Novel Cooperation Strategies for Free-Space Optical Communication Systems in the Absence and Presence of Feedback

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Abstract. In this paper, we investigate cooperative diversity as a fading mitigation technique for Free-Space Optical (FSO) communications with intensity modulation and direct detection (IM/DD). In particular, we propose two novel one-relay cooperation strategies. The first scheme does not require any feedback and is based on selective relaying where the relay forwards only the symbols that it detected with a certain level of certainty that we quantify in both cases of absence or presence of background radiation. This technique results in additional performance enhancements and energy savings compared to the existing FSO cooperative techniques. The second scheme can be applied in situations where a feedback link is available. This scheme that requires only one bit of feedback results in significant performance gains over the entire range of the received signal level.

Keywords: Free-Space Optics (FSO), Cooperative Systems, Rayleigh Fading, Pulse Position Modulation (PPM), Diversity.

1 Introduction

Recently, Free-Space Optical (FSO) communications attracted significant attention as a promising solution for the "last mile" problem [1]. A major impairment that severely degrades the link performance is fading (or scintillation) that results from the variations of the index of refraction due to inhomogeneities in temperature and pressure changes [2]. In order to combat fading and maintain acceptable performance levels over FSO links, fading-mitigation techniques that were extensively investigated in the context of wireless radio-frequency (RF) communications were recently tailored to FSO systems. These techniques can be classified in two broad categories. (i): Localized diversity techniques where multiple apertures are deployed at the transmitter and/or receiver sides. In the wide literature of RF systems, this is referred to as the Multiple-Input-Multiple-Output (MIMO) techniques that can result in significant multiplexing and diversity gains over wireless links. (ii): Distributed diversity techniques where neighboring nodes in a wireless network cooperate with each other to form a "virtual" antenna array and profit from the underlying spatial diversity in a distributed manner. These cooperative

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techniques are becoming more popular in situations where limited number of apertures can be deployed at the transceivers [3, 4].

In the context of FSO communications, localized diversity techniques include aperture-averaging receiver diversity [5], spatial repetition codes [6], unipolar versions of the orthogonal space-time codes [7] and transmit laser selection [8]. However, these techniques suffer mainly from the channel correlation that is particulary pronounce in FSO systems. In fact, for RF systems, the wide beamwidth of the antennas and the rich scattering environment that is often present between the transmitter and the receiver both ensure that the signal reaches the receiver via a large number of independent paths. Consequently, the assumption of spatially uncorrelated channels is often valid for these systems. On the other hand, for FSO links, the laser's beamwidth is very narrow and these links are much more directive thus rendering the assumption of uncorrelated channels practically not valid for these systems. For example, the presence of a small cloud might induce large fades on all source-detector sub-channels simultaneously [6]. Consequently, the high performance gains promised by MIMO-FSO systems might not be achieved in practice and "*alternative means of operation in such environments must be considered*" [6].

On the other hand, while the literature on cooperation in RF networks dates back to about a decade [3,4], it was only recently that this diversity technique was considered in the context of FSO communications [9]. In [9], a simple decode-and-forward strategy was proposed in scenarios where one neighboring node (relay) is willing to cooperate with the source node. Based on the strategy proposed in [9], the relay simply decodes and retransmits all symbols it receivers independently from the quality of signal reception at this relay. This strategy was analyzed in the absence and presence of background radiation in situations where the channel state information (CSI) is not available neither at the transmitter nor at the receiver sides. [9] highlighted the utility of cooperation despite the non-broadcast nature of FSO communications where the message transmitted from the source to the destination can not be overheard by the neighboring relay.

In this work, we present an enhancement to the decode-and-forward strategy proposed in [9]. This enhancement will be referred to as selective relaying. Instead of decoding all symbols at the relay and automatically forwarding these symbols to the destination, the relay will now back off if the fidelity, with which the symbol is recovered, is judged to be low. In this context, we propose two novel rules based on which the relay forwards the corresponding symbol or not. One of these rules is suitable for the no-background radiation case while the other rule is applied in the presence of background radiation. In the absence of background radiation, we will prove that selective relaying results in exactly the same performance as the decode-and-forward scheme proposed in [9] while in the presence of background radiation selective relaying can result in significant performance gains compared to [9]. However, in both cases, the proposed relaying scheme results in significant energy savings since the relay is stopping its transmission (and thus saving energy) when it judges that a certain symbol will most probably be decoded erroneously whether it cooperates or not. Another contribution of the paper is that we propose a novel cooperation strategy that can be applied when the CSI is available at the receiver and when one feedback bit is available between the receiver and the transmitter. Under this scenario, significant error rate enhancements can be observed whether in the absence or presence of background radiation.

2 System Model

Consider the example of a FSO Metropolitan Area Network as shown in Fig. 1. Consider three neighboring buildings (1), (2) and (3) and assume that a FSO connection is available between each building and its two neighboring buildings. In FSO networks, each one of these connections is established via FSO-based wireless units each consisting of an optical transceiver with a transmitter and a receiver to provide full-duplex capability. Given the high directivity and non-broadcast nature of FSO transmissions, one separate transceiver is entirely dedicated for the communication with each neighboring building. We assume that the transceivers on building (2) are available for cooperation to enhance the communication reliability between buildings (1) and (3). By abuse of notations, buildings (1), (2) and (3) will be denoted by source S, relay R and destination D, respectively. It is worth noting that the transceivers at R are not deployed with the objective of assisting S. In fact, these transceivers are deployed for R to communicate with S and D; if R is willing to share its existing resources (and R has no information to transmit), then it can act as a relay for assisting S in its communication with D.

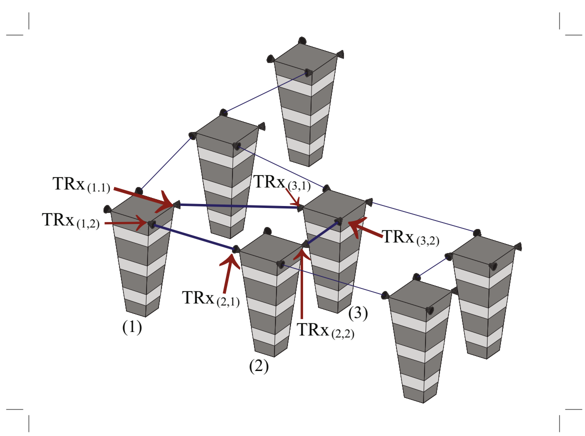

Fig. 1. An example of a mesh FSO network. Cooperation is proposed among the transceivers on buildings (1), (2) and (3) where the transceivers on building 2 can help in transmitting an information message from (1) to (3). Note how, given the non-broadcast nature of FSO transmissions, one couple of FSO transceiver units is dedicated for each link.

Denote by a_0, a_1 and a_2 the random path gains between S-D, S-R and R-D, respectively. In this work, we adopt the Rayleigh turbulence-induced fading channel model [6] where the probability density function (pdf) of the path gain ($a > 0$) is given by: $f_A(a) = 2ae^{-a^2}$. In the same way, denote by P_0, P_1 and P_2 the fractions of the total power dedicated to the S-D, S-R and R-D links, respectively. These parameters must satisfy the following relation: $P_0+P_1+P_2 = 1$ so that the cooperative system transmits the same total energy as non-cooperative systems.

We consider Q-ary pulse position modulation (PPM) with intensity modulation and direct detection (IM/DD) where each receiver corresponds to a simple photoelectrons counter. Denote by λ_s and λ_n the average number of photoelectrons per slot resulting from the incident light signal and background radiation (and/or dark currents), respectively. These parameters are given by [6]:

$$\lambda_s = \eta \frac{P_r T_s/Q}{hf} = \eta \frac{E_s}{hf} \quad ; \quad \lambda_n = \eta \frac{P_b T_s/Q}{hf} \tag{1}$$

where:

- η is the detector's quantum efficiency assumed to be equal to 1 in what follows.
- $h = 6.6 \times 10^{-34}$ is Planck's constant.
- f is the optical center frequency taken to be 1.94×10^{14} Hz (corresponding to a wavelength of 1550 nm).
- T_s is the symbol duration.
- P_r (resp. P_b) stands for the optical signal (resp. noise) power that is incident on the receiver.

Finally, in eq. (1): $E_s = P_r T_s/Q$ corresponds to the received optical energy per symbol corresponding to the direct link S-D.

As a first step in the cooperation strategy, a PPM symbol $s \in \{1, \dots, Q\}$ is transmitted along both the S-D and S-R links. Denote by $Z^{(0)} = [Z_1^{(0)}, \dots, Z_Q^{(0)}]$ and $Z^{(1)} = [Z_1^{(1)}, \dots, Z_Q^{(1)}]$ the Q-dimensional vectors corresponding to the photoelectron counts (in the Q slots) at D and R, respectively. In other words, $Z_q^{(i)}$ corresponds to the number of photoelectrons detected in the q-th slot along the S-D link for $i = 0$ and along the S-R link for $i = 1$ for $q = 1, \dots, Q$.

For $q \neq s$, no light signal is transmitted in the q-th slot and the only source of photoelectrons in this slot is background radiation. In this case, $Z_q^{(i)}$ can be modeled as a Poisson random variable (r.v.) with parameter [6]:

$$\mathrm{E}[Z_q^{(i)}] = \lambda_n \quad ; \quad q \neq s \quad ; \quad i = 0, 1 \tag{2}$$

where E[.] stands for the averaging operator.

For $q = s$, $Z_q^{(i)}$ can be modeled as a Poisson random variable with parameter:

$$\mathrm{E}[Z_s^{(i)}] = \beta_i P_i a_i^2 \lambda_s + \lambda_n \quad ; \quad i = 0, 1 \tag{3}$$

where $\beta_0 \triangleq 1$ and β_1 is a gain factor that follows from the fact that S might be closer to R than it is to D. In other words, the received optical energy at R corresponding to the energy $\beta_0 E_s = E_s$ at D is $\beta_1 E_s$. Performing a typical link budget analysis [6] shows that $\beta_1 = \left(\frac{d_{SD}}{d_{SR}}\right)^2$ where d_{SD} and d_{SR} stand for the distances from S to D and S to R, respectively.

Based on the decision vector $Z^{(1)}$ available at R, the maximum-likelihood (ML) detector corresponds to deciding in favor of the symbol $\hat{s}$ given by:

$$\hat{s} = \arg \max_{q=1\cdots Q} Z_q^{(1)} \tag{4}$$

Now, the symbol $\hat{s}$ (rather than the symbol s) is transmitted along the R-D link. In this case, the corresponding decision vector can be written as $Z^{(2)} = [Z_1^{(2)}, \ldots, Z_Q^{(2)}]$ where $Z_q^{(2)}$ is a Poisson r.v. whose parameter is given by:

$$\mathrm{E}[Z_q^{(2)}] = \begin{cases} \beta_2 P_2 a_2^2 \lambda_s + \lambda_n, & q = \hat{s}; \\ \lambda_n, & q \neq \hat{s}. \end{cases} \tag{5}$$

where $\beta_2 = \left(\frac{d_{SD}}{d_{RD}}\right)^2$ with d_{RD} corresponding to the distance between R and D.

3 Cooperation in the Absence of Background Radiation

In this section, we assume that $\lambda_n = 0$.

3.1 Cooperation Strategies in the Absence of Feedback

Next, we propose two cooperation strategies that will be referred to as scheme 1 and scheme 2, respectively. For both schemes, a sequence of symbols is first transmitted simultaneously to D and R. Given that $\lambda_n = 0$, therefore if symbol s was transmitted, then the $Q-1$ decision variables $Z_q^{(0)}$ for $q \neq s$ (at D) and the $Q-1$ decision variables $Z_q^{(1)}$ for $q \neq s$ (at R) will all be equal to zero. Consequently, two scenarios are possible at R. (i): $Z_s^{(1)} > 0$; in this case, R decides in favor of $\hat{s} = s$ and the decision it makes is correct. In fact, in the absence of background radiation the only source of this nonzero count at slot s is the presence of a light signal in this slot. (ii): $Z_s^{(1)} = 0$ implying that zero photoelectron counts are observed in all Q slots. In this case, the best that the relay can do is to break the tie randomly and decide randomly in favor of one of the slots $\hat{s}$ resulting in a correct guess with probability $1/Q$.

For scheme 1, the relay forwards the decoded symbol $\hat{s}$ automatically to D independently from whether $Z_s^{(1)} > 0$ or $Z_s^{(1)} = 0$. For scheme 2, the relay forwards the decoded symbol $\hat{s}$ only if $Z_s^{(1)} > 0$. In fact, if all counts are equal to zero ($Z_s^{(1)} = 0$), then most probably the relay will make an erroneous decision (with probability $\frac{Q-1}{Q}$). In order to avoid confusing D by forwarding a wrong estimate of the symbol, the relay backs off and stops its retransmission during the corresponding symbol duration.

For both schemes, the decision at D will be based on vectors $Z^{(0)}$ and $Z^{(2)}$. If one slot of $Z^{(0)}$ has a nonzero photoelectron count, then D decides in favor of this slot (and its decision will be correct). On the other hand, if all components of $Z^{(0)}$ are equal to zero, then the decision at D will be based on $Z^{(2)}$. If one component of $Z^{(2)}$ is different from zero, then D decides in favor of this component. For scheme 1, this decision is correct with probability $1 - p_e$ where p_e stands for the probability of error at R; for scheme 2, this decision will be correct with probability 1 since R forwards the message if and only if it decoded the transmitted symbol correctly. Finally, if all components of $Z^{(2)}$ are equal to zero, then D decides randomly in favor of one of the Q slots. Note that for scheme 1, this case occurs only because of fading and shot noise along the R-D link while for scheme 2 this case might occur because the relay backed off as well (in addition to fading and shot noise along the R-D link).

To summarize, for both schemes, D decides in favor of symbol $\tilde{s}$ according to the following rule:

$$\tilde{s} = \begin{cases} \arg_{q=1,\dots,Q}[Z_q^{(0)} \neq 0], & Z^{(0)} \neq \mathbf{0}_Q; \\ \arg_{q=1,\dots,Q}[Z_q^{(2)} \neq 0], & Z^{(0)} = \mathbf{0}_Q \ , \ Z^{(2)} \neq \mathbf{0}_Q; \\ \text{rand}(1,\dots,Q), & Z^{(0)} = Z^{(2)} = \mathbf{0}_Q. \end{cases} \tag{6}$$

where $\mathbf{0}_Q$ corresponds to the Q-dimensional all-zero vector while the function rand$(1,\dots,Q)$ corresponds to choosing randomly one integer in the set $\{1,\dots,Q\}$.

3.2 Performance Analysis

Proposition: Schemes 1 and 2 achieve the same conditional symbol error probability (SEP) given by:

$$P_{e|A} = \frac{Q-1}{Q} e^{-P_0 a_0^2 \lambda_s} \left[e^{-\beta_1 P_1 a_1^2 \lambda_s} + e^{-\beta_2 P_2 a_2^2 \lambda_s} - e^{-\beta_1 P_1 a_1^2 \lambda_s} e^{-\beta_2 P_2 a_2^2 \lambda_s} \right] \tag{7}$$

where the channel state is defined by the vector $A \triangleq [a_0, a_1, a_2]$.

Proof: the proof is provided in the appendix.

Averaging $P_{e|A}$ over the Rayleigh distributions of a_0 ,a_1 and a_2 results in the following expression of the SEP:

$$P_e = \frac{Q-1}{Q} \frac{1}{1+P_0\lambda_s} \left[\frac{1}{1+\beta_1 P_1 \lambda_s} + \frac{1}{1+\beta_2 P_2 \lambda_s} - \frac{1}{(1+\beta_1 P_1 \lambda_s)(1+\beta_2 P_2 \lambda_s)} \right] \tag{8}$$

Note that for non-cooperative systems the SEP is given by: $P_e = \frac{Q-1}{Q}\frac{1}{1+\lambda_s}$ which scales asymptotically as λ_s^{-1} (for $\lambda_s \gg 1$) . On the other hand, eq. (8) scales asymptotically as λ_s^{-2} showing the enhanced diversity order (of two) achieved by the proposed schemes.

Despite the fact that scheme 1 and scheme 2 result in exactly the same error performance, scheme 2 presents the additional advantage of reducing the overall transmitted energy since the relay backs off when the decision vector $Z^{(1)}$ it observes does not ensure a correct decision. In other words, the relay stops its transmission when $Z^{(1)} = \mathbf{0}_Q$ thus saving energy. This saved energy can be further invested in the transmission of other symbols (for which $Z^{(1)} \neq \mathbf{0}_Q$) along the R-D link (refer to section 5 for more details).

Finally, eq. (6) shows that the proposed cooperation strategies can be implemented without requiring any channel state information (CSI) at the receiver side (non-coherent detection).

3.3 Cooperation in the Presence of Feedback

In the case where there is no feedback from the receiver to the transmitter, the CSI is not available at the transmitter side and no preference can be made among the available links. In this case, the best choice is to distribute the transmit power evenly among the three links S-D, S-R and R-D by setting $P_0 = P_1 = P_2 = 1/3$. This will be referred to as the no-feedback case in what follows.

When one feedback bit is available, the transmitter can choose to transmit the entire optical power either along the direct S-D link ($P_0 = 1$ and $P_1 = P_2 = 0$) or along the indirect S-R-D link ($P_0 = 0$, $P_1 \neq 0$ and $P_2 \neq 0$). This transmission strategy in the presence of feedback is motivated by the fact that the conditional SEP in eq. (7) is equal to the product of two functions; one of them depending only on the direct S-D link (via the channel coefficient a_0) and the other one depending only on the indirect S-R-D link (via the channel coefficients a_1 and a_2). In what follows, we fix $P_1 = P_2 = 1/2$ if the information is to be transmitted along the indirect link. Note that the values of P_1 and P_2 along the indirect link can be further optimized to minimize $P_{e|A}$; however, we observed that this optimization results only in a marginal decrease in the SEP. Moreover, this approach requires much more than one bit of feedback to inform S and R about the fractions of the power that they need to allocate to the S-R and R-D links.

If the direct link S-D is chosen (with $P_0 = 1$ and $P_1 = P_2 = 0$), the resulting conditional SEP will be given by:

$$P_{e|A}^{(\mathrm{d})} = \frac{Q-1}{Q} e^{-a_0^2 \lambda_s} \tag{9}$$

If the indirect link S-R-R is chosen (with $P_0 = 0$ and $P_1 = P_2 = 1/2$), the resulting conditional SEP will be given by:

$$P_{e|A}^{(\mathrm{in})} = \frac{Q-1}{Q} \left[e^{-\beta_1 a_1^2 \lambda_s/2} + e^{-\beta_2 a_2^2 \lambda_s/2} - e^{-\beta_1 a_1^2 \lambda_s/2} e^{-\beta_2 a_2^2 \lambda_s/2} \right] \tag{10}$$

Given that the third term in the above expression is two orders of magnitude smaller than the first two terms, then $P_{e|A}^{(\mathrm{in})}$ can be approximated by:

$$P_{e|A}^{(\mathrm{in})} \approx \frac{Q-1}{Q} \left[e^{-\beta_1 a_1^2 \lambda_s/2} + e^{-\beta_2 a_2^2 \lambda_s/2} \right] \approx \frac{Q-1}{Q} 2 e^{-\min(\beta_1 a_1^2, \beta_2 a_2^2) \lambda_s/2} \tag{11}$$

where the second approximation follows from the strong monotonic behavior of the exponential function.

Finally, based on eq. (9) and eq. (11), the proposed cooperation strategy in the presence of 1 feedback bit corresponds to choosing (P_0, P_1, P_2) according to:

$$(P_0, P_1, P_2) = \begin{cases} (1, 0, 0), & a_0^2 \geq \frac{1}{2}\min(\beta_1 a_1^2, \beta_2 a_2^2) - \frac{\ln 2}{\lambda_s}; \\ (0, 1/2, 1/2), & \text{otherwise.} \end{cases} \tag{12}$$

Note that the inequality $a_0^2 \geq \frac{1}{2}\min(\beta_1 a_1^2, \beta_2 a_2^2) - \frac{\ln 2}{\lambda_s}$ is easier to be satisfied for smaller values of λ_s. In this case, the direct link is preferred over the indirect link since the performance of the indirect link will be severely degraded because of errors occurring at the relay for these small values of λ_s.

4 Cooperation in the Presence of Background Radiation

In this section, we assume that $\lambda_n \neq 0$. In this case, the background radiation results in nonzero photoelectron counts even in empty slots.

As in the case of no background radiation, we consider two cooperation schemes: scheme 1 where the relay decodes and forwards all symbols it receives without applying any kind of selection strategy and scheme 2 where the relay backs off over some symbol durations in order not to confuse the destination with noisy replicas of symbols it received. Unlike the case of no background radiation where the relay can be 100% sure that the symbol it detected is correct (when there is one nonempty slot in $Z^{(1)}$), the presence of photoelectrons in empty slots due to background radiation imposes a certain level of uncertainty on the decision made at the relay.

For both schemes, the decision at D will be based on the vector $Z = Z^{(0)} + Z^{(2)} = [Z_1, \dots, Z_Q]$. If the decoded symbol at R ($\hat{s}$) is equal to the transmitted symbol (s), then the parameters of the components of Z that follow the Poisson distribution are as follows:

$$\mathrm{E}[Z_q] = \begin{cases} P_0 a_0^2 \lambda_s + \beta_2 P_2 a_2^2 \lambda_s + 2\lambda_n, & q = s; \\ 2\lambda_n, & q \neq s. \end{cases} \quad ; \quad \hat{s} = s \tag{13}$$

On the other hand, if $\hat{s} \neq s$:

$$\mathrm{E}[Z_q] = \begin{cases} P_0 a_0^2 \lambda_s + 2\lambda_n, & q = s; \\ \beta_2 P_2 a_2^2 \lambda_s + 2\lambda_n, & q = \hat{s}; \\ 2\lambda_n, & q \neq s \ ; \ q \neq \hat{s}. \end{cases} \quad ; \quad \hat{s} \neq s \tag{14}$$

For scheme 1, R is always relaying $\hat{s}$ defined in eq. (4); consequently, equations (13) and (14) correspond to the only two cases that might arise at D. For scheme 2, R might back off resulting in $Z^{(2)} = \mathbf{0}_Q$; consequently, in addition to equations (13) and (14), the parameters of the components of Z might be as follows:

$$\mathrm{E}[Z_q] = \begin{cases} P_0 a_0^2 \lambda_s + 2\lambda_n, & q = s; \\ 2\lambda_n, & q \neq s. \end{cases} \quad ; \quad \text{R is backing off} \tag{15}$$

We next propose a metric based on which R will forward the message or not. We define the probability $p(q)$ as the probability that the symbol was transmitted in slot q along the S-R link. Following from equations (2) and (3), this probability can be written as:

$$p(q) = \frac{e^{-(\beta_1 P_1 a_1^2 \lambda_s + \lambda_n)} (\beta_1 P_1 a_1^2 \lambda_s + \lambda_n)^{Z_q^{(1)}}}{Z_q^{(1)}!} \prod_{q'=1\ ;\ q' \neq q}^{Q} \frac{e^{-\lambda_n} \lambda_n^{Z_{q'}^{(1)}}}{Z_{q'}^{(1)}!} \quad ; \quad q = 1, \ldots, Q \tag{16}$$

The highest probability in the set $\{p(q)\}_{q=1}^{Q}$ is $p(\hat{s})$ since the maximum likelihood decision at R corresponds to selecting $\hat{s} = \arg\max_{q=1,\ldots,Q}[p(q)] \equiv \arg\max_{q=1,\ldots,Q}[Z_q^{(1)}]$. Denote by $\hat{s}'$ the symbol associated with the next highest probability in $\{p(q)\}_{q=1}^{Q}$:

$$\begin{aligned} \hat{s}' &= \arg\max \left[p(1), \ldots, p(\hat{s}-1), p(\hat{s}+1), \ldots, p(Q)\right] \\ &\equiv \arg\max \left[Z_1^{(1)}, \ldots, Z_{\hat{s}-1}^{(1)}, Z_{\hat{s}+1}^{(1)}, \ldots, Z_Q^{(1)}\right] \end{aligned} \tag{17}$$

in other words, the probabilities in the set $\{p(q)\}_{q=1}^{Q}$ satisfy the following relation: $p(q)|_{q \neq \hat{s}, \hat{s}'} \leq p(\hat{s}') \leq p(\hat{s})$.

Based on what preceded, we define the index that quantifies the accuracy of the decision at the relay as the logarithm of the ratio of the most two probable events: $I = \log \frac{p(\hat{s})}{p(\hat{s}')}$. Simplifying the common terms in $p(\hat{s})$ and $p(\hat{s}')$ results in:

$$I = \left(Z_{\hat{s}}^{(1)} - Z_{\hat{s}'}^{(1)}\right) \log \left(1 + \frac{\beta_1 P_1 a_1^2 \lambda_s}{\lambda_n}\right) \tag{18}$$

Now, the relay participates in the cooperation effort if the index I is greater than a certain threshold level I_{th}. In other words, if $I \geq I_{th}$, R forwards $\hat{s}$ and the parameters of the components of the decision vector at D are as given in equations (13) and (14). Note that the relation $I \geq I_{th}$ does not imply that $\hat{s} = s$; however, it imposes some kind of selectivity on the symbols to be forwarded. On the other hand, if $I < I_{th}$, R does not forward $\hat{s}$ and the parameters of the components of the decision vector at D are as given in eq. (15). Finally, as in the case of no background radiation, scheme 2 results in an additional energy saving compared to scheme 1. Similarly, in the absence of feedback, we set: $P_0 = P_1 = P_2 = 1/3$.

Note that the conditional SEP does not lend itself to a simple analytical evaluation in the presence of background radiation. Therefore, we adopt the strategy given in eq. (12), and that was proposed in the no-background radiation case, for selecting one of the direct S-D path or indirect S-R-D path for the background radiation case in the presence of feedback. Even though this approach is not optimal, it is simple and it is expected to result in additional performance gains compared to the no feedback case. These expectations are confirmed in the next section.

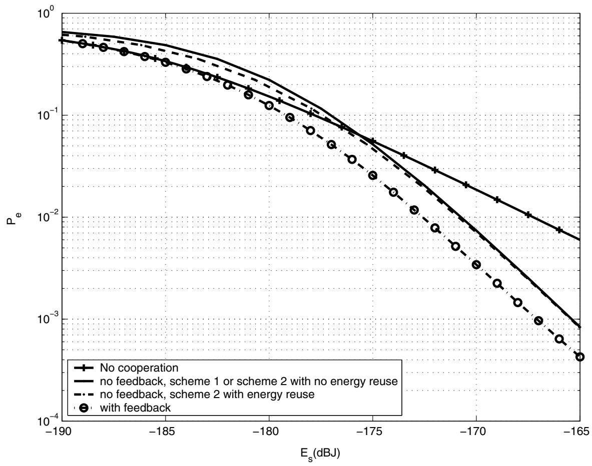

Fig. 2. Performance of 4-PPM in the absence of background radiation. In this figure, we fix: $\beta_1 = \beta_2 = 1$.

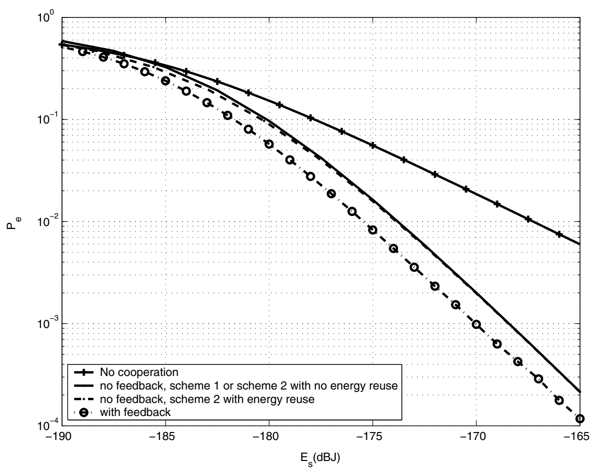

Fig. 3. Performance of 4-PPM in the absence of background radiation. In this figure, we fix: $\beta_1 = \beta_2 = 4$.

5 Numerical Results

Based on section 3, two variants of scheme 2 are possible. (i) Scheme 2 with no energy reuse: in this case the energy saved when the relay backs off is not used to transmit future symbols. This strategy achieves the same performance level as scheme 1 and the additional advantage over the latter can be quantified by a parameter $E_{s,s}$ that captures the amount of energy saved per symbol. (ii) Scheme 2 with energy reuse: in this case the energy saved when the relay backs off is used to transmit future symbols. This energy reuse allows this strategy to achieve a better performance than scheme 1; evidently, $E_{s,s} = 0$ in this case.

Fig. 2 shows the performance of 4-PPM in the absence of background radiation. In this figure, we assume that $d_{SD} = d_{SR} = d_{RD}$ resulting in $\beta_1 = \beta_2 = 1$. The slopes of the SEP curves indicate that cooperation results in an increased diversity order even in this extreme case where S is as far from R as it is from D. This figure also shows the impact of feedback on enhancing the performance. While cooperation with no feedback outperforms non-cooperative systems at values of E_s exceeding -176 dB approximately, cooperation with feedback outperforms non-cooperative systems for practically all values of E_s. This figure also shows that scheme 2 with energy reuse results in additional gains especially for small values of E_s. The energy reuse also decreases the value of E_s starting from which cooperation becomes useful (with no feedback) by about 1 dB. Similar results are obtained in Fig. 3 for $\beta_1 = \beta_2 = 4$.

Fig. 4 plots the variation of $E_{s,s}$ as a function of E_s for scheme 2 with no energy reuse. Two cases are considered: $\beta_1 = \beta_2 = 1$ and $\beta_1 = \beta_2 = 4$. This figure shows that significant energy savings are possible in both cases. Note that for $\beta_1 = 4$, the relay is two times closer to the source (compared to the case $\beta_1 = 1$).

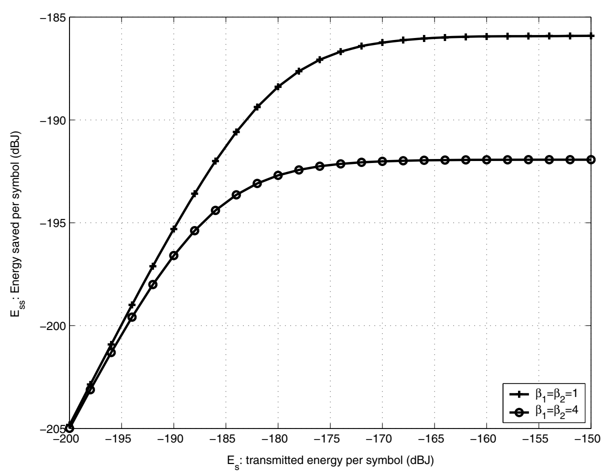

Fig. 4. Energy saved when scheme 2 is applied (with no energy reuse) in the absence of background radiation

In this case, the signal level at the relay is large implying that the relay backs off less often resulting in less energy savings.

Fig. 5 shows the performance of 4-PPM in the presence of background radiation where we fix: $P_b T_s/Q = -185$ dBJ. In this figure, we assume that $d_{SD} = 2d_{SR} = 2d_{RD}$ resulting in $\beta_1 = \beta_2 = 4$. This figure compares the performance of non-cooperative systems with scheme 1 and scheme 2. The results pertaining to scheme 2 are plotted in the case where the energy saved from certain symbols (for which $I < I_{th}$) is not reused for the transmission of subsequent

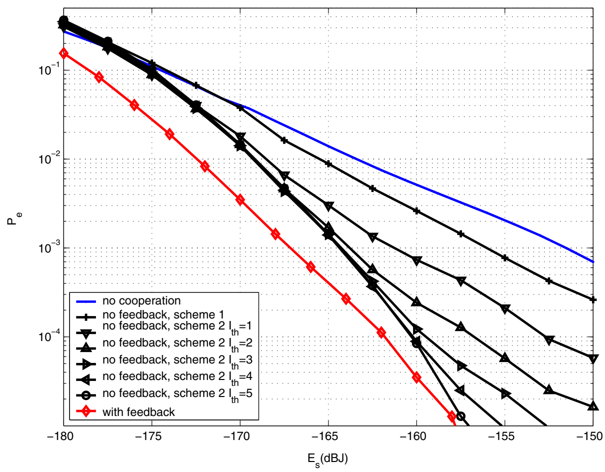

Fig. 5. Performance of 4-PPM in the presence of background radiation with $P_b T_s/Q = -185$ dBJ and $\beta_1 = \beta_2 = 4$. Scheme 2 is applied with no energy reuse

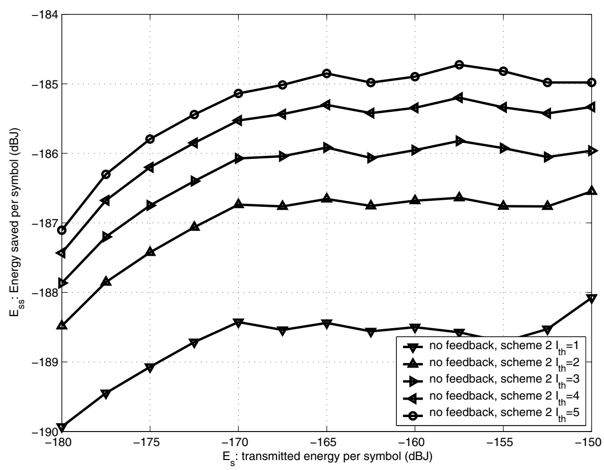

Fig. 6. Energy saved when scheme 2 is applied (with no energy reuse) with $P_b T_s/Q = -185$ dBJ and $\beta_1 = \beta_2 = 4$

symbols (no energy reuse). This figure shows that scheme 2 outperforms scheme 1 and that the performance gains increase with I_{th}. For example, as I_{th} increases from 1 to 2, the performance gain of scheme 2 compared to scheme 1 increases from 5 dB to about 8 dB. Moreover, these performance gains are achieved with additional energy savings that are shown in Fig. 6. Evidently, these savings are more significant for larger values of I_{th} since the relay will be backing off more often in this case.

6 Conclusion

We proposed novel cooperation strategies that are adapted to FSO systems. In the absence of feedback, selecting the cooperation intervals in an adequate manner can result in significant performance gains as well as energy savings that can be reinvested to further boost the error rate. In the presence of 1 feedback bit, a simple strategy based on selecting one link among the available direct or indirect links turned out to be very useful especially for low signal levels.

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Appendix: Performance in the Absence of Background Radiation with No Feedback

Assume that the symbol $s \in \{1, \dots, Q\}$ was transmitted. We recall that the decoded symbol at R is deonoted by $\hat{s}$ while the final decision at D is denoted by $\tilde{s}$.

Scheme 1: If $Z_s^{(0)} > 0$, then a correct decision will be made at D. Consequently, $P_{e|A}$ can be written as:

$$P_{e|A} = \Pr(Z_s^{(0)} = 0) \left[\Pr(Z_{\hat{s}}^{(2)} = 0) p_1 + \Pr(Z_{\hat{s}}^{(2)} > 0) p_2 \right] \tag{19}$$

where $p_1 = \frac{Q-1}{Q}$ since the case $Z_s^{(0)} = Z_{\hat{s}}^{(2)} = 0$ implies that $Z^{(0)} = Z^{(2)} = \mathbf{0}_Q$ resulting in a random decision taken at D. On the other hand, $p_2 = p_e$ (which is the probability of error at R). In fact when $Z_s^{(0)} = 0$ and $Z_{\hat{s}}^{(2)} > 0$, D will decide in favor of $\tilde{s} = \hat{s}$ resulting in an erroneous decision ($\tilde{s} \neq s$) when an erroneous decision is made at the relay ($\hat{s} \neq s$) with probability p_e. Given that p_e can be written as: $p_e = \frac{Q-1}{Q} \Pr(Z_s^{(1)} = 0)$, then eq. (19) reduces to:

$$P_{e|A} = \frac{Q-1}{Q} \Pr(Z_s^{(0)} = 0) \left[\Pr(Z_{\hat{s}}^{(2)} = 0) + \Pr(Z_{\hat{s}}^{(2)} > 0) \Pr(Z_s^{(1)} = 0) \right] \tag{20}$$

From eq. (3): $\Pr(Z_s^{(0)} = 0) = e^{-P_0 a_0^2 \lambda_s}$ and $\Pr(Z_s^{(1)} = 0) = e^{-\beta_1 P_1 a_1^2 \lambda_s}$ and from eq. (5): $\Pr(Z_{\hat{s}}^{(2)} = 0) = 1 - \Pr(Z_{\hat{s}}^{(2)} > 0) = e^{-\beta_2 P_2 a_2^2 \lambda_s}$. Replacing these probabilities in eq. (20) results in eq. (7).

For scheme 2, an error occurs with probability $\frac{Q-1}{Q}$ (tie breaking) only when $Z^{(0)} = Z^{(2)} = \mathbf{0}_Q$. Consequently, $P_{e|A}$ can be written as:

$$\begin{aligned} P_{e|A} &= \frac{Q-1}{Q} \Pr(Z^{(0)} = \mathbf{0}_Q) \Pr(Z^{(2)} = \mathbf{0}_Q) \\ &= \frac{Q-1}{Q} \Pr(Z_s^{(0)} = 0) \left[1 - \Pr(Z^{(2)} \neq \mathbf{0}_Q) \right] \end{aligned} \tag{21}$$

Now $Z^{(2)} \neq \mathbf{0}_Q$ if and only if R is forwarding the message (a nonzero photoelectron count was observed along the S-R link) and a nonzero photoelectron count was observed along the R-D link. In other words, $Z^{(2)} \neq \mathbf{0}_Q$ if and only if $Z_s^{(1)} > 0$ and $Z_{\hat{s}}^{(2)} > 0$ (where in this case $\hat{s} = s$ also). Consequently:

$$\begin{aligned} P_{e|A} &= \frac{Q-1}{Q} \Pr(Z_s^{(0)} = 0) \left[1 - \Pr(Z_s^{(1)} > 0) \Pr(Z_s^{(2)} > 0) \right] \\ &= \frac{Q-1}{Q} e^{-P_0 a_0^2 \lambda_s} \left[1 - (1 - e^{-\beta_1 P_1 a_1^2 \lambda_s})(1 - e^{-\beta_2 P_2 a_2^2 \lambda_s}) \right] \end{aligned} \tag{22}$$

which reduces to eq. (7).