Proportionality in the Sixth And Seventh Grades In The Lebanese Math Curricula

An Investigation of Its Teaching And Learning

by

ZEINA HARIZ

B.S., Math Education, Lebanese American University, 2002

Thesis submitted in partial fulfillment of the requirements for the Degree of Master of

Education/ Emphasis Math Education

Department of Education

LEBANESE AMERICAN UNIVERSITY

February 2009

LEBANESE AMERICAN UNIVERSITY

School of Arts and Sciences

Thesis approval Form

Student Name: Zeina Hariz I.D.: 199904240

Thesis Title : Proportionality in the sixth and seventh grades in the Lebanese math curricula: An investigation of its teaching and learning

Program : M.A. in Education / Emphasis Math Education

Division/Dept : Education

School : Arts and Sciences - Beirut

Approved by:

[redacted]

Iman Osta El Zein, Ph.D.
Associate Professor of Math Education

[redacted]

Samer Habre, Ph.D.
Associate Professor of Mathematics

[redacted]

Associate Professor of Applied Linguistics and TESOL

Date: February, 2009

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Name: Zeina Hariz

Signature: [redacted]

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ACKNOWLEDGMENTS

First I would like to express appreciation to the committee members, Dr. I. Osta for her constant support, encouraging words, guidance during the course of this work; Dr, Habre and Dr. Bahous for their help and for being committee members for this thesis.

Second I would like to dedicate this to my loving family for their support, encouragements, and prayers through all the work. I truly appreciate your understanding when I always needed to keep the laptop on and to work for long hours.

Third, I would like to thank all my friends for their support and help. Thanks for keeping me on track and helping me through this way.

Fourth I would like to thank all teachers and students who showed excitement about participating in this study.

Last and above all, I want to thank God for helping me to overcome all rough periods I passed through and for the joy of all the good times.

ABSTRACT

A relatively large number of sixth and seventh-grade school students have difficulty in using their computational knowledge and conceptual understanding to solve real-life word problems related to proportionality. The purpose of this study is to describe, analyze, and evaluate the teaching/learning of proportionality in Lebanese schools for grades six and seven. More specifically the study describes and analyzes the text material on proportionality in the Lebanese math textbooks and teacher's manuals, as well as their implementation in classrooms. This qualitative study was implemented in three schools in Lebanon following the Lebanese curriculum. Participants included 12 sixth- grade students, 12 seventh- grade students and six teachers, three for each grade level. Data were collected through reviewing the national math curriculum, textbooks and teacher's manual for grades six and seven, interviews with teachers regarding their approaches to teaching proportionality, classroom observations to explore how teachers teach the topic and related concepts, and clinical interviews with students to explore the techniques they use to solve proportionality related problems and the difficulties they face while working to solve them. The major findings of this study are as follows: Math teachers are guided by the curriculum and textbook to implement a constructivist approach to math education. Some teachers do indeed try to implement the techniques needed for this approach. At the same time though, the reality is that several factors prevent teachers from achieving this goal. Until the national textbooks are revised and modified in such a way they heed requirements of a constructivist approach to teaching math in general, and proportionality in particular, it is essential that schools provide supplementary and supporting materials that correspond to the needs of math teachers in each grade level. In addition, clinical interviews enabled the researcher to learn that students whose teachers are using techniques recommended by the constructivist approach to teaching were able to obtain better results when trying to solve proportionality problems. Moreover, it was unexpectedly noticed that there is no major difference between schools with different socioeconomic levels regarding the number of correct solutions obtained while solving proportionality problems.

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INTRODUCTION

This study aims to find out how proportionality and proportional reasoning are being taught in Lebanon. At the same time, it aims to explore students' understanding of proportionality and their ability to acquire proportional reasoning and solve proportionality problems.

The Problem

During the shopping festival in Beirut, a woman was shopping with her son in a period of discounts. Having decided to buy a 50, 000 L.L. T-shirt, the mother asked her son how much she should pay. Though the shop was on as simple as 50% sale on all items, yet the boy was puzzled with the question and was not able to answer. Such an example should draw math educator's attention to what is needed to be taught and emphasized in the classroom; in the middle grades, students don't recognize the importance of mathematics for their future and many real- life situations they face. Proportionality is one of the most commonly needed concepts in daily life. It is the subject of this study.

Curcio and Bezuk (1994) think that proportional reasoning is an important topic for someone to maximize his/her career options. Despite the fact that proportionality is a widely used topic in daily and academic situations, it is observed that students of various age levels face difficulties in solving proportionality tasks. Many factors could cause those difficulties such as lack of sufficient examples and real life problem situations presented in textbooks, or inefficiency of the techniques used by teachers to explain the subject in class. Some of the questions one may ask are: Do teachers stress more drill-and-practice or real-life examples? Do they emphasize memorization and rote application of formulas, procedures, and rules to perform different operations? Are students encouraged to use reasoning rather than direct application when solving proportional problems?

The Lebanese curriculum, reformed between the years 1997 and 2000, lists as one of its goals the development of thinking and reasoning. It advocates that since "there is no divorce between mathematics and everyday life", students' abilities of dealing with real-life situations should be extended (Educational Center for Research and Development ECRD, 1997). The 1997 Lebanese curriculum advocates the connection of mathematics to daily life through improving the formulation of objectives, remodeling the context, as well as reforming the methods of teaching. The present research aims at investigating whether this goal has been addressed throughout the development of the curriculum materials (especially textbooks) and through teaching, particularly for proportionality and proportional reasoning.

Purpose and Research Questions

The purpose of this study is to describe, analyze, and evaluate the teaching/learning of proportionality in Lebanese schools for grades six and seven. More specifically, the study will describe and analyze the text material on "proportionality" in the Lebanese math textbooks and teacher's manuals. The analysis will consider the teaching strategies proposed for presenting the lessons as well as the types of exercises and problems offered. Also, the study investigates the strategies and teaching techniques practically implemented by teachers, the types and sequencing of teaching activities, and their connections to real-life situations. On the other hand, the study attempts to evaluate students' learning of proportionality in grades six and seven.

More specifically, the study aims to address the following questions:

1. How does the Lebanese curriculum organize the teaching/ learning of proportionality, especially in grades six and seven?

2. What types of information, activities, exercises and problems are presented in math textbooks for grades six and seven in chapters on proportionality?
3. What teaching strategies/methods are suggested in textbooks and teacher's manuals?
4. What strategies/methods do teachers actually use in teaching proportionality in grades six and seven mathematics classrooms?
5. Which thinking strategies do students use when faced with real-life word problems related to proportionality?

Significance of the Study

Fisher (1988) contends that the reason for the complexity faced by students in solving proportionality tasks could be the insufficient development of the concept of proportionality in the classroom. Traditionally, proportionality is taught in grades 4, 5, 6, 7, 8 and 9. However, Sowder (2002) states that students are engaged in proportionality situations and activities even before they go to school. For example, they learn how to distribute certain objects like candies among their friends. Students bring those experiences with them to the classroom and connect them to what they learn.

In most of the curriculum, proportionality is dealt with according to two aspects: (a) proportionality as a topic in its own right in the curriculum. For example in the Lebanese math textbook developed by the ECRD (Educational Center of Research and Development), a chapter titled "proportionality" is found in each of grade six and grade seven textbooks, and (b) Proportional situations where proportionality is included indirectly or used as a tool or mode of reasoning in many other contexts, such as in multiplicative situations or speed-time-distance situations. Post (1989) explains that fractions, decimals and ratios play an important role in the development of proportional reasoning, which is an important intellectual topic in the secondary school. Proportionality is an essential topic that is related to several domains; for instance, math problems involving proportionality can constitute a big part of science as well as math textbooks. Rates can be found in most aspects of life including cooking, navigation, physics, earth science, economics, electronics, business, and industry. Therefore, students' ability to efficiently understand and use proportions is a major concern for the science and math educator. Yet again, little is done within the classroom to increase this proficiency.

Proportionality in the Lebanese curriculum is explicitly used in many subjects other than mathematics. For example, in physics proportionality is integrated in gravity, density, energy, speed; and in chemistry, it shows in pressure, density, molarity, chemical compositions, and saturation problems, just to name a few.

As reasoning in school mathematics topics like proportionality, ratio, and rate has proved to be problematic for a large percentage of students as stated by Wanda (2002), the determination of possible ways of successfully teaching the concept is of utmost importance. From early to mid-adolescence, students surprisingly "lack the ability to reason and to adopt methods for solving proportional tasks" (Bankov, 1999, p. 78- 86).

To recapitulate, numerous reasons could be tagged to why this topic is worth to be chosen: (a) proportionality is ranked as one of the most difficult math topics for teachers to teach and students to learn as Crucio and Bezuk (1994) state, (b) students' most common weaknesses in mathematics is proportionality and solving problems related to this topic, (c) Knowledge of proportionality is important in many other mathematics domains; thus, if students face difficulty in this topic, the difficulty will continue to further gaps in related topics and even other related disciplines, (d) the weakness students suffer in this topic is linked to many reasons, one of which could be the weakness of teachers' approaches in teaching basic concepts or computational methods to solve related problems, and (e) another reason could be the way mathematics curriculum and textbooks present the topic.

The results of the study could contribute to raising educators' and teachers' awareness of strengths and weaknesses in the teaching/ learning of proportionality and to the enhancement of the lessons on proportionality in the Lebanese mathematics curriculum. It may also improve the (a) teaching of proportionality, so teachers would use more effective approaches for teaching; and (b) learning of proportionality for better understanding and application in real-life problem situations.

On the other hand, this study is significant as much as it will contribute to the literature on the teaching and learning of proportionality, especially in the Lebanese context.

Definition of Terms

This study adopts the following interpretations of terms:

1. *Proportionality*: According to Rinehart and Winston (2007), a proportion is a statement of equality between two ratios. Symbolically, this is represented as $a: b = c: d$. In the Lebanese curriculum, proportionality is divided into five sections: Grade six includes percentage, rates, proportional sequences, and scale; while grade seven includes directly proportional magnitudes. Also, embedded in proportional reasoning in grade seven are the concepts of literal fractions and multiplicative relationship, which embraces proportionality in terms of equations in the form of $ax = b$.
2. *Proportional reasoning:* McLaughlin's (2003) definition of proportional reasoning will be adopted. "Proportional reasoning is the ability to compare ratios or the ability to make statements of equality between ratios." (p. 1) Particularly, the recognition of

proportional quantities in a situation is considered in this study as an important component of proportional reasoning.

3. *Math textbooks*: "Building Up Mathematics", the Lebanese national textbook for the official curriculum is adopted as the basic textbook in this study.
4. *Real-life problem situations*: Word problems that represent possible real-life situations that require proportional reasoning.

REVIEW OF LITERATURE

This chapter starts with a brief overview of basic notions of proportionality and proportional reasoning. The following section is dedicated to experts' explanations of difficulties that students face when learning proportional reasoning and their ideas as to what teachers can do to help students overcome these difficulties. The third part concerns experts' opinions regarding difficulties students face while solving proportionality problems and what can be done to avoid them. The last section focuses on the constructivist approach, as the most recommended approach to enhance teaching and learning of proportionality.

On Proportionality and Proportional Reasoning: Basic Ideas

Fractions, percents, ratios and proportions, all indicate a situation of proportionality. They all refer to a situation where two quantities (variables) are related to each other in such a way that if one varies then the other varies in a manner dependent on the first, in the relationship $\frac{A}{B} = k$ where k is the constant of proportionality. In such a case, one is talking about direct proportion. However, if, when one quantity increases, the other quantity decreases, one is talking about two quantities A and B that are inversely related i.e. an indirect proportion. The relationship would then be $A \times B = k$. The present study is only concerned with direct proportionality.

Ben-Chaim, Fey, Fitzgerald, Benedetto, and Miller (1998) determine three types of direct proportionality: (a) Comparison of two parts of a whole like ratio of boys to girls; (b) Comparison of magnitude of different quantities such as miles per gallon; and (c) Comparison of magnitude of different quantities that are related in a way like ratio of sides of a triangle.

For a student to understand proportionality and the related concepts, it is essential that s/he be able to undertake what is referred to as proportional reasoning. Ben-chaim, Illany, and Keret (1994) define proportional reasoning as the heart of mathematics. It involves different mathematical relationships that are multiplicative in nature. Norton (2005) and Ben-Chaim et al. (1998) define it as an approach used to describe the concepts and thinking required to understand rate, ratio, and proportionality including scale.

Children are able to grasp some aspects of proportionality from an early age. The investigation of how children understand proportionality by studying their reaction to several situations such as equilibrium and shadow size reported in Piaget and Inhelder (1958) is evidence to this reality. Furthermore, Piaget, Inhelder, and Szeminska (1960) show the developing ability of young children to partition different parts, lengths, and regions like dividing a pie into parts. Moore, Dixon, and Haines (1991) state that children's capacity to undertake proportional reasoning is nearly perfectly mastered by grade six.

Despite such encouraging observations however, the fact of the matter is that many students often face difficulties when learning proportionality and related concepts. Consequently, they find problems that involve using fractions, ratios, percentages and other proportional reasoning among the toughest to solve.

Difficulties in Learning Proportionality

Some experts have theorized about reasons behind difficulty in learning proportionality in general. The majority stipulated that students find proportionality and related concepts difficult to learn because of difficulties in understanding relationships.

Hasemann (1981) explains that it is difficult to teach the topic of proportionality in mathematics, and it is hard for students to learn this topic because students simply do not understand the relational aspect or the idea of it. He adds that students should deal with problem-solving to understand the topic clearly and be able to apply it in appropriate situations; otherwise, it will be difficult for them to apply it correctly. Children at many points can identify examples related to proportionality, but they cannot expand their ideas to real-life situations. As a result, the difficulties that students have with proportionality concept are theoretical.

According to Hiebert (1984) students' inability to understand is the main reason for many misconceptions in proportional reasoning. He talks about understanding as an instinct that helps students see how mathematics works. Understanding, in many cases, follows from daily experiences like sharing candies equally among friends or from school instruction like dividing money among people. Therefore, understandings are ideas that students have about mathematics that make sense to them. According to Hiebert (1984), it has been proven that students acquire much knowledge from understanding independently; thus, they cannot see the relationships between different ideas unless they understand them. Arguing along the same lines, Hiebert and Wearne (1985) explain that students fail to understand the proportionality concept when they lack real life application related to the topic.

In a study about the role of intuition in mathematical understanding, Moore, Dixon and Ahl (1992) explain students' behavior during temperature-mixing tasks when there was a variation of the presentation order of the intuitional versions (temperature presented pictorially as cold, hot etc.) and numerical versions of the tasks. They found that students can understand that when one adds hot water to cold water, the higher the temperature of the added water, the higher the overall temperature will be. Yet, they often cannot express this relation mathematically because they cannot turn it into a computational scheme. This shows that intuition plays a major role even in analytical understanding and can affect a student's ability to solve mathematical tasks. They contend that even incomplete intuitional understanding has an impact on students' ability to reach the correct answer of numerical task. In their opinion, this could explain difficulties students face when learning proportional reasoning. It is important that teachers realize that often when students cannot solve proportionality problems this could be very much that they understand the proportional relations intuitively, but they are not able to translate them into mathematical relations and schemes.

Recently, Norton (2005) following a study of student's inability to understand proportionality concepts, found that some students face difficulties in solving proportionality problems due to using subtractive and additive thinking instead of multiplicative thinking.

Along with theorizing about students' understanding of proportionality in general, a number of experts focused on studying students' difficulties in understanding the specific concepts of proportionality i.e. fraction, ratio, percent, and proportion. Below is an overview of some of their major findings.

Bigalke and Hasemann (1978) focus on fractions. They show that middle school students face difficulties with learning fractions, and proportionality in general, because: a) they do not use this topic in real life; b) fractions and proportionality are written in a complicated way; c) it is difficult for students to plot fractions in order on the number line; and (d) the rules required for calculating fractions are complex; hence, they are most often used mechanically without thinking or understanding the meaning.

Smith (2002) argues that when it comes to fractions i.e. when a number compares part of an object with the whole and is represented in the form of $\frac{a}{b}$ where "a" is the numerator and "b" is the denominator $\neq 0$, students make meaning of them by connecting to divided quantities.

According to him, students face a lot of problems when dealing with fraction equivalence, fractions that represent the same relative amount.

Smith (2002) also focuses on children's understanding of ratios which he defines as a relationship between two quantities expressed as the quotient of one divided by the other: for example the ratio of x to y is written x: y. It relates two quantities in one situation. In his opinion the difficulty in understanding this concept is due to the fact that the word "ratio" in everyday language is inaccurate, and its use in mathematical language can be imprecise and lead to serious errors. He argues that fraction-related misconceptions that some students have (e.g., 2/3 means 2 wholes divided into three parts) are a result of situations where the teacher does not use clear and consistent fraction language in the classroom.

In a study about young children's perception of proportionality, namely the ability to identify correspondences in spatial ratios, Sophian (2000) finds that children base their spatial proportionality judgments on relational information. She also argues that young children notice the ratio difference through visual discrimination without studying the idea of proportionality. For example, in the above mentioned study, children were able to discover the ratio by matching pictures that exaggerate the difference in area rather than thinking of it proportionally or explaining it.

Focusing on difficulties while learning percentages, Parker and Leinhardt (1995) argues that percentage is present in everyday life, in newspapers, in magazines, in the evening news, and in everyday business, and thus it is important to include it in the mathematics curriculum. To understand why students often find it hard to learn, however, it is essential to be aware of the following facts. Percent is a very old concept that has been in use since ancient civilizations, namely Chinese trading practices and Greek proportional geometry, the language of percent has misleading additive terminology for multiplicative meanings, and has multiple uses for the preposition "of".

Studying students' difficulties in learning percent several years later, Moss (2002) starts with a definition of percents as one part in a hundred; i.e., it is a fraction whose denominator is 100. In his opinion, students have a specific way to understand percents by classifying them into four parts: (a) 100 % is everything, (b) 99% is almost everything, (c) 50 % is exactly half, and (d) 1% is almost nothing.

Among researchers who attempted to explain difficulties students face while learning proportionality, a number of experts saw that the problem could also be in the teacher or the approach s/he is taking to explain the topics.

Fisher (1988) and Sophian (2000) argue that the problems faced by students in learning proportionality could be due to inappropriate strategies used while explaining the concept. Many teachers claim that students find it difficult to grasp or set up the formula while in reality, they find it difficult to explain proportionality and related concepts to students. While other experts like Fisher (1988) emphasize that teachers should learn to engage students in activities that allow reasoning of the proportionality concept in real-life situations.

Reasoning along the same lines, Howson (1981) emphasizes that teachers should not be servants to books. They should be creative and innovators of their own methods and strategies used in the classroom. If teachers use other material, it should be new and innovative. Schools can help teachers by introducing them to a new method, implement it for certain time, and then try to evaluate it.

Gore (2004) advises teachers to select different teaching and instructional methods that reach students and increase their level of learning and understanding. For example, to teach proportionality to grade 6, the teacher may explain the lesson beginning with real life examples about proportionality while in grade 7, a teacher might begin with a lecture explaining what proportionality is and state its definitions. The teacher could also use different interactive strategies such as cooperative learning. S/he could also resort to providing examples, using visual representations, or asking questions.

Inability to Solve Proportionality Problems: Some Reasons

For Freudenthal (1983) there are three categories of proportional reasoning problems: (a) Comparing two parts of a single whole, as in the 'ratio of girls to boys in a class, (b) Comparing magnitudes of different quantities with an interesting connection, as in 'miles per gallon', or 'people per square kilometer, and (c) comparing two whole quantities not parts, i.e. scaling, for example, the ratio of sides of two triangles.

According to Cramer, Post and Currier (1993), there are three groups of problems related to proportionality: (a) problems with a missing value, where three parts of data are given and the student has to find the fourth part, and (b) all four parts of the information are given and the students should compare ratios and rates, and (c) qualitative comparison problems where comparison is not related to numbers.

Experts have reflected about the recurrent phenomenon of students not being able, or at least finding it hard, to solve proportionality problems. Major reasons that have been identified to explain this reality are summarized below.

According to the National Assessment of Educational Progress (NAEP) (1973), generally speaking, the problem is that children tend to discard their analytic approaches by just memorizing rules and algorithms to solve any problem. They also underscore that the first and most common error regarding fractions is that only few students can correctly add fractions with different denominators. For Brown and Burton (1978), the problem is that children tend to memorize steps and procedures for solving problems and apply procedures that are not suitable for the problems given.

Hasemann (1981), however, observes that the problem is that students are skillful in following rules like addition and subtraction of whole numbers, yet they do not make sense of the rules. As a result, students can neither apply the rules in slightly different contexts nor apply different rules in the same context. He emphasizes that the problem is that students do not connect the rules to understanding that gives them meaning. In his opinion, the bulk of students memorize rules to find a solution to a problem, yet they are incapable of understanding the use of the rule and whether it works or not in a certain context. Students use fraction rules in a correct, easy and clear situation; however, they lack the exact understanding of the idea of fractions, which proves that students' understanding of the concept is "instrumental" rather than "relational". Reasoning along the same lines, Hiebert (1984) states that children simply do not connect the rules and concept they learn with real-life situations since such meaningful problems are not presented in the classroom teaching.

In a more recent study about children's difficulties with proportionality, Norton (2005) found that some students face difficulties in solving proportionality problems due to using subtractive and additive thinking instead of multiplicative thinking. He also adds that problems encountered while students solve proportionality problems are related to the different strategies that students use to solve them. Those strategies are (a) calculating external ratio such as price per unit or unit per price and compare ratios; (b) calculating internal ratio; (c) comparing price of same quantity by finding a common factor such as price per unit; (d) comparing amount of the same cost by finding a common factor such as the unit per price; (e) building up strategy; (f) looking at ratios of differences between the same variables; (g) responding to the numbers but not the context of a given problem; (h) relating to one variable by ignoring some data; and (i) finding effective responses to numerical data and questions.

Arends (1994) gives some recommendations for teachers to help students solve proportionality problems without difficulties. These recommendations are summarized as follows:

1) Use a direct instruction method that is controlled by the teacher and that promotes attainment of knowledge through step-by-step instruction. It is divided into five steps: (a) stating the objectives of the lesson; (b) organizing the lesson and giving examples; (c) helping students understand by practicing the information independently while monitored by the teacher; (d) giving feedback to students; and (e) encouraging practice.
2) Use simulation where students play roles of real-life situations to understand a certain concept or skill. An example could be monopoly. A teacher who uses this approach, (a) begins the lesson by explaining how the strategy will go; (b) teaches the students rules and goals of the strategy, (c) gives feedback and clarifies misconceptions, and (d) analyzes the experience and connects it to real life.
3) Emphasize concept thinking that helps students discover different concepts and develop advanced level thinking skills.
4) Emphasize discussion that can lead to many different opinions, answers, and sharing ideas about a certain problem because discussion is an open atmosphere for communication where students can ask, answer questions, and express their own ideas.
5) Resort to cooperative learning where students work in small groups.

6) Use problem-based instruction where students are presented with real and meaningful problems to examine and investigate for a certain amount of time in order to reach a conclusion.

Ben-Chaim et al. (1998) contend that math teachers should pay attention to the fact that the way problems are sorted can enhance students' understanding of the concept. They should also keep in mind that it is better to begin with concrete material rather than verbal textbooks, and then give students symbolic representations such as $\left(\frac{a}{b}=\frac{c}{d}\right)$.

Weinberg (2002) recommends that math teachers of proportionality always teach students different ways of reasoning. He asserts that it is important to aid students through many ways of solving proportionality problems related to real-life situations. In his opinion, this will make them (a) focus attention on connectivity to real life, (b) connect different concepts and skills in math, and (c) reinforce themes of problem solving, reasoning, and communication by identifying the strength and weakness of each method. This is in accordance with Wollman-Bonilla's (1997) recommendation to challenge students to reapply, modify, and set up their own formulas to solve proportionality problems.

In addition, Cantenbury (2007) states that teachers should always keep in mind that students need their help to use the right representations while attempting to solve a problem. This can be achieved through (a) encouraging students to use their own representations to communicate their mathematical thinking; and (b) helping students to understand and develop meaning of their own representations. Hence, teachers should supply students with a wide variety of representations in the classroom and help them recognize those that are suitable for problem solving. Huinker (1998) and Kamii and Clark (1995) add that students have fractions as diagrams and algorithms in their mind. Therefore, teachers should be careful that students mostly depend on the visual representation of a specific idea rather than on their operational knowledge of it. In order to be able to understand proportionality and related problems, students must be assisted to reach a point where they can begin reasoning with those visual representations to reach a stage where they can think more abstractly. Students at this level can solve problems, simplify and explain their mathematical parts or relate them to real world.

Recommendations to Enhance Teaching and Learning of Proportionality

Ben-Chaim et al. (1998) propose the Connected Mathematics Project (CMP). The idea is to develop students' knowledge and understanding in mathematics using a new curriculum that has rich connections with the core ideas of mathematics. It provides teachers with different strategies to explore proportionality at different grade levels. CMP encourages students to develop their own understanding and procedures, while the usual curriculum asks the teacher just to show a sample of a problem to each student before solving it. These same authors also underscore that more proportionality problems should be treated by the curriculum. They explain that there should be more extensive instruction in grades six and seven since students need more time to discover different formulas and strategies to solve different kinds of problems on their own.

Most experts, however, advocate a constructivist approach to education, including teaching mathematics. They are influenced by the philosophy of the social constructivist Vygotsky (1978) who states that a person should trust that cognitive learning originates from social interactions. For this school of thought, a learner constructs knowledge socially, in interaction with their environment. Clements (1997) underscores that constructivism assumes that learning entails self-instruction and that engagement in learning is a request for meaning. In other words, the value of constructivism lies in the fact that it is an approach that focuses on the learner rather than on the teacher. It allows students to be involved in their learning and helps them think at more critical levels. Woolfolk (2008) explains constructivists' view that all higher mental processes are first co-constructed during shared activities between the child and another person.

Experts have emphasized the importance of adopting a constructivist approach to math education since the middle of the twentieth century. For Polanyi (1958) constructivism serves as an aid for mathematics education. It is an approach to education that helps educators to view math from different perspectives, namely the eyes, mind, and hands of the child. It was not before the mid-1980s that teachers began agreeing on the constructivist view by placing the attention on the students' experiences rather than on the teachers' knowledge (Sharp & Adams, 2002). This change paved the way to classrooms where students are actively engaged in learning, and it came after many experimental studies and theoretical work in the education field that showed experts that constructivism is an important trend in teaching and learning mathematics (Simon, 1995).

The power of constructivism for mathematics education lies in the principle highlighted by von Glasersfeld (1984; 1987) namely that individuals constantly shape the knowledge they form (Jaworski, n.d.). According to her: "… we can *only* know what we ourselves have constructed, and modified according to further experience" (Jaworski, n.d., Definition, para. 1).

Nobles (1990) explains that constructivism in mathematics plays an active role in students' learning as they build their own knowledge and permits the teacher to follow up on students' thought processes by listening to them and triggering their meta-cognitive thinking.

For Von Glasersfeld (1995) constructivism offers teachers an approach to teach mathematics in order to solve the problem of underachievement and unsatisfactory performance in some math topics. It is an invitation for teachers to emphasize students' conceptual development through several activities to provide them with an epistemology to improve their learning. Those activities will help the students expand their conceptualization skills as they try to find a solution for a problem.

According to Draper (2002): "Constructivism as a theory of learning can provide the framework needed to help math teachers move from a transmission model to one in which the learner and the teacher work together to solve problems, engage in inquiry, and construct knowledge"(p 521).

Furthermore, the National Council of Teachers of Mathematics (NCTM) (2000) underscores that the importance of the constructivist approach to teaching mathematics stems from its capacity to help students solve the kinds of problems they will face in the future. "Students must learn mathematics with understanding by actively building new knowledge from experience and prior knowledge" (NCTM, 2000, p 20).

Constructivism remains the most recommended approach to education to this day. Although efforts to prove it through studies have receded, there are scholars who still make it a point to highlight its effectiveness in the teaching and learning of mathematics. In a recent study concerning mathematics and learning difficulties, Confrey and Kazak (2006) have underscored that constructivism has a remarkable effect on mathematics education, since "… it has propelled children into the forefront of activity … [and] documented a number of substantial considerations of student thinking about which all teachers need to know" (p.335), and it has produced many useful actions that led to the incorporation of technology enhancing the outcome of teaching mathematics.

With this observation about constructivism and math education we conclude this chapter.

Before proceeding to present the findings of the study, the following chapter is dedicated to identifying the methodology and units of analysis i.e. how the author investigated the extent to which the constructivist approach to teaching is implemented in the Lebanese math curriculum, textbook, and classrooms of grades six and seven.

METHOD

Triangulation of methods was observed in this study about the teaching and learning of proportionality in grades six and seven in three schools in Lebanon from a constructivist perspective. In order to enhance reliability and validity, several qualitative tools were used to collect data needed to answer the different research questions that this study adopted. Along with analyzing the content of the national math curriculum, textbook and teacher's guide regarding proportionality, the study entailed conducting interviews with sixth and seventh grade teachers, classroom observations of teachers teaching proportionality in these grades, and conducting clinical interviews with students while solving problems on proportionality. This chapter is dedicated to each of the methods used along with identifying the instruments used to collect data.

Participants

Three different schools participated in the study. The schools were chosen according to three different socioeconomic levels, based on the yearly fees of students' registration in each school. The schools are the following: (a) a public school with low socioeconomic level (S1), (b) a private school with middle socioeconomic level (S2), and (c) a high socioeconomic level private school (S3).

The study was conducted in six intermediate-level math classes of grades six and seven, two classes from each school. The six teachers of those classes and their students in the six classes are part of the participants of the study.

Analysis of the Curriculum, Textbooks, and Teacher's Guide

Given that this study seeks to investigate the content of Lebanese math curriculum and textbooks when it comes to teaching proportionality, part of this study entailed analyzing:

- The chapters related to proportionality in the national math textbooks for grades six and seven focusing on the topics presented, representations used, and problems offered (Appendix A).

At the same time, investigating how proportionality is being taught in schools of Lebanon entailed analyzing the content of the teacher's manuals to examine the teaching methods advocated and activities suggested for teaching proportionality (Appendix B).

Interviews with Teachers

Semi structured interviews is among the most useful tools in a qualitative study when the researcher is seeking to explore opinions of a target group. An interview was conducted with each of the six participating teachers to investigate their perception of the topic (proportionality) and its teaching. The interviews were conducted in the teachers' staff room of each school. Each interview consisted of seven open and semi-open questions (See Appendix C) and lasted for a maximum of ten minutes. The interviews were audio taped.

Classroom Observations

In order to analyze the teaching strategies and techniques that math teachers of grades six and seven use while teaching proportionality, the researcher conducted classroom observations. Observation took place in each of the three participating schools (S1, S2, and S3) in math classes for grades six and seven during which proportionality was being explained. Two sessions in each grade were observed, which makes a total of 12 observation sessions lasting two hours each.

Table 1

Number of Sessions Observed

Schools	Grade 6	Grade 7
S1	2 sessions	2 sessions
S2	2 sessions	2 sessions
S3	2 sessions	2 sessions

The researcher did not ask the students or the teachers any question. Observation rubrics were used (See Appendix D) to systematize the observation and focus it around the teacher's method of teaching, the representations used, and how teachers interacted with the students and how students interacted with each other. At the same time, special attention was paid to specific teacher and student actions related to solving and discussing real-life problems in class.

Clinical Interviews

Clinical interview is an open ended technique that employs naturalistic observation to evaluate competence. Clinical interviews can help teachers and researchers know more about how children think. According to Ginsburg (1997), the clinical interview develops one-to-one relationship between interviewer and interviewee, involves flexible and innovated construction and presentation of tasks or problems to be solved. It employs tasks which direct the subject's activity into particular areas and facilitates rich understanding which sheds light on underlying processes through asking students to express their thinking.

For the clinical interviews, two pairs of students were selected from each of the grade six and grade seven sections in the three schools participating in the study.

In total, 24 students were selected to be interviewed: eight students from each participating school, and four students from each grade (grades six and seven). Thirteen males and 11 females composed the overall group of students. Students were selected according to their achievement level and their teachers' recommendations. The first pair has one student with high level and one student with mid-high levels of achievement while the second pair has one student with low level and one student with mid-low level of achievement. Therefore, all in all, 12 clinical interviews were conducted.

Those students were selected based on their achievement in mathematics and their teachers' recommendations about their class interaction and work. From each class, four distinct level of achievement were considered: one high level, one mid-high, one mid-low, and one low level of achievement.

Table 2

Students selected for the clinical interviews

	Grade 6		Grade 7	
	Pair 1	Pair 2	Pair 1	Pair 2
S1	High (F) Mid-high (F)	Mid-low(M) Low (F)	High (M) Mid-high (M)	Mid-low(F) Low (F)
S2	High (M) Mid-high (M)	Mid-low(F) Low (M)	High (F) Mid-high (M)	Mid-low(M) Low (M)
S3	High (F) Mid-high (M)	Mid-low(M) Low (F)	High (F) Mid-high (M)	Mid-low(M) Low (F)

M= Male F = female

During the clinical interviews, students solved word problems handed to them on a paper. A set of five problems was given to students (Appendix E). The first four problems are based on the four semantic problem types listed below:

1. "Well-chunked measures" which involves the comparison of two executive measures.
2. "Part – Part – whole" which is the ratio of a subset of a whole in terms of two or more sub-subsets of which it is composed.
3. "Associated sets" which is a connection of elements whose relation is not defined except in a problem situation.
4. "Stretchers and Shrinkers" which is one-to-one continuous ratio-preserving mapping that exists between quantities representing a specific characteristic of an element like height, length, width, etc.

(Lamon, 1993, p. 41- 46)

The fifth problem is added to introduce percent as an additional type of proportionality.

During the clinical interviews students worked in pairs to encourage them to think aloud and to communicate their thinking to their peers. As the students set to solve the problems the interviewer asked questions, when necessary, to prompt students to explain their thinking or to justify their choice of a certain strategy. The interviewer (the researcher) was careful not to affect students' thinking: she showed no approval, disapproval, surprise or any other reaction for any solution or strategy used by the participants. She used only non-guiding questions such as: why did you do this? What do you mean by this? What are you thinking about? Why do you think so?

The sessions were videotaped for further analysis. During each interview, pupils were handed a pen and a worksheet containing five word problems related to proportionality to solve in a one- hour session. The interviews were conducted in English language, the work was written on the given worksheet, no scratch paper was allowed, and students were instructed not to erase or scratch their drafted solutions, even if they find out that they are wrong or contain mistakes. In case of a wrong solution, they would draw a line and start another solution path on the same paper.

RESULTS

After reviewing the literature regarding the teaching of proportionality, and having identified the research scheme that was used to investigate the teaching and learning of proportionality in schools in Lebanon, this chapter is dedicated to findings of the study. It includes an overview of the content of the math curriculum that was officially endorsed by virtue of decree No. 10227 of 8 March 1997 that stresses problem solving as one of the major goals to be emphasized in school mathematics (Educational Center for Research and Development (ECRD), 1997). It also includes an analytical description of the following: chapters related to proportionality in math textbooks that are currently used to teach math for sixth and seventh grades in schools of Lebanon; interviews with grade six and grade seven teachers in two schools of Lebanon regarding the teaching of the topic; class observation of these teachers teaching proportionality; and clinical interviews with a number of their students.

Proportionality in the Lebanese Math Curriculum

In general, a "curriculum defines: (i) why; (ii) what; (iii) when; (iv) where; (v) how; and (vi) with whom to learn" (Braslavsk, 1999, p.1). In other words, it must include objectives, methods, and assessment measures for each topic. The math curriculum, like the curriculum for other subject matters, is divided into four parts: an introduction, general objectives for the entire subject, scope and sequence for each year with each of the three cycles that constitute the schooling years in Lebanon, and objectives and a syllabus for each year within the cycles. Below is an analytical description of how the curriculum organizes the teaching and learning of proportionality, while adopting a constructivist approach to education.

Introduction and General Objectives in the Lebanese Math Curriculum

The introduction emphasizes the importance of math in society. It recognizes the importance of real life problem situations to explain different concepts and highlights the importance of building conceptual understanding and skill development through problem solving tasks. It advocates a constructivist approach to education. Specifically, the authors underscore that:

> Mathematics … is a fertile field for the development of critical thinking … for rigor and for precision. … [It] proves to be an ineluctable necessity to the life of societies and to their development. … [It] no longer consists of teaching already made Mathematics but of making it by oneself. Starting with real-life situations in which the learner raises questions, lays down problems, formulates hypotheses and verifies them, the very spirit of science is implanted and rooted. (ECRD, 1997, p.288)

The authors also affirm the importance of concrete mathematical activities that are related to students' cultural background and development. The emphasis is on having students solve exercise and problems that make them reflect about real life situations. This is reflected in statements like the following: "The recommended method consists of starting from real life situations, lived or familiar, to show that there is no divorce between mathematics and everyday life" (ECRD, 1997, p.288).

The constructivist approach is also evident in the focus on student-centered approach when teaching. There is stress on problem based learning, and on teaching students communication of reasoning, and integration of real tasks. This is reflected in the statement in this curriculum "mathematics constitutes an activity of the mind…" (ECRD, 1997, p.288).

In addition to the above, it is noteworthy that the authors stress in the introduction the use of calculators and computers as "two technological novelties which will have benefits on the formation" (ECRD, 1997, p.288) and may in a way benefit students in learning and help them in solving problems.

The general objectives that the Lebanese curriculum has set to achieve upon teaching math, follow from the philosophy and pedagogical foundations underscored in the introduction. They, too, display a commitment to the constructivist approach that stresses on learning through observation, analysis, and taking decisions. In them, there is evidence that the reformed Lebanese curriculum is committed to spreading independent and active learning among students in schools at all levels. Specifically, the aim is to help students to develop the ability to: emphasize mathematical reasoning; solve mathematical problems; establish relations between mathematics and the surrounding reality; value the role of mathematics in the technological, cultural and economical development; communicate mathematically; and value mathematics (ECRD, 1997).

Scope and Sequence in the Lebanese Math Curriculum

Teaching students math in the Lebanese educational system entails teaching them arithmetic and algebra, geometry, measurement, and statistics. Below is an examination of the systematic sequence of proportionality throughout the different grades.

Grades one to five. Proportionality is not taught to students in grades one and two. In grades three and four, the concept is implicitly introduced through fractions. These constitute a core component of teaching proportionality throughout the three cycles. Specifically, fractions are represented as $\frac{1}{n}$ in grade three and $\frac{a}{b}(a \leq b)$ in grade four. It is mainly through working with equivalent fractions and representing them with equal parts of wholes that proportions are manifested although there is no explicit intention of them. In addition, in the latter level, students are also expected to compare fractions and place fractions on the number line. It is mainly through working with equivalent fractions and representing them with equal parts of wholes that proportions are manifested, although there is no specific mention of them. In the fifth year, addition, subtraction of fractions and multiplying a number by a fraction are introduced. Adding fractions with the same and different denominator on the number line taking into consideration the need to find the least common multiple (LCM) of the two denominators is also integrated. In this grade students are also introduced to the concept of scale. In a chapter about dilation students are taught the reduction and magnification of figures on a grid by transporting the figures from one grid to another. Students in this chapter are asked to enlarge or reduce a figure represented on a grid three, four, five times. It should be noted here that the curriculum does not stipulate that scales should be addressed in this grade.

Grade six. Proportionality continues to be addressed indirectly at this level through fractions, but it also appears for the first time as a topic on its own. Actually, at this level, teachers should start by teaching students the first concepts and skills necessary to understand proportionality. They should start by teaching them how to compare fractions as well as solve problems involving addition, subtraction, multiplication, and division of fractions. Following this, proportionality at this level is addressed in three chapters that fall under arithmetic and algebra and that are taught over 20 hours: Percentage and Rates, Proportional Sequence, and Scale.

Grade seven. In grade seven, as in grade six, the math curriculum stipulates that students should learn arithmetic and algebra, geometry, and statistics. Numbers and mainly fractions appear in the area of ratios, proportions, and percent. Fractions in this grade are represented as reduction of fractions as a chapter by itself. It is linked to decimals in the chapter of decimals so students write the decimals as fractions. Actually the analysis of the book below will show that fractions appear more in the book and it is not shown in the curriculum.

Proportionality is presented as a topic by itself to be covered in ten hours of teaching. Proportionality shows the aspect of classifying different directly proportional magnitudes. The first major standard that addresses proportions is the number sense standard where proportions are used to solve several types of problems and exercises as drill and practice without including any word problems. For example, determine the value of N if $\frac{N}{4} = 7$. Students learn irreducible fractions, decimal fractions, multiplication and division of fractions.

In this grade too, real-life multi-step problems related to proportionality come in problems about velocity, length of shadows, and many others. For example, students learn that distance is speed multiplied by time or $d = v\ t$. In addition, although there is no chapter that focuses on ratios, understanding of proportionality is extended at this level to apply to solving percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. Students also learn about ratios and proportions in scale drawings like designing a room or an office, read road maps.

Objectives and Syllabus of the Three Cycles

As shown above, proportionality in this curriculum is mainly introduced through numbers and particularly through fractions, then through ratios, proportions, and percents. More importantly, the reformed national curriculum adopted in1997 aims to teach students how to think analytically and be able to clearly express themselves. This constructivist approach to teaching math in the Lebanese curriculum appears in the general objectives, specific objectives, and syllabus dictated for each of the three cycles too. For example, the curriculum states that students must be taught how to:

- "… [V]isualize situations and handle information" and "… [U]se and apply mathematics in various domains, especially in technology and other branches of learning" (ECRD, 1997, p. 299).

- "… [F]ind connections between the real world and mathematical models …. [and] construct a mathematical model associated with a situation" (ECRD, 1997, p. 302).

Statements of this sort show that the reformed Lebanese curriculum aims first and foremost to teach students how to take the initiative to choose the mathematical technique that fits the purpose of the problem and express themselves both orally and in writing. For example, cycle three cites the importance of recognizing the relationship between real world and mathematical problems. The constructivist approach to teaching students mathematics in general, and proportionality in particular, is revealed through using terms and concepts such as "measuring", "choosing a strategy", "construct a mathematical model" , "using calculating machines with memory", and "decomposing" (ECRD, 1997, p. 302).

Having overviewed what the national math curriculum stipulates about the teaching of proportionality to grades six and seven students, the following section focuses on how the national math textbook and teacher's guide address the teaching of proportionality.

Proportionality in the Math National Textbooks: Grades Six and Seven

In general, textbooks are viewed as a tool that supports students and teachers in achieving the requirements of learning goals and objectives set by the curriculum. As Van Zoest and Bohl (2002) underscore, mathematics textbooks can influence how and what teachers teach; they are the source for the exercises and various procedures that students have to solve in order to learn the mathematical principles. In textbooks, teachers can find the topics to be taught during the year in a suggested sequence. This sequence, along with the suggested activities, determines to a certain extent the way teachers teach. Hence, in order to assess the teaching of proportionality in schools in Lebanon, it is essential to take a close look at the chapters dedicated to teaching this topic in the math national textbooks.

Before embarking on the analysis of the chapters that teach proportionality in the two textbooks in question though, it is important to note the following. The title of the Lebanese national math textbooks, *Building up mathematics* indicates a commitment to a constructivist approach to education. It reveals an approach to education that considers students are active builders of the knowledge they end up acquiring and the role of the teacher in the classroom is restricted to a guide or facilitator. This is also illustrated in the introduction of the textbook. It is stated in the introduction to grade six textbook that the method of teaching that math teachers are expected to adopt " … challenges the students with actual problems taken from real life and the environment urging them to conjecture, experiment, and verify different solutions …." (ECRD, 1999a, Introduction). At the same time, the authors point out in the introduction to the grade seven textbook that the student " … will be confronted with situations which make him ask questions, hypothesize, exchange ideas with his classmates, defend his ideas, convince his peers, or be convinced by them" (ECRD, 1998a, Introduction).

Structure of the Textbook

Each chapter in grade six and grade seven textbooks starts with a list of objectives. They inform the students what they must have acquired and developed by the end of the chapter and after having done the exercises and solved the problems.

The objectives are followed by an introduction and a reminder of the concepts and skills that students have already learned about proportionality. For this purpose, before the teacher starts explaining the new lesson, students are expected to solve a series of exercises and problems referred to as activities. Unlike grade six however, this section in the grade seven textbook is divided into two parts. One is dedicated to "recall" activities for reviewing previous knowledge and the other focuses on preparing students for the lesson.

The teacher starts to teach the lesson after the students have completed the activities. In this section, the text is the theoretical part derived from the first activity and that comprises the mathematical core of the chapter.

Following the explanation of the lesson, the student is led to focus on the main concepts and skills in the chapter. Students are then given the chance to evaluate their knowledge in the "self evaluation" corner, before they move to solve problems. The chapter ends with a "just for fun" section that includes a non-routine problem. The exercises or problems included in this section are always more difficult than the ones included in the lesson, and thus more challenging. Students are encouraged to try to solve the problems, knowing that they should think of them rather as a game. Hence, they set to solve them without worrying about the result. They know that this is just sort of a brain teaser.

Content of Chapters

This section investigates the extent to which real-life problems are used in the textbook chapters. In other words, the aim is to explore the extent to which conceptual understanding and skill development are being built through problem solving tasks in grade six and grade seven math national textbooks.

Introduction in the chapters about proportionality. In grade six textbook, the students begin to implicitly learn about proportionality in the chapter on fractions that teach students about irreducible and decimal fractions. The objectives state that students ought to learn to identify an irreducible fraction, find the irreducible form of a given fraction, recognize a decimal fraction, write a decimal fraction in a form of fraction with ten as denominator, and write a fraction as a decimal and vice-versa. The chapter includes set of real life problems such as fractions of an hour and fraction of a road in a trip. Actually fractions are introduced early in the book in the chapter "Development of a decimal number in terms of powers of 10 and $\frac{1}{10}$ as well as the chapters of "signed number" and "multiplying and dividing fractions"

The next chapter that taps on proportionality is the one about quotient and ratio. The objectives set for students to achieve by the end of the chapter are to: a) "recognize and represent the quotient of a division", and b) "recognize and use the ratio of the two extensions" (ECRD, 1999a).

This chapter is followed by the one on percentage. By the end of this chapter students are expected to know how to calculate a percentage of a number and recognize, calculate, and compare percentages (ECRD, 1999a).

In the chapters about fractions, quotients, ratios and even percentages, there is no explicit mention of proportionality. As such, when they reach the chapter on proportionality, students have the impression that they will be learning something new. The objectives at the beginning of the chapter state that by the end of this chapter, the student must be able to: a) recognize and construct proportional chain (series) and b) to calculate the proportionality coefficient and the fourth proportional" (ECRD, 1997).

In grade seven textbook, proportionality is addressed in two chapters, one about fractions "Reduction of Fractions and Fractions and Decimals" and one on the topic of proportionality. The notion of fractions is introduced as "fundamental in mathematics. It represents symbolically real situations...and consequently to deal with them mentally" (ECRD, 1998a, p.85). Specifically, fractions in this grade are represented as reduction of fractions as a chapter by itself and it is linked to that of (Greatest Common Divisor of the numerator) GCD and (Lowest Common Divisor of the numerator) LCM where students need to use GCD in order to reduce the fractions. In this chapter, some new terms are introduced like "irreducible, simplify, and reduce" and the concept of $\frac{a}{a} = 1$ is also brought in. This chapter does not include any real life problems; all what it uses is drill and practice exercises and problems.

It should be noted here that fractions are connected to decimals in the chapter "decimals and fractions" where students "learn how to identify these fractions (called decimal fractions) and how o identify non-decimal fractions" (ECRD, 1998a, p.94). Also fractions are found in the chapter of powers on how to calculate them for example $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b_2}$ and $\frac{a^5}{a^3} = a^{5-3} = a^2$. Besides the book includes fractions in the chapter of multiplying and dividing signed numbers and excludes it from the chapter on adding and subtracting signed numbers. At last fraction in this grade are embraced in equations as part of the equation to be solved.

In the chapter about proportionality, the students are reminded at the beginning of the chapter of the relevance of proportionality to real life. Specifically, part of the introduction in the chapter on proportionality includes statements such as: "Proportionality is one of the most useful mathematical concepts; it applies in many fields of everyday life" and "It has direct applications to real life problems (purchasing, selling, duration, speed, pressure, distance, reduction problem, etc.)"(ECRD, 1998a, p. 150).

In the introduction to the chapter, the students are also reminded that up to this chapter they had learned how to identify proportional sequences, calculate the coefficient of proportionality, recognize ratios and quotient, and use percentages, rate, and scales. At the same time, they are informed that by the end of the lesson they ought to be able to identify a situation of proportionality, recognize a proportion, calculate the fourth proportion, and use calculations of the forth proportional to solve problems such as buying, selling, duration, speed, distance, and dimensions.

Other sections of the chapters. Below is the analytical description of the remaining sections of the chapters that address proportionality. Emphasis is on highlighting the extent to which the exercises and problems included lead to students' learning of the different concepts as stipulated by the constructivists, in other words through relating to real life situations that they do or that they might encounter in their daily lives. Tables 3 to 9 summarize the main findings about this issue.

The first part of the analysis focuses on the section titled "Activity" which is meant to introduce students to the lesson through activities. Table 3 highlights the extent to which real life problems are included.

Table 3

Number of Real Life Problems in the Section "Activity"

	Chapter title			
	Grade 6			Grade 7
	Quotients and Ratios	Percentage	Proportionality	Proportionality
One real life situation	X	X	X	
More than one real life situation				X
No real life situation				

The activities in the chapter on fractions are mostly real life examples that help students discover that the same part of a surface can be represented in different fractions that are equivalent. They also remind students of the fastest way to simplify a fraction to its simplest equivalent form through using the (Greatest Common Divisor) GCD of the numerator and the denominator. For example in one of the activities, students are asked to solve a problem in which they are told the prices of difference refrigerators shown in a catalogue, and they are guided by the teacher to answer the question whether the prices are proportional or not.

Activities at the beginning of the chapter on quotients and ratios aim to "gather information allowing students to write ratio of two quantities of the same type and use this ratio in the comparison of the two quantities" (ECRD, 1999a, p.127). The first activity is just drill and practice; students cannot reach the intended goals through this activity. Students are asked to write the quotient of a by b, arrange quotients in the appropriate order, enclose the quotient of 4/3, and exercises of the same sort that do not enable students to reach the objectives stated in the teacher's guide. In the second activity however, students are asked to compare two ratios through thinking about real-life situations namely, making a cake and speed of a bus on a trip. In other words, they are guided to reach the goals set at the beginning of the (Appendix A, p. 128 in the book).

In the chapter about percentage, the "Activities" section teaches students to solve a real life problem about price reduction if given the percent discount on a certain object. One of the activities goes as follows: "The T-shirt is yours Ziad; its price was $60 but I bought it for $45. The calculation shown in the following table will allow you to find the discount on this T-shirt" (Appendix A, p. 133 in the book). An activity like this one in this grade is difficult and tricky. Students are told to fill in the blanks in a given table, but they have not been taught how to calculate the discount on price.

The chapter on proportionality begins with a real life activity, which according to the teacher's guide, aims to make students become familiar with proportionality. In this activity, the student is supposed to construct a parcel (on top of each other) using three books then measure the height of the parcel. Then the student is asked to try to guess the height of a parcel made of seven books without actually putting them in a parcel. Following this, the student is asked to arrange several notebooks, all of the same thickness, and arrange them into parcel, complete a table and then find the multiplication operator (Appendix A, p. 139 in the book).

In the chapter dedicated to teaching about proportionality in grade seven textbook, the majority of the exercise in the "Activities" section included to help students recall what they have already learned about proportionality relate to real life situations. The second activity is related to real life while the first one is scientific and requires the student to calculate the coefficient of proportionality of two proportional sequences dealing with the mass and height of water in a graded cylinder. The activities involve proportional sequences, calculations with percentages, and problem situations of proportionality. The textbook also points out to the attention of students several places and situations where proportionality appears in real life. For example, the activities included to prepare students for the lesson include the following questions:

-How many kg can we buy with 3375L.L?

- A child needs 30 min to write his homework, how much time do 5 children need to write their homework?

- With a bike moving at 15 k/h, I need 20 min. to go to school. With a car moving at 60 km/h what will be the situation of the trip?

(ECRD, 1998a, p.151)

By way of ending this descriptive analysis of the "Activities" section in the different chapters on proportionality, and before moving to the descriptive analysis of the following section in the lesson, it is important to note that these activities lack in accuracy. For instance in the case of the problem where the student is asked to calculate how much time do 5 children need to write their homework, the fact of the matter is that the time it takes a child to finish his or her homework depends on several factors such as comprehension of the lesson and speed in understanding instructions. At the same time, there is the fact that the question does not tell the students are working in parallel or working together in a group or in sequence. The children could all finish at the same time if they are working in parallel, especially if they all understood the lesson well and they began working at the same time. If however the students are working in group, the five children together might need the same time it takes one of them to finish the problem. In other words, the answer is given in the question; one student could very well argue that it takes the group thirty minutes to finish. Also the question of the third example "what will be the situation of the trip?" is somehow confusing; it could have been much easier to ask the student about the time it takes to school with a car moving at 60 km/h.

The following part of the chapter focuses on the findings regarding the section titled "Texts" in the proportionality chapters summarized in Table 4:

Table 4

Number of Real Life Problems in the Section "Text"

	Chapter title			
	Grade 6			Grade 7
	Quotients and Ratios	Percentage	Proportionality	Proportionality
One real life situation				
More than one real life situation	X			X
No real life situation		X	X	

In the chapter on quotients and ratios, students are provided with real life situations. For example, for the introduction to explain the concept of ratio they are given the following example:

> If Wadad is 10 years old and Samir is 145 months old, you convert age of Wadad to months.
> - Wadad = 10x12 = 120 months.
> - Ratio of Wadad's age to that of Samir's is 120/145 = 24/29

(ECRD, 1999a, p.129)

An example of this sort is not really helpful for students to understand the lesson. The students are not explicitly told what it exactly means to find the ratio of one number to another. They simply read "… you convert age of Wadad to months" and there is no question or explanation whatsoever. In addition, this is not the best real-life example to teach children about ratios. It is not very often, if ever, that someone needs or is asked to calculate the ratio of one person's age to that of another in real life. Calculating "ratios of ages" is not, in any means, a suitable example for presenting the meaning of ratios and the situations in which they are used.

In the chapter on percentages, this section lacks severely on real life situations. The concept of percents is explained by giving examples of multiplying a percent by the number directly like 5% of a 340 is $\frac{5}{100}\times 340=17$. The student is not told why s/he needs to divide 5 by 100 to calculate this percentage; there is no explanation as to why the number 100 is used as the quotient and not any other number. Students are then taught to relate percents to fractions. No representations are used to make it easier for the students to understand the topic. For example, a question such as 25 is 12 percent of which number leaves the student to think hard in a way to work with the given numbers.

In the chapter on proportionality this section starts with a definition of the proportionality table with two numerical application examples. The table is defined as a numerical table with two lines, where a student multiplies the first line by a number "c" called the coefficient. In the first example, the student is told that the table is proportionality table because the products of the numbers given in the first line by "4" are the numbers given in the second line (row). In the second example however, the students are provided with a table and they are simply told that this is not a proportionality table (Appendix A, p.140 in the book). The authors should have explained to the student that no number can be put in the blank because there is no one number that can a coefficient for the three series at the same time.

Following the proportionality table, there is a definition of a proportional series, proportionality coefficient, and fourth proportion. No real life situations are stressed in this section. Hence, unless the teacher provides concrete examples, the student is left with confusing abstract statements.

In grade seven, in the chapters on fractions students learn what irreducible fractions are, use the property of $\frac{a}{a}=1$ for all non-zero natural numbers a, and calculate the reduced form of a fraction using several methods:

(a) dividing the numerator and denominator by the GCD,

(b) decomposing the numerator and denominator into prime factors and then simplifying them,

(c) proceeding with successive divisions by common divisors.

At the same time, students need to change a decimal number into a sum of successive fractions whose denominators are 10, 100, 1000, etc., as well as write a decimal fraction in decimal form, and compute an approximate form of non-decimal fraction by a decimal number.

In the only chapter allotted to proportionality in this grade the students are reminded of the relevance of proportionality to real life (distance, total price, interest, and real distance).

Proportionality is then introduced as a relationship between two quantities that are in a linear relation: y = a × x. The ratio of proportionality is defined as the ratio of the two quantities x and y where a = $\frac{y}{x}$. The definitions are a continuation of those elucidated in grade six textbook with the addition of one property, which is the addition property stating that $\frac{a}{b}=\frac{c}{d}=\frac{a+c}{b+d}$. In order to find the fourth proportion, the textbook encourages students to use cross multiplication. To explain to students the latter two concepts, examples related to real life situations are used (Appendix A, page 153 in the book).

The following part of the chapter focuses on the findings regarding real life problems in the section titled "Focus" which are summarized in Table 5:

Table 5

Number of Real Life Problems in the Section "Focus"

	Chapter title			
	Grade 6			Grade 7
	Quotients and Ratios	Percentage	Proportionality	Proportionality
One real life situation				
More than one real life situation		X		
No real life situation	X		X	X

In the chapter on quotients and ratios, the "focus" section is divided into two parts. In the first part the focus is on the quotient of two numbers a by b. The purpose is to draw students' attention to the difference between decimal or non-decimal fraction. In the second part of this section the focus is on the ratio of a to b; students are taught to distinguish between ratios when *a* and *b* are of the same and when *a* and *b* are of different units. In addition, students are also called to pay attention that when the ratio is a constant it is called an average.

In the chapter about percentages, this section provides students with one example to show them how to get a percentage out of two given numbers and one example to show them how to calculate the percent of a number (Appendix A, p.135 in the book). Students are provided with the two examples but they are not told the aim behind including them in this section. At this level, students cannot know by themselves the purpose from an example, and what they are supposed to learn from it, unless they are guided by the textbook and the teacher. The section should have included more explanation how the figure obtained was calculated and what it represents.

In the chapter about proportionality the student is provided with a new type of the proportionality table that has a proportionality coefficient on both sides and a statement at the top of the section saying: "We can go both ways" (Appendix A, p. 141 in the book). A close inspection of this example shows that rather than helping students, it tends to be problematic and confusing to students for the following reasons:

- Students are told prior to this example that a proportionality table has to have the multiplication sign to one side and the division sign to the other side (with two opposite arrows). This table has the multiplication sign on both sides (with two inverse coefficients) and there is no explanation why there is suddenly this change and how come both tables are referred to as proportionality tables.
- On one side the arrows go both ways from and to the coefficient, which is not possible; 15 times 0.2 is not equal to 75.

- The example is missing a statement that explains to students that the purpose behind this table is to know that two numbers like 15 and 75 in one proportionality table can have two coefficients at the same time such as 5 and 0.2. One can multiply 75 by 0.2 and get 15 and s/he can multiply 15 by 5 and get 75. If the teacher does not explain it to students, they are left to guess what is meant by "both ways".

In grade seven, the "focus" section in the only chapter about proportionality includes a summary of the property relations deduced from a proportion but it does not include any reference to real life situations. Students could find this summary useful when they face difficulties while practicing and solving problems, but this lack of real life situations is a major shortage that obstructs the implementation of the constructivist approach (Appendix A, p.155 in the book).

After highlighting the real life problems that are included in the "Focus" section, the following part of the chapter focuses on the findings regarding the section titled "Exercises" summarized in Table 6:

Table 6

Number of Real Life Problems in the Section "Exercises"

	Chapter title			
	Grade 6			Grade 7
	Quotients and Ratios	Percentage	Proportionality	Proportionality
One real life situation	X			
More than one real life situation			X	X
No real life situation		X		

In grade six, the exercises in the chapters on fractions, quotient, ratio, and percentages are mostly drill and practice. Only the chapter on quotients and ratios includes one real-life problem dealing with average speed. It requires the student to calculate the average speed per hour by a person who covers by bike a distance of 40 km in two hours, at a constant speed (ECRD, 1999a). In the chapter on proportionality, out of the twenty exercises that students are supposed to complete, real life instances are included in five problems as headings in five proportionality tables (Appendix A, p. 141 in the book). The exercises in which the terms and concepts that refer to real life situations are used are nothing but the usual drill and practice type. All the exercise requires the student to do is to say whether there is proportionality between the numbers given in the two rows of the table.

When it comes to grade seven, in the chapters on fractions most exercises are computational and depend on drill and practice. Students need only to memorize the way of solving by checking one of the examples in the lesson to be able to solve the exercise. In the chapter on proportionality, students are expected to solve 17 exercises (Appendix A, p. 157 in the book). One of the exercises is of the drill and practice type. 11 of the exercises are about real-life situations. One problem is about salary raise while the remaining of the real life situations in this section are limited to four topics: production- consumption, supply- speed, change of units, and approximations. The remaining five exercises deal with scientific subjects; in one of them the students are asked to calculate the area of a cube, in another s/he is asked to find the distance to Mars, and a third asks the student to find the speed of a turtle.

Before proceeding to the descriptive analysis of the following section, it is important to point out that a major weakness in the math textbook of grades six and seven in Lebanon is that it fails to familiarize students with the wide range of real life situations that involve proportionality. There should have been more problems for instance about taxes, discount, salary raises, perimeters.

The section "Exercises" in the chapter of the math textbook is followed by the section "Self-Evaluation". The findings regarding real life problems in this section are summarized in the following Table:

Table 7

Number of Real Life Problems in the Section "Self-evaluation"

	Chapter title			
	Grade 6			Grade 7
	Quotients and Ratios	Percentage	Proportionality	Proportionality
One real life situation	X	X		
More than one real life situation				X
No real life situation			X	

In the chapter on quotient and ratios students are asked to do a self evaluation of their understanding of the concepts explained in the chapter through two exercises. Only the second exercise is about real life matter (water and earth), while the first one asks students to use a calculator to find the approximate value of a quotient of two numbers (Appendix A, p.131 in the book).

In the chapter on percentages, the self evaluation section tests students in two parts. The first part includes drill and practice exercise. The second part, however, includes a real life problem on sales. As much as useful this problem is, it is placed in the wrong section. It is wrong to evaluate students for something they have not been taught. This is the first real life problem in the chapter. A similar problem should have been included in exercises before the self-evaluation.

In the chapter on proportionality however, the self-evaluation includes only drill and practice exercises. The students are asked to do strictly computational exercises (Appendix A, p. 142 in the book).

In grade seven, the "self-evaluation" section in the chapter about proportionality includes four problems about real life situations (Appendix A, p. 156 in the book). The first two problems ask students to respond with true or false. One of these two problems requires the student to find out whether when a girl who is 12 years reaches 24, her mother who is 37 years will become 49 years old. To answer the question, the students are given a hint that 12x2 = 24. This hint however is misleading. It could give the student the impression that to find the answer s/he should multiply 32 by 2 whereas the purpose behind this exercise is to teach students that age is not proportionally calculated. The students must realize that the question boils down to the following: how old will the mother who is 37 years old be after 12 years? Along with asking students to think about age, this section also focuses on calculating discounts.

The following part of the chapter focuses on the findings regarding real life problems in the section titled "Problems" in the proportionality chapters summarized in Table 8:

Table 8

Number of Real Life Problems in the Section "Problems"

	Chapter title			
	Grade 6			Grade 7
	Quotients and Ratios	Percentage	Proportionality	Proportionality
One real life situation				
More than one real life situation	X	X	X	X
No real life situation				

In the chapter on fractions, problems included are mostly real life problem situations. In the chapter on quotient and ratio, five out of nine problems are computational while the other four require the students to think about real–life situations (Appendix A, p. 132 in the book). The latter type of problems focus on the following: a bike ride, turtle's speed, ratio of mountain's heights, diameter of Pluto, and even ratios of mass of different parts in the human body.

In the chapter on percentages, all of the 14 problems that students are asked to solve are real-life problems. Students are asked to calculate the percentage of increase and decrease of a quantity. As much as it is important to include such exercises in math textbooks because students are bound to encounter such situations in their real life, they risk being confusing to students. Calculating percentage increase of a price for instance and then calculating the same percentage decrease of the result, does not lead students to the number they started from. Calculating a 20% increase, followed by a 20% decrease on a $200 coat is a good example to illustrate this point.

As for the chapter on proportionality, in "the problems" section students are asked to solve eight real life problems (Appendix A, p. 143, 144 in the book). The problems require students to reflect about situations that they can relate to. They require the students to think about proportionality in relation to real life situations in their environment. For instance, one problem is about two wheels, a big one and a small one, another is about a board game and another is about making a cake with specific quantities of ingredients.

In grade seven, problems in the chapters on fractions are computational and drill and practice (Appendix A, p. 92 in the book). All what students need to do is to memorize the way of solving, or check an example to be able to solve most of the problems. In the chapter on proportionality, all problems are related to Lebanese real life. For example, one of the problems talks about a worker's salary in Lebanese pounds. Other problems also include voting, accidents, percentage of success and failure in exams, painting, baking using different amounts of ingredients, and finding the best bets to buy certain things.

The last section in the chapter in the math text book is titled "Just for Fun". The following part of the chapter focuses on the findings regarding real life problems in this section summarized in Table 9:

Table 9

Number of Real Life Problems in the Section "Just for fun"

	Chapter title			
	Grade 6			Grade 7
	Quotients and Ratios	Percentage	Proportionality	Proportionality
One real life situation				X
More than one real life situation		X		
No real life situation	X		X	

No real-life situation is included in this section of the chapter on quotients and ratios (Appendix A, p. 132 in the book). The student is asked to find the area and the perimeter. This should have been an easy task had the student been provided with the length of each side.

In this section of the chapter on percentages, the student is asked to calculate the scale at which a football field is represented if the real field is a rectangle of dimensions 100m and 60m, whereas the statement introducing the problem states: "A football, a basketball, and tennis fields are represented below" (ECRD, 1999a, p.138), while there is only one picture (Appendix A, p. 138 in the book). Though it relates to real life, this problem risks being confusing to students because it assumes that all students know that all three fields can have the same dimension. In addition, the number next to the question gives the impression that there are more questions whereas in reality there is only one question. Details of this sort can turn a "just for fun" problem into a frustrating exercise.

As for the "just for fun" part in the chapter on proportionality, the student is asked to place three different objects in the sun and measure the length of the shadow of one of the objects and then find without measuring the length of the two other shadows. The question is missing much information and explanation and a student most likely cannot find an answer.

In grade seven, the "just for fun" section in the chapter about proportionality, like in the other chapters, is rather confusing. The first question begins with "_ought the chocolate by weight" and students need to guess what the missing letter "_" is and who bought the chocolate with no guidance or tips how to proceed to solve the problem (Appendix A, p. 157 in the book).

After describing the math textbooks of grades six and seven, as to their emphasis on real life situations, the next part of this paper is dedicated to highlighting what the teacher's guide has for teachers of math at these levels as far as proportionality is concerned.

Proportionality in the Teacher's Guide for Grades six and Seven

A teacher's manual should serve as a guide for teachers to facilitate teaching mathematics and connecting it to real life. As such, it should include (a) an introduction that shows the objectives of the unit or chapter; (b) a detailed explanation to help the teacher understand the concept; (c) connection to previous or future chapters; (d) lesson plan of the unit; (e) assessment of the chapter and questions asked by teacher to check students' understanding; (f) discussion on each problem solved and samples of some of students' responses besides suggestions on whether the problem should be solved individually or can be solved within a group; (g) extra work for students who need more challenge; (h) materials such as transparencies and technology; and finally (i) suggestions of exercises, games, and challenges for students with special needs…

Each of the textbooks described above has a teacher's manual, yet the manual does not suggest how teachers should use the textbook to meet the objectives of each lesson. The teacher is not guided in the process of teaching. It is left up to him or her to decide how to teach students while respecting the constructivist approach to education revealed in the curriculum and the textbooks. The teacher's guide at the disposition of math teachers in Lebanon fails to highlight the importance and necessity of training and change of beliefs in teachers so that they can apply constructivist approach. Teachers are not adequately prepared to adapt their teaching strategies to different situations and they are not guided to emphasize different activities to be performed by students for better learning and understanding of different concepts.

Grade Six

Grade six teacher's manual begins with the objectives of the chapter as stated in the book, in addition to an explanation about the objective behind each activity. For example, the book presents an activity where students need to find the decimal part of the fraction on each character's t-shirt and link it to the boat it will use; (Appendix B, p. 71 in the teacher's guide) the guide underscores that the objective of such an activity is to motivate the students to reach the beautiful products. The teacher's manual does not provide any explanation or ideas on how to enable students to reach this goal through the activity i.e. how to teach. It only includes solutions for the exercises and problems.

Grade Seven

Grade seven teacher's manual differs from that of grade six, in the sense that it starts with an explanation of the concept. For example, the chapter on proportionality in the teacher's manual explains the importance of proportionality in mathematics and in real life so people sometimes say, "we think through proportionality". Teaching proportionality is set to be called "magical potion" since all its notions like scale, rate, interest, and percents are used everywhere and in different situations and contexts.

Furthermore, the chapter on proportionality in the teacher's guide reminds the teachers of the concepts that students must already know from grade six (quotient, percentage, etc.) and emphasizes the important role of the teacher when it comes to refreshing the memory of students before starting the lesson on proportional sequences.

Another illustration of the key role that the teacher's guide plays in the teaching process is the following. When it comes to teaching fractions, the manual reminds the teacher that students have already seen fractions in previous years like simplifying fractions, adding and subtracting fractions, multiplying and dividing fractions, and sorting them. Teachers are encouraged to try to find different ways for solving other than the intrinsic methods used in the book. Furthermore, the chapter of decimals and fractions stresses the importance of decimals and heighten the awareness of students that some numbers cannot be represented in decimal form and prepare them to the infinite sequence which will be taught later on during higher grade levels.

In the following section, the focus will be on teachers' feedback. It will overview what a number of math teachers in a number of schools in Lebanon have to say about the curriculum, textbooks, teacher's guide, and the optimal approach to teach proportionality.

Teachers' Feedback

After analyzing the national curriculum, chapters on proportionality in the textbook, and the teacher's guide, this section focuses on the feedback provided by six teachers (grade six and grade seven) regarding their conceptions of the concept, their ideas as to how it should be taught, as well as their opinions about the national textbook and the teacher's guide. Names of the teachers were replaced with T and a number, for example T1 for the first teacher interviewed. T1, T3, and T5 are teachers of grade six students and T2, T4, and T6 are teachers of grade seven.

Teachers' Conceptions of Proportionality

Teachers' answers indicate that teachers who teach the same grade and the same topic have different understandings of the concept and do not necessarily have similar conceptions of the topic. T1 did not provide a clear answer. She stated that two things are proportional when all numbers are related to each other in a formula and she added that she doesn't know what it really means but thinks that many things fall under this topic. For her, proportionality is addressed when we need to price two different quantities in a supermarket or when we need to determine how much we need of a certain material to make 15 liters of a certain product when we know the amount it takes to make one liter of this product.

For T2 and T3 proportionality refers to the "equal relation between two pairs of numbers" and they pointed out that as they understand it, many topics come under this topic such as size, measurement, dimensions and degree of similarity between two objects. T4 started by stating that she knows nothing about the definition because she doesn't teach this topic usually. Thereafter she added that proportionality is like two variables related by or forming a relationship with other parts or quantities. T5 defines proportionality as two equal ratios; she mentioned that many topics fall under it such as percentages and scaling. For T6 proportionality refers to having two numbers related to each other in a way when one of the quantities varies, the other will vary in an equivalent manner. For her many topics fall under proportionality such as fractions, ratio, percentage, and scale drawing.

Teachers' Opinions Regarding the Textbook and the Teacher's Guide

When asked about the textbook and the teacher's guide T1 stated that she thought that many problems exist in the Lebanese math textbooks and said that she did not know anything about the guide since she did not know it existed. T2 stated that she found the textbooks to be satisfactory as they are, except for some ambiguities where she felt she could not do without referring to the teacher's manual. In addition, she stated that she thinks that more tables and drawings needed to be added to help students better understand and represent real life problems. Both T1 and T2 reported that in order to compensate for the missing elements in the textbooks and guide, they use supporting material from other textbooks, mainly American and French ones.

T3 reported that she does not use the teacher's guide because she does not feel the need to do so. As for the textbook, she thinks that although real life problems are available, they fail to be really helpful for students' learning because they tend to be abstract. In her opinion, more real life problems with more concrete situations chosen from students' daily experiences should be added (pizza problems for example).

T4 is not satisfied with the Lebanese curriculum textbook. She thinks that the chapters are short, and they do not include enough real-life situations. As such, she said that in order to compensate for the problems, she usually reminds students of concepts related to the new lesson that they had learned before; she uses supplementary material from old version American textbooks; and she uses the teacher's manual to check some objectives missing in the math textbook.

As for T5 she stated that in her opinion, real-life problems should be used to teach proportionality, but she is not satisfied with the textbook and the curriculum. She thinks that both should be re-written again. In order to compensate for the weaknesses in the textbooks she thinks that calculators can be used along with some real life problems.

Finally, for T6 more activities and examples should be used if the students are to be able to understand the lesson without over relying on the teacher as they currently do. In addition, T6 is not satisfied with the textbook and teacher's manual; in her opinion, they fail to equip the teacher with a strategy to adopt while explaining the lesson. In her opinion, using more real life problems of the sort used in the textbook is the best way to make up for these problems.

Teachers' Beliefs about Teaching Proportionality

When it comes to how teachers perceive teaching of proportionality, it seems that teachers in this study are adopting a constructivist approach to teaching. The majority of respondents state that they give students the chance to think and discover by themselves through using real life problem solving while they gave them hints and ways of solving and tricks for students to memorize in order to be able to solve drill and practice problems. Actually T6 stated that she uses newspaper ads as hands on activities while teaching students how to calculate percentages.

As for the way proportionality should be taught, T1's answer was not expected. While she did not emphasize using real life problems when expressing her understanding of proportionality, here she underscores the importance of using them along with tables. T2 emphasized relying on multiplication and division. As for the other interviewees, they affirmed that most imperative is that students use real life problems related to their environment. In their opinion, this is the best way to teach students the concepts and ideas in the lesson.

Concluding Notes

The analytical description of grade six and grade seven national math textbooks reveals the commitment to the constructivist approach to education in principle, but many gaps in practice. They include a noteworthy number of real-life word problems that are intended to enable students to become reflective individuals capable of reasoning and critical thinking. It is important to note though, that the learning process is obstructed at several points in these chapters due to mistakes such as the following: "We read on the *unitization notebook* of the car" while it should be the *user's manual.* Also some problems are missing the right partition of parts and include several misspelling mistakes such as the word "economicasitates". (ECRD, 1998a, p.157)

At the same time, the overview of the teacher's guide and interviews with teachers revealed that the teacher's guide plays for some of them a role in regulating the teaching process to secure education within a constructivist perspective. Still however it has shortcomings; it fails to provide teachers with essential guidelines and practical tips as to what they should do to reach the objectives of the curriculum and those of the lessons. In addition, it is disappointing that the teacher's manual failed to include details on organizing the lesson, constructing an explanation based on evidence and real-life situation, and even preparing a kind of assessment activities that help the teacher evaluate the students' understanding. Mostly, what the entire manual focused on was to provide teachers with answers to problems and exercises in the book.

Last but not least, feedbacks by math students for grades six and seven in the schools included in this study revealed a rather alarming reality. Most of the interviewees do not know exactly what proportionality means/refers to and some of them do not know the methods used in the textbook and in the teacher's guide. At the same time however, teachers interviewed in this study reported a promising reality when it comes to teaching math in general in schools of

Lebanon. Most of them use real life problems to explain proportionality in the classroom. This indicates that they are respecting the constructivist approach to teaching math stipulated in the national curriculum.

Teachers' Approaches to Teaching Proportionality

This section presents findings of the non-participant observation of the three grade six mathematics teachers (T1, T3, T5) and three grade seven mathematics teachers (T2, T4, T6) over sessions of two hours during which they taught proportionality. The observation revealed similarities and differences among teachers when it comes to implementing a constructivist approach while teaching math. The following analytical description of sequencing of teaching activities, teachers' strategies used, types of exercises and problems offered illustrates this reality.

Sequencing of Teaching Activities

Sequencing of activities refers to way the teacher begins the session, whether s/he uses a problem related to real life or s/he starts with a concept or an application, and the extent to which s/he keeps connecting to real life situations while s/he is explaining the lesson. Table 10 summarizes main patterns among teachers who participated in this study regarding the ordering of teaching activities.

T1 started with informing students. She described to them how learning mathematics used to be dull when she was a student. She also drew their attention to the fact that unless they paid attention and unless they did the activities and exercises, they were not going to succeed in learning the lesson. Following this information, she presented a real life problem through which she started explaining the lesson.

T2 started the session with the topic that she was going to explain. She asked students to open the book on the page where the chapter begins and told her what the lesson is going to be about. Then she asked students to come up with a real life example that relates to proportionality. When none of the students were able to answer the question, she explained to them that proportionality is found in percents, proportions, quotient and ratios, and she gave the review about each of these concepts that they had learned the previous year.

Table 10

Teachers' Ordering of Activities during Classroom Observations

	How does the teacher start the lesson?	The problem is about a real life situation	Teacher begins with the new concept then moves to the application	Teacher connects to real life during explanation
T1	Information	Not applicable	No	Yes
T2	Ask the students to read and explain	Yes	No	Yes
T3	Information Read title and objectives	Yes	Yes	Yes
T4	Problem about real life situation	Yes	Yes	Yes
T5	Exercise related to proportionality	No	No	No
T6	Information	Yes	No	Yes

T3 started by asking students to open the book on the page on which the chapter started, she read the title of the lesson and the objectives, and then asked one of the students to read the first activity which happens to be about a real life situation (Appendix A, p. 133 in the book). T3 then explained the activity and connected it to the lesson.

T4 is the only one who began the lesson with a question related to a problem about a real life situation related to proportionality. She began the session by asking the students about the price of five bagels if the price of eight bagels is $4.5. After solving this part and discussing the answer, she asked the students whether they know from the example given what is the lesson going to be about.

T5 started the lesson with the proportionality table below:

5		7	10
20	16		

She asked students to continue to fill in the blanks taking into consideration the first column. She, then, asked the students to turn their book to page 140, check the proportionality tables given and try to solve the exercises on page 141 and 142 after pointing out to them that they should pay attention to the fact that the third exercise deals with their real life although it is not shown in the way it is written.

T6 started the session by reminding students of what they learned in grade six. She asked them if they knew what is meant by calculating the percent of a number and to give her examples of situations in real life that involve percents.

When it comes to connecting to real life situations while explaining the lesson, some of the teachers did an effort to implement this technique. In general however, teachers do not seem to be able to give such examples naturally and in a consistent manner. For example, after asking the students to solve a problem, T2 jotted the answer "25,000L.L." on the board without explanation and asked the students who found the right answer on the way they got the answer. One of the students explained that 50% is half the price so half of 50,000 that means 25,000L.L. Without further explanation, T2 asked the class about the importance of such problem in real-life to drive students to conclude that knowing about percent is important because at many times there are sales in shops, and it is important to be able to calculate the discounted price of a product to be able to tell whether they can afford buying it or not. The session with T3 provides also an illustration of this reality. After asking students to read and tell them what the lesson is about, and then solving a couple of problems following in the book that are about real life situations, T3 went over the text of the chapter slowly explaining each detail without including any real-life problems.

Teaching Strategies

Assessing teachers' strategies boils down to observing whether they involve students in constructive learning i.e. whether they ask them questions, hence drive them to think, in addition to giving students an opportunity to interact with each other and with the teacher. Furthermore, this task also entails observing whether teachers make appropriate use of mathematical language, involve students in activities, and whether their teaching is student-centered. Table 11 below summarizes the observations noted down by the researcher during the classroom observations of teachers' approaches to teach proportionality.

Asking questions. Among all the teachers observed, T1 and T4 were most skillful in using the technique of asking students questions. Actually during the session T1 was teaching, almost every sentence was a question. T4 used the technique as follows; after the application with which she started the session, she wrote the answer on the board using cross multiplication and asked the students to check if the answer is right or wrong then explain. The teacher, then, began asking students different questions such as: "How should I write this in a ratio form?", "Should the numerator and denominator be the same unit? Why?" and "Should I put an equal in between? If yes what do we mean by it? Does it mean equivalent?" T4 asked the students several similar problems and asked many questions along with them to make sure that students understood the idea. T2 limited the questions to only to few instances. She started the lesson with a general question. She asked them if they knew what proportionality meant. When students started giving answers, she asked them a more specific one. From this question she started explaining the lesson. As to T3, asked questions only towards the end of the session as if to confirm students' understanding.

Table 11

Different Strategies Teachers Used during Classroom Observations

	Teacher uses questioning	Teacher Interacts with Students	Teacher's use of mathematical language	Use of Activities	Teaching is Student Centered	Teacher guides Students' Thinking
T1	Often	Always	Rarely	Often	Always	Often Gives each student the chance
T2	Often Ask about importance of the lesson	Often Ask students for examples	Always Many real life problems	Often	Often	Always
T3	Rarely (Just in the end of the lesson)	Rarely	Often A lot through explanation	Fairly What is in the book	Rarely	Fairly
T4	Often	Often	Rarely	Always	Always	Always
T5	Rarely	Rarely	Rarely	Rarely	Never	Rarely
T6	Often	Often	Often	Often + uses group work	Often	Often

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Interacting with students. While T5 rarely interacted with students, and T3 limited interaction to the end of the session when she asked students whether they knew what profit meant, the other teachers were more successful in creating situations that allowed the class to be more interactive. In general, in most classrooms teachers succeeded in maintaining an atmosphere of informality and casualness, which is essential for any kind of interaction to take place.

For example, in T1's session, when a student gave a wrong answer to a problem, one of the students began shouting that the answer is not right and that there is one more step to go. He went to the board and added what was missing in the solution.

After reminding students about types of proportionality they have already learned (fraction, percent, and ratio), then asking the questions with which she started the session, T2 asked students to give different examples of these types. One of the students referred to the sales season where they buy many clothes on sale like 50% sale. T2 asked two students to act the example in front of the class. One of the students said "How much is this shirt" while the other answered: "it is for 40,000L.L. but it is on 50% sale". Then T2 asked the students to solve the problem on a piece of paper. At the end she jotted the answer "20, 000L.L." on the board without explanation and asked one of the students who gave the right answer to explain how he got the answer. He explained that 50% is half the price so half of 40, 000 is 20, 000L.L.

T4 made an effort to guide students and communicate with them while they were learning. After starting the session by asking students about the price of five bagels if the price of eight bagels is $4.5, students directly tried solving the problem on their notebook while the teacher circulated asking them about the way they are using.

T6, on the other hand, succeeded in enhancing students' learning by making them interact together. After getting some answers to the questions with which she started the lesson, she divided the students into groups of four and gave each group a worksheet of four different problems. The first one deals with percents, the second with ingredients for baking a cake, the third is about Taxes in Lebanon and the last one about speed. This group work fostered discussion and interaction among the students.

Use of mathematical language. T2 and T3 come closest to the appropriate use of mathematical language by involving students in solving problems about real life situations or giving examples of real life situations during explanation. For example, T2, after asking students to calculate the price of a shirt on which there is 50% sale, she asked the class about the importance of such problem in real life. One of the students said that it is important because at many times there are sales in shops. Towards the end of the session, she asked the students to solve the same problem for the next day supposing that the shop was on 20%, 40% and 70% sale instead of 50%. It is important to note that the use of mathematical language is to help students understand mathematics better and exhibit their understanding of mathematical concepts in real life.

T6 is less successful than T2 and T3 in making use of mathematical language, while the other teachers, T1, T4, and T5 rarely made appropriate use of mathematical language.

Activities. While T5 rarely provides students with an opportunity to take part in activities, T2 emphasizes simulation of situations of buying/selling products, and T3 focuses on reading what is in the book. Activities are featured in the remaining observation sessions. For example, T1 asked the students to go with her to the shop in the school. She asked two students to go over and buy things. She had previously asked the shop keeper to do some discounts and ask the students some questions. Student one asked for a sandwich which is for 5000L.L. and has 40% sale on it; the second student asked for a piece of cake. The shop keeper asked the latter to guess the number of eggs in one piece of cake if it takes four eggs to do eight pieces. In the case of T4, the teacher relied on one real-life problem and added other problems to make sure students understood the example given without applying any hands-on or real activities. As for T6, she divided the students into groups of four and gave each group a worksheet of four different problems. Students were collaborative and constructive in reading problems to one another and cooperatively searching for solutions. This student-to-student interaction shifted the attention away from the teacher to students who helped each other in teamwork. The teacher's responsibility was only to guide and assist the students towards accomplishing their work.

Student centered teaching and raising students' thinking. T3 and T5 rely heavily on the textbook which is not student centered and is not successful in driving students to think for themselves (see section on national math textbooks). For example T3 asked the students how to write a percent in a fraction form; after getting several answers she wrote it on the board as "x % = x/100" so students began asking about the x till she changed it into a number without explaining. Going to the other part of the table in the book, she said that discount on the price means multiplying the price by the percent in fraction form and asked the students to do it using their calculators and write the answer. Furthermore, T3 moved on and asked the students to solve the first two drill and practice exercises in the book, she skipped all other sections, and asked students to solve two exercises in the "problems" section after she explained to them the meaning of the word "profit".

The rest of the teachers demonstrated more ability to make their teaching student-centered. They also showed more commitment to making the learning experience an opportunity for students to learn by thinking critically. This is evidenced in their skilful use of questions, creating situations of interaction in the classroom, and activities of the sort they involved their students in, whereby they had to think about real life situations and come up with solutions to problems they could encounter in their daily lives. It is also apparent through instances of the following sort.

For example, following taking students to the shop in the school, T1 asked a couple of them to stand up in front of the class and explain what they did in the shop and try to decide on how much to pay the shop keeper. Then T1 asked each student in the class to try to find an answer for both problems on a piece of paper while going around and checking their solutions without commenting. Following this, she asked a student who had obtained the right answer to solve the first problem on the board. He had to calculate the price of a sandwich that cost 5000 L.L. and on which there was a 40% sale. The equation he wrote was " $5000 \times \frac{40}{100} = \frac{5000 \times 40}{100} = \frac{200000}{100} = 2000$. When the answer was written on the board, one of the students claimed that the answer is not right and that there is one more step to go; he went to the board and added $5000 - 2000 = 3000$L.L., which is the sandwich price and explained to the class that his father taught him this way two weeks ago. The teacher then re-explained the reason of subtracting the percent price from the original price and told the students that in any case of percents, they have to use an equation which states:

$$percent\,(discount\,) = original\,(price\,) \times \frac{percent}{100}$$
$$New(price\,) = Original\,(price\,) - percent\,(discount\,)$$

It is important to note that the teacher should have asked a student who did not obtain the right answer to solve the problem on the board so that others who made the same mistake understand why they are mistaken.

After this problem, T1 asked a student to solve the second problem i.e. calculating how many eggs are there in a piece of cake knowing the number of eggs it takes to make 8 pieces. The student who was in the shop told the other students that he thinks what the teacher of last year taught them would help. His classmates looked at him telling him they did not understand what he was talking about. So he wrote on the board:

$\begin{array}{ll} 4eggs & 8\,pieces \\ \_?\_ & 1\,piece \end{array}$ and continued that they should cross multiply $\frac{1 \times 4}{8} = \frac{4}{8} = 0.5$

The teacher then tried explaining and discussing the way of solving; then she asked the students to try to find other real life problems similar to the ones they had from daily life for the next session.

Types of Exercises and Problems Offered

Analysis of the types of exercises and problems that teachers offer, boils down to finding out the extent to which students are given the opportunity to solve exercises and problems that require them to think about real life situations in order to be able to come up with the correct answer. Table 12 below highlights the prevalence of real life problems in the exercises and problems that teachers used during the classroom observations.

Table 12

Exercises and Problems Teachers Used during Classroom Observations

T1	T2	T3	T4	T5	T6
Real life	Real life	Real life but goes over the whole problems in the book one by one	Real life but not related to students	No	Real life and lot of exercises

As the table above shows, all the teachers in this study, except T5, showed an awareness of the importance of real life problems in teaching students math in general and proportionality in particular. The problems that T1, T2, T3, T4 and T6 asked students to solve during their respective observation sessions are all about real life situations. Among these teachers though, one could argue that the problem that T4 used was not really related to students' real life since it focused on the speed of trains, while in reality there are no trains in Lebanon.

Still, however, they are far from implementing this approach in a consistent way. For example, after reading the lesson's objectives, T3 asked her students to read what the first activity was about, she explained the exercise to them and then she asked them to solve the problem which is about finding a price and a discount percent. After solving this problem, T3 however, went slowly over the text of the chapter explaining each detail without including any real-life problems. Then she asked them to solve the first two exercises in the book as drill and practice and asked students to solve the first two problems in the last section of the chapter after reminding them of the meaning of profit as follows: "Remember, profit is the opposite of discount and sale… It means the price will increase in the end… so in the last step you have to add to the original price not subtract from it".

With the analytical description of the types of exercises and problems that teachers who participated in this study when teaching proportionality, we come to the end of the section on teachers' approaches to teaching proportionality. Far from claiming that these findings describe the reality in all schools of Lebanon, they nevertheless give an idea about what is happening and the issues that need to be addressed. The following section focuses on students, namely how they process the information and learning experiences they are exposed to while they are learning proportionality.

Summary of Observation Findings

When it comes to teachers' approaches to teaching proportionality, the non-participant observation of teachers showed that in general they are trying to commit to the constructivist theory one way or another. Their approach to teaching the topic reveals a conviction that students should learn by exploration rather than rules, formulas, and equations. There is a serious effort, at least on behalf of the aforementioned teachers to use problems about real life situations, which enables students to connect to their surrounding while solving problems. Actually, the constructivist theory stipulates that problems should make sense to students. This approach differs from that used by T5 which focuses on the transmission of information and knowledge to students. Students, on the other hand, should create their own ways of solving a problem and make their own hypothesis to explain what they are doing such as the students in the classes of T1 and T2.

In mathematics, a constructivist approach means less focus on drill and practice and more focus on problem-solving skills and application of real-life situations from the very beginning. As T1 and T2 did, students should be encouraged to participate in social interactive environment to be able to connect what is given in the classroom to their real surrounding. At the same time, the use of group work like T6 did, helps children learn different ideas, concepts and strategies of solving a problem from each other; besides it helps them learn the importance of collaboration.

Furthermore, teachers' interaction with the students showed that most of the teachers focused less on explaining the content of the lesson and more on the process students needed to learn, understand, and acquire. Teachers were more prone to ask students more questions for example "what do we mean by this?" "Where can we see this in real life?" "Give me more examples."

As for the use of mathematical language, activities dealing with real-life problem situations increase the opportunity that students speak in the classroom. This also helps teachers perceive what students are thinking and differentiate between those who have understood the concept and others who have memorized the steps to solve a drill and practice exercise.

As encouraging this reality is the truth of the matter is that teachers have not yet succeeded in fully implementing a constructivist approach in teaching proportionality and math in general.

More effort should be done on behalf of teachers so that students: have instructional activities in the form of problems to solve; develop answers to problems from their own point of view and experience; be encouraged to interact with the teacher and with students peers.

Students' Approaches to Solving Proportionality Problems

This section is dedicated to the analysis of data collected during clinical interviews with students solving problems on proportionality. Specifically, in order to investigate how students in grades six and seven approach solving problems related to proportionality, 12 clinical interviews were conducted with 24 students from these two grades in the three schools participating in this study. The participants were selected according to their achievement level and their teachers' recommendations. Each interview was conducted with two pairs of students, one pair consisting of a student with a high level of achievement and one student had a mid-high level of achievement, and the second pair of one student with low level of achievement and the second with mid-low level of achievement. During these sessions students were asked to solve five problems (Appendix E).

The analysis of how students solve proportionality problems focused on the four famous phases set by Polya (1957): what students did when they were first presented with the problem; what did they do while reading and rereading. In addition, there was emphasis on strategies that students used to solve problems, and whether they checked their answers once they finished solving the problem.

Reading the Problem

Reading the problem refers to understanding the meaning of words and symbols in the problem. Table 13 below identifies which students among those who participated in the clinical interviews read the problem with understanding before embarking on solving it.

Table 13

Number of Students who Read the Problem with Understanding

Achievement level	High socioeconomic level		Middle socioeconomic level		Low socioeconomic level	
	Grade 6	Grade 7	Grade 6	Grade 7	Grade 6	Grade 7
Low-mid-low	No	No	No	Yes	No	Yes
High-mid-high	No	Yes	Yes	Yes	No	No

Yes= read with understanding No = just picked up numbers

Out of the 12 students, seven took their time to read the problem carefully. The remaining five just took out the numbers, read the question or scanned the problem quickly, and then tried to connect and relate the numbers with operations in a way to get the answer. It is noteworthy that high and mid-high achievement level students tend to read the problem more carefully than those of low and mid-low achievers; they put more effort to make sure they did not miss any information. It is also remarkable that grade 7 students make the effort to read the problem and understand it much more than grade 6 do. This indicates that as students progress through their study of mathematics, they tend to adopt a more rational approach to problems, consisting of adopting the problem and trying to understand it rather than starting immediately to use the numbers in operations.

Rereading the Problem

Often students are supposed to reread the problem during the solution process to make sure they did not miss any important piece of information and decide on the best way to solve the problem. Rereading the problem indicates involvement and adoption of the problem, as well as perseverance to solve it. Table 14 identifies the students who used this technique in this study.

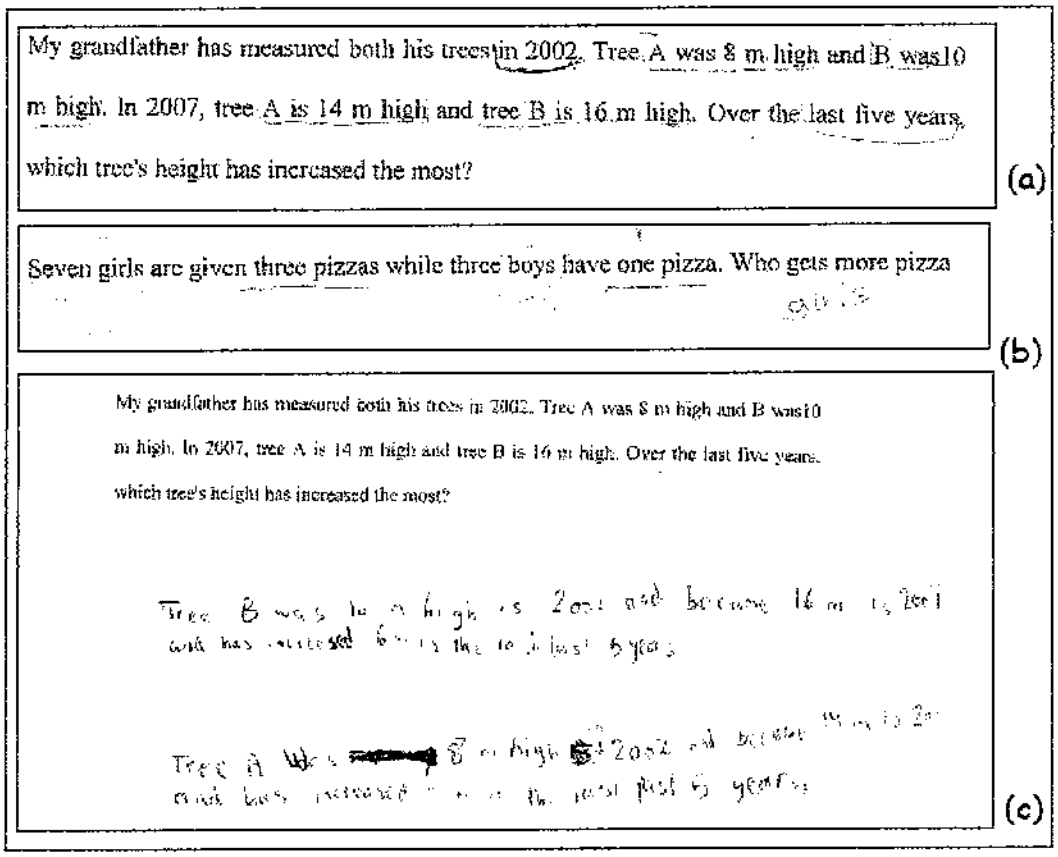
Table 14

Number of Students who Reread the Problem

Achievement level	High socioeconomic level		Middle socioeconomic level		Low socioeconomic level	
	Grade 6	Grade 7	Grade 6	Grade 7	Grade 6	Grade 7
Low-mid-low	No	No	No	Yes	No	Yes
High-mid-high	No	Yes	Yes	Yes	No	No

Yes=re-read with understanding No = Read the problem only once

When rereading the problem, two of the pairs of students in the low and mid-low achievement level group and three from the high and mid-high achievement level group sorted out the information needed by doing one or more of the following:



My grandfather has measured both his trees in 2002. Tree A was 8 m high and B was10 m high. In 2007, tree A is 14 m high and tree B is 16 m high. Over the last five years, which tree's height has increased the most?

Seven girls are given three pizzas while three boys have one pizza. Who gets more pizza

My grandfather has measured both his trees in 2002. Tree A was 8 m high and B was10 m high. In 2007, tree A is 14 m high and tree B is 16 m high. Over the last five years, which tree's height has increased the most?

Tree B was 10 m high in 2002 and became 16 m in 2007 and has increased 6 m in the [illegible] last 5 years

Tree A was 8 m high in 2002 and became 14 m in 20 and has increased [illegible] the [illegible] past 5 years

Figure 1: Actions done when re-reading the problem.

(a) underlining needed information or identifying it by writing it again,

(b) circling or underlining the unknown or what is required to find in the problem,

(c) scratching information that is not needed or making an organized list of what is needed from the problem.

In addition, two out of the three high and mid-high pairs of students re-stated the problem to their partners after re-reading it in a low voice to make sure they understand all the information.

Students' Use of Brainstorming

Brainstorming refers to coming up with ideas that might not be right and need to be revised or improved by the students themselves or their partners who take the idea to the next stage (Businessball.Com, n.d.). Brainstorming is known to increase the chances of solving a problem, especially in mathematics. Table 15 below identifies the students who used brainstorming while solving the problem with their partner.

Table 15

Number of Students who Used Brainstorming

Achievement level	High socioeconomic level		Middle socioeconomic level		Low socioeconomic level	
	Grade 6	Grade 7	Grade 6	Grade 7	Grade 6	Grade 7
Low-mid-low	Yes	Yes	No	Yes	No	Yes
High-mid-high	No	Yes	No	No	Yes	No

Yes=used brainstorming No = began writing directly

In this study, half of the pairs used brainstorming while the other half did not do so. It is noteworthy that the majority of students in the low and mid-low achievement level groups resorted to brainstorming while the majority in the high and mid-high achievement level group did not use this approach. This could be related to the fact that students in the high and mid-high groups have a better grasp of proportionality related concepts and are more adept at solving problems in general. It is most likely that, unlike low and mid-low level students, they are able to figure out the best approach to solve the problem at hand without needing to discuss it among themselves. They tend to be more at ease when they read the problem and they set to solve it and thus do not seek reassurance from their peers, or at least, they need less reassurance when it comes to the best approach to solve the problem at hand.

Strategies Used by Students in Solving the Problems

This section is dedicated to the analytical description of the strategies used by the 24 students who took part in the clinical interviews to solve the five problems mentioned above (Appendix E). After stating each problem, an analytical description of students' approaches is provided. From the many strategies available to solve problems, participants in this study used the following: writing a logical reasoning (LR), guess and check (GC), making a table (MT), working backwards (WB), using an equation (UE), drawing a picture (DP), using a simpler problem (SP), looking for a pattern (LP), and number relations (NR). Below is a detailed description of the 12 pairs of students' approaches to solving each of the five problems.

Problem one. My grandfather has measured both his trees in 2002. Tree A was 8 m high and B was10 m high. In 2007, tree A is 14 m high and tree B is 16 m high. Over the last five years, which tree's height has increased the most?

The problem is intended to test the student's understanding of the concept "increase" as a ratio of increase relative to the initial height. Though both trees' heights have increased by 6m., the ratio of increase should be considered: Tree A has increased by 6/8 (=75 %) in five years, and Tree B has increased by 6/10 (=60%) in 5 years.

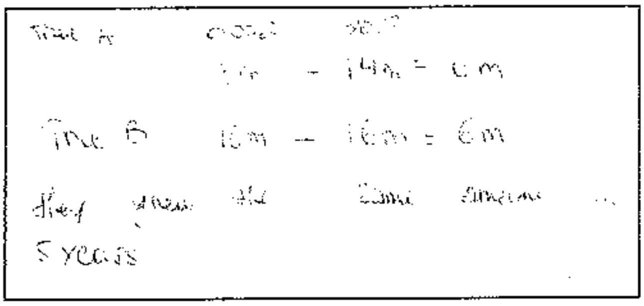
Table 16

Strategies Used to Solve Problem One

Grade	School Socio-economic level	Achievement Level	Strategies for Problem 1	Result
6	High	Low mid-low	NR	NC
		High mid- high	MT	NC
	Middle	Low mid-low	NR	NC
		High mid- high	LR	NC
	Low	Low mid-low	NR	NC
		High mid- high	LR	NC
7	High	Low mid-low	NR	NC
		High mid- high	NR	NC
	Middle	Low mid-low	NR	NC
		High mid- high	NR	NC
	Low	Low mid-low	NR	NC
		High mid- high	NR	C

C= correct answer NC = not correct answer
LR = logical reasoning MT = making a table NR = number relations

To begin with the first problem, grade six students used straightforward ways to get the answer by reasoning and explaining the problem in their own words and by relating numbers in the problem to get an answer, for example, by subtracting (see Figure 2). When subtracting, most of the students noticed that both answers are the same and concluded the answer.



Tree A [illegible] [illegible]
[illegible] − 14m = 6m
Tree B 16m − [illegible] = 6m
they grew the same amount ...
5 years

Figure 2: Subtraction used as a method to solve the first problem

Students' answers to this question revealed that most of grade six students do not understand what is meant by the concept "increased" and they do not understand how it is related to what they learned about proportionality. Most probably, they solved the problem using the following reasoning: since the height of each of the two trees increased by six meters, then none of the trees increased more than the other. This probably explains why most of the answers were wrong (5 out of 6 students), while only one pair of students used the number relation and somehow succeeded in getting the right answer. But as the figure below illustrates, most likely that students did not know how they got the answer or why they did a division (see figure 3).

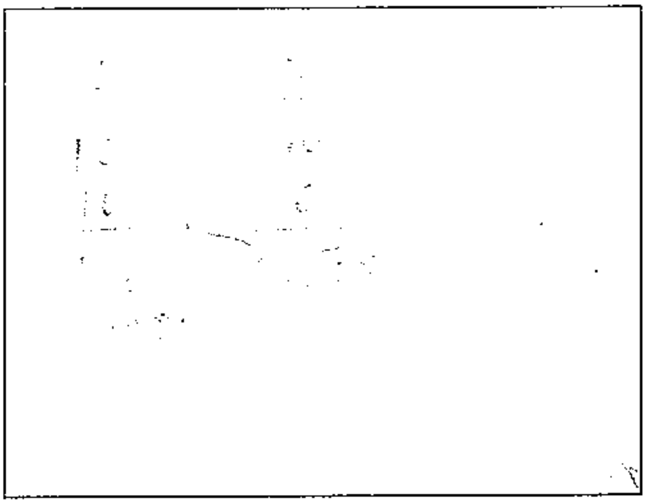


Figure 3: Solution to problem one using number relations

One of the papers tabulated the given information of the problem while they were reading the problem. The students took notes that they put in an organized form (see figure 4). Still, they were not able to find the answer in the end.

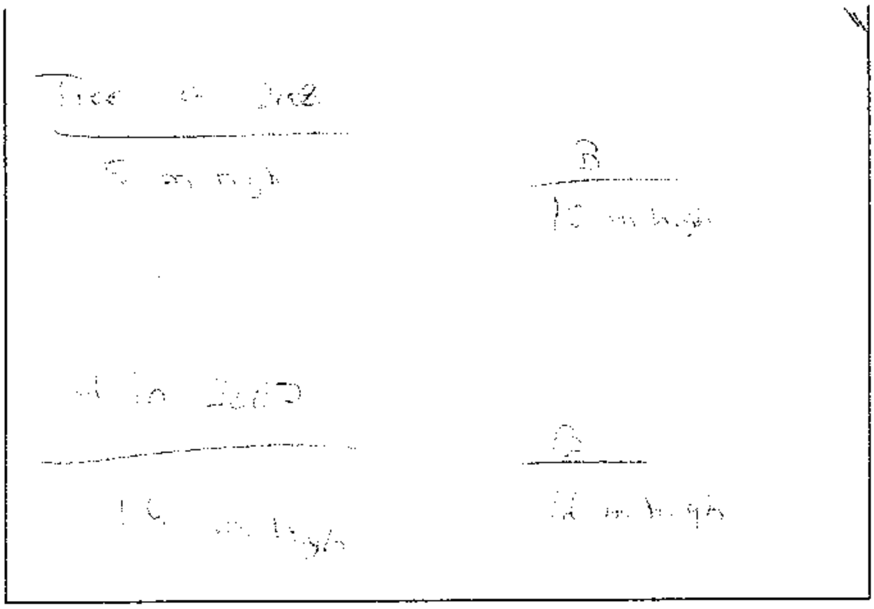


Figure 4: Explaining the given of the first problem in a table format.

Students who attempted to solve the problem using multiplication, division, addition, or some logical reasoning, that is they tried to guess the right answer, were not able to reach a solution either. (see figure 5).

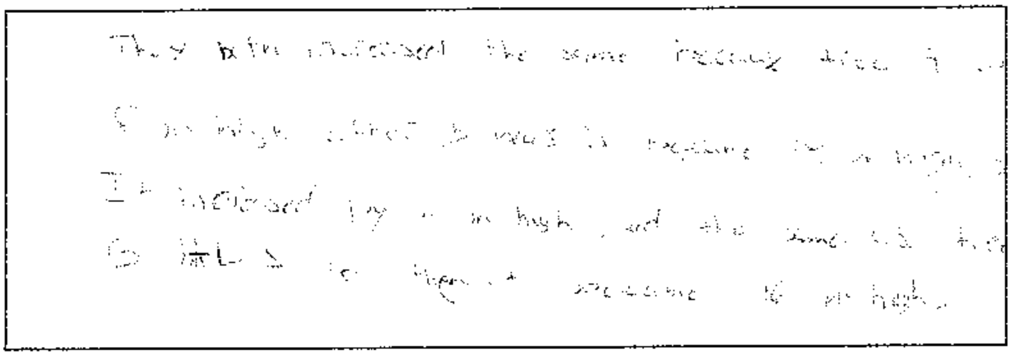


Figure 5: Logical reasoning in trying to solve the first problem.

Both mid-high and high achievement level students in grades six and seven in high socioeconomic level schools got the wrong answer when they set to solve the first problem in an algebraic manner combining numbers with each other to get an answer. As for the mid-low and low achievement level students in these schools, grade six students chose to solve the problem in a syntactic manner (writing the answer in words without using numbers), yet they also got the wrong answer while all grade seven students got the wrong answer, since they were just trying to relate numbers but they actually did not think about increase as a ratio.

In the middle socioeconomic level schools, both grade six pairs got wrong answers. One pair of students, however, used strictly algebraic approach to solve the problem, while the other pair chose to only use language. As for grade seven, none had a right answer.

The low socioeconomic level schools also showed incorrect answers in this problem, yet only one paper for high-mid high achievement student out of four had a correct answer. They explained that they solved the problem using number relations. Given, however, that they did not quite master this technique, they gave the teacher two answers that they reached using this technique. They left it to the teacher to point out to them which one is correct and which one is wrong, and give them a grade for the correct answer. All other papers showed an erroneous approach to solving the problem which lead to incorrect answers.

Problem two. If a certain school has 5 girls to every 6 boys in each class, how many girls are there for 24 boys?

It is intended to test whether students are able to identify the right relation (equation or equality of ratios) between the numbers. Students can solve the problem using cross multiplication, as well as proportion. The correct solution is explained below (see Figure 6)

5 girls → 6 boys

X girls → 24 boys

$$X = \frac{24 \times 5}{6} = 20 \text{ girls}$$

$$\frac{5girls}{\_\,girls} = \frac{6boys}{24boys}$$

so if we multiply 6 by 4

to get a 24, we should multiply 5 by

4 to get 20 girls

Figure 6: Solution of problem two using cross multiplication (left) or proportion (right)

Table 17 summarizes student's approaches to solve the problem.

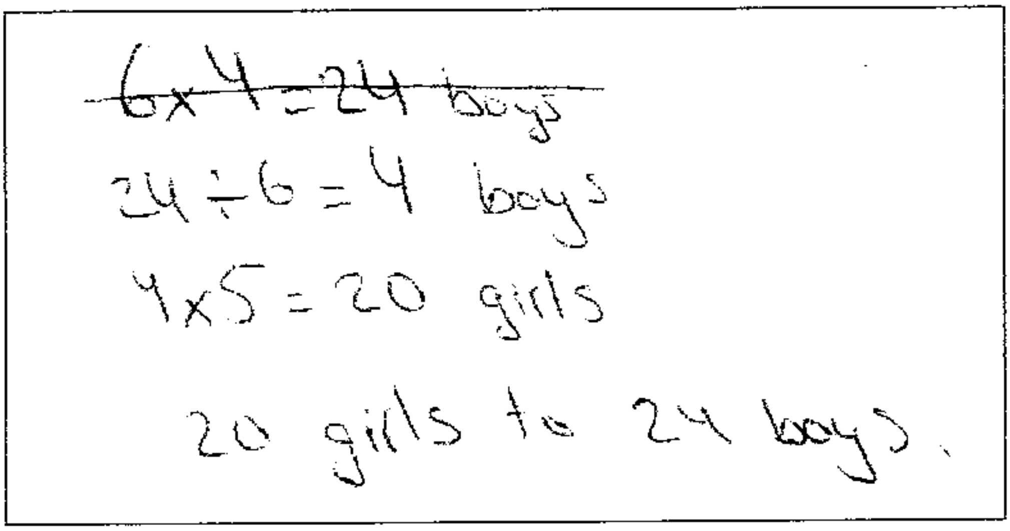
Table 17

Strategies Used to Solve Problem Two

Grade	School Socio-economic level	Achievement Level	Strategies for Problem 2	Result
6	High	Low mid-low	WB	C
		High mid- high	WB	C
	Middle	Low mid-low	WB	C
		High mid- high	LP	C
	Low	Low mid-low	WB	C
		High mid- high	LP	C
7	High	Low mid-low	NR	NC
		High mid- high	WB + GC	C
	Middle	Low mid-low	WB	C
		High mid- high	DP	NC
	Low	Low mid-low	LP	C
		High mid- high	GC	NC

C= correct answer NC = not correct answer
GC = guess and check WB = working backwards DP = drawing a picture LP = looking for a pattern
NR = number relations

Problem two assesses students' understanding of ratios. Half of the students from both grades and both levels of achievement worked backwards to solve the problem. They include four students in grade six from the three schools, specifically all those in the low-mid-low group and the students from the high and mid-high achievement level group in the high socioeconomic level school. As for grade seven students, only two of them used this technique. They are the students in the high socioeconomic level school in the high and mid-high achievement level group and the students in the middle socioeconomic level school in the low and mid-low achievement level group. These students began with the number of boys in class and divided it by the initial number of boys to get a number and multiplied the answer by the initial number of girls (see Figure 7).



~~6x4=24 boys~~

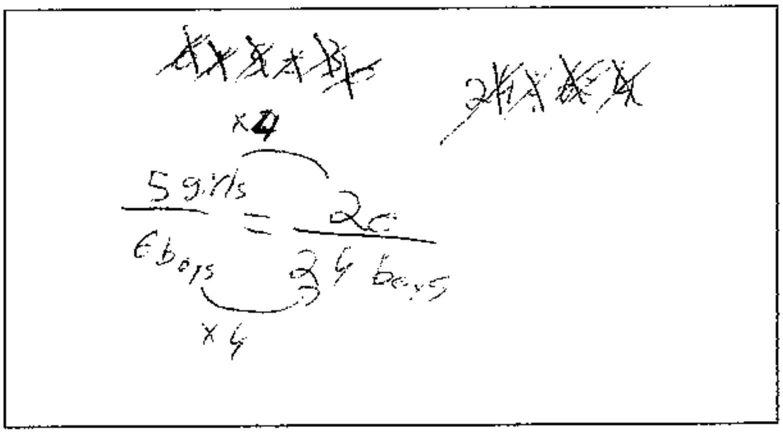
24÷6=4 boys

4x5=20 girls

20 girls to 24 boys.

Figure 7: Working backwards as a method to solve the second problem

Three students used the looking for a pattern (LP) strategy. They include two pairs of grade six students from the high and mid-high achievement level group, one from the middle and one from low socioeconomic level school. The third pair is in grade seven and belongs to the low-mid-low level in the low socioeconomic level school. They used pattern and ratio (see figure 8) to check the relation between numerators fill the missing umber in the denominator.



$$\frac{5\text{ girls}}{6\text{ boys}} \overset{\times 4}{=} \frac{20}{24\text{ boys}}$$

x 4

Figure 8: Using proportion to solve the second problem

One student from grade seven at a high-mid-high level belonging to the middle socioeconomic level school solved the problem by drawing tallies (see figure 9) for both boys and girls and then they added the excess number but actually were not able to continue and obtained wrong answer.

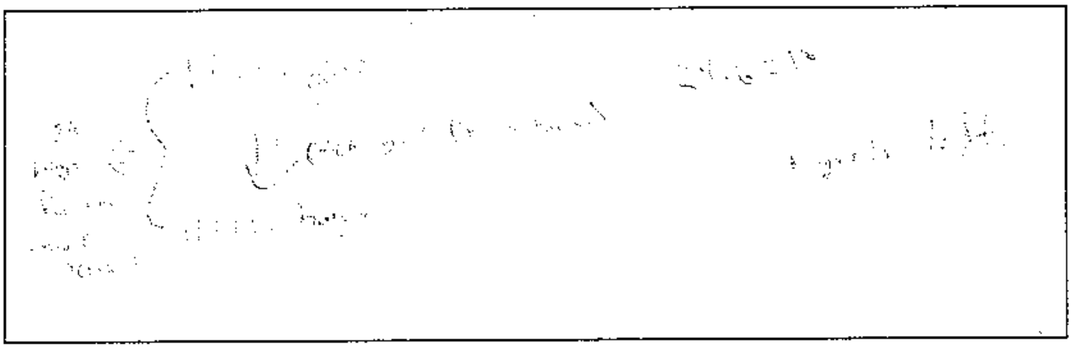
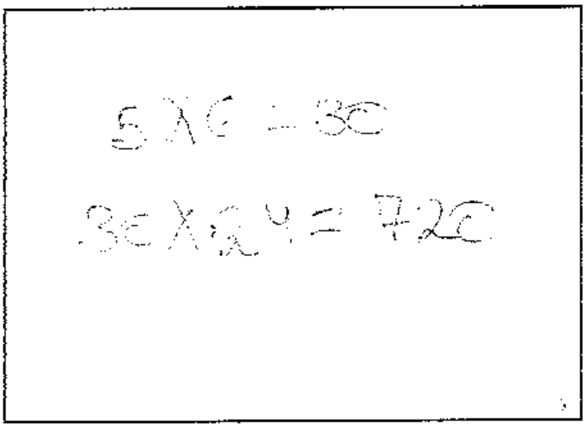


Figure 9: Trying to solve the second problem by drawing tallies

At the same time the students who just guessed the way to solve the problem like subtracting or multiplying the numbers (see figure 10) obtained a wrong answers.



5 X 6 = 30

30 X 24 = 720

Figure 10: An example of a wrong answer reached using number relations to solve the second problem

All papers of high socioeconomic level schools, except one, got a right answer, but they encountered some obstacles from arithmetic error due to speed; for example, one had the following equation written 6÷24 = 4 for the second problem. The interviewer asked one of the grade seven students at the high and mid-high achievement level to explain the way he solved the problem:

> Interviewer: Can you read this problem you are working on in a loud voice.
> Student: If a certain school has 5 girls to every 6 boys in each class, how many girls are there for 24 boys?
> I: How are you going to answer this?
> S: We will do a proportion. (While looking at the partner and then nodding his head)
> I: Ok … go ahead and solve it.
> S: I will put the number of boys over that of girls and on the other side of the equal the same.
> I: Ok so what do you get?
> St: I am thinking what we multiply 6 with to get 24. We multiply by four so we multiply also 5 by 4. Now we have 20.
> I: OK, so that is the solution to the problem?
> S: Yes 20 girls.
> (Grade seven student, high and mid-high achievement level, and middle socioeconomic level school)

The students from the middle socioeconomic level schools did the same; except for one of the students from grade six (mid-low and low level). He used tallies to count the number of boys and girls in class, and failed to reach a final answer. When the interviewer asked him to explain the method he used, here what he had to say:

> Interviewer: Please tell me how did you get the answer for this problem?
> Student: (counting the number of tallies drawn in the paper). The number of sticks is the number of girls and boys in each class. So we drew five sticks for girls and six for boys.
> I: Then what did you do?
> S: Here there should be one girl for each boy so there is one boy more (pointing to the tallies again) but I think we should only do 24 -6 to see how many girls we still need to be equal to boys.
> I: So?
> S: There should be 18 girls in the class for each 24 boys. (Silence) The same as 5 girls to every 6 boys.
> (Grade six student, mid-low and low achievement level, and middle socioeconomic level school)

Problem three. Seven girls are given three pizzas while three boys have one pizza. Who gets more pizza?

This problem assesses students' understanding of fractions as a first step. Students can solve the problem suing different ways of comparing fractions by cross multiplication, dividing fractions, numerical conversion when converting to same denominator, and even drawing pictures to show equivalent fractions. The correct solution of the problem goes as follows:

7 girls → each girl $= \frac{3}{7}$ pizza for each girl

3 boys → each boy $= \frac{1}{3}$ pizza for each boy

$\frac{3}{7} and \frac{1}{3} \Rightarrow \frac{9}{21} > \frac{7}{21}$ ➔ Girls have more pizza

Figure 11: Correct solution of problem three

Table 18 summarizes students' approaches to solve the problem.

Table 18

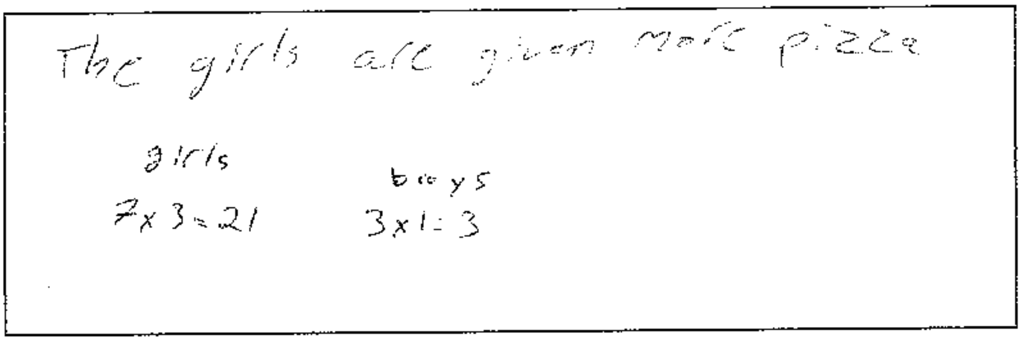
Strategies Used to Solve Problem Three

Grade	School Socio-economic level	Achievement Level	Strategies for Problem 3	Result
6	High	Low mid-low	NR	NC
		High mid- high	NR	NC
	Middle	Low mid-low	LR	C
		High mid- high	LR	C
	Low	Low mid-low	DP	NC
		High mid- high	LR	NC
7	High	Low mid-low	DP	C
		High mid- high	DP	C
	Middle	Low mid-low	LR	C
		High mid- high	NR	C
	Low	Low mid-low	LR	C
		High mid- high	NR	C

C= correct answer NC = not correct answer

DP = drawing a picture LR = logical reasoning NR = number relations

Problem three evaluates students' understanding of fractions. Four students used the number relations strategy. They include the two grade six pairs of students in the high socioeconomic level school and two grade seven pairs of students who belong to the high and mid-high achievement level group but one of them is in the middle socioeconomic level school while the second in the low socioeconomic level school (see figure 12).



The girls are given more pizza

girls
7x3=21

boys
3x1=3

Figure 12: Using number relations in problem three

They used number relations by constructing and examining divided quantities to find an answer. Half of the students had a wrong answer using this way while others got a right answer (see figure 13)

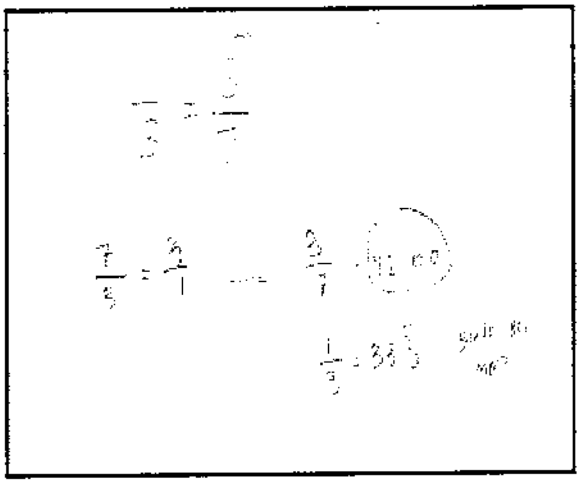
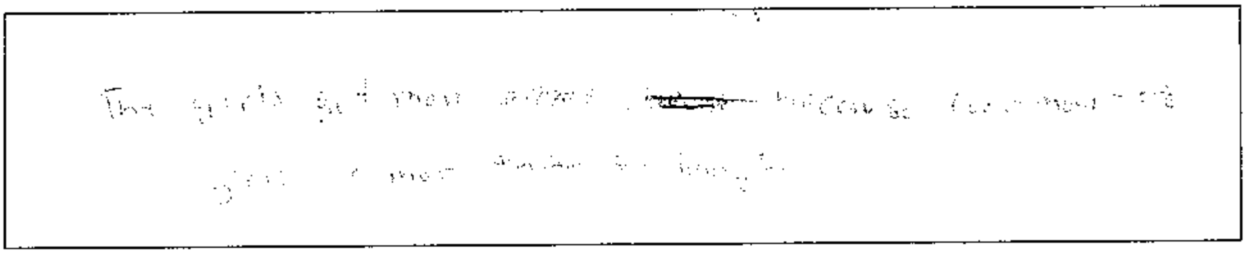


Figure 13: Using fractions to solve problem three

Five student pairs used logical reasoning (LR). They include the two grade six student pairs in the middle socioeconomic level school, the low and mid-low achievement level students in grade six in the low socioeconomic level school, and the two grade seven low and mid-low achievement level student pairs from the middle and low socioeconomic level schools. And although the logical reasoning is wrong yet the answer is right (see figure 14). The students found it logical for girls to have more pizza since their number is more than of boys. One of them however, did not succeed in finding the right answer (grade six, low socioeconomic level school, high-mid-high level).



The girls get more pizza because the number of girls is more than the boys.

Figure 14: Using Logical reasoning to solve problem three

The remaining students used the drawing a picture (DP) strategy. They drew pictures of pizzas, boys and girls and tried to give each a piece accordingly (see Figure 15).

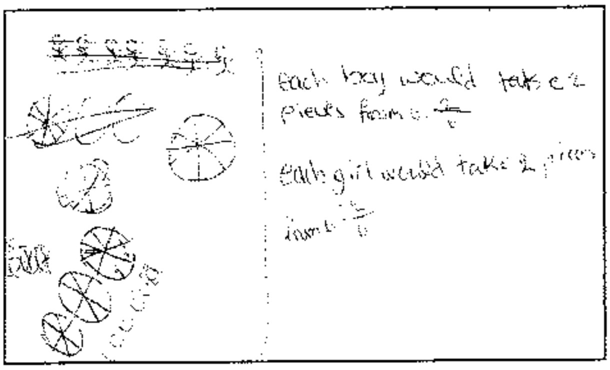


Figure 15: Drawing pictures as an approach to solve the third problem

All students in middle socioeconomic level schools got right answers but only one paper had a right proof while all others believed that since the number of girls is more, then they would have more pizza.

In the low socioeconomic level schools, three out of the four papers got the right answer for this problem although they showed a degree of discomfort when trying to solve it. Two used words to prove their answer right and two out of the three got the answer in the end by chance since their way of solving was completely wrong.

Before moving to the next problem, it is important to note that rarely any pair of students in the three schools was able to write an answer in a sentence form in the end, or at least write the word "girls" in the answer; all that was used is numbers combined to get an answer. Furthermore, while trying to solve this problem students resorted frequently to use of drawing; five out of 12 papers had pictures of people and pizza divided among them. In high socioeconomic level schools, two of the groups who resorted to drawing had the right answer. Still some students experienced confusion when trying to relate pictures to the right solution; this is shown through scratching some pictures and some answers to reach to the final answer.

Problem four. Your friend and you are using different road maps of Beirut. On your map, a road 6 cm long is really 24 km long. On your friend's map, a road 24 cm long is really 72 km long. Who is using a larger map of Beirut? Explain.

This problem stresses the importance of direct proportionality in scaling situations used in real life. It is a case that represents multiplicative relation between two quantities. The correct solution is included in figure 16.

6cm → 24 km $1:4 = 0.25$

24 cm → 72 km $1:3 = 0.3$

$1:4 > 1:3$

Then my friend is using a larger map of Beirut

Figure 16: Correct solution of problem four

As for the strategies used by students to solve this problem, they are summarized in table 19.

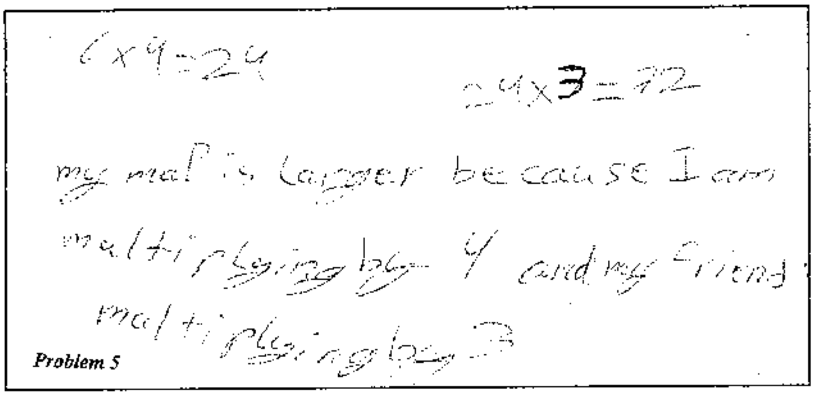
Table 19

Strategies Used to Solve Problem Four

Grade	School Socio-economic level	Achievement Level	Strategies for Problem 4	Result
6	High	Low mid-low	NR	NC
		High mid- high	WB	NC
	Middle	Low mid-low	WB	NC
		High mid- high	NR	C
	Low	Low mid-low	NR	C
		High mid- high	NR	NC
7	High	Low mid-low	NR	NC
		High mid- high	WB	NC
	Middle	Low mid-low	NR	C
		High mid- high	NR	NC
	Low	Low mid-low	NR	NC
		High mid- high	LR	C

C= correct answer NC = not correct answer
WB = working backwards LP = looking for a pattern NR = number relations

In problem four, the grade six students at the high and mid-high achievement level group in the high socioeconomic level school and all students in grade seven who belong to the high and mid-high achievement level group used one of the following three ways: (a) trying to find what is multiplied by 6 to get 24 and what is multiplied by 24 to get 72 and comparing to find the answer, (b) working backwards by dividing 24 by 6 and 72 by 24 then comparing the numbers (see figure 17), or (c) comparing proportions after decomposing fractions (see figure 18). The others just guessed an answer without any explanation and a few attempted to solve the problem using addition, multiplication, or some logical reasoning (see figure 19).

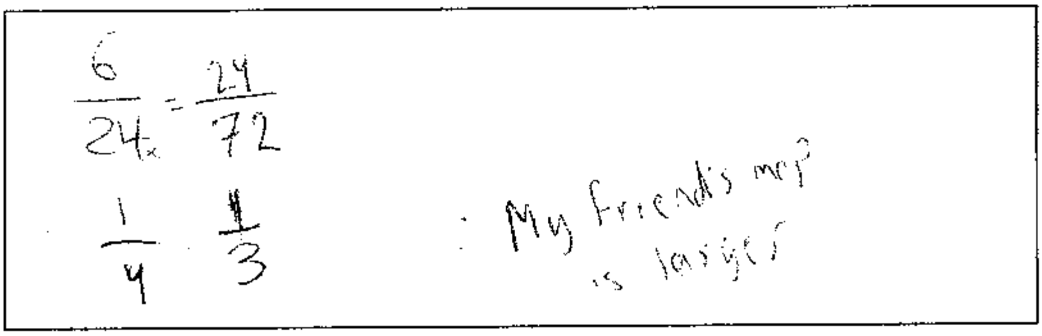


6×4=24 24×3=72

my map is larger because I am multiplying by 4 and my friend multiplying by 3

Problem 5

Figure 17: Working backwards to get the answer of the fourth problem

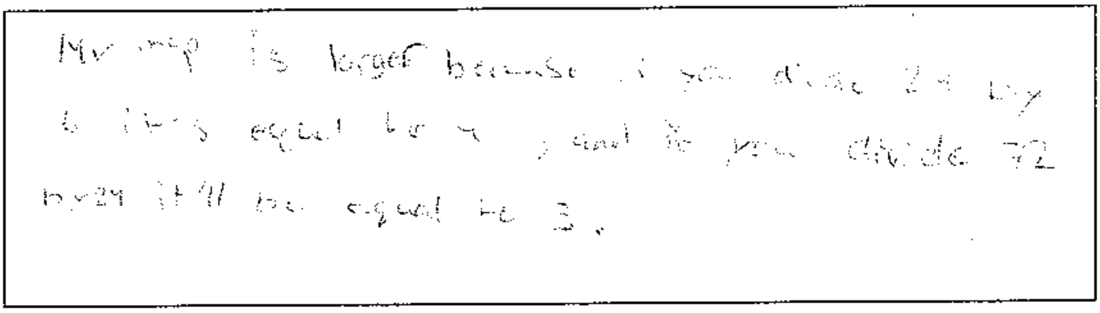


$\frac{6}{24} = \frac{24}{72}$

$\frac{1}{4}$ $\frac{1}{3}$

: My friend's map is larger

Figure 18: Using proportions and number relations in the fourth problem



My map is larger because if you divide 24 by 6 it's equal to 4, and if you divide 72 by 24 it'll be equal to 3.

Figure 19: Use of logical reasoning to solve problem four

In high socioeconomic level schools, no right answers were found. Two out of the three papers had the problem solved using the working backwards method. Students began with the real length of road and related it to that of the map. Two students suggested an inductive proof and they accurately described the solution when asked in the end stating: "Since 24 ÷ 6 = 4 and 72 ÷ 24 = 3, then, in my map each 1 cm is in reality 4km while for my friends, each 1 cm is 3km" (grade 6 students, high and mid-high level).

The same goes for the middle socioeconomic level schools i.e. they used backward solving method. Out of the two papers that had a right answer however, only one paper was able to show some explanation as to how students calculated the answer. The others either arrived at the answer by chance or provided an erroneous explanation. For example one student stated: "My friend is using a big map of Beirut more than you, and there is more km on your friend" (grade 7 students, mid-low and low achievement level).

As for the low socioeconomic level schools, although there were right answers, none of the students had a well stated explanation for the problem solution. Actually, one pair of students divided the numbers to get three and four and they could not connect them to get an answer. The others however, most probably they arrived at the answer by chance.

Problem five. A shirt is priced at 30,000 L.L. in a shop. Bilal wants to buy it, but he decides to wait for the sales season. Under sales, the shirt is 30% discounted. How much would Bilal pay to buy the shirt?

Problem five is a problem that students might face in their daily life while shopping. It stresses the thought of how students might solve the problem and in how many different ways. Following is the solution:

30% sales on 30,000

Discount = $30\% \times 30000 = 30000 \times \frac{30}{100} = 9000$

New price = Original price – Discount

New price = 30,000 – 9,000 = 21, 000 L.L.

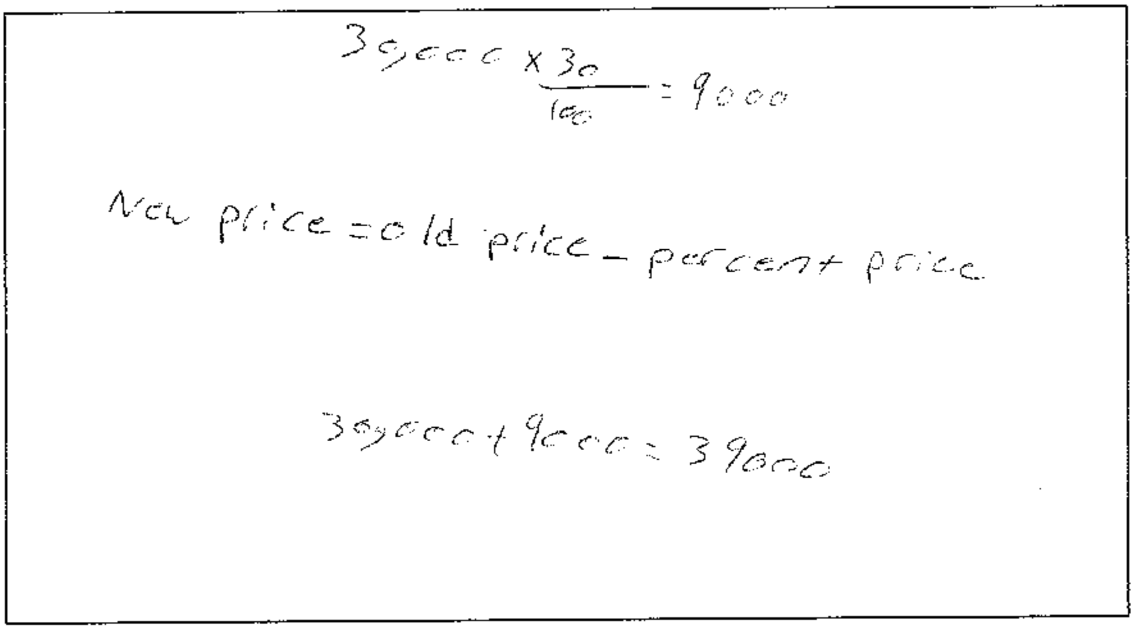
Table 20

Strategies Used to Solve Problem Five

Grade	School Socio-economic level	Achievement Level	Strategies for Problem 5	Result
6	High	Low mid-low	UE	C
		High mid- high	UE	C
	Middle	Low mid-low	NR	NC
		High mid- high	NR	NC
	Low	Low mid-low	UE	NC
		High mid- high	UE	NC
7	High	Low mid-low	UE	NC
		High mid- high	UE	NC
	Middle	Low mid-low	NR	NC
		High mid- high	NR	NC
	Low	Low mid-low	NR	NC
		High mid- high	SP	NC

C= correct answer NC = not correct answer
UE = using an equation SP = using a simpler problem NR = number relations

When solving problem five most students showed that they had memorized the method to use when solving a problem related to percent, i.e. multiplying the percent in fraction form by the price (see figure 20).



$30{,}000 \times \frac{30}{100} = 9000$

New price = old price − percent price

$30{,}000 + 9000 = 39000$

Figure 20: Using memorized equations to solve problem five

Most of the students did the first step of getting the percent price and neglected subtracting it from the original price to get the new price. Some added the number to the original price. One paper connected the problem to an easier one (see figure 21) related to daily life which is a 50 percent sale but could not connect the 50 percent to 30 percent in order to get the answer.

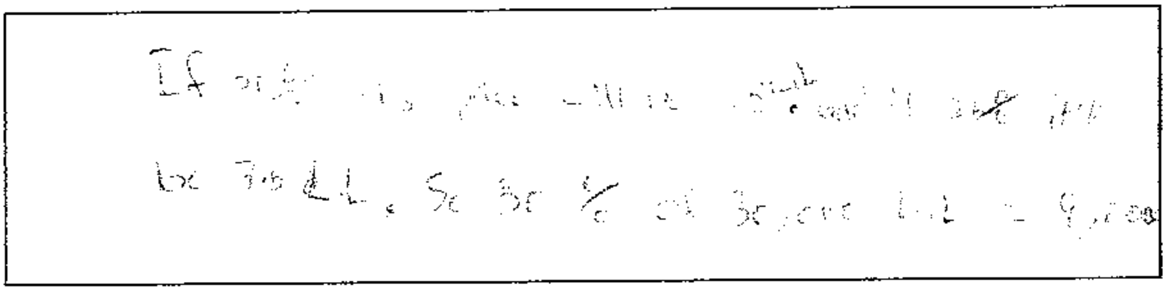


Figure 21: Relating the fifth problem to a real life problem to be able to solve it

Also, many students just tried to work with numbers to reach a solution, without reaching the final solution.

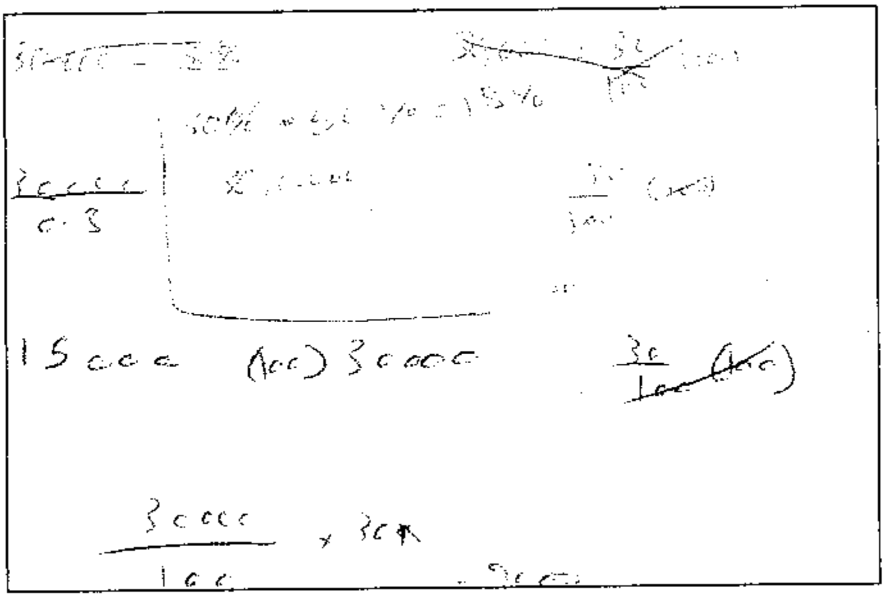


Figure 22: Number relation as an attempt to solve the fifth problem

It is noteworthy that when solving this problem, at the beginning most of the students showed easiness while solving it and did not show any difficulties. Most of the students in all three school types said that they found the problem as easy and can be solved in no time. While checking the papers however, it was noticed that only two papers from grade six in the high socioeconomic level schools had correct answers while seven other papers were only able to find the amount of the discount neglecting the other step of subtracting the amount of the discount from the original price to get the new price. In the middle socioeconomic level school, two papers from grades six and seven failed to find the way of solving the problem. Following is how one student answered the interviewer when she asked her to read the problem and explain the way to solve it:

> Interviewer: Can you please explain what this problem means.
> Student: Bilal wanted to buy a shirt that is 30, 000L.L. and has 30% sale. We should know how much Bilal paid.
> I: Ok… so what will you do?
> S: I think we should multiply the percent by the price. But we should put the percent over 100.
> I: Ok.

S: this is it... We will get 9000. One minute. Last week dad taught me about this ... I still remember ... we should still do something
I: Do what?
S: I don't know ... add the number to 30,000 or subtract it. Oh... subtract it yes subtract it because sale so we pay less
I: Just do anything you think is right. So...
S: I will subtract... it is 30, 000 – 9000 = 21,000
I: OK.
S: but you know... it is cheap (with a big smile) but I think it is right I don't know...
I: Ok. Thank you
(grade 6 student, high- mid- high level, High socioeconomic level school)

Check Action

Three out of the twelve couples who participated in the clinical interviews took their time in the end to recheck their answers and go over the solutions and the strategies they used. Other students simply went from one problem to another without looking back, and submitted their papers as soon as they finished.

Wrapping up on the Strategies Used to Solve the Problems

To reflect on major errors in solving the five problems, we can note that common and major errors fall in problem one, problem four and problem five. Most students in problem one had a misconception in understanding the word "increase"; they understood this word as how much each tree has increased and compared their answers so most answers were the same "both trees grew the same" or "both trees grew 6 cm". As for problem four, even when students had a right way to solve the problem, they compared their final answers and had 4 greater than 3 so concluded that my map is bigger than my friend's. Finally, in problem five, it was deduced that most of the students tried solving the problem by memorizing the equation of solving discount problems, and if they thought of another way, it was difficult for them to reach a final solution.

To end this section about strategies used by students while solving proportionality problems the following observations need to be pointed out. Identifying the methods and techniques students use when they have to solve such problems enabled the interviewer to discover misconceptions that could explain wrong approaches to solving proportionality related problems and reasons why they get wrong answers. For example in the problem of percents, most of the students did not go on to the last part of subtracting the discount from the original price. When asked about the reason, most answered that this is the price we should pay because this is the discount which is 30 percent. Furthermore, one student's calculations for the problem revealed different misconceptions. When the student was asked to explain his strategy, the following conversation took place:

> I: Can you tell me what are you doing in this problem?
> S: yes… it is easy… we can solve it easily with 50%
> I: How
> S: 50% of 30, 000L.L. is 15, 000 and 25% of it is 7500
> I: Ok… then
> S: so the 30% is 22,500 L.L.
> I: why?
> S: because it 15,000 + 7500 so it is 22, 500. This is the answer... I am sure
> (Grade 6 student, high and mid-high achievement level, and middle socioeconomic level school)

While the interview with another student went on as follows:

> I: So, what do you want to tell me about this problem?
> S: It is so easy
> I: really? Why?
> S: well I will tell you… when I buy a gum for 1000 and it has 50% then I get it for 500. Right?
> I: Ok
> S: because I multiply 1000 by 50%
> I: OK.
> S: so here it is the same we just multiply the 30, 000 by 30% and here is the answer 9000
> (Grade 6 student, high and mid-high achievement level, and low socioeconomic level school)

Moreover, the interviewer realized that although different representations can be useful in solving some problems, it was somehow difficult for students to complete the solution when using them. For example, one of the students discussed the representation in the pizza problem as follows:

I: Which problem are you solving?
S: the one of the pizza.
I: ok did you get the answer?
S: yes…I am trying to divide the pizza (showing the pizza drawn in the paper) into seven pieces but it is not working. So if we divide the pizza into 8 pieces (she divided it then scratched the figure) or into 6 pieces (she drew another pizza and divided it into 6 pieces) then we will have $6 \times 7 = 42$ pieces for the girls and $6 \times 3 = 18$ pieces for the boys.
I: then
S: sure the girls will get more pizza. It is easy
I: OK. Thank you
(Grade 6 student, mid- low and low level, high socioeconomic level school)

Furthermore, it was noticed that there is no major difference between schools with different socioeconomic levels regarding the number of right answers obtained. As noticed, 12 out of 20 problems are correct for high socioeconomic level schools in comparison to 10 out of 20 for both middle and low socioeconomic level schools. This indicates that the effect of the school's socioeconomic level on students' approaches to solving proportionality problems is not significant, if present at all. Furthermore, it is noteworthy that while brainstorming could have helped students reach the right solution, students in this study did not use it as the strategy to solve any of the problems they were asked to solve. Those who started by using brainstorming, mostly in grade seven, ended up dropping it and resorted to one of the strategies described above, strategies that lead to the direct answer.

Concluding Notes

To end this chapter, the following final observations come in place. The analytical description of grades six and grade seven national math textbooks reveals the commitment to the constructivist approach to education in principle. This shows for instance through the noteworthy number of real-life word problems that are intended to enable students to become reflective individuals capable of reasoning and critical thinking. It is important to note though, that the learning process is obstructed at several points in these chapters due to mistakes such as the following: "we read on the *unitization notebook* of the car" while it should be the *user's manual.* Also some problems are missing the right partition of parts and include several misspelling mistakes such as the word "economicasitates" (ECRD, 1998a, and p.157)

At the same time, the overview of the teacher's guide and interviews with teachers revealed that the teacher's guide plays for some of them a role in regulating the teaching process to secure education within a constructivist perspective. Still, however, it has shortcomings; it fails to provide teachers with essential guidelines and practical tips as to what they should do to reach the objectives of the curriculum and those of the lessons. In addition, it is disappointing that the teacher's manual failed to include details on organizing the lesson, constructing an explanation based on evidence and real-life situation, and even preparing a kind of assessment activities that help the teacher evaluate the students' understanding. Mostly, what the entire manual focused on was to provide teachers with answers to problems and exercises in the book.

At the same time, feedback from teachers of math for grades six and seven in the schools included in this study revealed a rather alarming reality. Most of the interviewees do not know exactly what proportionality means/refers to and some of them do not know the methods used in the textbook and in the teacher's guide. At the same time however, teachers interviewed in this study reported a promising reality when it comes to teaching math in general in schools of Lebanon. Most of them use real life problems to explain proportionality in the classroom. This indicates that they are respecting the constructivist approach to teaching math stipulated in the national curriculum. .

DISCUSSION AND RECOMMENDATION

The findings of this study about the learning and teaching processes of proportionality in grades six and seven in three schools in Lebanon presented in the previous chapter lead to the following discussion and corresponding recommendations (when applicable).

Textbooks and Teacher's Manual

Although the national math curriculum sets guidelines for teachers to implement a constructivist approach to teaching proportionality, several facts about the textbook and teacher's manual prevent teachers from achieving this goal. To start, there is the fact that math textbooks for grades six used in the schools of Lebanon includes three chapters (percentage, ratio and quotient, and proportionality), while grade seven math textbook includes only one chapter about proportionality. Accordingly, teachers expect grade seven students to know what proportional reasoning entails, to know what the concepts related to proportionality mean, and be familiar with appropriate ways to solve proportionality problems. The truth of the matter, however, is that when students reach grade seven, a sizeable number of students do not remember most of what they had learned in the previous year. As for those who do, they either remember vaguely what they learned, or they carry with them misconceptions that need to be rectified. As such, it is important that experts consider the including more chapters that address proportionality in grade seven math textbook.

When it comes to the extent to which the national math textbook is helpful, it was found that instructions in chapters about proportionality in grade six and seven textbooks are not clear; students cannot perfectly understand them if the teachers do not elaborate on the givens and explain what is required to do. In addition, the analyzed chapters revealed that math textbooks do not focus on examples as they should especially those real life examples most encountered in daily life in Lebanon namely, taxes, discount, salary raises, and perimeters.

At the same time, the analytical description of the textbooks revealed that although teachers are doing their best to implement what a constructivist approach to teaching entails, they fail to meet the high expectations for math learning that this approach sets for students and teachers. The chapters about proportionality seriously lack real life situation problems. This is especially the case in the grade six chapters. As such, students cannot realize the usefulness of learning proportionality, and are not enabled to become critical thinkers.

Furthermore, there is the fact that most problems are of the drill and practice type. All they require students to do is to apply specific equations without any need for reasoning or analysis, or any explanation or elaboration on behalf of the teacher. This is especially the case with the chapter on fractions which introduces students to proportionality and proportional reasoning. The problems about real life situations included in the different sections of the chapters (activities, text, exercise, self evaluation, problems, and just for fun), are not enough.

In addition, several of these real life problems have errors in them, or they include misspelled words, or they require much more time than students need to understand and solve and teachers to explain. For example, as pointed out above, asking student to calculate ratios of ages is not the best approach to explain to students the meaning of ratios and the situations in which they are used.

When it comes to the math teacher's manual, many teachers have to resort to other math textbooks (most often math textbooks used in Western countries such as France or the United States) to find essential illustrations and supporting material the absence of which prevents them from properly teaching the lesson. In addition, the teacher's manual fails to provide teachers with the support they need when it comes to using real life activities to help students understand the lesson and related concepts. The teachers' manual only makes available to teachers the objective they should realize in each lesson and provides them with the answers of the problems included in the textbook. This is far from what a teacher's guide is supposed to be. It should include all, if not most of the resources that teacher could need to appropriately teach the lesson.

Teachers' Approaches to Teaching Proportionality

The results of this study conform to the findings stressed in the paper by Gore (2004), namely that teachers tend to use different approaches and strategies to make sure that students understand the subject they are taught.

Furthermore, classroom observations conducted showed that most teachers are making an effort to promote classroom activities and constructivism in teaching. In this study, all teachers except T5, a grade six teacher in a low socioeconomic level school, tried hard to implement a constructivist approach to teaching mathematics. They all showed a serious effort to avoid using a traditional method of teaching. Hence, as Draper (2002) recommends, they adopt teaching methods that do not encourage memorization and they avoid imposing discrete and fixed knowledge on students.

It is equally noteworthy that the majority of teachers in this study, created a student centered learning environment using different strategies, and making a special effort to use the appropriate language while explaining the lesson and foster. All teachers, except T5, tried to create interactive situations in the class and they engaged students in real-life and hands-on activities. As such, they conformed to recommendations of Fisher (1988) and Howson (1981) who stated that teachers should not rely exclusively textbooks. They should in one way or another engage their students in creative activities. This secures better understanding and achievements among students.

All along the above mentioned reality when it comes to teachers' approaches to teach proportionality, there is the fact that some teachers failed to heed important instructions by constructivists. For example some teachers failed to give a fair chance to all students to build their knowledge and learn from their mistakes. Instead of asking students who obtained both a right and wrong answer to share their solution of problems, most of the time teachers only asked students with right answers to go to the board and solve the problem.

Students' Approaches to Learning Proportionality

The clinical interviews conducted with students support the conclusion that math teachers of grade six and grade seven are trying to implement the constructivist approach to teaching proportionality. This shows in the way students tried to solve the problems (Appendices E). For example, the high and mid-high achievement level students brainstormed with their partners and were able to give them ideas to be able to solve the problem in a correct way (especially problems one and two). This is also illustrated by the fact that students used different methods of varying levels of complexity to solve the same problem, although they have the same teacher.

The positive impact of the teacher's efforts to implement a constructivist approach to teaching math showed through the following; students who have encountered real life situations in the classroom showed more writing and reasoning on their papers through writing, drawing, etc. While students who have not had this chance did not try to solve the problem by scratching number; they resorted to remembering different theories or formulas used in the classroom to solve the problem.

At the same time, it is noteworthy that none of grade six students in the low socioeconomic level school read the problem carefully or re-read in a way or another. This is probably due to a failure on behalf of their teachers to teach them the appropriate skills in problem solving, namely, carefully understanding what the problem is about so as not to waste time with trying different methods. In addition, only one mid-low and low achievement level student was able to have a correct answer for problem two after drawing a picture and showing the way to solve it. This indicates that a student who showed an ability to choose the appropriate strategy to solve a problem can even fail to do so when trying to solve another problem.

In addition, it was noted that the students' interactions during clinical interviews helped them to solve the problems more efficiently and increased their chances of arriving at the correct answer. The more students interacted the better the results were. When they interacted with their classmates while solving the problems students learned various ways to solve the problem; not infrequently, they learned from their classmates' mistakes and incorrect solutions. This helped them eliminate wrong strategies they intended to use and focus on more appropriate ones.

At the same time, it is important to point out that clinical interviews conducted in this study indicate a predominant tendency among students that was emphasized by NAEP (1973) and Brown and Burton (1978); namely that students tend to memorize rules and equations to solve a problem, despite teachers' efforts to change this tendency. Actually, students of T5, the teacher who used traditional methods and techniques during the classroom observations, had the highest number of wrong answers.

Last but not least, the fact that students faced difficulties most when trying to solve percentage problems, supports recommendations by Parker and Leinhardt (1995) and Moss (2002) that percentage should be included in the curriculum and a chapter should be dedicated to teaching it in grade seven too.

Some Recommendations

Based on the above, the following recommendations come in place. The students' understanding and achievement of mathematics will be improved by designing a truly student-centered curriculum that stresses student interaction and students' own generation of methods and solution methods.

Until the national textbooks are revised and modified in such a way that they heed requirements of a constructivist approach to teaching math in general, and proportionality in particular, it is essential that schools provide supplementary/supporting materials that correspond to the needs of math teachers in each grade level. This is essential to give all students the chance to learn through real life situations and problems hence enable them to appropriately handle situations that involve proportionality in real life. At the same time, it is very important that the Lebanese authorities collaborate with education specialists to rectify the above listed problems in the national math textbooks and teacher's manuals. Also more math teachers should be involved in writing and correcting the textbooks.

When it comes to the learning environment, the teacher's observations indicate that students should have more opportunities for learning through problem solving of real life situations. Most of the students were not directed towards this in the classroom, as such they wasted time focusing on reasoning to get the right answer, showing how they got the answer by jotting different numbers and words to fill their papers, and support their final solution. Also it is recommended that teachers give students meta- cognition by using questioning, interaction as stated by Draper (2002). As he explains, the interaction with students helps them construct more knowledge and engage in inquiry more than when s/he is working alone. This also pushes the student to use mathematical language to emphasize the mathematical significance of ideas and connect it to real life situations' meaning of ideas, give students freedom of thinking, and group work . This would engage the class in sharing ideas and information so students will be able to learn form their peers' mistakes and get more ideas for solving problems. When it comes to teachers, as Arends (1994), Ben-chaim et al. (1998), Cantenbury (2007), and Weinberg (2002) suggest, teachers must always make sure to use various strategies.

In order to include the recommendations stated above, it is advised that schools provide professional development for teachers at the workplace since as Clement (2000) says, the place of work wields great influence on this development. This in the end will enhance students' achievement and progress.

Limitations of the Study

It is hoped that this study brought forth significant results. Still, it is important to highlight the fact that these results cannot be generalized to a larger population of students or to other schools.

An effort was made to increase the validity and reliability of the results by triangulating data collected. Results from various techniques and data collection tools concurred and confirmed each other. Yet, the following limitations should be kept in mind:

The validity of this study was limited by the following factors that could not be controlled: the small size of the sample (the number of participants does not allow generalization of findings) and the duration of the study (the study was conducted over a short period of time, only two weeks). In addition, more than one teacher took part in this study. The difference in personality, education background, teaching style, expectations and acquaintance with their students probably played a big role in the way students understood a lesson and therefore the outcomes of the study.

One must also pay attention to the fact that the test items or the problems given to students, although chosen from previous literature, could expose bias to the method used since some of the problems are not likely to happen in real life of students. For example one of the problems about the pizza pieces might involve concrete materials and would likely occur in everyday situation; but on the other hand it is doubtful that someone buying or dividing pizza will take out a pencil and paper to calculate the answer.

The need for this paper stemmed from the need to gain more knowledge about students' performance in many real life situations concerning proportionality. This study used many recommendations from literature concerning ways of teaching, curriculum, and integration with students in a constructivist mode. This could lead to better preparation by teachers, and higher achievement by students. The method used in his study focused on high quality thinking by students. Reliability was also affected by the fact that students were required to solve real life problem situations without prior knowledge of them; maybe some of them were given problems about situations they had never encountered before or heard about and thus during the study they had to go by trial and error to find for the best approach and strategies to solve them.

Perspectives for further research

The findings of this study indicate a need to review the national math curriculum, textbooks, as well as teacher's approaches to teaching proportionality in particular, and math in general. To validate the results, further research should investigate the issues explored in this paper using a bigger sample of schools involving more teachers and students. Furthermore, studies that decide to investigate how proportionality is being taught in schools in Lebanon, must also focus on professional development given that it is known to play a role in enhancing students' achievements.

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APPENDIX A

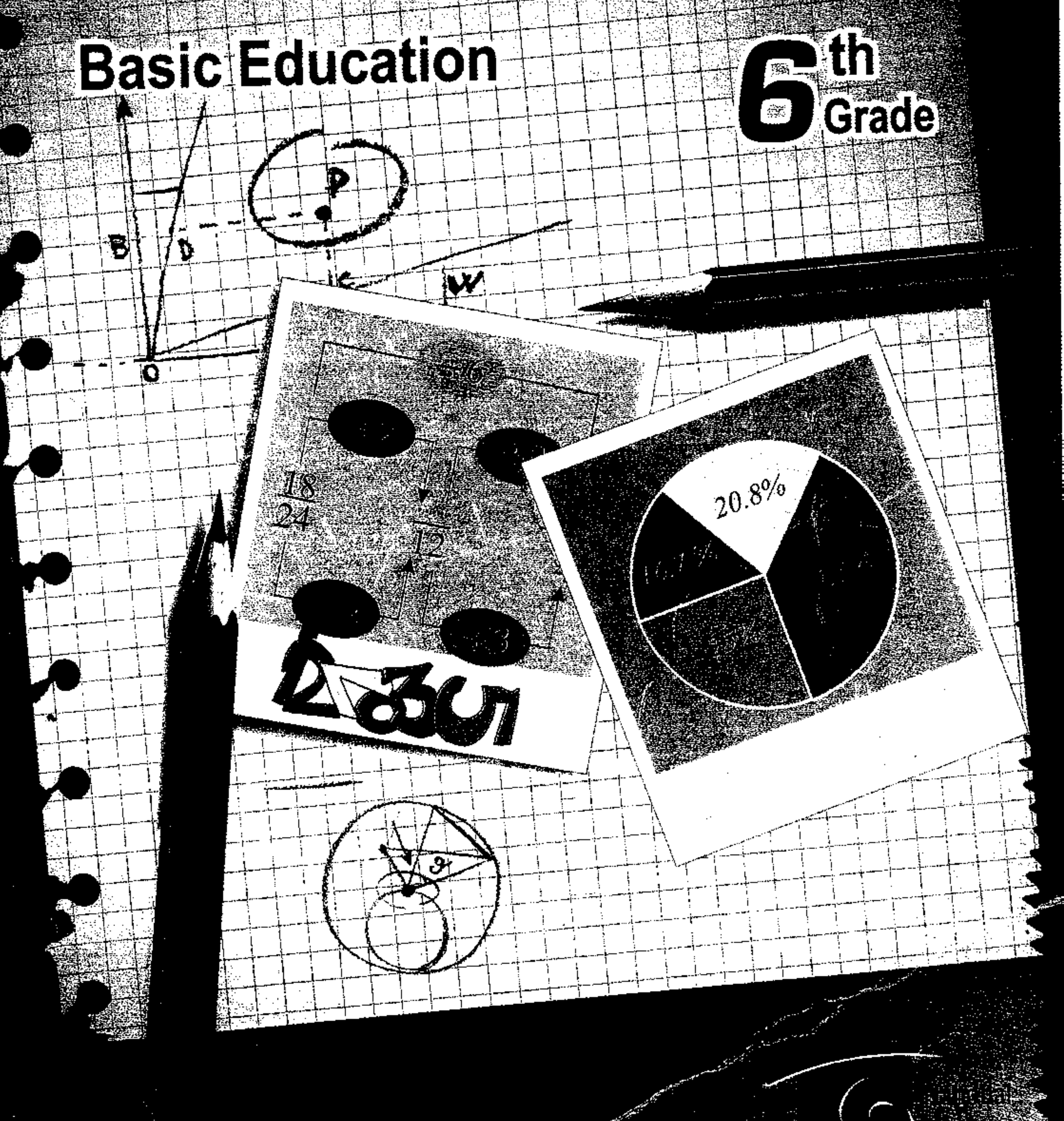
TEXTBOOKS

Building up

Mathematics

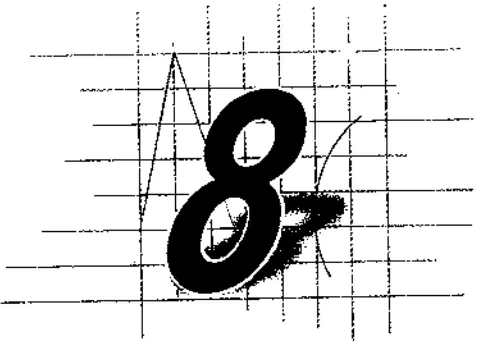
Basic Education

6th Grade



NEW CURRICULA

Irreducible fractions

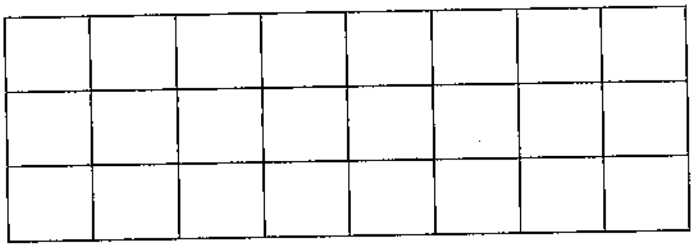


Objectives

- [illegible]
- [illegible]

Activities

A farmer divided his rectangular field into 24 equal parts but ploughed only 18 of them.



a) A child has reproduced the picture of the above rectangle representing the field, and colored 18 parts in brown.

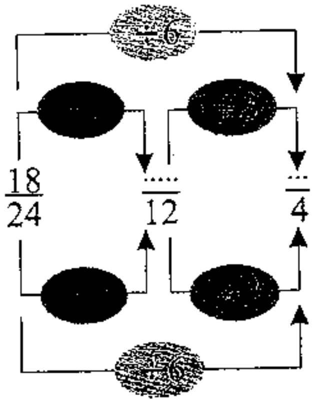
His neighbor did the same but divided the rectangle into 12 equal parts and colored only 9 of them.
When comparing the two drawings, it was found that the colored areas were equal.

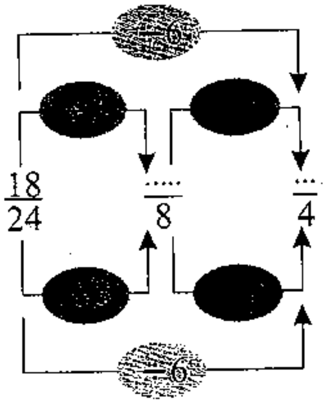
- Reproduce two equal rectangles. Divide the first one into 8 equal parts and the second into 4 equal parts. Then color in brown the parts representing the ploughed region of the field.

- Copy and complete:

  The ploughed part represents: $\frac{\dots}{24}$, or $\frac{\dots}{12}$, or $\frac{\dots}{8}$, or even $\frac{\dots}{4}$ of the field.

- Of these fractions, which is the simplest?





b) Starting from $\frac{18}{24}$, how can you reach $\frac{3}{4}$ in one step?

Do the numbers 3 and 4 have a common divisor other than 1?

What do you call these two numbers? Can you simplify $\frac{3}{4}$?

Text

$\frac{30}{24}$ is reducible

$\frac{30}{24} = \frac{30 \div 3}{24 \div 2} = \frac{15}{2}$.

e simplified $\frac{30}{24}$ to $\frac{15}{12}$.

ut $\frac{15}{12}$ is reducible.

$\frac{15}{12} = \frac{15 \div 3}{12 \div 3} = \frac{5}{4}$

he G.C.D of 5 and 4 is ne, therefore $\frac{5}{4}$ is the reducible fraction equal $\frac{30}{24}$.

riteria of divisibility :

by 2: numbers ending with : 0 - 2 - 4 - 6 - 8;

by 5: numbers ending with 0 or 5.

by 10: numbers ending with 0.

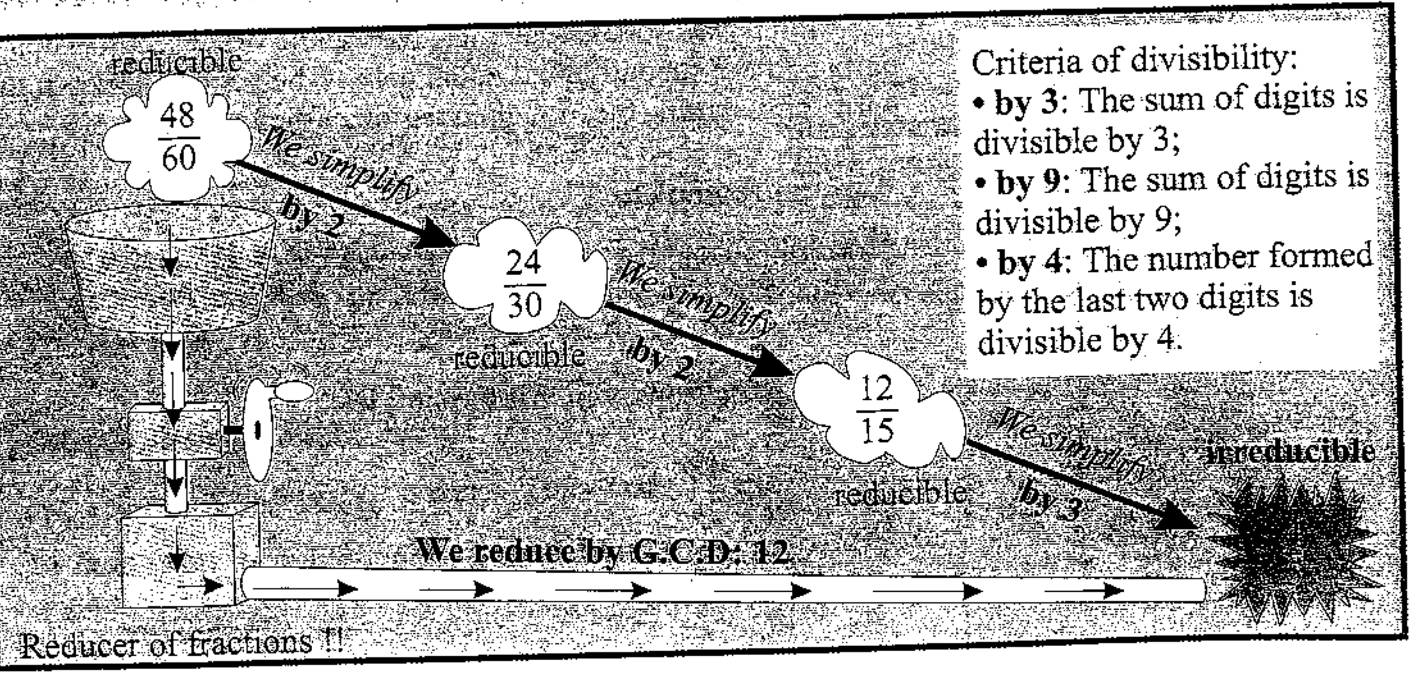
- A fraction $\frac{a}{b}$ is said to be
  - **reducible**, if a and b have a common factor. We can then simplify this fraction.
  - **irreducible**, if a and b are prime with each other (1 is their common factor).
- To simplify a fraction is to find an equivalent fraction where the denominator and numerator have less válues. Therefore, we should find the common factor to both terms of the fraction.
- To reduce a fraction is to find the irreducible fraction which is equivalent to it. This is reached:
  - either by the method of successive divisions, by finding the common divisors of the two terms of the fraction

$$\frac{48}{60} = \frac{48 \div 2}{60 \div 2} = \frac{24}{30} = \frac{24 \div 2}{30 \div 2} = \frac{12}{15} = \frac{12 \div 3}{15 \div 3} = \frac{4}{5};$$

  - or by the method of G.C.D:
    The divisors of 48 are: 1-2-3-4-6-8-12-16-24-48.
    The divisors of 60 are: 1-2-3-4-5-6-10-12-15-20-30-60.
    G.C.D (48 and 60) = 12

$$\frac{48}{60} = \frac{48 \div 12}{60 \div 12} = \frac{4}{5}.$$

Focus



Criteria of divisibility:
- **by 3**: The sum of digits is divisible by 3;
- **by 9**: The sum of digits is divisible by 9;
- **by 4**: The number formed by the last two digits is divisible by 4.

Exercises

1- Fill in the blanks:

$\frac{7}{8}=\frac{.....}{72}$; $\frac{11}{5}=\frac{.....}{35}$; $\frac{12}{7}=\frac{60}{.....}$; $\frac{24}{36}=\frac{2}{...}$; $\frac{5}{30}=\frac{1}{...}$;

$\frac{8}{72}=\frac{.....}{9}$; $\frac{123}{.....}=\frac{3}{8}$; $\frac{117}{.....}=\frac{9}{7}$; $\frac{330}{.....}=\frac{11}{2}$.

2- Find six fractions equivalent to $\frac{12}{18}$ and six other fractions to $\frac{30}{48}$.

3- Find the intruder

$\frac{21}{35}$ $\frac{180}{30}$ $\frac{270}{450}$ $\frac{12}{20}$

$\frac{39}{65}$ $\frac{3}{5}$ $\frac{24}{40}$ $\frac{210}{350}$

4- From the following fractions, frame the irreducible ones and circle the reducible ones.

$\frac{6}{9}$; $\frac{5}{5}$; $\frac{3}{17}$; $\frac{4}{7}$; $\frac{13}{29}$; $\frac{4}{9}$; $\frac{111}{423}$; $\frac{45}{12}$.

5- Complete the following equalities:

$\frac{45}{75}=\frac{9}{...}=\frac{...}{5}$; $\frac{48}{72}=\frac{12}{.....}=\frac{...}{9}=\frac{2}{...}$.

$\frac{84}{108}=\frac{.....}{54}=\frac{21}{.....}=\frac{...}{9}$.

6- Reduce the fraction $\frac{90}{126}$ using the method of successive divisions.

7- Find the irreducible fraction equivalent to each of the following fractions:

$\frac{40}{24}$; $\frac{49}{35}$; $\frac{200}{700}$; $\frac{64}{24}$; $\frac{36}{45}$; $\frac{66}{55}$.

8- a) Calculate the G.C.D of 144 and 312, then use the G.C.D to reduce the fraction $\frac{144}{312}$

b) Repeat the same procedures to reduce the fractions $\frac{81}{135}$ and $\frac{513}{180}$.

9- Find the irreducible fraction equivalent to each of the following fractions

$\frac{60}{24}$; $\frac{840}{490}$; $\frac{144}{108}$; $\frac{1400}{1050}$.

10- a) Complete the following equality $\frac{26}{65}=\frac{...}{5}$

b) Find a fraction equivalent to $\frac{26}{65}$ and which denominator is 100.

11- Draw a segment [*AB*] of length 8 *cm* and divide it into 4 equal segments.
On [*AB*], mark the point *I* such that the length of [*AI*] is $\frac{24}{32}$ that of [*AB*].

Self-evaluation

1- Indicate the irreducible fractions from the following:

$\frac{9}{7}$; $\frac{20}{65}$; $\frac{4}{16}$; $\frac{13}{15}$; $\frac{4}{13}$; $\frac{7}{28}$; $\frac{19}{3}$; $\frac{11}{121}$

2- Reduce using both methods (successive divisions and G.C.D):

$\frac{54}{42}$; $\frac{42}{96}$; $\frac{60}{195}$; $\frac{72}{468}$; $\frac{312}{26}$; $\frac{315}{280}$

Problems

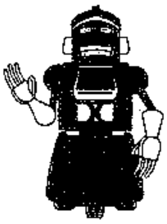
1- Write all the fractions that are equivalent to $\frac{3}{7}$ and that have a denominator inferior to 70.

2- a) Find the irreducible fraction equivalent to $\frac{60}{135}$.

b) Find the fraction that is equivalent to $\frac{60}{135}$ knowing that the sum of its terms is 65.

3- We entered the following fractions into "a reducing machine"

$\frac{12\,000}{8000}$; $\frac{7\,500}{3500}$; $\frac{270}{126}$; $\frac{675}{495}$; $\frac{81}{270}$; $\frac{480}{495}$; $\frac{375}{525}$.



Write the irreducible fractions that will come out. Copy and fill in the following table:

	$\frac{12\,000}{8000}$	$\frac{7\,500}{3500}$	$\frac{270}{126}$	$\frac{675}{495}$	$\frac{81}{270}$	$\frac{480}{495}$	$\frac{375}{525}$

4- a) What fractions of the hour represent:

12 min ? 35 min ? 40 min ?

45 min ? 50 min ?

52 min ? 58 min ?

b) A bus completes 21 times the entire tour of a city in 14 hours. Give in minutes the duration of a complete tour.

5- The area of a rectangular field is $792m^2$. Its length is $54m$.
Find its width. Give the answer in the form of an irreducible fraction.

6- An auditorium has 315 places, 180 of which are occupied. Give the fraction representing the number of places unoccupied with respect to the total number of places.

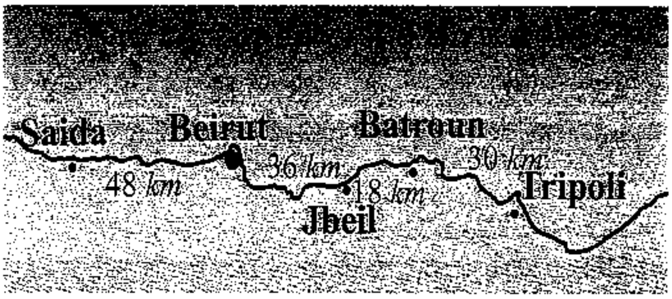
7- A library received 5 600 samples of a scholar by book A , 2 700 of a book B and 4 200 of a book C .
The library sold 4 800 samples of A 1 350 of B and 3 000 of C .

a) Give the fraction that represents the number of books A sold with respect to all samples of A .

b) Same question for B and C .

c) What fraction of the total number of books do the samples sold of book A represent? Of book B ? Of book C ?

8- Walid and his parents want to go from Tripoli to Sidon. They take the coastal road as indicated below:



What fraction of the road do they complete when they reach :

a) Batroun?

b) Jbeil?

c) Beirut?

Just for fun

Find the irreducible fractions equivalent to:

[redacted]

Decimal fractions - Fractional writing of a decimal number

10

Objectives

- [illegible]
- [illegible]
- [illegible]

Activities

Rules of the game

In order to get a boat, we must find first the quotient of a by b. $q = a \div b$.

- *If* $q \times b = a$, *then we take a boat.*
- *If* $q \times b \neq a$, *then we cannot take the boat.*

Five children went on a sea trip. Each had a fraction $\frac{a}{b}$ on his bathing-suit. To take a boat, they should respect the rules of the game given below.

a) Copy and complete the following table:

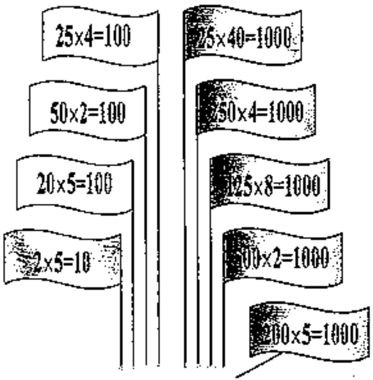
Fraction $\frac{a}{b}$	Quotient of a by b	$q \times b$	Can the child take the boat?
$\frac{1}{5}$	$1 \div 5 = 0.2$	$0.2 \times 5 = 1$	The child will take the boat having the number 0.2
$\frac{45}{60}$			
$\frac{27}{3}$			
$\frac{7}{33}$			
$\frac{34}{125}$			

One of the children will not go on the trip. Which one and why?

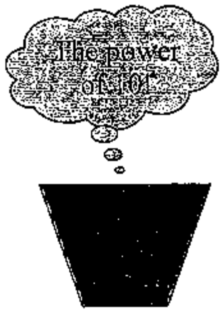
b) Complete:

$$\frac{1}{5} = \frac{.....}{10} \qquad \frac{27}{3} = \frac{.....}{1} \qquad \frac{34}{125} = \frac{.....}{1000} \qquad \frac{45}{60} = \frac{3}{.....} = \frac{.....}{100}$$

Try to find a fraction equal to $\frac{7}{33}$ and whose denominator is a power of ten. Did you succeed?



The beautiful products.



Text

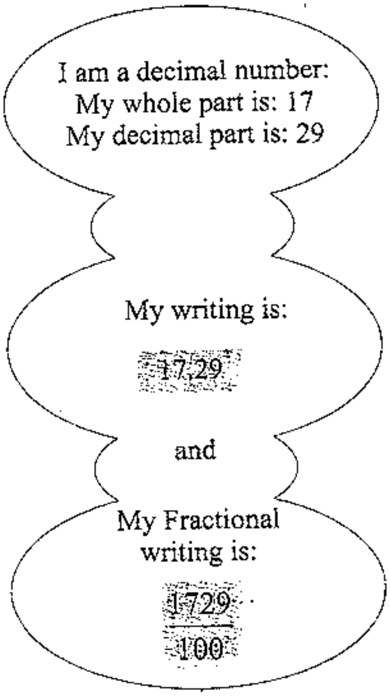
$\frac{12}{5}$ is a decimal fraction because:

$$\frac{12}{5} = \frac{12\times2}{5\times2} = \frac{24}{10}$$

5 ⟌ 12 = 2.4
- 10
20
- 20
00

$\frac{5}{6}$ is not a decimal Fraction, because :

6 ⟌ 50 = 0.833....
- 48
20
- 18
20
- 18
20



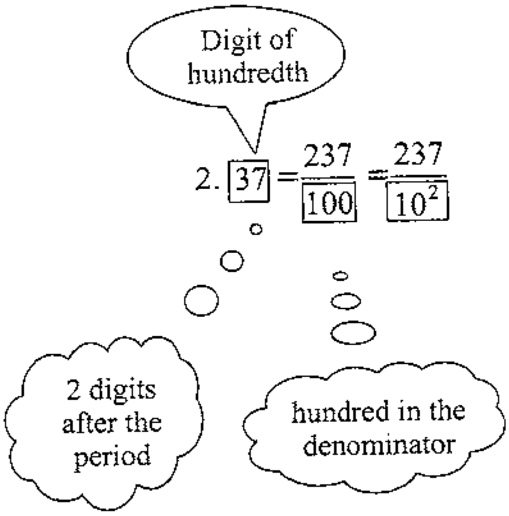
- A fraction $\frac{a}{b}$ is decimal if:
  - it can be written as a fraction in which the denominator is a power of 10;
  - the quotient of a by b is exact.

The decimal writing of $\frac{12}{5}$ is 2.4 .

- If the quotient of the division of a by b is not exact, then the fraction $\frac{a}{b}$ is not a decimal fraction.

$\frac{5}{6}$ is not a decimal fraction.

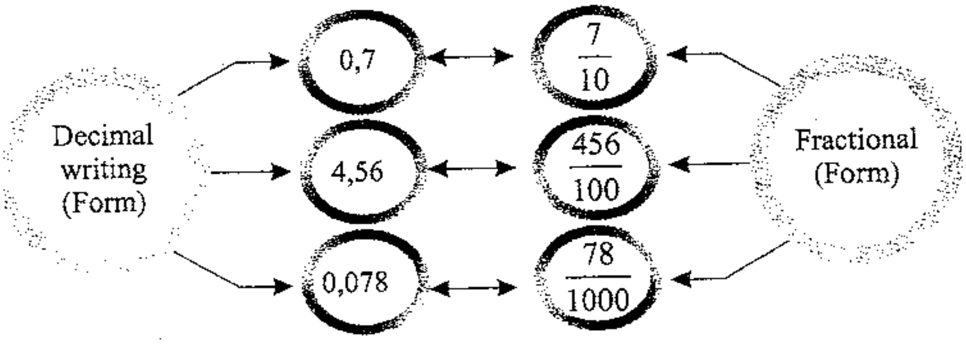
- A decimal number can be obtained from the division of an integer by 10, or 100 or 1000, or... Therefore, it can be written as a decimal fraction



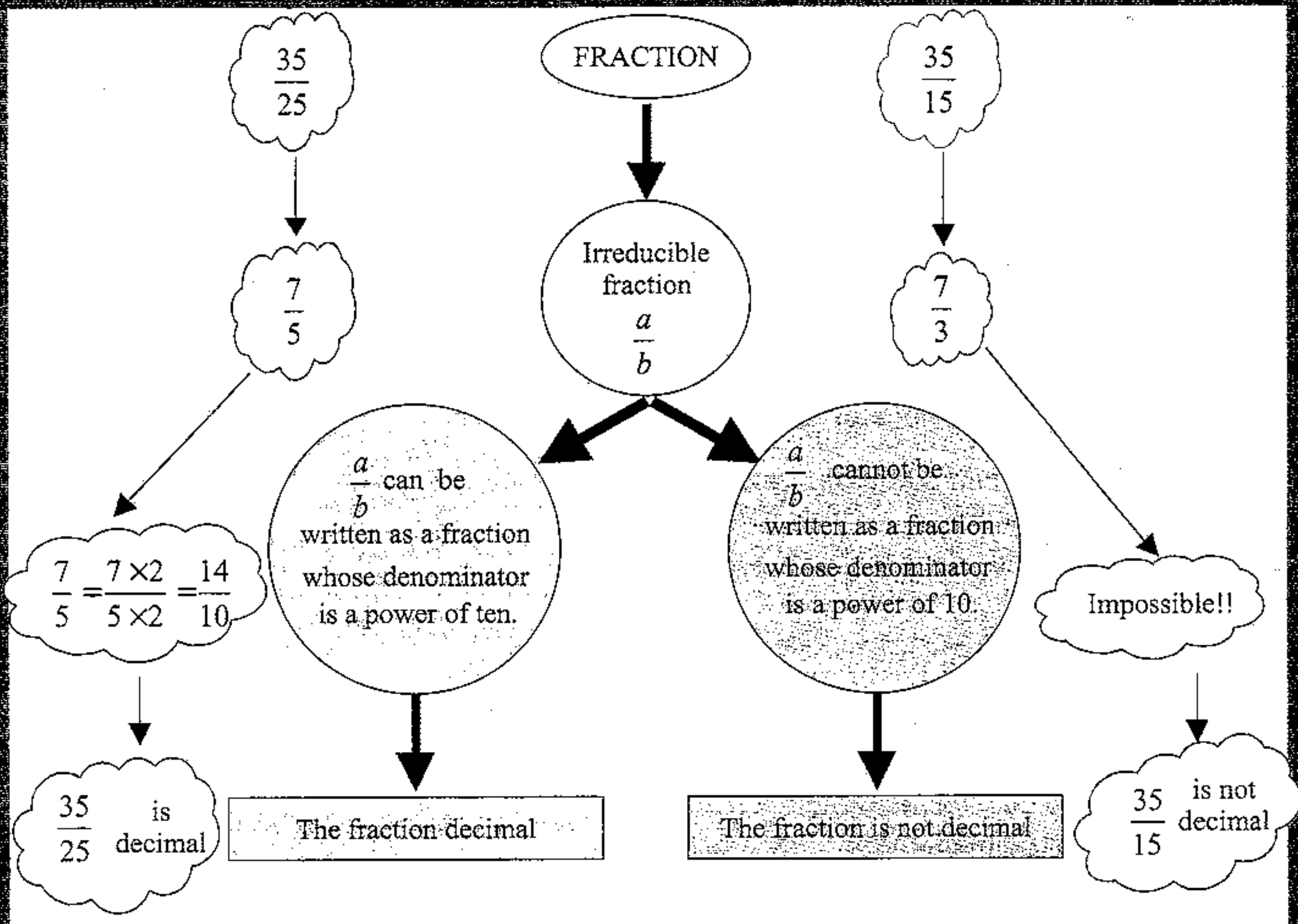
$$2.\boxed{37} = \frac{237}{\boxed{100}} = \frac{237}{\boxed{10^2}}$$

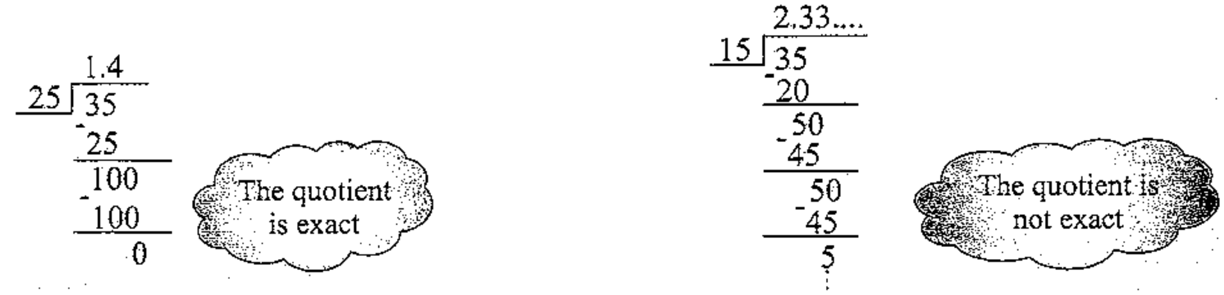
- Also, every decimal fraction represents a decimal number

$$\frac{49}{\boxed{10}} = 4.\boxed{9} \qquad \frac{1234}{\boxed{1000}} = 1.\boxed{234}$$



Focus





- Every decimal fraction represents a decimal number
- $\frac{14}{100}$ = 14 hundredths ⟶ 4 will be the digit of hundredth ⟶ $\frac{14}{\blacksquare}=0.\blacksquare$
- Every decimal number can be written as a decimal fraction

72.6 = 726 tenths ⟶ 6 is the digit of tenth so we write a fraction whose denominator is 10 ⟶ $72.\blacksquare=\frac{726}{\blacksquare}$

3.529 = 3 529 thousandths ⟶ 9 is the digit of thousandth so we write a fraction whose denominator is 1000 ⟶ $3.\blacksquare=\frac{3\ 529}{\blacksquare}$

Exercises

1- For the following quotients, circle the fractions:

$\frac{15}{8}$; $\frac{1.2}{4.5}$; $\frac{28}{12}$; $\frac{0.5}{6}$; $\frac{19}{2.7}$; $\frac{3}{50}$.

2- Write all the following as a fraction:

$\frac{51.7}{2.8}$; $\frac{6}{0.5}$; $\frac{0.24}{1.5}$; $\frac{16.3}{0.88}$; $\frac{3}{0.01}$; $\frac{4.035}{2.5}$.

3- a) Write all the fractions that are equivalent to $\frac{3}{4}$ and of denominator less than 45.

b) Is $\frac{3}{4}$ a decimal fraction?
Find a fraction equivalent to $\frac{3}{4}$ and whose denominator is 100.

4- Consider the fraction $\frac{12}{21}$.

a) Reduce this fraction, then find all the fractions that are equivalent to the reduced one but with a denominator less then 50.

b) Is $\frac{12}{21}$ a decimal fraction?

5- Circle the decimal fractions from the following :

$\frac{20}{28}$; $\frac{90}{100}$; $\frac{6}{4}$; $\frac{2}{6}$; $\frac{5}{8}$; $\frac{11}{3}$; $\frac{6}{25}$;
$\frac{45}{36}$; $\frac{5}{24}$; $\frac{24}{75}$.

6- Find the decimal fraction that is equivalent to each of the following fractions:

$\frac{11}{5}$; $\frac{13}{50}$; $\frac{63}{125}$; $\frac{9}{25}$;
$\frac{7}{8}$; $\frac{47}{20}$; $\frac{9}{4}$; $\frac{13}{2}$.

7- Copy and complete:

$\frac{3.5}{100}=\frac{35}{....}=\frac{7}{....}=0......$

8- Write as a decimal number:

$\frac{7}{10}$; $\frac{23}{100}$; $\frac{55}{1\,000}$; $\frac{37}{10}$; $\frac{4\,554}{10^2}$; $\frac{3\,004}{10^3}$.

9- Write as a decimal fraction

0.8 ; 0.45 ; 0.725 ; 0.03 ; 0.015 ; 3.09 ; 5.036 ; 1.2002 ; 0.007 ; 4.001.

10- Complete as shown below:

$\frac{3.8}{2}=\frac{38}{20}=\frac{19}{10}$; $\frac{0.45}{5}=\frac{.....}{500}=\frac{.....}{100}$

$\frac{9.5}{5}=\frac{95}{.....}=\frac{.....}{10}$; $\frac{5.6}{4}=\frac{.....}{40}=\frac{.....}{1\,000}$.

11- Give the equivalent decimal fraction (of denominator 10) to each of the following fractions, then give the decimal number that represents

$\frac{4}{5}$; $\frac{15}{2}$; $\frac{65}{50}$; $\frac{48}{60}$; $\frac{4.2}{2}$; $\frac{18.5}{5}$.

12- Write the decimal fraction of denominator 100 that is equivalent to each of the given fractions, then find its decimal number:

$\frac{14}{5}$; $\frac{15}{4}$; $\frac{16}{50}$; $\frac{43}{20}$; $\frac{37}{25}$; $\frac{94}{200}$; $\frac{15}{500}$;
$\frac{70}{1\,000}$; $\frac{13.5}{50}$; $\frac{35.2}{20}$.

13- Write the decimal fraction of denominator 1000 that is equivalent to each of the given fractions, then find its decimal number:

$\frac{19}{200}$; $\frac{3}{500}$; $\frac{17}{250}$; $\frac{6}{125}$; $\frac{7}{8}$; $\frac{8}{25}$; $\frac{45}{4}$;
$\frac{105}{40}$; $\frac{9}{5}$; $\frac{69}{3\,000}$; $\frac{634}{2\,000}$; $\frac{18.45}{50}$.

14- Write each of the fractions below as a decimal fraction, then give its equivalent decimal number

$\frac{1}{4}$; $\frac{2}{5}$; $\frac{30}{4}$; $\frac{98}{25}$; $\frac{528}{125}$; $\frac{72}{250}$.

15- Match each decimal number with its fraction:

[redacted]

- $\frac{805}{100}$
- $\frac{9}{1\,000}$
- $\frac{48}{100}$
- $\frac{145}{1\,000}$
- $\frac{5\,275}{100}$
- $\frac{23}{10}$
- $\frac{8}{10}$

16- Give the decimal number of the following fractions :

a) $\frac{7}{10}$; $\frac{18}{1\,000}$; $\frac{45}{100}$; $\frac{65}{10}$; $\frac{6\,475}{1\,000}$.

b) $\frac{1}{2}$; $\frac{7}{5}$; $\frac{13}{4}$; $\frac{13}{2}$; $\frac{1}{8}$.

c) $\frac{15}{24}$; $\frac{175}{125}$; $\frac{18}{40}$; $\frac{44}{20}$; $\frac{124}{32}$.

17- Copy and complete the following equalities:

$8.5 = \frac{\ldots\ldots}{10} = \frac{\ldots\ldots}{2}$.

$3.25 = \frac{\ldots\ldots}{100} = \frac{\ldots\ldots}{4}$.

$2.05 = \frac{\ldots\ldots}{100} = \frac{\ldots\ldots}{20}$.

$0.064 = \frac{64}{\ldots\ldots} = \frac{\ldots\ldots}{125}$.

18- Write each of the decimal numbers below as a decimal fraction, then reduce it if possible:
3.6 ; 0.03 ; 0.875 ; 0.72 ; 1.85 .

19- Write each of the following measures as a decimal fraction, then convert it to the wanted unit:

$45dm = \frac{\ldots\ldots}{\ldots\ldots} m = \ldots\ldots m$

$158cm = \frac{\ldots\ldots}{\ldots\ldots} m = \ldots\ldots m$

$39mm = \frac{\ldots\ldots}{\ldots\ldots} m = \ldots\ldots m$

$3485m = \frac{\ldots\ldots}{\ldots\ldots} km = \ldots km$

$96m = \frac{\ldots\ldots}{\ldots\ldots} hm = \ldots\ldots hm$

$7m = \frac{\ldots\ldots}{\ldots\ldots} dam = \ldots\ldots dam$

20- Write in the form of a decimal fraction each of the following masses, then convert it to the wanted unit:

$3dg = \frac{\ldots\ldots}{\ldots\ldots} g = \ldots\ldots g$

$326cg = \frac{\ldots\ldots}{\ldots\ldots} g = \ldots\ldots g$

$485mg = \frac{\ldots\ldots}{\ldots\ldots} g = \ldots\ldots g$

$634g = \frac{\ldots\ldots}{\ldots\ldots} kg = \ldots\ldots kg$

$9g = \frac{\ldots\ldots}{\ldots\ldots} hg = \ldots\ldots hg$

$137g = \frac{\ldots\ldots}{\ldots\ldots} dag = \ldots\ldots dag$

Self-evaluation

1- For the following fractions, find the decimal ones and give their decimal numbers:

$\frac{34}{25}$; $\frac{40}{75}$; $\frac{12}{8}$; $\frac{75}{35}$; $\frac{16}{4}$; $\frac{63}{72}$.

2- Copy and complete the following equalities:

a) $0.7 = \frac{\ldots\ldots}{10} = \frac{70}{\ldots\ldots} = \frac{\ldots\ldots}{1\,000} = \frac{70\,000}{\ldots\ldots}$

b) $5.36 = \frac{\ldots\ldots}{100} = \frac{\ldots\ldots}{1\,000} = \frac{\ldots\ldots}{10\,000} = \frac{536\,000}{\ldots\ldots}$

c) $1.8 = \frac{\ldots\ldots}{10} = \frac{\ldots\ldots}{5}$

d) $0.52 = \frac{\ldots\ldots}{100} = \frac{\ldots\ldots}{25}$

e) $0.032 = \frac{\ldots\ldots}{1\,000} = \frac{\ldots\ldots}{125}$

Problems

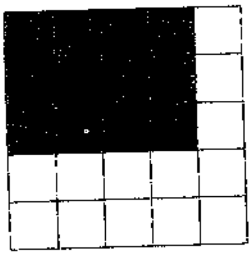
- I am a decimal number. My whole part is 9 and my decimal part is 47. Write me in he form of $\frac{\cdots}{100}$.

2- Given the decimal fraction $\frac{9}{10}$. We add 10 o both terms of the fraction.
) What is the new (obtained) fraction?
) Is it a decimal fraction? Why or why not?
) What is its decimal number?

3- Given the fraction $\frac{14}{5}$.
a) Is it a decimal fraction?
b) We add 10 to both terms of the fraction
- What is the fraction obtained?
- Is it a decimal fraction? Why or why not?

4- a) What does the fraction of the red rectangle represent to the square?

Transform this fraction into a decimal fraction.



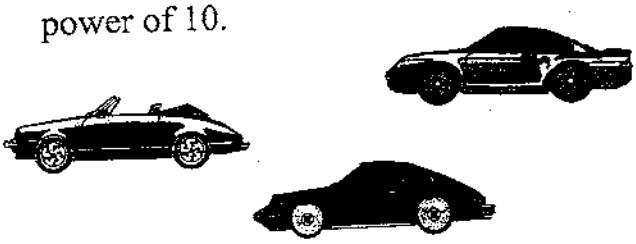
b) Knowing that the side of the big square is 5*cm*, what is its area?
Calculate the area of the red rectangle.

5- Samer walks 4 meters in 5 regular steps. Give in fractional number then in decimal one the length of his step.

6- The distance between two cities is 200*km*. A car Ⓐ runs50*km*, a car Ⓑ runs 120*km* and a car Ⓒ runs 185*km*.

a) What is the fraction of the total distance run by Ⓐ? by Ⓑ? by Ⓒ?

b) In each case, write the fraction as a decimal fraction its denominator is a power of 10.



7- An amount of 75 liters of oil has to be filled in 125 equal bottles.

a) Give the capacity of a bottle as a fraction, then transform it to a fraction whose denominator is a power of 10.

b) What is in *ml* the capacity of each bottle?

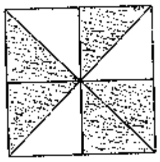
8- Mirna bought 250g of cheese at 3 600L.L.

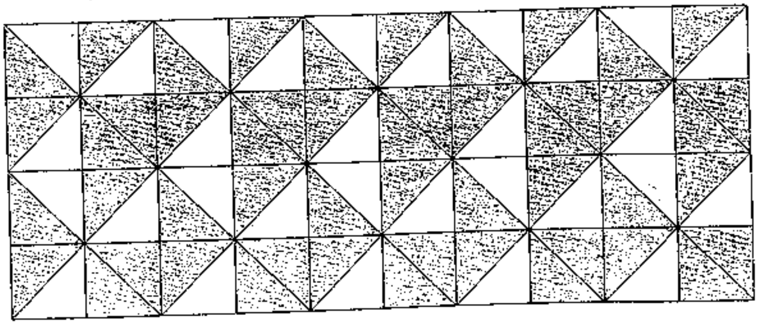
a) What is the price of one kilogram of cheese?
Give the result as a fraction number, a decimal number, then an integer.

b) Express this sum in the form of a decimal number of thousands of L.L.

Just for fun

What fraction of the area of the square is colored? Then what fraction of the area of the rectangle is colored?





Quotient
nd ratio

20

Objectives

[illegible]

- [illegible]
- [illegible]

Activities

Activity 1

a) In each of the following cases, write the quotient of a by b in the form of a fraction:

$a = 932$	$a = 5$	$a = 4$	$a = 42$	$a = 6$
$b = 23$	$b = 4$	$b = 3$	$b = 33$	$b = 5$

b) Copy the tables given below:
Arrange each of the quotients you have obtained in the appropriate place in the tables:

Natural Number	Decimal Fraction	Decimal Number	No decimal Fraction	Decimal Writing
....	....	....	....	....

c) Enclose the quotient $\frac{4}{3}$ between:

1) two consecutive natural integers.
2) two decimal numbers with one decimal part each.
3) two decimal numbers with two decimal parts each.
4) two decimal numbers with three decimal parts each.

d) Can you determine the 70th decimal part of the quotient of 4 by 3?

e) How can you represent a quotient and a writing that is useful in all cases?

$\frac{1}{3} = 0.33333333333...$

he quotient of 1 by 3 s a number made of a hole art and an unlimited ecimal part of epresentative number 3.

$.3 < 0.333...... < 0.4$

Is the enclosing of 0.3333...... between 2 decimal numbers of 1 decimal part each.

$\frac{46}{33} = 1.39393939......$

Activity 2

A) To make a chocolate cake we used the following ingredients:

Cocoa 75 g ; Sugar 260 g ; Flour 0.3 kg ; Butter 0.2kg

$\frac{5}{8}$ is the ratio of 5 to 8.50 ratio is a comparison of 2 numbers or two quantities of the same unit; otherwise, I should find the unit.

Eg: to find the ratio of 500 g to 2 *kg*, I should convert the 2 *kg* to 2000 g.

The Lebanese territory has a surface area of 10452 km^2. If the number of inhabitants is 3895000, then the average population is $\frac{389500}{10452}$ inhabitants/*km*^{2}.

Give the following ratios:

a) That of the mass of cocoa to the mass of sugar.
That of the mass of sugar to the mass of butter
Compare these two ratios.
What do you notice?
Do these ratios have units?

Speed: $\frac{\text{distance}}{\text{time}}$

b) A bus coverd a distance of 300 *km* in 6 *hours*, moving at a constant speed to visit Lebanese ruins.
This same school-bus had previously covered a distance of 400 *kms* in 8 hours also moving at a constant speed.
Find the average speed of the bus per hour in each of the two trips.
Compare the two answers showing your reasoning.

Text

The quotient:

To find the quotient of the division of two numbers *a* and *b*, we can apply the common method used in division. This quotient can be:

- a natural number

or

- a decimal one: if you can write $\frac{a}{b}$ as a decimal fraction.

See the examples given next.

a	*b*	quotient
225	25	$\frac{225}{25}=9$
22.5	2.5	$\frac{22.5}{2.5}=\frac{225}{25}=9$
2.25	2.5	$\frac{2.25}{2.5}=\frac{22.5}{25}=\frac{225}{250}=\frac{9}{10}=0.9$
2.25	25	$\frac{2.25}{2.5}=\frac{225}{2500}=\frac{9}{100}=0.09$
2.25	250	$\frac{225}{250}=\frac{225}{25000}=\frac{9}{1000}=0.009$

The division of 40 and 33 is not decimal because it is equal to 1.212121... and 21 repeats itself.

- a decimal number in which the decimal part is infinitely repetitive so $\frac{a}{b}$ is a non-decimal fraction, as is the case for $\frac{15}{11}$ since

  $\frac{15}{11} = 1.3636......$ 3 and 6 will be repeated continuously.
- Given an approximate value for example the quotient of 2 by 3 is 0.666..... so we can:
  - Enclose 0.666...... between two natural numbers or two decimal numbers within a range of 1 or 0.1 according to the required precision

Oh! How can I deal with such a number! 0.666666666.....

Example:

$0 < 0.666.... < 1$

$0.6 < 0.6666.... < 0.7$

$0.066 < 0.6666..... < 0.67$

$0.666 < 0.666..... < 0.667$

- Consider the first number in the enclosure as being an approximate value in deficit and the other an approximate value in excess.

- Written as $\frac{a}{b}$ to represent a quotient in all cases on condition that $b \neq 0$.

I assume that 0.66 is the approximate value of the division of 2 by 3

Ratio

- The fraction $\frac{4}{5}$ is the ratio of 4 to 5. In general, the ratio of a to b is the fraction $\frac{a}{b}$, with b different from zero.

- If the two numbers to be compared are of the same type (like two lengths, or two areas, or two ages, or two weights...).

Example: If the length of a bar is 3 cm, and the length of a piece of wood is 7cm, then the ratio of the length of the bar to that of the piece of wood is $\frac{3}{7}$.

- If the two numbers to be compared are of the same type but of different subdivisions of units like the cm and m for length, the *g* and *kg* for mass,.... you should convert them to the same unit then compare.

I assume that 0.67 is the approximate value of the division of 2 by 3

Example: If Wadad is 10 years old and Samir is 145 months old, you convert the age of Wadad to months.

$\Rightarrow$ Wadad $= 10 \times 12 = 120$ months.

$\Rightarrow$ ratio of Wadad's age to that of Samir's is $\frac{120}{145} = \frac{24}{29}$.

- The ratio is one of the methods used in comparison:

If $\frac{a}{b}$ is greater than a unit (one), this means that a is greater than b and vice-versa.

In this case, the ratio is unitless.

I simply say that the quotient of 2 by 3 is $\frac{2}{3}$

- If a and b are of different units (example if (a) is a measure of time and (b) a measure of length), then the unit of the ratio is the unit of (a) divided by the unit of (b).
- If the ratio is constant, then we call it average.

Example: If the price of 4 meters of fabric is 20000LL then:

$$\frac{\text{Price of fabric}}{\text{N° of meters of fabric}} = \frac{20\ 000\ \text{LL}}{4\ m} = 5\ 000\ {}^{\text{LL}}\!/_{m}$$

Focus

The quotient of a by b

$\frac{a}{b}$; $b\neq 0$

decimal fraction

The quotient is a natural number
or
The quotient is a decimal number

non-decimal fraction

The quotient is a decimal number in which the decimal part is repetitive like

$$\frac{13}{11} = 1.181818...$$

(1 and 8 are repetitive)

the average value is → 1.1 (default)
→ 1.2 (excess)

Ratio

ratio of a to $b = \frac{a}{b}$; $b \neq 0$

a and b are of the same unit

ratio is unitless

$\frac{a}{b} < 1 \Leftrightarrow a < b$

a and b are of different units

ratio has a unit

$$\text{unit} = \frac{\text{unit of } a}{\text{unit of } b}$$

if ratio is a constant then we call it average.

Exercises

1- Find the quotient of a by b in each of the following cases:

$a=1353 \qquad b=11$
$a=310.5 \qquad b=9$
$a=69.44 \qquad b=12.4$.

2- Give the repetitive numbers (frequency) in the quotient of a by b in each of the following:

$a=16 \qquad b=3$
$a=67 \qquad b=66$
$a=368 \qquad b=999$.

Check your answers using a calculator.

3- Find the approximate value for the quotient of a by b based on the following conditions:

$\frac{150}{26}$ to the nearest 0.1 in excess;

$\frac{123}{7}$ to the nearest 0.01 by default;

$\frac{25}{6}$ to the nearest 0.001 in excess.

4- Find the approximate value of the quotient of the division of 81458.23 by 7175.256 to the nearest 0.001 in excess. (you may use the calculator).

5- Give the quotient $\frac{a}{b}$ in fractional form and simplifyit if possible:

$$\frac{145}{120} \quad ; \quad \frac{68}{17} \quad ; \quad \frac{0.1}{0.01} .$$

6- Given a rectangle that is 60*mm* in length and 3 *cm* in width.
- Find the ratio of length to width;
- Find the ratio of width to length;
- Find the ratio of perimeter to surface.

7- Given a square with a side equal to 4 *cm*.
Find the ratio of the side of this square to its perimeter;
Find the ratio of the side of this square to its area.

8- Fadi covers by bike a distance of 40 *km* in two hours, at a constant speed. Find the average speed of Fadi per hour.

9- Look at the given example.
The speed of a turtle is equal to 108*m* per hour

$$\frac{108 \times 100\ cm}{3\ 600 \times 1\ s} = \frac{10\ 800}{3\ 600} = 3\ cm/\text{s}$$

If the speed of the eagle is 162 *km*/hour, convert this speed to *m*/s.

10- Circle the correct answer:

If $\frac{a}{b} = 0.9$, then:

$a > b$ $\qquad$ $a = b$ $\qquad$ $a < b$

Self-evaluation

- Find using your calculator the approximate value of the quotient of the division of the two numbers: 1 234 567 and 532.123 to the nearest 0.001 in excess.
- If the ratio of the area covered by water on Earth to the area of the land is equal to $\frac{71}{29}$ whose area is greater: that of land or that of water? Justify.

Problems

1- The ratio of the number of girls in the 6th grade to that of the boys in the same class is $\frac{5}{6}$. Which is greater, the number of girls or boys? Why?
Find the approximate value for the quotient of the division of two numbers 5 and 6 to the nearest 0.01 in default, then compare this value with a unity.

2- The diameter of Pluto is approximately $3 \times 10^3 km$ and that of Saturn is $120 \times 10^3 km$, approximately.
- a- Find the ratio of the length of Pluto's diameter to that of Saturn.
- b- Write this ratio as a decimal number, then as a decimal fraction.

3- If every 9 *kg* of sea water contains 288 *g* of salt, find the ratio of the mass of salt to water and transform this ratio into a decimal number.

4- If Sami is 156 months old and Samar is 21 years old, find the ratio of Sami's age to that of Samar's.

Find the average value of the quotient of the age of Samar to Sami to the nearest 0.1 in excess.

5- In 2 010, it is expected that the Lebanese's needs for water becomes 3 300 million m^3, of which 2 160 million m^3 will be used for irrigation.
- a- Find the ratio of water needed for irrigation to the total amount of water Lebanon will need in 2 010.
- b- Using your calculator, find the repetitive numbers of the quotient of the division of 3 300 million by 2 160 million.
- c- Find the approximate value of this quotient of the nearest 0.001, in excess.

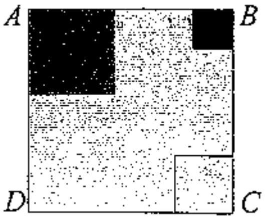
6- Sannine is 2 628 *m* high and Al-Kanaisa is 2 032 *m* high.
- a- Find the ratio of height of Sannine to that of Al-Kanaisa. Write this ratio as an irreducible fraction.
- b- Using your calculator, find the average value of the quotient of the division of 2628 by 2 032 to the nearest 0.001 by default.

7- The area of the Saudi territory is 2.2 million km^2, and the area of the Egyptian territory is 1 million km^2.
- a- Find the ratio of the area of the Egyptian territory to the Saudi.
- b- Find the quotient of the division of 2.2 million by one million and write it as a decimal fraction.

8- The human body is made up of 208 bones and 900 muscles.
- a- Find the ratio of the number of bones to muscles.
- b- Using your calculator, find the quotient of the division of 208 by 900 to the nearest 0.001 by default.

9- A train covers 1170 *km* in 9 hours.
- a- Find its average speed per hour.
- b- What distance will this train cover in 10 hours?
- c- Find its average speed in meters/minute.

Just for fun



1. Find the perimeter and area of each of the squares given next.
2. Find the ratio of the perimeter to the length of the side of the square.
3. Find the ratio of the perimeter of each square to the area of the same square.

Percentage

21

Objectives

[illegible]

- [illegible]
- [illegible]

Activities

After spending a day shopping, a mother goes back home and calls her two children:

a) "Sara, take this dress. Its price was $300 before the sale, but I profited from a twenty- percent discount on its price.
Complete the table, by following the steps indicated and using your calculator, then you will know how much I paid".

[illegible]	[illegible]	[illegible]	[illegible]

b) "The T-shirt is yours Ziad; its price was $60 but I bought it for $45. The calculation shown in the following table will allow you to find the discount on this T-shirt".

[illegible]	[illegible]	[illegible]	[illegible]
60	45	....	$\frac{\text{.....} - \text{.....}}{\text{.....}} = \text{.......}$

[illegible]		
a decimal number	a fraction of denominator 100	percentage
.... =	$\frac{\text{.......}}{100}$	= .... %

Text

We say 2.5% to indicate the operation $\frac{2.5}{100}$ and avoid writing $\frac{25}{1000}$

The multiplication with a positive number is compatible with the order.

Percent of a number

To calculate one percent, two percent, three percent,.... of a number is to multiply it respectively by:

$$\frac{1}{100}, \qquad \frac{2}{100}, \qquad \frac{3}{100}\ldots\ldots\ldots$$

Example Five percent of $340 = \frac{5}{100} \times 340 = 17$.

We write: "17" is equal to "5 percent of 340".
better yet:

5% of 340 is equal 17.

• Percentages

The multiplication operators , $\frac{1}{100}$, $\frac{2}{100}$, $\frac{3}{100}$, ...,
noted respectively as 1% , 2% , 3% , ...,
are percentages.

• Comparison using percentages

Starting from an inequality $620 < 734$

the operator "percent" $\frac{620}{100} < \frac{734}{100}$

makes an inequality having the same significance (meaning), from where: $8 \times \frac{620}{100} < 8 \times \frac{734}{100}$

The order is conserved when we apply a percentage.

• On the other hand, if you want to classify two numbers, *m* and *n*, of which 32 % are respectively 416 and 240, it is sufficient to compare 416 and 240. Since $416 > 240$, then $m > n$.

Practically, to classify two numbers, it is sufficient to classify their "percent".

Determination of a "percentage" of a number:

To find the "percentage" of m represented by n:

you calculate the quotient $\frac{n}{m}$,

then you write this quotient as a fraction of denominator 100.

The numerator of the obtained fraction is the percentage we are looking for.

$m = 3500 \quad n = 420$

$$\frac{420}{3\,500} = 0.12$$

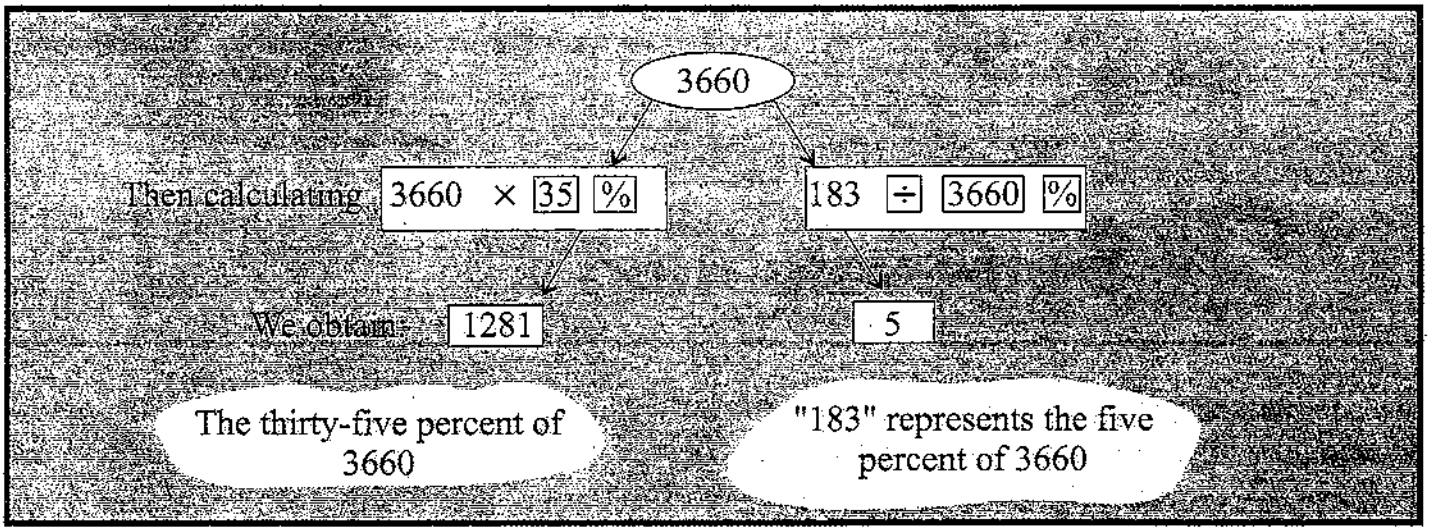
$$0.12 = \frac{12}{100}$$

420 = 12 % de 3 500.

Examples:

- $\frac{35}{50} = 0.7$, therefore 35 is equal to 70 % of 50.
- $\frac{1\,573}{5\,720} = 0.275$, therefore 1 573 represents the 27.5 % of 5 720

Focus



Exercises

1- Write as a percentage:

a) 5 of 10 ; 1 of 4 ;

2 of 25 ; 3 of 25 ;

7 of 10 ; 1 of 50 ;

b) 3 of 4 ; 9 of 20 ;

30 of 50 ; 15 of 75;

2- Write as a percentage:

a) $\frac{2}{40}$; $\frac{5}{16}$; $\frac{9}{32}$; $\frac{21}{20}$;

b) $\frac{24}{60}$; $\frac{18}{15}$; $\frac{27}{75}$; $\frac{35}{175}$.

3- Write as a "percent":

a) 0.2; 0.33; 0.15; 0.75; 0.80.

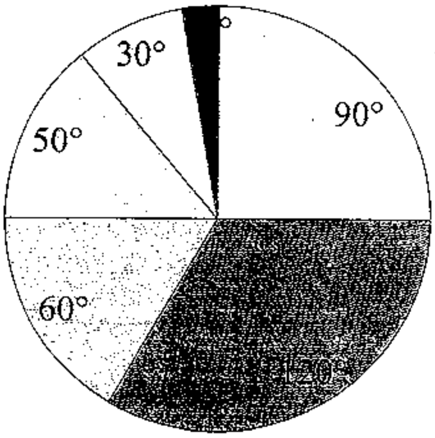
b) 1.03; 2.25; 3.5; 7.35; 9.2.

4- What percentage of a disc do the following represent?

- the semi-disc?
- the quarter of a disc?
- the three quarters of a disc?
- the four fifths of a disc?
- the seven-twentieths of a disc.

5- The full central angle of a disc is divided into 6 angles of measures: 90°; 50°; 60°; 30°; 10° and 120°.

What percentage of the whole (full) angle do the above cited angles represent?



6- What is the 15 % of each of the following numbers:

130 ; 2 453 ; 3 202

5 500 ; 673.5 ; 87.42 -

9.990 ; 1.882 ; 88 880.

7- Calculate the 20 %, and the 30% of 18000.

Compare the sum of the two answers with the half of 18000.

Self-evaluation

1- State whether the following is true or false:

- 25 % of a number is its quarter (one fourth).
- 30 % of a number is its one third.
- 20 % of 250 = $\frac{250}{5}$.
- The eighth of a number is equal to 12.5 % of this number.

2- During sales, Samer and Rima bought two pairs of pants at half price. Samer paid 39000 LL for his and Rima paid 45000 LL.

Which of the two pairs of pants was more expensive before the sale?

Problems

1- A property has been bought for 35 000 000LL. We want, when reselling it, to make a profit of 7 %. What will the profit be?

2- A man bought 35 lambs for 135000 LL each. Since he paid the total amount in cash, he got a discount of 6%.
How much did he pay?

3- A merchant bought 175 meters of fabric for 2880 LL per meter. He sold the $\frac{2}{5}$ for a profit of 16 % and the remainder with a profit of 10 %.

How much did he gain?

4- A grocer buys 100 kg of green coffee for 13000 LL per kg. Knowing that, when green coffee is roasted, it loses 18 % of its mass, calculate the price of a kg of roasted coffee.

5- A farmer picked 3600 *kg* of olives to extract oil. Knowing that every 100 *kg* of olives makes 20 *kg* of oil, calculate the mass of oil this farmer will obtain. What in liters is the quantity obtained if one liter of oil weighs 0.90 *kg*?

6- A salesman buys a quantity of wheat, of which he sells the 2/5 for a price of 480 000 LL per ton. He thus gains 24 % on the buying price. He sells the rest of the wheat for 832700 LL thus losing 5000 LL per ton.
How many tons of wheat did he buy?

7- What is the price of an object labeled 10000 LL if you can benefit from a 30 % discount on its price?

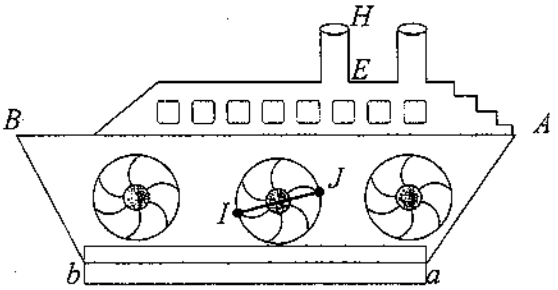
8- A grocer buys 420 kilograms of coffee at 7350000 LL. How much should he sell the 200 g to get a profit of 11 % on the buying price?

9- A piece of fabric measures 47.86 m. When rinsed it loses 3.2 % of its length. What will its length be after rinsing?

10- An editor gives a 23 % discount on the price of packages. What is the discount done on a package of 30 000 LL?
What is the price of a package on which we got a discount of 17250 LL?

11- Milk gives 12 % of its mass as cream and 35 % of the mass of the cream as butter. How many kg of butter will 80 kg of milk give?

12- Since we cannot draw a ship with real measurements, we draw it by dividing all the dimensions by 1000. So we constructed this reduced model:



Take the necessary measures in your notebook and complete the following table:

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]				
[illegible]				

"We say that this drawing is made to the scale $\frac{1}{1\ 000}$ which means that the measure of a length on this drawing is equal to $\frac{1}{1\ 000}$ of the real measure of this same length.

$$\text{Scale} = \frac{\text{mesure on the drawing}}{\text{real measure}}$$

13- A rectangular field is represented by a rectangle of dimensions 3.2 *cm* and 5 *cm*, in a drawing with the indication:

$$\text{Scale } "\frac{1}{5000}" \text{ .}$$

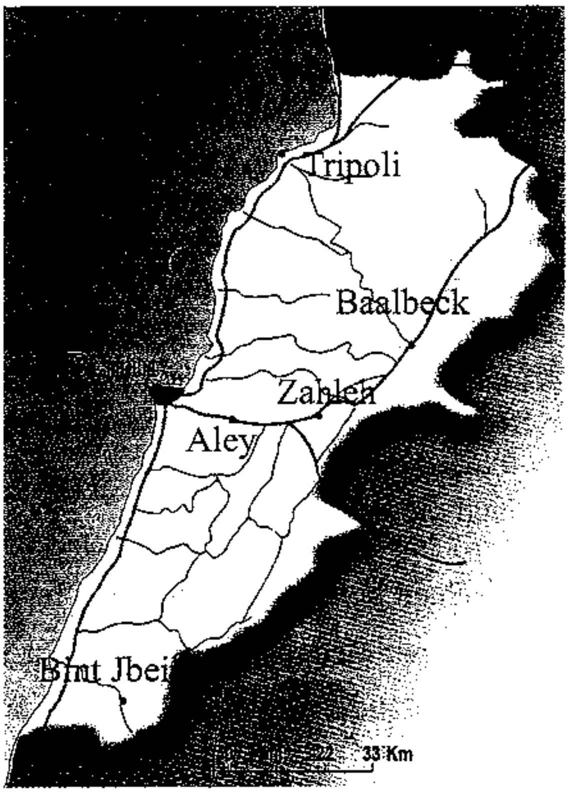
Calculate the real dimensions of this field.

14- Points *A* and *B* on a geographical map represent two positions on Earth. The distance *AB* on the map represents the real distance (bird flight) between these two points. Calculate the real distances of :

* Bint Jbeil to Tripoli.

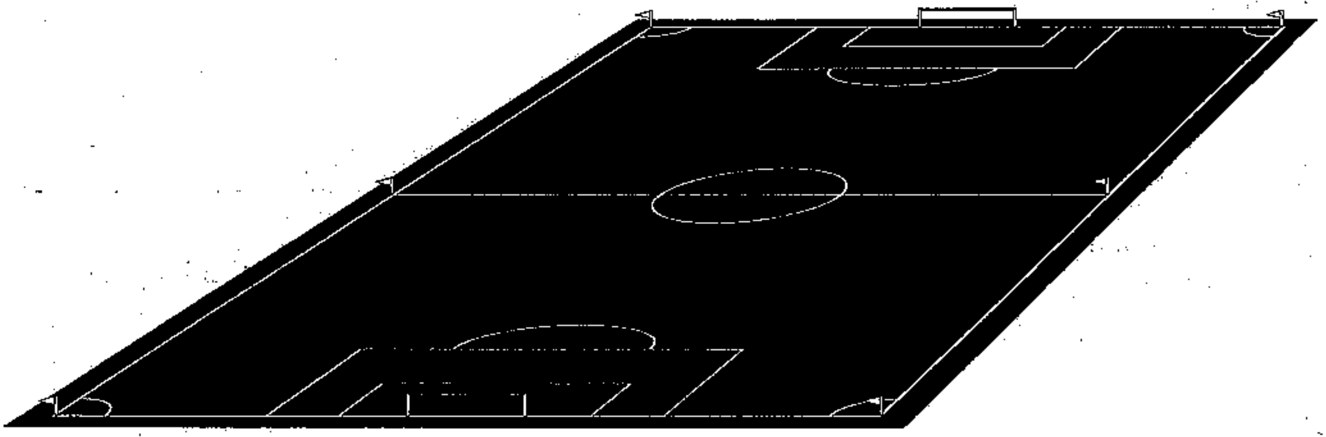
* Beirut to Baalbeck.

* Zahlé to Aley.



Just for fun

A football, a basketball and a tennis fields are represented below:



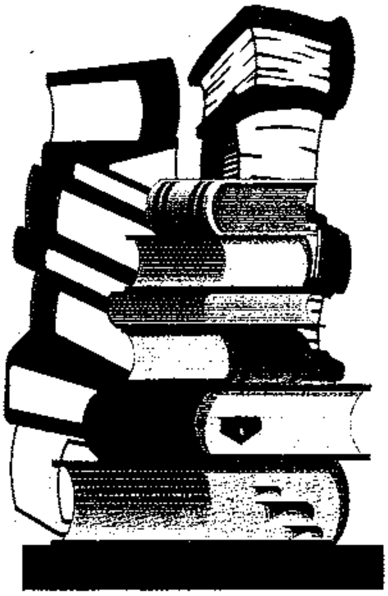
1) Knowing that a football field is a rectangle of dimensions 100*m* and 60*m* .
Calculate the scale at which we drew the above field.

Proportionality

22

Objectives

Activities



a) Make a parcel using two, three books and measure its height.

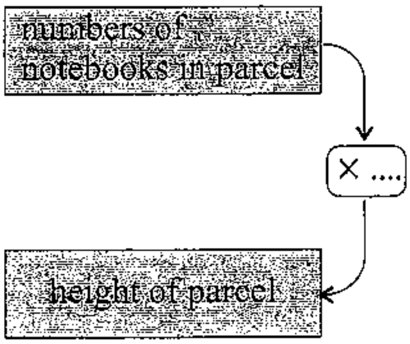
Using the above information, can you guess the height of the seven books without putting them in a parcel (on top of each other)?

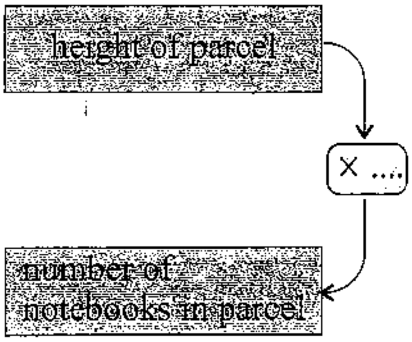
b) Take several notebooks, all of the same thickness, and arrange them into parcels. For each parcel, write the number of notebooks or the height of the parcel in *cm*.

Reproduce and complete the table:

Number of notebooks	10	20	25	...	48	...
Height of the parcel	8	...	...	25.6	...	52.8

Determine the multiplication operator corresponding to each of the following steps:





Text

4	5	6
8	10	12

2	20
3	30
7	70
11.2	112

The proportionality table

A numerical table with two lines is a proportionality table. If you multiply the numbers in one of the lines by a number "c" called the proportionality coefficient, you obtain the numbers of the other line.

Examples:

a- In the table given to the right, the products of the numbers given in the first line by "4" are the numbers given in the second line (row), therefore the given table is a proportionality table.

0.3	0.8	1.2	7	10
1.2	3.2	4.8	28	40

×4

b- As for the next table, no number can be put in the blank therefore this is not a proportionality table.

3	5	8
18	30	56

×...

If there are 2 distinct coefficients, therefore there exists none!

Proportional Series (Sequences)

If you reproduce the numbers of a proportionality table as they are and in the same order we obtain two sequences called "proportional series".

0.3	0.8	1.2	7	10
1.2	3.2	4.8	28	40

0.3 0.8 1.2 7 10

1.2 3.2 4.8 28 40

Also, when you have two proportional series you can arrange them in a proportionality table.

6 7 9 11

7.8 9.1 11.7 14.3

6	7	9	11
7.8	9.1	11.7	14.3

The proportionality coefficient

Given two proportional series, to calculate the proportionality coefficient is to divide one of the terms of the first series by the term having the same order in the second series.

6 7 9 11

7.8 9.1 11.7 14.3

$9.1 \div 7 = 1.3$

$11.7 \div 9 = 1.3$

Notice: If you have two proportional series, the coefficient will be the same no matter what you choose in terms of a different order.

The calculation of the coefficient can be a way to identify whether two series are proportional.

Examples:

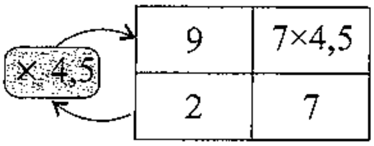
9 - 16 - 2 and 12.6 - 22.4 - 3

$\frac{12 .6}{9} = 1.4$ and $\frac{3}{2} = 1.5$

This indicates that the two series are not proportional.

9	
2	7

$9 \div 2 = 4,5$



9	7×4,5
2	7

In fourth proportional

- If you are dealing with the small proportionality table "2 rows and 2 columns" and if one number only is missing, then this number is called the fourth proportional of the 3 other numbers.

  In order to determine the fourth proportional, it is sufficient to calculate the proportionality coefficient and let it act as an operator.

- Practically and in the case of a table

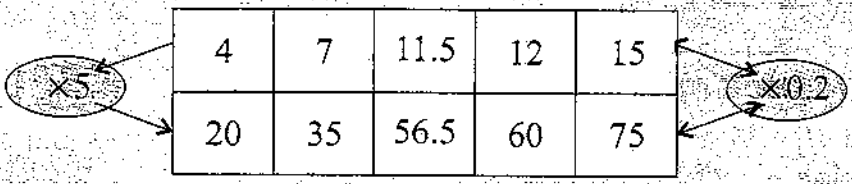
a	x
b	c

where a, b and c are known

$$\boxed{\text{the fourth proportional is: } x = \frac{a \times c}{b}}$$

Focus

We can go both ways



4	7	11.5	12	15
20	35	56.5	60	75

$5 \times 0.2 = 1$

Exercises

1- From the following table, choose the proportionality numbers:

5	6	7
12.5	15	17

3	5	7
3.6	6	8.4

10	15	40
4	6	16

4	3.2	7	8
12	9.6	21	24

18	30	9	105
3.6	6	1.8	21

14	18	21
15.4	19.8	23

2- Reproduce then complete the proportionality tables

× ......

[illegible]	[illegible]	......
[illegible]	......	[illegible]

× ......

[illegible]	......	[illegible]	......
[illegible]	[illegible]	......	[illegible]

× ......

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	......

3 %

[illegible]	[illegible]	[illegible]

[illegible]	[illegible]
[illegible]	......

......	[illegible]
[illegible]	[illegible]

3- Is there proportionality between the two lines in the following cases?

a- Age and length

[illegible]	10	11	12
[illegible]	132	140	145

b- Quantity bought and price

[illegible]	2	3	6
[illegible]	500	750	1 500

c- Consumption and course

[illegible]	5	12	16
[illegible]	40	96	128

d- Number of steps and height reached

[illegible]	4	7	10	12
[illegible]	60	105	150	180

e- Number of pages and price

[illegible]	230	540	680
[illegible]	3 000	7 000	10 000

Self-evaluation

a- Complete:

× ......

[illegible]	[illegible]	......	[illegible]	......
[illegible]	......	[illegible]	......	[illegible]

× ......

b- Find the fourth proportional:

[illegible]	......
[illegible]	[illegible]

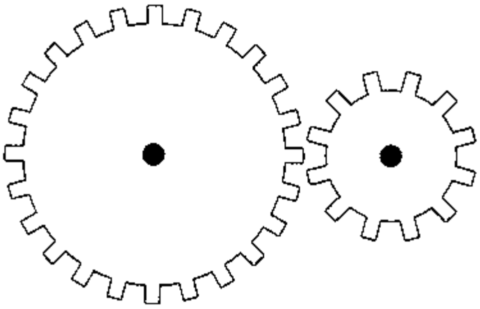
......	[illegible]
[illegible]	[illegible]

[illegible]	[illegible]
[illegible]	......

[illegible]	[illegible]
......	[illegible]

Problems

1- When the big wheel effects 2 complete rounds, the small one effects five.



How many rounds will the small wheel complete if the big one does 32 rounds? How many rounds will the big wheel effect if the small one does 65?

2- A game consists of going backwards 2 steps wherever you advance (go forward) 7 steps.

- How many steps do you advance if you go backwards 50 steps? 120 steps? 150 steps?
- How many steps do you go backwards if you advance 35 steps? 49 steps? 105 steps?
- Where would you end (n° of final steps) if you made 180 steps in total?

3- To make a cake, we follow the directions given in the recipe below:

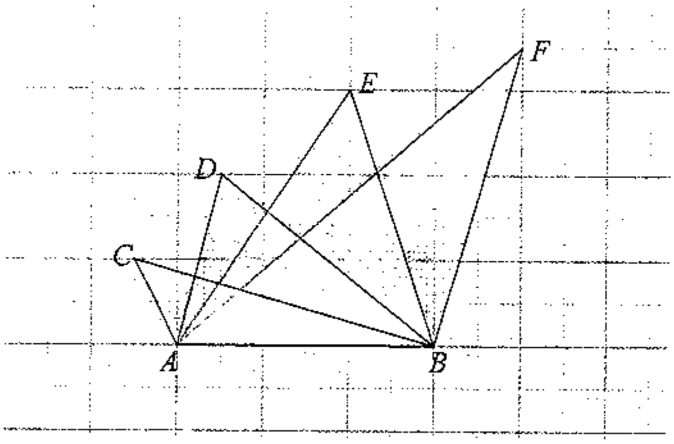
Sugar	Flour	Butter	[redacted]
100g	75g	40g	3

Calculate in grams the necessary quantities of each ingredient if we want to make the same type of cake but using 6 eggs; 9 eggs and 30 eggs.

4- When filling tanks with a capacity of 10 liters each, we spilled 0.25 liter on the ground.

- How much do we spill when we fill 20, 30 and 50 tanks of this kind?
- What is the number of tanks filled if you have spilled 15 liters?

5- • Calculate in cm^2 the areas of each of the triangles *CAB*, *DAB*, *EAB* and *FAB* then fill in the table:



Triangles	*CAB*	*DAB*	*EAB*	*FAB*
Height				
Area				

- What is the area of a triangle *MAB* (not drawn) of height 14 *cm*?

6- Draw five rectangles that have a width of 4 *cm* with different lengths.

Calculate for each rectangle, its perimeter and.

Reproduce and complete the table:

Length of rectangle	....	....	....	....	....
Perimeter	....	....	....	....	....
Area	....	....	....	....	....

- Is there proportionality between:
  - – The sequence of lengths perimeters?
  - – The sequence of lengths areas?
- What is the perimeter of a rectangle of 118 *cm* in length and 4 *cm* in width?

7- A bus can transport 36 students. Calculate the number of buses needed to transport 144 students going on a trip.
The consumption of fuel by a bus is 0.22 liters per *km*. How much fuel will one bus need if the trip is 230-*km* long? How much fuel will all buses need?
The price of 20 liters of octane (fuel) is 13000 LL. Calculate how much the trip will cost.

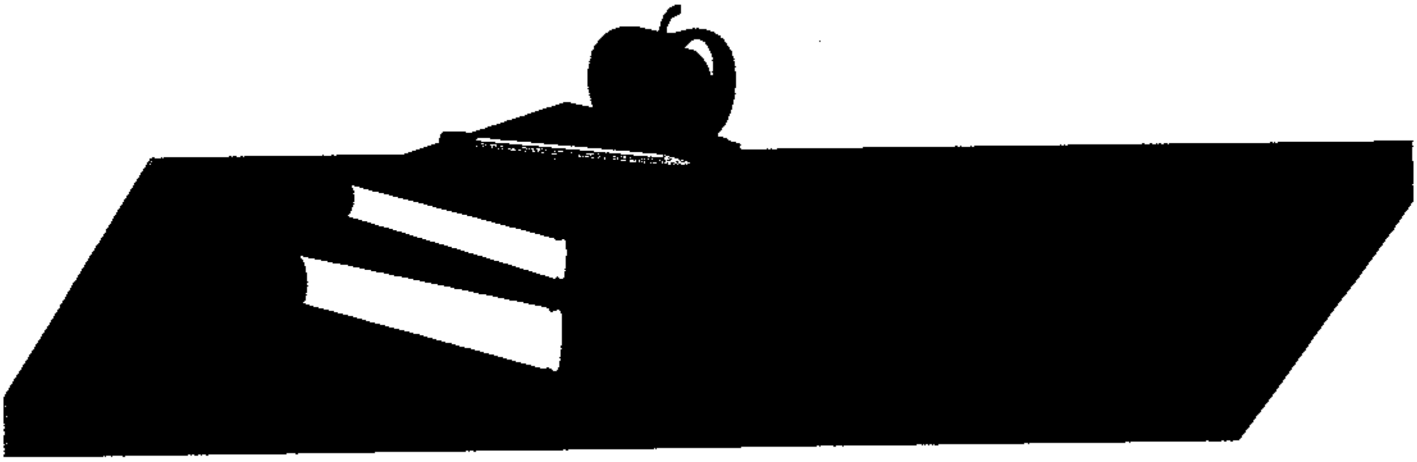
8- To make jam, mother uses 2.7 *kg* of sugar for 3.5 *kg* of fruit.

1) Calculate:

- The quantity of sugar needed for 15 *kg* of fruit.
- The quantity of fruit corresponding to 20 *kg* of sugar.

2) Knowing that the mixture fruit-sugar loses 70% of its mass when boiled, what are the obtained quantities of jam in the two cases of the previous question?

Just for fun



Place three different objects in the sun, knowing the height of the objects. Measure the length of the shadow of one of the objects.

Find without measuring the length of the two other shadows.

BUILDING UP MATHEMATICS

7th Grade

Basic Education

REVISED EDITION

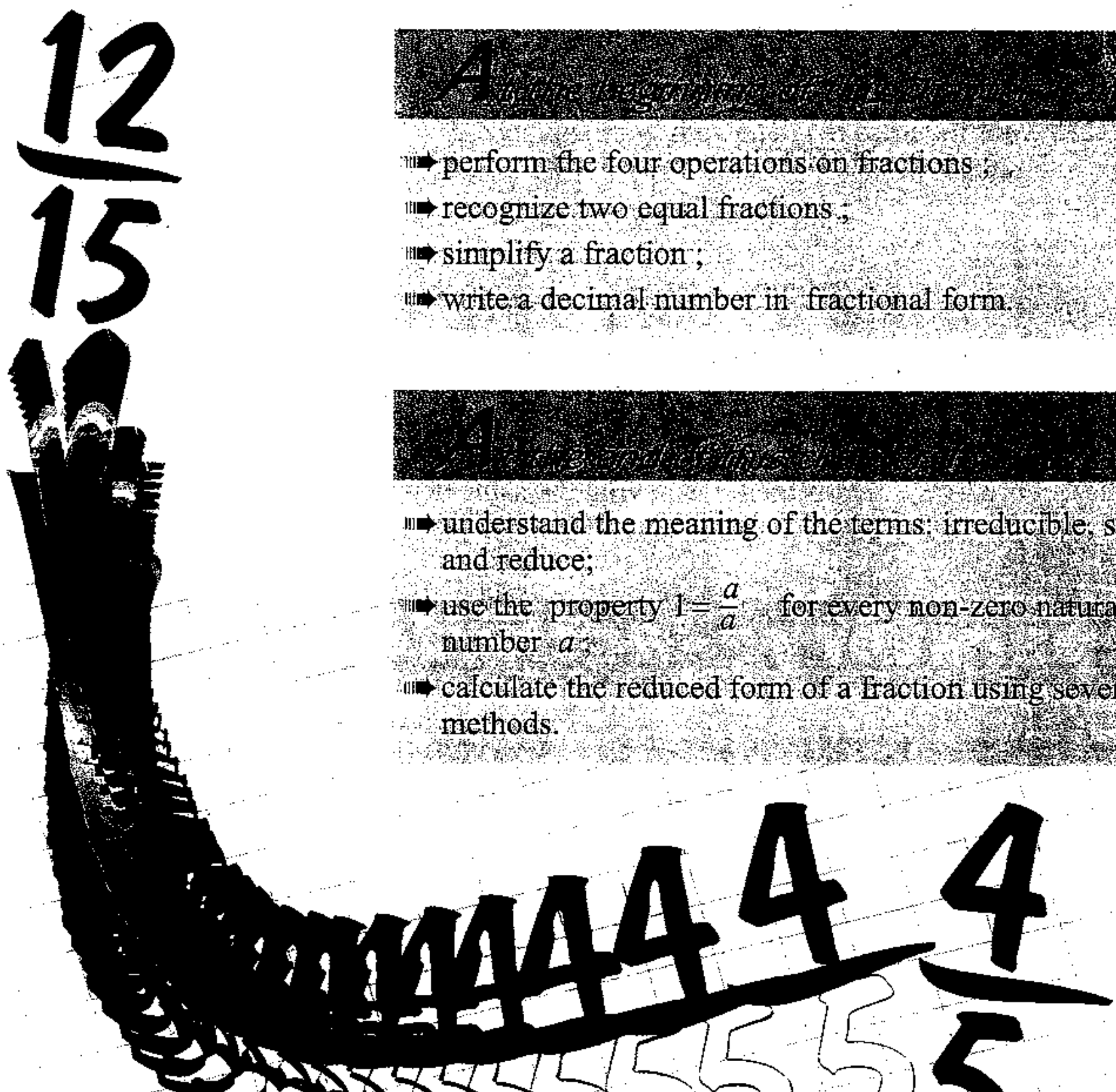
Reduction of fractions

8

introduction

Different fractions and yet equal!!

Since many years already, you learned how to manipulate fractions: definition, equality, simplification, operations of calculation. In fact, the notion of fraction is fundamental in mathematics. It represents symbolically real situations (we say also that it serves as a model for these situations) and consequently to deal with them mentally.



At the beginning of [illegible]

- perform the four operations on fractions ;
- recognize two equal fractions ;
- simplify a fraction ;
- write a decimal number in fractional form.

At the end of this [illegible]

- understand the meaning of the terms: irreducible, simplify and reduce;
- use the property $1 = \frac{a}{a}$ for every non-zero natural number a ;
- calculate the reduced form of a fraction using several methods.

Recall activities

Activity 1

Equal fractions or not ?!

1) How many seasons are there in a year ? Hence, what fraction of a year is represented by a season ?
2) What fraction of 12 months does a season represent ?
3) Recopy and complete: $\frac{1}{4}=\frac{...}{12}$.
4) Explain how to get a fraction equal to a given fraction .
5) Compare the two fractions : $\frac{5}{8}$ and $\frac{3}{8}$ Recall how to compare two fractions having the same denominator.
6) How do we compare the two fractions $\frac{5}{7}$ and $\frac{3}{8}$? Recall how to compare two fractions with different denominators

Activity 2

Calculation with fractions ?!

1) Perform : $\frac{3}{17}+\frac{5}{17}$; $\frac{8}{15}-\frac{2}{15}$. Recall the rules of addition and subtraction of two fractions having the same denominator.
2) Perform : $\frac{2}{7}\times\frac{3}{5}$; $\frac{5}{8}\div\frac{3}{8}$. Recall the rules of multiplication and division of two fractions .
3) Perform : $\frac{2}{5}+\frac{3}{4}$. Recall the rules of addition and subtraction of two fractions having the same denominator.

Preparatory activities

Activity 1

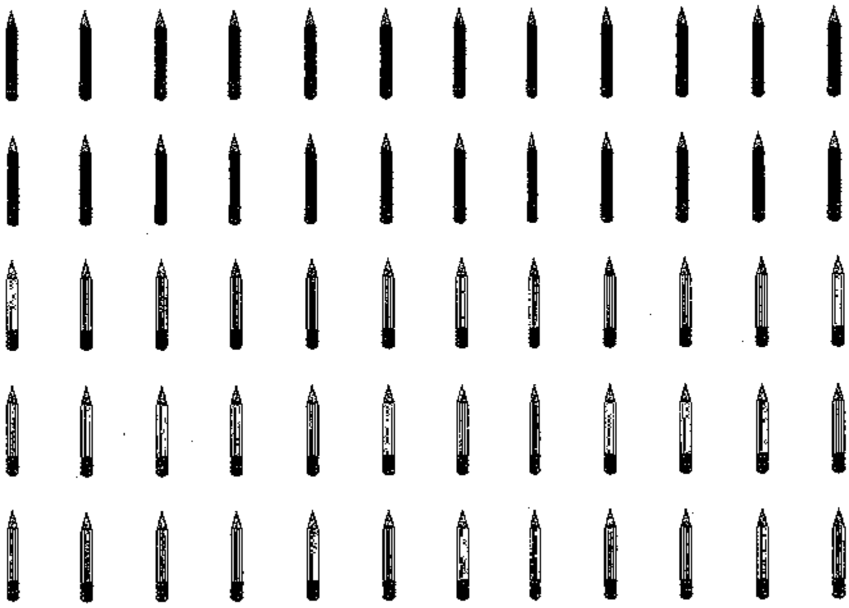
Divide to rule

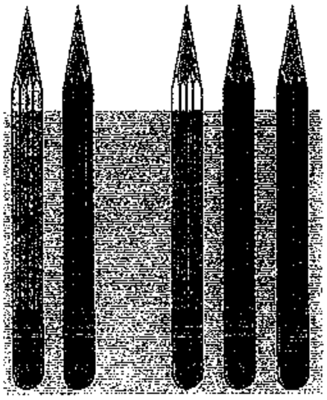
1) Write the fraction $\frac{2\times3}{5\times7}$ as a product of two factors. In how many ways, can you do that ?
2) Write the numerator and the denominator of the fraction $\frac{12}{35}$ as a product of two numbers, then deduce the writing of this fraction in the form of a product of two fractions .
3) Find a common divisor of the two numbers 12 and 15, and deduce a simple writing of the fraction $\frac{12}{15}$.

Activity 2

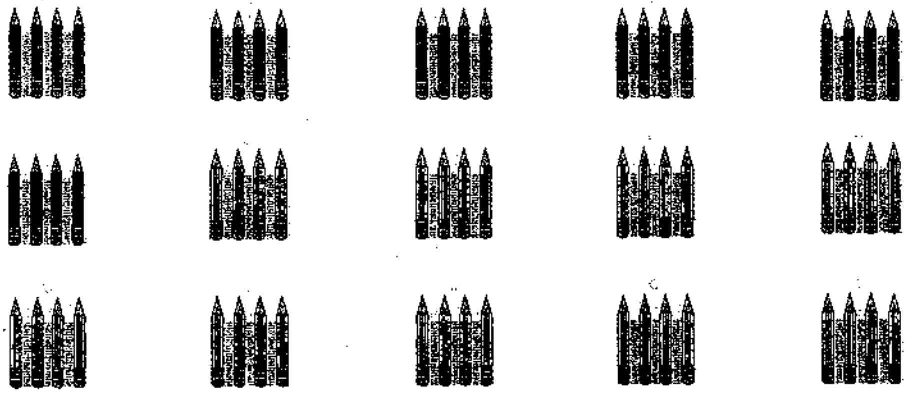
Wrap up in order to better calculate!

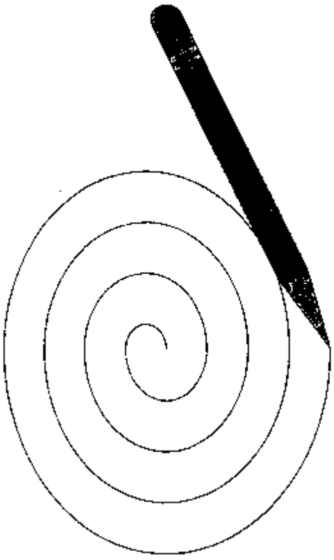
1) What fraction of the pencils is represented by the red pencils in the indicated figure?



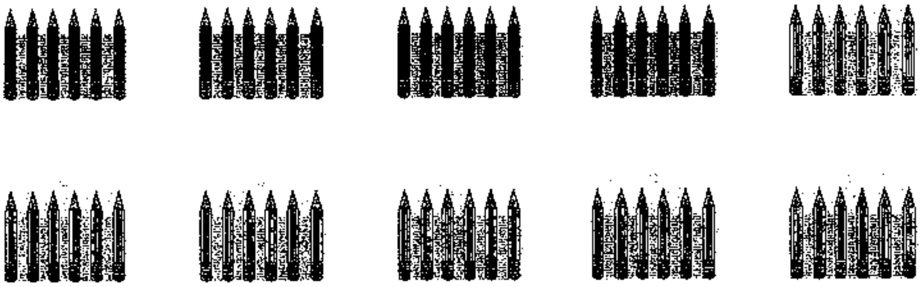


2) We decide to put the pencils in boxes of four. what fraction of all the boxes is represented by the boxes containing the red pencils?





3) And with boxes of six?



4) Can you do better?

5) Write the equalities of the different fractions obtained and translate the passing of the first fraction to each of the others in mathematical terms.

Text

I. Simplification

Examples : a) $\frac{10}{20}=\frac{5\times2}{5\times4}=\frac{5}{5}\times\frac{2}{4}=\frac{2}{4}$

b) $\frac{24}{18}=\frac{2\times12}{2\times9}=\frac{12}{9}$

In general, if $a = n\times c$ and $b = n\times d,$ then $\frac{a}{b}=\frac{n\times c}{n\times d}=\frac{c}{d}$

In this case, we get a fraction $\frac{c}{d}$ which is equal to $\frac{a}{b}$ and such that $c < a$ and $d < b$. We say then that we simplified $\frac{a}{b}$.

II. Reduction

Definition 1 A fraction is said to be **irreducible** if it cannot be simplified.

Examples :

1) the fraction $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ are irreducible.

2) evry fraction of numerator 1 is irreductible.

Property 1 If the numerator and the denominator of a fraction are relatively prime, then the fraction is irreducible.

Be reminded that two integers are prime between them if their GCD is equal to 1.

to reduce a fraction fraction $\frac{a}{b}$, is to find an irreducible simplification of this fraction . The fraction obtained is called the ***reduced form*** of $\frac{a}{b}$.

Example .

The fraction $\frac{2}{3}$ is the reduced form of $\frac{16}{24}$ since :

$\frac{2}{3}=\frac{16}{24}$ and GCD (2;3)= 1 .

Property 2 The reduced form of a fraction is obtained by dividing the numerator and the denominator by their GCD.

In the preceding example, we have GCD(16;24) = 8, we get the reduced form of $\frac{16}{24}$ by dividing these numbers by 8 .

Focus

1. How to simplify a fraction .

We simplify a fraction by dividing the two terms by a common divisor .

Example :

10 is a common divisor of the members of the fraction $\frac{100}{60}$.

Hence, simplify this fractions as follows :

$$\frac{100 \div 10}{60 \div 10} = \frac{10}{6}$$

2. How to reduce a fraction to an irreducible fraction .

There are several methods to reduce a fraction .

a) By dividing the numerator and denominator by their GCD .

Example:

GCD(180;100) = 20, Hence $\frac{180}{100} = \frac{180 \div 20}{100 \div 20} = \frac{9}{5}$.

b) By decomposing the two numbers into prime factors and then by simplifying.

Example:

$$\frac{180}{100} = \frac{\cancel{2}\times\cancel{2}\times3\times3\times\cancel{5}}{\cancel{2}\times\cancel{2}\times\cancel{5}\times5} = \frac{3\times3}{5} = \frac{9}{5}$$

c) By proceeding by successive divisions by common divisors:

Example :

$$\frac{180}{100} = \frac{180 \div 2}{100 \div 2} = \frac{90}{50} = \frac{90 \div 10}{50 \div 10} = \frac{9}{5}$$

where 9 and 5 are relatively prime.

We then get the reduced fraction $\frac{180}{100}$.

Exercises

Review and Practice

1. Perform :

a) $\frac{24}{12}+\frac{7}{18}$; $\frac{15}{8}-\frac{16}{9}$; $\frac{2}{5}\div\frac{3}{8}$.

b) $\frac{5}{9}+\frac{8}{20}$; $10-\frac{23}{6}$; $\frac{1}{5}\times\frac{3}{9}$; $\frac{5}{18}\div\frac{25}{45}$.

c) $\frac{5}{14}+8$; $\frac{45}{12}-2$; $\frac{18}{21}\times\frac{7}{6}$; $\frac{3}{63}\div\frac{9}{8}$.

d) $\frac{2}{7}+\frac{12}{32}+\frac{10}{16}$; $\frac{9}{7}-\frac{7}{9}$; $5\times\frac{12}{15}\times\frac{4}{16}$.

e) $8.5+\frac{5}{8}$; $3.5-\frac{14}{5}$; $\frac{3}{8}\times\frac{15}{20}$; $5\div\frac{3}{8}$.

f) $31\times\frac{52}{62}$; $\frac{4}{12}\div\frac{7}{6}$; $\frac{8}{18}\div 6$.

2. Complete the table:

$+$	$\frac{2}{3}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{4}{7}$
$\frac{1}{4}$					
$\frac{1}{6}$					
$\frac{1}{2}$					
$\frac{1}{7}$					
$\frac{1}{10}$					

3. Complete

$\frac{8}{14}=\frac{4}{\dots}=\frac{\dots}{28}=\frac{24}{\dots}$.

4. Write each of the following decimal numbers in the form of a fraction and simplify when possible :

a) 2.5 ; 1.4 ; 6.5 ; 1.2 ; 28.3.

b) 11.3 ; 3.25 ; 2.2 ; 0.4 ; 12.18 .

5. a) Can you find a fraction equal to $\frac{2}{3}$:

- whose numerator is 126?
- whose denominator is100 ?

b) Compare each of the fractions $\frac{14}{21}$ and $\frac{24}{36}$ with the fraction $\frac{2}{3}$. What relation is there between these fractions ?

6. List from the smallest to the largest :

a) $\frac{3}{4}$; $\frac{2}{5}$; $\frac{5}{7}$; $\frac{4}{9}$; $\frac{2}{3}$.

b) $\frac{2}{6}$; $\frac{1}{2}$; $\frac{3}{8}$; $\frac{5}{11}$; $\frac{7}{8}$.

7. Complete the table :

$\times$	$\frac{3}{2}$	$\frac{1}{7}$	$\frac{3}{8}$	$\frac{4}{6}$	$\frac{2}{7}$
$\frac{1}{2}$					
$\frac{1}{3}$					
$\frac{1}{4}$					
$\frac{1}{5}$					
$\frac{1}{10}$					

8. Simplify each of the following fractions :

a) $\frac{25}{30}$; $\frac{18}{40}$; $\frac{90}{120}$; $\frac{30}{75}$; $\frac{15}{81}$.

b) $\frac{16}{48}$; $\frac{36}{144}$; $\frac{64}{320}$; $\frac{14}{56}$; $\frac{36}{270}$;

9. Among the following fractions find the reduced ones :

$\frac{13}{15}$; $\frac{22}{33}$; $\frac{14}{31}$; $\frac{27}{72}$; $\frac{1}{3}$.

10. a) Find the GCD of 4235 and 9800.

b) Use the GCD in part a) to reduce the fraction $\frac{4235}{9800}$

11. Reduce each of the following fractions :

a) $\frac{180}{450}$.

b) $\frac{96}{144}$.

c) $\frac{252}{396}$.

d) $\frac{860}{900}$.

e) $\frac{350}{560}$.

f) $\frac{45}{35}$.

g) $\frac{42}{63}$.

h) $\frac{33}{55}$.

i) $\frac{333}{555}$.

j) $\frac{2323}{4545}$.

k) $\frac{2^4 \times 3^2 \times 13}{2^4 \times 3 \times 11}$.

ℓ) $\frac{25 \times 48 \times 30}{50 \times 12 \times 42}$.

m) $\frac{180}{204}$.

n) $\frac{1170}{8250}$.

o) $\frac{4375}{5625}$.

p) $\frac{4680}{6840}$.

12. Same exercices :

a) $\frac{4542}{4212}$.

b) $\frac{12600}{3300}$.

c) $\frac{1+3}{5+7}$.

d) $\frac{5 \times 6 + 7 \times 6}{20 \times 6}$.

e) $\frac{11 \times 38 - 11 \times 8}{11 \times 18}$.

f) $\frac{555}{999}$

g) $\frac{7 \times 8 + 3 \times 8}{3 \times 8}$

h) $\frac{11700}{18900}$

i) $\frac{650}{1000}$

j) $\frac{208}{153}$

A Among of the following fractions, determine those that are irreducible :

$\frac{14}{21}$; $\frac{13}{16}$; $\frac{12}{15}$; $\frac{7}{31}$; $\frac{16}{24}$.

B Compute $\frac{1155}{363}+1$; $\frac{345}{535}+1$.

C Using the GCD, reduce the fraction $\frac{3780}{6600}$.

D Reduce the fraction $\frac{520}{208}$ by the method of decomposing into prime factors.

Problems

1. I am a fraction .

a) Who am I if three fourth of me is $\frac{8}{5}$?

b) Who am I if my half is equal to $\frac{5}{3}$?

c) Who am I if my two third is $\frac{7}{4}$?

d) Who am I if by adding $\frac{5}{6}$, to me, Ibecome $\frac{31}{30}$?

2. A rectangular garden measures 9 m in length and 5 m in width . We pave alleys for pedestrians, of 1m of width as in the indicated figure, and we cultivate the remaining. What fraction represents the part cultivated of this garden?

3. a) Verify that each of the following fractions is equal to : $\frac{27}{63}$

$$\frac{9}{21} ; \frac{54}{126} ; \frac{3}{7} .$$

b) which is the reduced form of $\frac{27}{63}$? Justify your answer .

4. a) Find the GCD 12600 and 3300.

b) Reduce the fraction $\frac{12\,600}{3\,300}$.

5. find a fraction equal to $\frac{12}{20}$ and such that the sum of the terms is equal to 48 .

6. Find all fractions that are equal to $\frac{49}{84}$ and whose terms are less than 50 .

7. Simplify and compare :

a) $\frac{300}{400}$ and $\frac{536}{800}$

b) $\frac{225}{75}$ and $\frac{130}{50}$

c) $\frac{175}{50}$ and $\frac{60}{20}$

8. Reduce each of the following fractions :

a) $\frac{8 \times 3 \times 7}{28 \times 6 \times 5}$

b) $\frac{9 \times 16 \times 25}{10 \times 18 \times 5}$

c) $\frac{3^2 \times 5^3 \times 7}{3^3 \times 5^2 \times 7}$

d) $\frac{4^3 \times 15^3 \times 2^2}{6^2 \times 12^2}$

e) $\frac{3131}{4747}$

9. Decompose into a product of prime factors then reduce each of the following fractions:

a) $\frac{16\,750}{14\,700}$ b) $\frac{4\,235}{9\,800}$

10. We add 12 to the numerator of the fraction $\frac{28}{63}$. Can you find a number that can be added to the denominator so that the fraction we get is equal $\frac{28}{63}$?

Just For Fun

1. Math and poetry !

This is a poem of "Jean de la Fontaine" :

"To a choice lunch, a perverted school boy
invites two friends: they divide among themselves
this magic cake, object of their envy .
Since we are three, each one take a third ;
being master of my house, I take a half ;
since I am the author, I keep the sixth .
Divide then the remaining, into some parts,
said the master of the house to his unhappy friends ."

What remains for the others ?

2. Crossed numbers !

Horizontally

1. The numerator of the reduced fraction is $\frac{900}{252}$.
2. The denominator of the fraction is equal to $\frac{900}{252}$ and the numerator is 1800.
3. The numerator of the reduced fraction of $\frac{426}{180}$.
4. The L.C.M of 175 and 70 .

Vertically

1. A divisor of 504 .
2. The G.C.D of 250 and 350 .
3. The G.C.D of 940 and 130 .
4. The numerator of the reduced fraction of $\frac{4560}{456}$.

	1	2	3	4
1			■	■
2				■
3		■		
4	■			

Decimals and fractions

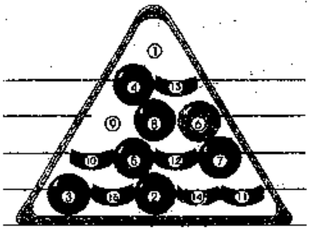
9

introduction

From a number of two "parts" to a "fraction"

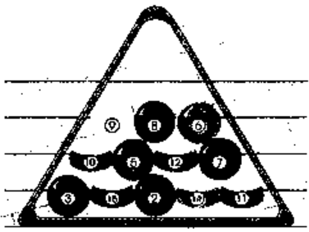
In the preceding classes, you have seen decimal numbers or numbers with decimal points. You learned how to calculate these numbers and you saw how to write a decimal number in a fractional form. Last year, you have seen in particular certain fractions equal to decimal numbers.

In the present chapter, you will learn how to identify these fractions (called decimal fractions) and how to identify non-decimal fractions.



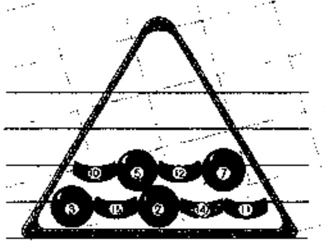
At the beginning of this chapter, I am able to:

- reduce a fraction ;
- write a decimal number in fractional form.



At the end of this chapter, I will be able to:

- write a decimal number as a sum of an integer and decimal fractions whose denominators are 10, 100, 1000, etc. ;
- write a decimal fraction in the form of a decimal number;
- write a non-decimal fraction in the form of a decimal number, in which the decimal part is unbounded and periodic;
- find the fractional writing of a number where the decimal part is unbounded and periodic ;
- compute an approximate value of a non-decimal fraction by a decimal number .



A Among the following fractions, indicate those that are not decimals:

$$\frac{8}{40} ; \frac{13}{65} ; \frac{5}{24} ; \frac{3}{21} ; \frac{15}{64} ; \frac{2}{125} ; \frac{14}{35} ; \frac{24}{30}$$

b)Write each of the following decimal fractions in the form of a decimal number, then in the form of a fraction whose denominator is a power of 10 :

$$\frac{11}{44} ; \frac{3}{24} ; \frac{27}{60} .$$

B Among the following fractions, indicate those that are not decimals and justify your answer :

$$\frac{4}{7} ; \frac{21}{35} ; \frac{16}{25} .$$

C Write each of the following non-decimal fractions in the form of a number with a point, whose decimal part is unbounded and periodic and give an approximation for them to the nearest 0.001 from below :

$$\frac{14}{9} ; \frac{171}{99} ; \frac{1219}{999} .$$

D Write a decimal fraction and a non-decimal fraction using the digits 2 ; 4 ; 5 and 7 .

E Write each of the following decimals in the expanded form according to the powers of 10 and $\frac{1}{10}$:

12.4502 ; 1.5001 ; 10.000 006 .

Problems

1. What are the approximations of order 0.0001 of the quotients:

$$\frac{121}{66}\ ;\ \frac{425}{70}\ ;\ \frac{73}{15}\ ;\ \frac{11}{3}\ ?$$

2. Complete the table

[illegible]

3. a) Write all the numbers with 3 decimals between 7.43 and 7.44 .

b) Write all the numbers with 4 decimals between 27.435 and 27.437.

Just For Fun

1. Can you find two distinct natural numbers *a* and *b* such that:

$$1=\frac{1}{a}+\frac{1}{b}\ ?$$

2. Can you find three distinct natural numbers *a, b* and *c* such that:

$$1=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\ ?$$

3. Can you find three distinct natural numbers *a, b, c* and *d* such that :

$$1=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\ ?$$

4. Continue...

Proportionality

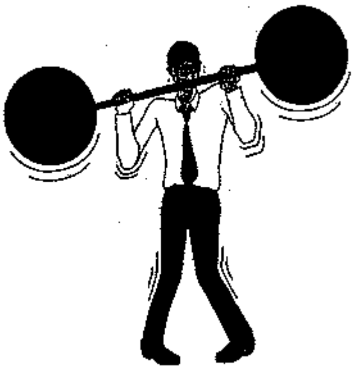
16

introduction

Proportionality is one of the most useful mathematical notions; it applies to various fields of everyday life.

It has direct applications to real life problems (purchasing, selling, duration, speed, pressure, distance, reduction problems, etc.)

You have already studied percentages, rates and scales. In this chapter, you will learn what a proportion is, and how to calculate the fourth proportional.



At the beginning of this chapter, I am able to:

- ➡ identify proportional sequences;
- ➡ identify and calculate the coefficient of a proportionality;
- ➡ identify ratios and quotient;
- ➡ use percentages, rates and scales.

At the end of this chapter, I will [illegible]

- ➡ identify a situation of proportionality;
- ➡ identify a proportion;
- ➡ calculate a proportion from a given proportion;
- ➡ calculate the fourth proportional;
- ➡ use the calculation of the fourth proportional in problems.

Recall *activities*

Activity 1

Experiment and deduce

In a graded cylinder, we pour water and we read the height reached. The results are written in the following table:

Mass of water in the cylinder (in g)	150	200	400	575	650	750
Height of water in the cylinder (in cm)	3	4	8	11.5	13	15

1) Are the two sequences proportional? If yes, calculate the coefficient of proportionality .

2) Can you calculate the height reached by 300 g of water poured in the cylinder?

Activity 2

The catalogue

A catalogue gives the price of some refrigerators:

Volume in litres	250	290	320	410
Price in L.L.	300 000	348 000	450 000	665 000

1) Are these two sequences proportional ?

2) Can you calculate the price of a refrigerator 500 liters in volume

Activity 3

Beware of greasy substances

It is written on a cheese box "35 % M.G." .

1) What does this inscription mean?

2) How can you calculate the mass of grease contained in 125 g of cheese ? in 220 g ?

Preparatory *activities*

Activity 1

Everyday problems

1) How many kg can we buy with 3 375 L.L. ?
2) A child needs 30 min to write his homework. How mach time do 5 children need to write their homework ?
3) With a bike moving at 15 km/h, I need 20 mn to go to school. With a car moving at 60 km/h what will be the duration of the trip?
4) The perimeter of a square *ABCD* is 16 . What is the perimeter of a square whose side is the double of *ABCD* ?
5) The area of a square *ABCD* is 16 . What is the area of a square whose side, is the double of *ABCD* ?

Which of the five preceding problems are situations proportionality?

Text

I. Proportional quantities

Examples

1) distance = $v \times$ duration, where v is the speed (assumed constant)

2) total price $= u \times$ quantity, where u is the unit price of an article

3) interest = $t \times$ capital, where t is the rate;

4) real distance = $e \times$ distance on the map, where e is the scale.

In all these formulas, there is a relation of the same type between two quantities. This relation can be written as:

$Y = a \times X$ where a is a constant.

Such situation is called **a proportionality situation. proportionnality.**

- We say then that the dimension Y is proportional to that of X or that Y and X are two **proportional quantities proportionnelles .**
- In this case, to each measure of one, we can calculate the corresponding measure of the other.
- The constant a is the **coefficient of proportionality** .

II. Ratio of proportionality

Let Y and X two proportional dimensions, of coefficient of proportionality a .

$y_1, y_2, y_3, \ldots$ are measures of the quantity Y and $x_1, x_2, x_3, \ldots$ are the corresponding measures X. we have :

$y_1 = ax_1$, $y_2 = ax_2$, $y_3 = ax_3 \ldots$ which gives:

$$\frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{y_3}{x_3} = \cdots = a = \text{ constant.}$$

Thus,

Property1

[redacted]

and conversely,

Property 2

[redacted]

III. Proportion

Definition

Proportion is the equality between two quotients

$$\boxed{\frac{a}{b} = \frac{c}{d}}$$

If $\frac{a}{b} = \frac{c}{d}$ is a proportion, then each of the numbers a, b, c and d is a ***term*** of that proportion .

The terms a and d are called the ***extremes*** and b and c are called the ***middle*** terms, or the **means**

IV. The fourth proportional

In a proportionality situation, we need to calculate the fourth term of a proportion $\frac{a}{b} = \frac{c}{d}$, knowing the other three. The missing term is

called **The fourth proportional.** We have to find d, knowing a, b and c , or to find c , knowing a, b and d .

$$\begin{array}{ccc} a \rightarrow b & \qquad & a \rightarrow b \\ c \rightarrow ? & \qquad & ? \rightarrow d \end{array}$$

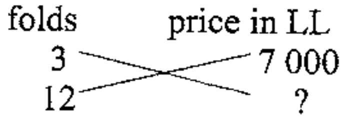
How to find the fourth proportional?

There are several methods to calculate the fourth proportional. Among which: the cross product rule and the use of equations

Example :

The price of three folds of wall paper is 7000 L.L., What is the price of 12 folds of the same wall paper ?

1. Solution using the cross product rule



To find the fourth missing term, we use a disposition analogous to that shown in the diagram and we put the terms according to the "diagonals" . The missing term is then the quotient of the product of the two terms divided by the third term.

Price of 12 folds : $\frac{12 \times 7000}{3} = 28\ 000$ LL.

The rule of cross product, called the golden rule is one by which we find a number to be in proportion with three given numbers."

Selection of the methodical mathematical Encyclopedia of 'Alembert (1784), article : Rule.

This rule is justified by the fact that the writing $\frac{a}{b} = \frac{c}{d}$ is equivalent to the writing $a \times d = b \times c$ (the product of the extreme terms is equal to that of the middle terms).

2. Solution using the unit

According to this method we find the price per unit, then we calculate the price of the 12 folds.

Price of the 12 folds = price of one fold 12

$$= \frac{7000}{3} \times 12 = 28\,000 \quad \text{LL.}$$

3. Solution by equation

Let x be the price of the 12 folds. We have : $\frac{x}{12} = \frac{7000}{3}$, which

$$x = \frac{7000 \times 12}{3} = 28\,000 \quad \text{LL.}$$

IV. Addition property

Example.

A cyclist moves at a constant speed. He covers a distance a in 2 hours, then a distance b in 3 hours , finally a distance c in 4 hours . What is the distance covered each time, knowing that the total distance is 180 km ?

In this problem, we have a proportionality situation because the speed is constant .

The distances a, b and c have the property : $\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$.

ratio is equal to the speed, and it is also equal to the ratio of the sum of the distances covered divided by the sum of the times. We have

$$\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{a+b+c}{2+3+4} = \frac{180}{9}$$

And the calculation of a, b and c is that of the fourth proportional.

$$a = \frac{2 \times 180}{9} = 40 \text{ km} \; ; \; b = \frac{3 \times 180}{9} = 60 \text{ km} \; ; \; c = \frac{4 \times 180}{9} = 80 \text{ km.}$$

Property 3

If x_1, x_2, ... are measures of a quantity X and y_1, y_2, ... the measures of a quantity Y proportional to X then we have :

$$\frac{y_1}{x_1} = \frac{y_1}{x_1} = \frac{y_1}{x_1} = \cdots = \frac{y_1 + y_2 + y_3 + \ldots}{x_1 + x_2 + x_3 + \ldots}$$

Focus

Relations deduced from a proportion

Let $\frac{a}{b} = \frac{c}{d}$ be a proportion. We have the following relations (common proportions)

a) $a \times d = b \times c$;

b) $\frac{a}{c} = \frac{b}{d}$;

c) $\frac{d}{b} = \frac{c}{a}$;

d) $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$.

Exercises

Review and practice

1. Specify whether the sequences in each line are proportional and state why?

a) $(3 ; 5 ; 7 ; 9)$ and $(2 ; \frac{10}{3} ; \frac{14}{3} ; 6)$

b) $(2 ; 4 ; 3 ; 5)$ and $(3 ; 5 ; 6 ; 7)$

c) $(4 ;12 ; 20 ;28)$ and $(1.6; 4.8; 8;9.2)$

d) $(4 ; 12 ; 28)$ and $(16 ; 48 ; 7)$.

2. In a town of 4000 voters, the participation rate in an election was 65%. How many voters participated in the election?

3. Three cars out of 75 were in an accident. What percentage does this represent?

4. Sixteen students out of 20 succeeded in the Brevet.
What is the failure percentage ?

5. In reality, the façade of a house is 25m by 10m. On a plane, a segment of 4cm represents a height of 10m.
How many centimeters are required to represent the width of the house? What is the scale of that representation?

6. The mortality rate in a country in 1996 was 1.5% . Knowing that the population was 5 millions.

a) How many dead people were there in 1996?

b) Knowing that there were 120000 births that year, what was the birth rate?

Production consumption

7. We read on the utilization notebook of a car: consumption of 8ℓ for every 100km at a speed of 90km/h, 40ℓ capacity.

a) What distance can this car cover with 40ℓ of gas at the speed of 90km/h?

b) What is the volume of gas needed to cover 350km with the same car and at the same speed?

8. With 3.5ℓ of paint, we can paint 14m^2.

a) What area can we paint with 12ℓ of paint?

b) What is the quantity needed to paint 23m^2.

9. With 30ℓ of milk, we can make 8.4kg of white cheese.

a) What is the quantity needed to make 10kg.
With 100ℓ of milk, what weight of white cheese can we make?

10. By working 6 days, a worker earned 150000 L.L.

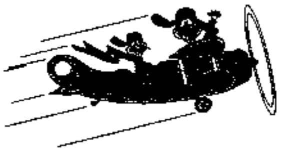
a) How much will he earn by working 21 days?

b) How many days does he need to work in order to earn500 000 L.L. ?

Supply - Speed

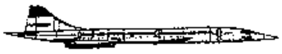
11. What is the capacity of a container that is filled in 15 minutes by a tap that supplies 150ℓ per hour?

12. An airplane flies at the average speed of 750km/h.
What is the duration of
a 1600km flight?,
of a 1125km flight?



13. At the entrance of Nahr-El Kalb, 3250 cars passed between 9 o'clock and 11:30. If the flow remains constant, how many cars would pass per hour?

Change of units



14. The average speed of the Concord is 1.7 Mach.
The Mach is a speed unit .

1Mach ≈ 1200 km/h.

What is the duration of a trip between France and the United States if the distance between them is 6000 km?

15. Evaluate in km/h the speed of the following animals:

Tortule	3cm/s ;
Whale	12m/s ;
Eagle	45m/s ;
Cangoro	3m/s .

Approximations

16. By selecting 600 grams of blond lentils, we found 12 pebbles.
a) How many pebbles will we find in selecting 1000 grams of lentils?
b) If, before finishing the picking of 1000 grams of lentils, we found 20 pebbles, can we affirm that there are no more pebbles?

17. A package of 240 papers has a thickness of 3.4cm.
a) What is the thickness of a 700 paper-package?
b) How many papers are there in a 13.5cm thickness package?

A **True or false?**

a) Mirna is 12 years old; her mother is 37 years old. When Mirna will be 12 × 2=24 years old, will her mother be 49? Justify your answer.

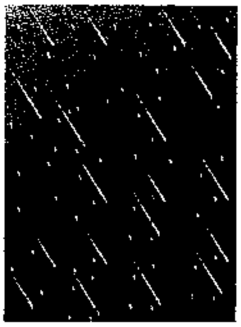
b) In 2 hours, Rola travels 40km on her bike. In 4 hours, she travels 80km. True or false ?

B The Alfa shop announces a reduction of 10% during the month of October, then a 20% tax payment for every purchased object.
The Beta shop makes a better announcement: during the same period, a reduction of 10% is made after calculating the 20% increase of the tax .
Which is the best proposal for a 250 000 L.L. object ?

C Sami, Tala and Mirna buy 4, 5 and 6 chocolate tablets respectively of fixed prices ; they paid 15 000 L.L. how much does each of them have to pay?

Problems

1. What film length do you use when you film during 5 minutes, knowing that the images have a height of 8mm and they are taken at the speed of 20 images per second ?



2. In a stormy night, Rima hears the thunder 5 seconds after seeing the lighting. Knowing that this cloud is at 1.7km, how far would another cloud be if Rima had heard the thunder 8 seconds after having seen the lighting?

3. Two secretaries write 20 pages in two hours. How many pages do 4 secretaries write in 8 hours?

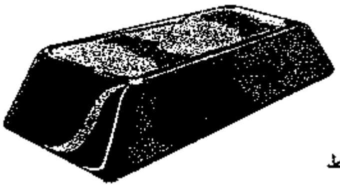
4. With 8.4kg of bread, we can feed 15 person. Knowing that 1.200kg of bread cost 1500 L.L.

a) how many person can we feed with 20000L.L?

b) how much money is needed to feed 50 person?

5. A gold alloy "18carats" contains 75% of pure gold, 12.5% of silver and 12.5% of copper.

How much of pure gold, of silver and copper is needed to make a gold ring "18carats" of 5.4g?



Pure gold is 24 carats.

الذهب الصافي عيار ٢٤ قيراط

6. A worker has a monthly salary of 450 000L.L. We proposed him a raise :

of 1% per month ;

or of 3.6% of the monthly salary per semester

or of 11% of the monthly salary per year.

Which is the solution most to his advantage?

7. Layla loves chocolates, but she is economicasitates between a 150g tablet at 900 L.L. and a 400g tablet at 2000 L.L.. Help her to make the best choice?

Just For Fun

1. _ought this chocolates by weight. My school colleagues tasted them and found them delicious and they bought me some. We paid the following amounts:

Nabila	7 200 LL	for	300g
Rami	20 800 LL	for	900g
Sali	18 000 LL	for	750g
Maher	10 800 LL	for	450g .

There was a mistake for one of us. Which one?

2. The confectioner adds 1000 L.L. to the chocolate price for the packaging, whatever is the mass of chocolate. The following table shows the price of the chocolate boxes in function of their weight:

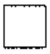
a) Complete the above table.

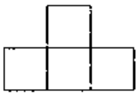
Chocolate weight in g.	100	150	200	250
Box price in L.L .	3400	4600	5800	

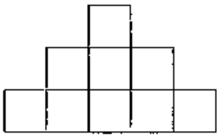
b) Is the price of selling a chocolate box proportional to its weight? Justify your answer.

3. Draw a square whose summits are on the sides of a given triangle.

4. Construction of staircases with cubes .
We make staircases with cubes according to the above model .


1 step


2 steps


3 steps

a) Can you anticipate how many cubes you need to make 20 steps?
b) Is the number of cubes proportional to the number of steps?
Justify your answer.

Once upon a time

In the 3rd century *B.C,* the great Greek mathematician Euclid explained the theory of proportion in his 5th book.

He deduces 25 theorems, establishing the properties of quantities and quantity ratios, still used today.

"Scale"

When the decimal system hasn't been invented yet, it was often inconvenient to calculate the price at the time of commercial transactions .

That's why the "Book of accounts", edited in 1688 by Mister Barème, had known a fantastic success during more than one century, to the extent that the name of its author became a genetic name (a "barem").

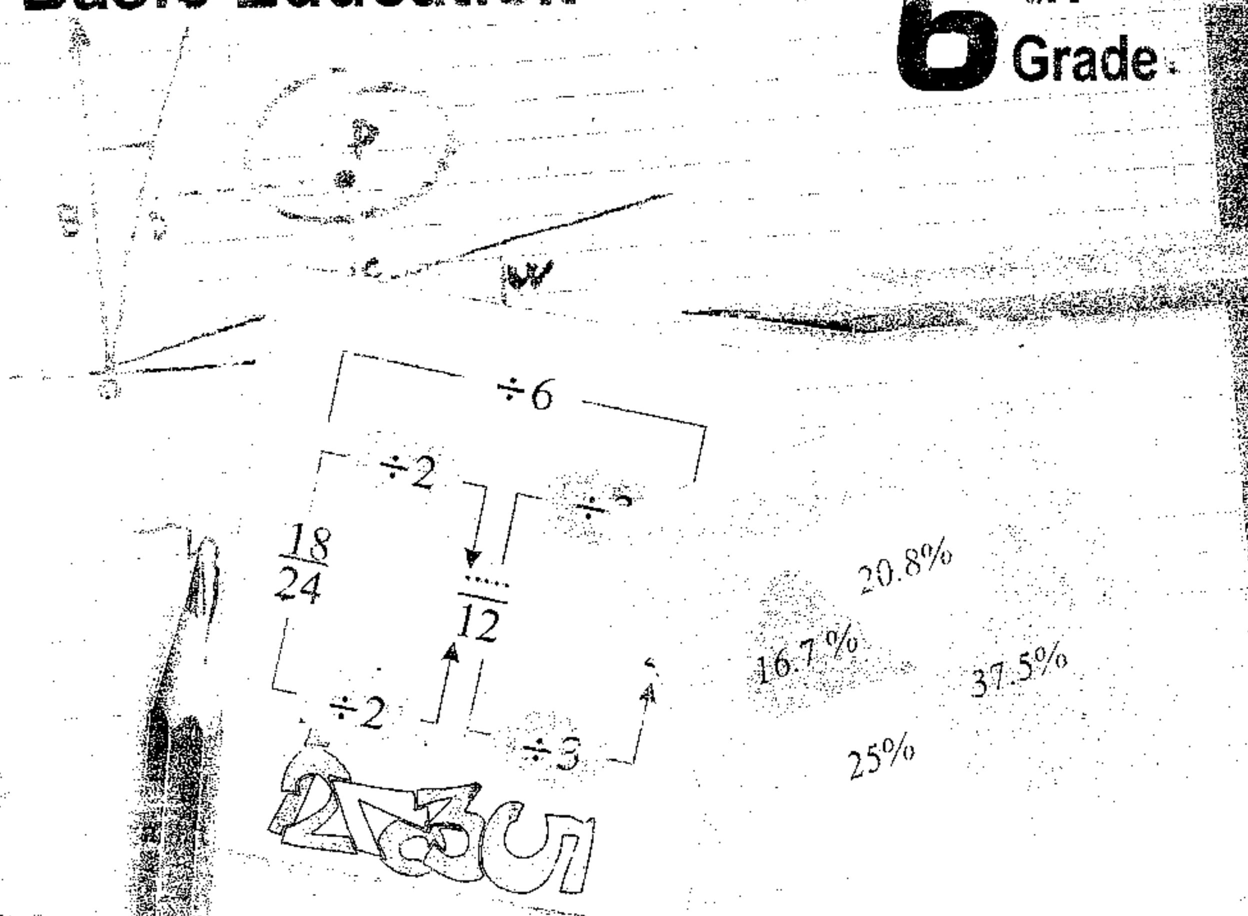
APPENDIX B

TEACHER'S MANUALS

Building up
Mathematics

Basic Education

6th Grade



Teacher's Guide

National
Textbook
NEW CURRICULA

Educational Center for Research and Development

8

Irreducible fractions

Objectives

- Recognize an irreducible fraction.
- Find an irreducible fraction equal to the given one.

Activities (30 MINUTES)

1- Discover that the same part of a surface can be represented by different fractions, but all thes fraction are equivalent. Then identify the simplest fraction.

- Discover the rule that allows students to move from one fraction to another that is equivalen to it several times until they reach the simplest one.

2- Find the shortest way to move from the fraction to its irreducible form: the G.C.D method.

- The student will learn that the terms of an irreducible fraction are prime with each other.

INSTRUCTIONS FOR SOLUTIONS

Exercises

1.

$$\frac{7}{8}=\frac{63}{72} \qquad \frac{11}{5}=\frac{77}{35} \qquad \frac{12}{7}=\frac{60}{35}$$

$$\frac{24}{36}=\frac{2}{3} \qquad \frac{5}{30}=\frac{1}{6} \qquad \frac{8}{72}=\frac{1}{9}$$

$$\frac{123}{328}=\frac{3}{8} \qquad \frac{117}{91}=\frac{9}{7} \qquad \frac{330}{60}=\frac{11}{2}.$$

2.

$$\frac{12}{18}=\frac{6}{9}=\frac{2}{3}=\frac{4}{6}=\frac{24}{36}=\frac{36}{54}=\frac{8}{12}.$$

$$\frac{30}{48}=\frac{15}{24}=\frac{5}{8}=\frac{10}{16}=\frac{20}{32}=\frac{40}{64}=\frac{60}{96}.$$

3.

$$\frac{21}{35}=\frac{270}{450}=\frac{12}{20}=\frac{39}{65}=\frac{24}{40}$$

$$=\frac{210}{350}=\frac{3}{5}$$

So: $\frac{180}{30}=6$

The intruder is: $\frac{180}{30}$.

4.

$$\frac{6}{9};\ \frac{5}{5};\ \frac{3}{17};\ \frac{4}{7};\ \frac{13}{29};\ \frac{4}{9};\ \frac{111}{423};\ \frac{45}{12}.$$

5. $\frac{45}{75}=\frac{9}{15}=\frac{3}{5}$

$\frac{48}{72}=\frac{12}{18}=\frac{6}{9}=\frac{2}{3}$

$\frac{84}{108}=\frac{42}{54}=\frac{21}{27}=\frac{7}{9}$.

6. $\frac{90}{126}=\frac{90\div 9}{126\div 9}=\frac{10}{14}=\frac{10\div 2}{14\div 2}=\frac{5}{7}$.

7. $\frac{40}{25}=\frac{40\div 5}{25\div 5}=\frac{8}{5}$

$\frac{49}{35}=\frac{49\div 7}{35\div 7}=\frac{7}{5}$

$\frac{200}{700}=\frac{200\div 100}{700\div 100}=\frac{2}{7}$

$\frac{64}{24}=\frac{64\div 4}{24\div 4}=\frac{16}{6}=\frac{16\div 2}{6\div 2}=\frac{8}{3}$

$\frac{36}{45}=\frac{36\div 9}{45\div 9}=\frac{4}{5}$

$\frac{66}{55}=\frac{66\div 11}{55\div 11}=\frac{6}{5}$.

8. a) G.C.D (144 and 12) = 24

$\frac{144}{312}=\frac{144\div 24}{312\div 24}=\frac{6}{13}$

b) G.C.D (81 and 135) = 27

$\frac{81}{135}=\frac{81\div 27}{135\div 27}=\frac{3}{5}$

G.C.D (513 and 180) = 9

$\frac{513}{180}=\frac{513\div 9}{180\div 9}=\frac{57}{20}$.

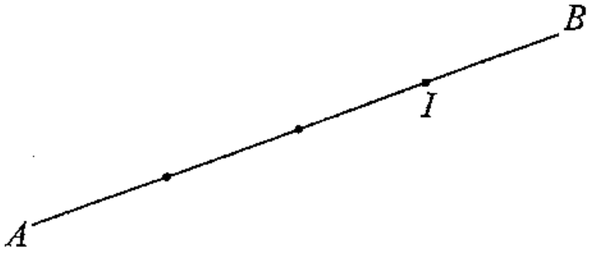
9. $\frac{60}{24}=\frac{5}{2}$ $\qquad$ $\frac{840}{490}=\frac{12}{7}$

$\frac{144}{108}=\frac{4}{3}$ $\qquad$ $\frac{1400}{1050}=\frac{4}{3}$.

10. a) $\frac{26}{65}=\frac{2}{5}$ (we divide both terms of $\frac{26}{65}$ by 13).

b) $\frac{26}{65}=\frac{2}{5}=\frac{2\times 20}{5\times 20}=\frac{40}{100}$.

11. $AB = 8\ cm$.



So: $\frac{24}{32}=\frac{3}{4}$

Therefore $AI=\frac{3}{4}\times AB$.

Problems

1. $\frac{3}{7}=\frac{6}{14}=\frac{9}{21}=\frac{12}{28}=\frac{15}{35}=\frac{18}{42}=\frac{21}{49}$

$=\frac{24}{56}=\frac{27}{63}=\frac{30}{70}$.

2. a) $\frac{60}{135}=\frac{60\div 5}{135\div 5}=\frac{12}{27}=\frac{12\div 3}{27\div 3}=\frac{4}{9}$.

b) $\frac{4}{9}=\frac{20}{45}$ and $20+45=65$.

3.

Fraction in	Fraction out
$\frac{12000}{8000}$	$\frac{3}{2}$
$\frac{7500}{3500}$	$\frac{15}{7}$
$\frac{270}{126}$	$\frac{15}{7}$
$\frac{675}{495}$	$\frac{15}{11}$
$\frac{81}{270}$	$\frac{3}{10}$
$\frac{480}{495}$	$\frac{32}{33}$
$\frac{375}{525}$	$\frac{5}{7}$

4. a) $\frac{12}{60}=\frac{1}{5}$ so 12 min = $\frac{1}{5}$ H.

35 min = $\frac{7}{12}$ H.

40 min = $\frac{2}{3}$ H.

45 min = $\frac{3}{4}$ H.

50 min = $\frac{5}{6}$ H.

52 min = $\frac{26}{30}$ H.

58 min = $\frac{29}{30}$ H.

b) $\frac{14}{21}=\frac{2}{3}$

The duration of a complete tour is $\frac{2}{3}$ H and 40 min.

5. $\frac{792}{54}=\frac{792\div 9}{54\div 9}=\frac{88}{6}=\frac{88\div 2}{6\div 2}=\frac{44}{3}$

The width of the rectangular fiel $\frac{44}{3}$ m.

6. $\frac{180}{315}=\frac{180\div 5}{315\div 5}=\frac{36}{63}=\frac{36\div 9}{63\div 9}=\frac{4}{7}$.

The occupied places represent fraction $\frac{4}{7}$

Therefore the unoccupied place is

which is $\frac{135}{317}=\frac{3}{7}$.

7. a) The fraction is the ratio of Ⓐ bo sold to the total number of Ⓐ boo

$\frac{4800}{5600}=\frac{48}{56}=\frac{12}{14}=\frac{6}{7}$

b) The fraction is the ratio of Ⓑ bo sold to the total number of Ⓑ boc

$\frac{1350}{2700}=\frac{135}{270}=\frac{27}{54}=\frac{1}{2}$

Similary for Ⓒ.

$\frac{3000}{4200}=\frac{30}{42}=\frac{10}{14}=\frac{5}{7}$.

c) The total number of books
5 600 + 2 700 + 4 200 = 1 200

for A: $\frac{5600}{12500}=\frac{56}{125}$

for B: $\frac{4200}{12500}=\frac{42}{125}$

for C: $\frac{1350}{12500}=\frac{27}{250}$.

8. The total length (distance)

$30 + 18 + 36 + 48 = 132$ *km*.

a) $\frac{30}{132} = \frac{15}{66} = \frac{5}{22}$.

$\frac{5}{22}$ is the fraction that represents the distance when Walid and his parents reached Batroun.

b) $\frac{30+18}{132} = \frac{48}{132} = \frac{16}{44} = \frac{4}{11}$.

$\frac{4}{11}$ is the fraction that represents the distance when Walid and his parents reached Jbeil.

c) $\frac{48+36}{132} = \frac{84}{132} = \frac{28}{44} = \frac{7}{11}$.

$\frac{7}{11}$ is the fraction that represents the distance when Walid and his parents reached Beirut.

Decimal fractions fractional writing of a decimal

10

Objectives

- Recognize a decimal fraction.
- Write a decimal fraction in the form of a fraction whose denominator is a power of ten.
- Write a decimal fraction in the form of a decimal number and vice versa.

Activities (30 MINUTES)

a- Using a certain game, the student can distinguish the fractions $\frac{a}{b}$ in which the quotient of a by b is exact from fractions $\frac{c}{d}$ in which the quotient is not exact.

Note: The teacher can reduce the fractions and show that their quotient is exact, and that there are only factors of 2 and 5 in the decomposition of the denominator of the reduced fraction. This is not the case for $\frac{7}{33}$ (optional).

b- *Note:* All fractions held by the children who will reach the boats can be written as fractions whose denominators are powers of ten.

- Do the same with the fraction $\frac{3}{4}$
- Motivate the student to reach "the beautiful products".

INSTRUCTIONS FOR SOLUTIONS

Exercises

1. $\frac{15}{8}$; $\frac{12}{4.5}$; $\boxed{\frac{28}{12}}$;

$\frac{0.5}{6}$; $\frac{19}{2.7}$; $\boxed{\frac{3}{50}}$.

2. $\frac{51.7}{2.8}=\frac{517}{28}$; $\frac{6}{0.5}=\frac{60}{5}$;

$\frac{0.24}{1.5}=\frac{24}{150}$; $\frac{16.3}{0.88}=\frac{1630}{88}$;

$\frac{3}{0.1}=\frac{30}{1}$; $\frac{4.035}{2.5}=\frac{4035}{2500}$.

3. a) $\frac{3}{4}=\frac{6}{8}=\frac{9}{12}=\frac{12}{16}=\frac{15}{20}=\frac{18}{24}$

$=\frac{21}{28}=\frac{24}{32}=\frac{27}{36}=\frac{30}{40}=\frac{33}{44}$.

b) $\frac{3}{4}$ is a decimal fraction

$\frac{3}{4}=\frac{3\times 25}{4\times 25}=\frac{75}{100}$.

4. a) $\frac{12}{21}=\frac{12\div 3}{21\div 3}=\frac{4}{7}$

and $\frac{4}{7}=\frac{8}{14}=\frac{12}{21}=\frac{16}{28}=\frac{20}{35}$

$=\frac{24}{42}=\frac{28}{49}$.

b) The fraction $\frac{12}{21}$ is not a decimal fraction.

5. $\frac{20}{28}$; $\boxed{\frac{90}{100}}$; $\boxed{\frac{6}{4}}$; $\frac{2}{6}$; $\boxed{\frac{5}{8}}$; $\frac{11}{3}$; $\boxed{\frac{6}{25}}$;

$\frac{45}{36}$; $\frac{5}{24}$; $\boxed{\frac{24}{75}}$;

6. $\frac{11}{5}=\frac{22}{10}$; $\frac{13}{50}=\frac{26}{100}$;

$\frac{63}{125}=\frac{63\times 8}{125\times 8}=\frac{504}{1000}$;

$\frac{9}{25}=\frac{36}{100}$;

$\frac{7}{8}=\frac{7\times 125}{8\times 125}=\frac{875}{1000}$;

$\frac{47}{20}=\frac{235}{100}$; $\frac{9}{4}=\frac{225}{100}$;

$\frac{13}{2}=\frac{65}{10}$.

7. $\frac{3.5}{100}=\frac{35}{1000}=\frac{7}{200}=0.035$.

8. $\frac{7}{10}=0.7$ $\frac{23}{100}=0.2$

$\frac{55}{1000}=0.055$ $\frac{37}{10}=3.7$

$\frac{4554}{10^2}=45.54$ $\frac{3004}{10^3}=3.$

9. $0.8=\frac{8}{10}$ $0.45=\frac{45}{100}$

$0.725=\frac{725}{1000}$ $0.03=\frac{3}{100}$

$0.015=\frac{15}{1000}$ $3.09=\frac{309}{100}$

$5.036=\frac{5036}{1000}$ $1.2002=\frac{120}{1}$

$0.007=\frac{7}{1000}$ $4.001=\frac{400}{10^3}$

10. $\frac{3.8}{2}=\frac{38}{20}=\frac{19}{10}$ $\frac{0.45}{5}=\frac{45}{500}=$

$\frac{9.5}{5}=\frac{95}{50}=\frac{19}{10}$ $\frac{5.6}{4}=\frac{56}{40}=\frac{14}{10}$

11. $\frac{4}{5}=\frac{8}{10}=0.8$ $\frac{15}{2}=\frac{75}{10}=$

$\frac{65}{50}=\frac{13}{10}=1.3$ $\frac{48}{60}=\frac{8}{10}=$

$\frac{4.2}{2}=\frac{42}{20}=\frac{21}{10}=2.1$

$\frac{18.5}{5}=\frac{185}{50}=\frac{37}{10}=3.7$.

12. $\frac{14}{5}=\frac{280}{100}=2.8$

$\frac{15}{4}=\frac{375}{100}=3.75$

$\frac{16}{50}=\frac{32}{100}=0.32$

$\frac{43}{20}=\frac{215}{100}=2.15$

$\frac{37}{25}=\frac{148}{100}=1.48$

$\frac{94}{200}=\frac{47}{100}=0.47$

$\frac{15}{500}=\frac{3}{100}=0.03$

$\frac{70}{1000}=\frac{7}{100}=0.07$

$\frac{13.5}{50}=\frac{27}{100}=0.27$

$\frac{35.2}{20}=\frac{176}{100}=1.76$.

13. $\frac{19}{200}=\frac{95}{1000}=0.095$

$\frac{3}{500}=\frac{6}{1000}=0.006$

$\frac{17}{250}=\frac{68}{1000}=0.068$

$\frac{6}{125}=\frac{48}{1000}=0.048$

$\frac{7}{8}=\frac{875}{1000}=0.875$

$\frac{8}{25}=\frac{320}{1000}=0.32$

$\frac{45}{4}=\frac{11250}{1000}=11.25$

$\frac{105}{40}=\frac{2625}{1000}=2.625$

$\frac{9}{5}=\frac{1800}{1000}=1.8$

$\frac{69}{3000}=\frac{23}{1000}=0.023$

$\frac{634}{2000}=\frac{317}{1000}=0.317$

$\frac{18.45}{50}=\frac{369}{1000}=0.369$

14. $\frac{1}{4}=\frac{25}{100}$

$\frac{2}{5}=\frac{4}{10}$

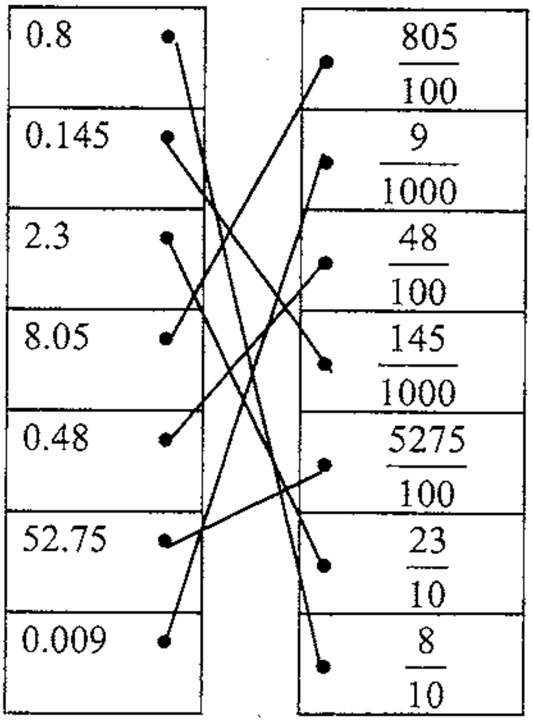
$\frac{30}{4}=\frac{750}{100}$

$\frac{98}{25}=\frac{392}{100}$

$\frac{528}{125}=\frac{4224}{1000}$

$\frac{72}{250}=\frac{288}{1000}$.

15.



16. a) $\frac{7}{10} = 0.7$ $\qquad \frac{18}{1000} = 0.018$

$\frac{45}{100} = 0.45$ $\qquad \frac{65}{10} = 6.5$

$\frac{6475}{1000} = 6.475$.

b) $\frac{1}{2} = \frac{5}{10} = 0.5$

$\frac{7}{5} = \frac{14}{10} = 1.4$

$\frac{13}{4} = \frac{325}{100} = 3.25$

$\frac{13}{2} = \frac{65}{10} = 6.5$

$\frac{1}{8} = \frac{125}{1000} = 0.125$.

c) $\frac{15}{24} = \frac{5}{8} = \frac{625}{1000} = 0.625$

$\frac{175}{125} = \frac{35}{25} = \frac{7}{5} = \frac{14}{10} = 1.4$

$\frac{18}{40} = \frac{9}{20} = \frac{45}{100} = 0.45$

$\frac{44}{20} = \frac{22}{10} = 2.2$

$\frac{124}{32} = \frac{31}{8} = \frac{3875}{1000} = 3.875$.

17. $8.5 = \frac{85}{10} = \frac{17}{2}$

$3.25 = \frac{325}{100} = \frac{13}{4}$

$2.05 = \frac{205}{100} = \frac{41}{20}$

$0.064 = \frac{64}{1000} = \frac{8}{125}$.

18. $3.6 = \frac{36}{10} = \frac{18}{5}$

$0.03 = \frac{3}{100}$;

$0.875 = \frac{875}{1000} = \frac{7}{8}$

$0.72 = \frac{72}{100} = \frac{18}{25}$

$1.85 = \frac{185}{100} = \frac{37}{20}$.

19. $45\ dm = \frac{45}{10}\ m = 45\ m$

$158\ cm = \frac{158}{100}\ m = 1.58\ m$

$39\ mm = \frac{39}{1000}\ m = 0.039$

$3485\ m = \frac{3485}{1000}\ km = 3.485\ km$

$96\ m = \frac{96}{100}\ hm = 0.96\ hm$

$7\ m = \frac{7}{10}\ dam = 0.7\ dam.$

20. $3\ dg = \frac{3}{10}\ g = 0.3\ g$

$326\ cg = \frac{326}{100}\ g = 3.26\ g$

$486\ mg = \frac{486}{1000}\ g = 0.486\ g$

$634\ g = \frac{634}{1000}\ kg = 0.634\ kg$

$9\ g = \frac{9}{100}\ hg = 0.09\ hg$

$137\ g = \frac{137}{10}\ dag = 13.7\ dag.$

Problems

1. a) $\frac{2947}{100}$.

If we add 10 to both terms of the fraction $\frac{9}{10}$ we obtain $\frac{19}{20}$.

b) $\frac{19}{20}$ is a decimal fraction, since $\frac{19}{20}=\frac{95}{100}$.

c) The decimal number is 0.95.

3. a) $\frac{14}{5}$ is a decimal fraction since $\frac{14}{5}=\frac{28}{10}$.

b) If we add 10 to both terms of the fraction $\frac{14}{5}$ we obtain a fraction equal to $\frac{24}{15}$.

- but $\frac{24}{14}$ is not a decimal fraction since its denominator cannot be written as a power of 10.

4. a) The fraction of the red rectangle represents the square which is $\frac{12}{25}$

- the decimal fraction is:

$$\frac{12}{25}=\frac{12\times4}{25\times4}=\frac{48}{100}$$

b) - the area of the square is: $5\times5=25\ cm^2$

- the area of the red rectangle is $25\times\frac{12}{25}=12\ cm^2$.

c) the length of Samer's step is $\frac{4}{5}=\frac{8}{10}=0.8\ cm$.

6. a) $\frac{50}{200}=\frac{5}{20}=\frac{1}{4}$.

$\frac{120}{200}=\frac{12}{20}=\frac{3}{5}$.

$\frac{185}{200}=\frac{37}{40}$.

- the distance run by (A) in fraction is $\frac{1}{4}$
- the distance run by (B) in fraction is $\frac{3}{5}$
- the distance run by (C) in fraction is $\frac{37}{40}$.

b) $\frac{1}{4}=\frac{25}{100}$.

$\frac{3}{5}=\frac{60}{100}$.

$\frac{37}{40}=\frac{925}{1000}$.

7. a) The capacity of a bottle is $\frac{75}{125}\ell$ which is $\frac{3}{5}\ \ell$ and $\frac{3}{5}=\frac{6}{10}$.

b) The capacity of each bottle is 0.6 ℓ which is 600 $m\ell$.

8. a) The price of one kilogram of cheese is $\frac{3600}{0.250}$ LL and $\frac{3600}{0.250}$

$$=\frac{360000}{25}=\frac{1440000}{100}=14400$$

Therefore the price of one kilogram of cheese is 14400 LL.

b) This sum is equal to 14.4 of thousands of Lebanese liras.

Quotient and ratio

20

Objectives

- Individual activity followed by gathering of information to get results. The activity aims at making the student capable of:
  - Finding the quotient of the division of two numbers.
  - Giving an approximate value for the quotient of two numbers.
  - Using $\frac{a}{b}$ to represent the quotient.

  The objectives of activities: Required time.

Activities

Activity 1 and 2:

A- Individual activity followed by gathering of information thereby allowing the student to:

* Write the ratio of two quantities of the same type and use this ratio in the comparison between these two quantities.

B- Individual activity followed by gathering of information that will lead to expressing the ratio of two quantities of different types, and identifying the notion of average.

INSTRUCTIONS FOR SOLUTIONS

Exercises

1. Find the quotient of the division of "a" by "b" in each of the following cases:

$a = 1353$
$b = 11$
quotient: $1353 \div 11 = 123$.

$a = 310.5$
$b = 12.4$
quotient: 34.5

$a = 69.44$
$b = 12.4$
quotient: 5.6

2. Indicate the repetitive numbers in the quotient of the division of "*a*" by "*b*" in each of the following cases:

- $a = 16$; $b = 3$

The repetitive number is: 3.

- $a = 67$; $b = 66$

The repetitive number is: 5 and 1.

- $a = 368$; $b = 999$

The repetitive number is: 3, 6 and 8.

3. Find the approximate value of the quotient of the division of "*a*" by "*b*" according to the given instructions.

- $a = 150$; $b = 26$

The approximate value to the nearest 0.1 in excess is 5.8.

- $a = 123$; $b = 7$

The approximate value to the nearest 0.01 in default is 17.57.

- $a = 25$; $b = 6$

The approximate value to the nearest 0.001 in excess is 4.167.

4. Calculate the approximate value of the quotient of division 81458.23 by 7175.256 to the nearest 0.001 in excess.
The approximate value to the nearest 0.001 in excess is 11.353.

5. Write the quotient of the division "*a*" by "*b*" as a fraction $\frac{a}{b}$ in each of the following cases, then simplify; if possible:

$a = 145$; $b = 120$

quotient: $\frac{a}{b} = \frac{145}{120} = \frac{29}{24}$.

$a = 68$; $b = 17$

quotient: $\frac{a}{b} = \frac{68}{17} = 4$.

$a = 0.1$; $b = 0.01$

quotient: $\frac{a}{b} = \frac{0.1}{0.01} = 10$.

6. A- the ratio of the length of the rectang

to its width is: $\frac{60\ mm}{30\ cm \times 10} = \frac{60}{30} = 2$

B- The ratio of its width to its length i

$\frac{30}{60} = \frac{1}{2}$.

C- The ratio of its perimeter to its area i

$\frac{(60mm + 30mm) \times 2}{60mm \times 30mm} = \frac{180}{1800} = \frac{1}{10}$

7. a) The ratio of the side length of a squa

to its perimeter is: $\frac{4}{4 \times 4} = \frac{1}{4}$

b) The ratio of its side length to its ar

is: $\frac{4}{4 \times 4} = \frac{1}{4}$.

8. The eagle's speed is:

$162km/\text{hour} = \frac{162km \times 1000}{1hr \times 3600} = 45m/s$

10. If the ratio of "*a*" to "*b*" is 0.9; it mea that "a" is smaller than "*b*".

Problems

1. A) The number of boys is greater than that of girls since the number of girls relative to that of boys is equal to $\frac{5}{6}$ a unity.

B) The approximate value of the quotie of the division of 5 by 6, to the neare 0.01 in default is $0.08 < 1$.

2. a) The ratio of the diameter length of Pluto to that of Saturn is:

$$\frac{3\times10^3 km}{120\times10^3 km}=\frac{3}{120}=\frac{1}{40}.$$

b) This ratio equals: $\frac{1}{40}=0.025$ (*a* decimal number).

$\frac{1}{40}=\frac{25}{1000}$ (*a* decimal fraction).

3. a) The ratio of the salt weight to that of water is:

$$\frac{288\text{ g}}{9kg\times1000}=\frac{19}{750}$$

b) This ratio is equal to the decimal number: 0.025333....

4. a) The ratio of Sami's age to that of Samar is: $\frac{156\text{months}}{21\text{years}\times12}=\frac{156}{252}=\frac{13}{21}$.

b) The average value of the quotient of the division of 156 by 252 to the nearest 0.1 by default is 0.6.

5. a) The ratio of the needed quantity of water for irrigation to the total quantity of water is:

$$\frac{2160\text{ millions de }m^3}{3300\text{ million de }m^3}=\frac{2160}{3300}$$
$$=\frac{36}{55}.$$

b) The repetitive numbers in the quotient of the division of the two numbers (using a calculator) are: 5 and 4.

c) The approximate value of this quotient to the nearest 0.001 excess is 0.655.

6. a) The ratio of height of Sannine mountain to that of Al-Konaïsa is: $\frac{2628}{2032}=\frac{657}{508}$ (irreducible fraction).

b) The approximate value of the quotient of the division of 2628 by 2032 to the nearest 0.001 in default is 1.293.

7. a) The ratio of the area of Egypt to that of Saudi Arabia is:

$$\frac{1\text{ million }km^2}{2.2\text{ million }km^2}=\frac{10}{22}=\frac{5}{11}.$$

b) The quotient of the division of 2.2 million km^2 by 1 million km^2 is: (as a decimal fraction): $\frac{11}{5}=\frac{22}{10}$.

8. a) The ratio of the number of bones to that of muscles is: $\frac{208}{900}=\frac{52}{225}$.

b) The approximate value of the quotient of the division of 208 by 900, to the nearest 0.001 in default, is 0.231.

9. a) The average speed of the train in one hour is

$$\frac{1170km}{9hrs}=130km/\text{hr}.$$

b) In 10 hours, this train can travel a distance = 130 × 10 = 1300 *km*.

c) The average speed of the train in

$$m/s=\frac{130km\times1000}{1hr\times3600}=\frac{325}{9}m/\text{s}.$$

21

Percentage

Objectives

- Calculate the percentage of a number.
- Recognize, calculate and compare percentage.

Activity (25 MINUTES)

- To be familiar with percentage.
- To simplify the percentage.
- To calculate the percentage of a certain number.

INSTRUCTIONS FOR SOLUTIONS

Exercises

1. a) 5 = 50 % of 10
1 = 25 % of 4
2 = 8 % of 25
3 = 12 % of 25
7 = 70 % of 10
1 = 2 % of 50.

b) 3 = 75 % of 4
9 = 45 % of 2
30 = 60 % of 50
15 = 20 % of 75.

2. a) $\frac{2}{40} = 5\ \%$ b) $\frac{24}{60} = 40\ \%$

$\frac{5}{16} = 31.25\ \%$ $\frac{18}{15} = 120\ \%$

$\frac{9}{32} = 28.125\ \%$ $\frac{27}{75} = 36\ \%$

$\frac{21}{20} = 105\ \%$ $\frac{35}{175} = 20\ \%.$

3. a) $0.2 = \frac{20}{100}$; $0.33 = \frac{33}{100}$

$0.15 = \frac{15}{100}; 0.75 = \frac{75}{100};$

$0.8 = \frac{80}{100}.$

b) 1.03 = 103 %; 2.25 = 225 %;

7.35 = 735 %; 9.2 = 920 %.

4. - The semi-disc represents 50 % of a disc.
- The quarter of a disc represents 5 % of a disc;
- Three quarters of a disc represent 75 % of a disc;
- $\frac{4}{5}$ of a disc represents 80 % of a disc;
- $\frac{7}{20}$ of a disc represents 35 % of a disc.

5. $90° = 25\ \%$ of $360°$

$50° = 13.88...\ \%$ of $360°$

$60° = 16.66...\ \%$ of $360°$

$30° = 8.333...\ \%$ of $360°$

$10° = 2.777...\ \%$ of $360°$

$120° = 33.33...\ \%$ of $360°$.

6. 15 % of 130 = 19.5

15 % of 2 453 = 367.95

15 % of 3 202 = 480.3

15 % of 673.5 = 101.025

$$\frac{15}{100} \times 87.42 = 13.113$$

$$\frac{15}{100} \times 9.990 = 1.4985$$

$$\frac{15}{100} \times 1.882 = 0.2823$$

$$\frac{15}{100} \times 88\,880 = 13\,332\,.$$

7. * 20% of 18 000 = 36 00

or $\frac{20}{100} 18\,000 = 3\,600$

* 30% of 18 000 = 5400

or $\frac{30}{100} \times 18\,000 = 5\,400$

50% of 1 800 = $\frac{50}{100} \times 1\,800 = 9\,000$.

Which is equal to 3 600 + 5 400.

Problems

1. The profit is:

$$\frac{7}{100} \times 35\ 000\ 000 = 2\,450\,000 \text{ LL.}$$

2. The price of 35 lambs is:

35 × 135 000 = 4 725 000 LL.

The discount is 6% so he paid 94%.

$$\frac{94}{100} \times 4\ 725\ 000 = 4\,441\,500 \text{ LL.}$$

3. The price of the fabric is:

175 × 2 880 = 504 000 LL.

- $\frac{2}{5}$ of 175 is:

  $\frac{2}{5} \times 175 = 70$ meter.

The price of 70*m* is: 70 × 2 880 = 201 600 LL.

He gains: $\frac{16}{100} \times 201\ 600 =$

32 256 LL. in 70 *m*.

105 *m* is the profit of the first amount.

The gain is $\frac{10}{100} \times\ 2\ 880 \times 105 =$

30 240 LL.

- The total profit this merchant made in 175 *m* is:

  32 255 + 30 240 = 62 495 LL.

4. The mass of roasted coffee:

$$100 \times \frac{82}{100} = 82\ kg.$$

- The price of green coffee is:
  $100 \times 13\ 000 = 1\ 300\ 000$
  So 1 300 000 is the price of $82kg$.
- The price of 1 kg of roasted coffee is:
  $1\ 300\ 000 \div 82 = 15\ 853.65$ LL.

5. Mass of oil:

$$3\ 600 \times \frac{20}{100} = 720\ kg.$$

The quantity of oil in liter:

$720 \div 0.90 = 800$ liters.

6. The buying price of a ton:

$$\frac{620\ 000}{124} \times 100 = 500\ 000 \text{ LL.}$$

The mass in tons of the rest is:

$13\ 365\ 000 \div 495\ 000 = 27$ tons.

The mass of the purchase wheat is:

$$27 \times \frac{5}{3} = 45 \text{ tons.}$$

7. The price is:

$$10\ 000 - \frac{30}{100} \times 10\ 000 = 7\ 000 \text{ LL.}$$

8. The total profit is:

$$7\ 350\ 000 \times \frac{11}{100} = 808\ 500 \text{ LL.}$$

The buying price is:

$7\ 350\ 000+808\ 500=8\ 158\ 500$ LL.

The buying price per kilogram is:

$8\ 158\ 500 \div 420 = 8\ 158\ 500$ LL.

The buying price of $200g$ is:

$19\ 425 \div 4 = 3\ 885$ LL.

9. The length of a price of fabric a
rinsing is:

$$\frac{96.8}{100} \times 47.86 = 46.32848\ m.$$

$(100 - 3.2 = 96.8)$

10. • $30\ 000 \times \dfrac{23}{100} = 6\ 900$ LL.

• $10\ 810 \times \dfrac{100}{23} = 47\ 000$ LL.

11. Mass of cream is:

$$\frac{12}{100} \times 80 = 9.6\ kg.$$

Mass of butter is:

$$\frac{35}{100} \times 9.6 = 3.36\ kg.$$

Proportionality

22

Objectives

- Recognize and construct two proportional sequences.
- Calculate the proportionality coefficient and the fourth proportional.

Activity (10 MINUTES)

To be familiar with proportions.

INSTRUCTIONS FOR SOLUTIONS

Exercises

1. Proportional tables are:

3	5	7
3.6	6	8.4

10	15	40
4	6	16

4	3.2	7	8
12	9.6	21	24

2.

× 0.2

4	7	9	11
0.8	1.4	1.8	2.2

× 4

18	10	12.5	15
72	40	50	60

× 0.002

7	8	11
0.014	0.016	0.022

3 %

77	50	38
4.31	1.5	1.06

5	4
0.15	0.12

3	7
9.3	21.7

3. a) no; b) yes; c) yes; d) yes; e) no.

Problems

1.

2	
5	32

When the big wheel effects 32 rounds, the small wheel rotates: $\frac{32 \times 2}{5} = \frac{64}{5} = 12.8$.

12 complete rounds and $\frac{8}{10}$ of the round.

2. a)

2	50	150
7	175	525

b)

10	14	30
35	49	105

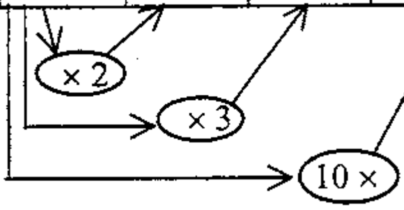
c)

5	110
7	184

Number of final steps is 110.

3.

sugar	100 *g*	200 *g*	300 *g*	1000 *g*
flour	75 *g*	150 *g*	225 *g*	750 *g*
butter	40 *g*	80 *g*	120 *g*	400 *g*
eggs	3	6	9	30



4.

10	20	30	50
0.25	0.50	0.75	1.25

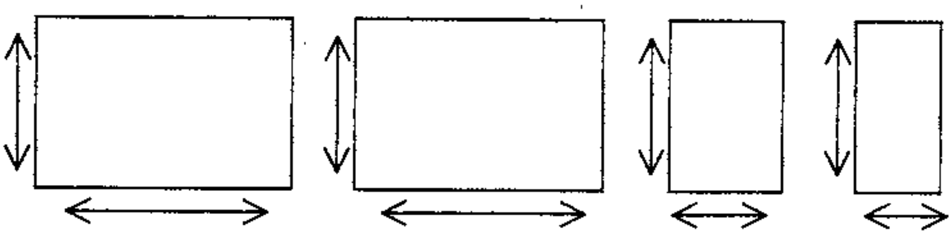
10	600
0.25	15

5.

Triangles	*CAB*	*DAB*	*EAB*	*FAB*
Height	1 *cm*	2 *cm*	3 *cm*	3.5 *cm*
Area	1.5 cm^2	3 cm^2	4.5 cm^2	6.25 cm^2

The area of triangle *MAB* is $\frac{14 \times 3}{2} = 21\ cm^2$.

6.



Length of rectangle	2	2.7	5.8	6	0.4
Perimeter	12	13.4	19.6	20	8.8
Area	8	10.8	23.2	24	1.6

$\times 4$

- There is a proportionality between the sequence of lengths and the sequence of areas.
- There is no proportionality between the sequence of lengths and the sequence of perimeters.
- Perimeter of rectangle is: $2(118 + 4) = 2 \times 122 = 244$ *cm*.

7. • $144 = 36 \times 4$ so we need 4 buses.

- $0.22 \times 230 = 50.6$ litters. Each bus needs 50.6 ℓ.
- The 4 bus needs $50.6 \times 4 = 202.4$ liters.
-

20	202.4
13 000	?

The cost of the trip is $\dfrac{13000 \times 202.4}{20} = 131\ 560$ LL.

8. 1) • The quantity of sugar needed for 15*kg* of fruit: $\dfrac{15 \times 2.8}{3.5} = 12$ *kg*.

- The quantity of fruit corresponding to 20*kg* of sugar is: $\dfrac{20 \times 3.5}{2.8} = 25$ *kg*.

2) First case: $15 + 12 = 27$ *kg* So: $30 \times \dfrac{27}{100} = 8.1$ *kg*.

Second case: $20 + 25 = 45$ *kg* So: $30 \times \dfrac{45}{100} = 13.5$ *kg*.

BUILDING

[illegible]

[illegible] Grade

Basic Education

Teacher's Guide

National
Textbook

ational Center for Educational Research and Development

NEW CURRICULA

Reduction of fractions

8

OBJECTIVES

1. Reduce a fraction by using many methods.

- Know the signification of the terms: irreducible, reduced, reduction, reduce and simplify.
- Use the property according to which, for every non-zero natural number a , $\frac{a}{a}=1$.
- Calculate the reduced form of a fraction by using the GCD of its two terms.
- Calculate the reduced form of a fraction by decomposing its terms into prime factors and simplifying.
- Calculate the reduced form of a fraction by performing successive divisions.

EXPLANATION

Arriving to this class, the student has already seen fractions in the previous years; he learned how to manipulate them in all sorts of calculations. He more or less had an idea on the equality of fractions and learned, consequently, to simplify a fraction.
Therefore the present lesson constitutes a final step for this subject as well as a synthesis of many notions already seen (decomposition into prime factors, GCD, Euclidean division and divisibility, simplification, equality of two fractions, etc.).
Although our intention is to teach the student systematic methods (also called intrinsic) for solving the reduction problem, we highly advise you to profit from the richness of this subject by applying quite various methods that are notably inspired from the nature of the given information (in fact, it is there that we have a particularity characterizing all problems on integers.) In fact, one of our intentions is to stimulate the imagination of the student to create hum what we call "the instinctive reflex": for example, it is by acquired reflex that the student sees number 1 in the form $\frac{5}{5}$ once it is added to the fraction $\frac{3}{5}$. Similarly, if the student sees a number such as 231 231 231, then he must immediately know, by the same reflex, that this number is divisible by 231 !
It is precisely these instinctive reflexes that always drive us to finding particular methods of resolution (short cut), even in the presence of systematic and intrinsic methods (which means independent of the given information).

ACTIVITIES

Recall activity 1

Objective Recall equal fractions, comparing two fractions with the same denominator, then different denominators.

Recall activity 2

Objective As its title indicates, the objective of this activity is to recall the rules of calculation on fractions.

Preparatory activity 1

Objective Prepare the students to the simplification of fractions.

Preparatory activity 2

Objective Prepare the students to the notion of reduced fraction.

EXERCISES

Review and practice

1. a) $\frac{43}{18};\frac{7}{72};\frac{16}{15}$. b) $\frac{43}{45};\frac{37}{6};\frac{1}{15};\frac{1}{2}$. c) $\frac{117}{14};\frac{7}{4};1;\frac{8}{189}$.

d) $\frac{9}{7};\frac{32}{63};1$. e) $\frac{73}{8};\frac{7}{10};\frac{9}{32};\frac{40}{3}$. f) $26;\frac{2}{7};\frac{2}{27}$.

2.

$+$	$\frac{2}{3}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{4}{7}$
$\frac{1}{4}$	$\frac{11}{12}$	$\frac{9}{8}$	1	$\frac{9}{20}$	$\frac{23}{28}$
$\frac{1}{6}$	$\frac{5}{6}$	$\frac{7}{24}$	$\frac{11}{12}$	$\frac{11}{30}$	$\frac{31}{42}$
$\frac{1}{2}$	$\frac{7}{6}$	$\frac{5}{8}$	$\frac{5}{4}$	$\frac{7}{10}$	$\frac{15}{14}$
$\frac{1}{7}$	$\frac{17}{21}$	$\frac{15}{56}$	$\frac{25}{28}$	$\frac{12}{35}$	$\frac{5}{7}$
$\frac{1}{10}$	$\frac{23}{30}$	$\frac{9}{40}$	$\frac{17}{20}$	$\frac{3}{10}$	$\frac{47}{70}$

3. $\frac{8}{14}=\frac{4}{7}=\frac{\mathbf{16}}{28}=\frac{24}{\mathbf{42}}$.

4. a) - $\frac{2}{3}=\frac{126}{189}$;

- no because 100 is not a multiple of 3.

b) $\frac{14}{21}=\frac{24}{36}=\frac{2}{3}$.

5. a) $\frac{2}{5}<\frac{4}{9}<\frac{2}{3}<\frac{5}{7}<\frac{3}{4}$;

b) $\frac{2}{6}<\frac{3}{8}<\frac{5}{11}<\frac{1}{2}<\frac{7}{8}$.

6.

×	$\frac{3}{2}$	$\frac{1}{7}$	$\frac{3}{8}$	$\frac{4}{6}$	$\frac{2}{7}$
$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{14}$	$\frac{3}{16}$	$\frac{1}{3}$	$\frac{1}{7}$
$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{21}$	$\frac{1}{8}$	$\frac{2}{9}$	$\frac{2}{21}$
$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{28}$	$\frac{3}{32}$	$\frac{1}{6}$	$\frac{1}{14}$
$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{35}$	$\frac{3}{40}$	$\frac{2}{15}$	$\frac{2}{35}$
$\frac{1}{10}$	$\frac{3}{20}$	$\frac{1}{70}$	$\frac{3}{80}$	$\frac{1}{15}$	$\frac{1}{35}$

7. a) $\frac{5}{6};\frac{9}{20};\frac{3}{4};\frac{2}{3};\frac{5}{27}$ b) $\frac{1}{3};\frac{1}{4};\frac{1}{5};\frac{1}{4};\frac{2}{15}$.

8. a) $2.5=\frac{25}{10}=\frac{5}{2}$; $1.4=\frac{14}{10}=\frac{7}{5}$; $6.5=\frac{65}{10}=\frac{13}{2}$;
$1{,}2=\frac{12}{10}=\frac{6}{5}$; $28{,}3=\frac{283}{10}$.
b) $11.3=\frac{113}{10}$; $3.25=\frac{325}{100}=\frac{13}{4}$; $2.2=\frac{22}{10}=\frac{11}{5}$;
$0.4=\frac{4}{10}=\frac{2}{5}$; $12.18=\frac{1218}{100}=\frac{609}{50}$.

9. Among the following fractions, the reduced fractions are: $\frac{13}{15}$; $\frac{14}{31}$; and $\frac{1}{3}$.

10. a) GCD(4235;9800) = 35 .

b) $\frac{4235}{9800}=\frac{4235\div35}{9800\div35}=\frac{121}{280}$.

11. a) $\frac{2}{5};\frac{2}{3};\frac{7}{11};\frac{43}{45};\frac{5}{8}$.

b) $\frac{9}{7};\frac{2}{3};\frac{3}{5};\frac{3}{5};\frac{23}{45}$.

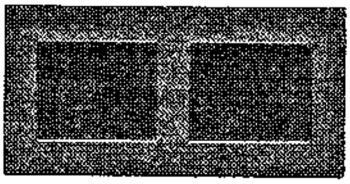
c) $\frac{26}{11};\frac{10}{7};\frac{15}{17}$.

d) $\frac{39}{275};\frac{7}{9};\frac{13}{19};\frac{757}{702};\frac{42}{11}$.

e) $\frac{1}{3}; \frac{3}{5}; \frac{5}{3}$.

12. a) $\frac{32}{15}$. b) $\frac{10}{3}$. c) $\frac{21}{8}$. d) $\frac{1}{5}$.

13. The area of the garden is: $9 \times 5 = 45\ cm^2$;
The area of the cultivated part is: $(9\text{-}3) \times (5\text{-}2) = 18\,cm^2$.
The cultivated part represents $\frac{18}{45} = \frac{2}{5}$ of the garden.



14. a) $\frac{9}{21} = \frac{9 \times 3}{21 \times 3} = \frac{27}{63}$;
$\frac{54}{126} = \frac{54 \div 2}{126 \div 2} = \frac{27}{63}$;
$\frac{3}{7} = \frac{3 \times 9}{7 \times 9} = \frac{27}{63}$.

b) $\frac{3}{7}$ is the reduced form of the fraction $\frac{27}{63}$ since 3 and 7 are prime themselves.

15. a) GCD(12 600;3 300) = 300 .
b) $\frac{12\,600}{3\,300} = \frac{12\,600 \div 300}{3\,300 \div 300} = \frac{42}{11}$.

16. The reduced form of $\frac{12}{20}$ is $\frac{3}{5}$. On the other hand, we have: $3 + 5 = 8$, hence $(3+5) \times 6 = 48$, which means $(3 \times 6) + (5 \times 6) = 48$. Then the required fraction is: $\frac{3 \times 6}{5 \times 6} = \frac{18}{30}$.

17. The fractions equal to $\frac{49}{84}$ and whose terms are less than 50 are:
$\frac{7}{12} = \frac{14}{24} = \frac{21}{36} = \frac{28}{48}$.

18. a) $\frac{536}{800} = \frac{268}{400} < \frac{300}{400}$.
b) $\frac{225}{75} = \frac{75}{25}$ and $\frac{130}{50} = \frac{65}{25}$; hence $\frac{225}{75} > \frac{130}{50}$.
c) $\frac{175}{50} = \frac{35}{10}$ and $\frac{60}{20} = \frac{30}{10}$; hence $\frac{175}{50} > \frac{60}{20}$.

19. a) $\frac{8 \times 3 \times 7}{28 \times 6 \times 5} = \frac{1}{5}$.

b) $\frac{9\times16\times25}{10\times18\times5}=4$.

c) $\frac{3^2\times5^3\times7}{3^3\times5^2\times7}=\frac{5}{3}$.

d) $\frac{4^3\times15^3\times2^2}{6^2\times12^2}=\frac{500}{3}$.

e) $\frac{3131}{4747}=\frac{31}{47}$.

f) $\frac{555}{999}=\frac{5}{9}$.

g) $\frac{7\times8+3\times8}{3\times8}=\frac{56+24}{24}=\frac{80}{24}=\frac{10}{3}$.

h) $\frac{11700}{18900}=\frac{13}{21}$.

i) $\frac{650}{1000}=\frac{13}{20}$.

j) $\frac{318}{153}=\frac{106}{153}$.

20. a) $\frac{16750}{14700}=\frac{2\times5^3\times67}{2^2\times3\times5^2\times7^2}=\frac{335}{294}$.

b) $\frac{4235}{9800}=\frac{5\times7\times11^2}{2^3\times5^2\times7^2}=\frac{121}{280}$.

21. $\frac{28}{63}=\frac{4}{9}$ and 28 + 12 = 40. We must find a fraction equal to $\frac{28}{63}$ and whose numerator is 40.

Now $\frac{28}{63}=\frac{4}{9}=\frac{40}{90}$ and we have: 90 = 63 + 27 . Therefore we must add 27 to the denominator and 12 to the numerator to get the required fraction.

Decimals and fractions

9

OBJECTIVES

1. Recognize a non-decimal fraction.
2. Write a fraction in decimal form (approximate calculation).

- Write a decimal fraction in the form of a decimal number.
- Define and recognize a non-decimal fraction.
- Know that a non-decimal fraction can be written in the form of a decimal number, in which the decimal part is unbounded and periodic.
- Write a number whose decimal part is unbounded and periodic in the form of a fraction.
- Know that every decimal is a fraction, but there are fractions which are not decimal numbers.
- Write a decimal number in the form of a sum of many decimal fractions whose denominators are, in the order, 10, 100, 1000, ...

EXPLANATION

Decimal numbers are the only version known in all calculators of real numbers. Therefore, decimals constitute the "practical" part of real numbers (rational or irrational). Also, one of the frequent uses of decimal numbers is the approximation of non-decimal (non-decimal fractions, and subsequently, irrational numbers).

Our intention is, from one point, to heighten the awareness of the student to the presence of numbers which cannot be represented by decimal form, from another, to initiate him to the imagination of an infinite sequence (the sequence of the priodic decimal part of a non-decimal rational number).

ACTIVITIES

Activity 1

Objective Propose to the stwo different writings of decimal numbers, in one (Stevin notation) is the one used now, whereas the other shows the extended decimal structure of decimal numbers (development according to the powers of 10 and $\frac{1}{10}$).

Activity 2

Objective Show the relationship between fractions and decimals.

Activity 3

Objective Prepare the students to the approximation of a non-decimal fraction by a decimal number.

Activity 4

Objective Introduce the students to decimal fractions.

EXERCISES

---

1. Among the given fractions, the decimal fractions are:

a) $\frac{13}{5}$; $\frac{19}{2^2 \times 5}$; $\frac{12}{3 \times 5}$; $\frac{2}{5^2}$.

b) $\frac{7}{40}$; $\frac{3}{25}$; $\frac{5}{16}$; $\frac{49}{35}$.

2. The fractions which give a decimal quotient are: $\frac{1}{20}$; $\frac{1}{25}$ and $\frac{1}{5}$.

3. $\frac{9}{16} = 0.5625 = \frac{5625}{10^4}$; $\frac{17}{20} = 0.85 = \frac{85}{10^2}$;

$\frac{7}{4} = 1.75 = \frac{175}{10^2}$; $\frac{5}{8} = 0.625 = \frac{625}{10^3}$;

$\frac{18}{45} = 0.4 = \frac{4}{10}$; $\frac{35}{112} = 0.3125 = \frac{3125}{10^4}$.

4. a) $25 \div 7 = 3.571428\ 571428 \ldots$.

b) The approximation of $\frac{25}{7}$ to the nearest 0.01 by default is 3.75.

The approximation of $\frac{25}{7}$ to the nearest 0.001 by approximation is 3.572 .

5. The decimal approximation to the nearest 0.01 by default is: of $\frac{22}{7}$: 3.14 ;

of $\frac{87}{126}$: 0.69 ;

of $\frac{335}{113}$: 2.96 .

6. The decimal approximation to the nearest 0.0001 by default is: of $\frac{2}{3}$: 0.6666 ;

of $\frac{5}{7}$: 0.8333 ;

of $\frac{11}{6}$: 1.8333 ;

of $\frac{7}{9}$: 0.7777 .

7. a) $34.023 = 3\times 10+4+\frac{2}{10^2}+\frac{3}{10^3}$;

$3.405 = 3+\frac{4}{10}+\frac{5}{10^3}$;

$27.01 = 2\times 10+7+\frac{1}{10^2}$.

b) $104.104 = 100+4+\frac{1}{10}+\frac{4}{10^3}$;

$27.101 = 2\times 10+7+\frac{1}{10}+\frac{1}{10^3}$;

$3001.001 = 3\times 10^3+1+\frac{1}{10^3}$.

8. a) The integer part:

of $\frac{17}{5}$ is 3 ; of $\frac{52}{12}$ is 4 ; of $\frac{121}{7}$ is 17 ; of $\frac{324}{36}$ is 9 ; of $\frac{125}{102}$ is 1 and of $\frac{345}{111}$ is 3.

b) The decimal part:

of $\frac{17}{5}$ is $\frac{17}{5}-3=\frac{2}{5}$;

of $\frac{52}{12}$ is $\frac{52}{12}-4=\frac{4}{12}=\frac{1}{3}$;

of $\frac{121}{7}$ is $\frac{121}{7}-17=\frac{2}{7}=\frac{1}{3}$;

of $\frac{324}{36}$ is $\frac{324}{36}-9=0$;

of $\frac{125}{102}$ is $\frac{125}{102}-1=\frac{23}{102}$;

of $\frac{345}{111}$ is $\frac{345}{111}-3=\frac{23}{111}$.

9.

	Approximation	
	to the nearest 1/10 by default	to the nearest 1/100 by excess
$\frac{355}{113}$	3.1	3.15
$\frac{47}{12}$	3.9	3.92
$\frac{151}{180}$	0.8	0.84

10. $3+\frac{1}{10}+\frac{5}{100}=3.15$; $17+\frac{1}{100}=17.01$;

$2+\frac{3}{10}+\frac{7}{100}+\frac{8}{1000}=2.378$; $\frac{1}{10}+\frac{1}{1000}=0.101$.

11. a) $\frac{78}{99}=0.\overline{78}$; $\frac{7}{9}=0.\overline{7}$; $\frac{5}{3}=1.\overline{6}$; $\frac{543}{999}=0.\overline{543}$.

b) $\frac{16}{9}=1.\overline{7}$; $\frac{125}{99}=1.\overline{26}$; $\frac{300}{99}=3.\overline{03}$

12. $\frac{121}{66}\cong 1.833$; $\frac{425}{70}\cong 6.071$; $\frac{73}{15}\cong 4.866$; $\frac{11}{33}\cong 3.666$.

13.

	Approximation by default	
	to the nearest 0.1	to the nearest 0.001
$\frac{25}{3}$	8.3	8.333
$\frac{18}{7}$	2.5	2.571
$\frac{5}{6}$	0.8	0.833

14. $0.\overline{45}=\frac{45}{99}$; $0.\overline{123}=\frac{123}{999}$;

$101.\overline{17}=101+0.\overline{17}=101+\frac{17}{99}=\frac{10016}{99}$; $23.\overline{234}=23+0.\overline{234}=23+\frac{234}{999}=\frac{23211}{999}$.

15. *a)* The numbers with 3 decimals between 7.43 and 7.44 are:

7.431 ; 7.432 ; 7.433 ; 7.434 ; 7.435 ; 7.436 ; 7.437 ; 7.438 ; 7.439 .

b) The numbers with 4 decimals between 27.435 and 27.437 are:

27.4351 ; 27.4352 ; 27.4353 ; 27.4354 ; 27.4355 ; 27.4356 ; 27.4357 ; 27.4358 ; 27.4359.

16.

a) $\frac{15}{99}-0.\overline{13}=\frac{2}{99}=0.\overline{02}$; b) $\frac{5}{3}-0.\overline{2}=\frac{13}{9}=1.\overline{4}$;

c) $2-0.\overline{33}=\frac{165}{99}=1.\overline{6}$; d) $\frac{5}{9}+0.\overline{3}=\frac{8}{9}=0.\overline{8}$;

e) $0.\overline{32} \times 3 = \frac{96}{99} = 0.\overline{96}$; f) $.4 \times 0.\overline{312} = \frac{1248}{999} = 1.\overline{249}$

g) $0.\overline{25} \div 4 = \frac{25}{396} = 0.06\overline{31}$; h) $0.\overline{302} \div 4 = \frac{302}{3996} = 0.0\overline{755}$.

17. a) $15.\overline{304} = 15 + 0.\overline{304} = 15 + \frac{304}{999} = \frac{15289}{999}$;

b) $23.\overline{103} = 23 + \frac{103}{999} = \frac{23080}{999}$;

c) $20.\overline{04} = 20 + \frac{4}{99} = \frac{3964}{99}$;

d) $10.\overline{31} = 10 + \frac{31}{99} = \frac{1021}{99}$;

e) $2.\overline{320} = 2 + \frac{320}{999} = \frac{23018}{999}$;

f) $2.\overline{32} = 2 + \frac{32}{99} = \frac{230}{99}$;

g) $123.\overline{321} = 123 + \frac{321}{999} = \frac{123198}{999}$;

h) $203.\overline{302} = 203 + \frac{302}{999} = \frac{203099}{999}$.

Proportionality

16

OBJECTIVES

1. Calculate the fourth proportional.
- Define a proportion.
- Identify the terms of a proportion (middle, extremes).
- Transform a proportion to obtain another.
- Know how to complete a proportion with a missing term (4th proportional).
- Express the calculation of the fourth proportional by cross multiplication.
- Use the calculation of the fourth proportional in problems (buying, selling, duration, speed, distance, dimensions, etc.)

EXPLANATION

Proportionality is one of the most important notions in Mathematics. Some people may go as far as saying «we think through proportionality» !
Through history, the notion of proportionality was considerably developed. Studies on proportionality, proportions and proportional sizes exist from the Ancient Times. In the *Elements* of Euclid, a whole book is dedicated to this subject. It is due to proportionality that the mathematicians discovered the rational numbers, then, later on, the imaginary numbers !
As regards to teaching proportionality, it has come a long way (and is still far from reaching the end): from the cross multiplication, to the fourth proportional, to the "passage to unity", to the coefficient of proportionality, to the linear function, etc.
Teaching this notion is clarified through different other notions. Although it is usually taught implicitly: scale, rate and interest, calculation of prices, calculation of distances, speed or distance traveled, alloys, etc. It is often shown as the "magical potion" which suddenly appears every time we are in distress! It is reviewed over and over again under different aspects but always in different contexts and to teach other notions and never to teach it explicitly.
In our program, we tried to stress on it a little, rather timidly, because, although it is constituted of a fertile method (very!) of resolution of problems, we think that this is a notion that is as difficult as it is important.
We start in the sixth year with the study of percentages and scales, to then pass onto the study of proportional sequences. In this year, we propose to study the directly proportional sizes, through which we introduce the notion of proportionality. The interest for the subject only appears through very diverse applications. The long process of the teaching of the proportionality does not end here: in the following class, we teach the inversely proportional sizes (in order to maintain the notion of proportionality) and in the ninth year we will make the link between this notion and that of linearity (and the study of proportionality continues ...).
If we are on this subject, this is to make the teaching corps aware of the great interest of this notion, because we think that we are far from reserving it the place it deserves.

ACTIVITIES

Review activities 1 and 2

Objective Remember the proportional sequences.

Review activity 3

Objective Remember calculations with percentages.

Preparatory activity

Objective Familiarize the students with situations of proportionality.

EXERCISES

Review and practice

1 a) The two sequences are proportional because $\frac{3}{2}=\frac{5}{\frac{10}{3}}=\frac{7}{\frac{14}{3}}=\frac{9}{6}=1{,}5$.

b) The two sequences are not proportional because $\frac{2}{3}\neq\frac{4}{5}$.

c) The two sequences are not proportional because $\frac{4}{1{,}6}\neq\frac{28}{9{,}2}$.

d) The two sequences are not proportional because $\frac{4}{16}\neq\frac{28}{7}$.

2 The number of participants is: $\frac{4000\times 65}{100}=2600$ voters.

3 Three cars out of 75 represent 4%.

4 We have: $\frac{4}{20}=\frac{20}{100}$. The failure percentage is 20% .

5 To go from 10 to 25, we must multiply by 2.5; hence we must represent the width of the house by par: $4cm \times 2.5 = 10cm$.
The scale of this representation is therefore: 4 per 1000 (4 ‰) .

6 a) The number of dead people in 1996 is: $\frac{5\,000\,000\times 1{,}5}{100}=75000$ people.

b) The birth rate was: $\frac{120\,000}{5\,000\,000}=\frac{24}{1000}$ (24 ‰) .

Production-consumption

7 a)

$$5\times\left[\begin{array}{l}8\ell\\ \rightarrow 40\ell\end{array}\right.\quad \text{hence}\quad \left.\begin{array}{r}100km\\ ?\ \leftarrow\end{array}\right]\times 5$$

The distance covered under these conditions is therefore: 100×5 = 500 *km* .

b)

$$3{,}5\times\left[\begin{array}{l}8\ell\\ \rightarrow\ ?\end{array}\right.\quad \text{hence}\quad \left.\begin{array}{r}100km\\ 350km\leftarrow\end{array}\right]\times 3{,}5$$

The volume of gas needed to cover 350*km* is therefore: $8\times3.5=28\ell$.

8 a) The surface we can paint with 12ℓ of paint is: $\dfrac{12\times14}{3.5}=48m^2$.

b) The quantity of paint needed to paint 23m^2 is: $\dfrac{3.5\times23}{14}=5.75m^2$.

9 a) The quantity of milk needed is: $\dfrac{10\times30}{8.4}\cong35.71\ell$.

b) The weight of white cheese we can make is: $\dfrac{100\times8.4}{30}=28kg$.

10 a) By working 21 days he earns: $\dfrac{21\times150000}{62}=525\ \ 000LL$.

b) In order to earn 500 000 LL, he needs to work: $\dfrac{6\times500\,000}{150\,000}=20$ days.

Supply - Speed

11 The capacity of the container is: $\dfrac{150\times15}{60}=37.5\ \ell$.

$$\begin{array}{cc}150\ell & 60' \\ ? & 15'\end{array}$$

12 The duration of a flight of 1600 *km* is: $\dfrac{1600\times60}{750}=2h\ 8'$.

$$\begin{array}{cc}750km & 60' \\ 1600km & ?\end{array}$$

The duration of a flight of 1125 *km* is: $\dfrac{1125\times60}{750}=1h\ 30'$.

$$\begin{array}{cc}750km & 60' \\ 1125km & ?\end{array}$$

13 The number of cars which pass in an hour is: $\dfrac{3250\times60}{150}$=1300 cars.

$$\begin{array}{cc}150' & 3250 \\ 60' & ?\end{array}$$

Change of units

14 Average speed : 1.7 mach = 1.7×1200 *km* = 2040 *km/h*.

Duration of the trip: $\dfrac{6000}{2040}\cong2h\ 56'\ 28''$.

15

	Average speed	
Animal	given	in *km/h*
turtle	3cm/s ;	0.108
eagle	45m/s ;	162
whale	12cm/s	0.432
hare	3m/s	10.8

Approximations

16 a) The number of pebbles found in selecting 1000 grams of lentils is: $\frac{12\times 1000}{600} = 20$ pebbles (approximately).

b) No.

17 a) The thickness of a 700 paper-package is: $\cong \frac{700\times 3.4}{240}\ 9.9\ cm$.

b) The number of papers in a package of thickness 13.5cm is: $\frac{13,5\times 240}{3,4} \cong 953$ papers.

PROBLEMS

1 The length of the film used is: $\overset{s}{(5\times 60)}\times \overset{mm}{8}\times \overset{images}{20} = 48\ 000\ mm = 480\ m$.

2 The cloud is at: $\frac{8\times 1.7}{5} = 2.72\ km$

3 In 2 hours, the four secretaries type: $\frac{4\times 20}{2} = 40$ pages.

In 8 hours they type: $\frac{8\times 40}{2} = 160$ pages.

4 a) With 20 000 LL, we can buy: $\frac{20\ 000\times 1.200}{1500} = 16\ kg$.

With 20 000 LL, we can feed: $\frac{16\times 15}{8.4} \cong 28$ people.

b) To feed 50 people, we need: $\frac{8.5\times 50}{15} \cong 28\ kg$ of bread;

hence, the cost is: $\frac{28\times 1500}{1.200} = 35\ 000$ LL.

5 Necessary weight of pure gold: $\frac{5.4\times 75}{100} = 4.05\ g$.

Necessary weight of silver: $\frac{5.4\times 12.5}{100} = 0.675\ g$.

Necessary weight of copper: 0.675 *g*.

6 First choice: 1 % per month, or 12 % per year.
Second choice: 3.6 % per semester, or 10.8 % per year.
Third choice: 11 % per year.
Hence the most advantageous choice is the first choice.

7 It is sufficient to compare the price of the gram:
for the tablet of 150 *g*, the price of the gram is 6 LL;
for the tablet of 400 *g*, the price of the gram is 5 LL.
The 400 *g* tablet is therefore the best choice.

APPENDIX C

QUESTIONS ASKED THE TEACHERS

1. What is proportionality in your opinion?
2. Which topics enter under proportionality?
3. What strategies should be used to teach proportionality?
4. In what way, should proportionality be presented in books?
5. Are you satisfied in what the Lebanese curriculum textbook is presenting regarding proportionality?
6. Do you sue the teacher's manual? Is it helpful?
7. What better idea can you provide to improve the textbook?
8. Do you use supplementary materials from other books for more exercises or problems?

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APPENDIX D

RUBRIC

Competency	T1	T2	T3	T4	T5	T6
Demonstrates understanding of subject matter and pedagogical knowledge of instruction.						
Plans effective activities						
Provides clear direction for activities						
Provides enthusiasm for the lesson						
Seeks innovative ways and promotes creativity to deliver the idea.						
Chooses effective ways to enhance students thinking						
Uses technology while explaining						
Identifies appropriate grouping to facilitate learning						
Helps students understand the relevance of the lesson to them						
Links new information to previous knowledge						
Let students generate hypothesis						
Promotes critical thinking and reasoning through brainstorming						
Helps students connect ideas						
Communicate effectively with students						
Logical sequence of explanation: examples and problems given						
Uses the language appropriately						
Demonstrates understanding of connection between math concepts and real life problems						
Give all students the chance to ask questions and answer them.						
Provides useful feedback for students questions						
Facilitates discussion between students and teachers.						

APPENDIX E

STUDENTS' PROBLEMS WORKSHEET

Problem 1 (Well – chunked measures)

My grandfather has measured both his trees in 2002. Tree A was 8 m high and B was10 m high. In 2007, tree A is 14 m high and tree B is 16 m high. Over the last five years, which tree's height has increased the most?

Problem 2 (Part – Part- whole)

If a certain school has 5 girls to every 6 boys in each class, how many girls are there for 24 boys?

Problem 3 (Associated sets)

Seven girls are given three pizzas while three boys have one pizza. Who gets more pizza?

Problem 4 (shrinkers /stretches)

Your friend and you are using different road maps of Beirut. On your map, a road 6 cm long is really 24 km long. On your friend's map, a road 24 cm long is really 72 km long. Who is using a larger map of Beirut? Explain.

Problem 5

A shirt is priced at 30,000 L.L. in a shop. Bilal wants to buy it, but he decides to wait for the sales season. Under sales, the shirt is 30% discounted. How much would Bilal pay to buy the shirt?