A Rate-1 2×2 Space-Time Code without any Constellation Extension for TH-UWB Communication Systems with PPM

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***Abstract*— In this paper, we propose for the first time a 2×2 Space-Time (ST) block code that can be applied with Pulse Position Modulation (PPM) without introducing any constellation extension. This encoding scheme is adapted to low cost carrier-less Time-Hopping Ultra-Wideband (TH-UWB) systems where information must be conveyed only by the time delay of the modulated signal that has a very low duty cycle. The proposed scheme permits to achieve full transmit diversity with M-ary PPM constellations for all values of M. Moreover, it permits to overcome the additional phase rotations or amplitude amplifications that are introduced by all the known ST codes such as the Alamouti code [1], the rate-1 codes proposed in [2], [3] for PSK and the diversity schemes proposed in [4] for TH-UWB systems with PAM or PPM.**

I. Introduction

The literature of Space-Time (ST) coding is huge [1]–[3]. However, ST coding was considered mainly with QAM and PSK constellations. Recently, ultra-wideband (UWB) emerged as a promising technology for wireless personal area networks. Given the very short duration of the transmitted UWB pulses, Pulse Position Modulations (PPM) that were for long time exclusive for optical communications can now be used over radio frequencies. However, all of the existing ST block codes will result in an additional constellation extension when associated with PPM.

Consider for example the Alamouti code [1] where the codewords take the following form: $C = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}$ where s_1 and s_2 are QAM or PAM symbols. For these constellations, the Alamouti code does not introduce a constellation extension since s^* (the complex conjugate of s) and $-s$ are both QAM (resp. PAM) symbols whenever s is a QAM (resp. PAM) symbol. Therefore, the Alamouti code does not introduce any kind of constellation extension with these modulations. This property will be referred to as the shape preserving constraint in what follows.

On the other hand, consider a M-ary PPM constellation. This is a M-dimensional constellation where each information symbol is represented by a M-dimensional vector that belongs to the following signal set:

$$\mathcal{C}_{\text{PPM}} = \{e_m \ ; \ m = 1, \dots, M\} \tag{1}$$

where e_m is the m-th column of the $M \times M$ identity matrix I_M. It can be easily shown that the Alamouti code permits to achieve a full transmit diversity order with PPM constellations. On the other hand, the Alamouti code is not shape preserving with these constellations since s and $-s$ can never belong simultaneously to the signal set given in eq. (1).

In the same way, the 2×2 rate-1 codes proposed in [2] can be expressed as: $C = \begin{bmatrix} s_1 & s_2 \\ \gamma s_2 & s_1 \end{bmatrix}$ where s_1 and s_2 are PSK symbols. Multiplying by γ corresponds to an appropriate phase rotation or amplitude amplification that permits to achieve full transmit diversity. Moreover, this code becomes shape preserving when γ is chosen such that γs is a M-ary PSK symbol whenever s is a M-ary PSK symbol for a certain value of M. However, phase rotations or amplitude amplifications are not admissible with PPM constellations and the code proposed in [2] is not shape preserving with PPM. From eq. (1), the only value of γ that results in a shape preserving code with PPM is $\gamma = 1$. However, the transmit diversity will be lost with this value of γ [2].

The same argument applies to the diversity schemes proposed for TH-UWB systems in [4] where a repetition code applied on the data stream of one antenna along with an alternation of the amplitudes of the pulses transmitted from the other antenna (between ± 1) permit to achieve a full transmit diversity order with two transmit antennas.

In this work, we study the problem of ST coding with two transmit antennas and PPM constellations. We take advantage from the structure of the constellation given in eq. (1) in order to construct an adapted coding scheme that does not necessitate any kind of phase rotations or amplitude amplifications. This code keeps the natural advantages of low cost TH-UWB where it is difficult to control the amplitudes and the phases of the transmitted pulses. Moreover, the proposed minimal-delay code permits to achieve a full transmit diversity order without any data rate reduction with all PPM constellations.

As for the decoding problem, we show that linear decoders can assure optimal detection with M-PPM for $M = 2$ and when the modulation delay is larger than the channel delay spread (the received constellation is orthogonal). For $M > 2$, the absence of the polarity inversions hinders any kind of orthogonality between the transmitted data streams necessitating the deployment of more sophisticated maximum-likelihood decoders (such as the sphere decoder in [5]).

II. SYSTEM MODEL

Consider the general case of a hybrid M-PPM-M'-PAM constellation where the input data is modulated onto both the amplitudes and the positions of the transmitted pulses. Each element of this constellation is represented by a M-dimensional vector that belongs to the set:

$$\mathcal{C} = \{(2m' - 1 - M')e_m \ ; \ m' = 1, \dots, M'; \ m = 1, \dots, M\} \quad (2)$$

The PPM constellation given in eq. (1) follows as a special case of the M-PPM-M'-PAM constellation.

Consider a single-user multi-antenna TH-UWB system where the transmitter and the receiver are equipped with $P = 2$ and Q antennas respectively. The signal transmitted from the p-th antenna can be expressed as:

$$s_p(t) = \sqrt{\frac{E_s}{PN_f}} \sum_{n=0}^{N_f-1} \sum_{m=1}^{M} a_{p,m} w(t - nT_f - (m-1)\delta) \quad (3)$$

where $w(t)$ is the pulse waveform of duration T_w normalized to have unit energy. E_s stands for the average energy per transmitted symbol and the multiplying factor $\frac{1}{\sqrt{P}}$ was introduced in order to have the same total transmitted energy as in the case of single-antenna systems. N_f is the number of pulses used to transmit one information symbol and T_f is the average separation between two consecutive pulses. No reference to the TH code is made in eq. (3) since all transmit antennas of the same user will share the same pseudorandom TH sequence. δ is the modulation delay and it is chosen to verify $\delta \geq T_w$. Finally, $a_p = [a_{p,1}, \dots, a_{p,M}]^T \in \mathcal{C}$ is composed of $M - 1$ zero components and one component that belongs to the M'-PAM constellation (this component is equal to 1 with PPM).

The received signal at the q-th antenna can be expressed as:

$$r_q(t) = \sum_{p=1}^{P} \sum_{n=0}^{N_f-1} \sum_{m=1}^{M} a_{p,m} h_{q,p}(t - nT_f - (m-1)\delta) + n_q(t) \quad (4)$$

where $n_q(t)$ is the noise at the q-th antenna and it is supposed to be real AWGN. For notational simplicity, the multiplying factor $\sqrt{\frac{E_s}{PN_f}}$ was removed since this term can be included in the expression of the noise variance. In this case, the variance of the noise term in eq. (4) is equal to $\frac{PN_f N_0}{2E_s}$. $h_{q,p}(t)$ is the convolution of $w(t)$ and $g_{q,p}(t)$ that stands for the impulse response of the frequency selective channel between the p-th transmit and the q-th receive antennas.

In order to take advantage from the multi-path diversity, a L-th order Rake is implemented at the receiver side. The finger delays are chosen as $\Delta_l = (l-1)MT_w$ for $l = 1, \dots, L$. This corresponds to combining the first arriving multi-path components. In the absence of Inter-Frame-Interference (IFI), the QLM decision variables collected at the receiver side during one symbol duration are given by:

$$\begin{aligned} x_{q,l,m} &= \int_0^{N_f T_f} r_q(t) \sum_{n=0}^{N_f-1} w(t - nT_f - \Delta_l - (m-1)\delta) \mathrm{d}t \\ &= \sum_{p'=1}^{P} \sum_{m'=1}^{M} a_{p',m'} r_{q,p'}((m - m')\delta + \Delta_l) + n_{q,l,m} \end{aligned} \quad (5)$$

where: $r_{q,p}(\tau) = \int_0^{T_f} h_{q,p}(t) w(t - \tau) \mathrm{d}t$. $n_{q,l,m}$ is a white Gaussian noise term. This follows from $\Delta_l = (l-1)MT_w$ and $\delta \geq T_w$. Equation (5) follows from the absence of IFI. This can be obtained by fixing:

$$T_f \geq \Gamma + (M-1)\delta + T_w \quad (6)$$

where Γ stands for the maximum delay spread of the underlying channel ($\Gamma >> T_w$).

Equation (5) can be expressed in matrix form as:

$$X = RA + N \quad (7)$$

where X is the decision vector of length QLM whose $((q-1)LM + (l-1)M + m)$-th component is equal to $x_{q,l,m}$ for $q = 1, \dots, Q$, $l = 1, \dots, L$ and $m = 1, \dots, M$. N is the noise vector that is constructed from the noise terms $n_{q,l,m}$ with $E[NN^T] = \frac{PN_0}{2E_s} I_{QLM}$. A is a MP-dimensional vector given by: $A = [a_1^T, \dots, a_P^T]^T$ where $a_p \in \mathcal{C}$ (or $a_p \in \mathcal{C}_{\text{PPM}}$) is the M-dimensional vector representation of the symbol transmitted from the p-th antenna for $p = 1, \dots, P$.

In eq. (7), $R = [R_1^T, \dots, R_Q^T]^T$ is the $QLM \times PM$ channel matrix. $R_q = [R_{q,1}^T, \dots, R_{q,L}^T]^T$ is a $LM \times PM$ matrix for $q = 1, \dots, Q$. $R_{q,l}$ is a $M \times PM$ matrix given by: $R_{q,l} = [R_{q,l,1} \ \cdots \ R_{q,l,P}]$ where $R_{q,l,p}$ is a $M \times M$ matrix whose (m, m')-th element is given by:

$$R_{q,l,p}(m, m') = r_{q,p}((m - m')\delta + \Delta_l) \quad (8)$$

Note that the impact of the interference between the different modulation positions is included in eq. (7). This interference is present when $\delta < \Gamma$. In the absence of Inter-Position-Interference (IPI), $R_{q,l,p}$ becomes a $M \times M$ diagonal matrix whose diagonal elements are all equal to $r_{q,p}(\Delta_l)$.

When a ST block code of length J is applied at the transmitter side, eq. (7) will take the following form:

$$X = RC + N \quad (9)$$

where X and N are now $QLM \times J$ matrices obtained from the horizontal concatenation of the decision variables and noise samples collected during the J symbol durations respectively. R has the same structure as before and C is a $PM \times J$ matrix whose $((p-1)M + m, j)$-th entry corresponds to the amplitude of the pulse (if any) transmitted at the m-th position of the p-th antenna during the j-th symbol duration for $p = 1, \dots, P$, $m = 1, \dots, M$ and $j = 1, \dots, J$.

III. Code Construction

For M-dimensional constellations, we propose the following structure for the minimal-delay $2M \times 2$ codewords:

$$C = \begin{bmatrix} s_1 & s_2 \\ \Omega s_2 & s_1 \end{bmatrix} \tag{10}$$

where $s_1, s_2 \in \mathcal{C}$ given in eq. (2) are the M-dimensional vector representations of the information symbols. For PPM constellations, $s_1, s_2 \in \mathcal{C}_{\text{PPM}}$ given in eq. (1). Ω is a $M \times M$ cyclic permutation matrix given by:

$$\Omega = \begin{bmatrix} \mathbf{0}_{1\times(M-1)} & 1 \\ I_{M-1} & \mathbf{0}_{(M-1)\times 1} \end{bmatrix} \tag{11}$$

where $\mathbf{0}_{m\times n}$ is the all-zero $m \times n$ matrix.

Evidently, $\Omega s \in \mathcal{C}_{\text{PPM}}$ given in eq. (1) whenever $s \in \mathcal{C}_{\text{PPM}}$ and the code is shape preserving with PPM. The same argument holds for the constellation given in eq. (2).

Proposition: The proposed code permits to achieve full transmit diversity with M-PPM-M'-PAM constellations for $M \geq 3$ and $\forall M'$ and with M-PPM constellations for $M \geq 2$.

Proof: Denote by $\Delta C(a_1, a_2)$ the difference between two codewords given by:

$$\begin{aligned}\Delta C(a_1,a_2) &= C(s_1 - s_1', s_2 - s_2') = C(s_1,s_2) - C(s_1',s_2') \\ &= \begin{bmatrix} a_{1,1} & \cdots & a_{1,M} & a_{2,\pi(1)} & \cdots & a_{2,\pi(M)} \\ a_{2,1} & \cdots & a_{2,M} & a_{1,1} & \cdots & a_{1,M} \end{bmatrix}^T\end{aligned} \tag{12}$$

where $\pi(.)$ stands for the cyclic permutation given by $\pi(i) = (i-2) \mod M+1$. $a_{i,m}$ is the m-th component of the vector a_i for $i = 1, 2$ and $m = 1, \ldots, M$. a_1 and a_2 belong to the set $\mathcal{A}$ that denotes the set of all possible differences between two information vectors:

$$\mathcal{A} = \{s - s' \ ; \ s, s' \in \mathcal{C}\} \tag{13}$$

where $\mathcal{C}$ must be replaced by $\mathcal{C}_{\text{PPM}}$ for PPM.

The proposed code is fully diverse if the matrix $\Delta C(a_1, a_2)$ has a full rank for $(a_1, a_2) \in \mathcal{A}^2 \backslash \{(0_M, 0_M)\}$ where 0_M stands for the M-dimensional all-zero vector. In other words, the code is fully diverse for a given value of M if all the non-zero M-dimensional vectors that result in a rank-deficient matrix $\Delta C(a_1, a_2)$ do not belong to the set $\mathcal{A}$ given in eq. (13).

As can be seen from the proposition, the properties of the code depend on the dimensionality of the signal set (M). Following from eq. (13), elements of $\mathcal{A}$ can have a maximum number of two non-zero components. The transmit diversity order is achieved because of this particular structure of $\mathcal{A}$.

$\Delta C(a_1, a_2)$ is a rank-deficient matrix if there exits a non-zero rational number k such that $\Delta C_2(a_1, a_2) = k \Delta C_1(a_1, a_2)$ where $\Delta C_i(a_1, a_2)$ stands for the i-th column of $\Delta C(a_1, a_2)$. From eq. (12) this implies that:

$$\frac{a_{2,1}}{a_{1,1}} = \cdots = \frac{a_{2,M}}{a_{1,M}} = \frac{a_{1,1}}{a_{2,\pi(1)}} = \cdots = \frac{a_{1,M}}{a_{2,\pi(M)}} = k \tag{14}$$

After some manipulations, eq. (14) implies that:

$$a_{2,M} = k^{2i} a_{2,M-i} \quad ; \quad i = 1, \ldots, M-1 \tag{15}$$

Since $k \neq 0$, eq. (15) implies that all the components of a_2 must be equal to zero or different from zero simultaneously. Since the elements of $\mathcal{A}$ can have a maximum number of two non-zero components, then for $M \geq 3$ at least one component of a_2 is equal to zero. Consequently, eq. (15) implies that $a_2 = 0_M$ when $a_2 \in \mathcal{A}$ and $M \geq 3$. From eq. (12), $a_2 = 0_M$ implies that $\text{rank}(\Delta C(a_1, a_2)) = 2$ unless when $a_{1,1} = \cdots = a_{1,M} = 0$. Therefore, the only rank-deficient matrix $\Delta C(a_1, a_2)$ associated with two elements a_1 and a_2 of $\mathcal{A}$ is the all-zero matrix. Since the above proof is independent from the value of M', then the code given in eq. (10) permits to achieve a transmit diversity order of 2 when associated with M-PPM-M'-PAM constellations for $M \geq 3$ and for all values of M'. In particular, a full transmit diversity order can be achieved with M-PPM for $M \geq 3$.

Consider the case of M-PPM with $M = 2$. In this case, eq. (15) implies that $a_{2,2} = k^2 a_{2,1}$. For $a_{2,1} = 0$, then an approach similar to the proof presented above for M-PPM-M'-PAM can be applied implying that all the non-zero matrices ΔC have a full rank. On the other hand, eq. (1) and eq. (13) imply that one of the components of a_2 must be equal to $+1$ while the other component must be equal to -1 when $a_2 \neq 0_M$ is an element of $\mathcal{A}$. Consequently, the relation $a_{2,2} = k^2 a_{2,1}$ will imply that $k^2 = -1$ which is not possible given that all the entries of the different codewords are real-valued. Consequently, the proposed code is fully diverse with M-PPM for $M = 2$ in addition to the values $M \geq 3$.

Note that for M-PPM-M'-PAM, and even though the proposed ST code is novel, it is not so interesting because the Alamouti code can be applied with M-PPM-M'-PAM and it is shape preserving when $M' \neq 1$. On the other hand, for M-PPM with $M \geq 2$, the proposed code is the first known rate-1 code that is shape preserving and fully diverse.

IV. Decoding

Equation (9) can be written as:

$$\mathcal{X} = (I_2 \otimes R)\, \Phi(\Omega) S + \mathcal{N} \tag{16}$$

where $\otimes$ stands for the Kronecker product. $\mathcal{X}$ and $\mathcal{N}$ are $2QLM$-dimensional vectors given by: $\mathcal{X} = \text{vec}(X)$ and $\mathcal{N} = \text{vec}(N)$ respectively where the function $\text{vec}(X)$ stacks the columns of the matrix X vertically one after the other. $S = [s_1^T \quad s_2^T]^T$ is the $2M$-dimensional information vector.

For a given $M \times M$ matrix $\mathcal{M}$, the $4M \times 2M$ matrix $\Phi(\mathcal{M})$ is defined as:

$$\Phi(\mathcal{M}) = \begin{bmatrix} I_M & \mathbf{0}_M \\ \mathbf{0}_M & \mathcal{M} \\ \mathbf{0}_M & I_M \\ I_M & \mathbf{0}_M \end{bmatrix} \tag{17}$$

where $\mathbf{0}_M$ is the all-zero $M \times M$ matrix.

From eq. (16), the information vector S can be determined based on the Maximum-Likelihood (ML) criterion:

$$\hat{S} = \arg \min_{S \in \mathcal{C}^2} ||\mathcal{X} - (I_2 \otimes R)\, \Phi(\Omega) S||^2 \tag{18}$$

When $QL \geq P$, the decoding algorithm proposed in [6] can be applied in order to assure a ML detection of S.

This algorithm assures the convergence towards the closest M-PPM-M'-PAM point (and not simply towards the closest lattice point as the sphere decoder [5]). In other words, this algorithm assures that the vector $\hat{S}$ contains two sub-vectors each having $M-1$ zero components and 1 component of the M'-PAM constellation. On the other hand, for binary PPM constellations in the absence of IPI, we will show in what follows that a simpler decoding technique based on linear processing can assure a ML detection.

Consider the binary PPM constellation. In this case:

$$\Omega s = -s + \mathbf{1}_M \quad \forall\ s \in \mathcal{C}_{\text{PPM}}\ ;\ M = 2 \tag{19}$$

where $\mathbf{1}_M$ stands for the M-dimensional vector whose components are all equal to 1.

Consequently, from eq. (17) and eq. (19):

$$\Phi(\Omega)S = \Phi(-I_M)S + I' \tag{20}$$
$$= (\phi \otimes I_M)\, S + I' \tag{21}$$

where $I' = [0\ 0\ 1\ 1\ 0\ 0\ 0\ 0]^T$ and $M = 2$ in the last two equations. The matrix ϕ is given by:

$$\phi = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}^T \tag{22}$$

Moreover, for M-PPM in the absence of IPI, the $QLM \times 2M$ channel matrix R can be written as:

$$R = R' \otimes I_M \triangleq \begin{bmatrix} R'_1 & R'_2 \end{bmatrix} \otimes I_M \tag{23}$$

where R'_p is a QL-dimensional vector for $p = 1, 2$. Its $((q-1)L + l)$-th element is equal to $r_{q,p}(\Delta_l)$ for $l = 1, \dots, L$ and $q = 1, \dots, Q$.

Consequently, for 2-PPM constellations in the absence of IPI, combining eq. (16), eq. (21), eq. (22) and eq. (23) results in ($M = 2$):

$$\mathcal{X} = [I_2 \otimes (R' \otimes I_M)]\, [(\phi \otimes I_M)\, S + I'] + \mathcal{N} \tag{24}$$

Following from the properties of the Kronecker product, the last equation implies that:

$$\mathcal{Y} \triangleq \mathcal{X} - [(I_2 \otimes R') \otimes I_M]\, I' \tag{25}$$
$$= [(I_2 \otimes R')\, \phi \otimes I_M]\, S + \mathcal{N} \triangleq [\mathcal{R} \otimes I_M]\, S + \mathcal{N} \tag{26}$$

From eq. (22) and eq. (23), the $2QL \times 2$ matrix $\mathcal{R}$ can be written as:

$$\mathcal{R} = (I_2 \otimes R')\, \phi = \begin{pmatrix} R'_1 & -R'_2 \\ R'_2 & R'_1 \end{pmatrix} \tag{27}$$

Consequently:

$$\mathcal{R}^T \mathcal{R} = \left(\sum_{q=1}^{Q} \sum_{p=1}^{P} \sum_{l=1}^{L} \left(r_{q,p}(\Delta_l)\right)^2 \right) I_2 \tag{28}$$

Consequently, the two constituent sub-vectors s_1 and s_2 of S can be decoded independently according to:

$$p_i = \arg\max_{m=1,2} \left(\mathcal{Y}'_{2(i-1)+m} \right) \quad ; \quad s_i = e_{p_i} \tag{29}$$

where $\mathcal{Y}'_j$ is the j-th component of the vector $\mathcal{Y}'$ given by:

$$\mathcal{Y}' = \left[\mathcal{R}^T \otimes I_2\right] \mathcal{Y} \tag{30}$$

As a conclusion, eq. (25), eq. (29) and eq. (30) describe the detection process that must be performed with 2-PPM constellations in the absence of IPI.

V. Simulations and results

The second derivative of the Gaussian pulse with a duration of $T_w = 0.5$ ns is used. In order to eliminate the inter-frame-interference and inter-symbol-interference, the frame duration (T_f) is chosen to verify eq. (6). At the receiver side, perfect channel state information is assumed. The antennas of the transmit and the receive arrays are supposed to be sufficiently spaced so that each one of the PQ sub-channels is generated independently from the other sub-channels using the standard IEEE 802.15.3a channel model recommendation CM2 that corresponds to non-line-of-sight (NLOS) conditions [7].

Fig. 1 compares the performance of single-antenna and of the ST coded TH-UWB systems with one receive antenna and a 5-finger Rake. The modulation delay is chosen to verify $\delta = T_w = 0.5$ ns and, consequently, the simulations are performed under the impact of IPI. In this case, the ML decoder proposed in [6] is used for detection. Results show the high performance levels achieved by the proposed coding scheme. The performance gains are visible with different orders of the PPM signal sets.

In Fig. 2, we fix $\delta = 100$ ns and the results show the performance of binary PPM in the absence of IPI. In particular, we compare 1×1, 2×1 and 2×2 systems with receivers that are equipped with L-finger Rakes for $L = 1, 15$. The linear detection process described in the last section is used for separating the two transmitted data streams. Results show the utility of the additional spatial degree of freedom. For a $2 \times Q$ ST coded system with a L-finger Rake, the diversity order is equal to $2QL$ (from eq. (28)). Results show that a 2×2 system whose decisions are based simply on the first arriving multi-path component (the diversity order is equal to 4) outperforms the single-antenna systems combining up to 15 multi-path components (having a diversity order of 15). Moreover, even for systems that profit from a relatively high multi-path diversity order ($L = 15$), applying the proposed coding scheme results in important performance gains. In particular for $L = 15$, the 2×1 system outperforms the single-antenna system by 3 dB at a bit error rate of 10^{-3}.

VI. Conclusion

We investigated the problem of constructing a ST coding scheme that is suitable for TH-UWB systems using PPM. The proposed construction solves the problem of the non-existence of shape preserving constructions for PPM. The shape preserving constraint renders this code applicable with optical wireless communications as well.

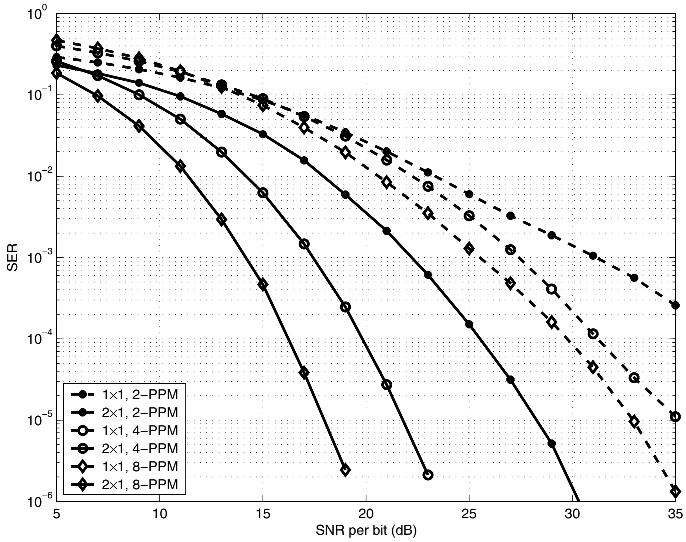

Fig. 1. Performance of single-antenna and 2×1 ST coded TH-UWB systems over CM2 with a 5-finger Rake and M-PPM constellations for $M=2,4,8$.

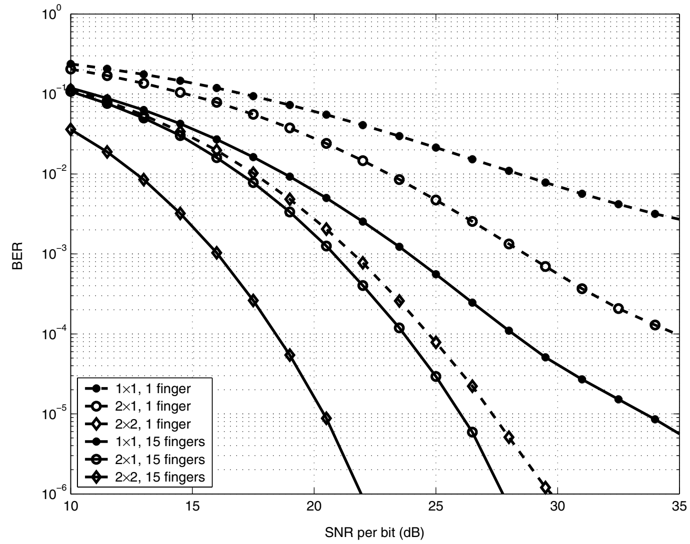

Fig. 2. Performance of single-antenna, 2×1 and 2×2 TH-UWB systems with orthogonal binary PPM and a L-finger Rake for $L=1,15$.

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