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Problem Posing: A Teaching/Learning Strategy to Enhance Problem Solving Abilities

By

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To my loving father, who planted the love of seeking knowledge in my heart…

As a child, you were my guide…

And my inspiration as an adult…

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ABSTRACT

Researchers argue that one of the main aims of mathematics education is to develop students' abilities to solve a wide range of complex problems. Thus, advocates of the reformed movements in mathematics education constantly research various methods which ought to enhance the ability to be a good problem solver. The current research explores the effectiveness of using problem posing as a teaching strategy to enhance students' problem solving abilities. It also tries to determine the effects of such a strategy on students with originally different achievement level. This is accomplished by using a mixed-method research design for data collection and analysis, consisting of an experimental study comparing two groups of grade-8 students, a control group and an experimental group. A pretest and a posttest are administered to both groups to determine the level of improvement after the study. The study evaluates the performance of students in these tests, according to a problem solving rubric which targets four main problem solving abilities: "understand a problem", "use information appropriately", "apply appropriate procedures/representation", and "answer a problem". On the other hand, clinical interviews are conducted with a selected sample consisting of three pairs from each group at different levels of achievement, then qualitatively analyzed to explore the effect of problem posing activities on students' problem solving abilities. The study shows that students who were subject to problem posing activities demonstrated more improvement than students who were subject to the usual method of solving word problems. The researcher also concludes that the average- and high- math achieving students were the most affected by the problem posing activities. Students who were subject to the problem posing activities were more successful in problems that require logical thinking and reasoning. On the other hand, students in the control group were more successful in problems of procedural nature. Regarding the assessed problem solving abilities, the whole sample, and in particular the high-achieving group, revealed a significant improvement in the ability to "understand a problem". The study concludes with recommendations for further research on the use of problem posing strategies at the intermediate level, with larger samples and more time dedicated for the implementation.

Keywords: Mathematical problem solving, Problem posing, Teaching strategy, Eighth grade, Conceptual understanding, Critical thinking.

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List of Codes

CG	Control group
EG	Experimental group
EG-L	Low and limited math achieving students of experimental group
CG-L	Low and limited achieving math achieving students of the control group
EG-A	Average math achieving students of the experimental group
CG-A	Average math achieving students of the control group
EG-H	High math achieving students of the experimental group
CG-H	High math achieving students of the control group
EP-L	Low achieving pair of the experimental group
CP-L	Low achieving pair of the control group
EP-A	Average achieving pair of the experimental group
CP-A	Average achieving pair of the control group
EP-H	High achieving pair of the experimental group
CP-H	High achieving pair of the control group

CHAPTER ONE

INTRODUCTION

1.1 Overview

One of the main goals of education is to prepare students to be good problem solvers which ought to help them become effective members of their societies. The problems offered by school mathematics curricula are often organized and well-structured to be solved through previously taught strategies. However, the problems that students will face later in their daily lives are rarely structured and often require creative and novel ways for solving. Kilpatrick (1987) believes that most of the problems in real life must be “created or discovered by the solver, who gives the problem an initial formulation” (p.124). Realistic situations mostly require critical reflection, identification of possible problems that may arise at later stages and generation of suitable methods of solution with whatever available resources. Being able to recognize and formulate mathematical problems can assist in coming up with adequate decisions (Singer, Ellerton & Cai, 2013).

This vital characteristic of an effective problem solver can be developed and nurtured by exposing learners to problem posing activities as a regular part of school curricula (Silver, 1994; Singer & Voica, 2012). This is emphasized by several teaching organizations, such as the National Council of Teachers of Mathematics, which stresses the need for exposing learners to problem posing activities in their classrooms (NCTM, 2000).

Mathematical problem posing is recognized by several leaders in Mathematics education as an important feature that should exist during the teaching and learning process of mathematical topics (Silver, 1994). These leaders believe that using such a strategy in class is advocated for by constructivist theories of teaching/learning. Silver (1994) states that the process of problem posing lies in the core of mathematics discipline and way of thinking. Stimulating learners to inquire about issues of concern in a strategic method should be one of the main goals of education, and this can be achieved by exposing them to problem posing situations (Stoyanova, 2003). Advocates of such a teaching and learning strategy believe that it is of great importance for students to see themselves as posers rather than mere receivers of tasks and problems suggested by instructors (Olson & Knott, 2012).

1.2 Statement of the Problem

Researchers argue that one of the main aims of mathematics education is to develop students' abilities to solve a wide range of complex problems. Thus, recent educational reform movements place realistic problem solving at the heart of mathematics education. However, most current mathematics curricula emphasize conventional ways of approaching real-life problems. Such ways can be summarized as follows: a) read and understand the problem, b) mentally transform the given and the question of the problem by choosing a suitable mathematical strategy then c) apply the involved algorithm called for by this strategy in order to provide an acceptable answer, and finally d) look back to reflect on and check the obtained results for the purpose of connecting it to the situation at hand. Usually students overlook this final step, and teachers don't often dedicate the sufficient time and considerable attention it deserves or requires.

Teacher-centered teaching in mathematics leads mainly to a procedurally oriented teaching whose aim is to teach students how to solve certain types of problems with minimal consideration to developing their problem solving abilities. Such abilities, however, would equip them for dealing with new situations and solving new types of problems. Studies reveal that in tasks that require high cognitive demands, realistic answers are hard to produce due to students' inflexibility in thinking processes and in applying appropriate models and strategies (Bayazit, 2013). This happens mainly because students learn to focus on the results they need to obtain from solving word problems without emphasis on the strategies used and on choosing the most appropriate one. They have rare occasions to be involved in formulating such strategies and processes, but rather they are mere recipients of these teacher imposed strategies.

Educators often expect students to use rote practice and drill in order to get an answer as fast as possible. Such teaching and learning methods produce learners who find problem solving tasks overwhelming and somehow ambiguous. They do not regard such tasks as useful tools for their everyday lives and try to avoid performing them as much as possible

1.3 Purpose

The purpose of this research is to study the effect of implementing problem posing as a teaching and learning strategy on the problem solving abilities of students when a problem posing approach is used during the teaching and learning process of problem solving skills at the intermediate level, specifically grade 8.

1.4 Significance of the Study

Researchers worldwide agree that the studies targeting problem posing in mathematics education are not sufficient. It is also agreed that there is a need for further research, especially about the effect of problem posing on students' problem solving abilities at the intermediate level, which this study explores. Current research about problem posing targets mainly students at the lower elementary level of schooling. Thus, the significance of this study lies in the fact that it will contribute to the current literature with quantitative and qualitative evidence that may reveal the nature of the effects that problem posing would have on the problem solving abilities of students at the intermediate level. If the current study shows improvement in mathematical problem solving abilities of intermediate students, this would have some significant implications to consider in terms of curriculum planning. It would raise, among curriculum developers and education administrators, the issue of considering the integration of mathematical problem posing in the Lebanese Mathematics curriculum in general. It would also motivate teachers to use problem posing as a strategy in their teaching approaches.

Another significance of this study lies in the fact that problem posing involves learners in higher-order thinking and active learning assignments which is important for the development of their problem solving abilities. It provides a link between ill- structured problem-based learning and some sort of scientific inquiry as it involves learners in higher-order questioning skills. The study targets learners' conceptual understanding and involves students in critical reflection as they shift away from only acquiring knowledge and work more on applying it (Nardone &Lee, 2011, Stoyanova, 2003). Thus, this research targets the teaching and learning of Mathematics with understanding rather than by mechanical procedures and memorized and drilled algorithms. This raises the expectations that students will retain life-long knowledge and understanding of mathematical concepts.

1.5 Definition of Terms

Many mathematicians define problem posing as the generation of previously unsolved problems. On the other hand, in empirical studies, posing problems is described as the process of formulating novel problems of unknown solutions or the reformulation of existing problems, mainly ill-structured ones (Pelczer & Rodriguez, 2011). Singer and Voica (2012) mention a variety of terms that refer to problem posing such as "problem finding, problem sensing, problem formulating, creative problem-discovering, problematizing, problem creating and problem envestigating".

A general definition of problem posing according to Silver is "both the generation of new problems and the re-formulation of given problems" (Silver, 1994, p.19). During a problem posing episode, the emphasis is on the situation of the problem itself and not on the final answer (Olson & Knott, 2012).

The present research study will compare two different strategies for teaching and learning problem solving: a strategy that handles problem solving situations using the conventional method of teaching problem solving, and a strategy that teaches problem solving situations using problem posing techniques. The first strategy will be referred to as a *nonposing* strategy, and the second as a *posing* strategy. In the *nonposing* strategy, problems are posed by the teacher, discussed and solved as s/he would do in a regular problem solving class. In the *posing* strategy, both the teacher and the students will share the responsibility of posing problems of interest to be discussed and solved. Thus, for the current research, problem posing is a student-centered pedagogical approach where students are involved in the following activities for the purpose of learning:

a- Discussions and investigations that may arise upon changing one or more of the given data of a given problem.

b- Posing extension questions to previously defined problem situations, by relying on why, what if and what if not questions.

c- Posing their own problems that are similar to previously solved ones.

1.6 Research Questions

The proposed study assumes the following hypothesis: Using mathematical problem posing as a teaching and learning strategy to teach problem solving is expected to improve learners' problem solving abilities. The study aims to answer the following research question:

What impact does problem posing have on students' mathematical and problem solving capabilities?

This research will also attempt to answer the following supplementary research questions:

1- How do the problem solving abilities of students develop throughout implementing problem posing strategies while teaching problem solving in class?

2- How will the inclusion of problem posing activities affect differently students with originally different levels of problem solving abilities?

As an attempt to answer the above mentioned research question, an experimental study is conducted on two groups of eight graders, in which one group is the experimental group, and the other is the control group. A pretest is administered to both groups to determine the starting level of each group. Problem posing activities are implemented in the experimental group, whereas problem solving is taught using a conventional teaching and learning process. At the end of the study, a posttest is administered to determine the achievement levels of both groups. Furthermore, a qualitative study is performed on an assigned group for a deeper analysis.

The coming chapters discuss in more details the theoretical background and framework on which the currect study was based on. Furthermore, a description of the method used while conduting this study is detailed in Chapter 3. A thorough analysis of both the quantitative and qualitative data is presented in Chapter 4, and finally a comprehensive conclusion is discussed in the last chapter.

CHAPTER TWO

LITERATURE REVIEW

Recent studies suggest that using problem posing as a strategy for teaching and learning problem solving in mathematics have a positive effect on students' mathematical performance in general, and problem solving abilities in specific. However, researchers agree that there is a need for further research that explores the link between problem solving and problem posing, and in particular, the effect of problem posing tasks on students' problem solving skills. The current study focuses on the uses of problem posing as a strategy for teaching mathematical problem solving. This section of the study provides a general review about the use of problem posing in school curricula in general, with emphasis on the link between problem posing and problem solving, and the way problem posing activities can be implemented in mathematics classrooms. The review inquires into the advantages of using problem posing as a teaching and learning strategy, as well as describes the difficulties faced by teachers upon its implementation.

2.1 Problem solving in mathematics education

It is currently believed that a problem based approach in mathematics leads to better mathematical performance. A problem is a situation where the path to a desired goal is not direct, which causes cognitive conflicts. Problem solving is a dynamic process in which one has to understand the situation, make a plan, select or create methods and strategies to solve, and then apply them to reach a solution. Finally one has to look back and check the obtained results (Bayazit, 2013). The National Council of Teachers of Mathematics (1989, p.471) defines problem solving as follows:

> the process of applying acquired knowledge to new and unfamiliar situations… problem solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error.

The significance of problem solving is that it connects mathematical concepts to real life and other subjects in school curricula. Thus mathematical concepts should be regarded as tools that enable us to solve problems of interest. Usually, students and sometimes teachers treat concepts as isolated entities and fail to view these concepts as part of a greater body of integrated knowledge. Advocates of inquiry based learning believe that the focus should not be on merely getting a correct answer to a problem, but on acquiring it in the most effective way, and reflecting on how the involved teaching and learning processes would help later on. Arndt (2009) believes that learning actually happens after the problem is solved by reflecting on the solution process, assimilating what has been learnt and deciding how the solving processes would help in later experiences.

2.2 A more dynamic problem solving approach

The way most mathematics curricula are designed is to train students in getting correct answers using memorized algorithms during which an overall shallow understanding of concepts is obtained. Arndt (2009) explains that drilling an algorithm enables students to obtain faster answers; however, using the algorithm to investigate the connections in a given complex problem is more of a challenge and promotes deeper conceptual understanding. A more productive solving of problems requires students to use past knowledge to assist them in deciding where to begin and how to carry on with the solution process.

Recent educational reform movements advocate for a more active teaching learning approach which requires learners to take an active role in their learning experiences, and advise educators to encourage leaners to assume authority in building their own knowledge. Learners should be engaged in higher order thinking processes. Such higher order thinking processes can be nurtured by student-centered inquiry-based teaching of problem solving. In an inquiry oriented approach, students and teachers share the responsibility for formulating and solving problems. Thus, they depend less on textbook material and teacher proposed questions. Most research reveals that teacher assigned tasks are straight to the point and do not require high cognitive demands (Brown & Walter, 1983; Crespo, 2003; Hirashima, Nakano &Takeuchi, 2000; Silver, 1997). Tasks are remodeled to be less demanding in order to avoid conflict. This however gives students the wrong idea about Mathematics being disconnected from reality and does not allow them to connect mathematical concepts to their real-life application. Furthermore, research emphasizes that students learn more in classes that tackle tasks of a high level of cognitive demands.

NCTM (2000) clearly specifies that effective problem solving should not be introduced by treating word problems as situations that ought to target specific disintegrated mathematical concepts. Word problems should be presented as situations that provide opportunities for students to see mathematics as a whole body of knowledge that can also be integrated with other subjects. Thus NCTM (as cited by Silver, 1994) encourages teachers to avoid the use of traditional textbook problems and to hand the role of generating questions of interest to students (Silver, 1994). Thus reform movements stress that mathematics should be taught and learnt as a dynamic living subject. It should be viewed as a world of patterns and relationships / connections to be discovered and not only mere numbers and operations to be carried out. The Lebanese Mathematics Curriculum emphasizes, in its introduction and general objectives, personal construction of own mathematical knowledge (The Center for Educational Research and Development, 1997). Problem solving is highly called for, emphasizing that teaching and learning should be related to real life, and thus learning concepts should be by starting with real life situations. Students should then be encouraged to discover mathematical methods and algorithms, as well as to formulate, doubt and modify their own conjectures, instead of passively receiving teacher imposed algorithms and conjectures.

2.3 Problem posing in mathematics education

A good problem solver can be characterized as one who takes risks, develops good communication skills, understands basic facts and number sense, has knowledge of mathematical language and uses common sense. Furthermore, a competent solver forms connections to previous experiences, organizes thinking and written work, and is able to work constructively with others (Arndt, 2009). Problem posing provides a rich medium for developing these characteristics in students since problem posing as a teaching strategy is considered to be a “feature of inquiry-oriented instruction” (Silver, 1994, p.21).

Research about introducing problem posing as an important part of school curricula began to emerge in the 1980s and 1990s. Educational reform movements such as the NCTM call for the use of problem posing in teaching. However, long before that, Einsten and Infeld mentioned that viewing existing situations from different perspectives and formulating new questions are important for creative scientific progress (Ellerton, 2013). In Turkey, problem posing activities have been introduced in Mathematics curriculum for grades one to five since 2005 (Kilic, 2013b). Researchers in the field of problem posing called for integrating this aspect of mathematics in the process of learning since posing leads to a better appreciation and understanding of the origin of mathematical problems (Contreras, 2007); however, problem posing is not usually given as much importance as problem solving, when discussing curricula. Using problem posing as a teaching and learning strategy is not a popular one, and the reason for this unpopularity is because teachers view these activities as a heavy burden. This is mainly due to the high demands of such activities where great emphasis should be put on organizing a posing task that fits the educational objective. Thus, teachers find it difficult to implement such activities due to lack of sufficient resources, in comparison to those resources that support a more conventional and procedural view of mathematical problems. However, teachers should realize the potential of introducing posing activities as part of the teaching and learning sequence and as a rich medium for enhancing students' performance.

Advocates of introducing problem posing as a main feature in mathematics curriculum and as part of instructors' teaching and learning strategies focus less on teaching the conventional rigid mathematical topics and focus more on preparing learners to be able to use their mathematical knowledge in a more practical and realistic way (Silver, 1994). This enables learners to form connections between subject contents and the realistic world in which they will eventually have to survive, which is basically what schools should be preparing young learners to do. An effective ability of posing the right questions in specific and mainly unstructured situations proves to be of great value when students enter the workplace and are required to reflect, analyze and make decisions (Stoyanova, 2003).

2.4 Problem posing environment

Silver (1994) defined problem posing as the process of generating new problems or reformulating ill-defined ones for the purpose of exploring intended mathematical concepts and the relations that exist among them. Researchers exploring the link between problem solving and posing claim that teaching and learning problem solving through problem posing activities enhances problem comprehension. Thus, they regard problem posing as both a teaching activity and strategy (Kilic, 2013 a&b). Furthermore, Kilpatrick strongly claims that problem posing should be used as both, a way and a purpose of instruction (1987).

A successful problem posing task is based on several important elements: clarity, consistency, originality and the correctness of the mathematical concepts in the task. Kilic (2013a) provides a thorough description of the stages involved in a posing environment. These stages can be summarized as follows: Describing the content, defining the problem, personalizing the problem, and discussing alternatives by asking relevant and inductive questions.

There are three main categories of posing situations: free, semi-structured and structured situations (Stoyanova, 2003). In a free posing situation, students are asked to generate a problem on the basis of a naturalistic situation. An example on such a situation would be to ask students to make up a money problem. In a semi-structured posing situation, students would be given an open situation and asked to finish it using knowledge, skills and concepts from previous experiences. For example, students would be provided with some given information and asked to formulate questions, or to pose questions based on a given answer. In a structured posing situation, a well-structured problem is given and the task is to construct new problems that relate to the given problem or solution.

Stoyanova (2003) suggested some procedures in which students are guided towards formulating such related problems. These procedures are mentioned below:

1- Using what-if, what-if-not questions.

2- Changing unknowns and keeping the task, or vice-versa.

3- Posing classes of problems that are mathematically similar.

4- Posing a set of problems interconnected with specific problem posing fields, problem cycles or series.

5- Presenting a problem in a different information format.

6- Improving syntax and structural characteristics of a specific situation.

7- Posing problems that are based on changes in the problem statement that affect or do not affect the solution.

8- Restating the problem on the basis of its solution.

9- Reformulating a specific problem in one's own words without changing the mathematical nature.

10- Completing questions or identifying missing information.

2.5 The link between problem solving and problem posing

Researchers call for a need to explore the type of links that exist between problem posing and problem solving (Ellerton, 2013). Kontorovich, Koichu, Leiken and Berman (2012) see posing and solving to be different in nature yet complementary. Even more, they regard posing as a special case of solving where the initial given state is a posing task, and the goal is to construct a problem that meets the demands of the task. Bonotto (2010) considers that posing should accompany solving in mathematics education as both are of great importance when learning mathematical concepts since both lie at the heart of mathematical thinking.

In a study that explored the link between problem posing and problem solving, Cai and Hwang (2003) stressed the fact that posing represents an important aspect of mathematical research and education, and that it is believed to develop learners' mathematical thinking and problem solving abilities, and thus agreed with Polya (1957) that posing problems can be considered an effective strategy for teaching and learning problem solving. Their study reveals a link between the ability to pose extension problems and the skill to solve problems at the intermediate level. This comes in harmony with Polya's (1957) claim that complex problem solving processes require the formulation and solution of subsidiary problems. Contreras (2007) believes that extension problems provide the opportunity to dwell on such relations and to frame them in a larger more general context.

Researchers believe that activities in which students reflect on mathematical problems followed by posing new but similar problems may improve their problem solving abilities (English, 1997 a&b; Silver, 1997; Stoyanova, 2003). They view posing activities as a nurturing medium for stimulating problem solving actions. However, there is need for further research that studies the relationship between problem posing and solving in order to get a better understanding on how students' composed problems would lead to better solution activities (Cifarelli & Sheets, 2009).

Cifarelli and Sheets (2009) suggest that a dynamic relation exists between problem posing and solving. A student who is involved in a solving process might get a result that would challenge the initial goals. One of the ways such a student might react is to pose further questions which might lead to a reformulation of the prior problem. Here lays the significance of training students in the process of posing problems. Furthermore, posing in general is a more complicated task than solving, specifically to beginners. Problem posing is an activity that requires creativity and is widely recognized to be intellectually more demanding than problem solving. Thus Miwa, Terrai, Okamoto and Nakaike (2013) suggest that learning how to pose should follow problem solving learning. These two activities once linked tend to reinforce each other. Perrin (2007) believes that solving problems is essential for learning, and that it should be both an aim in the mathematics curriculum and an effective tool for learning mathematical concepts. The researcher believes that problem solving is "relative to the solver" (p.182). Depending on the cognitive level of students, what might be seen as a routine problem to one student might be a nonroutine task for another. The challenge is to provide a task that would be a challenge for all students. If students were to pose their own questions of interest, basically it can be assumed that all students would be involved in their personal nonroutine tasks. Perrin strongly believes that students learn better when they explore questions they formulate instead of solving teacher assigned problems. Perrin (2007) also claims that implementing posing activities would increase students' problem solving abilities, raise their interest, solidify concepts and develop understanding in a nontraditional way.

2.6 Advantages of Problem Posing in Math Education

2.6.1. Positive influence on problem solving.

More and more studies reveal that the use of problem posing as a teaching and learning strategy can have a positive effect on students' problem solving abilities (Akay & Boz, 2010; Ellerton, 2013; Kesan, Kaya & Guvercin, 2010; Silver, 1994; Singer et al, 2013). Silver (1994) believes that by involving students in posing activities, they become more aware of mathematical facts and connections among concepts. Ellerton's study (2013), in which the participants were pre-service teachers, used a problem posing approach while teaching problem solving. After the study, the participating pre-service teachers stated that they had a better understanding of the involved problem structures, after performing the assigned posing activities. Thus, they were more able to solve similar problems since then they understood the processes and elements involved. Furthermore, the study conducted by English (1997 a & b), which focused on developing the problem posing abilities of fifth graders, revealed that students, who were subjected to problem posing activities, displayed a better understanding of problem structure and situation, as well as recognizing problems of a similar structure, than those who did not participate in the study.

The findings of the study conducted by Kesan et al (2010) reveal that by using a problem posing approach in mathematics secondary classes, students' problem solving abilities, as well as general mathematical abilities, improved because problem posing activities foster logical thinking and reasoning. In her study about teachers' views about problem posing, Kilic (2013 a & b) explains that the advantages of using problem posing as a teaching and learning strategy lay in the fact that it develops mathematical skills and fosters critical thinking and creativity. It also helps in connecting concepts and enhancing reflection, flexibility, self-assessing and communication skills. Kilic concludes that most teachers in her study believe that problem posing contributes positively to students' learning. Kesan et al (2010) claim that "problem posing is a more intellectually demanding task than solving problem tasks" (p. 678), since during solving conventional problems from regular textbooks, students resort to previously learned methods. However, while working on problem posing tasks, students work on a larger scale and attempt to grasp the wider picture in order to relate mathematical concepts together and with the real world. When students grapple with concepts and the cognitive demands required by a problem, the learning outcome is deeply rooted and their understanding is emphasized (Nardone & Lee, 2011). Thus, they delve deeper into concepts and deal with intellectual contexts which ought to enrich their higher-order mathematical abilities. Such activities are meant to foster conceptual understanding as well as communication and reasoning abilities. In addition, problem solving usually requires the formulation of further questions during the solving process. Thus a good problem poser has more robust problem-solving strategies (Cai, et al., 2012).

English (1997 a&b) discusses her experience while exposing her students to a problem posing program. She claims that exposing students to diverse problem posing activities will foster problem solving abilities and will lead to a more diverse and flexible mathematical mind, which should broaden mathematical perception and solidify concepts. She explains that learners need to be able to identify various meanings in formal operations in order to deeply grasp the concept within these operations. Usually school curricula do not focus on this, and according to research, students are found to be inflexible in this area (English, 1997 a&b). For example, they always assign 'take away' to the operation '_'. Posing activities increase students' mathematical flexibility, and thus they are more able to link operations and formal symbols to a variety of meanings in real life context. Thus, problem posing activities lead to more divergent thinking (Ghasempour, Bakar, & Jahanshahloo, 2013). English also suggests that involving students in posing activities should draw their attention to both conceptual and structural complexities of problems they create and solve. This ought to enhance their problem solving abilities, as was emphasized by Stoyanova (2003) who perceives problem posing as a strategy with potential for aiding students to become better problem solvers.

The study conducted by Cai and Hwang (2003), on the link between problem posing and problem solving abilities, reveals that students who were able to pose subsidiary problems were able to use complex problem solving strategies. This result implies that integrating posing activities in instructional processes may have a positive influence on students' learning in terms of using diverse complex solution strategies. Furthermore, the study by Limin, Van Dooren and Verschaffel (2013), exploring the relationship between students' problem solving and problem posing abilities, reveals that there exists a strong relationship between problem posing and problem solving abilities, as well as general mathematical abilities. This indicates that problem posing has an important role in mathematics learning.

2.6.2. Medium for inquiry, conjecturing and reflecting.

Mathematical explorations allow students to grasp a clearer idea of the nature of mathematics as a field of inquiry and investigation, rather than a mere imitation of imposed algorithms and strategies. Thus, problem posing provides a medium for such explorations to occur in a mathematics classroom. Involving students in posing activities allows them to formulate conjectures, and the need to justify these conjectures arises when they try to explain them. Thus the habit of testing, modifying or reformulating conjectures is created (Silver, 2013).

Wilson, Fernandez and Hadaway (1993) insist that what is learnt after the problem is what matters and not just the implicit answer. They also claim that the “looking back stage” of Polya’s problem solving stages is not emphasized enough in school curricula in general. This stage emphasizes: a) reflecting on given and solution, b) interpreting or reinterpreting and checking results, c) reflecting on the possibility of applying the proposed strategy or acquired results in another situation, and d) posing new problems. Integrating problem posing as a teaching strategy provides the opportunity to dwell more on the “looking back stage”. Reflection makes students realize that they need to reject their roles of receivers of knowledge and to assume more constructivist roles in the process of learning (Nardone & Lee, 2011). Kontorovich et al. (2012) explain that problem posing activities provide a medium where uncertainties occur, which demands the use of reasoning. This involves students in reflecting on causes and reasons of solutions, instead of focusing only on mathematical processes. Cankoy (2014) agrees with this point and claims that even when a student posed question may be unsolvable, a positive learning outcome can still exist by reflecting on the mathematical inquiries that arise as well as by exploring the elements that make the solution impossible to be achieved. This is crucial for developing conceptual understanding. Based on the study by Da Ponte and Henriques (2013), which investigated the problem posing activities that university students experienced while exploring unconventional activities, students were found to realize the importance of reflecting on answers. Through this reflection, they were able to test, modify, refute or justify conjectures. Thus, they were engaged in a cycle which reinforced their view of mathematical activity as a recursive process of inquiry and exploration.

2.6.3 Nurturing a medium for intellectual and cognitive development.

Studies reveal that students undergoing problem posing and solving activities perform substantially better in math than those undergoing only problem solving activities. In her study about problem posing in a Calculus classroom, Perrin (2007) advises that encouraging students to formulate and solve questions of their interest ought to help them acquire a better and deeper understanding of desired concepts. In addition, students are more likely to value what they are learning because they are constructing their own knowledge, instead of simply receiving it from someone else. The study conducted by Kesan et al (2010) reveals a significant increase in the scores of the group that was exposed to problem posing tasks when compared to the scores of the control group. The researchers conclude that when students are exposed to problem posing activities, their verbal and analytical abilities are enriched. Furthermore, this leads to better flexibility in thinking and increased motivation to generate challenging problems.

Stoyanovas's study about teaching problem solving through problem posing activities reveals that students also seemed to be able to integrate math with other subjects as well as to develop innovative writing skills. Making up questions and creating applications help students link the concrete situations with abstract concepts. In her article about extending students' mathematical understanding by problem posing, Stoyanova (2003) emphasizes that it is important to solve the problem and reach a correct answer. However, it is also important to understand the solution process, method and situation, and also to appreciate the importance of reporting exact written explanations. To improve students' mathematical language, it would be useful to provide them with a poorly written solution and ask them to improve it. Stoyanova also suggests presenting different solution formats and discussing which makes a better solution presentation. Stoyanova also recommends starting with students' mistakes to build further discussions.

Problem posing activities also link personal interests with mathematics, and promote independent thinking processes. During problem posing tasks, the emphasis should be on presenting and discussing mathematical concepts rather than regarding posing as an end product by itself (Pelczer & Rodriguez, 2011). Bonotto (2010) believes that when students are working on situations that they posed from their surroundings, they find a more relevant meaning to mathematics in terms of sense making and usefulness in daily experiences. Consequently, problem posing nurtures the students' intellectual and cognitive development.

2.6.4. A chance to monitor students' understanding.

Problem posing activities enable teachers to get a better understanding of students' learning and understanding of mathematical concepts, and thus they are more capable of assessing understanding and improving the teaching process. As students work on posing activities, teachers have the opportunity to monitor their progress and to detect any misconceptions or failure in linking mathematical concepts to realistic situations. Stoyanova (2003) thinks that when students create their own mathematical problems, they express mathematical concepts the way they see them, and thus educators may have a clearer insight about students' understanding, development of concepts, and their perception about mathematics. Teachers will then be able to plan for any required reteaching. As students become more experienced in problem posing, they become more able to mathematize real-life situations, which will increase their motivation to learn. Dealing with mathematical topics may become less intimidating since the learning process becomes more personal as it shifts away from just applying learnt algorithms and memorization. Problem posing, however, is an open-ended activity which allows students to share authority in posing their own problems. This means, there is no definite right or wrong. Each student poses personal problem situations which leads to different solutions, making it rather hard for teachers to give implicit feedback for each individual.

2.6.5. A social context for learning.

Crespo (2013) describes problem posing as an authentic activity that is important for "learning to think and act in a community of practice" (p.246). As students generate their problems of interest, a socio-emotional commitment towards solving these problems is created, which gives them purpose and motivation for the learning processes involved (Lijnse, 2005). Through the use of problem posing as a teaching and learning strategy, students are involved in building up their knowledge, which becomes more personal to them. Thus they become more interested and motivated to learn. Since problem posing is based on inquiry, the learning setting provides many opportunities for discussion among learners, where they learn to evaluate and criticize the problems that are offered by their peers. Consequently, a valuable social element of learning is emphasized in such a classroom where ideas are exchanged among learners to synthesize new meanings (Singer, Ellerton & Cai, 2013; Olson & Knott, 2013).

2.6.6. Mathematical modeling.

Downton (2013) explains that a variety of resources can be used for posing activities such as diagrams, definitions, statements, concrete material, and real world situations or artifacts which link the formal mathematics with its informal aspect. Downton argues that problem posing not only influences problem solving abilities positively, but also lays ground for mathematical modeling. Modeling is a process in which a real world situation is problematized, translated, worked out, and whose solution is translated back to the original situation to be evaluated and communicated. Often modeling such situations tends not to be so neat and organized and may require posing further questions. The findings of the study by Downton convey that young students (5-6 years) are able to pose questions that are suitable for further inquiry and modeling even though they had no prior training on posing. Furthermore, the findings in English's study (1997a, 1997b) reveal a significant improvement in students' modeling of new problems that are parallel to an existing problem structure. Thus the research suggests that since students are of an inquisitive nature, it would be beneficial to develop this nature, and to provide them with more opportunities to pose, model and solve their own problems.

2.6.7. Fostering creativity.

Silver (1994, p.20) regards problem posing as “a characteristic of creative activity or exceptional talent”. Furthermore, student-posed problems develop students’ creativity (Cai, et al., 2012). Christou, Mousoulides, Pittalis, Pitta-Pantazi and Sriraman (2005) explain that problem posing is an important element in mathematical modeling of real life situations, and that posing questions is the nature of scientific inquiry since it is the way science progresses. They relate creative imagination and the development of mathematical concepts with the posing of new questions, as well as looking for original opportunities and revisiting old problems from a different perspective. Thus they value problem posing and solving as central activities in Mathematics curricula.

Previous studies focused on the possibility of using problem posing as a way of identifying gifted students and found that talented learners are more able to pose problems of a higher-level than average learners (Van Harpen & Presmeg, 2013; Pelczer & Rodriguez, 2011; Silver, 1994; Kesan et al., 2010). Thus, creativity and posing original problems are related, and so novel problem posing can be both a sign of mathematical creativity and a way of enhancing it (Singer et al., 2013). Silver (1994) argues that creativity can be seen during the cycle of formulating, trying to solve, reformulating and then solving a question. As students work on ill-defined situations that they posed, they develop their representational skills and become more fluent and creative while planning solution strategies, especially novel ones. Thus, upon integrating problem posing tasks in the curriculum, students’ temperament towards mathematics may tend to take a more creative nature, since problem posing and solving activities provide a rich environment for developing creativity and flexibility of thought.

This chapter presented a summary of the main theoretical background about problem posing and its effect on the problem solving abilities. This summary situated problem posing at the heart of the constructivist theory of mathematics education, where students are encouraged to assume ownership of constructing their knowledge. Through purposeful problem posing tasks, students are involved in critical thinking, logical reasoning, analysis and inquiry processes, which ought to develop their conceptual understanding.

CHAPTER THREE

METHOD

The purpose of the research is to study the effect of using problem posing as a teaching and learning strategy on the problem solving abilities of students at the intermediate level, specifically grade eight. The study assumes the following hypothesis: using problem posing as a teaching strategy improves students' problem solving abilities.

The study adopts a mixed-method approach, using quantitative and qualitative techniques for data collection and analysis. The sample that was chosen for this research is considered to be relatively a small one, thus, qualitative techniques for analysis were employed to support the qualitative data. Interpretation of both, the quantitative and qualitative data, should prove to be complementary to each other. Qualitative data was also required to provide a better description of the effects that problem posing may have on problem solving abilities of the involved sample.

3.1 Participants

The proposed study targets learners at the middle school level in general. The intended classes were conveniently chosen from a Lebanese, English speaking, private school, and consisted of two grade-8 sections, each consisting of 30 students. Both sections were starting with approximately the same overall math averages, ranges and standard deviations. It is important to note that usually the participating school distributes students in sections based on similar levels of achievement in the two sections. Both groups have had no experience in posing problems of their own for the purpose of exploring mathematical concepts.

3.2 Procedures and Instruments

For the purpose of exploring the effect of implementing problem posing activities on the problem solving abilities of students at the level of grade eight, the following procedure was adopted. One of the two sections was referred to as Experimental Group or EG and the other as Control Group or CG, where Experimental Group was the section in which the problem posing unit plan was implemented, and Control Group was the section in which a non-posing unit plan was implemented. The researcher was the mathematics teacher for Experimental Group, whereas another mathematics teacher was in charge of the Control Group. The researcher coordinated with the other teacher throughout the study.

Furthermore, the researcher collected the participants' math grades from the previous year. The grades were averaged out in each of the two groups, and the ranges and standard deviation of the grades were also calculated to check the similarity of baseline achievement. The participants' previous math average was used to categorize all students in EG and CG into three categories according to their level of achievement: high math achievers (H), average math achievers (A) and limited and low math achievers (L). The categories in each group were referred to as EG-H, EG-A, and EG-L, in Experimental Group, and CG-H, CG-A and CG-L, in Control Group. The scores of the students in each of these categories were analyzed and compared as well.

3.2.1. Pretest.

The two math teachers administered a pretest (Appendix A) to both groups to determine the starting level of problem solving abilities for both groups. The test consisted of three word problems which can be solved using various strategies. The duration of the assessment was 60 minutes for each of the two groups and was done in two consecutive hours, meaning once the test was over in the first group, it started in the second group to prevent the dissemination of the test items.

The test was piloted to ensure validity of questions. The test was administered to a similar sample, a different grade eight section (30 students) that was not participating in the current study. Questions were modified in a way which would help in assessing the different areas specified by the problem solving rubric that was adopted for this study. Tasks related to representing given data and justifying students' solving processes and strategies were added to the original word problems. Students' work was assessed according to a prepared problem solving rubric (Appendix B).

At the same time as the pretest in each group, three interviewers assisted the researcher to conduct clinical interviews, which were video-taped, with 12 chosen students (3 pairs from each group). The interviews took place in a separate room where students solved the test out loud in pairs. This is discussed in more details in a later section in this paper.

3.2.2. Assigned activities.

The two teachers then performed a series of activities as described in pre-prepared unit plans (Appendices C & D). These activities revolved around solving various problem situations. The word problems included in the unit plans for both sections did not involve complex mathematical concepts or procedures, but rather they targeted problem solving skills at the grade eight level. Thus, the focus was on evaluating and analyzing students' abilities instead of their knowledge of rigid mathematical algorithms. In CG, problem solving activities were carried out, explored and solved using the usual non-posing method of teaching problem solving. Problems were introduced, explained by the teacher who highlighted the important elements of the solving process. Students were given some time to explore different solution strategies, and then their answers were discussed and checked for validity. In EG, the teacher used problem posing as a strategy for teaching problem solving. The same problem situations were used in both groups, however with the necessary modifications on the problems for EG to encourage posing activities to take place. The teacher followed previously prepared lesson plans that described both the solving and posing activities to be carried out. A more detailed description of the unit plans prepared for the current study is provided in a later section of this paper.

3.2.3. Posttest.

Once the unit plan was completely implemented, a posttest (Appendix E) was administered to assess the students' problem solving abilities at the end of the study. The posttest was similar to the pretest, in nature and mathematical concepts involved. The problems in the posttest were of a higher level of difficulty than the parallel ones in the pretest. The same procedure as the one followed during the pretest was followed during the posttest. Clinical interviews were also conducted during the posttest with the same 3 pairs of students from each group, as described earlier, to detect changes in students' mathematical problem solving abilities.

After the analysis and evaluation of the posttest, the scores of the two groups were compared and analyzed. Furthermore, the scores of corresponding categories from each group, for example EG-H and CG-H, were compared and analyzed to detect the gradual changes in terms of problem solving abilities for groups of different achievement levels with or without the intervention. This quantitative analysis is discussed further in a later section of this paper

3.2.4. Clinical interviews.

During the pretest and posttest, clinical interviews were conducted with 3 pairs of students from each group to analyze their work and problem solving approaches in the test. The students were video-taped as they solved the tests out loud; however, to respect the privacy of the interviewees, no faces were shown. The video-taping focused on the students' work on paper and recorded their discussions.

Clinical interviewing is a research method which is popular among researchers who are interested in exploring the cognitive processes and strategies that learners use while attempting to work on a given task (Ginsburg, 1997). Such interviews are flexible and usually start with a common question. The interviewer, then, customizes the interview according to how the interviewed student reacts, probing more deeply into student's thinking and misconceptions, if any. The interviewer should ask prompting but neutral and objective questions, without giving away hints on solutions. Clinical interviews are usually student-centered, where the interviewer observes a student's responses and tries to interpret them while following the student's chain of thoughts and cognitive processes.

The twelve students (3 pairs from each group), who were clinically interviewed, were chosen according to their general math average of the previous year and to the previous year's teacher's assessment of their problem solving skills, two high (H), two average (A) and two low (L) achievers. The students were referred to as EP-H for the high achieving pair of students of the experimental group (EG), and EP-A for the average achieving pair of the students of EG, and EP-L for the low and limited achieving pair of students of EG. The students of the similar categories from CG were referred to in a similar way. For the purpose of categorizing the students into the three achievement levels (high, average and low), the researcher assumed that a student's global math achievement level is usually reflective of this student's problem solving achievement level.

Three other teachers helped with the interviews. Each interviewer clinically interviewed a pair of students of the same achievement level as they solved the test together out loud. The researcher sat ahead of time with the interviewers to decide on the type of prompting questions that were asked. Some examples of the questions that were asked are: "Can you explain to me why you started from this specific point? Why did you choose this representation? Why did you choose this solving strategy? Do you think it's possible to find the answer in a shorter way?"

3.2.5. Problem solving rubric.

Problem solving abilities of students were evaluated according to a pre-prepared problem solving rubric (Appendix B). The rubric used in this study was adopted from the Utah Education Network (www.uen.org/Rubric/rubric.cgi?rubric_id=13) with necessary modification to suit the criteria and objectives of the present research. Evaluation of students' problem solving abilities addressed 4 areas of their solving approaches: understanding the situation, using the appropriate provided information, applying correct solving strategies, and answering the problem. Students' work was graded on a scale from 1 to 4 in each of the above mentioned areas, where 1 indicated the lowest assigned grade and 4 indicated the highest grade (figure 1)

.

Math Problem Solving Rubric

Name: ___________________________

	Distinguished - 4	Proficient - 3	Apprentice - 2	Novice - 1
Understands the Problem	Able to interpret all necessary information needed for a complete and correct solution	Able to interpret most given information necessary to start the solving process.	Able to interpret some given information to solve part of the problem or to get part of the solution	Unable to interpret enough given information to get started or make progress
Uses Information Appropriately	Uses all appropriate information correctly	Uses most appropriate information correctly	Uses some appropriate information correctly	Uses inappropriate information
Applies Appropriate Representations/Procedures	Uses correct and advanced procedures/ representations which lead to a correct solution	Applies completely appropriate procedures/representations	Applies some appropriate procedures	Applies inappropriate procedures
Answers the Problem	Correct solution of problem with answer statement and correct labeling of answer.	Correct solution with minor technical error (calculation mistake, copy mistake) with answer statement and correct labeling of answer.	partial answer for problem (due to major mathematical error), no answer statement, answer labeled incorrectly	No answer or wrong answer based upon an inappropriate plan

Figure 1: math problem solving rubric

3.3 Quantitative Data analysis

Data was entered into a Microsoft Excel sheet, and then transferred to the Statistical Package for Social Sciences (SPSS), version 22, which was used for data cleaning, management and analyses.

A database was structured based on the data collected for the study. It was divided into 4 main sections:

1- Baseline information: Intervention related information was added, such as the group to which the student belongs and the achievement level (the three levels mentioned earlier).

2- Pretest data: The scores were divided over the three problems of the pretest and entered individually, after which they were averaged out.

3- Posttest data: The scores were divided over the three problems of the posttest, entered individually, after which they were averaged out.

Data analyses was carried out at three levels:

1- Descriptive level: Data was summarized and presented as mean and standard deviation (SD) for numerical data.

2- Bivariate level: the association between scores between the two groups was assessed using the Independent student's t-test. The difference between the different scores of students within the same group was assessed using the paired t-test.

Moreover, students in each of the two groups were divided into three categories based on their achievement level (mentioned above). Accordingly, stratified analyses were carried out according to these groups to identify whether the intervention had more or less effect in one of these groups. This should give the researcher a clearer idea on how problem posing activities affected students with different achievement levels, targeting the second supplementary research question. For example, did teaching by problem posing improve problem solving abilities of high achievers more than it improved the abilities of average or low achievers?

Statistical significance was identified at a p-value of <0.05.

3.4 Methods of Qualitative Analysis

In preparation for the qualitative analysis of the work of the interviewed pairs, the researcher prepared two table templates (Appendices G and H) to be filled out, and upon which the qualitative analysis was based. The first kind of tables, which was referred to as Template 1 (Appendix G), presented a comparison of the work of each pair of interviewed students between the pretest and posttest. One table was used for each interviewed pair to record significant shifts and patterns in problem solving abilities in one of the three problems from the pretest and its parallel problem from the posttest. For example the work of EP-L, in the pretest and the posttest for the pre-equation and post-equation problems, was recorded in a table. The comparison of the students' work was based on the four abilities specified in the problem solving rubric. Figure 2 below presents a template of the tables used for this comparison.

Group name and category	Pre-name of problem	Post-name of problem
Understands the problem		
Uses information appropriately		
Applies appropriate representations/procedures		
Answers the problem		
Conclusion		
Understands the problem		
Uses information appropriately		
Applies appropriate representations/procedures		
Answers the problem		

Figure 2: Template 1

As the figure reveals, the table also included a conclusion which summarized the shift in each of the four problem solving abilities based on the students' performance in the pretest and the posttest.

The second table template (Appendix H), which was referred to as Template 2, presented a comparison between corresponding achievement categories from EP and CP, per ability for the three problems (figure 3). For example, a comparison of the work of the interviewed pairs EP-L and CP-L was recorded in one table. This table summarized the shifts in the pairs' problem solving abilities for each of the assessed abilities in all the problems.

Comparing EP with CP(per achievement category) per ability (for the 3 problems)		
	EP (Pre-equation/Pre-pattern/Pre-reasoning)	CP (Post-equation/Post-pattern/Post-reasoning)
Understands the problem		
Uses information appropriately		
Applies appropriate representations/procedures		
Answers the problem		

Figure 3: Template 2

The researcher then used Template 2 for the qualitative analysis which described and compared the shifts in problem solving abilities for same achievement level pairs from EP and CP

3.5 Unit Plan Development

Two parallel unit plans (Appendices C &D) were developed to be implemented in EG and CG respectively. The current mathematics curriculum in the school in which the study was conducted did not include problem solving in a specific separate unit. Rather, teachers had the freedom of choosing the kind of word problems they wanted to solve in class, as well as the solution approach they followed. Usually, teachers follow a non-posing teaching strategy where the teacher assumes authority of posing the problems and questions as well as imposing the solution algorithms and processes on their students. This kind of teaching does not encourage students' creation of strategies and processes; neither does it foster the proper justification of steps involved.

The unit plans that were developed for the current study included a set of activities that should be completed in a period of 8 weeks. Since the groups undergoing the study need to complete the assigned curriculum for the year, the researcher assigned two sessions per week to perform the designed activities. Each session lasted 50 minutes. The units focused on word problems that require algebraic thinking, logical reasoning, as well as patterns and combinations.

The two unit plans were similar in nature, with emphasis on improving students' problem solving abilities in both groups; however, different teaching strategies were followed in each of the groups. The researcher used problem posing as a strategy during the teaching and learning of problem solving in Experimental Group. The Control Group math teacher followed the usual non-posing approach of teaching and learning of problem solving. Both the researcher and teacher acted as facilitators and guides in both groups;

however, only the teacher assumed authority of posing problems in Control Group. On the other hand, in Experimental Group, the researcher shared the responsibility of posing problems with the students. Thus, the teaching and learning processes in Experimental Group were based on inquiry and student-centered discussions.

The unit plan (Appendix C) which was used in Experimental Group included two major phases per lesson plan: a problem solving phase and a problem posing phase. The unit plan (Appendix D) used in Control Group involved solving similar word problems; however without the problem posing phase. To ensure both groups were provided the same time allocated for problem solving situations, Control Group was supplied with more teacher-posed problem situations to be solved to compensate for the additional time that Experimental Group spent on problem posing and solving student-posed problems.

3.5.1. Unit plan: Experimental Group.

The unit plan for this group consisted of 10 lesson plans (Appendix C). Each activity consisted of two major phases: Phase I: Solving, and Phase II: Posing. In the first phase, the instructor introduced a certain problem situation to be solved. The instructor worked with students on solving the problem, emphasizing the important steps of problem solving. In the second phase, the instructor guided learners while performing various problem posing strategies starting from the original problem situation presented in the first phase. Students were encouraged to:

- Vary the given information, the question or the situation of the problem
- Make up extension problems to the original given situation.
- Make up their own problems that would be solved in a similar way.
- Create their own solution strategies.

- Work on problems created by their peers and give their own feedback.

3.5.2. Unit plan: Control Group.

The unit plan for this group also consisted of 10 lesson plans (Appendix D). It consisted of only one phase, longer in duration, which was the problem solving phase. This phase was almost similar to the problem solving phase for Experimental Group except for a few minor differences that were altered to fit the posing phase in Experimental Group. The instructor used the usual way of teaching problem solving, and stressed the importance of following the steps of a problem solving process. Another difference in the unit plan for this group was that each lesson plan contained more word problems of similar type and nature to the main word problem, to compensate for the extra time that Experimental Group spent working on student-posed problems in Phase II of each lesson plan.

3.6 Description of the Activities in the unit plans

The mathematical concepts that were involved in the unit plans are evaluating numerical expression, equations of the first degree with one or more unknowns, patterns, combinations, and logical reasoning. The unit plans that were developed for the current study included problems that were within students' zone of proximal development of learning (Wass, Harland & Mercer, 2011). Students at the level of grade eight have not yet formally studied systems of equations, inequalities, patterns or combinations; however, the problems were developed and approached in a way to meet students' level of mathematical knowledge and experience. The problems satisfied the purpose of the study, in which students were going to deal with them as non-routine problems, where they explored methods for solution. Students relied on their prior knowledge in order to construct new knowledge through active learning. Students involved in the study had proper knowledge of basic mathematical operations as well as solving first degree equations with one unknown, and thus had the necessary prior knowledge needed to work effectively on the assigned problem situations. It needs to be mentioned that the suggested problems were presented in a social context which ought to get students more involved in the solution process. The posing phase that was included in the unit plan for Experimental Group also provided the opportunity for students to work on problems of their own social context.

The problems included in the unit plans were categorized according to the mathematical concepts involved, and the solution process was addressed and analyzed per category. There were two main categories in the unit plans and both included mathematical logical reasoning.

The first category of problems dealt with combinations, patterns and logical reasoning (lesson plans 4, 5, &6). In these types of problems, students were expected to organize their given data using an effective representation, such as tables. They might use symbols to easily refer to the given information. For example, in the "mothers and daughters" problem (lesson plan 4), students could arrange the given data in a table, and then use elimination in order to complete the matching assignment. Some students might use guess and check in order to match the persons involved.

In the "Ice cream flavors" problem (lesson plan 6), students might find it easier to use symbols that would represent the different flavors. A sophisticated solution process would be to notice that a pattern is developing upon combining the different flavors while satisfying the given conditions. A less sophisticated solution process would be to list all the possible combinations that satisfy the given conditions and then count them.

In the “Lego” problem (lesson plan 5), students might use diagrams to help them visualize the situation, or they may use only mathematical expressions to calculate the number of Lego pieces found in each level and then adding them all. Another solving procedure is to discover that starting with the first level, which has 2 Legos, 4 Legos are added to complete level 2, and then 6 to complete level 3, then eight and so on, by adding consecutive even numbers to each level. A sophisticated problem solver might notice that the situation led to a more advanced pattern to be discovered which would permit a faster calculation of the number of pieces. The number of Lego pieces in each level is equal to the product of the number of the current level by the number of the next level, taking the topmost level to be level 1 and the one beneath it to be level 2 and so on. Meaning, if we are at level 12 then then number of Legos needed for that level is $12\times13=156$. Using this pattern, students are able to calculate the number of pieces found at the 100^{th} level for example.

The second category of problems dealt with equations and logical reasoning (lesson plans 1,2,3,7,8,9 &10). These problems required accurate translation of mathematical relations in either expressions or equations. Students might use symbols and diagrams to properly organize the given data. In “The Bank Accounts” problem (lesson plan 1), students could use repeated addition and subtraction until they reached the week specified by the given information. A more achieving problem solver might translate the given data into mathematical expressions to calculate the accounts. In lesson plans 2 and 3, “House Chores” and “Cookie Sale”, the situation required the use of a system of two equations with two unknowns. However, students at this level haven’t yet been introduced to this topic. An advanced solution would involve translating the given relations into two equations, then merging them into one equation with one unknown. A less advanced solution would involve using the guess and check strategy. Students might guess one of the unknowns and then check if their guess will satisfy the conditions.

Lesson plan 7 presents an age problem written in a riddle form which ought to raise students' curiosity. An advanced solution procedure may include translating the given data into an equation with one unknown and then solving it. A less advanced solution procedure would be working backwards to reach the original age. Students might also use *guess and check* in order to reach a solution.

Lesson plan 8 included the "Ship Problem", in which students needed to work with average speeds, schedules and distances travelled. This problem could be solved using the concept of proportionality. Students were introduced to this concept in the previous scholastic year, but this problem provided a more complex situation, in which average speeds varied, and the solution required more than one step. Students might represent the given data on a timeline, on which they represent the distances travelled at each interval and the time elapsed or remaining. In this strategy, they would rely on their mathematical logical reasoning to complete the solution. A more advanced solution would be to use proportional reasoning to calculate the distances and time using the speed formula.

The "Farm Animals" problem (Lesson plan 10) included the use of systems of equations with several unknowns. Students needed to translate the given data into mathematical equations. This problem required the use of mathematical logical reasoning in order to reach a solution. Some students might start by guessing and checking different combinations, however this would take too much time, since there were three unknowns.

The best way would be to eliminate one of the unknowns. After translating the given data, we end up with these equations:

$P+G+S=100$ and $21P+8G+3S=\mathbf{600}$.

If we merge these two together, we end up with $18P+5G=300$. At this stage, students should use their logic to figure out that the number of pigs can't exceed 16, because the price of the pigs and goats becomes more than 300. The fact that the number of P is even, leaves fewer choices to pick from. Students must find a number which is even and which, when multiplied by 18 then subtracted from 300 yields a number that should be divisible by 5. Ten is the only number. If the number of pigs wasn't even, then 15 would have been a possible solution.

Lesson plan 9 presented a situation where solving inequalities was needed to reach an answer. Students, however, were not familiar with this mathematical concept. They might resort to guessing and checking until they reach an answer. They could tabulate their calculations for a better representation. A more advanced way would be to solve the situation as if it were an equality situation and then use logical reasoning in order to find a possible solution.

3.7 Conceptual Analysis of the Pretest and Posttest

For the purpose of evaluating students' work in the pretest and posttest, a more thorough, yet not exclusive, conceptual analysis is presented below. As explained before, both tests included problems of parallel nature, and thus were named according to their nature. The first problem was referred to as the equation problem, the second as the pattern problem and the third as the reasoning problem. Following this sense, the equation problem in the pretest was referred to as pre-equation problem, and that in the posttest was referred to as post-equation problem. The other problems were named in a similar way.

3.7.1. Pretest items

The pre-equation problem presented a situation where four friends commute to school. They were at different distances from their school, but the total distance that was travelled was given. There were four unknowns in this problem; however they were related to one another, meaning three of the unknowns could be written in terms of the fourth. The task in this problem was to find the distance travelled by each person. The mathematical concept, which was depicted in this problem, is solving first degree equations. Students are familiar with this concept; however, many students find it hard to deal with several unknowns that are related to one another. The first step in the solving process of this word problem is to properly organize the given data in a way that would provide a clearer picture of the situation. There are four friends: Lara, Mohammad, Karim and Sam. Students can refer to the distances travelled by each as L, M, K and S. They each travel a different distance to school.

M travels $L/2$; K travels $M+L$; S travels $3L$

All together they travel 36 Km, so $L+M+K+S=36$

Noticing that M, K and S can be written in terms of L, students should replace each unknown with its value in terms of L: $L+L/2+M+L+3L=36$; but $M=L/2$ so substituting again should give us the following equation: $L+L/2+L/2+L+3L=36$. After simplifying and solving the equation, the distance travelled by Lara can be calculated and it turns out to be 6 Km. The other distances can be calculated by substituting the value for L in the given relations. So, the distance travelled by Mohammad is 3 Km, by Karim 9 km, and by Sam 18 Km. It is important so check whether the calculated distances are correct by verifying that they add up to 36 Km. The answers should be presented in full sentences which gives real-life meaning to the calculated numbers. The solution presented above is considered a sophisticated solution strategy in which a student should receive a full mark. A less sophisticated strategy would be to use guess and check. Students might start by trying out different distances, starting with the distance Lara travels. For example, if Lara travels 8 Km, then Mohammad travels 4, Karim 12 and Sam 24 Km, but adding all distances gives more than 36, so a smaller distance needs to be chosen.

The pre-pattern problem presented a situation where six teams were going to play with one another, each team would play with the other team only once, disregarding who won, and the task was to find out how many games will be played. The mathematical concept that is presented in this word problem is combinations. Students are not familiar with this concept or with the general formulas that can be used to find a fast answer. Thus, students should rely purely on their problem solving skills in order to come up with a solution. They might refer to each team with a letter or a number, and start matching who plays with whom. For example, T1 represents the first team and T2 represents the second team and so on. Thus, the following list shows the games to be played.

T1-T2; T1-T3; T1-T4; T1-T5; T1-T6→ 5 games that first team will play in.

T2-T3; T2-T4; T2-T5; T2-T6 →4 games that second team will play in. T2 can not play with T1 again and of course can not play with itself.

T3-T4; T3-T5; T3-T6→ 3 games.

T4- T5; T4-T6 → 2 games.

T5-T6 → 1 game.

Adding up all the games gives a total of 15 games.

An advanced problem solver might not need to do the whole list, and is satisfied with only the first two or three rows before such a solver notices that there is a pattern here, one less game for each row. Students could use various representation strategies to solve the problem, but mostly they are similar to the above list. They might use a table or matching up among two columns.

The pre-reasoning problem presented a situation that would require mathematical reasoning, specifically numerical reasoning, to complete the task, which was to fill a tank, of maximum capacity of 10 Liters, with 6 Liters of water using only 5 Liter container and 8 Liter container. The word problem provided an extra given data which is the maximum capacity of the drinking tank. Students were required to check if there was extra information in the given data. This was used to check whether students use given information appropriately. A simple diagram would help students get a clearer idea about the situation at hand. The solution which requires the smallest number of steps is to fill the 8 Liter container with water and then pour this water into the 5 Liter container. This leaves 3 liters in the 8 Liter container. This water should be poured into the tank, and the procedure is repeated another time for the remaining 3 liters. A solution with more steps is to fill the 5 Liter container and pour the water into the 8 Liter container. Fill the 5 Liter container again, and pour the water into the 8 Liter container. Two liters would be left in the 5 Liter container, since 3 liters were poured into the 8 Liter container. Pour these 2 liters into the tank, and repeat the above steps three times to fill the tank with 6 liters.

3.7.2. Posttest items

The post-equation problem was similar, in the nature of the mathematical concept it depicts, to the pre-equation problem. It could be solved by using equations. However, this problem was of higher difficulty level, since it required the use of a system of equations of four unknowns. Students are not familiar with this concept; however, the problem could be solved by using other strategies. The problem presented a situation where different numbers of cows were found in pastures connected by bridges. The given data was represented by a given figure, on which the number of cows in two adjoining pastures was provided. Students were asked to interpret the data presented by the given figure, and would be asked to check for any unnecessary given data. This would help in evaluating how students handle given information. A sophisticated solution strategy would start by interpreting the given data into the following equations: $A+B=29$; $A+C=25$; $B+D=32$; $C+D=28$; $B+C=26$, where A is the number of cows in the top left pasture, B in the top right pasture, C in the bottom left pasture, and D in the bottom right pasture. After examining the first two equations, a student may notice that the difference between the number of cows in pastures B and C is the difference between 29 and 25 which is 4. Thus, B and C are two numbers whose difference is 4 and sum is 26, according to the last equation. Students can use guess and check to find that $B=15$ and $C=11$, or they can find this answer by solving the equations $B+C=26$ and $B-C=4$. Once B and C are found, students can then use the other equations to find that $A=14$ and $D=17$. Some students may not be able to interpret the given data into the above equations, and will rely only on guess and check from the very beginning.

The post-pattern problem was similar in nature to the second item in the pretest, yet higher in its difficulty level. It could be solved using a similar strategy; however, the given number of cells (30) was intentionally high in order to force students to try and come up with an effective solving strategy. Instead of listing all the possibilities, which is a tedious job, they should start to notice the pattern that this problem presents. To connect cell1 to every other cell, 29 passages are needed, to connect cell2 with every other cell, except to itself and to cell1 again, 28 passages are needed, and so on…. Adding up all these cells gives 29+28+27+…+2+1=435 passages. Part 'a' of this item, which is to explain why no three cells are collinear, will help in evaluating students' understanding of problem conditions.

The post-reasoning problem also presented a situation that would need mathematical logical reasoning to be solved. Students were asked to justify their work which helped the researcher in evaluating students' level of understanding the problem situation and solving processes. The word problem stated that there were a total of 4 dozens dogs that were distributed among six differently colored kennels: blue, red, orange, yellow, green and purple. To find the number of dogs in each kennel, some clues were given. In order to solve this problem effectively, the given data needs to be properly represented. Students might resort to using a diagram that shows the six kennels, with the given clues written next to each kennel. The easiest to start with is the orange kennel, because it is clearly given that it is the largest kennel with ten dogs in it. Since the orange kennel has the largest number of dogs, students should keep in mind that no other kennel should contain ten or more dogs. The second clue states that the smallest kennel has six dogs in it, but the color is not known. The yellow and green kennels has the same number of dogs, so students may use a variable to represent this number, for example D. A third clue states that there are 13 dogs in both the red and blue kennels which should contain the smallest number of dogs. So now, students should reason that either the red or the blue is the smallest kennel and thus one of them should contain 6 dogs, as stated in the given information, and the other should has 7 in order to add up to 13. However, it is not yet known which color is for each kennel. The last clue should help in determining this. The purple kennel has 2 more dogs than the blue kennel, so it can either have 6+2=8 dogs or 7+2=9 dogs. The students should then consider each of the two possibilities in order to determine what the correct answer is. If there were 8 dogs in the purple kennel then the sum of all the dogs in all kennels would be represented by this equation: 6+7+10+D+D=48, which when solved would give D=8.5 dogs which is not a possible answer. However, if there were 9 dogs in the purple kennel, the solution would be D= 8 dogs in each of the yellow and green kennels. This is a valid answer. Thus, the red kennel has 6 dogs, and the blue has 7 dogs, the orange has 10, the yellow and green has 8 in each, and the purple has 9 dogs. It has to be mentioned here that students do not need to use an equation to solve this problem. A less advanced strategy would be to add up the known number of dogs which is 32 dogs, and then subtract this from the total of 48 dogs which would give 16 dogs to be distributed equally among the yellow and green kennels and so 8 dogs in each.

3.8 Time Frame

The time required to implement the intervention in this study was 10 weeks. The first week was allocated to acquire approval of the IRB as well as that of the school administration to conduct the study, and to have consent forms signed by students' parents. Also, the students to be clinically interviewed, and whose work should be analyzed, were identified during that first week. After that, the researcher and teacher needed eight weeks to administer the pretest for both groups during which the clinical interviews for the 12 students were conducted, and to implement the prepared activities in both groups. The last week was needed to administer the posttest and the final clinical interviews with the twelve chosen students.

3.9 Validity and Reliability

In this experimental design, each of the two groups were randomly assigned to one of the two instruction practices, by writing EG and CG on two pieces of paper and blindly drawing one. All instruments except a part of the unit plans were equally assigned to both groups; the units, however, were parallel in the nature of activities, sequence and time allocated for the implementation of activities. To ensure that both groups were exposed to an equal time, Control Group was assigned extra teacher-posed word problems to compensate for the time Experimental Group spends on the posing phase of the activities. The extra problems were of a similar nature to the original problems of the activity at hand.

The pretest and posttest were prepared by the researcher in a way to match the requirements of the study as well as the grade level of the targeted population. The items included in both tests were adopted from the following websites http://learn.fi.edu/school/math and http://www.figurethis.org and were adapted to address regional requirements in terms of syntax and language. The tests were shown to the intermediate Math coordinator and an intermediate math teacher to ensure that the level of the tests matched the level of grade eight, and that the time allocated for the students to finish the task was appropriate as well.

The rubric for evaluating problem solving abilities of students was adopted and modified from the Utah Education Network. It was ensured to evaluate each required area of problem solving separately to facilitate quantitative and qualitative analysis.

All instruments and activities were piloted before their use to ensure their validity. The tests were administered to a similar sample in the same school. Misleading terms were modified. As for the activities, they were chosen from a larger collection of problem solving activities that were also performed with a similar group of grade eight and modified to address the aim the study more precisely.

Chapter Four

FINDINGS

The purpose of this experimental research is to study the effect of implementing problem posing as a teaching and learning strategy on the problem solving abilities of students when a problem posing approach is used while teaching problem solving skills at the intermediate level.

The current study adopts a mixed-method approach, using quantitative and qualitative techniques for data collection and analysis. The sample that participated in this study consists of two sections at the intermediate level (Grade 8), which were randomly assigned to one of two groups: Experimental Group (EG) and Control Group (CG). In the Experimental Group a unit plan, which includes problem posing activities, was implemented. At the same time, in the Control Group, a parallel unit plan was also implemented, however the same activities were handled using the usual problem solving approach. A pretest and a posttest were administered to both groups to determine the effect of using a problem posing approach on students' problem solving abilities. The results of these assessment were used for quantitative analysis (Appendix F). Furthermore, clinical interviews were conducted, during the pretest and posttest, with a sample that was chosen according to students' math achievement level. The work of this sample is used for further qualitative analysis.

4.1 Quantitative Analysis

The scores of both the Control Group (CG) and the Experimental Group (EG), in the pretest and the posttest are used for the quantitative analysis that is performed in this research. Data is entered into a Microsoft Excel sheet, and then transferred to the Statistical Package for Social Sciences (SPSS), version 22, which is used for data cleaning, management and analyses. Statistical significance is identified at a p-value of <0.05.

In this section, the quantitative analysis is discussed first for the whole sample, and then for the different math achievement levels per group.

4.1.1 Whole sample analysis

The following section presents an analysis of the results of both groups EG and CG, based on the performance of the whole sample in the pretest and the posttest. The analysis is presented according to: 1- overall performance, 2- performance per problem, and 3- performance per ability.

4.1.1.1 Whole sample analysis-Overall Performance

After analyzing the overall results of the pretest and the posttest, it was found that the improvement in students' problem solving abilities in the Control Group (CG) was 1.1% (sd =13.7%); however, the improvement in students' problem solving abilities in the Experimental Group (EG) was 9.4 % (sd =15.8%). The association among EG and CG for the overall pre- and posttest was found to be of borderline significance with a p-value of 0.05.

4.1.1.2 Whole Sample Analysis- Performance per Problem

The scores of the participants in both groups were also analyzed for each of the three problems in the pretest and the posttest (refer to Table 1). Both groups revealed improvement in terms of their problem solving abilities in the equation problem; however, CG's improvement, which is 15% (sd=26.8%), was higher than that of EG, whose performance improved by only 4.5% (sd=24.4%), with p-value of 0.15.

The analysis that was performed on the pattern problem reflected a decrease in students' problem solving abilities in both groups. However, the regression in EG was less than that revealed by CG with a highly significant p-value of 0.003.

With respect to the reasoning problem in both pretest and posttest, a significant association was found (p-value=0.01) between the performance of both groups. CG's performance was found to have improved by 12.5% in comparison to the 29.8% improvement in problem solving performance reflected by EG.

Table 1: Association between improvement of EG and CG in overall test and per problem

Variables		**EG**	**CG**	**P value**
Total sample n=50		**n=25**	**n=25**	
Grades difference equation problem	(Mean, ±sd)	4.5% (±24.4%)	15.0% (±26.8%)	0.15
Grades difference pattern problem	(Mean, ±sd)	-6.3% (±16.8%)	-24.3% (±23.3%)	0.003
Grades difference reasoning problem	(Mean, ±sd)	29.8% (±29.7%)	12.5% (±15.1%)	0.01
Grades difference Overall test	(Mean, ±sd)	9.4% (±15.8%)	1.1% (±13.7%)	0.05

4.1.1.3 Whole Sample Analysis- Performance per Ability

The same analysis was also made on each of the assessed abilities to compare the improvement of the assessed abilities in each of the two groups (refer to Table 2). It was found that CG's ability to "understand the problem" (Ability 1) regressed by 0.67% (sd=19.5%). On the other hand, EG's performance for the same ability improved by 16.3% (sd=16.4%), and thus a significant association was found (p=0.002) between the performance of both groups.

As for ability 2, which is to "use information appropriately", there was no significant association (p-value=0.13) between the performances of the two groups. Both groups showed improvement, where EG's performance improved by 10% (sd=19.2%), and CG showed less improvement which was 2.3% (sd=15.7%).

The performance of the two groups also showed a slight improvement for Ability 3 and Ability 4, which assessed the ability to "use appropriate representations/procedures", and the ability to "answer the problem" respectively; however no significant associations (p-values=0.32 and 0.65 respectively) were found between the performances of the two groups. EG's performance improved by 6% (sd=17.4%), whereas CG's performance improved by 2.3% (sd=15.7%) for the third ability. As for the fourth ability, CG improved only by 1% (sd=14.1%), and EG improved by 3% (sd=16.5%).

Table 2: Association between improvement of EG and CG per ability

Variables		EG	CG	P value
Total sample n=50		**n=25**	**n=25**	
Grades difference ability 1	(Mean, ±sd)	16.3% (±16.4%)	-0.67% (±19.5%)	0.002
Grades difference ability 2	(Mean, ±sd)	10.0% (±19.2%)	2.3% (±15.7%)	0.13
Grades difference ability 3	(Mean, ±sd)	6.0% (±17.4%)	2.0% (±12.8%)	0.32
Grades difference ability 4	(Mean, ±sd)	3.0% (±16.5%)	1.0% (±14.1%)	0.65

4.1.1.3 Interpretation of the whole-sample analysis

The purpose of this study is to study the effect of using problem posing as a teaching strategy on the problem solving abilities of students at the intermediate level. In general, the Experimental Group (EG) showed more improvement than the Control group (CG), as the analysis revealed a borderline significance of the associations among the groups (p-value=0.05). Thus, it can be deduced, in this study, the students in EG benefitted from using problem posing as a teaching strategy in terms of improved problem solving abilities.

EG demonstrated more improvement in the four assessed abilities than CG did, especially in the ability to "understand the problem", in which the improvement was significant. EG improved by 16.3% while CG regressed by 0.67% with a highly significant p-value of 0.002. This improvement in understanding a problem is logical, since students in EG were involved in activities that required a lot of reasoning and analyzing given information. Some of the posing activities that EG was exposed to required students to change given information or to construct problems of a certain nature, and these activities target the students' understanding of a problem's conditions. Thus, if the effect of using problem posing proves to be positive, it would seem logical to notice a significant improvement in students' understanding of a problem.

EG showed more improvement (10%, 6% and 3 %) than CG (2.3%, 2% and 1%) in Abilities 2, 3 and 4 respectively; however, this improvement can not be considered to be highly significant. Due to the limited time interval of implementing the problem posing strategy, it is logical that if any improvement is to be detected, then this improvement should start at the first level of problem solving, which is understanding a given problem. To achieve more progress in the other problem solving abilities, more time should be dedicated to implement this strategy.

Regarding the performance of the groups in each of the three problems, EG showed a more significant improvement in the reasoning problem than CG did, where the improvements were 29.8% and 12.5% respectively with p-value 0.01. This improvement could be as a result of the progress in the four assessed problem solving abilities that EG reflected while working on the problems.

On the other hand, CG showed a slightly more improvement than EG in the equation problem, where the improvement was 15% in comparison to EG's improvement of 4.5%. CG, however, revealed a highly significant regression in the reasoning problem than EG did in the same problem.

It has to be emphasized that CG showed improvement only in the equation problem, whereas EG improved in the pattern and reasoning problems. This can be attributed to the nature of these problems as well as the solving procedures the problems depict. Both pattern and reasoning problems require logical reasoning and critical thinking. These aspects are involved in the problem posing activities that EG was subjected to. On the other hand, the equation problem requires procedural thinking which is emphasized in the unit plan that was implemented in CG. Thus, when CG attempted to work on the post problems, which are designed to be of the same nature as those in the pretest but of a higher difficulty level, the group was more successful in the problem that required procedural methods of solving. On the other hand, EG was more successful in the problems which required more analysis and reasoning.

This section presented this study's results based on the quantitative analysis for the scores of the whole sample. It was found that the experimental group revealed more progress than the control group in terms of their problem solving abilities, especially in the ability to "understand a problem" where the improvement was significant. Furthermore, it was found that the students in the experimental group were more successful in the pattern and reasoning problems, whereas students in the control group were more successful in the equation problem.

4.1.2 Stratified analysis done on groups according to achievement levels

The same analysis, which was performed on the original sample, was also repeated with the three groups that were classified according to different achievement levels: high, average, and limited and low. The analysis targets the performance of each achievement level first by problem, and then by ability. It has to be noted that the sample sizes are small, and thus the associations among the different groups do not reflect statistical significance. Instead, educational significance is determined according to the degree of improvement achieved by each group.

4.1.2.1- Limited and Low Math Achievers

The sample sizes for the low achieving groups is 7 for the Limited and Low Math Achievers in the Experimental Group (EG-L) and 6 for the Limited and Low Math Achievers in the Control Group (CG-L). In general, CG-L's problem solving abilities regressed slightly by 2.4% (sd=13%), where on the other hand, EG-L showed an improvement of 4.2% (sd=16.9%) in their problem solving abilities. However, the association between the two groups was not found to be statistically significant (p-value= 0.45).

The following results were found.

Performance per problem

Regarding the performance of the two groups, EG-L and CG-L, in each of the three problems of the pretest and posttest, the following results were found (refer to Table 3). With respect to the equation problem, EG-L's performance regressed slightly by 0.9% (sd=14.2%) while CG-L's performance improved by 12.5% (sd=25.3%) with statistical association of p-value=0.25.

As for the pattern problem, the performance of both groups showed some regression. CG-L's problem solving abilities regressed by 22 % (sd=22.9%), whereas the students in EG-L revealed less regression than those in CG-L, represented by 8% (sd=13.8%). The association among the groups for this problem has a p-value of 0.19.

Problem solving abilities of both groups improved in the reasoning problem; however, EGL's improvement was higher than that of CG-L. The students' performance in EG-L was found to have improved by 21.4% (sd=38.3%), whereas CG-L's performance improved only by 3.1% (sd=10.3%). Due to the small sample size, the association among the groups for this problem does not have statistical significance (p-value=0.28); however, it does have educational significance where EG-L did show more improvement that CG-L.

Table 3: Association between improvement of EG-L and CG-L in overall test and per problem

Variables		**EG-L**	**CG-L**	**p-value**
Total sample n=13		**n=7**	**n=6**	
Grades difference equation problem	(Mean, ±sd)	-0.9% (±14.2%)	12.5% (±25.3%)	0.25
Grades difference pattern problem	(Mean, ±sd)	-8.0% (±13.8%)	-22.9% (±24.3%)	0.19
Grades difference reasoning problem	(Mean, ±sd)	21.4% (±38.3%)	3.1% (±10.3%)	0.28
Grades difference Overall test	(Mean, ±sd)	4.2% (±16.9%)	-2.4% (±13.0%)	0.45

Performance per ability

Upon analyzing the scores for EG-L and CG-L (refer to Table 4), it was found that students in EG-L showed a slight improvement in Ability 1 and Ability 2, where the analysis found an improvement in these abilities by 10.7% (sd=13.4%) and 1.2% (sd=18.9%). On the other hand, CG-L showed a slight regression of 2.8% (15.5%) and 4.2% (sd=16.5%) respectively for the same abilities. The associations among EG-L and CG-L for Ability 1 and Ability 2 have p-values of 0.12 and 0.6 respectively.

As for Ability 3, EG-L showed no improvement at all, but CG-L's performance regressed slightly by 1.4% (sd=11.1%). The association among the groups has a p-value of 0.88.

Finally, the scores of both groups reflected a slight decrease in Ability 4, represented by 3.6% (sd=15.9%) and 1.4% (sd=16.2%) for EG-L and CG-L respectively with a statistical association of p-value of 0.81.

Table 4: Association between improvement of EG-L and CG-L per ability

Variables		**EG-L**	**CG-L**	**p-value**
Total sample n=13		**n=7**	**n=6**	
Grades difference ability 1	(Mean, ±sd)	10.7% (±13.4%)	-2.8% (±15.5%)	0.12
Grades difference ability 2	(Mean, ±sd)	1.2% (±18.9%)	-4.2% (±16.5%)	0.60
Grades difference ability 3	(Mean, ±sd)	0.00% (±19.2%)	-1.4% (±11.1%)	0.88
Grades difference ability 4	(Mean, ±sd)	-3.6% (±15.9%)	-1.4% (±16.2%)	0.81

4.1.2.2- Average Achievers

The sample sizes for the Average Math Achievers in the Experimental Group (EG-A) and the Average Math Achievers in the Control Group (CG-A) were 14 and 16 respectively. In general, EG-A showed an improvement by an average of 8.9% (sd=14.7%), which was slightly higher than CG-A's improvement of 2.1% (sd=13.3%). The analysis did not reveal a statistical significance where the p-value is 0.19; however, the higher improvement displayed by EG-A reflects an educational significance.

Performance per Problem

A deeper analysis (refer to Table 5) revealed that CG-A's problem solving abilities, in the equation problem, improved by 14.1% (sd=28.3%). This improvement was slightly higher than EG-A's improvement of 7.1% (sd=22.2%), and the statistical association has a p-value of 0.47. Furthermore, it was found that both groups regressed in the pattern problem; however, EG-A's regression (9.8%, sd=17.6%) was less than CG-A's decrease in performance (19.9%, sd=20.2%) for the same problem with a p-value of 0.16. On the other hand, both groups showed some improvement in the reasoning problem, where EG-A's problem solving abilities improved by 29% (sd=27.6%). This improvement was significantly higher than CG-A's improvement of 12.1% (sd=14%), and thus revealing a statistically significant association with p-value of 0.05.

Table 5: Association between improvement of EG-A and CG-A in overall test and per problem

Variables		**EG-A**	**CG-A**	**p-value**
Total sample n=30		**n=14**	**n=16**	
Grades difference equation problem	(Mean, ±sd)	7.1% (±22.2%)	14.1% (±28.3%)	0.47
Grades difference pattern problem	(Mean, ±sd)	-9.8% (±17.6%)	-19.9% (±20.2%)	0.16
Grades difference reasoning problem	(Mean, ±sd)	29.0% (±27.6%)	12.1% (±14.0%)	0.05
Grades difference Overall test	(Mean, ±sd)	8.9% (±14.7%)	2.1% (±13.3%)	0.19

Performance per Ability

The analysis of the performance per ability revealed the following results (refer to Table 6). A significant association (p-value=0.05) was detected between the groups' performance regarding Ability 1. CG-A showed no improvement at all, whereas EG-A improved by 13.7% (sd=12.9%).

Regarding Ability 2, EG-A showed an improvement of 11.3% (sd=19.8%), which was slightly higher than CG-A's improvement of 4.2% (sd=15.5%). The statistical association among EG-A and CG-A for this ability has a p-value of 0.28. As for Abilities 3 and 4, EG-A's performance improved by 5.4% (sd=16.5%) for each of the assessed abilities. On the other hand, CG-A revealed a slight improvement of 2.6% (sd=11.3%) and 1.6% (sd=13.7%) for the third and fourth abilities respectively with p-values 0.59 and 0.5 respectively.

Table 6: Association between improvement of EG-A and CG-A per ability

Variables		**EG-A**	**CG-A**	**p-value**
Total sample n=30		**n=14**	**n=16**	
Grades difference ability 1	(Mean, ±sd)	13.7% (±12.9%)	0.00% (±21.5%)	0.05
Grades difference ability 2	(Mean, ±sd)	11.3% (±19.8%)	4.2% (±15.5%)	0.28
Grades difference ability 3	(Mean, ±sd)	5.4% (±16.5%)	2.6% (±11.3%)	0.59
Grades difference ability 4	(Mean, ±sd)	5.4% (±16.5%)	1.6% (±13.7%)	0.50

4.1.2.3 High Math Achievers

The sample sizes for the high achieving groups were relatively small, mainly 4 and 3 for the High Math Achievers in the Experimental Group (EG-H) and the High Math Achievers in the Control Group (CG-H) respectively. In general, EG-H showed more improvement than CG-H, in terms of problem solving abilities. The analysis revealed that EG-H improved by 20.3% (sd=16.3%), whereas CG-H improved by only 2.8% (sd=21%). The association among EG-H and CG-H did not reveal a statistical significance (p-value=0.27); however the researcher can detect an educational significance reflected by the higher level of improvement of EG-H's problem solving abilities

Performance per Problem

A more detailed analysis for the groups' performance in each of the three problems revealed the following (refer to Table 7). With respect to the equation problem, CG-H improved by 25% (sd=28.6%); however, EG-H improved only by 4.7% (sd=46.3%). The association among the groups for this ability has p-value of 0.54. On the other hand, in the pattern problem, a significant association between the two groups' performance was found, with a p-value of 0.01. In this problem, EG-H's performance improved by 9.4% (sd=12%); however, CG-H's performance regressed by 50% (sd=28.6%). As for the reasoning problem, CG-H improved by 33.3% (sd=9.5%), but EG-H showed more improvement, where the analysis reflected an improvement of 46.9% (sd=16.5%). The p-value for this association was 0.27.

Table 7: Association between improvement of EG-H and CG-H in overall test and per problem

Variables		**EG-H**	**CG-H**	**p-value**
Total sample		**n=4**	**n=3**	
Grades difference equation problem	(Mean, ±sd)	4.7% (±46.3%)	25.0 (±28.6%)	0.54
Grades difference pattern problem	(Mean, ±sd)	9.4 % (±12.0%)	-50.0 (±28.6%)	0.01
Grades difference reasoning problem	(Mean, ±sd)	46.9% (±16.5%)	33.3 (±9.5%)	0.27
Grades difference Overall test	(Mean, ±sd)	20.3% (±16.3%)	2.8 (±21.0%)	0.27

Performance per Ability

The analysis that was performed on the groups' performance regarding the assessed abilities revealed the following results (refer to Table 8). It was found that EG-H showed an improvement of 35% (sd=21.9%) in Ability 1, where on the other hand CG-H showed no improvement at all. The association between the groups' performance in this ability was found to be of borderline significance with p-value of 0.09.

The analysis also revealed that EG-H's performance reflected more improvement than CG-H's performance regarding the other three abilities. EG-H's performance in Abilities 2 and 3 improved by 20.8% (sd=14.4%) and 18.8% (sd=14.2%) respectively, which was higher than CG-H's improvement by 5.6% (sd=17.3%) and 5.6% (sd=25.5%) respectively in each of the mentioned abilities. The p-values for the associations among EG-H and CG-H for these abilities were 0.26 and 0.42 respectively. As for Ability 4, EG-H's performance improved by 6.3% (sd=18.5%) while CG-H improved by 2.8% (sd=17.3%).

Table 8: Association between the improvement of EG-H and CG-H per ability

Variables		**EG-H**	**CG-H**	**p-value**
Total sample		**n=4**	**n=3**	
Grades difference ability 1	(Mean, ±sd)	35.4% (21.9%)	0.00% (22.0%)	0.09
Grades difference ability 2	(Mean, ±sd)	20.8% (14.4%)	5.6% (17.3%)	0.26
Grades difference ability 3	(Mean, ±sd)	18.8% (14.2%)	5.6% (25.5%)	0.42
Grades difference ability 4	(Mean, ±sd)	6.3% (18.5%)	2.8% (17.3%)	0.81

4.1.2.4 Interpretation of the stratified analysis on achievement groups

Regarding the effect of problem posing on different achievement groups, the above reported analysis reveals the following. In general, the experimental groups of all achievement levels showed more improvement, in the overall posttest, than the control groups. EG-L and EG-A's improvement (4.2% and 8.9% respectively) was slightly more than the improvement reflected by CG-L and CG-A which showed a regression by 2.4% and an improvement of 2.1% respectively. Furthermore, CG-H showed what the researcher considers to be an educationally significant improvement. The progress this group reflected (20.3%) was more than CG-H's progress (2.8%), but because of the small sample size, the association among EG-H and CG-H is statistically insignificant.

As for the four assessed problem solving abilities, the analysis revealed the following results. Regarding Ability 1, the experimental groups EG-L, EG-A and EG-H improved more than the control groups of the different achievement categories. In fact, EG-H's improvement (35.4%) is found to be significant when compared to CG-H's progress (0%) with a p-value of 0.09. Furthermore, where EG-L displayed a slight improvement (10.7%) in their ability to "understand the problem", CG-L's work reflected a regression of 2.8%. Again, it can be inferred that the three achievement groups of the experimental group benefitted from the problem posing strategy in terms of understanding a given problem.

With respect to Ability 2, "use information appropriately", all experimental groups of different achievement categories reflected more improvement than the control groups, where the improvement was 1.2%, 11.3% and 20.8% for EG-L, EG-A and EG-H. In comparison, CG-L regressed by 4.2%, CG-A improved by 4.2% and CG-H improved by 5.6%. Thus, it can be deduced that the experimental groups were more able to "use information appropriately", especially the high achieving experimental group. This can be justified by the improvement in understanding problems that the experimental groups reflected while solving. Improved interpretation of necessary information should lead to a better and more appropriate use of this interpreted information.

As for Ability 3, “Applies appropriate procedures/representations”, the low and limited math achieving groups of the control and experimental groups showed no progress in this ability: 0% progress for EG-L and 1.4% regression for CG-L. However, EG-A and EG-H’s work reflected more improvement (5.4% and 18.8% respectively) than the control groups of corresponding achievement levels, where the improvement was 2.6% and 5.6% for CG-A and CG-H respectively. Thus, it can be assumed, in this study, that the problem posing strategy affects positively the ability of applying the appropriate procedure, of average and high math achieving students; however, more time should be dedicated for posing activities in order to notice a significant improvement. One can also notice, that the problem posing strategy affected the high achievers more than it did the average achievers, and that the effect was minimal with the low and limited achievers.

Finally, regarding Ability 4, “answer the problem”, the average and high achieving groups of the experimental group, EG-A and EG-H, progressed slightly more than the corresponding control groups. The improvement was 5.4% and 6.3% respectively, in comparison to CG-A and CG-H’s improvement of 1.6% and 2.8% respectively. The same ability regressed in both EG-L and CG-L; however, EG-L showed less regression than the control group. Thus it can be deduced that, in this study, the problem posing strategy revealed its least effect on the ability to provide an answer, where the effect was minimal for the low and limited achieving groups. This can be explained by the fact that problem posing activities involve students in dissecting the different elements that make up a problem, as well as understanding how different conditions may or may not work together, instead of focusing only on the final answer.

The improvement or regression of the four problem solving abilities that are discussed above was reflected by the students' work in the three problems of both pretest and posttest. The following interpretations can be made, regarding the effect of problem posing activities on students of different achievement levels.

The groups EG-L, EG-A and EG-H were more successful in the reasoning problem, where their improvement was 21.4%, 29%, & 46.9% respectively, and exceeded the improvement reflected by the corresponding achievement groups of CG (3.1%, 12.1%, & 33%). In specific, EG-A revealed a significant improvement in this problem, where the association among EG-A and CG-A for this problem is statistically significant with p-value of 0.05. One can notice that even EG-L showed an improvement in the reasoning problem due to the students' improved ability of understanding the problem.

As for the equation problem, CG-A and CG-H showed more improvement (14.1% and 25% respectively) than the experimental groups which improved by 7.1% for EG-A and 4.7% EG-H. The progress is not found to be statistically significant. Again this conforms to the interpretation made for the whole sample, where CG improved more than EG in problems of procedural nature such as the equation problem.

In the pattern problem, the groups EG-A, CG-A, EG-L and CG-L showed regression in the post-pattern problem, due to the complexity of the situation where the number of cells to be connected was larger than the number of teams to be combined in the pre-pattern problem. However, the regression in the experimental groups was less than that of the control groups. EG-A and EG-L regressed by 9.8% and by 8% respectively whereas CG-A and CG-L regressed by 19.9% and 22.9% respectively.

On the other hand, in the pattern problem, CG-H showed a noticeable regression of 50%, whereas the students in EG-H were more successful (9.4% improvement) than the students of CG-H while working on this problem. This can be justified by the relapse in CG-H's assessed problem solving abilities.

Again it can be noted that the problem posing activities affected the high achieving students more than it did the other two achievement groups, especially in problems that require critical thinking and logical reasoning.

4.2 Qualitative Analysis

The intent of this study is to study the impact of the use of problem posing as a teaching/learning strategy on the problem solving abilities of students. To achieve this aim, a qualitative analysis was conducted on the work of the 12 students who were chosen for the clinical interviews as described in Chapter Three: low and limited achieving pair of the experimental group (EP-L), average achieving pair of the experimental group (EP-A), the high achieving pair of the experimental group (EP-H), and the corresponding pairs CP-L, CP-A, and CP-L from the control group. Each pair of the above mentioned sample, for example EP-L, was videotaped and clinically interviewed while sitting for the pretest and the posttest. The interviews were transcribed and then analyzed to detect any shifts in terms of problem solving abilities between the pretest and the posttest for each pair of students.

4.2.2 Qualitative analysis for EP-L and CP-L

The pairs EP-L and CP-L were clinically interviewed while working on both the pretest and the posttest. The researcher arrived to some inferences in relation to their problem solving abilities. The analysis is presented based on the Qualitative Comparison table of the low and limited achieving groups (Appendix I) for the four assessed abilities.

4.2.2.1 Ability1: Understands the Problem

The qualitative analysis regarding the first ability revealed that EP-L showed more progress than CP-L. In general, EP-L's ability to "understand the problem" improved. The group's ability progressed from "unable to interpret enough given information to get started or make progress", in the pre-equation problem, to "able to interpret all necessary information needed for a complete and correct solution", in the post-equation problem as revealed in the figures 4 and 5 below.

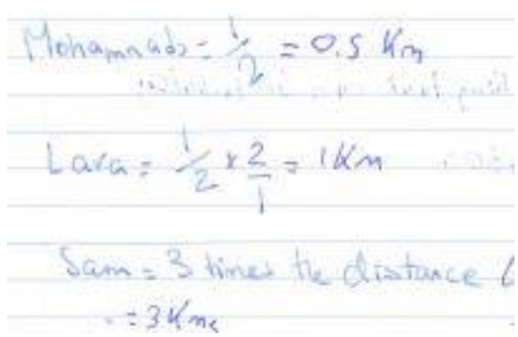

Figure 4: pre-equation EP-L

Figure 5: post-equation EP-L

Figure 4 shows EP-L's interpretation of the given information in the pre-equation problem. This problem requires students to find the distances travelled by each of four friends, when the total distance is given as well as the relations between different distances. When asked to justify their interpretation of the information, EP-L explained, "It's given that Mohammad's distance is one half as Lara. So we can tell that it's equal to half, and then we will double Lara's distance." Clearly, the group misunderstood the condition given on Mohammad's distance to school, and did not realize that the distances need to be written in terms of one another. However, in the post-equation problem (Figure 5), where the students are given the number of cows given in every two adjacent pastures, and the task is to find the number of cows found in each pasture separately, the students in this group explained, "we know that the number of cows in the first cube, which is the pasture, and the second cube is 29". When prompted to give further explanation about what this means, the students provided an example which demonstrated that EP-L completely understood the given situation. The following is an excerpt from the interview conducted with this group.

> - for example, if in cube 1 we have 16, then we will have 13 in cube 2….but we also need to keep in our mind, that if cube 1 has 16, then cube 3 will have 25-16=9 cows.
> - Interviewer: ok.. And can we have this?
> - Yes, we can check… but no, 9 is less than 10…. And in the problem they specified that we need to have more than 10 cows.

On the other hand, in CP-L, the same ability regressed, where the members were "unable to interpret enough given information", in the post-equation and post-pattern problems, after being able, in the parallel problems of the pretest, "to interpret most given information necessary to start the solving process. Figures 6 and 7 below shows this relapse which occurred between the pretest and the posttest. Figure 6 shows CP-L's interpretation of the situation in the pre-equation problem, which is correct. The group explained, "Mohammad's distance is one half Lara's so whatever Lara's distance is, Mohammad will be half. For example, if Lara's distance is 20 Km, then Mohammad's distance is 10 Km, and Sam will be 60 Km." The group only overlooked the given about the total distance.

Figure 7 shows the interpretation of the given information in the post-equation problem, which is wrong. When asked why they had 5 pastures in their solution, the members of CP-L explained, “we have 5 numbers given, they must be the number of the cows in the pastures.

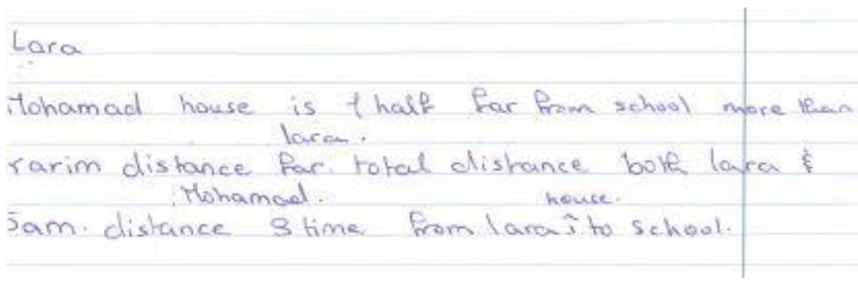
Lara
Mohamad house is 1 half far from school more than lara.
Karim distance far. total distance both lara & Mohamad. house.
Sam. distance 3 time from lara's to school.

Figure 6: pre-equation CP-L

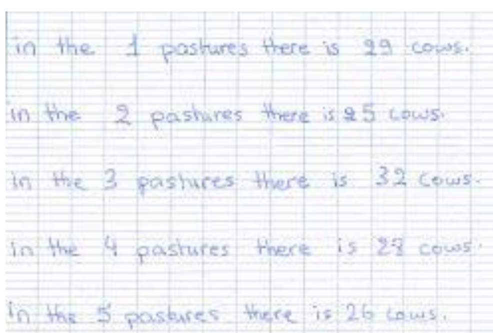
in the 1 pastures there is 29 cows.
in the 2 pastures there is 25 cows.
in the 3 pastures there is 32 cows.
in the 4 pastures there is 27 cows.
in the 5 pastures there is 26 cows.

Figure 7: post-equation CP-L

EP-L also showed some progress in the post-pattern and post-reasoning problems by being “able to interpret some given information to solve part of the problem or to get part of the solution”. On the other hand, CP-L maintained the same level of being “able to interpret most given information” in the pre- and post-reasoning problems.

4.2.2.2 Ability 2: Uses information appropriately

Upon comparing the ability to "use information appropriately", EP-L showed more improvement than CP-L, where the progress in this ability was reflected in two out of three problems with EP-L and in only one problem with CP-L.

EP-L was able to show good improvement, in the post-equation problem, where the group was able to "use all appropriate information correctly" after not being able to "use appropriate information correctly" in the pre-equation problem. Figure 8 shows EP-L's use of information in the pre-equation problem. In the pre-equation, the only information that was used correctly was Sam's distance, where members of EP-L decided to multiply Lara's distance by 3 as specified by the problem; however, all other information was misused.

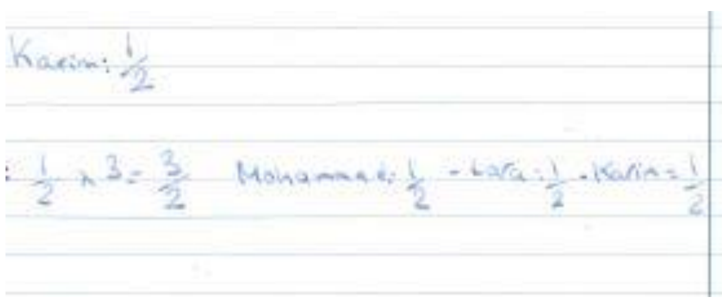

Figure 8: pre-equation EP-L

On the other hand, in the post-equation problem, EP-L was able to "use all appropriate information correctly" regarding the number of cows found in each two adjacent pastures while solving. Figure 9 shows how the students were using the information correctly.

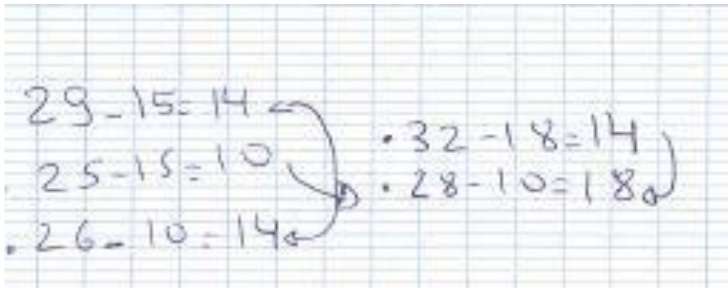

Figure 9: post-equation EP-L

The following is an excerpt from the interview conducted with EP-L while working on the post-equation problem.

- Interviewer: Can you explain to me what these calculations are?
- We are trying to find the number of cows in each of these cubes, so we are picking a random number, and checking if it works with the other numbers.
- Interviewer: Can you tell me how this is working with you?
- We are going to start with 15 cows for the cube here (pointing to the top left corner)… we do 29-15=14 in this cube (pointing to the top right), also 25-15=10 in this cube (pointing to the bottom left), then the 28-10=18 in this cube (pointing to the bottom right).
- Interviewer: Are you done?
- We can use these to check too… for example here (pointing to the top right with the bottom right) we have 32 in both. We can check by doing 32-18 from the one before to get 14, which is what we got before. I think we are done.

The group did not notice that they made a calculation mistake while subtracting 10 from 26.

The same ability regressed in both groups in the post-pattern problem, but improved slightly in the post-reasoning problem.

4.2.2.3 Ability 3: Applies appropriate representations/procedures

The qualitative analysis showed progress in EP-L's ability to "apply appropriate representations/procedures" to solve problems. The group was able to "apply completely appropriate representations/procedures" in the post-equation problem, whereas it was unable to do so in the pre-equation problem. The members "applied inappropriate procedures" to solve the post-pattern problem. In the post-pattern problem, the task is to find out the minimum number of tunnels needed to connect each of 30 cells with every other cell. The group was unable to devise an appropriate plan for solving, but they refrained from using the same wrong procedure they followed in the pre-pattern problem. The group members explained, "We can't divide by 2, even if every two cells are connected, because this is wrong. It means that not all are connected, but we don't know what else to do." The group faced a similar task in the pre-pattern problem where the students has to find out the number of games to be played, if six teams are competing such that every team plays with every other team only once. In that problem, members of EP-L were confused between two inappropriate procedures: 1-dividing 6 teams into groups of 2 to get 3 games, and 2- raising 2 to the power of 3.

The group's work in the post-reasoning problem also revealed some improvement in their ability to "use appropriate representations/procedures", though only slightly in this case. In the post-reasoning problem, students are given some clues to try and find out the number of dogs found in different kennels. EP-L made a diagram (Figure 10) which represented the different colored kennels. The group tried to represent differently colored kennels, where the students actually used different colors to keep track of some of the given information; however, they did not use all information appropriately as the group believed that one of the kennels is red and blue, instead of having two separate kennels for the mentioned colors.

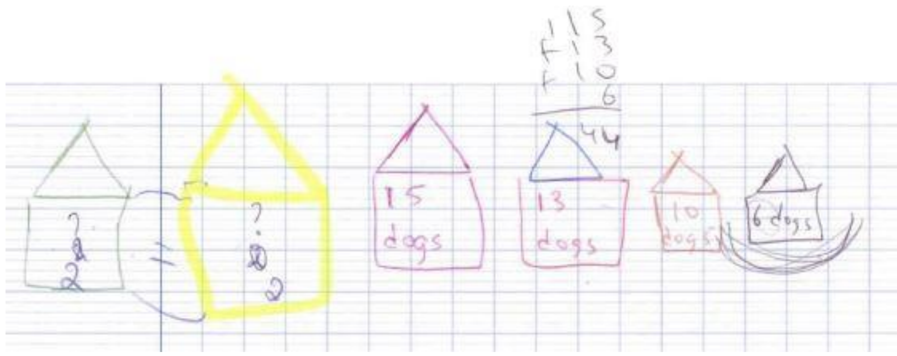

Figure 10: post-reasoning EP-L

In comparison, CP-L was not able to "use appropriate procedures" in the post-equation and post-pattern problems, where in the post-pattern problem the group divided the 30 cells by 2 to get 15 tunnels. However, the post-reasoning problem revealed a slight improvement in the group's ability to "use appropriate procedures", where the group used a table to fill in the number of dogs found in each kennel.

It should also be noted that, after the intervention, EP-L developed an ability to rely more on representations while solving. The group used some representations in the post-equation and post-reasoning problems to assist them while solving, while representations were poor in their pretest. Figure 11 shows a sample of the representations used in EP-L's posttest while working on the post-equation problem.

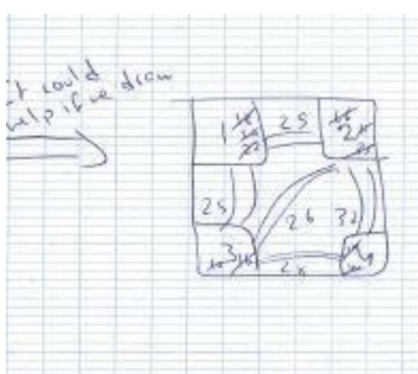

Figure 11: post-equation EP-L

CP-L, on the other hand, did use some representations in both tests; however, the representations were not useful while solving. For example, Figure 12 shows CP-L's representation while working on the pre-reasoning problem, where the students had to figure out a way to fill 6 liters of water in a drinking tank using only a 5L and an 8L container. The representation is poor and did not help the group while solving.

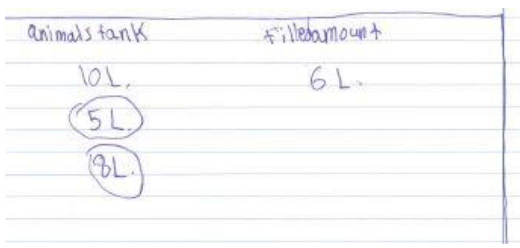

Figure 12: pre-reasoning CP-L

4.2.2.4 Ability 4: answers the problem

The improvement in the ability to "answer the problem" was more noticeable in EP-L's work. The group's ability progressed from submitting a "wrong answer", in the pre-equation and pre-reasoning problems, to giving "a correct solution with a minor technical error", and to "a partial answer to the problem", in the post-equation and post-reasoning problems respectively. CP-L's ability did not show any progress at all.

4.2.3 Qualitative analysis of EP-A and CP-A

Upon analyzing EP-A and CP-A's work in both the pretest and the posttest, the researcher deduced the following interpretations, basing the analysis on noticeable changes in the four assessed problem solving abilities. Refer to Appendix J for the Qualitative Comparison table of the average achieving groups.

4.2.3.1 Ability 1: understands the problem

EP-A displayed improvement in the ability to "understand the problem", as was revealed in the post-pattern and post-reasoning problems. Figures 13 and 14 below show the difference in interpreting given information between the pre-reasoning (Figure 13) and post reasoning (Figure 14) problems. The interpretation in the post-equation problem is more informative and detailed than the interpretation of information in the pre-equation problem, which aided the group in solving the problem. Figure 13 shows that EP-A provided a very short description of the situation presented in the pre-reasoning problem, where they had to fill a tank using two different kinds of containers. The group wrote "He has 5 liter", instead of a "5L container", which caused them some confusion while solving. On the other hand, Figure 14 reveals that EP-A provided a completely accurate description of the information provided by the post-reasoning problem.

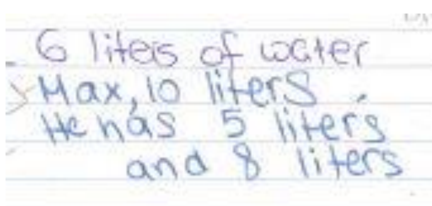

Figure 13: pre-reasoning EP-A

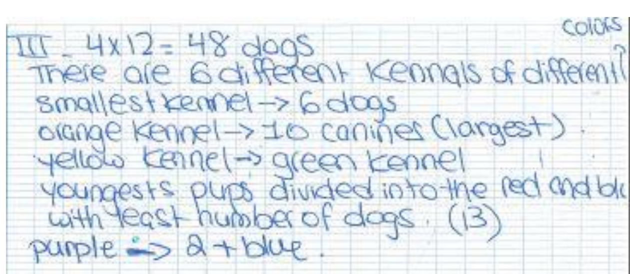

Figure 14: post-reasoning EP-A

Furthermore, upon asking the group members about the given conditions in the pre-pattern and post-pattern problems, a clear improvement can be depicted. EP-A interpreted the given information in the pre-pattern problem as follows "…every team will play only once". Thus the group overlooked two necessary conditions: 1- a team will play with every other team and 2- only once. However, in the post-pattern problem, EP-A was able to interpret all necessary conditions accurately, explaining "… so we need to connect each cell with every other cell, but since we need the minimum number of tunnels, then each cell will be connected only once to another cell, still we need to make sure all cells will be connected together"

The group maintained the same level of being "able to interpret all necessary information needed for a complete and correct solution" in the pre-equation and post-equation problems.

In comparison, CP-A maintained the same level of being "able to interpret most given information", as was revealed in the equation problem, and of being "able to interpret all necessary information" in the reasoning problem. The same ability regressed in the post-pattern problem as the group was "able to interpret some given information" after being "able to interpret all necessary information' in the pre-pattern problem. Figure 15 below reveals CP-A's correct interpretation of the conditions in the pre-pattern problem.

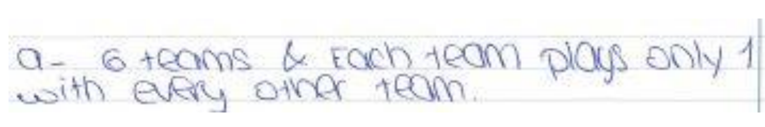

Figure 15: pre-pattern CP-A

After some prompting from the interviewer, the group explained the meaning behind the sentence they used to summarize the given. The following is an excerpt from the interview conducted with the group members while working on this problem.

- Six teams, and each teach plays only once with every other team. I feel they played 5 games. If 1 played with 2, this is 1 game, then 2 with 3, then 3 with 4, then 4 with 5, then 5 with 6. So 5 games.
- Interviewer: Does that fit the given you have?
- Yes, since six teams, and each teach plays only once with every other team.

On the other hand, Figure 16 shows the inaccurate interpretation of the given information in the post-pattern problem, where the group did not interpret correctly the condition of "fewest number of passages". Instead, they justified, "we need to connect each cell to every other cell.", and thus counted the combination of two cells more than once.

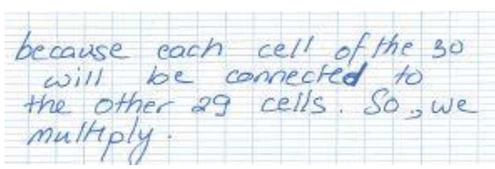

Figure 16: post pattern CP-A

4.2.3.2 Ability 2: Uses information appropriately

Upon analyzing the group's use of information while solving, EP-A revealed more progress than CP-A. EP-A was able to improve this ability in all three problems, whereas CP-A was able to do so in only two of the problems. An example of this progress is revealed in the pre-pattern and post-pattern problems for both groups. EP-A was not able to use all information and conditions presented in the pre-pattern problem, where the group members simply divided the 6 teams by 2, justifying that "Since there are 6 teams and in the tournament, 2 play against each other, so in 1 game there are 2 teams, so we divided 6 by

2 to give us 3." On the other hand, in the post-pattern problem, EP-A was able to "use all appropriate information correctly", justifying to the interviewer, "we need to connect every cell to all remaining cells, but we need the fewest number of tunnels, so we shouldn't count same connections again… for example we can't do cell 1 and cell 2, then cell 2 and cell 1, since we'll be counting it twice."

4.2.3.3 Ability 3: Applies appropriate representations/procedures

The qualitative analysis done on EP-A's work revealed the following. In general, the group showed improvement in the ability to "apply appropriate representation/procedure" while solving. More representations were used and relied on, especially when the members faced obstacles while solving, in contrast to the pretest, where the representations were poor. The group showed flexibility while switching between representations. Figures 17, 18 and 19 shows the representations used by EP-A in each of the three problems of the posttest.

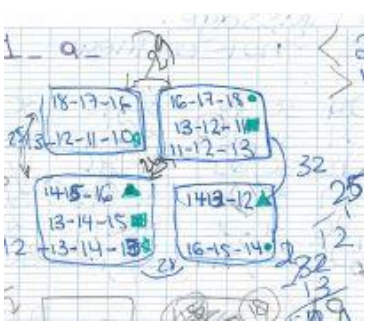

Figure 17: post-equation EP-A

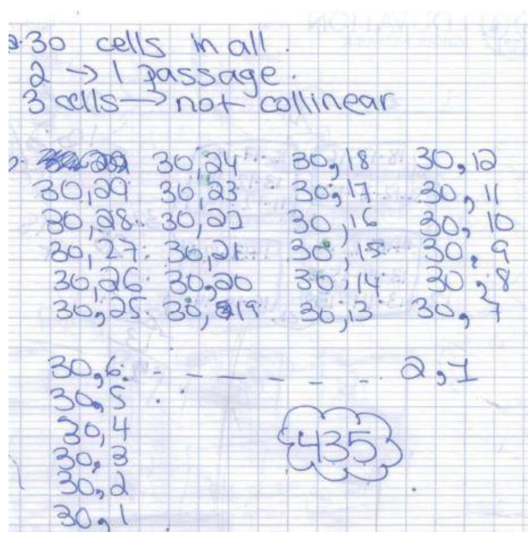

Figure 18: post-pattern EP-A

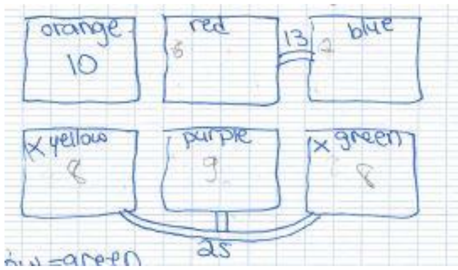

Figure 19: post-reasoning EP-A

As for the solving procedures, EP-A showed progress in all three problems, especially in the post-pattern problem, where the group revealed distinguished improvement in the ability to "apply appropriate representation/procedure". In the pre-pattern problem, the group failed to use an appropriate procedure and did not use any representation; however, they were able to use an advanced procedure to solve the post-pattern problem, which is similar in nature but considerably harder (Figure 17). The group also used an appropriate representation for the given situation. The group started by listing the possible connections among cells, then realized that this is time consuming, so the group recognized that a pattern was developing when the different combinations of connected cells are being listed (Figure 18). The group explained, "We started by listing all possible the possible combinations for cells that can be connected with cell number 30. For example 30, 29… 30,28…, and so on like this… then we did the same with cell 29, but we did not write it because it's too many numbers, so we counted them in our mind while referring to the list we did for cell 30. We wrote the number we found aside which is: 28 cells." The group repeated this for a couple more cells, and after a while, they noticed that a pattern existed among the numbers they counted. The group explained, "We noticed that first time it was, 29 then 28 then 27 so we deduced the other numbers, and kept subtracting till 1, which is the last." Finally, they added up the numbers they counted to reach a final answer.

In the post-reasoning problem (Figure 19), the group represented the situation in a diagram, which was informative and proved to be helpful to the students while solving. The students coded their diagram with the interpreted information given by the problem. When asked to explain their procedure, EP-A explained, "Each square is a different kennel. We will put the given on the figure. We placed an 'x' in the kennels which have the same number of dogs, yellow and green. We also connected the blue and red kennels and put 13 in the middle to know the sum of dogs in both kennels. So we have 48 dogs, we subtract the 13 and the 10 to get 25 dogs in the yellow, purple and green. We are going to use guess and check now to find the dogs found in these kennels."

In contrast, in the pattern problem, and after coming up with a correct solving procedure in the pretest, CP-A were not able to do so in the post-pattern problem.

Furthermore, EP-A displayed more organized and systemized solving procedures in the posttest than CP-A did. Referring Figure 17, one can notice that even when EP-A resorted to less advanced procedures such as Guess and Check, the group was organized by keeping track of the guesses. The group then decided this was time consuming, and so decided to focus on three connected pastures to find the number of cows in these three pastures first, and then calculate the fourth, and finally check. This is evident in Figure 20 below, where the group focused their guesses on the two bottom pastures and the top right one.

Figure 20: post-equation EP-A

On the other hand, CP-A did show some improvement in their solving procedures, mainly in the equation and reasoning problems; however, their procedures were unorganized and time consuming. For example, in the post-reasoning problem, the group made errors while guessing, and thus were unable to reach a correct and complete solution. The group also resorted to Guess and Check in the post-equation problem, but the guesses were random and unorganized, and did not take into consideration the condition that was set to narrow down the options, and thus wasted time trying guesses such as 9 cows.

It also has to be noted that the representations used by CP-A were not as informative and effective as those used by EP-A, and did not given much assistance to the group during the solution process as is shown in Figure 21 below.

Kennel	nb of Dogs
orange	10 Dogs
yallow	6 dogs
Green	6 dogs
Red	6 dogs
Blue	7 dogs
Purple	9 dogs

Figure 21: post-reasoning CP-A

4.2.3.4 Ability 4: answers the problem

EP-A was able to provide correct answers to all three problems in the posttest, after not being able to do so in the pretest. On the other hand, CP-A was able to give one correct answer and a partial answer in the posttest after submitting wrong answers or partial answers in the pretest. Furthermore, CP-A presented a wrong answer for the post-pattern problem after being able to "submit a partial answer" in the pre-pattern problem.

4.2.4 Qualitative analysis of EP-H and CP-H

Upon analyzing the work EP-H and CP-H in both the pretest and the posttest, the researcher deduced the following interpretations, basing the analysis on noticeable changes in the four assessed problem solving abilities. Refer to Appendix K for the Qualitative Comparison table of the high achieving groups.

4.2.4.1 Ability 1: Understands the problem

Upon analyzing both groups' work, there was no significant change in the ability to "understand the problem". Both groups maintained the same level of being "able to interpret all necessary information needed for a complete and correct solution" in the equation and reasoning problems, in both the pretest and the posttest. However, EP-H showed some regression in this ability, in the post-pattern problem, where the group members were unable to interpret all necessary conditions, and this hindered the solution process. The group misinterpreted the condition about every cell being connected to every other cell. Figure 22 below shows EP-H's representation of the situation presented in the post-pattern problem.

Figure 22: post-pattern EP-H

When the group was asked to explain their diagram, the members explained, "Every two cells will be connected to one another, to get the fewest number of tunnels…" This interpretation of the situation is inaccurate and incorrect.

The same group was "able to interpret all necessary information needed for a complete and correct solution" in the pre-pattern problem. The group explained that, " every team will play with every other team, but only once…there are 6 teams, so each team will play only 5 games, but we will not count all five each time, since we should only count the common games only once…"

On the other hand, CP-H also showed some regression in the ability to "understand the problem" where their ability regressed from "able to interpret most given information" in the pre-pattern to "able to interpret some given information" in the post-pattern problem. In the pre-pattern problem, the group explained the given information as follows, "The football tournament has 6 teams and then each team will play with every team once, so each team plays 5 times."; however, the group was not able to accurately interpret the "play only once" condition as is evident in Figure 23 below, where the students counted duplicate combinations of teams.

Figure 23: pre-pattern CP-H

In the post-pattern problem, the group misinterpreted the conditions set on connecting the cells, where the group only connected each two cells together.

4.2.4.2 Ability 2: Uses information appropriately

In general, CP-H's ability to "use information appropriately" regressed in the posttest, as was revealed in the three problems. The group was able to "use all or most appropriate information correctly" in the pretest, but was unable to do so in the posttest, where only part of the given information was used appropriately. For example, in the post-reasoning problem, the group was "able to interpret all necessary information", but used only some of this information correctly, as is evident in Figure 24 below.

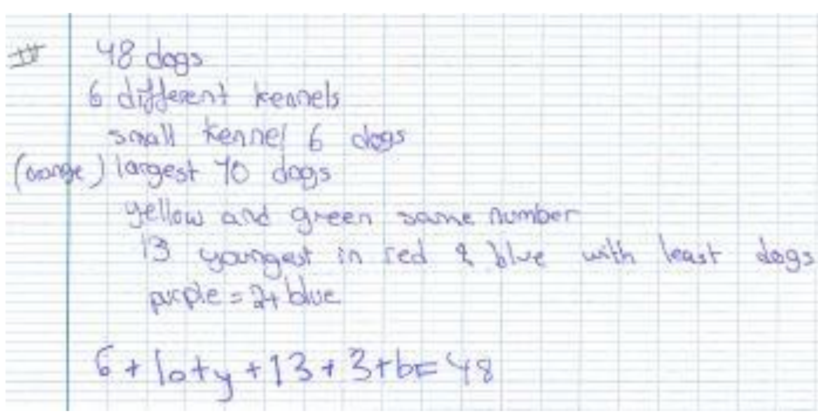

Figure 24: post-reasoning CP-H

The group decided to form an equation which sums up the total of dogs in all kennels, that is given to be 48; however, while substituting for the number of dogs in each kennel, the group misused some of the information, such as the number of dogs in the purple kennel, which is supposed to be 2+blue not 3+blue. In addition, the group did not account for the green kennel, justifying, “since they have the same number of dogs.”

On the other hand, EP-H maintained the same level of “using all appropriate information correctly” in the equation and reasoning problems in both tests. However, the group showed some regression in the pattern problem as a result of misinterpreting some of the necessary information, due to lack of complete understanding of all necessary conditions.

4.2.4.3 Ability 3: Applies appropriate representations/procedures

Concerning the ability to “apply appropriate representations/procedures”, EP-H displayed more improvement than CP-H. In general, EP-H maintained the same level of this ability in the equation problem, regressed in the pattern problem, but showed improvement in the reasoning problem. On the other hand, CP-H showed regression in the post-pattern and post-reasoning problems, but maintained the same level in the pre- and post-equation problems.

CP-H revealed no significant improvement in the post-equation problem. The group used the same procedure, which is equations, that was used in the pretest. The procedure is advanced, but the group was unable to follow the procedure throughout the solution. The group faced algebraic difficulties while solving the equation in the pretest, and was not able to form a correct equation in the posttest due to difficulties in manipulating the relations between the 4 unknowns. Figure 25 below shows the unsolved equation that the group used while attempting to solve the pre-equation problem.

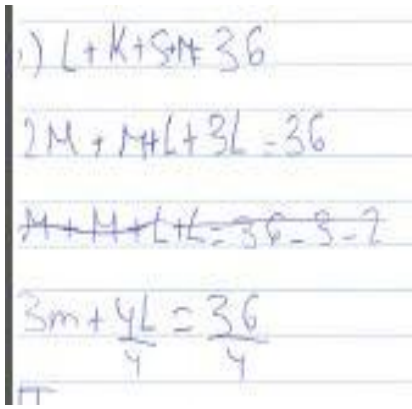

Figure 25: pre-equation CP-H

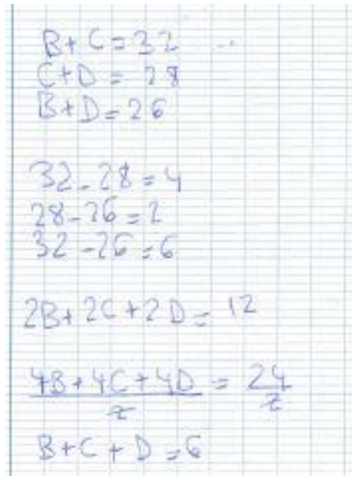

Figure 26: post-equation CP-H

Figure 26 above reveals the procedures that were used by CP-H to solve the post-equation problem, where they had to find the number of cows in different pastures given the number of cows found in each two adjacent pastures. The group used equations to represent relations between connected pastures, and then tried to compare the amount of cows found in different pastures. The group argued that if pastures B and C had 32 cows and pastures C and D had 28 cows, then there are 4 more cows in pasture B than in D. This analysis was represented by the following equations: B+C=32 and C+D= 28 so 32-28=4, so 4 more cows in B than in D.

CP-H used the same argument with the other pastures. When the group could not reach an answer this way, they decided to add the original equations (B+C=32; C+D=28; B+D=26) to get 2B+2C+2D=12. When asked about the value 12 that the students use, they explained, "This is the sum of the differences we calculated here." Referring to the series of numerical calculations shown in Figure 26. This is clearly an inappropriate solving procedure.

EP-H was able to "use completely appropriate procedures" in the equation problem of each test. The group solved the pre-equation problem by relying completely on an advanced procedure, which is formulating and solving equations. Figure 27 below displays the group's formulation of the main equation, which is an appropriate as well as advanced solving procedure.

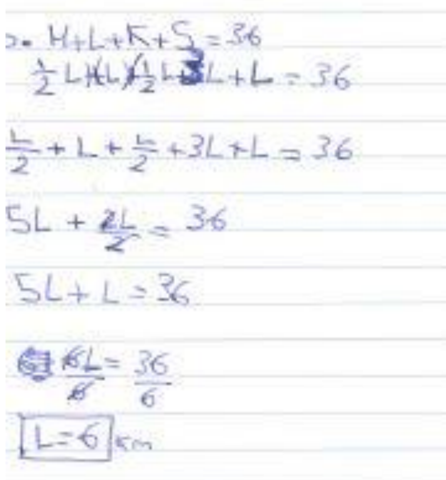

Figure 27: pre-equation EP-H

When asked to justify their work, EP-H explained, "We know that this problem needs an equation, which we will solve… since all the distances can be written in terms of Lara's distance, we will substitute these in the equation… so it is Lara's distance we will get first, then we can get the others.

In the post-equation problem, the group relied on guess and check in addition to formulating equations to reach a correct answer. Figure 28 below shows EP-H's work in the post-equation problem, where the group tried to formulate equations that relate each two adjacent pastures, for example x+y=29. The group members then argued that if they added the 2 equations x+a=25 and y+b=32, they can get the total number of cows, but then the group was unable to proceed. It has to be noted that even though the procedure used by EP-H in the posttest was not as advanced as the one used in the pretest, the group revealed some flexibility while switching among representations. When the group was unable to reach a solution by solving the formulated equations, the group members organized the information in a table and then used guess and check to solve the problem as is revealed in figure 27 below. The students still used the total number of cows that they calculated previously in addition to the given relations while checking their guesses.

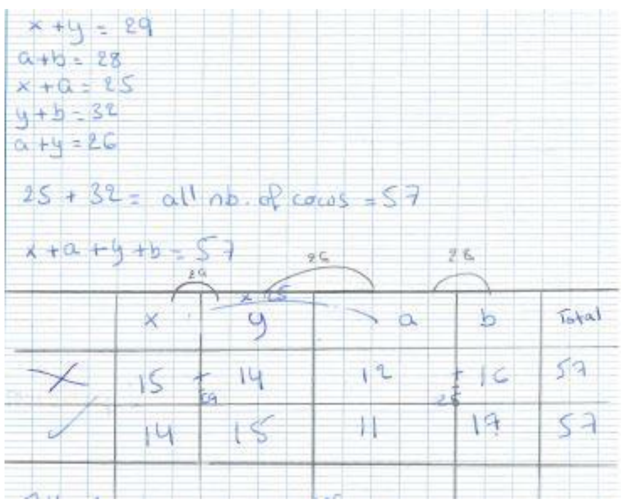

Figure 28: post-equation EP-H

Both groups regressed in the post-pattern problem mainly because their ability to "understand the problem" regressed in this problem which affected their ability to "apply appropriate procedure"

After comparing the groups' work in the reasoning problems of both tests, EP-H showed some improvement in the ability to "apply appropriate procedures", whereas CP-H revealed regression in the same ability. EP-H was able to represent all interpreted information in an equation, which the group simplified as much as possible relying on the given information. The group then depended on organized guess and check to get the final answer. On the other hand, CP-H attempted to formulate an equation which the members were unable to solve mainly due to their improper use of some of the given information as revealed in Figure 29.

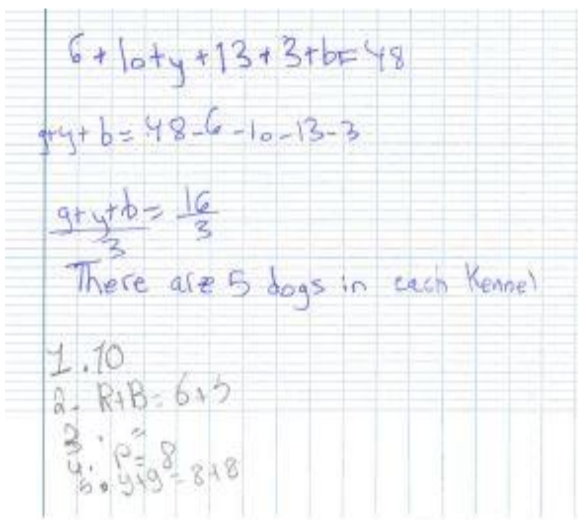

Figure 29: post-reasoning CP-H

It has to be noted that CP-H was able to use appropriate and advanced procedures to solve the pre-reasoning problem. The following is an excerpt from the interview conducted with the group while working on this problem.

- Teacher: So what does the worker have to do?
- To fill a drinking tank with 6 L.
- Teacher: did the problem specify any conditions?
- Yes. He needs to use the 5L and 8L containers.
- Teacher: so, how can he do that?

- We don't need the 10 L.
- Teacher: having these 5 L and 8 L containers, how can I fill the tank with 6
  The group took a few moments to think about this.
- Teacher: Can you explain to me what you are thinking of?
- So we thought of adding the 5 and the 8 to get 13 L, but 6 is between 5 and 8.. and we are trying to see how can we get 6 from 5 and 8
- Teacher: so how can you?
- Ok. We can fill the 8 then empty from it into the 5, we will have 3, so we repeat this 2 times.

The above dialogue reveals that the students of CP-H were able to use an advanced procedure in the pre-reasoning problem, where the students reasoned with the given numbers in order to fulfill the assigned task of filling the drinking tank with a certain amount of water.

4.2.4.4 Ability 4: Answers the problem

CP-H was not able to "provide a correct solution" in the equation and pattern problems of both tests. In the pre-equation and post-equation problems, the group provided "a partial answer" due to algebraic difficulties while solving, so one can deduce that the group's ability did not improve in this area. In both pattern problems, the group was not "able to interpret all necessary information", which lead them to a wrong answer. Furthermore, the group showed regression in the post-reasoning problem, where the members provided a wrong answer in the posttest after being able to provide a correct answer in the pretest.

EP-H maintained the same level of being able to "answer the problem" in the pre-equation and post-equation problems, where the group provided "a correct solution" to both problems. The group was unable to reach a correct answer in the post-pattern problem after being able to do so in the pre-pattern problem. EP-H showed some improvement in the post-reasoning problem, where the group was able to reach a correct answer, as revealed in Figure 30 below.

Figure 30: post-reasoning EP-H

On the other hand, in the pre-reasoning problem, the group was not able to put the answer in words that would reflect the real life situation presented by the problem. Figure 31 below reveals EP-H's answer in the pre-reasoning problem, where the group explained, " I know that the answer has to do with the following number relations, 8-5=3 and then 3×2=6, or maybe 8+5=13 and 13 -10=3, but I can't really explain this answer.

Figure 31: pre-reasoning EP-H

In general, the qualitative analysis of EP-H and CP-H's work in the pretest and posttest, reveals no significant shifts or progress in their problem solving abilities. In fact, a slight regression can be detected in CP-H's overall performance.

This section presented a stratified qualitative analysis of the performance of the interviewed pairs of the two groups. The analysis compared the progress per ability revealed by each pair. In general, it was found that the low and average achieving pairs of the experimental group revealed more improvement in the four assessed problem solving abilities, than the corresponding pairs of the control group.

A global analysis, which focuses on the improvement in these abilities and the effect reflected on each of the three problems in the posttest, reveals that the low achieving pair of the experimental group showed more progress in the equation and reasoning problems than the corresponding pair of the control group. Furthermore, the average achieving pair of the experimental group showed a significant improvement in the pattern problem. Also, the high achieving pairs of the experimental and control groups showed no progress in the equation and pattern problem; however, the experimental pair showed more improvement in the reasoning problem than the high achieving control pair.

The coming and final chapter discusses the results that are presented in this chapter, globally and in answer to the main research question and supplementary questions. Chapter 5 also compares the results of this study with similar and previous studies conducted on the effect of problem posing on students' problem solving abilities. Furthermore, the chapter also describes the limitations on the current study, as well as suggests some perspectives for future studies.

Chapter Five

Discussion and Conclusion

The current study aims to to study the effect of implementing problem posing as a teaching and learning strategy on the problem solving abilities of students at the level of grade-8. The study was conducted in a private English speaking school, where problem posing activities were implemented in one of two classes, with the other class being the control group. Problems of a parallel nature were implemented in the control group at the same time as the implementation of problem posing strategy in the experimental group. A pretest and a posttest were administered to both groups in order to determine the level of improvement in the problem solving abilities of the control and experimental groups. Furthermore, clinical interviews were conducted with a sample of students from both groups, for qualitative analysis that targeted the effect of problem posing activities on each of the four assessed problem solving abilities.

5.1 Discussion of results

In the following section, the discussion of results is presented based on the main research question and the two supplementary questions.

5.1.1 *What impact does problem posing have on students' mathematical and problem solving cabalities?*

The quantitative analysis shows that problem posing activities have an overall positive influence on the problem solving abilities of students. The quantitative analysis reveals a significant association (p-value=0.05) among the experimental and control groups, where the experimental group showed an improvement of 9.4% which is higher than control group's improvement of 1.1% after the study.

5.1.2 How do the problem solving abilities of students develop throughout implementing problem posing strategies while teaching problem solving in class?

The quantitative analysis revealed that students in the group that implemented problem posing activities, showed improvement in problems that require logical reasoning, critical thinking and analysis, more specifically problems that require conceptual understanding. This was evident since the quantitative analysis revealed a significant association between the two groups in the pattern and reasoning problems (p- values=0.003 and 0.01 respectively), in which the experimental group showed more improvement than control group.

On the other hand, the study revealed that students who were not subject to problem posing activities, were more successful in problems that require procedural understanding. This is revealed by the progress that the control group revealed in the equation problem. This supports the notion that problem posing actitivies target students' critical thinking and conceptual understanding of mathematical concepts. However, the usual, or procedural way problem solving is taught, focuses on concepts that are more of a procedural nature.

Furthermore, the study showed that problem posing activities improves significantly the ability to "understand the problem", where the quantitative analysis revealed that the experimental group showed more progress, in this ability, than the control group did. The experimental group revealed an improvement of 16.3% in this ability in comparison to a slight regression of 0.67% in the control group.

The experimental group showed better improvement than the control group in the other three abilities as well; however, the significance of this improvement decreases with the other higher-level abilities. The researcher believes this is because of the limited duration of implementing the problem posing strategy. Given more time, other abilities would probably show more development.

5.1.3 *How will the inclusion of problem posing activities affect differently students with originally different levels of problem solving abilities?*

The researcher relied on both qualitative and quantitative analyses to answer this supplementary question.

5.1.3.1 Conclusions based on the qualitative analysis

To qualitatively determine which groups showed more improvement in each of the achievement groups, the researcher counted the number of problems in which each group showed improvement, for a particular ability. The conclusion is presented in the following sections based on each of the four abilities

The ability to "understand the problem"

Out of three problems, the low achieving pair of the experimental group improved in all problems, while low achieving pair of the control group did not show improvement; the average achieving pair of the experimental group improved in 2, while the average achieving pair of the control group improved in none; both high achieving pair of the experimental and control groups did not face problems in this ability from the beginning of the study, so there was no noticeable change.

Thus, in general, it can be determined that the low and limited achieving and the average achieving students of the experimental group benefitted from the problem posing activities.

The ability to "Use information appropriately"

Out of 3 problems, the low achieving pair of the experimental group improved in 2, while the corresponding control pair reflected no improvement; the average achieving pair of the experimental group improved in 3, while the corresponding control pair improved in 2 problems; the high achieving pair of the experimental group regressed in 1, but at the same time the corresponding control pair regressed in 3 problems.

The researcher inferred that the problem posing strategy was more beneficial to the low and limited achieving and the average achieving students of the experimental group.

The ability to "Apply appropriate procedures/representations"

Out of 3 problems, the low achieving pair of the experimental group improved in 2 while the corresponding control pair improved in only 1; the average achieving pair of the experimental group improved in 3, but the corresponding control pair improved in only 2; the high achieving pair of the experimental group regressed in 1 problem and improved in 1 problem, but the corresponding control pair regressed in 2.

The researcher decided that problem posing activities were beneficial to students of different achievement levels of the experimental group with respect to this ability.

The ability to "answer a problem"

Out of 3 problems, the low achieving pair of the experimental group improved in 2 while the corresponding control pair did not show improvement; the average achieving pair of the experimental group improved in 3, whereas the corresponding control pair improved in 2; the high achieving pair of the experimental group improved in 1 problem, while the corresponding control pair did not show improvement in any of the problems.

The researcher deduced that problem posing activities were more effective with the limited and low achieving students and to a lesser effect with the average and high achieving students of the experimental groups.

In conclusion, the qualitative analysis which focused on the work of the sample chosen for the clinical interviews revealed the following inferences:

1- Regarding the four assessed abilities, problem posing activities are found to be effective with students of all achievement categories.

2- The effect of problem posing on students' ability to "understand a problem" and to "use information appropriately" is more evident in the low and limited achieving and average achieving groups.

3- Problem posing actitvities seem to assist high achieving students with their use of procedures and representaions, as well as providing a correct answer to a problem.

5.1.3.2 Conclusions based on the quantitative analysis

The stratified quantitative analysis, which assessed the performance of different achievement categories in each of the two groups, is discussed in this section per ability, after which a general conclusion is presented.

"Understands the question"

The low, average and high achieving experimental groups were more successful than the corresponding control groups, with the analysis revealing a significant association between the high achieving experimental and control groups (p-value=0.002), where the students' ability to "understand a problem improved by 35% in the high achieving experimental group, while those in the control group showed no improvement.

"Uses information appropriately"

The low, average and high achieving experimental groups showed more improvement than the corresponding control groups. It has to be noted that the average and high achieving students improved slightly more than the limited and low achieving students.

"Applies appropriate procedures/representations"

The analysis revealed a slight improvement for the average and high achieving experimental groups over the corresponding control groups.

"Answers a problem"

The analysis reveals a slight improvement for the average and high achieving experimental groups over the corresponding control groups. The analysis also shows a regression for the low achieving experimental group; however, the low achieving control group showed more regression than the corresponding experimental group.

According to the stratified quantitative analysis done on the achievement groups, the researcher reached the following conclusions:

1- Regarding the four assessed abilities, problem posing activities seem to have a positive effect on students of different achievement groups.

2- The high and average achieving students showed more improvement in the abilitiy to “understand a problem” than the low and limited achieving students.

3- In general, problem posing activities had more effect on the average and high achievers of the experimental group than the low and limited achievers. The analysis revealed a more significant statistical association for the improvement of the average achieving experimental group over the correstponding control group (p-value=0.19) than the association for the improvement of high achieving experimental group over the corresponding high achieving control group (p-valule=0.27). However, the researcher decided that the improvement of the high achieving experimental group is educationally significant due to the smaller sample size of the high achievers in comparison to the average achievers.

After crossing the results of the quantitative and qualitative analyses, the study revealed the following conclusions:

Implementing problem posing activities, during the learning/teaching of problem solving, has positive effects on the problem solving abilities of students with different achievement levels, as is reflected by the qualitative and quantitative analyses. However, quantitative analysis revealed that the improvement in the ability to "understand a problem" is more evident with average and high achieving students. On the other hand, the qualitative analysis revealed that this improvement was more evident with limited and low achieving and average achieving students. Since the sample sizes for the qualitative analysis was small, and that for the quantitative was relatively higher, then in the current study, the researcher feels that the quantitative results were more reflective of the effect of problem posing on students' problem solving abilities. Thus, the researcher will adopt the results of the quantitative analayis to determine which achievement group was affected most by the problem posing approach. In general, the high and average achieving students reflected more improvement in the posttest. Statistically, the average group showed a more significant improvement than the high achieving group; however, due to the considerable difference in the sample sizes of these two groups, the improvement made by the high achieving group is also taken into account.

After taking the limitations that were present in this study into consideration, the researcher concludes that problem posing has a positive effect on students' problem solving abilities, specially in terms of prompting understanding, flexibility, and creativity.

5.2 Conclusion

The results of this study concur with previous studies conducted by pionneers in the area of mathematical problem posing and solving. Several studies reveal that problem posing has positive effects on students' problem solving abilities (English,1997a,1997b; Silver, 1994; Singer et al., 2013). For example, the study conducted by Kesan et al. (2010), which explored the effect of problem posing activities on students' performance, reveals that students who were subjected to problem posing activities showed a better performance than the students in the control group. The results of the current study agrees with such studies, where the group exposed to the problem posing activities revealed a higher level of improvement, in terms of problem solving abilities, than the students in the control group.

Advocates of implementing problem posing in the teaching/leraning process believe that problem posing activities develop students' critical thinking and conceptual understanding. In the study conducted by Ellerton(2013), in which pre-service teachers were subjected to problem posing activities, the participants reported that they had a better understanding of the involved problem structures, after being involved in these activities. Furthermore, the study conducted by English (1997a, 1997b), which focused on developing the problem posing abilities of fifth graders, revealed that students who were involved in problem posing activities displayed a better understanding of problem structure and situation than those who did not participate in the study. Also, in the study conducted by Cankoy and Darbaz (2010), which explored the effect of problem posing activities on third graders' ability to understand a problem, reveals that this ability showed more improvement with the students in the experimental group than the control group. The results of the current study concurs with these studies, since the analysis reveals that students of the group which implemented problem posing had an improved ability of understanding a problem.

5.3 Assumptions, Scope and Limitations

This research aims to give mathematics educators a clear idea about the effect of using problem posing as a teaching strategy while teaching problem solving in a mathematics intermediate class. It will be a starting point for the implementation of this strategy to improve students' conceptual understanding of mathematical concepts at grade-eight level. On the other hand, the sudden implementation of instruction based on problem posing in one of two classes who should basically follow the same curriculum for a specific unit presents both a challenge and a complexity for the concerned teacher. Teachers face difficulties such as planning time, time allotted for activities in the classroom, and changing teaching styles in the middle of the year which can be reduced through proper and continuous training and resources. The study presents some challenges for the group of students who are not accustomed to posing their own problems in Math. This is controlled by a well-trained teacher who can guide and assist students throughout the assigned activities.

A limitation of the current study towards generalization of the results is the low number of the chosen participants. Thus, in the smaller groups, the statistical associations between groups do not reflect any significance. Therefore, the researcher determined the significance of the effect of problem posing depending on the difference of improvement between the two concerned groups.

Another limitation to this study is that the experimental and control groups are taught by different mathematics teachers. This adds another variable to account for in this study, since different teachers have different teaching strategies, classroom management techniques, etc… Furthermore, a major drawback is the fact that problem solving and posing activities are usually best applied in small groups. This is to ensure that feedback on students' work and approaches is provided in adequate time. However, the researcher is compelled to conduct the study in classes with an average of 30 students per section, and thus the teacher should exert extra effort so that implicit feedback is provided to all students.

Location and implementation threats might be possible in this study since the two groups were working in two different rooms where different discussions might arise among students. Furthermore, the teacher, who was also the researcher in this study, might be somehow biased while implementing the unit plan in Experimental Group. Thus, there was a threat of subjectivity in this case while delivering the material. However, the teacher was aware of such threats and strived to abide by the rules of control set throughout the design of the study to maintain objectivity and fair treatment.

It should also be mentioned that the teacher who is implementing the unit with the posing activities did not receive professional training in such teaching strategies. Rather, the teacher is self-trained and educated in these strategies, after referring to experiences and a wide body of literature by pioneers in this area.

5.4 Perspective for further research

The current study reveals that implementing problem posing as a teaching/learning strategy has positive effects on students' problem solving abilities. However, the results can not be generalized due to the small sample size. This calls for further research regarding the implementation of problem posing activities in the mathematics curriculum. Such reseach should be conducted with larger samples for both quantitative and qualitative analyses. Future studies should also put more emphasis on qualitative analysis of a larger sample than the one adopted in this study. Furthermore, the implementation duration should be longer to gain more significant information regarding the progress of problem solving abilities when students are subjected to problem posing activities.

Moreover, this study revealed that problem posing activities benifitted students in problems that requires critical thinking and logical reasoning more than it did in problems that requires procedural understanding. Thus, it might be also useful to research a model/process which starts with a nonposing problem solving approach and proceeds with problem posing strategies which ought to benefit all achievement levels, as well as address different types of word problems.

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Appendix A

Pretest

Name: **Date:**

Grade 8: **Time:60 minutes**

Instructions:

- **You are given 3 independent problems to solve. You are free to use any strategy to find a solution to each of the problems.**
- **Write all necessary justification, diagrams, drawings etc… on this paper.**
- **If you feel a need to explain why you chose a specific step, please do so next to the related step.**
- **Relax and be creative!**

I- Lara, Mohammad, Karim, and Sam are friends who live in neighboring cities. They commute to school every day by their parents' cars. Mohammad's house is one half as far from school as Lara's house. Karim's parents drive as far as the total distance that both Lara and Mohammad's parents drive together. The distance from Sam's house to school is 3 times the distance from Lara's house to school.

a- Rewrite the necessary given in your own words.

b- How many kilometers does each friend travel to school if the friends together travel 36 kilometers?

II- Six teams are involved in a football tournament. In such a tournament, each team plays with every other team once and only once. It does not matter who wins the games.

a- How would you represent the above given information to start your solution?

b- How many games in all are played?

III- A circus worker needs to fill an animal's drinking tank with exactly 6 liters of water. The drinking tank can be totally filled with 10 liters of water. He has a 5-liter container and an 8-liter container.

a- Is there an extra given in this problem?

b- Use a proper representation (drawing, diagram, sketch, etc.) to represent the above given information.

c- How can he use these containers to fill the drinking tank?

d- Is there a simpler way (smallest number of steps)?

Appendix B
Math Problem Solving Rubric

Name: ______________________________

	Distinguished - 4	**Proficient - 3**	**Apprentice - 2**	**Novice - 1**
Understands the Problem	Able to interpret all necessary information needed for a complete and correct solution	Able to interpret most given information necessary to start the solving process.	Able to interpret some given information to solve part of the problem or to get part of the solution	Unable to interpret enough given information to get started or make progress
Uses Information Appropriately	Uses all appropriate information correctly	Uses most appropriate information correctly	Uses some appropriate information correctly	Uses inappropriate information
Applies Appropriate Representations/Procedures	Uses correct and advanced procedures/ representations which lead to a correct solution	Applies completely appropriate procedures/representations	Applies some appropriate procedures	Applies inappropriate procedures
Answers the Problem	Correct solution of problem with answer statement and correct labeling of answer.	Correct solution with minor technical error (calculation mistake, copy mistake) with answer statement and correct labeling of answer.	partial answer for problem (due to major mathematical error), no answer statement, answer labeled incorrectly	No answer or wrong answer based upon an inappropriate plan

Appendix C

Unit Plan: Experimental Group

Grade: Intermediate level grade 8

Number of sessions: 17 sessions- 50 minutes each

Unit title: Problem Solving and Posing

Objective: the objective of this unit is to use problem posing in Mathematics as a teaching strategy while working on various mathematical word problems.

By the end of this unit students will be able to:

- identify the important elements of a problem situation
- restate the problem in their own words
- organize given data in an effective way
- use various problem solving strategies
- check the validity of their answers
- pose extension problems to original problems posed by the teacher
- pose similar problems to those posed by the teacher
- pose problems of their own

Note: while implementing this unit, the teacher should pay special attention to the following points:

- stress the significance of each of the provided given as well as the question at hand
- encourage students to discuss their ideas, strategies as well as concerns regarding the problem situation at hand
- emphasize the social element of solving such problems by frequently providing students with opportunities to question themselves as well as their peers while discussion
- encourage students to constantly validate their answers by checking the situation at hand
- encourage students to justify significant steps of their solution strategies as this ought to help them organize their work, thought and approach

Lesson Plan 1

Duration: 2 non-consecutive sessions (50 minutes each)

Objective: students are expected to:

- Identify the important elements in the given data which permits solution of a problem
- Assign symbols to represent different variables in a word problem
- Translate given data into a mathematical expression
- To communicate appropriately a valid answer

Solution: the difference between the two accounts is $51

Phase I : solving

Stage 1: introducing the problem (10 minutes)

- The following problem is introduced, and the given data is explained and discussed (there is a missing given).
  "Sara and Tala each have bank accounts. Sara withdraws $15 each weekend while Tala deposits $12. At the end of 13 weeks, what is the difference in their bank accounts?
- Guide learners towards discovering that there is actually some given missing. Then add it. "Sara has 500$ and Tala has 200$"
- Discuss possible starting points that would allow solving this problem.

Stage 2: solving the problem: (10 minutes)

- Allow students to work on their own to solve the problem while rotating among them.

Stage 3: discussing solution: (15 minutes)

- Discuss different solution strategies of students.
- Reflect on what might be a better strategy.

Phase II: Posing

Objective: students are expected to:

- Identify the important elements in the given data which permits solution of a problem
- Assign symbols to represent different variables in a word problem
- Translate given data into a mathematical expression
- Vary different given data in a problem, and to modify the problem accordingly
- Manipulate given data algebraically
- Make up their own version of such problem situations
- To communicate appropriately a valid answer

Stage 1: vary the problem given : (15minutes)

- Take students' suggestions on how the given information can be varied,
  For example: if we don't know what Sara's initial account is, what might we need to add as given information to be able to solve. If we don't know how much Tala withdraws? Can you find a week when there is only a $3 difference?
- Assign the following homework: "Pose your own problem similar to the one we worked on during this session. Try changing some elements in the problems such as the given or the question."

Note: stages 2 and 3 take place in the second session.

Stage 2: posing a problem: (25 minutes)

- Ask students to swap their posed problems with a friend.
- Allow students to work on their friends' problems on their own.
- Students are asked to try and solve their friends' posed problem and in case it can't be solved to try and modify it.

Stage 3: discussing the assignment (25 minutes)

- This time is used to discuss the posed problems especially those that needed modification.
- Stress the factors that helped in solving interesting problems as well as those that hindered solution.
- Make sure students always check if their answers are valid.

Lesson Plan 2

Duration: 1.5 sessions (75 minutes)

Objective: after the 2 phases, students are expected to:

- Identify important given data for the situation at hand
- Organize the given data using an effective representation
- Predict a possible question to the situation at hand
- Check if the suggested question can be solved or not, then modify the given situation accordingly
- Present a meaningful and valid answer to their questions
- Make up a similar problem to an original given situation

Phase I: solving

Stage 1: introducing the problem (10minutes):

- The following problem is introduced and discussed:

  "Fadi and his parents are a little unusual. If he does an acceptable job of doing his chores, he gets paid $3.33 for that day. If he does an outstanding job, he gets $3 more. During a 10-day period, Fadi received $42.30 for his work. "

  (The problem is missing a question)

  - Ask student to organize the given data in a way that would make the situation clearer to them.

Phase II: posing

Stage 2: students' work (15 minutes)

- Students are encouraged to pose a relevant question to be answered by the above situation.
- Discuss with students on what might work as a relevant and meaningful question in this case.
- Ask students to work on solving the questions they posed.
- Rotate among them to monitor and guide.

Stage 3: class discussion (15 minutes)

- Discuss the different questions posed along with the strategies used.

stage 4: posing a problem(15 minutes)

- Ask students to make a similar problem in which a similar situation is to be solved.
- Encourage them to build a problem in which a different question is going to be posed.
- Rotate among them and offer guidance when needed

<u>Stage 5: class discussion (20 minutes)</u>

- Discuss their problems and reflect on any gaps or on significant posed problems.

Lesson Plan 3

Duration: 1.5 sessions (75 minutes)

Note: students at this level should be able to translate a problem into equations and to solve a first degree equations; however, they are not obliged to use equations to reach a solution.

Objective: students are expected to:

- Translate given data into mathematical expression
- formulate an equation of the first degree
- use algebraic reasoning to solve an equation of the first degree
- present a meaningful and valid solution to the given situation

Solution: Tala made $12.8, Lili made $51.2, Samar made $25.6 and Jad made $110.4.

Phase I: solving

Stage 1: introduce the problem (5 minutes)

Lili, Tala, Samar and Jad were selling cookies in their school's food sale. At the end of the sale, each had made a different amount of money. The profits that each made were as follows: Samar made half of what Lily earned. Jad earned double what Lily made after it was increased by $4. Tala made half what Samar made. Lily noticed that the total profit of her and her friends was $200 more than what she made. How much money did each of the three friends make?

Stage 2: students' individual work (15 minutes)

- Allow students some time to work on the above problem.
- Encourage students to organize the given data.
- Rotate among students to offer guidance.

Stage 3: class discussion (15 minutes)

- Ask some students to discuss their solution strategies with the class.
- Stress on important factors that helped in solving the problem or hindered the solution.
- Encourage students to reflect on the importance of each given provided in the problem

Phase II: posing

Objective: in addition to the above mentioned objectives, students are expected to:

- Write a situation based on a given mathematical expression
- Make up a word problem based on a given mathematical expression
- Check the validity of a solution to a given answer
- Present a meaningful answer to a given situation

Phase 1: introduce the task (5 minutes)

- Make up your own money problem that can be solved using the following equation $3(x+2) = 2x - 5$
- Note: the variable turns out to be negative. The purpose of this is to raise in students' mind the issue of loss and gain, or similar situations such as height and depth, moving forward and backward, etc..

Phase 2: students' work (15 minutes)

- Allow students some time to work on their problems. Rotate among them to offer guidance.
- Encourage them to check if their problems can be solved using the given equation
- Suggest to include more than one variable in their problems

Phase 3: class discussion (20 minutes)

- Ask some students to pose their questions to the whole class.
- Point out significant factors that would allow or hinder the solution.
- Ask groups to swap problems and to try and solve them. An important element here is to check whether the situation matches the given equation.
- Rotate among groups to check their progress.

Invite groups with significant work to discuss their approaches with the whole class

Lesson Plan 4

Duration: 1 session (50 minutes)
Objective: this is a matching word problem, in which students are expected to:

- organize the given data in an effective way
- identify the conditions that are necessary to solve the problem
- use an appropriate representation that will facilitate the solution
- To communicate appropriately a meaningful solution

Solution:
Mrs. Anderson and Terry, Mrs. Blake and Janine, Ms. Allen and Beth, Mrs. Murphy and Jessica

Phase I: solving

<u>Stage 1: introducing the problem:</u> (5 minutes)

- The following problem is introduced:
  "Match the mothers and daughters.
  Clues:
  No mother is exactly twice the age of her daughter.
  No mother has the same hobby as her daughter.
  No mother likes the same music as her daughter.
  Terry is 20 and enjoys photography and rock music.
  Janine is 22 and enjoys golfing and the blues.
  Beth is 19 and enjoys cooking and jazz.
  Jessica is 25 and likes dancing and classical music.
  Ms. Blake is 40 and likes cooking and classical music.
  Ms. Anderson is 50 and likes golfing and jazz.
  Ms. Murphy is 44 and enjoys working in the kitchen.
  Ms. Allen is 42. "
- Allow a brief discussion on possible ways to start the solving process.

Stage 2: solving the problem: (10 minutes)

- Allow individual work on the solution of problem.
- Encourage the use of any strategy students prefer.
- Rotate to offer guidance when needed.
- Stress the importance of checking the answer.

Stage 3: discussing the problem(10 minutes)

- Allow some students to present their solutions and discuss the different strategies used.
- Reflect on the most effective strategy.

Phase II: posing

Objective: students are expected to :

- Identify important elements required to make up a solvable matching problem
- Revise and modify their work
- Reflect on necessary conditions that allow such problems to be solved
- To communicate appropriately a meaningful solution

Stage 1: posing a problem: (25 minutes):

- Ask students to make up a similar problem. Encourage them to vary the setting of the problem in any way they wish to.
- Allow them around 10 minutes of individual work to make up their problems.
- Rotate to check progress and to provide assistance.
- Discuss whether their problems can be solved or not.
- The remaining time is allocated for class discussion and emphasis on any missing elements of the proposed problems.

Lesson Plan 5:

Duration: 2 sessions (50 minutes each)

Objective: this is a pattern problem. Students are expected to:

- Organize given data in an effective way
- Discover a pattern in a given situation
- Extend the pattern to reach the desired solution

Solution: they will need a total of 728 legos to make a 12-level building.

Phase I: solving

Stage 1: introduce the problem (5 minutes)

- Introduce the following problem:
  "Jody, Amy, and Tim are building a model out of Legos. The top level is a 1 by 2 rectangle. The level directly underneath is a 2 by 3 rectangle. The level underneath is a 3 by 4 rectangle, and the pattern continues. How many Legos will they need if their building is to be 12 levels high? "

Stage 2: students' individual work (15 minutes)

- Allow students some time to work on the above problem.
- Encourage them to organize the given data using a representation that may help them understand the situation.
- Rotate among students to offer guidance.

Stage 3: class discussion (15 minutes)

- Ask some students to discuss their solution strategies with the class.
- Stress on important factors that helped in solving the problem or hindered the solution.
- Encourage students to reflect on the importance of each given provided in the problem.

Phase II: posing

Objective: in addition to the above mentioned ones, students are expected to:

- Vary some given data in a problem situation and then modify the solution accordingly
- Make up a pattern problem that can be solved by a specific given pattern

Stage 1: varying the given (15 minutes)

- Encourage students to choose some given of their choice to be varied. What would happen to the answer in each case?

Stage 2: introduce the task (10 minutes)

Given the following pattern: 1, 3, 5, 7, etc… make up a word problem in which the given sequence will be used. Pose your own questions to your problem.

Phase 2: students' work (15 minutes)

- Allow students some time to work on their problems. Rotate among them to offer guidance.
- Encourage them to check if their problems can be solved using the given pattern

Phase 3: class discussion (25 minutes)

- Ask some students to pose their questions to the whole class.
- Point out significant factors that would allow or hinder the solution.
- Ask groups to swap problems and to try and solve them. An important element here is to check whether the situation matches the given equation.
- Rotate among groups to check their progress.
- Invite groups with significant work to discuss their approaches with the whole class.

Lesson Plan 6:

Duration: 2 sessions (50 minutes each)

Objective: this is a pattern/combination word problem, in which students are expected to:

- organize the given data in an effective way
- identify the conditions that are necessary to solve the problem
- use an appropriate representation that will facilitate the solution
- recognize a pattern which is formed depending on the given conditions
- To communicate appropriately a meaningful solution

Solution: there are 36 children.

Phase I :solving

Stage 1: introducing the problem (5 minutes):

The following problem is introduced and discussed:

"An ice cream stand has 9 different flavors. A group of children come to the stand and each buys a double scoop cone with 2 different flavors. If no pair of children chooses the same combination of flavors, and every possible combination is chosen, how many children are there?"

Stage 2: students' work (20 minutes)

- Encourage students to organize the given data in way that would help them understand the situation.
- Ask students to work on solving the questions they posed.
- Rotate among them to monitor and guide.

Stage 3: class discussion (15 minutes)

- Discuss the different strategies used.

Phase II: posing

Objective: in addition to the above mentioned objectives, students are expected to:

- Identify the significance of each given condition
- Vary the given conditions and check the effect on the solution
- Modify given conditions when one condition is varied
- Make up their own problem similar to the original given problem and can be solved using a similar strategy.

stage 1: varying the given(15 minutes)

- Ask students to vary some of the given data in the above problem.
- Check how the answer may differ.

<u>Stage 2: posing a similar problem (20 minutes)</u>

- Ask students to make up their own problem similar to the original posed problem.
- Instruct them to make sure all necessary given is provided to ensure their problems can be solved.

<u>Stage 3: classwork (25 minutes)</u>

- Ask students to swap their problems with a friend and to work on them.
- Ask students to try and modify any problem that can't be solved.
- Collect students' work to be analyzed.

Lesson Plan 7

Duration: 2 non-consecutive sessions (50 minutes per session)

Objective: this is an algebraic problem which requires knowledge of basic mathematical operations and fractions. Students are expected to:

- Identify significant given data
- Organize given data effectively
- Explain the situation in their own words
- Translate given word problems into mathematical expressions
- Make up and solve an equation
- Communicate a valid and meaningful answer to the problem

Solution: the age of the young lady is 11 years old

Phase I: solving

Stage 1: introduce the problem (5 minutes)

- Introduce the following problem and explain any ambiguous words:

"Mathematical curiosities and puzzles have fascinated people throughout the ages. These were often expressed in verse or as riddles. Here is one of these.

A lady being asked her age by young man , gave the following reply:

If to my age there added be;

One half, one third my age and three times three

Six score and ten the sum you'd see

Now pray tell what my age may be?

(a score equals to 20 years, and it is a term that was used long time ago.)"

Stage 2: students' individual work (25 minutes)

- Ask students to rewrite the problem in their own words to acquire a better understanding of the situation.
- Ask students to solve the problem individually.
- Rotate for assistance.

Stage 3: class discussion (20 minutes)

- Encourage individuals to explain interesting solution strategies.
- Point out factors which might have hindered solution of the problem in some cases.
- After discussing the solution, ask to try to make up a similar problem about a similar or different situation which can be solved in a similar way, to be submitted in a following assigned session.

Phase II: posing

Objective: in addition to the above mentioned objectives, students are expected to:

- Examine a given word problem and check if it can be solved or not
- Modify a word problem with missing or wrong given data
- Make up a mathematical riddle which requires a similar solving procedure as the situation presented in the original problem

Stage 1: students' individual work (20 minutes)

- Ask students to exchange their riddles with a friend and to try and solve.
- Encourage them to modify riddles that can't be solved for some reason.

Stage 2: class discussion (30 minutes)

- Choose some interesting riddles to be discussed with students. They may be solvable or unsolvable.
- Point out interesting factors involved in solving the chosen riddles.

Lesson Plan 8

Duration 1 session (50 minutes)

Objective: this problem requires proportional thinking and basic mathematical operations. Proportionality was explained in grade-7. Students are expected to:

- Organize given data in an effective way
- Justify the steps they take as this should help them organize their thinking
- Translate given data into mathematical expressions
- Apply proportional reasoning while dealing with proportional quantities
- Present a meaningful and valid answer.

  Solution: the ship's speed must be 26 km/hr for the remaining 6 hours in order to compensate for the delay.

Phase I: solving

Stage 1: introduce the problem (5 minutes)

- Introduce the following problem:

  "A ship must average 22 kilometers per hour to make its ten-hour run on time. During the first four hours, bad weather caused it to reduce speed to 16 kilometers per hour. What should its speed be for the rest of the trip to keep the ship to its schedule?"
- Students may not be aware of the practical meaning of average speed so make sure the term is explained.

Stage 2: students' individual work (20 minutes)

- Allow students some time to work on the above problem.
- Encourage them to organize the given data in an effective way.
- Rotate among students to offer guidance.

Stage 3: class discussion (10 minutes)

- Ask some students to discuss their solution strategies with the class.
- Collect work for analysis.

Phase II: posing: 15 minutes

Objective: in addition to the above mentioned objectives, students are expected to:

- Modify a solution to a situation when a certain given data is varied
- Vary the question to a given problem situation
- Make up extension problems to a given situation
- Check the validity of the questions they pose

Class discussion

- Stress on important factors that helped in solving the problem or hindered the solution.
- Encourage students to reflect on the importance of each given provided in the problem.
- Ask the following questions:

1- What if the trip needed 12 hours?

2- What if ship averaged 20 kilometers/hour or 30 kilometers/hour?

3- What if the ship reduced its speed to 15 instead of 16 km/hr

- Ask students to pose 3 extension problems to be added to the above problem wich may be solved with the given data, or additional given data, to be submitted in an assigned time. Ask them to solve their problems as well.

Lesson Plan 9

Durations: 2 sessions (50 minutes each).

Objective: This problem can be solved using inequalities however the chapter hasn't been explained yet at this stage. Students are expected to:

- organize the given data in an effective way
- identify the conditions that are necessary to solve the problem
- use an appropriate representation that will facilitate the solution
- translate given data into mathematical expressions
- restate the situation in their own words
- To communicate appropriately a meaningful solution

Solutions: the least number of DVDs which makes offer A better is 4 DVDs

Phase I: solving

Stage 1: introducing the task (5 minutes)

- the following problem is given:

"A DVD rental shop offers its customers the following packages:

Package A: a monthly fee of 15$ and then 2$ for each rented DVD.

Package B: 7$ for each rented DVD.

What is the minimum number of DVDs that you should rent so that Package A suits you best?"

- Discuss the different elements of the problem.

Stage 2: classwork (20 minutes)

- Ask students to explain the problem in their own words or by a representation of their choice.
- Discuss their explanations/representations.
- Allow students to work individually on solving the problem.
- Rotate to offer guidance.

Stage 3: class discussion (15 minutes)

- Ask some students to present their solutions and discuss the strategies used.
- Stress on the given that allowed us to solve the problem.

Phase II : posing

Objective: in addition to the above mentioned objectives, students are expected to:

- Modify a problem situation when a certain given data is changed
- Track changes that would occur to a situation when the question is changed

- Identify what types of problems requires the use of equations.
- Make up a similar problem to the original given situation

- **Stage 1: varying the given (10 minutes)**
- Ask for suggestions on how we can change the problem: for example how can we change the variable? How will this affect the questions asked at the end? If we don't know the monthly fee, how would we modify the question?(we will need the number of DVDs to be rented). How can we make it an equation problem?

Stage 2: posing a problem (20 minutes)

- Ask students to pose a similar problem: they can choose their own setting, and own variable.
- Encourage them to check if their problems can be solved.

Stage 3: class discussion (30 minutes)

- Discuss interesting problems with the class with emphasis on the elements involved.

Lesson Plan 10:

Duration: 2 sessions (50 minutes each)

Objective: this problem requires knowledge of formulating and solving simple equations, evaluating numerical expressions, and knowledge of divisibility rules. Students are expected to:

- Organize given data in an effective method
- Translate given data into mathematical expressions and equations
- Make use of some given conditions to reduce the variables in the problem
- Use logic, numerical reasoning as well as knowledge of divisibility rules
- Check the validity of their answer
- Present a meaninful solution

Solution: he bought 10 pigs, 24 goats and 66 sheep.

Phase I: solving

Stage 1: introduce the problem (5 minutes)

"Mr. Melhim bought some pigs, goats and sheep. Altogether he bought 100 animals and spent $600. Mr. Melhim paid $21 for each pig, $8 for each goat and $3 for each sheep. There was an even number of pigs. How many of each of the animals did he buy?"

Stage 2: students' individual work (15 minutes)

- Encourage students to interpret the given data using a representation that best helps them to understand the problem.
- Allow students some time to work on the above problem.
- Rotate among students to offer guidance.

Stage 3: class discussion (20 minutes)

- Ask some students to discuss their solution strategies with the class.
- Stress on important factors that helped in solving the problem or hindered the solution.
- Encourage students to reflect on the importance of each given provided in the problem.

Phase 2: posing

Objective: in addition to the above mentioned objectives, students are expected to:

- Replace a given data with another and then track the changes that would happen to the situation at hand
- Make up their own problem similar to the original given one.

Stage 1: varying the given (10 minutes)

- Ask the following questions:
  1- Why is the even number of pigs important?
  2- What if the number of pigs was odd? What would change?
  3- If the total of $600 was removed. What given might be added that would enable you to solve this problem? Would the question change and how?
- Assign the following homework to be submitted on a following assigned session: "Make up 3 versions of this problem, each having a certain condition removed or added. Offer them some examples such as: omit the price for a pig, and encourage them to see how would this problem be modified so that it remains solvable?"
- Students' assignment should be collected and analyzed.
- Choose interesting problems to be discussed in the following session. The chosen problems may or may not be solved.

Stage 2: posing questions (20 minutes)

- Ask students to swap their posed problems with a friend and to try and solve them.
- Encourage students to modify a problem if it seems unsolvable.
- Rotate among students to offer guidance when needed.

Stage 3: class discussion (30 minutes)

- Ask some students to present their problems to be discussed in class.
- Put emphasis on important factors which helped in solving or hindered it, as well as interesting solution strategies used.

Appendix D

Unit Plan: Control Group

Grade: Intermediate level grade 8

Number of sessions: 17 sessions- 50 minutes each

Unit title: Problem Solving

Objective: the objective of this unit is to teach solving nonroutine mathematical word problems using a traditional teaching instruction.

By the end of this unit students will be able to:

- identify the important elements of a problem situation
- restate the problem in their own words
- organize given data in an effective way
- use various problem solving strategies
- check the validity of their answers

Note: while implementing this unit, the teacher should pay special attention to the following points:

- stress the significance of each of the provided given as well as the question at hand
- encourage students to discuss their ideas, strategies as well as concerns regarding the problem situation at hand
- encourage students to constantly validate their answers by checking the situation at hand
- encourage students to justify significant steps of their solution strategies as this ought to help them organize their work, thought and approach

Lesson Plan 1

Duration: 2 sessions (50 minutes each)

Objective: students are expected to:

- Identify the important elements in the given data which permits solution of a problem
- Assign symbols to represent different variables in a word problem
- Translate given data into a mathematical expression
- To formulate a first degree equation from a given data
- To communicate appropriately a valid answer

Solution: - Solution of the first problem : the difference between the two accounts is $51

- Solution of the second problem: after 10 turns both girls will have the same score

Stage 1: introducing the problem (10 minutes)

- The following problem is introduced, and the given is explained and discussed

  “Sara and Tala each have bank accounts. Sara has 500$ and Tala has 200$. Sara withdraws $15 each weekend while Tala deposits $12. At the end of 13 weeks, what is the difference in their bank accounts?

- Discuss possible starting points that would allow solving this problem.

Stage 2: solving the problem: (20 minutes)

- Allow students to work on their own to solve the problem while rotating among them.

Stage 3: discussing solution: (20minutes)

- Discuss different solution strategies of students.
- Reflect on what might be a better strategy.

Second session

Note: this problem requires formulating an equation and solving it, after determining what the variable is, which is the number of turns played. However, it can be solved by repeating patterns of addition and subtraction.

Similar problem (50 minutes): repeat the same stages for the following problem.

"Lea and Nour are playing a game of cards. The rules for this game are the following: you gain 3 points for each win, and lose 2 points for each lost turn. Right now, Lea has 100 points and Nour has 50 points. How many turns does Nour have to win so that the scores of the girls become equal?"

Lesson Plan 2

Duration: 1.5 sessions (75 minutes)

Objective: students are expected to:

- Identify important given data for the situation at hand
- Organize the given data using an effective representation
- Translate given data into mathematical expressions and equations
- Solve a first degree equation
- Present a meaningful and valid answer to their questions

Solution: - The solution to the first problem is: Fadi did an outstanding job for 3 days.

- The solution to the second problem is: Tala sold 8 large mugs and 4 small mugs.

First session

Stage 1: introducing the problem (5 minutes):

- The following problem is introduced and discussed:

"Fadi and his parents are a little unusual. If he does an acceptable job of doing his chores, he gets paid $3.33 for that day. If he does an outstanding job, he gets $3 more. During one 10 day period, Fadi received $42.30 for his work. How many days did Fadi do an outstanding job?"

Stage 2: students' work (15 minutes)

- Ask students to work on solving the problem.
- Rotate among them to monitor and guide.

Stage 3: class discussion (20minutes)

- Discuss the different solution strategies used.

Second session

Stage 1: introducing the problem (5 minutes):

- The following problem is introduced and discussed:

Tala sells mugs in Beirut. She sells two sizes of mugs: a small mug that she sells for $2.50 and a large mug that she sells for $5.75. Yesterday Tala made $56. Before she opened her shop in the morning, she had 200 mugs in her inventory. At the end of the day she had 188 mugs left. How many mugs of each price did she sell?

Stage 2: students' work (15 minutes)

- Ask students to work on solving the problem.
- Rotate among them to monitor and guide.

Stage 3: class discussion (15minutes)

- Discuss the different solution strategies used.

Lesson Plan 3

Duration: 1.5 sessions (75 minutes)

Note: students at this level should be able to translate a problem into equations and to solve a first degree equations; however, they are not obliged to use equations to reach a solution.

Objective: students are expected to:

- Translate given data into mathematical expression
- formulate an equation of the first degree
- use algebraic reasoning to solve an equation of the first degree
- present a meaningful and valid solution to the given situation

Solution of the first problem:

- Tala made $12.8, Lili made $51.2, Samar made $25.6 and Jad made $110.4.

Solution of the second problem: Ali will take $120, Karim will take $60 and Sara will take $20.

Stage 1: introduce the problem (5 minutes)

"Lili, Samar and Jad were selling cookies in their school's food sale. At the end of the sale, each had made a different amount of money. The profits that each made were as follows: Samar made half of what Lily earned. Jad earned double what Lily made after it was increased by $4. Lily noticed that the total profit of her and her friends was $200 more than what she made. How much money did each of the three friends make?"

Stage 2: students' individual work (15 minutes)

- Allow students some time to work on the above problem.
- Rotate among students to offer guidance.

Stage 3: class discussion (20minutes)

- Ask some students to discuss their solution strategies with the class.
- Stress on important factors that helped in solving the problem or hindered the solution.

- Encourage students to reflect on the importance of each given information provided in the problem

Similar problem

Repeat the same steps for the following problem: (35 minutes)

"Three friends, Sarah, Ali and Karim want to divide $ 200 among themselves. Sara is going to take a third of what Karim takes. Ali will take double of what Karim takes. How much did each of the friends take?"

Lesson Plan 4

Duration: 1 session (50 minutes)

Objective: this is a matching word problem, in which students are expected to:
- organize the given data in an effective way
- identify the conditions that are necessary to solve the problem
- use an appropriate representation that will facilitate the solution
- To communicate appropriately a meaningful solution

Solution:
- the solution for the first problem is: Mrs. Anderson and Terry, Mrs. Blake and Janine, Ms. Allen and Beth, Mrs. Murphy and Jessica
- the solution for the second problem is: the four games are as follows: Brazil vs. Croatia, Italy vs. Argentine, Holland vs. Spain, and France vs. Germany

Stage 1: introducing the problem: (5 minutes)

- The following problem is introduced:

"Match the mothers and daughters.

Clues:

No mother is exactly twice the age of her daughter.

No mother has the same hobby as her daughter.

No mother likes the same music as her daughter.

Terry is 20 and enjoys photography and rock music.

Janine is 22 and enjoys golfing and the blues.

Beth is 19 and enjoys cooking and jazz.

Jessica is 25 and likes dancing and classical music.

Ms. Blake is 40 and likes cooking and classical music.

Ms. Anderson is 50 and likes golfing and jazz.

Ms. Murphy is 44 and enjoys working in the kitchen.

Ms. Allen is 42. "

- Discuss the problem and the given it provides

Stage 2: solving the problem: (10 minutes)

- Allow individual work on the solution of problem.
- Encourage the use of any strategy students prefer.
- Rotate to offer guidance when needed.
- Stress the importance of checking the answer.

Stage 3: discussing the problem(15minutes)

- Allow some students to present their solutions and discuss the different strategies used.
- Reflect on the most effective strategy.

Stage 4: a similar problem (20 minutes)

- Repeat the work with this problem:

"There are 4 football games scheduled for tonight as part of the World Cup eliminations.

Three sports writers predict the winners in the morning paper.

The first picks: Brazil, Italy, Germany and Spain.

The second picks: Argentine, Holland, Germany and Brazil.

The third picks: France, Italy, Holland and Brazil.

No one picked the Croatia.

Figure out who played with whom."

Lesson Plan 5

Duration: 2 sessions (50 minutes each)

Objective: this is a pattern problem. Students are expected to:

- Organize given data in an effective way
- Discover a pattern in a given situation
- Extend the pattern to reach the desired solution

Solution of the first problem: they will need a total of 728 legos to make a 12-level building.

Solution of the second problem: 19 guests will enter on the 10^{th} ring.

Stage 1: introduce the problem (5 minutes)

- Introduce the following problem:

"Jody, Amy, and Tim are building a model out of Legos. The top level is a 1 by 2 rectangle. The level directly underneath is a 2 by 3 rectangle. The level underneath is a 3 by 4 rectangle. And the pattern continues. How many Legos will they need if their building is to be 12 levels high? "

Stage 2: students' individual work (20 minutes)

- Ask students to explain the given data in their own words, or to represent it in a more effective way
- Allow students some time to work on the above problem.
- Rotate among students to offer guidance.

Stage 3: class discussion (25 minutes)

- Ask some students to discuss their solution strategies with the class.
- Encourage students to reflect on the importance of each given provided in the problem.

Similar problem

Repeat the same steps for the following problem (50 minutes)

“Sally is having a party. The first time the doorbell rings, 1 guest enter.

The second time the doorbell rings, 3 guests enter.

The third time the doorbell rings, 5 guests enter.

The fourth time the doorbell rings, 7 guests enter. And the pattern continues.

How many guests will enter on the 10th ring?”

Lesson Plan 6:

Duration: 2 sessions (50 minutes each)

Objective: this is a pattern/combination word problem, in which students are expected to:

- organize the given data in an effective way
- identify the conditions that are necessary to solve the problem
- use an appropriate representation that will facilitate the solution
- recognize a pattern which is formed depending on the given conditions
- To communicate appropriately a meaningful solution

Solution:

- Solution of the first problem is: there are 36 children.
- The solution of the second problem is: there are 252 choices he can pick from.

First session

Stage 1: introducing the problem (5 minutes):

The following problem is introduced and discussed:

"An ice cream stand has 9 different flavors. A group of children come to the stand and each buys a double scoop cone with 2 flavors. If none of the children chooses the same combination of flavors and every combination is chosen, how many children are there?"

Stage 2: students' work (20 minutes)

- Encourage students to interpret the given data using an effective representation of their own.
- Ask students to work on solving the questions they posed.
- Rotate among them to monitor and guide.

Stage 3: class discussion (25 minutes)

- Discuss the different strategies used.

Second session

Stage 1: introducing the problem (10 minutes):

The following problem is introduced and discussed:

Adam wants to get his mother a card, a pin, and a bag of coffee. There are nine cards to choose from, four pins, and seven flavors of coffee. How many different choices does he have?

Stage 2: students' work (15 minutes)

- Encourage students to interpret the given data using an effective representation of their own.
- Ask students to work on the given problem.
- Rotate among them to monitor and guide.

Stage 3: class discussion (25 minutes)

- Discuss the strategies used.
- Collect students work to be analyzed.

Lesson Plan 7

Duration: 2 sessions (50 minutes each)

Objective: this is an algebraic problem which requires knowledge of basic mathematical operations and fractions. Students are expected to:

- Identify significant given data
- Organize given data effectively
- Explain the situation in their own words
- Translate given word problems into mathematical expressions
- formulate and solve an equation
- Communicate a valid and meaningful answer to the problem

Solution:

- The solution of the first problem is : the age of the young lady is 11 years old.
- The solution of the second problem is: Mira paid $2,700 for the car.

First session

<u>Stage 1: introduce the problem (5 minutes)</u>

- Introduce the following problem and explain any ambiguous words:

  "Mathematical curiosities and puzzles have fascinated people throughout the ages. These were often expressed in verse or as riddles. Here is one of these.

  A lady being asked her age by young man gave the following reply:

  If to my age there added be;

  One half, one third my age and three times three

  Six score and ten the sum you'd see

  Now pray tell what my age may be?

  (A score equals to 20 years, and it is a term that was used long time ago.)"

<u>Stage 2: students' individual work (25 minutes)</u>

- Ask students to explain the given data in their own words, or to represent it in a more effective way
- Ask students to solve it individually.
- Rotate for assistance.

Stage 3: class discussion (20 minutes)

- Encourage individuals to explain interesting solution strategies.

Second Session

A similar problem (50 minutes):

Repeat the same steps for the following problem

"Haya bought a car for $5400. She sold it to Rami for 5/6 the price she paid for it. Rami sold it to Lara for 1/5 less than he paid for it. Lara sold it to Mira for 3/4 what she paid. How much did Mira pay for the car?"

Lesson Plan 8

Duration 1 session (50 minutes)

Objective: this problem requires proportional thinking and basic mathematical operations. Proportionality was explained in grade-7. Students are expected to:

- Organize given data in an effective way
- Justify the steps they take as this should help them organize their thinking
- Translate given data into mathematical expressions
- Apply proportional reasoning while dealing with proportional quantities
- Present a meaningful and valid answer.

  Solution:

  - The solution of the first problem is: the ship's speed must be 26 km/hr for the remaining 6 hours in order to compensate for the delay.

  - The solution of the second problem is: the express will overtake the freight train after 12 PM.

Stage 1: introduce the problem (5 minutes)

- Introduce the following problem:

  "A ship must average 22 kilometers per hour to make its ten-hour run on time. During the first four hours, bad weather caused it to reduce speed to 16 kilometers per hour. What should its speed be for the rest of the trip to keep the ship to its schedule?"

- Students may not be aware of the practical meaning of average speed so make sure the term is explained.

Stage 2: students' individual work (20 minutes)

- Ask students to explain the given data in their own words, or to represent it in a more effective way
- Allow students some time to work on the above problem.
- Rotate among students to offer guidance.
- Collect students' work for analysis.

Stage 3: class discussion (10 minutes)

- Ask some students to discuss their solution strategies with the class.

Stage 4: a similar problem (15 minutes)

- Introduce the following problem:

  "A freight train left Beeville at 5 AM at 30 miles per hour. At 7 AM an express train traveling 50 miles per hour left the same station. When did the express overtake the freight?"
- Repeat the same work with the fist problem

Lesson Plan 9

Durations: 2 sessions (50 minutes each).

Objective: This problem can be solved using inequalities however the chapter hasn't been explained yet at this stage. Students are expected to:

- organize the given data in an effective way
- identify the conditions that are necessary to solve the problem
- use an appropriate representation that will facilitate the solution
- translate given data into mathematical expressions
- restate the situation in their own words
- To communicate appropriately a meaningful solution

Solutions:

- The solution to the first problem is: the least number of DVDs which makes offer A better is 4 DVDs.
- The solution to the second problem is: the least number of kilometres that he should travel so that Charlie's Taxi is the better offer for him is 74 Km

First Session

Stage 1: introducing the task (5 minutes)

- the following problem is given:

" a DVD rental shop offers its customers the following packages:

Package A: a monthly fee of 15$ and then 2$ for each rented DVD.

Package B: 7$ for each rented DVD.

What is the minimum number of DVDs that you should rent so that Package A suits you best?"

- Discuss the different elements of the problem.

Stage 2: classwork (20 minutes)

- Ask students to explain the given data in their own words, or to represent it in a more effective way.
- Allow students to work individually on solving the problem.
- Rotate to offer guidance.

Stage 3: class discussion (25 minutes)

- Ask some students to present their solutions and discuss the strategies used.
- Stress on the given that allowed us to solve the problem.

Second session

Stage 1: introducing the task (5 minutes)

- the following problem is given:

"Sam needs to rent a car for his upcoming trip. Allo Taxi charges $20.25 per day plus $0.14 per kilometer. Charlie's Taxi charges $18.25 a day plus $0.22 for each extra kilometer. Sam plans to do a lot of driving on his 3-day trip. Sam went with Charlie's Taxi. What is the least number of kilometers that he should cross so that his choice is the best for him?"

Discuss the different elements of the problem.

Stage 2: classwork (20 minutes)

- Ask students to explain the given data in their own words, or to represent it in a more effective way.
- Allow students to work individually on solving the problem.
- Rotate to offer guidance.

Stage 3: class discussion (25 minutes)

- Ask some students to present their solutions and discuss the strategies used.
- Stress on the given that allowed us to solve the problem.

Lesson Plan 10:

Duration: 2 sessions (50 minutes each)

Objective: this problem requires knowledge of formulating and solving simple equations, evaluating numerical expressions, and knowledge of divisibility rules. Students are expected to:

- Organize given data in an effective method
- Translate given data into mathematical expressions and equations
- Make use of some given conditions to reduce the variables in the problem
- Use logic, numerical reasoning as well as knowledge of divisibility rules
- Check the validity of their answer
- Present a meaninful solution

Solution of the first problem: he bought 10 pigs, 24 goats and 66 sheep.

Solution of the second problem: Mr. Watson weighed 68 kg, Mrs. Watson weighed 56 kg, and their son weighed 84 kg.

Stage 1: introduce the problem (5 minutes)

"Mr. Daley bought some pigs, goats and sheep. Altogether he bought 100 animals and spent $600. Mr. Daley paid $21 for each pig, $8 for each goat and $3 for each sheep. There was an even number of pigs. How many of each of the animals did he buy?"

Stage 2: students' individual work (15 minutes)

- Ask students to explain the given data in their own words, or to represent it in a more effective way
- Allow students some time to work on the above problem.
- Rotate among students to offer guidance.

Stage 3: class discussion (30 minutes)

- Ask some students to discuss their solution strategies with the class.

- Encourage students to reflect on the importance of each given provided in the problem.

Second session

Repeat the same steps with the following problem (50 minutes)

“Mr. and Mrs. Watson and their son work on the dairy farm. Mrs. Watson decided that they were all getting a little overweight so she wanted to weigh the family. Unfortunately the only scales they had were some old ones that had once been used to weigh cows and these couldn’t weigh anything less than 100kg. So Mr. Watson said that they should get on the scales two at a time.

When they weighed Mr. and Mrs. Watson the scales showed 124 kg. When they weighed Mrs. Watson and her son they showed 140 kg. When they weighed Mr. Watson and his son they showed 152 kg. How heavy was each member of the family?”

Appendix E

Posttest

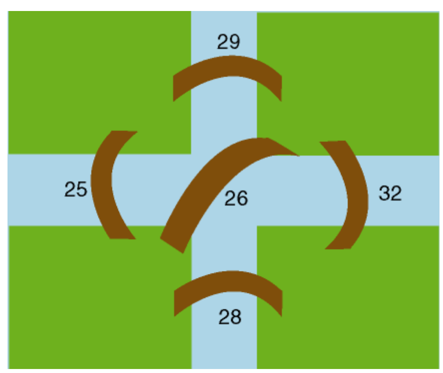
Name: **Date:**

Grade 8: **Time: 60 minutes**

Instructions:

- **You are given 3 independent problems to solve. You are free to use any strategy to find a solution to each of the problems.**
- **Write all necessary justification, diagrams, drawings etc… on this paper.**
- **If you feel a need to explain why you chose a specific step, please do so next to the related step.**
- **Relax and be creative!**

I- There are cows in each of these pastures. The pastures are connected by bridges, shown in brown color. The number on each bridge tells the total number of cows in the 2 connecting pastures.



a- Represent the given information in your own way.

b- How many cows are found in each pasture? Hint: no pasture has more than 20 or less than 10 cows.

c- Was there an extra given information that you did not use while solving the problem?

II- Lara is reading about Windemere Castle in Scotland. Many years ago, when prisoners were held in various cells in the dungeon area, they bEG-An to dig passages connecting each cell to each of the other cells in the dungeon. There were 30 cells in all, and no three cells are collinear,

a- Rewrite the necessary given in your own words.
b- What is the fewest number of passages that had to be tunneled out over the years?
c- Why do you think that the condition that "no three cells are collinear" is important while solving this problem?

III- Four dozen dogs live in 6 different colored kennels. The smallest kennel has 6 dogs and the orange kennel is the largest with 10 canines. The yellow kennel and the green kennel are the only ones with the same number of dogs. The 13 youngest pups are in the red and blue kennels with the least number of dogs. The purple kennel has 2 more dogs than the blue kennel.

a- Represent the above given information using a representation of your choice.
b- How many dogs are in each kennel? Justify your work.

Appendix F
Results of Quantitative Analysis
Quantitative Results Table 1

Table 1: Association between the percentage of grades in pretest problems, Posttest problems, overall test, pre ability, post ability, and difference in grades between pretest problems, posttest problems, and in abilities for the whole sample

Variables		EG	CG	P value
Total sample		**n=25**	**n=25**	
Previous Math average	(Mean, ±sd)	68.2% (±12.1%)	66.1% (±14.3%)	0.58
	Low achievers Average achievers High achievers	7 (28.0%) 14 (56.0%) 4 (16.0%)	6 (24.0%) 16 (64.0%) 3 (12.0%)	0.84
Grades in Pre-equation	(Mean, ±sd)	50.5% (±27.4%)	38.3% (±9.3%)	0.04
Grades in Pre-pattern	(Mean, ±sd)	47.5% (±16.8%)	54.5% (±22.8%)	0.22
Grades in Pre-reasoning	(Mean, ±sd)	38.3% (±19.7%)	32.0% (±6.0%)	0.14
Grades in overall Pretest	(Mean, ±sd)	45.3% (±17.1%)	41.6% (±10.0%)	0.35
Grades equation-activity	(Mean, ±sd)	51.5%(±22.0%)	42.5% (±13.3%)	0.09
Grades pattern-activity	(Mean, ±sd)	76.0% (±18.0%)	59.3% (±26.0%)	0.01
Grades reasoning-activity	(Mean, ±sd)	75.5% (±22.8%)	43.8% (±9.0%)	<0.0001
Grades in Post-equation	(Mean, ±sd)	55.0% (±21.2%)	53.3% (±26.5%)	0.80
Grades in post-pattern	(Mean, ±sd)	41.3% (±11.1%)	30.3% (±5.9%)	<0.0001
Grades in post-reasoning	(Mean, ±sd)	68.0% (±22.3%)	44.5% (±13.0%)	<0.0001
Grades In overall posttest	(Mean, ±sd)	54.8% (±14.1%)	42.7% (±10.4%)	0.001
Grades in pre ability 1	(Mean, ±sd)	59.3% (±17.1%)	60.7% (±15.9%)	0.78
Grades in pre ability 2	(Mean, ±sd)	44.0% (±17.1%)	40.7% (±10.6%)	0.41

Grades in pre ability 3	(Mean, ±sd)	42.3% (±18.8%)	33.3% (±8.7%)	0.04
Grades in pre ability 4	(Mean, ±sd)	38.0% (±16.9%)	31.7% (±9.3%)	0.11
Grades in post ability 1	(Mean, ±sd)	75.7% (±14.8%)	60.0% (±12.3%)	<0.0001
Grades in post ability 2	(Mean, ±sd)	54.0% (±18.5%)	43.0% (±13.5%)	0.02
Grades in post ability 3	(Mean, ±sd)	48.3% (±14.8%)	35.3% (±11.4%)	0.001
Grades in post ability 4	(Mean, ±sd)	41.0% (±12.0%)	32.7% (±11.3%)	0.02
Grades difference equation-problem	(Mean, ±sd)	4.5% (±24.4%)	15.0% (±26.8%)	0.15
Grades difference pattern-problem	(Mean, ±sd)	-6.3% (±16.8%)	-24.3% (±23.3%)	0.003
Grades difference reasoning-problem	(Mean, ±sd)	29.8% (±29.7%)	12.5% (±15.1%)	0.01
Grades difference Overall test	(Mean, ±sd)	9.4% (±15.8%)	1.1% (±13.7%)	0.05
Grades difference ability 1	(Mean, ±sd)	16.3% (±16.4%)	-0.67% (±19.5%)	0.002
Grades difference ability 2	(Mean, ±sd)	10.0% (±19.2%)	2.3% (±15.7%)	0.13
Grades difference ability 3	(Mean, ±sd)	6.0% (±17.4%)	2.0% (±12.8%)	0.32
Grades difference ability 4	(Mean, ±sd)	3.0% (±16.5%)	1.0% (±14.1%)	0.65

Quantitative Results Table 2

Table 2: Association between the percentage of grades in pretest problems, posttest problems, abilities, and difference in grades between pretest problems, posttest problems, ability with the groups stratified by achievement according to previous math average

	Variables		EG	CG	P value
Low achievers (n= 13)	**Total sample**		**n=7**	**n=6**	
	Grades in Pre-equation	(Mean, ±sd)	40.2% (±20.0%)	31.3% (±4.0%)	0.31
	Grades in Pre-pattern	(Mean, ±sd)	45.5% (±13.8%)	49.0% (±23.5%)	0.75
	Grades in Pre-reasoning	(Mean, ±sd)	36.6% (±28.1%)	30.2% (±6.1%)	0.60
	Grades in overall pretest	(Mean, ±sd)	40.8% (±18.0%)	36.8% (±8.2%)	0.63
	Grades equation activity	(Mean, ±sd)	42.0% (±17.9%)	34.4% (±6.6%)	0.35
	Grades pattern activity	(Mean, ±sd)	70.5% (±21.6%)	55.2% (±24.2%)	0.25
	Grades reasoning activity	(Mean, ±sd)	83.0% (±20.6%)	44.8% (±10.8%)	0.002
	Grades in Post-equation	(Mean, ±sd)	39.3% (±10.0%)	43.8% (±25.9%)	0.68
	Grades in post-pattern	(Mean, ±sd)	37.5% (±10.2%)	26.0% (±2.6%)	0.03
	Grades in post-reasoning	(Mean, ±sd)	58.0% (±21.6%)	33.3% (±7.6%)	0.02
	Grades In overall posttest	(Mean, ±sd)	44.9% (±11.1%)	34.4% (±7.5%)	0.07
	Grades in pre ability 1	(Mean, ±sd)	53.6% (±19.2%)	52. 8% (±10.1%)	0.93
	Grades in pre ability 2	(Mean, ±sd)	41.7% (±16.7%)	34.7% (±8.2%)	0.38
	Grades in pre ability 3	(Mean, ±sd)	39.3% (±20.2%)	29.2% (±7.0%)	0.27
	Grades in pre ability 4	(Mean, ±sd)	36.9% (±14.3%)	30.6% (±10.1%)	0.38
	Grades in post ability 1	(Mean, ±sd)	64.3% (±14.2%)	50.0% (±13.9%)	0.10
	Grades in post ability 2	(Mean, ±sd)	42.9% (±14.8%)	30.6% (±10.1%)	0.11
	Grades in post ability 3	(Mean, ±sd)	39.3% (±13.4%)	27.8% (±6.8%)	0.08

	Grades in post ability 4	(Mean, ±sd)	33.3% (±6.8%)	29.2% (±10.2%)	0.40
	Grades difference equation problem	(Mean, ±sd)	-0.9% (±14.2%)	12.5% (±25.3%)	0.25
	Grades difference pattern problem	(Mean, ±sd)	-8.0% (±13.8%)	-22.9% (±24.3%)	0.19
	Grades difference reasoning problem	(Mean, ±sd)	21.4% (±38.3%)	3.1% (±10.3%)	0.28
	Grades difference Overall test	(Mean, ±sd)	4.2% (±16.9%)	-2.4% (±13.0%)	0.45
	Grades difference ability 1	(Mean, ±sd)	10.7% (±13.4%)	-2.8% (±15.5%)	0.12
	Grades difference ability 2	(Mean, ±sd)	1.2% (±18.9%)	-4.2% (±16.5%)	0.60
	Grades difference ability 3	(Mean, ±sd)	0.00% (±19.2%)	-1.4% (±11.1%)	0.88
	Grades difference ability 4	(Mean, ±sd)	-3.6% (±15.9%)	-1.4% (±16.2%)	0.81
Average achievers (n=30)	**Total sample**		**n=14**	**n=16**	
	Grades in Pre-equation	(Mean, ±sd)	52.2% (±26.9%)	39.1% (±8.4%)	0.10
	Grades in Pre-pattern	(Mean, ±sd)	51.3% (±18.5%)	51.6% (±19.6%)	0.98
	Grades in Pre-reasoning	(Mean, ±sd)	41.5% (±17.8%)	33.2% (±5.9%)	0.12
	Grades in overall pretest	(Mean, ±sd)	48.2% (±17.5%)	41.3% (±9.3%)	0.18
	Grades equation activity	(Mean, ±sd)	52.7% (±22.8%)	45.3% (±14.7%)	0.30
	Grades pattern activity	(Mean, ±sd)	75.4% (±17.1%)	59.8% (±27.3%)	0.07
	Grades reasoning activity	(Mean, ±sd)	75.9% (±24.1%)	41.8% (±6.7%)	<0.0001
	Grades in Post-equation	(Mean, ±sd)	59.4% (±21.8%)	53.1% (±24.7%)	0.47
	Grades in post-pattern	(Mean, ±sd)	41.5% (±12.2%)	31.6% (±6.2%)	0.01
	Grades in post-reasoning	(Mean, ±sd)	70.5% (±23.6%)	45.3% (±11.3%)	0.002
	Grades In overall posttest	(Mean, ±sd)	57.1% (±14.9%)	43.4% (±8.2%)	0.006
	Grades in pre ability 1	(Mean, ±sd)	63.7% (±15.2%)	62.0% (±16.9%)	0.77
	Grades in pre ability 2	(Mean, ±sd)	45.8% (±18.7%)	40.6% (±9.6%)	0.34

	Grades in pre ability 3	(Mean, ±sd)	45.2% (±19.5%)	32.8% (±7.7%)	0.04
	Grades in pre ability 4	(Mean, ±sd)	38.1% (±19.0%)	29.7% (±7.4%)	0.14
	Grades in post ability 1	(Mean, ±sd)	77.4% (±13.2%)	62.0% (±10.1%)	0.001
	Grades in post ability 2	(Mean, ±sd)	57.1% (±20.6%)	44.8% (±10.9%)	0.05
	Grades in post ability 3	(Mean, ±sd)	50.6% (±15.8%)	35.4% (±9.4%)	0.003
	Grades in post ability 4	(Mean, ±sd)	43.4% (±13.1%)	31.2% (±9.9%)	0.007
	Grades difference equation problem	(Mean, ±sd)	7.1% (±22.2%)	14.1% (±28.3%)	0.47
	Grades difference pattern problem	(Mean, ±sd)	-9.8% (±17.6%)	-19.9% (±20.2%)	0.16
	Grades difference reasoning problem	(Mean, ±sd)	29.0% (±27.6%)	12.1% (±14.0%)	0.05
	Grades difference Overall test	(Mean, ±sd)	8.9% (±14.7%)	2.1% (±13.3%)	0.19
	Grades difference ability 1	(Mean, ±sd)	13.7% (±12.9%)	0.00% (±21.5%)	0.05
	Grades difference ability 2	(Mean, ±sd)	11.3% (±19.8%)	4.2% (±15.5%)	0.28
	Grades difference ability 3	(Mean, ±sd)	5.4% (±16.5%)	2.6% (±11.3%)	0.59
	Grades difference ability 4	(Mean, ±sd)	5.4% (±16.5%)	1.6% (±13.7%)	0.50
High achievers (n= 7)	**Total sample**		**n=4**	**n=3**	
	Grades in Pre-equation	(Mean, ±sd)	62.5% (±39.9%)	47.9% (±13.0%)	0.53
	Grades in Pre-pattern	(Mean, ±sd)	37.5% (±13.5%)	81.3% (±27.2%)	0.04
	Grades in Pre-reasoning	(Mean, ±sd)	29.7% (±3.1%)	29.2% (±3.6%)	0.85
	Grades in overall pretest	(Mean, ±sd)	43.2% (±16.9%)	52.8% (±11.8%)	0.44
	Grades equation activity	(Mean, ±sd)	64.1% (±23.6%)	43.8 (±10.8%)	0.23
	Grades pattern activity	(Mean, ±sd)	87.5% (±12.5%)	64.6 (±30.8%)	0.23
	Grades reasoning activity	(Mean, ±sd)	60.9% (±19.3%)	52.1 (±14.4%)	0.54
	Grades in Post-equation	(Mean, ±sd)	67.2% (±21.9%)	72.9% (±36.1%)	0.80

	Grades in post-pattern	(Mean, ±sd)	46.9% (±8.1%)	31.3% (±6.3%)	0.04
	Grades in post-reasoning	(Mean, ±sd)	76.6% (±17.2%)	62.5% (±6.3%)	0.21
	Grades In overall posttest	(Mean, ±sd)	63.5% (±5.0%)	55.6% (±13.9%)	0.43
	Grades in pre ability 1	(Mean, ±sd)	54.2% (±19.8%)	69.4% (±17.3%)	0.34
	Grades in pre ability 2	(Mean, ±sd)	41.7% (±15.2%)	52.8% (±12.7%)	0.36
	Grades in pre ability 3	(Mean, ±sd)	37.5% (±16.0%)	44.4% (±9.6%)	0.54
	Grades in pre ability 4	(Mean, ±sd)	39.6% (±17.2%)	44.4% (±9.6%)	0.66
	Grades in post ability 1	(Mean, ±sd)	89.6% (±4.2%)	69.4% (±9.6%)	0.01
	Grades in post ability 2	(Mean, ±sd)	62.5% (±4.8%)	58.3% (±14.4%)	0.67
	Grades in post ability 3	(Mean, ±sd)	56.3% (±4.2%)	50.0% (±16.7%)	0.49
	Grades in post ability 4	(Mean, ±sd)	45.8% (±10.8%)	47.2% (±12.7%)	0.88
	Grades difference equation problem	(Mean, ±sd)	4.7% (±46.3%)	25.0 (±28.6%)	0.54
	Grades difference pattern problem	(Mean, ±sd)	9.4 % (±12.0%)	-50.0 (±28.6%)	0.01
	Grades difference reasoning problem	(Mean, ±sd)	46.9% (±16.5%)	33.3 (±9.5%)	0.27
	Grades difference Overall test	(Mean, ±sd)	20.3% (±16.3%)	2.8 (±21.0%)	0.27
	Grades difference ability 1	(Mean, ±sd)	35.4% (21.9%)	0.00% (22.0%)	0.09
	Grades difference ability 2	(Mean, ±sd)	20.8% (14.4%)	5.6% (17.3%)	0.26
	Grades difference ability 3	(Mean, ±sd)	18.8% (14.2%)	5.6% (25.5%)	0.42
	Grades difference ability 4	(Mean, ±sd)	6.3% (18.5%)	2.8% (17.3%)	0.81

Appendix G

Table 1: Qualitative Table Template 1

Group name and category	Pre-name of problem	Post-name of problem
Understands the problem		
Uses information appropriately		
Applies appropriate representations/procedures		
Answers the problem		
Conclusion		
Understands the problem		
Uses information appropriately		
Applies appropriate representations/procedures		
Answers the problem		

Appendix H

Table 2: Qualitative Table Template 2

Comparing EG with CG (per category) per ability (for the 3 problems)		
	EG (Pre-equation/Pre-pattern/Pre-reasoning)	CG (Post-equation/Post-pattern/Post-reasoning)
Understands the problem		
Uses information appropriately		
Applies appropriate representations/procedures		
Answers the problem		

Appendix I

Qualitative Comparison tables of the low and limited achieving groups

EG-L: **progress per ability throughout pretest- -posttest (problem 1: Equation)**

EG-L	Pre-equation	Post-equation
Understands the problem	- Failed to understand and interpret most given information - Assumed that Mohamad's distance is ½ and calculated the other distances accordingly	- Able to interpret given information required to start the solution process correctly
Uses information appropriately	- Uses some appropriate information correctly such as some of the relations between the distances - Failed to make use of the given total distance.	- Used interpreted information appropriately
Applies appropriate representations/ procedures	- Applies inappropriate procedures: started by assuming that Mohamad's distance is ½. And worked accordingly.	- Able to represent the given situation: used 2 representations->diagram and word relations between the number of cows in adjacent pastures. - Used a correct and appropriate though not advanced procedure: guess and check.
Answers the problem	- Wrong answer based on misinterpreting important information needed to reach a correct solution, and on inappropriate solving procedure.	- Correct solution with minor technical error (calculation mistake)
Conclusion		
Understands the problem	- Progressed from not understanding enough information to solve correctly to understanding and interpreting all given information necessary to start the solution process.	
Uses information appropriately	- Progressed from using only some of the interpreted information appropriately to using all given information appropriately	

Applies appropriate representations/ procedures	- Progressed from using inappropriate procedure to using a correct and appropriate procedure. - Relied on representation while solving in the posttest.
Answers the problem	- Progress from wrong answer to correct solution with minor technical error.

EG-L: **progress per ability throughout pretest- -posttest (problem 2: pattern)**

EG-L	Pre-pattern	Post-pattern
Understands the problem	- Able to interpret some of the given information, but not enough to devise a solving procedure, and to reach a correct solution.	- Able to interpret enough information which should help them to start a solution
Uses information appropriately	- Was not able to use most information appropriately due to misinterpreting the information.	- Was not able to use any of the information
Applies appropriate representations/procedures	- Applies inappropriate information. - Divided the 6 teams by 2 to get 3 games. - Represented 2x3 as 2 power 3	- Were not able to devise a solution procedure
Answers the problem	- Wrong answer based upon inappropriate plan and misinterpreting information	- Did not provide an answer to the problem

Conclusion	
Understands the problem	- Progressed from understanding some of the given information to understanding enough information to start solving
Uses information appropriately	- Regressed from not being able to use most of the given information to not being able to use any information appropriately
Applies appropriate representations/procedures	- In the pretest, they used a wrong and inappropriate procedure, and in the posttest they were not able to come up with a solving procedure. (when asked they explained that they know that dividing 30 by 2 is wrong, but they did not know what else to do)
Answers the problem	- Wrong answer in the pretest, no answer in the posttest

EG-L: **progress per ability throughout pretest- -posttest (problem 3: reasoning)**

EG-L	Pre-reasoning	Post-reasoning
Understands the problem	- Failed to understand necessary information that is required to reach a correct solution. - Did not grasp that the containers are not graduated	- Understood some given information that is necessary to start the solving procedure. - Failed to understand that the red and blue kennels are separate
Uses information appropriately	- Did not use the necessary information correctly due to misinterpreting the given information	- Uses some given information appropriately.
Applies appropriate representations/procedures	- Failed to represent the situation - Used a wrong procedure mainly due to misinterpreting the given information.	- Represented the situation by a diagram. - Applied some appropriate procedures: some guess and check along with applying the given relations that were interpreted correctly
Answers the problem	- Provides a wrong answer	- Provides a partial answer to the problem due to misinterpreting part of the given information
Conclusion		
Understands the problem	- Progressed from not being able to interpret most information necessary to start a solution procedure to being able to interpret some given information that should help to start the solving process.	
Uses information appropriately	- Progressed from not using information appropriately to using some of the interpreted information appropriately	
Applies appropriate representations/procedures	- Progress in representations - Progress in procedures from using wrong procedures to using some appropriate procedures	
Answers the problem	- Progressed from wrong answer to partial answer	

CG-L: **progress per ability throughout pretest--posttest (problem 1: Equation)**

CG-L	Pre-equation	Post-equation
Understands the problem	- Was able to understand most given information and to interpret into mathematical relations	- Showed understanding of some given but was not able to interpret enough information to reach a correct solution (did understand that the number on the bridge represents the number of cows in 2 pastures)
Uses information appropriately	- Did not use information appropriately	- Did not use information appropriately
Applies appropriate representations/procedures	- Did not apply correct and appropriate procedures - Wanted to divide 36 km by 3 (according to them since Sam's distance is triple Lara's, and it's the greatest.	- Did not use an appropriate representation (did not even number the pastures) - Did not use an appropriate procedure: assumed a value for first pasture then calculated the others according to some of the given without checking the other conditions
Answers the problem	- Did not reach an answer	- Provided a wrong answer based on using some information incorrectly and an inappropriate procedure
Conclusion		
Understands the problem	- Regressed from understanding and interpreting most given information to understanding some given but not being able to accurately interpret the information	
Uses information appropriately	- No progress: not able to use information appropriately	
Applies appropriate representations/procedures	- No progress: did not use appropriate representations and procedures	
Answers the problem	- No progress: did not provide an answer in the pretest, and in the posttest, the answer came out wrong	

CG-L: **progress per ability throughout pretest- -posttest (problem 2: pattern)**

CG-L	Pre-pattern	Post-pattern
Understands the problem	- Able to understand and interpret most given information. - Did not interpret the only once condition correctly	- Understood some given information but not enough to reach a correct answer. - Did not interpret correctly the given about every cell being connected to every other cell.
Uses information appropriately	- Used most interpreted information appropriately(except for the only once condition)	- Used some given information appropriately
Applies appropriate representations/ procedures	- Applies some appropriate procedures - Multiplied 6 teams with 5 games each to get a total of 30 games (due to misunderstanding the only once condition)	- Did not use any representation. - Applied inappropriate procedure due to misinterpreted information: divided the 30 cells by 2 to get 15 passages (justified the division by 2 due to the condition of no 3 cells are collinear)
Answers the problem	- Provides a wrong answer to the problem due to misinterpreting one of the conditions.	- Wrong answer due to misinterpreting given and inappropriate procedure

Conclusion	
Understands the problem	- Regressed from understanding and interpreting most given information to understanding and interpreting some given information but not enough to reach a correct solution
Uses information appropriately	- Regressed from using most information appropriately to using some information appropriately.
Applies appropriate representations/ procedures	- No representations in both - Regressed from applying some appropriate procedures to applying inappropriate representations
Answers the problem	- No progress: wrong answer due to misinterpreting given

CG-L: **progress per ability throughout pretest- -posttest (problem 3: reasoning)**

CG-L	Pre-reasoning	Post-reasoning
Understands the problem	- Able to understand and interpret most given information necessary to start the solution process.	- Able to understand and interpret most given information - Was not able to understand that the smallest kennels is either the red or the blue
Uses information appropriately	- Were not able to use interpreted information appropriately	- Used some information appropriately - Misused the information about the red and blue kennels
Applies appropriate representations/ procedures	- Represented the situation with a diagram - Did not use a correct procedure	- Arranged kennels and their respective number of dogs in a table, but did not respect the given conditions. - Tried to use guess and check, but they ignored some of the conditions.
Answers the problem	- Reached a wrong answer (each insisted on a different approach :use the 8L since the 6L can fit in it but not in the 5 L- use the 5 L because they would know that they still need 1 more liter, but when using the 8L we don't know when to stop)	- Provided a wrong answer based on misusing some of the interpreted information
Conclusion		
Understands the problem	- Same level: able to understand and interpret most given information	
Uses information appropriately	- Progressed from not being able to use information appropriately to using some information appropriately	
Applies appropriate representations/ procedures	- Did use a representation in both tests, however it was not helpful during the solution - Procedures: showed some progress from using an inappropriate procedure in the pretest to using some appropriate procedures in the posttest	
Answers the problem	- No progress: wrong answer	

Comparing EG-L with CG-L per ability (for the 3 problems)		
	EG-L (P1/P2/P3)	CG-L(P1/P2/P3)
Understands the problem	- Progressed from not understanding enough information to solve correctly to understanding and interpreting all given information necessary to start the solution process.	- Regressed from understanding and interpreting most given information to understanding some given but not being able to accurately interpret the information
	- Progressed from understanding some of the given information to understanding enough information to start solving	- Regressed from understanding and interpreting most given information to understanding and interpreting some given information but not enough to reach a correct solution
	- Progressed from not being able to interpret most information necessary to start a solution procedure to being able to interpret some given information that should help to start the solving process.	- Same level: able to understand and interpret most given information
Uses information appropriately	- Progressed from using only some of the interpreted information appropriately to using all given information appropriately	- No progress: not able to use information appropriately
	- Regressed from not being able to use most of the given information to not being able to use any information appropriately	- Regressed from using most information appropriately to using some information appropriately
	- Progressed from not using information appropriately to using some of the interpreted information appropriately	- Progressed from not being able to use information appropriately to using

		some information appropriately
Applies appropriate representations/procedures	- Progressed from using inappropriate procedure to using a correct and appropriate procedure. - Relied on representation while solving in the posttest.	- No progress: did not use appropriate representations and procedures
	- In the pretest, they used a wrong and inappropriate procedure, and in the posttest they were not able to come up with a solving procedure. (when asked they explained that they know that dividing 30 by 2 is wrong, but they did not know what else to do)	- No representations in both - Regressed from applying some appropriate procedures to applying inappropriate representations
	- Progress in representations - Progress in procedures from using wrong procedures to using some appropriate procedures	- Did use a representation in both tests, however it was not helpful during the solution - Procedures: showed some progress from using an inappropriate procedure in the pretest to using some appropriate procedures in the posttest
Answers the problem	- Progress from wrong answer to correct solution with minor technical error.	- No progress: did not provide an answer in the pretest, and in the posttest, the answer came out wrong
	- Wrong answer in the pretest, no answer in the posttest	- No progress: wrong answer due to misinterpreting given
	- Progressed from wrong answer to partial answer	- No progress: wrong answer

Appendix J

Qualitative Comparison tables of the average achieving groups

EG-A: progress per ability throughout pretest- -posttest (problem 1: Equation)

EG-A	Pre-equation	Post-equation
Understands the problem	- Show good understanding of problem given and conditions - Were able to interpret all necessary information needed for a complete and correct understanding - Understood the assigned task	- Revealed good understanding of problem given and information by representing the given information needed for a complete solution. - Understood and interpreted the condition of more than 10 less than 20 cows.
Uses information appropriately	- Able to use most information correctly - Used variables to represent the values of each of the distances, and to determine the relations that exist between different unknowns - Forgot to include Lara's distance in the overall equation that sums up the total distances - Unable to use the same given condition more than once (after determining that M=1/2 L and substituting this in the overall equation at the beginning, they failed to substitute again for M, at a later point during the solution)	- Used all appropriate information correctly. - Used the numbers on the bridges to find out the number of cows in each pasture. - Used the condition on the number of cows to narrow down the options
Applies appropriate representations/ procedures	- Used a correct and advanced procedure: use of algebraic equations. - Able to substitute the unknown distances using	- Used guess and check - The strategy was random at first, however abiding with the condition set on the number of cows to narrow down the guesses

	the relations they interpreted. - Faced some algebraic difficulties while solving the equation (replacing the value of M again with ½ L, and the order with which an equation is solved	- Kept track of the guesses by writing the numbers inside the pastures. - Decided this was time consuming, and so decided to focus on three connected pastures, find the number of cows in these 3 first, and then calculate the fourth, then checking.
Answers the problem	- Able to provide a partial answer due to the algebraic difficulties while solving the equation	- Able to reach a correct and complete solution of the problem

Conclusion	
Understands the problem	- Maintained same level of understanding all given information
Uses information appropriately	- Improved their use of information appropriately where in the pretest, members were not able to use all the information appropriately; however, they were able to do so in the posttest
Applies appropriate representations/ procedures	- Used appropriate procedures/ representations in both pretest and posttest; however the procedure(equations) in the pretest didn't lead the group to a correct solution due to algebraic difficulties faced when solving the equation. The solution procedure(organized guess and check) used in the posttest was not advanced as the equations used in the pretest; however, it did lead to a complete and correct solution. - Reflected some creativity while tracking the guesses they were making - Relied more on representations
Answers the problem	- Partial answer in the pretest vs. a complete and correct solution in the posttest. - Checked their answer

EG-A: progress per ability throughout pretest-posttest (problem 2: patterns)

EG-A	Pre-pattern	Post-pattern
Understands the problem	- Failed to understand most problem given and conditions	- Revealed distinguished ability to understand and interpret all necessary information needed for a complete and correct solution
Uses information appropriately	- Thought that each team will play only once - Used inappropriate information	- Used all appropriate information correctly
Applies appropriate representations/procedures	- Inappropriate procedure due to failure to understand given situation (divided 6 teams by 2) - No attempt to make a representation	- Used correct and advanced procedures and representation which lead to a correct solution - Were able to draw upon past experience by identifying the pattern that existed when the cells are being connected (they started by listing the possible connections among cells, then realized that this is time consuming, so they recognized that a pattern was developing, and used that
Answers the problem	- Wrong answer based upon an inappropriate procedure.	- Correct solution of the problem
Conclusion		
Understands the problem	- Improved ability of understanding and interpreting all necessary information and conditions that are needed for a complete and correct solution	
Uses information appropriately	- Improved ability of using the appropriate information correctly	
Applies appropriate representations/procedures	- Distinguished improvement in the ability to apply appropriate representation/procedure: - The group failed to use the appropriate procedure and did not use any representation for such kind of problems in the pretest; however, they were able to use an advanced procedure to solve a similar but harder problem in the posttest, as well as to use an appropriate representation for the given situation	
Answers the problem	- Improved ability to answer the problem: - Failed to answer the problem correctly in the pretest - Able to give a correct answer in the parallel problem in the posttest	

EG-A: progress per ability throughout pretest-posttest (problem 3: reasoning)

EG-A	Pre-reasoning	Post-reasoning
Understands the problem	- Failed to understand the given situation - Did not understand that the containers were not graduated. - Confused about the required task	- Able to understand and to correctly interpret given information and conditions
Uses information appropriately	- Did not use most information appropriately	- Appropriate use of information throughout the solution process.
Applies appropriate representations/procedures	- Inappropriate procedure due to limited understanding of the problem situation - Inaccurate representation	- Applied appropriate procedures and representations - Started by representing the given information in short sentences, then switched to a more appropriate representation that assisted them in reaching a correct solution. - Used a diagram on which the group represented all the given conditions - Applied arithmetic operations and number reasoning to solve part of the problem, and then used guess and check to solve the remaining part
Answers the problem	- Wrong answer based upon misinterpretation of the given information, as well as inappropriate plan	- Able to produce a correct answer to the problem
Conclusion		
Understands the problem	- Improved ability of understanding and interpreting the necessary information which is needed to reach a solution	
Uses information appropriately	- Improved ability to use the appropriate information correctly, due to better understanding of problem given and conditions	
Applies appropriate procedures/ representations	- Improved ability of applying appropriate procedures and representations - Better representation techniques - More flexibility while switching between representations depending on appropriateness to the situation at hand - More organized solving procedures	

Answers the problem	- Improved ability of answering the problem correctly - Reflecting on and checking the final answer

CG-A: progress per ability throughout pretest-posttest (problem 1: equation)

CG-A	Pre-equation	Post-equation
Understands the problem	- Revealed good understanding of most given information - Were able to correctly interpret this information into relations which could lead into a correct solution - Failed to understand and interpret the given information about the total distance traveled by the 4 friends	- Showed good understanding of problem given - Able to interpret given information correctly - Could not interpret the condition on the number of cows
Uses information appropriately	- Couldn't use interpreted information in mathematical relations or equation that would help in solving the problem - Didn't use the total distance - Couldn't use the relations they interpreted while solving	- Used most information appropriately - Could not use the condition on the number of cows appropriately
Applies appropriate representations/procedures	- Used words and phrases to represent the relations in equations - Used an ineffective solution procedure- they assumed one	- Applied an effective but time consuming procedure: guess and check. - The guessing was unorganized and random

	distance and calculated the others according to the relations they interpreted - They calculated some distances wrong misusing the relations they interpreted	- Did not use the condition on number of cows to narrow down the options - Represented the given situation in another diagram where the order of pastures and bridges is in a different orientation of that in the given
Answers the problem	- Did not provide a correct answer due to lack of ability to use interpreted information correctly - Did not check	- Was able to provide an answer to the problem
Conclusion		
Understands the problem	- No significant improvement in terms of understanding. - Were able to understand and interpret most relations in the pretest (except for the total distance), and were able to understand and interpret most given conditions in the posttest (except for the condition on the number of cows	
Uses information appropriately	- Improved ability of using the information appropriately where in the pretest, the group was unable to use the given information and the relations they interpreted to solve the problem; whereas in the posttest, the group was able to use most given information appropriately (did not use the condition on the number of cows, tried numbers that were less than 9 and more than 10)	
Applies appropriate representations/procedures	- Improved ability of applying an appropriate procedure, where in the pretest the group was not able to use an effective procedure for solution; however, in the pretest, the group was able to use an effective though not advanced solving procedure	
Answers the problem	- Improved ability, where in the pretest the group was unable to provide a correct answer to the problem whereas in the posttest the group was able to do so.	

CG-A: progress per ability throughout pretest- posttest (problem 2: patterns)

CG-A	Pre-pattern	Post-Pattern
Understands the problem	- Showed good understanding of most problem given and conditions (with much hesitation, discussion and prompting from the interviewer)	- Able to interpret some given information but not enough to reach a solution - Understood the condition of no 3 cells collinear.
Uses information appropriately	- Misinterpreted and misused some of the given conditions (such as the played with every other team only once) - Able to use given information appropriately (after prompts)	- Unable to use the interpreted given appropriately - Did not use the condition which narrows down the number of passages to the fewest possible number.
Applies appropriate representations/procedures	- Started with multiplying 6 teams with 6 games, then realized they will play 5 games each, so they switched to 6 teams by 5 games each. - After prompts from the interviewer, the group tried to represent the situation using a list - They counter the number of games played by each team then wrote it down in a list. They decreased the number of games played by 1, for each team. - Switched from inappropriate procedures to appropriate after prompts from interviewer to explain the conditions and situation.	- Used inappropriate solving procedure due to lack of complete understanding of problem conditions as well as inappropriate use of interpreted information. - Multiplied the 30 cells by 29 passages (fewest number of passages disregarded here)

Answers the problem	- Able to answer most parts of the problem. - Calculated the number of games played by each team - Did not provide the total number of games - Decided that Team 6 played with all teams to be part of the answer.(did not understand that the numbers they wrote in answer to the number of games played by each team, is in fact the number of games to be counted once, since all teams would be playing 5 games but some would be counted only once.	- Gave a wrong answer due to proper use of information and inappropriate plan.
Conclusion:		
Understands the problem	- Regressed from understanding most given information to understanding some of the given information	
Uses information appropriately	- No improvement: in the pretest, group required many prompts from interviewer in order to use the information appropriately (specially the played only once), in the posttest (minimal prompts) the group also misused the condition "fewest passages possible"	
Applies appropriate representations/procedures	- No improvement: in the pretest, group started with inappropriate plan and then switched to an appropriate one only after interviewer prompts which helped the members to gain a better understanding of the situation, whereas in the posttest (minimal prompts), the members were not able to use an appropriate procedure(keeping in mind they used it in the pretest), and instead relied on the inappropriate plan they started with in the pretest	
Answers the problem	- No improvement: in pretest, group was able to give a partial answer to the problem (after interviewer prompts), but in the posttest, group reached a wrong answer.	

CG-A: progress per ability throughout pretest-posttest (problem 3: reasoning)

CG-A	Pre-reasoning	Post-reasoning
Understands the problem	- understood given information and conditions - able to explain what the required task was	- Understood problem given and conditions necessary to start the solution procedure
Uses information appropriately	- Unable to use the given information while attempting to solve - Unable to use the suggestion of 13 L of water (see below)	- Used most information appropriately and correctly (relied on guessing in some stages of solution)
Applies appropriate representations/ procedures	- Represented the situation in a diagram - Suggested combining the 2 containers to get 13 L of water altogether - Did not come up with an appropriate solving procedure	- Represented some information in a table, but did not record some conditions (on the number of dogs in each kennel on paper, which forced them to reread the given every while) - Used appropriate though not advanced solving procedure- guess and check - Made errors while guessing, did not check the final answer
Answers the problem	- Could not present an answer to the problem	- Partial answer, due to calculation mistake
Conclusion		
Understands the problem	- No change in this ability as the group faced no problem in it in the pretest	
Uses information appropriately	- Ability improved from not being able to use information appropriately to being able to use most information appropriately	
Applies appropriate representations/ procedures	- Use of representations did not change significantly: diagrams are not informative and does not assist in solution procedure - Solving procedures improved from inappropriate to appropriate though not advance procedures: in the pretest the group was unable to come up with an appropriate solving procedure, whereas in the posttest, they were able to develop one.	
Answers the problem	- Improved ability: where in the pretest they were unable to provide an answer, but in the posttest they were able to provide a partial answer.	

Comparing EG-A with CG-A per ability (for the 3 problems)		
	EG-A (P1/P2/P3)	CG-A (P1/P2/P3)
Understands the problem	- Maintained same level of understanding all given information	- No significant improvement in terms of understanding. - Were able to understand and interpret most relations in the pretest (except for the total distance), and were able to understand and interpret most given conditions in the posttest (except for the condition on the number of cows
	- Improved ability of understanding and interpreting all necessary information and conditions that are needed for a complete and correct solution	- Regressed from understanding most given information to understanding some of the given information
	- Improved ability of understanding and interpreting the necessary information which is needed to reach a solution	- No change in this ability as the group faced no problem in it in the pretest
Uses information appropriately	- Improved their use of information appropriately where in the pretest, members were not able to use all the information appropriately; however, they were able to do so in the posttest	- Improved from not being able to use necessary information to reach a solution to being able to use most information
	- Improved ability of using the appropriate information correctly	- No improvement: in the pretest, group required many prompts from interviewer in order to use the information appropriately (specially the played only once), in the posttest (minimal prompts) the group also misused the condition "fewest passages possible"
	- Improved ability to use the appropriate information correctly, due to better	- Improved: from not being able to use information appropriately to being able to

	understanding of problem given and conditions	use most information appropriately
Applies appropriate representations /procedures	- Relied more on representations - Improved creativity and flexibility - Procedures: both are appropriate, but in pretest it was advanced but did not lead to a correct solution, however in the posttest procedure was not advanced but it did lead to a correct solution	- Improved ability: in pretest, group was unable to use an appropriate procedure, but in the posttest, group was able to use an appropriate though not advanced procedure. - No change in representations
	- Distinguished improvement from wrong procedure to an appropriate and advanced procedure	- No improvement. Same inappropriate procedure
	- Better representation techniques - More flexibility while switching between representations - More organized solving procedures	- Representations did not improve significantly: not informative or effective - Procedures: improved from inappropriate to appropriate but not advanced
Answers the problem	- Partial answer in the pretest vs. complete correct answer in the posttest	- Improved from wrong answer to correct and complete solution
	- Improved: wrong answer to complete and correct answer	- Regressed from partial to wrong
	- Improved from wrong answer to correct answer - Reflecting and checking on final answer	- Improved from wrong answer to partially correct answer

Appendix K

Qualitative Comparison tables of the high achieving groups

CG-H: progress per ability throughout pretest- -posttest (problem 1: Equation)

CG-H	Pre-equation	Post-equation
Understands the problem	- Understands all problem given and conditions - Able to interpret the given information appropriately	- Revealed good understanding of problem given and information - Able to interpret all given information correctly
Uses information appropriately	- Able to use all given information appropriately	- Used only part of the information appropriately
Applies appropriate representations /procedures	- Use correct and advanced procedures/representations which should lead to a correct solution - Used equations to solve - Able to replace unknowns in terms of given information - Faced algebraic difficulties while solving the equation	- Represented the given information in a table that shows the number of cows in each pair of pastures. - Applied some appropriate procedures: used equations to represent relations between connected pastures- tried to compare the amount of cows found in different pastures (B+C=32 and C+D= 28 so 32-28=4 so 4 more cows in B than in D) - Added the following 3 equations : B+C=32; C+D=28; B+D=26 to get 2B+2C+2D=12 where the 12 was the sum of the differences the group calculated between B&D; 2 between C&B and 6 between C&D.
Answers the problem	- Not able to present a correct answer due to algebraic difficulties while solving the equation	- Was not able to provide a correct answer to the problem due to inappropriate procedures and misuse of some information.

Conclusion	
Understands the problem	- Good level of understanding and good interpretation of given information in both the pretest and the posttest
Uses information appropriately	- Regressed: in the pretest the group was able to use all information appropriately, but in the posttest the group was able to use only part of the information appropriately
Applies appropriate representations /procedures	- No improvement: same procedures used, equations: advanced but the group was unable to follow the procedure throughout the solution. The group faced algebraic difficulties while solving the equation in the pretest, and was not able form a correct equation in the posttest due to difficulties in manipulating the relations between the 4 unknowns.
Answers the problem	- No improvement: not able to answer the questions correctly in both pre and post tests

CG-H: **progress per ability throughout pretest- -posttest (problem 2: pattern)**

CG-H	Pre-pattern	Post-pattern
Understands the problem	- Understood most given information - Able to interpret most given information necessary to start the solving procedure	- Understood most given information - Failed to understand that all cells would be connected together - Was not able to interpret the "no 3 cells are collinear" condition
Uses information appropriately	- Used most information correctly - Did not consider the condition that every team will play only once with every other team (counted matches like 2,3 and 3,2)	- Did not use information appropriately due to lack of complete understanding of problem given information
Applies appropriate representations /procedures	- Applies some appropriate procedures - First procedure: multiply 6 x5= 30 games, justifying the each team of the 6 will play with every other team, so 5 games. - Decided this was wrong, switched to a table that matched between teams playing against one another. Crossed out the	- Applies inappropriate procedures: divided 30 by 2 to get 15 tunnels - Justified that each 2 cells will be connected

	intersection of the same teams (1,1;2,2; etc…) then counted the remaining intersections to get 24 games	
Answers the problem	- Wrong answer based on misinterpreting the condition of 'every team will play only once with every other team'	- Wrong answer due to lack of complete understand and inappropriate solving procedure
Conclusion		
Understands the problem	- No improvement: Maintained same level of understanding most given information	
Uses information appropriately	- Regressed from using most interpreted information appropriately to did not use information appropriately	
Applies appropriate representations /procedures	- Regressed from applying some appropriate procedures to applying inappropriate procedures (in the pretest, 6 x 5 reveals that some of the conditions are met, however, in the pretest the 30÷2=15, reveals that even the conditions of every cell being connected to every other cell was not met.	
Answers the problem	- No improvement: wrong answers in both pretest and posttest - The wrong answer in the pretest violated only 1 condition; whereas in the posttest, the wrong answer violated 2 conditions.	

- CG-H: **progress per ability throughout pretest- -posttest (problem 3: Equation)**

CG-H	Pre-reasoning	Post-reasoning
Understands the problem	- Shows good understanding of problem given conditions - Able to interpret the given information	- Understood and interpreted all the given information
Uses information appropriately	- Able to use all given information appropriately	- Used only part of the information correctly - Misused the number of dogs in the purple kennel, as well as the equal number of dogs in the yellow and green kennels
Applies appropriate representations /procedures	- Applied appropriate and advanced procedures: number reasoning->empty the 8 into the 5 to get 3, then repeat twice.	- Represented information by stating the relations between the number of dogs in each kennel - Applied some appropriate procedures: equations

		- Faced the following difficulties: replacing unknowns with their values (blue kennel), misusing the information about the green and yellow kennels, algebraic difficulties while solving.
Answers the problem	- Provides a completely correct answer	- Wrong answer due to the difficulties mentioned above.
Conclusion		
Understands the problem	- Maintained same level of understanding and interpreting all given information	
Uses information appropriately	- Regressed from using all information correctly to using only part of the information correctly	
Applies appropriate representations /procedures	- Regressed from applying appropriate and advanced procedure which lead to a correct answer to applying some appropriate procedures which did not lead to a solution	
Answers the problem	- Regressed from a completely correct answer to a wrong answer	

EG-H: **progress per ability throughout pretest- -posttest (problem 1: Equation)**

EG-H	Pre-equation	Post-equation
Understands the problem	- Revealed complete understanding of all problem given and conditions - Able to interpret the given into mathematical relations among the different unknowns	- Showed good understanding of given information and conditions. - Interpreted all information necessary for a complete and correct solution - Interpreted given into mathematical relations
Uses information appropriately	- Used all appropriate information correctly - Substituted unknowns correctly in the main equation. - Used the interpreted information to calculate all the unknown distances	- Used all appropriate information correctly - Came up with an equation that sums up the total number of cows found in all pastures.

Applies appropriate representations /procedures	- Advanced procedure: equations. - Used correct and advanced procedure which lead to a correct solution - Represented unknowns by using equations to relate unknowns according to the given.	- Applied correct and appropriate procedure which depended on both equations which is advanced and on trial and error. - Represented number of cows in pastures with letters, then organized their guesses in a table. They used the total number of cows in all pastures, which they calculated first through the use of equations, to check if their guess was correct. - Organized information on the table
Answers the problem	- Provided a correct solution of problem with answer statement and correct labeling of answer.	- Provided a correct solution
Conclusion		
Understands the problem	- Maintained same level: understood all given necessary information	
Uses information appropriately	- Maintained same level: uses all information correctly and appropriately	
Applies appropriate representations /procedures	- Were able to use correct and appropriate procedures in both pre and post; used advanced procedures in both but in the post test they also used guess and check	
Answers the problem	- Provided a correct answer in both	

EG-H: **progress per ability throughout pretest- -posttest (problem 2: Pattern)**

EG-H	Pre-pattern	Post-pattern
Understands the problem	- Understood all given information - Able to interpret all given information necessary to start the solving procedure	- Able to interpret some given information to get part of the solution. - Did not understand that every cell would be connected to every other cell

		- Understood the condition of no 3 cells collinear
Uses information appropriately	- Uses all information appropriately and correctly	- Uses some interpreted information appropriately. - Did not understand that every cell would be connected with every other cell and thus were not able to use this condition
Applies appropriate representations/ procedures	- Represented the situation b y a figure at the beginning of the solution procedure, then at the end to check if their work was correct. - Started by saying that t1 will play 6 times, but after representing the teams, they discovered its 5 times not 6. - Procedure: listing possible games for team 1 then for team 2 and then were able to discover the pattern behind the problem, and extended it to the other teams. - Explained that the number of games that they presented for each team was not the number of games played by that team, since all will be playing 5 times, but that the number is actually the number of counted games, after removing the common games with other teams.	- Applied inappropriate procedures due to misinterpreting some given conditions. - Drew 30 dots and connected them consecutively.
Answers the problem	- Provides a complete and correct answer with statement and label.	- Provides a wrong answer due to misinterpreting an important condition that was necessary to devise a solution procedure
Conclusion		
Understands the problem	- Regressed from understanding and interpreting all necessary information to some information	

Uses information appropriately	- Regressed due to misinterpreting part of the necessary information in the post
Applies appropriate representations/ procedures	- Regressed due to misinterpreting part of the information in the post which lead the group to an inappropriate procedure
Answers the problem	- Regressed from a correct answer to a wrong answer

EG-H: **progress per ability throughout pretest- -posttest (problem 3: reasoning)**

EG-H	Pre-reasoning	Post-reasoning
Understands the problem	- Showed complete understanding of problem given and situation. - Were able to interpret given information.	- Understood all given information that was needed for a complete and correct solution.
Uses information appropriately	- Used most given information correctly	- Used all given information appropriately and correctly
Applies appropriate representations/ procedures	- Applies correct procedures; however, they were not able apply the mathematical solution they reached on the real life situation. - Were able to see that as numbers, there is a relation between the given numbers, and represented as follows: 8-5=3; 3x2=6 and 8+5=13; 13-10=3 then 3x2=6. - Able to mathematize the problem	- Applied advanced and correct procedures - Used letters to represent unknowns - Formed mathematical relations between unknowns - Organized information and relations in a table. - Came up with an equation to sum up all dogs. - Then used guess and check to find the final answer (when they reached that the sum of the dogs in the yellow, green and blue kennels is 23.
Answers the problem	- Presented an almost complete answer since they were not able to interpret the procedure into an actual solution. - Were not able to put the answer in words that would	- Provides a correct answer with label

	reflect the real life situation they have.	
Conclusion		
Understands the problem	- Maintained same level: understood and interpreted all necessary information	
Uses information appropriately	- Maintained same level: used information appropriately	
Applies appropriate representations/ procedures	- Showed improvement: able to represent all interpreted information in a table and in an equation, then resorted to guess and check to get the final answer. - Revealed flexibility in switching between representations.	
Answers the problem	- Showed slight improvement: was able to provide a complete and correct solution in the post test, where as they were not able to provide a complete answer in the pretest	

Comparing EG-H with CG-H per ability (for the 3 problems)		
	EG-H (P1/P2/P3)	CG-H (P1/P2/P3)
Understands the problem	- Maintained same level: understood all given necessary information	- Good level of understanding and good interpretation of given information in both the pretest and the posttest
	- Regressed from understanding and interpreting all necessary information to some information	- No improvement: Maintained same level of understanding most given information
	- Maintained same level: understood and interpreted all necessary information	- Understood and interpreted all the given information
Uses information appropriately	- Maintained same level: uses all information correctly and appropriately	- Regressed: in the pretest the group was able to use all information appropriately, but in the posttest the group was able to use only part of the information appropriately
	- Regressed due to misinterpreting part of the necessary information in the post	- Regressed from using most interpreted information appropriately to did not use information appropriately
	- Maintained same level: used information appropriately	- Regressed from using all information correctly to using only part of the information correctly
Applies appropriate	- Were able to use correct and appropriate procedures in	- No improvement: same procedures used, equations:

representations /procedures	both pre and post; used advanced procedures in both but in the post test they also used guess and check	advanced but the group was unable to follow the procedure throughout the solution. The group faced algebraic difficulties while solving the equation in the pretest, and was not able form a correct equation in the posttest due to difficulties in manipulating the relations between the 4 unknowns.
	- Regressed due to misinterpreting part of the information in the post which lead the group to an inappropriate procedure	- Regressed from applying some appropriate procedures to applying inappropriate procedures (in the pretest, 6 x 5 reveals that some of the conditions are met, however, in the pretest the 30÷2=15, reveals that even the conditions of every cell being connected to every other cell was not met.
	- Showed improvement: able to represent all interpreted information in a table and in an equation, then resorted to guess and check to get the final answer. - Revealed flexibility in switching between representations.	- Regressed from applying appropriate and advanced procedure which lead to a correct answer to applying some appropriate procedures which did not lead to a solution
Answers the problem	- Correct answer in both	- No improvement: not able to answer the questions correctly in both pre and post tests
	- Regressed from a correct answer to a wrong answer	- No improvement: wrong answers in both pretest and posttest - The wrong answer in the pretest violated only 1 condition; whereas in the posttest, the wrong answer violated 2 conditions.
	- Showed slight improvement: was able to provide a complete and correct	- Regressed from a completely correct answer to a wrong answer

	solution in the post test, where as they were not able to provide a complete answer in the pretest	