A New Full Rate Full Diversity ST Code with Nonvanishing Determinant for TH-UWB Systems

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Abstract— **In this paper, we propose a new Space Time (ST) coding scheme for Time Hopping (TH) Ultra Wideband (UWB) systems. The proposed scheme encodes the pulses used to transmit one information symbol. This permits to achieve full rate and full diversity with a non-vanishing coding gain for all number of transmit antennas. The proposed code shows to be the optimal extension of the codes constructed from cyclic division algebras when inter-pulse coding is performed.**

I. INTRODUCTION

Carrier-less impulse radio (IR) UWB uses very short and low duty-cycle impulses occupying a single band of several GHz. It was shown recently that applying ST coding on such transmission schemes can result in important performance gains [1], [2], [3]. Several full rate and full diversity codes have been proposed for narrowband communications over the Rayleigh channel [4], [5], [6], [7]. However all of these codes are complex-valued and therefore not adapted to carrier-less UWB systems. Consequently all proposed ST coded UWB systems were limited to have a rate of 1 symbol per channel use (PCU) [1], [2] or were real-valued versions of the TAST code [3]. However the use of real-valued Diophantine numbers results in an unbalanced energy distribution among the different layers of the codewords limiting the system performance.

On the other hand, in TH-UWB systems, each information symbol is conveyed by a train of pulses which renders inter-pulse coding possible. Profiting from this advantage, we propose a new TH-UWB-specific ST code. It can be associated with pulse position modulation (PPM), pulse amplitude modulation (PAM) or a combination of the two. An explicit expression of the coding gain that depends uniquely on the dimensionality of the transmitted constellation is given and the design is related to the class of codes constructed in [5].

II. SYSTEM MODEL

The symbols transmitted from the P antennas are modulated in amplitude and position resulting in a combined PPM-PAM constellation. This constellation is constructed from M-ary PPM constellations by including M' amplitude modulated signals in each dimension. Each symbol can be represented by the coordinates (a_p, d_p) such that $a_p \in \{(2m'-1-M'); m' = 1, ..., M'\}$ and $d_p \in \{0, ..., M-1\}$.

The signal transmitted from the p-th transmit antenna is:

$$s_p(t) = \frac{1}{\sqrt{P}} \sum_{n=0}^{N_f-1} \sum_{m=0}^{M-1} a_{p,m} b_{p,n} w(t - nT_f - m\delta) \qquad (1)$$

where $w(t)$ is the monocycle pulse waveform of duration T_w normalized to have unit energy. The normalization by P assures that the total transmitted energy is independent from the number of transmit antennas. N_f pulses are used to convey each information symbol. Each one of these pulses is transmitted during one time frame of duration T_f resulting in a data rate of $P\log_2(MM')/T_s$ bits/s; T_s is the symbol duration and is equal to $N_f T_f$. δ is the modulation delay and is chosen to satisfy $\delta \geq T_w$. $a_p = [a_{p,0}, ..., a_{p,M-1}]^T$ is the vector representation of the symbol transmitted from the p-th antenna. It is composed of $M-1$ zero values and one entry that belongs to the M'-ary PAM constellation. Finally the $P \times N_f$ matrix B whose elements are $b_{p,n}$ corresponds to the amplitude spreading matrix.

The received signal at the q-th antenna of a receiver equipped with Q antennas is given by:

$$r_q(t) = \frac{1}{\sqrt{P}} \sum_{p,n,m} a_{p,m} b_{p,n} h_{q,p}(t - nT_f - m\delta) + n_q(t)$$

where $n_q(t)$ is the noise at the q-th antenna which is supposed to be real AWGN with double sided spectral density $N_0/2$. $h_{q,p}(t)$ is the convolution of $w(t)$ and $g_{q,p}(t)$ which stands for the impulse response of the frequency selective channel between the p-th transmit and the q-th receive antenna. ISI can be eliminated by choosing $T_f \geq \max_{q,p}(T_{q,p}) + (M-1)\delta + T_w$, where $T_{q,p}$ is the maximum delay spread of $g_{q,p}(t)$.

In order to take advantage of the multi-path diversity with moderate complexity, a simplified version of an L-th order Rake receiver [8] is adopted by choosing the finger delays as $\Delta_l = lMT_w$ for $l = 0, ..., L-1$. This corresponds to combining the first arriving multi-path components. For the l-th Rake finger and at the n-th frame, the received signal at the q-th antenna is correlated with the M following reference signals:

$$\tilde{s}_{l,m,n} = w(t - nT_f - \Delta_l - m\delta) \qquad (2)$$

This results in the $QLMN$ decision variables:

$$x_{q,l,m,n} = \int_{nT_f}^{(n+1)T_f} r_q(t) \tilde{s}_{l,m,n}(t) \mathrm{d}t$$
$$= \frac{1}{\sqrt{P}} \sum_{p',m'} a_{p',m'} b_{p',n} r_{q,p'}((m-m')\delta + \Delta_l) + n_{q,l,m,n}$$

where: $r_{q,p}(\tau) = \int_0^{T_f} h_{q,p}(t) w(t-\tau) \mathrm{d}t$ and the second equation follows from the condition of no ISI. $n_{q,l,m,n} =$

$\int_{nT_f}^{(n+1)T_f} n_q(t)\tilde{s}_{l,m,n}(t)\mathrm{d}t$ is a white Gaussian noise which follows from the choice $\delta \geq T_w$ and $\Delta_l = lMT_w$.

The last equation can be expressed in matrix form as:

$$X = \frac{1}{\sqrt{P}} RAB + N \tag{3}$$

X and N are the $QLM \times N_f$ decision and noise matrices respectively. $A = \mathrm{diag}(a_1, ..., a_P)$ is the $PM \times P$ matrix corresponding to stacking the vector representations of the transmitted symbols on the principal diagonal. Finally, B is the amplitude spreading matrix and R is the corresponding channel matrix. $R = [R_1^T, \ldots, R_Q^T]^T$ is a $QLM \times PM$ matrix. $R_q = [R_{q,0}^T, \ldots, R_{q,L-1}^T]^T$ is a $LM \times PM$ matrix corresponding to the q-th receive antenna. The $M \times PM$ constituent matrices $R_{q,l}$ take the form $R_{q,l} = [R_{q,l,1} \ \cdots \ R_{q,l,P}]$. $R_{q,l,p}$ is a $M \times M$ matrix whose (m, m')-th element is given by: $R_{q,l,p}(m, m') = r_{q,p}(\Delta_l + (m - m')\delta)$.

III. Code Construction

To profit from the existing transmit diversity, the following coding scheme is proposed. Given an information vector $a = [[a_{1,0}, ..., a_{1,M-1}], ..., [a_{P,0}, ..., a_{P,M-1}]]^T$, we apply the $P \times P$ rotation M_P on each one of the M dimensions of the PPM-PAM constellation. In other words, we construct the vector s:

$$s = (M_P \otimes I_M)a \tag{4}$$

where $\otimes$ stands for the Kronecker product and I_M is the $M \times M$ identity matrix. The rotation matrix M_P is constructed in such a way that it achieves full modulation diversity [9], [10]. In other words, if a is a n-dimensional vector and $s = M_n a$ is transmitted over n parallel channels, a can be reconstituted unless all these channels suffer from sever fading. Moreover for some given dimensions, M_n can be constructed to maximize the minimum product distance defined as:

$$d_{min} = \min_{\substack{a \neq a' \\ a, a' \in \mathbb{Z}^n}} \prod_{i=1}^{n} |s_i - s'_i| \tag{5}$$

Designate by $s_{p,m}$ the $((p-1)M + m)$-th element of s. The next step in the coding scheme consists of applying the spreading matrix B resulting in the following codewords:

$$C(a) = \begin{bmatrix} s_{1,1} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ s_{1,M} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & s_{P,1} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & s_{P,M} \end{bmatrix} B = S(a)B \tag{6}$$

$C(a)$ is a $PM \times N_f$ matrix such that the $((p-1)M + m, n)$-th element of $C(a)$ corresponds to the amplitude of the pulse emitted by the p-th antenna during the m-th position of the n-th time frame. In what follows, B is chosen to be unitary.

Following the design criterion proposed in [11], the diversity gain is equal to the minimum rank of $C(x)$ over all values of $x \neq 0$ where $x = a - a'$. We will now show that the proposed code is fully diverse. In fact, $C(x)C^T(x) = S(x)BB^TS^T(x) = S(x)S^T(x)$ since B is unitary. Therefore, the matrix $C(x)C^T(x)$ possesses P eigenvalues λ_p that take the values ($p = 1, ..., P$):

$$\lambda_p = \sum_{m=1}^{M} y_{p,m}^2 \tag{7}$$

where $y = (M_P \otimes I_M)x$.

λ_p is equal to zero if and only if $y_{p,m} = 0$ for $m = 1, ..., M$. For a given value of m, we have $y_{p,m} = M_{P,p}[x_{1,m}, ..., x_{P,m}]^T$ where $M_{P,p}$ stands for the p-th row of M_P. Since M_P generates a fully diverse constellation, $y_{p,m} = 0$ implies that $x_{p,m} = 0$ for $p = 1, ..., P$. Therefore, λ_p can not be equal to zero unless when $x_{p,m} = 0$ for $p = 1, ..., P$ and $m = 1, ..., M$, in other words when $a = a'$. Therefore, $C(x)$ admits P non-zero eigenvalues and thus it achieves the maximum transmit diversity of P.

The coding gain of $C(x)$ corresponding to a M-PPM-M'-PAM constellation $\mathcal{C}$ is given by [11]:

$$\delta_{min} = \min_{\substack{a \neq a' \\ a, a' \in \mathcal{C}}} \left(\prod_{p=1}^{P} \lambda_p \right)^{\frac{1}{P}} = \min_{\substack{a \neq a' \\ a, a' \in \mathcal{C}}} \left(\prod_{p=1}^{P} \left(\sum_{m=1}^{M} y_{p,m}^2 \right) \right)^{\frac{1}{P}} \tag{8}$$

For one dimensional constellations ($M = 1$), the sum in eq. (7) contains only one summand and eq. (8) reduces to $\delta_{min} = 4d_{min}^{2/P}$. The factor 4 comes from the fact that unlike eq. (5) that is defined over $\mathbb{Z}$, eq. (8) is defined over the PAM constellation $\{\pm 1, \pm 3, ..., \pm (M' - 1)\}$. For multi-dimensional constellations, suppose that the sum in eq. (7) contains n non zero summands. For $n = 1$ we obtain $\delta_{min} = 4d_{min}^{2/P}$. If $n > 1$, δ_{min} is increased n times but it will be further reduced 4 times since the difference between 2 data symbols now belongs to $\{m'\}^M$ rather than $\{2m'\}$ as in the case of PAM symbols for $m' \in [-M' \ M']$. So, the minimum value of the coding gains will be obtained for data vectors a_p and a'_p that differ in exactly one position ($n = 2$) for all values of p. It follows that $\delta_{min} = 2d_{min}^{2/P}$. From [9] and [10], $d_{min} = d_{\mathbb{K}}^{-1/2}$ where $d_{\mathbb{K}}$ is the discriminant of the field extension $\mathbb{K}/\mathbb{Q}$ over which the rotation matrix is constructed.

Note that the proposed encoding scheme has the property that the coding gain does not vanish with the size of the signal set. Moreover, the proposed code has a data rate of P symbols PCU. Thus it permits to increase the spectral efficiency while profiting from the full spatial and multi-path diversity offered by the underlying channel. In comparison with the previous full-rate and full-diversity codes [4], [5], [6], the proposed scheme presents the following advantages. Each codeword includes P symbols rather than P^2 symbols which results in a lower decoding complexity. Finally, eq. (6) suffers from no decoding delays (1 instead of P symbol durations).

IV. Relation with Previously Proposed Codes

In this section, we will show that the code proposed in Section III is the optimal version of the codes proposed in

[4] and [6] when the receiver is capable of separating the transmitted data streams (which is achieved by choosing B to be unitary). For ST codes that extend over J symbol durations, eq. (3) can be generalized to:

$$X = \frac{1}{\sqrt{P}} RA(I_J \otimes B) + N \quad (9)$$

where I_J is the $J \times J$ identity matrix. $A = [A(1), ..., A(J)]$ and $A(j) = \text{diag}(a_1(j), ..., a_P(j))$ with $a_p(j)$ being the vector representation of the symbol transmitted from antenna p during the j-th symbol duration. We will next consider the case of PAM constellations, the extension to combined PPM-PAM can be done in the same way as in the preceding section.

The various $P \times P$ codes proposed in [4], [6], [7] take the following generic form ($n = P$):

$$C(\gamma) = \begin{bmatrix} f_0 & f_1 & \cdots & f_{n-1} \\ \sigma(f_{n-1}) & \sigma(f_0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \sigma^{n-2}(f_1) \\ \sigma^{n-1}(f_1) & \cdots & \sigma^{n-1}(f_{n-1}) & \sigma^{n-1}(f_0) \end{bmatrix} \quad (10)$$

where $f_i = \sqrt{c_n(\gamma)}\gamma^{\frac{i}{n}}\alpha k_i$ and $k_i = \sum_{j=0}^{n-1} a_{ni+j+1}\theta^j$ for $i = 0, ..., n-1$. $\{1, \theta, ..., \theta^{n-1}\}$ is an integral basis of the cyclic number field extension $\mathbb{K}/\mathbb{Q}$ with Galois group $Gal(\mathbb{K}/\mathbb{Q}) = \langle\sigma\rangle$ ($\sigma^n = 1$). a_j are the information symbols belonging to a PAM constellation for $j = 1, ..., n^2$.

$c_n(\gamma) = n / \sum_{i=0}^{n-1} \gamma^{\frac{2i}{n}}$ is a normalization factor. $\alpha \in \mathbb{K}$ assures that the transmitted constellation is a rotation of the original signal set [6]. Finally, γ is appropriately chosen to insure full diversity. It is taken to be transcendental in [4], [5] and a "non Norm" algebraic element in [6], [7]. From eq. (9), combining the decision variables corresponding to the same symbol results in:

$$Y = X(I_J \otimes B)^T = \frac{1}{\sqrt{P}} RA + N' \quad (11)$$

The noise vector N' is still white. The code matrix in eq. (11) will now be denoted by $A(\gamma)$ and will take the form:

$$A(\gamma) = \left[\ \text{diag}(C_1(\gamma)) \ \cdots \ \text{diag}(C_n(\gamma)) \ \right] \quad (12)$$

where $C_i(\gamma)$ is the i-th column of eq. (10). The matrix $A(\gamma)A(\gamma)^T$ admits n eigenvalues. These eigenvalues are given by $\lambda_i = \sigma^i(\lambda)$ for $i = 0, ..., n-1$ and $\lambda = \sum_{i=0}^{n-1} f_i^2$. Therefore, the coding gain of $A(\gamma)$ becomes:

$$g_{min}(\gamma) = d_{\mathbb{K}}^{-\frac{1}{n}} c_n(\gamma) \min \left(\text{N}_{\mathbb{K}/\mathbb{Q}} \left(\sum_{i=0}^{n-1} \gamma^{\frac{2i}{n}} k_i^2 \right) \right)^{\frac{1}{n}} \quad (13)$$

$$\geq d_{\mathbb{K}}^{-\frac{1}{n}} c_n(\gamma) \min \left(\sum_{i=0}^{n-1} \gamma^{2i} \text{N}_{\mathbb{K}/\mathbb{Q}}(k_i)^2 \right)^{\frac{1}{n}} \quad (14)$$

where eq. (13) follows the fact that the algebraic norm of α verifies $\text{N}_{\mathbb{K}/\mathbb{Q}}(\alpha) = d_{\mathbb{K}}^{-\frac{1}{2}}$ [9], [10]. Note that $\text{N}_{\mathbb{K}/\mathbb{Q}}(k_i) \in \mathbb{Z}$.

Considering the case $\gamma \geq 1$, the minimum of the right hand side of eq. (14) is obtained when $k_i = 0$ for $i = 1, ..., n-1$ and $\text{N}_{\mathbb{K}/\mathbb{Q}}(k_0) = \pm 1$. The value of γ that maximizes the coding gain is $\gamma = 1$ since $c_n(\gamma)$ is a decreasing function of γ for $\gamma \geq 1$. For $\gamma \leq 1$, the minimum of the right hand side of eq. (14) is obtained for $k_0 = \cdots = k_{n-2} = 0$ and $\text{N}_{\mathbb{K}/\mathbb{Q}}(k_{n-1}) = \pm 1$. Maximizing over γ results in $\gamma = 1$ since now $c_n(\gamma)\gamma^{\frac{2(n-1)}{n}}$ is an increasing function of γ. The optimal choice $\gamma = 1$ shows that since the transmitted streams are separated at the receiver, the best strategy is to evenly distribute the total energy among the different data streams. Arranging the columns of eq. (12):

$$A = \text{diag}\left(\alpha, ..., \sigma^{n-1}(\alpha)\right) \begin{bmatrix} \text{diag}(k_0, ..., \sigma^{n-1}(k_0)) \\ \vdots \\ \text{diag}(k_{n-1}, ..., \sigma^{n-1}(k_{n-1})) \end{bmatrix}^T \quad (15)$$

Equation (15) shows that coding between adjacent symbols is not needed since $k_0, ..., k_{n-1}$ are decoupled, and so the temporal length of the code J can be chosen to be equal to 1. Finally, the structure of the code performing inter-pulse coding and achieving a coding gain of $d_{\mathbb{K}}^{-\frac{1}{n}}$ (over $\mathbb{Z}$) is:

$$C = \text{diag}(\mathcal{M}[a_1, ..., a_n]^T)B \quad (16)$$

where B is any unitary matrix and $\mathcal{M}$ is any one of the rotation matrices constructed in [9] or [10] and whose (i, j)-th element is given by $\sigma^{i-1}(\alpha\theta^{j-1})$. Finally, expanding eq. (16) results in eq. (6) and the coding gain is $4d_{\mathbb{K}}^{-\frac{1}{n}}$ for PAM and $2d_{\mathbb{K}}^{-\frac{1}{n}}$ for combined PPM-PAM.

V. Simulations and Results

The pulse waveform $w(t)$ is chosen to be the second derivative of the Gaussian pulse with a duration of 0.5 ns normalized to have unit energy. The transmit and the receive arrays are supposed to be sufficiently spaced so that the PQ sub-channels are generated independently using the standard IEEE 802.15.3a channel model CM2 that corresponds to non line of sight (NLOS) conditions [12]. Orthogonal PPM modulations are used with $\delta = 0.5$ ns and to eliminate ISI, the frame time was fixed to $T_f = 100$ ns.

Fig. 1 shows the performance of the ST code for $P = 2, 3, 4$ and 6 transmit antennas with 4PPM-2PAM constellations. The receiver is equipped with 1 antenna and a 1 finger Rake. This figure shows that the ST code is capable not only of increasing the spectral efficiency with P, but it also results in important performance gains since the diversity is multiplied by P for the same number of Rake fingers.

Fig. 2 shows the importance of rotations in achieving diversity with $Q = 1$, 2 PAM modulations and $L = 4$, 12. Orthogonal codes attributed to the transmit antennas are capable of separating the transmitted data streams and increasing the data rate, but they fail to assure any diversity gain. For $L = 12$, the 2×1 and the 4×1 unrotated systems show the same performance even though the latter has a higher spectral efficiency. Introducing rotations will now result in 4 dB and 5.5 dB gains at 10^{-4} respectively.

Fig. 3 compares the new ST code with the codes "STC 1" and "STC 2" proposed by Giannakis and Yang in [1] for $Q = 1$ and $L = 1$. The comparison is made at the same

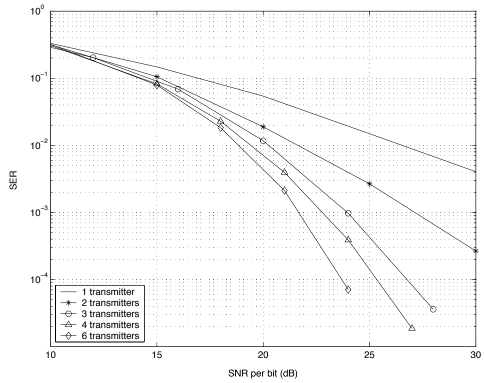

Fig. 1. Performance with $Q = 1$, $L = 1$ and 4PPM-2PAM.

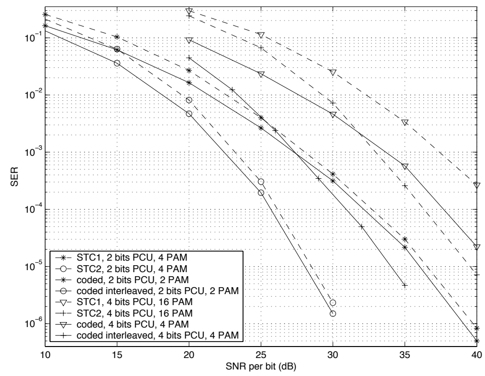

Fig. 3. The proposed code vs. Giannakis-Yang codes with $Q = 1$ and $L = 1$.

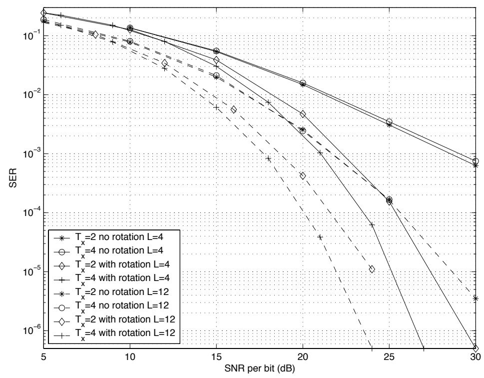

Fig. 2. Effect of rotation in achieving diversity with 2 PAM and $Q = 1$.

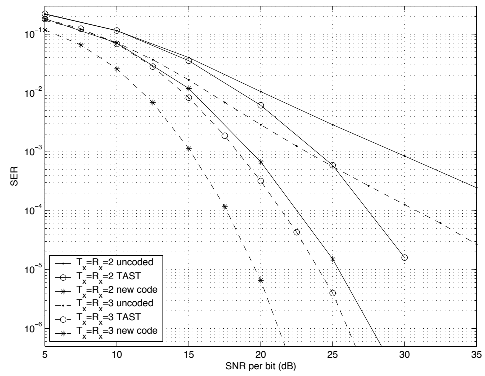

Fig. 4. The proposed code vs. the TAST code with $L = 1$ and 2 PAM.

spectral efficiency and with constellations that have the same dimensions resulting in receivers that have the same number of correlators. To compare our code with "STC 2", interleaving over 2 fading blocks was performed. This figure shows that the proposed code outperforms "STC 1" and "STC 2" especially at high spectral efficiencies.

Fig. 4 compares the new code with the TAST code [4] for $L = 1$ and 2 PAM. For the TAST code, the Diophantine number is taken to be equal to 2 since this choice maximizes the coding gain for 2 PAM [3]. Results show that the proposed scheme shows better performance.

VI. Conclusion

A new totally-real, full rate, fully diverse ST code with non-vanishing determinant was proposed for TH-UWB systems. It can be associated will all combined PPM-PAM constellations for any number of transmit antennas. Moreover, it has many appealing properties such as reduced complexity decoding, low peak-to-average-power-ratios (PAPR) and small decoding delays. Simulations over realistic indoor highly frequency selective channels showed that the proposed scheme outperforms all previously proposed ST-TH-UWB schemes.

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