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Chapter 4

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Chapter 4

The Keynesian Model II: The IS-LM Model

"We are all Keynesians now"

Richard Nixon

We have shown in chapter 4 that investment demand in the capitalist economy plays the central role in determining equilibrium output. It was stated that the amount of investment forthcoming as a result of the decision of private investors will not, most of the time, fill the gap needed to maintain full employment. Hence, the capitalist economy generates underemployment equilibrium. In this chapter we elaborate on Keynes's theory of investment demand. In addition, the Hicksian interpretation of the Keynesian model is presented. It is called the IS-LM model.

The Marginal Efficiency of Capital

For Keynes, when an investor buys a capital–asset, he is purchasing "the right to a series of prospective returns which he expects to obtain from selling its output, after deducting the running expenses of obtaining that output, during the life of the asset" (Keynes, GT, p. 135). These series of expected returns: $Q1, Q2 \ldots\ Qn$ are the prospective yields of the investment. The present values of each of the annuities are

$$PV = \frac{Q_1}{1+r} + \frac{Q_2}{(1+r)^2} \ldots + \frac{Q_n}{(1+r)^n}$$

where r = the market interest rate that gives the present value of the yields.

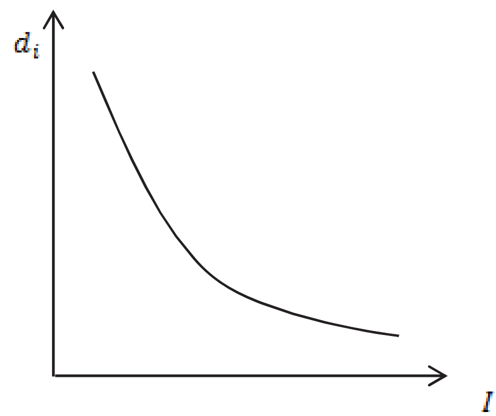
The capital- asset has a supply price P_k which is the price that would induce the supplier of capital- assets to produce an additional unit of such assets. The marginal efficiency of capital is defined as the discount rate, d, such that;

$$P_k = PV$$

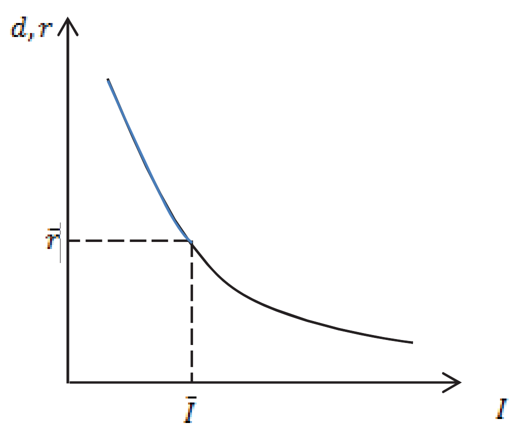
or

$$P_k = \frac{Q_1}{1+d} + \frac{Q_2}{(1+d)^2} \ldots + \frac{Q_n}{(1+d)^n}$$

In addition, for any type of asset, the MEC of that type will decrease as more investment is done. The reasons are; first, the prospective yield will decrease as the supply of capital increases and second, the supply price of capital will increase. Hence, for every type of capital, we can draw an investment schedule.



If we aggregate all schedules ∀i then we get the aggregate investment-demand schedule



If market interest rate is $\bar{r}$, then for all d > $\bar{r}$, $I > 0$ and ∀ d < $\bar{r}$, $I = 0$.

The total amount of investment is the sum of all capital investments in assets whose MEC = $d \geq \bar{r}$ or $\bar{I}$. Hence, investment depends on the market rate of interest r where the investment function is

$I = I\,(\sum Q\,, Pk, r)$

If we assume that $\sum Q$ & P_k are held constants for various types of capital then

$I = I(r)$

Solved Problem 1 Suppose there is a type of capital asset that has a life-time of two years with prospective yields $Q_1 = \$100$ and $Q_2 = \$200$. Assume $P_k = \$150$.

a) Calculate the MEC

$P_k = PV$

$$\$150 = \frac{100}{1+d} + \frac{200}{(1+d)^2}$$

$$150 = \frac{100(1+d)}{(1+d)^2} + \frac{200}{(1+d)^2}$$

$=> 100(1 + d) + 200 = 150(1 + d)^2$

$=> 100 + 100d + 200 = 150(1 + d)^2$

$=> 300 + 100d = (1 + d^2 + 2d)$

$=> 750d^2 + 300d - 100d = 300 - 150$

$=> 150d^2 + 200d = 150$

$=> d_1 = 0.535$

b) If the market interest rate r is 0.3, will an investor buy this capital asset.

Since $d > r$
=> Yes

c) Suppose $Q'_1 = 100$; $Q'_2 = 100$

Repeat a & b

a) The quadratic equation is then

$\Rightarrow 200 + 100d = 150\,(1 + d^2 + 2d)$

$150\, d^2 + 300d - 100d = 200 - 150$

$150\, d^2 + 200d - 50 = 0$

$d = 0.215$

b) Since $d < r \Rightarrow No$

d) Suppose $P_k = \$175$; repeat a & b

a) The quadratic equation is then
$300 + 100d = 175\,(1 + d^2 + 2d)$

$300 + 100d = 175 + 175d^2 + 350d$

$175d^2 + 350d - 100d = 300 - 175$

$175\, d^2 + 250d - 150 = 0$

$d = 0.392 > r \Rightarrow Yes$

The Liquidity Preference Theory

The level of investment demand in the economy, as we have seen, depends on the market interest rate. What determines the market interest rate will then play a major role in determining the investment demand in the economy. Keynes rejected the classical theory of interest. Remember that the classical theory of interest states that the interest rate is determined by the equilibrium in Savings-Investment market.

Keynes advanced that individuals have two decisions to make. One decision relates to their time preference for current vs. future consumption governed by the marginal propensity to consume. This decision gives the savings function $S = -\bar{C} + mpsY$. Second, individuals decide on which form they want to keep their savings. Do they want to hold it in form of money or in other forms (such as commodities, debts, etc…)? This is the aspect of savings decisions that the classical economists did not address.

Keynes defined the liquidity-preference as the schedule of amount of money individuals would want to hold under different circumstances. Hence, the rate of interest cannot be a return to savings since money pays no interest. In this view, the interest rate becomes the price for parting with 'liquidity' and not the price of 'waiting' to consume. In market analysis, the interest rate is not as the classical economists contended the price that equilibrates the investment demand with the individuals desire to abstain from current consumption but the price that equilibrates the demand for money with the money supply. The liquidity preference function can be written as

$$M^d = L(r)$$

$$\frac{dL}{dr} < 0$$

For Keynes, there are three motives for liquidity holding based on the two functions of money as both a medium of exchange & a store of value. The first motive is the transactions-motive which the same as the classical theory of money demand. However, Keynes adds two motives that are based on the uncertainty regarding the future rate of interest.

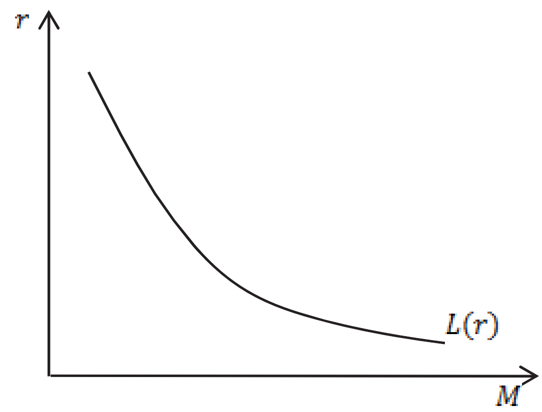
The precautionary-motive is the desire to obtain a future cash equivalent of a certain proportion of total resources.

The speculative-motive arises from the uncertainty of future interest rates given organized debt markets. In the speculative model, the choice of individuals is between holding M or holding bonds (B) that pay interest. We know that the price of bonds is inversely related to the interest rate.

$$P_B \propto \frac{1}{r}$$

In organized markets, each individual has what can be called a preconceived idea about the normal interest rate r_N. If it happens that the market rate $r_M > r_N$, he expects interest rates to go down and if $r_M < r_N$ he expects interest rates to go up. In the first case he will hold bonds because as r decreases, P_B increase and in the second case he will hold cash.

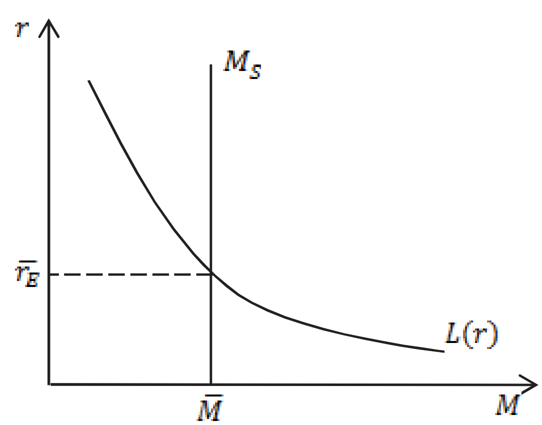
If we have N individuals with different perceptions of what constitutes the natural rate of interest ($r_N^1, r_N^2, r_N^3, \ldots, r_N^N$) then the liquidity function is downward sloping.



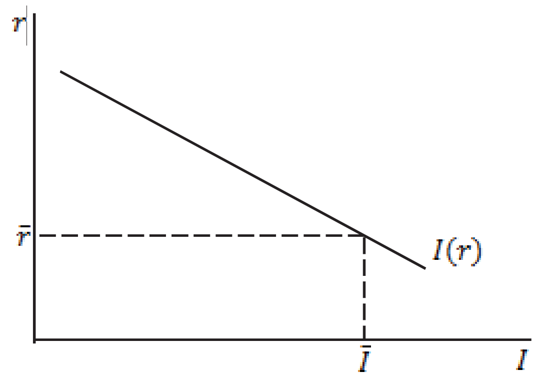
As the interest rates go down, more and more individuals will sell bonds and hold cash as the market interest rate crosses their own r_N^i. Hence, the existence of a variation in r_N^i makes the liquidity preference function smooth. This is very important for the way in which changes in the money supply affect economic activity. As r declines the reward for parting with liquidity gets smaller & individuals move into cash. The concept of liquidity preference is close to the concept of hoarding. Keynes said that seen in this way "interest has been usually regarded as the reward of not spending, whereas, in fact it is the reward of not hoarding" (p. 174, GT)

Equilibrium in Money Markets

The market interest rate is determined by the interaction between liquidity – preference $L(r)$ and the money supply in the economy. Graphically, this can be represented,



Given the equilibrium in money markets, then the r_E will determine the amount of investment given the MEC schedule.



Hence, given the multiplier model, the equilibrium output is dependent on the rate of interest.

$Y_E = \frac{1}{1-mpc}\ (\bar{C} + \bar{I}\ )$

In the Keynesian model, the system's determinants are the mpc, MEC and r. They are independent as Keynes said "in the sense that their values cannot be inferred from one another." (p. 184)

The IS–LM Model

The IS–LM model is one and the most dominant interpretation of Keynes's theories. It started with an article by John Hicks in 1937 and became the prototype model for small and large macroeconometric models developed after WWII and the main way of presenting Keynes ideas in textbooks. In this chapter, we develop the IS-LM model given P is fixed.

The LM Curve

Given Keynes's speculative motive for liquidity and the transaction motive for liquidity, the money demand function can be written as;

$M^d = L(y, r)$

In linear form the demand function can be represented by

$M^d = c_1Y - c_2r$

where c_1 and c_2 are > 0.

Given that the money supply is exogenous, and determined by monetary authority then,

$M^s = M^d = c_1Y - c_2r$

is the equilibrium condition in money markets

or

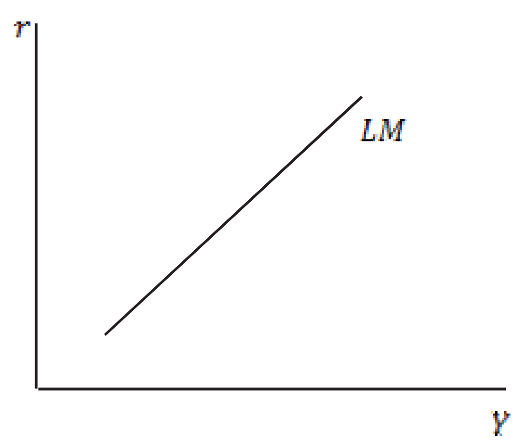
$\bar{M} = c_1Y - c_2r$

This can be rewritten as,

$\bar{M} - c_1Y = c_2r$

$\Rightarrow r = \frac{c_1}{c_2}y - \frac{\bar{M}}{c_2}$

This is called the LM curve which represents combinations of y & r such that the money market is in equilibrium. The LM curve can be drawn as;



The IS Curve

In the goods market, we have the following Keynesian functions,

$I = \bar{I} - \beta r$

$S = -\bar{C} + mpsY$

We know that in equilibrium,

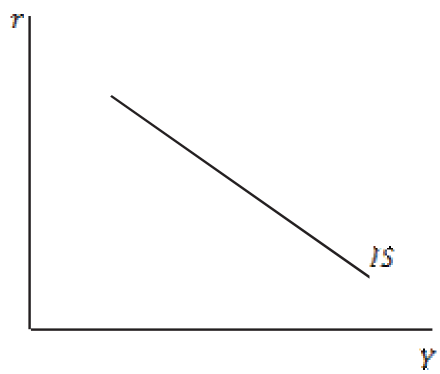
$S = I$

$-\bar{C} + mpsY = \bar{I} - \beta r$

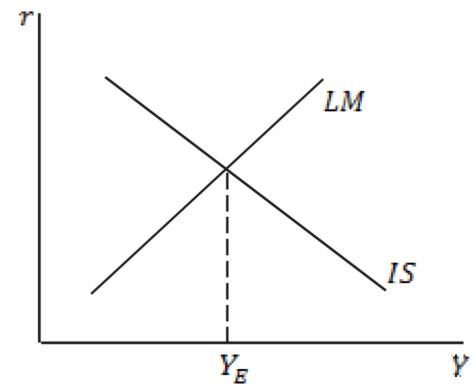
$\beta r = \bar{I} + \bar{c} - mpsY$

$r = \frac{\bar{C} + \bar{I}}{\beta} - \frac{(1-mpc)}{\beta} Y$

which is the equation for the IS Curve. The IS curve represents all combinations of Y & r such that the goods market is in equilibrium. The graph of the IS



The superimposition of the IS & LM curves gives the (Y_E, r_E) such that both goods & money markets are in equilibrium.



The IS–LM model hence provides a model of the economy in which the equilibrium output & interest rate are determined. It preserves the main Keynesian idea that $Y_E < Y_f$ in the normal state of affairs.

Algebraic Formulation of IS–LM

The simultaneous equilibrium in money and goods markets is the intersection of the IS and LM curves. Algebraically, this can be written as

$$r|_{IS} = r|_{LM}$$

$$\frac{\bar{C} + \bar{I}}{\beta} - \frac{(1 - mpc)}{\beta} Y = \frac{c_1}{c_2} Y - \frac{\bar{M}}{c_2}$$

$$\left(\frac{c_1}{c_2} + \frac{(1-mpc)}{\beta}\right) Y = \frac{\overline{C+I}}{\beta} + \frac{\bar{M}}{c_2}$$

or

$$Y_E = \frac{1}{\beta\left(\frac{c_1}{c_2} + \frac{1 - mpc}{\beta}\right)} (\bar{C} + \bar{I}) + \frac{\bar{M}}{c_2\left(\frac{c_1}{c_2} + \frac{(1 - mpc)}{\beta}\right)}$$

$$\lim_{\beta \to 0} Y_E = \frac{1}{1 - mpc} (\bar{C} + \bar{I})$$

Hence, when investment is not a function of the interest rate ($\beta = 0$), the IS-LM model collapses to the simple Keynesian multiplier model.

Monetary and Fiscal Policies in the IS-LM Model

Fiscal Policy

Assume now that government is present where $G > 0$ & for simplicity, $T = 0$. The IS curve can be derived as follows

$Y = C + I + G$ equilibrium condition

$Y = \bar{C} + mpcY + \bar{I} - \beta r + G$

then

$$r = \frac{\bar{C} + \bar{I} + G}{\beta} - \frac{(1-mpc)}{\beta}Y \qquad \text{IS}$$

Deriving the equilibrium output from the condition

$r|_{IS} = r|_{LM}$

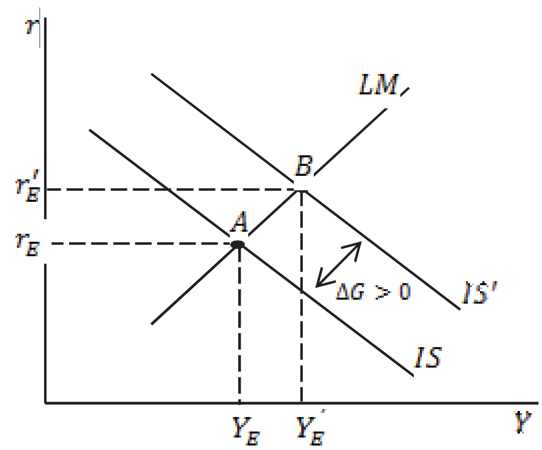
we get

$$Y_E = \frac{1}{\beta\left(\frac{c_1}{c_2} + \frac{1-mpc}{\beta}\right)}(\bar{C} + \bar{I} + G) + \frac{\bar{M}}{c_2\left(\frac{c_1}{c_2} + \frac{(1-mpc)}{\beta}\right)}$$

Policy

Suppose government increases expenditures ($\Delta G > 0$), what happens to Y & r?

Graphically,



ΔG > 0 => IS shifts to the right b/c ∀r, y that equilibrates the goods market is higher.

The new equilibrium point B gives $Y_E' > Y_E$; $r_E' > r_E$. Algebraically, (for output; do it for interest rate)

$$\Delta Y_E = \frac{1}{\beta\left(\frac{c_1}{c_2} + \frac{1 - mpc}{\beta}\right)} \Delta G$$

$$\Rightarrow \text{If } \Delta G > 0 \Rightarrow \Delta Y_E > 0$$

where $\frac{1}{\beta\left(\frac{c_1}{c_2}+\frac{1-mpc}{\beta}\right)}$ is the expenditures multiplier.

Notice that,

$$\frac{1}{\beta\left(\frac{c_1}{c_2} + \frac{1 - mpc}{\beta}\right)} < \frac{1}{1 - mpc}$$

As output increases, the interest rate r increases causing a dampening of investment. Hence, the multiplier is smaller than the simple Keynesian multiplier.

Monetary Policy

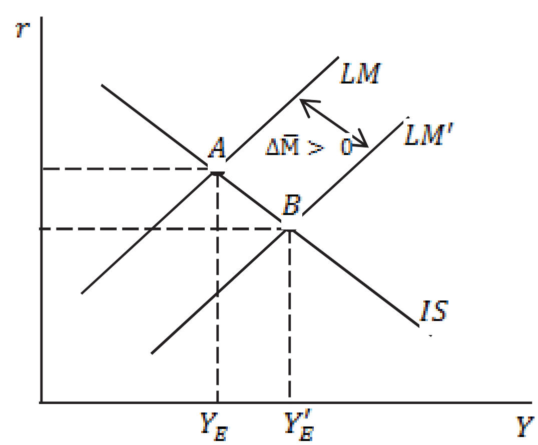
Suppose the central bank increases money supply ($\Delta\bar{M} > 0$), what happens to output & interest rates? Algebraically,

$$\Delta Y_E = \Delta\bar{M}\left(\frac{1}{C2\left(\frac{c_1}{c_2} + \frac{1-mpc}{\beta}\right)}\right)$$

$$\Rightarrow \text{if } \Delta\bar{M} > 0 \Rightarrow \Delta Y_E > 0.$$

where $\frac{1}{C2\left(\frac{c_1}{c_2}+\frac{1-mpc}{\beta}\right)}$ is the multiplier for the money supply.

Graphically,



When $\bar{M}$ increases, the LM curve shifts to the right because ∀y, the interest rates that equilibrate money markets are lower.

$Y_E \longrightarrow Y_E' > Y_E$

$r_E \longrightarrow r'_E < r_E$

Solved Problem 2 Suppose there is economy with the following,

$(mpc = 0.75, \beta = 2, c_1 = 1, c_2 = 0.5, \bar{I} = 100, \bar{C} = 50, \bar{M} = 100)$

a- Write down the equation of the IS curve,

$$r = \frac{100+50}{2} - \frac{1-0.75}{2}Y$$

$$r = 75 - 0.125Y$$

b- Write down the equation of the LM curve,

$$r = \frac{1}{0.5}Y - \frac{100}{0.5}$$

$$r = 2y - 200$$

c- Calculate equilibrium output and interest rate.

$r|_{IS} = r|_{LM}$

$$75 - 0.125y = 2y - 200$$

$$=> 2.125y = 275$$

$$=> Y_E = \frac{275}{2.125} = 129.4$$

$$r_E = 2y - 200$$

$$= 2\,(129.4) - 200$$

$$= 258.8 - 200$$

$$r_E = 58.8$$

d- Suppose investors become pessimistic about prospective yields and $\bar{I}$ --> $\bar{I}'$ = 75; what happens to output (show also graphically)?

From formula for Y_E, we have

$$\Delta Y_E = \frac{1}{\beta\left(\frac{C1}{C2} + \frac{1-mpc}{\beta}\right)}\,\Delta\bar{I}$$

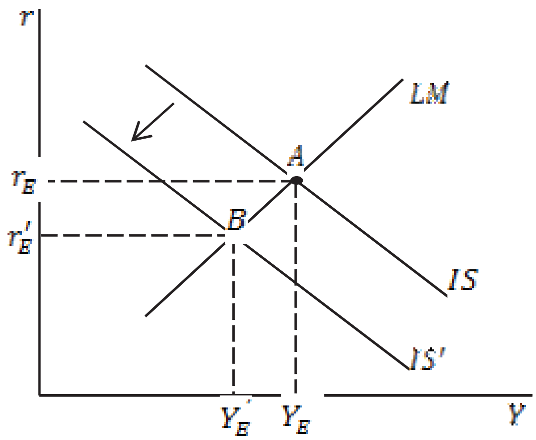
$$= \frac{1}{2\left(\frac{1}{0.5} + \frac{1-0.75}{2}\right)}\,(-25)$$

$$= \frac{-25}{2(2+0.125)}$$

$= -\frac{25}{4.25}$

$\Delta Y_E \cong -6$

Graphically, when $\Delta\bar{I} < 0$ then IS curve shifts to the left



=> ΔY_E < 0

Solved Problem 3 Suppose in problem 2, there is government ($G = 50, T = 0$)

a- Repeat a, b, c in 5.2

a') $r = \frac{100+50+50}{2} - \frac{1-0.75}{2Y}$

$r = 100 - 0.125y$ (IS)

b') The LM curve is given as before

$r = 2y - 200$ (LM)

c') $r|_{IS} = r|_{LM}$

$100 - 0.125y = 2y - 200$

$=> 2.125y = 300$

$$y = \frac{300}{2.125}$$

$$Y_E = 141.2$$

b- What happens to equilibrium output if $\Delta G = 50$

$$\Delta Y_E = \left(\frac{1}{\beta\left(\frac{C1}{C2} + \frac{1-mpc}{\beta}\right)}\right) \Delta G$$

$$\Delta Y_E = \left(\frac{1}{2\left(\frac{1}{0.5} + \frac{1-0.75}{2}\right)}\right) 50$$

$$\Delta Y_E = \frac{50}{4.25}$$

$$\Delta Y_E = 12$$

Box 4.1 Macroeconometric Models

The Keynesian IS-LM model gave a new impetus to macroeconometric models in the 1950s and 1960s. Macroeconometric models were first implemented by Jan Tinbergen in Holland in 1936. They are statistically estimated models of the economy using historical data. They are made up of simultaneous equations models. They are used in forecasting output, inflation and unemployment and in gauging the impact of monetary and fiscal policies. A simple Keynesian two equation Keynesian macroeconometric model with consumption and investment as variables is

$$C_t = 17.5 + 0.2Y_t + 0.7C_{t-1} \qquad \text{(B1)}$$

$$I_t = 100 - 3r_t + 0.2\Delta Y_t \qquad \text{(B2)}$$

$$Y_t = C_t + I_t + G_t \qquad \text{(B3)}$$

Consumption, investment and output are endogenous variables that are determined from within the system. Government expenditures and interest rates are exogenous or policy variables. The model (B1)-(B3) can then simulated to study the effects of government expenditures G on output or the effect of interest rates on output. For example, the above model with initial conditions $at\ t = 0, \{Y_0 = 445, C_0 = 285, I_0 = 70, G_0 = 90, r_0 = 6\}$. If we solve the model for t=1 with two different values for G and then we compare the solutions to get the multiplier. For $G_1 = 110, r_1 = 5.5$, we get for the variables the following values at t=1, $\{Y_1 = 535.8, C_1 = 324.1, I_1 = 101.7\}$. For $G_1 = 100, r_1 = 5.5$, we get $\{Y_1 = 519.1, C_1 = 320.8, I_1 = 98.3\}$. As expected in a Keynesian model, output, consumption and investment increased as a result of the increase in government expenditures and reduction of interest rates. The multiplier in the first period is calculated $\frac{\Delta y}{\Delta G} = \frac{535.8-519.1}{110-100} = 1.67$.

Source: Lawrence R. Klein and Richard M. Young. *An Introduction to Econometric Forecasting and Forecasting Models*. 1980. Lexington Books. For a more complicated macroeconometric model with many equations that can also be solved online, see the Fair model at www.fairmodel.com.

Problems

1. The following data is for a fictitious economy:

$C = 100 + 0.5\, Y_d$

$G = 100$

$T = 0$

$I = 100 - 0.05\, r$

$M^d = 300 - 10000\, r$

$M^s = 200$

a. Derive the IS curve for this economy. Plot it.

b. Derive the LM curve for this economy. Plot it.

c. Determine the equilibrium GDP and interest rate.

d. Determine the new equilibrium GDP and interest rate if the central bank increases the money supply to 250.

e. Does the result in (d) agree with the early Keynesian economists belief that monetary policy would be ineffective in curing a depression in the economy?

f. By how much did the transactions demand for money change? the speculative demand for money?

g. Calculate the new equilibrium GDP if G increases to 100. Compare it to the GDP predicted by the simple Keynesian multiplier model. Is there a difference? Why or why not?

h. Calculate the change in the velocity of money as a result of (g).

2. The following data is for a fictitious economy:

$C = 50 + 0.75 Y_d$

$G = 100$

$I = 50$

$T = 0.2y$

$M^d = 100 + 0.01\,y - 10\,r$

$M^s = 50$

a. Derive the IS curve for this economy. Plot it.
b. Derive the LM curve. Plot it.
c. Determine the equilibrium output and interest rate.