**	1	0	0	**1**	**10**
A11	1	1	1	**3**	1	2	1	1	**5**	1	1	0	**2**	**10**
A12	1	1	1	**3**	1	1	1	1	**4**	1	0	0	**1**	**8**
A13	1	1	1	**3**	0	0	0	0	**0**	1	0	1	**2**	**5**
A14	1	1	1	**3**	0	0	1	2	**3**	1	0	0	**1**	**7**
A15	1	1	1	**3**	0	1	1	1	**3**	1	0	0	**1**	**7**
A16	1	1	1	**3**	0	1	1	1	**3**	0	0	0	**0**	**6**
A17	1	1	1	**3**	1	2	1	2	**6**	0	0	0	**0**	**9**
A18	1	1	1	**3**	0	1	0	0	**1**	1	1	0	**2**	**6**
A19	1	1	1	**3**	1	2	1	2	**6**	1	1	0	**2**	**11**
A20	1	1	1	**3**	0	0	1	1	**2**	1	0	0	**1**	**6**
A21	1	1	1	**3**	0	1	1	0	**2**	1	0	0	**1**	**6**
A22	1	0	1	**2**	0	1	1	0	**2**	1	1	1	**3**	**7**
A23	1	1	1	**3**	0	0	1	0	**1**	1	0	0	**1**	**5**
A24	1	1	1	**3**	1	0	1	1	**3**	0	0	0	**0**	**6**
A25	1	1	0	**2**	0	1	0	1	**2**	1	0	0	**1**	**5**
A26	1	1	1	**3**	1	2	1	1	**5**	0	0	0	**0**	**8**

Table H-A-C-1 Specific results of group **A** students in the Conceptual Understanding section of the posttest

	Part 1	**Part 2**	**Part 3**	**Average**
	2 pts.	**2 pts.**	**2 pts.**	
Average of correct answers	2.88	3.12	1.19	7.19
Percentage of correct answers	48%	52%	20%	40%
Standard deviation	0.3	1.6	0.9	1.8

Table H-A-C-2 Cumulative results of group **A** students in the Conceptual Understanding section of the posttest

	Part **I**			**Part II**		**Sum**
	Draw an isosceles triangle	Only one angle is 70°	Explanation	Draw a scalene triangle	Explanation	
	Item 11		**Item 12**	**Item 13**	**Item 14**	
	2 pts.	**2 pts.**	**2 pts.**	**2 pts.**	**2 pts.**	**10 pts.**
A1	2	0	1	2	1	**6**
A2	2	0	1	2	1	**6**
A3	1	0	0	1	0	**2**
A4	2	0	1	2	1	**6**
A5	2	0	1	1	1	**5**
A6	0	0	0	0	0	**0**
A7	0	0	0	1	0	**1**
A8	2	2	2	2	2	**10**
A9	2	0	1	1	1	**5**
A10	1	0	1	0	1	**3**
A11	2	2	2	2	1	**9**
A12	2	0	1	2	1	**6**
A13	0	0	1	1	0	**2**
A14	1	0	1	2	1	**5**
A15	2	0	1	2	1	**6**
A16	2	0	1	2	0	**5**
A17	2	0	1	2	1	**6**
A18	2	0	1	2	0	**5**
A19	2	0	1	2	1	**6**
A20	2	2	1	1	1	**7**
A21	1	0	1	2	1	**5**
A22	2	0	1	2	1	**6**
A23	2	0	1	1	1	**5**
A24	2	2	2	2	2	**10**
A25	2	0	1	1	1	**5**
A26	2	0	1	2	1	**6**

Table H-A-S-1 Specific results of group **A** students in the Skills section of the posttest

	Skills					
	Part **I**			**Part II**		**Average**
	Item 11		**Item 12**	**Item 13**	**Item 14**	
Number of correct answers	23	4	23	24	20	
Percentage of correct answers	88%	15%	88%	92%	77%	**53%**
Standard deviation	0.7	0.7	0.5	0.6	0.5	**2.4**

Table H-A-S-2 Cumulative results of group **A** students in the Skills section of the posttest

	Problem Solving								
	Problem 1				Problem 2				
	Understand the concept of angle and angle measures.	Make calculations.	Find the correct answer.	Sum	Subtract.	Divide.	Find the correct answer.	Sum	Sum
	Item 15				Item 16				
	2 pts.	2 pts.	2 pts.	6 pts.	2 pts.	2 pts.	2 pts.	6 pts.	12 pts.
A1	2	2	2	**6**	2	2	2	**6**	**12**
A2	0	0	0	**0**	0	0	0	**0**	**0**
A3	2	2	2	**6**	2	1	2	**5**	**11**
A4	2	2	1	**5**	2	2	2	**6**	**11**
A5	2	2	1	**5**	0	0	0	**0**	**5**
A6	0	0	2	**2**	2	2	1	**5**	**7**
A7	0	0	0	**0**	0	0	0	**0**	**0**
A8	2	2	2	**6**	0	0	0	**0**	**6**
A9	2	2	2	**6**	2	2	1	**5**	**11**
A10	2	2	2	**6**	2	2	1	**5**	**11**
A11	2	2	2	**6**	0	0	0	**0**	**6**
A12	2	2	2	**6**	2	2	1	**5**	**11**
A13	0	0	0	**0**	1	0	0	**1**	**1**
A14	2	2	2	**6**	0	0	0	**0**	**6**
A15	0	0	0	**0**	2	2	1	**5**	**5**
A16	2	2	2	**6**	1	1	0	**2**	**8**
A17	0	0	0	**0**	2	1	0	**3**	**3**
A18	2	2	2	**6**	2	2	1	**5**	**11**
A19	2	2	1	**5**	2	2	1	**5**	**10**
A20	2	2	2	**6**	1	0	0	**1**	**7**
A21	0	2	2	**4**	2	2	1	**5**	**9**
A22	2	2	0	**4**	2	0	0	**2**	**6**
A23	0	0	0	**0**	2	2	1	**5**	**5**
A24	0	0	0	**0**	0	0	0	**0**	**0**
A25	0	0	0	**0**	0	0	0	**0**	**0**
A26	2	2	2	**6**	0	0	0	**0**	**6**

Table H-A-P-1 Specific results of group **A** students in the Skills section of the posttest

	Problem 1	Problem 2	Average
	2 pts.	2 pts.	
Average of correct answers	3.73	2.73	6.46
Percentage of correct answers	62%	46%	54%
Standard deviation	2.7	2.4	4.0

Table H-A-P-2 Cumulative results of group **A** students in the Skills section of the posttest

	Knowledge			
	Item 1	Item 2	Item 3	Sum
	2 pts.	2 pts.	2 pts.	6 pts.
B1	2	2	2	**6**
B2	2	2	2	**6**
B3	2	2	2	**6**
B4	2	2	2	**6**
B5	2	2	2	**6**
B6	2	2	2	**6**
B7	2	2	2	**6**
B8	2	2	2	**6**
B9	2	2	2	**6**
B10	2	2	2	**6**
B11	2	2	2	**6**
B12	2	2	2	**6**
B13	2	2	2	**6**
B14	2	2	2	**6**
B15	2	2	2	**6**
B16	2	2	2	**6**
B17	2	2	2	**6**
B18	2	2	2	**6**
B19	2	2	2	**6**
B20	2	2	2	**6**
B21	2	2	2	**6**
B22	2	2	2	**6**
B23	2	2	2	**6**
B24	2	2	2	**6**
B25	2	2	2	**6**

Table H-B-K-1 Specific results of group **B** students in the Knowledge section of the posttest

	Item 1	Item 2	Item 3	Average
	2 pts.	2 pts.	2 pts.	
Number of correct answers	25	25	25	
Percentage of correct answers	96%	96%	96%	**100%**
Standard deviation	0.0	0.0	0.0	**0.0**

Table H-B-K-2 Cumulative results of group **B** students in the Knowledge section of the posttest

	Part **I**			Sum	Part **II**				Sum	Part **III**				Sum
					Yes/No Response	Explanation	Yes/No Response	Explanation		Yes/No Response	Draw the obtuse triangle	Demonstrate an understanding of obtuse traingle	Sum	
	Item 4	**Item 5**	**Item 6**		**Item 7**		**Item 8**			**Item 9**	**Item 10**			
	2 pts.	**2 pts.**	**2 pts.**	**6 pts.**	**1 pt.**	**2 pts.**	**1 pt.**	**2 pts.**	**6 pts.**	**2 pts.**	**2 pts.**	**2 pts.**	**6 pts.**	**18 pts.**
B1	2	2	2	**6**	0	1	1	2	**4**	2	2	0	**4**	**14**
B2	2	2	2	**6**	0	1	1	1	**3**	0	0	0	**0**	**9**
B3	2	2	2	**6**	0	1	1	0	**2**	0	2	0	**2**	**10**
B4	2	2	2	**6**	0	0	1	2	**3**	2	2	0	**4**	**13**
B5	2	2	2	**6**	0	0	1	2	**3**	2	0	0	**2**	**11**
B6	0	0	0	**0**	0	0	1	1	**2**	2	2	0	**4**	**6**
B7	2	2	2	**6**	1	2	0	0	**3**	0	2	0	**2**	**11**
B8	2	2	2	**6**	0	1	1	0	**2**	2	2	0	**4**	**12**
B9	2	2	2	**6**	0	1	1	0	**2**	0	0	0	**0**	**8**
B10	2	2	2	**6**	0	1	1	1	**3**	2	2	0	**4**	**13**
B11	2	2	2	**6**	1	2	1	2	**6**	2	2	2	**6**	**18**
B12	2	2	2	**6**	0	0	0	0	**0**	0	0	0	**0**	**6**
B13	2	2	2	**6**	0	1	1	2	**4**	2	2	2	**6**	**16**
B14	2	2	2	**6**	0	1	1	0	**2**	2	2	0	**4**	**12**
B15	2	2	2	**6**	1	0	1	1	**3**	2	0	0	**2**	**11**
B16	2	2	2	**6**	1	2	1	0	**4**	0	0	0	**0**	**10**
B17	2	2	2	**6**	1	1	1	0	**3**	2	0	0	**2**	**11**
B18	2	2	2	**6**	0	1	1	0	**2**	2	2	0	**4**	**12**
B19	2	2	2	**6**	0	1	1	2	**4**	2	2	0	**4**	**14**
B20	2	2	2	**6**	1	1	1	1	**4**	2	2	2	**6**	**16**
B21	2	2	2	**6**	1	2	1	2	**6**	2	2	2	**6**	**18**
B22	2	2	2	**6**	0	1	1	0	**2**	0	0	0	**0**	**8**
B23	2	2	2	**6**	0	1	1	0	**2**	0	0	0	**0**	**8**
B24	2	2	2	**6**	1	1	1	0	**3**	2	2	2	**6**	**15**
B25	2	2	0	**4**	0	1	1	1	**3**	2	2	0	**4**	**11**

Table H-B-C-1 Specific results of group **B** students in the Conceptual Understanding section of the posttest

	Part 1	**Part 2**	**Part 3**	**Average**
	2 pts.	**2 pts.**	**2 pts.**	
Average of correct answers	5.68	3.00	3.04	11.72
Percentage of correct answers	95%	50%	51%	65%
Standard deviation	1.2	1.3	2.2	3.3

Table H-B-C-2 Cumulative results of group **B** students in the Conceptual Understanding section of the posttest

	Skills					
	Part I			Part II		Sum
	Draw an isosceles triangle	Only one angle is 70°	Explanation	Draw a scalene triangle	Explanation	
	Item 11		Item 12	Item 13	Item 14	
	2 pts.	2 pts.	2 pts.	2 pts.	2 pts.	10 pts.
B1	2	0	1	1	1	**5**
B2	2	0	1	2	1	**6**
B3	2	0	1	1	0	**4**
B4	2	0	1	2	1	**6**
B5	2	0	1	2	1	**6**
B6	1	0	1	2	1	**5**
B7	2	2	1	1	1	**7**
B8	2	0	0	0	0	**2**
B9	2	0	1	2	1	**6**
B10	2	0	1	1	0	**4**
B11	2	2	2	2	2	**10**
B12	0	0	0	2	0	**2**
B13	2	2	2	2	2	**10**
B14	2	1	1	1	1	**6**
B15	2	2	1	2	0	**7**
B16	2	0	1	2	1	**6**
B17	2	2	1	1	0	**6**
B18	2	0	1	2	1	**6**
B19	2	2	2	2	1	**9**
B20	2	2	2	2	1	**9**
B21	2	0	1	2	1	**6**
B22	2	0	1	2	2	**7**
B23	2	2	2	2	2	**10**
B24	2	2	2	2	1	**9**
B25	2	0	1	2	1	**6**

Table H-B-S-1 Specific results of group **B** students in the Skills section of the posttest

	Skills					
	Part I			Part II		Average
	Item 11		Item 12	Item 13	Item 14	
Number of correct answers	24	10	23	24	19	
Percentage of correct answers	92%	38%	88%	92%	73%	**64%**
Standard deviation	0.4	1.0	0.6	0.6	0.6	**2.2**

Table H-B-S-2 Cumulative results of group **B** students in the Skills section of the posttest

	Problem Solving								
	Understand the concept of angle and angle measures.	Make calculations.	Find the correct answer.	Sum	Subtract.	Divide.	Find the correct answer.	Sum	
	Item 15				Item 16				
	2 pts.	2 pts.	2 pts.	6 pts.	2 pts.	2 pts.	2 pts.	6 pts.	12 pts.
B1	2	2	2	**6**	2	2	2	**6**	**12**
B2	0	0	0	**0**	0	0	0	**0**	**0**
B3	2	2	2	**6**	2	2	2	**6**	**12**
B4	2	2	2	**6**	2	0	0	**2**	**8**
B5	2	2	2	**6**	2	0	0	**2**	**8**
B6	2	2	0	**4**	2	0	0	**2**	**6**
B7	2	2	2	**6**	2	2	2	**6**	**12**
B8	2	2	2	**6**	0	0	0	**0**	**6**
B9	0	0	0	**0**	0	0	0	**0**	**0**
B10	2	0	0	**2**	2	2	2	**6**	**8**
B11	2	2	2	**6**	2	2	2	**6**	**12**
B12	0	0	0	**0**	0	0	0	**0**	**0**
B13	2	2	2	**6**	2	2	2	**6**	**12**
B14	2	2	2	**6**	0	0	0	**0**	**6**
B15	2	2	2	**6**	2	0	0	**2**	**8**
B16	2	2	2	**6**	2	0	0	**2**	**8**
B17	2	2	2	**6**	0	0	0	**0**	**6**
B18	2	2	2	**6**	2	2	2	**6**	**12**
B19	2	2	2	**6**	2	2	2	**6**	**12**
B20	2	2	2	**6**	2	2	2	**6**	**12**
B21	2	2	2	**6**	2	2	2	**6**	**12**
B22	2	2	2	**6**	2	0	0	**2**	**8**
B23	2	2	2	**6**	2	2	2	**6**	**12**
B24	2	2	2	**6**	2	2	2	**6**	**12**
B25	0	2	2	**4**	2	0	0	**2**	**6**

Table H-B-P-1 Specific results of group **B** students in the Problem Solving section of the posttest

	Problem 1	Problem 2	Average
	2 pts.	2 pts.	
Average of correct answers	4.96	3.44	**8.40**
Percentage of correct answers	83%	57%	**70%**
Standard deviation	2.1	2.6	**4.0**

Table H-B-P-2 Cumulative results of group **B** students in the Problem Solving section of the posttest

Appendix I - Statistical Tables of Pretest and Posttest

Knowledge	Conceptual Understanding	Skills	Problem Solving	Total
60	78	100	42	279
80	89	60	50	279
20	33	100	50	203
60	78	80	42	259
80	22	80	42	224
60	33	60	0	153
0	11	40	0	51
80	78	80	50	288
60	78	80	0	218
60	67	100	42	268
60	22	80	0	162
60	22	60	58	201
80	78	40	50	248
60	78	80	42	259
60	67	100	42	268
60	56	60	50	226
80	100	40	50	270
40	67	80	8	195
60	89	20	50	219
60	44	80	33	218
60	33	80	17	190
40	22	40	17	119
80	22	0	25	127
60	89	80	42	271
80	100	20	17	217
60	67	80	50	257

Table I-E-A Scores of the pretest of group **A**

Knowledge	Conceptual Understanding	Skills	Problem Solving	Total
80	56	80	42	257
60	0	80	33	173
80	78	80	42	279
100	44	80	50	274
40	78	80	42	239
80	0	0	0	80
60	22	60	50	192
60	22	0	0	82
60	22	0	0	82
100	78	40	33	251
80	22	60	42	204
80	0	60	42	182
80	89	60	58	287
80	11	40	42	173
100	78	60	42	279
80	89	60	25	254
60	0	60	42	162
80	22	60	50	212
80	89	80	33	282
100	22	60	50	232
80	22	60	42	204
80	0	60	42	182
80	78	80	75	313
80	22	40	42	184
60	11	60	33	164

Table I-E-B Scores of the pretest of group **B**

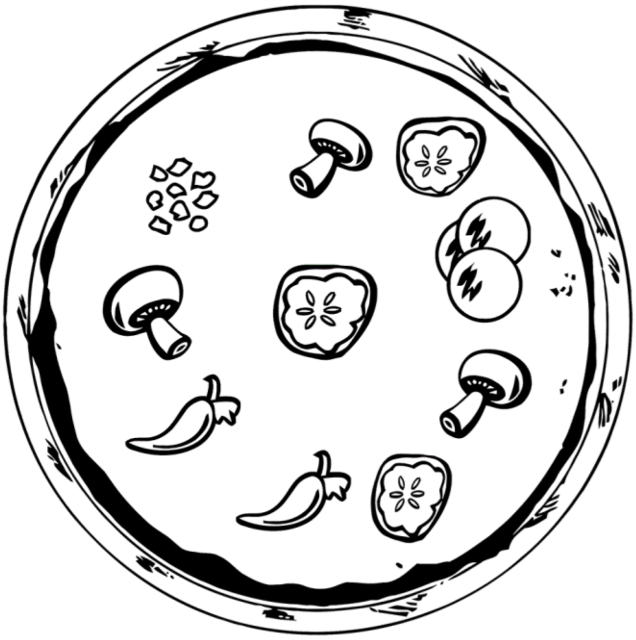
Knowledge	Conceptual Understanding	Skills	Problem Solving	Total
100	56	60	100	316
67	22	60	0	149
100	44	20	92	256
100	50	60	92	302
100	33	50	42	225
100	39	0	58	197
67	39	10	0	116
100	44	100	50	294
67	33	50	92	242
100	56	30	92	277
100	56	90	50	296
100	44	60	92	296
67	28	20	8	123
100	39	50	50	239
100	39	60	42	241
100	33	50	67	250
100	50	60	25	235
100	33	50	92	275
100	61	60	83	304
100	33	70	58	262
100	33	50	75	258
67	39	60	50	216
67	28	50	42	186
67	33	100	0	200
67	28	50	0	144
100	44	60	50	254

Table I-O-A Scores of the posttest of group **A**

Knowledge	Conceptual Understanding	Skills	Problem Solving	Total
100	78	50	100	328
100	50	60	0	210
100	56	40	100	296
100	72	60	67	299
100	61	60	67	288
100	33	50	50	233
100	61	70	100	331
100	67	20	50	237
100	44	60	0	204
100	72	40	67	279
100	100	100	100	400
100	33	20	0	153
100	89	100	100	389
100	67	60	50	277
100	61	70	67	298
100	56	60	67	282
100	61	60	50	271
100	67	60	100	327
100	78	90	100	368
100	89	90	100	379
100	100	60	100	360
100	44	70	67	281
100	44	100	100	344
100	83	90	100	373
100	61	60	50	271

Table I-O-B Scores of the posttest of group **B**

Appendix J - Before Unit Problem



A restaurant sells round and square pizzas. It also gives customers pieces of cartons for the bases of the pizza slices. Customers at each table should cut the pizza into equal slices.

1. At table **1**, three friends order a round pizza. At table **2**, five friends order a round pizza. Both pizzas are of the same size. Help the waiter identify the type of angles of the carton pieces (acute, obtuse, or right) and their measures. Justify your answer.
2. At table **3**, four friends order a medium round pizza. At table **4**, six friends order a large round pizza. The waiter says the sum of the angles of the carton pieces for table 3 is different from the sum of angles of carton pieces of table 4. Do you agree with him? Why or why not?
3. At table **5**, two friends order a square pizza. At table **6**, another two friends order a square pizza. The two pizzas are of the same size. Table 5 customers cut from a corner to another corner passing through the center. Table 6 customers cut through the center of the pizza, parallel to its side. Help the waiter identify the forms of the needed carton pieces for each table.

Appendix K - After Unit Problem

At an exhibition, companies are given boards to display their posters. The boards are all of the same height but of different widths. They come in colors. The widths of the boards are: 3 meters for the red, 4 meters for the blue, and 5 meters for the green. Each company uses three boards only to stick them in a triangular prism model. Each model is built on a wooden base which sides are traced exactly with the bottom of the boards.

Company **A** model uses boards of colors: red, blue, and green.

Company **B** model uses boards of colors: red, blue, and blue.

Company **C** model uses boards of colors: blue, green, and green.

Company **D** model uses boards of colors: red, red, and red.

1. If an angle between two boards of company **A** model is 90°, what is the type of the base? If another angle between two boards of company **A** is half the first angle, what is the type of the base? How do you know?
2. Do the bases for companies **B** and **C** have the same type? The same size? The same sum of angles? Explain each answer.
3. Can the base for company **D** be a right triangle? If yes, explain why. If not, what is its type? Explain.
4. Tell whether it is *always*, *sometimes*, or *never* that the bases for companies **C** and **D** have the same sum of angles. Explain.