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Short-Term Financing of Economic Order Quantity (EOQ) Inventory Model With Probabilistic Quality

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In this paper, the classical economic order quantity (EOQ) inventory model assumption that all items of a certain product received from a supplier are of perfect quality is relaxed. Another basic assumption that the payment for the items is made at the beginning of the inventory cycle when they are received is also eased. We consider an inventory situation where items received from the supplier are of two types of quality, perfect and imperfect, and a short deferral in payment is allowed. The split between perfect and imperfect quality items is assumed to follow a known probability distribution. Both qualities of items have continuous demands, and items of imperfect quality are sold at a discount. A mathematical model is developed using the net present value of all cash flows involved in the inventory cycle. A numerical method for obtaining the optimal order quantity is presented, and the impact of the short-term financing is analyzed. An example is presented to validate the equations and illustrate the results.

Keywords: economic order quantity (EOQ), imperfect quality items, continuous demand, allowed deferred payment, net present value, financing policies, operations planning

Introduction

The classical economic order quantity (EOQ) model describes an inventory situation where the demand rate for a certain item is D. At the beginning of each inventory cycle, an order of size Q is received from a supplier at a unit cost C to meet the demand and an ordering cost of K. Let h be the holding cost per unit per unit time, then the total inventory cost per unit time function is given by:

$$TCU(Q) = KD/Q + CD + hQ/2$$

The optimal order quantity or the EOQ is:

$$Q^* = \sqrt{(2KD/h)} \quad (1)$$

This model is established based on several basic assumptions that are seldom encountered in practice. Ever since the EOQ model and the closely related economic production quantity (EPQ) model were introduced in the

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early 20th century, researchers have been studying the EOQ/EPQ models extensively under real-life situations (Bedworth & Bailey, 1987; Simpson, 2001). A vast literature on inventory and production models generalized the EOQ/EPQ models in numerous directions by modifying or relaxing the underlying assumptions of the models. One of the assumptions of the classical EOQ model is that all items received from a supplier are of the perfect quality type. Due to deterioration, shifting production process, or other factors, some items may be of imperfect quality. Good examples of such situations are found in the electronics industry. Another basic assumption is that payment for the items is made at the beginning of the inventory cycle when the order is received. A survey of papers from literature tackling inventory models in which the above mentioned assumptions are considered is presented below.

Cheng (1991) presented an EPQ model with imperfect production process and demand-dependent unit production cost. Porteus (1986) investigated the effects of defective items on the basic EOQ model. Schwaller (1988) proposed an EOQ model with fixed and variable costs for screening and removing defective items. Rosenblatt and Lee (1986) assumed that defective items can be reworked instantaneously at a cost and concluded that the presence of defective products motivates smaller lot sizes.

Salameh and Jaber (2000) initiated a new line of research in the field of inventory management that ensures quality. They proposed an EOQ model where items received from the supplier contain imperfect items that are not necessarily defective and can be salvaged at a discounted price. Hayek and Salameh (2001) studied the production lot sizing with the reworking of imperfect quality items. Chiu (2003) determined the optimal lot size for an EPQ model with random defective rate, rework, and backlogging. Y. S. P. Chiu, S. W. Chiu, and Chao (2006) developed an EPQ model in which defective items are reworked or scraped and presented a numerical method for determining the optimal lot size. Ozdemir (2007) examined an EOQ model with defective items that allows shortages and backorders. El-Kassar, Dah, and Salameh (2008) developed an EPQ model with different quality items having continuous demands. Khan, Jaber, Guiffrida, and Zolfaghari (2011) presented an extensive survey of such articles.

In a different direction, numerous papers incorporating financial factors, such as time value of money, inflation, and credit facilities, into the classical inventory models can be found in the literature. Salameh and El-Kassar (1999) investigated the effects of time value of money and credit facility on the optimality of the single period inventory model. Salameh, Abboud, El-Kassar, and Ghattas (2003) presented a continuous review inventory model with delay in payments. The effects of time discounting on the EOQ and EPQ models were examined in Salameh, Abdul-Malak, and El-Kassar (1999) and Salameh and El-Kassar (2003). Dah and El-Kassar (2008) developed a uniform replenishment inventory model with payment credit facilities.

Salameh and El-Kassar (2007) introduced an EPQ model that accounts for the cost of raw material. El-Kassar and Dah (2009) extended that model to include the time value of money. El-Kassar, Yassin, and Maknieh (2010) generalized the model to a multi-stage production process. Time discounting was incorporated into the multi-stage model (El-Kassar & Yassin, 2011). El-Kassar, Salameh, and Bitar (2012a) investigated the effects of imperfect quality items of raw material on the EPQ model. El-Kassar, Salameh, and Bitar (2012b) extended the model to account for the time value of money. Mikdashi, El-Kassar, and Joudi (2012) studied the effects of having a probabilistic percentage of imperfect quality items of raw material on the optimal production lot size. Yassine (2014) analyzed the optimal order quantity for two qualities of products when payment delay is allowed; the results showed that the payment delay yields a lower optimal order quantity compared to the classical model.

This paper expands the model of Yassine (2014) by considering the time value of money of receipts and disbursements in the ordering process. Therefore, we calculate the net present value of total profits, a more realistic performance measure than the regular total profits. It considers an inventory situation where items received from a supplier are of two types of quality: perfect and imperfect. The percentage of perfect quality items is a random variable having a known probability distribution. It is assumed that demands of both types of items occur continuously. A screening process conducted at the beginning of the inventory cycle is used to detect the imperfect quality items. Items of perfect quality are sold at a regular price while the imperfect ones are sold at a discount. Also, this paper assumes that the supplier offers a delay of payment for the items received at the beginning of the cycle. Since we directly extend Yassine (2014), the notations here follow the previous notations closely, with a couple of new variables added.

Notations

To develop the proposed model, the following notations will be used throughout this paper:

Q = Quantity ordered;

D_p = Demand rate of items that are of perfect quality;

D_i = Demand rate of items that are of imperfect quality;

D = Demand rate of both perfect and imperfect quality items, $D = D_p + D_i$;

x = Screening rate;

q = Percentage of perfect quality items received;

$f(q)$ = Probability density function for q;

μ = Expected percentage of perfect quality items received;

σ = Standard deviation of the percentage of perfect quality items received;

K = Ordering cost per inventory cycle;

h = Holding cost per unit per unit time;

C = Unit purchasing cost;

C_s = Unit screening cost;

S_r = Regular selling price of one perfect quality item;

S_d = Discounted selling price of one imperfect quality item;

T = Total inventory cycle length;

T_p = Perfect quality items inventory cycle length;

T_i = Imperfect quality items inventory cycle length;

T_s = Screening period ($T_s = Q/x$);

i = Simple interest rate;

M = Delay in payment period offered by the supplier;

$E[.]$ = Expected value of an expression.

Cash Flow Discounting

In this section, we derive the formulae to discount future cash flows in the three cases below. We use P to denote the present value of a sum, whereas F is used for the future cash flow. The simple interest rate is considered, because banks use it in the short-term discounting.

For a single cash flow, F, occurring N periods in the future, the relationships between P and F are:

$$F = P(1+iN)$$

$$P = \frac{F}{1+iN} \tag{1}$$

In the case of a uniform continuous cash flow A occurring between time 0 and N, we have:

$$F = AN + \int_0^N iA(N-t)dt = AN + \frac{iAN^2}{2} = AN(1+iN/2)$$

$$P = \frac{AN(1+iN/2)}{1+iN} \tag{2}$$

The net present worth of a linear gradient continuous cash flow G extending from 0 to N is obtained as follows:

$$F = \frac{GN^2}{2} + \int_0^N iG(N-t)^2\,dt = \frac{GN^2}{2} + \frac{iGN^3}{3} = \frac{GN^2}{6}(3+2iN)$$

$$P = \frac{GN^2(3+2iN)}{6(1+iN)} \tag{3}$$

The Mathematical Model

We consider the inventory situation where an order of size Q is received from a supplier at a unit purchasing price C and an ordering cost K. It is assumed that each order received, at the beginning of the inventory cycle, contains both perfect and imperfect quality items. The percentage of perfect quality items is a random variable q having a known probability density function $f(q)$. Items of imperfect quality are detected through a screening process conducted at a rate of x units per unit time. The perfect quality items are sold at a regular unit price S_r and the imperfect quality items are sold at a discounted unit price of S_d, where $S_d < S_r$. Most EOQ models with imperfect quality assume that the imperfect quality items are sold as a single batch at the end of the screening period. In this paper, we assume that both perfect and imperfect items have continuous demands, of rates D_p and D_i, respectively. In order to meet the demands of both types of items, we assume that the screening rate x is greater than both D_p and D_i. Figures 1 and 2 illustrate the behaviors of the inventory levels.

The order of size Q received from the supplier contains qQ perfect quality items and $(1 - q)Q$ imperfect quality items, we have that the cycle lengths for the two types of items are given by:

$$T_p = qQ/D_p \tag{4}$$

$$T_i = (1-q)Q/D_i \tag{5}$$

The combined inventory cycle length is $T = \max\{T_p, T_i\}$. From $t = 0$ to $T_m = \min\{T_p, T_i\}$, both types of items are sold. In order to avoid shortages of perfect quality items, we assume that $T_p \geq T_i$. In this case:

$$T = \max\left\{T_p, T_i\right\} = T_p \tag{6}$$

and the imperfect quality items will be sold out at time $T_m = \min\{T_p, T_i\} = T_i$.

To find the optimal order quantity, the total profit per unit time function, $TPU(Q)$, is maximized. This function is determined by accounting for the cost and revenue components incurred or received during the inventory cycle. During the inventory cycle, the revenue components received are as follows. Revenues from sales of perfect quality items sold at a regular price can be considered as a uniform continuous cash flow of size S_rD_p extending from time 0 to T_p.

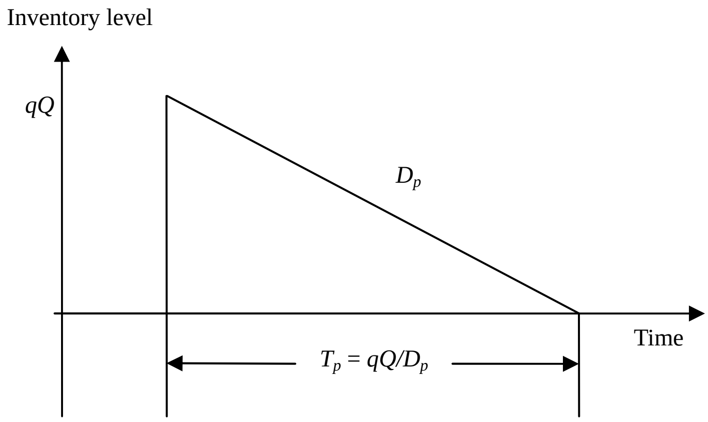

Figure 1. Perfect quality items inventory level.

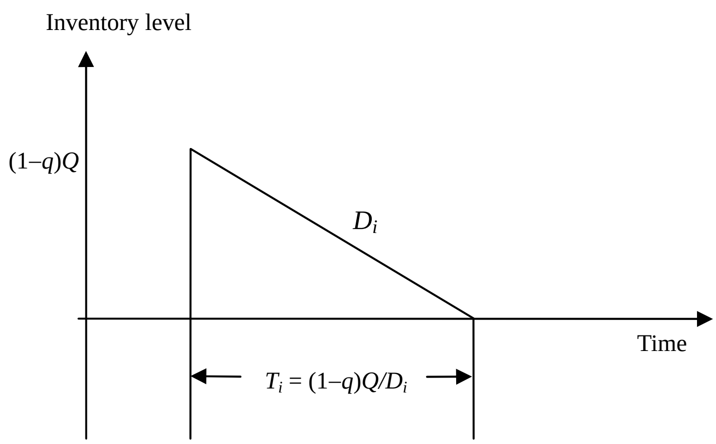

Figure 2. Imperfect quality items inventory level.

From Equation (1), we have that the present worth of perfect quality items sold at a regular price is:

$$
\begin{aligned}
PV(Perfect\,Quality\,Items\,Sold) &= \frac{S_r D_p Tp(1+iTp/2)}{1+iTp} \\
&= \frac{S_r D_p (qQ/Dp)(1+i(qQ/Dp)/2)}{1+i(qQ/Dp)} \\
&= \frac{S_r qQ(D_p + iqQ/2)}{D_p + iqQ}
\end{aligned}
\tag{7}
$$

Similarly:

$$PV(Imperfect\,Quality\,Items\,Sold) = \frac{S_d(1-q)Q(D_i + i(1-q)Q/2)}{D_i + i(1-q)Q} \tag{8}$$

Hence, the total revenue function per inventory cycle is given by:

$$TR(Q) = \frac{S_r qQ(D_p + iqQ/2)}{D_p + iqQ} + \frac{S_d(1-q)Q(D_i + i(1-q)Q/2)}{D_i + i(1-q)Q} \tag{9}$$

The total cost function per inventory cycle, $TC(Q)$, consists of four components: purchasing cost, screening cost, setup cost, and holding cost.

Present worth of purchasing cost is as follows:

$$Present\ worth\ of\ purchasing\ cost = \frac{CQ}{1+iM} \tag{10}$$

Present worth of screening cost is as follows:

$$\begin{aligned} Present\ worth\ of\ screening\ cost &= \frac{C_s x T_s(1+iT_s/2)}{1+iT_s} \\ &= \frac{C_s Q(1+iQ/(2x))}{1+iQ/x} = \frac{C_s Q(2x+iQ)}{2(x+iQ)} \end{aligned} \tag{11}$$

Present worth of setup cost is as follows:

$$Present\ worth\ of\ setup\ cost = K \tag{12}$$

The holding cost for perfect quality items is a gradient cash flow starting at $hqQ = (hD_p)(qQ/D_p)$ and decreasing at a rate hD_p. From Equation (3), the present worth of the holding cost for perfect quality items is:

$$\begin{aligned} Present\ worth\ of\ the\ holding\ cost &= \frac{hD_p(qQ/D_p)^2(3+2iqQ/D_p)}{6(1+iqQ/D_p)} \\ &= \frac{hq^2Q^2(3D_p+2iqQ)}{6D_p(D_p+iqQ)} \end{aligned} \tag{13}$$

Similarly, the present worth of the holding cost for imperfect quality items is:

$$Present\ worth\ of\ the\ holding\ cost = \frac{h(1-q)^2Q^2(3D_i+2i(1-q)Q)}{6D_i(D_i+i(1-q)Q)} \tag{14}$$

Hence, the total cost function per inventory cycle is given by:

$$\begin{aligned} TC(Q) = K + \frac{CQ}{1+iM} + \frac{C_s Q(2x+iQ)}{2(x+iQ)} + \frac{hq^2Q^2(3D_p+2iqQ)}{6D_p(D_p+iqQ)} \\ + \frac{h(1-q)^2Q^2(3D_i+2i(1-q)Q)}{6D_i(D_i+i(1-q)Q)} \end{aligned} \tag{15}$$

From Equations (9) and (15), we have that the total profit function per inventory cycle is:

$$TP(Q) = \frac{S_r qQ(D_p + iqQ/2)}{D_p + iqQ} + \frac{S_d(1-q)Q(D_i + i(1-q)Q/2)}{D_i + i(1-q)Q} - K - \frac{CQ}{1+iM} - \frac{C_s Q(2x+iQ)}{2(x+iQ)} - \frac{hq^2Q^2(3D_p + 2iqQ)}{6D_p(D_p + iqQ)} - \frac{h(1-q)^2Q^2(3D_i + 2i(1-q)Q)}{6D_i(D_i + i(1-q)Q)} \tag{16}$$

The total profit per unit time function is obtained by dividing the total profit function per inventory cycle by the cycle length, i.e., $TPU(Q) = TP(Q)/T$. By the renewal reward theorem, the expected profit per unit time is approximated by $E[TPU(Q)] = E[TP(Q)]/E[T]$.

To evaluate $E(TP(Q))$, we need the following:

$$E[q] = \int_{-\infty}^{\infty} qf(q)dq = \mu \tag{17}$$

$$E[q^2] = \int_{-\infty}^{\infty} q^2 f(q)dq = \sigma^2 + \mu^2 \tag{18}$$

$$\gamma = E\left[\frac{1}{D_p + iqQ}\right] = \int_{-\infty}^{\infty}\left(\frac{1}{D_p + iqQ}\right)f(q)dq \tag{19}$$

$$\lambda = E\left[\frac{1}{D_i + i(1-q)Q}\right] = \int_{-\infty}^{\infty}\left(\frac{1}{D_i + i(1-q)Q}\right)f(q)dq \tag{20}$$

Note that γ and λ are functions in Q. From Equations (4), (6), and (17), we have:

$$E[T] = E[T_p] = E[qQ/D_p] = E[q]Q/D_p = \mu Q/D_p \tag{21}$$

and from Equations (15) to (20), we get:

$$E[TP(Q)] = S_r D_p/2i - \gamma D_p^2 S_r/2i + \mu S_r Q/2 - \mu S_d Q/2 - \lambda S_d D_i^2/2i + \frac{(D_i + iQ)S_d}{2i} - K - \frac{CQ}{1+iM} - \frac{C_s Q(2x+iQ)}{2(x+iQ)} + \frac{hD_p}{6i^2} - \frac{h\mu Q}{6i} - \frac{h(\sigma^2+\mu^2)Q^2}{3D_p} - \frac{\gamma hD_p^2}{6i^2} - \frac{h(\sigma^2+\mu^2)Q^2}{3D_i} + \frac{h\mu Q(D_i + 4iQ)}{6iD_i} - \frac{\gamma hD_i^2}{6i^2} - \frac{h(-D_i^2 + D_i iQ + 2i^2Q^2)}{6i^2 D_i} \tag{22}$$

Since $E[TPU(Q)] = E[TP(Q)]/E[T]$, the expected profit per unit time is given by:

$$E[TPU(Q)] = \frac{D_p}{\mu Q}\left\{S_r D_p/2i + \gamma D_p^2 S_r/2i + \mu S_r Q/2 - \mu S_d Q/2 - \lambda S_d D_i^2/2i + \frac{(D_i + iQ)S_d}{2i} - K - \frac{CQ}{1+iM} - \frac{C_s Q(2x+iQ)}{2(x+iQ)} + \frac{hD_p}{6i^2} - \frac{h\mu Q}{6i} - \frac{h(\sigma^2+\mu^2)Q^2}{3D_p} - \frac{\gamma hD_p^2}{6i^2} - \frac{h(\sigma^2+\mu^2)Q^2}{3D_i} + \frac{h\mu Q(D_i + 4iQ)}{6iD_i} - \frac{\gamma hD_i^2}{6i^2} - \frac{h(-D_i^2 + D_i iQ + 2i^2Q^2)}{6i^2 D_i}\right\} \tag{23}$$

The optimal solution can be obtained by numerical methods. This will be illustrated in the following example.

A Numerical Example

Assume that the executive officer of an enterprise determined the parameters of the optimal ordering problem by examining the book-keeping records. The cost of placing an order was \$1,085, and the purchasing cost of one unit of this product was \$25. The imperfect quality items can be detected via a screening process at a rate which is considerably higher than the demand rate, and at a cost of \$0.5 per unit. The percentage of perfect quality items received in pervious orders was found to be uniformly distributed between 70% and 90%. Based on the enterprise's policy that sets the annual holding cost rate of any item at 29.2% of its unit purchasing cost, the executive officer estimates the cost of keeping one single unit of this product in stock for one day to be \$0.02. One unit of perfect quality item can be sold at a regular price of \$50, and the discounted selling price per unit is \$30. The enterprise's required rate of return is 7.7% annually.

The executive officer summarizes the parameters of the problem as follows: D_p = 40 units/day, D_i = 20 units/day, C = \$25, C_s = \$0.50, K = \$1,085, x = 100 units/day, h = \$0.02/unit/day, S_r = \$50, S_d = \$30, a = 0.7, b = 0.9, i = 0.077/365 daily interest rate, and M = 15 days (Note that $T_p \geq T_i$). For a uniformly distributed random variable q over the interval [a, b], the expected value is $E[q] = \mu = (a + b)/2$ and the variance is $\sigma^2 = (b - a)^2/12$. Also:

$$\gamma = \frac{Ln(D_p + biQ) - Ln(D_p + aiQ)}{iQ(b-a)}$$

$$\lambda = \frac{Ln(D_i + i(1-a)Q) - Ln(D_i + i(1-b)Q)}{iQ(b-a)}$$

Thus, for a percentage of perfect quality items uniformly distributed over [70%, 90%], we get: μ = 80%, $\sigma^2 = (0.9 - 0.7)^2/12 = 0.00333$, and $\sigma = 0.057735$. Also, $\gamma = 0.0247908$ and $\lambda = 0.04979$.

Using Equation (23), the function is maximized for the optimal order quantity and the corresponding maximum daily profit is found to be Q^* = 2,000 units. One can obtain the optimal solution via a numerical search. Figure 3 illustrates the behavior of the expected daily profit function $E[TPU(Q)]$.

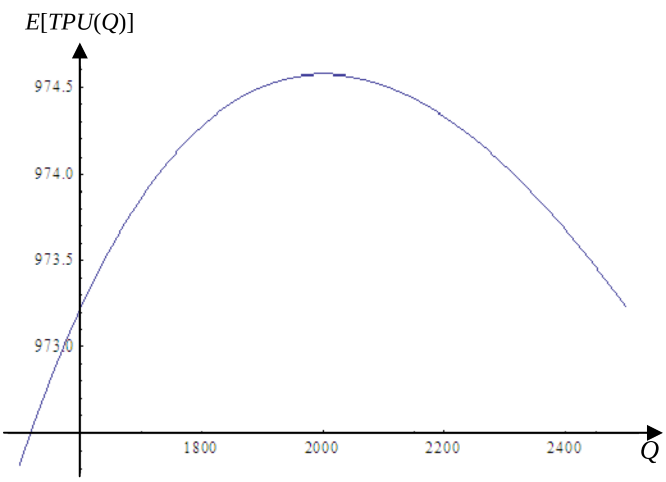

Figure 3. The total annual profit function $TPU(y)$.

Analyzing the optimal solution, the officer determines that for an order size of Q^* = 2,000 units, one expects μQ^* = 1,600 units of perfect quality and $(1 - \mu)Q^*$ = 400 units of imperfect quality. The expected length of the perfect quality items inventory cycle is $\mu Q^*/D_p$ = 40 days, and that of the imperfect quality items is $(1 - \mu)Q^*/D_i$ = 20 days. After 20 days, all imperfect quality items will be sold out and 800 units of perfect quality items will remain in stock to be sold over the rest of the inventory cycle. The expected present value of revenues from selling perfect quality items at a regular price using Equation (7) above is $79,663.6. As for the imperfect quality items, the present value of sales at a discounted price using Equation (8) above is $11,972.7. Hence, the expected total revenue per inventory cycle is $91,636.3. The present value of purchasing cost per cycle is determined using Equation (10), which is $49,842.3, and the present value of screening cost using Equation (11) above is equal to $997.899. The present value of holding costs for perfect items per inventory cycle is calculated by evaluating Equation (13) above at these parameter values yielding $641.521, and the present value of holding cost for imperfect items is similarly calculated by evaluating Equation (14) above to get $86.5267 per unit time. Thus, the total cost per inventory cycle from Equation (15) is $1,085 + $49,842.3 + $997.899 + $641.521 + $86.5267 = $52,653.2; and the expected total profit per inventory cycle using Equation (16) is $38,983.1. Dividing by the inventory cycle length, the officer finds the expected daily profit to be $974.576.

To evaluate the new ordering policy, the executive officer checks the store's current policy which is based on the classical EOQ model. The order quantity obtained from Equation (1) is $Q = 2{,}551.47 \approx 2{,}551$ units, and the corresponding expected daily profit is $561.33. Hence, the new model results in an additional annual profit of $413.25, which corresponds to an additional annual profit over the classical model of $150,836.

Conclusions

The model considered in this paper extends the classical EOQ model to account for the quality and time value of money. The orders received from a supplier contain both perfect and imperfect quality items. The percentage of perfect quality is a random variable having a known probability distribution. The imperfect quality items are to be sold at a discounted price and the demands for both perfect and imperfect quality items occur continuously during the inventory cycle. The inventory model accounted for the case when the supplier allows a delay in payment for items received at the beginning of the cycle. A mathematical model was developed based on the total profit per unit time. This function is maximized through a numerical search technique. A numerical example illustrating the model was presented. Taking into account the time value of money in the total profit calculation reduces the optimal order quantity and could lead to a substantial increase in the daily profits over the classical order quantity model. The following are suggestions for future research:

(1) Analyzing the effects of the various parameters;

(2) Comparing this discounted cash flow model to the one where regular cash flows were calculated;

(3) Considering the case of settling the outstanding amount after the permissible delay in payment period where an interest would be charged;

(4) Incorporating the effects of compounded interest into this model.

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