# A Film Thickness Correction Formula for Double-Newtonian Shear-Thinning in Rolling EHL Circular Contacts

W. Habchi[1,2*] , S. Bair[2**] , F.Qureshi[3] and M. Covich[3]

[1] Lebanese American University, Dep. of Ind. & Mech. Eng., Byblos, Lebanon
[2] G.W. Woodruff School of Mech. Eng., Center for High Pressure Rheology, Georgia Institute of Technology, Atlanta, GA 30332-0405
[3] Lubrizol Corporation

***********
******

*At the time this work was done, the first author was holding a visiting scholar position at Georgia Institute of Technology
** Corresponding author

## Abstract

Lubricants which contain a polymeric thickener will often display a second Newtonian plateau in measured flow curves. Like other manifestations of shear-dependent viscosity, this shear response will lead to an inaccurate prediction when the classical film thickness formulas are employed. A correction formula has been developed from numerical experiments for a range of parameters of the double-Newtonian modified Carreau equation. The parameters of this shear-thinning model were selected from measurements for real lubricants obtained in Couette viscometers and a capillary viscometer. Additionally, a full EHL film thickness formula has been derived from the same numerical experiments. The correction formula and the full formula were successfully validated using published film thickness data and published viscosity data for an EHL reference liquid, a polymer solution. Clearly, viscometer measurements of shear dependent viscosity which contain the inflection leading to the second Newtonian are essential for a film thickness calculation when a high-molecular-weight component of the lubricant is present.

Keywords: elastohydrodynamic, film-thickness, non-Newtonian, rheology, pressure-viscosity coefficient

## Introduction

Early on, it was recognized that the shear dependence of the viscosity of polymer-thickened lubricants must influence film thickness [1] in elastohydrodynamic lubrication (EHL). The development of the classical Newtonian film thickness formulas for EHL circular contacts [2] was one of the shining achievements of the field. In the ensuing enthusiasm it was often overlooked that these formulas lacked precision for all conditions and were accurate only for mineral oils and other low-molecular-weight base oils under mild conditions. Experimental measurements indicated that, for polymer blended mineral oil or high molecular weight silicone oil, the predicted film thickness may be about twice the measured value [3]. This trend could be observed when employing a pressure-viscosity coefficient obtained from a viscometer [4] rather than a pressure-viscosity coefficient which had been adjusted [5] to yield agreement with the same Newtonian formula.

The shear stress in a steady shear flow for a non-Newtonian liquid is related to the shear rate by

$$\tau = \eta\dot{\gamma} \qquad (1)$$

where the generalized viscosity, $\eta$, is some function of the invariants of stress and strain rate. Many empirical functions have been derived for $\eta$ and most employ a parameter, $\mu_2$, representing a limit to the viscosity at infinite shear rate or simply a second plateau apart from the first Newtonian plateau, $\mu$, at low shear rate.

$$\eta = \mu_2 + \left(\mu - \mu_2\right) F\left(\dot{\gamma}\right) \text{ or } \eta = \mu_2 + \left(\mu - \mu_2\right) F\left(\tau\right) \qquad (2)$$

The function $F$ goes to 1 as the argument goes to zero and $F$ goes to 0 as the argument goes to infinity. The expectation is that, for a polymer solution, the contribution of the solvent to the viscosity will be unaffected by shear within the inlet of the EHL contact. This contribution of the solvent is expected to result in a second plateau or, at least, an inflection in the flow curve which will be produced by $\mu_2 > 0$. An extensive list of models which have served the role of equation (2) may be found in [6].

The shear dependence of the viscosity of polymer solutions has been of extreme importance to Rheology [7] and considerable research effort has been devoted to understanding the shear response. It must be stressed that the generalized Newtonian approach which is used here and throughout tribology is incomplete. In addition to shear dependent viscosity other, even more profound, effects such as normal stress differences result from the shearing of the polymer solutions [8] which are ubiquitous as lubricants.

Within the tribology literature, it is often recommended to set $\mu_2$ equal to the viscosity of the base oil or the viscosity of the oil less the polymer [9]. In the rheology literature, where the second Newtonian viscosity has been measured with precision, the second Newtonian viscosity generally is greater than the viscosity of the solvent [10] and occasionally less, but seldom equal. There has been some work to determine the relationship between the intrinsic second Newtonian viscosity and the intrinsic first Newtonian viscosity [11]; however the principals involved do not lead to an estimation of $\mu_2$.

It is usually difficult to observe both the first and second Newtonian plateaus in a single experimental flow curve. Experience also indicates that the approach to the second Newtonian plateau is often interrupted by mechanical degradation of the polymer or the onset of shear-dependence of the base oil. Gear oils are less likely to display a clear inflection than motor oils. When a second Newtonian appears, the ordinary meaning of $n$, $= \partial\tau/\partial\dot{\gamma}$ in the power-law regime, does not apply unless $\mu_2/\mu < 0.03$.

An example for which the first Newtonian appears, as well as an inflection which may be characterized by a second Newtonian viscosity, is shown in Figure 1. This flow curve for a multigrade motor oil was obtained with a pressurized, thin-film Couette viscometer [6]. The curves plotted in Figure 1 represent a remarkably useful modification of the Carreau [12] equation

$$\eta = \mu_2 + (\mu - \mu_2)\left[1 + \left(\frac{\tau}{G}\right)^2\right]^{\frac{1-\frac{1}{n}}{2}} \qquad (3)$$

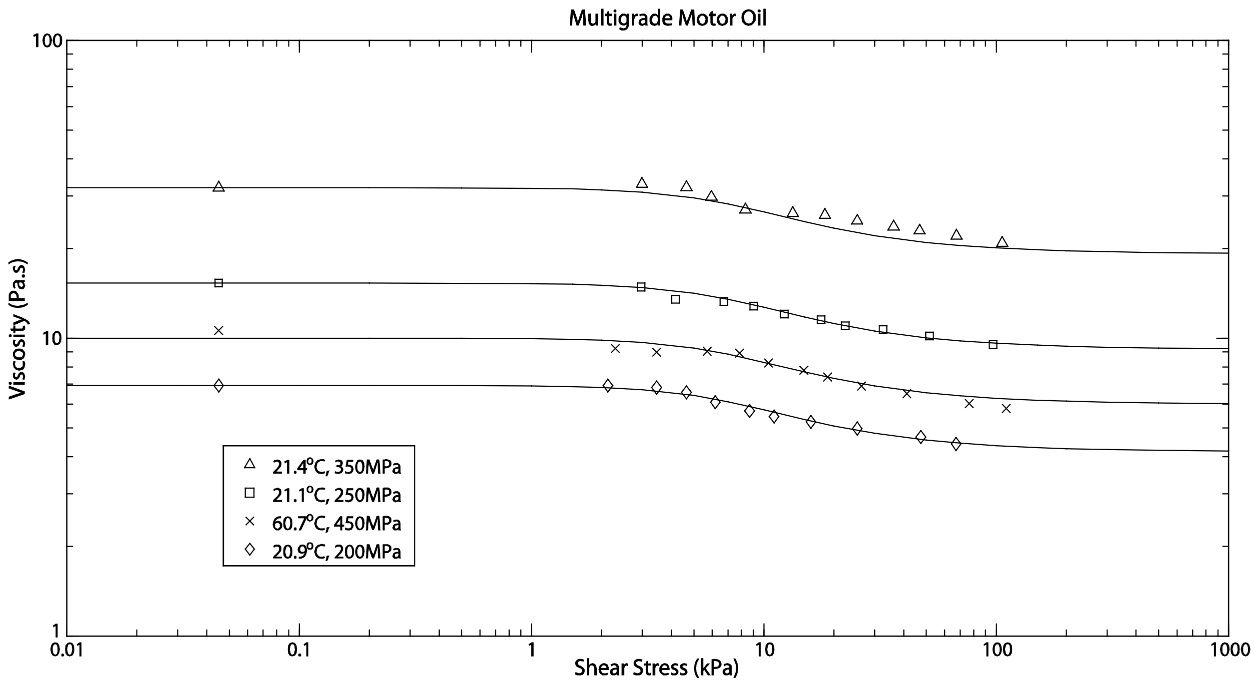

Figure 1. Flow curves for a motor oil obtained with a pressurized, thin-film Couette viscometer.

Notice that in Figure 1, the value of $G$, which establishes the limit of Newtonian response has been held constant, a useful and simple application of the time-temperature-pressure superposition principal. The curves can be superimposed by shifting vertically. A useful relation for estimating the Newtonian limit, $G$, for a polymer solution [13] is

$$G \approx \frac{c\rho R_g T}{M} \qquad (4)$$

where $M$ is the molecular weight of a monodisperse polymer of concentration, $c$, in a solution of mass density, $\rho$, and absolute temperature, $T$. In Figure 1, the value of $\mu_2/\mu$ has also been held constant with good result. The effective shear modulus, $G$ and the power-law exponent, $n$, only have the usual meanings [6] when $\mu_2 \ll \mu$, that is $1-1/n$ equals the slope on a log–log plot of viscosity versus shear stress.

Reynolds equations for double Newtonian shear-thinning have been analytically derived for the one-dimensional case [14] and the two-dimensional case [15]. Correction formulas have already been derived from numerical experiments for single-component liquids for which a second Newtonian plateau is not expected [16][17][18][19][20][21]. One of these, [21], is unusual in that the viscosity function takes the form of an empirical expression for thermal softening from viscous heating rather than for shear-thinning. Corrections are generated by calculating film thicknesses for the Newtonian and shear-thinning rheologies over some range of load, geometry and rolling velocity. The correction factor is the ratio of Newtonian to shearthinned film thickness. Here, the same approach is taken for double-Newtonian shear-thinning lubricants.

## Numerical Experiments

In this section, the global numerical procedure employed in this work is described. The authors employ the finite element full-system approach described in [22] for solving the EHL problem. The goal is to model a lubricated contact between a sphere and a plane under a prescribed external load. Both contacting bodies are elastic and have constant surface velocities. Surface separation is ensured by a complete lubricant film. In this work, only pure rolling conditions are considered under mild mean entrainment speeds. Therefore, thermal effects are neglected.

In the Full-System approach, the generalized Reynolds, linear elasticity and load balance equations are solved simultaneously. The Reynolds equation for a steady state point contact lubricated with a generalized Newtonian lubricant under unidirectional surface velocities in the x-direction is given by Yang and Wen [23]:

$$\frac{\partial}{\partial x}\left[\left(\frac{\rho}{\eta}\right)_e h^3\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial y}\left[\left(\frac{\rho}{\eta}\right)_e h^3\frac{\partial p}{\partial y}\right]=12\frac{\partial}{\partial x}\left(\rho^* U_m h\right) \qquad (5)$$

Where:

$$U_m=\frac{u_p+u_s}{2} \qquad \left(\frac{\rho}{\eta}\right)_e=12\left(\frac{\eta_e\rho'_e}{\eta'_e}-\rho''_e\right)$$

$$\rho^*=\frac{\left[\rho'_e\eta_e\left(u_s-u_p\right)+\rho_e u_p\right]}{U_m} \qquad \rho_e=\frac{1}{h}\int_0^h \rho\,dz$$

$$\rho'_e=\frac{1}{h^2}\int_0^h \rho\int_0^z \frac{dz'}{\eta}dz \qquad \rho''_e=\frac{1}{h^3}\int_0^h \rho\int_0^z \frac{z'dz'}{\eta}dz$$

$$\frac{1}{\eta_e}=\frac{1}{h}\int_0^h\frac{dz}{\eta} \qquad \frac{1}{\eta'_e}=\frac{1}{h^2}\int_0^h\frac{z\,dz}{\eta}$$

Note that this equation accounts for the variations of viscosity across the film thickness as can be seen in the integral terms. In fact, the changes in viscosity stem from shear rate variations across the lubricant film. Moreover, both density and viscosity are allowed to vary with pressure as described in the following section. Indices p and s correspond to the plane and the sphere respectively and $\eta$ is the generalized Newtonian viscosity. The film thickness *h* is defined from the film thickness equation:

$$h(x,y)=h_0+\frac{x^2+y^2}{2R}-\delta(x,y) \qquad (6)$$

Where $h_0$ corresponds to the rigid body separation and δ the equivalent elastic deformation of both contacting solids obtained by solving the linear elasticity equations on a large solid representing a semi-infinite medium as described in [22].

Finally, the load balance equation ensures the correct external load $L$ is applied to the contact by balancing it with the integrated pressure field over the contact area:

$$\int p \, d\Omega = L \qquad (7)$$

This equation ensures load balance by monitoring the value of the rigid body separation variable $h_0$. The generalized Reynolds, linear elasticity and load balance equations are solved simultaneously using a finite element discretization and a non-linear damped Newton resolution procedure. For more details about the technical implementation of the numerical scheme employed in this work, the reader is referred to [22].

**Selection of rheological Parameters**

Two representations of the pressure dependence of viscosity of lubricating oils were used in this analysis. Both are based upon the Doolittle free volume relation and utilized the Tait equation of state to supply the specific volume of the liquid. The pressure dependence of the density also comes from the Tait equation. The parameters of the Tait equation of state are the universal parameters proposed in reference [6], page 70. The two sets of Doolittle parameters are the model strong liquid and model fragile liquid in reference [6], page 123. The temperature was assumed to be 60°C for the model strong liquid and 80°C for the fragile liquid. The reference viscosity was 0.3 Pa·s, resulting in ambient pressure viscosities of 0.0302 and 0.0123 Pa·s for the strong and fragile liquids, respectively, and reciprocal asymptotic isoviscous pressure coefficients of 14.6 and 18.3 $GPa^{-1}$, respectively. The fragility classification of glass-forming liquids [24] provides a useful means of describing the viscosity dependence on temperature and pressure at high pressure.

The shear dependence of viscosity was given by equation (2). The various combinations of parameters of this viscosity function should be representative of the behavior of real lubricants. The combinations of $G$, $n$ and $\mu_2/\mu$ plotted as the solid points in Figure 2 were obtained from curve fitting of flow curves of motor oils and gear oils. Twelve motor oils were investigated in this work and eight clearly showed an inflection which allowed the determination of $\mu_2/\mu$. The remaining data were not used. Five additional oils were included in Figure 2 from references [14][25][26][27] and these include two gear oils. The combinations chosen for the numerical experiments are shown as the open points in Figure 2. A total of 25 individual combinations of $G$, $n$ and $\mu_2/\mu$ (as summarized in Table 1) were investigated numerically.

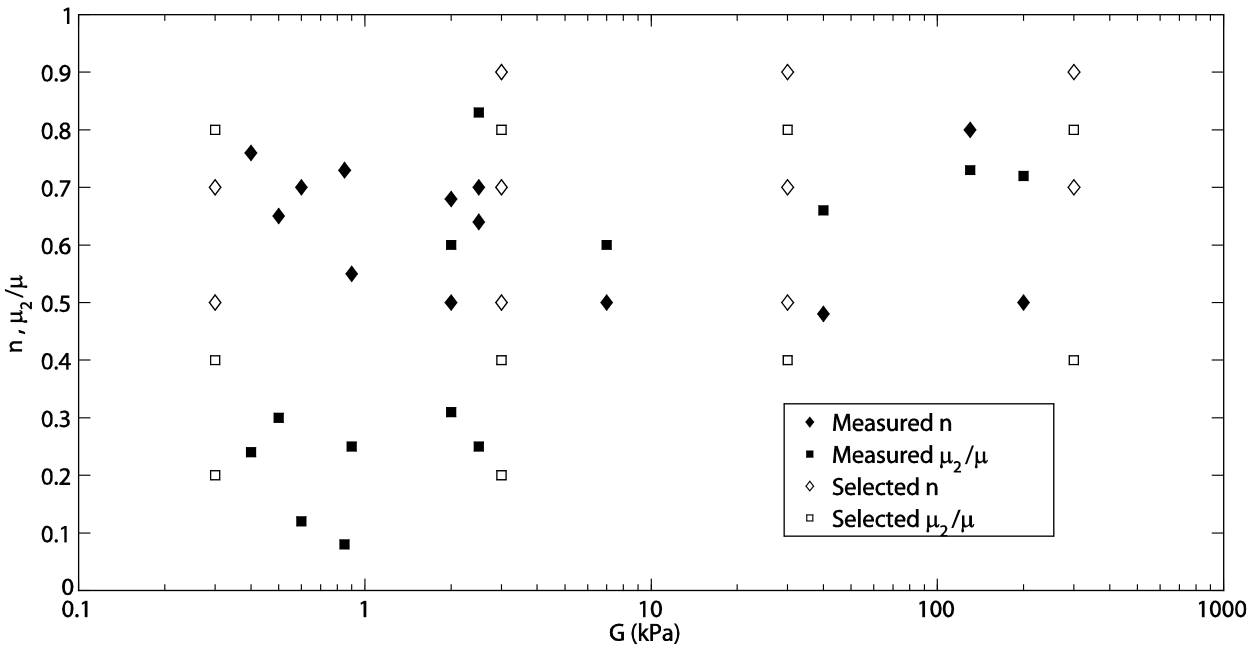


Figure 2. Experimentally measured values of the second to first Newtonian viscosity ratio and the power-law exponent versus Newtonian limit plotted as solid points and values selected for analysis plotted as open points.

Both Newtonian and non-Newtonian solutions for central and minimum film thicknesses were generated for 36 permutations involving rolling velocities of 0.1, 0.3, 1 and 3 m/s, reduced radii of 0.005, 0.015 and 0.05 m and Hertz pressures of 0.5, 1 and 1.5 GPa. For the non-Newtonian case, each of these permutations was investigated for 25 combinations of the shear dependent viscosity parameters shown in Table 1.

Table 1. Combinations of parameters of equation (2) employed in the numerical experiments.

| $G$ /kPa | $n$ | $\mu_2/\mu$ |
|---|---|---|
| 0.3 | 0.5 | 0.2, 0.4, 0.8 |

| | | |
|---|---|---|
| 0.3 | 0.7 | 0.2, 0.4, 0.8 |
| 3 | 0.5 | 0.2, 0.4, 0.8 |
| 3 | 0.7 | 0.2, 0.4, 0.8 |
| 3 | 0.9 | 0.2, 0.4, 0.8 |
| 30 | 0.5 | 0.4, 0.8 |
| 30 | 0.7 | 0.4, 0.8 |
| 30 | 0.9 | 0.4, 0.8 |
| 300 | 0.7 | 0.4, 0.8 |
| 300 | 0.9 | 0.4, 0.8 |

**Film thickness Results and Derivation of Correction Formula**

Each of the 900 results for central and minimum film thicknesses using the pressure-viscosity response of the model strong liquid were treated by dividing into the corresponding Newtonian result to yield values of $\varphi$.

$$\varphi = \frac{h_{Newt}}{h_{nonNewt}} \qquad (8)$$

The task of deriving a correction formula amounts to finding an expression that approximates $\varphi$. In many past works [16][17][18][19] an inlet Weissenberg number was used to quantify the shear stress of the inlet flow relative to the Newtonian limit for the liquid.

$$\Gamma = \mu_0 \bar{u} / \left( h_{cNewt} G \right) \qquad (9)$$

where $h_{cNewt}$ is the Newtonian solution for central film thickness. In the past [16][17][18][19], it has been useful to employ, as the correction formula, the functional form of the viscosity law, with $\Gamma$ substituted for the usual Weissenberg number. This is not surprising since the film thickness should vary roughly with viscosity raised to the 2/3 power. This technique is used here. Seven trial functional forms were tested and the form which yielded a combined low standard deviation and simplicity is

$$\frac{1}{\varphi} = \left( \frac{\mu_2}{\mu} \right)^a + \left( 1 - \left( \frac{\mu_2}{\mu} \right)^a \right) \left[ 1 + b \cdot \Gamma \right]^{(n-1)} \qquad (10)$$

Where $n$ is simply the power-law exponent in the constitutive law. The two parameters $a$ and $b$ and the standard deviations are listed in Table 2 for central and minimum thicknesses obtained from a least squares regression.

The formula (10) was derived for the case of the strong liquid. Applying it to the results for the fragile liquid resulted in standard deviations of 5.0% and 6.1% for central and minimum film thicknesses respectively.

Table 2. Parameters and standard deviations of equation (10).

| Type | Central | Minimum |
|---|---|---|
| $a$ | 0.7469 | 0.8930 |
| $b$ | 1.678 | 1.543 |
| Standard Deviation | 3.2% | 4.2% |

Another approach is taken to benefit from the extensive data obtained from the numerical experiments. A complete expression for the film thickness can be written as the product of a Newtonian solution and the correction equation (10). If the Newtonian solution is put in terms of the three Blok [28] dimensionless numbers

$$H = \frac{h}{R}\left(\frac{ER}{2\mu_0 u}\right)^{\frac{1}{2}},\; M = \frac{F}{ER^2}\left(\frac{ER}{2\mu_0 u}\right)^{\frac{3}{4}},\; L = \alpha E\left(\frac{2\mu_0 u}{ER}\right)^{\frac{1}{4}} \qquad (11)$$

The full formula reads

$$H_{Newt} = A \cdot L^B M^{-C} \qquad (12)$$

$$H = A \cdot L^B M^{-C}\left\{\left(\frac{\mu_2}{\mu}\right)^a + \left(1 - \left(\frac{\mu_2}{\mu}\right)^a\right)\left[1 + b\cdot\Gamma\right]^{(n-1)}\right\} \quad (13)$$

The same 900 results for central and 900 results for minimum film thicknesses were employed in a least squares regression. The parameters and the standard deviations are listed in Table 2 for central and minimum thicknesses. Although equation (13) yields greater standard deviations, it has the advantage of not requiring an independent Newtonian prediction.

Table 3. Parameters and standard deviations of equation (13).

| Type | Central | Minimum |
|---|---|---|
| *A* | 2.233 | 2.805 |
| *B* | 0.4664 | 0.4791 |
| *C* | 0.1061 | 0.2702 |
| *a* | 0.7589 | 0.9069 |
| *b* | 1.929 | 2.189 |
| Standard Deviation | 5.6% | 7.9% |

## Experimental Validation

For experimental validation of the new correction formulas both rheological data and film thickness data are required for the same material and there are few examples available. Fortunately, the film thicknesses have been measured for one of the reference liquids of reference [29], a solution of 15% by weight cis-polyisoprene ($M = 4 \times 10^4$ Daltons) in squalane. New viscosity data at 450 MPa pressure from [30] are shown in Figure 3 along with data from reference [29]. Curves plotted in Figure 3 are equation (2) with $G$=23 kPa, $n$=0.65 and $\mu_2/\mu$=0.28.

Film thicknesses for this liquid, obtained from optical measurements, have been reported in reference [31] for a circular contact formed by a 12.7 mm radius ball against plane with combined elastic modulus of 124 GPa. The load was 23 N to give a maximum Hertz pressure of 0.47 GPa. The test temperature, at 40°C, results in $\mu_0$=0.0711 Pa·s and $\alpha$=18.53 GPa$^{-1}$.

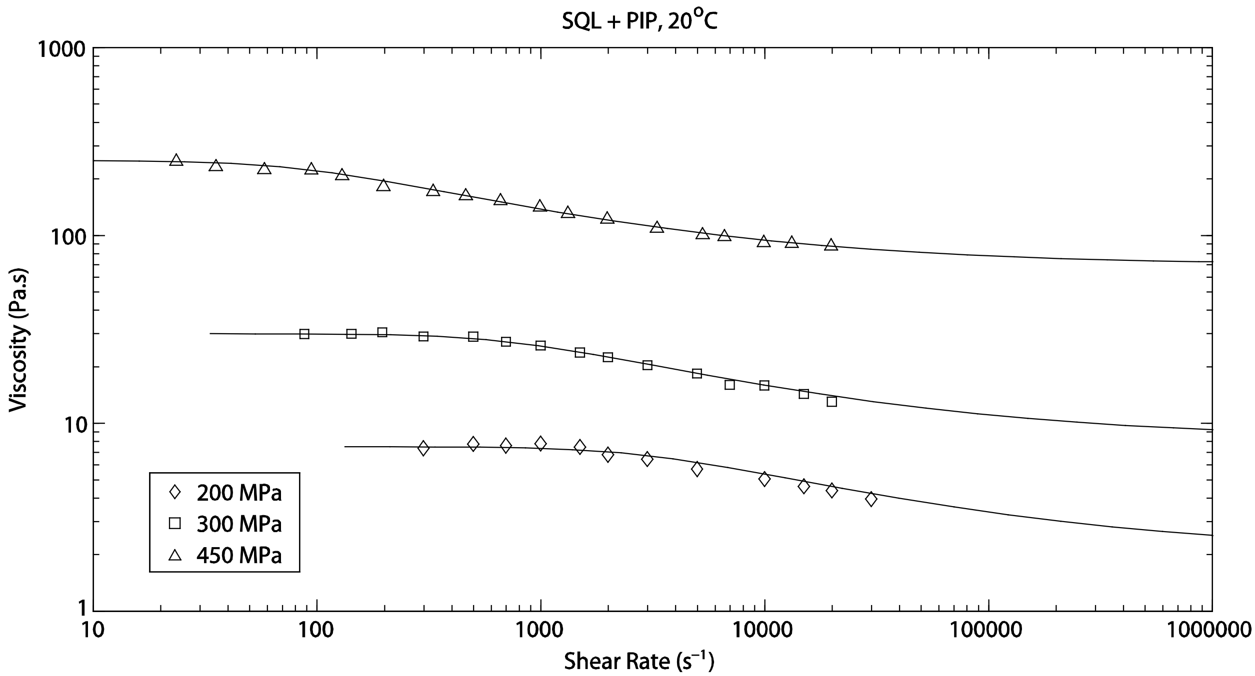

Figure 3. Experimentally measured values of the viscosity of SQL+PIP compared with equation (2).

The Hamrock and Dowson Newtonian film thickness formulas [2] were used for validation of the correction formulas (10) in Figure 4 and 5. The corrected film thicknesses are shown to improve the film thickness predictions. The central prediction improved from an average deviation of 41% to -9% and the minimum prediction improved from an average deviation of 44% to -11%.

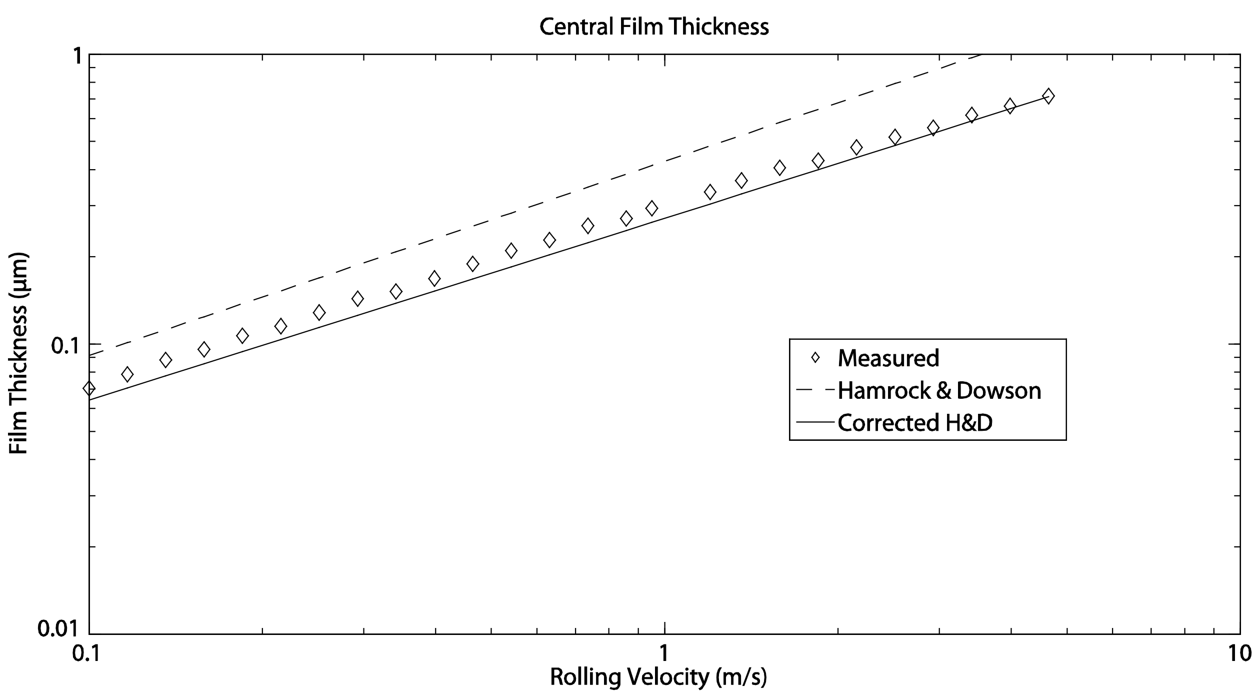

Figure 4. Experimentally measured central film thicknesses compared to the Hamrock and Dowson prediction and the corrected prediction.

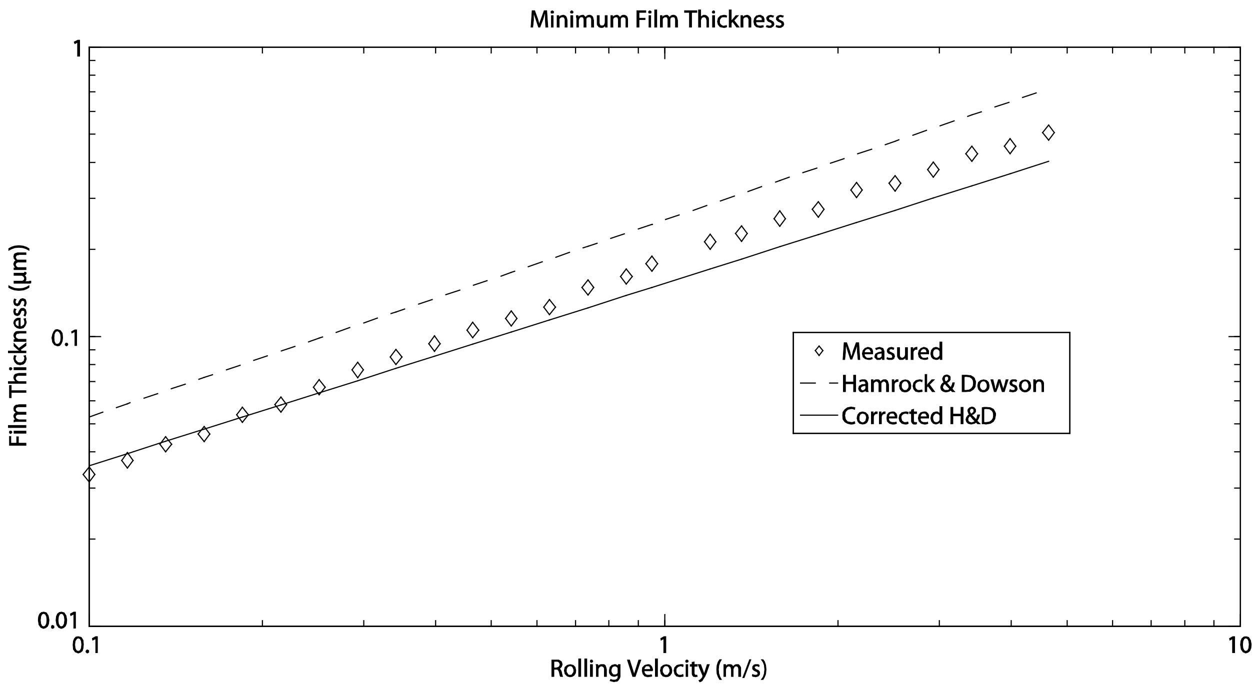

Figure 5. Experimentally measured minimum film thicknesses compared to the Hamrock and Dowson prediction and the corrected prediction.

Next, the full film thickness formulas (13) are compared to the Newtonian formulas (12) in Figures 6 and 7. The full film thickness formulas are shown to improve the film thickness predictions over the Newtonian predictions. The central prediction improved from an average deviation of 49% to -6% and the minimum prediction improved from an average deviation of 74% to 1%.

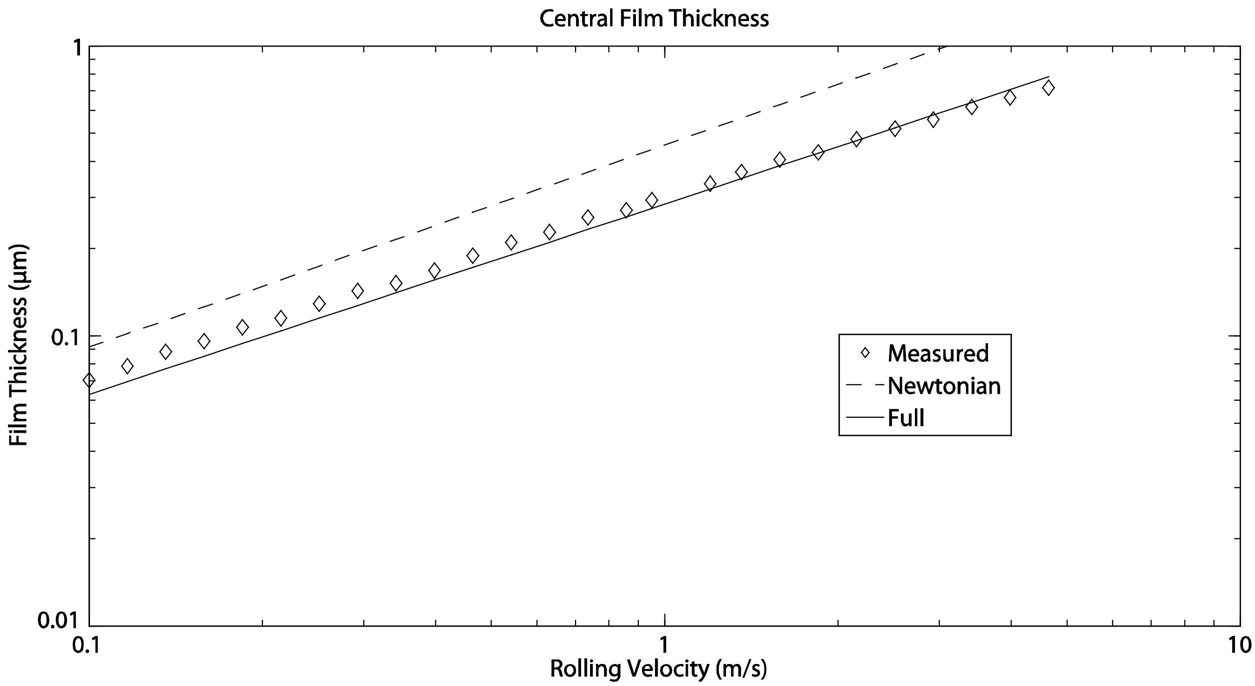


Figure 6. Experimentally measured central film thicknesses compared to the present Newtonian prediction and the present full prediction.

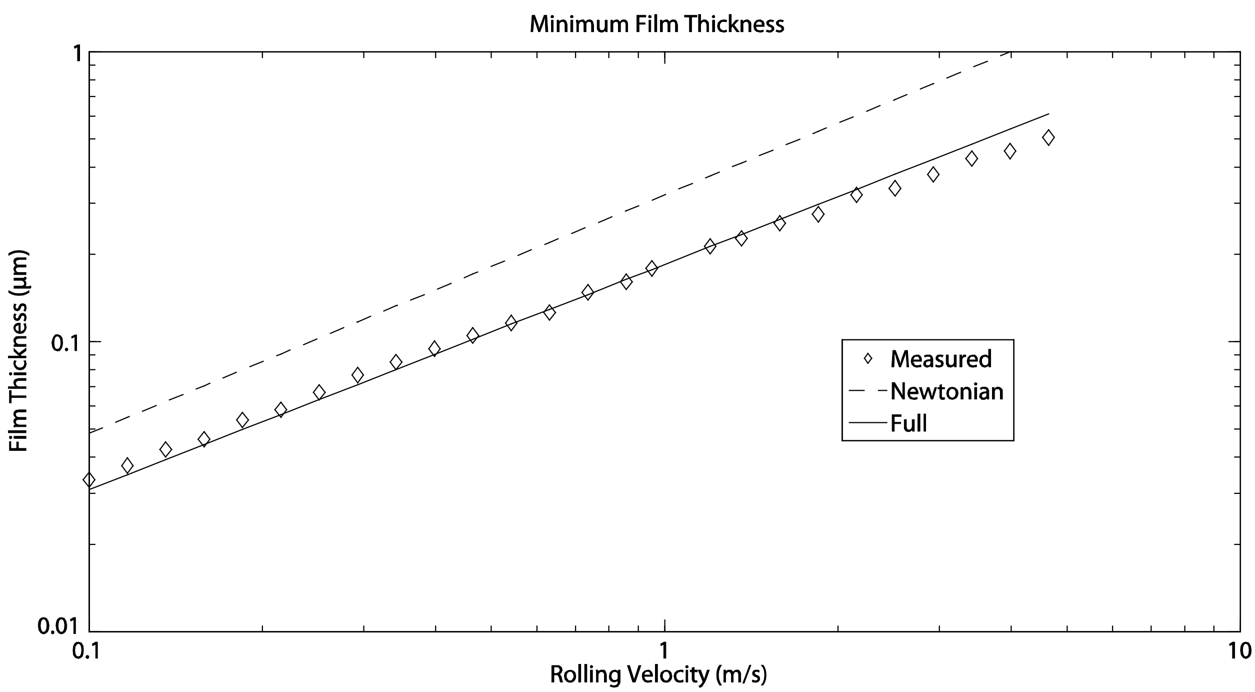


Figure 7. Experimentally measured minimum film thicknesses compared to the present Newtonian prediction and the present full prediction.

## Conclusions

1. A correction formula has been developed from numerical experiments for a range of parameters of the double-Newtonian modified Carreau equation. The parameters of this shear-thinning model were selected from measurements for real lubricants obtained in Couette viscometers and a capillary viscometer. Additionally, a full EHL film thickness formula has been derived from the same numerical experiments.

2. The correction formula and the full formula were successfully validated using published film thickness data and published viscosity data for an EHL reference liquid, a polymer solution.

3. Clearly, viscometer measurements of shear dependent viscosity which contain the inflection leading to the second Newtonian are essential for a film thickness calculation when a high-molecular-weight component of the lubricant is present.

## Acknowledgements

Bair was supported by the Lubrizol Corporation and by the Center for Compact and Efficient Fluid Power, a National Science Foundation Engineering Research Center funded under cooperative agreement number EEC-0540834. The first author wishes to thank the Council for International Exchange of Scholars (CIES) and the Fullbright commission for funding his visiting scholar program at the Georgia Institute of Technology.

## Nomenclature

| | |
|---|---|
| $\mu$ | low shear (first Newtonian) viscosity |
| $\mu_0$ | ambient pressure, low shear viscosity |
| $\mu_2$ | second Newtonian viscosity |
| $\eta$ | generalized (non-Newtonian) viscosity |
| $\tau$ | shear stress |
| $\dot{\gamma}$ | shear rate |
| $\alpha$ | reciprocal asymptotic isoviscous pressure coefficient |
| $p$ | pressure |
| $\Gamma$ | Inlet Weissenberg number |
| $G$ | effective liquid shear modulus associated with $\lambda$ |
| $h$ | film thickness |
| $n$ | power-law exponent |

$M$ molecular weight of polymer

$c$ weight fraction of polymer

$\rho$ mass density

$R_g$ universal gas constant

$T$ temperature

$D$ polydispersity index

$h_{cNewt}$ Newtonian solution for central film thickness

$E$ combined elastic modulus of the rollers.

## References


[1] Westlake, F. J. & Cameron, A., "Optical Elastohydrodynamic Fluid Testing," *ASLE Transactions*, Volume 15, Issue 2, 1972, pp. 81-95.

[2] Hamrock, B. J., Dowson, D.: Isothermal Elastohydrodynamic Lubrication of Point Contacts. Part III-Fully Flooded Results. *Journal of Lubrication Technology* **99**, 264-276 (1977).

[3] Dalmaz, G. and Chaomleffel, J.P., "Elastohydrodynamic lubrication of point contacts for various lubricants," *Fluid Film Lubrication-Osborne Reynolds Centenary*, D. Dowson, et al., (Editors), The University of Leeds, England, Elsevier, Amsterdam, 1987, pp.207-218.

[4] Villechaise, B., Dalmaz, G., Makala, J. and Chaomleffel, J.P., "On the Traction Fluid Behavior in Concentrated Contacts", *Boundary and Mixed Lubrication: Science and Applications*, D. Dowson, et al., (Editors), Elsevier, 2002, pp. 401-412.

[5] Aderin, M., Johnston, G.J., Spikes, H.A., Caporiccio, G., "The elastohydrodynamic properties of some advanced non hydrocarbon-based lubricants," Lubrication Engineering. Vol. 48, no. 8, 1992, pp. 633-638.

[6] Bair S. – High Pressure Rheology for Quantitative Elastohydrodynamics, Elsevier Science, Amsterdam, 2007, pp. 29, 143-150, 70, 123.

[7] Tanner, R.I., *Engineering Rheology*, second edition, Oxford Univ. Press, Oxford, 2000, pp.129-131.

[8] Bair, S., Lockwood, F.E., Anderson, W.B., Zhang, Z. and Dotson, D., "Measurements of Elasticity in Multigrade Motor Oil at Elevated Pressure", *STLE Trib. Trans.*, Vol.50, No.3, 2007, pp. 407-414.

[9] Dobson, G.R., "Analysis of high shear rate viscosity data for engine oils," *Tribology International*, Vol.14, Issue 4, 1981, pp.195–198.

[10] Talbot, G.R., "High shear viscometry of concentrated solutions of poly (alkylmethacrylate) in a petroleum lubricating oil," Rheologica Acta, Vol.13, No.2, 1974, pp.305-317.

[11] Ram, A. and Siegman, A. "Intrinsic viscosity of polymer solutions at high shear rates," *Journal of Applied Polymer Science*, Vol.12, Issue 1, 1968, pp.59-69.

[12] Carreau, P.J., "Rheological Equations from Molecular Network Theories", *Trans. Soc. Rheology*, Vol. 16, No. 1, 1972, pp. 99-127.

[13] Bird, R.B., Armstrong, R.C. and Hassager, O., *Dynamics of Polymeric Liquids Volume 1 Fluid Mechanics*, second edition, Wiley, New York, 1987, p.144.

[14] Bair, S., Khonsari, M. M., "Generalized Reynolds equations for line contact with double-Newtonian shear-thinning," Tribology Letters, Volume 18, Number 4 (2005), pp.513-520.

[15] Charles, P., Elfassi, M. and Lubretch, A.A., "'Double-Newtonian' rheological model for the two-dimensional Reynolds equation," *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, vol. 225 no. 6, 2011, pp.335-346.

[16] Bair, S. and Winer, W. O., 1996, "A Simple Formula for EHD Film Thickness of Non-Newtonian Liquids," In Elastohydrodynamics - '96 Fundamentals and Applications in Lubrication and Traction, Elsevier Tribology Series, Vol. 32, 1997, pp. 235-241.

[17] Bair, S., "Shear-Thinning Correction for Rolling/Sliding Elastohydrodynamic Film Thickness", *Proc. I. Mech. E., Part J, Journal Engineering tribology*, Vol. 219, 2005, pp. 69-74.

[18] Kumar, P. and Khonsari, M.M., "EHL Circular Contact Film Thickness Correction Factor for Shear-Thinning Fluids," *ASME Trans. J. Tribol*. 130, 2008, 041506-1-7.

[19] Jang, J.Y., Khonsari, M.M. and Bair, S. "Correction Factor Formula to Predict the Central and Minimum Film Thickness for Shear-Thinning Fluids in EHL", *ASME, J. Tribology*, Vol.130, 2008, 024501.

[20] Anuradha, P. and Kumar, P., "New film thickness formula for shear thinning fluids in thin film elastohydrodynamic lubrication line contacts," *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, vol. 225 no. 4, 2011, pp. 173-179.

[21] G E Morales-Espejel and A W Wemekamp, "Ertel-Grubin methods in elastohydrodynamic lubrication - a review," *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology* , Vol.222, 2008, pp.15-34.

[22] Habchi W., Eyheramendy D., Vergne P. and Morales-Espejel G. – Stabilized Fully-Coupled Finite Elements for Elastohydrodynamic Lubrication Problems, *International Journal of Advances in Engineering Software*, 2012, vol. 46, pp. 4-18.

[23] Yang, P., Wen, S. - A Generalized Reynolds Equation for Non-Newtonian Thermal Elastohydrodynamic Lubrication, *ASME J. Tribol.*, 1990, vol. 112, pp. 631-636.

[24] Drozd-Rzoska, A., Rzoska, S.J., Roland, C.M. and Imre, A.R., "On the pressure evolution of dynamic properties of supercooled liquids," *J. Phys.: Condens. Matter*, Vol.20, 2008, 244103.

[25] Novak, J.D. and Winer, W.O., "Some Measurements of High Pressure Lubricant Rheology", *ASME J. Lubr. Techn.*, Vol.90, Ser.F, No.3, 1968, pp.580-591.

[26] Bair, S., Qureshi, F., and Kotzalas, M., "The Low Shear Stress Rheology of a Traction Fluid and the Influence on Film Thickness", *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, Vol. 218, 2004, pp. 95-98.

[27] Bair, S. and Qureshi, F., "The high pressure rheology of polymer-oil solutions," *Tribology International*, Vol.36, Issue 8, 2003, pp. 637–645.

[28] Greenwood, J. A., "Film thicknesses in circular elastohydrodynamic contacts," *Proceedings of the Institution of Mechanical Engineers, Part C, Mechanical engineering science*, Vol. 202, No1, 1988, pp. 11-17.

[29] Bair S. – Reference Liquids for Quantitative Elastohydrodynamics: Selection and Rheological Characterization, *Tribology Letters*, 2006, vol. 22, pp. 197-206.

[30] Bair, S., "Rheology and high-pressure models for quantitative elastohydrodynamics," *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, Vol. 223, no. 4, 2009, pp.617-628.

[31] Habchi. W., Eyheramendy D., Bair S., Vergne P. and Morales-Espejel G. – Thermal Elastohydrodynamic Lubrication of Point Contacts Using a Newtonian / Generalized Newtonian Lubricant, *Tribology Letters*, 2008, vol. 30 (1), pp. 41-52.