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# Performance Analysis of UWB Systems over the IEEE 802.15.3a Channel Model

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**Abstract**—In this paper, we propose an accurate method for evaluating the performance of Ultra-Wideband systems over the IEEE 802.15.3a channel model. Depending on the severeness of fading, the method consists of approximating the distribution of the captured channel energy by either a Coxian, gamma-mixture or lognormal distribution based on a least square fitting criterion. The proposed approximations lend themselves to simple mathematical analysis and turn out to be useful in evaluating the performance, in terms of average bit-error-rate (BER) and outage probability, over the IEEE 802.15.3a channels.

**Index Terms**—Ultra-Wideband, UWB, IEEE 802.15.3a, performance analysis, Inter-Pulse-Interference.

## I. INTRODUCTION AND PROBLEM FORMULATION

A widely accepted model for ultra-wideband (UWB) channels was developed by the IEEE 802.15.3a standards body [1]. According to this model, the impulse response of the UWB channel can be expressed mathematically as:

$$g(t) = \sum_{l=0}^{L_{max}-1} \beta_l \alpha_l \delta(t - \tau_l) \quad (1)$$

where  $\delta(t)$  stands for the Dirac delta function. The parameter  $\beta_l \in \{\pm 1\}$  with equal probability models the random pulse inversion that can occur due to reflections. The random variables (r.v.s)  $\tau_l$  and  $\alpha_l$  stand for the arrival time and the fading amplitude of the  $l$ -th multipath component (MPC) respectively. According to [1],  $\tau_l$  encompasses the cluster arrival time and the arrival time of the MPC within its cluster. On the other hand,  $\alpha_l$  is modeled as a lognormal r.v. whose parameters depend on the arrival time  $\tau_l$  according to an exponential-decay power law. In eq. (1),  $L_{max}$  corresponds to the maximum number of MPCs.

A major particularity of the IEEE 802.15.3a model that distinguishes it from other multi-path fading channel models resides in the fact that the different arriving MPCs can overlap in time thus resulting in what is referred to as intra-pulse interference (IPI). If IPI is neglected, the signal-to-noise ratio (SNR) at the output of a receiver that combines all the MPCs arriving within a duration  $T_i \leq \tau_{L_{max}-1}$  is proportional to the random variable (r.v.)  $h$  defined as:

$$h = \sum_{l=0}^{L-1} \alpha_l^2 \quad (2)$$

where  $L$  corresponds to the number of MPCs in the interval  $[0 T_i]$ :  $\tau_l \leq T_i$  for  $l = 0, \dots, L - 1$ .

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On the other hand, for realistic UWB systems having a finite temporal resolution  $T_p$  (that is often limited by the receiver's sampling frequency or the width of the UWB pulses), the r.v.  $h$  takes the following value:

$$h = \sum_{l=0}^{\lfloor T_i/T_p \rfloor - 1} \gamma_l^2 \triangleq \sum_{l=0}^{\lfloor T_i/T_p \rfloor - 1} \left[ \sum_{i \in \mathcal{I}_l} \beta_i \alpha_i \right]^2 \quad (3)$$

where  $\mathcal{I}_l$  is the set given by:  $\mathcal{I}_l = \{i \mid lT_p \leq \tau_i < (l+1)T_p\}$ . Note that eq. (3) models the case of IPI since  $\gamma_l$  corresponds to a linear combination of the MPCs arriving in the interval  $[lT_p (l+1)T_p]$ . Note that the no-IPI case follows from the IPI case by assuming a very high temporal resolution:  $T_p \ll 1$ . In this case,  $\gamma_l \rightarrow \beta_l \alpha_l$  and eq. (3) boils down to eq. (2).

In this paper, we propose a framework for analyzing the performance of UWB systems by first approximating the r.v.  $h$  by a convenient distribution. Despite the fact that the path gains follow the lognormal distribution, the problem under consideration differs significantly from the lognormal-sum approximation (LSA) problem that was considered extensively in the literature [2]–[6]. This problem corresponds to approximating the distribution of the sum of lognormal r.v.s, that is not known in closed form, by another distribution. The proposed solutions are either based on the lognormal approximation [2], [3] or they might resort to non-lognormal approximations [4]–[6]. The adopted approximating techniques can be classified into many categories including the Moment Matching Approximation (MMA) [7], the Moment Generating Function (MGF)-based matching [8] and the Least-Squares (LS) approach [9]. The main reasons that render the problem of approximating the r.v.  $h$  (in eq. (2) or eq. (3)) very different from the well known LSA problem are as follows. (i): While the number of summands in the LSA problem is fixed (and known prior to performing the approximation), the number of summands in either (2) or (3) is not constant and it depends on the specific channel realization. (ii): While the parameters of the lognormal summands in the LSA problem are the same (and fixed), the parameters of the summands in  $h$  are not constant since they depend on the clusters arrival times and on the MPCs arrival times (within their clusters) that both vary from one channel realization to another.

Despite the fact that IPI is a well recognized problem in the UWB context, most previous contributions neglect this phenomenon only for the sake of simplicity [10]. Other contributions adopt simplistic tap-delay line channel models (with no IPI) that exclude any clustering phenomenon and that assume that the number of MPCs per integration window is constant [11], [12]. The adoption of such models signifi-

cantly simplifies the problem and directly links it to the LSA problem. For example, [11] evaluates the performance of UWB systems by approximating the lognormal-sum r.v.  $h$  by another lognormal r.v. based on the technique proposed in [2].

The performance of UWB systems over the IEEE 802.15.3a model is more challenging to analyze theoretically if IPI is to be taken into consideration. Hence, most of the previous studies were based on Monte-Carlo methods (refer to [13] and the references therein). Recently, an analytical framework for evaluating the performance of UWB systems in the presence or absence of IPI was proposed in [14]. In this reference, the type-IV Pearson distribution, that was initially proposed to approximate the lognormal-sum distribution in [4], was adopted for approximating the r.v.  $h$  in the no-IPI and IPI cases. Despite the breakthrough made in [14], the limitations of the type-IV Pearson approximation are as follows: (i) the approximating pdf and cdf expressions are complicated and do not lend themselves to simple mathematical analysis. (ii) The complexity of the approximating pdf (eq. (14) in [14]) results in complicated and non-intuitive expressions of the bit-error-rate (BER) and outage probability. For instance, the average BER does not admit a closed-form solution and was thus evaluated using the Gauss-Legendre method. Moreover, the BER can not be upper bounded by any simple closed-form bounds. (iii) The approach proposed in [14] corresponds to a tap-delay channel model where the time axis is divided into time bins. The implication of this model is that it results in a constant number of summands in eq. (3) which is not the case for the IEEE 802.15.3a model where the number of summands depends on the specific channel realization. Recently, the technique proposed in [14] was improved in [15] where a MMA in the logarithmic domain was used to determine the parameters of the approximating type-IV Pearson distribution in a simpler way. However, this simplification does not change the resulting pdf, cdf, BER, and outage probabilities that remain the same as in [14].

The objective of this paper is to evaluate the performance of UWB systems over the IEEE 802.15.3a model without resorting to typical simplifications in system assumptions. This was achieved by approximating the r.v.  $h$  (that models the energy captured over the UWB channels) by either a Coxian, gamma-mixture or lognormal distribution depending on the severeness of fading. The advantages of the proposed approximation approach are as follows: (i) they are suitable for both the no-IPI and IPI cases. (ii) The proposed approximations are accurate for different channel profiles and over the entire range of integration times. (iii) The approximating closed-form pdfs are simple. (iv) The proposed approximations provide a satisfactory accuracy for estimating both the BER and the outage probability. (v) The resulting BER and outage probability expressions are simple and are thus suitable for analyzing the achievable diversity orders over the IEEE 802.15.3a channels. The proposed approach outperforms [14] by its simplicity (in terms of pdf, cdf, BER and outage probability expressions) and by its suitability to the IEEE 802.15.3a model. However, this comes at the expense of a reduced accuracy. Note that while [14] is based on a MGF-based matching approach for fitting the parameters of the approximating Pearson type-IV

distribution to those of the exact distribution, our approach is based on a least-squares fitting technique for minimizing the Mean Square Error (MSE) between the exact pdf and one of the proposed approximating pdfs. A similar approach for minimizing the MSE between the cdfs was proposed in [6] in the context of the lognormal-sum approximation problem. Despite the fact that the adopted approach is computation-demanding, it is simple, straight-forward and results in tractable results. The adoption of this approach is further motivated by the fact that non of the existing approximation techniques (MMA, MGF, LS ...) in [7]–[9] (in the LSA context) is unquestionably better than the other, and each one of these techniques can be seen as achieving different levels of compromise between complexity and accuracy.

## II. PERFORMANCE ANALYSIS

1) *Preliminaries:* In this paper, we evaluate the BER and outage probability of UWB systems with BPSK. The average BER is given by:

$$P_e = \int_0^{+\infty} Q\left(\sqrt{SNR \cdot x}\right) f_h(x) dx \quad (4)$$

where  $f_h(\cdot)$  corresponds to the pdf of the r.v.  $h$  and the function  $Q(x)$  is defined by:  $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-t^2/2} dt$ . For evaluating the asymptotic BER behavior,  $P_e$  can be upper-bounded by:

$$P_e \leq \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} \frac{1}{\sqrt{SNR \cdot x}} e^{-\frac{SNR \cdot x}{2}} f_h(x) dx \quad (5)$$

where this bound follows since  $Q(x) \leq \frac{1}{\sqrt{2\pi}} \frac{e^{-x^2/2}}{x}$ .

The outage probability is defined as [14]:

$$P_{out} = \text{Prob}(h \leq h_{th}) = \int_0^{h_{th}} f_h(x) dx = F_h(h_{th}) \quad (6)$$

where  $F_h(\cdot)$  stands for the cumulative distribution function (cdf) of the r.v.  $h$ .

2) *Least-Squares Fitting:* No exact closed-form solution to the distribution of  $h$  is available in the literature. In this paper, we propose to approximate the exact pdf  $f_h(x)$  by an approximate pdf  $f_{app}(x)$  based on a minimum mean-square-error (MSE) criterion:  $MSE = \int_0^{+\infty} [f_h(x) - f_{app}(x)]^2 dx$ . The degree of fading was captured by the coefficient of variation of the r.v.  $h$  defined as:  $c_v = \frac{\sigma_h}{\mu_h}$  where  $\mu_h$  and  $\sigma_h$  stand for the mean and standard deviation of  $h$ , respectively. Note that  $c_v$  depends on  $T_i$ ,  $T_p$  and the channel profile (LOS or NLOS propagation). We investigated the Coxian, gamma-mixture and lognormal approximations and found out that the Coxian distribution is best adapted to the case of “severe” fading ( $c_v > 1$ ) while the gamma-mixture and lognormal distributions can approximate  $f_h(x)$  with a higher accuracy in the cases of “normal” fading ( $0.8 \leq c_v \leq 1$ ) and “non-severe” fading ( $c_v < 0.8$ ), respectively.

The solution to the above least-squares nonlinear curve fitting problem can be obtained numerically using the Gauss-Newton numerical algorithm for example.

3) *Coxian Distribution*: When  $c_v$  is larger than 1, a two-component Coxian distribution was found suitable for this case of “severe” fading where the captured signal energy is characterized by large variations near zero. The corresponding pdf is given by ( $\lambda_1 \geq 0$  and  $\lambda_2 \geq 0$ ):

$$f_{\text{Cox}}(x) = p\lambda_1 e^{-\lambda_1 x} + (1-p)\lambda_2 e^{-\lambda_2 x} ; x \geq 0 \quad (7)$$

When the exact pdf  $f_h(x)$  is replaced by the approximating pdf  $f_{\text{Cox}}(x)$  in eq. (5), direct calculations show that the corresponding upper-bound can be written as:

$$P_e \leq \frac{p}{\left(\frac{2\text{SNR}}{\lambda_1}\right)^{\frac{1}{2}} \left(1 + \frac{1}{2} \frac{\text{SNR}}{\lambda_1}\right)^{\frac{1}{2}}} + \frac{1-p}{\left(\frac{2\text{SNR}}{\lambda_2}\right)^{\frac{1}{2}} \left(1 + \frac{1}{2} \frac{\text{SNR}}{\lambda_2}\right)^{\frac{1}{2}}} \quad (8)$$

For sufficiently large values of the SNR, eq. (8) can be written as  $P_e \approx \frac{p\lambda_1 + (1-p)\lambda_2}{\text{SNR}}$  implying that  $P_e$  scales asymptotically as  $\text{SNR}^{-1}$  implying that the diversity order of the UWB system in the case of “severe” fading is equal to 1.

Based on the Coxian approximation, eq. (6) reduces to:

$$P_{\text{out}} = 1 - pe^{-\lambda_1 h_{th}} - (1-p)e^{-\lambda_2 h_{th}} \quad (9)$$

4) *Mixture of Gamma Distributions*: In the case where the coefficient of variation  $c_v$  is not much smaller than 1 (in particular,  $0.8 \leq c_v \leq 1$ ), we found (based on a numerical analysis) that a mixture of gamma distributions approximates the exact distribution of  $h$  with the highest accuracy. The pdf of a  $N$ -component mixture of gamma distributions can be written as:

$$f_\gamma(x) = \sum_{n=1}^N \gamma_n \frac{e^{-x/\theta_n} x^{k_n-1}}{\Gamma(k_n) \theta_n^{k_n}} ; x \geq 0 \quad (10)$$

where  $\sum_{n=1}^N \gamma_n = 1$ . The parameters  $k_n$  and  $\theta_n$  are positive for all values of  $n$  and  $\Gamma(x) = \int_0^{+\infty} e^{-t} t^{x-1} dt$  corresponds to the Gamma function ( $x \geq 0$ ).

Replacing  $f_h(x)$  by  $f_\gamma(x)$  in eq. (5) results in:

$$P_e \leq \frac{1}{\sqrt{2\pi}} \sum_{n=1}^N \gamma_n \frac{\Gamma(k_n - \frac{1}{2})}{\Gamma(k_n) (\theta_n \text{SNR})^{\frac{1}{2}} \left(1 + \frac{\theta_n \text{SNR}}{2}\right)^{k_n - \frac{1}{2}}} \quad (11)$$

which, for sufficiently large values of the SNR, can be written as:

$$P_e \approx \frac{1}{\sqrt{\pi}} \sum_{n=1}^N \gamma_n 2^{k_n-1} \frac{\Gamma(k_n - \frac{1}{2})}{\Gamma(k_n)} (\theta_n \text{SNR})^{-k_n} \quad (12)$$

Equation (12) shows that the dominant term that has the major influence on the asymptotic-BER is the smallest term among  $k_1, \dots, k_N$ . In this case,  $P_e$  scales asymptotically as  $(\theta_m \text{SNR})^{-k_m}$  where  $m \triangleq \arg \min_{n=1 \dots N} (k_n)$ . In other words, the diversity order in this case of “normal” fading is equal to  $k_m$ . Interestingly, the numerical results show that  $k_m$  is an increasing function of  $T_i$ . In the same way,  $k_m$  takes larger values in LOS conditions (for example, CM1) compared to NLOS conditions (for example, CM2).

Replacing eq. (10) in eq. (6) results in:

$$P_{\text{out}} = \sum_{n=1}^N \gamma_n \frac{\gamma(k_n, h_{th}/\theta_n)}{\Gamma(k_n)} \quad (13)$$

where  $\gamma(s, x)$  corresponds to the lower incomplete gamma function:  $\gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt$ .

5) *Lognormal Distribution*: In the case of “non-severe” fading ( $c_v < 0.8$ ), we found that the best approximating distribution is the lognormal distribution. The pdf of a lognormal distribution with parameters  $\mu$  and  $\sigma$  is given by:

$$f_{LN}(x) = \frac{1}{\sqrt{2\pi}\sigma x} \exp\left[-\frac{(\ln(x) - \mu)^2}{2\sigma^2}\right] ; x \geq 0 \quad (14)$$

A closed-form solution for eq. (5) is unfortunately not available when  $f_h(x)$  is replaced by  $f_{LN}(x)$ . On the other hand, using the looser bound  $Q(x) \leq \frac{1}{2} e^{-\frac{x^2}{2}}$  implies that  $P_e$  in eq. (4) can be upper-bounded by:

$$P_e \leq \frac{1}{2} \int_0^{+\infty} \exp\left[-\frac{\text{SNR} \cdot x}{2}\right] \frac{1}{\sqrt{2\pi}\sigma x} \exp\left[-\frac{(\ln(x) - \mu)^2}{2\sigma^2}\right] dx \quad (15)$$

The above equation can be written as:

$$P_e \leq \frac{1}{2} \text{Fr}\left(\frac{1}{2} \text{SNR} e^{\mu + \frac{\sigma^2}{2}}, 0; \frac{\sigma}{2}\right) \quad (16)$$

where  $\text{Fr}(a, 0; b)$  is the lognormal density frustration function given by [16]:

$$\text{Fr}(a, 0; b) = \int_0^{+\infty} \frac{1}{\sqrt{2\pi}b^2 x} \exp(-ax^2) \exp\left[-\frac{(\ln(x) + b^2)^2}{2b^2}\right] dx \quad (17)$$

A closed-form evaluation of the frustration function does not exist. Consequently, the lognormal approximation does not result in a closed-form expression of the asymptotic BER when  $Q(x)$  is upper-bounded by either  $\frac{1}{\sqrt{2\pi}} \frac{e^{-x^2/2}}{x}$  or  $\frac{1}{2} e^{-\frac{x^2}{2}}$ .

In what follows, we will show that an infinite diversity order can be achieved in the case of “non-severe” fading. Since  $\exp(-ax^2) = \left(\sum_{n=0}^{+\infty} \frac{a^n x^{2n}}{n!}\right)^{-1} \leq \frac{n!}{a^n x^{2n}} \forall n \in \mathbb{N}$ , then eq. (17) can be upper-bounded by:

$$\begin{aligned} \text{Fr}(a, 0; b) &\leq \frac{n!}{a^n} \int_0^{+\infty} x^{-2n} \frac{1}{\sqrt{2\pi}b^2 x} \exp\left[-\frac{(\ln(x) + b^2)^2}{2b^2}\right] dx \\ &= \frac{n!}{a^n} \text{E}[x^{-2n}] = \frac{n!}{a^n} \exp(2(n^2 + n)b^2) \end{aligned} \quad (18)$$

where the mean  $\text{E}[\cdot]$  in the last equation is evaluated over a lognormal distribution with parameters  $-b^2$  and  $b$ . Finally, replacing eq. (18) in eq. (16) results in:

$$P_e \leq \frac{2^{n-1} n!}{\text{SNR}^n} \exp\left(n^2 \frac{\sigma^2}{2} - n\mu\right) \forall n \in \mathbb{N} \quad (19)$$

Since the above bound holds for all integer values of  $n$ , then, in particular, it holds for very large values of  $n$  implying that the  $P_e(\text{SNR})$  curve is infinitely steep at high SNRs implying that the UWB system profits from an infinite diversity order in this case of “non-severe” fading.

Based on the lognormal approximation, the outage probability takes the following value:

$$P_{\text{out}} = Q\left(\frac{\mu - \ln(h_{th})}{\sigma}\right) \quad (20)$$

TABLE I  
BEST APPROXIMATING DISTRIBUTIONS FOR CM1 AND CM2 IN THE ABSENCE OF IPI

$T_i$	1 ns	2 ns	3 ns	5 ns	7 ns	10 ns	20 ns	30 ns	$t_{max}$
CM1: $c_v$	1.071	0.9458	0.8847	0.8336	0.8136	0.7992	0.7815	0.7782	0.7778
CM1: dist.	GM-3	GM-3	GM-3	GM-3	GM-3	GM-3	lognormal	lognormal	lognormal
CM1: MSE	$1.665 \cdot 10^{-3}$	$9.135 \cdot 10^{-5}$	$2.978 \cdot 10^{-5}$	$2.985 \cdot 10^{-5}$	$2.777 \cdot 10^{-5}$	$3.693 \cdot 10^{-5}$	$4.293 \cdot 10^{-5}$	$2.902 \cdot 10^{-5}$	$3.133 \cdot 10^{-5}$
CM2: $c_v$	1.3383	1.1804	1.0774	0.9561	0.8831	0.8289	0.7828	0.7776	0.7767
CM2: dist.	Coxian	Coxian	GM-3	GM-3	GM-3	GM-3	lognormal	lognormal	lognormal
CM2: MSE	$1.673 \cdot 10^{-3}$	$3.22 \cdot 10^{-4}$	$1.357 \cdot 10^{-3}$	$7.973 \cdot 10^{-5}$	$1.474 \cdot 10^{-5}$	$2.426 \cdot 10^{-5}$	$2.374 \cdot 10^{-5}$	$2.419 \cdot 10^{-5}$	$2.539 \cdot 10^{-5}$

GM-3: 3-component gamma-mixture;  $t_{max}$ : maximum delay-spread of the channel.

TABLE II  
BEST APPROXIMATING DISTRIBUTIONS FOR CM1 AND CM2 IN THE PRESENCE OF IPI

$T_i$	1 ns	2 ns	3 ns	5 ns	7 ns	10 ns	20 ns	30 ns	$t_{max}$
CM1: $c_v$	1.537	1.236	1.104	0.984	0.932	0.893	0.831	0.815	0.809
CM1: dist.	Coxian	Coxian	GM-3	GM-3	GM-3	GM-3	GM-3	lognormal	lognormal
CM1: MSE	$8.429 \cdot 10^{-3}$	$6.869 \cdot 10^{-3}$	$9.015 \cdot 10^{-5}$	$8.382 \cdot 10^{-5}$	$7.368 \cdot 10^{-5}$	$8.373 \cdot 10^{-5}$	$8.359 \cdot 10^{-5}$	$7.45 \cdot 10^{-5}$	$7.041 \cdot 10^{-5}$
CM2: $c_v$	1.714	1.449	1.289	1.106	1.002	0.915	0.854	0.846	0.844
CM2: dist.	Coxian	Coxian	Coxian	GM-3	GM-3	GM-3	GM-3	GM-3	GM-3
CM2: MSE	$2.35 \cdot 10^{-2}$	$2.54 \cdot 10^{-3}$	$1.13 \cdot 10^{-3}$	$1.536 \cdot 10^{-4}$	$6.958 \cdot 10^{-5}$	$8.269 \cdot 10^{-5}$	$8.346 \cdot 10^{-5}$	$7.6537 \cdot 10^{-5}$	$8.029 \cdot 10^{-5}$

GM-3: 3-component gamma-mixture;  $t_{max}$ : maximum delay-spread of the channel.

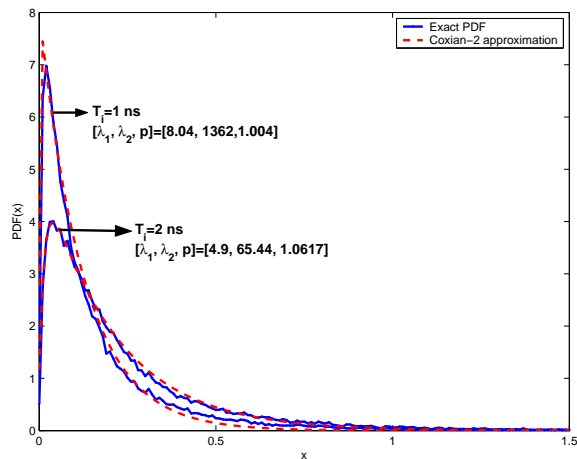


Fig. 1. PDFs of the Coxian-2 approximations over CM2 in the absence of IPI.

### III. NUMERICAL RESULTS

We performed an extensive numerical analysis that showed that the proposed approximations are accurate for different channel profiles and integration times in the presence or absence of IPI. Regarding the gamma-mixture approximation, numerical results showed that increasing the number of components in the mixture ( $N$  in eq. (10)) beyond three resulted only in a marginal decrease in the MSE. Consequently, throughout this paper, 3-component mixtures of gamma distributions are used for modeling the energy capture in the case of “normal” fading. In what follows, the minimum UWB channel multi-path separation is set to 10 ps. IPI is taken into consideration by fixing the temporal resolution of the receiver to  $T_p = 0.5$  ns. The best approximating distributions and their corresponding MSEs are given in table-I and table-II for the channel profiles CM1 and CM2 in the no-IPI and IPI cases, respectively. Similar results were obtained for the profiles CM3 and CM4 and were not presented here for brevity.

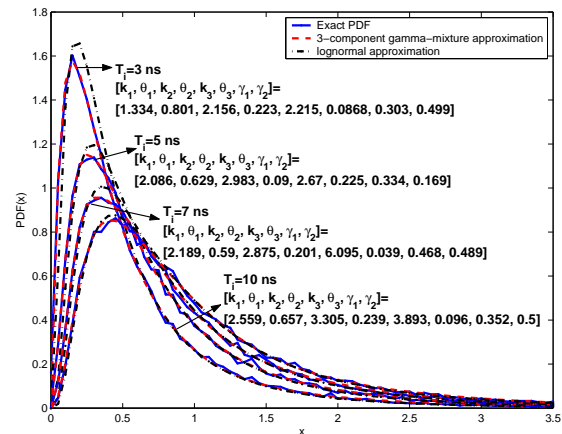


Fig. 2. PDFs of the 3-component gamma-mixture approximations over CM2 in the presence of IPI.

Fig. 1 compares the exact pdfs with their corresponding Coxian approximations for different integration times over CM2 when IPI is neglected. Results show the close match between the exact and approximate pdfs. Similar results were obtained in Fig. 2 in the case of gamma-mixture approximation over CM2 in the presence of IPI. This figure also shows the lognormal approximations that are found to be far from the exact pdfs in this case of “normal” fading. Finally, Fig. 3 shows the accuracy of the lognormal approximation for the “non-severe” fading case over CM2 in the absence of IPI.

Fig. 4 shows the BER performance over CM1 in the absence of IPI. The approximating distributions used for evaluating the exact BERs and their corresponding upper-bounds are given in table-I. Results show the suitability of the proposed approach for BER calculations. For the 3-component gamma-mixture approximation, results also show that the proposed upper-bound given in eq. (12) turns out to be very close to the exact BER for SNRs that are not excessively very large. Moreover, all the proposed bounds can be used to predict the diversity

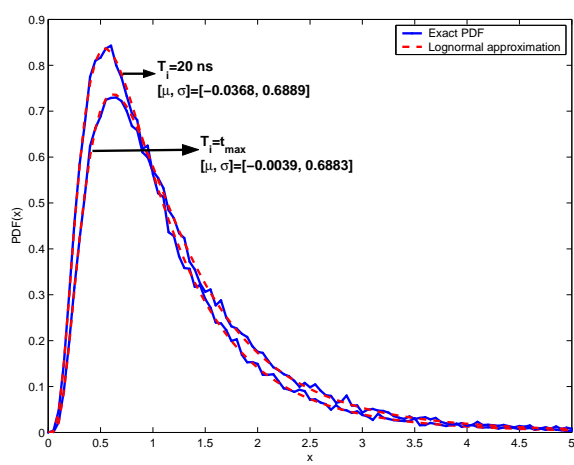


Fig. 3. PDFs of the lognormal approximations over CM2 in the absence of IPI.  $t_{max}$  stands for the maximum delay-spread of the channel.

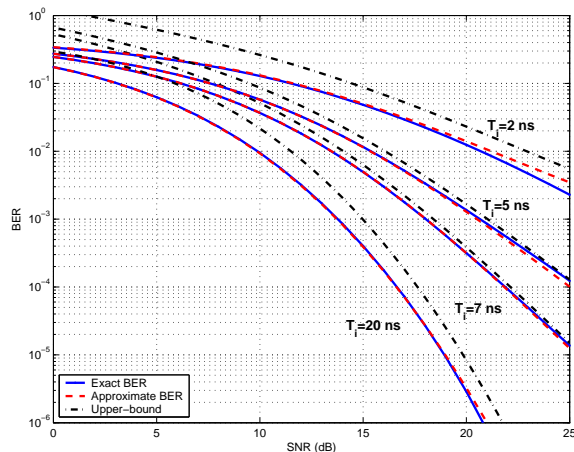


Fig. 4. BER performance over CM2 in the absence of IPI.

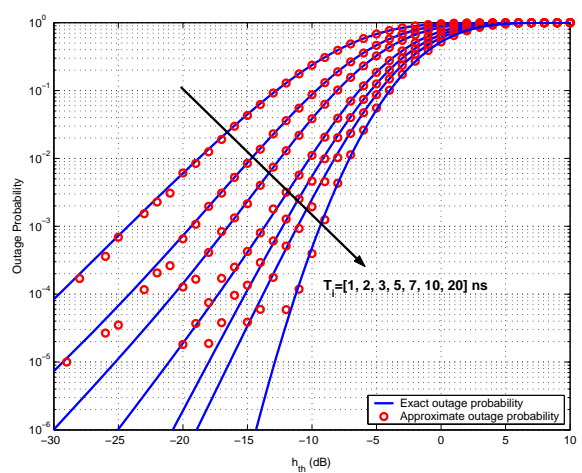


Fig. 5. Outage probability over CM1 in the absence of IPI.

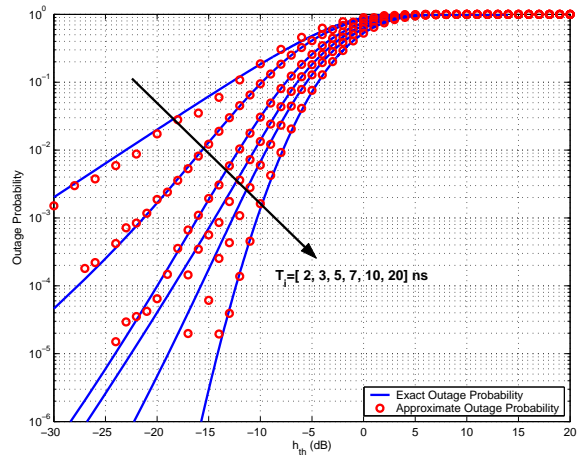


Fig. 6. Outage probability over CM1 in the presence of IPI.

order (related to the steepness of the BER curves that increases with the integration times).

Figures 5 and 6 compare the exact outage probabilities with the approximate expressions given in eq. (9), (13) and (20) over CM1 in the no-IPI and IPI cases. The approximating distributions used for evaluating the outage probabilities are given in table-I for the no-IPI case and in table-II for the IPI case. Results show that the outage probabilities can be accurately approximated by the proposed expressions over the entire ranges of  $h_{th}$  and  $T_i$ . Comparing the results in Fig. 5 and Fig. 6 shows the degradations introduced by IPI. For example, for  $T_i = 2$  ns and  $h_{th} = -20$  dB, the presence of IPI results in an increase in the outage probability from  $6 \times 10^{-4}$  to  $2 \times 10^{-2}$ . Variations in the outage probability are less pronounced for larger integration times.

#### IV. CONCLUSION

We presented a framework for evaluating the performance of UWB systems over indoor wireless channels. This framework is based on proposing convenient approximating distributions for the energy captured over these channels. The proposed approximations result in simple pdf expressions with acceptable

levels of accuracy over the entire range of parameters. These approximations are useful in evaluating the BER and outage performance and they capture the diversity orders that can be achieved over the IEEE 802.15.3a channel model.

#### REFERENCES

- [1] J. Foerster, "Channel modeling sub-committee Report Final," IEEE, Document IEEE P802.15-02/490r1-SG3a, 2003.
- [2] N. C. Beaulieu, A. A. Abu-Dayya, and P. J. McLane, "Estimating the distribution of a sum of independent lognormal random variables," *IEEE Trans. Commun. Technol.*, vol. 43, pp. 2869–2873, December 1995.
- [3] N. C. Beaulieu and Q. Xie, "An optimal lognormal approximation to lognormal sum distributions," *IEEE Trans. Commun. Technol.*, vol. 53, pp. 479–489, March 2004.
- [4] H. Nie and S. Chen, "Lognormal sum approximation with type IV Pearson distribution," *IEEE Commun. Lett.*, vol. 11, no. 10, pp. 790–792, October 2007.
- [5] F. W. Z. Liu, J. Almhana and R. McGorman, "Mixture lognormal approximations to lognormal sum distributions," *IEEE Commun. Lett.*, vol. 11, pp. 711–713, September 2007.
- [6] Z. Liu, J. Almhana, and R. McGorman, "Approximating lognormal sum distributions with power lognormal distributions," *IEEE Trans. Veh. Technol.*, vol. 57, pp. 2611–2617, July 2008.
- [7] M. Pratesi, F. Santucci, and F. Graziosi, "Generalized moment matching for the linear combination of lognormal R.V.s: application to outage analysis in wireless systems," *IEEE Trans. Wireless Commun.*, vol. 5, pp. 1122 – 1132, May 2006.

- [8] N. B. Mehta, J. Wu, A. F. Molisch, and J. Zhang, "Approximating a sum of random variables with a lognormal," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 2690 – 2699, July 2007.
- [9] L. Zhao and J. Ding, "Least squares approximations to lognormal sum distributions," *IEEE Trans. Veh. Technol.*, vol. 56, pp. 991–997, March 2007.
- [10] J. A. Gubner and K. Hao, "A computable formula for the average bit-error probability as a function of window size for the IEEE 802.15.3a UWB channel model," *IEEE Trans. Microwave Theory Tech.*, vol. 54, no. 4, pp. 1762–1768, June 2006.
- [11] H. Liu, "Error performance of a pulse amplitude and position modulated ultra-wideband system over lognormal fading channels," *IEEE Commun. Lett.*, vol. 7, pp. 531–533, Nov. 2003.
- [12] M. D. Renzo, F. Graziosi, and F. Santucci, "An exact framework for performance analysis of IRUWB systems: The need for approximations," *IEEE Commun. Lett.*, vol. 11, no. 10, pp. 769 – 771, October 2007.
- [13] M. Eslami and X. Dong, "RAKE-MMSE-equalizer performance for UWB," *IEEE Commun. Lett.*, vol. 9, no. 6, pp. 502–504, June 2005.
- [14] M. Di-Renzo, F. Graziosi, and F. Santucci, "Approximating the linear combination of log-normal rvs via Pearson type iv distribution for UWB performance analysis," *IEEE Trans. Commun.*, vol. 57, no. 2, pp. 388–403, February 2009.
- [15] M. D. Renzo, F. Graziosi, and F. Santucci, "Further results on the approximation of log-normal power sum via Pearson type IV distribution: a general formula for log-moments computation," *IEEE Trans. Commun.*, vol. 57, no. 4, pp. 893 – 898, April 2009.
- [16] S. Halme, B. Levitt, and R. Orr, "Bounds and approximations for some integral expression involving lognormal statistics," *MIT Res. Lab. Electron. Quart. Prog. Rept.*, 1969.