A Scalable Family of Unitary and Differential Space-Time Codes for UWB Communications

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***Abstract*—In this paper, we consider the problem of differential space-time (ST) block coding in the context of impulse-radio ultra-wideband (IR-UWB) communications with two transmit antennas and Pulse Position Modulation (PPM). We propose a novel family of codes where the information is encoded differentially through the number of transmitted pulses, the positions of these pulses and the locations of the non-idle symbol durations within each ST block. In order to maintain a low complexity of the UWB transmitter, unipolar transmissions are ensured by avoiding all forms of amplitude-domain encoding. The proposed scheme provides a full scalability capability where depending on the value taken by the scale parameter Θ, different levels of compromise between data rates and error rates (along with the decoding complexity) can be achieved. Finally, the proposed scheme lends itself to optimal detection with cross-correlation receivers that can be implemented in the analog domain in the absence of all forms of channel state information at the transmitter and receiver sides.**

***Index Terms*—Ultra-Wideband, UWB, differential, space-time, Pulse Position Modulation, PPM, unipolar, scalable.**

I. Introduction

There has been a growing interest in impulse-radio ultra-wideband (IR-UWB) communications as a strong candidate solution for short range high data rate applications. Following from the high frequency selectivity of the UWB channels, the transmitted signal energy is spread over a very large number of resolvable multi-path components. This fact renders the coherent solutions, that are based on estimating and combining a number of multi-path components, extremely complex. Consequently, suboptimal noncoherent detectors were proposed as simple alternatives that bypass the tedious UWB channel estimation task. Noncoherent solutions include differential modulation [1], [2], transmitted reference communications [3]–[5] and energy-based detection [6]–[8]. On the other hand, multiple-input-multiple-output (MIMO) techniques have been extensively studied as powerful tools for leveraging the range, reliability and data rate of IR-UWB systems [9]–[15]. While most research in this direction targeted coherent space-time (ST) coding [9]–[11], differential MIMO-UWB systems were studied in [12], [13] while transmitted reference and energy-based MIMO-UWB systems were proposed in [14] and [15], respectively.

The existing differential single-antenna systems [1], [2] and MIMO systems [12], [13] are based on differential binary phase shift keying (DBPSK) where the information is encoded in the polarity inversion of the pulse (resp. pulses) in one symbol duration (resp. ST block) with respect to the previous symbol duration (resp. ST block) in the case of single-antenna (resp. MIMO) systems. Given the high cost of the UWB generators that transmit pulses with very low duty cycles, such amplitude-domain encoding is not desirable and time-domain encoding is often preferred since it is easier to control the positions of the sub-nanosecond UWB pulses. In this context, pulse position modulation (PPM) is an appealing modulation scheme for IR-UWB systems.

This work revolves around differential MIMO-UWB ST coding in the case where the transmitter is equipped with two antennas. Unlike the existing differential MIMO-UWB schemes [12], [13] that require the transmission of more than one amplitude level, the proposed ST code is unipolar where all the transmitted pulses have the same amplitude. In this context, a single pulse generator is required at the transmitter and all forms of amplitude-domain encoding are avoided. In a more detailed manner, the proposed ST code encodes the data differentially in the time domain through (i): the relative shifts of the PPM slots within a symbol duration, (ii): the relative shifts of the symbol durations within a ST block and (iii): the number of UWB pulses per symbol. The proposed family of codes is scalable via a parameter that is denoted by Θ that can assume any value between 1 and $M-1$ where M stands for the number of PPM positions. While the cardinality of the proposed codebook increases with Θ, large values of Θ incur a higher decoding complexity that is associated with some performance losses. In this context, the value of Θ can be adjusted to meet the system requirements in terms of data rate, error rate and complexity. The proposed scheme involves the transmission of up to Θ pulses per symbol. Consequently, for $\Theta = 1$ the proposed code is shape-preserving with PPM where exactly one UWB pulse is transmitted per symbol in a way that is completely analogous to single-antenna PPM systems. On the other hand, for $\Theta > 1$ the proposed scheme incurs a constellation expansion of the PPM signal set while always respecting the constraint of unipolar transmissions. In addition

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to being unipolar and scalable, the proposed scheme profits from a number of desirable features as follows. The proposed code is fully-diverse with M-PPM constellations for all values of M. Moreover, the receiver can be implemented in the analog domain avoiding all forms of Nyquist-rate sampling. In this context, maximum-likelihood (ML) detection is based on cross-correlation receivers that correlate the signal received in a certain ST block with the signal received in the previous block in a way that is completely analogous to differential single-antenna systems based on DBPSK. Finally, it is worth noting that despite the numerous families of differential ST codes in the narrow-band context (for example, refer to [16] and the references therein), these codes can not be applied in the considered scenario since they are not adapted to PPM.

II. Transmission and Encoding

A. General System Parameters

Consider a single-user MIMO time-hopping (TH) UWB system where the transmitter is equipped with $P = 2$ antennas and the receiver is equipped with Q antennas. M-ary PPM is deployed where the information symbols are represented by M-dimensional vectors carved from the following set:

$$\mathcal{S}_{\text{PPM}} = \{e_m \ ; \ m = 1, \ldots, M\} \tag{1}$$

where e_m stands for the m-th column of the $M \times M$ identity matrix I_M.

Since single-user transmissions are assumed, no reference to the pseudo-random TH sequence is made in what follows. The symbol duration T_s is divided into M PPM slots each of duration δ. In order to avoid inter-pulse interference that follows from the excessive delay spreads of the UWB channels, the PPM duration δ is chosen to be larger than the channel delay spread..

B. ST Codebook

The proposed ST scheme is a block scheme where each encoded block extends over two symbol durations. The data will be encoded in a differential manner and this data will be reconstructed at the receiver side by comparing the $(k-1)$-th and k-th blocks. Denote by $S^{(k)}$ the ST codeword transmitted in the k-th block. This codeword is a $(2M \times 2)$-dimensional matrix whose $((i-1)M + m, p)$-th element stands for the amplitude of the pulse (if any) transmitted by p-th transmit antenna during the m-th PPM slot of the i-th symbol duration of the k-th block for $p = 1, 2$, $i = 1, 2$ and $m = 1, \ldots, M$.

The proposed family of ST codes can be parameterized by the integer Θ that can take any value in the set $\{1, \ldots, M-1\}$. For a given selected value of Θ, the codebook will comprise a total of $2M\Theta$ codewords. For scalability and enumeration purposes, the codebook $\mathcal{C}$ will be written as the union of Θ sub-codebooks: $\mathcal{C} = \bigcup_{\theta=0}^{\Theta-1} \mathcal{C}^{(\theta)}$ where the sub-codebook $\mathcal{C}^{(\theta)}$ contains $2M$ codewords denoted by $C_{0,0}^{(\theta)}, C_{0,1}^{(\theta)}, \ldots, C_{M-1,0}^{(\theta)}, \ldots, C_{M-1,1}^{(\theta)}$.

The differential encoding scheme is as follows. In order to differentially encode the information symbol $\Delta_k \in \{0, ..., 2M\Theta - 1\}$, the codeword transmitted in the k-th block takes the following form:

$$S^{(k)} = C_{m_k,u_k}^{(\theta_k)} \tag{2}$$

where the integers θ_k, m_k and u_k are determined in two steps as follows.

- *Step 1*: Find the integer c_k such that:

$$c_k = c_{k-1} + \Delta_k \mod 2M\Theta \tag{3}$$

  where $c_0 = 0$ stands for the index of the reference codeword. In (3), c_k stands for the index of the codeword transmitted in the k-th block that depends on the index of the codeword in the previous block and on the current information symbol.
- *Step 2*: Write the integer c_k under the following form:

$$c_k = 2M\theta_k + 2m_k + u_k \tag{4}$$

  where $\theta_k \in \{0, \ldots, \Theta-1\}$ stands for the index of the sub-codebook while $m_k \in \{0, \ldots, M-1\}$ and $u_k \in \{0, 1\}$ stand for the indices of the codeword in the corresponding sub-codebook.

The codewords are constructed as follows:

$$C_{m,u}^{(\theta)} = \frac{1}{\sqrt{\theta+1}} \begin{bmatrix} \sum\limits_{i=0}^{\theta} \Omega^{m+i} e_1 & \mathbf{O}_M \\ \mathbf{O}_M & \sum\limits_{i=0}^{\theta} \Omega^{m+i} e_1 \end{bmatrix} \ ; \ u \text{ even} \tag{5}$$

and:

$$C_{m,u}^{(\theta)} = \frac{1}{\sqrt{\theta+1}} \begin{bmatrix} \mathbf{O}_M & \sum\limits_{i=0}^{\theta} \Omega^{m+1+i} e_1 \\ \sum\limits_{i=0}^{\theta} \Omega^{m+i} e_1 & \mathbf{O}_M \end{bmatrix} \ ; \ u \text{ odd} \tag{6}$$

where $\mathbf{O}_M$ stands for the M-dimensional all-zero vector while Ω is the $M \times M$ cyclic permutation matrix given by:

$$\Omega = \begin{bmatrix} \mathbf{O}_{M-1}^T & 1 \\ I_{M-1} & \mathbf{O}_{M-1} \end{bmatrix} \tag{7}$$

Equations (5)-(6) show that the codeword $C_{m,u}^{(\theta)}$ corresponds to the transmission of $\theta + 1$ unipolar UWB pulses per symbol where the normalization by $\sqrt{\theta+1}$ ensures the same transmit power as in single-antenna systems. Moreover, the elements of the codewords (excluding the normalization) can be equal to either 0 or 1 indicating the absence or presence of an UWB pulse, respectively. In this context, the proposed encoding scheme is unipolar where no polarity inversions or amplitude scalings are required. Moreover, the structure of the permutation matrix in (7) ensures that $\Omega^m e$ is an element of the set $\mathcal{S}_{\text{PPM}}$ in (1) whenever e is an element of this set for any integer value m. Finally, from (5)-(6), it can be observed that the codewords are unitary:

$$\left[C_{m,u}^{(\theta)}\right]^T C_{m,u}^{(\theta)} = I_2 \ ; \ \theta \in \{0, \ldots, \Theta - 1\} \ ; \ m \in \{0, \ldots, M-1\} \ ; \ u \in \{0, 1\} \tag{8}$$

As an example on (5)-(7), for $\Theta = 0$ and $M = 2$, the four codewords are given by: $C_{0,0}^{(0)} = \begin{bmatrix} e_1 & \mathbf{O}_M \\ \mathbf{O}_M & e_1 \end{bmatrix}$, $C_{1,0}^{(0)} = \begin{bmatrix} \Omega e_1 & \mathbf{O}_M \\ \mathbf{O}_M & \Omega e_1 \end{bmatrix}$, $C_{0,1}^{(0)} = \begin{bmatrix} \mathbf{O}_M & \Omega e_1 \\ e_1 & \mathbf{O}_M \end{bmatrix}$ and $C_{1,1}^{(0)} = \begin{bmatrix} \mathbf{O}_M & \Omega^2 e_1 \\ \Omega e_1 & \mathbf{O}_M \end{bmatrix}$ where $\Omega e_1 = e_2$ and $\Omega^2 e_1 = e_1$ highlighting the fact that the code is shape-preserving for $\Theta = 0$.

The main feature of the proposed scheme resides in the fact that the information will be encoded by the relative shifts (or delays) of the UWB pulses in the k-th block with respect to the pulse positions in the $(k-1)$-th block. For example, consider the case $M = 4$ and $\Theta = 2$ and assume that the symbols $\{1, 7, 10, \ldots\}$ are to be encoded differentially. In this case, $c_1 = 1$ implying that $(\theta_1, m_1, u_1) = (0, 0, 1)$ and $S^{(1)} = \begin{bmatrix} \mathbf{O}_4 & e_2 \\ e_1 & \mathbf{O}_4 \end{bmatrix}$; $c_2 = 8$ implying that $(\theta_2, m_2, u_2) = (1, 0, 0)$ and $S^{(2)} = \frac{1}{\sqrt{2}} \begin{bmatrix} e_1 + e_2 & \mathbf{O}_4 \\ \mathbf{O}_4 & e_1 + e_2 \end{bmatrix}$; $c_3 = 2$ implying that $(\theta_3, m_3, u_3) = (0, 1, 0)$ and $S^{(3)} = \begin{bmatrix} e_2 & \mathbf{O}_4 \\ \mathbf{O}_4 & e_2 \end{bmatrix}$.

C. Rate and Scalability

The proposed encoding scheme is a scalable scheme that comprises a total of $2M\Theta$ codewords. Given that each codeword extends over two symbol durations, then the rate of the proposed scheme is given by:

$$R_\Theta = \frac{\log_2(2M\Theta)}{2T_s} \quad \text{bits/s} \tag{9}$$

In this context, the proposed encoding scheme can be applied with any value of Θ in the set $\{1, \ldots, M-1\}$. Evidently, the smallest cardinality is obtained for $\Theta = 1$ while the largest cardinality (and thus the highest rate) can be achieved with $\Theta = M-1$ which offers a rate scalability of the proposed scheme. On the other hand, as will be highlighted in the following section, while the rate increases with Θ, this will incur an increased decoding complexity.

For $\Theta = 1$, the proposed code is shape-preserving with PPM. In other words, it does not introduce any constellation expansion to the PPM signal set where each antenna can transmit only one PPM pulse per symbol in a manner that is completely analogous to single-antenna PPM systems. In this case, the ST codebook takes the following simple form:

$$\mathcal{C} = \left\{ \begin{bmatrix} e_{m+1} & \mathbf{O}_M \\ \mathbf{O}_M & e_{m+1} \end{bmatrix} ; \begin{bmatrix} \mathbf{O}_M & e_{(m+1 \mod M+1)} \\ e_{m+1} & \mathbf{O}_M \end{bmatrix} ; \; m = 0, \ldots, M-1 \right\} \tag{10}$$

where the transmitted symbols are all carved from the PPM signal set in (1).

For $\Theta > 1$, the proposed scheme is unipolar but not PPM shape-preserving where more than one pulse need to be transmitted in consecutive PPM slots.

D. Diversity Order

The proposed code achieves a full transmit diversity order of two based on the rank criterion that is used for the construction of coherent and differential ST codes [17]. In a more formal way, the following proposition holds.

Proposition 1: For any $M \geq 2$ and $\Theta \in \{1, \ldots, M-1\}$, the proposed codewords satisfy the following relation:

$$\text{rank}\left(C_{m,u}^{(\theta)} - C_{m',u'}^{(\theta')} \right) = 2 \quad \text{for} \quad (\theta', m', u') \neq (\theta, m, u) \tag{11}$$

where the above relation holds for all values of θ and θ' in $\{0, \ldots, \Theta-1\}$, of m and m' in $\{0, \ldots, M-1\}$ and of u and u' in $\{0, 1\}$.
Proof: The proof is provided in the appendix.

III. Reception and Decoding

A. Signal Reception

Based on the transmission scheme described in the previous section, the signals transmitted from the transmit antennas can be written as (for $p = 1, 2$):

$$s_p(t) = \sum_{k=0}^{+\infty} \sum_{i=1}^{2} \sum_{m=1}^{M} S_{(i-1)M+m,p}^{(k)} w(t - k2T_s - (i-1)T_s - (m-1)\delta) \tag{12}$$

where $w(t)$ stands for the UWB pulse-shape while $S_{i,j}^{(k)}$ stands for the (i, j)-th element of the transmitted matrix $S^{(k)}$.

The signal received at the q-th receive antenna can be written as:

$$r_q(t) = \sum_{p=1}^{P} s_p(t) * h_{q,p}(t) + n_q(t) \tag{13}$$

where $*$ stands for convolution and $n_q(t)$ stands for the filtered noise at the q-th receive antenna. In (13), $h_{q,p}(t)$ stands for the impulse response of the frequency-selective channel between the p-th transmit antenna and the q-th receive antenna.

The receiver corresponds to a correlation receiver that cross-correlates the $2M$ signals received in the PPM slots of the k-th block with the $2M$ signals received in the PPM slots of the $(k-1)$-th block. This receiver is implemented in the analog domain thus avoiding Nyquist-rate sampling. Moreover, the receiver can be implemented without the knowledge of the channel neither at the receiver nor at the transmitter sides. The decision variables at the q-th receive antenna will be constructed as follows:

$$x_{m,m'}^{(i,i')}(q) = \int_0^{T_i} r_q(t - (k-1)2T_s - (i-1)T_s - (m-1)\delta) \times r_q(t - k2T_s - (i'-1)T_s - (m'-1)\delta) \mathrm{d}t \tag{14}$$

where $r_q(t)$ is given in (13).

The decision variable in (14) corresponds to the comparison between the signal received in the m-th position of the i-th symbol of the $(k-1)$-th block with the signal received in the m'-th position of the i'-th symbol of the k-th block for $i, i' \in \{1, 2\}$ and $m, m' \in \{1, \ldots, M\}$. Finally, T_i stands for the integration time that must take values that are neither very small nor very large in order to collect a sufficient energy of the UWB channel without integrating an excessive amount of noise [1]–[5].

Finally, the decision at the receiver will be based on the following $(2M)^2$ decision variables:

$$x_{m,m'}^{(i,i')} = \sum_{q=1}^{Q} x_{m,m'}^{(i,i')}(q) \ ; \ i,i' \in \{1,2\} \ ; \ m,m' \in \{1,\dots,M\} \tag{15}$$

B. Decoder

We define the cyclic permutation function of order m as:

$$\sigma^{(m)}(\lambda) = (\lambda + m - 1) \mod M + 1 \tag{16}$$

in this context, the vector $\Omega^m e_\lambda$ can be written as $e_{\sigma^{(m)}(\lambda)}$.

In order to offer more insights on the operations performed by the decoder, we first start with the special case of $\Theta = 1$. The corresponding codebook (which is equal to the first sub-codebook in this case) is given in (10).

Now, the decoder decides in favor of $\hat{c}_k = 2\hat{m}_k + \hat{u}_k$ where:

$$(\hat{m}_k, \hat{u}_k) = \arg \max_{\substack{m=0,\dots,M-1 \\ u=0,1}} \{Z_{m,u}\} \tag{17}$$

where $Z_{m,u}$ is determined from the decision variables in (15) as follows.

If $u = 0$, (5) shows that the first transmit antenna is transmitting in the same symbol duration of each of the $(k-1)$-th and k-th blocks and that the same holds for the second transmit antenna. Moreover, the pulses transmitted in both blocks are shifted by m positions. Based on the above, $Z_{m,u}$ can be written as:

$$Z_{m,u} = \sum_{\lambda=1}^{M} \left[x_{\lambda,\sigma^{(m)}(\lambda)}^{(1,1)} + x_{\lambda,\sigma^{(m)}(\lambda)}^{(2,2)} \right] \ ; \ u = 0 \tag{18}$$

If $u = 1$, (6) shows that the first transmit antenna is transmitting in different symbol durations of each of the $(k-1)$-th and k-th blocks and that the same holds for the second transmit antenna. Moreover, if any pulse is transmitted from the first (resp. second) antenna, this pulse will be shifted by m (resp. $m+1$) positions. Consequently, in this case, $Z_{m,u}$ can be written as:

$$Z_{m,u} = \sum_{\lambda=1}^{M} \left[x_{\lambda,\sigma^{(m)}(\lambda)}^{(1,2)} + x_{\lambda,\sigma^{(m+1)}(\lambda)}^{(2,1)} \right] \ ; \ u = 1 \tag{19}$$

Equations (17)-(19) show that the maximum-likelihood decoder needs to select among $2M$ decision variables.

Next, we extend the ML decoder to the general case $\Theta > 1$. In this case, the detection rule will change as a function of the integers θ_{k-1} and θ_k that denote the indices of the sub-codebooks that contain the matrices transmitted in the $(k-1)$-th and k-th blocks, respectively. In fact, if $\theta_k > \theta_{k-1}$, the number of pulses in $S^{(k)}$ will exceed the number of pulses in $S^{(k-1)}$ and vice versa. This difference in the numbers of transmitted pulses will drastically affect the expression of the decision metric for the sake of collecting the entire signal energy in the blocks $k-1$ and k. In this context, and unlike the case $\Theta = 1$, there is no unified decision metric that holds in the two cases $\theta_k > \theta_{k-1}$ and $\theta_k \leq \theta_{k-1}$.

For this general case, the decoder decides in favor of:

$$\hat{c}_k = 2M(\hat{\theta}_k - \hat{\theta}'_k) + 2\hat{m}_k + \hat{u}_k \mod 2M\Theta \tag{20}$$

where:

$$(\hat{\theta}_k, \hat{\theta}'_k, \hat{m}_k, \hat{u}_k) = \arg \max_{\substack{(\theta,\theta') \in \{0,\dots,\Theta-1\}^2 \ ; \ u \in \{0,1\} \\ m \in \{0,\dots,M-1\}}} \{Z_{\theta,\theta',m,u}\} \tag{21}$$

Note that while m and u are relative quantities corresponding to the pulse permutation and codeword structure of the matrix transmitted in the k-th block with respect to the matrix transmitted in the $(k-1)$-th block, the integers θ and θ' are not relative quantities. In this case, θ denotes the index of the sub-codebook of $S^{(k-1)}$ while θ' denotes the index of the sub-codebook of $S^{(k)}$.

In a way similar to (18), the decision variable $Z_{\theta,\theta',m,u}$ takes the following expression for $u = 0$:

$$Z_{\theta,\theta',m,u} = \sum_{i_1=0}^{\theta} \sum_{i_2=0}^{\theta'} \sum_{\lambda=1}^{M} \left[x_{\sigma^{(i_1)}(\lambda),\sigma^{(m+i_2)}(\lambda)}^{(1,1)} + x_{\sigma^{(i_1)}(\lambda),\sigma^{(m+i_2)}(\lambda)}^{(2,2)} \right] \ ; \ u = 0 \tag{22}$$

and the following form for $u = 1$:

$$Z_{\theta,\theta',m,u} = \sum_{i_1=0}^{\theta} \sum_{i_2=0}^{\theta'} \sum_{\lambda=1}^{M} \left[x_{\sigma^{(i_1)}(\lambda),\sigma^{(m+i_2)}(\lambda)}^{(1,2)} + x_{\sigma^{(i_1)}(\lambda),\sigma^{(m+i_2+1)}(\lambda)}^{(2,1)} \right] \ ; \ u = 1 \tag{23}$$

where (22) and (23) follow from the fact that, for a given PPM position λ, the pulses will occupy the positions $\{\sigma^{(i_1)}(\lambda)\}_{i_1=0}^{\theta}$ in the $(k-1)$-th block and the positions $\{\sigma^{(i_2)}(\lambda)\}_{i_2=0}^{\theta'}$ in the k-th block. Moreover, in (22)-(23), the following relation is invoked $\sigma^{(m)}\left(\sigma^{(i_2)}(\lambda)\right) = \sigma^{(m+i_2)}(\lambda)$.

Equations (20)-(23) show that $2M\Theta^2$ possibilities need to be checked where this number reduces to $2M$ (the decoding complexity in (17)-(19)) for $\Theta = 1$. Note that (20)-(23) reduce to (17)-(19) for the special case of $\Theta = 1$ where in this case $\theta = \theta' = 0$.

IV. Simulations and Results

In this section we present some numerical results that show the variations of the symbol error rate (SER) as a function of the signal-to-noise ratio (SNR) per information bit. For the proposed scheme, $\frac{\log_2(2M\Theta)}{2}$ information bits are transmitted per symbol duration while for the single-antenna systems $\log_2(M)$ information bits are transmitted per symbol duration. The UWB channels between the different transmit and receive antennas are generated independently according to the IEEE 802.15.3a NLOS channel model recommendation CM2 [18]. A Gaussian pulse with a duration of $T_w = 0.5$ ns is used and the modulation delay is set to $\delta = 100$ ns in order to eliminate the interference between the different PPM slots. In this context,

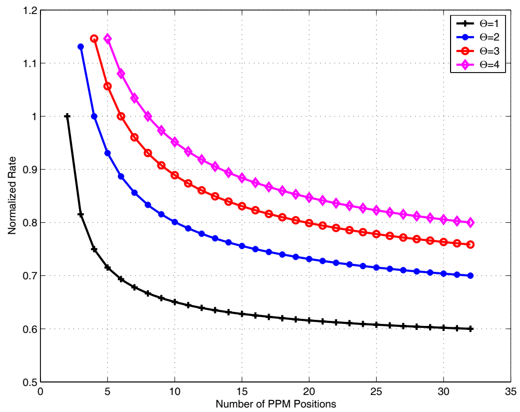

Fig. 1. Normalized rate with respect to single-antenna systems.

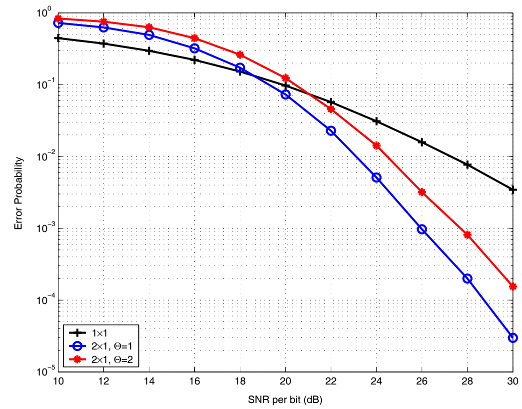

Fig. 3. Performance with 3-PPM for $T_i = 5$ ns.

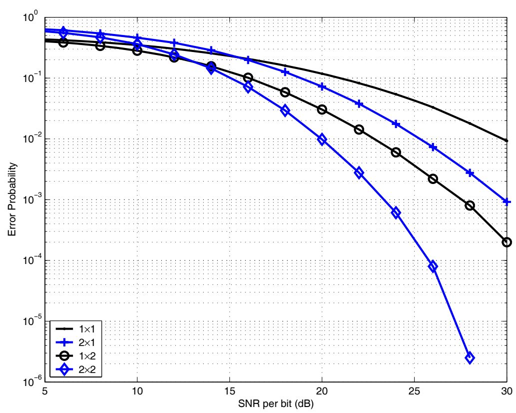

Fig. 2. Performance with 2-PPM for $T_i = 1$ ns.

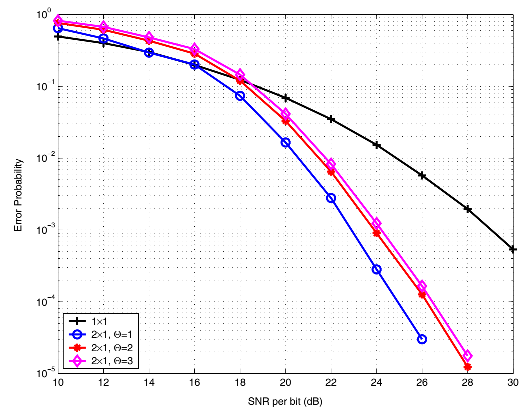

Fig. 4. Performance with 4-PPM for $T_i = 10$ ns.

$T_s = M\delta$ increases with the order of the PPM modulation in order to avoid interference. Note that the integration time T_i can be chosen independently form M and T_s.

Fig. 1 compares the normalized rates of the proposed scheme for different values of Θ. The normalized rate is equal to the ratio between the rate of the MIMO system given in (9) and the rate of the single-antenna system that is equal to $\frac{\log_2(M)}{T_s}$. Results show that this normalized rate is always less than or equal to 1 for the case $\Theta = 1$ implying that the code of order 1 will incur a data rate reduction with respect to single-antenna systems. On the other hand, for $\Theta > 1$, the normalized rate can exceed 1 especially for small values of M. Finally, the normalized rate is a decreasing function of M for any value of Θ and, consequently, it is more advantageous to apply the proposed scheme with small values of M.

Fig. 2 shows the performance with $M = 2$ for an integration time of $T_i = 1$ ns. In this case, the only possible value of Θ is $\Theta = 1$ and the rate of the proposed scheme is the same as the rate of the single-antenna systems. Results show the enhanced performance levels and diversity orders that can be achieved by the proposed scheme. With one receive antenna, the performance gain is about 4.5 dB at an error rate of 10^{-2}. With two receive antennas, the performance gain is about 4 dB at an error rate of 10^{-3}. The error curves of the 1×2 and 2×1 systems are parallel to each other indicating that the diversity orders of these systems are the same. This shows that the proposed scheme achieves a full transmit diversity order.

Fig. 3 shows the performance with $M = 3$ and $T_i = 5$ ns. The two codes obtained for $\Theta = 1$ and $\Theta = 2$ are compared. Results show that significant SER improvements can be observed for average-to-large value of the SNR. While the code obtained for $\Theta = 2$ transmits at a higher rate (it comprises twice the number of codewords compared to the case $\Theta = 1$), this comes at the expense of some performance losses. In particular, results in Fig. 3 highlight a performance loss in the order of 2 dB for large SNRs. Results also show that both codes achieve the same diversity order where the corresponding error curves are practically parallel to each other for large values of the SNR.

Fig. 4 shows the performance with $M = 4$ for an integration time of $T_i = 10$ ns. In this case, Θ can range between 1 and 3 and the three obtained codes are compared. The findings are similar to those obtained in Fig. 3. In this case, the performance gap between the cases $\Theta = 1$ and $\Theta = 2$ is in the order of 1 dB while the performance gap between the cases $\Theta = 2$ and $\Theta = 3$ is very small and in the order of 0.3 dB. The improvements with respect to single-antenna systems are evident for average-to-large values of the SNR.

V. Conclusion

We proposed a novel unipolar, unitary, differential, fully-diverse and scalable family of ST codes for UWB communications with two transmit antennas and M-ary PPM for all values of M. This family of codes is associated with cross-correlation receivers that can be implemented in the analog domain without the need to estimate the underlying UWB channel. The proposed solution is appealing since it renders the extension of the single-antenna systems to the MIMO scenario simple and cost-effective without imposing any additional constraints on the RF circuitry to control the phase or the amplitude of the very low duty cycle sub-nanosecond pulses.

VI. Appendix

From (5)-(6), we define the M-dimensional vectors U and V as $U = \frac{1}{\sqrt{\theta+1}} \sum_{i=0}^{\theta} \Omega^{m+i} e_1$ and $V = \frac{1}{\sqrt{\theta+1}} \sum_{i=0}^{\theta} \Omega^{m+1+i} e_1$ which is equal to ΩU. Vectors U' and V' are defined in the same way based on the integers m' and θ'. Vector U (and similarly U') comprises $\theta+1$ nonzero elements that occupy the position $m+1, \dots, M$ and $1, \dots, m+1-M+\theta$. This property is of significant importance for achieving the full rank.

Define the matrix A as $A = C_{m,u}^{(\theta)} - C_{m',u'}^{(\theta')}$ and denote by A_i the i-th column of A for $i = 1, 2$. In what follows, we will prove that the relation

$$k_1 A_1 + k_2 A_2 = \mathbf{O}_{2M} \tag{24}$$

will hold if and only if $k_1 = k_2 = 0$ implying that A has a full rank of 2. The following four cases need to be considered in our proof.

Case 1: $u = u' = 0$. In this case:

$$A = \begin{bmatrix} U - U' & \mathbf{O}_M \\ \mathbf{O}_M & U - U' \end{bmatrix} \tag{25}$$

In this case, (24) will hold if $k_1(U - U') = \mathbf{O}_M$ and $k_2(U - U') = \mathbf{O}_M$. For the vector U that comprises θ non-zero elements to be equal to the vector U' that comprises θ' non-zero elements, θ must be equal to θ'. Moreover, m must be equal to m' so that these non-zero elements will occupy the same positions. Consequently, $U \neq U'$ for $(\theta, m, u) \neq (\theta', m', u')$ resulting in $k_1 = k_2 = 0$.

Case 2: $u = u' = 1$. In this case:

$$A = \begin{bmatrix} \mathbf{O}_M & V - V' \\ U - U' & \mathbf{O}_M \end{bmatrix} \tag{26}$$

In this case, (24) will hold if $k_1(U-U') = \mathbf{O}_M$ and $k_2(V - V') = \mathbf{O}_M$ where the last equation implies that $k_2(U-U') = \mathbf{O}_M$. Since $U = U'$ only for $(\theta, m, u) = (\theta', m', u')$ following from the analysis in case 1, then $k_1 = k_2 = 0$.

Case 3: $u = 0$ and $u' = 1$. In this case:

$$A = \begin{bmatrix} U & -V' \\ -U' & U \end{bmatrix} \tag{27}$$

In this case, (24) will hold if $k_1 U = k_2 V' = k_2 \Omega U'$ and $k_1 U' = k_2 U$. If $(k_1, k_2) \neq (0, 0)$, combining these equations results in $k_1^2 U = k_2^2 \Omega U$ which is not possible since the vector U, that comprises at least one zero element, can not be proportional to a shifted version of itself. Consequently, at least one value among k_1 and k_2 is zero which will imply that the other term is also zero following from $k_1 U' = k_2 U$.

Case 4: $u = 1$ and $u' = 0$. The proof follows directly from case 3 by interchanging the roles of the vectors U and U' on one hand and of V and V' on the other hand.

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