Diversity Orders and Coding Gains of Repetition Coding and Transmit Laser Selection over MIMO Free-Space Optical Links

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***Abstract*—In this paper, we analyze the repetition coding and transmit laser selection schemes over multiple-input-multiple-output free-space optical communication systems with intensity modulation and direct detection over gamma-gamma atmospheric turbulence channels. We derive novel closed-form expressions for the outage probabilities achieved by the above schemes. Through an asymptotic analysis of the outage probabilities, we prove that both schemes achieve the same diversity order that is equal to the product of the number of transmit apertures, number of receive apertures and a channel related parameter. We also prove that the superiority of the transmit laser selection scheme over the repetition coding scheme resides in a coding gain that we quantify analytically in this work.**

I. Introduction

Recently, Free-Space Optical (FSO) communication systems attracted a significant amount of attention as a license-free and cost-effective wireless solution to the last mile problem. Optical transceivers placed on the rooftops of buildings, for example, ensure a directive optical connectivity capable of delivering very high data rates. However, FSO links suffer from significant levels of fading (or scintillation) that result mainly from the variation of the index of refraction with humidity and temperature. Consequently, analogously to Radio Frequency (RF) systems, diversity methods were proposed and analyzed in the context of FSO communications. In a manner analogous to the well known Multiple-Input-Multiple-Output (MIMO) systems, several lasers (transmit apertures) can be placed at the transmitter side while several photodetectors (receive apertures) can be placed at the receiver side [1]–[13]. In this context, the existing research effort targeted Pulse Position Modulation (PPM) as well as On-Off Keying (OOK) that constitute the most popular and adapted modulation schemes to pulse-based optical communications.

Early stages of research on diversity methods for FSO communications targeted the receive diversity and Space-Time Block Coding (STBC) techniques. In [1], the receive diversity based on aperture averaging was analyzed for OOK over gamma-gamma turbulence induced fading. In [2] and [3], STBCs for MIMO FSO systems were proposed and analyzed in the cases of OOK and PPM, respectively. The more recent MIMO FSO solutions proposed in the literature revolved around the Repetition Coding (RC) and Transmit Laser Selection (RLS) schemes. These solutions are better adapted to the FSO links where, unlike the RF propagation channels, the path gains are real and non-negative. For the RC scheme, the same PPM or OOK symbol is transmitted from all transmit apertures simultaneously [4]–[10]. On the other hand, for the TLS scheme, a single laser is selected to transmit at a given time [11]–[13]. While the implementation of TLS necessities acquiring the channel state information (CSI) at the transmitter unlike RC, the TLS solution is superior to RC. On the other hand, it was proven in [4] that RC in its turn outperforms STBCs over MIMO FSO channels thus highlighting the research interest in investigating the RC and TLS techniques.

In [5], the Bit Error Rate (BER) of RC with OOK was provided over lognormal fading channels where it was proven that the effect of spatial diversity manifests itself as a decrease of the channel variance. The advantage of RC with OOK was also demonstrated in [6] through a BER analysis over K-distributed atmospheric turbulence channels. The BER and channel capacity achieved by RC with PPM were studied in [7] over exponential and lognormal fading channels. This work was further extended to multi-pulse PPM in [8] and to receivers equipped with a single optical amplifier in [9]. In [10], an optimization method for the gain of the avalanche photodetectors was proposed for 2×2 MIMO FSO systems deploying PPM over lognormal and exponential channels.

The TLS scheme based on the selection of the laser that ensures the maximum path gain was proposed in [11]. Through a BER analysis over K-distributed atmospheric turbulence channels, the authors of [11] demonstrated the superiority of TLS compared to STBCs and RC. Closed-form BER expressions for TLS MISO systems over exponential atmospheric turbulence channels were subsequently derived in [12]. The authors of [13] tackled the laser selection scheme as a power allocation problem and proved the optimality of the TLS scheme in the sense of minimizing the conditional BER. A joint selection-repetition protocol was also proposed in the case of limited feedback.

In this paper, we compare the RC and TLS schemes over the

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widely approved gamma-gamma turbulence-induced fading channel model. By appropriately approximating the gamma-gamma probability density function near the origin by a simpler alternative function, we derive closed-form asymptotic expressions of the outage probabilities achieved by the above schemes. The contributions of the paper are two fold. First, we prove that the power margin (or equivalently, the signal-to-noise ratio) exponent is the same for both schemes showing that the diversity orders achieved by these schemes are the same. Second, we provide a simple closed-form expression that relates the coding gain of TLS (over RC) to the number of transmit and receive apertures as well as the distance of the FSO link.

II. System Model

A. General Parameters

Consider a MIMO FSO communication system where the transmitter is equipped with P transmit apertures or lasers and the receiver is equipped with Q receive apertures of photo-diodes. In this work, we adopt the gamma-gamma channel model. For this model the irradiance, which is the square of the path gain, between any pair of transmit and receive apertures is modeled by a gamma-gamma random variable whose probability density function (pdf) is given by ($I \geq 0$):

$$f_{\gamma\gamma}(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{(\alpha+\beta)/2-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta I}\right) \quad (1)$$

where $\Gamma(.)$ is the Gamma function and $K_c(.)$ is the modified Bessel function of the second kind of order c.

An FSO channel is completely characterized by the two parameters α and β that are related to the link distance d by:

$$\alpha(d) = \left[\exp\left(0.49\sigma_R^2(d)/(1+1.11\sigma_R^{12/5}(d))^{7/6}\right) - 1\right]^{-1} \quad (2)$$

$$\beta(d) = \left[\exp\left(0.51\sigma_R^2(d)/(1+0.69\sigma_R^{12/5}(d))^{5/6}\right) - 1\right]^{-1} \quad (3)$$

where $\sigma_R^2(d)$ is the Rytov variance given by:

$$\sigma_R^2(d) = 1.23 C_n^2 k^{7/6} d^{11/6} \quad (4)$$

where $k = \frac{2\pi}{\lambda}$ is the wave number (where λ is the wavelength) and C_n^2 denotes the refractive index structure parameter.

Binary Pulse Position Modulation (BPPM) constitutes a popular modulation scheme for FSO systems with Intensity-Modulation and Direct-Detection (IM/DD). In this case, one of the two symbols $[1 \ \ 0]^T$ or $[0 \ \ 1]^T$ is transmitted where the 0 and 1 indicate the absence and presence of a light signal, respectively. The electrical signal at the output of the q-th photodetector corresponding to the optical signal transmitted by the p-th laser can be written as [14]:

$$\mathbf{r}_{p,q} = RT_b\left(\eta I_{p,q} P_t \mathbf{s}_p + P_b\right) + \mathbf{n}_q \quad (5)$$

where $\mathbf{s}_p \in \{[1 \ \ 0]^T, [0 \ \ 1]^T\}$ corresponds to the BPPM symbol transmitted by the p-th laser. R is the photodetector's responsivity and T_b stands for the bit duration. P_b stands for the power of background radiation (and dark currents) that gets superimposed with the signal and non-signal parts of the PPM symbol. P_t stands for the total transmitted optical power that affects only the signal part of $\mathbf{s}_p$. In (5), $\mathbf{n}_q$ stands for the two-dimensional noise vector at the q-th receive aperture. As in [14], we assume background noise limited receivers implying that each of the signal and non-signal components of $\mathbf{n}_q$ can be modeled as a signal-independent additive white Gaussian noise (AWGN) with zero mean and variance $N_0/2$. Finally, η is a power normalization factor that depends on the implemented transmission strategy as will be explained later.

In (5), $I_{p,q}$ stands for the irradiance between the p-th transmit and q-th receive apertures. Elements of $\{I_{p,q} \ , \ p = 1,\dots,P \ ; \ q = 1,\dots,Q\}$ are identically distributed according to the gamma-gamma pdf in (1). We also assume that the elements of the above set are independent which corresponds to the scenario where the elements of the transmit and receive arrays are sufficiently spaced.

B. Repetition Coding

For the repetition coding (RC) diversity scheme, the same BPPM symbol is transmitted from all transmit apertures and $\mathbf{s}_p$ can be written as $\mathbf{s}$. Moreover, the transmit power is evenly split among the transmit apertures and the coefficient η in (5) is set to $\frac{1}{P}$. Unlike RF systems where space-time coding techniques need to be implemented, the RC scheme is better adapted to FSO systems with IM/DD following from the non-negativity of the path gains (or irradiances) which ensures non-destructive interference at the receiver side. This scheme is particularly appealing because of its simplicity since non-coherent detection can be implemented at the receiver without necessitating any kind of channel estimation nor feedback.

Setting $\mathbf{s}_p = \mathbf{s}$ and $\eta = \frac{1}{P}$ in (5), the decision at the receiver will be based on the following signal:

$$\mathbf{r} = \sum_{q=1}^{Q} \mathbf{r}_{p,q} = RT_b\left(\frac{1}{P}\mathcal{I}_{RC} P_t \mathbf{s} + P_b\right) + \mathbf{n} \quad (6)$$

where $\mathbf{n} \triangleq \sum_{q=1}^{Q} \mathbf{n}_q$ is the AWGN vector with its components in the signal and non-signal slots having a variance of $QN_0/2$.

The equivalent RC irradiance in (6) is given by:

$$\mathcal{I}_{RC} = \sum_{q=1}^{Q}\sum_{p=1}^{P} I_{p,q} \quad (7)$$

After removing the constant bias RT_bP_b from both BPPM slots in (6), the signal-to-noise ratio (SNR) with RC can be written as [14]:

$$\gamma_{RC} = \frac{R^2 T_b^2 \mathcal{I}_{RC}^2 P_t^2}{P^2 Q N_0} \quad (8)$$

The MIMO system is said to be in outage if the SNR falls below a ceratin decoding threshold value necessary for ensuring the detection of the information symbols with an arbitrarily small probability of error. Denoting this threshold SNR by γ_{th}, the outage probability of MIMO systems deploying RC can be obtained from (8) as follows:

$$P_{RC} \triangleq \Pr\left(\gamma_{RC} < \gamma_{th}\right) = \Pr\left(\mathcal{I}_{RC} < \frac{P\sqrt{Q}}{P_M}\right) \quad (9)$$

where $P_M \triangleq \frac{RT_bP_t}{\sqrt{N_0\gamma_{th}}}$ denotes the power margin.

C. Transmit Laser Selection

For the transmit laser selection (TLS) diversity scheme, only one laser out of the P total lasers is selected to transmit the BPPM symbol while the remaining $P-1$ lasers remain idle. The selected laser ensures the maximum irradiance at the receiver side. In this case, the decision at the receiver will be based on the following signal:

$$\mathbf{r} = \sum_{q=1}^{Q} \mathbf{r}_{p,q} = RT_b\left(\mathcal{I}_{TLS}P_t\mathbf{s} + P_b\right) + \mathbf{n} \tag{10}$$

where:

$$\mathcal{I}_{TLS} = \max_{p=1,\dots,P} \sum_{q=1}^{Q} I_{p,q} \tag{11}$$

and thus $\mathbf{s}$ is transmitted by the laser ensuring the maximum value of $\sum_{q=1}^{Q} I_{p,q}$. Since only one laser is transmitting, η is set to 1 in (5) while the noise vector $\mathbf{n}$ is the same as in (6).

From (7) and (11), $\mathcal{I}_{TLS} \geq \mathcal{I}_{RC}$ and TLS achieves higher performance levels compared to RC. However, this enhanced performance is associated with an increased complexity. In fact, unlike the RC scheme, the receiver needs to estimate the channel irradiances $I_{p,q}$ in the case of TLS. Moreover, a feedback link is needed in order to update the transmitter on the laser to be activated depending on the specific channel realization. On the other hand, the coherence time of FSO links is much larger than the coherence time of RF systems. Consequently, relatively longer training sequences can be used for accurately estimating the channel irradiances and the feedback link needs to be triggered less frequently.

In a way similar to (8), the SNR with TLS is given by:

$$\gamma_{TLS} = \frac{R^2T_b^2\mathcal{I}_{TLS}^2P_t^2}{QN_0} \tag{12}$$

resulting in the following expression of the outage probability:

$$P_{TLS} \triangleq \Pr\left(\gamma_{TLS} < \gamma_{th}\right) = \Pr\left(\mathcal{I}_{TLS} < \frac{\sqrt{Q}}{P_M}\right) \tag{13}$$

Finally, in a way similar to (9) and (13), the outage probability of Single-Input-Single-Output (SISO) systems is:

$$P_{SISO} = \Pr\left(I < P_M^{-1}\right) \tag{14}$$

where I is the path irradiance between the single transmit aperture and the single receive aperture.

III. Performance analysis

A. Repetition Coding

The random variable $\mathcal{I}_{RC}$ in (7) corresponds to the summation of PQ gamma-gamma random variables and no closed-form solutions for such summations are available in the literature. In an attempt to derive closed-form analytical expressions for the diversity order and coding gain of RC, we resort to approximating the pdf in (1) by the following function as in [15]:

$$f_{SISO}(I) \approx aI^{\beta-1} \quad ; \quad I \geq 0 \tag{15}$$

where the constant a is related to the channel parameters α and β in (2)-(3) by:

$$a = \frac{(\alpha\beta)^{\beta}\Gamma\left(\alpha-\beta\right)}{\Gamma\left(\alpha\right)\Gamma\left(\beta\right)} \tag{16}$$

where the approximation in (15) is close to the exact pdf in (1) near the origin and is thus suitable for an asymptotic analysis of the outage probability where the performance is dominated by the extremely small path irradiances.

From (14) and (15), the outage probability of SISO systems can be approximated by:

$$P_{SISO} = \frac{a}{\beta}P_M^{-\beta} = \frac{\alpha^{\beta}\beta^{\beta-1}\Gamma\left(\alpha-\beta\right)}{\Gamma\left(\alpha\right)\Gamma\left(\beta\right)}P_M^{-\beta} \tag{17}$$

showing that the diversity order over a SISO link is equal to the parameter β of the gamma-gamma channel and, hence, is related to the distance of the FSO link.

Following from (7) and (15), the pdf of $\mathcal{I}_{RC}$ can be approximated by:

$$f_{RC}\left(I\right) \approx a^{PQ}(I^{\beta-1} \underbrace{* \cdots *}_{PQ-1 \text{ times}} I^{\beta-1}) \tag{18}$$

where $*$ stands for convolution.

The convolution between two functions of the form I^{β_m-1} and I^{β_n-1} (defined for positive values of I) can be written as:

$$y(I) = \int_{-\infty}^{+\infty} \tau^{\beta_m-1}(I-\tau)^{\beta_n-1}\mathrm{d}\tau = \int_{0}^{I} \tau^{\beta_m-1}(I-\tau)^{\beta_n-1}\mathrm{d}\tau \tag{19}$$

since the function τ^{β_m-1} is non-zero for $\tau > 0$ and the function $(I-\tau)^{\beta_n-1}$ is non-zero for $\tau < I$.

The integral in (19) can be written as $y(I) = B(\beta_m,\beta_n)I^{\beta_m+\beta_n-1}$ [16, 3.191.1] where $B(x,y)$ is the beta function (Euler's integral of the first kind). Given that the beta function can be related to the gamma function by $B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$ [16, 8.384.1], then evaluating the $PQ-1$ convolutions in (18) in a recursive manner results in:

$$f_{RC}\left(I\right) \approx a^{PQ}\left[\prod_{i=1}^{PQ-1}\frac{\Gamma(i\beta)\Gamma(\beta)}{\Gamma((i+1)\beta)}\right]I^{PQ\beta-1} \tag{20}$$

$$= \frac{1}{\Gamma(PQ\beta)}\left[\frac{(\alpha\beta)^{\beta}\Gamma(\alpha-\beta)}{\Gamma(\alpha)}\right]^{PQ} I^{PQ\beta-1} \tag{21}$$

where the product in (20) reduces to $\frac{\Gamma(\beta)^{PQ}}{\Gamma(PQ\beta)}$ while (16) was invoked in (21).

Following from (21), the outage probability in (9) can be written as:

$$P_{RC} = \frac{1}{\Gamma(PQ\beta)}\left[\frac{(\alpha\beta)^{\beta}\Gamma(\alpha-\beta)}{\Gamma(\alpha)}\right]^{PQ}\int_{0}^{\frac{P\sqrt{Q}}{P_M}} I^{PQ\beta-1}\mathrm{d}I \tag{22}$$

$$= \frac{1}{\Gamma(PQ\beta+1)}\left[\frac{(\alpha\beta)^{\beta}\Gamma(\alpha-\beta)}{\Gamma(\alpha)}\right]^{PQ}\left[\frac{P_M}{P\sqrt{Q}}\right]^{-PQ\beta} \tag{23}$$

Equation (23) shows that the diversity order of $P\times Q$ MIMO systems deploying RC is equal to $PQ\beta$. Hence, a PQ-fold increase in the diversity order is obtained with respect to SISO systems whose outage probability is as in (17).

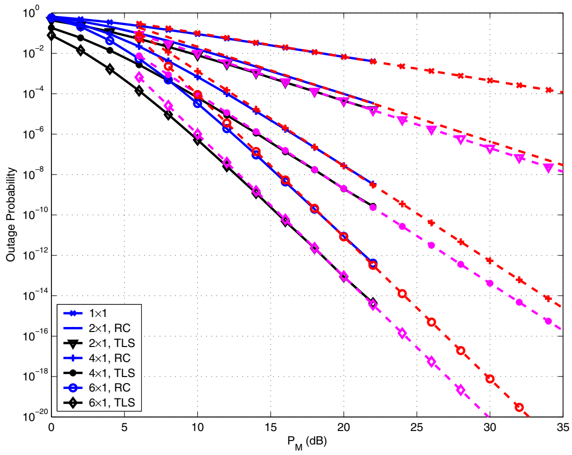

Fig. 1. Outage probability over a 5 km link. Solid and dashed lines correspond to the exact and approximate outage probabilities, respectively.

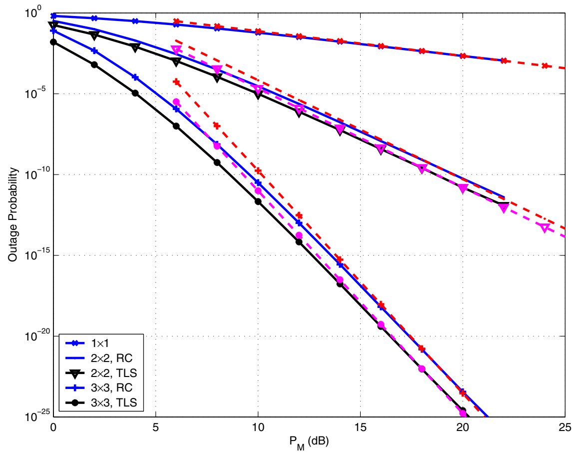

Fig. 2. Outage probability over a 3 km link. Solid and dashed lines correspond to the exact and approximate outage probabilities, respectively.

B. Transmit Laser Selection

The equivalent irradiance with TLS is given in (11). The term $\sum_{q=1}^{Q} I_{p,q}$ appearing in (11) corresponds to the summation of Q independent and identically distributed gamma-gamma random variables and, consequently, its pdf can be approximated in a way similar to (21) as follows:

$$f_{\sum_q I_{p,q}}(I) \approx \frac{1}{\Gamma(Q\beta)}\left[\frac{(\alpha\beta)^{\beta}\Gamma(\alpha-\beta)}{\Gamma(\alpha)}\right]^{Q} I^{Q\beta-1} \tag{24}$$

From (11) and (13), the outage probability with TLS can be written as:

$$P_{TLS} = \Pr\left(\mathcal{I}_{TLS} < \frac{\sqrt{Q}}{P_M}\right) = \prod_{p=1}^{P} \Pr\left(\sum_{q=1}^{Q} I_{p,q} < \frac{\sqrt{Q}}{P_M}\right) \tag{25}$$

where the second equality follows from the independence of the random variables $\sum_{q=1}^{Q} I_{p,q}$ for different values of p. On the other hand, given that these P random variables are identically distributed, then from (24) and (25):

$$P_{TLS} = \left[\frac{1}{\Gamma(Q\beta)}\left[\frac{(\alpha\beta)^{\beta}\Gamma(\alpha-\beta)}{\Gamma(\alpha)}\right]^{Q}\int_0^{\frac{\sqrt{Q}}{P_M}} I^{Q\beta-1}\mathrm{d}I\right]^{P} \tag{26}$$

$$= \frac{1}{\Gamma(Q\beta+1)^{P}}\left[\frac{(\alpha\beta)^{\beta}\Gamma(\alpha-\beta)}{\Gamma(\alpha)}\right]^{PQ}\left[\frac{P_M}{\sqrt{Q}}\right]^{-PQ\beta} \tag{27}$$

Note that (23) and (27) reduce to (17) in the case of SISO systems where $P = Q = 1$.

Equation (27) shows that $P \times Q$ MIMO systems deploying TLS also achieve a diversity order of $PQ\beta$. In other words, both RC and TLS achieve the same diversity order over gamma-gamma channels and the only advantage of TLS will reside in a coding gain that will be evaluated in what follows.

Writing the outage probability in (23) under the form $(\Lambda_{RC}P_M)^{-PQ\beta}$ and the outage probability in (27) under the form $(\Lambda_{TLS}P_M)^{-PQ\beta}$, the coding gain (in dB) of TLS over RC can be quantified as $10\log_{10}\left(\frac{\Lambda_{TLS}}{\Lambda_{RC}}\right)$. After straightforward manipulations, the coding gain can be expressed as:

$$g_c = \frac{10}{PQ\beta}\log_{10}\left(\frac{\Gamma(Q\beta)^{P}}{\Gamma(PQ\beta)}(Q\beta)^{P-1}P^{PQ\beta-1}\right) \tag{28}$$

which directly relates the coding gain to the parameters P, Q and β. To the author's best knowledge, expressions similar to (28) were not reported previously in the open literature on MIMO FSO systems. Evidently, the coding gain in (28) reduces to 0 dB in the case of SIMO systems ($P = 1$) where the single transmit aperture is always activated.

IV. Numerical Results

We next present some numerical results that compare the performance of the RC and TLS protocols over gamma-gamma fading channels. The refractive index structure constant is set to $C_n^2 = 1\times10^{-14}$ $\mathrm{m}^{-2/3}$ and the wavelength to 1550 nm.

Fig. 1 compares the performance of SISO systems with that of 2×1, 4×1 and 6×1 MIMO systems for a 5 km link. Results show that the asymptotic expressions derived in (17), (23) and (27) can accurately predict the asymptotic performance of SISO, MIMO-RC and MIMO-TLS systems, respectively. As predicted by the calculations performed in the previous section, RC and TLS achieve the same diversity order where the corresponding outage probability curves are practically parallel to each other for large values of P_M. As predicted by (28), the codings gains of TLS with respect to RC are equal to 1.4 dB, 2.37 dB and 2.78 dB for the 2×1, 4×1 and 6×1 systems, respectively.

Fig. 2 compares the performance of SISO systems with that of 2×2 and 3×3 MIMO systems for a 3 km link. As in Fig. 1, the theoretical findings of the paper are supported by the results of Fig. 2 as well. Moreover, the performance gains offered by the MIMO techniques are huge whether with the implementation of the RC or the TLS schemes.

In Fig. 3, we plot the coding gain in (28) as a function of the number of transmit apertures P for various values of

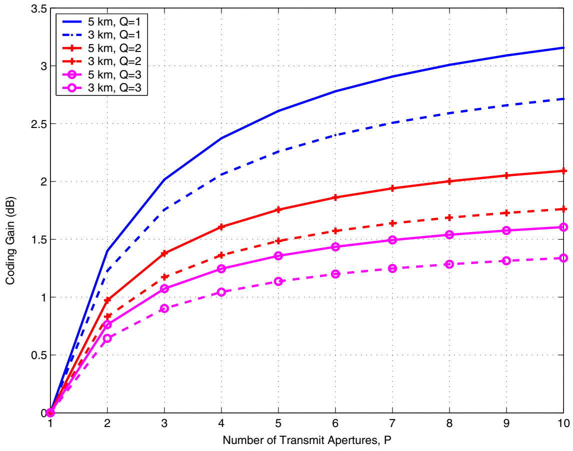

Fig. 3. Coding gain of TLS with respect to RC.

the link distance and the number of receive apertures Q. The results show that the coding gain is an increasing function of both P and the link distance while it is a decreasing function of Q. For example, in the presented results, the highest coding gain is observed with one receive aperture for a link distance of 5 km.

V. Conclusion

A comprehensive analysis of the two most widely studied MIMO FSO diversity schemes, namely RC and TLS, was provided. Through a well-tailored calculation methodology that targeted an asymptotic evaluation of the outage probability, we derived the diversity orders and coding gains that can be achieved over gamma-gamma atmospheric turbulence channels. The dependence of the coding gain on the different parameters of the MIMO link was established through a novel closed-form expression. Future work will target the impact of channel correlation on the achievable diversity orders and coding gains.

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