A 4×4 Unipolar Space-Time PPM-Code with Simplified Decoding for IR-UWB Transmissions

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***Abstract*— In this paper, we propose a novel 4×4 Space-Time (ST) code that can be associated with Impulse-Radio Ultra-Wideband (IR-UWB) systems using Pulse Position Modulation (PPM). The proposed rate-1 and fully diverse code can be applied with unipolar M-PPM constellations for all even values of M without introducing any constellation expansion. In other words, as in single-antenna IR-UWB systems, information is conveyed only by the time delays of the modulated sub-nanosecond pulses without introducing any amplitude amplification or phase rotation. An adapted simple Maximum-Likelihood (ML) decoder and diversity-preserving suboptimal decoders that take the structure of the proposed code into consideration are also presented.**

I. Introduction

Time-Hopping Ultra-wideband (TH-UWB) systems are often associated with Pulse Position Modulation (PPM) which is appealing since it does not necessitate any polarity inversion or amplitude scaling of the very short UWB pulses. On the other hand, there is a growing interest in applying the Space-Time (ST) coding techniques on TH-UWB systems [1]–[3].

In this context, ST codes can be classified into two categories depending on wether they are shape-preserving [3]–[6] or non shape-preserving with PPM [1]. A PPM shape-preserving code does not introduce any expansion to the PPM constellation. In other words, each antenna of a multi-antenna system transmits only one unipolar pulse during each symbol duration. Such codes are appealing since they do not impose any additional constraints on the RF circuitry to control the amplitudes and the phases of the UWB pulses.

Denote by P the number of transmit antennas and by M the cardinality of the PPM signal set. A unipolar PPM-specific code was first proposed in [4] for $(M,P) = (2,2)$. This code was extended to the case of $P = 4, 8$ with 2-PPM in [5]. While [4], [5] are linearly decodable, their main limitation is that they are limited to binary PPM. A linearly decodable code for M-PPM was also proposed in [6]; however, this code is limited to two-antenna systems. Finally, a family of unipolar ST codes was proposed in [3] for a wide range of (M,P); however, these codes do not lend themselves to simple Maximum-Likelihood (ML) decoding.

The first contribution of this paper is the proposition of a rate-1, fully diverse and shape-preserving ST block code for unipolar PPM with four antennas. The advantage over [5] is that the proposed scheme can be associated with M-PPM for all even values of M. The advantage over [3] is that the proposed code lends itself to simpler ML decoding. In fact, the complexity of the optimal decoding procedure associated with the proposed code scales with $(M/2)^3$ rather than M^4 as for the other 4×4 M-PPM codes such as [3]. An additional advantage over [3] is that, unlike the code that we propose, [3] can not be applied for $M = 2, 4$ when the transmitter is equipped with four antennas. The second contribution is the proposition of a suboptimal diversity-preserving decoder when $M > 8$. The complexity of this decoder increases as $(M_d/2)^3$ where M_d is an even number verifying: $8 \leq M_d < M$.

II. System Model

Consider a M-ary PPM constellation where all the modulated pulses have the same amplitude level and can occupy one out of M modulation positions. This is a M-dimensional constellation where each information symbol is represented by a M-dimensional vector belonging to the following signal set:

$$\mathcal{S}_{PPM} = \{I_{M,m} \ ; \ m = 1, \dots, M\} \tag{1}$$

where $I_{M,m}$ stands for the m-th column of the $M \times M$ identity matrix I_M.

Consider a MIMO TH-UWB system where the transmitter and the receiver are equipped with P and Q antennas respectively. Assume that each receive antenna is followed by an L-order Rake that combines the first L arriving multipath components. For M-dimensional constellations, the linear dependence between the baseband inputs and outputs of the channel can be expressed as:

$$Y = HC + N \tag{2}$$

where Y and N are $(QLM \times T)$-dimensional matrices that stand for the decision and noise matrices respectively where T stands for the number of symbol periods occupied by each ST codeword. The $((q-1)LM + (l-1)M + m, t)$-th component of Y (resp. N) stands for the decision variable (resp. noise term) collected by the l-th Rake finger of the q-th receive antenna during the m-th modulation position of the t-th symbol duration for $q = 1, \dots, Q$, $l = 1, \dots, L$, $m = 1, \dots, M$ and $t = 1, \dots, T$.

In eq. (2), C is the $PM \times T$ codeword whose $((p-1)M + m, t)$-th entry corresponds to the amplitude of the pulse (if any) transmitted by the p-th antenna during the m-th position of the t-th symbol duration for $p = 1, \dots, P$, $m = 1, \dots, M$ and $t = 1, \dots, T$. In what follows, we consider minimal-delay codes that extend over $T = P$ symbol durations. H

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stands for the $QLM \times PM$ channel matrix that can be written as: $H = [H_1^T \cdots H_Q^T]^T$ where H_q is the $LM \times PM$ matrix corresponding to the q-th receive antenna for $q = 1, \ldots, Q$. H_q can be written as: $H_q = [H_{q,1}^T \cdots H_{q,L}^T]^T$ where $H_{q,l}$ is a $M \times PM$ matrix that is given by $H_{q,l} = [H_{q,l,1} \cdots H_{q,l,P}]$ for $l = 1, \ldots, L$. $H_{q,l,p}$ is a $M \times M$ matrix for $p = 1, \ldots, P$. The (m, m')-th element of $H_{q,l,p}$ corresponds to the impact of the signal transmitted during the m'-th position of the p-th antenna on the m-th correlator (corresponding to the m-th position) placed after the l-th Rake finger of the q-th receive antenna. This term can be written as [3]:

$$H_{q,l,p}(m, m') = r_{q,p}((m - m')\delta + \Delta_l) \tag{3}$$

where δ stands for the modulation delay and Δ_l stands for the l-th finger delay. $r_{q,p}$ corresponds to the frequency selective channel between antennas p and q.

III. Code Construction

In this section, we propose a minimal-delay ST code for $P = 4$ transmit antennas. For M-PPM, the codewords correspond to $4M \times 4$ matrices having the following structure:

$$C(s_1, s_2, s_3, s_4) = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ \Omega s_2 & s_1 & \Omega s_4 & s_3 \\ \Omega s_3 & s_4 & s_1 & \Omega s_2 \\ \Omega s_4 & \Omega s_3 & s_2 & s_1 \end{bmatrix} \tag{4}$$

where $s_1, \ldots, s_4 \in \mathcal{S}_{PPM}$ given in eq. (1) are the M-dimensional vector representations of the information symbols. In what follows, we limit ourselves to even values of M so that Ω will correspond to the following $M \times M$ permutation matrix:

$$\Omega = I_{M/2} \otimes \Omega_0 \triangleq I_{M/2} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{5}$$

where $I_{M/2}$ is the $(M/2)$-dimensional identity matrix and $\otimes$ stands for the Kronecker product.

Evidently, $\Omega s \in \mathcal{S}_{PPM}$ given in eq. (1) whenever $s \in \mathcal{S}_{PPM}$ implying that the code is shape-preserving with PPM. In fact, as can be seen from eq. (4), the proposed code does not introduce any amplitude scaling or phase rotation and is based simply on introducing appropriate permutations to the PPM symbols. In other words, during each symbol duration, each antenna transmits exactly one unipolar pulse during one out of the M available modulation positions. Eq. (4) also implies that the proposed code is a rate-1 code that transmits at the rate of 1 symbol ($\log_2(M)$ bits) Per Channel Use (PCU).

Proposition 1: The proposed 4×4 code achieves a full transmit diversity order with M-PPM constellations for all even values of M.

Proof: Based on the rank criterion proposed in [7], the proposed code is fully diverse if all matrices $C(a_1, a_2, a_3, a_4)$ have a full rank (of 4) for $(a_1, a_2, a_3, a_4) \neq (0_M, 0_M, 0_M, 0_M)$ where 0_M is the M-dimensional vector having all of its components equal to zero. Symbols $a_1, \ldots, a_4$ correspond to the difference between two M-PPM symbols and they belong to the following set:

$$\mathcal{A} = \{s - s' \ ; \ s, s' \in \mathcal{S}_{PPM}\} \tag{6}$$

Based on eq. (1), elements of $\mathcal{A}$ (that are M-dimensional vectors) have the following structure: (1) either they have all of their components equal to zero or (2) they have exactly two non-zero components; moreover, one of these components must be equal to $+1$ while the other component must be equal to -1. Note that the transmit diversity order is achieved because of this particular structure of $\mathcal{A}$. For example, if $a_1, \ldots, a_4$ have all of their components equal to 1 then the corresponding codeword will be rank-deficient. However, from what preceded, these vectors do not belong to the set $\mathcal{A}$ for any value of M.

In order to prove that the code is fully diverse, we fist construct a $2M \times 4$ matrix $C_0(a_1, a_2, a_3, a_4)$ such that its n-th row is chosen to be the difference between rows $2n - 1$ and $2n$ of $C(a_1, a_2, a_3, a_4)$ for $n = 1, \ldots, 2M$. Following from eq. (4) and from the structure of the matrix Ω given in eq. (5), this matrix can be written as:

$$C_0(a_1, a_2, a_3, a_4) = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ -c_2 & c_1 & -c_4 & c_3 \\ -c_3 & c_4 & c_1 & -c_2 \\ -c_4 & -c_3 & c_2 & c_1 \end{bmatrix} \tag{7}$$

where $\{c_i\}_{i=1}^4$ are vectors whose lengths are equal to $M' \triangleq M/2$ and whose m'-th components are given by:

$$c_{i,m'} = a_{i,2m'-1} - a_{i,2m'} \quad ; \quad m' = 1, \ldots, M' \tag{8}$$

Since C_0 is a sub-matrix of a matrix that is obtained by performing linear combinations on the rows of C, then the rank of C is at least equal to that of C_0. In what follows, we will prove that all non-trivial matrices C_0 have a rank of 4. By permuting the rows of C_0, this matrix can be written as:

$$C_0(a_1, a_2, a_3, a_4) = \begin{bmatrix} C_{0,1}^T & \cdots & C_{0,M'}^T \end{bmatrix}^T \tag{9}$$

where $C_{0,m'}$ is a 4×4 matrix whose j-th row is equal to the $((j-1)M' + m')$-th row of C_0 for $m' = 1, \ldots, M' = M/2$ and $j = 1, \ldots, 4$. From eq. (7), matrices $\{C_{0,m'}\}_{m'=1}^{M'}$ can be written as:

$$C_{0,m'} = \begin{bmatrix} c_{1,m'} & c_{2,m'} & c_{3,m'} & c_{4,m'} \\ -c_{2,m'} & c_{1,m'} & -c_{4,m'} & c_{3,m'} \\ -c_{3,m'} & c_{4,m'} & c_{1,m'} & -c_{2,m'} \\ -c_{4,m'} & -c_{3,m'} & c_{2,m'} & c_{1,m'} \end{bmatrix} \tag{10}$$

Eq. (10) shows that $C_{0,m'}$ is equal to the 4×4 codewords of the real orthogonal ST codes [8]. Consequently, $C_{0,m'}$ has a rank of 4 unless when $c_{1,m'} = \cdots = c_{4,m'} = 0$. Since m' can take any value between 1 and M', then eq. (8) and eq. (9) show that C_0 (and consequently C) has a full rank of 4 unless when:

$$a_{i,2m'-1} = a_{i,2m'} \text{ for } i = 1, \ldots, 4 \text{ and } m' = 1, \ldots, M' \tag{11}$$

On the other hand, following from the structure of the set $\mathcal{A}$ given in eq. (6), the elements $a_{i,2m'-1}$ and $a_{i,2m'}$ belong to the set $\{0, -1, +1\}$ and they can not be equal simultaneously to $+1$ or -1. Consequently, eq. (11) can hold only when $a_{i,2m'-1} = a_{i,2m'} = 0$ for $i = 1, \ldots, 4$ and $m' = 1, \ldots, M'$. In other words, eq. (11) can hold only when $a_1, \ldots, a_4$ are all equal to the all-zero vector. This proves that the rank of $C(a_1, a_2, a_3, a_4)$ is equal to 4 unless when $a_1 = \cdots = a_4 = 0_M$ proving that the code is fully diverse.

IV. Optimal and Suboptimal Detection

Denote by Γ the maximum delay-spread of the UWB channel. The modulated pulses interfere with each other at the receiver when $\delta < \Gamma$ where δ stands for the modulation delay. This interference will be referred to as Inter-Pulse-Interference (IPI) in what follows. Since the code that we proposed is based on the rank criterion [7] and is independent from the value taken by δ, then this code can achieve a full transmit diversity order either in the presence or in the absence of IPI.

In this section we present a simple optimal ML detector that can be applied only in the absence of IPI. In the presence of IPI, the above code must be associated with more sophisticated ML decoders such as the PPM-extension of the sphere decoder proposed in [9]. At the end of this section, we also present a simpler suboptimal decoder that is appealing when M takes large values.

A. Optimal ML Decoder

Following from the linearity of the proposed code, eq. (2) can be written as:

$$\mathcal{Y} = (I_P \otimes H)\, \Phi(\Omega) S + \mathcal{N} \tag{12}$$

where $\mathcal{Y}$ and $\mathcal{N}$ are $QPLM$-dimensional vectors given by: $\mathcal{Y}=\text{vec}(Y)$ and $\mathcal{N}=\text{vec}(N)$, respectively, where the function $\text{vec}(X)$ stacks the columns of the matrix X vertically one after the other. S is the PM-dimensional vector obtained from the vertical concatenation of $s_1,\dots,s_4$ ($P=4$ in what follows). Finally, $\Phi(\Omega)$ is the $P^2M \times PM$ matrix that satisfies the relation: $\text{vec}(C) = \Phi(\Omega)S$. Note that $\Phi(\Omega)$ depends on the value taken by Ω in eq. (5).

In the absence of IPI, the channel matrix H can be written as:

$$H = H^{(0)} \otimes I_M \tag{13}$$

where $H^{(0)}$ is the $QL \times P$ matrix whose $((q-1)L+l,p)$-th element is equal to $r_{q,p}(\Delta_l)$ (refer to eq. (3)).

From eq. (1), each one of the vectors $s_1,\dots,s_4$ can be written as:

$$s_i = I_{M,P_i} \quad ; \quad i=1,\dots,4 \tag{14}$$

where $P_i \in \{1,\dots,M\}$ indicates the position of the i-th pulse and can be written as:

$$P_i = 2(p_i-1)+p'_i \quad ; \quad i=1,\dots,4 \tag{15}$$

where $p_i \in \{1,\dots,M'=M/2\}$ and $p'_i \in \{1, M''=2\}$. Eq. (15) can be seen as partitioning the M positions into $M' \triangleq M/2$ slots containing $M'' \triangleq 2$ positions each. In this case, p_i (resp. p'_i) will refer to the slot (resp. the position within each slot) in which the i-th pulse is present.

The reason behind performing the above partitioning is the observation that the M-dimensional entries Ωs_i in eq. (4) can be written as:

$$\Omega s_i = -s_i + I_{M',p_i} \otimes 1_2 \tag{16}$$

where $1_2 \triangleq [1 \;\; 1]^T$. As will be explained later, eq. (16) will simplify the ML decoder by relating the decoding process of the proposed code to that of the 4×4 orthogonal ST code [8].

At the receiver side, the $4QLM$-dimensional decision vector $\mathcal{Y}$ given in eq. (12) will be partitioned into M' vectors $\mathcal{Y}^{(1)},\dots,\mathcal{Y}^{(M')}$ having dimensions $4QLM''$ each. In this case, $\mathcal{Y}^{(k)}$ will be composed from the decision variables collected during the k-th slot of the four symbol durations.

Because of the absence of IPI and the structure of the matrix Ω given in eq. (5), eq. (12) can be separated into M' separate equations as follows:

$$\mathcal{Y}^{(k)} = (I_P \otimes H')\, \Phi(\Omega_0) S^{(k)} + \mathcal{N}^{(k)} \quad ; \quad k=1,\dots,M' \tag{17}$$

where Ω_0 is given in eq. (5) and $\Phi(\Omega_0)$ is the $P^2M'' \times PM''$ matrix that is constructed from Ω_0 in the same way as $\Phi(\Omega)$ is constructed from Ω. In a way similar to eq. (13), H' is the $QLM'' \times PM''$ matrix given by:

$$H' = H^{(0)} \otimes I_{M''} \tag{18}$$

In eq. (17), the vector $\mathcal{Y}^{(k)}$ (resp. $\mathcal{N}^{(k)}$) is the $4QLM''$ vector composed from the components $(p-1)QLM+(q-1)LM+(l-1)M+2(k-1)+m''$ of $\mathcal{Y}$ (resp. $\mathcal{N}$) for $p=1,\dots,4$, $q=1,\dots,Q$, $l=1,\dots,L$ and $m''=1,2$.

In eq. (17), $S^{(k)}$ is the $4M''$-dimensional vector given by:

$$S^{(k)} \triangleq \begin{bmatrix} (s_1^{(k)})^T & (s_2^{(k)})^T & (s_3^{(k)})^T & (s_4^{(k)})^T \end{bmatrix}^T \tag{19}$$

where $s_i^{(k)} \triangleq [s_{i,2k-1} \;\; s_{i,2k}]^T$ where $s_{i,m}$ stands for the m-th component of the M-dimensional vector s_i for $i=1,\dots,4$.

Following from the proposed partitioning, eq. (16) implies that:

$$\Omega_0 s_i^{(k)} = -s_i^{(k)} + \delta_{k,p_i} 1_2 \tag{20}$$

where $\delta_{i,j}$ stands for Kronecker's delta function ($\delta_{i,j}=1$ for $i=j$ and $\delta_{i,j}=0$ for $i \neq j$).

Given that the matrix Ω multiplies only the symbols s_2, s_3 and s_4 in eq. (4), then the matrix $\Phi(\Omega_0)S^{(k)}$ in eq. (17) can be written as:

$$\Phi(\Omega_0)S^{(k)} = \Phi(-I_{M''})S^{(k)} + \sum_{i=2}^{4} \delta_{k,p_i} \mathcal{I}^{(i)} \tag{21}$$

where $\mathcal{I}^{(2)},\dots,\mathcal{I}^{(4)}$ are P^2M''-dimensional vectors that satisfy the relation:

$$\mathcal{I}^{(i)} \triangleq \text{vec}\left(C^{(i)}\right) \quad ; \quad i=2,3,4 \tag{22}$$

where $C^{(2)},\dots,C^{(4)}$ are $PM'' \times P$ matrices given by:

$$C^{(2)} = \begin{bmatrix} 0_2 & 0_2 & 0_2 & 0_2 \\ 1_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 1_2 \\ 0_2 & 0_2 & 0_2 & 0_2 \end{bmatrix} ; \; C^{(3)} = \begin{bmatrix} 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 0_2 \\ 1_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & 1_2 & 0_2 & 0_2 \end{bmatrix}$$

$$C^{(4)} = \begin{bmatrix} 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 1_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 0_2 \\ 1_2 & 0_2 & 0_2 & 0_2 \end{bmatrix} \tag{23}$$

where $0_2 \triangleq [0 \;\; 0]^T$.

From eq. (4) and eq. (23), it can be observed that the matrix $C^{(i)}$ is obtained by replacing the occurrences of Ωs_i in the codeword C by 1_2 while replacing all the remaining entries by 0_2. Note that the vectors $\mathcal{I}^{(2)}, \mathcal{I}^{(3)}, \mathcal{I}^{(4)}$ are constant vectors that do not depend on the values of the transmitted PPM symbols.

Now it can be observed that $\Phi(-I_{M''})$ can be written as:

$$\Phi(-I_{M''}) = \phi \otimes I_{M''} \tag{24}$$

where ϕ is the $P^2 \times P$ orthogonal matrix that depends uniquely on the structure of the orthogonal codes [8] and that satisfies the relation: $\text{vec}(C_{\text{orth}}(x_1, \dots, x_4)) = \phi[x_1 \cdots x_4]^T$ where $x_1, \dots, x_4$ are scalars and C_{orth} stands for the 4×4 orthogonal ST codeword.

Finally, replacing equations (18), (21) and (24) in eq. (17) results in:

$$\mathcal{Y}^{(k)} = \left[I_P \otimes \left(H^{(0)} \otimes I_{M''}\right)\right] \cdot \left[\left(\phi \otimes I_{M''}\right) S^{(k)} + \sum_{i=2}^{4} \delta_{k,p_i} \mathcal{I}^{(i)}\right] + \mathcal{N}^{(k)} \tag{25}$$

Following from the properties of the Kronecker product, the last equation can be written as:

$$\mathcal{Y}^{(k)} = \left[\mathcal{H} \otimes I_{M''}\right] S^{(k)} + \mathcal{H}^{(0)} \sum_{i=2}^{4} \delta_{k,p_i} \mathcal{I}^{(i)} + \mathcal{N}^{(k)} \tag{26}$$

where: $\mathcal{H} \triangleq (I_P \otimes H^{(0)})\phi$ and $\mathcal{H}^{(0)} \triangleq I_P \otimes (H^{(0)} \otimes I_{M''})$. It can be observed that $\mathcal{H}$ satisfies the relation $\mathcal{H}^T\mathcal{H} = \sum_{q=1}^{Q} \sum_{p=1}^{P} \sum_{l=1}^{L} r_{q,p}^2(\Delta_l)$. Note that this relation follows from the structure of the orthogonal codes that is embedded in the matrix ϕ.

Conditioned on the slot indices $\tilde{p}_2 = p_2$, $\tilde{p}_3 = p_3$ and $\tilde{p}_4 = p_4$, the encoded symbols can be determined from the following decision vectors:

$$\mathcal{X}^{(k)}(\tilde{p}_2, \tilde{p}_3, \tilde{p}_4) \triangleq \left[\mathcal{H}^T \otimes I_{M''}\right] \left[\mathcal{Y}^{(k)} - \mathcal{H}^{(0)} \sum_{i=2}^{4} \delta_{k,\tilde{p}_i} \mathcal{I}^{(i)}\right] \tag{27}$$

This relation follows from the orthogonality of the matrix $\mathcal{H}$ that implies that the right hand side of the above equation is proportional to the vector $S^{(k)}$ (note also that the factor of proportionality is positive).

Consequently, conditioning on $(\tilde{p}_2, \tilde{p}_3, \tilde{p}_4)$ (which is equivalent to assuming that the i-th pulse is in slot $\tilde{p}_i$ for $i = 2, 3, 4$), the positions $P_2, \dots, P_4$ can be determined from:

$$\hat{P}_{i|\tilde{p}_2,\tilde{p}_3,\tilde{p}_4} = 2(\tilde{p}_i - 1) + \arg\max_{m''=1,2} \left[\mathcal{X}^{(\tilde{p}_i)}_{2(i-1)+m''}(\tilde{p}_2, \tilde{p}_3, \tilde{p}_4)\right] \; ; \; i = 2, 3, 4 \tag{28}$$

In the same way, the position of the first pulse can be determined from:

$$\hat{P}_{1|\tilde{p}_2,\tilde{p}_3,\tilde{p}_4} = \arg\max \left[\mathcal{X}^{(1)}_{1:2}(\tilde{p}_2, \tilde{p}_3, \tilde{p}_4) \; \cdots \; \mathcal{X}^{(M')}_{1:2}(\tilde{p}_2, \tilde{p}_3, \tilde{p}_4)\right] \tag{29}$$

where $\mathcal{X}^{(k)}_{1:2}(\tilde{p}_2, \tilde{p}_3, \tilde{p}_4)$ stands for the first two elements of the $4M''$-dimensional vector $\mathcal{X}^{(k)}(\tilde{p}_2, \tilde{p}_3, \tilde{p}_4)$ for $k = 1, \dots, M'$.

Finally, the ML decoder decides in favor of the vector $\hat{S}$ such that:

$$\hat{S} = \arg \min_{(\tilde{p}_2,\tilde{p}_3,\tilde{p}_4) \in \{1,\dots,M'\}^3} \left\|\mathcal{Y} - (I_P \otimes H)\, \Phi(\Omega) S(\tilde{p}_2, \tilde{p}_3, \tilde{p}_4)\right\|^2 \tag{30}$$

where:

$$S(\tilde{p}_2, \tilde{p}_3, \tilde{p}_4) \triangleq \left[\begin{array}{ccc} (I_{M,\hat{P}_{1|\tilde{p}_2,\tilde{p}_3,\tilde{p}_4}})^T & \cdots & (I_{M,\hat{P}_{4|\tilde{p}_2,\tilde{p}_3,\tilde{p}_4}})^T \end{array}\right]^T \tag{31}$$

To summarize, the ML receiver consists of (1): partitioning the decision variables into $M/2$ vectors $\mathcal{Y}^{(1)}, \dots, \mathcal{Y}^{(M')}$ as shown in eq. (17), (2): determining the $(M/2)^3$ candidate vectors according to equations (27)-(29) where each candidate vector is conditioned over three slot indices, (3): deciding in favor of the candidate vector that satisfies eq. (30). Consequently, the complexity of the proposed ML decoder increases as $(M/2)^3$ which is much smaller than the complexity of the non-simplified ML decoder that must perform M^4 comparisons for decoding one ST codeword.

Finally, note that the above decoder is optimal since no approximations or information non-preserving operations were made. Note also that for $M = 2$, $(M/2)^3 = 1$ and the decoder reduces to the linear decoder proposed in [5] for $M = 2$ PPM.

B. Suboptimal Decoder

Despite the fact that the approach proposed in the previous subsection reduces the complexity of the ML decoding procedure by a factor of $8M$, the complexity of this approach can be prohibitive for large values of M. In what follows, we present a simple suboptimal decoding strategy that can be applied when $M > 8$.

This decoder is based on the fact that the four encoded M-PPM symbols and their permuted replicas can not occupy more than eight positions. In other words, the vectors $s_1, \dots, s_4$ and $\Omega s_2, \dots, \Omega s_4$ in eq. (4) can occupy eight PPM positions at most while the remaining $M - 8$ positions will be empty.

Based on what preceded, the simplified decoder that we propose corresponds to assuming that the conditional slot indices $\tilde{p}_2, \dots, \tilde{p}_4$ span $M_d/2$ slots rather than $M/2$ slots where $8 \leq M_d < M$ and M_d is even.. In other words, instead of considering all of the sub-vectors $\mathcal{Y}^{(1)}, \dots, \mathcal{Y}^{(M/2)}$ given in eq. (17), we only consider the $M_d/2$ vectors having the highest Frobenius norms. In fact, given the absence of any signal in the remaining $M/2 - M_d/2$ empty slots, their corresponding decision variables (elements of $\mathcal{Y}$) are expected to have small values resulting in small Frobenius norms. Note that it is evident that this approach is suboptimal since, even in the occupied slots, the pulses from the different antennas might combine destructively resulting in decision variables having small magnitudes.

In other words, the suboptimal decoder will be based on the assumption that the transmitted pulses fall within the slots $n_1, \dots, n_{M_d/2} \in \{1, \dots, M/2\}$ whose corresponding vectors $\mathcal{Y}^{(n_1)}, \dots, \mathcal{Y}^{(n_{M_d/2})}$ have the highest Frobenius norms. In this case, eq. (27) must be evaluated only for $k \in \{n_1, \dots, n_{M_d/2}\}$ while the conditioning in equations (27)-(31) must be limited

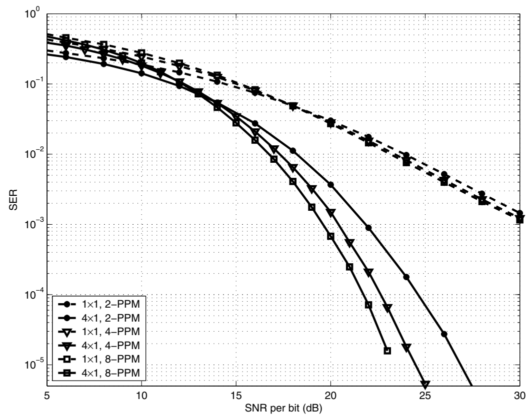

Fig. 1. Performance with the optimal decoder and a 5-finger Rake.

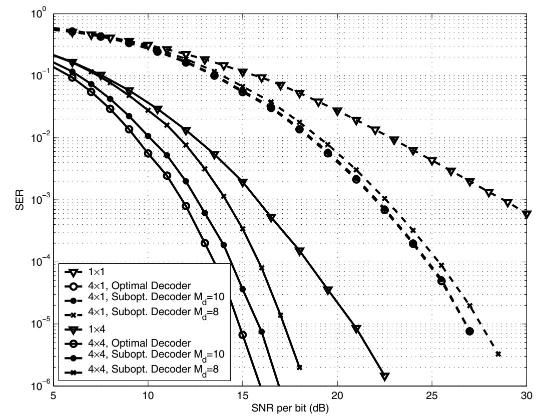

Fig. 2. Performance with 12-PPM and a 1-finger Rake.

to $(\tilde{p}_2, \tilde{p}_3, \tilde{p}_4) \in \{n_1, \dots, n_{M_d/2}\}^3$ resulting in a complexity that increases as $(M_d/2)^3$ rather than $(M/2)^3$. For this simplified decoding procedure, eq. (29) must be written as:

$$\hat{P}_{1|\tilde{p}_2,\tilde{p}_3,\tilde{p}_4} = \arg\max \Big[\mathcal{X}_{1:2}^{(n_1)}(\tilde{p}_2, \tilde{p}_3, \tilde{p}_4) \ \cdots \ \mathcal{X}_{1:2}^{(n_{M_d/2})}(\tilde{p}_2, \tilde{p}_3, \tilde{p}_4) \Big] \quad (32)$$

V. Simulations and Results

The PQ channels between the different antennas are generated according to the 802.15.3a channel model recommendation CM2 [10]. The modulation delay is fixed to $\delta = 100$ ns which is larger than the channel delay spread thus eliminating IPI. At the receiver side, perfect channel state information is assumed and the decoders presented in section IV are applied.

Fig. 1 compares the performance of single-antenna systems and of the 4×1 encoded systems. The receiver is equipped with a 5-finger Rake and the optimal decoder presented in subsection IV-A is used for detection. Results show the high performance level and the enhanced diversity order achieved by the proposed code with different PPM constellations. It is worth noting that the proposed ML decoder shows exactly the same performance as the optimal PPM-extension of the sphere decoder proposed in [9]. Note that for limited values of the constellation's cardinality (M), the proposed ML decoder has smaller decoding times as compared to [9] especially at low Signal-to-Noise Ratios (SNR). For example, with 8-PPM, the proposed ML decoder is approximately 6.77 and 2.39 times faster than [9] at SNRs of 5 dB and 10 dB respectively.

Fig. 2 shows the performance of the optimal and suboptimal decoders with 12-PPM. The main observation is that the suboptimal decoding procedure preserves diversity since the Symbol-Error-Rate (SER) curves of the optimal and suboptimal decoders are parallel to each other at high SNRs. The second observation is that associating the proposed code with the simple suboptimal decoder always outperforms the single-antenna systems at high SNR. Varying the value of M_d permits to achieve different tradeoffs between complexity and performance. For example, for $M_d = 10$ and $M_d = 8$, the suboptimal decoder performs 1.2 and 1.5 times less comparisons than the optimal decoder respectively. With 4×4 systems, this reduced complexity is associated with a performance loss of about 0.8 dB and 2.3 dB at a SER of 10^{-3} for $M_d = 10$ and $M_d = 8$ respectively.

VI. Conclusion

By replacing the phase rotations in the orthogonal codes with convenient pulse permutations, we proposed the first known shape-preserving PPM-specific 4×4 ST code that can be associated with M-PPM for all even values of M. In the presence of IPI, the proposed code must be associated with sequential decoders. In the absence of IPI, simple optimal and suboptimal decoders can insure the separation of the transmitted data streams. Future work will consider the sequential implementation of these decoders.

References

[1] L. Yang and G. B. Giannakis, "Analog space-time coding for multi-antenna ultra-wideband transmissions," *IEEE Trans. Commun.*, vol. 52, pp. 507–517, March 2004.

[2] L. C. Wang, W. C. Liu, and K. J. Shieh, "On the performance of using multiple transmit and receive antennas in pulse-based ultrawideband systems," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 2738–2750, November 2005.

[3] C. Abou-Rjeily and W. Fawaz, "Space-time codes for MIMO ultra-wideband communications and MIMO free-space optical communications with PPM," *IEEE J. Select. Areas Commun.*, vol. 26, no. 6, pp. 938–947, August 2008.

[4] M. K. Simon and V. A. Vilnrotter, "Alamouti-type space-time coding for free-space optical communication with direct detection," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 35–39, January 2005.

[5] C. Abou-Rjeily, "Orthogonal space-time block codes for binary pulse position modulation," *IEEE Trans. Commun.*, to appear in the March 2009 issue, available at http://services.sea.lau.edu.lb/faculty/carjeily.

[6] C. Abou-Rjeily and M. Bkassiny, "Alamouti-type space-time coding for TH-UWB systems with unipolar pulse position modulations," in *Proceedings IEEE Int. Conf. on Personal, Indoor and Mobile Radio Commun.*, September 2008, pp. 1–5.

[7] V. Tarokh, N. Seshadri, and A. Calderbank, "Space-time codes for high data rate wireless communication : Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744–765, 1998.

[8] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-Time block codes from orthogonal design," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456–1466, July 1999.

[9] C. Abou-Rjeily, "A maximum-likelihood decoder for joint pulse position and amplitude modulations," in *Proceedings IEEE Int. Conf. on Personal, Indoor and Mobile Radio Commun.*, September 2007.

[10] J. Foerster, "Channel modeling sub-committee Report Final," Technical report IEEE 802.15-02/490, IEEE 802.15.3a WPANs, 2002.