

# Differential Space-Time Ultra-Wideband Communications

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**Abstract**—In this paper, we present two space-time schemes for exploiting diversity in multi-antenna Impulse Radio Ultra Wideband (IR-UWB) systems when neither the transmitter nor the receiver has access to channel state information. The first scheme encodes the pulses of the same data symbol by a combination of differential phase shift keying and permutation codes and can be associated with any number of transmit and receive antennas. The second scheme encodes different symbols with two transmit antennas. These schemes are associated with Rake receivers and achieve full spatial and multi-path diversity with no data rate loss for constant-modulus multi-dimensional constellations. Adaptive versions of these schemes are also presented and shown to approach the performance of coherent receivers in stationary indoor channels.

## I. INTRODUCTION

Recently differential and non-coherent impulse radio ultra-wideband (IR-UWB) systems have drawn considerable attention as a means of obtaining a good compromise between performance and complexity [1]-[4]. The interest of such approach is evident in cases where accurate channel estimation is questionable because of the need of extensive training at low signal to noise ratios or high multi-user interference levels especially in fast varying channels.

Differential transmitted reference (DTR) systems proposed in [1] and [3] are extensions of differential binary phase shift keying (DBPSK) to UWB systems where the data is conveyed through the phase difference between two consecutive symbols. On the other hand, pulse position modulation (PPM) was associated uniquely with non-coherent energy detectors [4] and with simple transmitted reference systems (STR) [2] where each data pulse is preceded by a reference pulse which carries no information. Performance can be ameliorated by averaging the reference pulses prior to detection in STR and by applying decision-directed algorithms in DTR systems.

On the other hand, multiple input multiple output (MIMO) systems can support very high data rates with low error probabilities even in cases where no channel state information (CSI) is available at the receiver [5]-[7]. This motivates the extension of the UWB differential techniques for MIMO systems. The first approach in this domain was made in [8] where a full diversity space-time (ST) code was proposed for the case of two transmit antennas. It was also shown that the proposed code keeps its diversity advantage when associated with non-coherent energy detectors and orthogonal PPM constellations.

In this paper, we will propose two differential ST codes for Time Hopping (TH) UWB systems. The first scheme encodes the different pulses used to convey one data symbol and is capable of achieving full spatial diversity for all number of transmit antennas with no data rate loss. Unlike energy detectors that can be associated only with PPM [8], the proposed scheme can be associated with all constant modulus real constellations such as  $M$ -ary PPM, BPSK and combined  $2M$ -ary bi-orthogonal PPM. The latter constellation permits to increase the spectral efficiency while compromising complexity and performance as in [9]. For the above constellations, information will be conveyed through the phase difference and/or the time shift between 2 consecutive symbols. An additional advantage with respect to energy detectors is that we can profit from the long coherence time of indoor channels to develop decision-feedback receivers that can ameliorate performance by reducing the noise level in the reference signal.

The second coding scheme is the extension of [6] and [7] to multi-dimensional constellations and to highly frequency selective UWB channels. It permits to achieve full diversity with no data rate loss for 2 transmit antennas and unlike the first ST code, it can not be associated with PPM constellations.

The rest of the paper is organized as follows. The channel model, encoding scheme and receiver structure of the two ST code are presented in sections II and III respectively. Results are presented in section IV while section V concludes.

## II. DIVERSITY SCHEME I

### A. Encoder/decoder structure

Consider the single user TH-UWB system consisting of  $P$  transmit and  $Q$  receive antennas. The same data stream will be emitted from the  $P$  transmit antennas. At the  $p$ -th antenna, each information symbol is conveyed through  $N_f$  pulses transmitted repeatedly with an average period of  $T_f$  and with different polarities. This symbol belongs to a  $2M$ -ary bi-orthogonal PPM constellation (also referred to as  $M$ -PPM-2-PAM) which comprises the  $M$ -PPM and the BPSK constellations as special cases. Each symbol can be represented by the coordinates  $(a, d)$  where  $a \in \{-1, +1\}$  and  $d \in \{0, \dots, M-1\}$  correspond to the amplitude and the position of the transmitted pulse respectively. The data rate is equal to  $\log_2(2M)/N_f T_f$  bits/s and is the same as for single antenna systems.

Taking the beginning of the  $k$ -th symbol as the time origin, the signal transmitted from the  $p$ -th transmit antenna during the  $k$ -th symbol duration can be expressed as:

$$s_{k,p}(t) = \frac{1}{\sqrt{PN_f}} a_k \sum_{n=0}^{N_f-1} b_{p,n} w(t - nT_f - \delta d_k) \quad (1)$$

where the factor  $1/P$  assures that the total transmitted power is the same as in the case of single antenna systems.  $w(t)$  is the transmitted pulse waveform of duration  $T_w$  normalized to have unit energy and  $\delta$  is the modulation delay.  $b_p = [b_{p,0} \dots b_{p,N_f-1}]$  is the sequence used to encode the pulses of the  $p$ -th transmit antenna with  $b_{p,n} = \pm 1$ . In what follows we will consider the case of  $N_f$  even and  $P \leq N_f$ . The sequences  $b_1, \dots, b_P$  are fixed independently from the considered user and are chosen to be orthogonal to each other. No reference to the TH code was made since all the transmit antennas of the same user will share the same pseudorandom TH sequence and thus the interference to other users is the same as in SISO systems.

The signal in (1) can be also expressed as:

$$s_{k,p}(t) = \frac{1}{\sqrt{PN_f}} \sum_{n=0}^{N_f-1} \sum_{m=0}^{M-1} a_{k,m} b_{p,n} w(t - nT_f - m\delta) \quad (2)$$

where  $a_{k,m} = a_k \delta(d_k - m)$  and  $\delta(\cdot)$  is the Dirac function.

In a system with no inter symbol interference (ISI), the received signal at the  $q$ -th receiving antenna takes the form:

$$r_{k,q}(t) = \sum_{p=1}^P s_{k,p}(t) * g_{q,p}(t) + n_{k,q}(t) \quad (3)$$

where  $*$  stands for convolution and  $n_{k,q}(t)$  is the Gaussian noise at the  $q$ -th antenna during the  $k$ -th symbol duration. The correlation introduced by the band-pass filter that constitutes the first step of the receiver will be neglected. This corresponds to choosing a filter with very large bandwidth and thus this noise term will be supposed to be white with double sided spectral density  $N_0/2$ .  $g_{q,p}(t)$  stands for the impulse response of the frequency selective channel between the  $p$ -th transmit and the  $q$ -th receive antenna. Each sub-channel comprises  $L_{q,p}$  multi-path components that can arrive at any time within the pulse duration; the  $l$ -th component arrives at instant  $\tau_{q,p,l}$  with a real amplitude that equals to  $\alpha_{q,p,l}$ :

$$g_{q,p}(t) = \sum_{l=0}^{L_{q,p}-1} \alpha_{q,p,l} \delta(t - \tau_{q,p,l}) \quad (4)$$

The multi-path delays satisfy:  $\tau_{q,p,0} < \tau_{q,p,1} < \dots < \tau_{q,p,L_{q,p}-1}$ .  $T_{q,p} = \tau_{q,p,L_{q,p}-1}$  denotes the maximum delay spread of the  $(p,q)$ -th sub channel and is very large compared to the pulse duration  $T_w$  [11].

To keep the orthogonality between the different positions, the modulation delay is chosen to satisfy:  $\delta \geq \max(T_{q,p}) + T_w$  and to eliminate the ISI the time frame is chosen to satisfy  $T_f \geq \max(T_{q,p}) + (M-1)\delta + T_w$ .

Let  $h_{q,p}(t) = w(t) * g_{q,p}(t)$ , replacing (2) in (3) gives:

$$r_{k,q}(t) = \frac{1}{\sqrt{PN_f}} \sum_{p,n,m} a_{k,m} b_{p,n} h_{q,p}(t - nT_f - m\delta) + n_{k,q}(t) \quad (5)$$

To take advantage of the multi-path diversity offered by the highly frequency selective channel, the first stage of reception will consist of a Rake of order  $L$ . The construction of such receiver requires no particular CSI since the finger delays will be chosen to be multiples of the pulse-width  $T_w$  independently from the specific channel realization. This corresponds to what is referred to as partial Rake receivers (PRakes) [12].

For a given finger delay, the signal at the output of the  $q$ -th antenna is correlated with the  $PML$  reference signals:

$$\hat{s}_{p,l,m}(t) = \sum_{n=0}^{N_f-1} b_{p,n} w(t - nT_f - lT_w - m\delta) \quad (6)$$

From (5) and (6), the output of each correlator can be expressed as:

$$y_{k,q,p,l,m} = \int_0^{N_f T_f} r_{k,q}(t) \hat{s}_{p,l,m}(t) dt \quad (7)$$

$$= x_{k,q,p,l,m} + n_{k,q,p,l,m} \quad (8)$$

where:  $n_{k,q,p,l,m} = \int_0^{N_f T_f} n_{k,q}(t) \hat{s}_{p,l,m}(t) dt$ .  $n_{k,q,p,l,m}$  has zero mean and variance  $N_f N_0/2$  since  $w(t)$  has unit energy. Since the modulation duration and the finger delays are chosen to be greater than the pulse duration and because of the orthogonality between the transmit antennas, we can conclude that this noise term is white.

$$x_{k,q,p,l,m} = \frac{1}{\sqrt{PN_f}} \sum_{p',n',m',n} a_{k,m'} b_{p,n} b_{p',n'} r_{q,p'}(lT_w + (n - n')T_f + (m - m')\delta) \quad (9)$$

where  $r_{q,p}(\tau) = \int_0^{T_f} h_{q,p}(t) w(t - \tau) dt$ . Since  $T_f$  and  $\delta$  are chosen to be larger than the maximum delay spread and since the sequences attributed to each one of the transmit antennas are orthogonal, (9) will now simplify to:

$$x_{k,q,p,l,m} = \sqrt{\frac{N_f}{P}} a_{k,m} r_{q,p,l} \quad (10)$$

where  $r_{q,p,l} = r_{q,p}(lT_w)$ .

(8) can be expressed in matrix form as:

$$Y_k = \sqrt{\frac{N_f}{P}} A_k R + N_k \quad (11)$$

where  $A_k$  is the  $M$  dimensional vector corresponding to the transmitted symbol:  $A_k = [a_{k,0}, \dots, a_{k,M-1}]^T$  where  $(\cdot)^T$  stands for matrix transposition.  $A_k$  comprises  $M-1$  zeros and one nonzero element that takes the value  $\pm 1$ . The  $PQL$  channel coefficients are stacked in the  $1 \times PQL$  matrix  $R = [R_1, \dots, R_Q]$ .  $R_q = [r_{q,1,0}, \dots, r_{q,1,L-1}, \dots, r_{q,P,0}, \dots, r_{q,P,L-1}]$  and it comprises the  $PL$  channel parameters at the  $q$ -th receiver.  $Y_k$  is the  $(M \times PQL)$  decision matrix whose  $(m, (q-$

1) $PL + (p - 1)L + l + 1$ -th entry is equal to  $y_{k,q,p,l,m}$ .  $N_k$  is the noise matrix and is constructed in the same way as  $Y_k$ .

Denoting by  $s_k$  and  $\Delta_k$  the amplitude and the position of the  $k$ -th information symbol respectively, the multi-dimensional differential encoding scheme corresponds to transmitting the vector  $A_k$  such that:

$$A_k = s_k \Omega^{\Delta_k} A_{k-1} \quad (12)$$

where  $A_0$  corresponds to the first column of the  $M \times M$  identity matrix  $I_M$  and  $\Omega$  is the permutation matrix defined as:

$$\Omega = \begin{bmatrix} \Theta^T & 1 \\ I_{M-1} & \Theta \end{bmatrix} \quad (13)$$

where  $\Theta$  is the  $M - 1$  dimensional null vector.

The encoding scheme in (12) can be viewed as a combination of DPSK and permutation codes [13]. The special cases of PPM and BPSK constellations can be obtained by fixing  $s_k = 1$  and  $\Delta_k = 0$  respectively in (12).

The differential decoder employs two consecutive code-words and is given by:

$$(\hat{s}_k, \hat{\Delta}_k) = \arg \max_{\substack{s=\pm 1 \\ p \in \{0, \dots, M-1\}}} (\max_{\text{diag}}(sY_k Y_{k-1}^T \Omega^{M-p})) \quad (14)$$

where the function  $\max_{\text{diag}}(X)$  returns the maximum of the diagonal of matrix  $X$ . This decoding strategy shows to be superior to the one employing the trace of the decision matrix in (14) since the  $(M-1)$  noisy diagonal elements are excluded from the sum. For PPM constellations,  $s$  must be fixed to 1 in (14) while for BPSK signals the restored polarity corresponds to the sign of the scalar  $Y_k Y_{k-1}^T$ .

In the absence of noise, the maximum of the right hand side of (14) takes the value of  $\sum_{q,p,l} r_{q,p,l}^2 / P$  (by excluding the multiplying factor  $N_f$  since it is also present in the noise variance). This means that the captured energy is small only if the magnitudes of all of the  $PQL$  quantities  $\{r_{q,p,l}\}$  are small. In other words, all of the  $PQ$  sub-channels must suffer from fading during a duration of  $LT_w$ . So, the coded system enjoys a better immunity against fading and exploits the spatial and multipath diversity of the underlying channel to attain a diversity gain of  $PQL$ . To this diversity gain, we must also add the increase in energy capture offered by the Rake receiver and by the receive antenna array but not by the transmit array.

### B. Nonorthogonal modulations

The proposed code can be readily adapted to nonorthogonal modulations where the modulation delay is chosen to be smaller than the channel delay spread. This approach is appealing since it results in shorter time frames but at the same time interference between the modulation positions will result in some performance losses.

To assure that the noise samples at the output of the correlators remain white, the modulation delay is chosen to verify  $\delta \geq LT_w$ . Under this assumption, (11) becomes:

$$Y_k = \sqrt{\frac{N_f}{P}} R(I_{PQL} \otimes A_k) + N_k \quad (15)$$

where  $Y_k$ ,  $A_k$  and  $N_k$  have the same structure as in the case of orthogonal constellations and  $\otimes$  stands for the Kronecker product.  $R$  is now a  $(M \times MPQL)$  matrix  $R = [R_1, \dots, R_Q]$ . Where  $R = [R_{q,1,0}, \dots, R_{q,1,L-1}, \dots, R_{q,P,0}, \dots, R_{q,P,L-1}]$  and  $R_{q,p,l}$  is the  $(M \times M)$  lower triangular matrix constructed from (9) as:

$$R_{q,p,l} = \begin{bmatrix} r_{q,p,l,0} & 0 & \cdots & 0 \\ r_{q,p,l,1} & r_{q,p,l,0} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ r_{q,p,l,M-1} & \cdots & r_{q,p,l,1} & r_{q,p,l,0} \end{bmatrix} \quad (16)$$

with  $r_{q,p,l,m} = r_{q,p}(lT_w + m\delta)$ .

Disregarding the factor  $N_f/P$  that can be included in the noise variance and in the absence of noise, the transmitted message can be decoded from:

$$sY_k Y_{k-1}^T \Omega^{M-p} = s \left( \sum_{q,p,l} R_{q,p,l} A_k A_{k-1}^T R_{q,p,l}^T \right) \Omega^{M-p} \quad (17)$$

Following from the structure of  $A_k$  and  $R_{q,p,l}$ , the diagonal elements of (17) take the form  $\sum_{q,p,l} r_{q,p,l,m} r_{q,p,l,m'}$  for  $m$  and  $m' \in \{0, \dots, M-1\}$ . And so the optimal value of  $\sum_{q,p,l} r_{q,p,l}^2$  (for  $m = m' = 0$ ) corresponding to the amplitude and the position of the transmitted symbol can be exceeded even in the absence of noise depending on the specific channel realization. So error floors are expected especially in non line of sight (NLOS) channels. Statistically, the last quantity becomes more predominant with increasing  $P$ ,  $Q$  or  $L$  and thus MIMO systems are expected to reduce these error floors. Moreover, given that the symbol duration increases with  $L$  it is interesting, from this point of view, to increase the dimensions of the antenna arrays rather than the number of fingers.

### C. Decision feedback receiver

The differential receiver in (14) is based on comparing two consecutive symbols, and thus the  $(k-1)$ -th symbol will act as a reference for the detection of the  $k$ -th symbol. Performance can be ameliorated if this noisy reference is replaced by another one based on more than one symbol. This is referred to as decision feedback or decision directed receivers [3],[14]. This principle can be readily adapted to multi-dimensional constellations; (14) can now be written as:

$$(\hat{s}_k, \hat{\Delta}_k) = \arg \max_{\substack{s=\pm 1 \\ p \in \{0, \dots, M-1\}}} (\max_{\text{diag}}(sY_k R_{k-1}^T \Omega^{M-p})) \quad (18)$$

The reference signal  $R_k$  is updated during the channel coherence time based on previously detected symbols as:

$$R_k = Y_k + \hat{s}_k \Omega^{\hat{\Delta}_k} R_{k-1} \quad (19)$$

with  $R_0 = Y_0$ . It was shown in the literature that such receivers do not suffer from error propagation and that the performance converges to that of a coherent receiver especially at high signal to noise ratios [14].

An equivalent system to that proposed in section (A) can be obtained by performing separate differential codec on the amplitude and non-coherent codec on the position. The

encoding scheme in (12) will change to  $A_k = a_k I_{M, \Delta_k + 1}$  such that  $a_k = s_k a_{k-1}$  and  $I_{M, n}$  is the  $n$ -th column of the identity matrix  $I_M$ .

The decoder will now perform separate decisions on the amplitudes and the positions of the transmitted symbols:

$$\hat{\Delta}_k = \arg \max_{p \in \{0, \dots, M-1\}} (Y_{k, p+1} Y_{k, p+1}^T)$$

$$\hat{s}_k = \text{sign}(Y_{k, \hat{\Delta}_k + 1} Y_{k-1, \hat{\Delta}_k - 1 + 1}^T)$$

where  $Y_{k, n}$  is the  $n$ -th row of the  $M \times PQ L$  matrix  $Y_k$ .

In the absence of feedback, this system shows negligible loss with respect to differential systems. However, feedback can not be performed for the position detection, and thus performance will be limited by the energy detector irrespective from the possible amplitude feedback. In other words, feedback can not ameliorate the performance of such systems.

### III. DIVERSITY SCHEME 2

Unlike the first ST code that encodes the pulses of the same symbol, the code that will be proposed in this section encodes two consecutive symbols while the intra symbol pulse coding is chosen to be the repetition code. While the first diversity scheme achieves full diversity for all number of transmit antennas and for all kinds of constellations, the second diversity scheme can be associated only with two transmit antennas and with constellations that include amplitude modulations. This code, which was proposed for narrow band communications in [6] and [7] is known to be optimal in the case of one receive antenna but its extension to any number of transmit antennas may result in data rate reduction [10].

The transmitted signal from the two transmit antennas during two consecutive symbol durations is:

$$s_{k,1}(t) = \frac{1}{\sqrt{2N_f}} \sum_{n,m} [a_{k,1,m} w_{n,m}(t) - a_{k,2,m} w_{n,m}(t - N_f T_f)]$$

$$s_{k,1}(t) = \frac{1}{\sqrt{2N_f}} \sum_{n,m} [a_{k,2,m} w_{n,m}(t) + a_{k,1,m} w_{n,m}(t - N_f T_f)]$$

where:  $w_{n,m}(t) = w(t - nT_f - m\delta)$ ,  $k$  is now the index of two symbol durations with  $a_{k,p,m} = a_{k,p} \delta(d_{k,p} - m)$ ,  $(a_{k,1}, d_{k,1})$  and  $(a_{k,2}, d_{k,2})$  are the coordinates of 2 consecutive symbols.

The reference signals in (6) are now independent from the corresponding transmit antenna and take the form:

$$\hat{s}_{l,m}(t) = \sum_{n=0}^{N_f-1} w(t - nT_f - lT_w - m\delta) \quad (20)$$

Assuming that the channel remains invariant for a duration of  $2N_f T_f$  seconds, the  $q$ -th receiver starts by calculating:

$$y_{k,q,1,l,m} = \int_0^{N_f T_f} r_{k,q}(t) \hat{s}_{l,m}(t) dt \quad (21)$$

$$y_{k,q,2,l,m} = \int_{N_f T_f}^{2N_f T_f} r_{k,q}(t) \hat{s}_{l,m}(t - N_f T_f) dt \quad (22)$$

$y_{k,q,1,l,m}$  and  $y_{k,q,2,l,m}$  stand for the outputs of the  $l$ -th finger of the  $q$ -th receive antenna corresponding to the  $m$ -th position during odd and even symbol durations respectively.

For orthogonal modulations, (21) and (22) can be represented in matrix form as:

$$\begin{bmatrix} y_{k,q,1,l} \\ y_{k,q,2,l} \end{bmatrix} = \sqrt{\frac{N_f}{2}} \begin{bmatrix} A_{k,1} & A_{k,2} \\ -A_{k,2} & A_{k,1} \end{bmatrix} \begin{bmatrix} r_{q,1,l} \\ r_{q,2,l} \end{bmatrix} + \begin{bmatrix} n_{k,q,1,l} \\ n_{k,q,2,l} \end{bmatrix} \quad (23)$$

where  $A_{k,1}$  and  $A_{k,2}$  are  $M$  dimensional vectors corresponding to the data symbols. The noise terms are the same as in (8) and  $y_{k,q,p,l} = [y_{k,q,p,l,0}, \dots, y_{k,q,p,l,M-1}]^T$  for  $p = 1, 2$ .

To simplify the analysis, (23) can be expressed as [7]:

$$Y_{k,q,l} = \sqrt{\frac{N_f}{2}} A_k R_{q,l} + N_{k,q,l} \quad (24)$$

where the previous matrices along with their dimensions are:

$$[Y_{k,q,l}]_{2M \times 2} = \begin{bmatrix} y_{k,q,1,l} & -y_{k,q,2,l} \\ y_{k,q,2,l} & y_{k,q,1,l} \end{bmatrix},$$

$$[A_k]_{2M \times 2} = \begin{bmatrix} A_{k,1} & A_{k,2} \\ -A_{k,2} & A_{k,1} \end{bmatrix}, [R_{q,l}]_{2 \times 2} = \begin{bmatrix} r_{q,1,l} & -r_{q,2,l} \\ r_{q,2,l} & r_{q,1,l} \end{bmatrix}$$

and  $N_{k,q,l}$  is constructed in the same way as  $Y_{k,q,l}$ .

Stacking the decision, noise and channel matrices we obtain:

$$Y_k = \sqrt{\frac{N_f}{2}} A_k R + N_k \quad (25)$$

with  $R = [R_{1,0} \dots R_{1,L-1}, \dots, R_{Q,0} \dots R_{Q,L-1}]$ ;  $Y_k$  and  $N_k$  are  $2M \times 2QL$  matrices constructed in the same way as  $R$ .

If  $s_{k,p}$  and  $\Delta_{k,p}$  stand for the amplitude and the position of the  $p$ -th symbol for  $p = 1, 2$ , the differential encoding scheme corresponds to transmitting the matrix  $A_k$  such that:

$$a_k = s_k a_{k-1} = \begin{bmatrix} s_{k,1} & s_{k,2} \\ -s_{k,2} & s_{k,1} \end{bmatrix} \begin{bmatrix} a_{k-1,1} & a_{k-1,2} \\ -a_{k-1,2} & a_{k-1,1} \end{bmatrix} \quad (26)$$

$$A_k = \begin{bmatrix} a_{k,1} I_{M, d_{k,1}} & a_{k,2} I_{M, d_{k,1}} \\ -a_{k,2} I_{M, d_{k,2}} & a_{k,1} I_{M, d_{k,2}} \end{bmatrix} \quad (27)$$

where:  $d_{k,p} = \Delta_{k,p} + d_{k-1,p} \pmod{M}$  for  $p = 1, 2$ .

In other words, the amplitude is encoded differentially by the coding scheme given in [7]. Moreover, during odd and even symbol durations the two antennas transmit pulses that have the same positions which are encoded differentially.

The maximum-likelihood receiver will start by calculating the  $M^2$  matrices for  $p_1, p_2 \in \{0, \dots, M-1\}$ :

$$G_{p_1, p_2} = Y_k Y_{k-1}^T \begin{bmatrix} \Omega^{M-p_1} & \Theta \\ \Theta & \Omega^{M-p_2} \end{bmatrix} \quad (28)$$

where  $\Theta$  stands now for the  $M \times M$  null matrix.

Let  $f_1$  (resp.  $f_2$ ) correspond to the index of the maximum magnitude of the first (resp. last)  $M$  diagonal elements of  $G_{p_1, p_2}$  with  $1 \leq f_1 \leq M$  and  $M+1 \leq f_2 \leq 2M$ . The next step consists of the construction of the  $2 \times 2$  matrix  $g_{p_1, p_2}$  whose  $(m, n)$ -th element is the  $(f_m, f_n)$ -th element of  $G_{p_1, p_2}$ . Finally, the receiver decides in the favor of:

$$(\hat{s}_k, \hat{\Delta}_k) = \arg \max_{\substack{s_1, s_2 = \pm 1 \\ p_1, p_2 \in \{0, \dots, M-1\}}} \text{trace}(s^T g_{p_1, p_2}) \quad (29)$$

$s$  is constructed from  $s_1$  and  $s_2$  in the same way as  $s_k$  in (26).

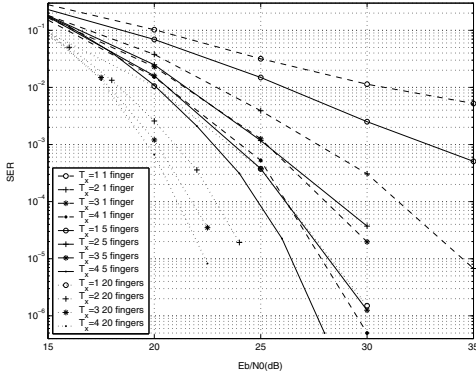


Fig. 1. Performance of 4PPM-2PAM with one receive antenna.

The function "trace" is used rather than "maxdiag" since decision must be performed jointly on 2 consecutive symbols. In the absence of noise,  $g_{p_1, p_2} = s_k \sum_{q,p,l} r_{q,p,l}^2 / 2$  if  $p_1$  and  $p_2$  correspond to the transmitted positions and will have one or more zero entries otherwise; and hence the ST-code achieves full diversity. The extension to nonorthogonal modulations and decision feedback receivers can be done in the same way as in paragraph 2 and thus will be omitted.

#### IV. SIMULATIONS AND RESULTS

The pulse waveform  $w(t)$  was chosen to be the second derivative of the Gaussian pulse with a duration of 0.5ns. The transmit and the receive arrays are supposed to be sufficiently spaced so the  $PQ$  sub-channels are generated independently using the standard IEEE 802.15.3a channel model CM2 which corresponds to non-line-of-sight (NLOS) conditions [11]. Orthogonal PPM modulations are used with  $\delta = 100$  ns,  $N_f = 16$  and the frame time was fixed to  $T_f = M\delta$ . The first results consist of comparing the two diversity schemes with two transmit antennas. As the analytical behavior in paragraphs II and III showed the same asymptotic behavior and diversity advantage, simulations show that the two proposed codes show very close performance (less than 0.1 dB of difference) for all signal to noise ratios and for all constellation dimensions. So in what follows, no distinction will be made between the codes.

Figure 1 shows the performance gain of the proposed code for bi-orthogonal 4-PPM-2-PAM constellations.  $Q = 1$  while  $P$  varies from 1 to 4. The performance improvement is the highest in situations where the single antenna systems suffer initially from an insufficient multipath diversity as in the case of  $L = 1$ . Moreover, the performance gain is still present with large number of fingers.

Figure 2 compares different constellations with a 1 finger Rake receiver. The transmit and receive arrays are chosen to have the same number of antennas which varies from 1 to 3. Single antenna systems suffer from severe fading and in these conditions the BPSK shows better performance. For (2,2) systems, the diversity order is multiplied by 4 and the energy capture is enhanced by 3 dB. In these favorable conditions, 4 PPM and 16 PPM constellations start outperforming BPSK at

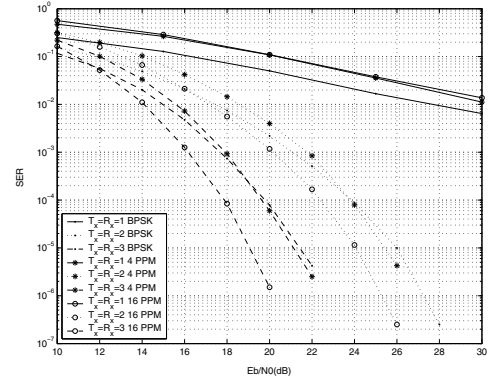


Fig. 2. Performance of different constellations with a 1 finger Rake.

24 dB and 16 dB respectively. The choice of high dimensional constellations becomes more evident for (3,3) systems.

Figure 3 compares systems with the same overall spatial and multipath diversity for 8-PPM-2-PAM constellations with one receive antenna. The diversity gain which is equal to  $PQL$  takes the value of 6 and 18. Results show that exploiting multipath diversity by increasing the number of Rake fingers can be more beneficial at low SNRs where performance is dominated by noise since larger number of fingers will increase the energy capture. For high SNRs, performance is dominated by fading and transmit diversity becomes more beneficial even though it does not increase the energy capture. This follows from the fact that consecutive rays of the same sub-channel can be simultaneously faded because of cluster and channel shadowing [11]. For example, at an error rate of  $10^{-3}$ , the coded system with 3 transmit antennas and 2 fingers presents an advantage of about 5.5dB with respect to a single antenna system with 6 fingers Rake.

Figure 4 shows the performance of decision feedback receivers for 8-PPM-2-PAM constellations with 2 transmit and one receive antenna.  $T_{coh}$  stands for the channel coherence time normalized by the symbol duration and the length of the decision feedback vector was fixed to  $T_{coh}$ . For comparison, the performance of maximum ratio combining (MRC) under the assumption of perfect channel estimation is also shown. Results show that decision feedback can ameliorate performance at all signal to noise ratios and can approach MRC especially at high SNRs and low fading ( $L = 20$ ).

In figure 5, the performance of nonorthogonal 4-PPM-2-PAM constellations with one receive antenna and 5 fingers Rake is shown for different modulation delays with no ISI ( $T_f = 3\delta + 100$ ). These results show the ability of multi-antenna systems to reduce error floors induced by the overlapping of different positions caused by the channel. For example, error rates in the order of  $10^{-5}$  are achievable with  $\delta = 10ns$  by employing 4 transmit antennas; while it was impossible to achieve this value with SISO systems even with  $\delta = 50ns$  for practical values of the SNR. From figure 2 and figure 5 we conclude that ST-coding allows to profit from the SER advantage of high order constellations with no high penalty on

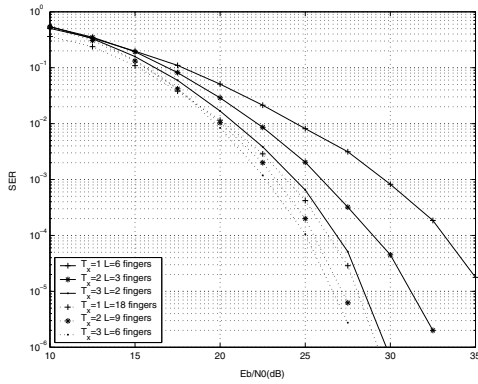


Fig. 3. Transmit versus multi-path diversity for 8-PPM-2PAM and  $Q=1$ .

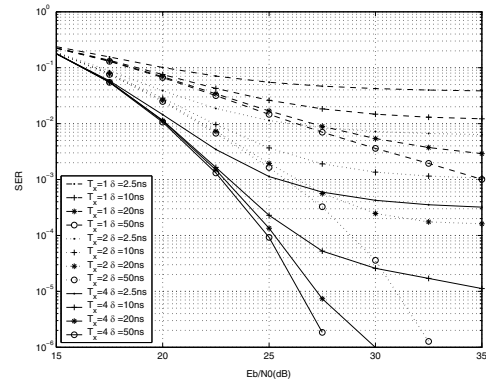


Fig. 5. Performance of 4-PPM-2PAM with one receive antenna.

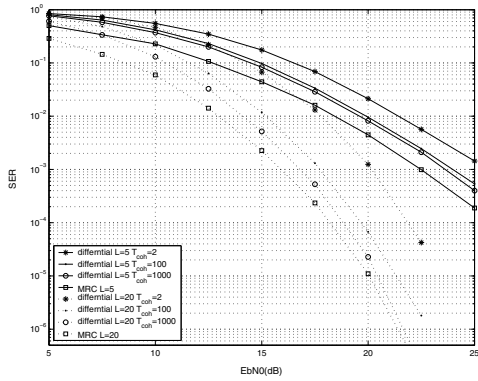


Fig. 4. Performance comparison of decision feedback and MRC receivers for 8-PPM-2-PAM constellations with 1 receive and 2 transmit antennas.

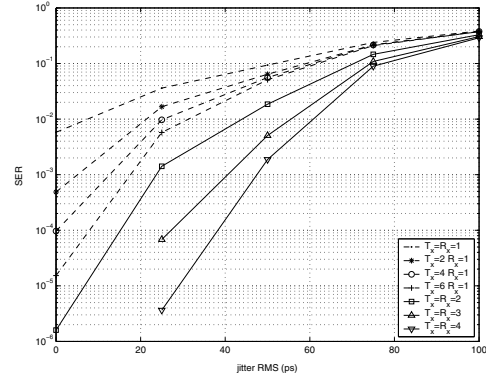


Fig. 6. Effect of timing jitter with BPSK signals and 5 fingers Rake at 26dB.

the symbol duration thus leading to better performance with higher data rates.

Figure 6 shows the performance loss incurred by timing jitter for BPSK with 5 fingers Rake at  $E_b/N_0 = 26dB$ . The jitter is modelled as a zero-mean normally distributed random variable. Results show that the coded systems enjoy better immunity against timing jitter.

## V. CONCLUSION

Taking advantage of both spatial and multipath diversity leads to the possibility of achieving high performance levels with low complexity differential receivers. To achieve these gains, two schemes were considered and were found to have the same performance with two transmit antennas. Simulations showed the ability to reduce error floors caused by timing jitter and small modulation delays and to approach coherent reception with adaptive receivers over slow varying channels.

## REFERENCES

- [1] Y. Chao and R.A. Scholtz, "Optimal and suboptimal receivers for ultra-wideband transmitted reference systems", Global telecommunications conference, vol. 2, pp. 759-763, March 2004.
- [2] J.D. Choi and W.E. Stark, "Performance of ultra-wideband communications with suboptimal receivers in multipath channels", IEEE journal on sel. areas in comm., vol. 20, pp. 1754-1766, 2002.

- [3] G. Durisi and S. Benedetto, "Performance of coherent and non-coherent receivers for uwb communications", IEEE international conference on communications, vol. 6, pp. 3429-3433, June 2004.
- [4] Y. Souilmi and R. Knopp, "On the achievable rates of ultra-wideband ppm with non-coherent detection in multipath environments", IEEE international conf. on communications, vol. 5, pp. 3530-3534, 2003.
- [5] B.L. Hughes, "Differential space-time modulation", IEEE transactions on information theory, vol. 46, pp. 2567-2578, November 2000.
- [6] V. Tarokh and H. Jafarkhani, "A differential detection scheme for transmit diversity", IEEE journal on selected areas in communications, vol. 18, pp. 1169-1174, July 2000.
- [7] C. Gao, A.M. Haimovich and D. Lao, "Bit error probability for space-time block code with coherent and differential detection", Vehicular technology conference, vol. 1, pp. 410-414, September 2002.
- [8] L. Yang and G.B. Giannakis, "Analog space-time coding for multi-antenna ultra-wideband transmissions", IEEE transactions on Communications, vol. 52, pp. 507-517, March 2004.
- [9] H. Zhang and T.A. Gulliver, "Pulse position amplitude modulation for time hopping multiple access uwb communications", IEEE WCNC, vol. 2, pp. 595-900, March 2004.
- [10] V. Tarokh, H. Jafarkhani, and A.R. Calderbank, "Space-time block codes from orthogonal design", IEEE transactions on Information Theory, vol. 45, pp. 1456-1466, July 1999.
- [11] J. Foerster (editor), "Channel modeling sub-committee Report Final", IEEE 802.15-02/490.
- [12] D. Cassioli and M.Z. Win, F.Vatalaro, and A.F. Molisch, "Performance of low-complexity Rake reception in a realistic UWB channel", IEEE int. conference on communications, vol. 2, pp. 763-767, May 2002.
- [13] A. Nordin and E. Viterbo, "Permutation modulation for fading channels", IEEE international conference on communications, vol.2, pp. 1177-1183, March 2003.
- [14] F. Adachi and M. Sawahashi, "Decision feedback differential phase detection of M-ary DPSK signals", IEEE transactions on vehicular technology, vol. 44, pp. 203-210, May 1995.