Novel High-Rate Transmit Diversity Schemes for MIMO IR-UWB and Delay-Tolerant Decode-and-Forward IR-UWB Transmissions

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***Abstract*— In this paper, we consider the problem of Space-Time (ST) coding for Multiple-Input-Multiple-Output (MIMO) and Decode-and-Forward (DF) Impulse-Radio Ultra-Wideband (IR-UWB) systems. In particular, we propose three novel transmission schemes that are suitable for such systems. The first scheme is a multiplexing scheme that transmits at a rate higher than that of spatial multiplexing with the same complexity. The second scheme is a unipolar fully diverse and totally-real scheme that is suitable for Pulse Position Modulation (PPM). The rate of this scheme exceeds that of full-rate ST codes with a lower decoding complexity. The third scheme corresponds to a 2×2 full-rate and fully diverse ST code that has a non-vanishing coding gain when associated with Pulse Amplitude Modulation (PAM). These diversity schemes are appealing since they are tolerant to the asynchronization between the relays in DF networks.**

I. Introduction

Multiple-Input-Multiple-Output (MIMO) techniques and cooperative systems are merging as candidate solutions for enhancing the data rate, performance and communication distance of Impulse-Radio Ultra-Wideband (IR-UWB) systems [1]–[3]. In this context, several Space-Time (ST) coding techniques were proposed for collocated MIMO-UWB systems [4], [5] and for distributed cooperative UWB networks [6]. The nature of the UWB transmissions imposes an additional constraint on the ST code design. In fact, it is difficult to control the phase of the sub-nanosecond UWB pulses that occupy several GHz of bandwidth resulting in the constraint that ST codes constructed for UWB must be totally-real. In this context, the conventional ST codes constructed for QAM [7] and that are based on phase rotations can not be applied with UWB. Consequently, to be suitable for IR-UWB, the different solutions that we propose in this paper are all totally-real.

IR-UWB systems are often associated with either Pulse Position Modulation (PPM) or Pulse Amplitude Modulation (PAM). Consider first the problem of ST coding with M-PPM in the case where the transmitter is equipped with P antennas. In this context, the existing PPM codes can be classified into two categories. The first category of codes is unipolar and, hence, can be applied with PPM without introducing any constellation expansion [4]. While such solutions are appealing since, as in single-antenna systems, only one unipolar pulse is transmitted from each antenna during each symbol duration, the main disadvantage is that these codes are rate-1 codes that transmit $\log_2(M)$ bits Per Channel Use (PCU). The second category of codes corresponds to full-rate codes that transmit at the rate of $P\log_2(M)$ bits PCU [5]. However, the algebraic rotations used in such solutions break the structure of PPM constellations necessitating non-unipolar transmissions.

The first contribution of this paper is that we propose a very high data-rate fully-diverse PPM-specific ST code that transmits at a rate of $M\log_2(P)$ bits PCU while maintaining unipolar transmissions. The advantage over [4] is that the proposed scheme has a higher rate since, in practical systems, the number of transmit antennas P takes only limited values because of the cost constraints while the number of modulation positions M can take relatively large values. The advantage over the existing full-rate codes [5] resides in the fact that the proposed scheme maintains unipolar transmissions. An additional advantage is that the proposed scheme admits a reduced decoding complexity since the decoding procedure involves the joint decoding of PM real dimensions rather than P^2M real dimensions as in [5]. Finally, unlike [4], [5], the proposed code can be applied for all values of P and M.

The advantages of this proposed scheme are rendered possible because of the use of Hermite pulses at the transmitter side. MIMO systems using Hermite pulses were previously reported in [3], [8]. In [3] two orthogonal pulses used for the channel estimation and data transmissions are transmitted simultaneously thus reducing the decoding delays at the receiver. In [8], associating the use of Hermite pulses with real algebraic rotations resulted in a fully-diverse scheme that achieves a rate of $M\log_2(P)$ bits PCU. The advantage of the proposed scheme over [3], [8] resides mainly in the enhanced data rate. Moreover, unlike [8], this scheme is unipolar. In the presence of only one pulse generator at the transmitter, this diversity scheme reduces to a multiplexing scheme that outperforms Spatial-Multiplexing (SM) since it transmits at a higher rate with the same transceiver complexity.

On the other hand, neighboring UWB terminals equipped with a single antenna can cooperate with each other in order to benefit from the spatial diversity in a distributed manner. In this context, the Decode-and-Forward (DF) protocol constitutes an interesting cooperation strategy [9]. DF cooperation is composed of two phases. During the first phase, the source terminal broadcasts its message to the neighboring relays. At

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a second time, these relays decode this message and transmit their corresponding encoded data streams simultaneously. During this second phase, each relay can simply transmit one row of a full-rate ST codeword.

Consequently, the codes proposed in [5] can be readily applied with DF systems. In this context, the superiority of the proposed PPM-specific code resides in the fact that it is delay-tolerant. According to the definition given in [10], a DF scheme is delay-tolerant if it keeps its diversity advantage even if the cooperating relays are asynchronous. While the codes proposed in [10] are complex-valued and extend over more than P symbol durations, the PPM-specific proposed scheme is real-valued and extends over only one symbol duration.

The second contribution of the paper is that we propose a 2×2 ST code for PAM constellations. This code satisfies a large number of construction constraints: it is totally-real, it has a full rate and it achieves a full transmit diversity order with a non-vanishing coding gain. Moreover, unlike the codes constructed from cyclic division algebras [11], the proposed PAM code is delay-tolerant.

II. SYSTEM MODEL

A. Transmitter Structure

Assume that N pulse generators are available at the transmitter and denote by $w_n(t)$ the waveform of the n-th generator for $n = 1, \dots, N$. As will be explained later, the transmitter can take advantage from the presence of these generators to enhance the diversity order of the UWB system. The case of more conventional transmitters equipped with one pulse generator will be treated as a special case by fixing $N = 1$.

Consider a MIMO IR-UWB system where the transmitter is equipped with P antennas. Assume that M positions (or time slots) are available for data modulation within each symbol duration. M is often fixed by the complexity constraints on the transceiver since the dimensionality of the position-modulated constellation as well as the number of matched filters at the receiver both scale with M [5]. We also assume that the amplitudes of the transmitted pulses can take M' possible values. In what follows, we present a general system model that can be applied for all values of M and M'. This general model can be applied in the special case of M-PPM (resp. M'-PAM) constellations by fixing $M' = 1$ (resp. $M = 1$).

Consider a transmit diversity scheme that extends over J symbol durations. The corresponding expression of the signal transmitted from the p-th antenna can be written as:

$$s_p(t) = \frac{1}{\sqrt{PN}} \sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{m=1}^{M} a_{p,n,j,m} w_n(t-(j-1)T_s-(m-1)\delta) \quad (1)$$

where the normalizing factor $\frac{1}{\sqrt{PN}}$ insures the same transmission level as with single-antenna systems equipped with one pulse generator. δ is the PPM modulation delay and T_s is the symbol duration. $a_{p,n,j,m}$ stands for the amplitude of the pulse having the waveform $w_n(t)$ and emitted from the p-th transmit antenna during the m-th PPM position of the j-th symbol duration. The encoding diversity scheme is completely determined from the mapping of the information symbols to $a_{p,n,j,m} \in \{2m' - 1 - M' \ ; \ m' = 1, \dots, M'\}$ for $p = 1, \dots, P$, $n = 1, \dots, N$, $j = 1, \dots, J$ and $m = 1, \dots, M$.

Note that δ can be much smaller than the channel delay spread resulting in Inter-Pulse-Interference (IPI). On the other hand, we assume that T_s is larger than the channel delay spread resulting in no Inter-Symbol-Interference (ISI). No reference to the time-hopping sequence was made in eq. (1) since the proposed encoding schemes do not depend on the number of time-hopped pulses used to transmit one information symbol. Moreover, it is assumed that all the transmit antennas of the same user share the same pseudo-random TH sequence resulting in the same average multi-user interference as in the Single-Input-Single-Output (SISO) case.

B. Receiver Structure

We assume that the receiver is equipped with Q antennas and that each antenna is followed by an L-th order Rake that combines the first L arriving multi-path components.

In what follows, the indices $q \in \{1, \dots, Q\}$, $p \in \{1, \dots, P\}$ and $l \in \{1, \dots, L\}$ will correspond to the receive antenna, transmit antenna and the Rake finger respectively. In the same way, $j \in \{1, \dots, J\}$ stands for the symbol index, $n \in \{1, \dots, N\}$ for the waveform index and $m \in \{1, \dots, M\}$ for the PPM position index.

The signal received at the q-th antenna can be written as:

$$r_q(t) = \sum_{p=1}^{P} \sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{m=1}^{M} a_{p,n,j,m} h'_{q,p,n}(t-(j-1)T_s-(m-1)\delta) + n_q(t) \quad (2)$$

where $h'_{q,p,n}(t)$ stands for the convolution of $w_n(t)$ with the impulse response of the channel between antennas p and q. In eq. (2), $n_q(t)$ is the noise at the q-th antenna and it is assumed to be real AWGN. Note that the normalizing factor $\sqrt{PN}$ in eq. (1) was omitted for simplicity; in fact, this term can be included in the variance of $n_q(t)$ which is equal to $PNN_0/2$ (where N_0 stands for the noise power spectral density).

The receiver consists of a bank of correlators that collects $JQLNM$ decision variables that are given by:

$$y_{j,q,l,n,m} = \int_0^{T_s} r_q(t) \widetilde{w}_{j,l,n,m}(t) \mathrm{d}t \quad (3)$$

where $\widetilde{w}_{j,l,n,m}(t)$ is a reference signal given by:

$$\widetilde{w}_{j,l,n,m}(t) = w_n\left(t - (j-1)T_s - \Delta_l - (m-1)\delta\right) \quad (4)$$

where $\Delta_l \triangleq (l-1)T_w$ is the delay of the l-th Rake finger.

Replacing eq. (2) in eq. (3) results in:

$$y_{j,q,l,n,m} = \sum_{p'=1}^{P} \sum_{n'=1}^{N} \sum_{m'=1}^{M} a_{p',n',j,m'} \cdot h_{q,p',n',n}\left(\Delta_l + (m-m')\delta\right) + n_{j,q,l,n,m} \quad (5)$$

where $n_{j,q,l,n,m} \triangleq \int_0^{T_s} n_q(t) \widetilde{w}_{j,l,n,m}(t) \mathrm{d}t$ and:

$$h_{q,p,n',n}(\tau) \triangleq \int_0^{T_s} h'_{q,p,n'}(t) w_n(t-\tau) \mathrm{d}t \quad (6)$$

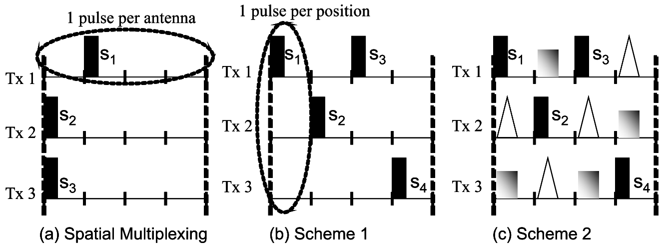

Fig. 1. Schematic representation with $P = 3$ transmit antennas and $M = 4$ modulation positions. (a): Spatial-Multiplexing with $s_1 = 2$ and $s_2 = s_3 = 1$; note that $s_p \in \{1, \dots, M\}$ for $p = 1, \dots, P$. (b): Scheme 1 with $s_1 = 1$, $s_2 = 2$, $s_3 = 1$ and $s_4 = 3$; note that $s_m \in \{1, \dots, P\}$ for $m = 1, \dots, M$. (c): Scheme 2 with $s_1 = 1$, $s_2 = 2$, $s_3 = 1$ and $s_4 = 3$; rectangles, triangles and squares correspond to the first, second and third waveforms respectively.

The ST encoding scheme will be determined by the choice of the $PMN \times J$ codeword $\mathcal{A}$ whose $((n-1)PM + (p-1)M + m, j)$-th entry is equal to $a_{p,n,j,m}$. Based on the above notations, the linear dependence between the baseband inputs and outputs of the channel can be expressed as:

$$\mathcal{Y} = \mathcal{H}\mathcal{A} + \mathcal{N} \tag{7}$$

where $\mathcal{Y}$ is the $QLNM \times J$ decision matrix whose $((q-1)LNM + (l-1)NM + (n-1)M + m, j)$-th element is equal to $y_{j,q,l,n,m}$. $\mathcal{N}$ stands for the noise matrix that is constructed in the same way as $\mathcal{Y}$.

In eq. (7), $\mathcal{H}$ stands for the $QLNM \times PNM$ channel matrix. This matrix can be written as: $\mathcal{H} = [\mathcal{H}_1^T \ \cdots \ \mathcal{H}_Q^T]^T$ where $\mathcal{H}_q = [\mathcal{H}_{q,1}^T \ \cdots \ \mathcal{H}_{q,L}^T]^T$ is the $LNM \times PNM$ matrix corresponding to the q-th receive antenna for $q = 1, \dots, Q$. $\mathcal{H}_{q,l}$ is a $NM \times PNM$ that can be written as: $\mathcal{H}_{q,l} = [\mathcal{H}_{q,l,1} \ \cdots \ \mathcal{H}_{q,l,P}]$ where $\mathcal{H}_{q,l,p}$ is a $NM \times NM$ matrix whose $((n-1)M + m, (n'-1)M + m')$-th element is equal to $h_{q,p,n',n}(\Delta_l + (m - m')\delta)$.

III. Codes Constructions

As indicated before, the diversity scheme is completely determined by the choice of the $PNJM$ symbols $a_{p,n,j,m}$ for $p = 1, \dots, P$, $n = 1, \dots, N$, $j = 1, \dots, J$ and $m = 1, \dots, M$. In an equivalent way, the diversity scheme is determined by the choice of the $PMN \times J$ codeword $\mathcal{A}$. In what follows, we propose three schemes that can be applied with MIMO IR-UWB systems. The first two schemes are adapted to unipolar transmissions ($M' = 1$) since the information is conveyed by the presence or absence of UWB pulses and these schemes can be applied even with $M = 1$. The third scheme is specific to M'-ary PAM ($M = 1$). The extension to DF systems as well as the delay tolerance of the proposed schemes will be discussed in the next section. The proposed scheme-1 and scheme-2 are described schematically in Fig. (1). In this figure, we also show the schematic representation of Spatial Multiplexing (SM) for comparison.

A. Scheme 1: Pulse Antenna Modulation

This scheme is specific to mono-pulse systems ($N = 1$) and it extends over $J = 1$ symbol duration. Consequently, symbols $a_{p,n,j,m}$ will be denoted by $a_{p,m}$ for simplicity.

Designate by $s_1, \dots, s_M \in \{1, \dots, P\}$ M P-ary symbols. The first scheme that we propose is given by:

$$a_{p,m} = \delta_{p,s_m} \quad ; \quad p = 1, \dots, P \ ; \ m = 1, \dots, M \tag{8}$$

where $\delta_{i,j}$ stands for Kronecker's delta function ($\delta_{i,j} = 1$ for $i = j$ and $\delta_{i,j} = 0$ for $i \neq j$).

The scheme proposed in eq. (8) corresponds to transmitting a pulse from antenna s_m during the m-th PPM position. In other words, instead of pulsing each transmit antenna during exactly one modulation position (as in the case of single-antenna systems and SM), scheme 1 corresponds to pulsing exactly one transmit antenna during each modulation position.

The interest of scheme 1 resides in its capability of achieving very high transmission rates. In particular, the number of bits transmitted PCU is:

$$\mathcal{R}_1 = M \log_2(P) \tag{9}$$

This rate has to be compared with the rate of M-PPM single antenna systems (that is equal to $\log_2(M)$ bits PCU) and to the rate of MIMO SM systems (that is equal to $P \log_2(M)$ bits PCU). Given that practical MIMO systems are equipped with a limited number of antennas, then M is often much greater than P and scheme 1 presents the main advantage of an enhanced data rate with respect to SM. For example, for modulation over 16 positions with 2 transmit antennas, scheme 1 transmits two times faster than the SM scheme.

Note that due to the absence of any symbol repetitions in eq. (8), scheme 1 can be viewed as a multiplexing scheme that does not take advantage from the transmit diversity (which is also the case of SM). Finally, since for scheme 1, in the average, M/P pulses are transmitted from each antenna, the signal in eq. (1) must be normalized by $\sqrt{P/M}$ to ensure the same transmission level compared to SISO M-PPM systems.

B. Scheme 2: Pulse Antenna Modulation Using Hermite Pulses

Scheme 2 extends over $J = 1$ symbol duration and is specific to transmitters that are equipped with $N = P$ pulse generators. The n-th waveform is chosen to be the modified Hermite polynomial of order n [12]:

$$w_n(t) = (-1)^n \exp\left(\frac{t^2}{4\Gamma}\right) \frac{\mathrm{d}^n}{\mathrm{d}t^n} \left(\exp\left(-\frac{t^2}{2\Gamma}\right)\right) \tag{10}$$

where Γ regulates the width of the pulses and it is chosen to be independent from n. This choice implies that higher order pulses will have larger durations but the orthogonality between the P pulses is conserved. Multiplying eq. (10) by a sinusoidal waveform along with an appropriate choice of Γ [12] results in pulses respecting the FCC mask.

For simplicity, symbols $a_{p,n,j,m}$ will be denoted by $a_{p,n,m}$ that will be encoded according to the following relation:

$$a_{p,n,m} = \delta_{p,\sigma^{n-1}(s_m)} \tag{11}$$

where $s_1, \dots, s_M \in \{1, \dots, P\}$ and $\sigma^k(.)$ stands for the cyclic permutation of order k over P elements and it is given by:

$$\sigma^k(i) = (i + k - 1) \mod P + 1 \tag{12}$$

It is evident that scheme 2 transmits at the rate given in eq. (9) and that it reduces to scheme 1 for $N = 1$.

Eq. (11) can be interpreted as follows: during the m-th modulation position, each one of the antennas $s_m, \sigma(s_m), \ldots, \sigma^{P-1}(s_m)$ transmits a unipolar pulse having the waveform $w_1(t), w_2(t), \ldots, w_P(t)$ respectively. Since the functions $\{\sigma^k(p)\}_{k=0}^{P-1}$ span the entire set $\{1, \ldots, P\}$ for all values of p, this means that the same information symbol (s_m) will be transmitted from all of the antennas. Since, during each modulation position, the different antennas are forced to transmit orthogonal pulses this means that P independent replicas of s_m will be available at the receiver side by simply projecting the received signal over the orthogonal basis formed from the set of different pulse waveforms (as indicated in eq. (3)). Consequently, scheme 2 transmits at the high rate of $M \log_2(P)$ bits PCU while profiting from a full transmit diversity order.

Note that the above argument was made assuming that the waveforms are orthogonal. While this assumption holds at the output of the transmitter, this orthogonality will not be complete at the receiver side mainly because of the interference between the different modulation positions (IPI). In the presence of IPI, the P replicas that scheme 2 provides to the receiver will not be completely independent, however, they will always be useful to enhance the overall diversity of the system. Since the system model of MIMO IR-UWB systems given in section II is intractable and does not lend itself to a simple analytical analysis, the above claim will be verified through a numerical analysis since the effect of IPI is included in all the simulation results that we present.

C. Scheme 3: Full-Rate ST Code

Scheme 3 can be applied with mono-pulse systems ($N = 1$). It corresponds to a minimal-delay ($J = P$) ST code that can be applied with M'-PAM constellations for all values of M'.

Denote by $a_1, \ldots, a_4$ four M'-PAM symbols. The encoding scheme is given by:

$$\mathcal{A} \triangleq DA = \frac{1}{\sqrt{3}} \begin{bmatrix} \left(1+\theta^2\right)^{-1/2} & 0 \\ 0 & \left(1+\theta_1^2\right)^{-\frac{1}{2}} \end{bmatrix} . \begin{bmatrix} a_1 + \theta a_2 + \sqrt{2} a_3 + \sqrt{2}\theta a_4 & -\left(a_1 + \theta a_2 - \sqrt{2} a_3 - \sqrt{2}\theta a_4\right) \\ a_1 + \theta_1 a_2 + \sqrt{2} a_3 + \sqrt{2}\theta_1 a_4 & a_1 + \theta_1 a_2 - \sqrt{2} a_3 - \sqrt{2}\theta_1 a_4 \end{bmatrix} \tag{13}$$

where $\theta \triangleq \frac{1+\sqrt{5}}{2}$ and $\theta_1 \triangleq \frac{1-\sqrt{5}}{2}$ stand for the Golden number and its conjugate respectively. In eq. (13), D is a diagonal matrix introduced for normalization purposes.

Proposition 1: The code proposed in eq. (13) is fully diverse and achieves the non-vanishing coding gain of $\frac{8}{3\sqrt{5}}$ with M'-PAM constellations for all values of M'.

Proof: From eq. (13), the codeword $\mathcal{A}$ has the same rank as the matrix A. This matrix can be written as:

$$\begin{aligned} A &= U A_0 V \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} a_1 + \theta a_2 & a_3 + \theta a_4 \\ 2\left(a_3 + \theta_1 a_4\right) & a_1 + \theta_1 a_2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ \sqrt{2} & \sqrt{2} \end{bmatrix} \end{aligned} \tag{14}$$

implying that A can be obtained from A_0 by performing linear combinations of its rows and its columns. Since such operations do not change the rank of a matrix, then A and A_0 have the same rank.

Now inspecting the matrix A_0, we observe that it has the same structure as the codes constructed from cyclic division algebras [11]. The corresponding cyclic algebra is denoted by $\mathcal{D}(\mathbb{K}/\mathbb{Q}, \sigma, \gamma)$ where $\gamma = 2$ and where the second order cyclic field extension $\mathbb{K} = \mathbb{Q}(\theta)$ has a Galois group $Gal(\mathbb{K}/\mathbb{Q}) = \langle\sigma\rangle$ with $\sigma^2 = 1$ and $\sigma(\theta) = \theta_1$. Non-zero codewords A_0 have a full rank if the algebra $\mathcal{D}$ is a division algebra. This can be realized if there is no element in $\mathbb{K}$ whose algebraic norm is equal to $\gamma = 2$ [11]. Using KANT software [13], we find that the ideal $2\mathcal{O}_{\mathbb{K}}$ is prime implying that 2 is a non-norm element and proving that the proposed code is fully diverse ($\mathcal{O}_{\mathbb{K}}$ stands for the ring of integers of $\mathbb{K}$).

Note that the minimum non-zero determinant of A_0 is equal to 1 following from the fact that $\gamma = 2$ is an algebraic element [7]. Consequently, eq. (13) and eq. (14) imply that the coding gain over any constellation carved from $\mathbb{Z}$ is equal to $\frac{2}{3\sqrt{d_{\mathbb{K}}}}$ where $d_{\mathbb{K}} = 5$ corresponds to the absolute discriminant of $\mathbb{K}$. Since the difference between two PAM symbols belongs to $2\mathbb{Z}$, then the coding gain with M'-PAM constellations is equal to $\frac{8}{3\sqrt{5}}$. Finally, note that since $\gamma = 2$ is algebraic, then the coding gain is non-vanishing and it keeps the same value for all values of M' [7].

IV. Delay Tolerance for DF cooperation

In this section, we consider the use of the proposed schemes for DF cooperation. This cooperation strategy is composed of two phases. At a first time, the source broadcasts a message to the relays and during the second phase the different relays transmit encoded messages simultaneously to the destination. For DT ST cooperation, each relay transmits one row of a ST codeword [9]. In what follows we only consider schemes 2 and 3 since scheme 1 does not result in any diversity advantage.

The relative delay between the p-th relay and the first relay (whose signal is assumed to arrive first at the destination) can be written as:

$$\tau_p = \tau'_p T_s + \tau''_p \quad ; \quad p = 2, \ldots, P \tag{15}$$

where τ'_p is an integer, $\tau''_p < T_s$ and $\tau'_1 = \tau''_1 = 0$.

In the case of perfect synchronization among the relays, diversity schemes 2 and 3 can be readily applied to take advantage from the spatial diversity in a distributed manner.

On the other hand, given the fine temporal resolution of UWB systems (the Rake delay Δ_l is much smaller than the symbol duration T_s), the proposed systems can easily tackle with synchronization errors or delays that do not exceed T_s (the part modeled by $\{\tau''_p\}_{p=2}^{P}$ in eq. (15)). In fact, given the very large number of multi-path components in typical UWB channels, such delays can be compensated for simply by delaying the sampling (or the Rake matched filtering) by an amount that is given by: $\tau''_{max} = \max_{p=2\cdots P}(\tau'_p)$ to compensate for such delays. In other words, the reference signal in eq. (4) must be simply shifted by τ''_{max}. Moreover, it is expected that this shift does not result in significant performance losses especially in Non-Line-Of-Sight (NLOS) scenarios since, in this case, the first arriving multi-path components do not necessarily have the highest power.

The problem of handling synchronization errors at the symbol level (the part modeled by $\{\tau_p'\}_{p=2}^{P}$ in eq. (15)) is much more challenging and requires the code to be delay-tolerant [10]. We first start by analyzing scheme 3 that can be applied with two relays. Eq. (13) can be written as:

$$\mathcal{A} = \begin{bmatrix} x_1 & -x_3 \\ x_2 & x_4 \end{bmatrix} \tag{16}$$

where $x_1, \ldots, x_4$ correspond to rotated versions of the PAM information symbols $a_1, \ldots, a_4$:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \frac{1}{\sqrt{3}}\mathcal{D} \begin{bmatrix} 1 & \theta & \sqrt{2} & \sqrt{2}\theta \\ 1 & \theta_1 & \sqrt{2} & \sqrt{2}\theta_1 \\ 1 & \theta & -\sqrt{2} & -\sqrt{2}\theta \\ 1 & \theta_1 & -\sqrt{2} & -\sqrt{2}\theta_1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \tag{17}$$

where $\mathcal{D}$ is a 4×4 diagonal normalizing matrix related to matrix D in eq. (13) by: $\mathcal{D} = I_2 \otimes D$ where $\otimes$ stands for the Kronecker product and I_2 stands for the 2×2 identity matrix.

Based on the definition given in [10], the ST code is delay-tolerant if all non-trivial codewords retain their full rank even when their columns are arbitrarily shifted. Consider the shifted replica of $\mathcal{A}$ given by:

$$\mathcal{A}_s = \begin{bmatrix} 0 & x_1 & -x_3 \\ x_2 & x_4 & 0 \end{bmatrix} \tag{18}$$

One needs to guarantee that non-trivial codewords $\mathcal{A}_s$ have full rank. Eq. (17) shows that the $x_1, \ldots, x_4 \in \mathbb{Q}(\theta, \phi)$ where the Golden number $\theta = \frac{1+\sqrt{5}}{2}$ is a degree-2 algebraic number that is a solution to the minimal polynomial $f_1(x) = x^2 - x - 1$ and $\theta_1 = \frac{1-\sqrt{5}}{2}$ is its conjugate. In the same way $\phi = \sqrt{2}$ is a degree-2 algebraic number that is a solution to the minimal polynomial $f_2(x) = x^2 - 2$ and $\phi_1 = -\sqrt{2}$ is its conjugate. Since $f_1(x)$ is irreducible over $\mathbb{Q}(\phi)$ and $f_2(x)$ is irreducible over $\mathbb{Q}(\theta)$, then $\mathbb{Q}(\theta, \phi)$ is a degree-4 algebraic extension of $\mathbb{Q}$ implying that the set $\{1, \theta, \phi, \phi\theta\}$ (as well as all of its conjugates) forms a basis over $\mathbb{Q}$. Consequently, $x_i = 0$ if and only if $a_1 = \cdots = a_4 = 0$ for $i = 1, \ldots, 4$.

Consider the submatrix formed from the first two rows and first two columns of matrix $\mathcal{A}_s$ given in eq. (18). The determinant of this matrix is equal to $-x_1 x_2$. This determinant is equal to zero if $x_1 = 0$ or $x_2 = 0$. Based on what preceded, it follows that this determinant is equal to zero if and only if $a_1 = \cdots = a_4 = 0$. This shows that all non-trivial matrices $\mathcal{A}_s$ have a rank two implying that the proposed code remains fully diverse even when one of the two relays is delayed by one symbol duration with respect to the other one.

Note that the proposed codeword $\mathcal{A}$ was related to the codeword A_0 (related to cyclic division algebras) by eq. (14). However, unlike $\mathcal{A}$, A_0 is not delay-tolerant. In fact if we set $a_3 = a_4 = 0$ and shift the columns by one, then the resulting matrix will be rank deficient with a rank of one.

We next analyze the delay tolerance of scheme 2. This scheme corresponds to transmitting orthogonal data streams from the different relays (refer to eq. (11) and Fig. 1.c). Since this orthogonality is realized by the use of different pulse shapes (and not by ST coding), it will always be maintained even with symbol level asynchronization. Consequently, scheme 2 is delay-tolerant to any delay profile and with any number of relays.

V. Simulations and Results

The PQ channels between the different antennas (or between the relays and the destination) are generated according to the 802.15.3a channel model recommendation CM2 [14]. The modulation delay is fixed to: $\delta = 0.5$ ns and it is chosen to be equal to the width of the highest order pulse $w_P(t)$. In order to eliminate ISI, we fix $T_s = 100$ ns. Since δ is very small compared to the channel delay spread, all of the presented results take IPI into consideration. At the receiver side, perfect channel state information is assumed and a modified PPM-specific version of the sphere decoder is applied [8].

Fig. 2 compares MIMO-UWB systems deploying SM, scheme 1 and scheme 2 respectively. Simulations are performed with 8-PPM and the receiver is equipped with a 2-finger Rake. The first observation is that SM outperforms scheme 1. This is justified by the fact that scheme 1 (and also scheme 2) transmits at a rate that is 1.33, 1.41 and 1.33 times higher than the rate of SM with 2, 3 and 4 transmit antennas respectively. Consequently, scheme 1 is capable of achieving very high multiplexing gains at the expense of some performance losses. Note that these losses decrease with the number of transmitters; for example, at a Symbol-Error-Rate (SER) of 10^{-3}, the losses are 2 dB, 1 dB and 0.8 dB for 2, 3 and 4 transmit antennas respectively. Note also that these multiplexing gains are achieved without increasing the complexity of the receiver (for example by increasing the number of modulation positions). Results also show the enhanced diversity order of scheme 2 that outperforms scheme 1 and SM in all scenarios. In fact, using the Hermite pulses and encoding them according to eq. (11) permits to achieve high multiplexing gains and diversity orders simultaneously.

Fig. 3 shows the performance of cooperative UWB systems with two and three relays using scheme 2. Simulations are performed with 4-PPM and the destination is equipped with one antenna and a 4-finger Rake. Results show that scheme 2 can still achieve a full transmit diversity order under different delay profiles since the SER curves of synchronous and asynchronous transmissions are parallel to each other at high signal to noise ratios (SNR). For example, synchronous cooperation with three relays results in a very large gain of 7 dB at a SER of 10^{-3}. In this context delaying the transmission of the third relay by one symbol duration (delay profile of $[0, 0, 1]$) and delaying the transmissions of the second and third relays by one and two symbol durations respectively (delay profile of $[0, 1, 2]$) reduce the gains to 3.5 dB and 2 dB respectively.

The performance of two-relay cooperation using scheme 3 with 2-PAM is shown in Fig. 4. A 7-finger Rake is used at the destination that is equipped with either one antenna or two antennas. The SER curves under synchronous transmissions

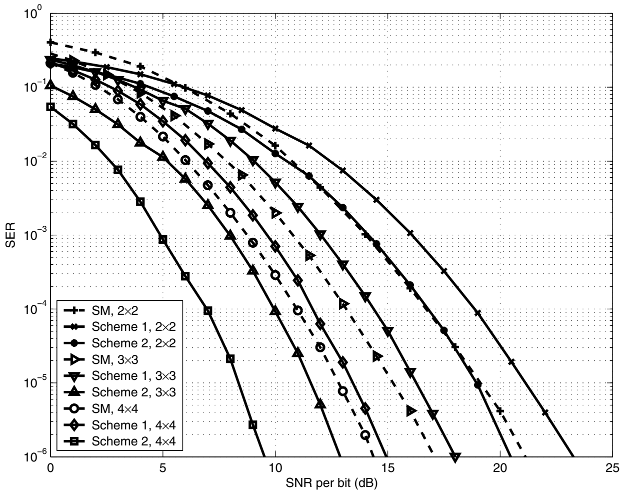

Fig. 2. SM vs. scheme 1 and scheme 2 with 8-PPM and a 2-finger Rake.

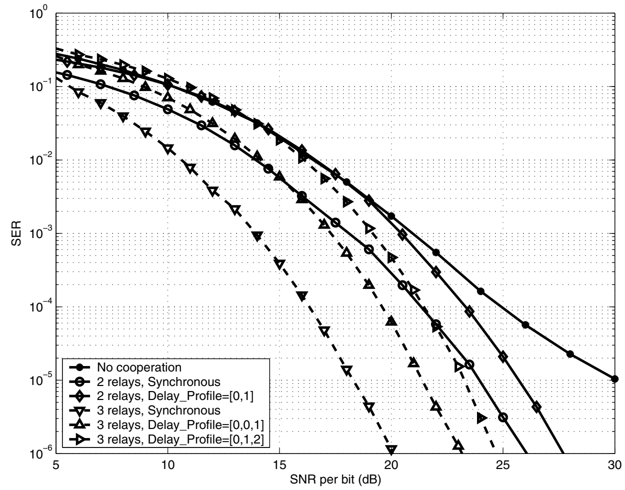

Fig. 3. Performance of DF systems using scheme 2 with 4-PPM. The destination is equipped with one antenna and a 4-finger Rake.

(using eq. (16)) and under asynchronous transmissions (using eq. (18)) are parallel to each other at high SNR showing that scheme 3 is delay-tolerant. Note that in the synchronous case, scheme 3 transmits at a rate of 2 bits PCU while the rate is equal to $4/3$ bits PCU in the case of asynchronous transmissions. Results also show the superiority of the proposed full-rate scheme with respect to 2-PAM non-cooperative transmissions (rate of 1 bit PCU) and with respect to 4-PAM non-cooperative transmissions (rate of 2 bits PCU) especially at high SNR.

VI. Conclusion

We presented novel transmission strategies for MIMO and DF UWB systems using PPM and PAM. The proposed PPM-specific construction solves the problem of the non-existence of unipolar codes for any number of transmit antennas and signal-set dimensionality. The proposed PAM ST code is the first known minimal-delay, totally-real and delay-tolerant code.

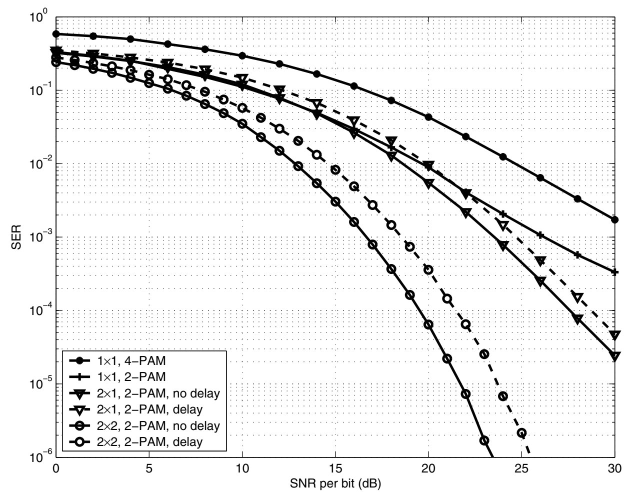

Fig. 4. Performance of scheme 3 with 2-PAM and a 7-finger Rake.

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