IEEE 802.11 WIRELESS MESH NETWORKS: ACCURATE MAC MODELING AND NOVEL ROUTING METRICS AND PROTOCOL DESIGN PROPOSALS

by

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Abstract

The IEEE 802.11 Medium Access Control (MAC) gained widespread popularity as a layer-2 protocol for Wireless Local Area Networks (WLANs). While the continuous lack of accurate analytical 802.11 MAC models as well as the non-existing state of the art IEEE 802.11-based Wireless Mesh Networks (WMNs) simulation and analysis tools attract our attention, the adaptation of the existing Mobile Ad-Hoc Networks (MANETs) routing metrics and protocols to 802.11s-based WMNs remains a major source of performance degradation of those latter. This work addresses these three voids by first providing two distinct mathematical models for evaluating the queuing delays in an IEEE 802.11-based WMN. In the first model each node is modeled as a discrete time *G*/*G*/1 queue characterized by general arrival patterns and service time distribution, whereas in the second model the service time is represented by a combination of Erlang-*k* and Coxian-2 distributions, thus nodes are modeled as either *G*/*E*-*k*/1 or *G*/*C*2/1 queues. Both models account for arbitrary packet size distributions, number of nodes in the network, channel access time resulting from the random access mechanism, collision avoidance and exponential back-off mechanism of 802.11, delays in channel access due to other nodes transmitting and delays caused by collisions. Second, this manuscript presents two novel MAC-AWARE and BUFFER-AWARE routing protocols based on a new IEEE 802.11-specific routing metrics used to provide Quality of Service (*QoS*) routing for both delay-sensitive and packet-loss sensitive traffic generated simultaneously in an 802.11-based network in DCF mode. A new IEEE 802.11-MAC based network-specific simulator that we developed allowed us to verify our models and assert the correctness and accuracy through extensive simulations.

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Chapter 1

Introduction

Wireless Mesh Networks (WMNs) emerged as one of the most promising design paradigm and concepts for the next generation self-organizing and auto-configurable wireless networks. The IEEE 802.11 Wireless Local Area Network (WLAN) is an international standard, which specified the Medium Access Control (MAC) sub-layer and Physical (PHY) layer [1]. Operating on radio frequencies ranging from 2.45 GHz in IEEE 802.11b, to 5 GHz in IEEE 802.11a/g, they can provide data rates in the range of 11 to 54 Mbps respectively [2]. Moreover, as soon as the 802.11n becomes a final amendment, data rates around the 600 Mbps will be available, [3]. However, this may cause the transmission range to emerge as a limiting factor since channels are limited from 20 to 40 MHz and transmission power has a maximum of 100 mW. The market's demands shift towards ubiquitous wireless connectivity at high speed but to meet such expectations, dense deployment of Access Points (APs) is an essential requirement. However, to interconnect, APs rely on a fixed infrastructure. Cost-wise, APs are cheap but the deployment and maintenance of the wired backbone drastically increases this cost. Therefore a wireless solution is the key, [3 – 5]. The initial specifications for the IEEE 802.11 popular standard for WLANs were completed by IEEE in 1999 and successively extended in 2003, [6]. By that time, all the family of IEEE 802.11 standards was specified for one-hop communications, making it unsuitable for multi-hop, multichannel, and multi-radio operations, [7]. This is why the IEEE set up a new working group: 802.11s task group, for the installation, configuration and operation of 802.11-based mesh networks, where all necessary functions to form such WMNs are described. While at start, this was restricted to APs only, the succeeding changes to its Project Authorization Request (PAR), gave the IEEE 802.11s a remarkable flexibility in providing multi-hop wireless Internet connectivity to mobile users.

Despite the already existing potential application scenarios of WMNs such as backhaul support for cellular, home, enterprise, community and intelligent transport system networks, WMNs development continues to be a hot and attracting topic for researchers on both academic and industrial levels since it deals with challenging architectural and protocol design factors and has an enormous set of open research topics. Many researches and projects in this field are ongoing in different universities and industrial research laboratories. Despite the ongoing researches and the already proposed works [20 – 37], our attention is attracted by the continuous lack of an accurate 802.11 MAC model. Accurate mathematical modeling for IEEE 802.11-based WMNs and thus correct performance metrics analysis for delays contributed by queuing, channel, collisions and transmissions would allow better performance assessment of such WMNs in addition to giving the ability to specify delay guarantees and enhance the overall network performance thus providing better support for both delay-sensitive (real-time) and packet-loss sensitive (non real-time) traffics, simultaneously. Moreover, the development of state of the art analysis and simulation tools which continue to be increasingly desirable not only because of the wide deployment of 802.11 but because the CSMA/CA mechanism with exponential random back-off used by the 802.11 MAC protocol to control the wireless channel access plays an essential role in new protocol designs and standard proposals such as IEEE 802.11e, [8].

It also comes to our attention that there is a major gap resulting from the lack of 802.11s-specific routing metrics and routing protocol designs that take into account the peculiar characteristics and properties 802.11s-based WMNs as opposed to adaptations of the routing strategies and schemes used for Mobile Ad-hoc Networks[1] (MANETs) where major characteristics and properties of those latter are no longer valid for 802.11s-based WMNs [9], such as:

1. **Mobility**: It is very rational that wireless devices with fixed locations will represent the vast majority of WMNs. This reasoning is as a matter of fact based on the way by which humans use computers. A mobile computer can be transported from any location to another such as a lounge, different areas of a certain campus, office, airport … etc. However, this computer will use a fixed wireless hot spot to gain access to the Internet.
2. **Dynamic topology**: In MANETs, nodes are highly mobile whereas in an IEEE 802.11s-based WMN only mesh client stations have the possibility to be mobile but their mobility is quite limited to tight geographic areas, which does not provoke variations in the network's topology.
3. **Node services**: In MANETs, each node can serve as both a router and a host, whereas in IEEE 802.11s-based WMNs there is a clear distinction between two device classes:
    a. *Mesh Routers*: Such form the wireless backbone and support mesh services such as mesh routing selection and forwarding and interwork with the wired networks.
    b. *Non-Mesh Nodes*: Simple client stations.

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[1] Mobile Ad-Hoc Networks represent series of mobile platforms that communicate using wireless transmission and can move in and out of a geographic area in a random or arbitrary manner, [9].

4. **Power:** Since in MANETs nodes are highly mobile, they are most likely to be operating on batter power. However, in WMNs, nodes are fixed and therefore can be connected to power sources. Moreover, the IEEE 802.11 series of WLAN standards are designed in such a way those nodes can be in one of two modes: active or sleep mode. When in sleep mode, the node periodically wakes up and listens to the wireless spectrum to check if any other node has data destined for it. If so, the node stays in active mode, otherwise it goes back to sleep mode. This design in fact reduces power consumption in such networks.
5. **Network scale**: Ad-Hoc networks can be formed of hundreds to thousands of nodes within a certain geographic area. However, the number of nodes supported by IEEE 802.11-based WMNs ranges from thirty-two in small/medium scale networks and up to a maximum of fifty nodes in large scale networks, [10].

In view of the above listed major differences between MANETs and IEEE 802.11s-based WMNs, the adaptation of existing MANET models and protocols to WMNs would result in a severe unfairness, [11]. Also using the IEEE 802.11 MAC as the wireless protocol when simulating wireless Ad-Hoc networks, is quite a bad practice frequently observed in the literature, [12 – 17]. In WMNs, there should be a close interaction between the routing and the MAC layers in order to maximize performance. Therefore the integration of adaptive performance metrics from layer-2 into routing protocols or merging certain operations of the MAC and routing protocols is of course a promising approach, [7], [18].

In this manuscript, we refine and correct major design problems and mistakes of the IEEE 802.11 MAC model based on discrete time *G/G/*1 queues proposed in [37] making it more realistic. Particularly, the expression of the packet service time probability, the key issue in that model, is corrected. Based on this the refined model is therefore able to perfectly account for channel access time resulting from the random access mechanism, collision avoidance and exponential back-off mechanism of 802.11, delays in channel access due to other nodes transmitting and delays caused by collisions. Thus such results can provide more accurate data used for the evaluation of the number of connections that can be supported for a given delay or loss constraint. We also propose a new model which is basically characterized by its simplicity, generality and accuracy where the packet service time in IEEE 802.11-based networks is modeled by a combination of Erlang-*k* and Coxian-2 distributions, always taking into account the general traffic arrival patterns. Thus our new analyses are based on *G/E-k/*1 and *G/C*2/1 queues. Moreover, this manuscript presents two novel MAC-AWARE and BUFFER-AWARE routing protocols based on a new IEEE 802.11-specific routing metrics used to provide Quality of Service (*QoS*) routing for both delay-sensitive and packet-loss sensitive traffic generated simultaneously in an 802.11-based network in DCF mode.

The rest of this manuscript is organized as follows. Section II, describes a survey of related works in the fields of IEEE 802.11 MAC modeling and routing metrics and protocol designs. Section III, provides an overview of the IEEE 802.11 architecture. Section IV discusses the IEEE 802.11 MAC and its 802.11s extensions. In section V, we will provide our new IEEE 802.11 MAC detailed queuing models and refined mathematical expression for the service time probability. In section VI we propose new IEEE 802.11s-specific routing metrics and their corresponding novel routing protocols. Simulations and results used to verify our models and protocol designs are presented and discussed in section VII. Finally Section VIII ends with concluding remarks.

Chapter 2

Review of Literature

Mathematical modeling of the IEEE 802.11 MAC has been the subject of numerous empirical studies. However, an accurate model simultaneously valid for both saturated and unsaturated network scenarios is still lacking. Most of the already published works in the field are centered on a saturated network throughput and capacity where the Markovian, mean value and fixed-point models and analysis were proposed [20-24]. We noticed that works devising IEEE 802.11 MAC Markovian models faced a key difficulty in the large number of states that exponentially increases with the number of nodes, [20], [21].

In the seminal work of [20], this difficulty was addressed by the assumptions that every station always has a packet to be transmitted, the packet collision probability is constant regardless of the state or station and the transmission error is a result of packets colliding and is not caused by medium errors. This model is justified for a certain subset of possible arrival processes and only in corresponding saturation scenarios that are in a way not always true in such 802.11-based WMNs since Internet applications such as browsing, e-mail and VoIP have bursty or on-off traffic characteristics. In addition, having fixed collision probabilities is absolutely not an accurate assumption since a traffic intensity decrease would also cause the probability of collision to decrease. Otherwise, packets arriving during low traffic periods will be subject to severe penalties from a service time perspective which is immediately dependent on the probability of collision, which means that they will be waiting so long on the top of the queues while they can be transmitted if not immediately, within a much shorter period of time. Of course this will have a considerable negative impact on the overall performance of the network.

The work in [21] is based on the work of [20], however, it relaxes the restriction of saturated operation and develop a Markovian model of the IEEE 802.11 MAC under non-saturated conditions in an attempt to retain most of the attractive simplicity of [20], particularly in deriving analytic relationships. This work would've been accurate enough for unsaturated conditions; however it falls for the same error of assuming constant collision probabilities.

Other researches were conducted on improving the 802.11 MAC using the channel adaptive back-off mechanisms on one hand [25-26] while on the other hand investigations on the impact of such schemes on traffic characteristics were done in [27]. Efficiency of polling techniques used in parallel with the PCF to provide support for voice services in 802.11-based WLANs in the presence of access points, has been investigated in [28-31], while [32] discusses the same problem in scenarios where access points are absent. Simulations were conducted in [33] to compare the delays in 802.11b and 802.11e in DCF mode. A lower limit on the delay exhibited in a saturated network where DCF is used has been theoretically expressed in [34] while channel access time in a saturated network as well is evaluated in [35]. Delay modeling and analysis in PCF mode have been reported in [28], [30] and [36]. Moreover, such modeling and analysis in DCF mode have been proposed in [37], which quite attracted our attention in their attempt to devise an IEEE 802.11-MAC model for all the set of possible arrival processes and hence are using *G*/*G*/1 queues. However, this model falls for several major mistakes, the first of which is the assumption that the saturated conditions provide good approximations to certain unsaturated scenarios. This causes the model to be erroneous in terms of queue characteristics and thus not producing accurate results for wireless networks operating in random access mode with DCF.

The routing layer, one of the key communication protocol layers aiming at efficient resource utilization in WMNs, [7], is another topic that has been investigated by many researchers, though not as intensively as MAC modeling. However, it remains a key problem for the newly developing standards such as IEEE 802.11s where we've been attracted by the non-existence of routing metrics and protocols designed to take into account the characteristics and properties of such networks, rather an adaptation of the existing routing metrics and protocol designs to IEEE 802.11-based WMNs has been observed in the literature. Moreover, rare are the protocol designs that are MAC-aware. The majority of the protocols found in the literature are PHY-aware, whereas a cross-layer MAC/PHY routing protocol design is a very promising approach as we will prove in this manuscript. We shall first have a look over the existing mostly known routing metrics and discuss their relative disadvantages and whether they would be appropriate for WMNs.

The traditional approach to routing in MANETs is the Shortest Path also known as the Minimum-Hop (MH) routing, [7], [38, 39].Indeed the hop count is the simplest cost metric however it is highly applied in networks where mobility is high which is not the case of IEEE 802.11s-based WMNs where only stations are poorly mobile and mesh routers forming the backbone of the network are strongly static. Thus when adapted to such networks, the minimum hop count fails to have satisfactory performance.

The effect of performance metrics on a routing protocol in static multi-hop wireless networks was studied in [19] where an empirical study was conducted to compare the performance of three link-quality metrics using a Dynamic Source Routing-based (DSR) routing protocol in a wireless test bed:

1. **Expected Transmission Count (ETX)**: Proposed by [40] for IEEE 802.11-based radios employing link-layer retransmissions to recover from frame losses where the ETX of a radio link was defined to be the estimated average number of transmissions necessary to transfer a packet successfully over the wireless link. This is quite an appealing cost metric because minimizing the total number of transmissions maximizes the overall throughput. However, it is uncommon that a link with lower ETX metric may in fact lead to higher observed loss rate at the transport layer because good link-layer protocols do not keep infinitely retrying to send lost packets but would drop this packet after a given upper limit. This is actually the case in IEEE 802.11-based networks where the re-transmission counts are bounded by the Short and Long Retry Counts, (SRC) and (LRC) respectively, [27].
2. **Per-hop Round-Trip-Time (RTT)**: A delay-based link cost metric using the measured average RTT seen by unicast probes between neighboring nodes, [41]. A fundamental problem associated with using RTT as a routing metric, is the fact that it varies according to the load, [7]. This leads to an oscillatory behavior or even instability. In case the delay at a certain node decreases due to reduced load, more paths tend to pass through this node, thus pulling the delay and hence the RTT metric back to a high value. This phenomenon is known as the *self-interference*. It was proven by the work done in [19] that RTT results in drastic performance degradation as compared to minimum-hop count. This is not to mention the high overhead associated with measuring the

RTT as well as the huge queuing delay as compared to transmission time in overall cost.

3. **Per-hop Packet Pair (PktPair)**: Built in the work of [19] in an effort to modify the Per-hop RTT. It is based on sending a short probe packet ahead of a long one and using it to set a time reference where each neighboring node keep the time difference between the reception of these two packets and feeds it back to the source which a record of those values. Their average is assigned as the cost metric for the link. Indeed this approach suppresses the queuing and processing delay portions from the overall delay and does not cause an increase in the metric. However, an increase in the load still causes the metric to increase and the improvement for PktPair is not good enough to outperform the simple hop-count metric in addition to its overhead, which is even greater than Per-hop RTT, [7].

Through multiple channels, it is possible to improve network throughput by simultaneously using the available non-overlapping channels defined by IEEE 802.11. This technique however needs to deal with two main issues to become effective. First, the *intra-flow* interference occurs when different nodes transmitting packets from the same flow interfere with each other. Minimization of the number of channels is not that trivial considering the fact that the nodes must maintain connectivity. Second, the *inter-flow* interference is suffered among concurrent flows. In [42], a new metric for routing in multi-radio multi-hop wireless networks, the Weighted Cumulative Expected Transmission Time (WCETT) was introduced, which explicitly accounts for the interference among links that use the same channel. At first glance, this metric appears to be attractive. However it does avoid *inter-flow* interference, [43]. In order to be loop-free it is of course necessary for Link-State routing protocols to have minimum-cost paths. By not avoiding inter-flow interference, WCETT may choose congested paths.

The Metric of Interference and Channel-switching (MIC), [43], deals with the problems of WCETT. With MIC, each node takes into account the number of interfering nodes in its neighborhood which helps it evaluate the inter-flow interference and uses virtual nodes to guarantee minimum-cost path computation based on ETT metric.

A critical issue of wireless networks is the fast link-quality variation which some metrics based on average values computed on a time-window interval may not be able to follow and therefore produce prohibitive control overhead. The iAWARE metric proposed by [44] considers such variations and uses Signal-to-Noise Ratio (SNR) and Signal-to-Interference-and-Noise Ratio (SINR) to evaluate neighboring interference variations. iAWARE estimates the average time the medium is busy from interfering neighbor transmissions. Thus iAWARE not only considers inter and intra-flow interference, but also medium instability and data-transmission time. However, all these variables are related to the physical layer. Therefore it is clear that the MAC layer in all those metrics is being neglected.

In IEEE 802.11s, the default routing metric is referred to as the airtime-cost. This metric takes into account PHY and MAC overhead and accounts for frame payload and packet error rate to reflect the radio link condition, [7].

At this stage, having looked at the various routing metrics already proposed in the literature, it is of our interest to look at the routing protocols where those metrics are deployed. Existing routing protocols can be classified into three main categories:

1. **Proactive**: Such routing protocols operate exactly similar to those on wired networks where routers usually keep at least one route to any destination in the network.
2. **Reactive**: These are protocols that request a route to a destination only when a node has packets to send to that particular destination, otherwise the node will never request a route to it.
3. **Hybrid**: This is the class or routing protocols that merge both proactive and reactive routing together.

While many of the existing routing protocols use similar strategies,they are mostly adapted to the particularities of WMNs, such as when using quality-aware routing metrics. WMN routing protocols can be classified into four subclasses: Ad-Hoc-based, controlled- flooding, traffic-aware and opportunistic, [7], [9]. Those subclasses' major differences are path discovery and maintenance procedures.

WMN Ad-Hoc-based protocols adapt Ad-Hoc routing protocols to deal with link-quality variations. Routers continuously update their outgoing-link metrics and propagate them to other routers.In [42], the Link Quality Source Routing protocol (LQSR) combines link-state proactive routing with the reactive strategy from Ad-Hoc networks. It uses a complete view of the network topology to compute shortest paths through a reactive path discovery procedure aiming at reducing the routing overhead resulting from medium instabilities and user mobility. During path discovery, LQSR obtains updated link state information of the traversed links, which reduces the periodicity of regular link-state advertisements.

Source Reactive Routing (SrcRR), [45], is another Ad-Hoc-based protocol extending the ETX routing metric by predicting the best 802.11 transmission bit-rate on each link and monitoring the loss rates of the links in each path it is using as well as the loss rates of the nearby alternate links, to ensure that it continues to use the best path. SrcRR switches to a new route only if the new route's metric is significantly better than that of the existing route. It performs more aggressive link-level retransmission than 802.11 and does not switch routes in response to modest numbers of lost packets. It also uses its own transmit bit-rate selection algorithm based on the medium loss rate measurements.

Physical aware techniques usually aim at improving the overall efficiency of routing protocols. In [42] as well, the Multi-Radio Link Quality Source Routing (MR-LQSR) adapts LQSR to operate over multiple channels and multiple interfaces, using the WCETT metric, which does not guarantee minimum-cost paths as already discussed. However MR-LQSR is loop-free because its property of being a source routing protocol.

The design of Controlled-Flooding protocols aims at reducing the control overhead. Network flooding with route updates causes scalability problems especially under high medium conditions variations. Such problems can be solved through either, setting the periodicity of path update while taking into consideration the position relatively to the source, or, limit the number of nodes that flood the network and therefore reduce redundancies because it is known that the farther the nodes are from the source, the less accurate information they are going to get from it. Moreover, only local-scope routing information is disseminated because most communications in WMNs happen between nearby nodes, meaning that packet control to far nodes is not as necessary as for the nearby ones. This can be achieved by finding the minimum set of nodes required to route information to all destinations in the networks.

A routing protocol that attributes long and short-term costs to links proposed in [46] is the Localized On-demand Link State (LOLS) using ETX and ETT as routing metrics. Long and short-term costs respectively represent the usual and the current cost of a link. Short-term costs are frequently sent to neighbors while long-term costs are sent using larger periods. This results in the reduction of control overhead.

A routing protocol that does not specify a routing metric is the Mobile Mesh Routing Protocol (MMRP) developed by MITRE Corporation. Similarly to the Open Short Path First (OSPF) protocol, MMRP assigns an age to routing messages that it extracts using the estimated time required to forward those messages.

The Optimized Link State Routing protocol (OLSR) discussed in [7] and (RFC 3626) was adapted to use ETX as a link metric in WMNs using the fraction of HELLO messages lost in a given interval of time. OLSR can be classified as an Ad-Hoc-based protocol but it uses Multi-Point Relays (MPRs) to control network flooding by limiting the number of nodes in charge of propagating control packets in order to reduce redundancies. Selector nodes select their MPR set as being the minimum number of one-hop neighbors needed to reach all two-hop neighbors.

Traffic-aware or tree-based routing protocols are deployed in infrastructure WMNs where backbone access is the common-case application. Such protocols consider a tree-like network topology. The Ad-Hoc On-demand Distance Vector-Spanning Tree (AODV-ST) proposed in [47], adapts the AODV protocol from Ad-Hoc networks where the gateway, being the root of the tree, periodically requests routes to every node in the network to update its routing table. Communications not including the gateway use the original AODV. It is clear therefore that deploying such a protocol in IEEE 802.11s-based WMNs will be a bad approach since no direct communication happens between stations. A station has the Mesh Portal (MPP) as its ultimate destination and the MPP has any of the stations as its ultimate destination. Finally, AODV-ST supports ETX and ETT as its routing metrics.

The work in [48] proposes a routing algorithm based on the spanning tree used in wired networks where route maintenance is done with join and leave requests and minimum-hop as well as other load-balancing metrics are used.

Opportunistic protocols aim at improving classical routing schemes based on cooperative diversity schemes. While classical routing protocols compute a sequence of hops to the destination by either using hop-by-hop or source routing before sending their packets, opportunistic protocols decide, on the fly, which next hop offers the best throughput therefore guaranteeing packet forwarding whenever at least one next hop exists. Moreover, in classical routing, if any link failure occurs, successive link-layer retransmissions are performed until successful reception at the next hop neighbor or a maximum number of retransmission attempts is reached thus causing packet drop. In contrast, the chosen routes in opportunistic protocols are more likely to use the best quality links while considering short-term variations, which certifies that the packet is transmitted with minimum retransmission. Since wireless links need time to recover from failures, such an approach may result in considerable performance degradation and high delays. Cooperative strategies, which are adapted to IEEE 802.11 transceivers, however, take advantage of the broadcast nature of radio-frequency transmission by setting multiple paths towards a certain destination. Suitable transceivers are then required to correctly process the relayed signals by choosing one of them or a combination of all. The ExOR protocol proposed in [49], combines routing with MAC-Layer functionality where routers, in order to reduce protocol overhead, broadcast packets in batches with no anterior route calculation. This broadcasting technique improves reliability since only one intermediate node is required to overhear a transmission. Thus only one node retransmits at a time. This node is actually determined by the source router through a forwarding list added to the packet's header and which contains the addresses of neighbors ordered by forwarding priority. This priority is determined by the distance of this node relatively to the destination and computed using a similar metric to ETX. However, packet arrivals are not guaranteed since they are not acknowledged, something that requires additional procedures to determine the successful reception of these packets.

In [50], the Resilient Opportunistic Mesh Routing protocol (ROMER) combines long-term, shortest-path or minimum latency routes with on-the-fly opportunistic forwarding to provide resilient routes and deal with short-term medium quality variations. ROMER calculates long-term paths using minimum hop count or minimum average delay and opportunistically expands or shrinks them at runtime to fully exploit short-term higher-quality links. Table 1 summarizes all the discussed routing protocols and lists the used routing metric by each.

Table 1: WMN Protocols and their routing Metrics.

Class	**Protocols**	**Metrics**
Ad-Hoc-Based	LQSR	ETX
	SrcRR	ETX
	MR-LQSR	WCETT
Controlled-Flooding	LOLS	ETX or ETT
	MMRP	Not Applicable
	OLSR	Hop, ETX, ETT
Traffic-Aware	AODV-ST	ETX or ETT
	Spanning-Tree-Based	Hop, Load-Balancing
Opportunistic	ExOR	Forward ETX
	ROMER	Hop, Delay

Chapter 3

IEEE 802.11 Architecture

The most basic entity in an 802.11-based network is a station (STA), [51]. Basically, the set of all devices that satisfy an 802.11-MAC and PHY layers requirements are referred to, in this context, as stations, [3]. Some of those stations, the Access Points (APs), are equipped with extended capabilities and serve as central devices for all the other stations in a WLAN. In one scenario, client stations authenticate and associate with an AP in order to obtain access to the network (i.e. stations rely on APs for communication and each client station has at least one link to the AP). This way, every AP along with its successfully associated clients (i.e. those client stations that were able to successfully join the network), form a star topology, which in IEEE 802.11, is referred to as a Basic Service Set (BSS).In another scenario, clients may communicate with each other directly. Therefore, the set of all clients that can directly communicate with each other without the intervention of an AP is referred to as an Independent Basic Service Set (IBSS). In other words, Ad-Hoc networks where no infrastructure is required. In this manuscript, our main focus is only on Basic Service Sets since they are the most commonly deployed.

In 802.11, the term link is defined from a MAC layer viewpoint as the single physical path over the Wireless Medium (WM) enabling two stations to communicate and thus exchange MAC Service Data Units (MSDUs). In all IEEE 802.11 standards, MSDUs are acknowledged except in 802.11e where acknowledgment is optional, [3]. An Acknowledgment (ACK) is a short frame usually transmitted using robust Modulation and Coding Schemes (MCS) with a Packet Error Rate (PER) is much smaller than the acknowledged frame itself. This means that in wireless communication, the successful transmission of a packet from a source S to a destination D does not guarantee the reverse, [52]. However, despite the fact that it is not explicitly stated, IEEE 802.11 assumes all links are bidirectional, [3].

As defined, infrastructure BSSs form wireless single-hop networks where all participating stations communicate via APs that operate as relays between them. The Distribution Service (DS) allows multiple APs to interconnect their BSSs to form Extended Service Sets (ESSs) using the Distribution System Service (DSS). To provide the DSS, APs rely on the Distributed System Medium (DSM), which is a non-802.11 network, [3]. It may either be a logical entity within the AP itself, or typically based on an 802.3 LAN segment. It is worthwhile noting at this stage, those APs, even if in mutual communication range, they do not use the WM to exchange packets. The entire area covered by all interconnected BSSs is referred to as the Extended Service Area (ESA) where it is possible for all stations within it to roam from one AP to another.

Nowadays, APs are most likely to be collocated with portals given that those latter provide integration services that deliver MSDUs to the non-802.11 networks. Portals allow APs to gain access to the 802.3 LANs that build the DSM. Through the DS, the Logical Link Control (LLC) layer sees the ESS as a single logical network. Therefore, the DSS allows handoff within the ESS and seamless packet forwarding between APs, portals and client stations. Four address fields are provided by IEEE 802.11 to allow station addressing within different BSSs. Those address field are namely the Source Address (SA) that holds the MAC address of the station generating a packet, the ultimate and final receiver's address in the Destination Address (DA), the Transmitter's Address (TA) used by APs when forwarding packets and finally the Receiver Address (RA) indicating the next intended receiving AP in the ESS.

IEEE 802.11s defines the Mesh where the Mesh Point (MP) is its basic element, [53]. However, unlike regular 802.11, MPs in an 802.11s have the possibility to exchange frames in a wireless multi-hop manner, something which enables them not only to communicate with other MPs within their mutual communication range, but also outside. In addition, similarly to regular APs and portals, MPs have the relaying capability. Thus MPs may act as application end-points (i.e. source or sink of data traffic) or they may forward packets in communication scenarios that they are not involved in and thus relay packets within the 802.11 networks. However, unlike APs, MPs do not provide AP services while a Mesh Portal (MPP) bridges the 802.11 with the non-802.11 networks.

Parallel to the 802.11 definition of a BSS, a set of MPs is defined as a Mesh BSS (MBSS). The IEEE 802.11 standard clearly states that successfully joining a BSS does not imply possible communication with all other members of that BSS, [3]. This also applies to MBSSs as well with the only difference that members of an MBSS have the ability to communicate with each other in a multi-hop fashion if and only if the Mesh Path exists between them. Therefore, the next step is to explain how the topology formation process takes place in an IEEE 802.11s-based network. At boot up, initialization occurs first after which the MP uses a Simple Channel Unification Protocol (SCUP) to perform active or passive neighbors' scanning. If no neighbors are detected, then the MP establishes itself as the initiator of a mesh network by selecting a channel precedence value based upon its boot time plus a random number. If two disjoint (i.e. using two distinct channels) mesh networks are discovered, then the channel is chosen according to the highest precedence value.

Note that if the mesh is in the 5 GHz band, it is therefore required to conform to the regulatory requirements of the Dynamic Frequency Selection (DFS) and radar avoidance to conform with FCC UNII-R regulation. Now given a certain Mesh Network, MPs that are not yet members of the mesh, discover candidate neighbors based on new Information Elements (IEs) – matching profiles where a profile usually consists of a Mesh ID (i.e. the name of the mesh) and WLAN Mesh Capability Element including path selection protocol identifier and link metric identifier – in beacons and probe response frames. If the beacon contains a nonzero peer link value, then the Mesh Link (ML) is established through a secure protocol and the exchange of MAC Service Data Units (MSDUs) between both MPs is now possible. After all MPs discover their neighbors and establish communication links with them, it is said that a new MBSS is born. Therefore all MPs in that MBSS can communicate with each other either through one-hop or multi-hops. In other words, mesh paths (i.e. concatenated set of MLs) exist from each MP to any other MP in the MBSS. This is more illustrated in Figure 1.

It is worthwhile mentioning that the 802.11s transparently supports any higher layer protocols, [54]. The MBSS therefore must support all types of unicast, multicast and broadcast traffic, [3]. The IEEE 802.11s defines the Mesh header field consisting from four to sixteen bytes, the first of which holds the Mesh Flags field. The first bit of this field indicates the presence of Address Extension (AE) and all the remaining bits are reserved. If the AE flag is set, an MP therefore uses the six-address scheme where additional address fields identify intermediate MPs on the Mesh Path. AE may be used in tree-based networks where a root MP is present, [3]. The second byte holds the Mesh Time-To-Live (MTTL), required to prevent endless packet looping. Therefore, each MP that forwards a certain packet decrements its MTTL counter.

Bytes three and four provide Mesh End-to-End (ME2E) sequence numbering based on a per-hop sequence control in 802.11. MPs use the ME2E sequence number field to avoid unnecessary retransmissions in packet flooding scenarios. The ultimate destination uses the ME2E sequence field to eliminate duplicates.

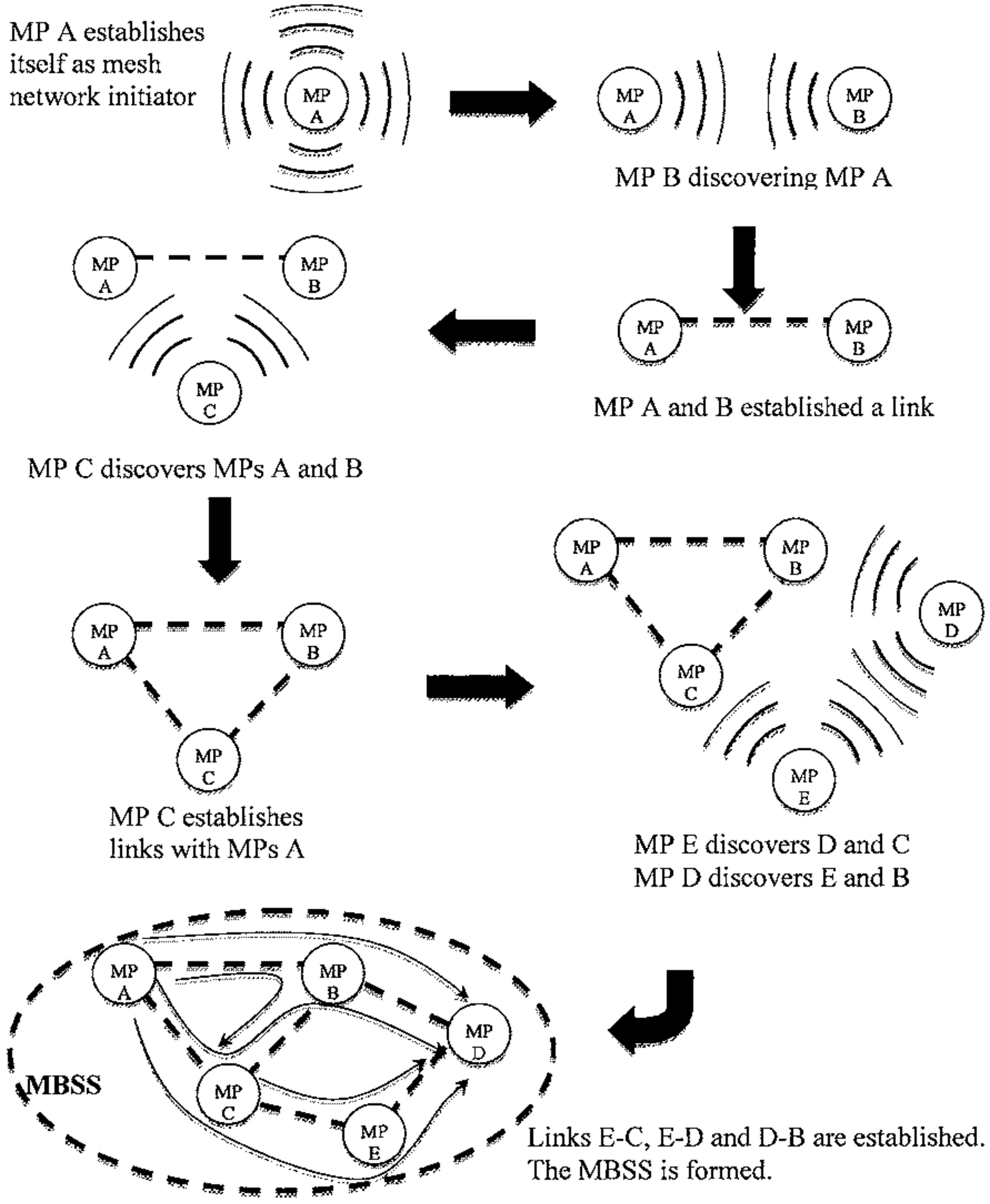


Figure 1: MP A can communicate with MP D through the mesh paths A-B-D, A-C-E-D, A-C-B-D, A-B-C-E-D.

Chapter 4

Medium Access Control in IEEE 802.11

The IEEE 802.11 MAC layer is responsible for a structured channel access scheme [37]. It was implemented using two different coordination functions, namely the Distributed Coordination Function (DCF) and the Point Coordination Function (PCF). While the DCF is based on the Listen Before Talk (LBT) mechanism, in other words the Carrier Sense Multiple Access (CSMA), [3], PCF is quite similar to a polling technique used to determine which of the existing nodes in the network has the right to transmit at a certain instant of time, [37]. DCF supports no prioritization. However, a superset that allows different medium access priorities is the Enhanced Distributed Channel Access (EDCA) where stations may also be allowed to send multiple packets after contention. This number of MSDUs is bounded by the Transmission Opportunity (TXOP) limit. EDCA was proven to operate more efficiently than DCF when using the Block Acknowledgement, [55], and is a mandatory coordination function in 802.11s, [3]. However it suffers from major problems that will be discussed later in this section.

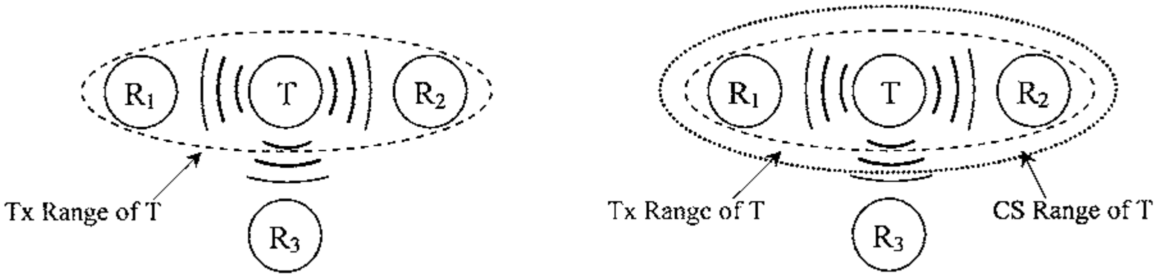
The Clear Channel Assessment (CCA) combines the input of two Carrier Sense (CS) mechanisms, namely the Physical Carrier Sense (P-CS) and the Virtual Carrier Sense (V-CS). In P-CS, every station senses the Wireless Medium (WM) for energy. If the energy level exceeds some threshold, a value that depends on the 802.11 Physical Layer (PHY), this means that the channel is busy and the station will not try to transmit packets. The V-CS informs the stations about pre-planned transmissions. Stations that are not in sleep mode will constantly monitor the WM and thus retrieve channel reservation information from any packet they may decode. In 802.11, such information is provided in the Duration field and according to it stations will set their Network Allocation Vector (NAV), which works as a countdown timer. A non-zero NAV value indicates a busy WM. Updating the NAV may occur at any time causing the NAV duration to be either extended or foreshortened.

Carrier Sense Multiple Access/Collision Detection (CSMA/CD), a mechanism used in wired networks to detect collisions, is no more applicable to wireless communication. To ensure proper sharing of the wireless medium, a technique called Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) is used, where before each transmission, a node has to perform a certain random back-off procedure, [3]. That is, each node having a packet to transmit first senses the medium to check if other nodes are using it. If the medium is idle for a minimum period of time denoted by the Distributed Inter-Frame Space (DIFS), then the node initializes its *back-off counter* with a randomly selected *back-off interval* drawn from the range (1, CW) where the Contention Window (CW) indicates the upper bound of this interval. The minimum contention window size is $CW_{\min}$ and it is doubled after each unsuccessful transmission, thus reducing the collision probability of a retransmission. The *back-off counter* is decremented by one every time the WM is sensed idle for a time-slot duration denoted by aSlot. The aSlot value depends on the PHY layer. A busy WM detection before the timer becomes zero causes this timer to freeze. The node will have to wait for another DIFS period of idle WM in order to resume decrementing its timer from where it left off. Once the timer reaches zero, the node is then allowed to start its transmission. After a successful transmission, the CW is rest to its initial size, $CW_{\min}$.

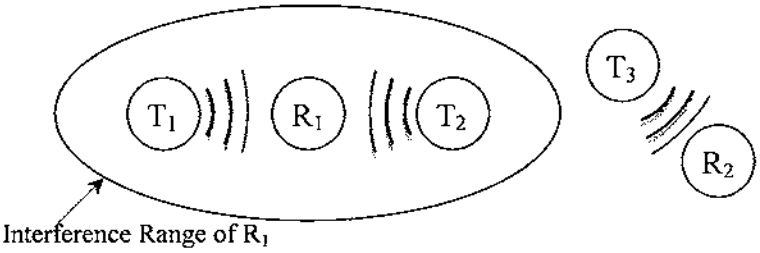
In parallel to Collision Avoidance, 802.11-based network nodes use a positive acknowledgement scheme (ACK) where a receiving node, after receiving each and every packet, waits for a short period of time denoted by the Short Inter-Frame Space (SIFS) and then transmits an ACK message to assert the proper reception of the packet transmitted to it.

Two severely limiting problems of IEEE 802.11-based WMNs are the *Hidden Terminal Problem* and the *Exposed Terminal Problem.* Three important characteristics of an 802.11 compliant device are as depicted in Figure 2 and described below:

1. **Transmission Range**: The range within which a transmitted packet is successfully received in the absence of interference. This range depends on the transmission power and attenuation.
2. **Carrier Sensing Range**: The range within which a transmitter triggers carrier sense detection. This range is greater than the transmission range and depends on the antenna sensitivity.
3. **Interference Range**: The range within which receivers will be interfered with and suffer losses.



(a) Transmission Range **(b)** Carrier Sensing Range



(c) Interference Range

Figure 2: In this figure T is a transmitter and R is a receiver. This figure represents the three main important transmission, carrier sensing and interference ranges of an IEEE 802.11-compliant node.

The *Hidden Terminal Problem* is encountered when two or more nodes, unaware of other ongoing transmissions, transmit their packets and thus cause collisions at the receiving nodes, [7]. Such a scenario is illustrated in Figure 3 where three nodes, A, B and C are considered with B being a common node to both the transmission ranges of A and C. The small dashed oval represents the transmission ranges of the nodes A and C whereas the larger dotted oval represents the carrier sensing range of A and the largest solid oval represents the interference range of B. In the figure, node A is already in packet exchange mode with B. Node C, which is outside of both transmission and carrier sensing range of A, has a packet to transmit. C senses the medium to be idle since it cannot hear A's transmission to B. Therefore C starts transmitting and hence causes a collision at B.

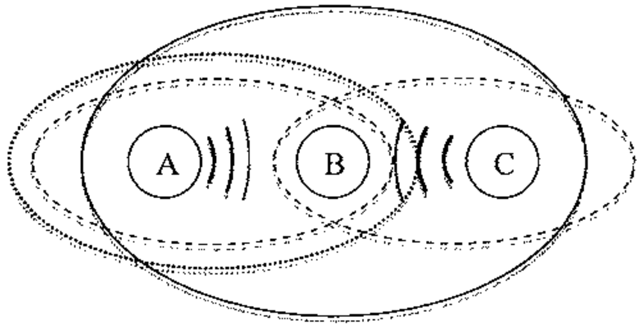


Figure 3: Illustration of the *Hidden Terminal Problem.* Any node within the Interference range of the receiver and outside the transmission range of the transmitter is a potential hidden node. In this figure, node C is not captured by A's P-CS.

To tackle this problem, IEEE 802.11 MAC uses the so-called reservation scheme, where each node with packets to transmit would send a Request-To-Send (RTS) packet to its destination, which, if willing to receive the packets would reply to the source by a Clear-To-Send (CTS) packet and hence the source may start transmitting the DATA. After complete reception of the DATA, the destination replies back to the source with an acknowledgement message to certify that the DATA was received successfully. This is known to be the basic operation of the CSMA/CA protocol as is illustrated in Figure 4.

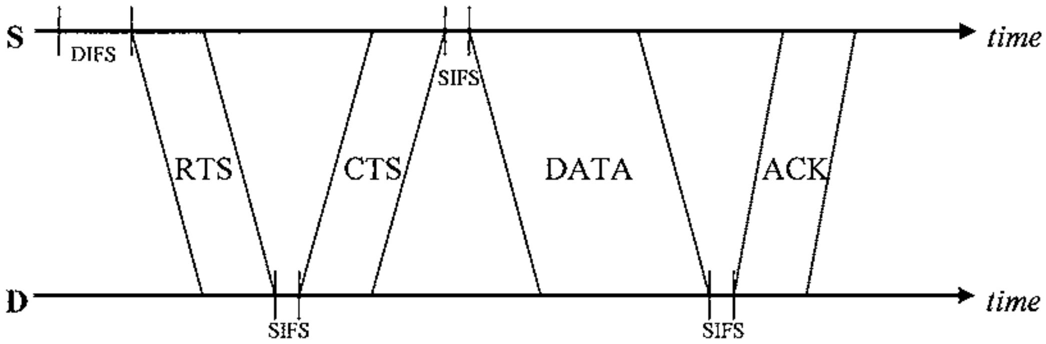


Figure 4: Basic CSMA/CA protocol operation.

The Request-To-Send (RTS) and Clear-To-Send (CTS) are short control frames. Their transmission duration depends on the Modulation and Coding Scheme (MCS). They are usually transmitted using the lowest MCS in order to maximize their robustness. RTS/CTS packets indicate in their Duration Field the duration of the following packet exchange. Therefore when successfully decoded by other neighboring nodes, these nodes will refrain their transmissions till after the current transmission is successfully made, [3]. This being a solution to the *Hidden Terminal Problem*, however, in some scenarios it is not efficient and therefore the problem persists such as the scenario depicted in Figure 5. Node B is within the transmission range of node A. However, node C is outside all of the transmission ranges of nodes A and B and the carrier sensing range of node A. Therefore, assuming a scenario where A is already transmitting packets to B. Node C, will not hear A's transmission to B (C cannot hear the RTS sent from A to B). Moreover, being outside the transmission range of B, node C will also not able to receive and decode the CTS sent from B to A and will not be able to update its NAV and reschedule its transmission after the end of A-B communication as specified by the IEEE 802.11 standard. Moreover, C is still covered by the interference range of B. Therefore in case C transmits to any of the nodes that lay within its transmission range at the same time as A is transmitting to B, a collision will occur at B. Resulting losses from the collision in this case are going to be detected at A. The communication between C and any of its neighbors will not be penalized.

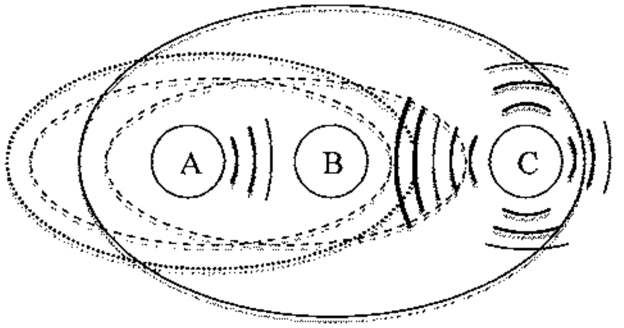


Figure 5: *Hidden Terminal Problem* scenario where a node, C, that is outside all of the transmission range of the transmitter and receiver and carrier sensing range of the transmitter but is still inside the interference range of the receiver is a hidden node. The A-B communication is penalized.

This scenario indicates that the RTS/CTS solution to the *Hidden Terminal Problem* is not valid and the as a matter of fact the problem persists. Moreover, in all today's Wireless Fidelity (Wi-Fi) products, as factory default configuration, the RTS/CTS threshold is set to its maximum and is not user customizable making this solution almost never used, as indicated by the measurements conducted in large wireless networks made by [57]. The work in [58] indicates that RTS/CTS has no impact on the network's performance and it was concluded by [59] and [60] that RTS/CTS only adds extra overhead.

Another severely limiting problem in 802.11 is known as the *Exposed Terminal Problem*. The more the sensitivity level of P-CS is decreased, the larger the CS range will increase. It is true that P-CS almost succeeds in preventing the existence of hidden nodes, [3], however, the physical carrier sensing range in 802.11 is extremely large as compared to the transmission range as illustrated in Figure 6 where the P-CS range of A is larger than its transmission range. Nodes C, D and E refrain their transmissions during the transmission from A to B. Node F cannot hear A's transmission and initiates a packet exchange with E that cannot reply as it detects a busy WM by A. This applies to F as well that cannot initiate a transmission to D. This may limit concurrent independent transmissions. As a matter of fact, in WMNs, for each node, the set of neighbors' neighbors includes more nodes than the set of neighbors itself. Therefore, a single transmission from a certain node may involve a large number of other stations through multi-hop and thus a lot of stations become exposed. Those nodes can reuse the WM for independent transmissions without causing interference. However, a relatively sensitive P-CS restricts spatial frequency reuse. Being widely used in EDCA, it causes this channel access scheme to achieve poor performance in an MBSS since EDCA has been developed for single hop WLAN where stations interpret the absence of a response frame (ACK, CTS, RTS) as a transmission failure which causes them to double their CW, increase their retry counter and perform an additional back-off procedure in an attempt to retry sending the packet again. In an MBSS large idle gaps exist due to the unpredictable medium access due to the fact that stations cannot detect their neighbor's availability.

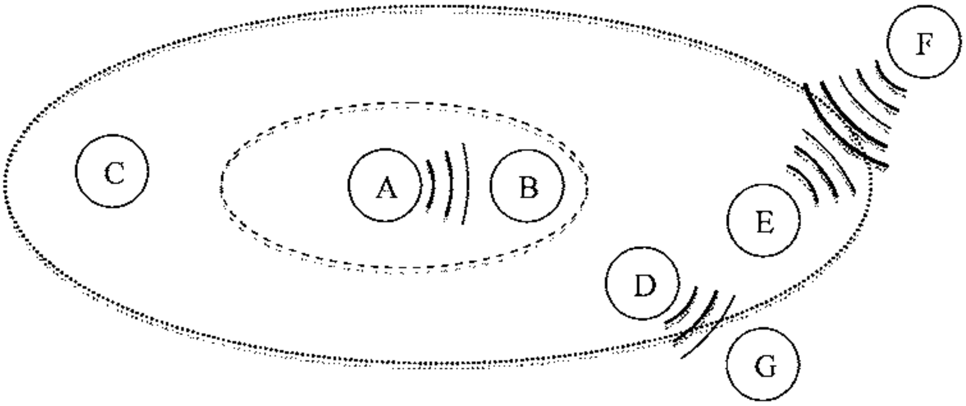


Figure 6: *Exposed Terminal Problem.*

According to the Wi-Mesh Alliance (WiMA) proposal, The IEEE 802.11s offers an optional congestion control mechanism and coordination function in an attempt to mitigate the problems of EDCA. With the Mesh Deterministic Access (MDA), MPs become aware of the poor radio environment. While the V-CS provides instantaneous medium reservation after successful transmissions, MDA separates the negotiation process for medium reservation, a mechanism that is very similar to the Distributed Reservation Protocol (DRP) defined in [57]. MDA defines an MBSS's wide periodic super frame. MDA capable MPs negotiate with their neighboring MPs on the WM reservation through the MDA Opportunity setup messages (MDAOP). Each MDA capable MP maintains and broadcasts in its beacon packets both, a list of all MDAOPs during which it is a transmitter or receiver, and a list of neighboring MDAOPs as an interference report, [3]. Thus neighboring MPs are able to avoid setting up overlapping MDA reservations. An MP that receives an MDAOP performs Clear Channel Assessment (CCA) and accesses the WM with the highest priority causing neighboring MPs to refrain their transmissions during that period.

Chapter 5

Accurate MAC Modeling of 802.11 under DCF

Several researchers attempted to mathematically model queue characteristics in wireless networks operating in the random access mode and analyze their ability to support real time traffic. However, general, accurate and relatively simple such models are still lacking. The work in [37] attracted our attention since it attempted to address this void and proposed a discrete time *G*/*G*/1 analytical queuing model for random access networks based on the IEEE 802.11 MAC. The basic idea behind this work is appealing as its objective is to develop a generalized flexible model that is suitable for all traffic arrival patterns, arbitrary number of users, packet size distributions and number of nodes while assuming a general packet service time distribution. Moreover, this model provides closed form expressions for delay and queue length characteristics thus providing probabilistic delay and packet loss guarantees as well as the possible number of supported connections under specific delay and loss constraints. However, the key to this model being the characterization of the packet service time distribution falls for so many serious mistakes and several alleviating assumptions the first of which is that the saturated conditions provide good approximations to certain unsaturated scenarios. This causes the model to be erroneous in terms of queue characteristics and thus not producing accurate results. Moreover, although supposed to be a generalized model, away from any kind of particularities, it turns out to converge to the particular case of *M*/*M*/1 queues when deriving the probability of an empty system in Eq. (6). Also assuming an exponential packet service time is by far away from reality and even constitutes a very bad approximation. In addition, the model fails to account for so many network states at the instant of packet arrivals such as the case of not backlogging an arriving packet to an idle node at instants where other nodes with non-empty queues are in back-off mode. Although the authors state that their simulations prove that this approximation leads to relatively accurate results, we believe that this assumption along with the so many similar assumptions made while developing this model such as neglecting the maximum number of allowed retransmission attempts and packet drops drive the results to be erroneous and inaccurate. Above all, the developed closed form equations leading to an ultimate packet service time distribution expression are based in our opinion on inappropriate and incorrect reasoning. Therefore the packet service time distribution given in this work is invalid.

In this section, we first try to refine and correct the back-off mechanism and queuing model based on the work proposed in [37] where we try to suppress all alleviating and erroneous assumptions and show the general aspect of the model. A major contribution of our work in mathematical modeling of such systems lies in the proposal of a new approach where we model the packet service time distribution in IEEE 802.11-based networks using a combination of Erlang-k and Coxian-2 distributions. General traffic arrival patterns are assumed. Therefore we will be talking about $G/E\text{-}k/1$ and $G/C2/1$ queues. The accuracy of our new model is revealed by our simulations.

5.1 IEEE 802.11 DCF Refined Back-off Mechanism

Assume an IEEE 802.11-based network with N nodes in DCF mode using RTS/CTS packets for channel reservation. The packet arrival process and packet length are assumed to be arbitrary where the channel capacity is C bits/sec. This model does not consider the *Hidden Terminal Problem.*

For correct modeling of the MAC layer queuing delays and losses, analysis of the exponential back-off mechanism of the IEEE 802.11 MAC protocol is necessary. For example, consider a scenario where three nodes are competing to hold the Wireless Medium (WM), which when sensed to be idle for a DIFS period, each of those nodes with a packet to transmit decrements its back-off counter (BC). The node which timer expires first starts transmission. All other nodes therefore hold their timers and postpone their transmissions up until the current transmitting node finishes successfully, the time at which the process repeats again and the nodes continue decrementing their counters from where they last held it off. This scenario is more illustrated in Figure 7.

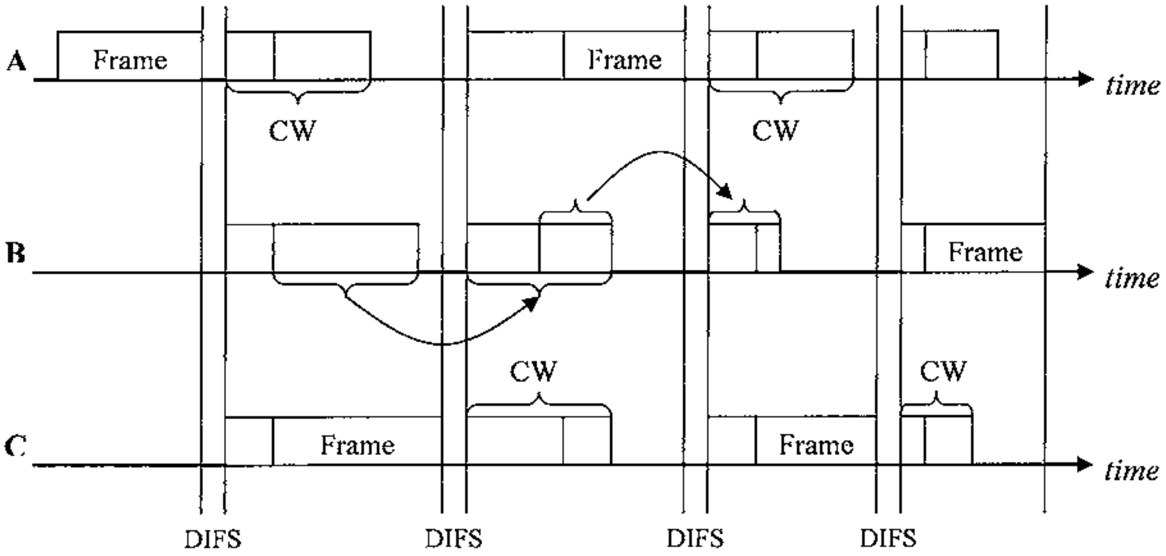


Figure 7: IEEE 802.11 MAC CA Backoff Scheme.

We start our analysis by representing the discrete time by slots where the size of each time slot is δ time units. Each time a node goes into back-off mode, it waits a certain uniformly distributed number of slots, beyond DIFS, before initiating a retransmission attempt in case of a previous collision or, a new transmission. We denote by $W_{\min}$ and $W_{\max}$ the minimum and maximum bounds of the Contention Window (CW). CW_k and W_k are respectively the CW and its upper bound at the k^{th} transmission attempt, $k = 1, 2, 3, \ldots$. Let BC be a uniformly distributed random variable representing the number of slots that a node has to wait before proceeding with the k^{th} transmission attempt, where $BC \in [1; W_k]$, [37]. Of course the first attempt occurs with $1 \leq BC \leq W_{\min}$, [6]. By the principle of collision avoidance and exponential back-off mechanism, each time an unsuccessful transmission attempt occurs, the upper bound of the CW is doubled up until it reaches the upper limit $W_{\max}$ specified by the standard. However, this upper bound cannot go beyond $W_{\max}$. This implies that when the k^{th} unsuccessful attempt occurs:

$$W_k = \min\left\{W_{\max}, 2^{k-1} W_{\min}\right\} \tag{1}$$

What remains to define is the probability of collision or unsuccessful transmission attempt. We denote this probability by p. Therefore the probability of a successful transmission is given by $(1-p)$. It is also worthwhile mentioning that the number of transmission attempts that would drive CW from its minimum $W_{\min}$ to its maximum $W_{\max}$, is given by $(m+1)$ such that:

$$m + 1 = \log_2\left(\frac{W_{\max}}{W_{\min}}\right) \tag{2}$$

Any retransmission attempt that occurs beyond m+1 would result in a CW such that $\{1 \le BC \le W_{\max} \mid k > m+1\}$. However, the IEEE 802.11 standard limits the number of retransmission attempts through the Long Retry Count (LRC) if RTS/CTS are used and the Short Retry Count (SRC) if RTS/CTS are not used. Without loss of generality, we denote this upper bound on the number of retransmission attempts by K. Therefore probabilistically speaking, if on one hand the k^{th} ($k \le m$) transmission attempt is successful, this means that all the k – 1 attempts resulted in a failure, thus cumulating an overall probability of collision of p^{k-1} and an upper bound on the CW, $W_{k-1} = 2^{k-2}W_{\min}$. At the k^{th} stage, $W_k = 2W_{k-1} = 2^{k-1}W_{\min}$ and since a successful transmission with a probability $(1-p)$ took place, then the probability of having $W_k = 2^{k-1}W_{\min}$ is equal to $p^{k-1}(1-p)$. On the other hand all the cases where $m < k \le K$ would result in the upper bound of CW to be equal to $W_{\max}$. However the probabilities of the node attempting the $(m+1)^{th}, (m+2)^{th}, ..., K^{th}$ transmissions are respectively $p^m(1-p), p^{m+1}(1-p), ..., p^{K-1}(1-p)$. Therefore the probability of the upper CW bound being $W_k = W_{\max}$ is given by:

$$\begin{aligned}
\Pr\left[W_k = W_{\max}\right] &= \sum_{k=m+1}^{K} p^{k-1}(1-p) = p^m(1-p) + p^{m+1}(1-p) + ... + p^{K-1}(1-p) \\
&= p^m(1-p)\left[1 + p + p^2 + ... + p^{K-m-1}\right] \\
&= p^m(1-p)\frac{1-p^{K-m-1}}{1-p} \\
&= p^m\left(1-p^{K-m-1}\right)
\end{aligned}$$

Therefore, generally, for all values of $k \in [1;K]$, the probability of having $W_k = W$ is given by:

$$\Pr[W_k = W] = \begin{cases} p^{k-1}(1-p) & , \quad 1 \le k \le m \Rightarrow W = 2^{k-1}W_{\min}. \\ p^m\left(1-p^{K-m-1}\right) & , \quad m < k \le K \Rightarrow W = W_{\max}. \end{cases} \qquad \textbf{(3)}$$

The probability of having a node waiting a certain number of slots, *n*, and $W_k = W$ before attempting the successful k^{th} transmission is given by the joint probability:

$$\begin{aligned} \Pr[BC = n, W_k = W] &= \Pr[BC = n \mid W_k = W] \cdot \Pr[W_k = W] \\ &= \begin{cases} \dfrac{p^{k-1}(1-p)}{W_k} & , \quad k \le m \\ \dfrac{p^m\left(1-p^{K-m-1}\right)}{W_{\max}} & , \quad m < k \le K \end{cases} \end{aligned} \qquad \textbf{(4)}$$

At an arbitrary back-off stage *k*, with a probability $\Pr[BC = n]$ to be determined, a node may initialize its BC to any random number of slots, *n*, that it waits before attempting a transmission. If $n > W_k$, then $\Pr[BC = n] = 0$. Figure 8 illustrates this reasoning where it is obvious that the first CW range $[1; W_{\min}]$ is common to all back-off stages, meaning that at any back-off stage from 1 to *k* $(k \le K)$, the node's BC may be equal to *n* where $1 \le n \le W_{\min}$. However for example, if at the first back-off stage, $W_{\min} \le n \le 2W_{\min}$, therefore the probability that the BC of the node be equal to *n* is zero. But at stages 2 to $k(k \le K)$, this is possible and so forth.

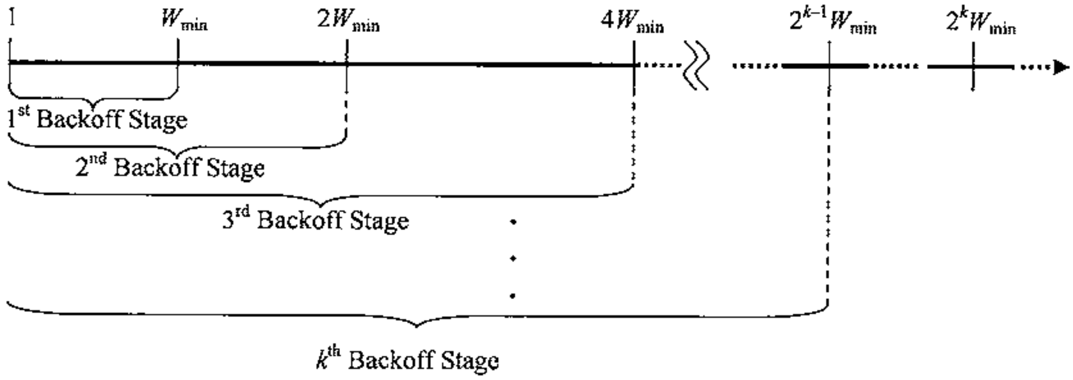


Figure 8: Contribution of CW ranges in the BC at different back-off stages.

Therefore $\Pr[BC = n]$ is given by:

$$
\Pr\left[BC = n\right] = \begin{cases}
\displaystyle\sum_{i=1}^{m} \frac{p^{i-1}\left(1-p\right)}{W_i} + \sum_{i=m+1}^{K} \frac{p^{i-1}\left(1-p\right)}{W_{\max}} & , \quad 1 \le n \le W_{\min} \\
\displaystyle\sum_{i=j}^{m} \frac{p^{i-1}\left(1-p\right)}{W_i} + \sum_{i=m+1}^{K} \frac{p^{i-1}\left(1-p\right)}{W_{\max}} & , \quad 2^{j-1}W_{\min} < n \le 2^{j} W_{\min} \\
 & \quad\;\; j = 2,3,\ldots,m \\
\displaystyle\sum_{i=m+1}^{K} \frac{p^{i-1}\left(1-p\right)}{W_{\max}} & , \quad n \ge W_{\max}
\end{cases}
$$

$$
= \begin{cases}
\displaystyle\frac{2}{W_{\min}} \frac{\left(1-p\right)\left[1-\left(\frac{p}{2}\right)^{m-1}\right]}{2-p} + \frac{1}{W_{\max}} p^{m}\left(1-p^{K-m-1}\right) & , \quad 1 \le n \le W_{\min} \\
\displaystyle\frac{1}{2^{j-2}W_{\min}} \frac{p^{j-1}\left(1-p\right)\left[1-\left(\frac{p}{2}\right)^{m-1}\right]}{2-p} + \frac{1}{W_{\max}} p^{m}\left(1-p^{K-m-1}\right) & , \quad 2^{j-1}W_{\min} < n \le 2^{j} W_{\min} \\
 & \quad\;\; j = 2,3,\ldots m \\
\displaystyle\frac{1}{W_{\max}} p^{m}\left(1-p^{K-m-1}\right) & , \quad n \ge W_{\max}
\end{cases}
$$

(5)

Now we respectively denote by $\overline{CW_k}$ and BC_k the average CW size and the BC of a node at stage *k*. Now since BC_k is initialized to a random uniformly distributed number of slots $n_k \in [1;W_k]$, then the average BC value, $\overline{BC_k}$, is given by:

$$\begin{aligned}\overline{BC_k} &= E[BC_k] = \sum_{n_k=1}^{W_k} n_k \cdot \Pr[BC_k = n_k] = \frac{1}{W_k}\sum_{n_k=1}^{W_k} n_k = \frac{1}{W_k}[1+2+\ldots+W_k] \\ &= \frac{1}{W_k}\frac{W_k(W_k+1)}{2} = \frac{W_k+1}{2} \simeq \frac{2^{k-1}W_{\min}+1}{2}\end{aligned} \tag{6}$$

Therefore a packet may be successfully transmitted from a first attempt with probability $(1-p)$ and $\overline{BC_1} = \frac{W_{\min}+1}{2}$, or in the second attempt with probability $p(1-p)$ and $\overline{BC_2} = \frac{2W_{\min}+1}{2}$, …, or in the m^{th} attempt with a probability $p^{m-1}(1-p)$ with $\overline{BC_m} = \frac{2^{m-1}W_{\min}+1}{2}$, or in the $(m+1)^{th}$ attempt with a probability $p^m(1-p)$ and $\overline{BC_{m+1}} = \frac{W_{\max}+1}{2} = \frac{2^m W_{\min}+1}{2}$, …, or in the K^{th} attempt with a probability $p^{K-1}(1-p)$ and $\overline{BC_K} = \frac{2^m W_{\min}+1}{2}$. After *K* attempts, if the packet is not successfully transmitted, it is dropped. Hence, the average contention window size $\overline{CW_k}$ is given by:

$$\overline{CW_k} = p^{k-1}(1-p)\overline{BC_k} = p^{k-1}(1-p)\frac{W_k+1}{2} \quad , \quad 1 \le k \le K \tag{7}$$

We now express the total average CW size for a packet that was backlogged and passed through *k* back-off stages as $\overline{CW_{1,k}}$ which is given by:

$$\overline{CW_{1,k}} = \sum_{i=1}^{k}\overline{CW_i} = \overline{CW_1} + \overline{CW_2} + \ldots + \overline{CW_k} \quad , \quad 1 \le k \le K \tag{8}$$

Under the assumption of a saturated network scenario where nodes continuously have packets waiting to be transmitted, any arriving packet is directly backlogged. The authors in [37] intended to develop a generalized model suitable for both saturated and unsaturated scenarios. However, in their work, the probability of collision p was derived under the assumption that saturated scenarios provide a good approximation for the unsaturated ones. Therefore that expression of p requires some refinements and corrections in order to comply with the idea of a generalized model. In an unsaturated network scenario, an arriving packet to a node will be backlogged at least once if either of the following two cases takes place at the instant of its arrival:

1. *At least one packet is found waiting in the queue*: In order to analyze this scenario, let p_0 denote the probability that a node is idle due to the fact that it got no packets to transmit. Therefore the probability that at least one packet is waiting in the queue, hence the arriving packet will be backlogged at least once is given by $(1-p_0)$.
2. *No packets are waiting in the queue, however the wireless medium happens to be busy*: Given that the network is composed of N nodes, the probability of a single node being idle (having no packets to transmit) is p_0, assuming that this probability is the same for all the nodes in the network and that the state of a node being idle or not is independent of all the other remaining nodes in the network, therefore the probability of having $N-1$ idle nodes is $(N-1)p_0$, meaning that the wireless medium is not busy, then the probability of having at least one node transmitting over the wireless medium (i.e. wireless medium is busy, so the arriving packet will be backlogged) is given by $1-(N-1)p_0$.

Given the above two arguments, we can therefore assert that the probability that an arbitrary arriving packet will be backlogged at least once can be given by:

$$\Pr[\text{Arbitrary arriving packet is backlogged at least once}] = (1-p_0) + [1-(N-1)p_0]$$
$$= 2 - Np_0$$

However for this probabilistic expression to be valid, it should yield values in the range $[0;1]$, therefore $0 \le 2 - Np_0 \le 1$. As shown in Figure 9, this allows us actually to come up with limiting bounds for the probability of an empty system (i.e. idle medium), p_0, as a function of, N, the number of nodes in the network, such that beyond those bounds, the backlogging probability will be either zero or one, thus independent of the number of nodes in the system. It is also important to mention that when deriving a closed form expression for p_0, it will be a function of the load. So this load is implicitly accounted for throughout this analysis. To wrap up, we have:

$$\frac{1}{N} \le p_0 \le \frac{2}{N} \quad , \quad N = 2,3,... \tag{9}$$

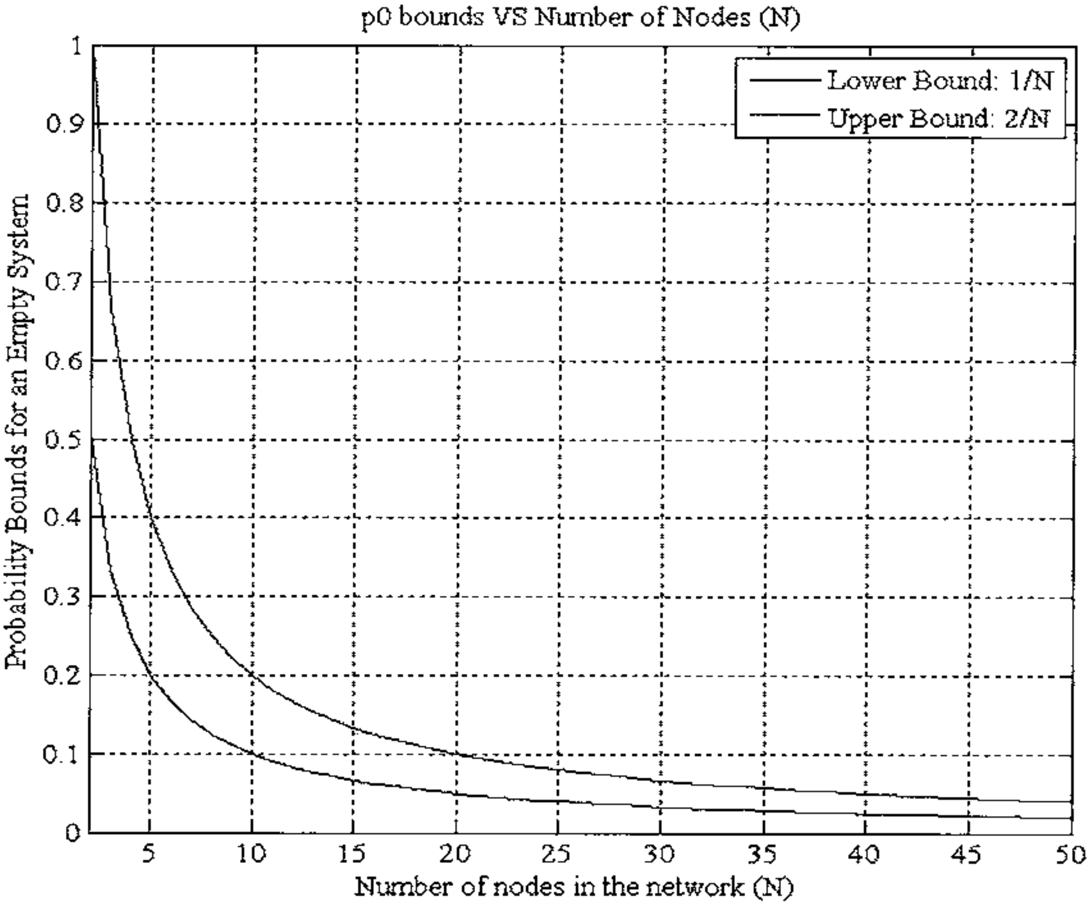


Figure 9: Limiting bounds of the probability of an empty system, p_0.

As a matter of fact, Eq. (6), on one hand, means that if either the network is formed of a small number of nodes, *N*, however the traffic load experienced by those nodes is very high, or the network comprises a large number of nodes with relatively low to medium traffic loads in such a way that the system is empty with a probability $p_0 \leq \frac{1}{N}$, then no matter what, any arbitrary arriving packet at a certain tagged node will find the wireless medium being occupied by at least one of the other $N-1$ nodes, and will therefore be backlogged. Therefore in this case, the probability that an arbitrary arriving packet is backlogged will always be equal to one. In other words the network in this case is saturated. On the other hand, the upper bound of p_0 in Eq. (6) means that if *N* is large enough however traffic load is very low, or *N* is small with a relatively low to medium traffic loads in such a way to have an idle system with a probability $p_0 \geq \frac{2}{N}$, then no matter what, any arbitrary arriving packet at a certain tagged node will never be backlogged. Thus in this case the probability that an arbitrary arriving packet is backlogged will always be equal to zero. Moreover, it is purely logical to consider the number of nodes in the network $N \geq 2$, since a single node cannot be in communication mode if it exists alone. Therefore, if we consider the case where a minimum number of nodes exist in a network ($N=2$), then we have $\frac{1}{2} \leq p_0 \leq 1$. Notice also that as the number of nodes in the network increases the bounding interval width of p_0 decreases, meaning that it is less likely to find an idle transmission medium due to the fact that all nodes are idle or in back-off mode. In addition, if the number of nodes in the network, *N*, becomes considerably large, then the bounding interval width become considerably small and very close to zero meaning that it is more and more likely that any arbitrarily arriving packet at any node will find the wireless medium to be occupied by at least one of the other $N-1$ nodes in the network, and will therefore be backlogged. However, it is important to mention that the maximum number of nodes that can be reached in a large scale IEEE 802.11-based network is $N=50$ for which we have $\frac{1}{50} \le p_0 \le \frac{2}{50}$. Therefore, with relatively low traffic, the wireless medium can be found empty with a maximum 2% probability which means that only the increase of the traffic load in this case will result in saturation, something which is very rare in such types of networks, [21]. Therefore, in a generalized way, accounting for both saturated and unsaturated scenarios, we let p_b denote the probability that an arbitrary arriving packet is backlogged, and we express it as:

$$p_b = \Pr\left[\text{An arbitrary arriving packet is backlogged }\right] \\ = \begin{cases} 0 & , \quad p_0 \ge \frac{2}{N} \\ 2 - Np_0 & , \quad \frac{1}{N} < p_0 < \frac{2}{N} \qquad , \quad N = 2,3,\ldots \\ 1 & , \quad p_0 \le \frac{1}{N} \end{cases} \tag{10}$$

Therefore an arbitrary arriving packet is immediately transmitted with a probability $(1-p_b)$ or is backlogged at least once with a probability p_b. Therefore the average contention window size for an arbitrary packet denoted by $\overline{CW}$ is given as follows:

$$\overline{CW} = p_b \overline{CW_{1,k}} \quad , \quad 1 \le k \le K \tag{11}$$

Now following the arguments of [24] and [27], and the fact that only nodes with non-empty queues can cause collisions if they simultaneously transmit with other nodes, the packet collision probability can be obtained by solving:

$$p = 1 - \left(1 - \frac{1-p_0}{\overline{CW}}\right)^{N-1} \tag{12}$$

5.2 Refined Queuing Model

To obtain the delays and losses experienced by packet at each node, the authors of [37] modeled the system as a discrete time *G*/*G*/1 queue. The service time of a packet was defined to be the difference between the instant at which the packet reaches the head of the queue in the node and the instant it successfully departs from the queue. Thus this service time has two components:

1. The time till the node successfully accesses and reserves the channel for use.
2. The time required to transmit the packet.

While point (2.) is essentially characterized by the packet length distribution, the first point needs a more detailed analysis. The determination of the time required to successfully grab the WM is done following the illustration made in Figure 10.

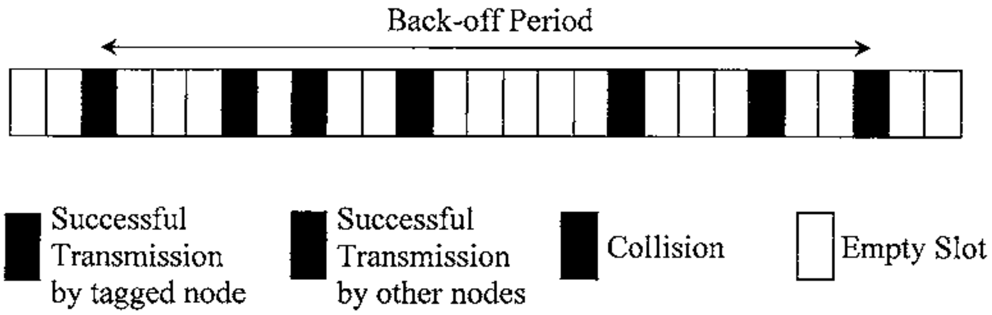


Figure 10: Interleaving of transmissions and collisions contributing to the service time.

Between any two successful transmissions by a tagged node, other nodes may successfully transmit a number of packets or may be involved in a number of collision, each of which add to the channel access time of the tagged node. Also, transmission attempts by the tagged node which result in collisions are also included in this access time characterization.

The authors of [37] start by characterizing the number of back-off slots (BO) that a certain tagged node has to wait between two successful transmissions. At an arrival instant, if the WM is idle, the node immediately starts transmitting the arriving packet without waiting any back-off slot. Therefore, the probability of having zero back-off slots, $\Pr[BO=0]=(1-p_b)(1-p)$. With a probability p_b, the node goes into back-off mode at least once. At this stage, it is noted that if the tagged node successfully transmits the packet in its first attempt (i.e. with a probability $1-p$), then the number of back-off slots is uniformly distributed in the range $[1;W_{\min}]$. If the node goes to the second back-off stage (i.e. a successful transmission after a single collision with probability $p(1-p)$), the probability mass function of the number of back-off slots is obtained through $U_{1,W_{\min}} * U_{1,2W_{\min}}$ and so forth, where $U_{a,b}$ denotes a uniform distribution between a and b, and $*$ represents the convolution operator. Thus for a packet passing through k back-off stages, we will have k convolutions. Note that if $k>m$, the first m terms are $U_{1,W_{\min}}, U_{1,2W_{\min}}, \ldots, U_{1,2^m W_{\min}}(i)$, while the remaining terms are all $U_{1,2^m W_{\min}}(i)$ since the contention window is constrained by $W_{\max}=2^m W_{\min}$. Therefore, the probability that the tagged node experiences i back-off slots ($i>0$) is given by:

$$\Pr[BO=i]=p_b\begin{bmatrix}(1-p)U_{1,W_{\min}}(i)+p(1-p)\left(U_{1,W_{\min}} * U_{1,2W_{\min}}(i)\right)+\ldots \\ +p^m(1-p)\left(U_{1,W_{\min}} * U_{1,2W_{\min}} * \ldots * U_{1,2^m W_{\min}}(i)\right) \\ +p^{m+1}(1-p)\left(U_{1,W_{\min}} * U_{1,2W_{\min}} * \ldots * U_{1,2^m W_{\min}}(i)\right)+\ldots \\ +p^K(1-p)\left(U_{1,W_{\min}} * U_{1,2W_{\min}} * \ldots * U_{1,2^m W_{\min}}(i)\right)\end{bmatrix} \tag{13}$$

At this stage therefore, it's clear that in order to determine the service time of a packet that has reached the top of the queue of a certain tagged node, we need to know how many of the back-off slots that such a node was supposed to initially wait, resulted in the expansion of the total back-off interval in order to account for collisions caused by other nodes trying to transmit as well as successful transmissions done by those nodes as shown in Figure 10. Such are called active slots.

Definition: A slot is said to be active if at least one of the N nodes in the network is attempting to transmit packets in it.

We denote the service time of a packet by a random variable S. Therefore the ultimate goal of this analysis is to obtain a closed form expression for the probability density function of S, $\Pr[S=s]$. For this reason we define the following:

- $j =$ The number of active slots out of the i initial back-off slots of the tagged node.
- $k =$ The number of active slots out of j that result in collisions.
- $\rho = \dfrac{\lambda}{\mu}$ The utilization factor for one node. That is the probability that a certain node has at least one packet to transmit.
- $\overline{CW} =$ The average CW size.
- $q =$ The probability that an arbitrary slot is active irrespective of the tagged node being either in back-off mode or not.

The probability that a node sends information in an arbitrary slot is given by $\dfrac{\rho}{\overline{CW}}$.

Therefore q can be seen as:

$$q = \Pr\big[\text{At least one of the other nodes is Tx in a slot AND the tagged node is not Tx in it}\big]$$
$$+$$
$$\Pr\big[\text{At least one of the other nodes is Tx in a slot AND the tagged node is also Tx in it}\big]$$

Denote by $\Pr[T_o]$ the probability that other nodes are transmitting in a slot, and by $\Pr[T_t]$ the probability that the tagged node is transmitting in a slot. Therefore q can be written as:

$$q = \Pr[T_o, \neg T_t] + \Pr[T_o, T_t] \qquad \textbf{(14)}$$

where $\neg$ denotes the "not" operator. However, the choices of slots in which nodes transmit packets are independent from node to node. Therefore q can be written as:

$$q = \Pr[T_o] \cdot \Pr[\neg T_t] + \Pr[T_o] \cdot \Pr[T_t] = \Pr[T_o](1 - \Pr[T_t]) + \Pr[T_o] \cdot \Pr[T_t] \qquad \textbf{(15)}$$

Now if the tagged node is in back-off mode, then it will never transmit. Therefore $\Pr[T_t] = 0$. This results in q being:

$$q = \Pr[T_o] \qquad \textbf{(16)}$$

The next step is therefore to determine $\Pr[T_o]$ as follows:

We know that $\Pr[\text{a node transmits in a given slot}] = \frac{\rho}{CW}$. Therefore a node doesn't transmit with probability $1 - \frac{\rho}{CW}$. Therefore the $N - 1$ node do not transmit in a slot with probability $\left(1 - \frac{\rho}{CW}\right)^{N-1}$. Therefore $\Pr[T_o]$ can be seen as the probability that at least one of the $N - 1$ nodes is transmitting in a slot, where $\Pr[T_o] = 1 - \left(1 - \frac{\rho}{CW}\right)^{N-1}$ Therefore the probability that an arbitrary slot is active is:

$$q = 1 - \left(1 - \frac{\rho}{CW}\right)^{N-1} \qquad \textbf{(17)}$$

Now given that the tagged node should wait i back-off slots, the next step is to determine the probability that j out them are active slots. This can be written as:

$$\Pr[j \text{ active slots} \mid BO = i] = \binom{i}{j} q^j (1 - q)^{i-j}, \forall j = 0, \ldots, i \qquad \textbf{(18)}$$

Next is to determine, q_c, the probability that a given active slot results in a collision irrespective if the tagged node is in back-off mode or not. However, we know that a collision can occur in one of the following two cases:

1. The tagged node and at least one of the other nodes transmit in the same slot. Therefore in this case we are looking at $\Pr[T_o, T_t] = \Pr[T_o] \,.\, \Pr[T_t]$. However we know that tagged node transmits with a probability $\frac{\rho}{CW}$. Moreover we have determined $P[T_o] = 1 - \left(1 - \frac{\rho}{CW}\right)^{N-1}$. Therefore we have:

$$\Pr[T_t, T_o] = \Pr[T_t] \cdot P[T_o] = \frac{\rho}{CW}\left[1 - \left(1 - \frac{\rho}{CW}\right)^{N-1}\right] \quad \textbf{(19)}$$

2. The tagged node does not transmit, but at least two of the other nodes transmit in the same slot. Then we're looking at:

$$\Pr[\neg T_t, T_{o_2}] = \Pr[\neg T_t] \cdot \Pr[T_{o_2}] = \left(1 - \frac{\rho}{CW}\right) \cdot \Pr[T_{o_2}].$$

Therefore the aim now is to determine $\Pr[T_{o_2}]$. Now we know that $N - 1$ node transmit with probability $\frac{(N-1)\rho}{CW}$. Moreover, $N - 2$ nodes do not transmit with a probability $\left(1 - \frac{\rho}{CW}\right)^{N-2}$. Therefore the probability that exactly one of the $N - 1$ nodes is transmitting is $(N-1)\frac{\rho}{CW}\left(1 - \frac{\rho}{CW}\right)^{N-2}$. This implies that $1 - \left(1 - \frac{\rho}{CW}\right)^{N-1} - (N-1)\frac{\rho}{CW}\left(1 - \frac{\rho}{CW}\right)^{N-2}$ is the probability that two or more of the other nodes are sending. Therefore we have:

$$\Pr\left[\neg T_t \wedge T_{o_2}\right] = \left(1 - \frac{\rho}{CW}\right)\left[1 - \left(1 - \frac{\rho}{CW}\right)^{N-1} - (N-1)\frac{\rho}{CW}\left(1 - \frac{\rho}{CW}\right)^{N-2}\right] \quad \textbf{(20)}$$

Given the two above cases, we can therefore express q_c as:

$$q_c=\frac{\frac{\rho}{CW}\left[1-\left(1-\frac{\rho}{CW}\right)^{N-1}\right]+\left(1-\frac{\rho}{CW}\right)\left[1-\left(1-\frac{\rho}{CW}\right)^{N-1}-(N-1)\frac{\rho}{CW}\left(1-\frac{\rho}{CW}\right)^{N-2}\right]}{1-\left(1-\frac{\rho}{CW}\right)^{N-1}}$$

$$=\frac{1-\left(1-\frac{\rho}{CW}\right)^{N-1}-(N-1)\frac{\rho}{CW}\left(1-\frac{\rho}{CW}\right)^{N-1}}{1-\left(1-\frac{\rho}{CW}\right)^{N-1}}$$

(21)

The next step is to define the probability that k out of those j active slots cause collision. That is:

$$\Pr[k \text{ collisions} \mid j \text{ active slots}]=\binom{j}{k}q_c^k\left(1-q_c\right)^{j-k} \quad \textbf{(22)}$$

Now we define T_c to be the number of slots that a single collision occupies $T_c \sqcap DIFS+\tau_{RTS}$ where τ_{RTS} is the number of slots required to send an RTS packet. Note as well that each successful transmission made by other nodes will contribute in the addition of a certain number of slots that is proportional to the length of the transmitted packet by the other node in question including RTS/CTS and ACK exchange and depending on the bit rate of the channel. We denote by $l(v)$ the probability that a packet transmission takes v slots. Therefore x successful transmissions contribute if they occurred in a total of u slots. The probability of such an event is defined as:

$$\Pr\left[\sum_x \text{number of slots contributed by successful transmissions of other nodes} = u\right] \quad \textbf{(23)}$$
$$= l * l * l * \ldots * l(u) = l^x(u)$$

which holds if all nodes have same packet length distributions. Consider the situation where a tagged node experiences i back-off slots, j of which are active and k of the j

active slots result in a collision. Thus the $j-k$ successful transmissions made by other nodes contribute by u slots. Therefore the tagged node will never be able to grab the channel before all those successful transmissions are done given the above i, j, and k. Thus the conditional service time is as follows:

$$\Pr[S=s \mid i,j,k]=\begin{cases} l^{(j-k)}(u) & , s=i+kT_c+u \\ 0 & , \text{otherwise} \end{cases} \tag{24}$$

However the probability of having i backoff slots, j of which are active with k out of the j active slots causing collisions can be written as:

$$\begin{aligned} \Pr[i,j,k] &= \Pr[j,k \mid i]\cdot\Pr[i]=\Pr[j \mid i]\cdot\Pr[k \mid j]\cdot\Pr[BO=i] \\ &= \binom{i}{j} q^{j}\left(1-q\right)^{i-j}\cdot\binom{j}{k} q_c^{k}\left(1-q_c\right)^{j-k}\cdot\Pr[BO=i] \end{aligned} \tag{25}$$

Therefore the closed form expression of the pdf of S is given by:

$$\begin{aligned} \Pr[S=s] &= \sum_{i,j,k}\Pr[S=s \mid i,j,k]\cdot\Pr[i,j,k] \\ &= \sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k=0}^{j} l^{(j-k)}(u)\cdot\Pr[j,k \mid i]\cdot\Pr[i] \\ &= \sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k=0}^{j} l^{(j-k)}(u)\cdot\binom{i}{j} q^{j}\left(1-q\right)^{i-j}\cdot\binom{j}{k} q_c^{k}\left(1-q_c\right)^{j-k}\cdot\Pr[BO=i] \end{aligned} \tag{26}$$

which is true only when $s = i + kT_c + u$. This is why I define an indicator function $V(s)$ such that:

$$V(s)=\begin{cases} 1 & , s=i+kT_c+u. \\ 0 & , \text{otherwise.} \end{cases}$$

Finally,

$$\Pr[S=s]=\sum_{i=1}^{\infty}\sum_{j=0}^{i}\sum_{k=0}^{j} l^{(j-k)}(u)\cdot\binom{i}{j} q^{j}\left(1-q\right)^{i-j}\cdot\binom{j}{k} q_c^{k}\left(1-q_c\right)^{j-k}\cdot\Pr[BO=i]\cdot V(s) \tag{27}$$

Note that the above expression should be evaluated for all values of i, j, k which result in $s = i + kT_c + u$. Moreover, I'd rather replace the infinity in the summation over i by the maximum allowed number of retrials.

5.3 Modeling the Packet Service Time using E_k and C_2 distributions

In the work of [37], an IEEE 802.11 node was modeled as a single server queue with general traffic arrival patterns and general service time distribution, i.e. a *G/G/*1 queue. In our new model, the generality of the traffic arrival patterns is preserved. However, our aim is to obtain a more accurate and simple representation of the service time distribution of packets in such nodes while always maintaining its general aspect and away from any fitting or approximations by known distributions under a considerable amount of alleviating assumptions, something which will drive the model away from reality.

5.3.1 The Coxian-k Distribution

It was stated in [58 – 59] that any distribution with a rational Laplace Transform (LT) can be represented by a sequence of fictitious stages as shown in Figure 11 where a single queue is considered to which packets arrive at a certain arrival rate λ. The server serves packets at with a service rate μ.

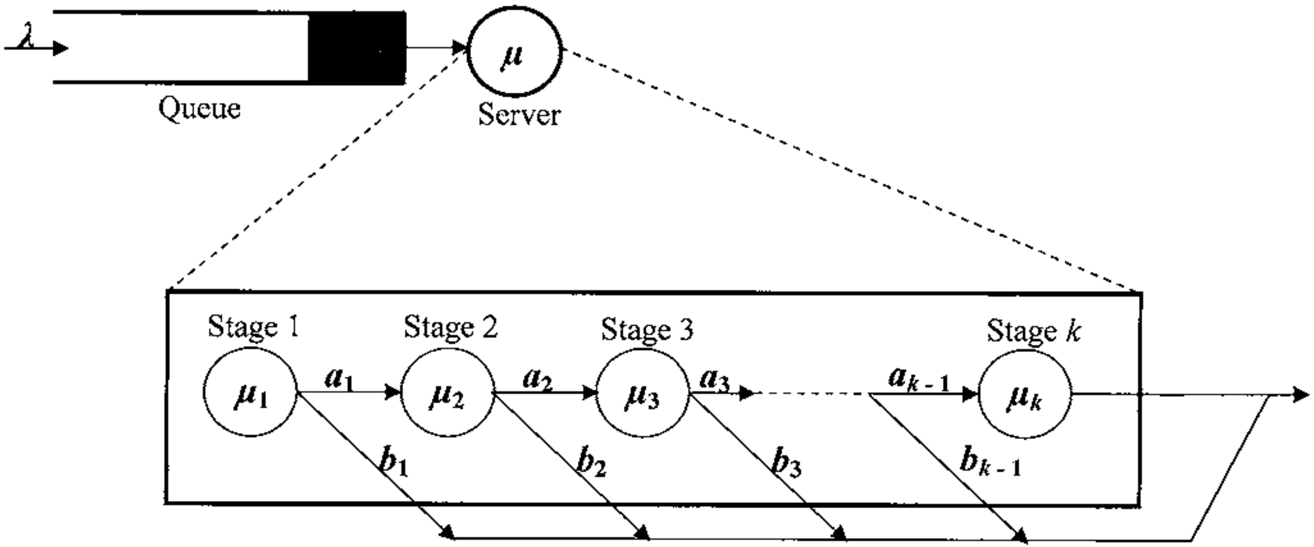


Figure 11: Packet service time divided into fictitious stages of a C_k distribution.

In such a queuing system, the service time a packet receives consists of one or more service stages. Those stages can be thought as each being a sub-server with its own service rate. In general, the service rate at an arbitrary stage is different from the ones at the other stages. Therefore we denote by μ_i the service rate at the i^{th} stage, $i = 1,2,...,k$. A packet arriving at the i^{th} stage, may, with a certain probability, a_i, proceed to the $(i+1)^{th}$ stage if it requires more service, or it may depart from the server with a probability $b_i = 1 - a_i$, meaning that it got all the service that it requires. Note that only one packet is allowed to be in the server. In other words, no packet is allowed to penetrate the server unless the packet that was already inside departs. Packets always start their service at the first stage and move on. The time a packet spends at the i^{th} stage is exponentially distributed with a mean $\frac{1}{\mu_i}$. This type of service distribution is known as the Coxian distribution and is denoted by C_k where *k* is the number of stages. Therefore, the queue that we are in the process of analyzing is known as the $G/C_k/1$ queue.

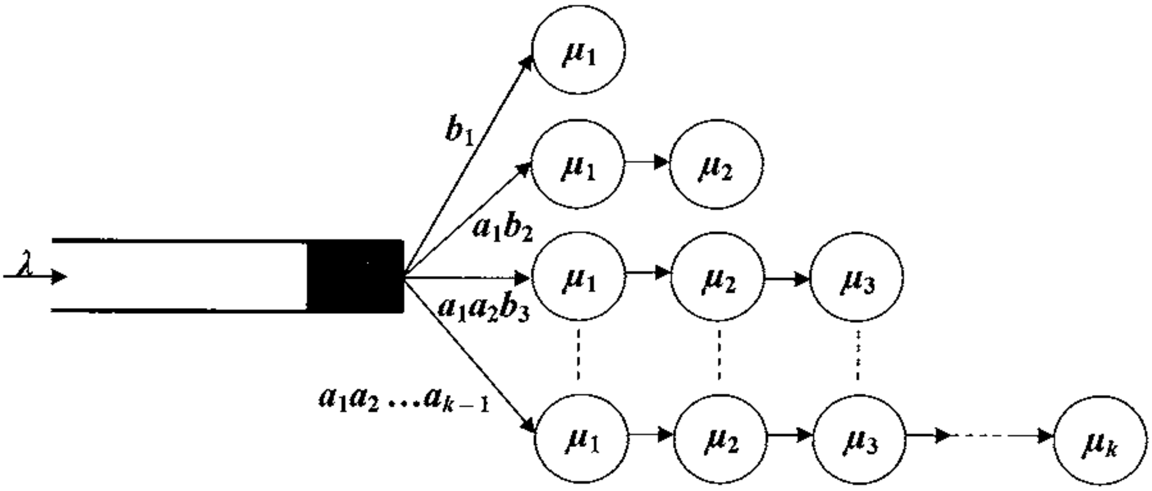


Figure 12: Probability that a packet goes through *i* service stages, $i = 1, 2\ldots k$.

As shown in Figure 12, a packet that enters the server will, with a probability b_1, go through only service stage one and then depart from the server. With a probability a_1b_2, the packet will pass through only service stages one and two. With a probability $a_1a_2b_3$, it will pass through stages one, two and three, and so forth. A packet may require service from all stages, 1 to k, with a probability $a_1a_2a_3\ldots a_{k-1}$. Let S_i be an exponentially distributed random variable with a mean $\frac{1}{\mu_i}$ representing the service time that a packet obtains at the i^{th} stage. Without loss of generality, let f_i be the probability density function of S_i. Also, let S_T be a random variable that follows a Coxian-k distribution and represents the total service time that a packet may require. We denote by f the probability density function of S_T. Therefore, based on Figure 12, S_T may assume values such as $S_1, S_1+S_2, S_1+S_2+S_3, \ldots, S_1+S_2+\ldots+S_k$, each with a certain probability given by:

$$\Pr\left[S_T = \sum_{j=1}^{i} S_j = S_1 + \ldots + S_i\right] = \begin{cases} b_i & , \quad i = 1 \\ b_i \cdot \prod_{j=1}^{i-1} a_j & , \quad 2 \le i \le k-1 \\ \prod_{j=1}^{i} a_j & , \quad i = k \end{cases} \tag{28}$$

However, for each value of i, the sum $\sum_{j=1}^{i} S_j$ has a probability density function that is given by $f_1 * \ldots * f_i$ which is nothing but a convolution of i exponential density functions. Therefore the probability density function of S_T is given by:

$$\begin{aligned} f(S_T) &= \sum_{i=1}^{k} \Pr\left[S_T = \sum_{j=1}^{i} S_j\right] \cdot [f_1 * \ldots * f_i] \\ &= b_1 f_1 + a_1 b_2 [f_1 * f_2] + a_1 a_2 b_3 [f_1 * f_2 * f_3] + \ldots + a_1 a_2 \ldots a_k [f_1 * \ldots * f_k] \end{aligned} \tag{29}$$

Let f^* and f_i^* denote respectively the Laplace Transforms (LTs) of f and f_i. Since f_i is an exponential density function, then its LT is given by:

$$f_i^*(s) = \frac{\mu_i}{s + \mu_i} \tag{30}$$

Knowing that the LT of a convolution is the product of the individual LTs, then we have:

$$[f_1 * \ldots * f_i]^*(s) = f_1^* \cdot \ldots \cdot f_i^* = \frac{\mu_1}{s + \mu_1} \cdot \ldots \cdot \frac{\mu_i}{s + \mu_i} = \prod_{j=1}^{i} \frac{\mu_j}{s + \mu_j} \tag{31}$$

Also given that the LT of a sum is the sum of the individual LTs and putting together Eqs. (29) and (31), the LT of the pdf of S_T is given by:

$$\begin{aligned} f^*(s) &= b_1 f_1^*(s) + \sum_{i=2}^{k-1} b_i f_i^*(s) \prod_{j=1}^{i-1} a_j f_j^*(s) + \prod_{j=1}^{k} a_j f_j^*(s) \\ &= b_1 \frac{\mu_1}{s + \mu_1} + \sum_{i=2}^{k-1} b_i \frac{\mu_i}{s + \mu_i} \prod_{j=1}^{i-1} a_j \frac{\mu_j}{s + \mu_j} + \prod_{j=1}^{k} a_j \frac{\mu_j}{s + \mu_j} \\ &= \frac{b_1 \mu_1 \prod_{j=2}^{k} (s + \mu_j) + \sum_{i=2}^{k-1} b_i \mu_i \prod_{j=1}^{i-1} a_j \mu_j \prod_{n=i}^{k} (s + \mu_n) + \prod_{j=1}^{k} a_j \mu_j}{\prod_{i=1}^{k} (s + \mu_i)} \end{aligned} \tag{32}$$

It can be easily seen in Eq. (32) that the maximum degree of the numerator can be $k - i$, thus less than k, whereas the degree of the denominator is k. Therefore $f^*(s)$ is said to be a rational polynomial. Note that the degree of the denominator, k, is the number of stages of the Coxian-k distribution.

The next step is to determine the moments of such C_k distributed random variables. In general, a packet that requires service from the first i stages has a total service time equal to the sum $S_1 + S_2 + \ldots + S_i$, where each term S_j, j = 1, 2... i, is exponentially distributed with a mean $\frac{1}{\mu_j}$. Therefore, the first moment, in other words the expected value of this particular sum is given by:

$$E\left[S_1 + S_2 + \ldots + S_i\right] = E\left[S_1\right] + E\left[S_2\right] + \ldots + E\left[S_i\right] = \sum_{j=1}^{i} E\left[S_j\right] = \sum_{j=1}^{i} \frac{1}{\mu_j} \qquad \textbf{(33)}$$

Therefore putting together equations (28) and (33), the expected value of S_T is computed over all possible stages and is expressed as:

$$\begin{aligned}
\overline{S_T} = E\left[S_T\right] &= \sum_{i=1}^{k}\left(\sum_{j=1}^{i}\frac{1}{\mu_j}\right)\Pr\left[S_T = \sum_{j=1}^{i}\frac{1}{\mu_j}\right] \\
&= b_1\frac{1}{\mu_1} + \ldots + b_i\prod_{j=1}^{i-1}a_j\left(\frac{1}{\mu_1} + \ldots + \frac{1}{\mu_i}\right) + \ldots + \prod_{j=1}^{k}a_j\left(\frac{1}{\mu_1} + \ldots + \frac{1}{\mu_k}\right) \qquad \textbf{(34)} \\
&= b_1\frac{1}{\mu_1} + \sum_{i=2}^{k-1}\left[\left(b_i\prod_{j=1}^{i-1}a_j\right)\cdot\sum_{n=1}^{i}\frac{1}{\mu_n}\right] + \left(\sum_{i=1}^{k}\frac{1}{\mu_i}\right)\cdot\prod_{j=1}^{k}a_j
\end{aligned}$$

Let $A_i \sqcap \sum_{j=1}^{i}\frac{1}{\mu_j}$ and $r_i \sqcap \prod_{j=1}^{i}a_j$ therefore Eq. (34) is simplified to:

$$\overline{S_T} = E\left[S_T\right] = \sum_{i=1}^{k} A_i \Pr\left[S_T = A_i\right] = \frac{b_1}{\mu_1} + \sum_{i=2}^{k-1} b_i r_{i-1} A_i + r_k A_k \qquad \textbf{(35)}$$

Using similar analysis to the above, the n^{th} moment of the Coxian-*k* distribution can be computed using:

$$E\left[S_T^n\right] = \sum_{i=1}^{k}\left(A_i - \overline{S_T}\right)^n \Pr\left[S_T = A_i\right] \qquad \textbf{(36)}$$

The complexities behind such computations are clearly obvious. However, the Laplace Transform offers a very important property. In fact the LT of a discrete time function *f* is expressed as:

$$f^*(s) = \sum_{x} e^{-sx} f_X(x) \qquad \textbf{(37)}$$

In particular, if *f* is the probability density function of a random variable *X*, then equation (37) becomes:

$$f^*(s) = \sum_x e^{-sx} \Pr[X = x] = E\left[e^{-sx}\right] \tag{38}$$

Property: The n^{th} moment of a random variable *X* with a probability density function $f(x)$ can be expressed as:

$$E\left[X^n\right] = (-1)^n \cdot \frac{d^n}{ds^n}\left[f^*(s)\right]\Big|_{s=0} \tag{39}$$

where $f^*(s)$ represents the LT of $f(x)$.

Proof: Let $n = 1$. Therefore using Eq. (37), we have:

$$\begin{aligned}(-1)\frac{d}{ds}\left[\sum_x e^{-sx} f(x)\right]_{s=0} &= -\sum_x \frac{d}{ds}\left[e^{-sx} f(x)\right]\Big|_{s=0} = -\sum_x -xe^{-sx} f(x)\Big|_{s=0} \\ &= \sum_x xe^{-sx} f(x)\Big|_{s=0} = \sum_x xf(x) = E[X]\end{aligned}$$

For $n = 2$, we have:

$$\begin{aligned}(-1)^2 \frac{d^2}{ds^2}\left[f^*(s)\right]\Big|_{s=0} &= \frac{d^2}{ds^2}\left[f^*(s)\right]\Big|_{s=0} = \frac{d}{ds}\left[\frac{d}{ds}\left[f^*(s)\right]\right]_{s=0} \\ &= \frac{d}{ds}\left[\sum_x -xe^{-sx} f(x)\right]_{s=0} = \sum_x x^2 e^{-sx} f(x)\Big|_{s=0} \\ &= \sum_x x^2 f(x) = E\left[X^2\right]\end{aligned}$$

By induction, assume this is true for $n = k$. That is:

$$\begin{aligned}(-1)^k \frac{d^k}{ds^k}\left[f^*(s)\right]\Big|_{s=0} &= \begin{cases} -\sum\limits_x -x^k e^{-sx} f(x)\Big|_{s=0} &, \quad k \text{ is odd} \\ \sum\limits_x x^k e^{-sx} f(x)\Big|_{s=0} &, \quad k \text{ is even} \end{cases} \\ &= \sum_x x^k f(x) = E\left[X^k\right]\end{aligned}$$

Prove it's true for n = k + 1:

$$
\begin{aligned}
(-1)^{k+1}\frac{d^{k+1}}{ds^{k+1}}\left[f^{*}(s)\right]\Big|_{s=0} &= (-1)^{k+1}\frac{d}{ds}\left[\frac{d^{k}}{ds^{k}}\left[f^{*}(s)\right]\right]_{s=0} \\
&= \begin{cases} \dfrac{d}{ds}\left[\sum\limits_{x} -x^{k}e^{-sx}f(x)\Big|\right]_{s=0} = \sum\limits_{x}\dfrac{d}{ds}\left[-x^{k}e^{-sx}f(x)\right]\Big|_{s=0} & ,\quad k \text{ is odd} \\ -\dfrac{d}{ds}\left[\sum\limits_{x} x^{k}e^{-sx}f(x)\Big|\right]_{s=0} = -\sum\limits_{x}\dfrac{d}{ds}\left[x^{k}e^{-sx}f(x)\right]\Big|_{s=0} & ,\quad k \text{ is even} \end{cases} \\
&= \begin{cases} \sum\limits_{x} x^{k+1}e^{-sx}f(x)\Big|_{s=0} & ,\quad k \text{ is odd} \\ -\sum -x^{k+1}e^{-sx}f(x)\Big|_{s=0} & ,\quad k \text{ is even} \end{cases} \\
&= \sum_{x} x^{k+1}e^{-sx}f(x)\Big|_{s=0} = \sum_{x} x^{k+1}f(x) = E\left[X^{k+1}\right]
\end{aligned}
$$

C.Q.D.

5.3.2 The Coxian-2 Distribution

The C_2 distribution is a special case of C_k as shown in Figure 13 where we only have two service stages. All other properties of a C_k apply to the C_2 distribution.

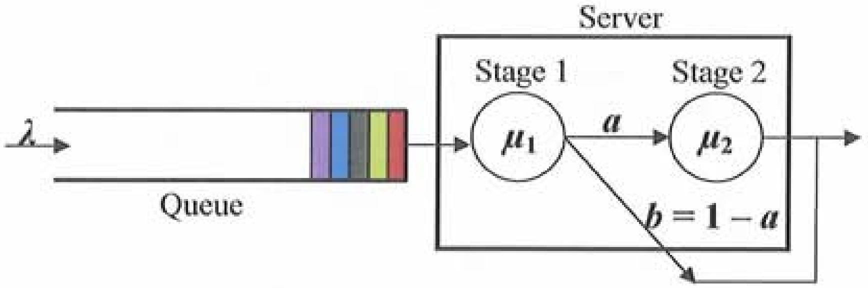


Figure 13: Queuing system where the server has a C_2 distributed service time.

Using Eq. (32), the LT of the total service time for a C_2 distribution is given by:

$$\begin{aligned} f^*(s) &= bf_1^*(s) + af_1^*(s)f_2^*(s) \\ &= (1-a)\frac{\mu_1}{s+\mu_1} + a\frac{\mu_1}{s+\mu_1}\cdot\frac{\mu_2}{s+\mu_2} \end{aligned} \tag{40}$$

By the principle of Partial Fractions Expansion (PFE), the second term of the sum in Eq. (40) can be written as:

$$\begin{aligned} a\frac{\mu_1}{s+\mu_1}\cdot\frac{\mu_2}{s+\mu_2} &= \frac{C_1}{s+\mu_1} + \frac{C_2}{s+\mu_2} = \frac{C_1(s+\mu_2)+C_2(s+\mu_1)}{(s+\mu_1)(s+\mu_2)} \\ &= \frac{C_1 s + C_1\mu_2 + C_2 s + C_2\mu_1}{(s+\mu_1)(s+\mu_2)} = \frac{(C_1+C_2)s + C_1\mu_2 + C_2\mu_1}{(s+\mu_1)(s+\mu_2)} \end{aligned}$$

Equating the numerators leads to:

$$(C_1+C_2)s + C_1\mu_2 + C_2\mu_1 = a\mu_1\mu_2 \Rightarrow \begin{cases} C_1 + C_2 = 0 \\ C_1\mu_2 + C_2\mu_1 = a\mu_1\mu_2 \end{cases}$$

Solving for C_1 and C_2 we obtain:

$$C_1 = \frac{a\mu_1\mu_2}{\mu_2-\mu_1}$$
$$C_2 = \frac{a\mu_1\mu_2}{\mu_1-\mu_2}$$

Therefore Eq. (40) can be rewritten as:

$$\begin{aligned} f^*(s) &= (1-a)\frac{\mu_1}{s+\mu_1} + \frac{C_1}{s+\mu_1} + \frac{C_2}{s+\mu_2} = (1-a)\frac{\mu_1}{s+\mu_1} + \frac{\frac{a\mu_1\mu_2}{\mu_2-\mu_1}}{s+\mu_1} + \frac{\frac{a\mu_1\mu_2}{\mu_1-\mu_2}}{s+\mu_2} \\ &= (1-a)\frac{\mu_1}{s+\mu_1} + \frac{a\mu_2}{\mu_2-\mu_1}\cdot\frac{\mu_1}{s+\mu_1} + \frac{a\mu_1}{\mu_1-\mu_2}\cdot\frac{\mu_2}{s+\mu_2} \\ &= \left[(1-a) + \frac{a\mu_2}{\mu_2-\mu_1}\right]\frac{\mu_1}{s+\mu_1} + \frac{a\mu_1}{\mu_1-\mu_2}\cdot\frac{\mu_2}{s+\mu_2} \\ &= \left[1 + \frac{a\mu_1}{\mu_2-\mu_1}\right]\frac{\mu_1}{s+\mu_1} + \left[\frac{a\mu_1}{\mu_1-\mu_2}\right]\frac{\mu_2}{s+\mu_2} \end{aligned} \tag{41}$$

By Inverse Laplace Transform (ILT) of Eq. (41), we obtain the probability density function of the total service time for a C_2 distribution as:

$$f(S_T)=\left[1+\frac{a\mu_1}{\mu_2-\mu_1}\right]\mu_1 e^{-\mu_1 S_T}+\left[\frac{a\mu_1}{\mu_1-\mu_2}\right]\mu_2 e^{-\mu_2 S_T} \quad , \quad \mu_1 \neq \mu_2 \qquad \textbf{(42)}$$

Using Eq. (35), the first moment of a C_2 distributed random variable is expressed as:

$$E[S_T]=\frac{1}{\mu_1}+\frac{a}{\mu_2} \qquad \textbf{(43)}$$

Using Eq. (39), the second moment of a C_2 distributed random variable is given by:

$$E\left[S_T^2\right]=\frac{2}{\mu_1^2}+\frac{2a}{\mu_1\mu_2}+\frac{2a}{\mu_2^2} \qquad \textbf{(44)}$$

5.3.3 The Erlang-k Distribution

The Erlang-*k* distribution denoted by E_k, is a special case of C_k as shown in Figure 14 where the service rates at all stages are equal. Also, the mean service times of all the *k* stages should sum up to the overall service time of the server, μ. That is, if we denote by $\frac{1}{\mu_i}$ the mean service time at the i^{th} stage, where μ_i is the service rate at that stage, then we have:

$$\frac{1}{\mu}=\frac{1}{\mu_1}+\frac{1}{\mu_2}+\ldots+\frac{1}{\mu_k}=\underbrace{\frac{1}{\mu_1}+\frac{1}{\mu_1}+\ldots+\frac{1}{\mu_1}}_{k \text{ times}}=\frac{k}{\mu_1} \qquad \textbf{(45)}$$

Eq. (45) means that $\mu_1=\mu_2=\ldots=\mu_k=k\mu$. In addition, a packet enters the server at the first stage and must pass through all the *k* stages and departs from the server only after finishing its service at the last stage, *k*. Therefore $a_1=a_2=\ldots=a_k=1$, and $b_1=b_2=\ldots=b_k=0$. We denote by S_i the random exponentially distributed service time that a packet obtains at the i^{th} stage, and by S_T the total service time a packet would have received after it departs from the server, where $S_T = \sum_{i=1}^{k} S_i$.

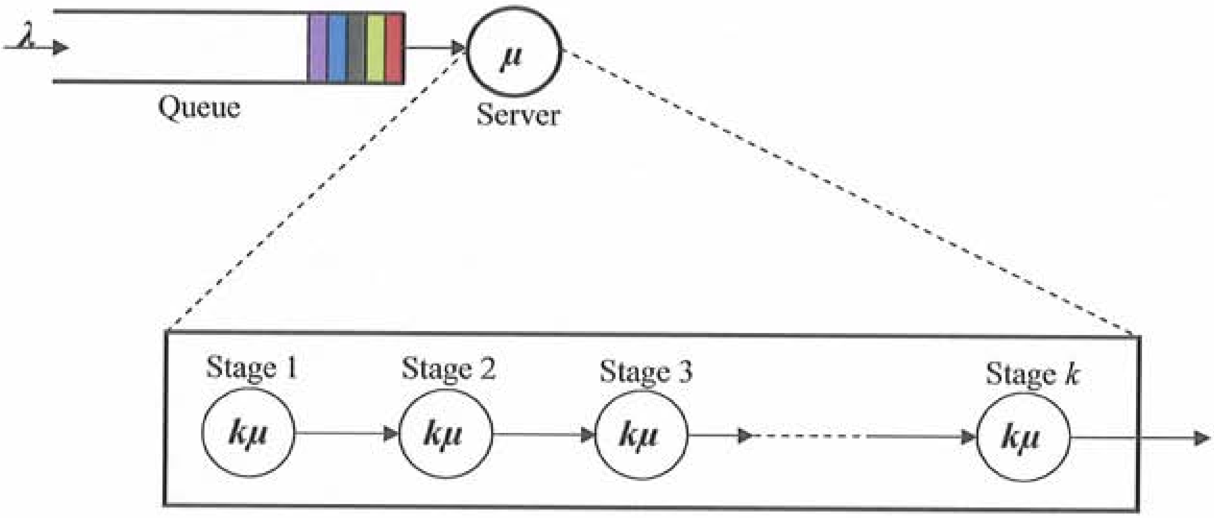


Figure 14: Packet service time divided into stages of an Erlang-*k* distribution.

Without loss of generality, let f_i and f denote the probability density functions of S_i and S_T respectively. Therefore we have:

$$f = f_1 * f_2 * \ldots * f_k \qquad \textbf{(46)}$$

The LT of f is given by f^* where

$$f^*(s) = f_1^*(s) \cdot f_2^*(s) \cdot \ldots \cdot f_k^*(s) = \prod_{i=1}^{k} f_i^*(s) = \prod_{i=1}^{k} \frac{\mu_i}{s+\mu_i} = \frac{(k\mu)^k}{(s+k\mu)^k} \qquad \textbf{(47)}$$

Therefore by the ILT of Eq. (47), the probability density function of S_T is:

$$f(S_T) = (k\mu)^k \cdot \frac{1}{(k-1)!} \cdot S_T^{k-1} \cdot e^{-k\mu S_T} = \frac{k\mu(k\mu S_T)^{k-1} e^{-k\mu S_T}}{(k-1)!} \qquad \textbf{(48)}$$

Similarly to a C_k distribution, using Eq. (39), we can compute the n^{th} moment of an E_k distributed random variable. In particular:

$$\begin{aligned} E[S_T] &= -\frac{d}{ds}\left[f^*(s)\right]\Big|_{s=0} = -\frac{d}{ds}\left[\frac{(k\mu)}{(s+k\mu)}\right]^k\Bigg|_{s=0} \\ &= -k\frac{-(k\mu)}{(s+k\mu)^2}\left[\frac{(k\mu)}{(s+k\mu)}\right]^{k-1}\cdot\Bigg|_{s=0} = \frac{1}{\mu} \end{aligned} \tag{49}$$

The second moment of S_T is given by:

$$\begin{aligned} E\left[S_T^2\right] &= (-1)^2\frac{d^2}{ds^2}\left[f^*(s)\right]\Big|_{s=0} = \frac{d}{ds}\left[k\frac{-(k\mu)}{(s+k\mu)^2}\left[\frac{(k\mu)}{(s+k\mu)}\right]^{k-1}\right]_{s=0} \\ &= \frac{2(k\mu)k(s+k\mu)}{(s+k\mu)^4}\cdot\left(\frac{k\mu}{s+k\mu}\right)^{k-1} + \frac{k(k-1)(k\mu)^2}{(s+k\mu)^4}\cdot\left(\frac{k\mu}{s+k\mu}\right)^{k-2}\Bigg|_{s=0} \\ &= \frac{k+1}{k\mu^2} \end{aligned} \tag{50}$$

5.3.4 Packet Service Time Model Motivation

Recall that the service time of a packet in an IEEE 802.11-based network is defined as the difference between the instant the packet reaches the top of the queue and the instant the packet either gets successfully transmitted or dropped in case the upper bound on the number of allowed retransmission attempts, *K*, as defined by the standard, is reached. Given the exponential back-off mechanism as explained in section A of this chapter, the packet service time will therefore consist of one or more back-off stages as shown in Figure 15. As already discussed, a packet will always start service at the first stage. After completing service at the i^{th} stage ($1 \le i \le K$), that is when the node's back-off counter at stage *i* reaches zero, the packet either successfully departs with a probability $(1-p)$ if the WM is not busy, or re-initializes its back-off counter to a new value and proceeds to the next stage with probability p, where p is the probability of collision. It is worthwhile to remember that in this scenario, the i^{th} service time, S_i, is uniformly distributed in the range $\left[1; 2^{i-1} W_{\min}\right]$ with an average of $\overline{S_i} = \frac{2^{i-1} W_{\min} + 1}{2}$, if $1 \le i \le m$, and in the range $\left[1; 2^{m} W_{\min}\right]$ with an average of $\overline{S_i} = \frac{2^{m} W_{\min} + 1}{2}$, if $m+1 \le i \le K$. We therefore denote by $\mu_i = \frac{1}{\overline{S_i}}$ the average service rate at the i^{th} stage. The total service time of the packet is denoted by S_T.

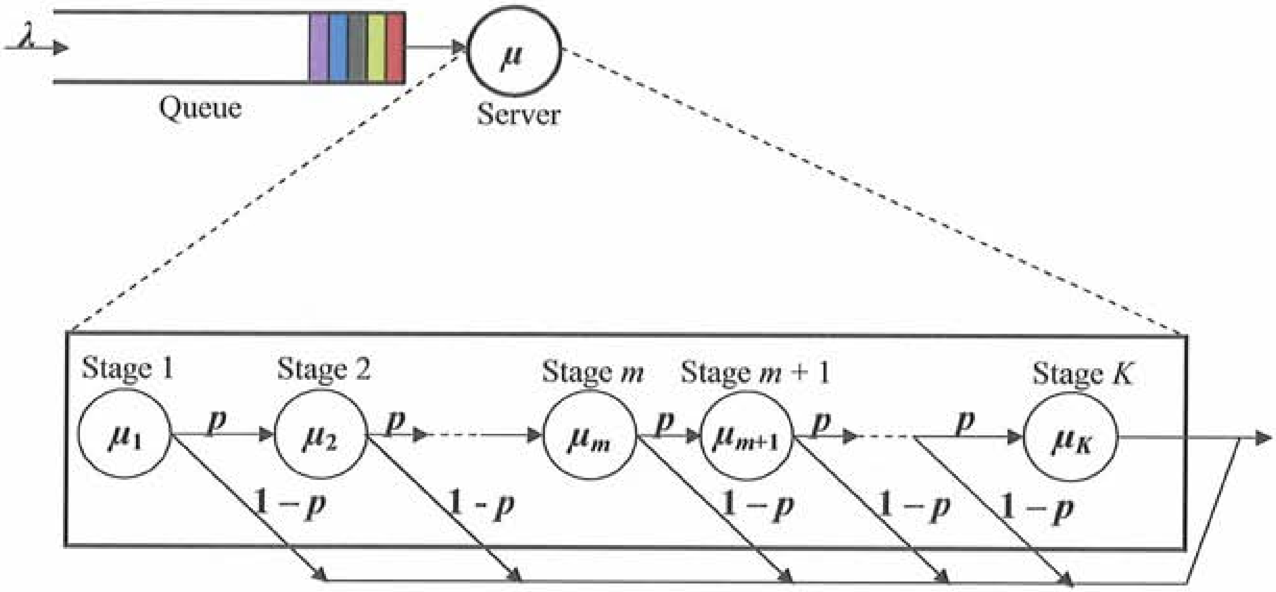


Figure 15: IEEE 802.11 packet service time as composed of several back-off stages.

It is clear from Figure 15, in compliance with the IEEE 802.11 standard, a packet that reaches the top of the queue, always passes initially through the first back-off stage, meaning that a node backs off after each successful transmission it does. With a probability $(1-p)$ the packet may then be successfully transmitted, or a collision occurs with a probability p and the packet proceeds to stage two, and so forth.

Therefore, the total service time, S_T, may, with a certain probability, be equal to the sum $S_1 + S_2 + \ldots + S_i$. This is more illustrated in Figure 16.

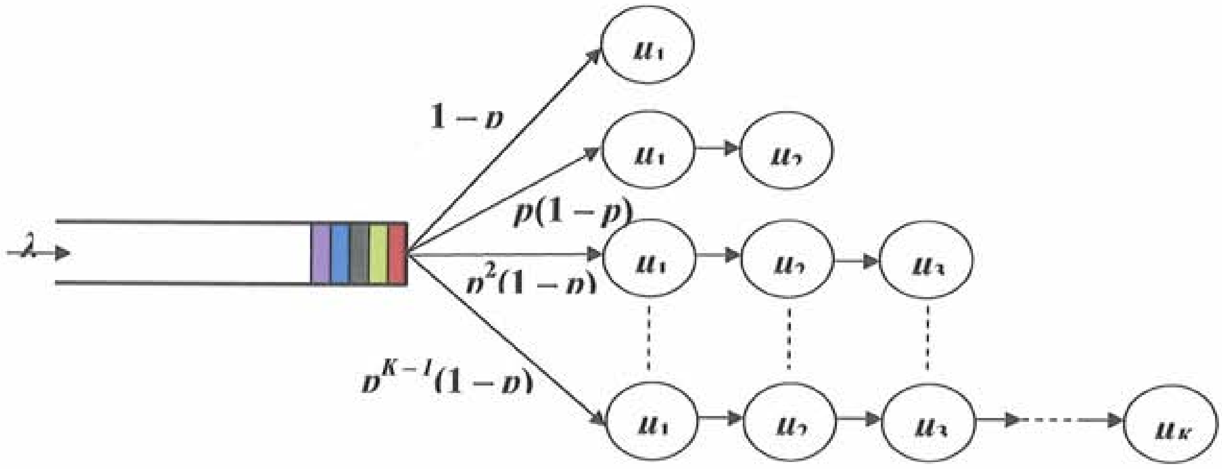


Figure 16: Probability that a packet goes through *i* back-off stages, $i = 1, 2\ldots K$.

From Figure 16, we can see that $S_T = S_1$ with a probability $(1-p)$, $S_T = S_1 + S_2$ with a probability $p(1-p)$, …, $S_T = S_1 + S_2 + \ldots + S_i$ with a probability $p^{i-1}(1-p)$, and so forth. Therefore we can write:

$$\Pr\left[S_T = \sum_{j=1}^{i} S_j\right] = p^{i-1}(1-p) \quad , \quad i = 1, 2, \ldots, K \tag{51}$$

For each value of *i*, the sum $\sum_{j=1}^{i} S_j$ has a probability density function that is given by $f_1 * \ldots * f_i$ which is nothing but a convolution of *i* uniform density functions such that:

$$f_i = \begin{cases} \dfrac{1}{2^{i-1} W_{\min}} & , \quad 1 \le i \le m \\ \dfrac{1}{2^{m} W_{\min}} & , \quad m+1 \le i \le K \end{cases} \tag{52}$$

Therefore using Eq. (29), the probability density function of S_T is expressed as:

$$\begin{aligned} f(S_T) &= \sum_{i=1}^{k} \Pr\left[S_T = \sum_{j=1}^{i} S_j\right] \cdot [f_1 * \ldots * f_i] \\ &= (1-p) f_1 + p(1-p)[f_1 * f_2] + p^2(1-p)[f_1 * f_2 * f_3] + \\ &\ldots + p^{K-1}(1-p)[f_1 * \ldots * f_K] \end{aligned} \qquad \textbf{(53)}$$

The LT of *f* is given by:

$$f^*(s) = \sum_{i=1}^{k} \Pr\left[S_T = \sum_{j=1}^{i} S_j\right] \cdot \prod_{j=1}^{i} f_i^*(s) \qquad \textbf{(54)}$$

The LT of f_i is :

$$f_i^*(s) = \begin{cases} \dfrac{\dfrac{1}{2^{i-1} W_{\min}}}{s} & , \quad 1 \le i \le m \\ \dfrac{\dfrac{1}{2^{m} W_{\min}}}{s} & , \quad m+1 \le i \le K \end{cases}$$

We define $\omega_i = \dfrac{1}{2^{i-1} W_{\min}}$ and $\omega_{m+1} = \dfrac{1}{2^{m} W_{\min}}$. Therefore $f_i^*(s)$ can be written as:

$$f_i^*(s) = \begin{cases} \dfrac{\omega_i}{s} & , \quad 1 \le i \le m \\ \dfrac{\omega_{m+1}}{s} & , \quad m+1 \le i \le K \end{cases} \qquad \textbf{(55)}$$

Putting together Eqs. (54) and (55) together we have:

$$\begin{aligned} f^*(s) &= \sum_{i=1}^{K} \Pr\left[S_T = \sum_{j=1}^{i} S_j\right] \cdot \prod_{j=1}^{i} f_i^*(s) \\ &= (1-p)\frac{\omega_1}{s} + p(1-p)\frac{\omega_1 \omega_2}{s^2} + \ldots + p^{K-1}(1-p)\frac{\omega_1 \ldots \omega_m \omega_{m+1}^{K-m}}{s^K} \\ &= \sum_{i=1}^{m} p^{i-1}(1-p)\frac{\prod_{j=1}^{i} \omega_j \cdot s^{K-i}}{s^K} + \sum_{i=m+1}^{K} p^{i-1}(1-p)\frac{\prod_{j=1}^{m} \omega_j \cdot \omega_{m+1}^{m-i} \cdot s^{K-i}}{s^K} \end{aligned} \qquad \textbf{(56)}$$

It is clear from Eq. (56) that the numerator of f^* has a degree which is less than the degree of its denominator. Therefore f^* is a rational function and can therefore be modeled by a C_K distribution.

5.3.5 Squared Coefficient of Variation and Distribution Fitting Analysis

Statistically speaking, the variance of a certain random variable, taken alone has no interpretable meaning. It only indicates the degree of variability of a random variable with respect to the mean of that variable.

Definition: The Squared Coefficient of Variation (SCV) is defined as the ratio of the variance, σ_X^2, of a given random variable, X, to the square of its mean, m_X^2. That is:

$$c^2 = \frac{\sigma_X^2}{m_X^2} \tag{57}$$

As it gives an insight to the dynamics of a random variable, X, the SCV is used as an indicator to which probability density function best fits a given set of numerical values that X can assume. For example, an exponentially distributed random variable X with a mean $m_X = \frac{1}{\lambda}$ and $\sigma_X^2 = \frac{1}{\lambda^2}$, the SCV is $c^2 = 1$. Whenever X is a constant equal to q, the mean $m_X = q$ and the variance $\sigma_X^2 = 0$, result in having $c^2 = 0$. The work in [60] shows that if the SCV of a certain given set of numerical data is greater than 0.5, then it is possible to determine a C_2 distribution with mean and variance equal to those of this provided set. However for values of SCV that are less than 0.5, [60] suggests an Erlang type distribution.

Coupled with mathematical techniques such as the n^{th}-Moment Fit method, the SCV allows the determination of the parameters of the distribution that best fits a given set of generated values. In an attempt to fit such values to a known distribution with n unknown parameters, computer software is used to numerically compute the first n moments of this given set of values, where the result is equated to the n moment expressions as obtained from Eq. (39). Accordingly the system of n moment equations is solved and the parameters of the distribution can be extracted. It is worthwhile to mention that in a C_k distribution, when k increases, the dimensionality problem emerges as the representation of the different states of the queue requires an extensive work. In particular, when the packet service time in an IEEE 802.11 network is exactly modeled as a C_K distribution where $K = 16$. Therefore the number of stages of such a distribution is relatively large. It was proven that a C_2 provides an excellent representation in such cases. The distribution fitting procedure is as follows:

1. Using Eq. (57), compute the SCV of the generated set, S, of packet service time values.
2. If $c^2 \leq 0.5$, then the distribution that best fits the values in S is an E_k with parameters computed as follows:
   a. $$k = \left\lceil \frac{1}{c^2} \right\rceil \tag{58}$$
   b. Compute the first moment, m_1, of the values in S using computer software. The mean μ and the variance of the E_k distribution are computed as:

$$\mu = \frac{1}{m_1} \tag{59}$$

$$\sigma^2 = \frac{1}{k\mu^2} \tag{60}$$

   c. The probability density function is given by Eq. (48) and its LT is given by Eq. (47).

   d. The n^{th} moments of this E_k distribution can be computed using computer software and equated to their corresponding expression obtained from Eq. (39).

3. If $0.5 < c^2 < 1$, then the distribution that best fits the values in *S* is a C_2 with the parameters computed using the 2-moments fit method and given as follows:

$$a = \frac{1}{2c^2} \tag{61}$$

$$\mu_1 = \frac{2}{m_1} \quad ; \quad \mu_2 = \frac{1}{m_1 c^2} \tag{62}$$

   Where m_1 is the mean of the values in *S* computed by computer software. The probability density function of this distribution as well as its LT are respectively given by Eqs. (42) and (41).

4. If $c^2 \geq 1$, then distribution that best fits the values in *S* is also a C_2. However its parameters are computed using the 3-moments fit method as follows:

   a. Using computer software, compute the first three moments respectively m_1, m_2 and m_3 of the values in *S*.

   b. Using Eq. (39), compute the first three moment expressions and equate them their corresponding values in step (4.a). That is:

      i. First moment:

$$E[S]=\frac{1}{\mu_1}+\frac{a}{\mu_2}=m_1 \qquad \textbf{(63)}$$

ii. Second moment:

$$E\left[S^2\right]=\frac{2b}{\mu_1^2}-\frac{2a\mu_1\mu_2-2a\left(\mu_1+\mu_2\right)^2}{\left(\mu_1\mu_2\right)^2}=m_2 \qquad \textbf{(64)}$$

iii. Third moment:

$$E\left[S^3\right]=\frac{6b}{\mu_1^3}-\frac{12a\mu_1\mu_2\left(\mu_1+\mu_2\right)-6a\left(\mu_1+\mu_2\right)^3}{\left(\mu_1\mu_2\right)^3}=m_3 \qquad \textbf{(65)}$$

c. Given the fact that $b=1-a$, solve Eqs. (63), (64) and (65) for a, μ_1, and μ_2 to obtain:

$$a=\frac{\mu_2}{\mu_1}\left(m_1\mu_1-1\right) \qquad \textbf{(66)}$$

$$\mu_1=\frac{X+\sqrt{X^2-4Y}}{2} \quad ; \quad \mu_2=X-\mu_1 \qquad \textbf{(67)}$$

Where X and Y are given by:

$$X=\frac{1}{m_1}+\frac{m_2Y}{2m_1} \quad ; \quad Y=\frac{6m_1-\frac{3m_2}{m_1}}{\frac{3m_2^2}{2m_1}-m_3} \qquad \textbf{(68)}$$

Note that for the 3-moments method to be successful, the condition $X^2-4Y\geq 0$ should hold. If this condition does not hold or if X happens to be negative, then even if $c^2\geq 1$, compute the parameters using the 2-moments fit method as described in step (3).

Chapter 6

Novel Routing Metrics and Protocol Designs

The routing layer, a topic that has been investigated by many researchers [7], [19], [27], [39 – 50], remains a key problem for the newly developing standards such as IEEE 802.11s. A major gap in this field lies in the non-existence of IEEE 802.11s-specific routing metrics and protocols. Instead, researches focused on the adaptation of those of MANETs to 802.11s-based network scenarios. In this section, we propose two novel routing metrics namely the Probability of Collision (PoC) and the Node Buffer Size (NBS). Accordingly, two novel routing protocols are proposed respectively a MAC-Aware Routing Protocol (MACARP) and Buffer-Aware Routing Protocol (BARP). Although specifically designed and tailored to IEEE 802.11s-based WMNs, however, with minor variations, those new metrics and protocols may also be adapted to other types of wireless as well as wired networks. For illustration purposes, we will consider the network shown in Figure 17 and therefore explain how the two novel metrics and protocols can be applied in the following sub-sections.

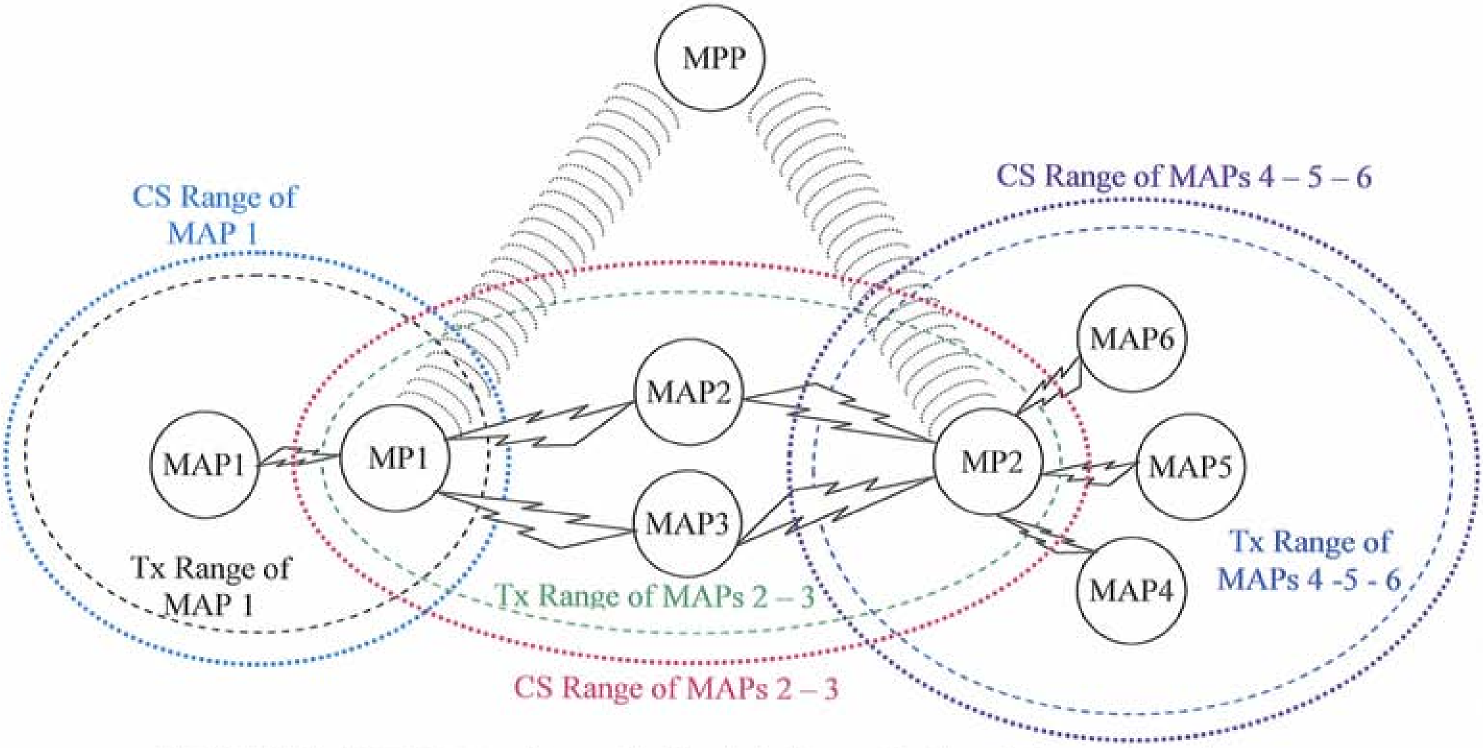


Figure 17: A network scenario used to illustrate the application of the novel routing metrics and protocols.

6.1 PoC Routing Metric and MAC-Aware Routing Protocol

In an IEEE 802.11-based wireless network, collisions are more likely to occur in either a dense network (i.e. there exists a large number of wireless nodes that are in the vicinity of each other and contending for the wireless medium) or in scenarios where nodes are subject to high load. In both cases simultaneous transmissions from different nodes are more likely to occur causing an increase in the probability of collision. From a tagged node viewpoint, the greater the probability of collision is, the more that node will be retrying to send a certain packet, and the maximum number of retransmission attempts specified by the standard, is more likely going to be reached causing the packet to be dropped. It logically follows, on one hand, that in such scenarios, packet losses will considerably increase. On the other hand, each time a collision occurs, the tagged node will go in back-off mode delaying its transmission by a number of back-off slots (BO) chosen from a range defined by the node's contention window (CW) size which will double with each consecutive collision that occurs as explained in section (V-*A*). However, if finally the node was able to successfully transmit its packet after a series of consecutive collisions, in this case, this packet would have experienced large delays. When added up together, delays as such occurring at the nodes the packet passes through on its path from the source to the destination, will significantly contribute in the increase of its end-to-end delay.

The Probability of Collision (PoC), a parameter extracted from the heart of the IEEE 802.11 MAC protocol, can be used as a routing metric based on which routing decisions can be made. To illustrate this, we consider the network scenario depicted in Figure 17. In this scenario, there are six Mesh Access Points (MAPs), two Mesh Points (MPs) and one Mesh Portal (MPP). MAP1's transmission and carrier sensing ranges only cover MP1. Therefore MAP1 can only communicate with MP1. MAP2 and MAP3 have transmission (Tx) and carrier sensing (CS) ranges that only cover MP1 and MP2. Therefore they can communicate with either MPs. MAP4, 5 and 6 have transmission and carrier sensing ranges that only cover MP2 thus can only communicate with this latter. It is therefore clear that this network suffers from the Hidden Terminal Problem as none of the MAPs lies in the transmission and carrier sensing range of the others meaning that collisions can easily occur. However, MAP1 is pretty far from MAPs 4, 5 and 6 therefore they do not contend with each other. However, when MAP2 and MAP3 transmit to MP1 on one hand, they will be contending with MAP1, thus a total of three nodes contending for the wireless medium. On the other hand, when MAP2 and MAP3 transmit to MP2, they will contend with MAP4, 5, and 6, therefore a total of five nodes contending for the wireless medium. Therefore, it is of course obvious that MAP2 and MAP3 will face a larger probability of collision when transmitting to MP2. As the network runs, MAP2 and MAP3 will start first forwarding packets in a random way to either MP1 or MP2. They will soon notice the fact that when forwarding their packets to MP2, they are experiencing more collisions. Therefore, as illustrated in Figure 18, they will dynamically reduce their transmission power thus reducing their transmission range such that it only covers the MP (i.e. MP1 in this case) which when transmitting packets to, those transmissions will experience the least number of collisions.

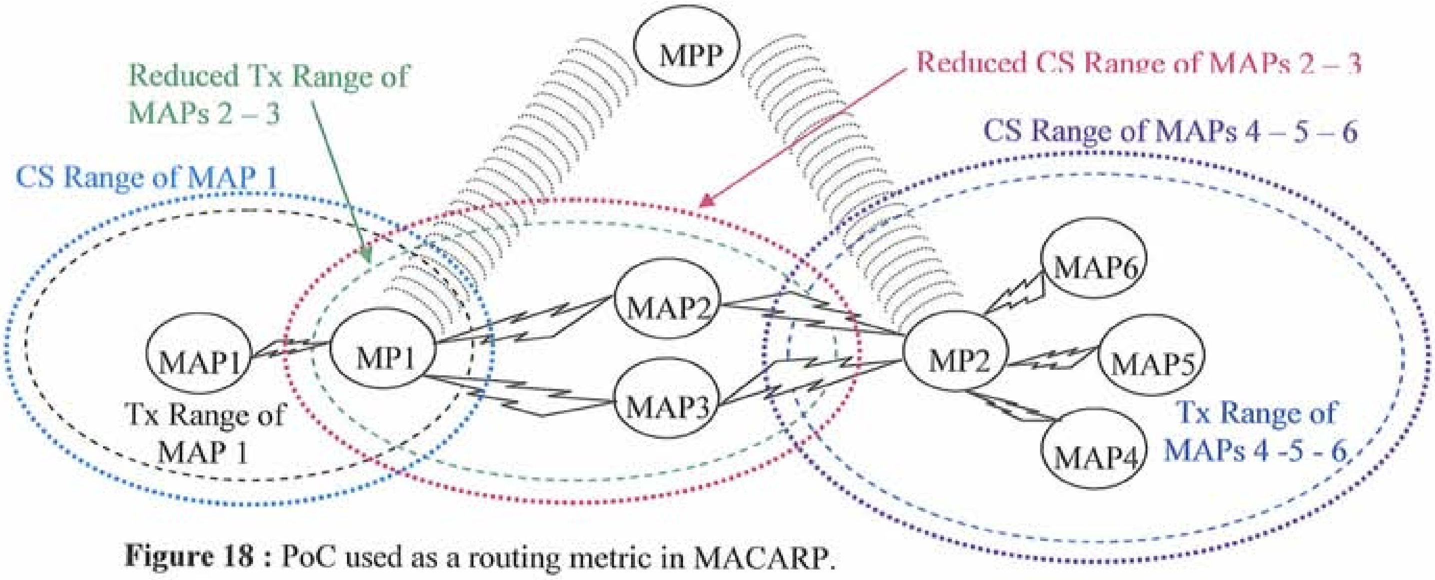
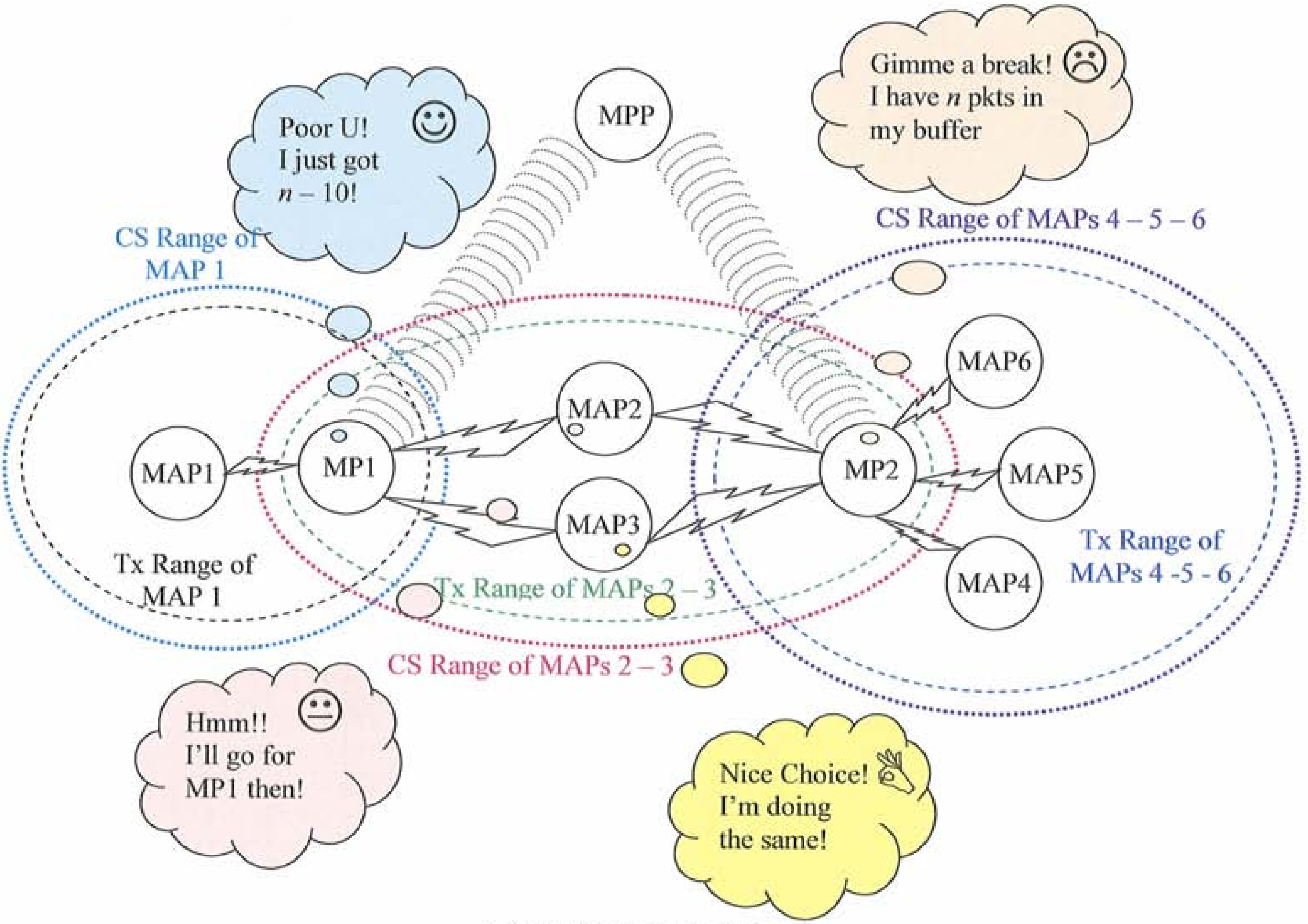


Figure 18 : PoC used as a routing metric in MACARP.

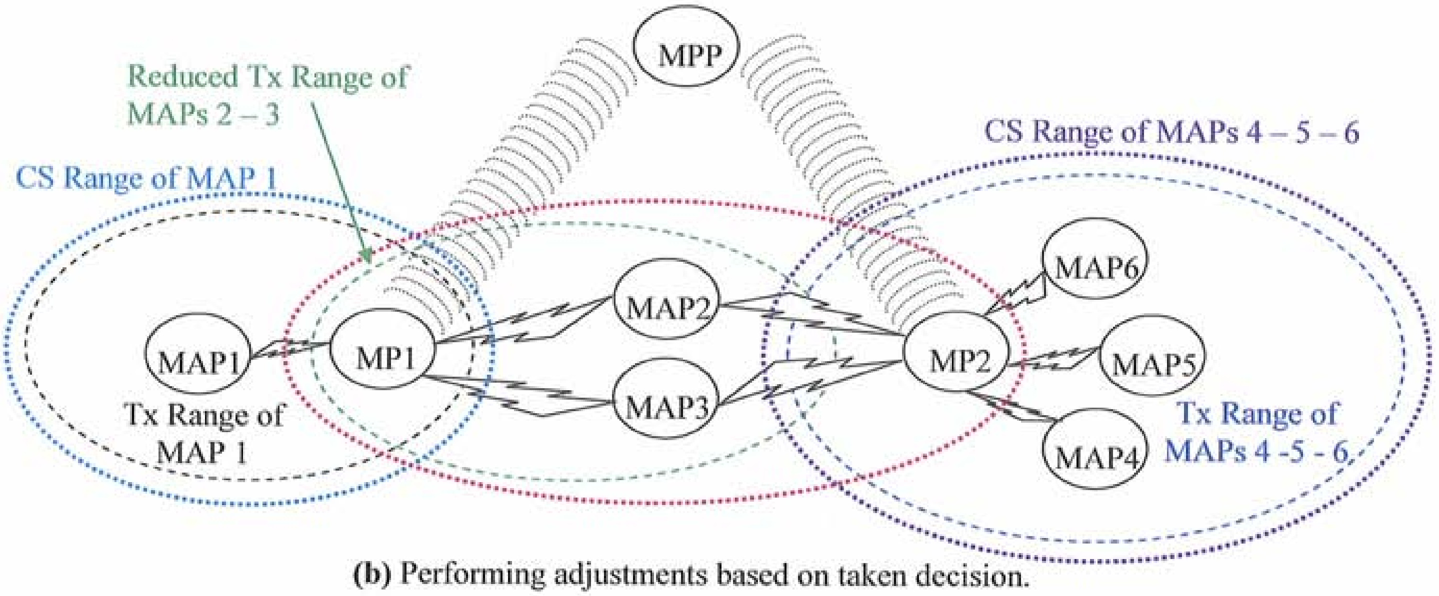
6.2 NBS Routing Metric and Buffer-Aware Routing Protocol

Generally, one of the major characteristics of a network node is its Buffer Size (BS) that can play an important role in boosting the performance of the network. The rationale behind this idea is the fact that a node with a full buffer will start dropping any new arriving packets to it which will result in a significant increase in packet losses. In addition, if any arbitrary arriving packet gets accepted by a certain arbitrary node having a relatively loaded buffer, may experience high queuing delays which can accumulate at each node this packet passes through along the path from the source to the destination, and therefore cause an increase in that packet's end-to-end delays. As a matter of fact, generally speaking, the three main factors that affect the state of a buffer are its size, the service rate of the node to which it belongs and the arrival rate at which packets are arriving to this node. A measure that combines both the arrival and service rates is the load, known as the ratio of the arrival rate to the service rate. In other words, a node that experiences medium to heavy traffic load is more likely to have its buffer filled up faster than other nodes that experience low load.

In IEEE 802.11-based wireless networks and especially such dense networks where the probability of collision is high due to the contention of a relatively large number of nodes for the wireless medium, nodes will more likely have to back-off more than once something which causes an increase in the service time of their packets. As a consequence, the node's service rate will decrease. Starting from relatively medium to high loads in the range of 4 to 20 Erlangs, IEEE 802.11-compliant nodes are witnessed to have fast buffer saturation. Packet losses and packet end-to-end delays become highly significant under loads from 6 to 20 Erlangs. However, with a load higher than 6 Erlangs, the packet arrival rate becomes greater than the node's service rate resulting in a normalized load greater than one, and thus an unstable queuing system. As a consequence, packets will start arriving faster than they are being served and the buffer will be full in a very short time. So, during the time it takes for a certain packet to complete its service, i.e. to get successfully transmitted or dropped due to a limited number of retransmission attempts, the node will have dropped a considerable amount of newly arriving packets due to saturated buffer. This is why, the Node Buffer Size (NBS), is seen as a critical parameter based on which nodes in an IEEE 802.11-based WMN can make routing decisions in such a way that a node that has the ability to forward packets to more than one neighboring nodes will choose among them the one that has the least number of packets in its buffer. To illustrate this concept we will also consider the scenario depicted in Figure 17. However, the only difference now is that MP1 and MP2 will always be broadcasting information about the number of packets they have in their respective buffers. MAP1 has no other choice but to always communicate to MP1. Same applies for MAPs 4, 5 and 6 which will always communicate with MP2. However, MAP2 and MAP3 in this case also have the choice to either communicate with MP1 or MP2. Therefore based on the broadcasted message from MP1 and 2, MAP 2 and 3 will then decide to forward their packets to the MP with the least number of packets in its buffer. Based on this decision and in order not to create a collision at the receiving MP in case another transmission from any the other hidden MAPs is taking place simultaneously, the MAP in question will reduce its transmission range so that it covers only the MP to which it decided to forward the packet to and will leave its carrier sensing range intact in such a way that it still keeps on receiving broadcast messages from both MPs declaring the number of packets in their buffers. This is more illustrated in Figure 19.



(a) Decision making phase.



(b) Performing adjustments based on taken decision.

Figure 19: First network scenario used to illustrate the use of NBS as a routing metric.

Chapter 7

Simulations and Results

After theoretically examining the packet service time, showing that it follows a Coxian distribution and conceptually introducing the two novel routing metrics, PoC and NBS, along with their corresponding routing protocols respectively MACARP and BARP, an important task is to verify the correctness of our model and the effectiveness of our newly proposed routing schemes through simulation. However, as we mentioned earlier, a state of the art simulation and analysis tool for IEEE 802.11s-based networks still does not exists. Therefore in this chapter we first introduce our newly developed simulator in section A and we use sections B and C to respectively verify our packet service time distribution model and simulate the MACARP and BARP routing protocols.

7.1 IEEE 802.11s Network Simulator Development

By the time this entire work was in progress, we had a major timing constraint. Therefore our aim was to develop a user friendly, accurate, reliable and easily customizable simulator that takes into account the specific properties of IEEE 802.11s-based WMNs while allowing us to integrate our changes and new proposals easily without the least time consumption. Simulators such as NS-2, Pythagor 1.2 and OPNET do support IEEE 802.11a/b/g which are standards used to implement Ad-Hoc networks and not WMNs. Therefore to build new libraries and/or modify the existing libraries and build upon them required a lot of code reading, understanding and extensive programming which is too much time consuming. However, MathWorks' MATLAB 2008, and specifically SIMULINK 2008 provide a very user friendly design environment where IEEE 802.11 nodes can be rapidly built from scratch by simply dragging and dropping required components from its different libraries. This is not to forget the flexibility, easiness and the so many options that MATLAB provides as a powerful technical computing language that one can use to create and program new components and functions that do not already exist in the libraries. Moreover, when simulating networks such as those based on the IEEE 802.11 standard, discrete event simulation techniques are the most suitable. SIMULINK provides the SimEvents toolbox, including different types of queues, servers, time-based and event-based entity generators, routing components such as switches, path combiners, splitters,… etc. Such a toolbox is specifically designed for such kinds of simulations.

Using the listed MATLAB/SIMULINK tools along with the SimEvents toolbox, we have constructed the three main components of an IEEE 802.11s-based WMN backbone, i.e. the Mesh Access Point (MAP), the Mesh Point (MP) and the Mesh Portal (MPP). Mesh Client Stations (STA) were not developed as stand-alone nodes since their only use is to generate packets that are going to be sent to the MAPs. Instead STAs were substituted, inside each MAP, by simple time-based entity generators that take as a parameter the different packet arrival rates based on which packet entities are going to be generated. Attributes such as the packet service time, drop time, transmission time, packet size, source and destination were given to each generated packet. At this stage the packet enters a simple FIFO queue. The top of the queue where each packet is supposed to spend its service time as defined in section (V-*B*) is represented by a single server with Time Out (TO) port enabled where based on the drop time attribute value as compared to the service time, the packet will either depart from the server through its output port, meaning that it has completed its service time and therefore got successfully transmitted by the MAP, or simply it will depart from the server's TO port to a packet sink where dropped packets are accumulated. The MP node is pretty similar to the MAP, however, clients do not connect to MPs. Therefore MP nodes do not have integrated packet generators such as the MAPs. Moreover, the MPP is supposed to forward packets to the wired network and receive packets from the wired network and forward them to any destination node inside the WMN in question. However, to prove the efficiency of our routing protocols, it is enough to consider one way communication where packets generated at the MAP level are sent out with an ultimate destination being the MPP. Therefore in this case the MPP would be nothing but a node where statistics such as packet loss percentages and packet end-to-end delays are collected and used therefore to evaluate the network performance. Now of course, MAP-MP communications and MP-MPP communications occur on two distinct channels. In our simulator, since the bit rate and packet size is provided, therefore the transmission time can be easily computed and given to the packet as an attribute. Therefore the channel is represented by just a single server where the packet will spend its transmission time and then depart to its destination. The simulator components are shown in Figure 20.

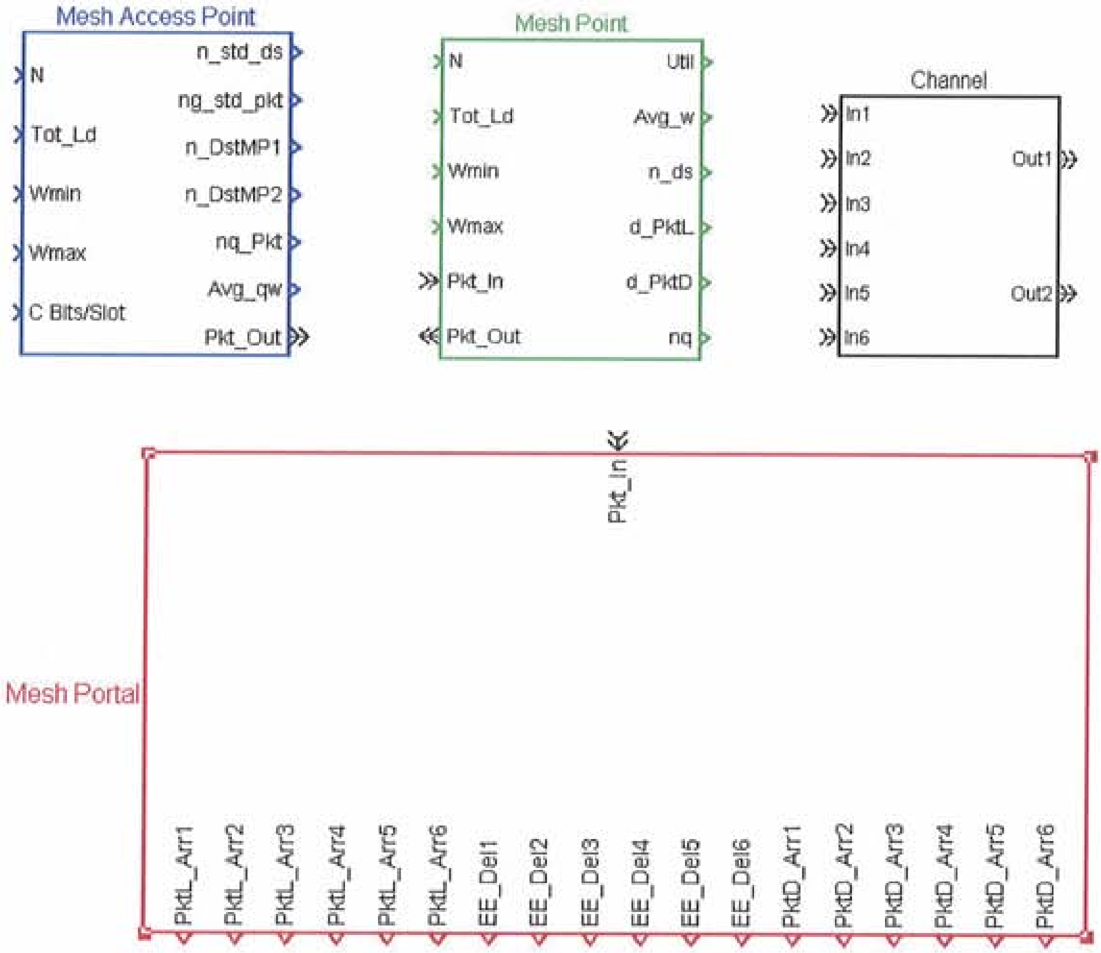


Figure 20: Simulator components as created by MATLAB/SIMULINK 2008.

It is worthwhile mentioning at this point that documentation of this simulator is outside the scope of this work. This is the first beta version this simulator still requires refinement, extension and exhaustive testing tasks that are left as a future work.

7.2 Verification of the Packet Service Time Coxian Modeling

In this section, we are going to verify the Coxian modeling of the packet service time distribution. For this reason, also using MATLAB 2008, we developed a simple procedure that takes as input parameters the probability of collision, the minimum and maximum contention window bounds, respectively 16 and 1024, and the maximum number of allowed retransmissions, $K = 16$, as specified by the IEEE

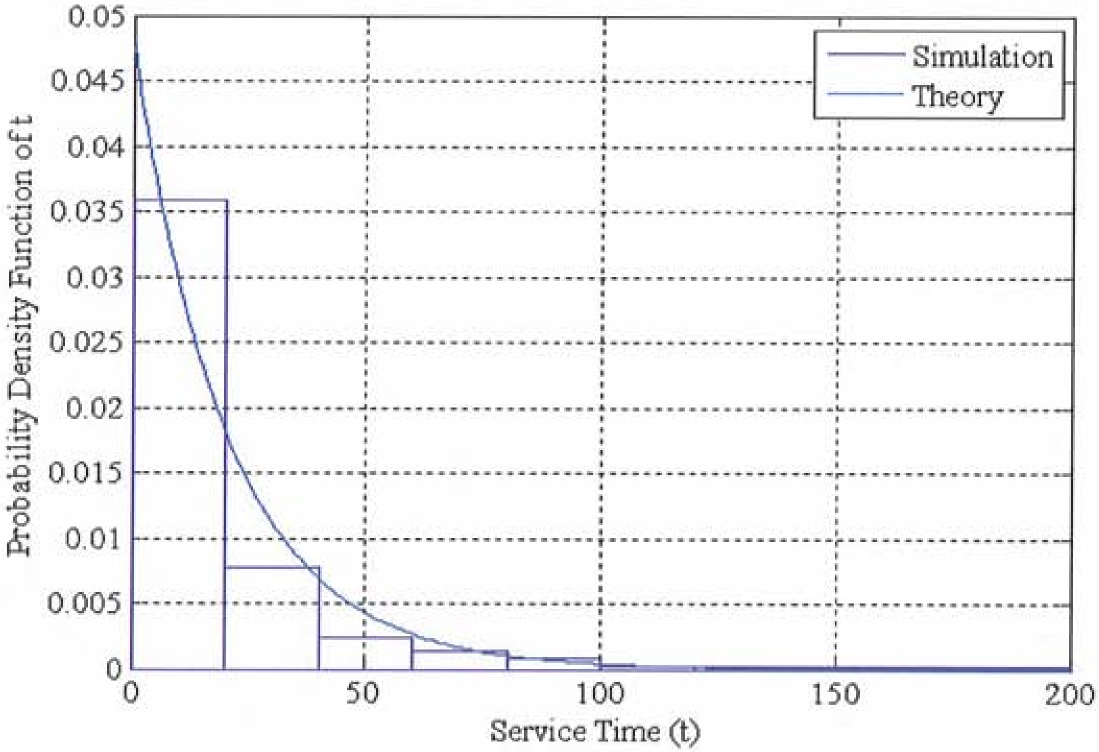
802.11 standard. This procedure will then generate a service time value for one packet following the back-off mechanism that was described in section (V-*A*). This being done, an extension was to generate, for different values of the probability of collision, a sample space of such service times of one million and hundred thousand elements, the first hundred thousand of which were considered as transient elements and were deleted and we are therefore left with a total of one million elements over which the Coxian distribution modeling is going to be verified. Such elements are saved in a vector S. Following the steps described in section (V-*C4*) and using the equations listed there, we computed the SCV and the first three moments of the elements in S and therefore extracted the C_2 parameters which were saved in a matrix. Table 2 shows the resulting parameters for chosen values of the probability of collision ranging between 0.35 and 1.

Table 2: Parameters of the C_2 distribution model.

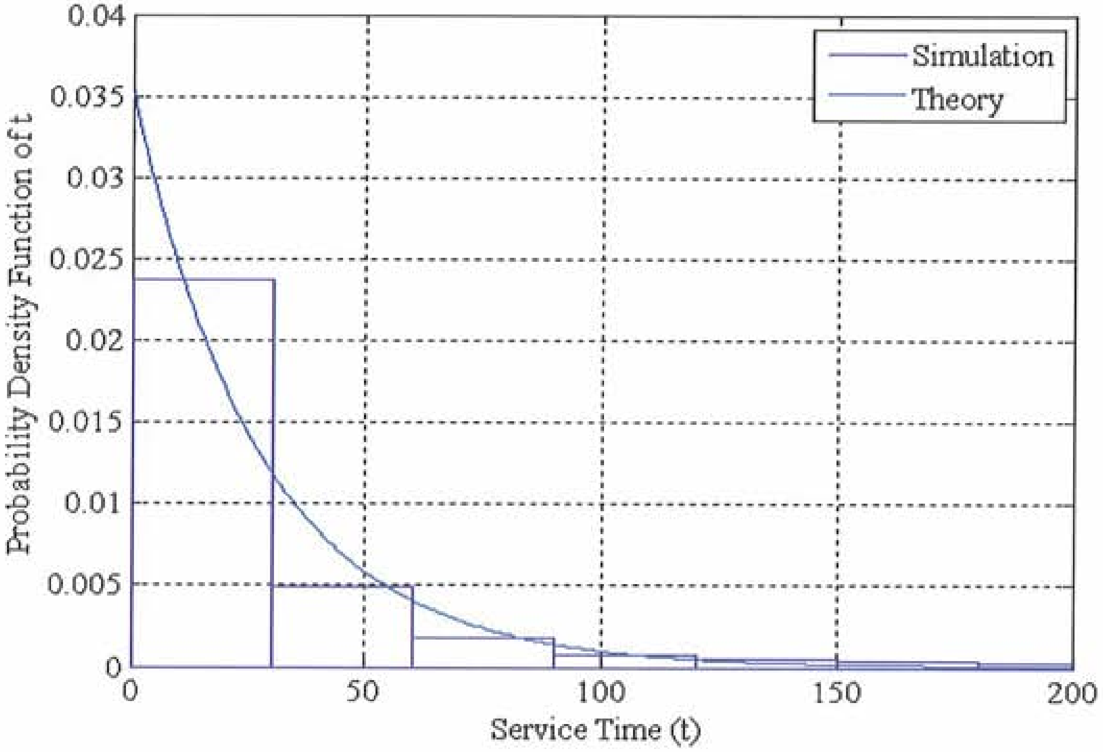
p	0.3500	0.4500	0.5500	0.6000	0.6500	0.7000	0.7500	0.8000	0.8500	1.0000
k	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	36.0000
a	0.0130	0.0287	0.0636	0.1066	0.1597	0.2444	0.1054	0.1517	0.2339	0.0000
$\mu 1$	0.0480	0.0361	0.0268	0.0262	0.0288	0.0522	0.0039	0.0025	0.0015	0.0000
$\mu 2$	0.0023	0.0016	0.0011	0.0010	0.0009	0.0008	0.0004	0.0004	0.0004	0.0000
μ	26.5592	46.1864	93.7682	145.4892	214.6060	328.9781	510.4964	811.8139	1293.1102	0.0002
c^2	7.7151	11.3851	11.1761	9.7415	8.1292	6.3740	4.7444	3.2953	2.1380	0.0283
X	0.0503	0.0377	0.0279	0.0272	0.0297	0.0529	-0.0271	-0.0044	-0.0008	0.0000
Y	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

At this stage, using the distribution fitting tool of MATLAB 2008, we were able to obtain the actual histogram of the elements in S corresponding to the values of the probability of collision in Table III. Each time we would generate a plot of the theoretical distribution as given in the equations in section (V-*C*3) and also using the corresponding parameters in the Table. Collapsing the histogram and the theoretical curves we were able to compare and verify our results which we illustrate in Figure 21.

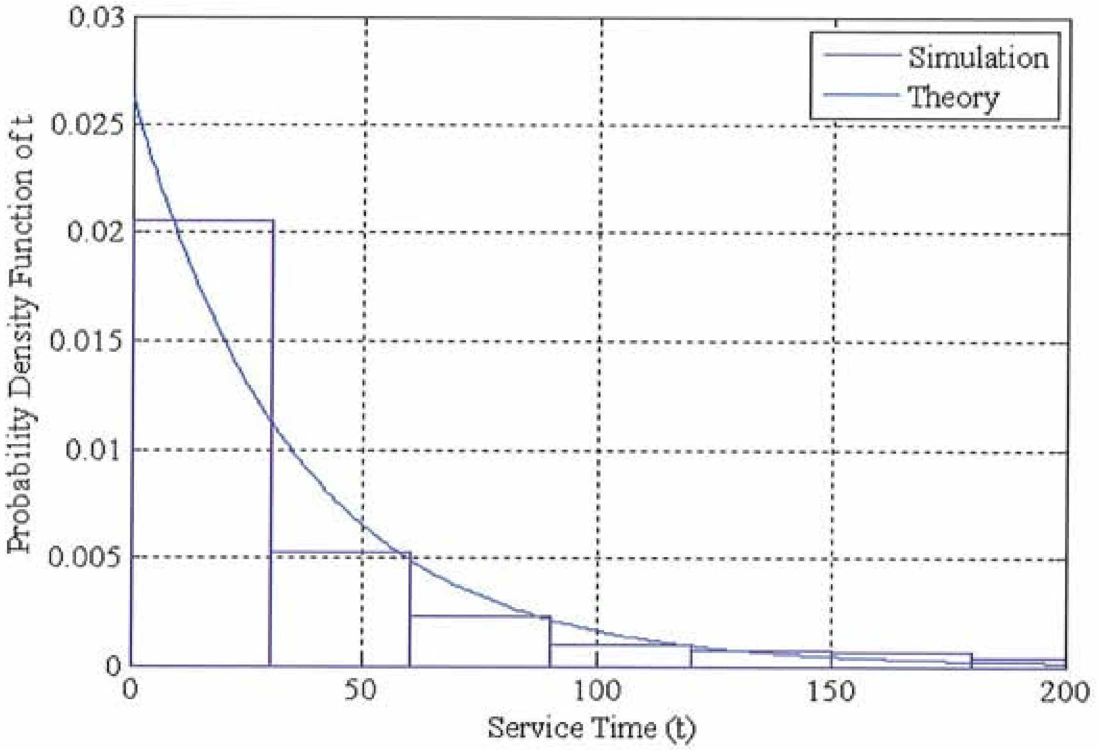
It is worthwhile mentioning, that the collision probabilities in the range [0; 0.3] are less likely to occur. This explains the absence of results corresponding to such probability values in this section. However, it is also true that a probability of collision equal to 1 is also less likely to occur in any real-life scenario of an IEEE 802.11 WMN. However, including the corresponding modeling results was just to show that the service time in this case has an Erlang-36 distribution which is accurately accounted for in our model. Thus modeling the service time using a combination of Erlang and Coxian distributions is accurate and preserves its general property.



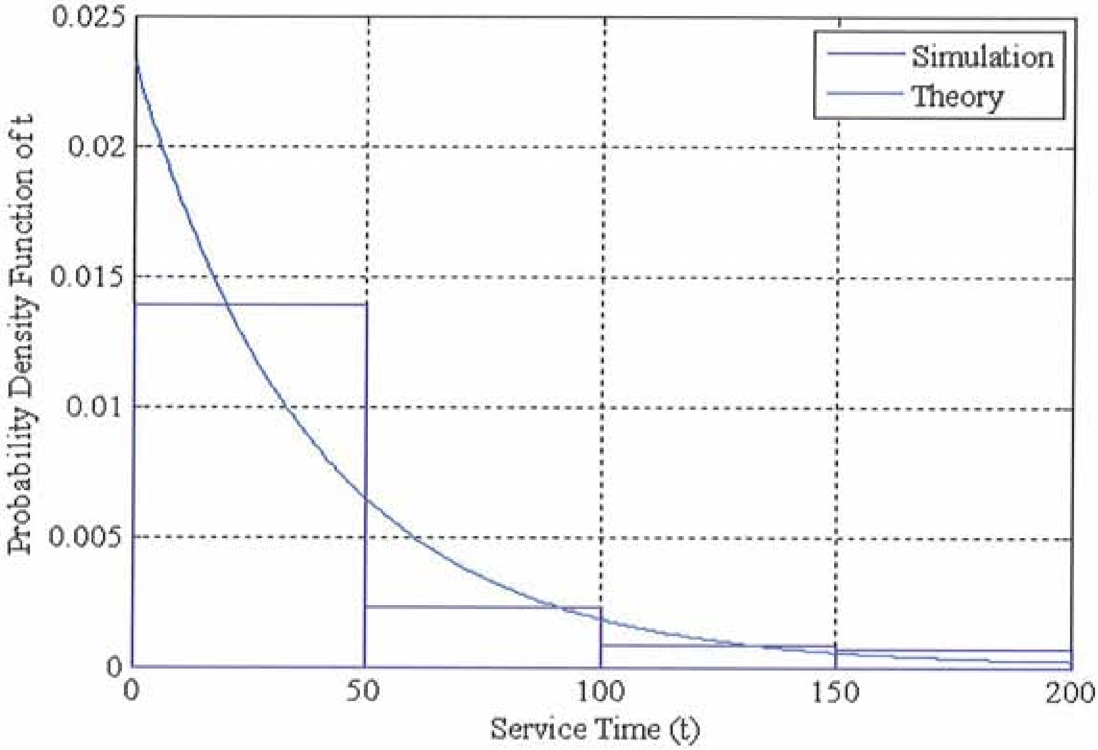
(a) Service time distribution model for a 0.35 probability of collision.



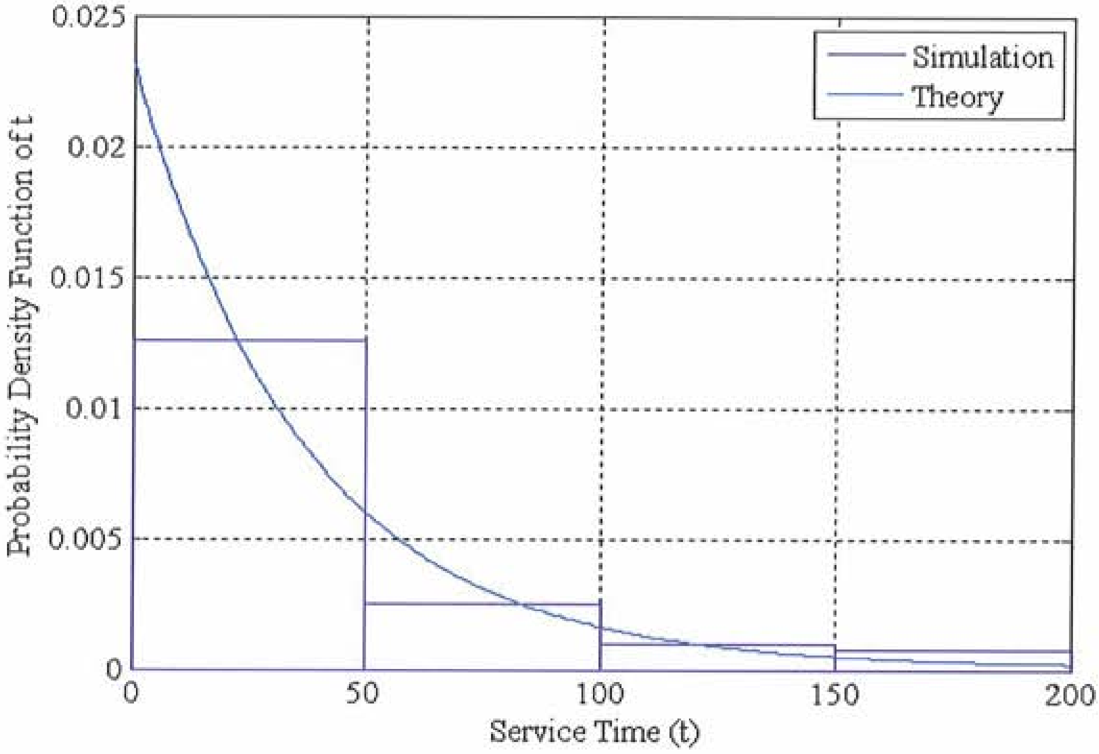
(b) Service time distribution model for a 0.45 probability of collision.



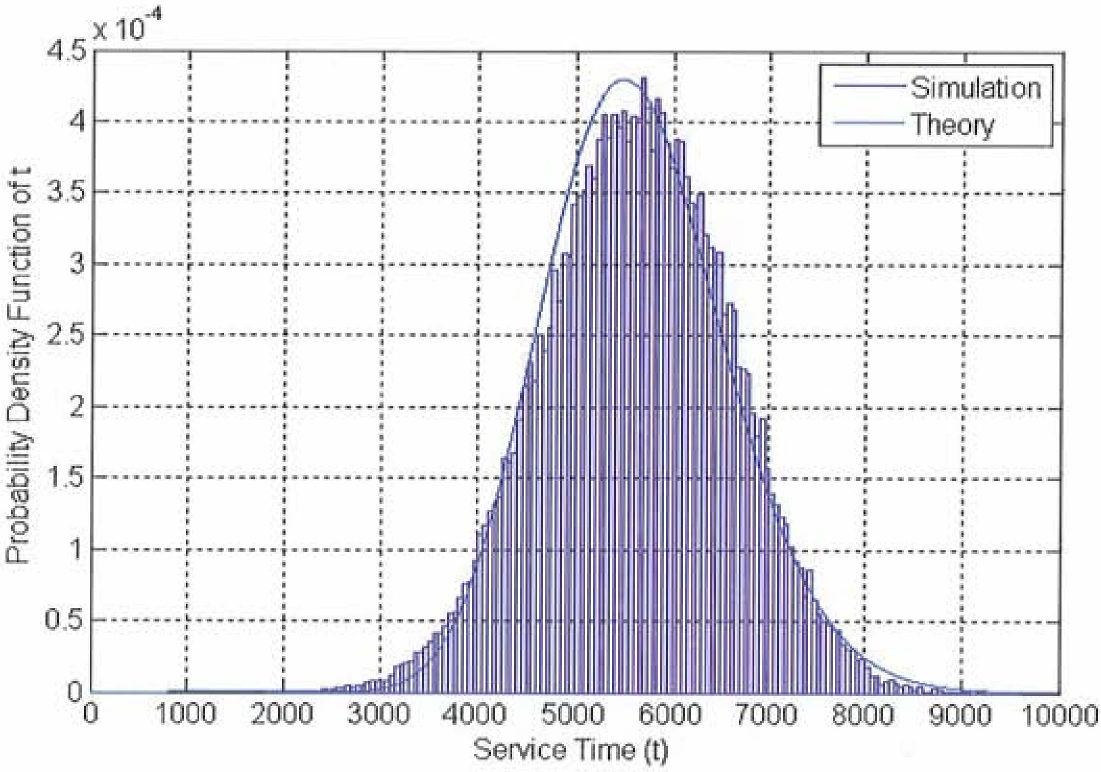
(c) Service time distribution model for a 0.55 probability of collision.



(d) Service time distribution model for a 0.60 probability of collision.



(e) Service time distribution model for a 0.65 probability of collision.



(f) Service time distribution model for a probability of collision equal to 1.

Figure 21: Packet Service Time Distribution modeling results.

7.3 Routing Protocol Simulations and Results

The developed simulator which we briefed in section A of this chapter was used in order to simulate the results and show the effectiveness of the new PoC and NBS routing metrics along with their respective corresponding protocols MACARP and BARP. However, our results cannot be compared to any other results published in the literature since first none of the already published works has tracked the network performance in terms of packet loss probability and end-to-end delays versus network load. Moreover, none of the works had already developed MAC and Buffer aware routing metrics and protocols. Therefore comparing our results to them will be inconsistent. For this reason, we have developed a routing scheme where packets are routed in an arbitrary random manner inside the network and we refer to such scheme as the Stupid Routing Scheme (SRS). The results of our newly proposed routing schemes were compared to those of SRS and conclusions were drawn accordingly.

The SRS scheme was applied using our developed simulator using different runs, each for a certain specific load. The parameters used are as shown in Table 3.

Table 3: Network Parameters

Wmin	16
Wmax	1024
Slot Size	20 *μsec.*
DIFS	50 *μsec.*
SIFS	10 *μsec.*
Bit Rate	11 Mbps
Medium	Wireless

The results of SRS are shown in Figures 22 and 23.

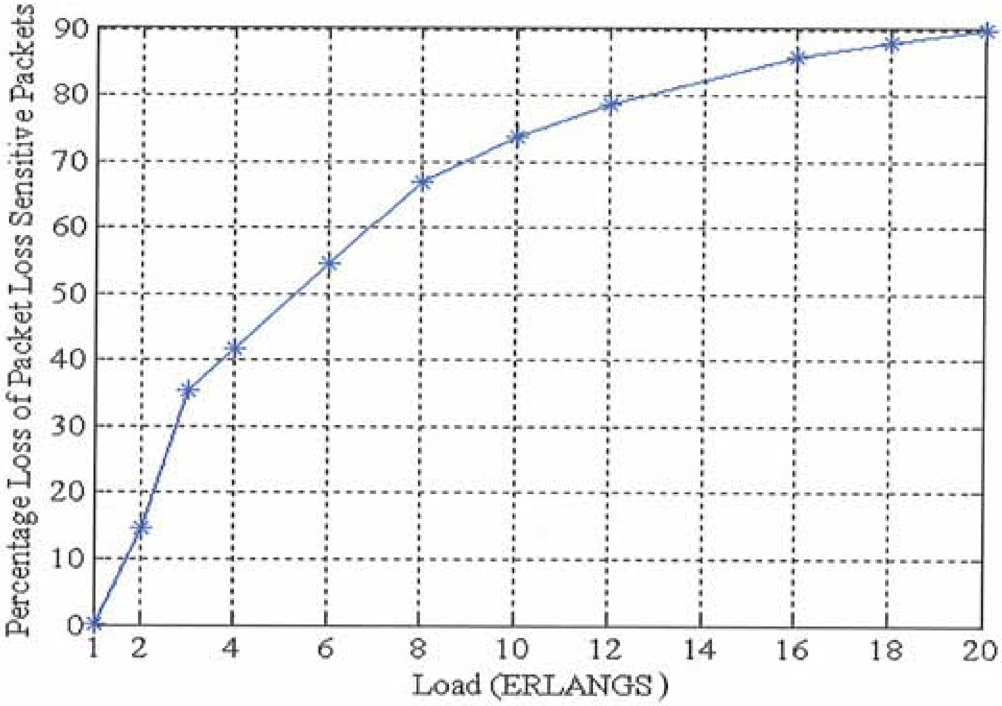


Figure 22: Stupid Routing, Packet Loss Probability VS Traffic Load.

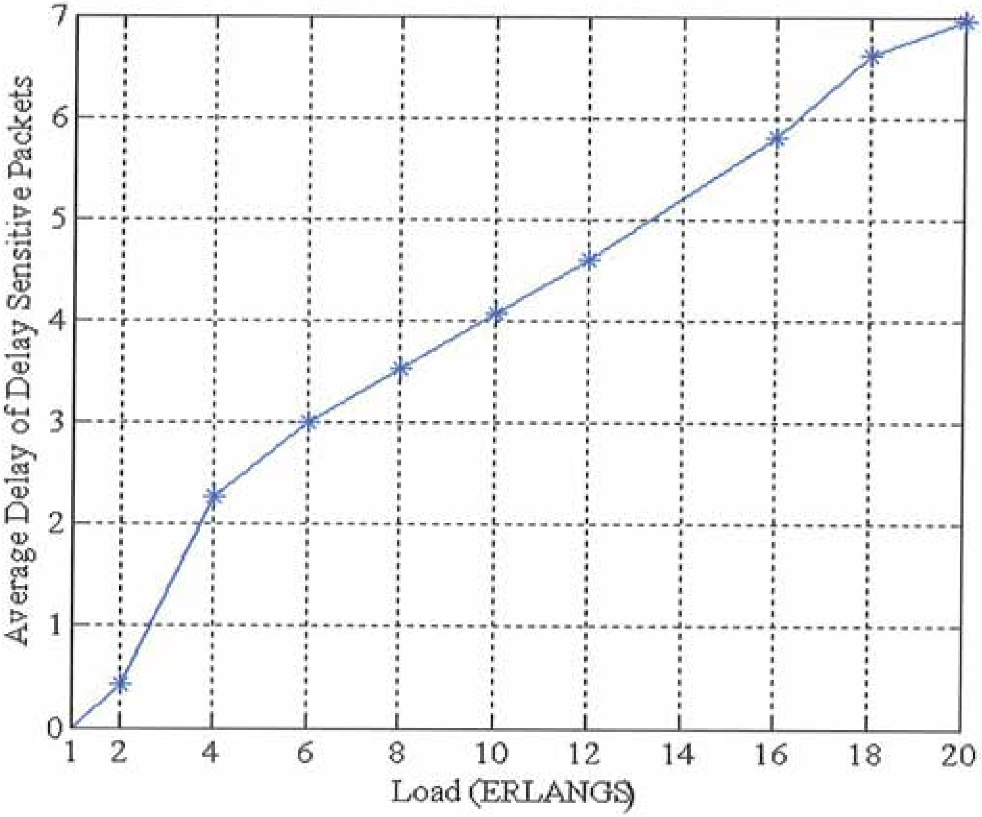


Figure 23: Stupid Routing Average Packet End-To-End Delay VS Traffic Load.

Results of the MACARP and BARP Protocols are respectively compared to those of SRS and shown in Figures 24, 25, 26 and 27.

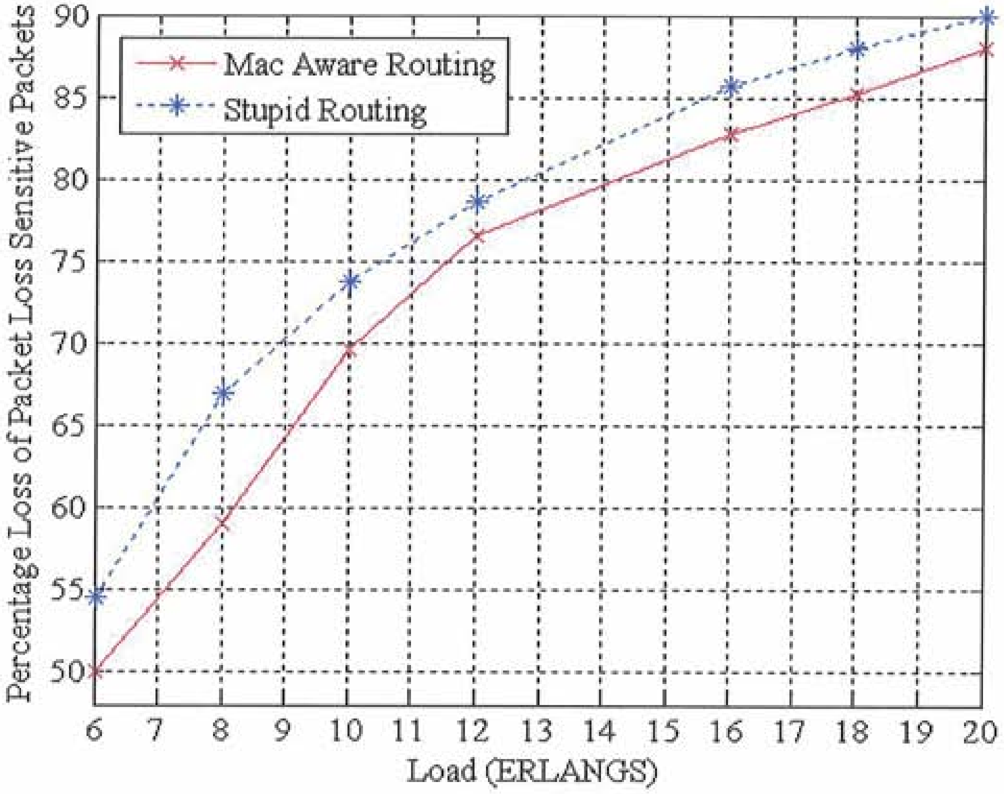


Figure 24: Comparison between SRS and MACARP Loss probabilities VS Load.

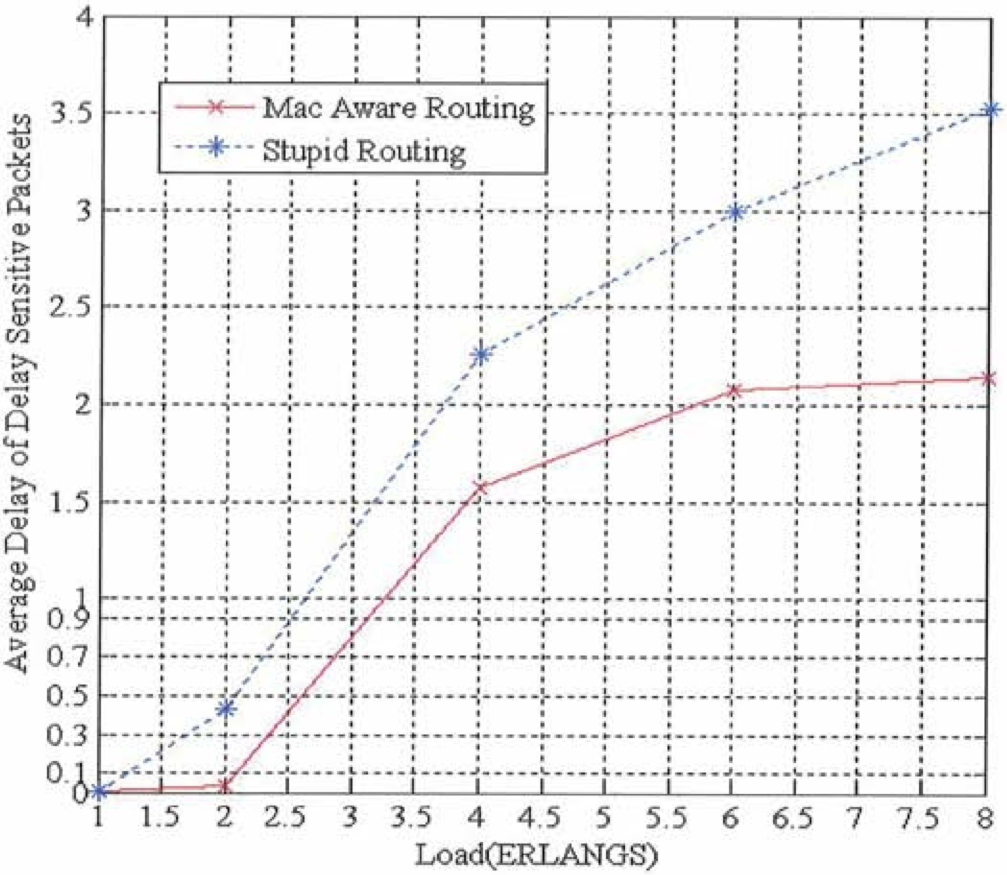


Figure 25: Comparison between SRS and MACARP Average Delay VS Load.

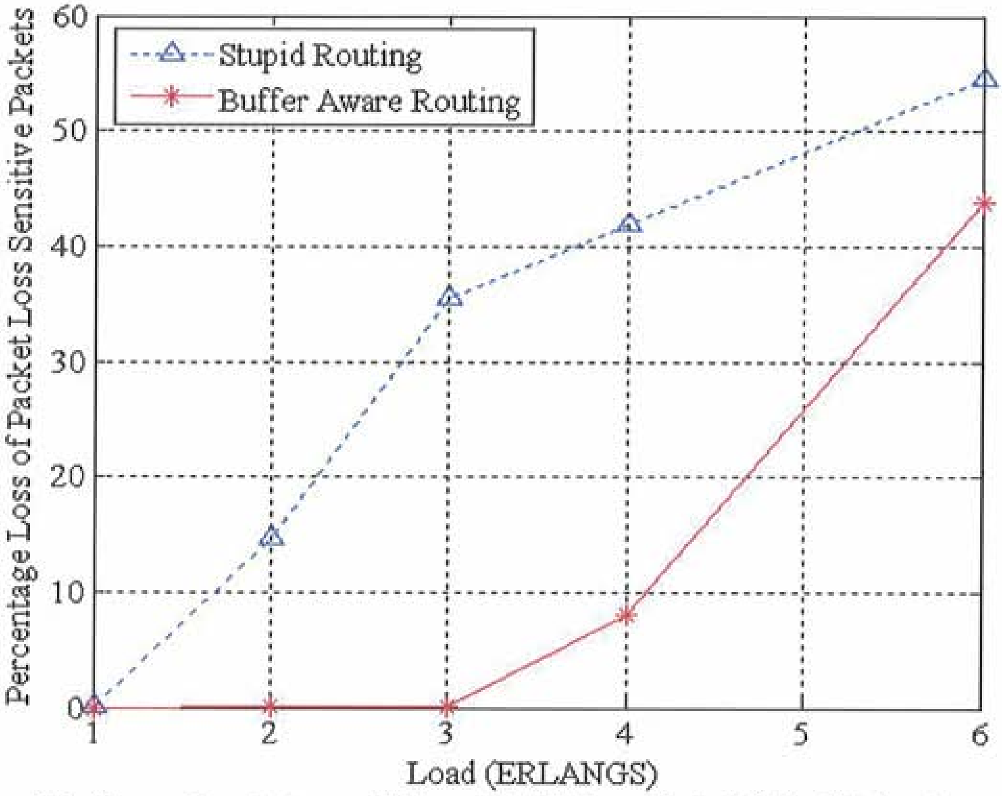


Figure 26: Comparison between SRS and BARP Loss Prob. VS Traffic Load.

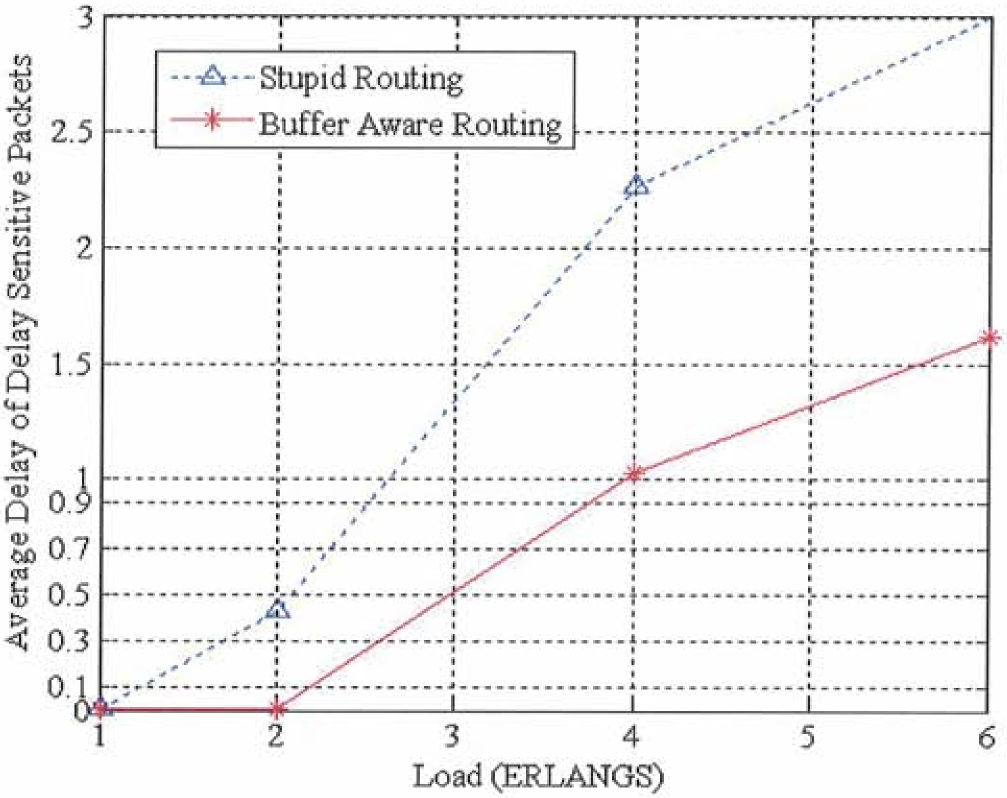


Figure 27: Comparison between SRS and BARP Packet Average Delay VS Load.

Conclusions

The MAC performance is a critical factor that should be accounted for when setting up an IEEE 802.11-based wireless mesh network that supports delay sensitive and packet loss sensitive applications. As a matter of fact, the MAC protocol can easily become the performance bottleneck when considering delays due to contentions and collisions. A key observation is that medium access and reservation, i.e. the packet service time, forms the primary contributor in the packet loss probability and packet end-to-end delay. This work was mainly concerned with accurately model service time probability distribution of packets in an IEEE 802.11 WMN using a combination of Coxian and Erlang distributions. General arrival patterns were considered and an arbitrary number of nodes were allowed. Thus nodes in such networks can now be modeled as $G/C_k/1$ and $G/E_k/1$ queuing systems. In addition, this work was concerned with filling the gap resulting from the non-existence of IEEE 802.11s-specific routing protocols and routing metrics that take into account the special characteristics of such WMNs. The two newly developed routing metrics, PoC and NBS, are essential parameters extracted from the heart of the IEEE 802.11 MAC protocol and there adaptation by the newly developed MACARP and BARP, which are IEEE 802.11s-specific routing protocols, is a major breakthrough in the field of routing metrics and protocol designs for IEEE 802.11s-based WMNs. Finally, a new simulator for such networks was developed which allowed us to verify our models and designs and assert their effectiveness in boosting the overall network performance through extensive simulations.

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