A Novel Selective-Relaying Protocol for Cooperative IR-UWB Communications with PPM

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***Abstract*—In this paper, we propose a novel cooperation strategy for Impulse-Radio Ultra-Wideband (IR-UWB) communication systems with binary Pulse Position Modulation (PPM). Unlike Decode-and-Forward (DF) strategies where a group of PPM symbols, or at least one PPM symbol, is decoded and forwarded to the receiver, the proposed scheme inspects the PPM slots in a sequential manner. In particular, only the UWB pulses that are received at the relay with a sufficient level of fidelity are retransmitted in subsequent PPM slots. This technique limits cooperation to only one symbol duration while avoiding any form of joint decoding at the relay and destination nodes. The proposed scheme corresponds to a selective-relaying protocol where the relay imposes a certain level of selectivity on the symbols to be retransmitted. The error performance of the proposed scheme is evaluated theoretically. Based on this performance analysis, we derived an optimal pulse selection strategy at the relay.**

***Index Terms*—Ultra-wideband, UWB, PPM, cooperation, relay, diversity, performance analysis, cooperative diversity.**

I. Introduction

Recently, cooperative communications in the context of Impulse-Radio Ultra-Wideband (IR-UWB) transmissions attracted a significant amount of attention where the additional spatial degree of freedom can be exploited in a distributed manner to leverage the performance of UWB sensor networks and Wireless Personal Area Networks (WPANs) [1]–[9]. In particular, a signal that is transmitted from a source node to a destination node can be overheard, processed and retransmitted by neighboring relay nodes thus enhancing the quality of signal reception at the destination. While the cooperation strategies in [1]–[5] and [6], [7] were considered with binary Pulse Amplitude Modulation (PAM) and Pulse Position Modulation (PPM) respectively, the cooperative schemes in [8], [9] can be applied with PAM, PPM and hybrid PPM-PAM constellations.

The existing UWB cooperation techniques can be classified into two broad categories. (i): Orthogonal techniques where cooperation is realized over two distinct time slots where in the first slot the message is transmitted from the source to the relays (and in some cases to the destination) and in the second slot the message is retransmitted from the relays to the destination [1], [4], [6]. For example, in [1] the symbol duration is doubled (compared to non-cooperative systems) where the first half of this duration is completely allocated to the transmission between the source and the relay while the second half is dedicated to the relay-destination link. Orthogonal techniques suffer from a data-rate reduction by a factor of $1/2$ compared to non-cooperative systems; however, they are characterized by a remarked simplicity where no joint decoding is required at the destination. (ii) Non-orthogonal techniques where the transmissions from the source and relays occur in the same TDMA slot (the one that is dedicated to the source) [7], [8]. These schemes transmit at the same rate as non-cooperative systems; however, sophisticated decoding techniques are required to decouple the numerous data streams embedded in the non-orthogonal space-time codewords.

In this paper, we propose a novel cooperation strategy that profits from the advantages of both orthogonal and non-orthogonal techniques. This is rendered possible because the proposed cooperation protocol is adapted to the structure of the PPM signal set. In particular, as in non-orthogonal systems, transmissions are limited to one TDMA slot while, as in orthogonal systems, the different symbols can be decoded separately. The above advantages come at the expense of a data-rate reduction by a factor of $\frac{2}{3}$ compared to non-cooperative binary-PPM systems. However, this ratio is better than the $1/2$ ratio resulting from the orthogonal schemes.

For the proposed cooperation strategy, an additional PPM time slot is added at the end of each symbol duration. In this case, the relay inspects the received UWB pulses in a sequential manner and if the level of an UWB pulse received within a certain slot exceeds a threshold level, then the relay will retransmit an UWB pulse in a subsequent PPM slot. In our work, we derive closed-form expressions for the error performance of the proposed scheme based on which we determine the optimal value of the above threshold level. Consequently, the proposed scheme can be perceived as an optimal selective-relaying protocol where the number of symbols that are retransmitted by the relay is determined by the quality of the links between the different nodes as well as by the level of noise. The selectivity imposed on the UWB pulses to be forwarded to the destination ensures high performance levels at the destination. Simulations performed over the IEEE 802.15.3a UWB channel model [10] are provided to support the theoretical results. Finally, unlike [9], the proposed scheme can be implemented independently from the number of time-hopped UWB pulses used to convey one information symbol.

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II. System Model and Cooperation Protocol

The proposed cooperation scheme extends over one symbol duration denoted by T_s. For IR-UWB communications with binary PPM, the signal transmitted by the source S over the corresponding symbol duration can be written as:

$$s_{\mathrm{s}}(t) = \sqrt{\frac{E_s}{2}} w(t-(a-1)\delta) \quad (1)$$

where $a \in \{1,2\}$ stands for the binary PPM symbol to be communicated. $w(t)$ is the UWB pulse waveform having a duration T_w and which is normalized to have a unit energy. δ stands for the modulation delay, that is the duration of each PPM slot, while E_s stands for the signal energy. The factor $\frac{1}{\sqrt{2}}$ was introduced in order to normalize the average energy transmitted by the source. In this case, the available energy is divided equally among the source S and relay R and the cooperative scheme transmits the same amount of energy as non-cooperative systems. No reference to the time-hopping (TH) sequence was made in (1) since we limit ourselves to the single-user case; moreover, the proposed cooperation strategy does not depend on the number of time-hopped pulses used to transmit one information symbol.

When the signal $s_{\mathrm{s}}(t)$ given in (1) is transmitted by S, the signal received at R can be written as:

$$r_{\mathrm{r}}(t) = \sqrt{\frac{\beta_{\mathrm{sr}} E_s}{2}} g_{\mathrm{sr}}(t-(a-1)\delta) + n_{\mathrm{r}}(t) \quad (2)$$

where $n_{\mathrm{r}}(t)$ stands for the additive white Gaussian noise (AWGN) with power spectral density N_0. In (2), $g_{\mathrm{sr}}(t)$ stands for the convolutions of $w(t)$ with the impulse response of the channel between S and R. Denote by d_{sd} and d_{sr} the distances from S to D (the destination) and from S to R, respectively. The term β_{sr} in (2) follows from the fact that d_{sr} might be different from d_{sd}. In other words, a signal-to-noise ratio (SNR) of $\frac{E_s}{N_0}$ at D will be equivalent to a SNR of $\beta_{\mathrm{sr}}\frac{E_s}{N_0}$ at R. Performing a typical link power budget analysis shows that $\beta_{\mathrm{sr}} = \left(\frac{d_{\mathrm{sd}}}{d_{\mathrm{sr}}}\right)^2$ assuming that the received power decreases with the square of the distance.

The detection at R will be based on a correlation receiver where R evaluates the following decision variables corresponding to the two PPM time slots:

$$y_{\mathrm{r}}^{(m)} = \int_0^{T_i} r_{\mathrm{r}}(t) g_{\mathrm{sr}}(t-(m-1)\delta)\mathrm{d}t \quad ; \quad m = 1,2 \quad (3)$$

where T_i stands for the duration of the integration window. In the above equation, it is assumed that R has acquired the value of the function $g_{\mathrm{sr}}(t)$ over a duration T_i via a channel estimation process that is assumed to be perfect in this case. Evidently, the complexity of this estimator increases with T_i.

In what follows, we consider the case of orthogonal PPM where all the multi-path components of the frequency-selective UWB channel arrive within one PPM slot of duration δ. In other words, we assume that $\delta \geq T_c + T_w$ where T_c stands for the maximum delay spread of the UWB channel ($T_c \gg T_w$). The last inequality ensures the elimination of Inter-Pulse-Interference (IPI). For the proposed cooperative system, and in order to be able to realize cooperation over only one symbol duration, we fix $T_s = 3\delta$ resulting in a data-rate reduction of $\frac{2}{3}$ compared to non-cooperative systems where it is sufficient to set $T_s = 2\delta$. This extension of the symbol duration will be justified in what follows as we explain the cooperation protocol.

The selective cooperation strategy at R is as follows. First, R evaluates the decision variable $y_{\mathrm{r}}^{(1)}$ corresponding to the first PPM slot. R then compares $y_{\mathrm{r}}^{(1)}$ with a ceratin threshold level I_{th}. If $y_{\mathrm{r}}^{(1)}$ exceeds I_{th}, then R will retransmit an UWB pulse in the second PPM slot. Note that in this case, there is no need to generate the decision variable $y_{\mathrm{r}}^{(2)}$ at R. On the other hand, if $y_{\mathrm{r}}^{(1)} < I_{th}$, the relay R proceeds to determining the value of $y_{\mathrm{r}}^{(2)}$ corresponding to the second PPM slot. If $y_{\mathrm{r}}^{(2)} < I_{th}$, then R will back off and no pulse will be retransmitted from R to D. Note that this backing off occurs when both $y_{\mathrm{r}}^{(1)}$ and $y_{\mathrm{r}}^{(2)}$ are smaller than I_{th} implying that the PPM symbol a was not reconstituted at R with a sufficient level of fidelity. In this case, R stops its transmission (during the corresponding symbol duration) in order not to confuse D with erroneous replicas of the noisy signal it received. On the other hand, when $y_{\mathrm{r}}^{(2)} \geq I_{th}$, R will retransmit an UWB pulse in the third (added) PPM slot.

Based on the above protocol, it can be observed that the transmissions from S fall within one of the intervals $[0\ \delta]$ (for $a = 1$) or $[\delta\ 2\delta]$ (for $a = 2$) while the retransmissions from R might occur only in the intervals $[\delta\ 2\delta]$ if $y_{\mathrm{r}}^{(1)} \geq I_{th}$ or $[2\delta\ 3\delta]$ if $y_{\mathrm{r}}^{(1)} < I_{th}$ and $y_{\mathrm{r}}^{(2)} \geq I_{th}$. These retransmissions are rendered possible by increasing the value of the symbol duration from 2δ (for non-cooperative systems) to 3δ. Finally, note that the causality of the cooperative system is respected and that the participation of R in the cooperation effort is determined by the value of I_{th}. The optimal value of the threshold level I_{th}, which minimizes the error probability, will be determined in Section IV.

For orthogonal PPM signal sets, the decision variables in (3) simplify to:

$$y_{\mathrm{r}}^{(m)} = \sqrt{\frac{\beta_{\mathrm{sr}} E_s}{2}} h_{\mathrm{sr}} \delta_{a,m} + n_{\mathrm{r}}^{(m)} \quad ; \quad m = 1,2 \quad (4)$$

where $\delta_{i,j} = 1$ for $i = j$ and $\delta_{i,j} = 0$ for $i \neq j$. In (4), $h_{\mathrm{sr}} \triangleq \int_0^{T_i} g_{\mathrm{sr}}^2(t)\mathrm{d}t$ and it stands for the channel energy captured along the S-R link over a duration T_i. On the other hand, $n_{\mathrm{r}}^{(m)} = \int_0^{T_i} n_{\mathrm{r}}(t) g_{\mathrm{sr}}(t-(m-1)\delta)\mathrm{d}t$ is a Gaussian random variable with variance $h_{\mathrm{sr}} N_0/2$. Note that the noise terms $n_{\mathrm{r}}^{(1)}$ and $n_{\mathrm{r}}^{(2)}$ are uncorrelated.

Based on the above cooperation strategy, the signal transmitted by R can be written as:

$$s_{\mathrm{r}}(t) = \sqrt{\frac{E_s}{2}} \sum_{m=1}^{M} \delta_{\hat{a},m} w(t-m\delta) \quad (5)$$

where:

$$\hat{a} = \begin{cases} 1, & y_{\mathrm{r}}^{(1)} \geq I_{th}; \\ 2, & y_{\mathrm{r}}^{(1)} < I_{th}\ ,\ y_{\mathrm{r}}^{(2)} \geq I_{th}; \\ 3, & y_{\mathrm{r}}^{(1)} < I_{th}\ ,\ y_{\mathrm{r}}^{(2)} < I_{th}. \end{cases} \quad (6)$$

Note that $s_{\rm r}(t)=0$ for $\hat{a}=3$ implying that no retransmissions from R will occur in this case. Moreover, while $w(t)$ in (1) is shifted by $(a-1)\delta$, the pulse shape $w(t)$ is now shifted by $\hat{a}\delta$ in (5) for $\hat{a}\neq 3$.

Based on the above notations, the signal received at D can be written as:

$$r_{\rm d}(t)=\sqrt{\frac{E_s}{2}}g_{\rm sd}(t-(a-1)\delta)+\sqrt{\frac{\beta_{\rm rd}E_s}{2}}\sum_{m=1}^{M}\delta_{\hat{a},m}g_{\rm rd}(t-m\delta)+n_{\rm d}(t) \quad (7)$$

where $n_{\rm d}(t)$ is the AWGN at the destination with variance $N_0/2$. As in (2), $\beta_{\rm rd}=\left(\frac{d_{\rm sd}}{d_{\rm rd}}\right)^2$ where $d_{\rm rd}$ stands for the distance between R and D. In (7), $g_{\rm sd}(t)$ and $g_{\rm rd}(t)$ stand for the convolutions of $w(t)$ with the impulse responses of the channels between S-D and R-D, respectively.

A bank of correlators is deployed at D in order to evaluate the following two decision variables (for $m=1,2$):

$$y_{\rm d}^{(m)}=\int_0^{T_i} r_{\rm d}(t)\left[g_{\rm sd}(t-(m-1)\delta)+g_{\rm rd}(t-m\delta)\right]{\rm d}t \quad (8)$$

where we assume that the functions $g_{\rm sd}(t)$ and $g_{\rm rd}(t)$ are perfectly estimated over a duration T_i at D.

In the absence of IPI and following from (7), the decision metrics in (8) simplify to (for $m=1,2$):

$$y_{\rm d}^{(m)}=\sqrt{\frac{E_s}{2}}h_{\rm sd}\delta_{a,m}+\sqrt{\frac{\beta_{\rm rd}E_s}{2}}h_{\rm rd}\delta_{\hat{a},m}+\left[\sqrt{\frac{E_s}{2}}\delta_{a,m+1}+\sqrt{\frac{\beta_{\rm rd}E_s}{2}}\delta_{\hat{a},m-1}\right]h_{\rm in}+n_{\rm d}^{(m)} \quad (9)$$

where $h_{\rm sd}\triangleq\int_0^{T_i}g_{\rm sd}^2(t){\rm d}t$ and $h_{\rm rd}\triangleq\int_0^{T_i}g_{\rm rd}^2(t){\rm d}t$. The interference between the channels S-D and R-D is reflected in the coefficient $h_{\rm in}$ given by: $h_{\rm in}\triangleq\int_0^{T_i}g_{\rm sd}(t)g_{\rm rd}(t){\rm d}t$. The noise terms are given by: $n_{\rm d}^{(m)}=\int_0^{T_i}n_{\rm d}(t)\left[g_{\rm sd}(t-(m-1)\delta)+g_{\rm rd}(t-m\delta)\right]{\rm d}t$. It can be proven that (for $m,m'\in\{1,2\}$):

$$\mathrm{E}\left[n_{\rm d}^{(m)}n_{\rm d}^{(m')}\right]=\frac{N_0}{2}\begin{cases}h_{\rm sd}+h_{\rm rd}, & m=m';\\ \sqrt{h_{\rm sd}h_{\rm rd}}, & m\neq m'.\end{cases} \quad (10)$$

where E[.] stands for the averaging operator. Equation (10) shows that the noise terms $n_{\rm d}^{(1)}$ and $n_{\rm d}^{(2)}$ are correlated.

III. Performance Analysis

For the sake of notational simplicity, we set $h_1=\sqrt{\frac{\beta_{\rm sr}E_s}{2}}h_{\rm sr}$, $h_2=\sqrt{\frac{E_s}{2}}h_{\rm sd}$ and $h_3=\sqrt{\frac{\beta_{\rm rd}E_s}{2}}h_{\rm rd}$ in what follows. The channel state is defined by the vector: $H=[h_1,h_2,h_3]$.

Assume first that the PPM symbol $a=1$ was transmitted by S. In this case, from (4), the decision variables at R are given by: $y_{\rm r}^{(1)}=h_1+n_{\rm r}^{(1)}$ and $y_{\rm r}^{(2)}=n_{\rm r}^{(2)}$. It follows that:

$$\Pr(y_{\rm r}^{(1)}<I_{th})=1-\Pr(y_{\rm r}^{(1)}\geq I_{th})\triangleq p_1 \quad (11)$$

$$\Pr(y_{\rm r}^{(2)}\geq I_{th})=1-\Pr(y_{\rm r}^{(2)}<I_{th})\triangleq p_2 \quad (12)$$

where it is straightforward to prove that the probabilities p_1 and p_2 are given by:

$$p_1=Q\left(\frac{h_1-I_{th}}{\sqrt{h_{\rm sr}N_0/2}}\right)\quad;\quad p_2=Q\left(\frac{I_{th}}{\sqrt{h_{\rm sr}N_0/2}}\right) \quad (13)$$

following from the fact that the noise terms $n_{\rm r}^{(1)}$ and $n_{\rm r}^{(2)}$ are uncorrelated Gaussian random variables with variances $h_{\rm sr}N_0/2$. In (13), the function $Q(x)$ is defined as: $Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{\infty}e^{-t^2/2}{\rm d}t$.

Consequently, from (6):

$$\Pr(\hat{a}=1|a=1)=1-p_1\quad;\quad\Pr(\hat{a}=2|a=1)=p_1p_2$$
$$\Pr(\hat{a}=3|a=1)=p_1(1-p_2) \quad (14)$$

In the same way, assuming that the symbol $a=2$ was transmitted by S, the decision variables in (4) can be written as: $y_{\rm r}^{(1)}=n_{\rm r}^{(1)}$ and $y_{\rm r}^{(2)}=h_1+n_{\rm r}^{(2)}$. In this case:

$$\Pr(y_{\rm r}^{(1)}\geq I_{th})=1-\Pr(y_{\rm r}^{(1)}<I_{th})=p_2 \quad (15)$$

$$\Pr(y_{\rm r}^{(2)}<I_{th})=1-\Pr(y_{\rm r}^{(2)}\geq I_{th})=p_1 \quad (16)$$

resulting in:

$$\Pr(\hat{a}=1|a=2)=p_2\quad;\quad\Pr(\hat{a}=2|a=2)=(1-p_2)(1-p_1)$$
$$\Pr(\hat{a}=3|a=2)=(1-p_2)p_1 \quad (17)$$

Next, we evaluate the performance of the proposed cooperation strategy assuming that $h_{\rm in}=0$. In fact, a numerical analysis performed over the IEEE 802.15.3a channel model [10] showed that the interference term $h_{\rm in}$ takes very small values and, hence, can be neglected. This can be justified by the randomness of the polarities of the multi-path components corresponding to $g_{\rm sd}(t)$ and $g_{\rm rd}(t)$.

According to (10), the noise terms $n_{\rm d}^{(1)}$ and $n_{\rm d}^{(2)}$ are correlated Gaussian random variables. We next prove the following preliminary result that will be very useful in our subsequent performance analysis.

Proposition: The probability $\Pr(x+n_{\rm d}^{(1)}<n_{\rm d}^{(2)})$ is given by:

$$\Pr(x+n_{\rm d}^{(1)}<n_{\rm d}^{(2)})=Q\left(\frac{x}{\sqrt{N_0(h_{\rm sd}+h_{\rm rd}-\sqrt{h_{\rm sd}h_{\rm rd}})}}\right) \quad (18)$$

Proof: The proof is provided in the Appendix.

We consider the following cases. Case (i.1): $a=1$ and $\hat{a}=1$ with probability $1-p_1$ from (14). In this case, the decision variables in (9) can be written as: $y_{\rm d}^{(1)}=h_2+h_3+n_{\rm d}^{(1)}$ and $y_{\rm d}^{(2)}=n_{\rm d}^{(2)}$. The probability of error in this case is given by:

$$P_1\triangleq\Pr(h_2+h_3+n_{\rm d}^{(1)}\leq n_{\rm d}^{(2)})=Q\left(\frac{h_2+h_3}{\sqrt{N_0(h_{\rm sd}+h_{\rm rd}-\sqrt{h_{\rm sd}h_{\rm rd}})}}\right)=Q\left(\frac{h_{\rm sd}+\sqrt{\beta_{\rm rd}}h_{\rm rd}}{\sqrt{h_{\rm sd}+h_{\rm rd}-\sqrt{h_{\rm sd}h_{\rm rd}}}}\sqrt{\frac{E_s}{2N_0}}\right) \quad (19)$$

where the second equality follows from (18).

Case (i.2): $a = 1$ and $\hat{a} = 2$ with probability $p_1 p_2$ from (14). In this case: $y_{\rm d}^{(1)} = h_2 + n_{\rm d}^{(1)}$ and $y_{\rm d}^{(2)} = h_3 + n_{\rm d}^{(2)}$ resulting in an error probability of:

$$P_2 \triangleq \Pr(h_2 + n_{\rm d}^{(1)} \leq h_3 + n_{\rm d}^{(2)}) = Q\left(\frac{h_{\rm sd} - \sqrt{\beta_{\rm rd}} h_{\rm rd}}{\sqrt{h_{\rm sd} + h_{\rm rd} - \sqrt{h_{\rm sd} h_{\rm rd}}}} \sqrt{\frac{E_s}{2N_0}}\right) \tag{20}$$

where the term $h_2 + h_3$ in the expression of P_1 was replaced by $h_2 - h_3$ for calculating P_2.

Case (i.3): $a = 1$ and $\hat{a} = 3$. From (9): $y_{\rm d}^{(1)} = h_2 + n_{\rm d}^{(1)}$ and $y_{\rm d}^{(2)} = n_{\rm d}^{(2)}$ resulting in an error probability of:

$$P_3 \triangleq \Pr(h_2 + n_{\rm d}^{(1)} \leq n_{\rm d}^{(2)}) = Q\left(\frac{h_{\rm sd}}{\sqrt{h_{\rm sd} + h_{\rm rd} - \sqrt{h_{\rm sd} h_{\rm rd}}}} \sqrt{\frac{E_s}{2N_0}}\right) \tag{21}$$

where, from (14), this event occurs with probability $p_1(1-p_2)$.

Evaluating the weighted sum of the probabilities in equations (19)-(21) results in:

$$P_{e|H}^{(1)} = (1 - p_1)P_1 + p_1 p_2 P_2 + p_1(1 - p_2)P_3 \tag{22}$$

where $P_{e|H}^{(1)}$ stands for the conditional error probability when the symbol $a = 1$ is transmitted.

Assume now that the symbol $a = 2$ is transmitted by S. Three cases are possible at D. Case (ii.1): $\hat{a} = 1$ with probability p_2 from (17). In this case, the decision metrics at D are given by: $y_{\rm d}^{(1)} = h_3 + n_{\rm d}^{(1)}$ and $y_{\rm d}^{(2)} = h_2 + n_{\rm d}^{(2)}$. This case is analogous to case (i.2) and the resulting error probability is P_2 given in (20). Case (ii.2): $\hat{a} = 2$ with probability $(1-p_1)(1-p_2)$ from (17). In this case: $y_{\rm d}^{(1)} = n_{\rm d}^{(1)}$ and $y_{\rm d}^{(2)} = h_2 + h_3 + n_{\rm d}^{(2)}$. Given the analogy between cases (ii.2) and (i.1), then the conditional error probability in this case is equal to P_1 given in (19). Case (ii.3): $\hat{a} = 3$ resulting in $y_{\rm d}^{(1)} = n_{\rm d}^{(1)}$ and $y_{\rm d}^{(2)} = h_2 + n_{\rm d}^{(2)}$ from (9). In this case, the probability of error will be equal to P_3 given in (21) and this event occurs with probability $p_1(1 - p_2)$ from (17).

Finally, combining the cases (ii.1)-(ii.3) results in:

$$P_{e|H}^{(2)} = p_2 P_2 + (1 - p_1)(1 - p_2)P_1 + p_1(1 - p_2)P_3 \tag{23}$$

where $P_{e|H}^{(2)}$ stands for the conditional error probability when the symbol $a = 2$ is transmitted.

Finally, combining equations (22) and (23) results in the following expression of the conditional error probability:

$$P_{e|H} = \frac{1}{2}\left[P_{e|H}^{(1)} + P_{e|H}^{(2)}\right] \tag{24}$$

$$= \frac{1}{2}\left[(1 - p_1)(2 - p_2)P_1 + (1 + p_1)p_2 P_2 + 2p_1(1 - p_2)P_3\right] \tag{25}$$

On the other hand, subtracting (22) from (23) shows that: $P_{e|H}^{(2)} - P_{e|H}^{(1)} = p_2(1-p_1)[P_2 - P_1]$. On the other hand, given that $h_{\rm sd} + \sqrt{\beta_{\rm rd}} h_{\rm rd} > h_{\rm sd} - \sqrt{\beta_{\rm rd}} h_{\rm rd}$ (since all involved quantities are positive), then $P_1 < P_2$ following from equations (19) and (20). This results in $P_{e|H}^{(2)} > P_{e|H}^{(1)}$ showing that it is more probable to make errors on the second PPM symbol $a = 2$. In other words, the symmetry of the PPM constellation is broken by the structure of the proposed cooperation strategy. This follows mainly from the fact that the decision metrics at R are inspected sequentially (and not simultaneously). In other words, forwarding the correct symbol $\hat{a} = a = 1$ is based solely on $y_{\rm r}^{(1)}$ that contains a signal part while forwarding the symbol $\hat{a} = a = 2$ is based on $y_{\rm r}^{(1)}$ that does not contain a signal part and on $y_{\rm r}^{(2)}$ that contains a signal part in this case.

Finally, the conditional error probabilities in equations (22)-(23) can not be integrated analytically over the IEEE 802.15.3a channel model. Consequently, we evaluate the error probability according to:

$$P_e = \frac{1}{2}\left[P_e^{(1)} + P_e^{(2)}\right] \tag{26}$$

where the average probabilities $P_e^{(1)}$ and $P_e^{(2)}$ are obtained by the numerical integration of $P_{e|H}^{(1)}$ and $P_{e|H}^{(2)}$, respectively.

IV. Optimizing the Threshold Level I_{th}

Increasing the value of I_{th} imposes a higher selectivity on the symbols to be forwarded to D while smaller values of I_{th} will imply that a larger number of symbols will be forwarded. Consequently, a compromise must be made on the choice of I_{th}. In this section, we determine the threshold level I_{th} that minimizes the conditional error probability in (25).

Ignoring the terms that correspond to the product of three probabilities in (25) implies that this equation can be approximated by:

$$P_{e|H} \approx \frac{1}{2}\left[2P_1 + 2p_1(P_3 - P_1) + p_2(P_2 - P_1)\right] \tag{27}$$

where, from (19)-(21), only the probabilities p_1 and p_2 depend on I_{th}.

Replacing p_1 and p_2 by their values from (13) and differentiating (27) with respect to I_{th} results in:

$$2\frac{{\rm d}P_{e|H}}{{\rm d}I_{th}} = 2(P_3 - P_1)\left(-\frac{1}{\sqrt{2\pi}}e^{-\frac{(h_1 - I_{th})^2}{h_{\rm sr}N_0}}\right)\left(-\frac{1}{\sqrt{h_{\rm sr}N_0/2}}\right) + (P_2 - P_1)\left(-\frac{1}{\sqrt{2\pi}}e^{-\frac{I_{th}^2}{h_{\rm sr}N_0}}\right)\left(\frac{1}{\sqrt{h_{\rm sr}N_0/2}}\right) \tag{28}$$

Solving the equation $\frac{{\rm d}P_{e|H}}{{\rm d}I_{th}} = 0$ results in the following unique solution:

$$I_{th} = \frac{h_1}{2} + \frac{h_{\rm sr}N_0}{2h_1}\ln\left(\frac{P_2 - P_1}{2(P_3 - P_1)}\right) \tag{29}$$

$$= \frac{h_1}{2} + \frac{N_0}{\sqrt{2\beta_{\rm sr}E_s}}\ln\left(\frac{P_2 - P_1}{2(P_3 - P_1)}\right) \tag{30}$$

showing that the optimal value of I_{th} shifts from the central value of $\frac{h_1}{2}$ by a quantity that depends mainly on the signal-to-noise ratio (SNR). Note that evaluating I_{th} at R necessitates the knowledge of the channel state vector H as well as the value of the noise variance.

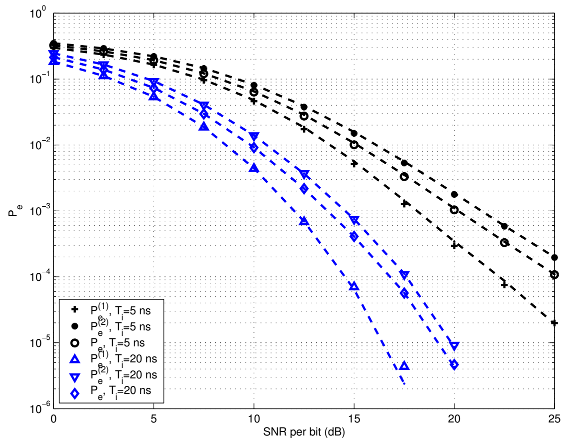

Fig. 1. The different error probabilities with $(\beta_{\text{sr}}, \beta_{\text{rd}}) = (4, 1)$. The marked points are obtained by simulations while the dashed curves correspond to the analytical results.

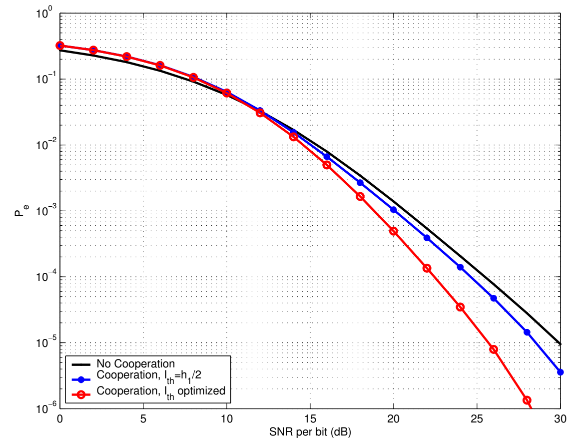

Fig. 3. Performance of the proposed scheme with $\beta_{\text{sr}} = 4$, $\beta_{\text{rd}} = 1$ and $T_i = 5$ ns.

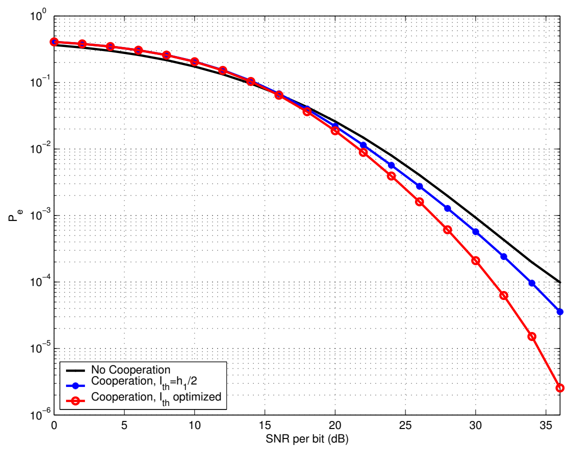

Fig. 2. Performance of the proposed scheme with $\beta_{\text{sr}} = 4$, $\beta_{\text{rd}} = 1$ and $T_i = 1$ ns.

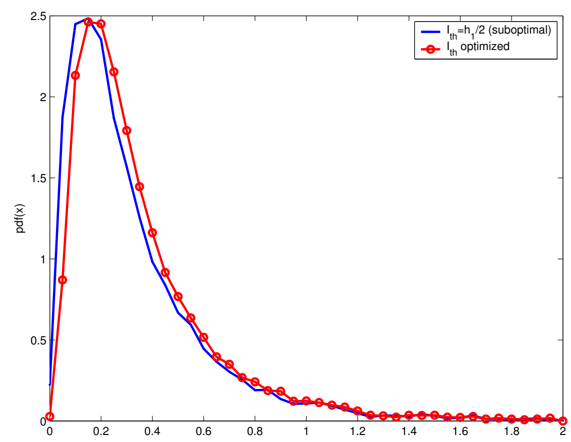

Fig. 4. Probability density functions (pdf) of I_{th} based on the optimal and suboptimal solutions.

For asymptotic values of the SNR, equations (19)-(21) show that P_1 can be neglected compared to P_2 and P_3. In the same way, P_3 is several orders of magnitude smaller than P_2. Consequently, the logarithm in (30) is positive. This will imply that the optimal value of I_{th} will increase with the noise variance which reduces the retransmission of unreliable symbols to D thus enhancing the performance of the cooperative system.

V. Numerical Results

The data-rate reduction (by a factor of 2/3) imposed by the proposed scheme is translated into a power penalty in all results that we present in this paper. In other words, the SNR per bit is set to $\frac{E_s}{N_0}$ for non-cooperative systems and to $\frac{3}{2}\frac{E_s}{N_0}$ for the proposed scheme resulting in an asymptotic loss of about 1.76 dB. Simulations are performed over the IEEE 802.15.3a channel model recommendation CM2 [10]. A Gaussian pulse with a duration of $T_w = 0.5$ ns is used. The modulation delay is chosen to verify $\delta = 100$ ns which is larger than the maximum delay spread of the UWB channel. As a benchmark, we compare the proposed solution corresponding to an optimal value of I_{th} that is determined from (30) with a suboptimal solution that consists of setting $I_{th} = h_1/2$ which corresponds to half the energy captured along the link S-R. Note that the latter solution constitutes an appealing simple solution since the corresponding threshold value depends only on the S-R link irrespectively from N_0 and the error probabilities along the other links. Finally, the obtained results correspond to average quantities evaluated over 10000 channel realizations.

Fig. 1 shows the variations of $P_e^{(1)}$, $P_e^{(2)}$ and P_e with $(\beta_{\text{sr}}, \beta_{\text{rd}}) = (4, 1)$ for different values of T_i. The results correspond to the suboptimal solution with $I_{th} = h_1/2$. This figure shows the close match between simulations and the theoretical analysis presented in section III despite the fact that the interference term h_{in} was neglected in the theoretical study. This shows that this term can be safely neglected without resulting in significant modifications of the results.

Fig. 2 shows the performance for $\beta_{\text{sr}} = 4$, $\beta_{\text{rd}} = 1$ and $T_i = 1$ ns. The obtained results show the huge gap between the optimal and suboptimal solutions. In particular, for $P_e = 10^{-3}$, performance gains in the order of 2.8 dB and 1.6 dB can be realized compared to non-cooperative systems and to the suboptimal solution, respectively. Similar results are obtained in Fig. 3 for $T_i = 5$ ns. The histograms of the corresponding optimal and suboptimal values of I_{th} are shown in Fig. 4 for a SNR of 15 dB. The results show a clear shift of I_{th} towards large values. This implies that the selectivity on the symbols to be forwarded by the relay is enhanced by the proposed optimal solution.

VI. Conclusion

We proposed a novel cooperation strategy for IR-UWB systems with binary PPM. We derived closed-form expressions for the conditional error probability that can be achieved by this cooperative scheme. Based on these expressions, we determined an optimal value of a threshold level that establishes the level of cooperation at the relay. This parameter, which depends on the specific channel realization and the noise variance, imposes a certain level of selectivity on the symbols to be forwarded by the relay node. This selective protocol boosts the performance level at the destination node. The proposed scheme is also characterized by a remarked decoding simplicity where joint decoding can be avoided at both the relay and destination nodes.

Appendix

We first apply the following transformation on $(n_{\text{d}}^{(1)}, n_{\text{d}}^{(2)})$ that turns out to be useful in simplifying the analysis:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} n_{\text{d}}^{(1)} \\ n_{\text{d}}^{(2)} \end{bmatrix} \tag{31}$$

This transformation results in the zero-mean independent Gaussian random variables u and v with variances:

$$\sigma_u^2 = \left(h_{\text{sd}} + h_{\text{rd}} + \sqrt{h_{\text{sd}} h_{\text{rd}}}\right) N_0/2 \tag{32}$$

$$\sigma_v^2 = \left(h_{\text{sd}} + h_{\text{rd}} - \sqrt{h_{\text{sd}} h_{\text{rd}}}\right) N_0/2 \tag{33}$$

The probability in (18) can be written as:

$$\Pr(x + n_{\text{d}}^{(1)} < n_{\text{d}}^{(2)}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{n_{\text{d}}^{(2)} - x} p(n_{\text{d}}^{(1)}, n_{\text{d}}^{(2)}) \mathrm{d}n_{\text{d}}^{(1)} \mathrm{d}n_{\text{d}}^{(2)} \tag{34}$$

where $p(n_{\text{d}}^{(1)}, n_{\text{d}}^{(2)})$ stands for the joint probability density function of the correlated Gaussian r.v.s $n_{\text{d}}^{(1)}$ and $n_{\text{d}}^{(2)}$:

$$p(x, y) = \frac{1}{2\pi\sigma_n^2\sqrt{1-\rho^2}} \exp\left(\frac{-x^2 + 2\rho xy - y^2}{2\sigma_n^2(1-\rho^2)}\right) \tag{35}$$

where, from (10), $\sigma_n^2 = \frac{N_0}{2}(h_{\text{sd}} + h_{\text{rd}})$ and $\rho = \frac{\sqrt{h_{\text{sd}} h_{\text{rd}}}}{h_{\text{sd}} + h_{\text{rd}}}$.

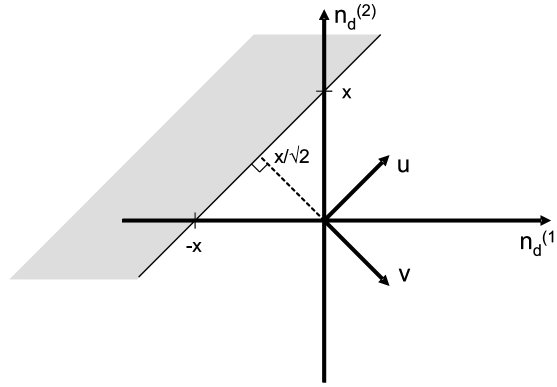

Fig. 5. The bivariate Gaussian vector $(n_{\text{d}}^{(1)}, n_{\text{d}}^{(2)})$, the r.v.s u and v and the integration area corresponding to equations (34) and (36).

Applying the transformation given in (31), the integral in (34) can be written as:

$$\Pr(x + n_{\text{d}}^{(1)} < n_{\text{d}}^{(2)}) = \int_{-\infty}^{+\infty} p(u) \mathrm{d}u \int_{-\infty}^{-\frac{x}{\sqrt{2}}} p(v) \mathrm{d}v \tag{36}$$

$$= 1 \times Q\left(\frac{x}{\sqrt{2\sigma_v^2}}\right) = Q\left(\frac{x}{\sqrt{N_0(h_{\text{sd}} + h_{\text{rd}} - \sqrt{h_{\text{sd}} h_{\text{rd}}})}}\right) \tag{37}$$

where $p(u)$ and $p(v)$ stand for the probability density functions of the independent r.v.s u and v defined in (31). The equivalence between the integrals in (34) and (36) is better illustrated in Fig. 5.

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