Problem solving in grade nine Lebanese math textbook

A study of curriculum-textbook coherence

by

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Abstract

Problem solving is highly important as students should learn how to find out solutions systematically and logically. It plays an important role in students' learning of new mathematical contents and in making connections between different areas of knowledge.

The purpose of this research is to study the implementation of the problem solving general objective mentioned in the Lebanese curriculum in instruction and in Grade Nine Lebanese math textbook. To accomplish this study an analysis of the Lebanese curriculum objectives in addition to three selected chapters from the grade nine textbook is conducted to investigate the agreement of the Lebanese problem solving objectives with the exercises and problems in each chapter. For more reliable results, two math teachers are interviewed to explore the difficulties they face with the book as a tool in teaching problem solving. Results showed that the Lebanese curriculum objectives are not completely reflected in the book, and that teachers are not using the book as tool for teaching problem solving strategies. In addition, the study showed that teachers need more guidance and workshops that would equip them with skills of teaching problem solving strategies and proofs.
Chapter one

Introduction

Problem solving is a broad term that refers to a complex of cognitive activities and skills. Math educators assert that problem-solving skill is very important as the students learn how to find solutions systematically and logically (Jones & Malloy, 1998). Thus, educators do not only focus on what to teach the students but also on guiding them as to how to learn and think.

The American National Council of Teachers of Mathematics (NCTM, 1989) suggests that in order to improve teaching and learning mathematics, problem solving must be an integral part of all mathematics learning. NCTM (2000) insists that in the intermediate level, to better understand content and process, one should go over problem posing and problem solving. Yet, the teacher's role in choosing useful problems and mathematical tasks is important. Indeed, "there are many math problems that are interesting and fun but that may not lead to the development of the mathematical ideas that are important for a class at a particular time" (NCTM 2000, p53). This idea was stressed by Malloy and Jones (1998) when they found that students need their teachers' help through setting problems that are appropriate to their knowledge and emphasizing the use of the appropriate method and processes of solving such as analyzing, deducing, looking for necessary information, constructing a mathematical model, choosing a strategy to find a solution and decomposing a problem into simpler ones.

In the Lebanese curriculum (Educational Center for Research and Development, ECRD, 1997) problem solving was considered as a main objective for mathematics in the introduction, the general objectives, and the instructional objectives for the third cycle. More
specifically, in its introduction, the curriculum insists that “the math book must offer to the students the necessary and sufficient knowledge that helps students understand problems whatever the domain is: physical, chemical, social, or computer” asserting that there must be “no divorce between mathematics and everyday life” (ECRD, 1997 p: 288)

In the general objectives “solving mathematical problems” is one of the main goals stated. Problem solving must be the most significant activity in the math domain; students must learn how to use different strategies to solve word problems. Moreover, the curriculum states that, in the intermediate level, students should be able to tackle different problem situations through “analyzing, deducing relevant elements, looking for necessary information, choosing the suitable strategy to find the solution and yet combine necessary data to reach a conclusion”. Thus, the Lebanese curriculum emphasizes “using various strategies to tackle difficulties in solving a problem, classifying, analyzing, serializing and quantifying” (ECRD, 1997 p: 289).

The math textbook must be a tool that helps students practice problem solving and develop the skills required. On the other hand, the textbook should help teachers in their task of developing students’ problem solving skills.

*Purpose*

The purpose of this study is to assess problem solving teaching/learning in grade-nine math through: analyzing the grade-nine math textbook (ECRD, 2000) in terms of its coherence with the problem solving goals and objectives in the curriculum (ECRD, 1997) and interviewing teachers to investigate the difficulties they and/or their students face in mathematical problem solving.
Key Research Questions

1. To which extent is the grade-nine math textbook coherent with the curriculum as to the problem-solving goal?

2. Is the textbook helpful to the teacher in terms of providing guidelines for developing students’ problem-solving skills?

3. What are the difficulties that teachers face in teaching problem-solving?

Rationale

One of the goals set by the NCTM (1989) for K-12 mathematics education was that students should become mathematical problem solvers, if they were to become productive citizens. It was perceived as important that students solve problems not only alone, but also by working cooperatively in small or large groups. Problems have to be varied, with some being open-ended and some in applied context. In grade-nine math Lebanese books, the focus of both the exercises and the problems is mainly on the content to be studied. The problems that students have to solve are more centered on drill-and-practice repetitive mathematical tasks.

Yet, the main significance of this study is to show the importance of teachers’ textbook and curriculum knowledge in producing student learning. For the teacher, curriculum knowledge involves awareness of how topics are arranged both within a school year and over time and ways of using curriculum resources, such as textbooks or the internet, to organize a program of study for students and avoid the textbook gaps.

For curriculum developers, the results of this study will help in evaluating the grade-nine math textbook and revising the current edition.
Method

More than one method are used to help achieve this project:

1. Content analysis of the Lebanese curriculum and textbooks
   a. The Lebanese curriculum objectives are analyzed trying to find out how problem solving objectives are developed through the intermediate level.
   b. Analysis of three chapters selected from the Grade nine math textbook. These chapters are:

   Algebra: Proportionality

   Geometry: Lines and circles

   Statistics: Statistical surveys.

2. Interview with teachers

   An interview is conducted with two grade-nine math teachers (T1 and T2) from two different schools (S1 and S2). The aim is to investigate their perceptions about problem solving in teaching math and about their experience with problem solving. The teacher’s understanding of mathematics and knowledge of the math objectives for the middle grades in the Lebanese curriculum contribute to gains in students’ mathematics achievement. For the interview questions refer to (Appendix I).

Project Outline

Chapter two of this project is a review of literature on three main topics: mathematical problem solving, curriculum-textbook coherence, and teachers’ use of the textbook. Chapter three consists of a curriculum analysis, in particular, an analysis of the objectives of the Lebanese curriculum regarding the problem solving objective, and analysis of three chapters
selected from the grade nine math textbook, looking at each chapter and fetching the book’s implementation of problem solving goal and general objectives of the curriculum, the chosen chapters are: "Lines and Circles", "Proportionality", and "Statistics". Last, the results of interviews with the teachers about their use of the textbook, its helpfulness, and the difficulties they face in teaching problem solving are declared. To conclude, some recommendations are asserted regarding the problem solving objective in Grade nine math textbook, and further open study options in this regard are suggested.
Chapter Two

Literature Review

The literature review comprises three topics. The first one is Problem solving. The research on problem solving covers four ideas. First, the definition of problem and problem solving according to various educators and sources are included. Second, previous literature is classified in two groups, a traditional one that considers problem solving a linear process, and a more developed view that believes that problem solving is an interactive activity of several phases that are not necessarily sequential stages. Third, literature is shown on the effect of the attitudes of students and teachers on their problem solving abilities.

The second topic of the review of literature is curriculum-textbook coherence. First, various definitions are included of a curriculum. Then the concept of a good or a coherent curriculum is introduced. Subsequently, different suggestions are included of developing the curriculum; then a criterion of a good curriculum is delineated. These criteria are very influential in the project. Namely, they are the number of topics covered in a curriculum, the sequential flow within and across grade levels, in addition to other criteria such as: focus and comprehensiveness, does it promote understanding rather than memorization, does it provide application of the mathematical concepts introduced, and are students engaged mentally and physically?

The third section of literature review is about teachers' use of the textbook. In this section, there are definitions of a textbook, a teacher, a teacher's job, and what a teacher is required to do. Then some references are quoted to say that the curriculum is the combination of textbook and instruction, thus different teachers make different enacted curricula. The
section also discusses teaching guidelines provided by the textbook. Most people find that the textbook provides guidance for inexperienced teachers; others add that it provides a good timetable for teachers, a good organization of ideas in a chapter, and the major number of problems. Many educators believe however those teachers should use other sources and instructional additional materials. Finally, suggestions for better use of the textbook recommended by several authors are included.

One of the main goals proposed by the NCTM (1989) Curriculum and Evaluation Standards for School Mathematics is that students become mathematics problem solvers. The importance of problem-solving in mathematics education has been an interest for several mathematics educators like Polya, Schoenfeld, Wilson, and others. Schoenfeld (1992) for example, believes that the core of teaching mathematics is problem-solving. Remillard (1991) considers problem solving an “authentic mathematical activity” that fosters one’s motivation to develop and apply mathematical skills.

**Problem Solving**

What is problem solving? Wilson (1993) defined a problem as an unknown item that satisfies given description; problem solving is the process of finding that unknown. He considered the existence of a goal a problem, and achieving this goal is problem solving. Schoenfeld (1985, as cited in Wilson 1993) pointed out that what is a problem is always relative to the individual.

Polya (1957) differentiated between routine and non-routine problems. Routine problems are those that can be solved by some rules or algorithms, non-routine problems are the challenging ones that require some degree of creativity and originality from the solver. It’s this sort of problems that develops problem solving ability of students (Polya, 1987 as
cited in Bahru, 2005). Thus, problem solving is not just about solving a mathematical problem; it is the process by which students learn how to use their skills (NCTM, 1989).

According to (Saleh, unknown year), the study of problem solving can be divided into three sections: first, algorithms and heuristics and the ability of using them in problem solving; second, the process that occurs during problem solving, and third, attitudes and conceptions of students and teachers regarding problem solving.

Algorithms are procedures applicable to a particular type of problems. Algorithms are important in mathematics, but carrying out an algorithm is not the core of problem solving (Wilson, 1993). Students should learn how to create their own algorithms (Wilson, 1993). Some problems can't be solved by a specific algorithm; heuristics are sort of strategies of solving such problems, such as "re-writing the problem in one's own words" or "making a diagram". According to Schoenfeld (1979 as cited in Wilson 1993), heuristics help a person use his/her knowledge effectively to solve problems. Schoenfeld (2000) found out that persons with good knowledge of the domain proved to be better problem solvers, or to better incorporate general heuristics in their work. Schoenfeld (1987) and Lesh (1981) pointed out some limitations of relevant research, suggesting taking into consideration other factors such as beliefs and past experiences. Most importantly, the student must have decisive skills to choose relevant heuristics and algorithms or make up new ones (Wilson, 1993).

The process of problem solving which was given different models historically is investigated. Constructivism and cognitive science maintain that problem solving involves exploration, pattern finding, and mathematical thinking (Wilson, 1993). Traditional textbooks view the problem solving as a linear process of four stages: read, decide, solve, examine. However, Polya (1957) suggested a genuine model of problem solving. It involves interaction among the various possible phases of the problem solving process: read, understand, plan, look back, and solve. Polya's stages are not necessarily steps. They fall into the "problem
posing" dynamics which involve rereading the problem, turning it into another problem or posing further related problems. Problem solving does not follow a typical memorized fashion. Rather, it is a deep process, which the students are largely unaware of according to Garofola and Lester (1985 as cited in Wilson, 1993). "Look back" is examining the solution, Polya(1957) suggested several ways of looking back, such as trying to get the result using different approach or changing a variable and checking the relative solution. Several educators like Latterell (2003) and Wilson (1993) warned that students as well as teachers don't emphasize looking back; teachers justify this fact by not having enough time and having difficulty with the students. "Solve" is trying to get to an answer. What really counts is not solving, but learning how to solve.

The relation between the attitudes of students and teachers on problem solving plays a role in the methods of both teaching and learning problem solving (Schoenfeld, 1992). The NCTM Curriculum and Evaluation Standards point out that students should believe in their ability of applying their problem solving skills to help solve problems in the world around them. Polya (1957) added that teachers wishing to have positive attitude students should have a positive attitude themselves. Similarly, math teachers should know that the goal of problem solving at school is not answer getting. A study by Schoenfeld (2000), as well as an assessment conducted by Latterell (2003), showed that students consider mathematics problem solving a memorization task and its goal is getting the answer. This may be in part due to what Thompson (2004) reported about the incongruence of teachers' awareness of the problem solving process and goals with practicing their beliefs whilst teaching because of curriculum and student pressure, despite their conceptual knowledge of the field.

Schoenfeld (2000) pointed out that mathematical instructional goals will be accomplished, when students become more engaged using inquiry-based, problem-solving learning strategies, particularly when joined with highly qualified teachers. All research
referenced in this context contends that learners and teachers should both be actively involved in the problem solving learning process, whereas main complaints by teachers regarding problem solving approach always melt down to length of curriculum (Wilson, 1993).

Curriculum-Textbook Coherence

Curriculum to Bobbitt (Wikipedia, 2008) is an ideal body of experiences the student has to acquire to become the person he/she ought to become. Contemporary theories disagree with Bobbitt; rather, they define curriculum as the actual course of experiences that shape persons (Wikipedia, 2008). In education, a curriculum is a learning program; it includes all aspects of the education process such as teaching, learning, and assessment materials (Wikipedia, 2008). As Taba (1962 as cited by Van Den Akker, 2003) puts it, a curriculum is a "plan of learning". The Third International Mathematics and Science Studies (TIMSS) distinguished between the achieved curriculum, expressing students' performance, and the enacted curriculum, referring to what is taught in the classroom (Li, 2007).

Research on the curriculum has dealt with two main issues: the contribution of the curriculum in students' performance, and the instructional features embedded in the curriculum (Li, 2007). The first was the focus of studies in the 1950s and 1960s with no consideration to all involved factors or control of variables and initial conditions Walker and Schaffarziék (1974 as cited by Li, 2007), while recent studies figured out the importance of instructional material and paid more attention to this concept (Li, 2007).

Coherence in this context can be discussed at various levels. Coherence within the curriculum itself should be vertical, between consecutive grades, and horizontal, between subjects of the same grade, coherence of the curriculum with the textbook refers to consistency of the course of study with the textbook which acts as a ground for both teachers
and learners (Cogan et al., 2005). The NCTM (2000) makes it clear that a curriculum is not just a set of problems and tasks. Porter (1989 as cited in Freeman, 1989) agrees that the curriculum should focus more on problem solving skills and higher order thinking rather than basic facts, and Resnick (1987 as cited in Freeman, 1989) rejects traditional trivial teaching methods, which proved to teach the students basic concepts without analyzing and higher order thinking skills (Athey & Singer, 1987; National Assessment of Education and Progress, 1987 as cited in Freeman, 1989). Recent calls for educational reform have increased, urging the adoption of more dynamic instruction and the expansion of curriculum beyond basics to cover conceptual understanding (Freeman 1989).

ChiWang et al. (2005) point out that the curriculum is examination oriented yet it doesn't assess problem solving strategies and conceptual understanding learned. However, Van Den Akker (2003) suggests three alternatives to help balance the curriculum: reducing the number of topics covered, acquiring more interaction between learning inside and outside school, and motivating learners upon moving from textbook dominated instruction to activity based approaches. "When teachers use a textbook as their sole guide for what to teach, the textbook becomes the curriculum" (Irujo, 2006).

With the release of the *Principles and Standards for School Mathematics*, more reform calls and efforts are expected. Reys, Trafton, and Wasman (2001) discuss six main examples of contrast between the new standards' curriculum and traditional textbook oriented curriculum. First, the newly suggested curriculum is comprehensive; it focuses on core mathematics and addresses general mathematics literacy goals to provide a good base and a framework for teaching various math strands such as algebra, arithmetic, geometry, and others, instead of teaching them as individual unrelated branches of mathematics. The Standards categorized school mathematics into five groups: number and operations, algebra,
geometry, measurement, and data analysis and probability. Second, the Standards-based curriculum materials are coherent and integrated as a whole; this can be fostered by promoting understanding rather than memorization. Third, the new curriculum develops different levels of each topic as the grade level goes up, instead of reiterating the same information virtually ten times in ten grade levels. In contrast, TIMSS describes textbooks as "a mile wide and an inch deep". Fourth, the newly suggested materials promote sense-making and understanding mathematics in the larger world context. Fifth, the new curriculum engages students intellectually and physically in an interesting classroom environment.

Finally, Standards-based material promote learning by incorporating applications in the learning process rather than simply including a few problem applications at the end of a chapter.

Evidently, Reys et al. (2001) agree with the previously mentioned studies in regarding the number/depth of topics covered; so does Watanabe (2007), who mentions two issues leading to an unfocused curriculum: the crowdedness of topics and the redevelopment of a topic in many grade levels yet in the same depth; in addition, Watanabe specifies the problem of the fear of removing a topic from the curriculum.

A study (Freeman, 1989) showed that state-level efforts in curriculum reform has mostly taken the form of in service programs, objectives statements for teachers, guidelines for developing local curricula, or statewide tests. The main factor, thus, that extends the "curriculum" further than the "textbook" is teacher's planning and activity, with particular importance of assessing students' performance (Freeman, 1989).

"Textbooks tend to define limits to the content of the curriculum" (McKnight et al., 1987, Brown, 1973 as cited in Flanders, 1994). While Amit and Freid (2002 as cited by Cruz,
Macias, & Santos, 2006) suggest that a basic purpose of the textbook is to convey the curriculum (Cruz, Macias & Santos, 2006). Johansson (2006) observed that textbooks are not playing their intended role in the curriculum because textbook production is affected by many factors. Whereas Sturino (2002) noticed that the role of textbook as proposed by the curriculum did not influence teacher planning directly because instruction was more influenced by the textbook itself and by department heads.

**Teachers' use of the textbook**

"A textbook is to students what software is to computers, something to be installed on them" (Westhues, 1991 as cited in Wikibooks, 2007). Not just for students, Johansson (2006) claims that Mathematics is, for both students and teachers, simply what is written in the textbook. They are one of the resources most used by teachers in classrooms (Freeman et al., 1989; Robitaille et al., 1989; Robitaille, 1995; Stodolsky, 1988; Moren, 1999; Schmidt et al., 1997; Pepin et al., 2001; Boaler, 1997 as cited in Cruz, Macias, & Santos, 2006) and "one of the most important means of conveying mathematics." (Cruz, Macias, & Santos, 2006).

A teacher, according to Jeppel (2004) is expected to interpret students' understandings and initiate students' involvement in investigating and generalizing. A good teacher, however, according to a questionnaire done by Jeppel (2004), should have three main characteristics, one of which is to free themselves from the textbook and introduce mathematics in many different ways.

The work of teaching Mathematics includes explaining concepts and terms to students, interpreting students, providing them with examples and solutions, and assessing students' work (Abrantas, 2001). Van Den Akker (2003) believes that the job of a teacher is to give students the chance to think and analyze logically, and transfer to them the basic
concepts as required in the curriculum. Wilson (1993), however, adds that one of the important goals of instruction is to develop skills and tasks that are not explicitly covered in the curriculum. Meanwhile, the textbook as the potentially enacted curriculum could serve as helping material or as obstacle (Johansson, 2006).

The teacher-textbook relation is embedded in the enacted curriculum (Johansson, 2006). Remillard (1991) agrees that the enacted curriculum depends on instruction, yet she suggested that what research viewed as teacher-textbook relation simplifies teacher-curriculum planning. Remillard (1991) and Li (2007) found that most curriculum materials never change unless the teacher activates his/her thinking and planning.

Instructional addition to textbook is one of the criteria one ought to look at when regarding the coherence of curriculum (Li, 2007). The implemented curriculum is greatly influenced by the teacher's conceptions about mathematics and education (Thompson, 2004). This was stressed by Trafton and his coauthors (2001) and Jeppel (2004). Cruz, Macias, and Santos (2006) found that not only different teachers make use of the same material in different ways, but also the same teacher used text differently when instructing different groups of people.

Although the textbook has an "enormous influence" on what is taught in class (Schmidt et al., 2001 as cited in Chi Wang, McKnight, & Schmidt, 2005), Jeppel (2004, p:250) found that the teacher has "the opportunity to introduce a new textbook ... as consistent with his own priorities", had he had the time or energy for that (Johansson, 2006).

A textbook is a helpful guide for inexperienced teachers and a time saver for all teachers (Irujo, 2006). It may provide information in a well-organized coherent context, yet the instructor is responsible to present this material in an appealing way (Li, 2007). Sturino
(2002) studied the use of textbooks in classrooms; he observed three main approaches of instruction: teacher-centered strategy where the use of textbook was limited to providing problem sets, computer/textbook centered strategy where the textbook/computer acted as a primary means for instruction, and calculator/activity led approach where the textbook was used in various means. Ball and Feiman-Nemse (1998, as cited by Cruz, Macías, & Santos, 2006) found that teachers used the textbook as a class organizer either because they are inexperienced or because school authorities stress on the use of the textbook.

Johansson (2006) also studied teacher's use of the textbook, but divided his study into two quantitative and qualitative parts. His main results were that the textbook was the only source for problem sets tackled by students whether in their private work, homework assignments, or class examples, the textbook also served in providing motivational background during class discussions (Johansson, 2006). Too much reliance on the textbook may sometimes get the teacher into trouble (Johansson, 2006). In this context, Flanders (1994) reported that about 90% of mathematics and science teachers rely solely on one textbook for instruction (Weiss 1987 as cited in Flanders, 1994). The “Wikibooks” online source (2007) pointed out that the textbook is only one possible resource among many.

The use of textbooks is to provide students with background knowledge and introduce mathematical terms and concepts in an organized framework (Johansson, 2006). The main purposes of the textbook are to convey the curriculum Amit and Freid (2002 as cited by Cruz, Macías, & Santos, 2006 ) and help teachers organize their classes, as well as being a source of activities and problem sets. Johansson (2006) contends that textbooks contribute in conveying knowledge, facilitating the work of teachers, and organizing mathematical topics and concepts in a suitable way for pupils.
"Life and learning can be more joyful than textbook world allows" Westhues (1991 as cited in Wikibooks, 2007). The factors preventing teachers from using the textbook as it was conceived by the author and the education authorities according to Cruz, Macías, and Santos (2006) are the teachers' deficient training and difficulty in mastering math contents, and the emphasis on problem solving activities more than conceptual understanding whether in textbooks or instruction.

The NCTM proposed that teaching strategies be adjusted towards less emphasis on textbooks and more use of other resources (Sturino, 2002). The idea is to use a textbook with knowing how to use it (Irujo, 2006).

Finally, several articles have suggested better ways of using textbooks. Some suggested that the text be used as an "anchor", with instructional supplementing (Irujo, 2006). Sasson (2006) advised the use of additional materials. Johansson(2006) suggested three possible teaching techniques that motivate students and involve them: (1) Preview-Connect-Predict (Preview the upcoming reading material, connect to prior information then predict what might come later.), (2) Think-Pair-Share (Students think in pairs, share what they understand, and evaluate other students' explanations.), and (3) Admit-Exit-Tickets (Students fill tickets/note cards with what they know before studying a unit so they be "admitted" into a lesson then they write a note card after explaining the unit to "exit").
Chapter Three

Results

Curriculum Analysis

As mentioned before, the next section of the project consists of a curriculum analysis and a book analysis. The curriculum analysis basically analyzes the compliance of the Lebanese curriculum with the criteria obtained in the literature review. A priori, the five general objectives of the curriculum are presented. Then the attainment of the criteria through these objectives are checked. To recall, the criteria are covered by the following questions: Is it focused? Is it comprehensive? Does it have a limited number of topics? Does it follow sequentially within and across? Does it promote understanding rather than memorization? Does it link the concepts introduced to other academic or real life application? And are students enjoying and engaged mentally and physically? The book analysis is a chapter-by-chapter analysis. The analysis examines basically the implementation of each of the related learning objectives mentioned in the textbook and the curriculum, the general objectives of the curriculum, and the criteria of curriculum analysis specified above through problem solving, and specifies some of the exercises of the textbook that support those implementations.

Crosswhite et al. (1986 as cited by Flanders, 1994) distinguished three types of curriculum: the intended curriculum which is expressed in the objectives, outline, and syllabus of a course, the implemented curriculum which is what the teacher does at the classroom level, and the attained curriculum which is what students learn. The objectives of the Lebanese curriculum, namely the intended curriculum, are hereby analyzed. As stated in the Lebanese curriculum, there are three main changes: formulation of objectives, remodeling contents, and method of teaching.
Formulation of objectives is the first aspect of the intended Lebanese curriculum. Main objectives are to teach students how to read mathematics, understand it and interpret it, use symbols, graphs, and tables, solve problems, and write and explain mathematics. Second, the curriculum aims at remodeling the contents so that a text is judged by its practical use rather than theoretical interest. Third, there is the method of teaching; the curriculum states that instruction should be organized and it is responsible for linking mathematics to everyday life and making sense of it among learners.

The main criteria of curriculum analysis can be inferred from the review of literature. A curriculum has historically been assessed upon checking if it is focused and challenging, if it is comprehensive or addresses general mathematics literacy goals, if it follows sequentially across and within grade levels, if it promotes understanding rather than memorization, and if it engages students mentally and physically in application during class.

The first objective is to develop critical thinking and mathematical reasoning. This is to be done by training students in constructing arguments and evaluating them. Students will develop critical thinking, and they will be repeatedly asked to observe, analyze, interpret, hypothesize, generalize, and demonstrate.

The second objective is to solve mathematical problems. "Solving mathematical problems is perhaps the most significant activity in the teaching of mathematics." (ECRD, 1997 p: 289) On one hand, problem solving is a way to link mathematics to real life. On the other hand, problem solving exposes students to various tasks. Students will learn how to classify given information and tasks, manipulate mathematical techniques, use algorithms and make new ones, verify solutions and apply mathematical skills.
The third objective is to improve research skills in addition to linking mathematics to all fields of life. Mathematics improves research skills. Research is needed in the development of various economic, technological, and cultural fields. This emphasizes the importance of mathematics in everyday life. The curriculum aims at making students value the role of mathematics in the modern society.

The fourth objective is to foster mathematical communication. The latter refers to encoding and decoding messages, and interpreting and demonstrating information, orally and in writing, with the help of mathematical tools and symbols.

The fifth objective is to value mathematics not only as a science but also as an art. In this context the intended curriculum is that students become confident in mathematical tools, application, and precision. They shall finally be able to develop intuition and imagination, and find pleasure in intellectual mathematical work.

Before moving attention to the mathematical content objectives, The point that the curriculum categorizes the material into five groups should be noted: algebra, geometry, analytic geometry, trigonometry, and statistics; it organizes a timetable with specific learning objectives for each group in details. Regarding this aspect, the curriculum is organized and focused. It's also noteworthy that the timetable and number of hours allotted to each mathematical area is a useful tool to help teachers achieve the goal of intended curriculum vis-à-vis teaching methods.

Read, interpret, communicate, solve, apply, and enjoy. This is the focus of the curriculum as expressed in the first, fourth, second, third, and fifth objective respectively. The curriculum challenges students' abilities when they develop critical thinking (first objective).
tackle different problems (second objective), improve research skills (third objective), and acquire creativity (fifth objective).

The categorization of the topics of mathematical subjects studied in Grade nine and the timetable included in the curriculum justifies the limited number of topics required for Grade Nine mathematics. A limited number of topics was a major recommendation of several studies on the curriculum, such as Van Den Akker (2003) and Bahru (2005).

Meanwhile, the curriculum is comprehensive in that it provides good math base in various areas of mathematics, thus it addresses general mathematics literacy problems; this is expressed in the first general objective. The first objective, as well as the second, promotes understanding mathematics rather than memorization. The first fosters understanding and interpreting. The second addresses solving problems with different tasks where memorization does not help students.

Linking mathematics to real life problems is witnessed in the second objective. Also, the third objective of the curriculum involves mathematical applications to the world in various fields.

Finally, the curriculum engages students physically and mentally in application in the classroom. As the fifth objective emphasizes, students are "theoretically" designated a joyful class environment where they find pleasure and foster imagination.

Book Analysis

To begin with a brief overview of Grade 9 mathematics textbook, in the chapter-by-chapter analysis, a focus is stressed on learning achievement and implementation of problem solving objectives and criteria. Through the study, a look is grabbed at other elements such as how mathematics arises naturally from everyday situations through updated and revised real-
life activities and the associated practice exercises, along with the activities and the exercises within the text. It was clear that all the chapters have the same structure and the book follows the same format for each chapter. For this reason, to analyze problem solving objective in the text book, three chapters are selected representing the three content domains: Lines and Circles (geometry), Proportion (algebra), and Statistical Surveys (statistics).

The first chapter of Grade nine mathematics textbook is about lines and circles. The objectives of this chapter as stated in the textbook are: being able to “draw a tangent to a circle at a point on the circle”, “determine the relative position of a line with respect to a circle”, “draw the tangents to a circle issued from a point outside this circle”, “show that the line joining the center of the circle and a point outside this circle is an axis of symmetry of the figure formed by this circle and the two tangents issued from this point” and “define the tangent to a circle of center O at a point A as the perpendicular to (OA) at A.” (Refer to appendix II, p: 12).

It is clear from the above that the textbook lays out the objectives explicitly, using expressions of students’ level and providing clear instructions. However, it does not help students in using different strategies and raising questions used in their real world in order to verify and grasp the new concept. Although activities are definitely aimed towards students, but they are not interesting enough and don’t provide valuable lessons that kids could use to learn (refer to appendix II, p: 13). Learning a new concept starting by a problem solving situation was a main goal mentioned in the Lebanese curriculum. It’s not only the concepts that are needed in order to learn mathematics, but the concepts behind the skills. This way the students can learn why things work and not just memorize them for the test. This is clear in the focus part of the chapter mentioned under the subtitle "Text" (refer to appendix II, p: 14). New knowledge isn’t introduced starting from the real situation.
On the other hand, the five general objectives of the Lebanese curriculum are partially fulfilled in this chapter. The first two objectives, "mathematical reasoning" and "solving mathematical problems", are not practiced in every problem the student reads on his/her own; in particular it is emphasized in the Lebanese curriculum that students must learn to use different strategies to tackle difficulties in solving a problem. Particularly, in exercise 11 (refer to Appendix II, p: 18) instructions are not as direct and simple as previous exercises. Students don't have the opportunity to practice the scientific knowledge in the real world. The second objective, "solving mathematical problem", is not sensed in almost all exercises and problems. The active style of the exercises doesn't develop students' problem solving skills mentioned, such as classifying and serializing. Considering exercise 20, opportunity is not given to the students to check, observe, or construct the moving point using different tools in order to have a clear decision about the proof (refer to appendix II, p: 19). Moreover, the application of mathematical ideas of the chapter is not suggested through the problems; the only instance is in the preparatory activity which points out that the meeting point is the closest point. This may be attributed to the assumption that geometry as an application rather than real life application is sufficient at students' current intellectual level. In problem 18 (refer to appendix II, p: 23) a moving circle is given tangent to two parallel lines and the locus of the center is required. This problem is not connected to real life; both teacher and student are not given the chance to make connections between real world problems and mathematics concepts.

Further, the consistency of the problems in this chapter is checked along with the criteria modeled in the curriculum analysis, but it's clear that chapter one is not very skill oriented. In exercise 20 (refer to appendix II, p:19) it's required to show that the difference between two segments is constant. This is a problem solving situation that encourages student to use technology for the proof, rather there is a big lack of using any technological tool in
this chapter such as computers. Exercise 17 (refer to appendix II, p:22) is a problem solving case that encourages student to use different strategies such as constructing and using different algorithms to find the locus of a moving point first and proving a constant perimeter second. Again, both teacher and student are not interested in solving such exercise through relating this problem to a real life situation. On the other hand, the chapter contains pictures for students to use while visualizing a problem the reason that helps the student to tackle difficulties when solving a problem and help to make material more interesting.

Unfortunately, there are no examples from real life except in the preparatory activity as mentioned earlier, and in the concept of symmetry visited in the beginning of the chapter (refer to appendix II, p:13). "Just for fun" is a problem that is used at the end of this chapter, it's a connecting problem to the real life that encourages student to serialize, verify and apply different strategies to find the answer. This problem is used in a useless corner as if it is teacher oriented and not student, further it contains no step by step instructions that help the instructor build students' critical thinking.

Chapter 2, titled "Proportion", is in Algebra as categorized by the Lebanese curriculum. According to the textbook, at the beginning of the chapter, the student is able to "Recognize two proportional sequences, calculate the fourth term of a proportion, and use the calculation of the fourth term of a proportion in problems." At the end of the chapter, the student should be able to "represent a linear situation using a proportion table or chart, represent a linear situation of the form y=kx, represent graphically a linear situation, and pass from one representation to another." The main instructions of the chapter don't contain the required strategies that help students observe, doubt, and generalize. The formulation of the objective doesn't start from the real world in order to help student raise different question and conjectures to deduce and verify and formulate hypothesis. (Refer to appendix III, p: 24).
and quantifying. (Refer to appendix III, p: 33). On the contrary, the exercise is teacher centered; located at the end of the chapter in a useless corner, it doesn’t contain the strategies that encourage students to solve it or the instructor to help students grasp the new knowledge using different problem solving strategies.

Finally, Statistical Surveys in Chapter 14 of the textbook. In the context of handling data, students are expected, according to the curriculum, to learn about distribution in one discrete variable and different representations and about mean and weighted mean. According to the textbook, students already know how to "organize data in a table," represent it in a bar graph or in pie chart. In this chapter, they will be able to "study statistical distributions and make different kinds of representation, pass from one representation into another, interpret a graphical representation of a statistical distribution, and calculate the mean of a statistical distribution."

The learning objectives of this chapter aren't stated in a way to make student connect what they are willing to learn to the real situation. No problem solving strategies are mentioned in the learning instructions (refer to appendix IV, p: 156).

The activities were mostly student-centered, giving them a chance to make connections between real world problems and mathematics concepts (refer to appendix IV, p: 157). On the contrary we can see no charts or real world drawings used in the second activity to facilitate and clear the ideas. The summary of the chapter is mentioned under the subtitle "Text", definitions are not introduced using a real life situation, and problem solving strategies are not mentioned as a main learning objective there are not enough charts and graphs that help students practice communicating about mathematics and problem solving (refer to appendix IV, p: 158).
Exercises are focused and organized; each part clearly requires one task. But on the contrary, to what the curriculum stresses in its general goals, most of them are theoretical and not real world cases such as exercises 1, 3, 4(refer to appendix IV, p: 161). In exercise one a statistical table is given and students are supposed to calculate frequency and represent results using a bar graph and a pie chart. This exercise is not encouraging student to think of various strategies to find the answer, question is not connected to the real world the reason that keep student away from exploring real world questions. Problems are comprehensive, representations and statistical means are included in the problems, and for instance, bar graph, pie chart, and table representations are practiced in problems 1, 3, and 5 respectively (refer to appendix IV p:162-163). Real world situations are practiced in most the problems such as 2, 3, 4 and 5(refer to appendix IV p: 162-163) this encourage students to observe, analyze and prove.

At the end of each chapter a challenge exercise is mentioned which is "just for fun" (refer to appendix IV, p: 163). Again it's a teacher oriented exercise, it can't help the student to apply what they already learn using different problem solving strategies.

After having a close look at each chapter of the book, some recommendations are recalled that passed indirectly through the book analysis. In brief, it's clear that the book is following the same format for all chapters. Each chapter consists of an introduction, a text section with a Focus for review, and a problem solving section with a self evaluation set of exercises, where in some places there should be more students-centered activities. The book doesn’t fulfill many of the goals set by the Lebanese curriculum such as developing problem solving skills and strategies through introducing new concept by a real situation. Activities should be supplemented by visual aid clip art, photos, and geometric figures to help students identify and apply math to the real world. As seen, instructions are easy, objectives are laid
out explicitly in simple vocabulary, yet need more illustration sometimes, real life and word problems should be increased. The textbook is not a good guidance for the teacher as to the order of ideas in the chapter and availability of examples and pop-up notes for students; however, more practice is needed in many chapters which leave for the instructor the task to find additional resources. On the other hand, time organization guidance is available in the curriculum.

Noteworthy, the preliminary pages of the textbook are not helpful and the analysis needs to be more concerned with problem solving which is about more than just its literal meaning; as asserted in the literature review, problem solving involves conceptual understanding and learning of techniques. The idea is not to get the solution; it is about learning how to solve.

*Interviews with Teachers*

The teachers are referred to as T1 and T2

The interviewed teacher T1 has been in the field of teaching Mathematics for seven years; she has taught grades 7, 8, 9, 10 and 11. Abiding by the coordination decision in the school S1 where she uses Building Up Mathematics, the Lebanese national mathematics textbook, for grades 7, 8 and 9, and another local book series for grades 10 and 11. T1 believes that problem solving is a "high mental activity" that requires attentive skills. She considers problem solving in Mathematics an act through which "students develop hypotheses and conjectures then justify or prove them. It is an activity that involves reasoning inductively or deductively."

When asked if she agrees with problem solving goals in the Lebanese curriculum, T1 answers that developing problem solving ability is a goal of the Lebanese mathematics
curriculum, yet it is rarely found in the curriculum for middle grades. TI says that more focus on problem solving should be included in the math curriculum of middle grades. When asked about what she agrees and doesn't agree with in this regard, she specified that she believes strongly in the importance of the problem solving goal of the curriculum; it is vital that students acquire this ability. The problem is that problem solving is not well developed in the Lebanese enacted curriculum; "many problems still recommend proofs and justifications" adds TI, pointing out that problems should be more "solution-directed" with less emphasis on justification abilities. Students should practice how to solve more problems than those provided by the textbook; many problems that require students to prove something are not very effective in teaching the problem solving techniques in middle grades. Students at this stage are more attracted by short answer questions and they benefit from them more with respect to the "solving" objective of the curriculum. TI believes that problem solving requires some mental abilities, particularly "analysis and synthesis". In her opinion, Analysis and synthesis contribute to problem solving; actually they are basic abilities, without which there is no problem solving attained. Problem solving is not about solving, it is about learning how to solve. After learning basic solving techniques through repetitive problems and simple practice exercises, students should understand the specific situation of the tackled problem through analysis, and then synthesize a solution by combining more than one learned technique or task, they synthesize an algorithm for that problem and then they might generalize it to problems of the same sort.

There are certain difficulties faced by each teacher. TI complains from problems that require students to prove and justify. "The main difficulty is when students are asked to build proofs." Even when students know the solution and have understood the concept of the chapter, they have difficulty writing their own proof; they need anchors to direct their abilities of writing mathematics, they don't know where to start, and it is not always the same
"where or how to start", so this problem is faced whenever given such exercises. T1 further spoke in details about problem solving during her mathematics class. "At the beginning of a new chapter," says T1, "students are sometimes given a problem in which they have to use their previous knowledge to construct or discover a new mathematical concept." This is how problem solving is involved in introducing a new topic to her students. Then "throughout the lessons, students are provided with some proof problems." So, some more problems are given after explaining the chapter. Problem solving however is not only included after explanation. T1 sometimes starts a new topic with problem solving. It is a good start to trigger problem solving abilities and linking previous mathematics knowledge to the newly introduced topic. When students are asked to solve they tend to notice these links more than when the teacher "reads" them in class.

On the other hand, T1 finds that the main difficulties student faces when solving problems are in "constructing new knowledge." The first set of problems of each kind is usually perplexing for students. As mentioned earlier, T1 stresses that the main difficulty faced by her and her students is in solving problems that require proving or require making up new algorithms, problems that require more than practice or repetitive tasks. When the chapter is at its end, although students do understand the topic and are familiar with practiced techniques, they still face difficulty in tackling new problems and constructing new solutions. According to T1, these difficulties should be dealt with. Students should be trained "to specify the necessary and sufficient data and then use this data in building new knowledge." Practice and instruction, adds T1, is what should be done to overcome those difficulties.

Finally, T1 was asked about the specific strategies, if any, she uses when teaching problem solving. The main four strategies T1 uses are specifying the given data, analyzing the given data, reasoning inductively or deductively and judging the correctness of the
solution. It is important to train students how to interpret the given and specify the given data, not only to turn text into math in case of word problems, but also to specify what the given data is and the given numbers mean. Also it is important that students learn how to analyze the given data, this is emphasized through instruction, it should be done explicitly and slowly by the teacher so that students learn the importance of this part of problem solving and recognize that it is a step in itself. Third, T1 uses both inductive and deductive reasoning in teaching problem solving; it is very significant that students observe both types of reasoning, this also fosters their conceptual understanding and their ability to reread and interpret the given problem; otherwise they tend to resort to memorization rather than understanding. Finally, checking the solution is just as important as other stages of problem solving; T1 includes judging the solution within problem solving; the teacher should note the importance of checking the solution and looking back at it carefully, sometimes a wrong technique may give a correct answer by accident, and when the answer is wrong it is important that the teacher goes over the solution carefully in front of students to teach them how to look back and find the mistake.

T2 who has been teaching for seven years, has taught grades 8 and 9. She teaches at another school S2, that uses the national mathematics textbook "Building Up Mathematics" for grades 8 and 9. According to T2, problem solving requires higher order thinking skills; "it means engaging students in solving a task for which they don’t know the solution".

T2 appreciates the emphasis on the problem solving objective of the curriculum. Yet, she says that some problems of the book still require proofs and ask students to build their own "long" proofs and justifications.

T2 believes that problem solving requires analysis and synthesis. "Although it plays a great role in introducing difficult concepts, problem solving requires analysis and
synthesis... to read the problem, analyze it and understand it carefully; to compare to other problems; ... these skills are not trivial for students," these are the mental abilities that T2 believes are involved in problem solving mental process.

T2 is also one of the many teachers who mainly complain from problems in which students are asked to build proofs. "The main difficulty is when the students are asked to build proofs", says T2. Students expect exercises to ask them for repetitive tasks and in middle grades they are still in dire need for practice and repetitive exercises; they tend to admire this sort of exercises, and thus be more enthusiastic about learning the solving technique of such problems which they can manage instead of having to solve many problems which they consider hard. They never learn to "figure out" or "propose" solutions for new constructive problems, they wait for instruction to lead them, also their attitudes towards this kind of problems is that these are "problems that they are not expected to know because each one has a different solution".

T2 resorts to problem solving through instruction, at the beginning of lessons that require introducing a new topic or constructing a new idea, "problem solving is effective in linking mathematical topics to previously learned concepts" says T2. Certainly, she uses problems throughout the lesson to teach students specific solving techniques.

Regarding students, T2 finds that the main difficulties student face are in "using problem solving approaches to explore and understand mathematical content", that is when they need to "adjust strategies" to acquire conceptual understanding rather than memorizing tasks, and in dealing with problems that relate the mathematical concept to real life situations or problems from other academic fields or even from unrelated mathematical subjects. "One of the main difficulties they struggle with is applying the modeling process of solving to new problems from outside mathematics or from real life." The main factor that helps in facing
these difficulties is within instruction; T2 believes that it is the teacher’s role to provide students with data and strategies they need to build the new knowledge understand the concept and apply it in any sort of problems. "Problem solving is an integral part of the process of learning mathematics."

The last question is about using any specific strategies when teaching problem solving. T2 says that she is a supporter of the view of Polya. "It is the strategy for problem solving suggested by Polya," says T2 gratefully. "I make sure students recognize the "Existence" of these steps and to try to abide by them as I do in class. First, the solver should read the problem, then understand it well and "translate it into mathematics", then devise a plan for solving, then carry out the plan and look back at the solution as a whole; "even when a solution is there, I try to go over the problem and the solution again in class to emphasize the importance of looking back, it is important that students understand and recognize each step of the algorithm used and understand the concept of it in order to be able to parallelize it for other problems."

It’s clear that the two teachers believe that problem solving is a good tool for introducing a new math concept, and for both it’s the best tool to link between new math knowledge and prerequisites. Further, they both complained that building a proof is the main issue when teaching new math knowledge. The textbook is not a good tool in providing easy step-by-step instructions for the instructor and student. According to T1 there should be an alternative math foreign textbook that fulfills the objectives mentioned in the Lebanese curriculum, whereas T2 mentioned that it’s the teacher role in providing extra problem solving exercises that serves the goals mentioned in the curriculum in order to help students really grasp the concept.
It's obvious that the two teachers show a weak experience in teaching problem solving and they need training sessions and workshops of how to teach problem solving strategies. This appears from their complaints about the exercises that require proofs or justifications.
Chapter Four

Discussion and Conclusion

Having analyzed and found the results, this chapter aims at comparing, contrasting and synthesizing those results in order to answer the research questions.

Just as reminder, the research questions are:

1. To which extent is the grade-nine math textbook coherent with the curriculum as to the problem-solving goal?
2. Is the textbook helpful to the teacher in terms of providing guidelines for developing students' problem-solving skills?
3. What are the difficulties that teachers face in teaching problem-solving?

Curriculum Text-book Coherence

The key research questions are covered throughout the paper. Briefly, the textbook is not completely coherent with the Lebanese curriculum, there are some gaps in the problems provided as can be seen from the book analysis. For, the third and fourth general objectives of the curriculum, "to apply the mathematical topic introduced" and "use it in the real situation", were not implemented sufficiently in the problems. Moreover, the problem sets included in each chapter are not enough and they don't engage students physically in any exercise. For such an engagement, including a problem or two where the variable is a real life object is suggested. For example, in Lines and Circles, no real life application is suggested. Also, promoting the use of tangency in communication was not enhanced. Tangent lines in real life are not found in many forms of real objects that attract students, for instance a problem about shadows would help. Also there is a need for more practice exercises prior to the challenging
problems with which students face difficulty as reported by the interviews. Moreover, it is concluded that problem solving is not used as a means to introduce or link topics to previously learned ones or to involve real world or other outside-the-field applications. Noteworthy, the exercises in some chapters don't cover all the objectives mentioned at the beginning of the chapter. However, the missing objectives are covered under the "Problems" section. Thus, some problems of the book should be included in the "exercises" section of the corresponding chapters.

The Analysis of the textbook showed that it is not helpful enough, neither for teachers nor for students. Although it provides the teacher ways for organizing time and sequence of ideas in a chapter, it doesn't offer them numerous ideas on what and how to teach problem solving strategies to their students. Activities are not helpful or interesting enough for students as well as teachers to introduce new concepts with real situations.

Also, there is much emphasis in the introduction of the curriculum on the role of teachers and on their responsibility of using additional instructional material, use other resources, and create methods to turn mathematics into a fun activity that helps students become problem solvers in real life. However, there is a lack of strategies for teachers to use in order to help students grasp each subject. For students, chapters are not student-centered, textbook pages are filled with nothing but numbers and problems, students may tend to feel anxious and overwhelmed by the concepts being taught. However, having pages partly filled with pictures and fun activities, help students be more likely to be relaxed and not as worried about whether or not they will understand the concepts. Notably, just as an example, in the "Statistics" chapter no direct question on plotting numbers on a number line is included. The fact that this task is easy and known by students doesn't mean that it could be neglected,
especially because it appeals to students. The skill of plotting on number line is important for students to learn in order to connect to their real life problems.

_Teachers' Difficulties_

Next, a brief review of the interviews with teachers, identifying significant comments, similarities and differences, is presented to find the difficulties they are facing in developing students' problem solving skills.

The difficulties faced by teachers are mainly in problems which require building proofs. This goes back to many factors such as students' weak synthesis abilities, conceptual understanding, etc. Problem solving, according to T1 and T2, is a good tool for introducing new topics. It is used in constructing new knowledge; and it proved to be the best way to link the new topic to the previously learned subjects. Both teachers say they resort to problem solving at the beginning of a chapter to introduce a new concept, they both use problem solving "sometimes" in introducing new chapters.

On the other hand, both teachers observe that the main difficulty faced by students is when they are asked to build new types of proofs. Although problem solving is emphasized in the curriculum, and practice exercises have proved efficiency in teaching many solving techniques, the textbook doesn't contain quite a number of exercises and problems that require students to come up with new strategies. Meanwhile, a good textbook is supposed to encourage students to come up with their own strategy of solving, but this skill usually depends on good enacted curriculum. The textbook should provide a set of exercises that cover the objectives at an elementary level, namely practice exercises. This doesn't mean however that the textbook should exclude problems that challenge students and encourage them to come up with their own learning strategies.
In this context, both teachers agree that the major burden to overcome difficulties faced by students lies on the teacher. Thus, the role of the teacher is indispensable, however efficient is the textbook used. It is important that the teachers themselves be aware of the importance of instruction and knowledgeable about the curriculum within the grade level and across grades, which highlights the importance of training and the job of coordinators at schools. Eventually it is important that students find solutions, yet what counts is that they learn the concept and the strategies of solving. The latter is why both teachers say they make sure they teach their students how to look back. Checking the solution is independent of its truth or falsity; students should witness that even the teachers need to go over the problem again to review the strategy and steps, to be able to repeat the work on similar kinds of problems.

One of the teachers adds that another difficulty faced by students is linking mathematical topics to problems outside the field or to real-life application problems. To overcome this difficulty, several factors play a role such as conceptual understanding, using visual material, instructional procedures, and teacher's knowledge about related instances in other academic fields within the grade level, as well as the abundance of practice exercises and real life links in the textbook.

Another difference between the two interviews is in the strategies used in teaching problem solving. It is a distinction of emphasis on certain stages of the problem solving process more than other stages. T1 for instance emphasizes the use of inductive and deductive reasoning, she stresses on the analysis part of the process, whereas T2 works equally on both mental abilities, analysis and synthesis; "to devise a plan" specifies T2. Meanwhile, both teachers point out the importance of students paying attention to separating the phases of the process and to the significance of each of these phases, precisely reading the
given data, analyzing it and looking back at the solution. To sum up, as asserted in the literature review, a good teacher can't avoid using additional instructional material and resources. Second, students and teachers should both be aware of the problem solving process incorporated, and of the goal of problem solving: it is not to solve, it is to learn how to solve.

Limitations of the Study

The study is limited to two interviewed teachers, with no classroom observation or students' examination. It did not cover analysis of the textbook and curriculum regarding their sequential flow across grade levels or, within grade nine, other textbook chapters; also, it did not give much attention to the use of other instructional material or technologies; moreover, more practical and precise recommendations could be made upon conducting observation of classrooms, interviewing more than two teachers, or interviewing students and examining their abilities to solve word problems.

Recommendations for improving the teaching of problem solving

Recommendations Concerning Curriculum Developers

The main problem of the curriculum as shown in the analysis is the inconsistency between the general objectives, and more specifically problem solving goals, and the more specific objectives and details of content. It is recommended to revise the Lebanese curriculum for more consistency and coherence (Van Den Akker, 2003) that would implement the general goals.

As for the textbook, the analysis showed that contrary to the claim in the introduction of the curriculum, not every new topic is introduced through problem solving. Students, on the other hand, are often not given the chance to observe, analyze, or make a synthesis. In order to improve the efficiency of the textbook, more help should be provided to
students in recognizing, identifying and applying different problem solving strategies, depending on the nature of the problem (Li, 2007). More examples from real life should also be added (Sturino, 2002).

Recommendations Concerning Teachers

Inexperienced teachers in problem solving must be guided by workshops and training sessions that would equip them with skills of how to enhance in their students problem solving skills, such as story problems using various problem solving techniques in many different ways (Jeppel, 2004). Moreover, teachers are recommended to supplement the textbook with materials providing various activities and real life links so that students have the chance to understand the new mathematical concept and practice problem solving strategies (Johansson, 2006).

Recommendations for Further Research

The conclusion of the project entails some recommendations deduced from the curriculum analysis, chapters’ analysis and the interviews, with few specific textbook reform suggestions, and suggestions for further research. It’s recommended that such further research could be done with different cultures and languages. Comparing the Lebanese math textbook, including the teacher’s manuals, to a foreign one may lead to more reliable and fruitful results. Noteworthy, for future studies it’s suggested to find the effect of using representations in problem solving involving technology such as computers and calculators.

Finally, there are many factors that contribute in the process of education and learning problem solving. Further work could have brought about more reliable results, such as taking into consideration personal past experience and mental abilities of students, general coherence of curriculum or textbook within and across grade levels, etc.
References


Appendix I

*Questions of the interview with teachers*

1. How long have you been in the field of teaching mathematics? Which grade levels have you taught?

2. Which books did you use?

3. What does problem solving mean to you in a math class?

4. Do you agree with the problem solving goals in the Lebanese curriculum? What are the aspects that you agree with, and those you don’t?

5. In your opinion what are the mental abilities that contribute to problem solving?

6. What difficulties do you face when teaching problem solving?

7. What proportion of your class work is problem solving?

8. Do you start a topic with solving problems?

9. What are the main students’ difficulties when solving problems?

10. What should be done to overcome those difficulties?

11. Do you use specific strategies when teaching problem solving?
Appendix II

Chapter 1

National Mathematics Textbook for grade 9

Lines and Circles
Lines and circles

Introduction

The meeting of a circle and a line...

Activity 1

The meeting point... The closest point

Activity 2

What is the point on a circle from where a tangent to the circle?

Activity 3

How do you go about the following?:

Activity 4

If you have a triangle, you can add a circle to it.

Activity 5

What does the line (AY) intersect in the following figure?

Activity 6

Place a tangent to a circle from a point outside the circle.

Conclusion

From a point outside the circle, draw a tangent to a circle, from a point on the circle, draw a tangent to the circle, from a point inside the circle, draw a tangent to the circle.

Activity

What is the meeting point of a circle and a line?

Diagram

Lines and circles
II. Relative positions and distances

1. Relative positions of a line and a circle:

- A line does not intersect a circle if it is parallel to the circle.
- A line intersects the circle in two points if it is located a secant to the circle.
- A line intersects the circle at exactly one point if it is a tangent to the circle.

II. Tangents through a point to a circle

1. From a point outside the circle two tangents can be drawn.
2. From a point inside the circle only one tangent can be drawn.

III. Properties

1. The joining the center of a circle to a point a outside it.

- If a tangent is drawn to a circle from a point outside, the tangent is perpendicular to the radius drawn to the point of contact.

Conversely, if a line drawn from the center of a circle is perpendicular to a line drawn from a point outside, the line is tangent to the circle.

Proof:

Consider a circle with center O and a point P outside the circle. Draw a line from O to P, which we'll call OP. Now draw a tangent from P to the circle, which we'll call PT. According to the Pythagorean theorem, in right triangle OTP, we have:

OP^2 = OT^2 + PT^2

Since OP is the distance from the center to the point outside, OT is the radius of the circle, and PT is the length of the tangent, we can see that PT is perpendicular to OT. Therefore, PT is a tangent to the circle.

II. Properties of the circle

1. If a line is tangent to the circle, the line segment from the point of contact to the center of the circle is perpendicular to the tangent.

Proof:

Consider a circle with center O and a tangent PT from point P outside the circle. Draw a line from O to P, which we'll call OP. According to the Pythagorean theorem, in right triangle OTP, we have:

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Appendix III
Chapter 2
National Mathematics Textbook for grade 9
Proportionality
A line can hide another.

**Activity**

The gear wheel

**Activity**

### Pre-Procedure

1. Complete the following table and then locate the center gear and each of the three left gears. Let's examine the number of teeth (cog) of each gear, and the distance between the teeth.

<table>
<thead>
<tr>
<th>Gear</th>
<th>Number of Teeth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

### Follow-Up Questions

1. If you are adding a pair of gear wheels, the total number of teeth must be the same. Why?

2. If you are removing a pair of gear wheels, the total number of teeth must be the same. Why?

### Recall

Proportionally, it is more than a definition. It is a way of thinking.
I. Proportion and Algebraic Relation

II. Proportion and Graphical Representation

III. Proportion and Table
Exercises

1. In each of the following cases, determine the equation of the line.

2. If \( y = mx + b \) is the equation of a line, determine the value of \( b \).

3. Solve the equation \( 2x + 3 = 7 \).

4. If \( f(x) = 3x - 2 \), find \( f(4) \).

5. Find the slope of the line passing through the points \( (1, 2) \) and \( (3, 6) \).

6. Graph the equation \( y = -2x + 4 \).

7. Determine the equation of the line passing through the points \( (2, 3) \) and \( (4, 7) \).

8. Solve the equation \( 3x - 5 = 10 \).

9. Find the x-intercept of the line \( y = 2x - 4 \).

10. Graph the equation \( y = mx + b \) for \( m = 2 \) and \( b = 3 \).

11. Solve the equation \( 2x^2 - 3x + 1 = 0 \).

12. Determine the equation of the line passing through the points \( (0, 0) \) and \( (2, 4) \).

13. Find the slope of the line \( y = 4x - 1 \).

14. Graph the equation \( y = x^2 \).

15. Determine the equation of the line passing through the points \( (1, 1) \) and \( (3, 3) \).

16. Solve the equation \( x^2 - 4x + 3 = 0 \).

17. Find the y-intercept of the line \( y = 3x + 2 \).

18. Graph the equation \( y = -x + 5 \).

19. Determine the equation of the line passing through the points \( (0, 5) \) and \( (2, 3) \).

20. Solve the equation \( 2x + 3y = 6 \).
Appendix IV
Chapter 3
National Mathematics Textbook for grade 9
Statistics
III. Mean of a Statistical Distribution

1. Given the following statistical distribution:

<table>
<thead>
<tr>
<th>% Raw Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>12</td>
</tr>
<tr>
<td>20</td>
<td>16</td>
</tr>
<tr>
<td>22</td>
<td>15</td>
</tr>
<tr>
<td>25</td>
<td>14</td>
</tr>
<tr>
<td>30</td>
<td>11</td>
</tr>
<tr>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>50</td>
<td>12</td>
</tr>
</tbody>
</table>

Calculate the mean of this distribution.

2. A statistical distribution is given below:

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
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<tr>
<td>7</td>
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<td>14</td>
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<tr>
<td>9</td>
<td>13</td>
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<td>11</td>
<td>10</td>
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<tr>
<td>12</td>
<td>9</td>
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<tr>
<td>13</td>
<td>8</td>
</tr>
<tr>
<td>14</td>
<td>7</td>
</tr>
</tbody>
</table>

(a) Calculate the mean of the distribution.
(b) Draw a frequency polygon for this distribution.
(c) Represent the frequency distribution in a bar graph.
(d) Represent the mean of this distribution.
(e) Draw an ogive of this distribution.

3. The following table represents a data set:

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
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<tr>
<td>6</td>
<td>12</td>
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<tr>
<td>7</td>
<td>15</td>
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<td>13</td>
<td>8</td>
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<tr>
<td>14</td>
<td>7</td>
</tr>
</tbody>
</table>

Calculate the mean of this data set.

4. The following distribution represents a sample of a population:

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
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<tr>
<td>6</td>
<td>12</td>
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<td>7</td>
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<td>13</td>
<td>8</td>
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<tr>
<td>14</td>
<td>7</td>
</tr>
</tbody>
</table>

(a) Calculate the mean of this distribution.
(b) Draw a frequency polygon for this distribution.
(c) Represent the frequency distribution in a bar graph.
(d) Represent the mean of this distribution.
(e) Draw an ogive of this distribution.

5. The following data set represents a population:

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
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<td>6</td>
<td>12</td>
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<td>7</td>
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<td>9</td>
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<td>13</td>
<td>8</td>
</tr>
<tr>
<td>14</td>
<td>7</td>
</tr>
</tbody>
</table>

(a) Calculate the mean of this distribution.
(b) Draw a frequency polygon for this distribution.
(c) Represent the frequency distribution in a bar graph.
(d) Represent the mean of this distribution.
(e) Draw an ogive of this distribution.

6. The following data set represents a sample of a population:

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
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<td>6</td>
<td>12</td>
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<td>7</td>
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<td>8</td>
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<tr>
<td>14</td>
<td>7</td>
</tr>
</tbody>
</table>

(a) Calculate the mean of this distribution.
(b) Draw a frequency polygon for this distribution.
(c) Represent the frequency distribution in a bar graph.
(d) Represent the mean of this distribution.
(e) Draw an ogive of this distribution.