

Alamouti-Type Space-Time Coding for TH-UWB Systems with Unipolar Pulse Position Modulations

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Abstract—In this paper, we consider the problem of Space-Time (ST) coding with unipolar Pulse Position Modulations (PPM). The proposed code satisfies a large number of construction constraints that render it superior to the existing PPM encoding schemes. In particular, the proposed 2×2 code achieves a full transmit diversity order while transmitting at a rate of 1 PPM-symbol per channel use. The proposed scheme can be associated with M -ary PPM constellations for all even values of M without introducing any constellation extension. This renders the proposed scheme suitable for low cost carrier-less Ultra-Wideband (UWB) systems where information must be conveyed only by the time delays of the modulated sub-nanosecond pulses without introducing any amplitude amplification or phase rotation. Finally, the proposed scheme is symbol-by-symbol decodable where the information can be reconstituted by performing simple linear operations at the receiver side. A possible extension to transmitters equipped with three antennas is also discussed in situations where a certain number of feedback bits is available.

I. INTRODUCTION

There is a growing interest in applying Space-Time (ST) coding techniques on Time-Hopping Ultra-WideBand (TH-UWB) systems [1], [2]. For these systems, Pulse Position Modulation (PPM) is appealing since it is difficult to control the phase and amplitude of the very low duty-cycle sub-nanosecond pulses used to convey the information symbols.

Two different approaches can be adopted for the construction of ST codes suitable for PPM constellations. The first approach consists of applying one of the numerous ST codes proposed in the literature for QAM, PAM or PSK [3], [4]. In this context, it can be easily proven that these codes remain fully diverse with PPM [2]. However, the disadvantage is that all of these codes introduce phase rotations or amplitude amplifications in order to achieve a full transmit diversity order and, consequently, they introduce an additional constellation extension when associated with PPM. For example, while single-antenna PPM systems transmit unipolar pulses, applying the Alamouti code [3] with PPM necessitates the transmission of pulses having positive and negative polarities.

In order to overcome the above disadvantage, the second approach consists of constructing shape-preserving PPM-specific unipolar codes [5]–[7]. However, all of these codes are exclusive to binary PPM (or OOK) and they permit to achieve a full transmit diversity order because of the structure of such binary constellations that are composed of a signal and its opposite defined as the signal obtained by reversing the roles of “on” and “off” [5]. Various extensions to M -ary

constellations were proposed in [8]. However, this was realized at the expense of an increased receiver complexity since these codes do not admit a simple symbol-by-symbol decodability.

The first contribution of this paper is the proposition of a rate-1, fully diverse and shape-preserving ST block code for unipolar PPM with two transmit antennas. The advantage over [5]–[7] is that the proposed scheme can be associated with M -PPM for all even values of M . The advantage over [8] is that the proposed code admits a simple symbol-by-symbol maximum-likelihood decodability. Note that unlike [3], this simple decodability is realized even though the proposed scheme is not orthogonal. Inspired from [9], the second contribution consists of extending the proposed scheme to the case of three antennas when 1, 2 or 3 feedback bits are available. In this case, a transmit diversity order of three can be achieved with simple decodability. Note that in the absence of feedback, the only existing solution is exclusive to M -PPM with $M = 3$ or $M \geq 5$ [8] and, moreover, this 3×3 code must be associated with non-linear lattice decoders [10].

II. TWO TRANSMIT ANTENNAS WITH NO FEEDBACK

Consider a TH-UWB system where the transmitter is equipped with $P = 2$ antennas and the receiver is equipped with Q antennas. In what follows, we propose a minimal-delay diversity scheme that extends over two symbol durations. Denote by $s_p(t)$ the signal transmitted from the p -th antenna for $p = 1, \dots, P$. We propose the following structure for the transmitted signals:

$$s_1(t) = w(t - (p_1 - 1)\delta) + w(t - T_s - (p_2 - 1)\delta) \quad (1)$$

$$s_2(t) = w(t - (\pi(p_2) - 1)\delta) + w(t - T_s - (p_1 - 1)\delta) \quad (2)$$

where $p_i \in \{1, \dots, M\}$ corresponds to the modulation position of the i -th information symbol for $i = 1, 2$. Note that two M -PPM symbols are transmitted during two symbol durations and the proposed scheme transmits at a rate of one symbol per channel use (PCU).

In eq. (1) and eq. (2), $w(t)$ is the monocycle pulse waveform of duration T_w normalized to have an energy of E_s/P where E_s is the energy used to transmit one information symbol and the normalization by P insures the same transmission level as in the single-antenna case. The modulation delay δ corresponds to the separation between two consecutive PPM positions while T_s stands for the symbol duration. No reference to the TH sequence was made in eq. (1) and eq. (2)

since all antennas of the same user are supposed to share the same TH sequence resulting in the same average multi-user interference as in the single-antenna case.

The permutation function $\pi(\cdot)$ in eq. (2) is defined by:

$$\pi(m) = (m \bmod 2) + 2 \left\lfloor \frac{m-1}{2} \right\rfloor + 1 \quad (3)$$

where the function $\lfloor x \rfloor$ rounds the real number x to the nearest integer that is less than or equal to it.

From eq. (1) and eq. (2), the pulses transmitted from the two antennas during two consecutive symbol durations occupy the positions p_1 , p_2 and $\pi(p_2)$. Since $\pi(\cdot)$ defines a mapping over the elements of the set $\{1, \dots, M\}$, then $p_1, p_2, \pi(p_2) \in \{1, \dots, M\}$. Moreover, the transmission strategy described in eq. (1) and eq. (2) does not introduce any amplitude scaling. Consequently, during each symbol duration, only one unipolar pulse occupying one out of M possible positions is transmitted. Therefore, the proposed scheme does not introduce any extension to the M -PPM constellation for all values of M . In what follows, M is limited to take even values.

In what follows, we prove that associating the permutation given in eq. (3) with the transmission strategy given in eq. (1) and eq. (2) permits to achieve a full transmit diversity order with a simple symbol-by-symbol decodability.

M -ary PPM constellations are M -dimensional constellations where the information symbols are represented by M -dimensional vectors that belong to the following signal set:

$$\mathcal{C} = \{e_m ; m = 1, \dots, M\} \quad (4)$$

where e_m is the m -th column of the $M \times M$ identity matrix.

Designate by $a_i \triangleq [a_{i,1} \dots a_{i,M}]^T = e_{p_i} \in \mathcal{C}$ the M -dimensional vector representation of the i -th information symbol for $i = 1, 2$. Equations (1) and (2) can be written as:

$$s_1(t) = \sum_{m=1}^M a_{1,m} w(t - (m-1)\delta) + a_{2,m} w(t - T_s - (m-1)\delta) \quad (5)$$

$$s_2(t) = \sum_{m=1}^M a_{2,\pi(m)} w(t - (m-1)\delta) + a_{1,m} w(t - T_s - (m-1)\delta) \quad (6)$$

where $a_{i,m}$ is the m -th component of a_i with $a_{i,m} = 1$ if $m = p_i$ and $a_{i,m} = 0$ otherwise for $m = 1, \dots, M$.

The received signal at the q -th antenna can be written as:

$$r_q(t) = \sum_{p=1}^P s_p(t) * g_{q,p}(t) + n_q(t) \quad (7)$$

where $*$ stands for convolution and $n_q(t)$ is the noise at the q -th antenna which is supposed to be real AWGN with double sided spectral density $N_0/2$. $g_{q,p}(t)$ stands for the impulse response of the frequency selective channel between the p -th transmit antenna and the q -th receive antenna.

In order to take advantage from the rich multi-path diversity of the UWB channels, a L -th order Rake is used after each receive antenna. Designate by $y_{q,l,i,m}$ the decision variable collected at the l -th Rake finger of the q -th receive antenna

during the m -th position of the i -th symbol duration for $q = 1, \dots, Q$, $l = 1, \dots, L$, $i = 1, 2$ and $m = 1, \dots, M$. Each one of these $2QLM$ decision variables is given by:

$$y_{q,l,i,m} = \int_{-\infty}^{+\infty} r_q(t) w(t - \Delta_l - (i-1)T_s - (m-1)\delta) dt \quad (8)$$

where $\Delta_l \triangleq (l-1)T_w$ stands for the delay of the l -th finger of the Rake.

Designate by T_c the delay spread of the UWB channel ($T_c \gg T_w$). Inter-Symbol-Interference (ISI) can be eliminated by choosing $T_s \geq T_c + T_w$. In the same way, the received PPM constellation is orthogonal if the modulation delay verifies $\delta \geq T_c + T_w$. In what follows, we consider orthogonal received PPM constellations in the absence of ISI since only in this case the proposed scheme is linearly decodable. In this case, the decision variables given in eq. (8) can be written as:

$$y_{q,l,1,m} = h_{q,1,l} a_{1,m} + h_{q,2,l} a_{2,\pi(m)} + n_{q,l,1,m} \quad (9)$$

$$y_{q,l,2,m} = h_{q,2,l} a_{1,m} + h_{q,1,l} a_{2,m} + n_{q,l,2,m} \quad (10)$$

where $n_{q,l,i,m}$ stands for the noise term during the i -th symbol duration:

$$n_{q,l,i,m} = \int_{-\infty}^{+\infty} n_q(t) w(t - \Delta_l - (i-1)T_s - (m-1)\delta) dt \quad (11)$$

moreover, since $\delta \geq 2T_w$ and $\Delta_l - \Delta_{l-1} \geq T_w$, it can be easily proven that these noise terms are white. In eq. (9) and eq. (10), the channel coefficients are given by:

$$h_{q,p,l} = \int_{-\infty}^{+\infty} h_{q,p}(t) w(t - \Delta_l) dt \quad (12)$$

where $h_{q,p}(t) \triangleq g_{q,p}(t) * w(t)$ for $p = 1, \dots, P = 2$ and $q = 1, \dots, Q$.

Despite the absence of orthogonality between the transmitted data streams (since unipolar signals are transmitted), the decoding procedure at the receiver side can be simplified by constructing the following QLM decision variables:

$$z_{q,l,i,n} = y_{q,l,i,2n-1} - y_{q,l,i,2n} ; n = 1, \dots, M/2 \quad (13)$$

Based on the permutation rule given in eq. (3), it follows that $\pi(2n) = 2n-1$ and $\pi(2n-1) = 2n$ for $n = 1, \dots, M/2$. Consequently, replacing eq. (9) and eq. (10) in eq. (13) results in:

$$z_{q,l,1,n} = h_{q,1,l} s_{1,n} - h_{q,2,l} s_{2,n} + n'_{q,l,1,n} \quad (14)$$

$$z_{q,l,2,n} = h_{q,2,l} s_{1,n} + h_{q,1,l} s_{2,n} + n'_{q,l,2,n} \quad (15)$$

where:

$$s_{i,n} \triangleq a_{i,2n-1} - a_{i,2n} ; i = 1, 2 ; n = 1, \dots, M/2 \quad (16)$$

and $s_{i,n} \in \{0, \pm 1\}$ since $a_{i,1}, \dots, a_{i,M} \in \{0, 1\}$ for $i = 1, 2$. In the same way, the noise terms are given by: $n'_{q,l,i,n} = n_{q,l,i,2n-1} - n_{q,l,i,2n}$ for $i = 1, 2$.

Note that the input-output relations given in eq. (14) and eq. (15) are similar to the input-output relations verified by the orthogonal Alamouti code [3]. However, the orthogonal-like behavior of the proposed scheme is achieved mainly

because of the position permutations described in eq. (3) and the associated decoding technique without necessitating any polarity inversion of the transmitted PPM pulses.

Finally, the decisions taken on the information symbols will be based on the following M decision variables (for $n = 1, \dots, M/2$):

$$Z_{1,n} = \sum_{q=1}^Q \sum_{l=1}^L [h_{q,1,l} z_{q,l,1,n} + h_{q,2,l} z_{q,l,2,n}] \quad (17)$$

$$Z_{2,n} = \sum_{q=1}^Q \sum_{l=1}^L [-h_{q,2,l} z_{q,l,1,n} + h_{q,1,l} z_{q,l,2,n}] \quad (18)$$

Replacing equations (14) and (15) in equations (17) and (18) results in (for $i = 1, 2$ and $n = 1, \dots, M/2$):

$$Z_{i,n} = \left[\sum_{q=1}^Q \sum_{p=1}^2 \sum_{l=1}^L h_{q,p,l}^2 \right] s_{i,n} + N_{i,n} \quad (19)$$

where $N_{1,n} = \sum_{q=1}^Q \sum_{l=1}^L [h_{q,1,l} n'_{q,l,1,n} + h_{q,2,l} n'_{q,l,2,n}]$ and $N_{2,n} = \sum_{q=1}^Q \sum_{l=1}^L [-h_{q,2,l} n'_{q,l,1,n} + h_{q,1,l} n'_{q,l,2,n}]$. It can be easily proven that these noise terms are still white.

Since the modified symbols $s_{1,n}$ and $s_{2,n}$ given in eq. (16) can be equal to zero, then the first step in decoding the i -th information symbol consists of calculating the integer \hat{n}_i such that:

$$\hat{n}_i = \arg \max_{n=1, \dots, M/2} |Z_{i,n}| \quad ; \quad i = 1, 2 \quad (20)$$

Following from eq. (20), the reconstituted position of the i -th PPM information symbol is given by:

$$\hat{p}_i = \begin{cases} 2\hat{n}_i - 1, & Z_{i,\hat{n}_i} \geq 0; \\ 2\hat{n}_i, & Z_{i,\hat{n}_i} < 0. \end{cases} \quad ; \quad i = 1, 2 \quad (21)$$

in other words, the vector representation of the i -th reconstituted information symbol will be given by: $\hat{a}_i = e_{\hat{p}_i} \in \mathcal{C}$ for $i = 1, 2$ where \mathcal{C} is given in eq. (4).

From eq. (19), it follows that at high signal-to-noise ratios $Z_{i,n} \ll 1$ if and only if $|h_{q,p,l}| \ll 1$ for $q = 1, \dots, Q$, $p = 1, \dots, P = 2$ and $L = 1, \dots, L$. In other words, the information symbols a_1 and a_2 are lost only when the PQ sub-channels $g_{q,p}(t)$ suffer from fading over a duration LT_w . Therefore, the proposed scheme achieves full transmit, receive and multi-path diversities. Moreover, the proposed system profits from a simple symbol-by-symbol decodability that can be achieved by applying the linear operations described in equations (8), (13), (17), (18), (20) and (21).

We next adopt a more rigorous approach for proving that the proposed scheme is fully diverse based on the design criteria of [11]. Designate by $C(a_1, a_2)$ the $2M \times 2$ codeword whose $((p-1)M + m, i)$ -th entry corresponds to the amplitude of the pulse (if any) transmitted at the m -th position of the p -th antenna during the i -th symbol duration for $p = 1, 2$, $m = 1, \dots, M$ and $i = 1, 2$. Based on eq. (5) and eq. (6), $C(a_1, a_2)$ can be written as:

$$C(a_1, a_2) = \begin{bmatrix} a_{1,1} & \cdots & a_{1,M} & a_{2,\pi(1)} & \cdots & a_{2,\pi(M)} \\ a_{2,1} & \cdots & a_{2,M} & a_{1,1} & \cdots & a_{1,M} \end{bmatrix}^T \quad (22)$$

Following from the linearity of the code and from [2], [11], the code is fully diverse if:

$$\text{rank}[C(a_1 - a'_1, a_2 - a'_2)] = 2 \quad \forall \quad (a_1, a_2) \neq (a'_1, a'_2) \quad (23)$$

where a_1, a'_1, a_2 and a'_2 belong to the set \mathcal{C} given in eq. (4).

In what follows, $C(a_1 - a'_1, a_2 - a'_2)$ will be denoted by C when there is no ambiguity. On the other hand, $\text{rank}(C) < 2$ if there exists a nonzero real number k such that $C_2 = kC_1$ where C_i stands for the i -th column of C for $i = 1, 2$. Moreover, given that the elements of C belong to the set $\{0, \pm 1\}$, then $k = \pm 1$. Let n be an odd integer that belongs to $\{1, \dots, M\}$. Investigating the n -th and $(M+n)$ -th rows of C respectively, the relation $C_2 = kC_1$ implies that:

$$a_{2,n} - a'_{2,n} = k(a_{1,n} - a'_{1,n}) \quad (24)$$

$$a_{1,n} - a'_{1,n} = k(a_{2,\pi(n)} - a'_{2,\pi(n)}) \quad (25)$$

Combining the last equations results in:

$$a_{2,n} - a'_{2,n} = k^2(a_{2,\pi(n)} - a'_{2,\pi(n)}) = a_{2,n+1} - a'_{2,n+1} \quad (26)$$

since $k^2 = 1$ and $\pi(n) = n+1$ when $n \in \{1, \dots, M\}$ is odd.

Consequently, C is rank deficient if and only if:

$$a_{2,n} - a_{2,n+1} = a'_{2,n} - a'_{2,n+1} \quad ; \quad n \in \{1, \dots, M\} \text{ is odd} \quad (27)$$

Given that $(a_{2,n}, a_{2,n+1}) \in \{(0, 0), (0, 1), (1, 0)\}$ and $(a'_{2,n}, a'_{2,n+1})$ belongs to the same set, then eq. (27) can be verified if and only if $a_{2,n} = a'_{2,n}$ and $a_{2,n+1} = a'_{2,n+1}$ for all odd integers n in $\{1, \dots, M\}$. Moreover, from eq. (24), $a_{2,n} = a'_{2,n}$ implies that $a_{1,n} = a'_{1,n}$. In the same way, from eq. (25), $a_{2,n+1} = a'_{2,n+1}$ implies that $a_{1,n+1} = a'_{1,n+1}$. Finally, $C(a_1 - a'_1, a_2 - a'_2)$ is rank deficient only when $a_1 = a'_1$ and $a_2 = a'_2$. Therefore, eq. (23) is verified proving that the code is fully diverse.

Note that for non-orthogonal constellations eq. (9) and eq. (10) do not hold and the advantage of symbol-by-symbol decodability will be lost. On the other hand, since eq. (23) is verified independently from the orthogonality of the constellation, then the proposed scheme achieves full transmit diversity with non-orthogonal constellations as well. However, in this case, the detection necessitates the implementation of more sophisticated algorithms such as those proposed in [10].

III. THREE TRANSMIT ANTENNAS WITH FEEDBACK

When a certain number of feedback bits is available, the scheme proposed in the previous section for orthogonal constellations with two transmit antennas can be extended to achieve full diversity with three transmit antennas. As in [9], the proposed coding extends over two symbol durations.

1) *One bit feedback*: In this case, the signals transmitted from the first and second antennas keep the same expressions as in eq. (5) and eq. (6) respectively. The signal transmitted from the third antenna is given by:

$$s_3(t) = \sum_{m=1}^M a_{1,\sigma(m)} w(t - (m-1)\delta) + a_{2,\sigma(m)} w(t - T_s - (m-1)\delta) \quad (28)$$

where the function $\sigma(\cdot)$ can be equal either to $\pi(\cdot)$ or to the identity function $\mathbf{1}(\cdot)$ depending on the channel realization.

In this case, the decision variables given in eq. (9) and eq. (10) will take the following values:

$$y_{q,l,1,m} = [h_{q,1,l}a_{1,m} + h_{q,3,l}a_{1,\sigma(m)}] + h_{q,2,l}a_{2,\pi(m)} + n_{q,l,1,m} \quad (29)$$

$$y_{q,l,2,m} = h_{q,2,l}a_{1,m} + [h_{q,1,l}a_{2,m} + h_{q,3,l}a_{2,\sigma(m)}] + n_{q,l,2,m} \quad (30)$$

On the other hand (for $i = 1, 2$):

$$a_{i,\sigma(2n-1)} - a_{i,\sigma(2n)} = \begin{cases} s_{i,n}, & \sigma(\cdot) = \mathbf{1}(\cdot); \\ -s_{i,n}, & \sigma(\cdot) = \pi(\cdot). \end{cases} \quad (31)$$

where $s_{i,n}$ is given in eq. (16).

Consequently, following from equations (29)-(31), the modified decision variables given in eq. (14) and eq. (15) will take the following values:

$$z_{q,l,1,n} = (h_{q,1,l} + bh_{q,3,l})s_{1,n} - h_{q,2,l}s_{2,n} + n'_{q,l,1,n} \quad (32)$$

$$z_{q,l,2,n} = h_{q,2,l}s_{1,n} + (h_{q,1,l} + bh_{q,3,l})s_{2,n} + n'_{q,l,2,n} \quad (33)$$

where:

$$b = \begin{cases} +1, & \sigma(\cdot) = \mathbf{1}(\cdot); \\ -1, & \sigma(\cdot) = \pi(\cdot). \end{cases} \quad (34)$$

Finally, replacing $h_{q,1,l}$ by $h_{q,1,l} + bh_{q,3,l}$ in eq. (17) and eq. (18), we conclude that the final decision variables $Z_{i,n}$ are related to the information symbols by the following relation:

$$\begin{aligned} Z_{i,n} &= \left[\sum_{q=1}^Q \sum_{l=1}^L [(h_{q,1,l} + bh_{q,3,l})^2 + h_{q,2,l}^2] \right] s_{i,n} + N_{i,n} \\ &= \left[\sum_{q=1}^Q \sum_{p=1}^3 \sum_{l=1}^L h_{q,p,l}^2 + 2b \sum_{q=1}^Q \sum_{l=1}^L h_{q,1,l}h_{q,3,l} \right] s_{i,n} + N_{i,n} \end{aligned} \quad (35)$$

To maximize the SNR, a convenient choice of the mapping function $\sigma(\cdot)$ given in eq. (28) based on the feedback bit is:

$$\sigma(\cdot) = \begin{cases} \mathbf{1}(\cdot), & \sum_{q=1}^Q \sum_{l=1}^L h_{q,1,l}h_{q,3,l} \geq 0; \\ \pi(\cdot), & \text{otherwise.} \end{cases} \quad (36)$$

2) *Two bits feedback*: In this case, the signals transmitted from the first two antennas are given in eq. (5) and eq. (6) respectively. Depending on the channel realization, the signal transmitted from the third antenna can either take the value given in eq. (28) or can take the following value:

$$\begin{aligned} s_3(t) &= \sum_{m=1}^M [a_{2,\sigma(\pi(m))}w(t - (m-1)\delta) \\ &\quad + a_{1,\sigma(m)}w(t - T_s - (m-1)\delta)] \end{aligned} \quad (37)$$

where $\sigma \equiv \mathbf{1}$ or $\sigma \equiv \pi$ depending on the channel realization.

When the signal transmitted from the third antenna takes the value given in eq. (37), the information symbols are determined from the following decision variables:

$$Z_{i,n} = \left[\sum_{q=1}^Q \sum_{p=1}^3 \sum_{l=1}^L h_{q,p,l}^2 + 2b \sum_{q=1}^Q \sum_{l=1}^L h_{q,2,l}h_{q,3,l} \right] s_{i,n} + N_{i,n} \quad (38)$$

where b is defined in eq. (34).

From eq. (35), it follows that when $s_3(t)$ takes the value given in eq. (28), then the first antenna is coupled with the third antenna. In this case, the value of the mapping function $\sigma(\cdot)$ is chosen so that the signals from these antennas will combine coherently. When $s_3(t)$ takes the value given in eq. (37), then the second antenna is coupled with the third antenna. Therefore, the third antenna must transmit the signal given in eq. (28) when $|\sum_{q,l} h_{q,1,l}h_{q,3,l}| \geq |\sum_{q,l} h_{q,2,l}h_{q,3,l}|$ and the signal given in eq. (37) otherwise. After this selection was made, the mapping function is chosen as $\sigma(\cdot) = \mathbf{1}(\cdot)$ when $\sum_{q,l} h_{q,i,l}h_{q,3,l} \geq 0$ and $\sigma(\cdot) = \pi(\cdot)$ otherwise where $i = 1$ or $i = 2$ depending on whether eq. (28) or eq. (37) was chosen.

3) *Three bits feedback*: In this case, the transmitter has the choice of transmitting the signals given in either eq. (6) and eq. (28), eq. (6) and eq. (37) or eq. (28) and eq. (6) from the second and third antennas respectively. In the last case, the first antenna is coupled with the second antenna. The signal transmitted from the first antenna always takes the value given in eq. (5).

The selection among these three possibilities depends on the value of $(i, j) \in \{(1, 3), (2, 3), (1, 2)\}$ that maximizes the quantity $|\sum_{q,l} h_{q,i,l}h_{q,j,l}|$. The mapping function is chosen as: $\sigma \equiv \mathbf{1}$ when $\sum_{q,l} h_{q,\tilde{i},l}h_{q,\tilde{j},l} \geq 0$ and $\sigma \equiv \pi$ otherwise where (\tilde{i}, \tilde{j}) is the value of (i, j) that maximizes $|\sum_{q,l} h_{q,i,l}h_{q,j,l}|$.

IV. SIMULATIONS AND RESULTS

Simulations are performed over the IEEE 802.15.3a channel model recommendation CM2 [12]. To insure the orthogonality of the received constellation, the modulation delay is chosen to verify $\delta = 100$ ns which is larger than the maximum delay spread of the UWB channel [12].

Fig. 1 shows the performance of the proposed scheme with 4-PPM. In particular, we compare 1×1 , 2×1 and 2×2 systems with receivers that are equipped with L -finger Rakes for $L = 1, 20$. Results show the high performance levels and the enhanced diversity order achieved by the proposed scheme. As expected, the performance gain is less significant for systems that profit from high multi-path diversity orders. For example, the gain of the 2×1 system over single-antenna systems drops from 5 dB at an error rate of 10^{-2} to 1 dB at 10^{-3} as the number of fingers increases from 1 to 20.

Fig. 2 shows the performance of 8-PPM with receivers that are equipped with a 10-finger Rake. As expected, increasing the number of feedback bits improves the error performance of the 3×1 systems. The highest improvement, with respect to the 2×1 system, results from the first feedback bit that results in a gain of 2.5 dB at 10^{-3} . Compared to this improvement, the additional feedback bits result in marginal gains.

To highlight the advantages of ST coding with UWB systems, Fig. 3 compares systems having the same overall diversity order that is equal to the product PQL (with $Q = 1$ in this case). Binary PPM is used and for a fair comparison, we plot the bit error rates (BER) as a function of PL for SNRs of 20 dB and 25 dB (note that the gap between the 1×1 and $P \times 1$ systems will increase if the BER was plotted as a function of

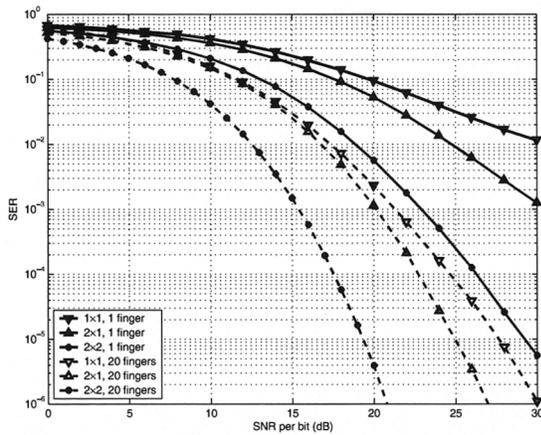


Fig. 1. The proposed scheme versus single-antenna systems with 4-PPM.

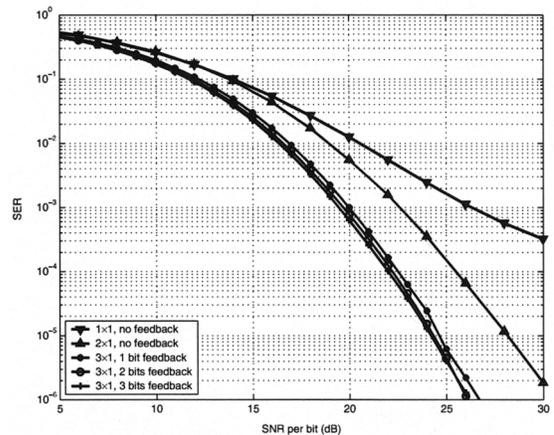


Fig. 2. Performance of 8-PPM with 1 receive antenna and a 10-finger Rake.

L). For example, a 1×1 system equipped with 60 fingers achieves a BER of 10^{-3} at a SNR of 20 dB. In this case, the 2×1 system with only 30 fingers achieves a better BER in the order of 3×10^{-4} while the 3×1 system with 3 bits of feedback and 20 fingers achieves a BER of 7×10^{-5} . Fig. 3 shows that exploiting the transmit diversity by increasing the number of transmit antennas can be more beneficial than enhancing the multi-path diversity by increasing the number of Rake fingers even though there is no increase in the energy capture. This follows from the fact that consecutive multi-path components of the same sub-channel can be simultaneously faded because of cluster and channel shadowing [12]. For SNR=20 dB, Fig. 3 shows that it is possible to achieve error rates smaller than 10^{-4} by increasing the number of transmit antennas while it was impossible to reach such error rates with single-antenna systems with any number of Rake fingers.

V. CONCLUSION

We investigated the problem of ST coding with TH-UWB systems using PPM. The proposed scheme has a full rate and is fully diverse resulting in high performance levels over the realistic indoor UWB channels. Moreover, this scheme is adapted to unipolar transmissions and, consequently, does not necessitate additional constraints on the RF circuitry to control the phase or the amplitude of the very low duty cycle sub-nanosecond pulses. At the receiver, a simple linear decoder assures a fast and optimal separation of the transmitted data streams. The shape preserving constraint renders this code applicable with optical wireless communications as well.

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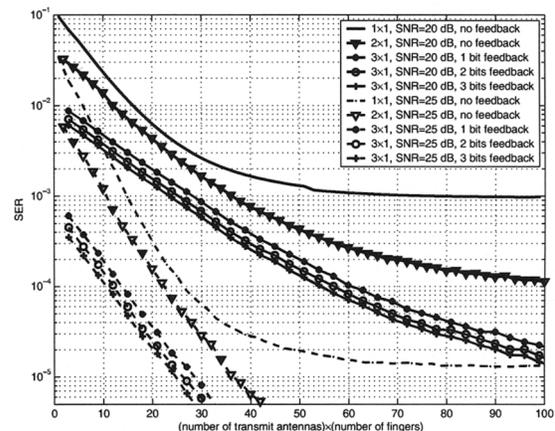


Fig. 3. Transmit diversity versus multi-path diversity with 2-PPM.

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