Evaluating the Alignment between a Mathematics Curriculum and the National Tests: The case of Lebanon

Secondary National Exams for the Life Sciences Section

By

WAEL AJWAD SAFA

A thesis
Submitted in partial fulfillment of the requirements for the degree of Master of Arts in Education

School of Arts and Sciences
June 2013
Thesis Proposal Form

Name of Student: Wael Safa I.D.#: 200700373

Department: Education

On (dd/mm/yy) May 17, 2013, has presented a Thesis proposal entitled:

Evaluating the Alignment between a mathematics curriculum and the national tests for student learning assessment: The case of Lebanon

in the presence of the Committee Members and Thesis Advisor:

Advisor Dr. Iman Osta
PhD, Mathematics Education

Committee Member Dr. Samer Habre
PhD, Mathematics

Committee Member Dr. Mona Nabhan
EdD, Educational Management and Leadership

Proposal Approved on (dd/mm/yy): May 17, 2013

Comments / Remarks / Conditions to proposal approval (if any):

Minor modifications that will be taken into consideration during the thesis work

Date: May 17, 2013 Acknowledged by (Dean, School of Arts and Sciences)

cc: Dean
    Chair
    Advisor
    Student
Thesis Defense Result Form

Name of Student: Wael Safa
Program / Department: Education
Date of thesis defense: June 7, 2013
Thesis title: Evaluating the Alignment between a mathematics curriculum and the national tests for student learning assessment: The case of Lebanon

Result of Thesis defense:
✓ Thesis was successfully defended. Passing grade is granted
☐ Thesis is approved pending corrections. Passing grade to be granted upon review and approval by thesis Advisor
☐ Thesis is not approved. Grade NP is recorded

Committee Members:
Advisor: Dr. Iman Osta, PhD, Mathematics Education
Committee Member: Dr. Samer Habre, PhD, Mathematics
Committee Member: Dr. Mona Nabhani, EdD, Education

Advisor's report on completion of corrections (if any):

Changes Approved by Thesis Advisor: Dr. Iman Osta
Date: 11/6/2013
Acknowledged by: [Signature]

cc: Registrar, Dean, Chair, Advisor, Student
Thesis approval Form

Student Name: Wael Safa
I.D. #: 200700373

Thesis Title: Evaluating the Alignment between a mathematics curriculum and the national tests for student learning assessment: The case of Lebanon

Program: MA in Education
Department: Education
School: School of Arts and Sciences

Approved by:
Advisor: Dr. Iman Osta, PhD, Mathematics Education
Committee Member: Dr. Samer Habre, PhD, Mathematics
Committee Member: Dr. Mona Nabhani, EdD, Education

Date: June 7, 2013
THESIS COPYRIGHT RELEASE FORM

LEBANESE AMERICAN UNIVERSITY NON-EXCLUSIVE DISTRIBUTION LICENSE

By signing and submitting this license, you (the author(s) or copyright owner) grants to Lebanese American University (LAU) the non-exclusive right to reproduce, translate (as defined below), and/or distribute your submission (including the abstract) worldwide in print and electronic format and in any medium, including but not limited to audio or video. You agree that LAU may, without changing the content, translate the submission to any medium or format for the purpose of preservation. You also agree that LAU may keep more than one copy of this submission for purposes of security, backup and preservation. You represent that the submission is your original work, and that you have the right to grant the rights contained in this license. You also represent that your submission does not, to the best of your knowledge, infringe upon anyone's copyright. If the submission contains material for which you do not hold copyright, you represent that you have obtained the unrestricted permission of the copyright owner to grant LAU the rights required by this license, and that such third-party owned material is clearly identified and acknowledged within the text or content of the submission. IF THE SUBMISSION IS BASED UPON WORK THAT HAS BEEN SPONSORED OR SUPPORTED BY AN AGENCY OR ORGANIZATION OTHER THAN LAU, YOU REPRESENT THAT YOU HAVE FULFILLED ANY RIGHT OF REVIEW OR OTHER OBLIGATIONS REQUIRED BY SUCH CONTRACT OR AGREEMENT. LAU will clearly identify your name(s) as the author(s) or owner(s) of the submission, and will not make any alteration, other than as allowed by this license, to your submission.

Name: Wael Safa

Signature: [Blank]

Date: June 2013
I certify that I have read and understood LAU’s Plagiarism Policy. I understand that failure to comply with this Policy can lead to academic and disciplinary actions against me.
This work is substantially my own, and to the extent that any part of this work is not my own I have indicated that by acknowledging its sources.

Name: Wael Safa
Signature: Date: June 2013
Dedication Page

To my loving parents
ACKNOWLEDGMENTS

This research would not have been possible without the help and assistance of many persons. First I would like to express my gratitude to my supervisor Dr Iman Osta for her great patience and continuous support as well as following up and guiding my work. I am also deeply grateful to Dr Samer Habre and Dr Mona Nabhani for being members in my thesis committee and for monitoring my progress. Thanks go also to Ms Liwa Sleiman for doing the same analysis of the test items in terms of their corresponding objectives and cognitive domains, and then meeting with me to agree on the unified analysis used in this study.
Evaluating the Alignment between a Mathematics Curriculum and the National Tests: The case of Lebanon
Secondary National Exams for the Life Sciences Section

Wael Ajwad Safa

Abstract

Alignment between various components of a curriculum is one major criterion to evaluate the curriculum. This paper aims to study the alignment between the Lebanese national tests for the “Life Sciences” track of the secondary level and the Lebanese math curriculum. The structure, content, and objectives of the Lebanese math national curriculum were qualitatively analyzed along with twelve national tests, and four model tests representing the intended math curriculum. The national tests and the model tests were analyzed quantitatively within a framework that accounted for their objectives and the TIMSS cognitive domains. Correlations between the test items of 6 sets of exams were calculated: The national test items and the model tests items, the national test items of the years 2001-2003 and those of the years 2010-2012, and the test items of the first session and the second session of the national tests. The quantitative analysis revealed an average correlation ($r = 0.50$) between the national tests and the model tests when the specific objectives and the cognitive domains were considered. However, a higher correlation was detected ($r = 0.87$) when the math domains and the cognitive domains were considered. A high correlation existed: between the model tests and the national tests of the years 2001-2003 ($r = 0.78$), between the model tests and the national tests of the years 2010-2012 ($r = 0.9$), between the national tests of the years 2001-2003 and those of the years 2010-2012 (0.88), and between the session-1 national tests and session-2 national tests ($r = 0.95$). However, it was found that the national tests and the model tests assess a narrow part of the curriculum and include stereotyped test items that emphasize the cognitive domains “knowing” and “applying” while relatively neglecting the cognitive domain “reasoning”, signifying a weak alignment.

Keywords: National Curriculum, National Assessment, Alignment, Mathematics, Secondary School Education, Lebanon.
CHAPTER ONE

INTRODUCTION

1.1 – Overview

Over the last 40 years, curriculum has been the focus of many education reform efforts. According to Porter (2004), there are several types of curricula: intended, enacted, learned and assessed. The intended curriculum has more to do with instructional content -i.e. student learning objectives and outcomes to be achieved by the end of the school year. On the other hand, the enacted curriculum consists of what is actually being taught in the classroom. The assessed curriculum comprises examinations that test student achievement and performance. Finally, the learned curriculum consists of students' actually acquired knowledge.

Curriculum alignment may be defined as the consistency between the various curricula: the intended, the enacted, the learned and the assessed curriculum. Similarly, Webb and National Institute for Science Education (1997) add that "Alignment is the degree to which expectations and assessments are in agreement and serve in conjunction with one another to guide the system toward students learning what they are expected to know and do” (p. 3).

The research on Alignment might possibly aid policymakers, tutors and assessment developers in establishing modifications so that the curriculum, instruction and assessment support each other (Roach, Niebling & Kurz, 2008).
1.2 – Context and Background

1.2.1 – Lebanese National Curriculum Reform

At the end of the Lebanese civil war, which originated in 1975 and lasted 14 years, the curriculum was set to be modified, after 30 years of using the same curriculum. Actually, guidelines for educational reform were included in the al-Taef Agreement which ended the war.

In 1994, the Lebanese Ministry of Education and Higher Education (MEHE) published a plan to improve the Lebanese curricula. The plan, developed by The Educational Center of Research and Development (ECRD), aimed to revise and improve the Lebanese educational objectives as pertains to their national, social, and intellectual aspects (ECRD, 1994).

The curriculum developers aimed to stress higher order thinking skills (such as problem solving) through constructive student centered methods in contrast to the “traditional” teaching styles that focused on basic skills and memorization of facts and procedures.

The curriculum was designed to fit the demands of the changing society. The reform of the new curriculum included designing a new syllabus, new textbooks, and new teacher’s guides (ECRD, 1997). According to the introduction of the Lebanese reference book of curriculum (referenced as Document I in Appendix A), mathematics helps us quantify objectively and precisely the qualitative description of reality. It is an essential tool that
enhances the development of societies in all domains and therefore, it must be used by all citizens.

The education ladder in Lebanon is divided into two parts: Basic Education and Secondary Education. The Basic Education includes the elementary and the intermediate levels. The elementary level is divided into two cycles: the first cycle is formed of the first three grades (grades 1, 2 and 3), and the second cycle is formed of the grades 4, 5 and 6. Grades 7, 8 and 9 form the third cycle at the intermediate level (by the end of which students sit for the national Brevet exams), and finally, grades 10, 11, and 12 constitute the fourth cycle at the secondary level (by the end of which students sit for the national Baccalaureate exams).

The national Lebanese exams (commonly known as official exams) play a central role in the promotion of students from one cycle to another. In June 2001, students sat for the first official exam built according to the new curriculum, at both, the Brevet level and the Baccalaureate level, with its four sections: Life Science (LS), General Sciences (GS), Literature and Humanities (LH) and Sociology-Economy (SE). The regular official exams are usually administered in June. However, a second session of official exams is usually administered in September to give a second chance to students who fail the regular June exam.

The reformed curriculum, issued by MEHE and ECRD, organizes the content, competences, objectives and sample official exams in the following texts:

1. The official text of the reformed curriculum for all grades of the 12 years of formal schooling as issued in 1997 (referenced as Document I in Appendix A): In addition to
the general and specific objectives, this text includes the syllabi and the scope and sequence of all disciplines.

2. The details of contents: These documents include the detailed content and the related objectives for each subject. In addition, relative comments are presented to clarify the inclusion or exclusion of some specific objectives. For each subject, three books were issued, respectively for the first, second, and third year of each cycle.

3. Evaluation Guides: they were issued to guide the teachers in implementing the curriculum for each subject of the 12 years of formal schooling. These guides list the competencies to be developed and assessed for each subject and grade. Moreover, the guides include model tests as a sample of the expected national tests.

1.2.2 – Participation in TIMSS advanced 2008 assessment

In an attempt to measure the improvement in the educational achievement in mathematics, Lebanon, in the year 2008, participated in the Trends in International Mathematics and Science Study (TIMSS), an international assessment of Math and Science. In the context of TIMSS, what is called “advanced mathematics assessment” is conducted, a project that aims at assessing the level of students with a special preparation in advanced mathematics and who are in their last secondary year. TIMSS advanced assessment gives the participating countries the opportunity to assess the performance of their leading students in Mathematics and Physics in an international context. Lebanon participated with students of grade 12 GS section.
A report (Mullis, Martin, Robitaille & Foy, 2009) of the findings from the TIMSS advanced 2008 assessment was published. The report presented substantial information about the contexts of teaching and learning for the contributing countries and the relevant aspects that influence students’ achievements. Below is a summary of the report focusing on Lebanon.

(a) Students in Lebanon, along with Iran and the Russian Federation, received more than 200 instructional hours of advanced mathematics per year which is higher than the average of the instructional hours of the 10 participating countries. (TIMSS Advanced 2008, n.d.).

(b) Lebanon was one of the three best performing counties in advanced mathematics. Alongside the Russian Federation and Netherlands, Lebanon’s average was 545, significantly higher than 500, the international scale average.

(c) Lebanese participating Females achieved better than males (the average achievement scores of females and males were 554 and 541 respectively). However, Lebanese male students scored higher averages than the males in half of the participating countries. In addition, when considering the cognitive domains in all the participating countries, males had higher scores than females in the reasoning cognitive domain.

(d) Compared to Lebanon’s low achievement in TIMSS 2007 for the eighth graders (The score was 449 which is below the TIMSS average 500), Lebanon has achieved significantly better in TIMSS advanced 2008. This was perceived as evidence for Lebanon’s success, in spite of socio economic conditions (Medium Human
Development Index), to raise the education of a particular group of students to high levels of achievements in international mathematics.

(e) Lebanese students performed well in Geometry (55% out of 21 test items were correct), but less well in Algebra (51% out of 25 test items were correct). In addition, students in Lebanon performed well in knowing (Out of the 27 test items on knowing, 65% were correct) and less well in applying (only 43% were correct out of the 27 test items on applying) and reasoning (51% out of the 17 items were correct on reasoning).

The above findings can be considered as indications of progress in Lebanese students’ achievements in Math. However, this progress is limited to students who had special preparations in advanced mathematics, mainly students at the GS track of the Secondary level.

1.3 – Purpose of the Study

This paper aims to study the extent to which the reformed math curriculum is aligned with the Lebanese national math exams for the LS section. In the year 2001, the new curriculum was fully implemented at all grades for the first time. This paper aims also to investigate the evolution of the official exams through the 13 years of implementation, by comparing the alignment in the first years (2001 to 2003) of implementation of the reformed curriculum to the latest years of implementation (2010 to 2012). In addition, this paper will investigate any differences in session 1 and session 2 of the official exams for the LS section, by studying their alignment with the reformed mathematics curriculum. Moreover, recommendations to strengthen the above alignments will be provided.
1.4 – Research Questions

The research questions are:

1. Are the Lebanese secondary-level official math exams for the LS section aligned with the national reformed curriculum over the years 2001-2012?

2. Is there any improvement in the alignment of the national exams over the years 2001-2003 and 2010-2012?

3. Are there differences between the exams in session 1 and session 2 for the LS section, in terms of content and cognitive domains addressed?

1.5 – Rationale and Significance of the Study

The aim of this thesis is to examine whether an alignment exists between the intended and the assessed national curriculum in Lebanon. The issue should concern teachers, curriculum developers, test developers, and administrators to recognize how classroom tutoring is supported or not by the testing methods. Thus certain changes may be done to the constituents of scholastic methods (Martone & Sireci, 2009).

If the national assessments do not align with the objectives of the curriculum, then teachers would not be teaching the intended curriculum. McGaw (2006) emphasized teachers’ tendency
to stress the ‘testable’ parts of the curriculum and to ignore the rest. The consequence, according to Osta (2007) is a “teaching to the test” practice by teachers resulting in “drill and practice” approaches by students.

This study will allow the mathematics Official Examinations Committee (OEC) to improve assessment on a national level. Moreover, the study will enable teachers to familiarize themselves with national assessment methods which might help them improve their teaching. The findings of this research will enable teachers to better understand how the national tests are developed and structured.
2.1 – Educational Reform and Change

One of the many purposes of education is social change. Because it has the potential to reproduce and transform roles in the society, educational reform is emphasized more and more in a changing society. However, according to UNESCO International Bureau of Education (2003) “Not all changes, superficial or more radical, transient or longer lasting, are worthy of the name reforms.” (p.23)

Several educational reforms were initiated as a consequence of a social desire to change. In the 1980s, many countries witnessed educational reforms (Wang, 2010). According to the TIMSS (the Third International Mathematics and Science Study), national mathematics curriculum reforms were ignited in 25 countries out of 36 in the early 1990s (Schmidt, Mcknight, Valverde, Houang & Wiley, 1997). The case of Lebanon exemplifies how a country at the end of a severe civil conflict could implement educational reform aiming to support social unity by shaping students from different religious backgrounds to be future citizens.
However, many challenges may confront the implementation of an educational reform (UNESCO International Bureau of Education, 2003). Below are some of these challenges:

1) Introducing a new curriculum

2) Specifying the new curriculum standards

3) Investigating the relevant instructional practices

4) Issuing new textbooks and the related material according to the new curriculum

5) Improving teachers’ instructional skills through training, and motivating them to participate dynamically in the reform

Over the years, curriculum was the focus of many educational reform efforts. Stimulated by the developments in information technology, countries adopted curricula as major tools for transforming the nationwide visions into intentions that give the children prospects for learning in schools (Anderson, 2002). However, curriculum development is not limited to curriculum designers. As the curriculum impacts the socio-economic level of the country, other stakeholders should be involved in the process of curriculum planning in a balanced way (UNESCO International Bureau of Education, 2003). According to Wiggins and McTighe (2005), teachers have a basic role in the process of curriculum reform; their profession relies mainly on crafting the curriculum and learning practices in order to reach specified objectives. Their role is to also make assessments that aim to identify the students’ needs and to verify whether the goals were achieved.
UNESCO International Bureau of Education (2003) presents principles and conditions for implementing a curriculum:

1) Regardless of the approval’s origins, the curriculum should be constantly revised
2) The implementations need to be locally interpreted and incorporated in the overall educational system
3) Teachers, through training, should emphasize the goal of the new curriculum
4) The curriculum needs to be implemented on a regular and slow pace

Teachers should strictly abide by the curriculum guidelines

2.1.1 – Curriculum Definitions

Curriculum definitions have evolved throughout the years, highlighting different aspects. A broad definition of curriculum was suggested by Gibson (2013, p.15): “a description of what teachers are supposed to teach and students are supposed to learn in each course of study…The curriculum describes what is taught but does not prescribe how the content is taught”.

A more detailed view of a curriculum was proposed by Anderson (2002), who stated that curriculum refers to aims and objectives, activities within instruction and material for
support, and assessment. Sowell (2005) argues that curriculum encompasses four main aspects: content, plan, experience and outcome.

The National Council of Teachers of Mathematics (NCTM), driven by the constructivist theory of learning, encompassed in its book *curriculum and evaluation for school mathematics*, what it called “context of learning” and defined a math curriculum as “an operational plan for instruction that details what mathematics students need to know, how students are to achieve the identified curricular goals, what teachers are to do to help students develop their mathematical knowledge, and the context in which learning and teaching occur.” (NCTM, 1989, p. 1)

Porter (2004) aimed to study the coherence of the different parts of a curriculum, a process he called curriculum assessment. In doing so, he differentiated between several types of curricula: the intended, the enacted, the learned and the assessed. The intended curriculum has more to do with instructional content - i.e. student learning objectives and outcomes to be achieved by the end of the school year. On the other hand, the enacted curriculum consists of what is actually being taught in the classroom. The assessed curriculum comprises examinations that test student achievement and performance. Finally, the learned curriculum consists of students’ acquired knowledge.

Gibson (2013) concluded that, prior to 1900, the idea of curriculum was simply describing it in terms of subjects, time allotted to these subjects, and when in years students would take these subjects. However, with the evolution of the theories of cognitive
development and learning theories and their implications on curriculum reforms, curriculum was viewed differently as more of a science with principles and methodology.

2.1.2 – Implications of the Learning Theories on Curriculum Development

The constructivist theory promotes student-centered curricula. In other words, the constructivist learning theory emphasizes the idea that students should construct their own knowledge based on previous learning experiences through engaging them in activities that rely on problem solving and real life situations. These concepts have become at the heart of the curriculum’s principles. (Ültanır, 2012)

According to Dewey (1961), experience is one of the main components of real education which relies on active participation by students who become self-directed and use their previous learning experiences in order to perform problem solving activities, which attributes to the student’s experience a greater importance compared to the curriculum’s subject.

Piaget (1953) emphasizes the role of adaptation and organization in the development of intelligence. Two major components of adaptation are assimilation and accommodation. Assimilation consists of integrating new knowledge to schemas that individuals have already built. Accommodation requires changing the schemas in order to conform to the new information (Piaget, 1953). In Piaget’s views, great importance is attributed to the role of each individual’s understanding of knowledge as well as his or her own pace of learning. (Powell & Kalina, 2009)
Vygotsky (1978) states that cognitive abilities rely mainly on social interactions. Vygotsky’s social mediation ideas were largely used, over time, in academic disciplines and metacognitive approaches. His model mainly emphasizes the idea that cognitive development and learning are originally social processes, which has opened the door to new ideologies for curriculum reform. The main message of these ideologies is to correct the past beliefs that stated that challenging subject matter is restricted to an elite group of students. It promotes instead the idea that all students are capable of learning.

These ideologies don’t only highlight the importance of making learning more interesting and motivating to students; they also stress on the importance of enhancing the potential to use knowledge in real life situations.

The theory of multiple intelligences of Howard Gardner has also its place in the field of education. According to Gardner, the human beings are capable of knowing the world through at least seven ways, known as “seven human intelligences” (Gardner, 1991, p.12). Gardner’s ideas are applied in the school curriculum not exclusively for cognitive intelligence, but also for different models of mastery. According to him, teachers should reinforce all types of intelligence. From this vision, all the individuals are capable of knowing the world through various intelligence types: a) verbal/linguistic, b) logical/mathematical, c) visual/spatial, d) musical/rhythmic, e) bodily/kinesthetic, f) interpersonal, g) intrapersonal, h) naturalistic. (Gardner, 1983)
2.2 – Assessment

Assessment is considered a basic element of an educational plan since it can enrich learning, endorse higher teaching standards, encompass motivation, and give valid educational decisions. Over the years, several assessment reforms were implemented. Attention was given not only to classroom assessment but also to large-scale standardized assessment (Hargreaves, Earl & Schmidt, 2002).

As a consequence of the reforms, different related terminologies evolved, each highlighting the diverse purposes of assessment (Gibson, 2013). In many cases, it has been assumed that assessment is synonymous with testing. According to Linn and Miller (2005), “Assessment is a general term that includes the full range of procedures used to gain information about student learning….A test is a particular type of assessment that typically consists of a set of questions administered during a fixed period of time under reasonably comparable conditions for all students” (p.26). Thus, assessments may include several methods that teachers use to gain insights about student learning. They also include paper-and-pencil tests, but incorporate other forms of gathering information about students such as interviews and observations (Webb 1997).

On the other hand, McMillan (2011) related evaluation to making judgments: “Evaluation is the process of making judgments about what is good or desirable. For example, judging whether a student is performing at a high enough level to move on…or whether to carry out a particular instructional activity requires judging” (p.168).
Assessment covers 4 basic areas, typically: 1) diagnosing the problem areas and strengths of students, 2) tracking how much students progress and improve, 3) assigning grades, and 4) analyzing decisions made in the instructional process and re-evaluating them (Popham, 2011). In the past years, a revolution in the role of assessment in education occurred. The shift from assessment of learning to assessment for learning was considered to be a change in the conceptual framework of assessment.

Shepard (2000) relates the reasons behind the paradigm shift in the view of assessment to the development of the cognitive and learning theories that prevailed. Central in these theories is the behavioristic perception of learning that views the total as the summation of its parts; that is, competence in a domain is the direct result of the competence in every module which constitutes that domain. Thus assessment should be reduced to the assessment of each module at the end of the allocated instruction period. With the advent of the constructivist theory of learning, the literature revealed a fundamental change in the view of assessment, the kind of assessment that is integrated within the teaching process to enhance learning. “Just as learning and the curriculum should be holistic and understood in relation to its constituent parts, assessment practices should be designed and practiced as an integrative whole to preserve the integrity of students’ learning” (Tan, 2011, p.11).

2.2.1 – Types of Assessment (AOL, AFL, AAL)
Many teachers are acquainted with assessments that occur after instruction and are not embedded within the teaching process (Tan, 2011). Assessment of learning (AOL), also known as summative assessment, limits its purpose to inferences about students’ knowledge, understanding and ability (McMillan, 2011). Witte (2012) explains that “summative assessment is a formal evaluation of progress and/or performance…so that students can be informed of what they still need to learn if they are to reach the intended learning targets” (p.11).

On the other hand, assessment for learning (AFL), also termed formative assessment or ongoing assessment, occurs during instruction with the intention to give students feedback and reflection during the whole instruction period (Gibson, 2013). Assessment for learning is defined by The Assessment Reform Group (2002) as “… the process of seeking and interpreting evidence for use by learners and their teachers to decide where the learners are in their learning, where they need to go and how best to get there”. Similarly, McMillan (2011) states, “It is a way of assessing students’ progress, providing feedback and making decisions about further instructional activities” (p. 6).

AFL detects students’ strengths and weaknesses. Therefore, assessment for learning supports both learning and teaching. In fact, any assessment that aims in its structure and practice to support learning can be labeled as an assessment for learning. (Black, Harrison, Lee, Marshall & Wiliam, 2003)
Advocates for AFL emphasize the fact that there are several levels of learning in which evaluation by summative tests is challenging (Tan, 2011). A practical hierarchy of learning was described by Salvia and Hughes (1990). The hierarchy includes five levels of learning: a) Acquisition, b) Fluency, c) Maintenance, d) Generalization, and e) Adaptation. Since AOL practices will not capture some of the “consequent learning outcomes”, AFL designs are considered more authentic to prompt such high levels of learning about a topic beyond what can be examined.

AFL can enrich learning, motivate, and sustain students’ satisfaction of learning inside and outside classrooms (Tan, 2011). Formal schooling has been criticized for its inability to equip students with authentic learning and assessment beyond the school contexts, i.e. in real life situations (Gulikers, Bastiaens & Kirschner, 2007), and its failure to prepare students to be life-long learners (Boud, 2000). AFL practices that are correlated with high levels of student motivation and enjoyment have a higher probability of arming students with authentic learning beyond school (Tan, 2011).

Singapore, labeled by Times Education Supplement (Boost to morale 1997, p.1) “as the most academically successful nation in the world”, experienced in the year 1997, a reform of the educational system under the vision termed ‘Thinking Schools Learning Nation’ (TSLN). At the basis of this reform, a significant change in the assessment practices involved shifting from assessment of learning to assessment for learning, encouraged by the influence of the latter on students’ learning (Tan, 2011).
Trends in assessment have recently included what is called “assessment as learning”. Assessment as learning (AAL) gives students the chance to evaluate their learning by teaching them the relative metacognitive processes. Barnett (2007) and Boud (2007) highlight the need to empower students with the skills of assessing their own learning. Moreover, Tan (2009) claims that a section of the curriculum and instruction in classrooms should be dedicated to teaching students self-evaluative skills that would be considered as important learning outcomes.

The rationale for such assessment was explained by Sadler (2007). When students have difficulties in acquiring a certain level of learning, teachers will provide scaffolds that are meant to be provisional structures to help students. However, students may use the scaffold automatically as a formula without considering any relative context. This overreliance on the scaffold may drive students to substitute the scaffold for the real understanding, and thus, students will be experiencing assessment deprived of learning.

2.2.2 – Setback of National Examinations

Educators suggest a balanced use of the two forms of assessments AFL (or formative assessment) and AOL (or summative assessment) (Tan, 2011). Webb and Jones (2009) emphasize that limiting assessment practices to formative without summative forms would be problematic. However, the indirect effects of summative testing, in particular national
examinations, must be considered. One indirect impact of national examinations is linked to learning. Students may study concepts that they label as relevant to these exams. As a consequence, a discrepancy between the intended curriculum and the learned curriculum will be noticed. This mismatch between the two curricula cannot be overlooked since it is recognized as a gap between what students are experiencing and what is intended for them (Tan, 2011).

This negative effect on learning weakens the consequential validity of the assessment method. Consequential validity is related to the influence of assessment on learning. Boud (1995) underlines the importance of having consequential validity in the consequences of an assessment practice on the society’s interpretation of the assessment results.

Another side effect of national examinations is related to teaching. William (1996) argues that when teachers teach the entire domain of the subject, students are expected to perform similarly in the ‘testable’ parts and the ‘untestable’ parts of the domain. Therefore, the exam is said to be valid for further academic predictions. In reality, assessment designers used such validity in defending their method of testing only portions of domains.

However, McGaw (2006) believes that summative assessment drives teachers to concentrate on the ‘testable’ parts of the domain. Teachers then, under the pressure of improving students’ results, will ignore some parts of the curriculum. Consequently, there will
be reduction in correlation between two parts of the domain: achievement in the taught and untaught (William, 1996).

In literature, this practice is perceived as ‘teaching to the test’. Studies reveal that ‘teaching to the test’ leads to spoon feeding instead of promoting independent thinking (Nuffield Foundation, 2006). However, Gipps (1994) remarks that “Teaching to the test is a relatively well understood activity in the UK, although here it might be called preparation for examinations” (p. 45). Gipps (1994) explains that teachers are pressured to apply this method: “It is not that teachers want to narrow their teaching, nor to limit unduly students’ educational experience, but if the test scores have significant effects on people’s lives, then teachers see it as part of their professional duty to make sure that their pupils have the best possible chance they can to pass the test” (p. 35).

Osta (2007) elaborated on the high-stakes Lebanese national exams. In Lebanon, national exams results are considered highly important since they are used to evaluate students, teachers and school achievements. In addition, results from national exams are used to promote students from one cycle to another or for their graduation from schools. In her study about the alignment between the Lebanese Math examinations and the Lebanese curriculum, Osta (2007) identifies “a mini curriculum from which specific topics are considered for test items” which encourages teachers to “teach to the test”.

The above research results concord with the results of a report dealing with the issue of UK National tests which asserts: “league tables turn the tests into high stakes assessment. The
unfortunate side effects of this can include teaching a narrow and shallow form of the curriculum tailored to the test” (The Advisory Committee on Mathematics Education, 2002, p.3).

### 2.2.3 – Curriculum Assessment

Curriculum assessment is a particular type of assessment that aims to sustain students’ learning by gathering and analyzing information to assess the coherence of a curriculum and to inform curriculum changes (Wolf, Hill & Evers, 2006). Many researchers elaborated on the effectiveness of curriculum assessment and its relation to curriculum coherence and alignment. For example, Porter (2004) defines curriculum assessment as “measuring the academic content of the intended, enacted, and assessed curricula as well as the content similarities and differences among them…. To the extent content is the same, they are said to be aligned” (p. 12). Alignment, on the other hand, is used to study the coherence of an educational system.

Schmidt and Prawat (2006), highlighting the role of TIMSS in the research about curriculum coherence, claim that the term “curriculum coherence” was defined as alignment in most of the studies that were conducted before the release of TIMSS results in 1997. Schmidt and Prawat (2006) conducted a study on 37 countries participating in TIMSS to investigate the relation between the national control of the curriculum and the curriculum coherence. Several types of alignment were measured: “Alignment between content standards and textbooks, alignment between textbooks and teacher coverage, and alignment between content standards
and teacher coverage” (p.4) concluding that “national control of the curriculum is not necessarily associated with greater curricular coherence”.

2.3 – Alignment between Curriculum and Assessment

According to Porter (2004), there are several types of curricula: intended, enacted, learned and assessed. The intended curriculum has more to do with instructional content -i.e. student learning objectives and outcomes to be achieved by the end of the school year. On the other hand, the enacted curriculum consists of what is actually being taught in the classroom. The assessed curriculum comprises examinations that test student achievement and performance. Finally, the learned curriculum consists of students' actually acquired knowledge.

Curriculum alignment may be defined as the consistency between the various curricula: the intended, the enacted, the learned and the assessed. Porter (2004) observes that many issues worth researching arise when considering matters related to curriculum instruction. For instance, is there a mismatch between the content being taught and what is being tested? Do teachers adhere to textbook material when teaching? Is there another mismatch between the material being tested and that in the intended curriculum? Finally, does the material being taught match what should be taught in the intended curriculum?
The increased importance of national exams underlines the value of alignment between curriculum and assessment (Fulmer, 2010). Alignment has been defined as the “extent to which curricular expectations and assessments are in agreement and work together to provide guidance for educators’ efforts to facilitate students’ progress toward desired academic outcomes” (Roach et al., 2008, p.1).

Similarly, Webb (1997) adds that "Alignment is the degree to which expectations and assessments are in agreement and serve in conjunction with one another to guide the system toward students learning what they are expected to know and do” (p. 3). To this end, researchers may use the alignment of instruction to textbooks to study the impact of using textbooks on the instruction material (Freeman & Porter 1989, as cited in Porter 2004). Additionally, Porter and Smithson (2001), observe that it is possible to use alignment of instruction to assessments to study the impact of assessment on the subject matter of instruction. Finally, it is also possible to use alignment of instruction to content standards to evaluate the effects of standards-based reform.

The NCTM’s *Curriculum and Evaluation Standards for School Mathematics (1989)* states:

When assessment instruments are aligned with the curriculum, the curriculum becomes the standard against which an assessment instrument should be judged. This alignment can be determinant by examining the extent to which the instruments that measure the content of the curriculum are consistent with its instructional approaches (1989, p.193).
Thus, proper alignment must be ensured by having the assessment tools match the goals and contents addressed in the curriculum.

Alignment closely relates to the concept of validity that Linn and Miller (2005, p. 68) define as a measure of how sufficiently and appropriately assessment results are interpreted and used. In addition, there are two types of validity of alignment: content validity and consequential validity. Content validity is defined as the extent to which a test measures the content it is supposed to measure. High content validity of a test means that the content of the test matches the testing purpose (Martone & Sireci, 2009). On the other hand, consequential validity is defined as the consequences of society’s interpretation of assessment results. Shepard (1997) views it as “the incorporation of test consequences into validity investigations”.

2.3.1 – The Value of Alignment

There are four reasons why curriculum alignment is a great concern for educators. The first reason is that it is essential to focus on what opportunities to learn are provided to students as a result of their educational experience. Students, according to Martone and Sireci (2009), will be given the opportunity to learn if the components: instruction, testing and the curriculum are aligned to give a consistent message of what should be tested, taught, and learned. Curtis McKnight and Bill Schmidt (1995) suggest that opportunities to learn should be provided to all students to avoid giving different education to different learners.
The second reason for the significance of curriculum alignment is the fact that suitable curriculum alignment permits recognizing the variances in the influence of schooling on learners’ accomplishments.

Reason number three for the significance of curriculum alignment is the fact that a weakly aligned curriculum causes our misjudging of the influence of instruction on learning. Even though teachers are investing all their efforts in their instruction, it remains ineffective if it is not aligned with the national standards or assessment (Anderson, 2002). This agrees with the view by Martone and Sireci (2009) that, in addition to the alignment of standards and assessments, an agreement on the teaching content given to students is needed. Otherwise, if teachers teach with no relation to a curriculum, students might achieve highly in the classroom and then miss on the assessments, without having the ability to pinpoint the gap (Martone & Sireci, 2009).

The fourth reason for the significance of curriculum alignment originates from the modern comprehension for “educational accountability”. Anderson (2002) suggests that it’s the responsibility of schools to provide students with the opportunity to learn the content of a test that is necessary for graduation, otherwise, denying diplomas to students would be illegitimate. Martone and Sireci (2009) argues that alignment research is at the base of accountability studies since it demonstrates a coherent message about why, how, and what should be learned at schools.
The coherent message of alignment is well valued and described by Porter (2002): “An instructional system is to be driven by content standards, which are translated into assessments, curriculum materials, and professional development, which are all, in turn, tightly aligned to the content standards” (p.5).

### 2.3.2 – Models for Alignment between Standards and Assessments

To study curriculum alignment, a state or district must choose from several proposed models the one that best meets its particular alignment goals, criteria, and resources. The widespread alignment models are: Webb’s alignment model (Webb, 2007), the Surveys of Enacted Curriculum model (SEC, as described by Porter, 2002), the Achieve model (cf. Rothman, Slattery, Vranek, & Resnick, 2002), and Council for Basic Education model (CBE).

According to Rothman et al., (2002), Webb’s model is based on four alignment criteria: Categorical Consistency, Depth-of-Knowledge Consistency, Range of Knowledge Correspondence, and Balance of Representation. Additionally, the Webb method may be used to compare alignment of assessment to a content standard.

Roach et al., (2008) note that the Porter alignment index analyzes the degree of alignment between two tables: one for the curriculum and the other for the test. The rows in both tables represent the content whereas the columns represent the emphasized cognitive skill. This analysis produces a single alignment index, ranging from 0 to 1, to indicate how closely the
distribution of points in the first table aligns with the second table. It is found using the formula:

\[ P = 1 - \frac{\sum_{i=1}^{n} |X_i - Y_i|}{2} \]

where ‘n’ represents the total number of cells in the table, and ‘i’ represents a certain cell such that ‘i’ varies from 1 to n.

Also, the Achieve alignment protocol uses both qualitative and quantitative methods of analysis to obtain the alignment of a state's assessment to its related content standards. The test items are first analyzed separately after which the whole test is analyzed. Certain criteria like content centrality, performance centrality, challenge, range and balance are used to examine the alignment between assessment items and standards (Rothman et al., 2002).

Finally, the CBE alignment method is straightforward and easy to use. The model works by identifying test items or framework specifications that fit the benchmarks, and then recording the degree of match in content and performance level. Measuring the alignment of standards, curriculum, and assessments is applied using the following criteria: (1) content, (2) content balance, (3) rigor, and (4) item response type. Reviewers then reach decisions on the degree of alignment according to an evaluation rubric (Council of Chief State School Officers, 2002).
According to Martone and Sireci (2009), conducting research on alignment can be challenging. Despite the numerous approaches to gather data about curriculum alignment, only a few comprehensive frameworks existed. Deprived from the suitable framework, the data interpretation would be difficult (Anderson, 2002).

One of the limited trials that have been done to develop a suitable methodological framework was the one designed by Osta (2007) to analyze “non-objective” type tests for studying the alignment of Math Exams with the curriculum in Lebanon. The study revealed a persistent “assessment culture” that is shaped by a steady structure of tests, stereotyped style of questions that cover a narrow part of the curriculum thus reducing the curriculum to a “mini curriculum”, and a “partition of the mathematical topics into specialized areas for each mathematical ability” (Osta, 2007).

Sleiman (2013) used the framework developed by Osta (2007) to study the alignment between the intended and assessed Lebanese curricula of the “Literature and Humanities” (LH) track of the secondary level. Sleiman (2013) diagnosed a similar fragile alignment between the Lebanese curriculum and the national math tests and a similar “mini curriculum” to the one described by Osta (2007). The results revealed a “teaching for the test” practice by teachers, leading to “drill and practice” approaches by students.
CHAPTER THREE
METHODOLOGY

3.1 – Design and Procedures

Techniques of content analysis were used to study the alignment between the national math tests and the national reformed Lebanese math curriculum. The curricular texts were analyzed quantitatively and qualitatively. They include:

1. The national text of the mathematics reformed curriculum for the secondary school level as issued in 1997 by the MEHE and ECRD (referenced as Document I in Appendix A), which includes the general and specific objectives as well as the scope and sequence and syllabus.

2. Curriculum of mathematics – Decree N°: 10227 – details of contents of the third year of each cycle, a document issued in May 1997 by MEHE and ECRD (referenced as Document II in Appendix A). It includes the detailed content along with the corresponding objectives and comments for the third year of each cycle. This study is concerned with the detailed content of grade 12, LS track.

3. Evaluation Guide for Mathematics for the Secondary Cycle, a document issued in October 2000 by the MEHE and ECRD (referenced as Document III in Appendix A). It consists of two parts. The first part includes the competencies for each year of the
secondary cycle along with sample test items for each competency. The competencies are classified in domains. There are three domains in LS section: “calculation processes”, “numerical functions (calculus)”, and “problem solving and communication”. The second part is a set of model tests for national test. It includes a set of criteria for the content and format of the national tests (see Appendix B) in addition to model tests for each of the four tracks in grade 12 and their corresponding “elements of solution and marking scheme”. The model tests for the LS track (see sample in Appendix C) are regarded in this study as representing the assessment philosophy in the reformed curriculum, while the actual national tests represent the practical implementation of that philosophy.

4. A sample of the national math tests for the LS section. Twelve tests (see sample in Appendix D) administered between 2001 and 2012 are considered. Those tests include 6 regular (first session) national tests usually administered in June, at the end of the academic school year, and 6 second-session national tests administered in September to give a second chance to students who failed or missed the regular June test.

Since the curriculum of the secondary level is split over three years (grades 10, 11, & 12), and because of the cumulative nature of mathematical knowledge and skills, it was needed at times to refer back to additional documents. The grade 12 math national tests included some test items addressing objectives from previous grade levels. As a result, the document containing the details of contents of grade 11 was also referred to in the analysis (referenced as Document IV in Appendix A).
The structure, content and objectives of the national curriculum were analyzed qualitatively, whereas the model tests and national tests were quantitatively analyzed and compared using Pearson Product-Moment coefficient. More specifically,

1. The national tests and the model tests are analyzed and compared quantitatively.

2. The national tests of the years 2001-2003 are analyzed and compared to those of the years 2010-2012 in order to check the evolution of the national tests under the reformed curriculum.

3. The session 1 national tests of the years 2001-2012 are analyzed and compared to those of session 2 in order to check their compatibility.

3.2 – Framework for Analyzing Tests

To study the alignment between assessment and curriculum, Osta (2007) developed a framework based on statistical tables for the model test and the national tests. Osta (2007) mapped the test items of the two types of tests according to their respective math content within the curriculum, and their cognitive level, using the National Assessment of Educational Progress (NAEP) mathematical abilities: Procedural Knowledge, Conceptual Understanding, and Problem Solving. These tables were then used to find the Pearson correlation between the items of the model tests and national tests, as classified in the tables.

Sleiman (2013) adopted the framework developed by Osta (2007) and classified the test items of each of the national tests and model tests using the same technique, according to the content and the cognitive levels that they address. Only, Sleiman (2013) considered the
cognitive domains: knowing, applying, and reasoning of TIMSS Advanced 2008 (Garden et al., 2006) Mathematics Framework (see Appendix E) instead of relating to the NAEP mathematical abilities. Two reasons justify the use of TIMSS cognitive domains: The first reason is that the TIMSS cognitive domains represent well the philosophy of the Lebanese reformed math curriculum delineated in the Introduction and general objectives and based on: critical thinking, use of math in everyday life, long life learning, and students constructing their own knowledge. The second reason is that Lebanon is one of the countries participating in TIMSS assessment, and adopting the TIMSS cognitive domains would shed light on the extent to which the national exams take into consideration the preparation of Lebanese students for TIMSS. This same framework will be adopted in the present paper.

3.2.1 – Definition of a Test Item

This paper adopts the definition of a test item by Osta (2007):

We define a “test item” as being any part of the test that requires a response from the student which entitles him/her to a part of the grade. A test item may take one of the two following forms:
- A question that requires an answer. For example, “What is the nature of triangle ABC?”
- An imperative sentence, such as “Calculate the coordinates of point I.”
In the case of many components required in one sentence, it is considered to stand for more than one test item. For example, “Plot the points A, B, C, and the straight line (D)” is counted for four items, because it stands for “Plot point A, plot point B, plot point C, and plot straight line (D)”.

33
3.2.2 – Qualitative Analysis

The structure and content of the curriculum, model tests, and national tests are qualitatively analyzed as follows:

1. The curriculum is described in terms of its structure, content, and objectives.
2. The model tests and national tests are described in terms of their structure and content.
3. The test items in both the model tests and national tests are classified according to topics: *Literal and numerical calculations*, *Numbers*, *Classical study*, *Definitions & Representations*, *Continuity and differentiation*, *Integration*, *Differential equations*, *Circular functions*, *Statistics* and *probability*. Then, a descriptive analysis follows in addition to supportive examples.

3.2.3 – Quantitative Analysis

Statistical tables are used to analyze the test items of the model test and national test according to the corresponding curriculum objectives, as well as the TIMSS Advanced 2008 cognitive domains that they measure.

3.2.3.1– Coding.

The national tests for the LS track are coded as LS011, LS012, LS021, LS022, LS031, LS032, LS101, LS102, LS111, LS012, LS121, and LS112. The letters represent the LS track,
the first two numbers specify the year of the test, and the last number specifies whether the test is first or second session.

The model tests for the LS track are coded as LSM1, LSM2, LSM3, and LSM4. The first two letters of the code represent the LS track, the “M” for “model” test, and the number distinguishes among the four model tests.

This study adopted the coding system of the details of contents of the national reformed mathematics curriculum for the LS track at the secondary school level (referenced as Document II in Appendix A). The Roman numbering i, ii, iii… are used to code the sub-objectives. This is represented in Appendix F.

The content of the three LS secondary years were assessed by the national tests at the end of the Grade 12 LS track. The items that were addressed in the model tests and national tests can be associated with Grade 10 or 11 curriculum content and are coded (A, B… TT). In addition, they are classified per topics (Arrangements and permutations, equations and inequalities, Complex numbers, Vectorial study, Geometry, Functions, and probability) (see Appendix G).

3.2.3.2– Mapping of Test Items

Osta’s (2007) technique in mapping the test items are adopted in this study. An extract from the table of the quantitative analysis of the first model tests (see appendix C) is
shown in Appendix H. In addition, Appendix I represents an excerpt from the quantitative table analyzing the national test LS121 (Appendix D).

3.2.4 – Validity of the Analysis

The Validation of the analysis of the tests in mapping the test items in an objective way was done by a judge. The judge is a Lebanese Math instructor in a private university. She has completed an MA in Math Education. The author asked the judge to perform the same analysis of the model tests and national tests. She also analyzed the test items as to their corresponding curriculum objectives, and the TIMSS Advanced 2008 cognitive domains they measure.
CHAPTER FOUR
FINDINGS

This chapter includes the qualitative and quantitative analysis of the documents and the data. The documents that are analyzed include:

1. The official text of the mathematics reformed curriculum for the secondary school level as issued in 1997 by the MEHE and ECRD (referenced as Document I in Appendix A), which includes the general and specific objectives as well as the scope and sequence and syllabus.

2. Curriculum of mathematics – Decree N°: 10227 – details of contents of the third year of each cycle, a document issued in May 1997 by MEHE and ECRD (referenced as Document II in Appendix A). It includes the detailed content along with the corresponding objectives and comments for the third year of each cycle. This study is concerned with the detailed content of grade 12, LS track.

3. Evaluation Guide for Mathematics for the Secondary Cycle, a document issued in October 2000 by the MEHE and ECRD (referenced as Document III in Appendix A). It consists of two parts. The first part includes the competencies for each year of the secondary cycle along with sample test items for each competency. The competencies are classified in domains. There are three domains in LS section: “calculation processes”, “numerical functions (calculus)”, and “problem solving and
communication”. The second part is a set of model tests for national test. It includes a set of criteria for the content and format of the official exams (see Appendix B) in addition to model tests for each of the four tracks in grade 12 and their corresponding “elements of solution and marking scheme”. The model tests for the LS track (see sample in Appendix C) are regarded in this study as representing the assessment philosophy in the reformed curriculum, while the actual official exams represent the practical implementation of that philosophy.

4. A sample of the official math exams for the LS section. Twelve exams (see sample in Appendix D) administered between 2001 and 2012 are considered. Those exams include 6 regular (first session) official exams usually administered in June, at the end of the academic school year, and 6 second-session official exams administered in September to give a second chance to students who failed or missed the regular June exam.

4.1 – Content Analysis

4.1.1 – Qualitative Analysis

The qualitative analysis includes the structure and content of the curriculum, model tests, and national tests.

4.1.1.1 – Qualitative Analysis of the Curriculum.

As presented in Document I (referenced in Appendix A), The mathematics curriculum includes five sections: introduction, general objectives, table of number of periods for each
subject per week / year, the table of scope and sequence, the specific objectives of each cycle along with the syllabus of every year of the cycle at both, basic education and secondary education. The following sections will be discussed.

1. Introduction. The introduction highlights the role of mathematics in a changing society. Mathematical knowledge provides an important key to understanding the whole world in which we live. It develops logical, critical, and creative thinking. Mathematics helps us quantify objectively and precisely the qualitative description of reality. It is an essential tool that enhances the development of societies in all domains and therefore, it must be used by all citizens. The introduction states that mathematics teaching is reformed through three axes: a) Formulation of objectives: the focus is on the “individual construction of Mathematics”. Students are provided with the opportunity to experience the spirit of science in real-life situations. Good communication skills such as reading, writing, explaining, and interpreting, will remain a fundamental principle in mathematics teaching; b) Remodeling contents: subjects are chosen according to their practical interests. The accent is on the use of appropriate technological devices as mathematical tools; c) Methods of teaching: The teaching of math is related to everyday life in an organized way leading the students to the intelligence of conceptual models.

2. General Objectives. The general objectives of the reformed math curriculum are: (a) Mathematical Reasoning through training students to formulate mathematical arguments, to doubt, conjecture, and abstract; (b) Solving Mathematical Problems through use of different strategies such as reading, interpreting, dealing with real life
problems, and using appropriate mathematical strategies and techniques; (c) 
*Developing the scientific spirit* by practicing the scientific approach and improving 
skills in research. Integrating Mathematics with other subjects and appreciating its role 
in “technological, economical and cultural development”; (d) *Communicating 
Mathematically* orally and in writing, and using mathematical tools in a variety of 
contexts; and (e) *Valuing Mathematics* through providing students with the opportunity 
to experience the beauty, elegance, and harmony of mathematical theories.

3. Secondary Education. The Secondary Education is divided into four tracks: LH, SE, 
GS, and LS. The section on Secondary Education comprises: (a) the objectives of each 
of the four tracks; (b) the scope and sequence of the math topics over the three years of 
each of the four tracks; and (c) the syllabus of each of the four tracks (LH, SE, GS, and 
LS) of the secondary cycle with the allocated time for each math topic.

This study will focus on the LS (Life Science) track over the secondary years. The 
objectives are: (a) *mathematical reasoning*, (b) *problem solving*, (c) *communication*, (d) 
*spacial*, (e) *numerical and algebraic*, (f) *calculus*, and (g) *statistics & probability*. The math 
domains of this cycle include *algebra, geometry, calculus* (numerical functions), *trigonometry*, 
and *statistics & probability*. The content of these domains is distributed over the three years of 
the LS secondary cycle.

Five periods per week and 150 periods per academic year are allocated for 
Mathematics at grade 12 LS track. The math content is distributed over five domains: *Algebra,*
Geometry, Calculus (Numerical Functions), Trigonometry, and Statistics & Probability. The syllabus of the LS track at the third secondary year is organized in the table below. Table 4.1 presents the main content under the five domains along with the allocated time for each (refer to Appendix F for the details of contents of the LS track).

Table 4.1
*The Math Topics in the LS Track of the Third Secondary Year*

<table>
<thead>
<tr>
<th>Code</th>
<th>Math Topics</th>
<th>Allocated Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>ALGEBRA</strong></td>
<td>35 hours</td>
</tr>
<tr>
<td>1.1</td>
<td>– Foundations</td>
<td>8 hours</td>
</tr>
<tr>
<td>1.1.1</td>
<td>→ Binary operations</td>
<td></td>
</tr>
<tr>
<td>1.1.2</td>
<td>→ Structure of group</td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>- Literal and numerical calculations</td>
<td>10 hours</td>
</tr>
<tr>
<td>1.2.1</td>
<td>→ Combinations: definition, notation, binomial formula, Pascal’s triangle.</td>
<td></td>
</tr>
<tr>
<td>1.3</td>
<td>- Equations &amp; Inequations.</td>
<td>7 hours</td>
</tr>
<tr>
<td>1.3.1</td>
<td>→ System of linear equations ((m \times n)): definition, Elementary operations on the rows, Gauss Method</td>
<td></td>
</tr>
<tr>
<td>1.4</td>
<td>- Numbers</td>
<td>10 hours</td>
</tr>
<tr>
<td>1.4.1</td>
<td>→ Module and argument of a complex number, properties</td>
<td></td>
</tr>
<tr>
<td>1.4.2</td>
<td>→ Trigonometric and exponential forms of a complex number.</td>
<td></td>
</tr>
<tr>
<td>1.4.3</td>
<td>→ Geometric interpretation of addition and multiplication of complex numbers and the passing to the conjugate.</td>
<td></td>
</tr>
<tr>
<td>1.4.4</td>
<td>→ De Moivre’s formula, applications.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td><strong>GEOMETRY</strong></td>
<td>15 hours</td>
</tr>
<tr>
<td>2.1</td>
<td>- Classical study</td>
<td></td>
</tr>
<tr>
<td>2.1.1</td>
<td>→ Components of the vector product. Mixed product</td>
<td></td>
</tr>
<tr>
<td>2.1.2</td>
<td>→ Equation of a plane and of a straight line in space</td>
<td></td>
</tr>
<tr>
<td>2.1.3</td>
<td>→ Orthogonality of two straight lines, of a straight line and a Plane; perpendicular planes.</td>
<td></td>
</tr>
<tr>
<td>2.1.4</td>
<td>→ Parallelism of straight lines and of planes.</td>
<td></td>
</tr>
<tr>
<td>2.1.5</td>
<td>→ Distance from a point to a plane, to a straight line</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td><strong>CALCULUS (NUMERICAL FUNCTIONS)</strong></td>
<td>65 hours</td>
</tr>
<tr>
<td>3.1</td>
<td>- Definitions &amp; Representations</td>
<td>25 hours</td>
</tr>
<tr>
<td>3.1.1</td>
<td>→ Inverse functions</td>
<td></td>
</tr>
</tbody>
</table>
3.1.2 → Inverse trigonometric functions
3.1.3. → Natural (Naperian) logarithmic function.
                      Logarithmic function to the base a.
3.1.4. → Exponential functions
3.2. → Continuity and derivation 15 hours
3.2.1. → Image of a closed interval by a continuous function
3.2.2. → Derivative of composite functions
3.2.3. → Derivative of an inverse function
3.2.4. → Second derivative, Successive derivatives
3.2.5. → L’Hopital’s rule
3.3. → Integration 15 hours
3.3.1. → Integral: definitions, properties
3.3.2. → Rules of integration
3.3.3. → Application of the integral calculations
3.4. → Differential equations 10 hours
3.4.1. → Definition
3.4.2. → Equations in separable variables
3.4.3. → Linear first order equations with constant coefficients.
3.4.4. → Linear second order equations with constant coefficients

4. TRIGONOMETRY 5 hours
4.1. - Circular functions
4.1.1. → Study of the circular functions of the form acos(bx+c)
                    and asin(bx+c)

5. STATISTICS AND PROBABILITY 30 hours
5.1. - Statistics 10 hours
5.1.1. → Measures of central tendency and measures of variability
                of a distribution of one (continuous or discrete) variable
5.2. - Probability 20 hours
5.2.1. → Conditional probability: definition, independence of
                two events.
5.2.2. → Formula for of probabilities
5.2.3. → Random real variable, law of associated probability,
                Distribution function. Characteristics.
5.2.4. → Bernoulli variable
5.2.5. → Binomial law.

However, the math curriculum at all the tracks of grade 12 was seen by MEHE and
ECRD over loaded and difficult to be taught in one year. Thus, the government, supported by
the suggestions of MEHE and ECRD to reduce the content of the math curriculum, publicized what it calls “reduction of the curriculum”. The details of contents that were omitted from the mathematics curriculum at the LS track are presented in Document V and referenced in Appendix A. The cancelled topics from the math curriculum in the grade 12 LS track are (see Appendix J): *binary operations, structure of group, system of linear equations (m × n): definition, elementary operations on the rows, gauss method, inverse trigonometric functions, logarithmic function to the base a, successive derivatives, Bernoulli variable, binomial law.*

The Evaluation Guide (Document III referenced in Appendix A) contains a section titled: “General principles about the guidelines and the way of developing the official exam questions in mathematics for the general secondary school certificate” (see Appendix B). This section includes the criteria for the selection of questions in all grade 12 tracks. The findings will be discussed based on these criteria.

4.1.1.2 – Qualitative Analysis of the Model Tests.

The Evaluation Guide (Document III referenced in Appendix A) includes four model tests for the LS track referred to in this research as LSM1, LSM2, LSM3, and LSM4 (refer to Appendix D presenting as a sample the first model test, coded as LSM1).
The math topics in the model tests for the LS track are presented in Table 1 in Appendix K. LSM1, LSM3, and LSM4 consist of three parts each, while LSM2 consists of four parts.

The parts included in LSM1 are based on the domains: Algebra, Calculus, and Geometry. However, the parts on Algebra includes test items on Literal and numerical calculations, but doesn’t include any test item about Numbers. In addition, all the test items on Calculus are on Definitions & Representations, and on Continuity and differentiation and thus, no test items occurred on Integration or Differential equations

However, LSM2 involves four parts based on the domains: Algebra, Calculus, and Statistics and Probability. The test items of the first part are on Numbers and Integration. The first part of the question demands to linearize a trigonometric expression, while the second part demands to integrate the linearized form of the trigonometric expression. The second part involves questions on Probability of two independent events. While the third problem is on Numbers, the fourth one tackles the concepts of inverse functions and composite functions. Its test items are on Definitions & Representations and Continuity and differentiation.

LSM3 has three parts: one on Algebra and Statistics and Probability, one part on Calculus and Algebra, and no test items on Geometry. The parts on Algebra includes test item on Literal and numerical calculations that are linked in content to the test items on Probability. Also, the test items on Numbers are integrated to the test items on Calculus.

LSM4 consists of three parts based on Statistics and Probability, Algebra, and Calculus. The first part is on statistics. Its test items are chosen from the objectives of grade
11. The part on *Algebra* contains test items that are related to *Numbers*. Moreover, the test items on *Calculus* are distributed over the topics: *Definitions & Representations, Continuity and differentiation*, and *Integration*.

Table 2 in Appendix K presents the distribution of grades over the math topics in the model tests for the LS track. The parts on *Calculus* occurred on all the model tests and are allocated the highest grades that range from 7 to 9 grades. The parts on *Algebra, Geometry*, and *Statistics and probability* are allocated similar grades that range from 4 to 6 grades.

4.1.1.3 – Qualitative Analysis of the National Tests.

Twenty-four national tests for the LS track administered between 2001 and 2012 are analyzed. They include twelve session-1 and twelve session-2 national tests. Those tests are referred to in this research as LS011, LS012, LS021, LS022, … LS111, LS112, LS121, and LS122 (refer to Appendix D presenting as a sample the session-1 LS national tests administered in 2012, coded as LS121).

Table 1 in Appendix K presents the codes of the math topics included in the national tests in the first and second sessions of the years starting from 2001 till 2012. The table shows that each national test consists of five parts; one part on each of the following topics: *Algebra, Geometry, Calculus, Trigonometry, and Statistics and probability*. However, the topic *Trigonometry* is not addressed in any of the official exams.
It is noticed that some topics are consistently addressed in most of the national tests. *Numbers, Geometry, Definitions & Representations, Integration, and Probability* are common topics in all exams. *Continuity and differentiation* is the second most occurring topic, the test items tackling this topic are integrated within a question on the study of functions.

The topic *Literal and numerical calculations* is included in all national tests except LS011, LS022, LS042, LS051, and LS081. The test items of this topic are integrated within a question on the study of *Statistics and Probability*. The official LS021, LS031, LS042, LS061, and LS062 exams include, each, questions on *Differential equations*. *Statistics* does not occur in any of the session-1 national tests. It occurs only in two session-2 national tests LS012 and LH032. *Circular functions* didn’t occur on any of the national tests.

Table 2 in Appendix K presents the distribution of grades over the math topics. The part on *Calculus* was assigned the highest grades that range between 8 and 9.5 out of 20, which are approximately half the grades assigned for the test. However, since the national test LS2071, 8 grade points were constantly assigned to this topic. The other three topics *Algebra, Geometry, and Statistics and probability* had similar patterns of distribution of grades that ranged between 2 and 6 grade points out of 20.

4.1.1.4 – Qualitative Analysis of the Test Items.
The qualitative analysis includes the topics as well as the test items considered in the model tests and the national tests. The topics assessed in both the model tests and the official exams are: *Literal and numerical calculations, numbers, classical study of geometry, definitions & representations, continuity and differentiation, integration, Statistics, and Probability.* One topic *differential equations* occurs only in the national tests.

Literal and numerical calculations

*Literal and numerical calculations* is a topic classified under *Algebra*. Table 3 in Appendix K presents the test items on *Literal and numerical calculations* as well as the tests where they occur.

The test items on *Literal and numerical calculations* in the official exam tests require basically calculating the number $^nC_p$ of all the combinations of $p$ elements of a set of $n$ elements to be used in calculating the probability of an event. Two exceptions to this type of test items occurred in each of the national tests (LS021) and (LS052) where the number $^nC_p$ was not used in calculating the probability of an event. All the test items under this topic in Table 3 in Appendix K go under the cognitive domain “knowing”.

The following is an example of a probability part retrieved from the national test LS122.

Consider two urns U and V. Urn U contains eight balls: four balls numbered 1, three balls numbered 2 and one ball numbered 4. Urn V contains eight balls: three balls numbered 1 and five balls numbered 2.

1) Two balls are selected, simultaneously and randomly, from the urn U. Consider the following events:

• A: « the two selected balls have the same number »
• B: « the product of the numbers on the two selected balls is equal to 4 ». 

47
Calculate the probability $P(A)$ of the event $A$, and show that $P(B)$ is equal to $\frac{1}{4}$.

On the other hand, \textit{Literal and numerical calculations} occurs in two model tests (LSM1) and (LSM3) where the test items go under the cognitive domain “knowing”. However, the test items in (LSM1) are not integrated within a question on probability. They are direct questions where the aim is to find a number of combinations of $p$ elements of a set of $n$ elements.

The following is the only part retrieved from the model test LSM1 under the topic \textit{Literal and numerical calculations}.

\begin{quote}
In a computer club of a school, there are four boys, numbered from 1 to 4, and five girls, numbered from 1 to 5. The manager of the club wishes to form a committee of three members.

1) How many committees of boys can be formed?
Deduce the possible number of committees having at least one girl.

2) How many committees having only one boy and a member numbered 2 can be formed?
\end{quote}

The test items under the topic \textit{Literal and numerical calculations} occurring in (LSM3) require finding the number of arrangements without repetition of a number of elements. The test items are integrated within a question on probability.

Note from the previous two examples that the test items under \textit{Literal and numerical calculations} in the national tests and those in the model tests are of different type: Most of such test items in national tests are integrated within a question on Probability, whereas it is not the case in the test items of the model tests. Also, there are no test items related to the number of arrangements in any of the national tests. Thus, they do not align. The following
objectives under the topic *Literal and numerical calculations* were never addressed in the official exam tests:

1.2.1.3. Construct the Pascal's triangle

1.2.1.4 Know and use the binomial formula

1.2.1.4.i Know and use the formula giving the number \( ^nC_p \) of all combinations of \( p \) elements of a set of \( n \) elements (\( p \leq n \))

1.2.1.4.ii Model situations by combinations

1.2.1.4.iii Know and use the binomial formula for expanding \( (a+b)^n \)

1.2.1.4.iv Know and use the formula \( ^nC_p = (n-1)C_p + (n-1) C (p-1) \)

1.2.1.5 Arrangements and permutations: Calculate \( n! \)

1.2.1.5.i Arrangements and permutations: Know and use the formulas that give the number of arrangements with and without repetition, and number of permutations

Numbers

The topic *numbers* is classified under *Algebra*. Table 4 in Appendix K presents the test items on *numbers* as well as the tests where they occur. The topic *numbers* involves basically the study of complex numbers in terms of: properties, geometric representation and applications. Most test items require writing a complex number in different forms: algebraic, trigonometric and exponential. In addition, many test items tackle the use of the properties of modulus and argument of complex numbers for establishing relations and solving problems of geometric or trigonometric nature such as: finding the set of points having a certain property,
or proving special triangles and quadrilaterals. Moreover, De Moivre’s formula and its applications are included in the study of this topic.

*The following is a part retrieved from the model test LSM4 under the topic numbers.*

1) Solve in $\mathbb{C}$ the equation $z^2 + \sqrt{3}z + i = 0$ \hspace{1em} (E)

We call $z_1$ and $z_2$ the roots of the equation \((E)\), the root $z_1$ is the one that has a positive imaginary part.

2) In the orthonormal plane let $A_1$ and $A_2$ be the points representing $z_1$ and $z_2$ respectively. Let $A$ be the point representing $z_A = i$.

   a- Prove that the points $A_1$, $A_2$ and $A$ are on a circle for which you should determine the center and the radius.
   b- Calculate $|z_A - Z_1|$. Deduce the type of triangle $OAA_1$.
   c- Specify the type of the quadrilateral $OAA_1A_2$.

It is noted that the test items under *numbers* do not consistently nor evenly occur in the model tests. The model tests LSM2 and LSM4 include a big part on *numbers* with multiple sections. LSM1 doesn’t include any test item on *numbers*, while LSM4 includes only one test item. On the contrary, almost all the national tests include a part on *numbers*. Moreover, the national tests LS011, LS021, LS082, LS102, LS112, and LS122 include a 2-by-2 table of questions and suggested multiple-choice answers where students are required to choose the right answer with justification for the questions given. Thus, although the contents of the test items are similar in Model tests and official exams, the structure and format of the test item varies considerably. In addition, some test items appear only in model tests, such as “Linearize simple trigonometric polynomials”, while the test items “Write in exponential form” and “write in algebraic form” appear only in national tests. Finally, the test items “Write a non-zero complex number, given in trigonometric form, in the algebraic form” and “Write a non-
zero complex number, given in exponential form, in the trigonometric form” didn’t occur in any model or national test.

The following is a part retrieved from the national test LS121 under the topic numbers.

The complex plane is referred to a direct orthonormal system \((O; \vec{u}, \vec{v}).\)

For every point \(M\) with affix \(z (z \neq 0)\), we associate the point \(M'\) with affix \(z'\)

\[ \text{such that } z' = \frac{2}{z}. \]

1) Let \(z = r \exp(i\theta)\) \((r > 0)\), write \(z'\) in exponential form.

2)  
   a- Show that \(OM \times OM' = 2\).
   
   b- If \(z = z'\), prove that \(M\) moves on a circle \((C)\) whose center and radius are to be determined.

3) Let \(z = 1 + iy\) where \(y\) is a real number.
   
   a- Prove that \(|z' - 1| = 1\).
   
   b- As \(y\) varies, show that \(M'\) moves on a circle \((C')\) whose center and radius are to be determined.

Classical study (geometry)

The topic classical study is classified under Geometry. This topic requires using knowledge of space geometry, plane geometry, scalar and vector product in the field of analytical geometry, to find equations of straight lines and planes in the space and to study their relative positions. Thus, solving problems on classical study of geometry may require visualizing geometric elements in space by sketching 3D drawings. Table 5 in Appendix K presents the test items on numbers as well as the tests where they occur. It is noted that the test items under classical study of geometry are distributed over all the official exams in a random way with no obvious pattern of occurrence.

The parts on classical study of geometry are structured in multiple sub-parts where most of the times the results of a preceding test item are used in solving the next one. The only
exception to this structure occurs in the official exam LS021, where the problem includes multiple-choice questions. Students are required to choose the right answer with justification for the questions given. Moreover, in the national tests LS031, LS041, LS042, LS052, and LS072 a figure is provided to support students’ visualization of 3D elements.

The following is a part retrieved from the national test LS042 under the topic *classical study of geometry*.

In the space referred to a direct orthonormal system, consider the cube OABCDEF such that: A(1; 0; 0), B(1; 1; 0) and F(1; 1; 1). Designate by P, Q and R the midpoints of the segments [DG], [DE] and [AE] respectively.

1) a- Show that \(2x + 2y + 2z - 3 = 0\) is an equation of the plane \((PQR)\).
   b- Prove that the plane \((PQR)\) passes through the midpoint of \([AB]\).
   c- Prove that the planes \((PQR)\) and \((BEG)\) are parallel.

2) a- What is the nature of quadrilateral \(EGCA\)?
   b- Let \(M\) be a variable point on the line \((AC)\).

Show that \(AM \times EF = AM \times GF\).

The test items on “classical study” of geometry occurred only in two model tests LSM1 and LSM3. In both tests, no figure was shown. However, the structure and the content of the problems are similar to that in the national tests.

The following is a part retrieved from the national test LSM3 under the topic *classical numbers*.

The space has the orthonormal system \((O; \vec{i}, \vec{j}, \vec{k})\).

Consider the planes \((P)\) and \((Q)\) of equations:
\[
(P) : 2x + 2y - z + 5 = 0
\]
\[
(Q) : 2x + y + 6z - 8 = 0
\]

1) Prove that \((P)\) and \((Q)\) are orthogonal. 2) Deduce the distance from the point \(A(2, 1, 4)\) to the line \((D)\), intersection of the two planes \((P)\) and \((Q)\).
3) Give an equation of the line (D).
4) Use the value found in question 2) to calculate the coordinates of the point H, orthogonal projection of the point A on the line (D).

Definitions & Representations

The topic definitions & representations is classified under Calculus (Numerical Functions). As previously mentioned, the topic Calculus was assigned the highest grades in the official exams. One part on Calculus is included in each of the national tests and the model tests. This reflects its importance. The part on Calculus includes all the test items of the following topics: “Definitions & Representations”, “Continuity and differentiation”, “Integration”, and “Differential equations”.

The topic Definitions & Representations deepens the study of exponential and trigonometric functions in terms of: Domain, variation, limits and asymptotes, graphical representations, derivative and primitive. This topic also involves studying composite functions and inverse function. Table 6 in Appendix K presents the test items on definitions & representations as well as the exams where they occur.

In the national tests, it can be observed that the questions addressing exponential functions are more frequent than those addressing logarithmic functions. Different structures of questions are used in addressing this topic. The most recurrent manner is giving the expression of the function followed by a set of questions as in the national test LS121. However, in the national test LS061 the table of variations of f’(x) is given, which helps student to set up the table of variations of f in order to graph it in a later stage. A graph of f’ is
displayed in the national tests LS042, LS022; whereas in LS031 a graph of f is presented. Moreover, a table of variation of h’ is given to determine the sign of f(x) in the national test LS032. Finally a table of f’(x) involving sign of f”(x) is presented in LS041.

The following is a part retrieved from the national test LS081 under the topic Definitions & Representations.

Let f be the function defined on IR by f(x) = (x – 1)e^x + 1 and designate by (C) its representative curve in an orthonormal system (O ; i , j ).

1) a- Calculate \( \lim_{x \to -\infty} f(x) \) and deduce an asymptote (d) of (C).
   b- Study, according to the values of x, the relative positions of (C) and (d).
   c- Calculate \( \lim_{x \to +\infty} f(x) \) and find f(2) in decimal form.

2) Calculate f’(x) and set up the table of variations of f.

4) a- Draw (d) and (C).
   b- Discuss graphically, according to the values of the real parameter m, the number of solutions of the equation \((m – 1) e^{-x} = x – 1\).

It is noted that question 3) is not included in the above example since the test items forming this question are considered under the topic Continuity and differentiation.

The test items occurring on Definitions & Representations in the model tests have similar content as the ones in the national tests. However, the structure of the problem addressing this topic varies. In all model tests, the expression of the function is always given and there are no tables, graphs, figures presented. In addition, in the model test LSM3, the test items on numbers are integrated within the test items on Definitions & Representations. Throughout the national tests and the model tests, the integration between two topics in the same part occurs only once.
The following is a part retrieved from the national test LSM3 under the topic

**Definitions & Representations.**

\[ f \text{ is the function defined on } ]0; + \infty [ \text{ by } f(x) = \frac{1}{2} x - 1 + \frac{x}{\ln x}. \]

1) **Study the limits of** \( f \) **at** \( 0 \) **and at** \( + \infty \). **Prove that the line** \((\Delta)\):

\[ y = \frac{1}{2} x - 1 \]  

**is an oblique asymptote of the graph** \((C)\) **of** \( f \). **Specify the relative positions of** \((C)\) **and** \((\Delta)\).

2) **a- Calculate** \( f'(x) \) **and then** \( f''(x) \). **Deduce the variations of** \( f \).
   **b- Calculate** \( f'(e^{1.5}) \) **and deduce the sign of** \( f' \).
   **c- Make the table of variations of** \( f \).

It is noted that the test items under this topic vary between the three cognitive domains:

“knowing”, “applying” or “reasoning”.

Continuity and differentiation

This topic is classified under “*Calculus*”. Table 7 in Appendix K presents the test items on *definitions & representations* as well as the tests where they occur. The test items on *continuity and differentiation* are regarded as tools for a better study of functions through the graphical interpretations of the answers. The study of this topic includes: Image of a closed interval by a continuous function, derivative of composite and inverse functions, Successive derivatives, and L’Hôpital’s rule. However, the objectives related to the study of derivatives of exponential and logarithmic functions are categorized under the topic “*definitions and representation*”.

55
The national test LS091 includes no test items under *continuity and differentiation*. Moreover, the following objective under the topic *continuity and differentiation* was never addressed in any of the official or the model tests:

3.2.1.1.i → Know that the image of an interval by a continuous function is an interval of the same nature.

The following are two questions retrieved from the national test LS052 under the topic *continuity and differentiation*.

1) Calculate \( \lim_{x \to +\infty} f(x) \). Prove that the line \((d)\) of equation \( y = x + 2 \) is an asymptote of \((C)\).

2) Write an equation of the line \((T)\) that is tangent to \((C)\) at the point \(A\) of abscissa 0.

3) Show that the equation \( f(x) = 0 \) has a unique root \( \alpha \) and verify that \( -0.5 < \alpha < -0.4 \)

On the other hand, *continuity and differentiation* occurs in two model tests (LSM1) and (LSM3). However, most of the test items in the model tests under *continuity and differentiation* address the objectives of grade 11.

This reflects a discrepancy in content alignment.

The following are two questions retrieved from the model test LSM1 under the topic *continuity and differentiation*.

Let \( f \) be the function defined by \( f(x) = \ln x - mx \) where \( m \) is a non-zero real number. Let \( C_m \) be the graph of \( f \) in an orthonormal system.

1) For which values of \( m \), the function \( f \) is strictly monotone increasing?

2) For which values of \( m \), \( C_m \) has a maximum or a minimum?
3) Find the coordinates of a point of $C_m$ at which the tangent to $C_m$ contains the origin.

Integration

The topic integration is classified under Calculus. This topic includes several methods of integral calculations to find the primitive of a function, and to use the primitive to calculate areas and volumes. Table 8 in Appendix K presents the test items on integration as well as the tests where they occur.

In the national tests, different approaches have been taken addressing the objectives of integration. In the national tests LS021, two graphs are presented where students should use the identification of the integration as the inverse operation of differentiation to choose the primitive of a function. Other methods include asking students to calculate the areas: a) under the graph of a function (LS091, LS092, LS112, LS121, LS122,....), b) under the graph of the derivative function (LS022), c) between a function and its asymptote (LS051). The following objectives under the topic integration were never addressed in the national tests:

3.3.3.1.ii. → Calculate volumes in the case of a usual solid of revolution with the help of integrals
3.3.3.1.iii. → Calculate the volume of a solid delimited by the rotation of a curve about coordinate axes.
3.3.3.1.iv. → Calculate an approximate value of an integral by the method of rectangles.

The following is a part retrieved from the national test LS061 under the topic integration.

Let $F$ be the function defined over the set of real numbers by $F(x) = (px^2 + qx + r)e^{-x}$.

a- Calculate $p$, $q$ and $r$ so that $F$ is an antiderivative of $f$. 

57
b- Calculate the area of the region bounded by (C), the axis of abscissas and the two lines with equations \( x = 0 \) and \( x = 1 \).

On the other hand, the model tests address the topic integration in three tests (LSM1, LSM2, LSM4). However, the questions are direct and address limited objectives.

The following is a question retrieved from the model test LSM1 under the topic integration.

Let \( f \) be a function defined by \( f(x) = \ln x - mx \) where \( m \) is a non-zero real number. Let \( C_m \) be the graph of \( f \) in an orthonormal system
- Calculate the area of the domain limited by \( C_1 \), the lines \( y = -x \), \( y = 1 \) and \( x = e \).

Differential equations

Differential equations is a topic classified under Calculus. It occurs only in the national tests LS021, LS031, LS042, LS061, and LS062, where the test items require solving first and second-order linear differential equations with initial conditions. Table 9 in Appendix K presents the test items on differential equations as well as the exams where they occur.

The following objectives under the topic differential equations were never addressed in the national tests:

3.4.3.1.i. \( \rightarrow \) Solve a differential equation of the form \( y' = f(x) \) where \( f \) is continuous on an interval \( I \).
3.4.3.1.ii. \( \rightarrow \) Solve a differential equation of the form \( y' = ay + b \) where \( a \) and \( b \) are given real numbers.
3.4.3.1.iii. \( \rightarrow \) Solve a differential equation of the form \( y' + ay = f(x) \), where \( f \) a simple function.
3.4.4.1.i. \( \rightarrow \) Solve a differential equation of the form \( y'' = f(x) \) where \( f \) is continuous on an interval \( I \)
3.4.4.1.iii. \( \rightarrow \) Solve a differential equation of the form \( y'' + w^2y = k \).
3.4.2.1.i. \rightarrow \text{Recognize a differential equation of the first order in separable variables}

3.4.2.1.ii. \rightarrow \text{Solve a differential equation of the form } y' + a(x) y = 0.

The following is an example of the \textit{differential equations} parts retrieved from the national test LS061.

\textit{Consider the differential equation}

\((E): y'' - 4y' + 4 y = 4x^2 - 16x + 10. \text{ Let } z = y - x^2 + 2x.\)

1) Write a differential equation \((E')\) satisfied by \(z\).
2) Solve \((E')\) and deduce the general solution of \((E)\).
3) Determine the particular solution of \((E)\) whose representative curve, in an orthonormal system, has at the point A \((0;1)\) a tangent parallel to the axis of abscissas

On the other hand, the topic \textit{differential equations} occurs only in the national tests. Hence, there is a lack of alignment between the national tests and the model tests under \textit{differential equations}.

Statistics

The topic \textit{statistics} is classified under \textit{statistics and probability}. It occurs only in two national exams session 2 and in one model test. It involves measuring the central tendency and variability of continuous or discrete variables. These ideas serve to develop new conclusions about given data. Table 10 in Appendix K presents the test items on \textit{statistics} as well as the tests where they occur. The test items in Table 10 in Appendix K go under the cognitive domains “knowing” and “applying”.

59
The following is one of the two parts on statistics retrieved from the official exam LS012.

The following table gives the distribution of monthly salaries (in thousands LL) of 40 workers in a factory.

<table>
<thead>
<tr>
<th>Salary</th>
<th>[ 400 ; 600 ]</th>
<th>[ 600 ; 800 ]</th>
<th>[ 800 ; 1000 ]</th>
<th>[1000 ; 1200]</th>
<th>[1200 ; 1400 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>8</td>
<td>5</td>
<td>9</td>
<td>15</td>
<td>3</td>
</tr>
</tbody>
</table>

1) Calculate the mean of this distribution, and give a meaning of this value.
2) Calculate the standard deviation and the variance.

The following is the only part on statistics retrieved from the model test LSM4.

The following table gives the quantity of calcium (in mg) taken daily by everyone of a group of 35 persons.

<table>
<thead>
<tr>
<th>879</th>
<th>1096</th>
<th>701</th>
<th>986</th>
<th>828</th>
<th>1077</th>
<th>703</th>
</tr>
</thead>
<tbody>
<tr>
<td>555</td>
<td>422</td>
<td>997</td>
<td>473</td>
<td>702</td>
<td>508</td>
<td>530</td>
</tr>
<tr>
<td>513</td>
<td>720</td>
<td>944</td>
<td>673</td>
<td>574</td>
<td>707</td>
<td>864</td>
</tr>
<tr>
<td>1099</td>
<td>743</td>
<td>1025</td>
<td>655</td>
<td>1043</td>
<td>599</td>
<td>1008</td>
</tr>
<tr>
<td>705</td>
<td>380</td>
<td>387</td>
<td>542</td>
<td>893</td>
<td>1052</td>
<td>473</td>
</tr>
</tbody>
</table>

1) Organize these data in a table of classes of amplitude 100, starting with the class [350, 450].
2) Calculate the median of the distribution obtained in question 1), and interpret this value.
3) Calculate the mean average $\bar{x}$ and the standard deviation $\sigma$ of these data.
4) We assume that the quantity of calcium taken daily is normal if:
   i- $750 < \bar{x} < 850$
   ii- $\sigma < 100$ mg
   iii- at least 95% of the total frequency belong to the class $[\bar{x} - 2 \sigma, \bar{x} + 2 \sigma]$. 
What do you think of the life standard of these persons?

The two examples have the same structure. A group of data is given, and the questions that follow require similar procedural methods. Therefore, the test item in the national tests and the model tests align.

Probability

The topic probability is also classified under statistics and probability. Probability involves the study of real random variables and distribution functions. In addition, probability includes the study of probability of events such as P(A), P(A and B), P(A or B), and P(A / B) with a focus on conditional probability and dependent events. Table 11 in Appendix K presents the test items on probability as well as the exams where they occur. The test items in Table 11 in Appendix K go under the cognitive domains “knowing”, “applying”, and “reasoning”.

All national exams include a problem on probability, and according to table 8, the question “Determine the probability distribution of X” is the most recurrent one. In addition, almost all problems on probability tackle the concept of conditional probability. However, problems on probability appear only in two model tests and the concept of conditional probability is not addressed in any of them. Moreover, both model tests contain questions about a random variable which is binomially distributed. However, as previously mentioned, the concept of binomial distribution was cancelled from the curriculum of the grade 12 for the
Life Sciences track. Thus, it can be deduced that there is a content mis-match or dis-alignment between the national tests and the model tests under *probability*.

The following is an example of the *probability* parts retrieved from the national exam LH121.

A shop sells two types of earphones $E_1$ and $E_2$ and three types of batteries $B_1$, $B_2$ and $B_3$. During the promotion period, some items are placed in two baskets U and V. Basket U contains 15 earphones of type $E_1$ and 5 earphones of type $E_2$; Basket V contains 8 batteries of type $B_1$, 10 batteries of type $B_2$ and 7 batteries of type $B_3$.

**A**- A customer selects, at random, one item from each basket.

1) **Show that the probability of obtaining an earphone $E_1$ and a battery $B_1$ is equal to** $\frac{6}{25}$.

2) **Calculate the probability that an earphone $E_1$ is among the two selected items.**

3) **The shop announces the following prices:**

<table>
<thead>
<tr>
<th>Item</th>
<th>Earphone $E_1$</th>
<th>Earphone $E_2$</th>
<th>Battery $B_1$</th>
<th>Battery $B_2$</th>
<th>Battery $B_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price in LL</td>
<td>40 000</td>
<td>15 000</td>
<td>30 000</td>
<td>25 000</td>
<td>50 000</td>
</tr>
</tbody>
</table>

$X$ is the random variable equal to the amount paid by the customer for buying the two selected items.

a- **Prove that the probability $P(X = 65 000)$ is equal to $\frac{37}{100}$.**

b- **Determine the probability distribution of $X$.**

**B**- In this question, a customer selects, at random, an earphone from basket U and selects simultaneously and at random two batteries from basket V. Calculate the probability that the customer pays an amount less than or equal to 70 000LL.

The following is an example of the *probability* problems retrieved from the model test LHM2.

In a factory where we make shirts, we notice that:

4% of the shirts have colour defect (called “defect C”),

2% of the shirts have defect in size (called “defect T”)

The existence of one defect in a shirt is independent from the existence or not of the other defect.

1) a- **Determine the probability that a shirt has the two defects C and T.**

b- **Determine the probability that a shirt has, at least, one defect.**
2) Let $X$ be the random variable representing the number of shirts having at least one defect in a set of 80 shirts.

a- What is the law of $X$?

b- What is the probability that each shirt of the set has at least one defect?

c- What is the mathematical expectation of $X$? Give an interpretation of the value you found.

The following objectives under the topic *probability* were never addressed in any of the official or model tests:

5.2.3.2.i. → Determine the distribution function $F$ of one random variable.
5.2.3.2.ii. → Represent the function $F$.
5.2.3.2.iii. → Interpret graphically $F(a)$ for a real constant.

In conclusion, the qualitative analysis shows that the topic *circular functions* was never addressed in the model tests and official exams. Also, the topic *differential equations* was never addressed in the model tests. In addition, the topics *numbers, geometry, calculus,* and *probability,* occurred in all the official exams. Similarly, topics like *differential equations and statistics* were rarely addressed in the official exams.

In the national tests, many specific objectives under various topics were never addressed. These objectives are: binomial formula and Pascal’s triangle, first order linear differential equations, and distribution function. They belong to the topics *Literal and numerical calculation, differential equations,* and *probability* respectively.

The qualitative analysis of the content of the official exams controverts the general principles about the guidelines that are included in the Evaluation Guide (Document III referenced in Appendix A) for the selection of Math questions in the official exams in terms of content (see Appendix B), in particular, the guideline emphasizing that the Math problems in
the national tests will be chosen from all the math topics, and no topic is consistently neglected. In addition, the guideline stating that no topic will be always occurring in the official exams is not abided by.

Finally, it can be observed from table 1 in Appendix K that some topics were only addressed in the first years of administering the official exams. For example, the topic Statistics was addressed only twice in session-2 official exams in the years 2001 and 2003 and never been addressed afterwards. Another example is the topic differential equations which was only addressed in the first years and was never addressed after the year 2006. Thus the topics Statistics and differential equations are getting more and more neglected.

4.1.2 – Quantitative Analysis

Statistical tables are used, in this part of the study, to analyze the test items, as defined by Osta (2007), of the model tests and official exam tests according to the corresponding curriculum objectives, as well as the TIMSS Advanced 2008 cognitive domains that they measure. The data in Table Mod (showing the statistical data of the four model tests), Table OffEx (showing the statistical data of the 12 national exam tests), Table OffEx1-3 (showing statistical data for the 6 national tests from 2001 to 2003, sessions 1 and 2), Table OffEx10-12 (showing statistical data for the 6 national tests from 2010 to 2012, sessions 1 and 2), and Table OffEx1 (showing statistical data for the 6 session-1 official exams), and Table OffEx2 (showing statistical data for the 6 session-2 official exams) were converted into percentages out of the total number of test items in each category, to create a unified base for comparison.
Pearson Product-Moment correlation coefficients between the percentages are calculated, presented and discussed.

4.1.2.1 – Correlations Between Model Tests and National tests.

The four model tests and the 12 national tests include 77 and 385 test items respectively. The distribution of the test items as to their corresponding cognitive domains and the math topics they address is presented in Table 4.2. The data in Table 4.2 are extracted from Tables Mod and OffEx.

The model tests and the official exam tests, as shown in Table 4.2, assess in a balanced way the different topics of the math curriculum. However, some discrepancies are observed. The percentages of the test items in the national tests are distributed over nine topics while those in the model tests cover only eight topics. This may be interpreted by the low number of model tests relative to the number of national tests. In addition, more than half of the test items are assigned to the topic calculus (rational functions) (57.15 %) in the model tests but only (45.2 %) in the national tests. The reason for this discrepancy is that there exists a part on the topic calculus in every model test and national test; however, three out of the four model tests consist of three parts while all the official exams consist of four parts.

The highest percentage out of the test items in the national tests are allocated to the topic calculus. Next are classical study of geometry (19.56), probability (16.17 %), numbers (15.8 %), and statistics (2.07 %). Huge discrepancies in the percentages out of the test items in
the model tests and the national tests occur under the topic *definitions and representations* (42.68 % and 33.1 % respectively). However, similar percentages of the test items of all the other topics in the model tests and the official exams reflect an acceptable balance between these topics. For instance, the percentages out of the test items in the model tests and the official exams under *Classical study of geometry* are 14.61% and 19.56% respectively. The topic “*differential equations*” occur only in the national tests with a percentage of 0.5%.

Table 4.2

*Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests and the National Tests of the LS Track at Grade 12.*

<table>
<thead>
<tr>
<th>The Topics of the Math Curriculum of the LS Track at Grade 12</th>
<th>Sum of Model Tests</th>
<th>Sum of Official Exams</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K %</td>
<td>A %</td>
</tr>
<tr>
<td>1.2. Literal and numerical calculations</td>
<td>1.95</td>
<td>0</td>
</tr>
<tr>
<td>1.4. Numbers</td>
<td>2.38</td>
<td>2.71</td>
</tr>
<tr>
<td>2.1. Classical study</td>
<td>5.84</td>
<td>6.17</td>
</tr>
<tr>
<td>3.1. Definitions &amp; Representations</td>
<td>13.64</td>
<td>27.92</td>
</tr>
<tr>
<td>3.2. Continuity and differentiation</td>
<td>4.11</td>
<td>3.46</td>
</tr>
<tr>
<td>3.3. Integration</td>
<td>1.52</td>
<td>1.52</td>
</tr>
<tr>
<td>3.4. Differential equations</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5.1. Statistics</td>
<td>3.25</td>
<td>0.65</td>
</tr>
<tr>
<td>5.2. Probability</td>
<td>6.49</td>
<td>2.6</td>
</tr>
<tr>
<td>Total</td>
<td>39.18</td>
<td>45.03</td>
</tr>
</tbody>
</table>

K = Knowing  
A = Applying  
R = Reasoning

Regarding the cognitive domains, the percentages out of the test items in the model tests and the official exams presented in Table 1 reflect a mismatch. Nearly half of the test items address the cognitive domain “applying” (45.03 %) in the model tests while about 1/3 of the test items (34.43 %) in the official exams. Next is the cognitive domain “knowing” (39.18 %) in the model tests and (44.54 %) in the national tests. Last is the cognitive domain
“reasoning” (15.81 %) in the model tests and (20.81 %) in the official exam tests. Thus, the curriculum as reflected in the national tests emphasize the cognitive domain “knowing” over “applying” and “reasoning”, while in the model tests, the cognitive domain “applying” is emphasized over “knowing” and “reasoning”.

As for both math topics and cognitive domains, the topic definitions & representations of Calculus have around double percentage out of test items in “applying” than “knowing” in the official exams (27.92% and 13.64% respectively). Numbers is the only topic having the highest percentages out of test items at the cognitive domain “reasoning” in the model tests and in the national tests, while the topics Statistics and probability have the highest percentage out of test items at the cognitive domain “knowing” (6.49% and 8.42% respectively) in the model tests and the national tests. Last, the cognitive domains are equally emphasized in the topic “differential equations” in the national tests where the percentage out of test items in “knowing”, “applying”, and “reasoning” is the same (0.17%).

Pearson Product-Moment coefficient is used to find the correlation between the national tests and model tests when considering all specific objectives and the three cognitive domains. The data in the Tables Mod and OffEx are correlated cell by cell. The overall correlation is average (r = 0.50). This value of correlation suggests that some of the specific objectives are addressed in both the model tests and in the official exam tests at the same cognitive level. This mirrors the findings in the tables Mod and OffEX where many specific objectives under the topics Algebra and Calculus were addressed in the model tests and in the national tests at the same cognitive domains.
However, when comparing the math domains rather than the specific objectives, correlations were calculated between the respective numbers in Table 4.2.

The correlations of the test items, in terms of the cognitive domains and math domains, between the national tests of the years 2001-2003 and 2010-2012, and the model tests for the LS track at grade 12 are calculated and presented in Table 4.3. Pearson Product-Moment coefficient is used to calculate the correlations between the respective numbers in the columns of Table 4.2.

Table 4.3

*Correlations between the National Tests and the Model Tests for Grade 12 LS Track*

<table>
<thead>
<tr>
<th>Overall correlation</th>
<th>in terms of cognitive domains</th>
<th>in terms of math content</th>
</tr>
</thead>
<tbody>
<tr>
<td>NT &amp; MT</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.87</td>
<td>0.95</td>
<td>0.97</td>
</tr>
<tr>
<td>K</td>
<td>K : Knowing</td>
<td>Alg. : 0.96</td>
</tr>
<tr>
<td>A</td>
<td>A : Applying</td>
<td>Geo : 0.86</td>
</tr>
<tr>
<td>R</td>
<td>R : Reasoning</td>
<td>Cal. : 0.91</td>
</tr>
<tr>
<td>Alg.</td>
<td>Alg. : Algebra</td>
<td>S.P. : 0.91</td>
</tr>
<tr>
<td>Geo</td>
<td>Geo : Geometry</td>
<td></td>
</tr>
<tr>
<td>Calc. : Calculus (Numerical Functions)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.P. : Statistics &amp; Probability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NT &amp; MT : Correlation between the national tests (NT) and the model tests (MT)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As shown in Table 4.3, the overall correlation between the national tests and model tests is 0.87 which reflects a good alignment. Moreover, the correlations between the national tests and model tests in terms of the cognitive domains “knowing”, “applying”, and “reasoning” are 0.95, 0.95, and 0.97 respectively. The reason for the very high positive correlations is the correspondence in the content coverage under each cognitive domain. This
correspondence can be inferred from the percentages of test items associated to each cognitive domain under the specific topic in the model tests and the national exams, as shown in Table 4.2.

In terms of the math domains, Table 4.3 presents the following correlations:

- A very high positive correlation (r = 0.96) is noticed under the Algebra domain. The domain Algebra includes two sub-topics literal and numerical calculations and numbers. These two together constitute 13.32% of the model tests and 16.8% of the national tests. This correspondence in the percentages leads to a high correlation. Moreover the correlation is high because the test items address similar objectives under the same sub-topics at the same cognitive domain. This is reflected in the correlation that was calculated between the respective numbers in the Tables Mod and OffEx under the domain Algebra (0.74%), which shows that, not only the subtopics under Algebra are well correlated, but also the specific objectives under these subtopics.

- A high positive correlation (r = 0.86) is noticed under the Geometry domain. The domain Geometry includes one sub-topic classical study of geometry. It constitutes 14.61% of the model tests and 19.56% of the national tests. The percentages are quite similar, and the correlation is high. A second reason for this high correlation is that the percentages out the test items in classical study of geometry are similarly distributed over the cognitive domains. The percentages out of the items on the subtopic Geometry in the model test at cognitive domains “knowing”, “applying”, and “reasoning” are
5.84%, 6.17% and 2.60% respectively. In the national tests, the percentages are 9.26%, 6.76%, and 3.54%.

- A very high positive correlation \( r = 0.91 \) is also noticed under the *Calculus* domain. The domain *calculus* includes four sub-topics *definitions & representations, continuity and differentiation, integration and simple & differential equations*. These four together constitute 57.15% of the models tests and 45.2% of the national tests. Although there exist discrepancy in the percentages of *Calculus* between the model tests and the national tests, the discrepancy is the result of the difference of one value: the percentage out of the items on the subtopics *definitions & representations* in the cognitive domain “applying”. All other percentages out of the test items on the subtopics under *Calculus* at all cognitive domains are quite similar. This can be inferred from the value of the correlation that was calculated between the respective numbers in the Tables Mod and OffEx under the domain *Calculus* (0.57%), which shows that there is an average balance between the specific objectives under Calculus at the cognitive domains.

- A high positive correlation \( r = 0.91 \) is noticed under the *Statistics & Probability* domain. The domain *statistics & probability* includes two sub-topics *statistics* and *probability*. These two together constitute 14.94% of the model tests and 18.24% of the national tests. The percentages are quite similar, and the correlation is high. On the other hand, the value of the correlation that was calculated between the respective numbers in the Tables Mod and OffEx under the domain *statistics & probability* (-0.31%) shows that there is an imbalance between the specific objectives under
statistics & probability at the cognitive domains. The reason for the negative correlation is that the test items under statistics & probability address different objectives in the model tests and the national tests. As previously explained in the qualitative analysis section, the objectives of conditional probability, random variable, and total probability were only addressed in the national tests and didn’t occur in the model tests.

4.2.2.2 – Correlations Between National Tests of the Years 2001-2003 and 2010-2012 Respectively and Model Tests.

The 4 model tests, the 6 national tests of the years 2001-2003, and the 6 national tests of the years 2010-2012 include 77, 185, and 210 test items respectively. The distribution of the test items as to their corresponding cognitive domains and the math topics they address is presented in Table 4.4. The data in Table 4.4 are extracted from Tables Mod, OffEx1-3, and OffEx10-12.

Table 4.4 shows that the model tests, the national tests of the years 2001-2003, and the national tests of the years 2010-2012 assess the topics of the math curriculum in a different way. The percentages of the test items in the official exams of the years 2001-2003 are distributed over 9 topics while those in the national tests of the years 2010-2012 are distributed over 7 topics. Those in the model tests cover only 8 topics. More than half of the test items are
allocated to the topic *Calculus* (57.14 %) in the model tests, (43.78 %) in the official exams of the years 2001-2003, and (46.53 %) in the official exams of the years 2010-2012.

Table 4.4
*Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests, and the National Tests of the Years 2001-2003 and 2010-2012 of the LS Track at Grade 12*

<table>
<thead>
<tr>
<th>The Topics of the Math Curriculum of the LS Track at Grade 12</th>
<th>Sum of Model Tests</th>
<th>Sum of 2001-2003 Official Exams</th>
<th>Sum of 2010-2012 Official Exams</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K %</td>
<td>A %</td>
<td>R %</td>
</tr>
<tr>
<td>1.2. Literal and numerical calculations</td>
<td>1.95</td>
<td>0.66</td>
<td>0</td>
</tr>
<tr>
<td>1.4. Numbers</td>
<td>2.38</td>
<td>3.78</td>
<td>3</td>
</tr>
<tr>
<td>2.1. Classical study</td>
<td>5.84</td>
<td>8.29</td>
<td>0.81</td>
</tr>
<tr>
<td>3.1. Definitions &amp; Representations</td>
<td>13.64</td>
<td>18.24</td>
<td>2.16</td>
</tr>
<tr>
<td>3.2. Continuity and differentiation</td>
<td>4.11</td>
<td>2.43</td>
<td>1.89</td>
</tr>
<tr>
<td>3.3. Integration</td>
<td>1.52</td>
<td>1.58</td>
<td>0.9</td>
</tr>
<tr>
<td>3.4. Differential equations</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
</tr>
<tr>
<td>5.1. Statistics</td>
<td>3.25</td>
<td>3.51</td>
<td>0.81</td>
</tr>
<tr>
<td>5.2. Probability</td>
<td>6.49</td>
<td>3.51</td>
<td>0.81</td>
</tr>
<tr>
<td>Total</td>
<td>39.18</td>
<td>47.95</td>
<td>20.28</td>
</tr>
</tbody>
</table>

K = Knowing
A = Applying
R = Reasoning

In the national tests of the years 2001-2003, the topic *Calculus* gets the highest percentage out of the test items. Next is *Statistics and Probability* (19.86 %), then *Algebra* follows (9.31 %) and *Geometry* (17.3%). However, in the national tests of the years 2010-2012, the pattern of the math domains changes, *Calculus* still has the highest percentage out of the test items; *Geometry* is second (21.65%), then *Probability and Statistics* (17.03%) and *Algebra* (14.8%). Furthermore, close correspondence in the percentages out of the test items in the national tests of the years 2001-2003, and the national tests of the years 2010-2012 occur under the topics *definitions & representations* (32.16 % and 33.96 % respectively), *continuity and differentiation* (6.48 % and 6.65 %, respectively), integration (4.06% and 5.92%)
respectively), and probability (15.54% and 17.03% respectively). The topic “differential equations” occur only in the years 2001-2003 with a percentage of 1.08%.

Regarding the cognitive domains, the percentages out of the test items in the model tests, the national tests of the years 2001-2003, and the national tests of the years 2010-2012 in Table 4.4 reflect a mismatch. Nearly half of the test items address the cognitive domain “knowing” (47.95%) in the national tests of the years 2001-2003, while about 1/3 of the test items (39.18%) in the model tests and (40.96%) in the national tests of the years 2010-2012. Next is the cognitive domain “applying” (45.03%) in the model tests, (29.87%) in the national tests of the years 2001-2003, and (38.77%) in the national tests of the years 2010-2012. Last is the cognitive domain “reasoning” (15.81%) in the model tests, (21.61%) in the national tests of the years 2001-2003, and (20.28%) in the national tests of the years 2010-2012. It can be inferred from the above results that, in studying the evolution of the official exams, more emphasis is given to the cognitive domain “applying” at the expense of the cognitive domain “knowing”. The curriculum, as demonstrated in the official exams of the years 2001-2003, and the national tests of the years 2010-2012 emphasize the cognitive domain “knowing” over “applying” and “reasoning”. However, the model tests emphasize the cognitive domain “applying” over “knowing” and “reasoning”.

As to both math topics and cognitive domains, numbers is the only topic having a higher percentage out of test items in “reasoning” than “knowing” and “applying” in the official exams of the years 2001-2003, in the national tests of the years 2010-2012, and in the model tests. The topic definitions & representations have a very low percentage out of test
items at the cognitive domain “*reasoning*” as compared to the percentages at the cognitive domains “knowing” and “applying” in the model tests, in the national tests of the years 2001-2003 and the years 2010-2012. Taking into consideration that the topic *definitions & representations* has a high percentage out of the test items in the said tests, and that high grades are allocated to it, this topic is considered the easiest for students. In addition, students can achieve the highest grades on this topic compared to other topics.

Correlations were also calculated between the corresponding numbers in the Tables Mod, OffEx 1-3, and OffEx 10-12. These correlations were found using Pearson Product-Moment coefficient under Microsoft Excel by correlating data in each of the two tables cell by cell. When considering all objectives and the three cognitive domains, the overall correlation between the first years of the national tests and the last years is average \( r = 0.53 \), between the first years of the national tests and model tests is \( r=0.45 \), between the last years of the national tests and the model tests is \( r= 0.36 \). These values of correlation suggest that some of the specific objectives are addressed in both the model tests and in the national tests at the same cognitive level. This mirrors the findings in the tables Mod, OffEX 1-3, and OffEX 10-12. Taking in consideration that two topics, *Statistics* and *Differential equations* are never addressed in the national tests of the years 2010-2012, the correlation between the first years of the national tests and the last years \( r = 0.53 \) suggests still a higher alignment between the specific objectives of the other topics.

However, when comparing the math domains rather than the specific objectives, correlations were calculated between the respective numbers in Table 4.4.
As shown in Table 4.5, the correlation between the national tests of the years 2001-2003 and the model tests is $r = 0.78$, while the correlation between the official exams of the years 2010-2012 and the model tests is $r = 0.90$. Therefore, the alignment between the national tests and the model tests is better over the years. The reason for not having a perfect alignment is the mismatch in the content coverage and discrepancy in the percentages out of the tests items in the national tests of the years 2001-2003 and the years 2010-2012 as compared to the model tests. However, the correlation between the national tests of the years 2001-2003 and those of the years 2010-2012 is $r = 0.88$. This high positive correlation shows that the national tests of the years 2001-2003 and those of the years 2010-2012 may be viewed as consistent.

Table 4.5

*Correlations Between the National Tests of the Years 2001-2003 and the National Tests of the Years 2010-2012 Respectively, between them and the Model Tests for Grade 12 LS Track*

<table>
<thead>
<tr>
<th></th>
<th>Overall Correlation</th>
<th>in terms of cognitive domains</th>
<th>in terms of math content</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>K</td>
<td>A</td>
</tr>
<tr>
<td>NT1-3 &amp; MT</td>
<td>0.78</td>
<td>0.98</td>
<td>0.94</td>
</tr>
<tr>
<td>NT10-12 &amp; MT</td>
<td>0.90</td>
<td>0.89</td>
<td>0.96</td>
</tr>
<tr>
<td>NT1-3&amp;NT10-12</td>
<td>0.88</td>
<td>0.93</td>
<td>0.99</td>
</tr>
</tbody>
</table>

K : Knowing  
A : Applying  
R : Reasoning  
Alg. : Algebra  
Geo : Geometry  
Calc. : Calculus (Numerical Functions)  
S.P. : Statistics & Probability  
NT1-3 & MT : Correlation between the national tests of the years 2001-2003 (NT1-3) and the model tests (MT)
In terms of the cognitive domains “knowing”, “applying”, and “reasoning”, refer to Table 4.5, the correlations between the national tests of the years 2001-2003 and the model tests are 0.98, 0.94, and 0.96 respectively; whereas, the correlations between the national tests of the years 2010-2012 and the model tests are 0.89, 0.96, 0.87 respectively. On the other hand, the correlation between the national tests of the years 2001-2003 and those of the years 2010-2012 are 0.93, 0.99, and 0.80 respectively. These high positive correlations in terms of the cognitive domains “knowing”, “applying”, and “reasoning” show that the national tests of the years 2001-2003 and those of the years 2010-2012 are consistent with each other and with the model tests.

In terms of the math content algebra, geometry, calculus, and statistics and probability, refer to Table 4.5, the correlation between the national tests of the years 2001-2003 and the model tests are 0.96, 0.84, 0.80, and 0.98 respectively; whereas, the correlation between the national tests of the years 2010-2012 and the model tests are 0.86, 0.88, 0.97, 0.78 respectively. Last, the correlations between the national tests of the years 2001-2003 and the national tests of the years 2010-2012 are 0.85, 0.75, 0.90, and 0.85 respectively. These high positive correlations in terms of the math content algebra, geometry, calculus, and statistics and probability show that the official exams of the years 2001-2003 and those of the years 2010-2012 are also consistent with each other and with the model tests.
4.2.2.3 – Correlations Between National Tests of Session-1 and Session-2 Respectively and Model Tests.

The four model tests, the six session-1 national tests and the six session-2 national tests of the years 2001-2003 and 2010-2012 include 77, 190, and 196 test items respectively. The distribution of the test items as to their corresponding cognitive domains and the math topics they address is presented in Table 4.6. The data in Table 4.6 are extracted from Tables Mod, OffEx1, and OffEx2.

Table 4.6 shows that the model tests, the session-1 official exams, and the session-2 official exams assess differently the topics of the math curriculum. Although the percentages of the test items in the session-1 national tests, in the session-2 national tests, and in the model tests are distributed over the same number of topics (8), yet, they occur on different topics. For instance, no test items in the session-1 national tests are assigned to the topic Statistics. In addition, the topic differential equation did not occur in any model test or on any session-2 national test. More than half of the test items are assigned to the topic Calculus (57.15 %) in the model tests, (43.68 %) in the session-1 national tests, and (46.68 %) in the session-2 national tests.

Table 4.6
Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests, and the Session-1 and Session-2 National tests of the LS Track at Grade 12
In the session-1 national tests, the topic *Calculus* gets the highest percentage out of the test items. Next is *Geometry* (21.23%), then *Statistics & Probability* (17.85%), and *Algebra* (16.63%). In the session-2 national tests, the topic *Calculus* gets the highest percentage out of the test items. Next is *Statistics & Probability* (18.93%), then *Geometry* (17.85%), and *Algebra* (16.54%).

It is noticed that the percentages out of the test items in the session-1 national tests and the session-2 national tests are similar, but different than those in the model tests. For instance, out of the test items in the session-1 national tests, in the session-2 national tests, and in the model tests, the percentages under the topics *numbers* (11.37 %, 16.05 %, and 15.56 % respectively), *classical study of geometry* (14.61 %, 21.32 %, and 17.85 % respectively), *definitions and representations* (42.86 %, 34.48 %, and 31.76 % respectively), and *probability* (11.04 %, 17.85 %, and 14.84 % respectively).

Regarding the cognitive domains, the percentages out of the test items in the model tests, in the session-1 national tests, and in the session-2 national tests in Table 4.6 reflect a mismatch. Nearly half of the test items address the cognitive domain “applying” (45.03 %) in
the model tests, while about 1/3 of the test items (36.62 %) in the session-1 national tests and (32.44 %) in the session-2 national tests. Next is the cognitive domain “reasoning” (15.81 %) in the model tests, (21.23 %) in the session-1 national tests, and (20.61 %) in the session-2 national tests. Last is the cognitive domain “knowing” (39.18 %) in the model tests, (41.63 %) in the session-1 national tests, and (46.95 %) in the session-2 national tests. It can be inferred from the above results that, in comparing the session-1 and session-2 national tests, more emphasis is given to the cognitive domain “knowing” in the session-2 national tests at the expense of the cognitive domain “applying”. The curriculum, as demonstrated in the session-1 national tests and the session-2 national tests emphasize the cognitive domain “knowing” over “applying” and “reasoning”. However, the model tests emphasize the cognitive domain “applying” over “knowing” and “reasoning”.

As to both math topics and cognitive domains, Table 4.6 shows high similarities between the percentages out of the test items is session-1 national tests and session-2 national tests. Moreover, continuity and differentiation and numbers have a higher percentage out of test items in “reasoning” than “knowing” and “applying” in the session-1 national tests. Whereas, numbers is the only topics in the session-2 national tests having a higher percentage out of test items in “reasoning” than “knowing” and “applying”.

Correlations were also calculated between the corresponding numbers in the Tables Mod, OffEx 1, and OffEx 2. These correlations were found using Pearson Product-Moment coefficient under Microsoft Excel by correlating data in each of the two tables cell by cell. When considering all objectives and the three cognitive domains, the overall correlation between the session-1 national tests and the session-2 national tests is slightly greater than
average (r = 0.65), between the session-1 national tests and model tests is (r=0.45), between the session-2 national tests and the model tests is (r= 0.44). These values of correlation suggest that some of the specific objectives are addressed in both the model tests and in the national tests at the same cognitive level. This mirrors the findings in the tables Mod, OffEX 1, and OffEX 2. In addition, these correlations support the previous result that there is a high similarity between the percentages of the test items of the session-1 and session-2 national tests in most of the topics at the cognitive domains.

However, when comparing the math domains rather than the specific objectives, correlations were calculated between the respective numbers in Table 4.6.

As shown in Table 4.7, the correlation between the session-1 national tests and the model tests is 0.85; whereas, the correlation between the session-2 national tests and the model tests 0.87. Therefore, session-2 national tests are slightly better alignment with the model tests. However, the correlation is r = 0.95 between the session-1 and session-2 national tests. This shows that the session-1 and session-2 national tests are highly aligned.

Table 4.7

*Correlations Between the National Tests of Session-1 and the National Tests of Session-2 Respectively, between them and the Model Tests for Grade 12 LS Track*

<table>
<thead>
<tr>
<th></th>
<th>Overall Correlation</th>
<th>in terms of cognitive domains</th>
<th>in terms of math content</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>K</td>
<td>A</td>
</tr>
<tr>
<td>NT1 &amp; MT</td>
<td>0.85</td>
<td>0.92</td>
<td>0.93</td>
</tr>
<tr>
<td>NT2 &amp; MT</td>
<td>0.87</td>
<td>0.97</td>
<td>0.96</td>
</tr>
<tr>
<td>NT1 &amp; NT2</td>
<td>0.95</td>
<td>0.96</td>
<td>0.97</td>
</tr>
</tbody>
</table>
K : Knowing
A : Applying
R : Reasoning
Alg. : Algebra
Geo : Geometry
Calc. : Calculus (Numerical Functions)
S.P. : Statistics & Probability
NT1 & MT : Correlation between the national tests of session-1 (OE1) and the model tests (MT)
NT2 & MT : Correlation between the national tests of session-2 (OE2) and the model tests (MT)
NT1 & NT2 : Correlation between the national tests of session-1 (OE1) and those of session-2 (OE2)

In terms of the cognitive domains “knowing”, “applying”, and “reasoning”, refer to Table 4.7, the correlations between the session-1 national tests and the model tests are 0.92, 0.93, and 0.94 respectively; whereas, the correlations between the second-2 national tests and the model tests are 0.97, 0.96, 0.98 respectively. On the other hand, the correlation between the session-1 national tests and the session-2 national tests are 0.96, 0.97, and 0.95 respectively. These high positive correlations in terms of the cognitive domains “knowing”, “applying”, and “reasoning” show that the session-1 national tests and the session-2 national tests are consistent with each other and with the model tests.

In terms of the math content algebra, geometry, calculus, and statistics and probability, refer to Table 4.4, the correlation between the session-1 official exams and the model tests are 0.96, 0.84, 0.80, and 0.98 respectively; whereas, the correlation between the session-2 official exams and the model tests are 0.86, 0.88, 0.97, 0.78 respectively. Last, the correlations between the session-1 official exams and the session-2 official exams are 0.85, 0.75, 0.90, and 0.85 respectively. These high positive correlations in terms of the math content algebra, geometry, calculus, and statistics and probability show that the session-1 official
exams and the session-2 official exams are also consistent with each other and with the model tests.

The quantitative results of this study are in agreement with the qualitative results. Each test includes four sections. Every section covers a domain: algebra (16.8 %), classical study of geometry (19.56 %), calculus (45.2 %), or statistics & probability (18.24 %). The highest percentage out of the test items in the national tests are allocated to the topic calculus (45.2 %). Next are classical study of geometry (19.56), probability (16.17 %), numbers (15.8 %), and statistics (2.07 %). Huge discrepancies in the percentages out of the test items in the model tests and the national tests occur under the topic definitions and representations (42.68 % and 33.1 % respectively). However, similar percentages of the test items of all the other topics in the model tests and the official exams reflect an acceptable balance between these topics. In addition, the curriculum as reflected in the national tests emphasize the cognitive domain “knowing” (44.54 %) over “applying” (34.43 %) and “reasoning” (20.81 %), while in the model tests, the cognitive domain “applying” (45.03 %) is emphasized over “knowing” (39.18 %) and “reasoning”(15.81 %).
CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 – Introduction

This paper aims to study the extent to which the reformed math curriculum is aligned with the Lebanese national math tests for the LS section. In other words, the purpose of this paper is to inspect the alignment between the assessed and the intended curriculum. In addition, this paper investigates the evolution of the national tests throughout the years of implementation and examines any differences between session 1 and session 2 of the national tests for the LS section, by studying their alignment with the reformed mathematics curriculum.

Techniques of content analysis are used to study the alignment between the national math tests and the national reformed Lebanese math curriculum.

The qualitative section examines the structure and content of the curriculum, model tests, and national tests. The qualitative analysis for the model tests and national tests includes the topics as well as the test items considered.

The model tests and national tests are quantitatively analyzed and compared using Pearson Product-Moment coefficient. Using statistical tables for each model test and national test, the test items, as defined by Osta (2007), are analyzed, using double-entry statistical
tables, as to their corresponding curriculum objectives, and to the cognitive domains they address, according to the TIMSS Advanced 2008 framework.

The analysis showed that the Lebanese national tests for the LS section are characterized by the following:

- All the national tests have the same structure. Each test includes four sections. Every section covers a domain: *algebra* (16.8 %), *Classical study* of geometry (19.56 %), *calculus* (45.2 %), or *statistics & probability* (18.24 %).

- The test items in general are repetitive. However, some test items under “Geometry” and “Numbers” are distributed over all the national tests in a way that there is no obvious pattern of occurrence. In addition, different structures of questions are sometimes used in addressing the specific objectives of Rational functions.

- The math domain *Trigonometry* is never tackled in the national tests. In addition, many objectives listed by the curriculum are also never addressed. The objectives are: binomial formula and Pascal’s triangle, first order linear differential equations, and distribution function. They belong to the topics *Literal and numerical calculation*, *differential equations*, and *probability* respectively.

- The math domain *Calculus* occurs on each national test and is allocated the highest grades in comparison to other domains. However, the topics *differential equations*, *literal and numerical calculations*, and *Statistics* are seldom tackled in the national tests (0.5 %, 1.00 % and 2.07% respectively).

- The national tests emphasize the cognitive domains differently. “Knowing” has the main emphasis (44.54 % out of the test items) then “applying” (34.43 % out of the test items)
and “reasoning” (20.81 %). However, compared to the national tests for the LH track (Sleiman, 2013), 8.41% of test items on reasoning, the emphasis on reasoning in the national tests for the LS track is significantly higher.

- Although some test items under the two domains Algebra and Calculus are integrated in one question in the model tests, this integration did not occur on any of the national tests. Instead, the national tests seem to relate each math domain to specific cognitive domains. The test items on Numbers stress the cognitive domain “reasoning” while the topics definitions & representations, geometry, and probability have a low percentage out of test items at the cognitive domain “reasoning” as compared to the percentages at the cognitive domains “knowing” and “applying”.

- In the session-2 national tests, more emphasis is put on the cognitive domain “knowing” at the expense of the cognitive domain “applying”.

- Some topics are neglected. Topics such as Statistics and Differential equations were only addressed in the first years of administering the national tests.

- The problems in the national tests are not presented in the context of real life situations.

The only exceptions are the problems under Probability.

This research complements the work done by Osta (2007) and Sleiman (2013) about evaluating the reformed Lebanese curriculum. The results of this paper are in agreement with the results of the previous research work; in particular, a steady structure of tests and a stereotyped style of questions that cover a narrow part of the curriculum which reduces it to a “mini-curriculum”. These are standard characteristics of a fragile alignment between the
intended and the assessed curriculum which leads to a “teaching to the test” practice by teachers, resulting in “drill and practice” approaches.

In Lebanon, school tests at the Brevet level and the Secondary level are meant to prepare students for national tests. The stable structure of the national tests and the stereotyped questions provide teachers with a model of the contents that are probably going to be included in the national test. Consequently, teaching would emphasize those contents and neglect the rest of the curriculum. As a result AFL practices will be ruled out leading to spoon feeding instead of promoting independent thinking.

The influence of national tests on teaching and learning has been discussed in the literature. McGaw (2006) emphasized teachers’ tendency to stress the ‘testable’ parts of the curriculum and to ignore the rest. According to Johnson (2007), prioritizing parts of the curriculum is a side effect of summative tests, and thus reallocation occurs “when teachers report shifting instructional time to focus more on the material emphasized by an important test” (p.10). Moreover, Students may only study contents that they label as relevant to these tests. As a consequence, a discrepancy between the intended curriculum and the learned curriculum will be noticed (Tan, 2011). School teachers should adopt AFL techniques since, according to Tan (2011), AFL practices can enrich learning, motivate, and sustain students’ satisfaction of learning inside and outside classrooms, and prepare students to be life-long learners. In this sense, assessment can be considered as a comprehensive base within which the different forms of learning can be bound (Tan, 2011).
The qualitative analysis of the national Lebanese tests reveals that these tests do not help the implementation of the reform of teaching Mathematics that is mentioned in the Lebanese math curriculum. The reform of teaching Mathematics, as written in the introduction of the math curriculum (referenced as Document I in Appendix A), is to be achieved through: 1) the formulation of objectives: The focus is on mental activities and mathematical reasoning. Students are provided with the opportunity to experience the spirit of scientific research in real-life situations. 2) Remodeling contents: subjects are chosen according to their practical interests. For instance, Statistics is used in adapting to socioeconomic problems. The emphasis is on the use of appropriate technological devices such as calculators and possibly computers which will help the learning process. 3) Method of teaching: the practice of Math should start from real life situations to highlight the relation between Math and everyday life.

However, in the Lebanese national tests, the use of calculators is limited to simple calculation that does not significantly enhance the development of solutions. The use of calculators in this sense might lead students to overdependence on them and performing lesser mental calculations in classrooms. Moreover, many topics that are known for their practical interests are either cancelled from the Math curriculum (e.g. system of linear equations \((m \times n)\), binomial law) or are rarely addressed in the national tests (Statistics and differential equations). Finally, as revealed by the qualitative analysis of the national tests, many sections are not addressed in real life situations in general contradicting the method of teaching mentioned in the reform. The only exception is the part on Probability and Statistics.
The findings of this study support the suggestions by Schmidt and Prawat (2006) that curriculum coherence is not an essential consequence of a nationally controlled curriculum. According to William (1996), the questions of national assessments should represent the whole intended Math domain and should be altered from one year to another to insure validity. Therefore, enduring curriculum assessment is essential for curriculum development. Only when the assessed, intended, and enacted curriculum are aligned, students are given an opportunity to learn (Martone & Sireci, 2009).

The Evaluation Guide (Document III referenced in Appendix A) contains a section titled: “General principles about the guidelines and the way of developing the national test questions in mathematics for the general secondary school certificate” (see Appendix B). This section includes the criteria for the selection of questions in all grade 12 tracks. The findings will be discussed based on these criteria. The national test should:

1. “Abide by the general and specific objective”. However, the specific objective to “formulate a problem based on situations studied in other sciences” is not addressed in any of the national tests, as very few of the national test items are presented in a real-life context.

2. “Balance between three hierarchies of learning: acquisition, application, and analysis”. Table 4.2 shows an imbalance among the cognitive domains in the national tests. “Knowing” has the main emphasis (44.54 % out of the test items) then “applying” (34.43 % out of the test items) and “reasoning” (20.81 %).
3. “Consider competences from all the domains and should include questions that test the competences from all aspects”. However, there are little chances that the national tests address the competencies comprehensively, considering the fact that each section of the national tests is associated with one domain: Algebra, Geometry, Calculus, and Probability and Statistics, and that some parts of the curriculum are neglected.

4. “Not follow a specific pattern, neglect any part of the curriculum, or consider continuously a certain topic”. This criterion is not respected in the national tests, as many topics are neglected while others are always adopted. In addition, the national tests follow a steady structure.

5. “Be clearly communicated to escape multiple interpretations”. Although this paper does not aim to investigate this criterion, it can be inferred from reading the Lebanese LS track national tests that they are well written and clearly communicated.

5.2 – Conclusions

This paper aimed to answer the following research questions: (a) are the Lebanese secondary-level national math tests for the LS track aligned with the national reformed curriculum over the years 2001-2012? (b) is there any improvement in the alignment of the national tests from the years 2001-2003 to the years 2010-2012?, and (c) are there differences
between the tests in session-1 and session-2 for the LS track in terms of content and cognitive domain addressed?

The three research questions will be discussed based on the results of this study.

5.2.1 – Research Question 1

Are the Lebanese secondary-level national math tests for the LS track aligned with the national reformed curriculum over the years 2001-2012?

The national tests and the model tests are highly correlated ($r = 0.87$) when considering the four math domains (Algebra, Geometry, Calculus, and Probability and Statistics) and the three cognitive domains (knowing, applying, and reasoning). However, when considering the specific objectives under each domain and the cognitive domains, the correlation is average ($r = 0.50$). Moreover, the curriculum as reflected in the national tests emphasize the cognitive domain “knowing” over “applying” and “reasoning” (with respective percentages 44.54 %, 34.43 %, and 20.81%), while in the model tests, the cognitive domain “applying” is emphasized over “knowing” and “reasoning” (with respective percentages 45.03 %, 39.18%, and 15.81 %).

Many reasons might have contributed to such average alignment between the Lebanese secondary-level national math tests for the LS track and the national reformed curriculum. First, both the model tests and the national tests do not assess the entire curriculum. The
domain Trigonometry is not addressed in both tests. Also, the topic differential equations is never addressed in the model tests. Similarly, topics like differential equations and statistics were rarely addressed in the national tests. Moreover, when considering the specific objectives under each domain and the cognitive domains, the value of correlation \((r = 0.5)\) suggests that some of the specific objectives are addressed in both the model tests and in the national tests at the same cognitive level. This can be considered a direct consequence of the use of analogous test items in both the model tests and the national tests.

Second, while the test items under some domains (Algebra) address similar objectives under the same sub-topics, the test items under other domains (Statistics & Probability) address different objectives in the model tests and the national tests. As previously explained in the qualitative analysis section, the objectives of conditional probability, random variable, and total probability were only addressed in the national tests and didn’t occur in the model.

Finally, the 4 model tests and the 12 national tests that are considered in the study include 77 and 385 test items respectively. This huge difference between the numbers of the test items weakens the alignment between the two tests.

5.2.2 – Research Question 2

Is there any improvement in the alignment of the national tests from the years 2001-2003 to the years 2010-2012?
The correlation between the national tests of the years 2001-2003 and those of the years 2010-2012 when considering the math domains and the cognitive domains is $r = 0.88$. This high positive correlation shows that the national tests of the years 2001-2003 and those of the years 2010-2012 may be viewed as consistent. In addition, when considering the specific objectives and the cognitive domains, the correlation is average ($r = 0.53$). A major reason for this value of correlation is revealed by the qualitative analysis; a stereotyped style of questions that occurs under almost all the domains and the topic of the math curriculum. However, it can be inferred from Table 4.4 that some topics (Statistics and differential equations) are getting more and more neglected as they are addressed only in the first years of the national tests.

Moreover, the qualitative analysis in this study shows an improved alignment between the national tests and the model tests over the years. As shown in Table 4.5, the correlation between the national tests of the years 2001-2003 and the model tests is $r = 0.78$, while the correlation between the national tests of the years 2010-2012 and the model tests is $r = 0.90$.

The improvement of alignment between the model tests and the national tests over the years is driven by more emphasis on the cognitive domain “applying” at the expense of the cognitive domain “knowing”. As a consequence, the distribution of the cognitive domains in the last years of the national tests is similar to that of the model tests. In addition, another reason for the improvement of the alignment is that some topics (differential equations) is addressed in both the last years and the model tests, but not addressed in the first years of the national tests.

Moreover, some specific objectives under the math topics are addressed only in the model tests but not in the first years (or vice versa). For instance, under Geometry, the specific
objective “Calculate the area of parallelogram” is addressed in the first years only but not in the model tests, while the specific objectives “use the mixed product to calculate the volume of a parallelepiped and that of a tetrahedron” and “know that the mixed product of three vectors is zero if, and only if, these vectors are coplanar” are addressed only in the model tests but not in the first years.

5.2.3 – Research Question 3
Are there differences between the tests in session-1 and session-2 for the LS track in terms of content and cognitive domain addressed?

The session-1 and session-2 national tests are highly aligned with each other. The correlation between them is $r = 0.95$; whereas, the correlation between session-1 national tests and the model tests ($r = 0.85$) is similar to that between the session-2 national tests and the model tests ($r = 0.87$) and thus the session-1 tests and the session-2 tests are similarly highly aligned with the model tests. One reason contributing to these values of alignment is that the percentages out of the test items in the session-1 national tests and the session-2 national tests are similar, but different than those in the model tests. In addition, another major reason is revealed by the qualitative analysis; a stereotyped style of questions that occurs under almost all the domains and the topic of the math curriculum on the session-1 and session-2 tests.

However, some differences are noticed in session-1 and session-2 national tests; more emphasis is given to the cognitive domain “knowing” in the session-2 national tests at the expense of the cognitive domain “applying”. Moreover, as shown in Table 4.6, session-1 tests
and session-2 assess different topics: The topic *statistics* occurs only in session-2 tests (LS012 and LS032). This agrees with the finding of the study conducted by Sleiman (2013) that “some topics that are not frequently addressed in the national tests are added in session-2 national tests just to send a message to teachers that all concepts are important and must be covered in classroom instruction”.

**5.3 – Recommendations**

The findings of this study reveal that the national tests have a stable structure and include stereotyped questions that target a narrow part of the curriculum, which leads to “teaching to the test” practice. In addition, the cognitive domain “knowing” is the most emphasized, then “applying” and “reasoning” follow. Johnson (2007) argues that even if “teaching to the test” is unavoidable in national assessment, the test structure, context, and the test item styles should be well considered.

Therefore, it is recommended that the design and content of the national tests be revised to include: 1) Different types of questions: short response questions and extended response questions that involve the integration of more than one math topic. 2) Questions written in the context of real life situations and integrated with other sciences. 3) Higher frequencies of non-routine questions tackling the cognitive domain “reasoning”. 4) Different types of tests where the graphical calculator is allowed in one paper, and is not permitted in the other. 5) Questions written in an increasing order of difficulty.
5.4 – Limitations of the Study

This paper has two limitations. The first limitation is the difference between the number of model tests and the number of national tests studied. Four model tests and twelve national tests are considered. The 4 model tests have lesser chances than the twelve national tests to cover the entire curriculum from different aspects.

The second limitation of this paper is the difference between applying and reasoning cognitive domains. Applying includes solving routine problems, which are problems similar to those students are likely to have encountered in class. Reasoning includes solving non routine problems. It is a limitation to know which problems are solved and which are not solved in class.

5.5 – Recommended Future Research

Recommendations for future research include:

1. A study comparing the alignment between the Lebanese secondary-level national math tests and the reformed math curriculum for the LS track to the alignment
between the International Baccalaureate Math High Level Diploma Program tests and the IB Math curriculum at the HL level.

2. A study whose aim would be to examine the influence of the use of graphical display calculators (GDC) in IB exams on alignment. The use of GDC in Math assessments will allow the inclusion of reasoning level and critical thinking test items, which require justification, graph interpretation and use of Math tools in different contexts, while limiting the purely procedural test items.

3. A study of the alignment between the Lebanese secondary national tests and classroom instruction. In other words, a study of the alignment between the intended curriculum and the enacted curriculum. The aim is to study the influence of the national tests on classroom instruction.
REFERENCES


APPENDIX A

The References of the Curriculum Documents

**Document I**


**Document II**


**Document III**

Document IV


مناهج التعليم العام وأهدافها. تعليم رقم 35 / 98. تفاصيل محتوى منهج مادة الرياضيات. السنة الثانية من كل حلقة ومرحلة. لبنان: وزارة التربية الوطنية والشباب والرياضة، والمركز التربوي للبحوث والإنماء.


Document V


**APPENDIX B**

**General Principles about the Guidelines and the Way of Developing the Official Exam Questions in Mathematics for the General Secondary School Certificate**

Retrieved from:


**مبادئ عامة حول أصول وطريقة وضع أسئلة الامتحانات الرسمية في الرياضيات للشهادة الثانوية العامة**

تهدف مسابقة الرياضيات في الامتحانات الرسمية إلى قياس مدى اكتساب التلاميذ للكفايات العائدة لهذه المرحلة (راجع لوائح الكفايات لمادة الرياضيات العائدة لصفوف الثالث ثانوي بفروعها الأربعة).

**الأسس المتبعة لاختيار الأسئلة في المضمون**

تبين أن تراعي أسئلة الرياضيات الأسّة التالية:

- التقييد بأهداف المادة (العامة والخاصة) وذلك من خلال احترام نظام التقييم الجديد وفلسفته (دليل المعلم للتقييم).
- التوازن بين مستويات المعرفة الأساسية الثلاثة (الاكتساب – التطبيق – التحليل).
- اختيار الكفايات من كافة المجالات وضمن الاختيار أسئلة تقييم كفايات متعددة تغطي عدة مواضيع من المناهج.
- الابتعاد عن نمط معين للاختيار، وذلك من خلال عدم إهمال أي جزء من المناهج بشكل دائم (معنى آلا يُستبعد دائم موضوع ما من أسئلة الاختبار)، وكذلك عدم اعتماد حتمية وجود موضوع ما في كافة الاختبارات.
- العناية بصياغة الأسئلة ووضوحها منعاً لكل التباس.

106

في الشكل

يتكون اختبار الرياضيات من عدة مسائل إلزامية (ليس هناك شرط على عدد المسائل).

تأتي الأسئلة في كراش (على الأقل أربع صفحات A3 مطوية).

ينبغي أن يكون الاختبار سهل القراءة لجهة اختيار نوع البنط (Font) وحجمه، والمسافات بين الأسطر والهوامش العامة أو الداخلية.

ترقم المسائل بالترقيم الروماني (I, II, III, etc.) (1, 2, etc.). ترقم الأسئلة للمسألة الواحدة بالأرقام العربية (1, 2, etc.). وترقم الأسئلة الفرعية بالأحرف اللاتينية (a- b- c- etc.) (أو internal).

تذكر علامة كل مسألة من المسائل الواردة في الاختبار دون تحديد العلامة لكل سؤال في المسألة الواحدة.

تخصص الصفحة الأولى من كراش أسئلة الاختبار لتوصيف الاختبار وتتضمن بعض الإرشادات العامة (انظر التفصيل لاحقا).

تتضمن الصفحة الأولى المعلومات التالية:

- الكتابة الرسمية (الجمهورية اللبنانية – وزارة التربية .. الخ).
- اسم الشهادة الرسمي.
- المادة.
- اللغة.
- عدد المسائل.
- مدة الاختبار.
- تعداد الأدوات اللازمة (أدوات الرسم الهندسي – آلة حاسبة غير قابلة للبرمجة أو لاختزان المعلومات أو لرسم البيانات – الخ).

APPENDIX C

Model Test 1 (LSM1)

Retrieved from:

الجمهورية اللبنانية
وزارة التربية والتعليم العالي

الشهادة الثانوية العامة

فرع علوم الحياة

اختبار الرياضيات

(الشيء)

المادة: 

عدد الأسئلة: ثلاث

مدة الاختبار: ساعتان

إرشادات عامة:

- يجب أن يكون مع المرشح: أدوات الرسم الهندسي - ألة حاسبة غير قابلة للبرمجة أو الحفظ
- المعلومات أو رسم البيانات.
- يجب أن يستخدم المرشح قلم أبيض (سهل أو دافئ) أزرق أو أسود بشكل عام. ويحق للمرشح استخدام أقلام متينة أو سجائر للرسم أو الإيضاح.
- يستحسن أن يقرأ المرشح كافة أسلة الاختبر قبل البدء بالجواب.
- يستطيع المرشح الإجابة بالتزامن الذي نسبته (دون الاستلام والالتزام المطلوب في الاختبار).
- عند إجابة الالتحام بال쯤ة خاصة للكل (المادة الرياضيات) والرجوع، بذلك ينصب المرشح بالكتابة بشكل واضح والالتزام في الإمكان، مع تجنب التشطيب.
I. (5 points)
In a computer club of a school, there are four boys, numbered from 1 to 4, and five girls numbered from 1 to 5. The manager of the club wishes to form a committee of three members.
1) How many committees of boys can be formed?
   Deduce the possible number of committees having at least one girl.
2) How many committees having only one boy, and a member numbered 2 can be formed?

II. (9 points)
Let \( f \) be the function defined by \( f(x) = \ln x - mx \) where \( m \) is a non-zero real number.
Let \( C_m \) be the graph of \( f \) in an orthonormal system.
1) Construct on the same sketch the graphs \( C_1 \) and \( C_4 \).
2) Calculate the area of the domain limited by \( C_1 \), the lines \( y = -x, x = 1 \) and \( x = e \).
3) For which values of \( m \), the function \( f \) is strictly monoton increasing?
4) For which values of \( m \), \( C_m \) has a maximum or a minimum?
5) In this question, we suppose that \( m > 0 \).
   a) Study, according to the values of \( m \), the sign of \( -1 - \ln m \).
   b) Use the variations of \( f \) to discuss, according to \( m \), the number of solutions of the equation \( f(x) = 0 \).
6) Find the coordinates of a point of \( C_m \) at which the tangent to \( C_m \) contains the origin.

III. (6 points)
In the orthonormal space \((O, \vec{i}, \vec{j}, \vec{k})\), we consider the points \( A(1,0,0), B(1,1,1), C(2,3,0) \) and \( D(2,0,3) \).
1) Verify that \( ABCD \) is a tetrahedron and calculate its volume.
2) Prove that \( (AB) \) is orthogonal to \( (CD) \).
3) Find the equation of the plane \( (ADC) \) and the coordinates of the point \( H \) orthogonal projection of \( B \) on the plane \( (ADC) \).
4) \( I \) is the midpoint of \( [CD] \). Prove analytically that \( A, H \) and \( I \) are collinear and give a geometric interpretation.
## Elements of solutions and marking scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Short answers</th>
<th>Note</th>
</tr>
</thead>
</table>
| L1       | Number of committees with no girls: $C_2^4 = 4$ \((I)\).  
Number of committees of at least one girl: $C_2^4 - C_1^4 = 80$ \((I)\). | 2 |
| L2       | We expect the student to distinguish between two cases:  
Case 1. The number of committees having the boy numbered 2 is $C_2^4 = 6$ (the girl numbered 2 is rejected);  
Case 2. The number of committees having the girl numbered 2 is $C_3^3 \times C_1^4 = 12$ (we must choose one boy, other than the number 2, and one girl among the remaining four).  
Total number of committees: 18. | 3 |
| II.1     | Argument based on the calculation of the derivative function: $m < 0$. | 1 |
| II.2     | Expected answer: $m > 0$. | 1 |
| II.3     | a. Expected answer: $m < 1/e$, \((I)\)  
b. Table of variations that shows the extreme values: \[
\begin{array}{c}
\frac{1}{m} - 1 - \ln m
\end{array}
\] \((I)\).  
Conclusion: a unique solution for $m = 1/e$, two solutions for $m < 1/e$, and no solutions for $m > 1/e$ \((I)\). | 3 |
| II.4     | Expected answer: $(e, 1/me)$. | 1 |
| II.5     | Construction of the representative curves of the two functions. | 2 |
| II.6     | Calculation of an integral. | 1 |
| III.1    | The student is expected to show that the four points are not on the same plane \((I)\). Calculation of the volume: it is equal to 1 \((I)\). | 2 |
| III.2    | Simple calculation of scalar product. | 1 |
| III.3    | Determination of the equation of a plane containing three points.  
Expected equation: $3x - y - z - 3 = 0$ \((I)\).  
Expected answer for $H$: $\begin{array}{c}
\frac{8}{11} \quad \frac{12}{11} \\
11 \quad 11 \quad 11
\end{array}$ \((I)\). | 2 |
| III.4    | Simple argument of geometric orthogonality and the use of Pythagoras allow to prove that $ADC$ is isosceles and to conclude that the three points are on the same line. | 1 |
## APPENDIX D

Session-1 Official Exam 2012 (LS121)

<table>
<thead>
<tr>
<th>الدورة العادية للعام 2012</th>
<th>امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة</th>
<th>وزارة التربية والتعليم العالي</th>
<th>المديرية العامة للتربية</th>
<th>دائرة الامتحانات</th>
</tr>
</thead>
<tbody>
<tr>
<td>اسم:</td>
<td>مسابقة في مادة الرياضيات</td>
<td>عدد المسائل: أربع</td>
<td></td>
<td></td>
</tr>
<tr>
<td>الرقم:</td>
<td>المدة ساعتان</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ملاحظات: 
- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

### I- (4 points)

In the space referred to a direct orthonormal system \((O; \vec{i}, \vec{j}, \vec{k})\), consider the following points:
\(A(4;0;1), B(2;1;2), C(2;0;3)\) and \(E(3;-1;0)\).

1) a- Write an equation of the plane \((P)\) determined by \(A, B,\) and \(C\).
   
   b- Show that \(A\) is the orthogonal projection of \(E\) on \((P)\).

2) a- Show that triangle \(ABC\) is right.
   
   b- Calculate the area of the triangle \(ABC\).
   
   c- Calculate the volume of the tetrahedron \(EABC\).

3) \((Q)\) is the plane with equation \(x - 2y - 2z - 2 = 0\).
   
   Show that \((Q)\) passes through \(A\) and is perpendicular to \((BE)\).

4) a- Write a system of parametric equations of the line \((BC)\).
   
   b- Let \(M\) be a variable point on \((BC)\). Prove that the distance from \(M\) to \((Q)\) remains constant as \(M\) moves on \((BC)\).

### II- (4 points)

A shop sells two types of earphones \(E_1\) and \(E_2\) and three types of batteries \(B_1, B_2\) and \(B_3\).

During the promotion period, some items are placed in two baskets \(U\) and \(V\).

Basket \(U\) contains 15 earphones of type \(E_1\) and 5 earphones of type \(E_2\);

Basket \(V\) contains 8 batteries of type \(B_1\), 10 batteries of type \(B_2\) and 7 batteries of type \(B_3\).

A- A customer selects, at random, one item from each basket.
1) Show that the probability of obtaining an earphone $E_1$ and a battery $B_1$ is equal to $\frac{6}{25}$.

2) Calculate the probability that an earphone $E_1$ is among the two selected items.

3) The shop announces the following prices:

<table>
<thead>
<tr>
<th>Item</th>
<th>Earphone $E_1$</th>
<th>Earphone $E_2$</th>
<th>Battery $B_1$</th>
<th>Battery $B_2$</th>
<th>Battery $B_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price in LL</td>
<td>4000</td>
<td>1500</td>
<td>3000</td>
<td>2500</td>
<td>5000</td>
</tr>
</tbody>
</table>

$X$ is the random variable equal to the amount paid by the customer for buying the two selected items.

a- Prove that the probability $P(X = 65,000)$ is equal to $\frac{37}{100}$.

b- Determine the probability distribution of $X$.

**B-** In this question, a customer selects, at random, an earphone from basket $U$ and simultaneously selects two batteries from basket $V$. Calculate the probability that the customer pays an amount less than or equal to 70,000 LL.

### III- (4 points)

The complex plane is referred to a direct orthonormal system $(O; \overrightarrow{u}, \overrightarrow{v})$.

For every point $M$ with affix $z$ ($z \neq 0$), we associate the point $M'$ with affix $z'$ such that $z' = \frac{2}{\bar{z}}$.

1) Let $z = r e^{i\theta}$ ($r > 0$), write $z'$ in exponential form.

2) a- Show that $OM \times OM' = 2$.

b- If $z = z'$, prove that $M$ moves on a circle $(C)$ whose center and radius are to be determined.

3) Let $z = 1 + iy$ where $y$ is a real number.

   a- Prove that $|z' - 1| = 1$.

   b- As $y$ varies, show that $M'$ moves on a circle $(C')$ whose center and radius are to be determined.

### IV- (8 points)
Consider the function $f$ defined over $\mathbb{R}$ by $f(x) = (x+1)^2 e^{-x}$ and denote by $(C)$ its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Determine $\lim_{x \to -\infty} f(x)$ and calculate $f(-2)$.
   
   b- Determine $\lim_{x \to +\infty} f(x)$ and deduce an asymptote to $(C)$.

2) Show that $f'(x) = (1-x^2)e^{-x}$ and set up the table of variations of $f$.

3) The line $(d)$ with equation $y = x$ intersects $(C)$ at a point with abscissa $\alpha$.

Verify that $1.4 < \alpha < 1.5$.

4) Draw $(d)$ and $(C)$.

5) Let $F$ be the function defined on $\mathbb{R}$ by $F(x) = (px^2 + qx + r)e^{-x}$.

   a- Calculate $p$, $q$ and $r$ so that $F$ is an antiderivative of $f$.

   b- Calculate the area of the region bounded by $(C)$, the axis of abscissas and the two lines with equations $x = 0$ and $x = 1$.

6) The function $f$ has over $[1; +\infty[$ an inverse function $h$. Determine the domain of definition of $h$ and draw its representative curve in the same system as $(C)$. 
Retrieved from:


http://timss.bc.edu/PDF/TIMSS_Advanced_AF.pdf

Advanced Mathematics
Cognitive Domains

To respond correctly to TIMSS test items, students need to be familiar with the mathematics content being assessed, but they also need to draw on a range of cognitive skills. Describing these skills is an essential aspect of developing the assessment of achievement in Advanced Mathematics because this ensures that the important cognitive goals of school mathematics education are surveyed across the content domains already defined.

A central aim of school mathematics programs at all levels is to have students understand the subject matter of the courses they are studying. Understanding a mathematics topic consists of having the ability to operate successfully in three cognitive domains. The first domain, knowing, covers the facts, procedures, and concepts students need to know, while the second, applying, focuses on the ability of students to make use of this knowledge to select or create models and solve problems. The third domain, reasoning, goes beyond the solution of routine problems to encompass the ability to use analytical skills, generalize, and apply mathematics to unfamiliar or complex contexts.
Each content domain will include items developed to address each of the three cognitive domains. For example, the algebra domain will include knowing, applying, and reasoning items, as will the other content domains.

**Knowing**

Facility in using mathematics or reasoning about mathematical situations depends on mathematical knowledge and familiarity with mathematical concepts. The more relevant knowledge a student is able to recall and the wider the range of concepts he or she has understood, the greater the potential for engaging in a wide range of problem-solving situations and for developing mathematical understanding.

Without access to a knowledge base that enables easy recall of the language and basic facts and conventions of number, symbolic representation, and spatial relations, students would find purposeful mathematical thinking impossible. *Facts* encompass the factual knowledge that provides the basic language of mathematics, and the essential mathematical facts and properties that form the foundation for mathematical thought.

*Procedures* form a bridge between more basic knowledge and the use of mathematics for solving routine problems, especially those encountered by many people in their daily lives. In essence, a fluent use of procedures entails recall of sets of actions and how to carry them out. Students need to be efficient and accurate in using a variety of computational procedures and tools. They need to see that particular procedures can be used to solve entire classes of problems, not just individual problems.

Knowledge of *concepts* enables students to make connections between elements of knowledge that, at best, would otherwise be retained as isolated facts. It allows them to make extensions beyond their existing knowledge, judge the validity of mathematical statements and methods, and create mathematical representations.
Behaviors Included in the Knowing Domain

Recall
Recall definitions, terminology, notation, mathematical conventions, number properties, geometric properties.

Recognize
Recognize entities that are mathematically equivalent (e.g., different representations of the same function or relation).

Compute
Carry out algorithmic procedures (e.g., determining derivatives of polynomial functions, solving a simple equation).

Retrieve
Retrieve information from graphs, tables, or other sources.

Applying

Problem solving is a central goal, and often a means, of teaching mathematics, and hence this and supporting skills (e.g., select, represent, model) feature prominently in the domain of applying knowledge. In items aligned with this domain, students need to apply knowledge of mathematical facts, skills, procedures, and concepts to create representations and solve problems. Representation of ideas forms the core of mathematical thinking and communication, and the ability to create equivalent representations is fundamental to success in the subject.

Problem settings for items in the applying domain are more routine than those aligned with the reasoning domain and will typically have been standard in classroom exercises designed to provide practice in particular methods or techniques. Some of these problems will have been expressed in words that set the problem situation in a quasi-real context. Though they range in difficulty, each of these types of “textbook” problems is expected to be sufficiently familiar to students that they will essentially involve selecting and applying learned procedures.

Problems may be set in real-life situations or may be concerned with purely mathematical questions involving, for example, numeric or algebraic expressions, functions, equations, geometric figures, or statistical data sets. Therefore, problem solving is included not only in the applying domain, with emphasis on the more familiar and routine tasks, but also in the reasoning domain.

Behaviors Included in the Applying Domain
<table>
<thead>
<tr>
<th>Select</th>
<th>Select an efficient/appropriate method or strategy for solving a problem where there is a commonly used method of solution.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Represent</td>
<td>Generate alternative equivalent representations for a given mathematical entity, relationship, or set of information.</td>
</tr>
<tr>
<td>Model</td>
<td>Generate an appropriate model such as an equation or diagram for solving a routine problem.</td>
</tr>
<tr>
<td>Solve Routine Problems</td>
<td>Solve routine problems, (i.e., problems similar to those students are likely to have encountered in class). For example, differentiate a polynomial function, use geometric properties to solve problems.</td>
</tr>
</tbody>
</table>

**Reasoning**

*Reasoning* mathematically involves the capacity for logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to non-routine problems. Non-routine problems are problems that are very likely to be unfamiliar to students. They make cognitive demands over and above those needed for solution of routine problems, even when the knowledge and skills required for their solution have been learned. Non-routine problems may be purely mathematical or may have real-life settings. Both types of items involve transfer of knowledge and skills to new situations, and interactions among reasoning skills are usually a feature. Problems requiring reasoning may do so in different ways. Reasoning may be involved because of the novelty of the context or the complexity of the situation, or because any solution to the problem must involve several steps, perhaps drawing on knowledge and understanding from different areas of mathematics.

Even though many of the behaviors listed within the reasoning domain are those that may be drawn on in thinking about and solving novel or complex problems, each by itself represents a valuable outcome of mathematics education, with the potential to influence learners’ thinking more generally. For example, reasoning involves the ability to observe and make conjectures. It also involves making logical deductions based on specific assumptions and rules, and justifying results.
### Behaviors Included in the Reasoning Domain

<table>
<thead>
<tr>
<th><strong>Analyze</strong></th>
<th>Investigate given information, and select the mathematical facts necessary to solve a particular problem. Determine and describe or use relationships between variables or objects in mathematical situations. Make valid inferences from given information.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Generalize</strong></td>
<td>Extend the domain to which the result of mathematical thinking and problem solving is applicable by restating results in more general and more widely applicable terms.</td>
</tr>
<tr>
<td><strong>Synthesize/Integrate</strong></td>
<td>Combine (various) mathematical procedures to establish results, results to produce a further result. Make connections between elements of knowledge and related representations, and make seen related mathematical ideas.</td>
</tr>
<tr>
<td><strong>Justify</strong></td>
<td>Provide a justification for the truth or falsity of a statement by reference to mathematical results or properties.</td>
</tr>
<tr>
<td><strong>Solve Non-routine Problems</strong></td>
<td>Solve problems set in mathematical or real-life contexts where students are unlikely to have encountered similar items, and apply mathematical procedures in unfamiliar or complex contexts.</td>
</tr>
</tbody>
</table>
APPENDIX F

Coding the Details of Contents of the Lebanese Reformed Math Curriculum
For the LS track at the Secondary School Level

Retrieved from:


Codes *Math Curriculum for the LH track at the Secondary School Level*

<table>
<thead>
<tr>
<th>1</th>
<th>ALGEBRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Foundations</td>
</tr>
<tr>
<td>1.1.1</td>
<td>Binary operations</td>
</tr>
<tr>
<td>1.1.1.1</td>
<td>Identify a binary operation.</td>
</tr>
<tr>
<td>1.1.1.1.i</td>
<td>→ Identify a binary operation on a set $E$ as a rule which associates to every pair $(x,y) \in E \times E$ an element $z \in E$.</td>
</tr>
<tr>
<td>1.1.1.2</td>
<td>Recognize the properties of a binary operation.</td>
</tr>
<tr>
<td>1.1.1.2.i</td>
<td>→ Identify an associative binary operation.</td>
</tr>
<tr>
<td>1.1.1.2.ii</td>
<td>→ Identify a commutative binary operation.</td>
</tr>
<tr>
<td>1.1.1.3</td>
<td>Recognize certain particular elements.</td>
</tr>
<tr>
<td>1.1.1.3.i</td>
<td>→ Identify a neutral element (an identity element) for a binary operation.</td>
</tr>
<tr>
<td>1.1.1.3.ii</td>
<td>→ Identify the symmetric element of an element for a binary operation.</td>
</tr>
<tr>
<td>1.1.2</td>
<td>Structure of group</td>
</tr>
<tr>
<td>1.1.2.1</td>
<td>Define a group and give examples of groups</td>
</tr>
<tr>
<td>1.1.2.1.i</td>
<td>→ Identify an Abelian group</td>
</tr>
<tr>
<td>1.1.2.1.ii</td>
<td>→ Identify a group.</td>
</tr>
<tr>
<td>1.2</td>
<td>Literal and numerical calculations</td>
</tr>
<tr>
<td>1.2.1</td>
<td>Combinations: definition, notation, binomial formula</td>
</tr>
<tr>
<td>1.2.1.1</td>
<td>Identify a combination of elements of a finite set</td>
</tr>
<tr>
<td>1.2.1.1.i</td>
<td>→ Identify a combination of $p$ elements of a set of $n$ elements ($p \leq n$) as a part of this set formed of $p$ elements</td>
</tr>
<tr>
<td>1.2.1.2</td>
<td>Calculate the number of combinations of $p$ elements of a set of $n$ elements ($p \leq n$)</td>
</tr>
<tr>
<td>1.2.1.2.i</td>
<td>→ Determine, in simple cases, all the combinations of $p$ elements of a set of $n$ elements ($p \leq n$)</td>
</tr>
</tbody>
</table>

120
1.2.1.3. Construct the Pascal’s triangle
1.2.1.4. Know and use the binomial formula
   → Know and use the formula giving the number nCp of all
   combinations of p elements of a set of n elements (p ≤ n)
1.2.1.4.i. Know and use the binomial formula for expanding (a+b)^n
1.2.1.4.ii. Model situations by combinations
1.2.1.4.iii. Know and use the binomial formula for expanding (a+b)^n
1.2.1.4.iv. Know and use the formula nCp = (n-1)Cp + (n-1) C (p-1)

1.3. Equations & Inequalities
   System of linear equations (m×n): definition, elementary operations on the
   rows, Gauss’ method

1.3.1. Identify a linear system (m×n)
1.3.1.2. Reduce a linear system (m×n) by successive applications of elementary
   operations
   → Apply an elementary operation on the equations of a linear system
   and know that it transforms it into an equivalent system
1.3.1.3. Solve a linear system (m×n) by the Gauss method
   → Recognize a solution of a linear system
   → Classify the linear systems into impossible systems, indeterminate
   systems, and determinate systems.
   → Recognize an impossible reduced linear system
   → Recognize a reduced linear system possessing a unique solution
   → Recognize a reduced linear system possessing an infinity of
   solutions and identify in this case the rank and the unknowns of the system
   → Solve a reduced linear system

1.4 Numbers
1.4.1. Modulus and argument of a complex number. Properties
   Calculate and Interpret geometrically the modulus (absolute value) and
   argument (amplitude) of a complex number
   → Calculate the modulus of a complex number written in an algebraic
   form
1.4.1.1. Know and use the formulas relative to the modulus of a complex numbers: mod(z)≥ 0 , mod(z) is a real number , [mod(z) = 0] ..
1.4.1.2. Know and use the following properties relative to the modulus of a
   complex number z
   → Know and use the following properties relative to the argument of a
   non-zero complex numbers: arg(-z)= π + arg(z) (2π)…..
1.4.2. Trigonometric and exponential form of a complex number
1.4.2.1. Write a complex number in the trigonometric form
   → Write a non-zero complex number z, given in algebraic form, in the
   trigonometric form z = r(cosθ +isinθ) where r, θ are real numbers, r>0
1.4.1.2. Write a complex number in the exponential form

1.4.1.2.i. Use the notation $e^{i\theta} = \cos \theta + i\sin \theta$

1.4.1.2.ii. Write a non-zero complex number $z$, given in trigonometric form, in the exponential form $z = re^{i\theta}$

1.4.1.3. Pass from one form of a complex number to another

1.4.1.3.i. Write a non-zero complex number, given in trigonometric form, in the algebraic form.

1.4.1.3.ii. Write a non-zero complex number, given in exponential form, in the trigonometric form.

### Geometric interpretation of addition, of multiplication of complex numbers and of the passage to the conjugate

1.4.3. Interpret geometrically the passage to the conjugate.

1.4.3.1. Construct the point of affix of $z$ conjugate from that of affix $z$

1.4.3.1.ii. Construct the point of affix $-z$ from that of affix $z$

1.4.3.1.iii. Construct the vector of affix $-z$ from that of affix $z$

1.4.1.2. Interpret geometrically the addition of two complex numbers

1.4.1.2.i. Know that the vector of affix $z + z'$ is the sum of vectors of affixes $z$ and $z'$

1.4.1.2.ii. Construct the vector of affix $z + z'$ from the vectors of affixes $z$ and $z'$

1.4.1.2.iii. Know that the affix of vector $AB$ is the complex number $z(b)-z(a)$

1.4.1.2.iv. Know that $AB = abs(z(b)-z(a))$

1.4.1.3. Interpret geometrically the multiplication of two complex numbers

1.4.1.3.i. Use a rotation and a homothety of center $O$ to construct the vector of affix $zz'$ from the vectors of affixes $z$ and $z'$

### De Moivre's formula. Applications

1.4.4. Know and use De Moivre's formula.

1.4.4.1. Know and use the formulas $\cos \theta = 1/2 (e^{i\theta} + e^{-i\theta})$

1.4.4.1.ii. Calculate $\cos n\theta$ and $\sin n\theta$ as a function of $\cos \theta$ and $\sin \theta$

1.4.4.2. Linearize simple trigonometric polynomials

1.4.4.2.i. Linearize $(\cos \theta)^n$, $(\sin \theta)^n$ and $(\cos \theta)^m$. $(\sin \theta)^n$

### 2 GEOMETRY

2.1. Classical study

2.1.1. Components of the vector product. Mixed product

2.1.1.1. Determine the components of the vector product of two vectors in a direct orthonormal system

2.1.1.1.i. Know and use the expressions of the components of the vector product $V \cdot V'$ or $V \times V'$ of the two vectors $V(X,Y,Z)$ and $V'(X',Y',Z')$

2.1.1.1.ii. Use the vector product to calculate the area of a parallelogram and that of a triangle.

2.1.1.1.iii. Know that the vector product of two vectors is zero, and only if, these two vectors are collinear.

2.1.1.2. Determine the mixed product of three vectors
2.1.1.2.i. → Recognize the mixed product of three vectors

2.1.1.2.ii. → Determine the anlytic expression of the mixed product in a direct orthonormal system

2.1.1.2.iii. → Use the mixed product to calculate the volume of a parallelepiped and that of a tetrahedron.

2.1.1.2.iv. → Know that the mixed product of three vectors is zero if, and only if, these vectors are coplanar.

2.1.2. Equation of a plane and a stright line in space

2.1.2.1. Determine the cartesian equation of a plane and a line defined by geometric elements in an orthonormal system.

2.1.2.1.i. → Recognize the equation $ux + vy + wz + r = 0$ as that of a plane perpendicular to the non-zero vector $V (u, v, w)$

2.1.2.1.ii. → Determine an equation of the plane passing through a given point and perpendicular to a non-zero vector.

2.1.2.1.iii. → Determine an equation of a plane passing through three non-collinear points.

2.1.2.1.iv. → Determine an equation of a plane passing through a given point and parallel to two non-parallel given directions.

2.1.2.1.v. → Know that the line of non-zero direction vector $V(a, b, c)$ and passing through a point $A(x_0, y_0, z_0)$ is the set of points $M(x, y, z)$ verifying the system of parametric equations: $x = at + x_0$, $y = bt + y_0$, $z = ct + z_0$ where $t$ ia a real parameter.

2.1.2.1.vi. → Determine a system of parametric equations of a line passing through two given points.

Additional Show that a given point lies in a plane

Additional Show that a line passes through a given point

Additional Show that a line lies in a plane

Additional Determine an equation of plane passing through a point and a line.

Additional Determine an equation of plane passing through 2 points and perpendicular to a plane.

Additional Determine an equation of plane containing 2 lines.

Additional Determine an equation of plane passing through a point and parallel to a plane.

2.1.3. Orthogonality of two straight lines, of a straight line and a plane; perpendicular planes

2.1.3.1. Characterize the orthogonality of two lines, of a line and a plane and of two planes, knowing their equations, in an orthonormal system.

2.1.3.1.i. → Know that two lines of respective direction vectors $V(a, b, c)$ and $V'(a', b', c')$ are orthogonal if, and only if, $aa' + bb' + cc' = 0$

2.1.3.1.ii. → Know that a line of a direction vector $V$ and a plane of normal vector $V'$ are orthogonal if, and only if, $V$ and $V'$ are collinear.

2.1.3.1.iii. → Know that two planes of respective normal vectors $V(u, v, w)$ and $V'(u', v', w')$ are orthogonal if, and only if, $uu' + vv' + ww' = 0$

2.1.4. Parallelism of straight lines and planes

2.1.4.1. Study the relative positions of two planes, two lines and of a plane and a line, knowing their equations, in an orthonormal system.
→ Know that two lines of respective direction vectors $V$ and $V'$ are parallel (or confounded) if, and only if, $V$ and $V'$ are collinear.

2.1.4.1.ii. → Know that a line of a direction vector $V$ and a plane of normal vector $V'$ are parallel if, and only if, $V$ and $V'$ are orthogonal

2.1.4.1.iii. → Know that two planes of respective normal vectors $V$ and $V'$ are parallel (or confounded) if, and only if, $V$ and $V'$ are collinear

2.1.4.1.iv. → Determine the system of parametric equations of the line of intersection of two secant planes

2.1.4.1.v. → Determine the intersection of two secant lines.

2.1.4.1.vi. → Determine the intersection of a line and a plane

Additional

2.1.5. → Determine the system of parametric equations of the line of intersection of two secant planes

Distance from a point to a plane, to a straight line.

2.1.5.1. → Know and use the relation $d = \text{abs}(ux0 + vy0 + wz0 + r)/\text{sqr}(u^2 + v^2 + w^2)$ expressing $d$ from a point $A(x0,y0,z0)$ to the plane $ux+vy+wz+r = 0$

2.1.5.1.i. → Calculate the distance from a point to a plane

2.1.5.1.ii. → Calculate the distance between two lines

Additional

3.1. Definitions & Representations

3.1.1. Inverse functions

3.1.1.1. → Determine the composite functions of two given functions.

3.1.1.1.i. → Recognize and calculate the composite function of two functions

3.1.1.2. → Characterize the functions having an inverse function.

3.1.1.2.i. → Recognize the reciprocal function $f^{-1}$ of a continuous and strictly monotonous function $f$

3.1.1.2.ii. → Know that the reciprocal function $f^{-1}$ of $f$ exists only if $f$ is continuus and strictly monotonous

3.1.1.3. → Compare graphically the graphs of a function and its inverse

3.1.1.3.i. → Determine the domain of definition of a reciprocal function

3.1.1.3.ii. → Know that a function and its reciprocal have the same sence of variation.

3.1.1.3.iii. → Calculate, if possible, the explicit expression of the reciprocal function.

3.1.1.3.iv. → Know that the graphs of a function and its reciprocal are symmetric to each other with respect to the first bisector of the orthonormal system

3.1.2. Inverse trigonometric functions

3.1.2.1. → Study the functions $\text{Arcsin}$, $\text{Arccos}$ and $\text{Arctan}$.

3.1.2.1.i. → Recognize the inverse function of the sine function over $[-\pi/2, \pi/2]$ and represent it graphically.

3.1.2.1.ii. → Recognize the inverse function of the cosine function over $[0, \pi]$ and represent it graphically.

3.1.2.1.iii. → Recognize the inverse function of the stangent function over $]-\pi/2, \pi]$ and represent it graphically.
3.1.3. **Natural (Naperian) logarithmic function. Logarithmic function to the base a**

3.1.3.1. Study and represent graphically the natural logarithmic function \( \ln \).
   → Recognize the domain of definition, variation and graph of the natural logarithmic function.
   → Know and use the properties of the natural logarithmic function: \( a \) and \( b \) are two strictly positive real numbers. \( \ln(ab) = \ln a + \ln b \)
   \( \ln(a/b) = \ln a - \ln b \), \( \ln \sqrt{a} = \frac{1}{2} \ln a \)

3.1.3.1.i. → Recognize the domain of definition, variation and graph of the natural logarithmic function.
3.1.3.1.ii. \( \ln \)ln(a/b)=lna-lnb, \( \ln \sqrt{a} = \frac{1}{2} \ln a \)
3.1.3.1.iii. → Know and use the properties of the natural logarithmic function: \( a \) and \( b \) are two strictly positive real numbers. \( \ln(ab) = \ln a + \ln b \)

3.1.3.1.iv. → Characterize the number \( e \)
   → Recognize the following limits: \( \lim \ln x (x \to 0^+) \), \( \lim \ln x (x \to +\infty) \), \( \lim \ln x/x (x \to 0) \)

3.1.3.2. Differentiate functions of the form \( \ln(u) \) and calculate the primitives of functions of the form \( u/u' \) where \( u \) is a function

3.1.3.2.i. → Recognize the derivative of \( \ln u \) where \( u \) is a function of \( x \) and a primitive of \( u'/u \) with \( u \neq 0 \).

3.1.3.3. Know the relation which links the function \( \ln \) to the logarithmic function to base \( a \) (\( a > 0 \) and \( a \neq 1 \)) and deduce the properties of the latter.

3.1.3.3.i. → Know that \( \log_a(x) = \frac{\ln x}{\ln a} \) with \( a > 0 \) and \( a \neq 1 \)
3.1.3.3.ii. → Know that the function \( \log_a \) is strictly increasing for \( a > 1 \) and strictly decreasing for \( 1 > a > 0 \).

3.1.3.3.iii. → Solve equations and inequalities that include the logarithmic function.

3.1.3.4. Compare the increases of the functions \( \ln \), \( e^x \), and \( x^\alpha \)

3.1.3.4.i. → Solve equations and inequalities that including logarithmic and
exponential functions.

<table>
<thead>
<tr>
<th>3.2.</th>
<th>Continuity and differentiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2.1.</td>
<td>Image of a closed interval by a continuous function</td>
</tr>
<tr>
<td>3.2.1.1.</td>
<td>Characterize the image of a closed interval by a continuous function</td>
</tr>
<tr>
<td>3.2.1.1.i.</td>
<td>Know that the image of an interval by a continuous function is an interval of the same nature</td>
</tr>
<tr>
<td>3.2.1.1.ii.</td>
<td>Know the fact that a continuous function on a closed interval reaches a maximum and a minimum on this interval and that it takes every intermediate value between the two extremes (theorem of intermediate values)</td>
</tr>
<tr>
<td>3.2.1.2.</td>
<td>Locate a root for a continuous function on a closed interval and justify the existence of this root.</td>
</tr>
<tr>
<td>3.2.1.2.i.</td>
<td>Know that if a function $f$ is continuous and strictly monotonous on an interval $I$, it defines a bijection of $I$ on $f(I)$</td>
</tr>
<tr>
<td>3.2.1.2.ii.</td>
<td>Know that if a function $f$ is continuous on the interval $[a,b]$ with $f(a)f(b) \leq 0$, it possesses at least one root in $[a,b]$</td>
</tr>
<tr>
<td>3.2.1.2.iii.</td>
<td>Know that if a function $f$ is continuous and strictly monotonous on an interval $[a,b]$ with $f(a)f(b) \leq 0$, it possesses one only root in $[a,b]$.</td>
</tr>
<tr>
<td>3.2.2.</td>
<td>Derivatives of composite functions</td>
</tr>
<tr>
<td>3.2.2.1.</td>
<td>Differentiate a composite function.</td>
</tr>
<tr>
<td>3.2.2.1.i.</td>
<td>Recognize and calculate the derivative of a composite function at a point.</td>
</tr>
<tr>
<td>3.2.2.1.ii.</td>
<td>Recognize and calculate the derivative of a composite function of two functions on an interval.</td>
</tr>
<tr>
<td>3.2.3.</td>
<td>Derivatives of an inverse function</td>
</tr>
<tr>
<td>3.2.3.1.</td>
<td>Differentiate an inverse function</td>
</tr>
<tr>
<td>3.2.3.1.i.</td>
<td>Use the formula $[f^{-1}]'(y_0) = 1/ f'(x_0)$, with $y_0 = f(x_0)$</td>
</tr>
<tr>
<td>3.2.3.1.ii.</td>
<td>Recognize the derivative of an inverse function on an interval.</td>
</tr>
<tr>
<td>3.2.4.</td>
<td>Second derivative, successive derivatives.</td>
</tr>
<tr>
<td>3.2.4.1.</td>
<td>Calculate the second derivative and the successive derivatives of a function.</td>
</tr>
<tr>
<td>3.2.4.1.i.</td>
<td>Calculate the second derivative of a function at a point and on an interval</td>
</tr>
<tr>
<td>3.2.4.1.ii.</td>
<td>Calculate the successive derivatives of a function at a point and on an interval.</td>
</tr>
<tr>
<td>Additional</td>
<td>Prove a point to be a point of inflection</td>
</tr>
<tr>
<td>Additional</td>
<td>Find the point of inflection</td>
</tr>
<tr>
<td>3.2.5.</td>
<td>L'Hopital's rule</td>
</tr>
<tr>
<td>3.2.5.1.</td>
<td>Use L'Hopital's rule when finding limits</td>
</tr>
<tr>
<td>3.2.5.1.i.</td>
<td>Use L'Hopital's rule to calculate limits</td>
</tr>
<tr>
<td>3.3.</td>
<td>Integration</td>
</tr>
<tr>
<td>3.3.1.</td>
<td>Integral: definition, properties</td>
</tr>
<tr>
<td>3.3.1.1.</td>
<td>Define the integral of a function $f$ continuous on an interval $[a,b]$</td>
</tr>
</tbody>
</table>
| 3.3.1.1.i. | Recognize the integral of a continuous function $f$ on the closed }
interval \([a,b]\) as the real number \(F(b) - F(a)\) where \(F\) is any primitive of \(f\) on \([a,b]\)

→ Know that the fundamental theorem of integration: if \(f\) is continuous on the interval \(I\) and if \(a\) is an element of \(I\), then \(\int f(t) \, dt\) from \(a\) to \(x\) is the unique

3.3.2.1.i. primitive of \(f\) on \(I\) which cancels at \(a\)

3.3.2.1.ii. Interpret graphically the integral of \(f\) on \([a,b]\)

3.3.2.1.iii. Demonstrate and use the properties of the integral

→ \(f\) being a continuous function on an interval \(I\), \(a\) and \(b\) elements of \(I\) \((a < b)\), know and use the properties of the integral

3.3.2. Methods of integration

3.3.2.1. Use the different methods of integration for the calculation of integrals

→ Use the inverse reading formulas of derivation (where the function is continuous on the interval considered)

3.3.2.1.i. Use the method of integration by parts.

3.3.2.1.ii. Decompose a rational fraction into simple elements.

3.3.2.1.iii. → Use the change of variable in simple cases.

3.3.2.1.iv. Use the trigonometric formulas allowing the linearization of some

3.3.2.1.v. trigonometric polynomials.

3.3.3. Application of the integral calculation.

3.3.3.1. Use the integral to calculate areas and volumes

3.3.3.1.i. Calculate areas with the help of integrals

3.3.3.1.ii. Calculate volumes in the case of a usual solid of revolution with the help of integrals

→ Calculate the volume of a solid delimited by the rotation of an arc of a curve about one of the coordinate axes.

3.3.3.1.iii. Calculate an approximate value of an integral by the method of rectangles.

3.3.3.1.iv. Calculate areas with the help of integrals

3.3.4. Differential equations

3.4.1. Definitions

3.4.1.1. Identify a differential equation and determine its order

3.4.1.1.i. → Recognize a differential equation of the first and second order

→ Identify the vocabulary associated with a differential equation (order, coefficient, equation with second member, a general solution, an implicit solution, an explicit solution

3.4.2. Equations in separable variables

3.4.2.1. Recognize and solve an equation in separable variables (simple cases)

→ Recognize a differential equation of the first order in separable variables as that which leads to the form \(\int f(x) \, dx = \int g(y) \, dy\)

→ Solve a differential equation of the form \(y' + a(x) \, y = 0\) where \(a\) is a simple function to integrate.

3.4.3. Linear first order equations with constant coefficients

3.4.3.1. Recognize and solve a linear first order equation with constant coefficients

→ Solve a differential equation of the form \(y' = f(x)\) where \(f\) is

127
continuous on an interval I
→ Solve a differential equation of the form \( y' = ay + b \) where \( a \) and \( b \) are given real numbers

3.4.3.1.ii. Solve a differential equation of the form \( y' + ay = f(x) \) where \( a \) is a given real number and \( f \) a simple function

3.4.3.1.iii. Solve a differential equation of the form \( y' = ay + b \) where \( a \) and \( b \) are given real numbers

3.4.4. Linear second order equations with constant coefficients

3.4.4.1. Recognize and solve a linear second order equation with constant coefficients.
→ Solve a differential equation of the form \( y'' = f(x) \) where \( f \) is continuous on an interval I

3.4.4.1.i. Solve a differential equation of the form \( ay'' + by' + cy = 0 \) where \( a \), \( b \), and \( c \) are given real numbers
→ Solve a differential equation of the form \( y'' + w^2y = k \) where \( w \) and \( k \) are given real numbers

4 TRIGONOMETRY
4.1. Circular functions
4.1.1. Study of circular functions of the form \( \cos(bx+c) \) and \( \sin(bx+c) \)

4.1.1.1. Differentiate in these functions the amplitude, frequency, period and phase.
4.1.1.2. Study and represent these functions graphically.
→ Study and represent graphically the function \( f \) defined by \( f(x) = a\cos(bx+c) \) where \( a \), \( b \) and \( c \) are real numbers.
→ Study and represent graphically the function \( f \) defined by \( f(x) = a\sin(bx+c) \) where \( a \), \( b \) and \( c \) are real numbers.

5 STATISTICS AND PROBABILITY
5.1. Statistics
Measures of central tendency and measures of variability of a distribution of one (continuous or discrete) variable

5.1.1. Calculate the measures of central tendency and measures of variability and know how to interpret them.
5.1.1.1. Recognize the median class.
5.1.1.1.i. Recognize the modal class(es).
5.1.1.1.ii. Identify and calculate analytically and graphically (if it can be done) the median and the mode(s).
5.1.1.1.iii. Identify and determine the range.
5.1.1.1.iv. Identify and calculate the mean, mean deviation, variance and standard deviation.
5.1.1.1.v. Compare and interpret two distributions of the same mean and of different standard deviations.

5.2 Probability
5.2.1 Conditional probability: definition, independence of two events
Define and calculate the probability of an event \( A \), knowing that an event \( B \) is realized.

5.2.1.1. Calculate \( P_B(A) \) by the formula \( P_B(A) = P(A/B) = P(A \cap B) / P(B) \).
→ Calculate \( P(A \cap B) \) by the formula: 
\[
P(A \cap B) = P(A/B) \times P(B) = P(B/A) \times P(A)
\]
where \( A \) and \( B \) are two non impossible events.

5.2.1.2. Define two independent events:
→ Recognize two independent events \( A \) and \( B \) by the fact that 
\[
P(A/B) = P(A)
\]

5.2.2. Formula of total probabilities

5.2.2.1. Recognize the formula for total probabilities.
→ Recognize a fundamental system of events (partition) 
\[
\Omega = \bigcup Bi / Bi \cap Bj = \emptyset , i \neq j
\]
→ Know that if an \( A \subseteq \Omega \), then, 
\[
A = \bigcup(A \cap Bk) k = 1,2,3,\
\]
→ Know and use the formula of total probability 
\[
P(A) = \sum P(Bi) \times P(A/Bi)
\]
where \( Bi \) is a fundamental system of events.

5.2.3. Characteristics.

5.2.3.1. Define a random real variable associated with a random trial
5.2.3.1.i. → Identify a random variable.
5.2.3.1.ii. → Recognize the set \( \Omega(X) \) of possible values of a random variable
→ Define a law of probability by determining the values of the variable
5.2.3.1.iii. \( X \) and the probabilities attached to each value
5.2.3.2. Characterize and represent graphically a distribution function.
5.2.3.2.i. → Determine the distribution function \( F \) of one random variable
5.2.3.2.ii. → Represent the function \( F \)
5.2.3.2.iii. → Interpret graphically \( F(a) \) for a real constant
5.2.3.3. Recognize the characteristics of a random variable
5.2.3.3.i. → Know and calculate the mathematical expectancy of \( X \)
5.2.3.3.ii. → Identify and calculate the variance of \( X \)
5.2.3.3.iii. → Identify and calculate the standard deviation of \( X \)
→ Interpret the two characteristics: mathematical expectancy and standard deviation.
5.2.3.3.iv. standard deviation.

5.2.4. Bernoulli variable

5.2.4.1. Recognize a bernoulli variable during a trial
5.2.4.1.i. → Recognize a variable associated with a Bernoulli trial
5.2.4.1.ii. → Determine the law of a Bernoulli trial
5.2.4.1.iii. → Calculate the characteristics of this variable.

5.2.5. Binomial law

5.2.5.1. Recognize a binomial law and determine its parameters and characteristics.
5.2.5.1.i. → Recognize a Bernoulli schema
5.2.5.1.ii. → Determine the parameters of a binomial law
→ Know and use the formula 
\[
P_k = P[X=K] = nCk p^k q^{n-k}
\]
for \( K = 0,1,\ldots,n \)
5.2.5.1.iii. → Calculate the characteristics of a binomial law
APPENDIX G

The curriculum content of Grade 10 or 11 that is associated with the items that were addressed in the model tests and official exams for the LS track:

A. Arrangements and permutations: Calculate n!

B. Arrangements and permutations: Know and use the formulas that give the number of arrangements and number of permutations

C. Polynomials, equations and inequalities of degree 2: Determine if a quadratic equation with real coefficients has real roots.

D. Polynomials, equations and inequalities of degree 2: Find the roots of a quadratic equation with real coefficients if they exist.

E. Complex numbers: Identify the real part and the imaginary part of a complex number.

F. Complex numbers: Determine the set of points that satisfy a given condition.

G. Complex numbers: Represent geometrically a complex number.

H. Complex numbers: Know and use the fact that the image of z and its conjugate are symmetric with respect to the real axis.

I. Complex numbers: Calculate the conjugate of a complex number and use its properties.

J. Complex numbers: Solve a quadratic equation with real coefficients and a negative discriminant.

K. Complex numbers: Characterize two equal complex numbers.
L. Complex numbers: Know the fact that the function from the set of points. \( p(x,y) \) to \( C \) which assigns \( p(x,y) \) to \( z=x+iy \) is a bijection.

M. Vectorial study: Find the coordinates of the midpoint of a segment.

N. Vectorial study: Know and use that the relations \( X(AB) = X(A) - X(B) \).

O. Geometry: Calculate the angle between vectors (using dot product).

P. Geometry: Prove ABC is right (Given 3 points).

Q. Geometry: Prove ABC is isosceles (use distance formula).

R. Geometry: Deduce circle is tangent to line.

S. Geometry: Deduce/prove nature of a quad.

T. Geometry: Know and use the properties of vector product.

U. Geometry: Prove E sym of B wrt W.

V. Geometry: Prove 3 points collinear.

W. Geometry: Prove w center of circumscribed circle.

X. Functions: Deduce V and/or H asymptotes using limits.

Y. Functions: Sketch an asymptote.

Z. Functions: Verify that a given line is an asymptote.

AA. Functions: Calculate coordinate of intersection of graph and asymptote/tangent.

BB. Functions: Study relative positions of C and asymptote/tangent.

CC. Functions: determine center of symmetry (by proving odd).

DD. Functions: Prove a point is a center of sym.

EE. Functions: discuss the number of roots \( f(x) = m \).

FF. Functions: interpret \( f'(0) \) graphically.
GG. Continuity and differentiation: Know that the derivative is the slope of tangent and know the equation of the tangent to a graph at a point.

HH. Continuity and differentiation: Find m so that f is strictly monotonic.

II. Continuity and differentiation: Find m so that C has an extremum.

JJ. Continuity and differentiation: Justify f is increasing using a given graph of f'(x).

KK. Continuity and differentiation: Justify f is increasing using a given table of f'(x).

LL. Continuity and differentiation: Study sign of f(x) using a table of variation of h(x).

MM. Continuity and differentiation: study sign of f'(x) given table of variations of f'(x).

NN. Continuity and differentiation: Find h'(x) (where h(x) = xf(x).

OO. Antiderivative: Identify the antiderivative as the inverse operation of differentiation.


QQ. Probability: Calculate the probability of an event using the basic properties of probability.

RR. Probability: Find P(A ∩ B) using formula when independent.

SS. Probability: Know that, for two events A and B, \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \).

TT. Probability: Know that if A and \( \bar{A} \) are complementary events then: \( P(A) + P(\bar{A}) = 1 \)
# APPENDIX H

Quantitative Analysis for Model Test 1 (LSM1)

<table>
<thead>
<tr>
<th>Curriculum of Mathematics - Decree No 10227 - Date: 08 May 1997</th>
<th>Details of Contents / Objectives of Grade 12 - LS section</th>
<th>Mathematics Framework - TIMSS Advanced 2008 - Cognitive Domains</th>
<th>Math - Model Test 1 - Grade 12 - LS Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowing</td>
<td>Applying</td>
<td>Reasoning</td>
<td>Test items</td>
</tr>
<tr>
<td>1.2.1.2</td>
<td>1</td>
<td>2</td>
<td>I1-I2</td>
</tr>
<tr>
<td>2.1.1.2.iii.</td>
<td>1/2</td>
<td>1/2</td>
<td>III1</td>
</tr>
<tr>
<td>2.1.1.2.iv.</td>
<td>1/3</td>
<td>1/3</td>
<td>III1</td>
</tr>
<tr>
<td>2.1.2.1.</td>
<td>1/4</td>
<td>1/4</td>
<td>III3</td>
</tr>
<tr>
<td>2.1.2.1.iii.</td>
<td>1/2</td>
<td>1/2</td>
<td>III3</td>
</tr>
<tr>
<td>2.1.3.1.i.</td>
<td>1/4</td>
<td>1/2</td>
<td>III2</td>
</tr>
<tr>
<td>2.1.4.1.vi.</td>
<td>1/4</td>
<td>1/4</td>
<td>III3</td>
</tr>
<tr>
<td>Grade 11</td>
<td>1/4</td>
<td></td>
<td>III2</td>
</tr>
<tr>
<td>Grade 11S</td>
<td>1/2</td>
<td></td>
<td>III4</td>
</tr>
<tr>
<td>3.1.3.1.i.</td>
<td>2</td>
<td></td>
<td>II1</td>
</tr>
<tr>
<td>3.1.3.3.iii.</td>
<td>1/3</td>
<td>1/3</td>
<td>II5a</td>
</tr>
<tr>
<td>Grade 11S</td>
<td>1/3</td>
<td>1/3</td>
<td>II5b</td>
</tr>
<tr>
<td>Grade 11S</td>
<td>1/3</td>
<td>1/3</td>
<td>II6</td>
</tr>
<tr>
<td>Grade 11S</td>
<td>1/3</td>
<td>1/3</td>
<td>II3</td>
</tr>
<tr>
<td>Grade 11S</td>
<td>1/3</td>
<td>1/3</td>
<td>II4</td>
</tr>
<tr>
<td>3.3.3.1.i.</td>
<td>1/3</td>
<td>1/3</td>
<td>II2</td>
</tr>
<tr>
<td>Total</td>
<td>5</td>
<td>5</td>
<td>17</td>
</tr>
</tbody>
</table>
## APPENDIX I

Quantitative Analysis for the Official Exam LS121

<table>
<thead>
<tr>
<th>Curriculum of Mathematics - Decree No 10227 - Date: 08 May 1997</th>
<th>Details of Contents / Objectives of Grade 12 - LS section</th>
<th>Mathematics Framework - TIMSS Advanced 2008 - Cognitive Domains</th>
<th>Math Official Exam - Grade 12 - LS Section - Year 2012 - Session 1 (LS121)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowing</td>
<td>Applying</td>
<td>Reasoning</td>
<td>Test items</td>
</tr>
<tr>
<td>1.2.1.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2.1.1.i.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2.1.2</td>
<td>1/6</td>
<td>1/2</td>
<td>IIB</td>
</tr>
<tr>
<td>1.4.1.1.i.</td>
<td>1/2</td>
<td>1/2</td>
<td>III3a</td>
</tr>
<tr>
<td>1.4.1.1.ii.</td>
<td>1/2</td>
<td>1/2</td>
<td>III2a</td>
</tr>
<tr>
<td>1.4.1.2</td>
<td>1/4</td>
<td>1/4</td>
<td>III1</td>
</tr>
<tr>
<td>1.4.1.2.iv.</td>
<td>1/4</td>
<td>1/4</td>
<td>III3b</td>
</tr>
<tr>
<td>Grade 11S</td>
<td></td>
<td>1/2</td>
<td>III2b-III3b</td>
</tr>
<tr>
<td>2.1.1.1.ii.</td>
<td>1/2</td>
<td>1/2</td>
<td>I2b</td>
</tr>
<tr>
<td>2.1.1.2.iii.</td>
<td>1/2</td>
<td>1/2</td>
<td>I2C</td>
</tr>
<tr>
<td>2.1.2.1.iii.</td>
<td>1/2</td>
<td>1/2</td>
<td>I2a</td>
</tr>
<tr>
<td>2.1.2.1.vi.</td>
<td>1</td>
<td>1</td>
<td>I2b-I3</td>
</tr>
<tr>
<td>Additional</td>
<td>1</td>
<td>1</td>
<td>I4b</td>
</tr>
<tr>
<td>2.1.3.1.i.</td>
<td>1/4</td>
<td>1/4</td>
<td>I2a</td>
</tr>
<tr>
<td>2.1.3.1.ii.</td>
<td>1</td>
<td>1</td>
<td>I4b</td>
</tr>
<tr>
<td>2.1.5.1.i.</td>
<td>1</td>
<td>1</td>
<td>I4b</td>
</tr>
<tr>
<td>Grade 11S</td>
<td>1/2</td>
<td></td>
<td>I2a</td>
</tr>
<tr>
<td>3.1.1.3.i.</td>
<td>1</td>
<td></td>
<td>IV6</td>
</tr>
<tr>
<td>3.1.1.3.iv.</td>
<td>1</td>
<td></td>
<td>IV6</td>
</tr>
<tr>
<td>3.1.3.1.i.</td>
<td>1</td>
<td></td>
<td>IV2-IV4</td>
</tr>
<tr>
<td>3.1.3.1.ii.</td>
<td>1</td>
<td></td>
<td>IV1a</td>
</tr>
<tr>
<td>3.1.3.1.iii.</td>
<td>1/2</td>
<td>1/2</td>
<td>IV1a</td>
</tr>
<tr>
<td>3.1.3.1.iv.</td>
<td>1/2</td>
<td>1/2</td>
<td>IV2</td>
</tr>
<tr>
<td>Grade 11S</td>
<td>1</td>
<td>1</td>
<td>IV1b</td>
</tr>
<tr>
<td>Grade 11S</td>
<td>1</td>
<td>1</td>
<td>IV4</td>
</tr>
<tr>
<td>3.2.1.2.</td>
<td>1/3</td>
<td>1/3</td>
<td>1</td>
</tr>
<tr>
<td>3.2.5.1.i.</td>
<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
</tr>
<tr>
<td>3.3.3.1.i.</td>
<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
</tr>
<tr>
<td>Grade11S</td>
<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
</tr>
<tr>
<td>5.2.3.1.iii.</td>
<td>1</td>
<td>1</td>
<td>IIA3b</td>
</tr>
<tr>
<td>grade 11 S</td>
<td>1</td>
<td>1</td>
<td>IIA1-IIA2</td>
</tr>
<tr>
<td>grade 11 S</td>
<td>1/2</td>
<td>2/3</td>
<td>2/3</td>
</tr>
<tr>
<td>grade 11 S</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>grade 11 S</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>13</td>
<td>10</td>
<td>6</td>
</tr>
</tbody>
</table>
APPENDIX J

Frozen Themes and Details of Contents of the Mathematics Curriculum

Retrieved from:


<table>
<thead>
<tr>
<th>CONTENT</th>
<th>OBJECTIVES</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4. Bemoulli variable.</td>
<td>1. Recognize a Bernoulli variable during a trial.</td>
</tr>
<tr>
<td>2.5. Binomial law.</td>
<td>1. Recognize a binomial law and determine its parameters characteristics.</td>
</tr>
</tbody>
</table>
# APPENDIX K

## Qualitative Analysis of the Model Tests and Official Exams

Table 1

<table>
<thead>
<tr>
<th>Math Topics</th>
<th>Model Tests</th>
<th>Official Exams of the LS Track at Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSM1</td>
<td>LSM2</td>
</tr>
<tr>
<td>1. ALGEBRA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2. Literal and numerical calculations</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>1.4. Numbers</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>2. GEOMETRY</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1. Classical study</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>3. CALCULUS (NUMERICAL FUNCTIONS)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.1. Definitions &amp; Representations</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>3.2. Continuity and differentiation</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>3.3. Integration</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>3.4. Differential equations</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>4. TRIGONOMETRY</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.1. Circular functions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. STATISTICS AND PROBABILITY</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.1 Statistics</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>5.2 Probability</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>
Table 2
Distribution of Grades by Math Topics in the Model Tests and Official Exams of the LS Track at Grade 12

<table>
<thead>
<tr>
<th>Math Topics</th>
<th>Model Tests</th>
<th>Official Exams of the LS Track at Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSM1</td>
<td>LSM2</td>
</tr>
<tr>
<td>1. ALGEBRA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2. Literal and numerical calculations</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>1.4. Numbers</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>2. GEOMETRY</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1. Classical study</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>3. CALCULUS (NUMERICAL FUNCTIONS)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.1. Definitions &amp; Representations</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>3.2. Continuity and differentiation</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3.3. Integration</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3.4. Differential equations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. TRIGONOMETRY</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.1. Circular functions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. STATISTICS AND PROBABILITY</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.1. Statistics</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>5.2. Probability</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>
### Table 3
Occurrences of Test Items on the Math Topic “Literal and numerical calculations” in the Model Tests and Official Exams of the LS Track at Grade 12

<table>
<thead>
<tr>
<th>Objectives of the test items on Literal and numerical calculations</th>
<th>Model Tests</th>
<th>Official Exams of the LS Track at Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSM1 LSM2 LSM3 LSM4</td>
<td>2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012</td>
</tr>
<tr>
<td>No of combinations</td>
<td>x</td>
<td>x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>No of combinations (specifications: at least...)</td>
<td>x</td>
<td>x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>Arrangements and permutations</td>
<td>x</td>
<td>x x x x x x x x x x x x x x x x x</td>
</tr>
</tbody>
</table>

138
<table>
<thead>
<tr>
<th>Objectives of the test items on Numbers</th>
<th>Model Tests</th>
<th>Official Exams of the LS Track at Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSM1</td>
<td>LSM2</td>
</tr>
<tr>
<td>Linearize</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>write in trigo form</td>
<td></td>
<td></td>
</tr>
<tr>
<td>write in exp form</td>
<td></td>
<td></td>
</tr>
<tr>
<td>write in alg form (from exp or trigo?)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Identify the real part of a complex number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Identify the imaginary part of a complex number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interpret geometrically the product zz'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate the argument of z</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate the modulus of z</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interpret geometrically the argument of z (prove collinear)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interpret geometrically the argument of z (u, OA)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interpret geometrically the modulus of z</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Determine the set of points that satisfy a given condition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Know that AB = abs ((z(b)-z(a))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deduce or prove the type of triangle, quadrilateral</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Know and use the properties of modulus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Represent geometrically a complex number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>express x' and y' in terms of x and y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recognize pure real</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recognize pure imaginary</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Know, use the fact that image of conj z sym of image of z</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate conjugate of a complex number and use properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solve a quadratic equation with complex roots</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Characterize two equal complex numbers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bijection p(x, y) and z = x+iy</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Table 5
Occurrences of Test Items on the Math Topic “Classical study of Geometry” in the Model Tests and Official Exams of the LS Track at Grade 12

<table>
<thead>
<tr>
<th>Objectives of the test items on Geometry</th>
<th>Model Tests</th>
<th>Official Exams of the LS Track at Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSM1 LSM2 LSM3 LSM4</td>
<td>2001 session 1 2002 session 1 2003 session 1 2004 session 1 2005 session 1 2006 session 1 2007 session 1 2008 session 1 2009 session 1 2010 session 1 2011 session 1 2012 session 1 2012 session 2</td>
</tr>
<tr>
<td>Show that a line lies/not in plane</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Show that (AB) lies in (P)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Show that a point belong/belong to plane</td>
<td></td>
<td>x x x</td>
</tr>
<tr>
<td>Show that a point belongs/not to a line</td>
<td></td>
<td>x x</td>
</tr>
<tr>
<td>Show that A sym A’ wrt plane</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Calculate the angle btw vectors (using dot product)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Calculate the distance from a point to a line</td>
<td></td>
<td>x x x</td>
</tr>
<tr>
<td>Calculate the distance from a point to a plane</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Calculate the distance from a A to the line of intersection</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Calculate the distance between two lines</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Calculate the distance between two planes</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Prove dist from A to (P) remains cst.</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Prove E is orth. Proj. of point on a line</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Prove E is orth. Proj. of point on a plane</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Find the orth proj. of a point on a plane</td>
<td></td>
<td>x x</td>
</tr>
<tr>
<td>Find the orth proj. of a point on a line</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Find eq. of a plane (passing A and perp to line)</td>
<td>x</td>
<td>x x</td>
</tr>
<tr>
<td>Find eq. of plane (contains line and a point)</td>
<td></td>
<td>x x</td>
</tr>
<tr>
<td>Find eq. of plane (2 points perp to a plane)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Find eq. of plane (A and parallel to two lines)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Find eq. of plane (containing two lines)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Find eq. of a plane (3 pts)</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Objectives of the test items on Geometry</th>
<th>Model Tests</th>
<th>Official Exams of the LS Track at Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSM1 LSM2 LSM3 LSM4</td>
<td>2001 session 1 2002 session 1 2003 session 1 2004 session 1 2005 session 1 2006 session 1 2007 session 1 2008 session 1 2009 session 1 2010 session 1 2011 session 1 2012 session 1 2012 session 2</td>
</tr>
<tr>
<td>Show that a line lies/not in plane</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Show that (AB) lies in (P)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Show that a point belong/belong to plane</td>
<td></td>
<td>x x x</td>
</tr>
<tr>
<td>Show that a point belongs/not to a line</td>
<td></td>
<td>x x</td>
</tr>
<tr>
<td>Show that A sym A’ wrt plane</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Calculate the angle btw vectors (using dot product)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Calculate the distance from a point to a line</td>
<td></td>
<td>x x x</td>
</tr>
<tr>
<td>Calculate the distance from a point to a plane</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Calculate the distance from a A to the line of intersection</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Calculate the distance between two lines</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Calculate the distance between two planes</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Prove dist from A to (P) remains cst.</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Prove E is orth. Proj. of point on a line</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Prove E is orth. Proj. of point on a plane</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Find the orth proj. of a point on a plane</td>
<td></td>
<td>x x</td>
</tr>
<tr>
<td>Find the orth proj. of a point on a line</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Find eq. of a plane (passing A and perp to line)</td>
<td>x</td>
<td>x x</td>
</tr>
<tr>
<td>Find eq. of plane (contains line and a point)</td>
<td></td>
<td>x x</td>
</tr>
<tr>
<td>Find eq. of plane (2 points perp to a plane)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Find eq. of plane (A and parallel to two lines)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Find eq. of plane (containing two lines)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Find eq. of a plane (3 pts)</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>
### Objectives of the test items on Geometry

<table>
<thead>
<tr>
<th>LSM1</th>
<th>LSM2</th>
<th>LSM3</th>
<th>Official Exams of the LS Track at Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find eq. of a plane (3 pts)</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Find eq. of plane (A and parallel to a plane)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove an expression is an eq. of plane (3 pts)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove an expression is an eq. of plane (1 pt and a line)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove an expression is an eq. of plane (1 pt parallel 2 line)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove a plane a mediator plane of a segment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find the mediator plane</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove two planes perpendicular</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove two planes parallel</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove two planes intersect</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find the line of intersection of two planes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find direction vector of line of inter. of two planes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find the line of intersection of two planes given A</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove a given line is inter. of two planes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find equation of a line (A and perp. to plane...)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find equation of a line (2 pts)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find equation of line tangent to a circle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove two line intersect at a given point</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove line perp. to a plane</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Determine m so that line perp. To plane</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove two lines are perp.</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove two lines are parallel</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove two lines are skew</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove point equidistant from two lines</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find E intersection of line and plane</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>deduce a line is bisector of an angle (btw two lines)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Determine the bisector of an angle (given one point)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Objectives of the test items on Geometry

<table>
<thead>
<tr>
<th></th>
<th>LSM1</th>
<th>LSM2</th>
<th>LSM3</th>
<th>LSM4</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine coordinates of pt E (E on line) AE= 5</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verify that ABCD is a tetrahedron</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate volume of tetrahedron</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate the area of triangle ABC</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate the area of quad ABCE</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove ABC is right (Given 3 pts)</td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove ABC is isosceles (use distance Formula)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deduce the dist. from A to a plane knowing volume</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deduce circle is tangent to line</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Deduce/prove nature of a quad</td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Know and use the properties of vector product</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove area/volume, distance is indep. Of…</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove distance indep. Of position of M</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Determine the intersection of plane with the axes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove E sym of B wrt W</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove 3 pts collinear</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find the coordinates of the midpoint of a segment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Know and use that the relations X(AB) = X(A)-X(B)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prove w center of circumscribed circle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>
Table 6
Occurrences of Test Items on the Math Topic “Definitions & Representations” in the Model Tests and Official Exams of the LS Track at Grade 12

<table>
<thead>
<tr>
<th>Objectives of the test items on Definitions &amp; Representations</th>
<th>Model Tests</th>
<th>Official Exams of the LS Track at Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSM1 LSM2 LSM3 LSM4</td>
<td>2001 session 1</td>
</tr>
<tr>
<td>Construct graph C1 (Given Cm)</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Study according to m the sign of f(m)</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>discuss using variations number of solutions</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Study variation of function (ln and exp)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Study variation of logarithmic function (base e)</td>
<td>x x x x x x</td>
<td>x</td>
</tr>
<tr>
<td>Study variation of exp function</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>Study variation of function (exp and lnx)</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>sketch graph of exp Function</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>sketch graph of logarithmic function (base e)</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>sketch graph of function (ln and exp)</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>Prove f admits an inverse fct f^(-1)</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>determine the explicit expression of f^(-1)</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>verify an expression to be the f^(-1)</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>Find domain of definition of f^(-1)</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>Graph f^(-1)</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>Study variation of f^(-1)</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Know and use that f(x) and f^(-1) are sym</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>Find limit of log function of base e</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>Find limit of exp function</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>Find limit of fct (exp and lnx)</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>Deduce V and/or H asymptotes using limits</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>Verify that a given line is an asymptote</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>calculate coordinate of inter. Of graph and asymptote</td>
<td>x x x x x x</td>
<td>x x x x x x x x x x x x x x x x x x x x x x x</td>
</tr>
<tr>
<td>Objectives of the test items on Definitions &amp; Representations</td>
<td>Model Tests</td>
<td>Official Exams of the LS Track at Grade 12</td>
</tr>
<tr>
<td>---------------------------------------------------------------</td>
<td>-------------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td>Study relative positions of C and asymptote/tangent</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>give table of variation without deriving</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Find ( f'(x) ) of log. Function of base e</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Find ( f'(x) ) of exponential Function</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Find ( f'(x) ) of function (lnx and exp)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Prove ( f'(x) ) positive (f is inc) from expression of ( f'(x) )</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Verify an expression to be the ( f'(x) )</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Deduce variation of ( f'(x) ) from expression of ( f'(x) )</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Sketch an asymptote</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Plot points</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>deduce the sign of a function (( f'(-1) ), ( f'(x) )) from table</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Calculate ( f(0) )</td>
<td>x x</td>
<td>x</td>
</tr>
<tr>
<td>Calculate ( f'(0) )</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Use the fact that ln x and Exp fct are bij and st. inc</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>determine center of symmetry (by proving odd)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Prove a point is a center of sym.</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>verify ( f(x)+f(-x) = 0 )</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>prove ( f(x) = g(x) ) has no roots</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>prove ( f(x) ) can be written as = …</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>discuss the number of roots ( f(x) = m )</td>
<td>x x</td>
<td>x</td>
</tr>
<tr>
<td>Solve ( f(x) = 0 )</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>interpret ( f'(0) ) graphically</td>
<td></td>
<td>x</td>
</tr>
</tbody>
</table>
Table 7
Occurrences of Test Items on the Math Topic “Continuity and differentiation” in the Model Tests and Official Exams of the LS Track at Grade 12

<table>
<thead>
<tr>
<th>Objectives of the test items on Continuity and differentiation</th>
<th>Model Tests</th>
<th>Official Exams of the LS Track at Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSM1</td>
<td>LSM2</td>
</tr>
<tr>
<td>Find m so that f is st monotone</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Find m so that C has an extremum</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Find f''(x) of log function</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Find limit using L'Hopital's rule</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Justify f is increasing using a given graph of f'(x)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Justify f is increasing using a given table of f'(x)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Study sign of f using a table of variation of h(x)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>study sign of f'(x) given table of variations of f'(x)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Prove C has a point of inflection using graph of f'(x)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Prove C has a point of inflection using table of f'(x)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Prove C has a point of inflection by calculating f''(x)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Find the coordinates of C at which tangent…</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>determine equation of tangent at a point</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Verify that a line is the tangent at a point</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Find the point of inflection</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Calculate the derivative of f''(-1)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>deduce/find slope(eq) of tangent using f''(-1)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Prove f(x)=0 has a (unique) root in [a,b]</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Find h'(x) (where h(x) =xf(x)</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>
Table 8
Occurrences of Test Items on the Math Topic “Integration” in the Model Tests and Official Exams of the LS Track at Grade 12

<table>
<thead>
<tr>
<th>Objectives of the test items on Integration</th>
<th>Model Tests</th>
<th>Official Exams of the LS Track at Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSM1</td>
<td>LSM2</td>
</tr>
<tr>
<td>Calculate area under a curve</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Calculate area between fct and asymptote</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate the area btw two graphs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>deduce the area between fct and asym. (from integral)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate the under f'(x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate a definite integral</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Calculate an indefinite integral</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Know which graph represents the primitive of f</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Know and use the properties of integrals</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Know and use the fundamental theorem of integration</td>
<td></td>
<td></td>
</tr>
<tr>
<td>use by parts to find a definite integral</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate a, b, c so that F is an antiderivative of f</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2011

2005

2006

2007

2008

2009

2010

2011

2012
Table 9
Occurrences of Test Items on the Math Topic “Differential equations” in the Model Tests and Official Exams of the LS Track at Grade 12

<table>
<thead>
<tr>
<th>Objectives of the test items on Differential equations</th>
<th>Model Tests</th>
<th>Official Exams of the LS Track at Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSM1</td>
<td>LSM2</td>
</tr>
<tr>
<td>Solve a linear second order diff equation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find a particular sol of second order diff</td>
<td></td>
<td></td>
</tr>
<tr>
<td>using a graph/passing through a point</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Write a diff equa satisfied by z</td>
<td></td>
<td></td>
</tr>
<tr>
<td>deduce general sol. of (E) form (E')</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 10
Occurrences of Test Items on the Math Topic “Statistics” in the Model Tests and Official Exams of the LS Track at Grade 12

<table>
<thead>
<tr>
<th>Objectives of the test items on Statistics</th>
<th>Model Tests</th>
<th>Official Exams of the LS Track at Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSM1</td>
<td>LSM2</td>
</tr>
<tr>
<td>Organize the data in classes of amplitude 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate the median</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interpret the median</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate the mean given classes and freq.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate the st. d. given classes and freq.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate variance given classes and freq.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Draw I.C.F polygon</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculate graphically n analytically the median</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

147
Table 11
Occurrences of Test Items on the Math Topic “Probability” in the Model Tests and Official Exams of the LH Track at Grade 12

<table>
<thead>
<tr>
<th>Objectives of the test items on Probability</th>
<th>Model Tests</th>
<th>Official Exams of the LS Track at Grade 12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LSM1</td>
<td>LSM2</td>
</tr>
<tr>
<td>P(event), 1 is chosen at a time (and)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>P(event), 1 is chosen at a time (or)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>p(A) use table</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P(only)using formulas</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>P(A or B) (1-p(A)) at least when ind.</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>P(event), more than 1 at a time (or)</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>P(event), more than 1 at a time (and)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P(A/B)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>P(A/B) from table</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>P(A/B) using formula</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find/ deduce P(A∩B)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Find P(A∩B) using formula when dependant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find P(A∩B) using formula when independant</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>P(A∩B) bar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P(A∩B) bar using formula</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total probability</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Find values of X</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Determine the probability distribution of X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find /prove P(X &gt;, &lt;, =)</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Find E(X)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>interpret E(X) /Use E(X) to estimate…</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>identify binomial dist</td>
<td></td>
<td></td>
</tr>
<tr>
<td>find E(X) when X binomial</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>p(event) basic properties of probability</td>
<td></td>
<td>x</td>
</tr>
</tbody>
</table>

148
APPENDIX L

Quantitative Analysis of the Model Tests and Official Exams

Table 1
Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests and the Official Exams of the LS Track at Grade 12 – Extracted from Table Mod and Table OffEx.

<table>
<thead>
<tr>
<th>The Topics of the Math Curriculum of the LS Track at Grade 12</th>
<th>Sum of Model Tests</th>
<th>Sum of Official Exams</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K %</td>
<td>A %</td>
</tr>
<tr>
<td>1.2. Literal and numerical calculations</td>
<td>1.95</td>
<td>0</td>
</tr>
<tr>
<td>1.4. Numbers</td>
<td>2.38</td>
<td>2.71</td>
</tr>
<tr>
<td>2.1. Classical study</td>
<td>5.84</td>
<td>6.17</td>
</tr>
<tr>
<td>3.1. Definitions &amp; Representations</td>
<td>13.64</td>
<td>27.92</td>
</tr>
<tr>
<td>3.2. Continuity and differentiation</td>
<td>4.11</td>
<td>3.46</td>
</tr>
<tr>
<td>3.3. Integration</td>
<td>1.52</td>
<td>1.52</td>
</tr>
<tr>
<td>3.4. Differential equations</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5.1. Statistics</td>
<td>3.25</td>
<td>0.65</td>
</tr>
<tr>
<td>5.2. Probability</td>
<td>6.49</td>
<td>2.6</td>
</tr>
<tr>
<td>Total</td>
<td>39.18</td>
<td>45.03</td>
</tr>
</tbody>
</table>

K = Knowing  
A = Applying  
R = Reasoning  
The sum of Totals is approximately equal to 100 because the percentages are rounded.
Table 2
*Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests, and the Official Exams of the Years 2001-2003 and 2010-2012 of the LS Track at Grade 12 – Extracted from Table Mod, Table OffEx1-3, and OffEx10-12*

<table>
<thead>
<tr>
<th>The Topics of the Math Curriculum of the LS Track at Grade 12</th>
<th>Sum of Model Tests</th>
<th>Sum of 2001-2003 Official Exams</th>
<th>Sum of 2010-2012 Official Exams</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K %</td>
<td>A %</td>
<td>R %</td>
</tr>
<tr>
<td>1.2. Literal and numerical calculations</td>
<td>1.95</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2.1. Classical study</td>
<td>5.84</td>
<td>6.17</td>
<td>2.60</td>
</tr>
<tr>
<td>3.1. Definitions &amp; Representations</td>
<td>13.64</td>
<td>27.92</td>
<td>1.3</td>
</tr>
<tr>
<td>3.2. Continuity and differentiation</td>
<td>4.11</td>
<td>3.46</td>
<td>2.81</td>
</tr>
<tr>
<td>3.3. Integration</td>
<td>1.52</td>
<td>1.52</td>
<td>0.87</td>
</tr>
<tr>
<td>3.4. Differential equations</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5.1. Statistics</td>
<td>3.25</td>
<td>0.65</td>
<td>0</td>
</tr>
<tr>
<td>5.2. Probability</td>
<td>6.49</td>
<td>2.6</td>
<td>1.95</td>
</tr>
<tr>
<td>Total</td>
<td>39.18</td>
<td>45.03</td>
<td>15.81</td>
</tr>
</tbody>
</table>

K = Knowing  
A = Applying  
R = Reasoning  
The sum of Totals is approximately equal to 100 because the percentages are rounded.
Table 3  
**Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests, and the Session-1 and Session-2 Official Exams of the LS Track at Grade 12 – Extracted from Table Mod, Table OffEx1, and OffEx2**

<table>
<thead>
<tr>
<th>The Topics of the Math Curriculum of the LS Track at Grade 12</th>
<th>Sum of Model Tests</th>
<th>Sum of Session-1 Official Exams</th>
<th>Sum of Session-2 Official Exams</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K %</td>
<td>A %</td>
<td>R %</td>
</tr>
<tr>
<td>1.2. Literal and numerical calculations</td>
<td>1.95</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.4. Numbers</td>
<td>2.38</td>
<td>2.71</td>
<td>6.28</td>
</tr>
<tr>
<td>2.1. Classical study</td>
<td>5.84</td>
<td>6.17</td>
<td>2.60</td>
</tr>
<tr>
<td>3.1. Definitions &amp; Representations</td>
<td>13.64</td>
<td>27.92</td>
<td>1.3</td>
</tr>
<tr>
<td>3.2. Continuity and differentiation</td>
<td>4.11</td>
<td>3.46</td>
<td>2.81</td>
</tr>
<tr>
<td>3.3. Integration</td>
<td>1.52</td>
<td>1.52</td>
<td>0.87</td>
</tr>
<tr>
<td>3.4. Differential equations</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5.1. Statistics</td>
<td>3.25</td>
<td>0.65</td>
<td>0</td>
</tr>
<tr>
<td>5.2. Probability</td>
<td>6.49</td>
<td>2.6</td>
<td>1.95</td>
</tr>
<tr>
<td>Total</td>
<td>39.18</td>
<td>45.03</td>
<td>15.81</td>
</tr>
</tbody>
</table>

**K** = Knowing  
**A** = Applying  
**R** = Reasoning

The sum of Totals is approximately equal to 100 because the percentages are rounded.