Integration of DGS Activities in Teaching/Learning Quadrilaterals According to the Lebanese Curriculum

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Abstract

Technology has become a part of most of our daily life activities. The new technological tools, such as computers and their software, provide educators with opportunities to teach in new and more constructive ways. The purpose of this project is to develop a guidebook for grade eight teachers to implement Dynamic Geometry Software, using Cabri-Geometre as a tool, in teaching the “Quadrilaterals” unit. The development of this unit has undergone several steps. First, a comprehensive literature review was conducted, covering three interdependent themes. These themes are DGS, teaching/learning proofs in geometry and Vygotsky’s social constructivist theory. Second, the Lebanese curriculum of grade eight was reviewed, analyzed and critiqued. Finally, the guide was developed through a combination of both Cabri-Geometre and paper-and-pencil activities. The design of the guide is based on Vygotsky’s social constructivist theory so that it bridges the gap between the content of the textbook and the introduction and general objectives of the Lebanese curriculum.
CHAPTER I

Introduction

Technology has, in the recent years, forced its way at all levels of mathematics teaching and learning, catalyzing the change from traditional pencil- and-paper learning to a mixture of calculator, computer and pencil-and-paper learning environments. It is agreed that, if used properly, technological tools enhance students' mathematics learning. Technology equipments are important tools in teaching and learning mathematics, and they assist students in developing visual techniques. Actually, one of the goals of teaching mathematics is to assist students in looking at problems and concepts in different ways in order to reach a meaningful understanding of concepts.

In this context, Dynamic Geometry Software (DGS) is becoming an essential component of mathematics education because it enables math teachers to present geometrical problem-solving contexts in unique and clever ways. DGS changes the way mathematics is done today as it allows the visualization of abstract concepts and ideas. Lecturing and textbooks are widely used in classrooms because they cover enormous amounts of material in no time, but they are weak in motivating students to engage in their own learning process. However, technological tools can be motivating to students and provide them with thinking environments if used by qualified teachers. Mathematics educators are the key to bring about reform of mathematics teaching with technology. If DGS were integrated properly, it would help create a learner-centered environment that is more effective in engaging and motivating students to learn. It should be introduced in the context of meaningful activities that address mathematical concepts and connect mathematics to real-world phenomena. These activities should take advantage of the capabilities of technology.
Integrating technology in math education in Lebanon would create a crucial change in the mathematics curriculum. Content would change from focusing on algorithmic skills to a content that emphasizes understanding of concepts. To use DGS efficiently in math teaching, teachers should be well trained to implement it in teaching so that they will have confidence in their abilities while using it in class. Mathematics teachers, who are today’s practitioners, did not have technology incorporated during their education years. So they should be assisted to use DGS properly, through professional development and follow-ups, in order to become more at ease and confident in integrating DGS inside their classrooms, which in turn will modify and improve their teaching skills.

This project will develop a teaching/learning module integrating DGS in the teaching of a mathematical unit, namely “Quadrilaterals”. Activities and suggestions will be provided on how to teach this unit using Cabri-Geometre (hereafter referred to as Cabri) software.

Purpose

The purpose of this project is to develop a mathematical unit on quadrilaterals integrating the use of dynamic geometry software. The software to be used is Cabri geometry. The design of the unit is aimed to impact students’ long-term conceptual understanding of concepts about quadrilaterals. It will consist of the part of the Lebanese curriculum about quadrilaterals but in an active and meaningful way with the help of Cabri. It will also provide teachers with guidelines on how to implement concepts and properties of quadrilaterals using Cabri, while they become facilitators and guiders instead of deliverers of information.
Method

The development of the Cabri-based unit will undergo several steps and procedures:

*Literature Review*

To collect and write a comprehensive review of literature on this topic, an extensive coverage of three themes that interact with each other and that are dependant on each other will have to be discussed in the literature. These themes are:

- Dynamic geometry software
- Teaching/learning proofs in geometry
- Vygotsky’s social constructivist theory.

*Review of the Lebanese Curriculum*

Considerable effort was put to reflect the constructivist view in the Lebanese math curriculum texts (CERD, 1997). This was well reflected in the “Introduction” to the curriculum, as well as the “General Objectives”. Yet, the content and design of the textbooks are hard to be delivered by a constructivist approach. The textbooks and their content are consistent neither with the introduction nor the general objectives of the Lebanese curriculum text (Osta, 2003). Since this project aims to assist Lebanese teachers in integrating DGS inside their classrooms, it should adopt the constructivism-based introduction, the general and the specific objectives of the curriculum.

*Injection of Activities under Cabri software*

The project will develop a variety of activities under Cabri. Of course, not every detail of the lessons will be implemented with Cabri, but Cabri will be dominant throughout the unit and it is going to be the main assistant to students in reaching a meaningful understanding of concepts and properties about quadrilaterals. This unit will be composed of Cabri-based lessons to teach
the different concepts and properties of quadrilaterals. It will contain exercises and problems that allow students to apply their understanding to new situations. The "Quadrilaterals" unit and its design are based on the following theoretical background:

**Theoretical Background.**

The way the unit is designed and should be implemented is based on Vygotsky's social constructivist theory. Vygotsky's sociocultural theory is given great attention and importance at present.

Vygotsky believed that culture and children's interaction with others are crucial in their overall development. According to him, learning depends on the support as well as the interaction with adults. The key to such interaction is scaffolding. Vygotsky believed that teachers should guide learning through explanation and demonstration whenever they feel that children will benefit from it. Teachers should scaffold students' learning by modeling strategies of learning then slowly shifting the responsibility to students. Vygotsky's sociocultural theory discusses cognitive development as dependant on the interaction with adults. A notion that is related to scaffolding is the concept of Zone of Proximal Development (ZPD). The zone of proximal development refers to the tasks that the child can't complete alone, but with the help of an adult he/she will be able to complete them. Applying this notion in the classroom, the teacher would provide a learning experience for a student that is at a level just past his/her performance level. By doing that, both the child and the teacher get involved in dialogues that enhance the learning process of the child (Gage & Berliner, 1998).

There are many educational implications in Vygotsky's theory of social constructivism. Students are active participants in the learning process. They engage in their own learning experience, construct their own knowledge under their teacher's guidance. The role of the
instructor changes from that of a dispenser of knowledge to that of a facilitator of learning. The instructor guides students through experiences in which they explore, conjecture, verify, generalize, and apply results to other settings and realistic problems (NCTM, 2000).

Significance of the project

DGS enhances the role of exploration, heuristics and visualization inside the mathematics classrooms (Hanna, 2000). With free exploration and manipulation, students can construct complex figures and in a short period of time perform a wide range of transformations, making them capable of having access to many examples impossible in non-computational environments (Marrades & Gutierrez, 2000). According to Mariotti (as cited by Hanna, 2000), DGS contributes to the understanding of theoretical geometry and to the transition from intuition to theory. Hadas, Hershkowitz and Shwartz (2000) stated that sometimes, when using DGS, students reach contradictions to their conjectures and thus arrive at a resolution. Research on the use of DGS has highlighted how diagrams play an ambiguous role in geometry. Not only do diagrams involve theoretical objects, but they also offer graphical and spatial properties.

One aspect that distinguishes DGS from other programs is the ability to specify the geometrical relationships between objects created on the computer screen and the ability to graphically explore the implications of the geometrical relationships established in constructing a figure. This is usually achieved through the “drag” function.

The issue of proof in teaching and learning mathematics with the help of DGS seems to be heavily dependent on the way the teacher manages the activities in the classroom. Cabri software in a classroom is the Cabri of the students where they can develop new meanings. What actually happens is that as students interact with each other when working on geometrical problems under
DGS, they name objects and describe properties using their own language and that helps them reach a meaningful understanding of the mathematical concepts they experienced using DGS when they are formally introduced to them (Jones, 1998).

Many educators experimented using DGS with students learning quadrilaterals. Jones (2000a) used problems of increasing difficulty on quadrilaterals to analyze students' interpretations and their evolving explanations when using DGS. Also, Hoyle & Jones (1998) together analyzed how children working under DGS discover the properties of a rhombus. De Villiers (2002) used quadrilaterals to explain and elaborate on the functions of proof. This project that will implement the unit of quadrilaterals and their properties and how they relate to each other under Cabri will hopefully benefit from the results of previous research and contribute to the field of teaching mathematics, according to the Lebanese curriculum.
CHAPTER II

Literature Review

In light of efforts to reform mathematics curricula, new teaching materials stress important, non-traditional higher level cognitive processes such as problem solving, reasoning, communication, and making connections amongst mathematical domains and to the real world (NCTM, 2000).

Constructivism as a Basis for Teaching Mathematics

Society is changing so rapidly that schools and educational institutions are failing to keep up (Hammond, 1992). According to Brown (2006), the problem with traditional curricula is that they ignore learners’ lives and are related neither to their interests nor to their needs. Brown believes that learners cannot relate with what they are being taught because they feel it is totally abstract to them. Ayers (1992) shares this view by stressing that the concepts and theories applied inside traditional classrooms do not reflect the real world in which students live. Traditional curricula prompt boredom, dependency, passivity, memorization and competition. Even when traditional schools attempt to improve themselves, they tend to purchase new packages without taking into account whether they suit students’ needs or not. Teaching simply comes down to delivering those packages. Traditional curricula are not able to prepare students for the challenging circumstances they are going to face in their future lives (Brown, 2006).

Sinclair (2004) revealed that in the traditional geometry classroom students are taught to ignore what they globally see in a diagram and only pay attention to what is given and what is required to prove. This way of teaching geometry combined with students’ lack of visual interpretation skills hold students back from being able to explain geometrical sketches. For students to attain complete understanding of mathematical concepts, Battista (2002) believes that
they have to construct their own knowledge from situations that personally mean to them. He advocates that the mathematics curriculum shouldn’t force students to memorize definitions and properties like the traditional one does. De Villiers (2004) joins this view by stating that students should be allowed to actively engage in developing and constructing the content they learn. Students should be active learners and participants, constructing new knowledge from already existing one to help in developing their mathematical understanding (Battista, 2002). To achieve that, Mariotti (2000) stresses that the development of meaning is attained through social environments in the classroom under the teacher’s guidance. Jones (2000b) elaborated on this point through the socio-cultural perspective. He stated that to be able to understand how students go about the process of learning, the focus should be on the employed resources that mediate learning. How students make sense of the mathematics they learn can be better understood if a socio-cultural perspective is adopted through the usage of pedagogical tools. Battista (2002) considers that if learning is to happen properly then inquiry, problem solving, communication and discussion among peers should be dominant in the mathematics classroom. This, Mariotti (2000) believes, would provide students with the opportunity to construct their own knowledge rather than being passive learners of mathematics.

Teachers are challenged to provide the appropriate input that would foster students’ reasoning and communication skills and make them actively involved in their learning process (Jones, 1998). Pierce and Kalkman (2003) held instructors responsible for effectively implementing the learner-centered approach inside their classrooms because according to Mariotti (2000), teachers play a significant role in directing class discussions and guiding students to comprehend the construction of a problem in a geometrical manner. But Belfort and Guimaraes (2004) couldn’t see teachers performing this role because many of them lack deep and thorough knowledge of
the subject matter they teach. They believe and suggest that educators should be provided with the opportunity to establish connections between their teaching practices and their learning experiences.

Hollebrands (2007) also suggests that instruction should be designed to help students attend to their reflections on mathematical relationships within a technological environment and in linking those relationships to their already existing mathematical understanding. Teachers should intervene at the right time and when they do, they assist students in moving from the procedure to explaining the procedure (Mariotti, 2000). NCTM (2000) emphasizes three significant mathematical activities that educators should practice inside their classrooms: problem solving, problem posing and conjecturing. It highlights that instruction should allow students to explore and test their conjectures to be able to make generalizations. For that to happen, Gage and Berliner (1998) suggest that teachers should guide learning through explanation and demonstration whenever they feel they are useful. Teachers should scaffold students by modeling strategies of learning then slowly shifting the responsibility to students. They strongly believe that instruction should be student-centered rather than teacher-centered. Laborde (2000) suggests that educators could prepare students for proof, for example, by providing them with activities that develop their awareness of properties that depend on each other. According to Battista (2002), working in such a constructivist geometry environment would assist students in building complex mental models of mathematical concepts, which are based on real mathematical understanding that has been attained through the active construction of knowledge by students. Jones (2000a) presents Dynamic software, as an example that offers a break from pencil-and-paper geometry, allowing students to construct their own knowledge by receiving immediate feedback from the software itself. Wares (2006) specified that construction in a
dynamic environment is a meaningful activity because it allows students to actively investigate properties and relationships between different geometric objects and fosters students’ reasoning skills under the teacher’s guidance.

A Global View on Dynamic Geometry Software

DGS provides a context where mathematical problems can be done in a different and a more meaningful way. Mogetta, Olivero and Jones (1999) explained that when a mathematical problem is first being tackled under DGS, an interpretation of the problem takes place from the menu items that are available in that software. Constructing the figure under DGS explicitly identifies the starting points and the connections between them. When students finish constructing and move on to use the dragging facility to generate hypotheses, they are actually switching between the empirical and the theoretical level. Jones (1998) specified that in Cabri software, the mediating influences on students involve the drag-mode, the behavior of a point, the ability to drag some points and the inability to drag others. Mogetta et al. (1999) elaborated on the drag-mode by saying that as students drag the movable points of a figure, the behavior of the figure represents its mathematical properties. So students dynamically experience the mathematical properties of a figure. The ability to measure the sides and angles of a mathematical object under Cabri assists students in shifting from reasoning about the figure to reasoning about the properties of the constructed figure (Hollebrands, 2007). Cabri provides students with the opportunity to interact with geometrical shapes through their mathematical features (Smith, 1999). Cabri software helps in providing a learning environment away from its "straightedge-and-compass counterpart" (Christou Mousoulides, Pittalis & Patanzi, 2004) because it allows students to describe what they observe through exploration and manipulation (Smith, 1999). Christou et al. (2004) listed the significant features that characterize DGS, which
are the ability to construct, manipulate and reshape figures as well as the ability to explore geometrical relationships between those figures. Hollebrands (2007) views the significance of DGS as a tool that empowers students to construct figures, explore them, measure and drag parts of the figure they constructed. With this free exploration and manipulation, Marrades and Gutierrez (2000) believe that students can construct complex figures and in a short period of time perform a wide range of transformations, making them capable of having access to many examples impossible in non-computational environments. The various capabilities of Dynamic Geometry Software support students in developing and enhancing their heuristics skills (Gawlick, 2002). According to Hanna (2000), Dynamic Geometry Software enhances the role of exploration, heuristics and visualization inside the mathematics classrooms. It actually encourages students to do some research in geometry where they come up with conjectures, test them and then modify them (Christou et al., 2004) through techniques of visualization and measurement that assist them in developing formal proofs (Wares, 2006). Mogetta et al. (1999) specify that the construction process is considered as a back up for the starting phase of the proving process. Mariotti (2000) believes that DGS contributes to the understanding of theoretical geometry where it facilitates the transition from intuition to theory. She provides Cabri software as an example of DGS that simply refers to classic constructions done by pencil and ruler. Yet, this software offers a dynamic environment where learners can manipulate figures and constructions on the screen and get immediate feedback on their manipulation. Smith (1999) views Cabri software as a motivating stimulus because it promotes students’ reflection on their learning process. With Cabri, he found that students feel that they are in control of the activity they are working on, which motivates them to go further in exploring and verifying. Hadas et al. (2000) also discovered that with Cabri students naturally authentically engage in a mathematical
activity. They are no longer passive learners of formal proofs but are actively engaged in constructing and evaluating arguments using their geometrical knowledge to verify their conjectures and overcome their uncertainty.

Proof in Geometry

“What is proof? This is one-question students and we mathematics teachers would normally never think of asking. We’ve seen proofs and we’ve done proofs. Proofs are what we’ve been watching and doing for the past 10 or 20 years.” (Hersh, 1993,p.389).

For decades, verifying mathematical statements has been done through formal proofs. Hadas et al. (2000) presented two main reasons for teaching proofs, which are: 1) proof is a vehicle used to verify that mathematical statements are universal, and 2) proof is a portion of human culture to teach deductive reasoning. Marrades and Gutierrez (2000) believe that the main objectives of proof are to verify, justify the correctness of a statement, explain the truth of a statement and transmit mathematical knowledge. According to Hanna (2000), the functions of proof are to verify, explain, systematize, discover, communicate, construct, explore and incorporate. Yet, Hanna emphasizes that students should start with the fundamental functions of proof, which are verification and explanation. Marrades and Gutierrez (2000) consider that proof is a way of surmounting contradiction and doubt while Hanna (2000) considers that proof is not only to see the validity of a mathematical statement but also to understand why a mathematical statement is valid. Hersh (1993) joins this view by stating that people want to know why a theory is correct, not just whether it is correct or not. They don’t want to be told that a proof exists; they want to comprehend what proof is. Hanna (2000) argues that proofs become convincing to mathematicians only when they lead to actual mathematical understanding. But Marrades and
Gutierrez (2000) believe that one of the most difficult tasks for mathematics educators is to assist students in achieving correct understanding of proof as well as to promote their proof techniques. Hoyle and Jones (1998) criticized the traditional mathematics curriculum because it presents proof as formal two-column statements where students are asked to believe the truth and validity of such a proof without asking any questions. Such kinds of proof cause conceptual difficulties for students. According to De Villiers (2004), the basic reason why the traditional geometry curriculum failed is because proof is presented at a level of understanding that is beyond students’ understanding level. Students are convinced that verifying a mathematical statement empirically is simply its proof. But Hoyle and Jones (1998) emphasize that there’s a difference between empirical and deductive arguments, and students always prefer the empirical approach. Jones (2000a) also discussed this issue and said that students don’t see the need for deductive proof. Moreover, they are unable to distinguish between explanation and verification on one hand and deductive proof on the other hand. Moore (1994) elaborates on this issue by saying that learners cannot connect with geometrical proof because they don’t understand its purpose or its function in the mathematical field. When students do not reach meaningful understanding of a mathematical concept, they can’t generate their own examples, which hinders them in writing a formal proof (Moore, 1994). Proof becomes meaningless to students after they become convinced of the correctness of a mathematical statement (Hoyle & Jones, 1998). Mariotti (2000) agrees with that point and explains that when students start to learn the deductive approach to geometry, they go through a great difficulty in grasping it with respect to their intuitive knowledge of properties. They don’t understand why they have to prove the validity of mathematical statements. According to Moore (1994), the reason for that is that students usually find difficulties in one or more of the following areas in proof: the mathematical language, the
logic of proof, understanding of a mathematical concept and problem solving skills. The reason for finding difficulty in these areas according to Marrades and Gutierrez (2000) is that students go through a slow transition phase from empirical to deductive reasoning and conjecturing and they suggest that this transition has to be rooted by empirical methods.

Yet, Hanna (2000) strongly believes that only proof provides a formal confirmation of a mathematical statement. Laborde (2000) joins Hanna’s belief and perceives proof as means to validate the truth of a conjecture and convince others of this validity. But Mogetta et al. (1999) think that constantly providing students with problems and exercises that start with “Prove that...” hinders the capacity of students to prove. They consider that those problems do not stimulate the necessary mental processes of students leaving them prisoners of the theoretical part of geometry and not allowing them to explore. But to Jones (2000a), that is not the only problem. He related the ability of students to develop a deductive proof to the classroom environment, the type of activities students tackle and the kind of interaction that takes place between students on one hand and between students and teachers on the other hand. Students should learn deductive proof through a variety of activities where they are able to demonstrate to others the reason behind the validity of a mathematical statement (Miyazaki, 2000). To achieve that Hoyles & Jones (1998) believe that mathematics educators face a challenge to make geometrical proofs possess exploratory and communicatory functions in addition to justification and verification. Inside the mathematics classroom, the main role of proof should foster students’ mathematical understanding and educators should adopt more effective ways to help students reach a meaningful understanding of mathematical concepts (Hanna, 2000).
How DGS Fosters Students’ Proving Skills:

Debate over the years has been created about the implementation of DGS; especially about whether integrating it in the mathematics curriculum would kill the need for proofs (Laborde, 2000). Hadas et al. (2000) found that the function of proof was questioned when DGS started to appear in the mathematical field. When students found that by using the dragging facility they could be sure of what is required, they couldn’t understand the need of a formal proof. Students are convinced that empirical evidence obtained from DGS is enough proof of a mathematical statement (Hollebrands, 2007). Yet, Christou et al. (2004) emphasize that empirical verification of mathematical statements is not sufficient because it doesn’t explain why a mathematical statement is true. Proof cannot be replaced by explorations and students should understand that. Without proof, De Villiers (2004) believes that the results of mathematical research are impossible to organize into axioms, postulates and theorems. According to Hanna (2000), mathematicians argue that intuition and heuristics are effective and useful at the preparatory stages of achieving mathematical results but they all agree that there is a difference between valid proofs and heuristic arguments and the correctness of a mathematical statement eventually should rest on proof. As a start De Villiers (2002) suggests that students shouldn’t be faced with verification in dynamic geometry contexts but it can be later developed to allow them reach a more meaningful understanding of the significance of deductive proof. One way to do that according to Christou et al. (2004) is to explore geometrical concepts under DGS because that will motivate and trigger students’ curiosity to justify their conjectures through formal proofs. Exploration in mathematics makes use of deductive reasoning, which is the basis of proof. Exploring and proving complement each other because exploration guides the way to discovery while proof leads the way to affirmation (Hanna, 2000). From another angle, Laborde
(2000) perceives that sometimes DGS makes students reach a cognitive conflict or surprise and that will trigger and motivate them to understand the reason behind this conflict. But in short, Hanna (2000) believes that students should acknowledge the fact that exploration does not substitute proof.

Mariotti (2000) highlights that the link between Cabri’s logic and Euclidean geometry is the basis for the relationship between constructions under Cabri and geometrical theorems. Introducing students to construction problems under Cabri is the key to enter the theoretical perspective. Laborde (2000) agrees and elaborates by saying that dynamic geometry environments promote the interaction between proof and construction as well as between conjecturing on the computer and using theoretical arguments as a means of justification. Jones (2000a) emphasizes that if students are to benefit from DGS to develop deductive proofs, then it is essential to know how they explain geometrical objects and relationships between these objects under DGS. Hoyles and Jones (1998) specify that the dragging facility in Cabri software, for example, allows students to test conjectures by focusing on the links between the geometrical objects they constructed. Mariotti (2000) elaborated on this idea by saying that as soon as the construction passes the dragging test, a justification of the solution is required. Justification is required because it is necessary to explain why a figure works and it has to refer to theory to be considered as a formal proof. When students explain the figures that they construct using DGS they are on the right track of the proving process (Laborde, 2000) because they actually perceive the axioms of plane geometry (Hoyles & Jones, 1998). De Villiers (2002) considers that it is not certainty that is needed but rather explanation. According to Olivero and Robutti (2007) students usually go back and forth between the graphical field and the theoretical field in the proving process. Moore (1994) argues that one of the reasons students fail to come up with formal proofs
is their inability to memorize and state mathematical definitions. He believes that students' ability to use definitions in a proof depends on how well they formally know the definition. But De Villiers (2004) disagrees and argues that knowing the definition of a mathematical concept does not imply or even mean that a concept is fully understood. Instead of memorizing facts and principles Battista (2002) suggests that dynamic geometry environments promote students' understanding and help them move to higher and more abstract geometrical thinking levels. Christou et al. (2004) join this view by saying that when students learn using DGS they acquire more meaningful and deep understandings, which are not possible to attain through traditional instruction. Hollebrands (2007) believes that DGS allows students to reflect on their work, which in turn enhances and develops their mathematical understanding because according to Christou et al. (2004), the stage before formal proof is useful for students since it allows them to comprehend the task given at their own pace and intellectual efforts. Jones (2000b) perceives DGS as a mediator that shifts students' explanations from pragmatic to conceptual. According to Marrades and Gutierrez (2000), DGS has a two-fold contribution to mathematical education: It allows students to explore and experiment freely and it is a non-traditional way to reach a meaningful understanding of mathematical concepts. When students are learning within a dynamic environment, interaction takes place between them and the software causing them to revise their previous knowledge and modify it to acquire new knowledge (Jones, 2000b).

**Challenges in Integrating DGS**

Gawlick (2002) doesn't believe that high expectations should be put on dynamizing the current curriculum. It goes beyond that. Educational environments should be altered where teachers become able to come up with novel teaching sequences so that DGS implementation inside the classroom becomes possible. Mitchell, Bailey and Monroe (2007) disagree and believe
that a radical change won't work. They suggest that starting should be gradual where small chunks should be tested and teaching methods should be gradually modified permitting the traditional teaching method to continue. But for change to be effective and productive, they emphasize that teachers should be supported and assisted in the process of technology integration because they face challenges and difficulties when shifting from traditional teaching to the technology integrated approach. Hanna (2000) highlights that another challenge for mathematics educators is to capitalize the excitement of exploration in order to encourage students to provide a proof. Their challenge also is to find and create problem settings where proof is an illumination into the validity of a result or a mathematical statement (Jones, 1999). Mogetta et al. (1999) suggest that one of the ways to do is to come up with open tasks that utilize the full potential of DGS in order to motivate students to prove. Jones (1999) supports this by saying that open problems provide students with the freedom to produce conjectures and in order to solve those open problems students have to explore, conjecture, validate the conjectures then prove them. Wares (2006) suggests another way and that is to face students with conjectures that are not found in their textbooks and ask them to prove those conjectures that would intellectually challenge them. According to Laborde (2000), proof appears in a variety of different contexts and the teacher's role is to play around with these contexts in order to motivate proof activities.

But Jones (2000b) believes that using mediation tools like Cabri depends on the context in which they are used as well as how the teacher deals with such tools inside the classroom. Yet, Hollebrands (2007) is convinced that students who learn mathematics within a technological environment have more chances to get involved in abstraction activities because of the variety of actions they can perform using the equipment, specifically computers. DGS should make mathematics educators reconsider the aim of going about formal geometric proofs in high school
mathematics curriculum (Pandiscio, 2002). According to Jones (1999) DGS calls for a radical change in the nature of mathematical problems because problems need to be more dynamic in order to promote students' productive thinking. But Mitchell et al. (2007) argue that it's too much to ask teachers to make this radical shift from traditional teaching to teaching using technology without giving them time to adapt. Baldin (2003) added that merely training teachers to manipulate technological equipment is not enough to implement that equipment as educational tools. They should be involved in the paradigm shift where they should contribute in evaluating the curriculum and suggest what needs to be altered (Mitchell et al., 2007). Baldin (2003) on the other hand considers that to be able to properly integrate technology inside mathematics classrooms, teachers should have a solid background in mathematics and they should know how to use technology-based activities in order to stimulate communication among their students. Moreover, De Villiers (2004) emphasizes that mathematics teachers should put an effort to make students appreciate the different functions of proof and provide more time and space for them to explore, conjecture, refute, reformulate and explain. Yet, he considers that if those teachers were never exposed to such constructivist methods, it is difficult to see them integrate these approaches inside their own classrooms without any help.

In conclusion, proof is an essential element of mathematical knowledge and its value goes beyond the confirmation of results. Dynamic geometry software does enable learners to attain certainty through empirical measurements, yet proof remains as essential as ever because it guides the way to new findings and discoveries and helps systemization (De Villiers, 2002). The mere existence of software packages does not imply that people will take advantage of what the software offers. Educational change cannot be established by the mere existence of technological tools. The use of technological equipment calls for the implementation of both mathematical
knowledge as well as the knowledge of the tool used (Olivero & Robutti, 2007). The best use of
technology is the ability it provides to visualize mathematical ideas easier and the ability it
provides to do things with technology that cannot be done without it, such as performing
thousands of trials of a certain operation or quickly graphing a function or transforming and
manipulating objects. Hoyles and Jones (1998) acknowledge that students need to use textbooks
and paper and pencil at times in order to practice and develop their skills. If software is used to
present proof in a traditional manner then no improvement will occur in students’ conceptual
understanding of mathematical concepts. Belfort and Guimaraes (2004) suggest that teachers
need to be provided with rich learning experiences within dynamic environments to comprehend
the potentials of such software. Mitchell et al. (2007) declare that problems and challenges will
definitely occur. That’s why, according to them, teachers should have a support group to turn to
in order to resolve their problems. Teachers may feel overwhelmed by the change that’s
occurring. That’s why they need guidance form people who are experts in technology integration
inside mathematics classrooms. Teachers should carefully consider when and how to use DGS
(Jones, 2000b) because teachers’ ability to appropriately use technology to make content easier is
the key to successfully integrating technology inside classrooms (Mitchell et al., 2007).

However, if technology is not incorporated effectively into the classroom, students may have
fun, but they probably won’t learn any math. Technology has the potential for many positive
benefits in the math classroom, but there is also the potential of technology being an impediment
to mathematical progress. Ultimately, the decision is up to the math teachers as to how to
effectively implement technology into their classroom.

The present project and the guide developed through it are based on the findings learned from
the above review of literature. Basically, one of the important facts learned is that although the
curriculum needs to be dynamized, yet it shouldn’t be radically dynamized. DGS is essential but can never replace formal proofs in the geometry material. As a result of that, lessons should be a combination of proper DGS integration and paper-and-pencil activities so that students are given the chance to explore, apply and develop their proving abilities.
CHAPTER III

Analysis of the Grade Eight Lebanese Curriculum Unit on “Quadrilaterals”

Substantial effort was put to apply the constructivist views in the Lebanese math curriculum texts (CERD, 1997). This was well displayed in the “Introduction” and the “General Objectives” of the curriculum. Yet, the content and instructional designs of the textbooks are difficult to be conveyed by a constructivist approach. Moreover, they are not consistent with the introduction and the general objectives of the Lebanese curriculum text (Osta, 2003). There’s a gap between the introduction and the objectives of the Lebanese curriculum on one hand and the content of the textbooks on the other hand. The introduction and the objectives of the curriculum present an ideal vision of how mathematics should be taught, but the content of the textbooks do not reflect those objectives and makes it impossible to achieve them creating a fear of going back to the old curriculum. In other words, the introduction, general objectives and the content of the textbook lack coherence and homogeneity. This project aims to assist Lebanese teachers in integrating DGS inside their classrooms. It should thus adopt the constructivism-based introduction, the general and the specific objectives of the curriculum. This chapter of the project will analyze the Lebanese mathematics Curriculum of grade eight concerning the “Quadrilaterals” unit and will describe the chapters of the grade eight textbook that investigate the different types of quadrilaterals.

Introduction and General Objectives:

In the introduction, the curriculum book (CERD, 1997) emphasizes that mathematics is a necessity to the life of societies and their development. The increasing demand for the learning of mathematics resulted in the modification of its use. The introduction highlights that teaching is reformed through the formulation of new objectives in which learning will begin with real life
situations and connections where learners become more active as thinkers, more involved in their own learning and not passively receiving information as it has been going on for the past 30 years before the development of this curriculum. Contents will be remodeled, as they will be accessible to all students. Teaching methods will be remodeled as well, where the teacher will act as a facilitator or a guide, helping students to discover information on their own through questions and activities that the teacher should relate to their previous knowledge and past experiences. This means that the teacher has to relate their questions and the activities they conduct to real life experiences.

This curriculum aims to achieve five general objectives:

1) Emphasizing students’ mathematical reasoning by developing and nurturing their critical thinking

2) Solving mathematical problems, where students should learn different strategies to tackle obstacles they may encounter upon solving a problem

3) Relating mathematics to technological, economical and cultural development

4) Training students to communicate mathematically, where they will be able to present their work not just to their teacher but also to their classmates

5) Helping students to develop their imagination and creativity as well as have self-confidence because that will provide them with the pleasure in learning and discovering mathematics.

*Intermediate Level Objectives:*

The objectives of the intermediate level emphasize that the students’ abilities in mathematical reasoning, problem solving, mathematical communication, spatial reasoning, numerical reasoning, measurements and statistics will be developed.
Concerning mathematical reasoning, students in grade eight will establish connections between the real world and mathematical models as well as between mathematical concepts and the mathematical models. They will be able to differentiate between a general statement and a specific one and will be able to carry out simple proofs as well as to recognize false ones. Concerning problem solving, students will be able to analyze situations, develop the ability to look for specific information, to clarify insufficient information as well as develop multiple strategies to solve problems. Students will also be able to decompose problems into simpler tasks. As for mathematical communication, students will be able to read, understand and use mathematical notations. They will be able to present their work orally or in writing with enough clarity. For spatial reasoning, students will be able to construct geometric figures based on the given properties, prove and apply properties of plane figures and be able to perform transformations on figures. All of the above constitute the specific objectives on geometry of grade eight, which were consistent with the general objectives of the curriculum, but naturally, more details were listed than the general objectives.

Objectives of "Quadrilaterals" Unit in Grade Eight:

In the scope-and-sequence table of the "Quadrilaterals" unit in grade eight, it is mentioned that the characteristic properties of a parallelogram will be covered in the "Plane Figures" part of the content. The "Details of Content" book (CERD, 1998) expands the objectives of what grade eight students cover about quadrilaterals. It elaborates that students will characterize the rectangle, rhombus and square. They will use properties of a parallelogram having to do with: sides, diagonals, opposite angles and center of symmetry. Students also will characterize the parallelogram as being a convex quadrilateral having different properties. Students will be able to characterize a rectangle as a quadrilateral with three right angles; a rhombus as a quadrilateral
with equal sides and characterize a quadrilateral as a rectangle or a rhombus based on the relation between diagonals. The rectangle and rhombus will be demonstrated as parallelograms with special properties and that a square is at the same time a rectangle and a rhombus. A relation is pointed out between the special quadrilaterals and the trapezoid. Students will acknowledge that the nature of a quadrilateral could be determined from its elements of symmetry.

It is worth to note that the topics to be addressed in this project are not totally new for students in grade eight. In grades four and five, students would have already learnt shapes such as squares, rectangles, rhombi and parallelograms, which were all referred to as quadrilaterals. However, students don’t know the features that define these shapes.

*Why should this unit be taught?*

This unit covers one of the most important areas of geometry. It is necessary for students to have a strong background in the topic of quadrilaterals so that they can further their understanding of size, shape and measurement of polygons in general. The students must have a strong grasp on quadrilaterals before they move on to future lessons in similarity and ratio. As students can better identify the important properties of quadrilaterals, they will be able to understand how the different applications of quadrilaterals affect every day life.

*What understandings will students develop?*

They will be able to grasp the differences between the different types of quadrilaterals by analyzing their properties. They will draw on previous developments of reasoning and proof to prove whether or not a given shape is a specific type of quadrilateral. Using alternate methods of proof will also strengthen students’ mathematical reasoning abilities.
Skills

This unit on quadrilaterals is designed to have students discover the properties of quadrilaterals using Cabri software. Students will explore and compare various types of quadrilaterals discovering their similarities and differences. Students will develop their reasoning skills. Also, they will be able to better classify shapes according to their properties. The load of discovery will often be put on students for this unit so they will strengthen their critical thinking skills as well as their communication skills.

Analyzing “Quadrilaterals” in Grade Eight Textbook

In the grade eight mathematics textbook, quadrilaterals are not given as one whole unit; instead each of square, rectangle, rhombus and parallelogram are considered in different non-consecutive chapters. Chapter four is dedicated to the square; chapter six is dedicated to the rectangle; chapter eight is dedicated to the rhombus and chapter 10 is dedicated to the parallelogram.

Chapter Structure

All chapters follow the same design. Below are the consecutive parts that constitute each chapter:

The “Recall” activities, which aim for reviewing concepts and properties previously learned about quadrilaterals. “Preparatory “ activities follow, preparing students for the specific type of quadrilateral the chapter is dedicated to, through a geometrical figure or through asking them a combination of transformations to imagine and provide answers, based on their imagination (see appendix, chapter six, p.53), or asking them to fill in a table of which properties of quadrilaterals they think are or are not preserved (see appendix, chapter eight, p. 85).
The “Text” part of the chapters start by defining the specific type of quadrilateral followed by its properties. The second part of the text is dedicated to showing the properties that are necessary and sufficient for a quadrilateral to be a square, a rectangle, a parallelogram or a rhombus.

The “Focus” part of the chapters sums up what was explained in the text in one page, including the properties of a quadrilateral and the necessary and sufficient properties to prove that a certain quadrilateral is a square, rhombus, rectangle or a parallelogram. The last part of the “Focus” shows students how to construct the studied quadrilateral.

The “Exercises” part of the chapter mainly asks students to construct a square, rectangle, rhombus or a parallelogram with certain characteristics. They are simple applications of the ideas given in the chapter.

The “Self-evaluation” part has a couple of activities asking students to construct a figure and asking a question based on their construction.

The “Problems” part of the chapter consists of asking students to construct the type of a quadrilateral as well as prove different properties related to that quadrilateral. The problems are of a higher level of difficulty than the exercises.

*Special Features of the “Parallelogram” Chapter*

It is worth noting that the “Text” part of the “Parallelogram” chapter contains an additional part titled “The parallelogram and its Descendants”. This part states the properties that relate the different types of quadrilaterals to each other. It states the necessary and sufficient properties that relate squares, rectangles, rhombi and parallelograms to each other (see appendix, chapter ten, p. 88). The “Focus” part of the same chapter also contains an additional part displaying through a map how the different types of quadrilaterals are related to each other (see appendix, chapter ten,
p. 89). Also in this chapter, the “Problems” part includes more complex figures, combining quadrilaterals with other geometrical objects such as circles or triangles (see appendix, chapter eight, p.93).

Pedagogical style of the chapters and their consistency with the general and specific objectives:

The recall activities do remind students of previous information about quadrilaterals and the preparatory activities try to prepare students for the concepts to be taught. Students’ exploration, which may start with these activities, increases their communication with each other through problem solving which makes them active thinkers and doers giving the teacher the guidance role.

However, the text of the chapters is packed with formal definitions and properties of quadrilaterals, without relating the content of the chapters to student’s experiences or personal knowledge. The “Text” part of chapters does not allow students to explore or develop different characteristics of quadrilaterals due to the extensive amount of information that exists in each chapter and hence the “Text” is not consistent, neither to the general nor to the specific objectives. The exercises are a variety of simple and direct applications, going deeper in the problems section. The “Problems” part in the chapters contains exercises that are more complex than what is found under the “Exercises” part. Both exercises and problems do not relate concrete experiences of students, making it harder for teachers to implement the learning cycle (exploration, development then application). If the teacher follows exactly the book, then the chapters have to be taught using the drill and practice style. In short, the textbook should be revised. The pedagogical style of these chapters does not reflect the general and specific objectives. More graphs should be included in the chapters in bigger sizes. Activities, exercises and problems should be rearranged so that they are related to students’ concrete and real life experiences. Also the text and explanations of chapters should be reconsidered so that more real
life examples should be included in order for students to relate better to the content. If the activities are student centered, if the content of the chapters has real life examples and exercises which students can discuss with each other and the teacher, then the learning cycle can be applied.

**Critique**

The content/material is appropriate for grade eight students. It is suitable for the age of the students, yet the way it has been designed to be delivered is difficult and mainly too abstract for students. Content should be more concrete and related to real life knowledge and experiences of the students. Chapters are designed in a dull manner. They contain lots of information and provide the chance neither for teachers nor for students to approach them in a constructive manner. Information and properties are simply listed one after the other. There is no room for exploration in any part of the chapters.

The “Text” part contains a set of topics listed one after another, giving students no time for exploration. The exercises are a set of drill and practice applications focusing on memory rather than understanding. For example, in the “Parallelogram” chapter exercises 1 till 4 test the same concept (see appendix, ch.10, p. 90). The way these exercises are located and phrased is for the purpose of testing students’ knowledge level only. Also in the “Exercises” part, numbers 6 till 9 are the same type of exercise asking students to construct and explain. Not only these exercises are not related to real life knowledge, but they also test nothing but the memory skills of students. After the “Exercises” part, which is similar in all chapters, students face questions like “Show that...” and “Prove that...” in the “Problems” part of the chapters. Students don’t face such questions except in the “Problems” part of each chapter. How can students prove when they don’t understand what proof is and the reasons for which they have to prove? The chapters
should be written to involve students more in their learning process so that they participate in exploring and developing the learnt information. The material is abstract, which makes it hard for students to retain knowledge when they need it. Even when a trial is made to make some relation with real life knowledge, it is done in the exercises or "Just for Fun" part, which is too late by then. Students do need real life knowledge in the problems, but more importantly they need it during the development phase of concepts, which is supposed to happen in the "Text" part of the chapters. In addition to that, chapters should contain few questions testing previous knowledge and concepts developed in previous chapters, so that spiral learning would be applied.

*Contribution of this project to the curriculum material of the “Quadrilaterals” Unit*

This project, though dedicated to the “Quadrilaterals” unit in grade eight, will try to bridge the gap between the introduction and the general objectives of the Lebanese curriculum with the textbook. The “Unit Plan” will distribute the load on both students and teachers. It will allow students to be more involved in their own learning process. It will include activities that allow students to explore and develop the different properties of the quadrilaterals with minimal help from the teacher. Students will be able to relate to what they’re learning by using DGS. They will be able to explore and actually see how properties of a figure can change. Through the said unit, this project will help students nurture their critical thinking skills, relate mathematics to technological development, communicate mathematically through DGS and develop their imagination and creativity.
Dynamize Your Quadrilaterals

A Guide for Using Cabri in a Geometry class
Preface:

For the past decades, technology has become a fundamental component of our daily experiences. Mathematics teachers are faced with various challenges in integrating technology, especially Dynamic Geometry Software (DGS), inside their classrooms. DGS can transform the teaching of mathematical concepts by engaging students in interactive demonstrations, constructions and explorations. It can also stimulate mathematical discussions and encourage mathematical thinking among pupils.

This is a guidebook for eighth grade mathematics teachers to integrate Dynamic Geometry in teaching the “Quadrilaterals” unit. It uses Cabri-Geometre (Belleman, 1992) as a tool. It is not addressed to teachers to teach this unit only through Cabri, but instead it will be a combination of Cabri and pencil-and-paper activities. One of the purposes of this guidebook is to help mathematics teachers familiarize themselves with the use of Cabri-Geometre in their geometry class. Another purpose of this guidebook is to be a bridge that connects abstract and concrete mathematics, which would result in a more meaningful understanding of concepts. The content of this guidebook is consistent with the Lebanese mathematics textbook for grade eight titled “Building up Mathematics” by the National Center for Educational Research and Development (NCERD, 1999). But the guide may as well be suitable for any curriculum unit on special quadrilaterals.

The guidebook is made up of four lessons each of which is dedicated to a specific type of a quadrilateral. Each of the lessons “Parallelogram”, “Rectangle” and “Rhombus” is divided into three parts. Part one includes activities that allow students to explore through using Cabri the properties of the quadrilateral. Part two develops students’ awareness of the interplay between
the definition and properties of the quadrilateral, where students explore partly using Cabri and partly using pencil-and-paper. Part three concentrates on the shift to formal proof where students should be able to prove theorems using pencil-and-paper after their exploration on Cabri. As for the “Square” lesson, it is made up of parts one and three only because it is identified as a special case of the other types of quadrilaterals. At the end of each lesson, some problems are suggested as a homework that should be corrected at the beginning of the next lesson. At the end of the unit a summative test of the unit is included, to be done using pencil-and-paper, which will test students’ understanding of the different properties of quadrilaterals, as well as their proving ability. Many of the exercises are taken from the grade eight Lebanese mathematics textbook, “Building up Mathematics” (NCERD, 1998).

For more clarity, the Cabri activities and the paper-and-pencil activities are distinguished by the following codes:

- Cabri activities
- Paper-and-pencil activities
Unit Plan

Lessons of the Unit

Lesson 1: Parallelogram

Lesson 2: Rectangle

Lesson 3: Rhombus

Lesson 4: Square

Goals of the Unit

By the end of this unit students will be able to:

- Classify quadrilaterals as parallelograms, rectangles, rhombuses, and squares.
- State the properties of each quadrilateral.
- Identify multiple names for one quadrilateral when relevant.
- Construct parallelograms, rectangles, rhombi and squares using Dynamic Geometry Software.
- Compare and contrast the following quadrilaterals: a parallelogram, rectangle, square and a rhombus.

Prerequisites:

- Definition of parallel lines.
- Different types of angles formed by parallel lines.
- Definition of perpendicular lines.
- Classification of quadrilaterals according to their sides and to their diagonals.
Parallelograms
Lesson 1: Parallelograms

Objectives of the lesson:

- Define a parallelogram
- Identify and prove the properties of a parallelogram
- Use suitable properties of a quadrilateral to prove that it is a parallelogram

Procedure:

Part I: Exploration of the Properties of a Parallelogram

Activity 1

- Teacher asks students to draw a quadrilateral (a four-sided shape) having both pairs of opposite sides parallel.

Steps of Construction:

- Create a segment AB
- Create a point “D” not belonging to line AB
- Through “D” construct a straight line parallel to AB
- Join points “A” and “D”
- Through point “B” construct a straight line parallel to [AD]
- Label point “C” as the point of intersection of the two straight lines
- Draw segments [DC] and [BC]
- Hide the straight lines (BC) and (CD)
Students should draw this four-sided figure having \([AB] // [CD] \) and \([AD] // [BC]\)

- Teacher asks students to drag the quadrilateral \(ABCD\)
  - Teacher should draw students' attention to the fact that when dragging point "A", point "C" will move sustaining parallelism (point C will move because it is symmetric to point A).
  - When dragging point "B", point "C" will move accordingly since a straight line passes through these two points.
  - When dragging point "D", point "C" will move in the same direction as point "D".
  - Teacher should draw students' attention to the fact that point "C" cannot be dragged because it is the intersection point of the two hidden straight lines and should explain that to students.

- Students should observe that the opposite sides will remain parallel to each other upon dragging.

- Teacher explains that such a quadrilateral is named a parallelogram. So \(ABCD\) is a parallelogram.

- Teacher asks students to measure the lengths of the sides of \(ABCD\)
  - Teacher should ask them what they notice about the lengths of the sides.

- Teacher asks students to drag the figure from any point they choose (they should already know that point "C" cannot be dragged).
- Teacher asks them what they notice; what changed and what remained constant.
- When dragging point “A” all side measures will change because all others depend on point “A” but in any position, opposite sides will remain equal.
- When point “B” is dragged only measures of [AB] and [CD] will change but remain equal.

Students should acknowledge upon measuring the sides of ABCD that the opposite sides of ABCD are equal and that this property is preserved upon dragging.

- Teacher helps students state the hypothesis of a property of a parallelogram:

  \[
  \text{The opposite sides of a parallelogram are equal}
  \]

- Teacher asks the students to find the measures of the angles of ABCD and answer the two questions:

  - What do you notice about opposite angles?
  - What do you notice about consecutive angles?

- Teacher asks students to find the sum of the angles of ABCD.
- Teacher asks students to choose a vertex of ABCD (other than “C”) and drag it.

  - What do you notice about the angles?
  - Do any of the measures of the angles stay invariant upon dragging?
- Did the sum of the angles of ABCD remain the same?

- Students should acknowledge that the opposite angles of ABCD are equal and that the pairs of consecutive angles are supplementary (i.e. the measures of angles BAD and ADC add up to 180° and so do angles ABC and BCD).

- So with this the teacher helps students state the hypothesis of a property of a parallelogram:

\[
\begin{align*}
\text{The opposite angles of a parallelogram are equal} \\
\text{The consecutive angles of a parallelogram are supplementary}
\end{align*}
\]

- Teacher asks students to draw the diagonals of the parallelogram and label their point of intersection as point O.

- Teacher asks students to find the lengths of [AO], [BO], [CO] and [DO].

  - What do you notice?

- Teacher asks students to choose a vertex (other than "C") of the parallelogram and drag it.

  - What happens to the segments formed by the diagonals?
  - What lengths change based on the vertex they choose to drag?
  - What lengths remain constant based on the vertex they choose to drag?
  - When the lengths of the segments vary, are there relationships between them that remain invariant?
Students should explore and find out that point O is the midpoint of the diagonals [AC] and [BD].

- Together teacher and students state the hypothesis of a property of a parallelogram:

\[
\text{The diagonals of a parallelogram bisect each other.}
\]

**Part II: Interplay definition/properties (Reciprocals)**

**Activity 2**

- Teacher asks students to draw two segments that bisect each other.

**Steps of Construction:**

- Create three non-collinear points E, O and F
- Construct the symmetricals of points “E” and “F” with respect to point “O”
- Label the new points as “G” and “H”
- Draw the segments [EG] and [FH] as well as the segments that make up the Quadrilateral EFGH
- Teacher asks students to check, based on what they have just learnt, whether EFGH is a parallelogram.
- Students should check whether both pairs of opposite sides are parallel (the definition of the parallelogram).

Together teacher and students state the hypothesis of a property of a parallelogram:

\[
\text{If diagonals of a quadrilateral bisect each other then the quadrilateral is a parallelogram.}
\]

**Activity 3**

Teacher asks students to draw a quadrilateral having one pair of opposite sides parallel and equal.

**Steps of Construction:**

- Use “Numerical Edit” to determine one value of segment length
- Create a ray having point “A” as its origin
- Create a point “D” not belonging to the ray
- Through Point “D” construct a line (L) parallel to the ray
- Construct a ray having “D” as its origin going in the same direction as the ray with origin “A”
- Hide the line (L)
- Use "Measurement Transfer" on both rays with the same measure entered as "Numerical Edit" and label the points "B" and "C"

- Draw the segments [AB], [BC], [CD] and [AD]

- Teacher asks students to check whether the quadrilateral ABCD is a parallelogram.

- Students should check whether the other pair of opposite sides is parallel i.e. using the definition of the parallelogram.

- Together teacher and students state the hypothesis of a property of a parallelogram:

\[
\text{If one pair of opposite sides of a quadrilateral is parallel and equal then the quadrilateral is a parallelogram.}
\]
Part III: Shift to Formal Proofs

The truth of a mathematical statement is not established until it is proved. Even when a property is observed to be true through many cases of the figure, a formal proof is the only mathematical way to establish it. Therefore, the following part of the lesson will have students practice writing deductive proofs using pencil-and-paper without any use of DGS.

Activity 4

Teacher asks students to prove the following theorem:

**If both pairs of opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram**

![Diagram](image)

Given: angle M = angle P and angle Q = angle N
Prove: MNPQ is a parallelogram

Activity 5

Teacher asks students to prove the following theorem:

**If the two pairs of opposite sides of a quadrilateral are equal then the quadrilateral is a parallelogram**

![Diagram](image)

Given: EF=GH and EH=FG
Prove: EFGH is a parallelogram
**Activity 6**

- Teacher asks students to prove the following theorem:

  \[
  \text{The opposite sides of a parallelogram are equal}
  \]

- Given: ABCD is a parallelogram
  
  Prove: AB = DC and AD = BC

**Activity 7**

- Teacher asks students to prove the following theorem:

  \[
  \text{The opposite angles of a parallelogram are equal}
  \]
  \[
  \text{The consecutive angles of a parallelogram are supplementary}
  \]

- Given: EFGH is a parallelogram
  
  Prove: a) \( \angle \text{HEF} = \angle \text{FGH} \);
  \angle \text{EHG} = \angle \text{EFG} \)
  
  b) Angles HEF and EFG are supplementary; angles EHG and FGH are supplementary
Activity 8

- Teacher asks students to prove the following theorem:

If diagonals of a quadrilateral bisect each other then the quadrilateral is a parallelogram

Given: [XZ] and [YW] bisect each other

Prove: XYZW is a parallelogram
Homework

1) Prove the following theorem:

**The diagonals of a parallelogram bisect each other**

Given: RAND is a parallelogram
Prove: [RN] and [AD] bisect each other

2) Prove the following theorem:

**If one pair of opposite sides of a quadrilateral are equal and parallel then the quadrilateral is a parallelogram**

Given: [MQ]=[NP] and MQ=NP
Prove: MNPQ is a parallelogram
Given: PILE is a parallelogram; [IN] is perpendicular to [EL]; [EM] is perpendicular to [IP]

a) Show that triangles PME and LNI are congruent.

b) The line (MN) cuts (LI) in point K and cuts (PE) in point J. Prove that triangles MIK and JEN are congruent.

c) Prove that JEKI is a parallelogram.
Rectangles
Lesson 2: Rectangle

Objectives:
- Define a rectangle
- Identify and prove the properties of a rectangle
- Use suitable properties of a quadrilateral to prove that it is a rectangle

Procedure:
- Together teacher and students correct the homework of Lesson 1.

Part 1: Exploration of Properties of a Rectangle

Activity 1

- Teacher asks students to construct a parallelogram ABCD using Cabri (use the "Steps of Construction" under Activity 1 in Lesson 1).
- Teacher asks students to measure under Cabri the angles of ABCD.
- Teacher asks students to drag slowly any vertex of parallelogram ABCD (other than "C") and observe the various looks of a parallelogram.
- Teacher asks: Is there a special position that you noticed?
  - Some students may find the position of the rhombus. But it is more likely that the shape that will be first identified is the rectangle. If the rhombus is also identified, the two special shapes are named, then the teacher suggests that in this session the properties of a rectangle will be studied.
- Teacher asks students two questions:
  - What do you notice about the measures of all angles? (Students should acknowledge that all the angles become 90°).
- Can one of the angles be 90° while others are not? Why? (A deductive argument must be given).

- Teacher and students provide the first definition of a rectangle:

\[ A \text{ rectangle is a parallelogram having four right angles} \]

- Teacher asks students:

  - Do all the angles have to be right for the parallelogram ABCD to be a rectangle?
  - What's the minimum number of angles that should be right for ABCD to be a rectangle? (Students should acknowledge that it is enough for one angle to be right so that parallelogram ABCD is a rectangle.

- Teacher and students provide the second definition of a rectangle:

\[ A \text{ parallelogram with one right angle is a rectangle} \]

- Through the above two definitions teachers and students deduce that a rectangle is also a parallelogram and that the rectangle has the same properties of a parallelogram plus others to be identified and verified.

- Teacher and students remember the properties of a parallelogram: opposite sides are parallel, opposite sides are equal, opposite angles are equal, consecutive angles are supplementary and diagonals bisect each other.

- Teacher asks students:

  - Does the rectangle have special properties that make it special and distinguished from any other parallelogram?
**Activity 2**

- Teacher asks students to construct a new rectangle using Cabri, such that the constructed figure will remain a rectangle upon dragging.

**Steps of Construction:**

- Construct [AB]
- Construct a perpendicular line (L1) to [AB] at point A
- Construct a perpendicular line (L2) to [AB] at point B
- Construct point “C” as point on object on (L2)
- Construct a perpendicular line (L3) at “C” to line BC
- Construct the intersection point of the two lines as point “D”
- Draw the segments [BC], [CD] and [AD]
- Hide the three lines (L1), (L2) and (L3)

- Teacher asks students to check whether ABCD is a rectangle (students should find the measures of the angles i.e. check the definition of the rectangle).

- Teacher asks students to drag any vertex of ABCD and check whether it remains a rectangle.
- Students should understand by now that they cannot drag point “D” because it not an initial object.
- Teacher asks students to explore the rectangle, try to verify that all properties of a parallelogram apply, and to detect other extra properties. Hopefully, when verifying that diagonals bisect each other, they will notice and conjecture that the diagonals are equal.

Students should acknowledge that $AC = BD$.

- Teachers and students state the hypothesis of a property of a rectangle:

  \[ \text{The diagonals of a rectangle are equal} \]

### Part II: Interplay Definition/Properties (Reciprocals)

#### Activity 3

- Teacher asks students to draw a quadrilateral having its diagonals bisect each other and equal at the same time.

**Steps of Construction:**

- Create a circle of center “O”
- Draw two straight lines (L1) and (L2) passing through the center “O”
- Label the points of intersection of (L1) with the circle as points “A” and “C” and the points of intersection of (L2) with the circle as points “B” and “D”
- Hide the two straight lines
- Draw segments [AC] and [BD] (Of course the center belongs to both segments)
- Teacher asks students: Are [AC] and [BD] equal? Why?
- Measure the lengths of the segments [AC], [BD], [AO], [BO], [CO] and [DO] to make sure that AB=CD and that "O" is the midpoint of these segments
- Draw segments [AB], [BC], [CD] and [AD]
- Hide the circle

- Teacher asks students to check whether ABCD is a rectangle.
  - Students should check whether the angles of ABCD are right or not (the definition of a rectangle).

- Together teacher and students state the hypothesis of a property of a rectangle:

  If the diagonals of a quadrilateral are equal and bisect each other then the quadrilateral is a rectangle
Part III: Shift to Formal Proofs

Activity 4
- Teacher asks students to prove the following theorem:

\[ \text{The diagonals of a rectangle are equal} \]

Given: MNOP is a rectangle
Prove: MO = NP

Activity 5
- Teacher asks students to prove the following theorem:

\[ \text{If the diagonals of a quadrilateral are equal and bisect each other then the quadrilateral is a rectangle} \]

Given: MK=AR and [MK] and [AR] bisect each other
Prove: MAKR is a rectangle
1) \hspace{1cm}

Given: PARC is a rectangle and point “O” is the point of intersection of the diagonals of PARC.
Suppose that CO = x+1 and PR = 3x, find x.

2) Prove the following theorem:

\begin{center}
\emph{The axes of symmetry of a rectangle are the perpendicular bisectors of its sides}
\end{center}

Given: EFGH is a rectangle; (TP) is the perpendicular bisector of [EH]
Prove: (TP) is the perpendicular bisector of [FG]

- Similarly, if (LM) is the perpendicular bisector of [EF] then (LM) is the perpendicular bisector of [FG].
Rhombus
Lesson 3: Rhombus

Objectives of the lesson:

- Define a rhombus
- Identify and prove the properties of a rhombus
- Use suitable properties of a quadrilateral to prove that it is a rhombus

Procedure:

- Together, teacher and students correct the homework of Lesson 2.

Part I: Exploration of the Properties of a Rhombus

Activity 1

- Teacher asks students to construct a parallelogram ABCD using Cabri (use the "Steps of Construction" under Activity 1 in Lesson 1).
- Teacher asks students to measure the lengths of the sides of ABCD.
- Teacher asks students to drag slowly any vertex of parallelogram ABCD (other than "D") and observe the various looks of a parallelogram.

  - Teacher should draw students’ attention to the fact that dragging point “A” will not help students in reaching a figure where all sides are equal because when point “A” is dragged in any direction, all the lengths of the sides change.

  - Teacher should draw students’ attention to the fact that students should either drag points “B” or “D” because that way only the length of one pair of opposite sides will change while the length of the other pair will remain invariant.
- Teacher asks students:
  
  - Can you find a special shape? The rectangle is already found but can you find yet another special shape?

- Together teacher and students provide the first definition of a rhombus:
  
  \[ A \text{ rhombus is a parallelogram having four equal sides} \]

- Teacher asks students:
  
  - Do all the sides have to be equal for the parallelogram ABCD to be a rhombus?
  
  - What's the minimum number of sides that should be equal for ABCD to be a rhombus? (Students should acknowledge that it is enough for two consecutive sides to be equal so that parallelogram ABCD is a rhombus because the parallelogram has two pairs of opposite sides equal).

- Teacher and students provide the second definition of a rhombus:
  
  \[ A \text{ parallelogram with two consecutive sides equal is a rhombus} \]

- Through the above two definitions teachers and students deduce that a rhombus is also a parallelogram and that the rhombus has all the properties of a parallelogram plus others to be identified and verified.

- Teacher and students remember the properties of a parallelogram: opposite sides are parallel, opposite sides are equal, opposite angles are equal, consecutive angles are supplementary and diagonals bisect each other.
- Teacher asks students:
  - Does the rhombus have special properties that make it special and distinguished from any other parallelogram?

Activity 2

- Teacher asks students to construct a new rhombus using Cabri, such that the constructed figure will remain a rhombus upon dragging.

Steps of Construction

- Construct [AB]
- Using the compass tool, construct a circle of center “A” and radius [AB]
- Using “Point on Object” construct a point “C” on the circle, not on the line (AB)
- Join points “A” and “C”
- Construct a line (L1) through “C” parallel to [AB]
- Construct a line (L2) through “B” parallel to [AC]
- Label point “D” as the intersection point of the two straight lines (L1) and (L2)
- Draw segments [CD] and [BD]
- Hide the circle and the two straight lines (L1) and (L2)
- Teacher asks students to check whether the quadrilateral is a rhombus.
- Students should check whether the lengths of the four sides are equal i.e. the definition of the rhombus.
- Teacher asks students to drag any vertex they choose and check whether the quadrilateral remains a rhombus.
- Students should know by now that they can drag points "A", "B" and "C" but not "D".
- Teacher should draw students' attention to the fact that point "C" can only move on the circumference of the hidden circle and not in any other direction.

- Teacher asks students to explore the rhombus, try to verify that all properties of a parallelogram apply, and to detect other extra properties. Hopefully, when verifying that diagonals bisect each other, they will notice and conjecture that the diagonals are perpendicular.
- Teacher asks students to choose a vertex (other than "D") and drag the rhombus ABDC and asks them:
  - What do you notice?
  - Do the angles formed by the diagonals remain 90° upon dragging?

- Together teacher and students state the hypothesis of a property of a rhombus:
  
  The diagonals of a rhombus are perpendicular

- Teacher asks students to measure angles BCD and BCA as well as the measures of angles CBA and CBD and asks them:
  - What do you notice?

- Teacher asks students to drag any vertex (other than "D") and asks them the question:
  - When the measures of the angles vary, are there any relationships between them that remain invariant?

- Teacher asks students to check what they just explored with the other pairs of opposite angles.

- Teacher and students state the hypothesis of a property of a rhombus:
  
  Each diagonal of a rhombus bisects a pair of opposite angles
Part II: Interplay Definition/Properties (Reciprocals)

Activity 3

- Teacher helps students construct a parallelogram whose diagonals are perpendicular.

Steps of Construction:

- Create a point “O”
- Draw two perpendicular lines (L1) and (L2) at point “O”
- Construct point “A” on (L1) and point “B” on (L2)
- Draw [AB]
- Reflect [AB] about the line (L2)
- Label the new point as “C”
- Reflect [BC] about line (L1)
- Label this new point as “D”
- Reflect segment [CD] about line (L2)
- Hide the two perpendicular lines (L1) and (L2)
- Join the (perpendicular) diagonals [AC] and [BD]
- Teacher asks students to check whether this quadrilateral is a parallelogram.
  - Students should check whether any property of a parallelogram holds.
- Teacher asks students to check whether this quadrilateral is a rhombus.
  - Students would check if the lengths of the sides of the quadrilateral are equal.

- Teacher and students state the hypothesis of a property of a rhombus:

  If the diagonals of a parallelogram are perpendicular then the parallelogram is a rhombus
Part III: Shift to Proof

Activity 4

Teacher asks students to prove the following theorem:

*The diagonals of a rhombus are perpendicular to each other*

Given: MNPQ is a rhombus

Prove: Diagonals [MP] and [NQ] are perpendicular

Activity 5

Teacher asks students to prove the following theorem:

*Each diagonal of a rhombus bisects a pair of opposite angles*

Given: RAST is a rhombus

Prove: Diagonal [RS] bisects the opposite angles TRA and AST

Activity 6

Teacher asks students to prove the following theorem:

*The diagonals of a rhombus are the axes of symmetry*

Given: ABCD is a rhombus

Prove: Diagonals (AC) and (BD) are the axes of symmetry
1) ABCD is a quadrilateral where the opposite sides are equal. Suppose AB = x and BC = 2x. Find x so that ABCD is a rhombus.

2) Prove the following theorem:

If the diagonals of a parallelogram are perpendicular then the parallelogram is a rhombus

Given: XYZW is a parallelogram with [XZ] perpendicular to [YW]
Prove: XYZW is a rhombus

3) Given: LIRE is a rhombus having the measure of angle LIR = 120° and IE = 6 cm

a) Calculate the measure of all angles in the figure

b) What is the length of each side of LIRE? Justify your answer

4) Prove the following theorem:

A quadrilateral in which the diagonals are axes of symmetry is a rhombus
Square
Lesson 4: Square

Objectives:
- Define a square
- Identify and prove the properties of a square
- Use suitable properties of a quadrilateral to prove that it is a square

Procedure:
- Together teacher and students correct the homework of Lesson 3.

Part I: Exploring Squares

Activity 1

- Teacher asks students to construct a rectangle ABCD using Cabri (use the "Steps of Construction" under Activity 2 in Lesson 2).

- Teacher asks students to find the lengths of the sides of ABCD.

- Teacher asks students to drag slowly a vertex of ABCD (other than point “D”).

- Teacher asks students:
  - Is there a special position that you noticed? (Students should find the position of a square).

- Teacher and students define:

  A square is a rectangle having all its sides equal

- Teacher explains to students that a square has all the properties of a rectangle plus others to be identified and verified.
- Teacher explains to students that since a rectangle is a parallelogram then so is a square. Therefore, a square has all the properties of a parallelogram plus others to be identified and verified.

- Teacher and students discuss the above definition of a square and emphasize that “having all its sides equal” makes it a rhombus.

- Therefore, teacher and students conclude that:

  \[ A \text{ square is a rhombus} \]

- Teacher explains to students that since a square is a rhombus then the square has all the properties of the rhombus and more.

- Teacher and students discuss the axes of symmetry of a square:
  - Since the square is a rectangle then the perpendicular bisectors of its sides are axes of symmetry.
  - Since the square is a rhombus then its diagonals are axes of symmetry.

- Therefore, teachers and students conclude that the square has four axes of symmetry.

\[ Activity\ 2 \]

- Teacher asks students to construct a square using Cabri such that the constructed figure will remain a square upon dragging.

\[ \text{Steps of Construction:} \]
  - Draw a circle with center “O”
  - Construct two perpendicular lines (L1) and (L2) through center “O”
  - Label the points “A” and “C” as the points of intersection of (L1) and the circle
  - Label the points “B” and “D” as the points of intersection of (L2) and the circle
- Draw the segments [AB], [BC], [CD] and [AD]
- Hide the lines (L1) and (L2) as well as the circle

- Teacher asks students to check whether the constructed figure is a square.

- Teacher asks students to explore the square and verify that the properties of a rectangle, a rhombus and a parallelogram apply.

Part II: Shift to Proof

Activity 3

- Teacher asks students to prove the following theorem:

A quadrilateral whose diagonals are equal, perpendicular and bisect each other is a square

Given: VS=AI, [VS] and [AI] bisect each other; [AI] is perpendicular to [VS]
Prove: VISA is a square
Homework

1)
   a) Let angle XNY be a right angle. Mark point "A" on [NX) and point "I" on [NY) such that NI=NA
   
   b) Construct the point J symmetric of N with respect (AI)
   
   c) What is the nature of quadrilateral NAJI? Justify your answer

2) Given: triangle RST is a right isosceles triangle at "S".

   Construct "U" the symmetric of "R" with respect to "S" and "V" the symmetric of "I"
   with respect to "S"

   Prove: RVUT is a square

3) Given: PUIS is a square

   a) Construct "L" the symmetric of "P" with respect to "U" and "O" the symmetric of "I"
      with respect to "U"

   b) What is the nature of the quadrilateral POLI?
Summative Test

1) Identify the type of quadrilaterals below and state the definition/postulate/theorem that supports your answer:

(a) ABCD is a quadrilateral having “O” as the point of intersection of its diagonals such that AO=CO and BO=DO. Name ABCD.

(b) HIJK is a quadrilateral having “L” as the point of intersection of its diagonals such that HL=LJ=IL=KL. Name HIJK.

(c) EFGH is a quadrilateral having “V” as the point of intersection of its diagonals such that EV=VG=HV=VF and [EG] is perpendicular to [HF]. Name EFGH.

(d) MNPQ is a quadrilateral such that MN=QP and MQ=NP and angle MQP=90°. Name MNPQ.

2) Draw diagrams to **disprove** the following statements that a quadrilateral is a parallelogram if it has:

i) One pair of sides parallel

ii) One pair of opposite sides equal

3) Given: [AB] [CD] and [AD] [BC] and angle BCD=120°

   Find: angle DAB and angle ABC

4) Given: MNPQ is a parallelogram. MN=6 cm and MQ=QP

   Find: NP and QP

5) Given: [EF] [HG] and EF=HG; angle EHG = 90° and FG = 3 cm

   Find: angle EFG and EH
6) Given: ABCD is a parallelogram and angle DBC = 45° and angle BDC = 30°.

Find: angle DAB

7) In a parallelogram ABCD, if the measurement of angle B exceeds the measurement of angle A by 50°, find the measurement of all angles in the parallelogram ABCD.

8)

ABCD is a rhombus such that angle BAC = 50°. DC is produced to point "P" such that and CP = BC. Draw [BP]. Let angle BPC = x. Draw the figure and find x, giving reasons for each step you provide.

9) Given: MNPQ is a parallelogram; from "M" drop a perpendicular to [QP] intersecting it at point "S"; Similarly, from "P" drop a perpendicular to [MN] intersecting it at point "T". Draw the figure and prove that MTPS is a rectangle

10) ABCD is a parallelogram. DC and AB are produced to intersect at point "P" such that CP = CD. [AP] intersects [BC] at point "Q". Draw the figure and prove that CQ = BQ.

What is the nature of quadrilateral ABPC?

11)

Given: MNPQ is a parallelogram. S is the midpoint of [MQ] and R is the midpoint of [NP]

Prove: MNRS is a parallelogram
12) Given: triangle ABC is equilateral. P, Q and R are the midpoints of [AB], [BC] and [AC] respectively. Draw the figure

Prove that: (i) triangle PQR is an equilateral triangle
(ii) How many parallelograms are there in the figure? Prove your answer.
(iii) Show that each of these parallelograms is a rhombus.

13) ABCD is a parallelogram. From point “C” draw a line that intersects [AB] at point “P” such that “P” is between points “A” and “B” and CP=BC. Draw the figure and show that angle ADC= angle PCD.

Source of exercises: http://www.geocities.com/mathematicsplus and “Building up Mathematics”

grade eight mathematics textbook
References


Brown, D.F. (2006). It's the curriculum, stupid: there's something wrong with it: educators, parents, and employers all seem to agree on the types of skills they believe students should be developing. But Mr. Brown finds that the traditional curriculum, divided up into separate subjects, neither engages students nor prepares them for productive lives. He believes that the answer to both problems is to have students design their own curricula. *Phi Delta Kappan* 87(10), 777. Retrieved on March 23, 2007, from Expanded Academic ASAP via Thomson Gale from http://find.galegroup.com/itx/infomark Doc Id= A148483331.
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Appendix: Quadrilateral chapters from “Building up mathematics” grade eight Lebanese textbook
introduction  A square? A good square!

The square is the only geometric shape that is easily recognized. It is quite regular, well symmetrical, and many of its properties are easily proven.

One question arises: what are the minimum properties necessary for a quadrilateral to be a square?

At the beginning of this chapter, I am able to:
- prove that the sum of the angles of a quadrilateral is 360°;
- recognize the fact that in a square:
  - the angles are right;
  - the sides are congruent (equal);
  - the diagonals are congruent;
  - the diagonals bisect each other;
  - the diagonals are perpendicular.

At the end of this chapter, I will be able to:
- identify a square as a quadrilateral having a right angle and congruent sides;
- identify a square as a quadrilateral in which the diagonals are perpendicular, congruent, and bisect each other;
- identify the elements of symmetry in a square.
Activity 1

Four .. Quadra .. Quadrilateral

1) What do you call the adjacent figure?
2) What do the points A, B, C, and D represent?
3) What do you call the segments [AB], [BC], [CD], and [DA]?
4) What are the angles of the figure?
5) What do you call the segments [AC] and [BD]?

Activity 2

There are degrees in every corner!

1) Draw a quadrilateral ABCD of your choice.
2) If you draw one of its diagonals, how many triangles will you obtain? Name them.
3) Using the sum of the angles in a triangle, calculate the sum of the angles in a quadrilateral.

Preparatory activities

Activity 1

Four sides against one angle!

1) Construct a quadrilateral having four equal sides and one right angle.
2) What is the shape of the obtained quadrilateral?

Activity 2

Well crossed diagonals...

1) Construct a quadrilateral whose diagonals are equal and bisect each other at right angles.
2) What is the shape of the obtained quadrilateral?
I. Definition and properties

**Definition 1** A square is a quadrilateral with equal sides and equal angles.

1. The square and its angles

   - The sum of the four angles of a square is 360° (since a square is a quadrilateral);
   - The four angles are equal (according to Definition 1);
   - So each angle measures 90° (360° / 4 = 90°).
   - Consequently:

**Property 1** In a square, the four angles are right.

2. The square and its sides.

   - Given a square ABCD.
   - [AD] ⊥ [AB] since BAD = 90°;
   - [BC] ⊥ [AB] since ABC = 90°;
   - So [AD] // [BC] because they are perpendicular to the same segment.

**Property 2** In a square, the adjacent sides are perpendicular and the opposite sides are parallel.

3. The square and its diagonals

   - Given a square ABCD.
   - Consider the triangles ABC and BAD. We have:
     - AD = BC: opposite sides of a square;
     - [AB]: common side;
     - ABC = BAD = 90°: angles of a square.
   - So the two triangles are congruent by S.A.S. and we have AC = BD, therefore:

**Property 3** In a square, the diagonals are equal.

**Property 4** In a square, the diagonals bisect each other at right angles and bisect the angles of the square.
II. When is a quadrilateral a square?

1. Starting from the sides and angles

Let \(ABCD\) be a quadrilateral in which the sides are equal and one angle \(\triangle DAB\) is right. Consider the two triangles \(BAD\) and \(BCD\).

We have:

\[BA = BC, \quad DA = DC, \quad \text{and } [DB] \text{ is a common side.}\]

So, the two triangles are congruent by S.S.S.

Then \(\triangle BCD = \triangle BAD = 90^\circ\) (corresponding angles).

Triangle \(ABD\) is right isosceles, so \(\overset{\frown}{ABD} = \overset{\frown}{ADB} = 45^\circ\).

Triangle \(CBD\) is right isosceles, so \(\overset{\frown}{CBD} = \overset{\frown}{CDB} = 45^\circ\).

Therefore, \(\overset{\frown}{ABC} = \overset{\frown}{ADC} = 90^\circ\).

Consequently, the quadrilateral \(ABCD\) is a square that has four equal angles and four equal sides. Thus:

**Property 5** A quadrilateral having four equal sides and one right angle is a square.

2. Starting from the diagonals

Let \(ABCD\) be a quadrilateral in which the diagonals \([AC]\) and \([BD]\) are equal, perpendicular, and bisect each other at \(O\).

Each diagonal is thus the perpendicular bisector of the other, and each is an axis of symmetry of the other.

We have: \(AB = BC = CD = DA\).

On the other hand, \(BAO\) is a right isosceles triangle, so \(\overset{\frown}{BAO} = 45^\circ\); and \(OAD\) is also a right isosceles triangle. So \(\overset{\frown}{OAD} = 45^\circ\).

Then, we have: \(\overset{\frown}{BAD} = \overset{\frown}{BAO} + \overset{\frown}{AOD} = 90^\circ\). Therefore:

**Property 6** A quadrilateral in which the diagonals are perpendicular, equal, and bisect each other is a square.
**Focus**

**In a square \(ABCD\):**
- the sides have the same length: \(AB = BC = CD = DA\);
- the opposite sides are parallel: \([AB] \parallel [DC]\) and \([BC] \parallel [AD]\);
- the diagonals and the perpendicular bisectors of the sides are axes of symmetry;
- the point of intersection of the diagonals is a center of symmetry.

![Diagrams of a square showing axes of symmetry and a center of symmetry.]

**To prove that a quadrilateral \(ABCD\) is a square, it is sufficient to prove that one of the following properties is satisfied:**
- starting from the definition:
  - the four sides are equal and the four angles are equal;
- starting from sides and an angle:
  - four equal sides and one right angle;
- starting from the diagonals:
  - the diagonals are perpendicular, equal, and bisect each other;
- starting from the diagonals and angles:
  - the diagonals are equal and bisect the angles of the quadrilateral.

**How to draw a square?**

1. I draw a circle
2. I draw two perpendicular diameters
3. I join the four extremities of the diameters
Exercises

1. Indicate which is a square among the following quadrilaterals:

2. Construct a square AMIS in which SA=4cm. Explain.

3. Construct a square ALOR given LR=6cm. Explain.

4. a) Draw a line (d) and take a point D not lying on (d).
   
b) Construct a square ANLD in which A and N lie on (d). Explain.

5. a) Consider two distinct points B and O.
   
b) Construct the square BILEN having O as center of symmetry. Explain.

6. a) Construct a square ANOR whose side is 3cm.
   b) Construct a square RIEN.

7. a) Observe the following figure and count the squares.
   
b) Reproduce the diagram on your copybook and indicate how many squares you have drawn.

8. Along the checked squares on your copybook, explain how to draw a square having the red dashes for a side and another square having the blue dashes for a diagonal.

Self-evaluation

A

a) Let $xN y$ be a right angle. Mark a point $A$ on $[Nx)$ and a point $l$ on $[Ny)$ such that $NI = NA$.

b) Construct the point $J$ symmetric of $N$ with respect to $(AI)$.

c) What is the nature of the quadrilateral $NAJl$? Justify your answer.

B

a) Draw a circle $C(O; 3cm)$.

b) Draw two perpendicular diameters $[RM]$ and $[AI]$.

c) What is the nature of quadrilateral $IMAR$? Justify your answer.
1. In the following figure, MARS is a square.

a) Show that [MR] and [AS] are the perpendicular bisectors of each other.

b) Show that the diagonals of a square are perpendicular and bisect each other.

2. a) Draw two lines perpendicular to each other at O; on one of these two lines, take a point E such that OE = 2 cm.

b) Construct a square MIRE having the two lines as axes of symmetry.

c) Construct another square with one of its sides passing through E and having the two lines as axes of symmetry.

3. a) Construct an isosceles triangle LOI right angled at O.

b) Construct N the symmetric of L and A the symmetric of I with respect to O.

c) Show that NALI is a square.

4. a) Construct a square PUIS of side 4 cm.

b) Construct L the symmetric of P and O the symmetric of I with respect to U.

c) Show that POLI is a square.

5. Given a point O, construct a square whose diagonal is 5 cm long and having O as a center of symmetry. Explain.

6. The following figure represents a square MAIN with its axes of symmetry and its center of symmetry B.

a) How many squares are there in the figure?

b) Show that PLUS is a square.

7. a) Construct an isosceles triangle ART right angle at A.

b) Construct I the symmetric of A with respect to (RT).

c) Show that the quadrilateral TARI is a square.

8. In the following figure, the red square has a side of 4 cm.

Calculate the area of the red region, the blue region, and the yellow region.
9. a) Reproduce the following figure.

b) What are the coordinates of R and N?

c) On the system \( x'Ox, y'Oy \), draw a square \( RIEN \) having \([RN]\) as a side and \( O \) as a center.

d) What are the coordinates of the points I and E?

e) How can you find the area of \( RIEN \)?

f) Locate the point \( D \) such that \( ROND \) is a square.

g) What are the coordinates of \( D \)? What is the area of \( ROND \)?

10. a) What are the properties indicated in the following figure?

b) Reproduce the figure.

c) What property is satisfied by the points \( A, I, \) and \( J \)? Prove this property.

**Fraction in a square!**

What fraction of the square does the red region represent?

*Extracted from Petit x n° 27 year 1990-1991. IREM de Grenoble.*
introduction  An elongated square!

The rectangle is a form that is less regular than a square. Some of its properties are evident and easily recognized and others are not.

The study of rectangles completes the one of squares. We will study exactly how to recognize and construct a rectangle.

At the beginning of this chapter, I am able to:

- recognize the fact that in a rectangle, angles are right;
- two opposite sides are parallel and equal;
- the diagonals are equal;
- the diagonals bisect each other.

At the end of this chapter, I will be able to:

- identify a rectangle as a quadrilateral having three right angles;
- identify a rectangle as a quadrilateral in which the diagonals are equal and bisect each other;
- identify the elements of symmetry in a rectangle.
Recall activities

Activity 1

Look and you will find..

1) What do you call the quadrilateral $ABCD$ in the adjacent figure?
2) What are its vertices?
3) What kind of angles does it have?
4) What do you call the segments $AC$ and $BD$?
5) What are the geometric properties of this figure?

Activity 2

Draw and learn!!

1) Draw any rectangle $ABCD$ of your choice.
2) Indicate all the properties you can see in the figure.

Preparatory activities

Activity 1

What if a square is stretched?

Imagine holding a square from two opposite, parallel sides and pulling them apart. Which properties are then preserved and which ones disappear?

<table>
<thead>
<tr>
<th>Property</th>
<th>preserved</th>
<th>not preserved</th>
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</thead>
<tbody>
<tr>
<td>right angles</td>
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<td>perpendicular diagonals</td>
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</tr>
<tr>
<td>other axes of symmetry</td>
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</tr>
</tbody>
</table>

Activity 2

Three angles quite right.. intersecting diagonals..

1) Construct a quadrilateral having three right angles.
2) Construct a quadrilateral having three right angles and two equal adjacent sides.
3) Construct a quadrilateral in which the diagonals are equal and bisect each other.
4) What is the shape of the quadrilateral obtained in each figure?
I. Definition and properties

**Definition 1**
A rectangle is a quadrilateral in which all angles are equal.

Here are the proofs of some properties of a rectangle:

1. **The rectangle and its angles**
   Given a rectangle $ABCD$.
   The sum of its four angles is $360^\circ$ (since it is a quadrilateral); its four angles are equal (according to the definition); so each angle measures $90^\circ$ ($360^\circ \div 4 = 90^\circ$). Therefore, we have:

   **Property 1**
   In a rectangle, the four angles are right.

2. **The rectangle and its sides**
   Given a rectangle $ABCD$.
   $[AD] \perp [AB]$ since $BAD = 90^\circ$;
   $[BC] \perp [AB]$ since $ABC = 90^\circ$;
   so $[AD] \parallel [BC]$ being perpendicular to the same segment. Thus, we have:

   **Property 2**
   In a rectangle, the two opposite sides are equal and parallel, and the two adjacent sides are perpendicular.

3. **The rectangle and its diagonals**
   Given a rectangle $ABCD$.
   Consider the triangles $ABC$ and $BAD$. We have:
   $AD = BC$ : opposite sides of a rectangle;
   $[AB]$ : common side;
   $ABC = BAD = 90^\circ$ : angles of a rectangle.
   So the two triangles are congruent by S.A.S. and we have : $AC = BD$.

   **Property 3**
   In a rectangle, the diagonals are equal.

   we similarly prove that:

   **Property 4**
   In a rectangle, the diagonals bisect each other.
II. What makes a quadrilateral a rectangle?

1. Starting from the angles

Let $ABCD$ be a quadrilateral in which the angles $A$, $B$, and $C$ are right.

The sum of the angles of $ABCD$ is $360^\circ$.

So, $\sum = 360^\circ - (A + B + C) = 360^\circ - (90^\circ + 90^\circ + 90^\circ) = 90^\circ$.

Therefore, $ABCD$ is a quadrilateral in which the four angles are equal and, consequently, it is a rectangle. Thus:

**Property 5**

A quadrilateral having three right angles is a rectangle.

2. Starting from diagonals

Let $ABCD$ be a quadrilateral in which the diagonals $[AC]$ and $[BD]$ are equal and bisect each other at $O$.

$OA = OC$ since $O$ is the center of symmetry of segment $[AC]$.  
$OB = OD$ since $O$ is the center of symmetry of segment $[BD]$.

Then, $O$ is the center of symmetry of the figure $ABCD$, and we have $AB = DC$, $AD = BC$, $\widehat{A} = \widehat{C}$, and $\widehat{B} = \widehat{D}$.

On the other hand, in triangles $ABD$ and $BAC$, we have:

$AC = BD$ : by hypothesis,

$AD = BC$ : by symmetry,


Therefore, these two triangles are congruent by S.S.S. and so $\widehat{A} = \widehat{B}$. Then we have $\widehat{A} = \widehat{B} = \widehat{C} = \widehat{D}$.

Consequently:

**Property 6**

A quadrilateral in which the two diagonals are equal and bisect each other is a rectangle.
In a rectangle $ABCD$:
- the opposite sides are equal: $AB = DC$ and $BC = AD$;
- the angles are right: $ABC = BCD = CDA = DAB = 90^\circ$;
- the point of intersection of the diagonals is a center of symmetry;
- the perpendicular bisectors of the sides are axes of symmetry.
- the opposite sides are parallel: $[AB] \parallel [DC]$ et $[BC] \parallel [AD]$;
- the diagonals $[AC]$ and $[BD]$ are equal and bisect each other: $AC = BD$; $AO = OC = BO = OD$;

To prove that a quadrilateral is a rectangle, it is sufficient to prove one of the following properties:
- starting from the definition:
  the four angles are equal;
- starting from the sides and an angle:
  the opposite sides are equal (or parallel) and one angle is right;
- starting from the diagonals:
  the diagonals are equal and bisect each other.
- starting from the angles:
  three angles are right.

How to draw a rectangle?

I draw a circle  
I draw two diameters (not perpendicular)  
I join the four extremities of the diameters
### Exercises

1. Among the following quadrilaterals, find the rectangles:

2. Construct a rectangle \( DARI \) in which \( RI = 4 \text{ cm} \) and \( ID = 6 \text{ cm} \). Explain.

3. a) Construct a rectangle \( NASO \) in which \( NS = 8 \text{ cm} \). Explain.

   b) Compare your rectangle with some of your classmates. Did you obtain equal rectangles?

4. a) Let \( BRAM \) be a quadrilateral in which \( \widehat{A} = \widehat{I} = x, \widehat{M} = 180^\circ - x, \) and \( \widehat{R} = 2x - 90^\circ \). What is the value of \( x \)?

   b) Mark three distinct non-collinear points \( N, O, \) and \( S \).

   c) Construct the quadrilateral \( NOIR \) in which \( S \) is a center of symmetry. Explain.

   d) What condition should \( N, O, \) and \( S \) satisfy for \( NOIR \) to be a rectangle?

6. Calculate all the angles in the following figure where \( RECT \) is a rectangle.

7. Construct a rectangle \( BLEU \) in each case:

   a) \( BL = 4 \text{ cm} \) and \( LU = 5 \text{ cm} \).

   b) \( LU = 6 \text{ cm} \) and \( EUL = 35^\circ \).

### Self-evaluation

**A**

a) On the sides \([Dx]\) and \([Dy]\) of a right angle, mark the points \( A \) and \( I \), respectively.

b) Through \( I \), draw the line parallel to \([Dx]\) and through \( A \) the line parallel to \([Dy]\). Let \( M \) be the point of intersection of these two lines.

C) What is the nature of the quadrilateral \( MADP \)? Why?

**B**

a) Construct an isosceles triangle \( LAO \) in which \( OA = OL \).

b) Construct the symmetric \( I \) of \( A \) and \( S \) of \( L \) with respect to \( O \).

C) What is the nature of the quadrilateral \( SALP \)? Why?

**C**

Why is a square a rectangle?
Problems

1. Given a rectangle $NUIT$ where $O$ is the point of intersection of the diagonals.
   a) Show that the triangles $ONU$ and $OIT$ are congruent.
   b) Deduce that the diagonals of a rectangle bisect each other.

2. Let $PARC$ be a rectangle of center $O$.

![Diagram of a rectangle with center O]

a) Assuming $CO = x + 1$ and $PR = 3x$, can you find $x$?

b) Construct another rectangle $PURE$ in which the diagonals are perpendicular. What is the nature of $PURE$? Justify your answer.

3. Using the squares in your copybook, explain how to construct a rectangle having the points shown in the figure as vertices.

4. a) Construct a quadrilateral $CIEL$ where $CL = IE$ and angles $\hat{L}$ and $\hat{E}$ are right.
   b) Prove that $\hat{LIE} = \hat{ILC}$.
   c) Prove that triangles $ILE$ and $ILC$ are congruent.
   d) Deduce that $CIEL$ is a rectangle.

5. a) Construct an isosceles triangle $LOI$ of vertex $O$ and in which $\hat{LOI} = 130^\circ$.
   b) Construct $N$ the symmetric of $L$ and $A$ the symmetric of $I$ with respect to $O$.
   c) Show that $INAL$ is a rectangle.
   d) Calculate the measure of all the angles in the obtained figure.

6. a) Draw two perpendicular straight lines $(\alpha)$ and $(\nu)$ intersecting at $O$. Let $P$ be a point outside $(\alpha)$ and $(\nu)$.
   b) Construct $L$ the symmetric of $P$ with respect to $(\alpha)$, $U$ the symmetric of $P$ with respect to $O$, and $S$ the symmetric of $P$ with respect to $(\nu)$.
   c) Show that $S$, $O$, and $L$ are collinear.
   d) What is the nature of the quadrilateral $PLUS$?
   Can you prove it?

7. a) In the following figure, $ZANE$ is a rectangle. What are the properties indicated in this figure?

![Diagram of a rectangle]

b) Reproduce this figure.
   c) Show that $(NI)$ is the bisector of angle $ANE$ and that $NI = EI$. 

\[58\]
8. In the following figure, $D$ varies on $[Ox]$ and $L$ varies on $[Oy]$ in such a way that $OD=2OL$. Let $I$ be a point such that $LIDO$ is a rectangle.

a) Reproduce this figure and construct the midpoints $M$ and $N$ of $[LI]$ and $[OD]$, respectively.
b) What is the nature of $OLMN$? Why?
c) What is the locus of $M$ as $D$ and $L$ vary?

9. Let $ITAR$ be a quadrilateral in which the opposite angles are equal.
   a) Show that any two adjacent angles (non-opposite) of $ITAR$ are supplementary.
   b) Draw the bisectors of the four angles of $ITAR$; they form a quadrilateral $LOUP$.
   Show that $LOUP$ is a rectangle.

10. In the following figure, $(xy)$ and $(rs)$ are two parallel lines and $(UP)$ is a transversal.

a) Reproduce the figure and draw the bisectors $(UF)$ and $(UE)$ of $PUy$ and $PUx$, respectively.
b) The bisector of $Ups$ cuts $(UF)$ in $L$ and the bisector of $UPr$ cuts $(UE)$ in $S$. Show that $PLUS$ is a rectangle.
c) Prove that $(SL) \parallel (EF)$.

11. Let $PLAT$ be a rectangle.
   a) Construct $E$ the symmetric of $P$ with respect to $L$ and $M$ the symmetric of $P$ with respect to $T$.
   b) Show that $E$, $A$, and $M$ are collinear.

**A fair division**

Show that the red rectangles have the same area.
introduction  A twisted square?!...

When four sticks of the same length are joined, forming right angles, they form a square. Pushing at one of the hinges to flatten the circle, the shape is no longer a square but a paternal shape: the rhombus.

At the beginning of this chapter, I am able to:

- recognize the fact that it is a rhombus;
- the sides are equal;
- two opposite sides are parallel;
- two opposite angles are equal;
- the diagonals are perpendicular;
- the diagonals bisect each other.

At the end of this chapter, I will be able to:

- identify a rhombus as a quadrilateral having four equal sides;
- identify a rhombus as a quadrilateral in which the diagonals are perpendicular and bisect each other;
- identify the elements of symmetry of a rhombus.
Recall activities

**Activity 1**

*Test your memory*

1) What do you call the quadrilateral $ABCD$ in the adjacent figure?
2) What are its vertices?
3) What can you say about its angles?
4) What do you call the segments $[AC]$ and $[BD]$?
5) What properties can you find in this figure?

**Activity 2**

*A sketch?.. An invader!!*

1) Draw a rhombus $ABCD$ of your choice.
2) Indicate all the properties you can find in the figure thus obtained.

Preparatory activities

**Activity 1**

*The twist..*

Imagine that you pull a square from a vertex, keeping its sides parallel: you have a rhombus. Which properties are lost and which ones are preserved? Examine the sides, the angles, the diagonals, and the elements of symmetry.

**Activity 2**

*The same for all sides..*

1) Construct a quadrilateral in which the four sides are equal.
2) What is the shape of the obtained quadrilateral?
3) Construct a quadrilateral in which the four sides are equal and one angle is right. What is the shape thus obtained?

**Activity 3**

*Diagonals.. and orthogonality..*

1) Construct a quadrilateral where the diagonals are perpendicular and bisect each other.
2) What is the shape of the obtained quadrilateral?
I. Definition and properties

**Definition 1** A rhombus is a quadrilateral in which the four sides are equal.

Here are proofs for some properties of a rhombus:

1. **The rhombus and its axes of symmetry**
   
   Let $ABCD$ be a rhombus.
   
   Since $AB = AD$, then $A$ belongs to the perpendicular bisector of $[BD]$.
   
   Since $CB = CD$, then $C$ belongs to the perpendicular bisector of $[BD]$.
   
   So, $(AC)$ is an axis of symmetry of the figure.
   
   Similarly, we show that $(BD)$ is an axis of symmetry of the figure. Thus:

   **Property 1** In a rhombus, the diagonals are axes of symmetry.

2. **The rhombus and its angles**
   
   Let $ABCD$ be a rhombus.
   
   $(AC)$ is an axis of symmetry, so $\widehat{ABC} = \widehat{ADC}$; and since $(BD)$ is an axis of symmetry, we have $\widehat{BAD} = \widehat{BCD}$ as well.
   
   Thus, we have:

   **Property 2** In a rhombus, the two opposite angles are equal.

3. **The rhombus and its sides**
   
   $(DB)$ is an axis of symmetry of the figure, so $\widehat{DAC} = \widehat{DCA}$.
   
   $(AC)$ is an axis of symmetry of the figure, so $\widehat{DAC} = \widehat{BAC}$.
   
   It follows that the alternate interior angles $\widehat{DCA}$ and $\widehat{BAC}$ are equal. Consequently, $(AB) \parallel (CD)$.

   **Property 3** In a rhombus, the two opposite sides are parallel.
4. The rhombus and its diagonals

Because of symmetry, each diagonal is an axis of

symmetry of the other, so it is its perpendicular

bisector.

Thus, we have:

**Property 4** In a rhombus, the diagonals are perpendicular and bisect each other.

II. When is a quadrilateral a rhombus?

1. Starting from axes of symmetry

Let $ABCD$ be a quadrilateral in which the
diagonals are axes of symmetry.

$(AC)$ is an axis of symmetry of the figure,
so $BA = DA$ and $BC = DC$.

$(BD)$ is an axis of symmetry of the figure,
so $BA = BC$ and $DA = DC$.

**Property 5** A quadrilateral, in which the diagonals are axes of

symmetry, is a rhombus.

2. Starting from the diagonals

Let $ABCD$ be a quadrilateral in which the
diagonals $[AC]$ and $[BD]$ are perpendicular and

bisect each other.

Each of the two diagonals is a perpendicular

bisector of the other. So each is an axis of

symmetry of the other.

$ABCD$ is then a quadrilateral in which the diagonals are axes of

symmetry; therefore, it is a rhombus. We have:

**Property 6** A quadrilateral, in which the diagonals are perpendicular

and bisect each other, is a rhombus.
In a rhombus $ABCD$:

- the four sides are equal:
  \[ AB = BC = CD = DA; \]
- the opposite sides are parallel:
  \[ [AB] \parallel [DC] \quad \text{and} \quad [BC] \parallel [AD]; \]
- the opposite angles are equal:
  \[ ABC = CDA \quad \text{and} \quad BCD = DAB; \]
- the diagonals $[AC]$ and $[BD]$ are perpendicular and bisect each other:
  \[ (AC) \perp (BD) \]
  \[ OA = OC \quad \text{and} \quad OB = OD; \]
- the diagonals are axes of symmetry and their intersection point is a center of symmetry.

To show that a quadrilateral $ABCD$ is a rhombus, it is sufficient to prove that one of the following properties is verified:

- starting from the definition:
  the four sides are equal;
- starting from the diagonals:
  the diagonals are perpendicular and bisect each other;
- starting from the elements of symmetry:
  the diagonals are axes of symmetry.

How to draw a rhombus?

I draw a segment

I draw another segment so that the two segments are perpendicular bisectors of each other.

I join the four extremities (endpoints).
Exercises

1. Among the following quadrilaterals, determine the rhombuses:

2. a) Construct a rhombus $AJIR$ such that $RA = 4\text{cm}$ and $RAJ = 60^\circ$.
   Explain.
   b) Evaluate the measure of each of the other angles of $AJIR$. Justify your answer.
   c) What is the length of each side?

3. a) Construct a rhombus $NALI$ in which $LI = 5\text{cm}$. Explain.
   b) Compare your rhombus with those of your friends. Did you all get equal rhombuses?

4. a) Construct a rhombus $LOSA$, draw its diagonals and let $T$ be its center.
   b) How many isosceles triangles can you find in the figure?
   c) How many congruent triangles can you find in the figure?
   d) How many right triangles can you find in the figure?

5. $LIRE$ is a rhombus such that $\angle LIR = 120^\circ$ and $IE = 2\text{cm}$. 
   a) Calculate the measure of all the angles in the figure.
   b) What is the length of each side of $LIRE$? Justify your answer.

---

Self-evaluation

A $ABCD$ is quadrilateral where the opposite sides are equal. Suppose $AB = x$ and $BC = 2x - 3$.
Evaluate $x$ so that $ABCD$ is a rhombus.

B $ABCD$ is a quadrilateral where the diagonals bisect each other.
Determine $x$ so that $ABCD$ is a rhombus.
1. Construct a rhombus \( MADI \) in each of the following cases:
   a) \( DM = 6 \text{ cm} \); \( AI = 4 \text{ cm} \).
   b) \( DA = 3 \text{ cm} \); \( DAI = 50^\circ \).
   c) \( DM = 8 \text{ cm} \); \( DI = 5 \text{ cm} \).
   d) \( DM = 10 \text{ cm} \); \( MDI = 25^\circ \).

2. a) In the figure below, \( FLAT \) is a rhombus. Which properties are indicated in the figure?

   b) Show that \( CO = CS = CI = CR \).

   c) What is the nature of the quadrilateral \( SOIR \)? Justify your answer.

   d) Determine the perpendicular bisectors of \([OS], [SR], [RI],\) and \([OI]\).

3. a) The following figure represents a square \( JOLI \). \( S \) is a point on the circle of center \( I \) and radius \( LJ \). Construct \( A \) the translate of \( S \) by the translation from \( J \) to \( O \).

   b) Show then that \( ISAL \) is a rhombus.

4. a) Draw a rhombus \( ALOR \).

   b) Through \( O \), draw the perpendicular to \( (LA) \) meeting it at \( M \).

   c) Through \( A \), draw the perpendicular to \( (OR) \) meeting it at \( N \).

   d) What is the nature of \( NAMO \)? What is its center? Justify your answer.

5. a) Draw an isosceles triangle \( DRA \) of base \( DA \).

   b) Construct \( P \) the translate of \( D \) by the translation from \( R \) to \( A \).

   c) Show that \( DRAP \) is a rhombus.

6. a) Draw an isosceles triangle \( SOI \) of base \( OI \) and construct \( R \) the symmetric of \( S \) with respect to \( (OI) \).

   b) Show that \( SORI \) is a rhombus.

7. \( CALK \) is a rectangle of center \( O \).

   a) What do \((PR)\) and \((UE)\) represent with respect to \( CALK \)?

   b) What is the nature of \( PURE \)? Justify your answer.

   c) Construct \( S \) the symmetric of \( O \) with respect to \( (CK) \). Prove then that \( COKS \) is the translate of \( PURE \) by a translation to be determined. Deduce that \( COKS \) is a rhombus.

   d) Compare the areas of \( COKS \) and \( CALK \).
8. a) Construct a square VISA whose diagonal is 6 cm. O is the center of VISA.
b) Locate M the midpoint of [ VO] and N the midpoint of [ OS].
c) What is the nature of MINA? 
d) Compare the areas of VISA and MINA.

9. In the following figure CHAT is a rhombus of center O, M is a point on [ CO], [ CG] is the perpendicular to [ AH], [ RL] is the parallel from M to [ CG], and [ MK] is the perpendicular to [ CH].
a) Show that K and R are symmetric with respect to (CA).

b) Show that KM + ML = RL.
c) Show that KM + ML = CG.
d) Construct the translates of T and M by the translation from C to H.
e) How would you deduce the translate of R by the same translation?

10. a) Let CIEL be a rhombus. Draw the perpendicular from I to (EL) to cut it in A, and the perpendicular from I to the side (CL) cutting it in T.
b) Draw the perpendicular from L to (CI) to cut it in U, and the perpendicular from L to (EI) to cut it in B.
c) Let O be the point of intersection of [ LA] and [ IT], and S the intersection point of [ IA] and [ LB]. Show that O and S belong to (CE).
d) Prove that LOIS is a rhombus.
e) Show that CT = CU = EA = EB.
f) What is the nature of the quadrilateral BATU?

1. Rhombus and circle well related
   What is the maximum number of intersection points of a rhombus and a circle?

2. Dividing the areas
   How would you choose a point M of [ BC] such that [ AM] divides the triangle into two triangles having equal areas?
**introduction**  
A stretched rhombus or a twisted rectangle?

The square, the rectangle, and the rhombus are quadrilaterals that have some common properties: the opposite sides are equal, the opposite angles are equal, the opposite sides are parallel, the diagonals bisect each other at their midpoint, which is the center of symmetry.

If we construct a quadrilateral possessing one of these properties, what do we obtain?

> **At the beginning of this chapter, I am able to:**
> - identify a square and its properties;
> - identify a rectangle and its properties;
> - identify a rhombus and its properties;
> - identify a parallelogram as a quadrilateral where the opposite sides are parallel.

> **At the end of this chapter, I will be able to:**
> - identify a parallelogram as a quadrilateral where the opposite sides are parallel;
> - identify a parallelogram as a quadrilateral where both pairs of opposite sides are equal;
> - identify a parallelogram as a quadrilateral where both pairs of opposite angles are equal;
> - identify a parallelogram as a quadrilateral where the diagonals bisect each other;
> - show that a square, a rectangle, and a rhombus are particular parallelograms.
Recall activity

Activity

Look .. think .. state ..
1) What do you call the quadrilateral $ABCD$ in the adjacent figure?
2) What are its elements?
3) What properties can you find in this figure?

Preparatory activity

Activity

The road from a square to a parallelogram

Starting from a square, we may get a rectangle then a parallelogram, or we may get a rhombus and then a parallelogram:

![Diagram showing the relationship between a square, a rhombus, a rectangle, and a parallelogram.]

Complete the table:

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<th>not preserved</th>
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<td>other axes of symmetry</td>
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I. Definition and properties

**Definition 1** A parallelogram is a quadrilateral whose opposite sides are parallel.

1. The sides and angles of a parallelogram

   Let $ABCD$ be a parallelogram. Consider the triangles $ABC$ and $ADC$. We have:
   
   - $[AC]$ common side;
   - $\overline{BAC} = \overline{DCA}$: alternate interior angles formed by the parallel lines $(AB)$ and $(CD)$ and the transversal $(AC)$;
   - $\overline{DAC} = \overline{BCA}$: alternate interior angles formed by the parallel lines $(AD)$ and $(CB)$ and the transversal $(AC)$;

   The two triangles are congruent by A.S.A., so:
   
   $AB = CD$, $AD = BC$ and $\overline{ABC} = \overline{ADC}$ corresponding elements.

   Similarly, prove $\overline{BAD} = \overline{BCD}$. We have:

   **Property 1** In a parallelogram, both pairs of opposite sides and opposite angles are equal.

2. The diagonals of a parallelogram

   Let $ABCD$ be a parallelogram.

   We can prove that the two triangles $AOB$ and $COD$ are congruent by A.S.A and conclude that $OA = OC$ and $OB = OD$.

   So, $O$ is the midpoint of $[AC]$ and $[BD]$ and, therefore, it is a center of symmetry. We have:

   **Property 2** In a parallelogram, the diagonals bisect each other, and their point of intersection is a center of symmetry.
II. When is a quadrilateral a parallelogram?

1. Starting from the two sides

Let $ABCD$ be a quadrilateral in which the sides $[AB]$ and $[CD]$ are parallel and equal.

$C$ is the translate of $D$ by the translation taking $A$ to $B$.

Consequently, $[BC]$ is the translate of $[AD]$, and the two segments are parallel; so, $ABCD$ is a parallelogram.

**Property 3**

A quadrilateral where two opposite sides are equal and parallel is a parallelogram.

2. Starting from the four sides

Let $ABCD$ be a quadrilateral where $AB = CD$ and $AD = BC$.

Triangles $ABC$ and $CDA$ are congruent by S.S.S. Consequently, $\angle BAC = \angle DCA$ and $\angle BCA = \angle CAD$.

So $(AB)$ and $(CD)$ are parallel because they form two equal alternate interior angles. Similarly, $(AD)$ and $(BC)$. Therefore, $ABCD$ is a parallelogram since two of its sides are equal and parallel.

**Property 4**

A quadrilateral whose opposite sides are equal is a parallelogram.

3. Starting from the diagonals

Let $ABCD$ be a quadrilateral whose diagonals $[AC]$ and $[BD]$ bisect each other at $O$.

In this case, $O$ is a center of symmetry, and then $AB = CD$ and $AD = BC$ by symmetry.

**Property 5**

A quadrilateral whose diagonals bisect each other is a parallelogram.

We can state this following property:

**Property 6**

A quadrilateral that admits a center of symmetry is a parallelogram.
4. Starting from angles

Let $ABCD$ be a quadrilateral where the opposite angles are equal.

Since the sum of the four angles is $360^\circ$, the sum of two adjacent angles is $180^\circ$ and, consequently, $\overline{a} + \overline{c} = 180^\circ$.

But $\overline{c} + \overline{e} = 180^\circ$; therefore, $\overline{d} = \overline{e} = \overline{b}$.

Also, $(AD)$ and $(BC)$ are parallel since they make two equal corresponding angles with $(DC)$.

Similarly, $(AB)$ and $(DC)$ are parallel, since they form two equal alternate interior angles with $(BC)$. So $ABCD$ has opposite parallel sides. Consequently:

### Property 7

A quadrilateral where both pairs of opposite angles are equal is a parallelogram.

---

II. The parallelogram and its descendants

We have seen that, in a square, in a rectangle, and in a rhombus the opposite sides are parallel. Thus:

### Property 8

The square, the rectangle, and the rhombus are particular parallelograms.

If a parallelogram has two equal adjacent sides, then the four sides are equal; therefore,

### Property 9

A parallelogram having two adjacent sides equal is a rhombus.

If a parallelogram has a right angle, then this parallelogram is a rectangle; thus,

### Property 10

A parallelogram having a right angle is a rectangle

Finally, we state the following property:

### Property 11

A square is a rectangle and a rhombus at the same time.
In a parallelogram $ABCD$:

- the opposite sides are equal: $AB = CD$ and $BC = AD$;
- the opposite angles are equal: $\angle ABC = \angle CDA$ and $\angle BCD = \angle DAB$;
- the point of intersection of the diagonals is a center of symmetry of $ABCD$.

- the opposite sides are parallel: $[AB] \parallel [DC]$ and $[BC] \parallel [AD]$;
- the diagonals $[AC]$ and $[BD]$ bisect each other: $OA = OC$ and $OB = OD$.

To prove a quadrilateral $ABCD$ is a parallelogram, it is sufficient to prove that one of the following properties is verified:

- starting from the definition:
  - the opposite sides are parallel;
- starting from the lengths of the sides:
  - the opposite sides are equal;
- starting from the position of the sides:
  - two opposite sides are equal and parallel;
- starting from the diagonals:
  - the diagonals bisect each other;
- starting from the element of symmetry:
  - the point of intersection of the diagonals is a center of symmetry.

Classification

A square is a rectangle is a parallelogram is a rhombus

How to draw a parallelogram?

- mark three non-collinear points.
- construct the symmetries of two points with respect to the third.
- join the endpoints of the segments obtained.
1. **CHAT** is a parallelogram.

![Diagram: Parallelogram CHAT with angles and points T, A, H, C]

Calculate its angles.

2. **LAME** is a parallelogram.

![Diagram: Parallelogram LAME with sides and points L, A, M, E]

Calculate $x$ and $y$.

3. **TROP** is a parallelogram.

![Diagram: Parallelogram TROP with angles and points T, R, O, P]

Calculate its angles.

4. **AUTO** is a parallelogram.

![Diagram: Parallelogram AUTO with sides and points A, U, T, O]

Calculate $x$ and $y$.

5. In each of the following cases, tell if **ROND** is a parallelogram (C is the point of intersection of the diagonals):
   
a) $RO = DN$ and $\overrightarrow{RO} = \overrightarrow{DN}$.
   
b) $RO = DN = RD = 4\text{cm}$ and $ON = 5\text{cm}$.
   
c) $OD = 6\text{cm} \; RN = 8\text{cm} \; CO = 3\text{cm} \; CR = 4\text{cm}$.
   
d) $(RO) \parallel (DN)$ and $\overrightarrow{RON} = \overrightarrow{RDN}$.
   
e) $(RD) \parallel (NO) \; RNO = NRD$.
   
f) $(RO) \parallel (DN) \; ON = RD \; \overrightarrow{ONR} = 70^\circ \; \overrightarrow{NRD} = 60^\circ$.
   
g) $OC = CD = 0 \; CR = CN = 0$.
   
h) $DRN = \overrightarrow{ONR} \; RD = \overrightarrow{ON} = 1$.

6. a) Construct a parallelogram with a diagonal of 7cm, a side of 5cm, and a side of 4cm.
   
b) Explain.

7. a) Mark two points $C$ and $O$, 3cm apart.
   
b) Construct a parallelogram **CUBE**, of center $O$ such that $CU = 8\text{cm}$ and $CE = 5\text{cm}$.
   
Explain your construction.

8. a) Construct a parallelogram given its sides are 4cm and 6cm, with an angle of 40°.
   
b) Explain.

9. a) Construct a parallelogram with a side of 5cm, an area of 20cm², and an angle of 130°.
   
b) Explain.
10. \( P \cdot L \cdot A \cdot T \) is a parallelogram where \( TA = 2AL = 6 \text{cm}, \ TPL = 75^\circ \), and \( S \cdot O \cdot L \cdot D \) is a rhombus.

13. In the following figure, \( R \cdot O \cdot M \cdot E \) is a parallelogram and \( RS = MA \).

\[ \begin{array}{c}
\text{a)} \text{ Show that } SO = AE. \\
\text{b)} \text{ Show that } MASR \text{ and } OSEA \text{ are parallelograms.} \\
\text{c)} \text{ Show that } [RM], [SA] \text{ and } [OE] \text{ intersect at the same point (concurrent).}
\end{array} \]

14. In the following figure:
\( LO = OC = 2LI \), \( (LO) \perp (LI) \) and \( (LO) \perp (OC) \).

\[ \begin{array}{c}
\text{a)} \text{ What is the nature of } JOLP \text{ of } LIEU? \\
\text{b)} \text{ Show that } E \text{ is the midpoint of } [LC]. \\
\text{c)} \text{ Show that the triangles } OIL \text{ and } COU \text{ are congruent.} \\
\text{d)} \text{ Prove that } CUI \text{ is an isosceles triangle.}
\end{array} \]

15. \( L \cdot O \cdot V \cdot E \) is a parallelogram where \( \overrightarrow{L} = x \) and \( \overrightarrow{O} = 2x - 90^\circ \).
Prove that \( L \cdot O \cdot V \cdot E \) is a rectangle.

16. \( L \cdot O \cdot R \cdot D \) is a parallelogram where \( \angle LOD + \angle RLD = 90^\circ \).
Prove that \( L \cdot O \cdot R \cdot D \) is a rhombus.
17. What fraction does the red region represent in each of the following cases?

a)  

b)  

c)  

18. a) In an orthonormal system of axes $x'Ox, y'Oy$, locate the points $A(-2;3)$ and $B(1;4)$.

b) Determine the point $C$, symmetric of $A$ with respect to $B$.

c) Determine the point $F$ so that $A, C, F,$ and $O$ form a parallelogram. Is $F$ unique?

19. In the following figure, $P4RC$ is a parallelogram, $(PH) \perp (AC)$, and $AC=3PH=9$ cm.

Calculate the area of $P4RC$.

20. a) Prove that the bisectors of the angles of a parallelogram form a rectangle.

b) What happens if the parallelogram is a rectangle?

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**Self-evaluation**

A  

**ELSA** is a trapezoid where $(EL)$ is parallel to $(AS)$ and $AS = 2EL$.

Let $M$ be the midpoint of $[AS]$.

Show that $ELMA$ is a parallelogram.

B  

Given in the adjacent figure, $(AL) \parallel (KD)$ and $(ML) \parallel (KJ)$.

a) Show that $KALM$ is a parallelogram.

b) Calculate the angles of the figure.

C  

Let $ABC$ be a triangle and $[AM]$ be the median issued from $A$.

a) Construct $F$ the symmetric of $A$ with respect to $M$.

b) Show that $ABFC$ is a parallelogram.
1. Let \( PILE \) be a parallelogram. 
From \( I \), draw a perpendicular to \( (PE) \) meeting it in \( N \). 
From \( E \), draw a perpendicular to \( (IL) \) meeting it in \( M \).

![Diagram of parallelogram PILE with points I, M, and L]

a) Show that triangles \( PIN \) and \( LEM \) are congruent. Give the corresponding elements.
b) What is the nature of quadrilateral \( MINE \)? Justify your answer.
c) The line \( (MN) \) cuts \( (LE) \) in \( K \) and \( (PI) \) in \( J \). Prove that triangles \( NEK \) and \( JIM \) are congruent. Give the corresponding elements.
d) Prove that \( IKE \) is a parallelogram.
e) Show that the lines \( (IE) \), \( (JK) \), and \( (PL) \) are concurrent.

2. Let \( BIEN \) be a parallelogram. Draw the heights \([EM]\) and \([BC]\) from \( E \) and \( B \) to \([IN]\).

a) Show that triangles \( BIC \) and \( MEN \) are congruent. Determine the corresponding elements.
b) Show that \( CEMB \) is a parallelogram.
c) Show that \([IN]\), \([BE]\), and \([MC]\) have the same midpoint.

3. \( NUIJ \) is a parallelogram of center \( J \). The circle has as diameter \([IN]\).

![Diagram of parallelogram NUIJ with circle and points]

a) Reproduce the diagram and show that \( V \), the symmetric of \( R \), and \( K \), the symmetric of \( S \), with respect to \( J \) belong to \((IT)\) and \((NT)\), respectively, and to the circle.
b) What is the nature of \( RSVK \)? Justify your answer.

4. \( SOIR \) is a parallelogram of center \( T \). The circle has \( T \) as its center.

![Diagram of parallelogram SOIR with circle and points]

a) Show that \( LUNE \) is a rectangle.
b) Show that \( JAMP \) is a rectangle.

5. Show that if \( ANGE \) is a quadrilateral where \((AN)\) is parallel to \((GE)\) and \( \angle ANG = \angle AEG \), then this quadrilateral is a parallelogram.
6. **PAIN** is a rectangle of center C. O and S are the symmetrics of A and N, respectively, with respect to I.

   ![Diagram of PAIN]

   a) Reproduce the diagram.
   b) What is the nature of **SONA**? Justify your answer.
   c) What is the nature of **PISA**? Justify your answer.
   d) The line (PI) meets [OS] in E. What is the nature of **CASE**? and that of **ONCE**? Justify your answers.
   e) Show that E is the midpoint of [OS].
   f) Show that (SN) meets [PO] at its midpoint.
   g) Deduce that I is the centroid (center of gravity) of triangle POS.

7. **FINE** is a parallelogram.

   a) Construct **FINE** so that IF = EF and FEI = FNI.
   b) What is the nature of the quadrilateral thus obtained? Justify your answer.

8. **CIEL** is a parallelogram and M and N are the midpoints of [CI] and [LE], respectively. Show that (LM) and (IN) cut the diagonal [CE] into three equal segments.

9. **FILM** is a parallelogram where $\overline{FM} = 90^\circ$. The perpendiculars from F and M to (LM) and (FL) respectively meet at A.

   ![Diagram of FILM]

   a) Sketch the diagram.
   b) Show that $\overline{FM} \perp \overline{LA}$.
   c) Let $U$ be the symmetric of L with respect to I. Prove that $MP = UI$.
   d) Show that **PLUF** is a rectangle.

10. **MODE** is a parallelogram where $MO = OE$ and $P$ is the midpoint of [ME] and [LO].

    ![Diagram of MODE]

    a) Sketch the diagram.
    b) What is the nature of **MOEL**.
    c) Show that $E$ is the midpoint of [DL].
    d) Let $I$ be the symmetric of $O$ with respect to $E$. What is the nature of the quadrilateral **IDOL**?
11. **GOLD** is a parallelogram of center $C$, $I$ is the midpoint of $[OC]$ and $E$ is the midpoint $[DC]$.

![Diagram of GOLD parallelogram]

- a) Prove that $GILE$ is a parallelogram.
- b) Show that triangles $OIL$ and $GED$ are congruent.

12. Let **MILK** be a parallelogram.
- a) Construct $N$ the symmetric of $M$ with respect to $I$. Prove that $NIKL$ is a parallelogram.
- b) Construct $E$ the symmetric of $M$ with respect to $K$. Prove that $LIKE$ is a parallelogram.
- c) Show that $N$, $L$, and $E$ are collinear.

13. a) Construct a parallelogram $MATH$ so that $MA = MT$.
- b) Construct $R$ the translate of $T$ by the translation from $M$ to $A$.
- c) What is the nature of quadrilateral $MART$? Justify your answer.
- d) Prove that $(MR)$ perpendicular to $(TA)$.
- e) Show that $MH = 2TO$, where $O$ designates the meeting point of the diagonals of $MART$.
- f) Let $I$ be the midpoint of $[MH]$. What is the nature of the quadrilateral $TOMI$? Justify your answer.

14. Let **CARE** be a square. $M$ is the symmetric of $C$ with respect to $A$ and $L$ is the symmetric of $R$ with respect to $E$.
- a) Show that the quadrilateral $ERMA$ is a parallelogram.
- b) Show that the quadrilateral $CLEA$ is a parallelogram.
- c) Prove that the quadrilateral $CMRL$ is a parallelogram and that $[CR]$, $[AE]$, and $[LM]$ have the same midpoint.

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**Just For Fun**

**The hidden parallelogram**

![Diagram of hidden parallelogram with points $O$, $(u)$, and $(v)$]

Construct a parallelogram $ABCD$ of center $O$ and such that $A$ belongs to $(u)$ and $C$ belongs to $(v)$. 

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