An Optimal 2×2 Space-Time Code for Time-Hopping Ultra-Wideband Systems with binary Pulse Position Modulation

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***Abstract*— The golden code [1] is a 2×2 space-time (ST) code that achieves the best known performance with all constellations carved from $\mathbb{Z}[i]$. On the other hand, impulse radio (IR) time-hopping (TH) ultra wideband (UWB) is a carrier-less baseband transmission technique. Therefore, being complex-valued, the golden code can not be associated with IR-UWB systems. In this paper, we propose a new ST code that satisfies all of the construction constraints of the golden code and that has the additional advantage of being totally real making it suitable for low cost carrier-less UWB terminals. Unlike the golden code that can be associated with all constellations carved from $\mathbb{Z}[i]$, the proposed code is optimal with binary pulse position modulation (PPM) which is a popular modulation scheme for TH-UWB. Moreover, the proposed scheme permits to achieve full transmit diversity with hybrid pulse position and amplitude modulations (PPAM) having two modulation positions.**

I. Introduction

The literature of ST coding is huge. However, associating ST coding with UWB is a recent and challenging research area [2], [3]. This approach can be a possible solution for very high data rate WPANs. However, an additional constraint related to the nature of the totally-real carrier-less transmissions must be added. Because of this constraint, the best known codes [1], [4] that can be associated with all narrow-band and wide-band CDMA and OFDM systems can not be applied with UWB. This motivates the construction of new totally-real ST coding schemes.

One possible solution of this problem is the application of the construction procedures that were adopted for constructing the last generation of full rate and fully diverse ST codes [1], [4]. In particular, cyclic division algebras (CDA) [5] are appealing because they result in a systematic code design. Moreover, the best known codes are based on CDA. The 2×2 codewords from CDA take the general form:

$$C = \begin{bmatrix} \alpha a_1 + \beta a_2 & \alpha a_3 + \beta a_4 \\ \gamma(\alpha_1 a_3 + \beta_1 a_4) & \alpha_1 a_1 + \beta_1 a_2 \end{bmatrix} \qquad (1)$$

This construction calls for the choice of a quadratic cyclic field extension $\mathbb{K} = \mathbb{P}(\theta)$ (where $\mathbb{P} = \mathbb{Q}$ for real constellations) whose Galois group is equal to $< \sigma >$ (with $\sigma^2 = 1$). $a_1, \dots, a_4$ are the information symbols that belong to the ring of integers $\mathcal{O}_{\mathbb{P}}$ of $\mathbb{P}$. γ is chosen such that there are no elements in $\mathbb{K}$ whose norms are equal to γ resulting in a fully diverse code. If $\gamma \in \mathcal{O}_{\mathbb{P}}$, then the resulting code has a non-vanishing determinant. α and β belong to $\mathcal{O}_{\mathbb{K}}$ and their choice results in a transmitted constellation that is a rotation of the initial signal set, $\alpha_1 = \sigma(\alpha)$ and $\beta_1 = \sigma(\beta)$. In other words, the generator matrix $G = \begin{bmatrix} \alpha & \beta \\ \alpha_1 & \beta_1 \end{bmatrix}$ must be unitary. For example, the golden code (GC) can be obtained from eq. (1) by setting $\gamma = i = \sqrt{-1}$, $\alpha = 1 + i(1-\theta)$, $\beta = \theta\alpha$ and $\theta = \frac{1+\sqrt{5}}{2}$ (the golden number). This code has the following properties:

- It achieves full rate and full diversity.
- It has the best known coding gain.
- It does not introduce any shaping losses (G is unitary).
- It is information lossless.
- It is an energy efficient code that has a constant average transmit energy per antenna ($|\gamma| = 1$).

For totally-real codes that can be used in conjunction with IR-UWB, γ, α and β must be real. It can be easily shown that for totally-real constructions, energy efficiency comes at the expense of shaping losses and vice versa (when $\gamma = -1$ is not a norm in $\mathbb{K}$, the generator matrix G is not unitary and vice versa [3]). Moreover, choosing $|\gamma| \neq 1$ results in energy-unbalanced codes that do not have a uniform average energy per transmit antenna resulting in performance losses. For example, $\gamma \in \{\frac{1}{2}, 2\}$ are the optimal choices; however the resulting coding gain is less than the coding gain of the GC.

Consider a hybrid 2-PPM-M'-PAM constellation obtained by modulating the data on M' possible amplitude levels during each modulation position. This constellation is given by:

$$\mathcal{C} = \{(2m' - 1 - M')I_1, (2m' - 1 - M')I_2\}_{m'=1}^{M'} \qquad (2)$$

where I_m is the m-th column of the 2×2 identity matrix I. The most popular modulations for TH-UWB are binary PPM ($M' = 1$) and bi-orthogonal PPM ($M' = 2$). Note that, eq. (1) can be associated with 2-PPM-M'-PAM constellations. In this case, the scalars $a_1, \dots, a_4$ in eq. (1) are replaced by their 2-dimensional vector representations taken from the set $\mathcal{C}$.

In this paper, instead of adopting the classical approach of constructing ST codes over the hypercubes carved from the lattice of rational integers [1], we exploited the structure of the

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constellation in eq. (2) in order to construct an adapted coding scheme. This code keeps the natural advantages of carrier-less impulse radio UWB (no need for frequency synthesizers, low cost, ...) and achieves better performance with higher spectral efficiency. In particular, the proposed solution outperforms all ST codes constructed from totally-real CDAs.

II. SYSTEM MODEL

Consider a single-user multi-antenna TH-UWB system with $P = 2$ transmit antennas and Q receive antennas. The antennas of the same user will share the same TH sequence, and since multiple access interference is not considered in this work, no reference to the TH sequence will be made hereafter. The signal transmitted from the p-th antenna can be expressed as:

$$s_p(t) = \sum_{n=0}^{N_f-1} \sum_{m=0}^{M-1} a_{p,m} w(t - nT_f - m\delta) \tag{3}$$

where $a_{p,m}$ is the $(m+1)$-th component of the M-dimensional vector A_p that belongs to the set $\mathcal{C}$ given in eq. (2) for $p = 1, \ldots, P$ and $m = 0, \ldots, M-1$ ($M = 2$ in what follows). In other words, $A_p = [a_{p,0}, \ldots, a_{p,M-1}]^T \in \mathcal{C}$ is composed of $M-1$ zero values and one component that belongs to the M'-ary PAM constellation. $w(t)$ is the monocycle pulse waveform of duration T_w normalized to have unit energy. N_f pulses are used to convey each information symbol. Each one of these pulses is emitted during one time frame of duration T_f. δ is the modulation delay and is chosen to verify $\delta \geq T_w$. In what follows, we suppose that T_f is larger than the channel delay spread and, consequently, the considered system does not suffer from inter frame interference (IFI) or inter symbol interference (ISI).

The signal received at the q-th antenna is given by:

$$r_q(t) = \sum_{p=1}^{P} \sum_{n=0}^{N_f-1} \sum_{m=0}^{M-1} a_{p,m} h_{q,p}(t - nT_f - m\delta) + n_q(t) \tag{4}$$

where $n_q(t)$ is the noise at the q-th antenna which is supposed to be real AWGN with double sided spectral density $PN_0/2$. $h_{q,p}(t)$ is the convolution of $w(t)$ and $g_{q,p}(t)$ which stands for the impulse response of the frequency selective channel between the p-th transmit and the q-th receive antenna.

In UWB systems, the interaction between the very short pulses is limited resulting in better immunity against multi-path fading. However, profiting from the multi-path diversity might necessitate the implementation of very high order Rake receivers given the very large delay spread of the UWB channels. In practice, TH-UWB systems are associated with receivers having a limited number of Rake fingers. Consider the case of a L-th order partial Rake (PRake) receiver that combines the L first arriving multi-path components [6]. Designate by Δ_l the delay of the l-th finger for $l = 0, \ldots, L-1$. At the receiver side, QLM decision variables are collected during each symbol duration. These decision variables are given by:

$$\begin{aligned} y_{q,l,m} &= \int_0^{N_f T_f} r_q(t) \widetilde{w}_{l,m} \mathrm{d}t \\ &= N_f \sum_{p'=1}^{P} \sum_{m'=0}^{M-1} a_{p',m'} r_{q,p'}((m - m')\delta + \Delta_l) + n_{q,l,m} \end{aligned} \tag{5}$$

where the reference signal is given by:

$$\widetilde{w}_{l,m} = \sum_{n=0}^{N_f-1} w(t - nT_f - \Delta_l - m\delta) \tag{6}$$

Equation (5) follows from the condition of no IFI and no ISI and $r_{q,p}(\tau) = \int_0^{T_f} h_{q,p}(t) w(t-\tau) \mathrm{d}t$. The correlation between the zero-mean noise terms is given by:

$$\begin{aligned} E\left[n_{q,l,m} n_{q',l',m'}\right] = &\frac{PN_f N_0}{2} \\ &\gamma_w\left(\Delta_l - \Delta_{l'} + (m - m')\delta\right) \delta(q - q') \end{aligned} \tag{7}$$

where $\gamma_w(\tau) = \int_0^{T_w} w(t) w(t-\tau) \mathrm{d}t$ and $\delta(.)$ is the Dirac delta function. From eq. (7), it follows that choosing $\Delta_l = lMT_w$ for $l = 0, \ldots, L-1$ results in white Gaussian noise terms since $\gamma_w(kT_w) = \delta(k)$. In what follows, the multiplying factor N_f will be removed from eq. (5) and eq. (7).

For a $P \times T$ space-time code ($T = P = 2$ in what follows), eq. (5) can be written in matrix form as:

$$Y = RA + N \tag{8}$$

where Y and N are $QLM \times T$ matrices corresponding to the decision variables and the noise terms respectively. A is the $PM \times T$ codeword whose $((p-1)M + m, t)$-th entry corresponds to the amplitude of the pulse (if any) transmitted at the m-th position of the p-th antenna during the t-th symbol duration for $p = 1, \ldots, P$, $m = 1, \ldots, M$ and $t = 1, \ldots, T$. R is the $QLM \times PM$ channel matrix. $R = [R_{q,p}]$ for $q = 1, \ldots, Q$ and $p = 1, \ldots, P$ where $R_{q,p} = [R_{q,p,l}]$ is an $LM \times M$ matrix. $R_{q,p,l}$ is an $M \times M$ matrix for $l = 0, \ldots, L-1$. The (m, m')-th element of $R_{q,p,l}$ corresponds to the impact of the signal transmitted during the m'-th position of the p-th antenna on the m-th correlator placed after the l-th Rake finger of the q-th receive antenna:

$$R_{q,p,l}(m, m') = r_{q,p}((m - m')\delta + \Delta_l) \tag{9}$$

III. CODE CONSTRUCTION

We propose the following $2M \times 2$ ST code for M-PPM-M'-PAM constellations ($M = 2$):

$$C = \begin{bmatrix} \sqrt{\alpha} I & 0 \\ 0 & \sqrt{\alpha_1} I \end{bmatrix} \begin{bmatrix} a_1 + \theta a_2 & a_3 + \theta a_4 \\ \Omega(a_3 + \theta_1 a_4) & a_1 + \theta_1 a_2 \end{bmatrix} \tag{10}$$

where $a_1, \ldots, a_4 \in \mathcal{C}$ given in eq. (2) are the 2-dimensional vector representations of the information symbols. $\theta = \frac{1+\sqrt{5}}{2}$ is the golden number and $\theta_1 = \sigma(\theta) = \frac{1-\sqrt{5}}{2}$ is the conjugate of θ. $\alpha = \frac{1}{1+\theta^2}$, $\alpha_1 = \sigma(\alpha)$ and I is the 2×2 identity matrix. While in eq. (1) full diversity is assured by an appropriate choice of the scalar γ, the scheme proposed in eq. (10) is based on a 2×2 unitary matrix Ω given by:

$$\Omega = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad (11)$$

The proposed code verifies a large number of interesting properties given by the following propositions.

Proposition 1: The proposed code achieves a full transmit diversity order with 2-PPM-M'-PAM constellations $\forall M' \geq 1$.

Proof: For this proof, we ignore the diagonal matrix in eq. (10) since it is common to all the codewords. Denote by ΔC the difference between two ST codewords given by:

$$\Delta C(x_1, x_2) = \begin{bmatrix} x_{1,1} & x_{1,2} & -\sigma(x_{2,2}) & \sigma(x_{2,1}) \\ x_{2,1} & x_{2,2} & \sigma(x_{1,1}) & \sigma(x_{1,2}) \end{bmatrix}^T \qquad (12)$$

where $x_{i,j}$ is the j-th component of the vector x_i for $i, j \in \{1, 2\}$. The vectors x_1 and x_2 belong to the set $\mathcal{A}$ given by:

$$\mathcal{A} = \{a - a' + (b - b')\theta \ ; \ a, a', b, b' \in \mathcal{C}\} \subset \mathcal{O}_{\mathbb{K}}^2 \qquad (13)$$

where $\mathcal{O}_{\mathbb{K}} = \mathbb{Z}(\theta)$ is the ring of integers of the quadratic field extension $\mathbb{K} = \mathbb{Q}(\sqrt{5})$. Based on the rank criterion [7], the proposed code achieves full transmit diversity if $\Delta C(x_1, x_2)$ has full rank for all values of $(x_1, x_2) \in \mathcal{A}^2 \backslash \{(\mathbf{0}, \mathbf{0})\}$ where $\mathbf{0}$ is the 2-dimensional vector having zero components.

We first consider the case where at least one component of x_1 or (and) x_2 is equal to zero. Without loss of generality, we suppose that $x_{1,1} = 0$. Considering the first row of the matrix given in eq. (12), if $\Delta C(x_1, x_2)$ is rank-deficient, $x_{1,1} = 0$ implies that $x_{2,1} = 0$. This implies that $\sigma(x_{2,1}) = 0$ since $\{1, \theta\}$ defines an integral basis over $\mathcal{O}_{\mathbb{K}}$. Considering the last row of $\Delta C(x_1, x_2)$ in eq. (12) this implies that $\sigma(x_{1,2}) = 0$ and consequently $x_{1,2} = 0$. Considering the second row of $\Delta C(x_1, x_2)$ this implies that $x_{2,2} = 0$. Therefore, if one of the vectors x_1 and x_2 has at least one zero component, then the only rank deficient matrix $\Delta C(x_1, x_2)$ is the all-zero matrix ($x_1 = x_2 = \mathbf{0}$).

Suppose that all the components of x_1 and x_2 are different from zero. $\Delta C(x_1, x_2)$ is rank-deficient if:

$$\frac{x_{2,1}}{x_{1,1}} = \frac{x_{2,2}}{x_{1,2}} = -\frac{\sigma(x_{1,1})}{\sigma(x_{2,2})} = \frac{\sigma(x_{1,2})}{\sigma(x_{2,1})} = k \qquad (14)$$

where $k \in \mathbb{K}$ since all the components of $\Delta C(x_1, x_2)$ belong to $\mathbb{K}$. Equation (14) implies that:

$$\begin{aligned} x_{2,1} &= k x_{1,1} = -k\sigma(k) x_{2,2} \\ &= -k^2 \sigma(k) x_{1,2} = -(k\sigma(k))^2 x_{2,1} \\ &= -(\mathrm{N}_{\mathbb{K}/\mathbb{Q}}(k))^2 x_{2,1} \end{aligned}$$

implying that:

$$\left(1 + (\mathrm{N}_{\mathbb{K}/\mathbb{Q}}(k))^2\right) x_{2,1} = 0 \qquad (15)$$

where $\mathrm{N}_{\mathbb{K}/\mathbb{Q}}(k)$ is the algebraic norm of k. Since $\mathrm{N}_{\mathbb{K}/\mathbb{Q}}(k) \in \mathbb{Q}$, then eq. (15) implies that $x_{2,1} = 0$. Since we previously showed that the nonzero matrix $\Delta C(x_1, x_2)$ has full rank whenever one (or more) of the components of x_1 and x_2 is equal to zero, this implies that $\Delta C(x_1, x_2)$ has full rank for all values of $(x_1, x_2) \in \mathcal{A}^2 \backslash \{(\mathbf{0}, \mathbf{0})\}$ and the proposed scheme permits to achieve a transmit diversity order of 2 with $P = 2$ transmit antennas.

Unlike the codes from cyclic division algebras that are constructed from an algebraic non-norm integer, $\det\left((\Delta C)^T \Delta C\right)$ is not a rational integer and therefore the proposed code does not have a non-vanishing determinant. For the special case of binary PPM constellations, the following proposition holds:

Proposition 2: The proposed code has the same coding gain as the golden code with 2-PPM constellations.

Proof: For binary PPM constellations, $\mathcal{C} = \{[1\ 0]^T, [0\ 1]^T\}$ and the set $\mathcal{A}$ in eq. (13) consists of the columns of the following matrix multiplied by ± 1:

$$\begin{bmatrix} 0 & 1 & \theta & 1+\theta & 1-\theta \\ 0 & -1 & -\theta & -1-\theta & -1+\theta \end{bmatrix} \qquad (16)$$

Therefore, the components of vectors x_1 and x_2 are related to each other by: $x_{i,2} = -x_{i,1}$ for $i = 1, 2$. Let $a = x_{1,1}$ and $b = x_{2,1}$, introducing the diagonal matrix from eq. (10), eq. (12) can be written as:

$$\Delta C(a, b) = \begin{bmatrix} \sqrt{\alpha} a & -\sqrt{\alpha} a & \sigma(\sqrt{\alpha} b) & \sigma(\sqrt{\alpha} b) \\ \sqrt{\alpha} b & -\sqrt{\alpha} b & \sigma(\sqrt{\alpha} a) & -\sigma(\sqrt{\alpha} a) \end{bmatrix}^T \qquad (17)$$

Designate by $\Delta C_{i,j}$ the 2×2 matrix composed from the i-th and j-th rows of ΔC. The following relation holds:

$$\begin{aligned} \det\left((\Delta C)^T \Delta C\right) &= \sum_{i=1}^{4} \sum_{j=i+1}^{4} \left(\det\left(\Delta C_{i,j}\right)\right)^2 \\ &\geq \sum_{i=1}^{2} \sum_{j=3}^{4} \left(\det\left(\Delta C_{i,j}\right)\right)^2 \\ &= 4\mathrm{N}_{\mathbb{K}/\mathbb{Q}}(\alpha) \left((\mathrm{N}_{\mathbb{K}/\mathbb{Q}}(a))^2 + (\mathrm{N}_{\mathbb{K}/\mathbb{Q}}(b))^2\right) \\ &= \frac{4}{5} \left((\mathrm{N}_{\mathbb{K}/\mathbb{Q}}(a))^2 + (\mathrm{N}_{\mathbb{K}/\mathbb{Q}}(b))^2\right) \geq \frac{4}{5} \end{aligned} \qquad (18)$$

where the last inequality holds since $\mathrm{N}_{\mathbb{K}/\mathbb{Q}}(a)$ and $\mathrm{N}_{\mathbb{K}/\mathbb{Q}}(b)$ are rational integers and since a and b can not be equal to zero simultaneously (for nonzero matrices $\Delta C(a, b)$). The last two equalities follow from direct calculations. This implies that the coding gain of the proposed code with binary PPM constellations is given by:

$$\delta_{min} = \min_{(a,b) \in \mathcal{C}'^2 \backslash \{(0,0)\}} \left| \det\left((\Delta C(a, b))^T \Delta C(a, b)\right) \right|^{\frac{1}{2}} = \frac{2}{\sqrt{5}} \qquad (19)$$

where, from eq. (16), $\mathcal{C}' = \{a + b\theta \ ; \ a, b = 0, \pm 1\}$.

The golden code has a coding gain of $\frac{4}{\sqrt{5}}$ with QAM and PAM constellations [1] and a coding gain of $\frac{2}{\sqrt{5}}$ with M-PPM-M'-PAM constellations for $M \geq 2$ [8]. From eq. (19), this implies that, for 2-PPM constellations, the proposed real-valued code has the same coding gain as the golden code.

For 2-PPM-M'-PAM with $M' > 1$, there is no closed form expression of the coding gain. By numerical evaluation, we find that the optimal coding gain of $\frac{2}{\sqrt{5}}$ can be achieved for $M' \in \{2, 4, 8\}$. Note that high order PAM constellations have

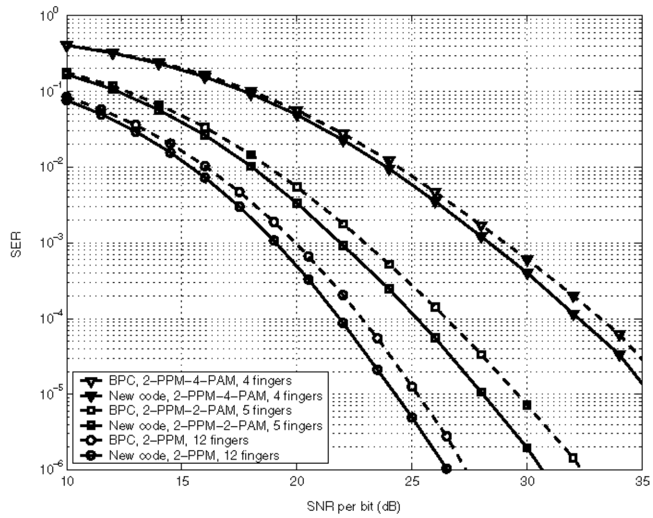

Fig. 1. The proposed code vs. the best previously known totally-real code (BPC) [3] with 1 receive antenna.

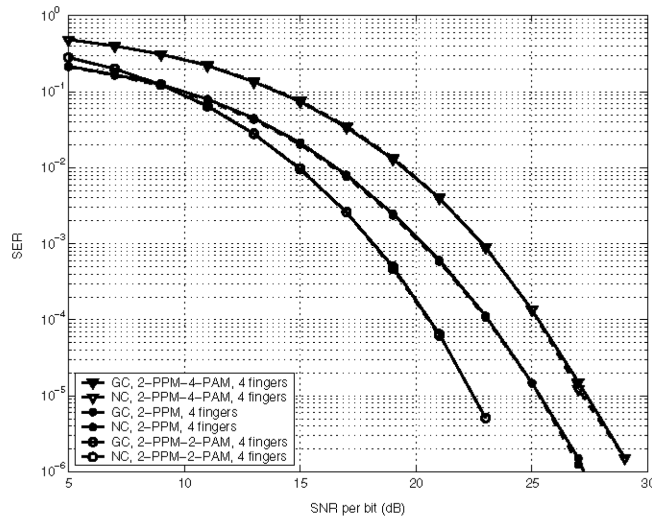

Fig. 2. The new code (NC) vs. the golden code (GC) [1] with 2 receive antennas.

very poor performance and TH-UWB systems are generally associated with PPM ($M' = 1$) or bi-orthogonal PPM ($M' = 2$). This shows the interest of the proposed code with these practical constellations.

Proposition 3: The proposed code is information lossless.

Proof: Concatenating the columns of the decision matrix Y in eq. (8) vertically one after the other, this equation can be written in an equivalent manner as:

$$\text{vec}(Y) = (I_2 \otimes R)\Phi[a_1^T\ a_2^T\ a_3^T\ a_4^T]^T + \text{vec}(N) \qquad (20)$$

where, from eq. (10), $\text{vec}(C) = \Phi[a_1^T\ a_2^T\ a_3^T\ a_4^T]^T$ and:

$$\Phi = \begin{pmatrix} \sqrt{\alpha}I & \sqrt{\alpha}\theta I & 0 & 0 \\ 0 & 0 & \sqrt{\alpha_1}\Omega & \sqrt{\alpha_1}\theta_1\Omega \\ 0 & 0 & \sqrt{\alpha}I & \sqrt{\alpha}\theta I \\ \sqrt{\alpha_1}I & \sqrt{\alpha_1}\theta_1 I & 0 & 0 \end{pmatrix} \qquad (21)$$

where, in this case, $\mathbf{0}$ is the 2×2 all-zero matrix. We can prove that Φ is unitary which follows from the fact that the basis $\sqrt{\alpha}\{1, \theta\}$ is orthonormal and $\Omega^T\Omega = I$. The fact that Φ is unitary is sufficient for C to be information lossless [9]. In an equivalent way, C verifies the shaping constraint (the matrix Ω only permutes the pulses transmitted during two consecutive modulation positions and inverses the polarity of the second pulse). Finally, since Ω is unitary, the proposed code has a uniform average transmitted energy per antenna and per position.

IV. Simulations and Results

The PQ channels between the different antennas are generated independently according to the channel model recommendation CM2 [10] which corresponds to non-line-of-sight (NLOS) conditions. The pulse waveform $w(t)$ is chosen be the second derivative of the Gaussian function with a duration of 0.5 ns and we fix $\delta = T_w$. The frame duration T_f is chosen to be $T_f = 100$ ns which is larger than the maximum delay spread of CM2 [10]. N_f has no effect on the performance in single user situations and it is fixed to 1.

Fig. 1 compares the proposed code with the best previously known totally-real code (inter symbol coding scheme in [3]). One receive antenna is used with L-th order PRakes and 2-PPM-M'-PAM constellations for different values of L and M'. Performance gains in the order of 1 dB are obtained at a symbol error rate of 10^{-3} with different constellation sizes.

In Fig. 2, the proposed code is compared with [1]. In this figure, for comparison reasons and even though it may seem practically infeasible, the UWB receivers are supposed to be equipped with IQ front ends. Results show the high performance levels achieved by the proposed scheme. Even though it is real-valued, it shows exactly the same performance as the best known 2×2 code.

V. Conclusion

We investigated the problem of constructing ST coding schemes suitable for UWB systems using 2-PPM-M'-PAM. The proposed construction solves the problem of the non-existence of energy-balanced, information lossless and totally-real constructions. The proposed solution outperforms the best known totally-real ST code based on cyclic division algebras.

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