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# DEVELOPING AND PILOTING A MATHEMATICAL MODULE FOR TEACHING DYSCALCULIC ADULTS

by

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# DEDICATION

This project is dedicated to the following special persons in my life:

To my Mother who always believed in me; although long time departed, her spirit remains with me, guiding me, encouraging me, and protecting me.

To my daughter Tamara and my son Salam; I hope this degree will inspire them to always aspire for being the best in everything they do.

To my husband who was a great support; I shall always appreciate his patience and understanding when I had to be away taken by my studies and work.

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To all my friends; I thank you for standing by me and encouraging me.

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# ABSTRACT

The purpose of this study was to develop and pilot an instructional module called *From 9 to 99 Math Helper: For Dyscalculia and Other Math Difficulties* designed for individual tutoring.

To that end this module was implemented over twelve sessions with an adult female diagnosed with math disability among others. Several constructivist based teaching strategies were used. At the end of the sessions, the participant demonstrated an understanding of concepts that she needed in her daily life such as: place value, addition, subtraction, multiplication, division, basic fractions, units of length, mass, time, capacity, percentage, and problem solving.

Posttest results showed significant improvement in calculation, math fluency and problem solving. It is expected that this module is beneficiary for adults in similar situations if implemented properly.

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# INTRODUCTION

*The Issue*

Math teachers are often challenged with students who have dyscalculia i.e. who have learning difficulties with math. Some are not able to understand basic concepts; others understand concepts but fail to apply them, while others cannot retrieve acquired knowledge when they need it. Parents of such students usually spend countless evenings tutoring their children, sometimes with little or no success.

As a result of such difficulties, children become insecure about mathematics and often carry these difficulties through their school years. More importantly, such students grow up with negative attitudes towards mathematics and they carry them throughout their adult lives, which often impairs their ability to become mathematically literate. Most of these adults fail to handle, plan, and monitor money or time, regardless of their occupations.

Experts have started studying this learning disability during the last decades of the twentieth century. So far they have identified several reasons behind it. According to Sherman, Richardson, and Yard (2005), these reasons can be categorized in general as environmental or personal factors. Environmental factors include:

- Instruction: When learning relies too heavily on rote memorization, students find difficulty recognizing and retaining math concepts.

- Curricular materials: Frequently assigning repetitious and uninteresting skill and drill work each year in order to teach underachieving students the basics limits opportunities to reason and solve problems.

- The gap between learner and subject matter: Serious achievement gaps result when mathematics content is not connected to students' ability.

As for the personal factors, they include:

- Locus of control: Students with math difficulties believe that their achievement is attributable to factors beyond their control, such as luck.

- Memory ability: Students lack mental strategies for remembering facts, formulas, or procedures.

- Attention span: Students usually face difficulty focusing on procedures and multistep problems.

- Understanding the language of mathematics: Memorizing mathematical terms without understanding seriously hampers students' ability to understand algorithmic operations and problem solving.

Chinn (2007) adds other factors that could explain students' mathematical difficulties, such as anxiety, low motivation, technical words or symbols used when talking about arithmetic, and the learner's thinking or cognitive style. He also explains that one of the reasons behind a student's failure to learn mathematics is the mismatch between the teaching methods and students' learning styles where the learner is expected to do all the adaptations (Chinn, 2007).

Aschroft and Chinn (2007) found that there is a need for teachers to be flexible in teaching and take into consideration the thinking style and the responsiveness of the students. They also state that poor attitude of students should be a major concern for the teacher because most often it is partly due to the inflexible system of school math where students are supposed to learn how to use formulas or preset methods, even if it is not their favorite way of doing math.

In addition to the above, difficulties with mathematics may be due to:

- The nature of the subject;

- The language of mathematics; specifically, the collection of symbols that make mathematics complex and formal;

- The vocabulary used by teachers as well as the vocabulary of word problems;

- Students having to progress to more difficult tasks before mastering the prerequisites;

- Poor understanding of numbers and the variety of situations where they can be used.

- Failure to develop counting skills, which is an important part of mastering numbers since they form the building blocks for learning to add, subtract, multiply and divide.

Based on the above, it becomes clear that many factors can explain dyscalculia or poor performance in mathematics among students, and the apprehension of the subject and numbers that such individuals keep on having throughout their life.

Given that experts are still uncovering aspects of this learning disability however, it is necessary to keep on exploring it since there is no doubt that sufficient mathematical skills and understanding enhance one's ability to make critically important educational, life and career decisions. Mathematics is a way of thinking. To be mathematically literate provides a person with strategies for organizing, analyzing, and synthesizing information. It has become an essential part of the world we live in, a world dominated by technological tools and devices. Unfortunately, persons with dyscalculia reach the adult age without having developed these essential skills. A dyscalculic person suffers the consequences of being mathematically illiterate throughout his or her life.

*Operational Definition*

According to Adler (2001), " the name 'Dyscalculia' is a contemporary derivative of the Latin 'dys' which means a form of special difficulties [...] and the Greek word 'calculus' which means counting stones hence difficulties in counting" (p.11).

The definition used in this project is adopted from the International Statistical Classification of Diseases, Injuries and Causes of Death. It refers to "disorders in which the main feature is a serious impairment in the development of arithmetical skills which is not explicable in terms of general intellectual retardation or of inadequate schooling" (as cited in Adler, 2001, p.64).

*Purpose of the Study*

This project was inspired by a dyscalculic graduate student's question: "Is it possible to find someone who would teach me to overcome my mathematical illiteracy?" Hearing the woman's complaint, the researcher felt a need to help reduce her frustration as well as that of other individuals with the same problem. She set to explore the possibility of teaching adult persons the skills they could not learn all through their school years. As such, she set to design a math didactical module titled *From 9 to 99 Math Helper: For Dyscalculia and Other Math Difficulties* to help a young woman overcome her mathematical difficulties.

The purpose of this study is to develop and test the effectiveness of the above mentioned pilot module aiming at helping dyscalculic individuals learn basic mathematical skills. Specifically, the study aims to assess the extent to which the module is capable of offering remedial teaching of basic computational skills and mathematical literacy to dyscalculic individuals. Concurrently, it seeks to evaluate the extent to which the module is effective in helping such individuals overcome their mathematical fear, hatred, and illiteracy.

*The Participant*

This is a case study. The participant is a 28 year-old student diagnosed with dyscalculia namely, problems in math fluency, math reasoning (problem solving) and computation skills (procedural knowledge), whose request to find someone to teach her mathematical skills initiated the study. She will be referred to hereafter with the name MC.

At the time the study was undertaken, MC was completing the requirements of a Masters degree (M.A.) in education following a Bachelor degree (B.A.) in social work and a teaching diploma in Teaching English as a Foreign Language (TEFL). She achieved all of the above despite concentration difficulties, restlessness and inattention which prompted her to ask to be assessed for Attention Deficit Disorder (ADD). She also reported a long history of anxiety, forgetfulness, lack of direction, lack of organization, and mild arthritis which makes her struggle with writing.

The participant's overall intellectual ability, as measured by the Woodcock Johnson III Standard General Intellectual Ability test ( WJ III GIA (STD) ), is in the low average range. When compared to others at her age level, the participant's performance is low average in math calculation skills and math reasoning.

*Assumptions*

The researcher embarked on this project with the following assumptions:

- There is growing awareness of the key role that mathematical skills play in our daily lives. There are more and more individuals who wish to have a second chance to learn the basic mathematical skills needed in everyday life situations such as the need to use a bank account, to balance a checkbook, to calculate the discount at the mall and many other situations that make adults rethink their mathematical skills.

- Since the participant was able to become a graduate student despite all of the above mentioned difficulties, then she should be able to learn the math she needs for everyday.

- Math as a subject is not difficult. It is made difficult by others. Those others could be teachers, parents, peers, curriculum or simply the way we are born. Thus, even learners with math difficulties are able to learn math if taught in suitable ways.

Hence, the global assumption in this case study is that if a tutor works with MC at her own pace using a variety of strategies that meet her needs, then everyday math will make sense to her.

*Significance of the Study*

To our knowledge, there is no research in Lebanon related to adult learning difficulties, particularly dyscalculia, and there are no programs tailored for the Lebanese curriculum developed for pupils with learning difficulties. As such, there is a genuine need for special programs to help individuals with this difficulty.

# LITERATURE REVIEW

The first part of this section presents a summary of different characterizations of dyscalculia found in the literature. It identifies major aspects of this learning disability. The second part sheds light on the constructivist approach to teaching, which shaped and provided theoretical background to the development of the pilot module tested in this study. The third and last part focuses on the essential role of metacognition in math learning.

*Characteristics and reasons of dyscalculia*

Mathematical skills are essential to solve problems and make decisions not only in the classroom but in real life situations as well. Many individuals however, spend their life unable to perform even the simplest mathematical tasks because they never had the chance to become mathematically literate. Carpenter and Lehrer (1999) define mathematical literacy in terms of 5 components: 1) mathematical content knowledge, 2) mathematical reasoning, 3) understanding of the social impact and utility of mathematics, 4) understanding the nature and historical development of mathematics and 5) mathematical disposition.

According to Abeel (2007), some people are born with an innate sense of math. They just have got it, while others have not. Society, the teachers, the school system make individuals' life difficult for a disability they were born with. Several studies show that there are parts of the brain responsible for mathematical abilities. For instance, O'Hare, Brown, and Aitken (1991) found right-hemisphere dysfunction in one case of childhood dyscalculia (as cited in Aschroft & Chinn, 2007). This awareness is essential for those adults who had enough from math and are convinced that whatever they do they will never be able to succeed in it. On the other hand, some scholars like Butterworth (1999) suggest that dyscalculia is inherited without knowing so far which genes are responsible for this.

The name "developmental dyscalculia" was first proposed by Kosc in the 1960s (Adler, 2001). According to Butterworth and Yeo (2004), it was first recognized however by the Department For Education and Skills (DFES) in the UK in 2001. It is defined as: " ... a condition that affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers, and have problems learning number facts and procedures [...]" (DFES, 2001, p.2).

Geary (1993, 1994, 2000, 2004) a leading expert and researcher on dyscalculia, describes it as numerical and arithmetical deficits following overt brain injury. As such, he uses the term "mathematical learning difficulties" to refer to it rather than other terms.

Chinn (2007) however, argues that dyscalculia is like dyslexia but with numbers instead of words. For him it seems that some people's brains are not efficiently wired for math. He uses dyscalculia to describe someone who has severe difficulties with math. In other words, when all possible social and environmental factors are ruled out, the child is still performing below the teacher's expectations. This could be manifested sometimes by an inability to know the value of a number, to recall a basic mathematical fact or simply deal with money. It can also mean an inability to follow certain directions, or to calculate change from a purchase or the tip one has to leave in a restaurant. Moreover, dyscalculics get very anxious about doing any math and become impulsive when they do, rather than analytical.

Bird (2007) suggests that dyscalculic students may be recognized when being competent in subjects other than mathematics but have a surprising level of difficulty with ordinary numeric operations and rely on finger-counting for all operations, while other students in the class have progressed to more efficient strategies. These students encounter great difficulties dealing with numbers especially counting backwards. Often they stand out as being unable to do a small estimate, not having a feel for numbers, and not being able to tell if an answer for an arithmetic problem is reasonable or not. Moreover, dyscalculics experience memory weakness both in long-term and short-term memory ability. They have great difficulty in remembering facts and procedures consistently or accurately and are unable to stay on track when they are applying a procedure in solving any mathematical task.

Dyscalculia is not widely recognized compared to its literacy counterpart dyslexia. It is sometimes referred to as number blindness in the ability to learn mathematical skills. Numerical processing however is so complex which makes defining dyscalculia difficult. Another main problem related to this disability is that there has been no single test for identifying it. One noteworthy tool is the one developed by Butterworth (2003) referred to as the "Dyscalculia Screener" test of basic numerical capacities. This tool is based on the idea that pupils are born with an understanding of "numerosity", and specific capacities for basic numerical tasks. Hence, pupils who are below their age level in this area show the lowest levels of arithmetical attainment and are the unhappy kids during math hours. This may lead to severe emotional side effects.

This supports the statement that "overlying all areas of difficulty are the emotional issues of self-esteem, self-concept, expectations, mathematics anxiety" (Buxton, 1981, as cited by Ashcroft & Chinn, 2007, p. 30).

There have been some attempts to address the mathematics curriculum, instruction, testing, and remediation especially customized for dyscalculic students. Most of these efforts however, have been directed toward primary school dyscalculic pupils. Butterworth and Yeo (2004) advocate that older dyscalculics who have very poor number skills, also need to acquire these same basic skills.

*Constructivism and teaching math*

Originally, constructivism is a philosophy of learning that considers that " … humans construct the meanings they understand" ("Building an Understanding", 1994, ¶ 2). Out of this basic epistemology and based on the views of John Dewey and Jean Piaget about how humans learn, moderate (or *simple* or *trivial*) constructivism, also known as cognitive constructivism, developed. According to Dewey individuals, including children, learn only when in a situation where they have to draw knowledge out of experiences that are meaningful to them. These situations have to occur in a social context such as the classroom (Dewey, 1916 as cited in "Building an Understanding", 1994). On the other hand, Piaget emphasizes that teachers, especially when teaching young children, must know that the setting where learning takes place shapes the learning process; the mind develops in stages and thus determines what a student can or cannot learn; and that learning happens by discovery and through assimilation and accommodation (Atherton, 2005; "Building an Understanding", 1994). As for the main contention of the theory it is best summarized as follows: Individuals are active learners i.e. they construct meanings using their mind to make sense of material presented to them (Boudourides,1998; Selden, 1998).

The moderate version of constructivism was further developed. Out of Lev Vygotsky's challenge to Piaget's emphasis on learning as an individual process grew social constructivism. This theory stresses the primary role of communication and social life in meaning formation and cognition i.e. it views learning primarily as a social process, which enables students to learn more advanced concepts and ideas that they cannot understand on their own with help from other students or adults. Moreover, ideas of Ernst von Glasersfeld (1984; 1987) about human learning resulted in radical constructivism (Boudourides, 1998; Jaworski, n.d.; Selden 1998). This theory is different in that it adds to the basic ideas of cognitive constructivism the emphasis that cognition is adaptive i.e. for this theory, learning is a process of adaptation based on and constantly modified by a learner's experience of the world (Jaworski, n.d.).

Over the last decade of the 21st century, interest among experts and scholars in constructivism as an approach to teach mathematics intensified. Constructivism as a learning theory and perspective became central to the research reported about mathematics education and was advocated as a perspective of teaching and learning math more than ever before (Cobb, Perlwitz, & Underwood-Gregg, 1998; Boudourides,1998; Noddings, 1993; Simon, 1995; Zazkis, 1999).

Jaworski (n.d.) explains that the power of constructivism for mathematics education lies in the principle highlighted by von Glasersfeld (1984; 1987) namely that individuals constantly shape knowledge they form. As she puts it, " ... it seems to say that *if* there is some independent, pre-existing body of mathematical knowledge we cannot know it except through our own experience, and we can *only* know what we ourselves have constructed, and modified according to further experience" (Jaworski, n.d., Definition, ¶ 1).

According to Davis, Maher and Noddings (1990) math " … requires construction, not passive reception, and to know mathematics requires constructive work with mathematical objects in mathematics community"( p.2).

Englert, Tarrant, and Mariage (1992), as well as Englert, Mariage, Garmon, and Tarrant (1998) suggest that in order for teachers to be adopting a constructivist approach with their students, they should embed instruction in meaningful activities, promote dialogue for self-regulated learning, demonstrate instructional responsiveness, and establish classroom learning communities (as cited in Polloway, Patton & Serna, 2005).

Huetink and Munshin (2000) add that the constructivist approach to teaching encourages teachers to work together with the students to deepen their understanding of mathematics. Arguing along the same lines, Draper (2002) states: "Constructivism as a theory of learning can provide the framework needed to help math teachers move from a transmission model to one in which the learner and the teacher work together to solve problems, engage in inquiry, and construct knowledge" (¶ 4).

In conclusion, as the National council of Teachers of Mathematics (2000) underscores the constructivist approach to teaching mathematics enables students to solve the new kinds of problems they will inevitably face in the future. It is important that teachers who use it keep remembering that "Students must learn mathematics with understanding actively building new knowledge from experience and prior knowledge"(NCTM, 2000, p. 20).

*Metacognition and learning math*

Broadly defined, metacognition (also referred to as metacognitive knowledge) is "one's knowledge concerning one's own cognitive processes and products ... [and] the active monitoring and consequent regulation and orchestration of these processes" (Flavell, 1976, p. 232). Or as Polloway, Patton, and Serna (2005) put it, it is the ability to reflect on and talk about one's own learning. Hence, as Roberts and Erdos (1993) note, because it is conceptually closely linked to consciousness, metacognition can dramatically enhance learning.

According to Siraj-Blatchford and Petayeva (n.d.) scholars interested in information processing theory first used the concept metacognition; their aim was "… to construct a model of cognitive control processes to differentiate actual strategic functioning in problem solving" (¶ 2). Studies have shown that solvers are said to be proficient in problem solving when they use metacognitive skills, namely planning, monitoring, and evaluating their thinking, during problem solving; and when they do so more efficiently than poor or novice problem solvers (Bookman, 1993; Cai,1994; Desoete, 2001; Flavell, 1976; Lucangeli, Coi & Bosco, 1997). In other words as David (2001) underscores, "Metacognition appears to function as a vital element contributing to successful problem solving by allowing an individual to identify and work strategically" (Background on writing and metacognition, ¶.2).

Over the past few decades, there was more and more agreement among scholars, researchers, and experts that metacognition plays a key role in solving mathematical problems and improving students' mathematical achievement in general (Artzt & Armour-Thomas, 1992; Desoete, 2001; Garrett, Mazzocco & Baker, 2006). Practices with a potential to promote the development of more effective metacognitive knowledge and skills among students are summarized below:

- Students learn strategies more effectively when those strategies are taught within the context of specific subject domains and actual academic learning tasks;

- Students can only use sophisticated learning strategies when they have a knowledge base to which they can relate new material;

- Students should learn a wide variety of strategies, as well as the situations in which each one is appropriate;

- Effective strategies should be practiced with a variety of tasks and on an ongoing basis;

- Strategy instruction should include covert as well as overt strategies;

- Teachers can model effective strategies by thinking out loud about new material;

- Teachers should scaffold students' initial attempts at using new strategies, gradually phasing out that scaffolding as students become more proficient;

- Students must understand why the skills they are taught are helpful;

- Students must believe that, with sufficient effort and appropriate strategies, they can learn and understand challenging material;

(Siraj-Blatchford and Petayeva (n.d.), Metacognition and study strategies, para. 7).

To this day, however, there is not enough literature on the relationship between metacognition and teaching learners with mathematics difficulties, namely the strategies to be used. Teachers have at their disposal at best general observations such as the following: "Effective teaching combines direct instruction (teacher-directed tasks, discussion, and concrete models) with strategy instruction (teaching ways to learn, such as memorization techniques for arithmetic facts, study skills and metacognition – learners identify strategies that help them to learn) (Sharma, 2003, What forms of instruction are most effective?, ¶ 6). As well as general models such the one developed by Bowens, Hawkins, and King (1997). They developed an approach that focuses on helping students discover how they naturally attend to learn. Their approach entails:

1- Recognizing and recalling information in order to gain knowledge.
2- Translating and summarizing/paraphrasing to gain comprehension.
3- Interpreting through generalizing, defining, and making connections between facts.
4- Solving problems through identifying them and the skills necessary for solving them.
5- Analyzing all the parts of a problem to discover what is similar and dissimilar.
6- Synthesizing, or solving a problem through original, creative thinking.
7- Evaluating or comparing and discriminating between ideas.

(Polloway, Patton, & Serna, 2005, p.220).

# METHOD

*Research Design*

The design used in this math didactic module titled *From 9 to 99 Math Helper* is a qualitative case study with a single participant.

*Method*

The study consisted of the following phases:

- A cognitive and educational assessment of the participant (pre-assessment) administered by Alphabète Center, a center for psycho-education assessment and remediation for students with learning difficulties;

- Development of the module;

- Implementation of the module;

- Follow-up assessment to evaluate the participant's progress.

*Data Collection*

The following data collection tools were used during the study:

- Video tape of the pre-assessment to provide a description of MC's attitude and behavior while taking the test.

- Formal reports related to the participant's assessment results.

- Written work by the participant to form the basis for error analysis and the evidence for concept learning.

- Notes taken by the researcher during the assessments and the tutoring sessions.

- Video tape of the post-assessment session to be able to compare MC's attitude before and after the sessions.

- Written feedback by the participant and her father.

*Ethical Considerations*

The research did not cause physical or psychological harm to anyone. All data collected about or from the participant were held in strict confidence. The videotapes and all other data collected are held in strict confidence.

# IMPLEMENTATION AND RESULTS

This section is dedicated to describing the phases of the pilot study, the procedures, and the results of implementing the pilot module *From 9 to 99 Math Helper*.

*Cognitive and Educational Assessment of the Participant*

The study started with a diagnosis of the case, i.e. identification of the disability experienced by the participant. Specifically, psycho-educational assessment was conducted during May 2007 based on *WJ III Tests of Cognitive Abilities* and *WJ III Tests of Achievement*. These tests provided measurement of the participant's overall intellectual ability, specific cognitive abilities, and academic achievement. Results of the cognitive assessment showed low average intellectual ability. Results of the achievement test ranged between low and low average on the math cluster.

To help identify specific strengths and weaknesses, a criterion-reference test, the *Criterion Test of Basic Skills-2 (CTOBS-2)* (Evans, Lundell, & Brown, 2002) was administered. This test is an instrument that assesses specific academic areas in more detail than the general achievement measures. It covers the domains of one-to-one correspondence, numbers and numerals, addition, subtraction, multiplication, division, measurement, telling time, symbols, fractions, decimals and percents, geometric concepts, pre-algebra, and rounding and estimation. The results indicated that the participant was at the mastery level in some domains, notably correspondence, telling time, symbols and pre-algebra. However, the participant demonstrated 'frustration level' in some categories, particularly those related to numbers and numerals, addition and subtraction with regrouping, multiplication, division, fractions, decimals and percents, understanding perimeters and estimations.

*Development of the Module*

Based on the assessments results, the researcher prepared worksheets for one-to-one tutoring during 10 to 15 sessions. Most of the exercises in the module on numerical skills were selected from the *Key Skills in Number for Ages 7-9 & 9-12* (Patilla, 1996 a, 1996 b). The reason is that math is a cumulative subject typically taught in a sequence i.e. it introduces certain skills before others. Hence, a remedial module that seeks to teach essential mathematical skills, namely skills needed for everyday life, should start from the basics. For example addition should be mastered before moving to multiplication which is repeated addition.

At the same time, a set of word problems were selected from *Mathematics for Elementary School Teachers* by Bassarear (2005), as they cover all four operations and the different contexts in which each operation can be encountered.

The rest of the exercises and problems were developed by the researcher to cover place value for large numbers, a skill important for dealing with topics such as money and population, and to bridge gaps which became evident during tutoring on topics such as the role of 1 and 0 in the four basic operations.

The following observations by experts regarding mathematical difficulties, particularly dyscalculia and effective approaches for teaching mathematics played a key role in the development of the part of the module related to problem solving and strategies:

- Problem solving is a very important skill, yet so difficult to teach since it requires many complex skills. Clearly defining the terms used, teaching strategies to identify significant words and to ignore nonessential information, using verbal mediation strategies to work through to the solution of a given problem, and providing procedures for systematically solving math problems, are all essential to guarantee effective ways to develop and enhance reasoning skills in arithmetic involves (Polloway, Patton, & Serna, 2005).

- There are three barriers to solving word problems, interest and motivation, deficiencies in basic skills, and lack of facility in cognitive skill areas (Moses, 1983). Hence, in order to help students succeed in problem solving, the tutor should teach them strategies and their use.

- Computation skills of individuals who attend remedial classes in math improved a great deal (Miller, Butler, & Lee, 1998). They become able to attack problems and solve them with self confidence.

During the sessions the following strategies were used to implement the module:

- Use of the semantic approach, by teaching the terminology in relation to meaning.

- Use of manipulatives such as counters and base-ten blocks to teach counting and place value, Cuisenaire rods to teach addition and subtraction, as well as paper folding and fraction bars and burgers for teaching fractions.

- Focus on mental math strategies to find sums and differences such as doubles, doubles plus one, make-a-ten, and compensation techniques.

- Practice of "counting on" and "counting back" strategies to teach addition and subtraction of whole numbers.

- Use of modeling and a variety of mathematical representations such as number lines, tallies, diagrams, and tables in solving word problems.

- Drill and practice activities to improve mental computation and reinforce computational techniques, using properties of operations: commutative, associative and distributive properties.

- Use of combinations of previous strategies.

Since there is no single strategy to apply, the researcher decided to resort to tutoring to secure effectiveness of the module. The role of the tutor in selecting the suitable strategies for each situation is very important.

The researcher also improved the development of the module by asking the participant to verbalize the procedure she was employing in solving a given task and this in order to determine the nature of the problems the participant was experiencing in math. This is based on the observation by Polloway, Patton, and Serna (2005) that the metacognitive strategies students need in order to succeed in school involve learning how they learn, and how to manage their learning problems or breakdowns.

*Procedures*

Following the formal results of the pre-assessment tests, the researcher set a preliminary work plan. Twelve one-to-one tutoring sessions took place. During each session the researcher used a combination of the strategies listed above to teach the participant basic mathematical operations and skills.

The selection of strategies depended on the difficulties faced by the participant. The tutor was aware that when using any given strategy, it may take several attempts to see positive results, and that with some students a certain strategy may not be effective. At the same time, the researcher had to know how to deal with the variety of exercises in each particular case by taking into consideration the suitable strategy for the situation, the learning style and pace of the learner.

Throughout the different sessions, the researcher was very friendly and provided a relaxing environment where learning can be enjoyable. She was careful, patient, supportive, and understanding of MC's thinking style and difficulties. She also made it very clear to MC that she believes in her ability to learn and that she will be able to achieve what she aimed to. Furthermore, throughout the implementation phase MC was encouraged to:

- Think mathematically;
- Master basic facts;
- Talk about her own learning and develop several metacognitive skills throughout the sessions.

Along with the teaching strategies, the researcher introduced concrete materials and explained to the participant that their importance is to make math visible, and to develop insight and intuition. More specifically, their importance stems from the fact that they would allow her to make meaning for herself and to create a model for understanding math that she can internalize. Using concrete materials supports the participant's progression to abstract higher order thinking.

Manipulatives were used whenever and as long as the participant needed them and not only in the introduction phase. They were not used in a mechanical way, simply to find an answer, but to help the participant acquire a stronger sense of numbers, get a better understanding of the relations among numbers, and make connections between the action she is performing on the manipulative materials and the mathematical symbolism used to record that action. Besides, they helped her move from the concrete stage to the semi-concrete or representational stage, to be able to reach the abstract concept.

Homework was often given at the end of the sessions, either for the participant to practice the acquired skills, or to reflect on problems. Some of the work of the participant is found in Appendix B.

Last but not least, it is important to note that throughout the implementation of the module, MC was asked to perform speedy calculations at the beginning of each session and this in order to encourage her to use the computation strategies she has been learning.

*Sessions Proceedings*

This section describes proceedings of each session. A table highlights the concepts, challenges faced by the participant, and strategies used to help her. The researcher's comments on the proceeding of the sessions are presented after each table.

*Session 1:*

| Session | Concept/Skills | Challenges/Difficulties | Strategy used |
|---|---|---|---|
| 1 | Digits & numbers | Not knowing the difference between them | - Compare to English alphabet<br>- List them |
| | Parity of a number | Confounding using odd & even | - Semantic approach i.e teach math terminology<br>- Use counters |

The researcher set the ground by identifying the digits 0 through 9 explaining to MC that they form the mathematical alphabet just as the letters of the English alphabet. She also explained to her the difference between digits and numbers. She made sure to explain to MC the importance of differentiating between digits and numbers.

Furthermore, during this session, the researcher had to explain to MC the difference between even and odd numbers. Again the researcher tried to relate the words to the use of these terms in spoken language that is when you say we are even we should be two, but something is odd when it stands alone in certain situations.

*Session 2:*

| Session | Concept/Skills | Challenges/Difficulties | Strategy used |
|---|---|---|---|
| 2 | Place value | Reading large numbers | - Use of base-ten blocks<br>- Use of place value chart |

In this session, the researcher moved to reading numbers and identifying place value in a given number. MC started identifying the place value in a given number from left to right. So she could not read large numbers correctly. The researcher used tables and base-ten blocks to clarify this misconception. Many exercises, some of which using the abacus, were done until MC finally was able to read large numbers correctly. Homework was given at the end of the session to make sure she is able to solve it on her own.

*Session 3:*

| Session | Concept/Skills | Challenges/Difficulties | Strategy used |
|---|---|---|---|
| 3 | Rounding | Inability to estimate | - Use of number line<br>- Use of Cuisenaire rods |
| | Addition | From left to right | - Use of base-ten blocks |

The first part of this session was dedicated to the concept of rounding numbers. Number lines were used to see the position of one-digit numbers with respect to five then two-digit numbers with respect to 15, 25 and so on. The importance of rounding was demonstrated in estimation, when we need to do some approximations. The example of window shopping was given.

Following rounding the researcher asked MC to perform simple additions. She started performing from left to right. This worked until she had to regroup numbers. She put the numbers in vertical form and performed. Here she noticed that she had to start from right to left to be able to regroup. But she had to stop at each partial addition to use her fingers to add the simple ones such as 8 + 5 that she did wrong. Here counting on and number line strategies were introduced. The researcher provided MC with counters and base-ten blocks.

*Session 4:*

| Session | Concept/Skills | Challenges/Difficulties | Strategy used |
|---|---|---|---|
| 4 | Subtraction | - Using fingers<br>- Retrieval from Memory | - Number line<br>- Count on<br>- Doubles<br>- Doubles plus one |

This session was dedicated to teaching MC subtraction. The work here was similar to the work done during the addition session. A variety of strategies were used including counting back, making a ten, expanding numbers to enhance mental math skills. Then the researcher helped MC find answers using doubles and doubles plus one. Simple subtraction equations involving missing terms were still very difficult to MC. She asked for help. So the researcher used drawing objects and backward counting strategies to explain the method. After many examples and a variety of ways MC was able to solve the rest.

*Session 5:*

| Session | Concept/Skills | Challenges/Difficulties | Strategy used |
|---|---|---|---|
| 5 | Addition and subtraction | Inability to identify missing operation signs | - Make a ten<br>- Expand numbers |

In this session, MC was given both addition and subtraction operations. She was stuck when she needed to find the missing operation sign in some equations. So the researcher explained to MC the method of comparing members on each side of the equation to guess the kind of operation; then by breaking numbers as well as using guess and check.

*Session 6:*

| Session | Concept/Skills | Challenges/Difficulties | Strategy used |
|---|---|---|---|
| 6 | Word problems | - Guessing<br>- Fear of problems | - Use different types of representations: drawing, graphs, and tables<br>- Use a calculator |

When given word problems, MC's first response was that one should add or subtract. The researcher had to ask her to read the problem again, identify the needed information, and what was required to do. After some concentration, MC was able to identify what should be done but she found difficulty dealing with large numbers. So the researcher gave her a calculator and asked her to think about her steps imagining the problem was about her. Would she add money with sandwiches? What is the problem talking about? What do we have to find? The researcher devised a plan with MC to follow and she was able to continue on her own. The plan is as follows:

1) Restating the problem : Have MC restate the problem in her own words. This verbalization helps her to structure the problem for herself and also shows whether she understands the problem or not.

2) Simplifying: Have her substitute smaller and easier numbers for problems with larger or more complex numbers so that she can understand the problems and verify the solutions more readily.

3) Determining the question: Have MC decide what is to be discovered and what is the problem to be solved.

4) Using visual reinforcements: Use concrete objects, drawings, graphs, or other visual reinforcements to clarify the problem, demonstrate solutions, and verify the answers. Have MC act out the problem.

5) Time for thinking: Allow MC enough time to think. Try to understand how she thought about the problem and went about solving it. Ask for alternative methods for solving the problems.

*Session 7:*

| Session | Concept/Skills | Challenges/Difficulties | Strategy used |
|---|---|---|---|
| 7 | Multiplication | - Inability to remember multiplication facts | - Repeated addition<br>- Find a pattern<br>- Make groups |

When presented with multiplication facts, MC relied on her memory to recall some of the facts. She admitted however that she did not know any pattern to help her recall them. As for multiplying a multiple-digit number by another, she remembered the technique but she could not perform the operation correctly because she could not recall facts. It was noteworthy that MC was able to complete any pattern adding 2s or 5s without being able to relate them to the multiplication by 2 or by 5. As such, repeated addition with the patterns was introduced to work on the concept of multiplication.

*Session 8:*

| Session | Concept/Skills | Challenges/Difficulties | Strategy used |
|---|---|---|---|
| 8 | Multiplication | -Inability to determine pattern for multiplication by 10<br><br>-Technique of multiplying a multiple-digit number by one-digit number | - Use multiplication tables of 2 & 5<br>- Use base-ten blocks & place value table<br>- Mental math strategies using distributive property<br>- Multiplication chart |

This session was dedicated to multiplication too. MC was not able to determine a pattern for multiplication by 10 and she found difficulty with the technique of multiplying a multiple–digit number by one-digit number. The researcher used tables of 2 & 5, base-ten blocks, place value table and mental math strategies using distributive property, and multiplication chart. MC ended up realizing that multiplying by 10 could be a pattern of writing a zero to the right of the number we are dealing with.

*Session 9:*

| Session | Concept/Skills | Challenges/Difficulties | Strategy used |
|---|---|---|---|
| 9 | - Division<br>- The role of 0 and 1 in the 4 basic operations | - Inability to relate it to multiplication<br>- Inability to remember the techniques | - Counters,<br>- Cuisenaire rods<br>- Base-ten blocks |

The real frustration appeared when MC had to perform long division. She said that in the past she had a private tutor and she was able to do them. Most probably, at that time MC learned the technique without understanding the concept. So the researcher had to model division by distributing candies equally among some friends, breaking down numbers using exchange of base-ten blocks until she was capable to solve such operations by herself.

When the researcher moved to highlighting the role of 0 and 1 in the 4 basic operations, she found out that MC was not able to deal with 0 and 1 in different positions in the four operations. She did not know for example the difference between 6 – 0; 0 – 6; 6 + 1; 1+6; 6: 1; 1: 6 . So the researcher resorted back to counters, modeled the operations, tried to read them while explaining them to make them meaningful.

*Session 10:*

| Session | Concept/Skills | Challenges/ Difficulties | Strategy used |
|---|---|---|---|
| 10 | Fractions and operations on fractions. | -Confusion between numerator and denominator<br>- Inability to see the same fraction in different representations | - Fraction bars<br>- Fraction burgers<br>- Cuisenaire rods<br>- Paper folding |

Fractions were introduced by shading half of a figure. MC could shade part of a figure horizontally only. She was not able to do so in any other direction (See Appendix B). Equivalent fractions were explained using the fraction bars, paper folding and Cuisenaire rods. When presented with addition of fractions having different denominators MC said she did know they have to be the same but she did not remember how to do them. Fraction bars as well as number lines were used to model the situations.

*Session 11:*

| Session | Concept/Skills | Challenges/Difficulties | Strategy used |
|---|---|---|---|
| 11 | - Measurement<br>- Percent | - Estimating measures with the appropriate unit<br>- Confusion between perimeter and area<br>- Inability to calculate a percentage | - Relate to actual situations<br>- Use the meter to measure the length of the desk<br>- Use of calculator to find taxes, discounts, original price, etc. |

The importance of fractions was related to situations involving measurement and percentages. MC found it hard to estimate measures with the appropriate unit. She was confused between perimeter and area, and she was not able to calculate a percentage. To treat these problems, the researcher taught MC to relate to actual situations, use the meter to measure the length of the desk, and use her calculator to find taxes, discounts, original price. By the end of the session the units of measurement were tackled covering only the basic ones that were needed at home for cooking, in the grocery store for shopping, in the work place to keep track of the events and the agenda, and to be able to estimate distances when moving from one place to another. The researcher also taught MC how to use a calculator to calculate a discount or a raise.

*Session 12:*

| Session | Concept/Skills | Challenges/Difficulties | Strategy used |
|---|---|---|---|
| 12 | Mixed word problems | | - Used representations: graphs, and drawings<br>- Number lines |

In the final session MC was given a variety of problems using the four main operations in different contexts. MC used representations, graphs, drawings, number lines and she was able to solve most of the problems correctly. The most important thing was that she has gained confidence in herself, she was explaining loudly what she has to do, and she had learned strategies to help her reach the correct answer instead of trying to guess it. MC was eager to take the post assessment because she was sure that she has improved.

*Analysis of Sessions' Proceedings*

The main aim of teaching the participant was to help her relearn, to allow her understand and internalize the basic mathematical skills so that she feels capable of reasoning, doing problem solving, and discovering associations that will lead to mathematical understanding in order to establish the following skills:

- Numerical literacy, which is in part reading and writing numbers fluently.
- Fast performance in the calculation while doing simple arithmetic.
- Fast assessment of relative number value.
- Selection of the appropriate operation for a problem.
- Recognition of mathematical symbols.

Based on the proceedings of the sessions described in detail above, the researcher concludes that the twelve one-to-one sessions yielded quite satisfactory results. As the sessions evolved, MC became increasingly able to perform mental math, and do problem solving correctly. She understood concepts needed in daily life such as, place value, addition, subtraction, multiplication, division, basic fractions, units of length, mass, time, capacity, percentage, and problem solving.

Furthermore, by the end of the twelve sessions, MC had stopped engaging in the following: showing a high level of anxiety; answering problems spontaneously without really thinking about the answer; use her fingers to perform simple additions and subtractions; memorize temporarily the techniques of performing divisions with no understanding of the conceptual meaning of division; guess answers with no strategy to use.

*Results*

To confirm the above observations regarding the impact of the module on the participant, parts of the *WJ III Tests of Cognitive Abilities and Achievement* administered during the pre-assessment phase were repeated, namely the Math fluency test, calculation and applied problems, using the same concept but different numbers.

During the test (post assessment) MC was sure of herself. She was not shaking her legs, did not use her fingers for counting, did not show restlessness, and was at ease. She was amazed at her ability to understand the problems this time. More importantly, MC's improvement in basic mathematical skills was clear. This is evidenced in the results presented in the table below:

Table 1

*Comparison of Results of Pre and Post Assessment Tests*

| Test | Result | | | | | |
|---|---|---|---|---|---|---|
| | Standard Score (Mean = 100) (SD = 15) | | Age Equivalent | | Grade Equivalent | |
| | Pre | Post | Pre | Post | Pre | Post |
| Calculation | 81 | 87 | 10-5 | 12-4 | 4.9 | 6.9 |
| Math Fluency | 85 | 94 | 13-4 | 15-10 | 7.9 | 10.3 |
| Applied Problems | 75 | 105 | 10-1 | >30 | 4.4 | 16.9 |

There was a major difference between the summaries of test results in the reports prior to and after the implementation of the module. The report of the pre-test states the following: "When compared to others at her age level, MC's performance is low average in math calculation skills and math reasoning. Her mathematics calculation skills standard score is in the low average range. Her mathematics calculation skills are limited, and math calculation skills above age 13-6 level will be quite difficult for her". The report of the post-test reads: "When compared to others at her age level, MC's standard scores are average in broad mathematics, math calculation skills, and brief mathematics".

Before concluding this section of the study, it is important to note the following:

- MC had to travel following the test. When she returned, she admitted that she was happy having confidence with numbers. She was able to handle money conversion by using the mental calculation strategies.

- A letter from the participant and another from her father to the researcher's advisors revealed their satisfaction from the results. These letters are found in appendices C & D.

# CONCLUDING NOTES

*Limitations*

This case study provides insights into the effectiveness of the pilot module titled *From 9 to 99 Math Helper* to help individuals with dyscalculia, and as Burns (2000) points out, with respect to case studies, it has reliability and external validity albeit to a limited extent.

Furthermore, the project was delayed due to the participant's personal circumstances. She had to travel many times and she was not always consistent in scheduling the sessions. The irregularity of the sessions' affected the consistency of the work and required repeating some work to reinforce some mathematical concepts before proceeding to others. Hence, some topics were not covered as planned. Due to the participant's obligations, the researcher had to stop the sessions to be able to do a post assessment of the covered material.

Finally, the researcher thinks that a study like the one presented in this work will provide better results with more cases. More systematic research is still needed.

*Implications*

Reflecting on the implications of this study, the following needs to be underscored:

- This approach i.e. using the pilot model enabled the researcher to obtain insightful and follow-up details of the precise improvement of MC's mathematical cognitive level.

- Satisfactory achievement can be attained when lessons and remediation are designed in terms of developmental instructional strategies, to meet the needs of students with learning difficulties.

- Regular math programs should be revised taking into consideration students with special needs.

- Program content should be set in a way to prepare students for using math at a later point in their lives to solve every day problems. Moreover, a major part of the curriculum should include math-related life skills. These involve concepts like estimation, measurement, time, and monetary skills.

The above-mentioned recommendations concern ways in which Lebanon can keep up with international changes and enhance the mathematical level of Lebanese students with math difficulties. There is an urgent need for a math curriculum that would meet those students' needs and help them learn the math they will need in their everyday life and at their work place as adults. There should be a campaign to raise awareness about the plight of these kids so we can improve their condition and make a difference in their lives.

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# APPENDIX A

## COGNITIVE AND EDUCATIONAL EVALUATION

Name: MC
Date of Birth: 01/01/1979
Age: 28 years, 6 months
Sex: Female
Dates of Testing: 05/26/2007
06/29/2007

School: LAU
Grade: 16.9
Examiner: Katia Hazoury

### TESTS ADMINISTERED

*WJ III Tests of Cognitive Abilities* (administered on 05/26/2007)
*WJ III Tests of Achievement* (administered on 06/29/2007)

These tests provide measures of MC's overall intellectual ability, specific cognitive abilities, and academic achievement. Relative strengths and weaknesses among her cognitive abilities are described in this report. A description of each ability is provided. MC's performance in each broad category is compared to age peers using a standard score range. Her proficiency is described categorically, ranging from very limited to average; MC's test performance can be generalized to similar, non-test, age-level tasks.

### INTELLECTUAL ABILITY

MC's general intellectual ability, as measured by the WJ III GIA score, is in the low average range of others her age. There is a 68% probability that her true GIA score would be included in the range of scores from 80 to 82.

### COGNITIVE ABILITIES

When compared to others of her age, MC's cognitive abilities are in the average range in Visual-Spatial Thinking (the ability to perceive, analyze, synthesize, and think with visual patterns, including the ability to store and recall visual representations).

Cognitive Efficiency is an index of MC's ability to process information automatically. MC's cognitive efficiency standard score is within the low average range (percentile rank range of 11 to 19; standard score range of 81 to 87) for her age. Her automatic cognitive processing is limited; she will probably find similar age-level tasks very difficult.

Thinking Ability is an overall measure of different thinking processes that MC could use when information in her short-term memory cannot be processed automatically. Although MC's overall

Thinking Ability standard score is within the low average range, her performance varied on two different types of thinking tasks. MC's performance is average on tasks involving synthesis of speech. Her performance is limited on tasks requiring associative and meaningful memory.

Comprehension-Knowledge is a measure of the breadth and depth of MC's language-based knowledge. It includes the ability to verbally communicate her verbal knowledge and comprehension. MC's comprehension-knowledge standard score is within the low average range (percentile rank range of 11 to 16; standard score range of 82 to 85) for her age. Her verbal knowledge and comprehension is very limited; it is likely that she will find age-level verbal communication, knowledge, and comprehension tasks extremely difficult.

Verbal Ability is a measure of MC's language development that includes the comprehension of individual words and relationships among words. MC's verbal standard score is within the low to low average range (percentile rank range of 7 to 12; standard score range of 78 to 83) for her age. Her word knowledge and comprehension is very limited; this suggests that she will find age-level verbal communication, knowledge, and comprehension tasks extremely difficult.

Long-Term Retrieval is the ability to store and retrieve information. Although MC's overall long-term retrieval standard score is within the low average range, she performed differently on two types of storage and retrieval tasks. MC's performance is average on tasks requiring fluent retrieval of previously-learned information. Her performance is limited on tasks requiring associative and meaningful memory.

Fluid Reasoning is the ability to reason, form concepts, and solve problems using unfamiliar information or novel procedures. MC's fluid reasoning standard score is within the low range (percentile rank range of 5 to 8; standard score range of 75 to 79) for her age. Her fluid reasoning ability is very limited; it is predicted that she will find age-level tasks requiring identifying categories and relations, drawing and generalizing inferences, recognizing and forming concepts, and drawing conclusions extremely difficult.

ACHIEVEMENT

Math Calculation Skills measures MC's computational skills and automaticity with basic math facts. MC's mathematics calculation skills standard score is within the low to low average range (percentile rank range of 8 to 15; standard score range of 79 to 84) when compared to others of her age. Her mathematics calculation skills are limited; math calculation tasks above the age 13-6 level will be quite difficult for her.

Mathematics Reasoning includes mathematical knowledge and reasoning. Although MC's overall mathematics reasoning standard score is within the low average range, her performance was different for two types of math reasoning tasks. MC's performance is limited on tasks requiring knowledge of mathematical concepts, symbols, and vocabulary. Her performance is negligible on tasks requiring the ability to analyze and solve applied mathematics problems.

## SUMMARY

MC's overall intellectual ability, as measured by the WJ III GIA (Std), is in the low average range.

MC's thinking ability (intentional cognitive processing) is in the low average range when compared to others at her age level. Her cognitive efficiency (automatic cognitive processing) is also in the low average range. No discrepancies were found among MC's cognitive abilities.

When compared to others at her age level, MC's performance is average in visual-spatial thinking; low average in comprehension-knowledge and long-term retrieval; and low in fluid reasoning.

When compared to others at her age level, MC's performance is low average in math calculation skills and math reasoning.

To help determine if any ability/achievement discrepancies exist, comparisons were made between MC's cognitive and achievement scores. No significant discrepancies were found between MC's overall intellectual ability and her measured mathematics achievement. Based on a mix of cognitive tasks associated with performance in each area, MC is performing at predicted levels in mathematics.

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TABLE OF SCORES: *Woodcock-Johnson III Tests of Cognitive Abilities and Tests of Achievement*
Report Writer for the WJ III, Version 1.1
COG norms based on age 28; ACH norms based on age 28

| CLUSTER/Test | Raw | AE | EASY to DIFF | | RPI | PR | SS(68% BAND) | GE |
|---|---|---|---|---|---|---|---|---|
| GIA (Std) | - | *9-6* | *8-0* | *11-11* | *26/90* | *10* | *81 (80-82)* | *4.2* |
| VERBAL ABILITY (Ext) | - | *10-10* | *9-1* | *13-1* | *13/90* | *13* | *83 (82-85)* | *5.4* |
| THINKING ABILITY (Std) | - | *8-5* | *6-9* | *11-8* | *54/90* | *13* | *83 (81-85)* | *2.9* |
| COG EFFICIENCY (Std) | - | *11-8* | *10-4* | *13-5* | *30/90* | *15* | *84 (81-87)* | *6.3* |
| COMP-KNOWLEDGE (Gc) | - | *10-10* | *9-1* | *13-1* | *13/90* | *13* | *83 (82-85)* | *5.4* |
| L-T RETRIEVAL (Glr) | - | *7-4* | *5-3* | *12-9* | *70/90* | *9* | *80 (78-82)* | *1.9* |
| VIS-SPATIAL THINK (Gv) | - | *19* | *9-5* | *>25* | *89/90* | *47* | *99 (95-103)* | *11.4* |
| FLUID REASONING (Gf) | - | *7-5* | *6-7* | *8-8* | *16/90* | *6* | *77 (75-79)* | *2.1* |
| BROAD MATH | - | *10-9* | *9-6* | *12-4* | *22/90* | *7* | *77 (76-79)* | *5.2* |
| MATH CALC SKILLS | - | *11-4* | *9-9* | *13-6* | *50/90* | *11* | *82 (79-84)* | *5.8* |
| MATH REASONING | - | *11-1* | *9-11* | *12-7* | *15/90* | *9* | *80 (78-82)* | *5.4* |
| Verbal Comprehension | - | *10-7* | *8-11* | *12-8* | *11/90* | *9* | *80 (78-83)* | *5.0* |
| Visual-Auditory Learning | *32-D* | *5-7* | *4-6* | *7-0* | *34/90* | *3* | *71 (68-73)* | *K.5* |
| Spatial Relations | *70-D* | *16-5* | *8-8* | *>25* | *88/90* | *44* | *98 (93-102)* | *9.6* |
| Sound Blending | *24* | *21* | *12-11* | *>26* | *89/90* | *49* | *99 (97-102)* | *12.9* |
| Concept Formation | *11-C* | *6-8* | *5-10* | *7-6* | *6/90* | *3* | *72 (69-75)* | *1.5* |
| Visual Matching | *47-2* | *13-1* | *11-10* | *14-11* | *47/90* | *28* | *91 (89-94)* | *7.9* |
| Numbers Reversed | *11* | *9-3* | *7-10* | *11-5* | *18/90* | *12* | *82 (78-86)* | *3.8* |
| General Information | - | *11-2* | *9-4* | *13-7* | *14/90* | *14* | *84 (81-86)* | *5.8* |
| Retrieval Fluency | *95* | *>30* | *8-4* | *>30* | *91/90* | *64* | *106 (101-111)* | *>18.0* |
| Picture Recognition | *52-D* | *24* | *10-3* | *>25* | *90/90* | *51* | *100 (94-106)* | *13.0* |
| Analysis-Synthesis | *20-D* | *8-6* | *7-4* | *10-10* | *35/90* | *16* | *85 (82-88)* | *3.2* |

Form A of the following achievement tests was administered:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Calculation | *18* | *10-5* | *9-5* | *11-10* | *34/90* | *11* | *81 (78-85)* | *4.9* |
| Math Fluency | *92* | *13-4* | *10-8* | *17-9* | *67/90* | *15* | *85 (83-86)* | *7.9* |
| Applied Problems | *33* | *10-1* | *9-3* | *11-1* | *2/90* | *5* | *75 (72-77)* | *4.4* |
| Quantitative Concepts | - | *13-0* | *11-2* | *15-4* | *62/90* | *27* | *91 (88-94)* | *7.0* |

| | STANDARD SCORES | | | DISCREPANCY | | Significant at |
|---|---|---|---|---|---|---|
| DISCREPANCIES | Actual | Predicted | Difference | PR | SD | + or - 1.50 SD (SEE) |
| *Intra-Cognitive* | | | | | | |
| VERBAL ABILITY (Std) | *80* | *89* | *-9* | *17* | *-0.96* | *No* |
| THINKING ABILITY (Std) | *83* | *85* | *-2* | *42* | *-0.19* | *No* |
| COG EFFICIENCY (Std) | *84* | *88* | *-4* | *35* | *-0.37* | *No* |

| | STANDARD SCORES | | | DISCREPANCY | | Significant at |
|---|---|---|---|---|---|---|
| DISCREPANCIES | Actual | Predicted | Difference | PR | SD | + or - 1.50 SD (SEE) |
| *Intellectual Ability/Achievement Discrepancies** | | | | | | |
| BROAD MATH | *77* | *85* | *-8* | *18* | *-0.90* | *No* |
| MATH CALC SKILLS | *82* | *87* | *-5* | *30* | *-0.52* | *No* |

| STANDARD SCORES | DISCREPANCY | | | Significant at | | |
|---|---|---|---|---|---|---|
| DISCREPANCIES | Actual | Predicted | Difference | PR | SD | + or - 1.50 SD (SEE) |
| *Intellectual Ability/Achievement Discrepancies** | | | | | | |
| BROAD MATH | 77 | 85 | -8 | 18 | -0.90 | No |
| MATH CALC SKILLS | 82 | 87 | -5 | 30 | -0.52 | No |
| MATH REASONING | 80 | 84 | -4 | 32 | -0.46 | No |

**These discrepancies compare GIA (Std) with Broad, Basic, and Applied ACH clusters.*

| | STANDARD SCORES | | | DISCREPANCY | | Significant at |
|---|---|---|---|---|---|---|
| DISCREPANCIES | Actual | Predicted | Difference | PR | SD | + or - 1.50 SD (SEE) |
| *Predicted Achievement/Achievement Discrepancies** | | | | | | |
| BROAD MATH | 77 | 82 | -5 | 33 | -0.44 | No |
| MATH CALC SKILLS | 82 | 85 | -3 | 40 | -0.26 | No |
| MATH REASONING | 80 | 80 | 0 | 50 | -0.01 | No |

**These discrepancies compare predicted achievement scores with Broad, Basic, and Applied ACH clusters.*

Name: MC
Date of Birth: 01/01/1979
Age: 29 years, 2 months
Sex: Female
Date of Testing: 02/22/2008

School: LAU
Examiner: A. Oueini

TESTS ADMINISTERED

*WJ III Normative Update Tests of Achievement*

SUMMARY OF STANDARD SCORES

When compared to others at her age level, MC's standard scores are average in broad mathematics, math calculation skills, and brief mathematics.

TABLE OF SCORES
*Woodcock-Johnson III Normative Update Tests of Achievement (Form A)*
WJ III NU Compuscore and Profiles Program, Version 3.0
Norms based on age 29-2

| CLUSTER/Test | Raw | W | AE | Development | RPI | SS (68% Band) | GE |
|---|---|---|---|---|---|---|---|
| BROAD MATH | - | 529 | 17-5 | age-approp | 86/90 | 97 (96-99) | 11.9 |
| BRIEF MATH | - | 537 | 18-3 | age-approp | 86/90 | 98 (96-100) | 12.6 |
| MATH CALC SKILLS | - | 517 | 13-5 | mild del-app | 75/90 | 90 (87-93) | 8.0 |
| Calculation | 23 | 520 | 12-4 | mild delayed | 63/90 | 87 (82-92) | 6.9 |
| Math Fluency | 108 | 514 | 15-10 | age-approp | 85/90 | 94 (92-96) | 10.3 |
| Applied Problems | 53 | 554 | >30 | approp to adv | 96/90 | 105 (102-109) | 16.9 |

# APPENDIX B

## Sample Exercises from Sessions

## Problem solving

a) Rawad has 10825 L.L. He buys 2 sandwiches each for 2750 L.L. , and 3 drinks each for 1150 L.L.

- How much money does he have left?

~~6,925~~ 1,875

With the money left, how many chocolate bars can he buy if each one costs 85 L.L.?

22 chocolates

b) In grade 5, there are 27 students.
Can their Math teacher seat them in groups of 2? of 3? of 4?
Explain your answer.

~~9~~ 27 ÷ 3 = 9 groups of 3

Bonus

30, 36, 42, 48, 54

I am between 25 and 55.
If you subtract 5 from me, I'll be divisible by 6.
If you add 4 to me, I'll be divisible by 5.
Who am I?

__________

Complete

$0 \times 6 = 0$ $\quad$ $0 + 6 = 6$

$0 - 6 = 6$ $\quad$ $6 - 0 = 6$

$6 \div 0 = 0$ $\quad$ $0 \div 6 = 0$

$0 + 7 = 7$ $\quad$ $7 + 0 = 7$

$0 \times 7 = 0$ $\quad$ $7 \times 0 = 0$

$0 - 7 = 7$ $\quad$ $7 - 0 = 7$

$7 \div 0 = 0$ $\quad$ $0 \div 7 = 0$

$1 \times 0 = 1$ $\quad$ $0 \times 1 = 0$

$1 \div 0 = 0$ $\quad$ $0 \div 1 = 0$

$1 + 0 = 1$ $\quad$ $0 + 1 = 1$

$1 + 7 = 8$ $\quad$ $7 + 1 = 8$

$1 - 0 = 1$ $\quad$ $0 - 1 = 1$

$1 - 7 = 6$ $\quad$ $7 - 1 = 6$

$1 \times 7 = 7$ $\quad$ $7 \times 1 = 7$

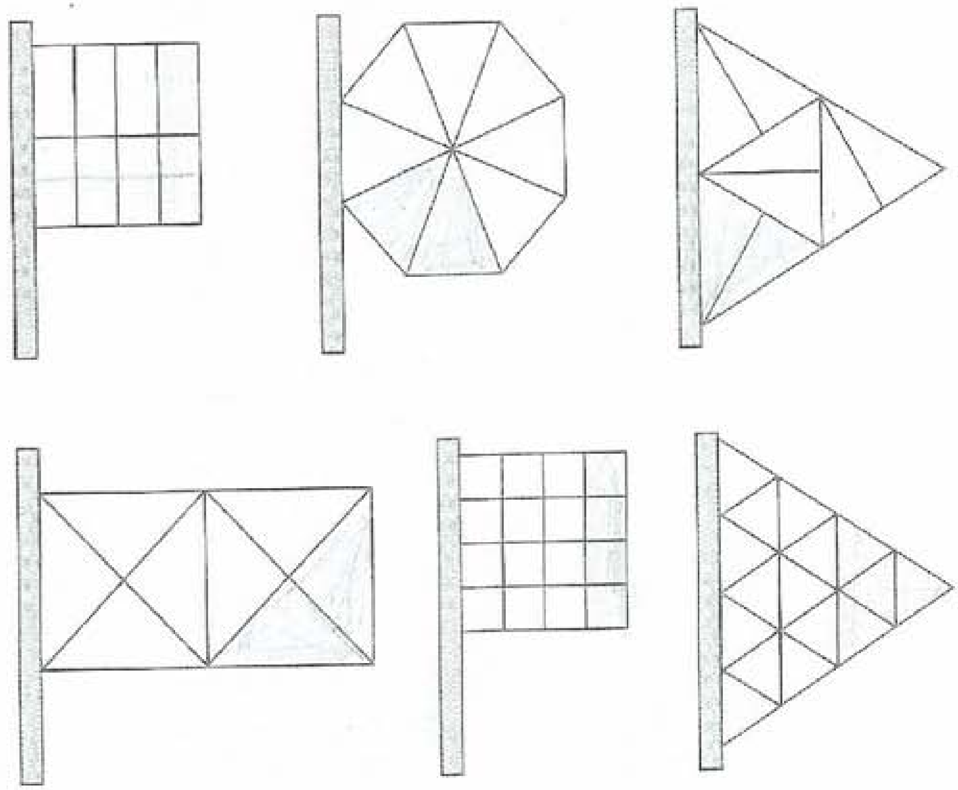
$7 \div 1 = 7$ $\quad$ $1 \div 7 = 0.14$

$7 + 7 = 14$ $\quad$ $7 - 7 = 0$

$7 \times 7 = 49$ $\quad$ $7 \div 7 = 1$

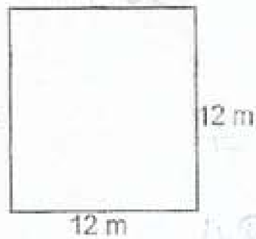
## Flags

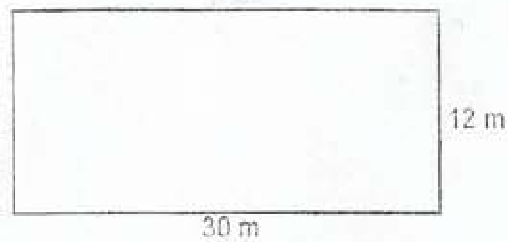
**A** Colour a quarter of each flag.



**B** Draw your own flag.
Colour a quarter of the flag.

1- Find the perimeter and area of each of the following figures:



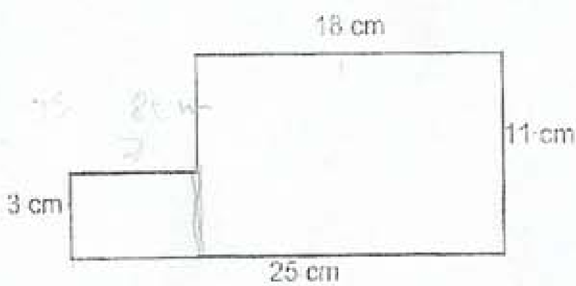




P = 48 m

A = 144 m²

P = 84 m

A = 360 m²




P = 72 cm

A = 198 + 21 = 219 cm²

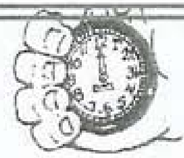
2- Draw a rectangle having a perimeter of 18 cm.

- Find its length and width.
- Find its area.

____________________

____________________

## Speedy sums



How fast are you at answering sums?
Time yourself answering these.

**A**

| 1 | 2 | 3 |
| --- | --- | --- |
| 3 × 4 = 12 | 6 × 10 = 60 | 2 × 2 = 4 |
| 4 × 5 = 20 | 3 × 5 = 15 | 3 × 4 = 12 |
| 5 × 7 = 25 | 7 × 3 = 27 | 5 × 4 = 20 |
| 2 × 7 = 14 | 6 × 2 = 12 | 5 × 10 = 50 |
| 5 × 5 = 25 | 8 × 5 = 40 | 4 × 4 = 16 |
| 10 × 8 = 80 | 9 × 4 = 36 | 3 × 1 = 3 |
| 3 × 6 = 18 | 9 × 10 = 90 | 2 × 4 = 8 |
| 2 × 3 = 6 | 3 × 2 = 6 | 2 × 10 = 20 |
| 10 × 4 = 40 | 0 × 5 = 0 | 10 × 10 = 100 |
| 3 × 0 = 0 | 8 × 2 = 16 | 5 × 2 = 10 |
| 3 × 9 = 27 | 7 × 10 = 70 | 4 × 10 = 40 |
| 4 × 7 = 28 | 6 × 5 = 30 | 4 × 6 = 24 |
| 5 × 9 = 45 | 8 × 4 = 32 | 2 × 5 = 10 |
| 3 × 8 = 24 | 10 × 3 = 30 | 4 × 9 = 36 |
| 2 × 9 = 18 | 3 × 3 = 9 | 3 × 7 = 21 |

**B**

| 1 | 2 | 3 |
| --- | --- | --- |
| 20 ÷ 2 = 10 | 80 ÷ 10 = 8 | 100 ÷ 10 = 10 |
| 6 ÷ 3 = 2 | 45 ÷ 5 = 9 | 20 ÷ 2 = 10 |
| 4 ÷ 2 = 2 | 16 ÷ 4 = 4 | 6 ÷ 2 = 3 |
| 27 ÷ 3 = 9 | 12 ÷ 3 = 4 | 15 ÷ 3 = 5 |
| 32 ÷ 4 = 8 | 10 ÷ 2 = 5 | 30 ÷ 5 = 6 |
| 25 ÷ 5 = 5 | 28 ÷ 4 = 7 | 20 ÷ 4 = 5 |
| 70 ÷ 10 = 7 | 90 ÷ 10 = 9 | 60 ÷ 10 = 6 |
| 16 ÷ 2 = 8 | 20 ÷ 5 = 4 | 5 ÷ 5 = 1 |
| 8 ÷ 4 = 2 | 4 ÷ 4 = 1 | 24 ÷ 3 = 8 |
| 3 ÷ 3 = 1 | 18 ÷ 3 = 6 | 36 ÷ 4 = 9 |
| 21 ÷ 3 = 7 | 14 ÷ 2 = 7 | 10 ÷ 5 = 2 |
| 12 ÷ 2 = 6 | 40 ÷ 4 = 10 | 18 ÷ 2 = 9 |
| 50 ÷ 10 = 5 | 15 ÷ 5 = 3 | 40 ÷ 5 = 8 |
| 35 ÷ 5 = 7 | 9 ÷ 3 = 3 | 8 ÷ 2 = 4 |
| 24 ÷ 4 = 6 | 30 ÷ 5 = 6 | 30 ÷ 3 = 10 |

# APPENDIX C

**Letter from the Father of Participant**

**To Whom It May Concern**

I am the father of MC. I have known MC to have had difficulty in mathematics through her early childhood and later in her elementary and secondary education. Only once she had a break when she was fortunate enough to have received special attention and had for an educator Mr. Simhari ( math. teacher at Eastwood College) during her last two high school years and managed to graduate with passing grades in Mathematics.

Recently during her graduate study year MC faced again some difficult times struggling with mathematics, and I got to a point I thought my daughter will never make it when it comes to calculating and problem solving. Once again she was fortunate to have had Ms. Rula Bayram to support her in her math requirements. I watched MC over a relatively short period (4 weeks) change from a completely lost person when it comes to counting percentages and calculating simple mathematical operations to a confident and much improved student ready to solve and handle practical day to day math operations. This has convinced me that the educational approach which was used by Ms. Bayram was the key factor in assisting MC to improve sufficiently to continue in her future calculating needs.

I would like to take this opportunity to thank and congratulate Ms. Bayram for being able to use that special approach for passing information like mathematics to many students like my daughter who badly need that special methodology. This proves that teaching techniques are instrumental in developing and building a student's knowledge particularly in complicated subjects such as mathematics.

# APPENDIX D

### Letter from the Participant

Dear Dr. Oueini

Before I started the math lessons with Rola, I expected not to understand anything because of my weak basics in Math. Rola was able to help me right from the beginning; she used manipulative techniques, and explained certain topics in Math numerous times if I did not understand the concept. Rola was patient throughout all the sessions.

I was diagnosed with ADHD as a child and I was reassessed again as an adult. The methods my math teachers used when I was a child were not helpful to me at all. I have failed Math all the way through high school and college, until Rola intercepted. I am now able to add, divide, multiply and round numbers in my head. I am able to understand and solve basic problem solving. I was never able to understand word problems no matter how the teacher explained it. During the course of research methods I found great difficulty in understanding basic statistics, but with the help of Rola I am now able to improve.

Rola used certain techniques and methods that helped me understand and solve Mathematical problems. I have always avoided Math, but now I feel I can calculate numbers in my head without counting the numbers using my fingers, which can be embarrassing in public at my age. I would like to thank Rola for all her help, support, and patience, and with constant practice and problem solving at home, I hope to be able to recall certain math techniques and solve problems on my own. While I was taking the post-test, I did not fear the word problems, I was at complete ease. During the part of Math fluency I completed a page and a half in 3 minutes. Last year when I did the Math fluency part I completed half a page in 3 minutes. I would like to thank Rola for helping me go through process.