THE SECOND LOW BID METHOD

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ABSTRACT

The low bid method has been the most common competitive bid selection approach used for public projects in the U.S. construction industry and worldwide. This method is usually coupled with a prequalification process to ensure that the lowest bidder has the financial capacity, the necessary experience, and enough bonding capacity to take charge of the project and to perform the work according to the project's requirements. However, driven by their bad financial status or by their urgent need for work, some contractors tend to abuse the free and price-directed competitive nature of the low bid method by deliberately submitting extremely low bid prices in order to enhance their chance of winning and to at least cover their general and administrative expenses. Thus it is possible for the project to be awarded to an accidental or deliberate unrealistic low bid. This often leads to cost overruns, schedule delays, claims and further disputes between parties during construction.

Alternatively, an owner could have paid a little more and reduced these risks by eliminating the lowest bid and rather awarding the contract to the second lowest bidder. This paper uses Monte Carlo simulation approach to analyze and model the second low bid method where the contractor whose bid is the second lowest one among submitted bids is awarded the contract. The simulation results are presented in nomographs that are used to determine optimal markup and optimum profit for a given project and to compare the merits and disadvantages of the low and second low bidding methods.

Keywords: bidding models, optimum markup, probability of winning, low bid, profit.

1. INTRODUCTION

In the construction industry, and particularly in the public sector, open competitive bidding has been the main bid selection process used in the U.S. and in most countries around the world. The low bid method is the most common form of competitive bidding where the lowest responsive and responsible bidder is awarded the project. It aims at promoting transparency in the bidding process and ensuring fairness in the construction market where all contractors have the opportunity to bid on projects and thus it removes any suspicion of unequal treatment by the procuring entity. Additionally, it promises owners the best returns on investments by having their projects built according to the designed plans and specs for the lowest possible price.

However, the lowest bid price might not always be the most economical one for the owner because it might not always result in the lowest possible final cost after project completion. It is possible for a bidder to submit the lowest bid because of an innovative cost-saving technique or well-experienced management and planning teams and in this case the lowest bidder is indeed the most competent one. On the other hand, an owner should also be aware of a possible risk of selecting a bidder who deliberately submits an unrealistically low bid price only to apply for excessive change orders after winning the project that can significantly raise the cost of construction above the original contract price. Poor economic environments with low cashflow and slow economic growth and a contractor's desperate need for work to stay in business might induce such bidding strategy where a contractor views change orders as a means to expand the scope of work and widen his profits which were in part given up for the sake of a lower bid price. Thus, awarding the job to the lowest bidder allows deliberate unrealistic low bids to win the project which increases the chance of cost overruns, schedule delays, claims and adversarial relationship between parties during construction (Grogan 1992; Holt et al. 1995; Clough and Sears 2005). Alternatively, an owner could have paid a little more and reduced these risks by eliminating the lowest bid and rather awarding the contract to the second lowest bidder. This can be considered as a truncated low bid method.

This paper models and analyzes the second-low bid method where the contractor whose bid is the second lowest one among submitted bids, is awarded the contract. An analytic derivation of the probability of winning under the second-low bid method is presented and Monte Carlo simulation is used to model the aforementioned bidding method. The merits and shortcomings of the low and second low bidding methods are analyzed and compared relative to each other through produced nomograms that depict the winning probability, the optimum markup and the optimum expected profit under each.

2. ANALYTIC DERIVATION OF WINNING PROBABILITY

This section presents the analytic derivation of the probability of winning of contractor A_0 when competing against n opponents under a second-low bid awarding method where the contractor whose bid is ranked as the second lowest one wins the job. Because of its ranking nature, the use of order statistics to define the event "Contractor A_0 wins" is of great help in this case. Let $B_1, B_2, ..., B_n$ be the random variables that represent the bids submitted by n opponents. The ordered values $B_{(1)}, B_{(2)}, ..., B_{(n)}$ are defined as the order statistics corresponding to the random variables $B_1, B_2, ..., B_n$. In other words, $B_{(1)}$ is the smallest among the submitted bids and $B_{(n)}$ is the largest.

It is worth noting that the second-low bid method is studied in this paper based on the same two assumptions for Friedman's low bid method model (Friedman 1956) in order to compare both methods on an equal basis. The first of these assumptions states that each opponent's bid B_i (i =1, 2,..., n) is standardized by taking its ratio to contractor A_0's cost estimate c_0. The resulting ratios X_i (i = 1, 2,..., n) are assumed to be mutually independent. The second assumption states that the probability distribution of each apparent bid-to-cost ratio X_i (i= 1, 2,..., n) is independent of the values of the markup x_0 and the cost estimate c_0 that A_0 chooses for the project at hand.

Let $X_{(1)}, X_{(2)}, ..., X_{(n)}$ be the order statistics corresponding to the apparent bid-to-cost ratios of opponents $X_1, X_2, \ldots, X_n$. For contractor A_0 to win the project under a second-low bid method, its bid b_0 has to fall between $B_{(1)}$ and $B_{(2)}$ and equivalently, its bid-to-cost ratio x_0 has to lie between $X_{(1)}$ and $X_{(2)}$. Accordingly, the formal definition for the event "Contractor A_0 wins" can be expressed as follows:

$$\{A_0 wins|b_0\} = \{B_{(1)} < b_0 < B_{(2)}\}$$

$$\{A_0 wins|b_0\} = \left\{\left(\bigcap_{B_i \neq min_{i=1}^{n} B_i} B_i > b_0\right) \cap \ (b_0 > min_{i=1}^{n}(B_i))\right\}$$

Hence, in order for A_0 to win the project, it is necessary for one of the n values $X_1, X_2, \ldots, X_n$, to be less than x_0 and $(n-1)$ of them to be greater than x_0. Given that X_i are assumed to be independent and identically distributed with a distribution function $F_X(x_i)$, the probability that any given set of $(n-1)$ of the X_i are greater than x_0 and the remaining one value is lower than x_0 equals $[F_X(x_0)][1-F_X(x_0)]^{n-1}$. And since there are $n!/(n-1)!1!$ different partitions of the n random variables $X_1, X_2, \ldots, X_n$ into the two sets (the first set containing $(n-1)$ values above x_0 and the second set containing only one value below x_0), the probability of winning of contractor A_0 given a certain bid-to-cost ratio x_0 can be expressed as follows:

$$P[A_0 wins|x_0, c_0] = \frac{n!}{(n-1)!\,1!}[F_X(x_0)]^1[1-F_X(x_0)]^{n-1}$$
$$P[A_0 wins|x_0, c_0] = n[F_X(x_0)][1-F_X(x_0)]^{n-1}$$

Since the probability of winning under a second-low bid method has a closed-form solution, the optimum expected profit (V) can have a closed-form expression too in function of x_0, c_0, and n:

$$E[V|x_0, c_0, n] = P[A_0 wins|x_0, c_0, n](x_0 - 1)c_0$$
$$E[V|x_0, c_0, n] = n[F_X(x_0)][1-F_X(x_0)]^{n-1}(x_0 - 1)c_0$$

Hence, in order to determine the value of x_0 that maximizes the expected profit $E[V|x_0, c_0, n]$ for contractor A_0, the first derivative of the profit expression with respect to x_0 should be determined and set equal to zero. It follows that the optimum markup x_0^* should be equal to:

$$x_0^* = 1 - \frac{F_X(x_0)[1-F_X(x_0)]}{f_X(x_0)[1-nF_X(x_0)]}$$

As mentioned previously, $F_X(x_0)$ represents the normal distribution for apparent bid-to-cost ratios X_i with a mean m_X and a standard deviation σ_X determined based on the database of previous projects that contractor A_0 has bid on.

3. PROBABILITY OF WINNING – SIMULATION PROCEDURE

This section determines the probability of winning using Monte Carlo simulation approach. In order to generalize the solution, we introduce the standardized bid-to-cost ratio x'_0 and the standardized mean for apparent bid-to-cost ratios m'_X where:

$$x'_0 = \frac{x_0 - m_X}{\sigma_X} \quad \text{and} \quad m'_X = \frac{m_X - 1}{\sigma_X}$$

Described below are the steps of the simulation procedure followed to determine the probability of winning as a function of the standardized bid-to-cost ratio x_0':

Step 1: Initialize the number of opponents n.

Step 2: Initialize the simulation set counter k = 1.

Step 3: In simulation set k, sample m vectors $x'_{1\,jk}, x'_{2\,jk}, \ldots, x'_{n\,jk}$ from the standard normal distribution. Each vector represents one of the m projects in simulation set k. The n values in each vector represent the standardized apparent bid-to-cost ratios of the n opponents in the j^{th} project.

Step 4: For each sampled vector $x'_1, x'_2, \ldots, x'_n$, sort these values in increasing order to form the corresponding order statistics $x'_{(1)}, x'_{(2)}, \ldots, x'_{(n)}$.

Step 5: For every x'_0 in the interval [-3,3] using an increment of 0.001:

1. Initialize to zero the number of projects won at the current values x'_0 and n for the current simulation set k, $w_k(x'_0, n) = 0$.
2. Consider the next project j (out of the m projects in simulation set k). If x'_0 belongs to its winning region $[x'_{(1)\,j}, x'_{(2)\,j}]$, then project j is won: increase $w_k(x'_0, n)$ by 1; otherwise do nothing. Repeat this sub-step for all m projects in the current simulation step k.

3. Compute the ratio $p_k(x'_0, n)$ of the total number of projects won $w_k(x'_0, n)$ over the total number of projects m within simulation set k:

$$p_k(x'_0, n) = \frac{w_k(x'_0, n)}{m}$$

This is the k^{th} estimate of the probability of winning at x'_0 and n.

Step 6: If the index k equals the total number of simulation sets s, then go to Step 7. Otherwise, increase k by 1 and go back to Step 3.

Step 7: Calculate the final estimate for the probability of winning at x'_0 and n by averaging $p_k(x'_0, n)$ over all simulation sets s:

$$P[Win|x'_0, n] = p(x'_0, n) = \frac{1}{s}\sum_{k=1}^{s} p_k(x'_0, n)$$

The results obtained from running the above described simulation algorithm are presented and discussed in the next section. The code was executed for 1000 simulation sets, each consisting of 1000 projects. The assigned values for the number of opponents, n, are 2, 4, and 8.

4. SIMULATION RESULTS AND COMPARISON WITH LOW BID METHOD

This section presents the simulation nomograms showing the probability of winning, the optimum markup and the optimum expected profit for both the low and the second low bidding methods. Figure 1 shows the probability of winning corresponding to a standardized bid-to-cost ratio x'_0 in the range [-3, 3]. Once the probability of winning $p(x'_0, n)$ is known from Figure 1, we can determine the optimum standardized bid-to-cost ratio, $x_0'^*$, which is the value of x_0' that maximizes the standardized expected profit $E[V]/c_0\sigma_X$ for a given value of the standardized bid-to-cost mean m'_X:

$$\frac{E[V|x_0, c_0, n]}{c_0, \sigma_X} = p(x'_0, n)\frac{x_0 - m_X + m_X - 1}{\sigma_X} = p(x'_0, n)(x'_0 + m'_X)$$

Then, Figure 2 shows the standardized optimum markup corresponding to a specific value of m'_X in the range [-1, 3], and Figure 3 shows the standardized expected profit. Clearly, there are many differences between the low bid and the second-low bid methods as shown in the graphs below. The first concerns the probability of winning given a standardized bid-to-cost ratio, x'_0, as shown in Figure 1. The curves for the low bid method indicate a probability of winning approaching 1 as x'_0 decreases regardless of the value of n, and that is expected since the lower a contractor bids, the higher is his chance of winning. However, this is not the case for the second low bid method curves which indicate a probability of winning approaching zero as x'_0 decreases. And that is expected again because the second low bid method does not allow a very low submitted bid to be awarded the contract and rather assigns the second lowest bidder as the winner. Therefore, as x'_0 takes large negative values, x_0 gets further below the mean bid-to-cost ratio and loses the chance of being the second lowest bidder, which translates into a zero probability of winning. Hence, the second-low bid method safeguards both owner and contractor against unreasonably low bids.

Note that the curve for the second-low bid method for n = 2 is symmetrical around $x'_0 = 0$ with a maximum probability of winning of 0.5 occurring at $x'_0 = 0$. This is expected since in the case of two opponents, and in order to win, contractor A_0 has to bid between the two competitors' bids to be ranked as the second lowest bidder. This means that A_0's bid will also be the closest to the average and therefore it will win under an average bid method too.

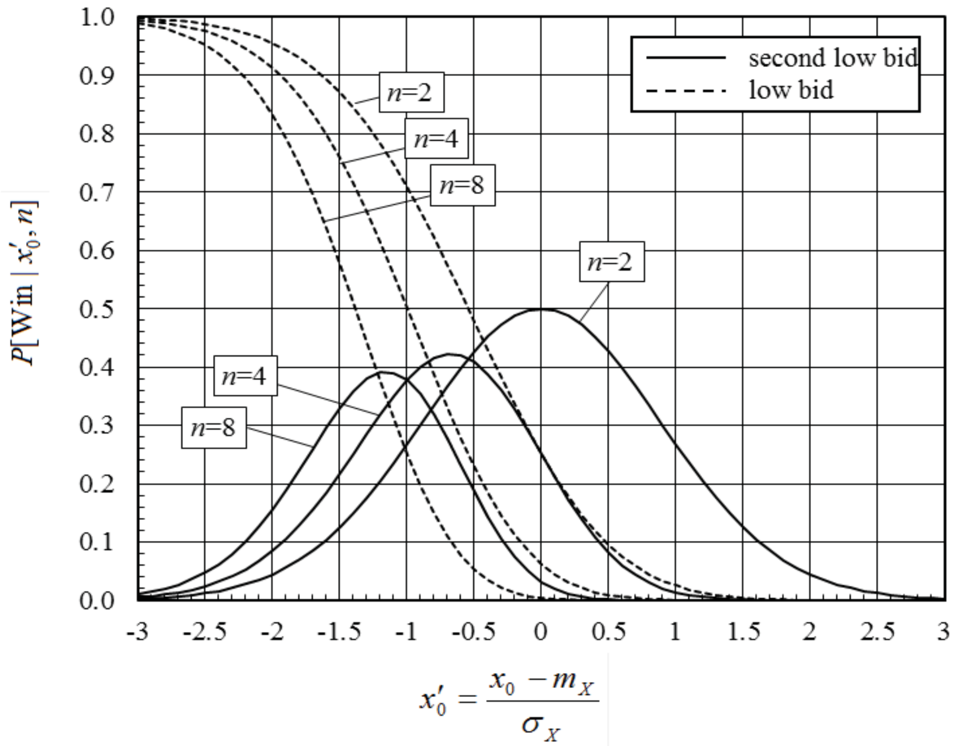

Figure 1: Probability of Winning as a function of x'_0, n.

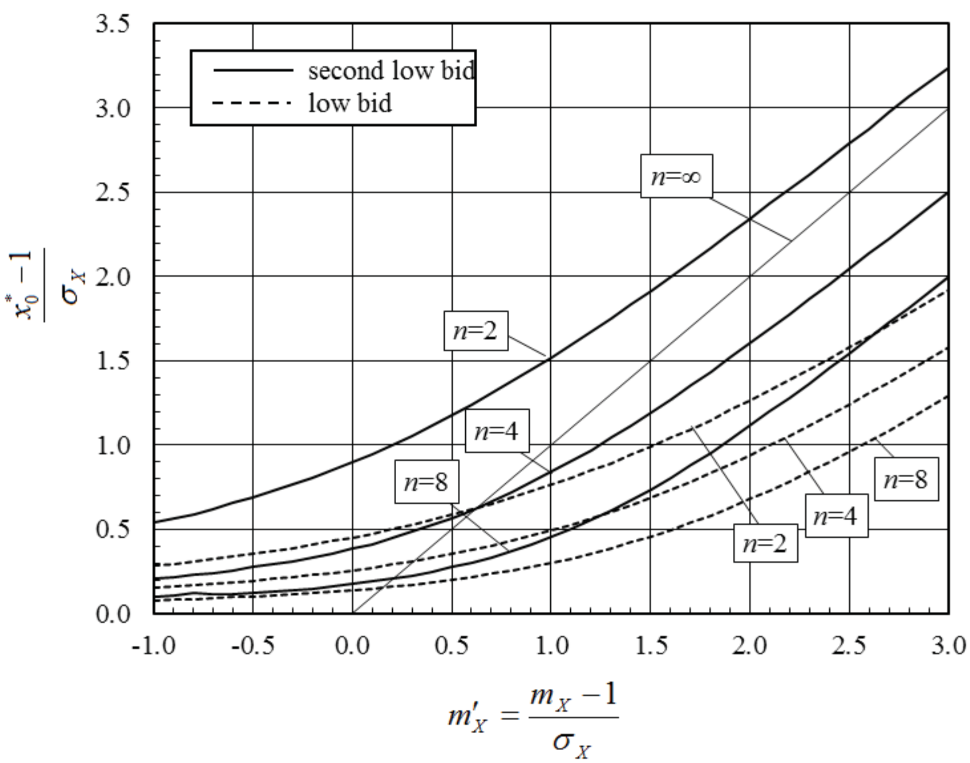

Figure 2: Optimum Markup as a function of m'_X, n.

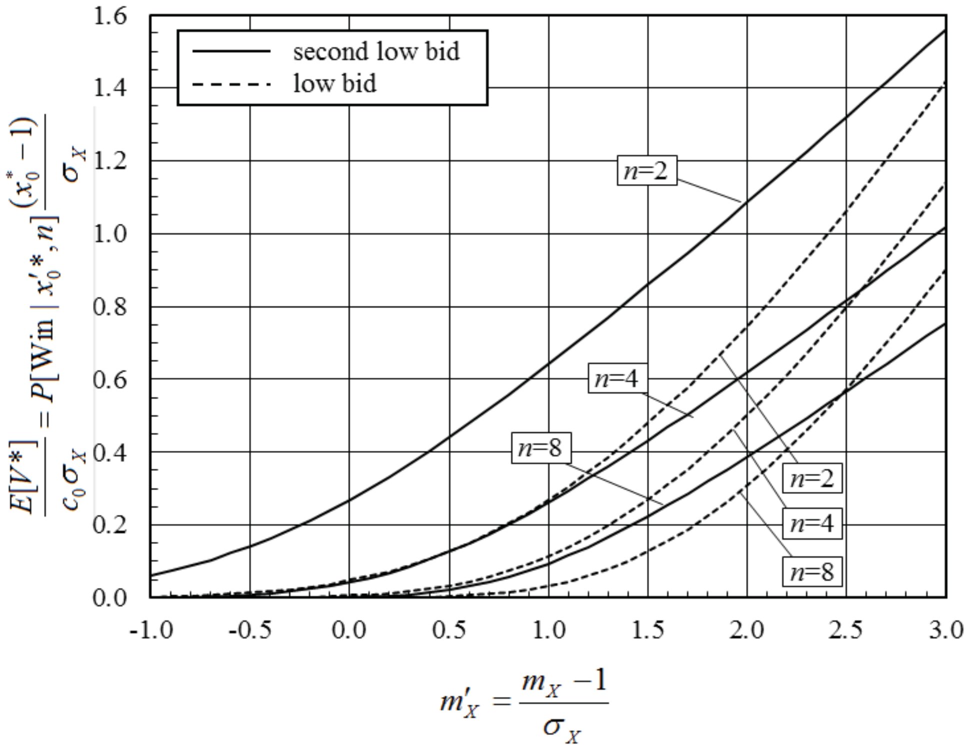

Figure 3: Optimum Expected Profit as a function of m'_X, n.

On the other hand, the second-low bid method curves for n = 4 and n = 8 remain bell-shaped and symmetrical but as n increases they become more shifted to the left. This is explained by the fact that as the number of opponents increases, the second lowest bid is expected to be further below the average and that explains why the maximum probability of winning occurs at a lower negative value of x'_0.

Moreover, for the same number of opponents n, the curves for the low bid method are above those for the second-low bid method for low negative values of x'_0 and then they cross each other and reverse their order as x'_0 increases. This is again as expected because as x'_0 decreases, it has a higher chance to be ranked as the lowest and hence win under a low bid tendering scenario. However, as x'_0 increases, it has better chance to be ranked as the second lowest rather than the lowest and therefore it has a higher winning probability under a second-low bid tendering scenario.

We shall now turn our attention to Figure 2 which depicts the optimum bid-to-cost ratio given by $(x_0^* - 1)/\sigma_X$ as a function of the standardized mean m'_X . Given the same m'_X and n, this ratio is always larger when using the second-low bid method. That is because the latter bidding method relaxes the low bid method constraint “the lowest wins” and hence, it results in a higher optimum standardized bid-to-cost ratio for any value of m'_X. Another observation is that, similarly to the low bid method curves, the second-low bid method curves rise monotonically as m'_X increases. This is again as expected, because, as the mean of the opponents’ bids increases, so should the optimal markup for contractor A_0.

Furthermore, both bidding methods, given a certain standardized mean m'_X, indicate a larger optimum bid-to-cost ratio as the number of opponents decreases. That is explained by the fact that as the number of bidders increases, the competition gets more intense and contractor A_0 has to bid lower to remain in the competitive zone. Similarly to the low bid method, the curves for the second-low bid method corresponding to different values for n become parallel straight lines as m'_X increases.

Figure 2 also includes a 45° straight line through the origin which indicates the limit where the optimum bid-to-cost ratio x_0^* is equal to the mean bid-to-cost ratio m_X. An interesting observation is that as

m'_X increases, all curves in the figure drop below the 45° straight line except for the second low bid method curve that corresponds to n = 2. This is expected since the second low bid method is identical to an average bid method in the case of two opponents.

On the other hand, when m'_X takes small values, the curves for both bidding methods fall above the 45° line and thus indicate optimum bid-to-cost ratios that are greater than the mean m_X. A negative m'_X means that the expected value of the competitors' bids is below the cost estimate of A_0. In this case, to avoid losing money, A_0 has no other choice but to bid above cost, which means to select a bid-to-cost ratio x_0* above the mean m_X. Also note that as the number of opponents increases and given a negative m'_X, the two bidding methods match almost perfectly and approach the X-axis; as indicated by the case for n =8 in Figure 2. In such conditions, the second-low bid method behaves similarly to the low bid method and expects the contractor to bid the project at cost.

Figure 3 shows the standardized expected profit as a function of m'_X. For n=2, the expected profit for the second-low bid method is always greater than for the low bid method. But this is not the case for larger values of n such as n =4 and n = 8 where for small values of m'_X, the expected profit of the second-low bid method is greater than for the low bid method but for large values of m'_X , the situation is reversed. This is because when m'_X takes large values, it means that there is small variance in the opponents' bids and hence they are clustered around their mean. In such a case, it is much easier to submit a very low bid and win under the low bid method than to rank as the second lowest bidder which requires A_0 to bid within the cluster of opposing bids around the mean. But this does not apply to n = 2 where the second-low bid method curve is always greater than that for the low bid method. That is because the optimum markup for the second low bid method is constantly above m_X and is much greater than that for the low bid method which is always below m_X , as shown in Figure 2. Therefore, contractors would favor the second-low bid method most of the time because it means higher expected profit; however, in the case of a very small variance in the opponents' bids or a high m_X, and with $n \geq 4$, contractors would rather opt for the low bid method.

The most important observation about Figure 3 is that similarly to the low bid method, the optimum profit approaches zero as the number of opponents increases under the second low bid tendering scenario. This is expected since as n increases, the optimum markup and the probability of winning for the second-low bid method tend to zero. Hence, as the number of opponents increases, the second-low bid method has the same drawback as the low bid method in the sense that the contractor is at a disadvantage of signing the project at cost with no expected profit, and the owner is at the risk of having to face a large amount of claims and disputes during construction of the project due to cost overruns.

5. CONCLUSION

The second-low bid method remedies an important shortcoming of the low bid method which is awarding the project to unrealistically low bid prices that are likely to cause problems during construction. Owners using the second-low bid method should expect to sign a contract for a higher price than using the low bid method, and contractors have the advantage of higher expected profits. However, in some cases where the project has a small variance in its submitted bids, the low bid method provides the contractor with higher expected profit than the second-low bid method due to a higher winning probability. That is because when the opposing bids are narrowly grouped around their mean, it becomes easier to underbid them which favors winning under the low bid method.

The simulation results also showed that the second low bid method has the same drawback as the low bid method when the number of competitors is relatively high because the owner should again expect to sign the contract at cost which often leads to cost and time overruns.

Last but not least, the second-low bid method could be open to unethical collusion among bidders. It is possible that when bidding on a project, a contracting company will form a dummy partner and will let it submit an extremely low bid on the same project the real company is bidding on. This will lead to exclusion of the dummy bid since it ranks as the lowest bid, and eventually will guarantee the original contracting firm a higher chance of winning the project no matter how low its bid is, as long as it is higher than the fake bid by the affiliated dummy company. Therefore, it is crucial to monitor and limit such behavior through clear bidding rules and strict prequalification processes.

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