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Evaluation of EPQ models with Imperfect
Quality items of Raw Material

By

Arwa Joudi

A thesis

Submitted in partial fulfillment of the requirements
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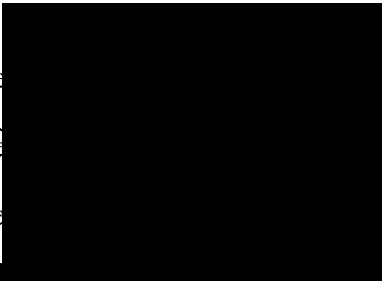
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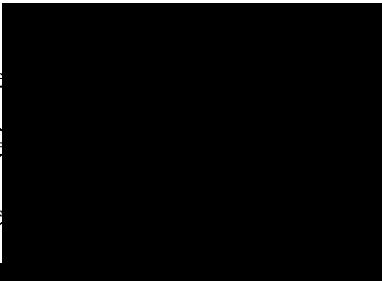
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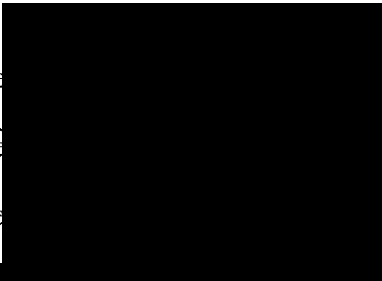
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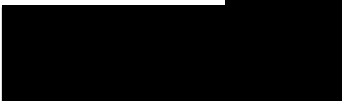
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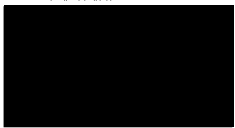


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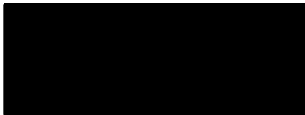
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*To My Baby Girl, who has not seen the light yet,
I dedicate this work...*

Evaluation of EPQ models with Imperfect Quality items of Raw Material

Arwa Joudi

Abstract

The main objective of the research is to evaluate the recently developed economic production models with imperfect quality items of raw material and consider their extensions to the case where the percentage of imperfect quality items is a random variable having a known probability density function. The deterministic models will be evaluated by examining the effects of the parameters on the optimal solution. The probabilistic models will be simulated using various density functions for the percentage of imperfect quality items to compare the theoretical and the computational solutions. The results will be used to develop an economic production model having an optimal ordering policy of raw material.

Keywords: Inventory Model, Economic Production Quantity, Inventory Cost Function, Raw Material, Imperfect Quality Items, Probabilistic Percentage.

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Chapter One

INTRODUCTION

In this chapter, the classical Economic Order Quantity Model and Economic Production Quantity Model are introduced. The presence of imperfect quality items within the raw materials used is also discussed. Several studies related to this topic are surveyed in a literature review. The chapter ends by stating our research questions, which will be answered throughout the whole thesis.

1.1 Classical EOQ and EPQ Models

Due to globalization and openness of global trade, customer demand for goods and services has been escalating worldwide. This is causing an ever growing challenge for companies to be able to supply products sufficiently and on time. However, demand is not that easy to predict. Many factors cause demand fluctuations such as seasonal changes, economic/political crisis, natural disasters...etc. For that reason, companies fall into the dilemma of whether to stock inventory or not. Under stocking causes shortages while overstocking increases total cost.

In a classical Economic Order Quantity model (EOQ), the aim is to minimize the inventory carrying costs and ordering costs in order to find out the most cost efficient quantity to be requested. In the case where the product is purchased, this is denoted

by order quantity, but when the product is actually manufactured, it is referred to as the production lot size. The EOQ model illustrates the case of ordering a specific item so that the demand is met. As the total inventory cost is minimized, the optimal order quantity will be established, see figure 1.1. The three main costs of inventory models are:

- 1) **Unit purchasing cost:** This is the actual cost per unit paid to the supplier.
- 2) **Ordering Cost:** This the one-time cost incurred whenever an order is placed.
- 3) **Stock holding costs:** These are the costs associated with maintaining the inventory and paying the interests charged on the capital borrowed to buy these stocks. Stock holding costs increase as the amount of inventory and stocking time increase. These cost of renting storage space, the cost of deteriorated items, obsolescence cost (the cost incurred when items become outdated), and also the administrative and insurance cost are all included under this cost type.

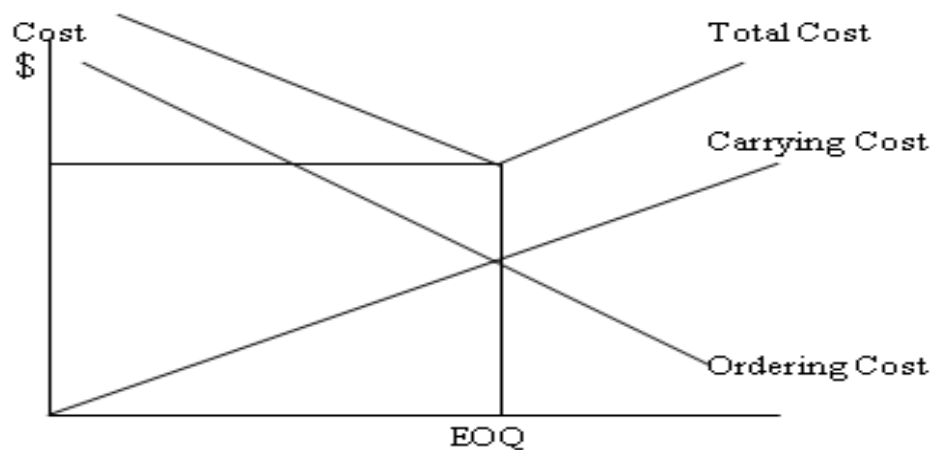


Figure 1.1: *The Costs involved in an EOQ Model*

As for the classical economic production quantity model (EPQ), it involves producing the items demanded and not purchasing them. The rate at which

production occurs exceeds that of the demand. The cost components of this model are:

- 1) **Unit production cost:** This is the actual cost paid by the manufacturer to be able to produce a single unit.
- 2) **Setup Cost:** These are the fixed costs realized when placing an order and when maintaining the machinery and equipments used in production. Examples on such costs are purchasing, ordering, supervising quality control, etc...
- 3) **Stock holding costs:** Same as EOQ.

The EPQ model has been the subject of study for many researchers over the past years. This study has been extended to include many elements and obstacles that are experienced daily by businesses and people using this model. One of the newly discussed topics in the EPQ model is the presence of items of imperfect quality and the study of its consequences. The traditional EPQ model supposes that the production process results in final products that are all perfect in quality. However, not all products end up as perfect as expected, since some causes lead to imperfection in the quality of manufacturing.

In this chapter, we introduce the idea that not all final products are of the same quality. Throughout manufacturing, two processes will be taking place: production and screening. After that, every final product is categorized as of perfect quality or imperfect quality. Let us consider q as the percentage of imperfect items generated. Then $1-q$ will refer to the percentage of perfect items produced. So, we notice that the production of items with perfect and imperfect quality takes place at a stable rate. Regarding the demand for those products during the cycle, we consider it as constant.

As production continues, inventory of both qualities perfect and imperfect will accrue, and they will be used up until the end of the inventory period. In our analysis, we suppose that both the perfect and imperfect quality items are of known and constant demand as they are systematically stocked. Yet it is impossible to accurately forecast demand, this is why it is expressed in terms of probability. Product replenishment is determined based on the differences between the rates of production and demand. Moreover, production rates should always exceed demand rates so that finished items can build up in stock and for the whole system to work efficiently.

In the EOQ model, it is taken for granted that suppliers only provide perfect quality raw materials. In our study, we will introduce an inventory model with raw materials with imperfect quality, yet they are not defective. Such items may be traded at a lowered price since they are imperfect. When raw materials acquired contain both qualities are acquired, the effect of having imperfect items will be incorporated in the EPQ formula. As we develop the mathematical model, we further explain it through a numerical example, whereby we determine the optimal policy.

In the classical economic production model, raw material costs and quality are not considered. We will introduce an inventory model that shows the different costs, such as those of production, acquiring, quality and holding the raw material. After that we attain the final product and develop a profit function, which will be maximized to reach the optimal production quantity.

1.2 Literature Review

Economic production models have been the topic of many thorough studies for many years. Poretus (1986) acknowledged the existence of defective items and demonstrated their effect on the classical EOQ model. Scwaller (1988) suggested an economic order quantity model whereby the amount of imperfect quality items received from suppliers is known. Even old traditional studies of EOQ models such as those just mentioned suggested that all production lots contained a specific amount of defectives, which creates fixed and variable inspection costs to be able to detect and eliminate the defective items.

Cheng (1991) depicted an EPQ model having imperfect production processes where the demand depends on the unit production cost. A precise mathematical approach was developed by Salameh and Abdul-Malak (1994) that highlights the importance of having an optimum order quantity, taking into account time inflation. Salameh and Kassar (1999) studied an instant replenishment model where they inspected the effects of credit facilities in payment on the economic order quantity model.

The contribution of Salameh and Jaber (2000) has added much to the EOQ model for imperfect items. Their aim was to set up the optimal order size of raw materials that includes imperfect quality items within. They considered that every lot received from the supplier contains imperfect items with an established probability density function. Imperfect quality items present randomly within raw materials are removed through a screening process, and then sold at a lower price than that of perfect raw material items. Numerous researchers who followed have extended this model to

include management of the supply chain, improvement of quality and measurement of the effect of human mistakes on manufacturing and systems of inventory. Hayek and Salameh (2001) also calculated the optimal lot size of production while taking into account items of imperfect quality.

Chang et al. (2002) not only studied delays in payment, but also expanded their research to a new model that considers time discounting and deterioration. Chang et al. (2003) concluded that the rate of deterioration of items stays a function of the quantity ordered. Chiu (2003) constructed a new EPQ model and generalized it by taking a production process whereby the rate at which items are found defective is random, and those units that are found to be imperfect are reworked and unsatisfied demand is backlogged.

On the other hand, Ozdemir (2007) assumed in his EOQ model with defective items that any deficiency in raw materials is backordered. Dah and Kassab (2008) focused on the effect of the holding and ordering costs of raw materials on the EOQ of the restricted production model. They considered the case of a semi-finished product received from a supplier and then produced at a certain rate while keeping the demand at a constant rate. They came up with the conclusion that the finished products are more likely to increase during the production period whereby towards the end of that phase, and until the inventory stage stops, items that have accumulated are used up gradually but a constant rate of demand. They assumed that the demand for the stocked units is precise and invariable. However, in real life this barely happens. Demand is predicted in probabilistic terms. Dah and Kassab (2008) concluded that the rate of production ought to be greater than the rate of demand.

Hayek and Salameh (2008) also studied an EPQ model where items of imperfect are reworked.

The movement of a product within the supply chain is directly affected by the presence of items of imperfect quality among the raw material requested and throughout manufacturing. El-Kassar (2009) on the other hand aimed to have a continuous demand for items of perfect and imperfect items during the inventory cycle.

The classical economic production model simplifies many assumptions, ignoring many issue faced during daily operations. This has been changing lately, since many researchers started to let go of the postulation of perfect quality items. Khan et al. (2011) also presented an extensive review of EOQ /EPQ models with imperfect quality items.

El Kassar et al. (2012 a.) extended the classical EPQ model in various ways to integrate factors faced in real-life situations. In their paper, they portrayed an EPQ model whereby the cost of raw material needed for producing the finished products is taken into consideration. The raw material obtained from the supplier is also thought to include a percentage that is of imperfect quality. As the inventory cycle initiates, the raw material is received instantly, and items of imperfect quality are discovered using a 100% screening process. After this stage, two scenarios are considered. The imperfect quality items of the raw material are sold at a discount towards the end of the screening period in the first scenario. In the second one, the items of imperfect

quality should remain in stock until the inventory cycle comes to an end and those units are returned to the supplier when the next order is received.

El Kassar et al. (2012 b.) examined the case where raw materials of imperfect quality items are used in the production process. At the beginning of the inventory cycle, a supplier provides the raw material needed for the manufacture of the final product, and a proportion of this material is thought to be of imperfect quality. Those items are distinguished by a 100% screening process, yet all the raw material items are utilized in production. This will result in two types of finished products, one with perfect and another with imperfect quality. The two finished products are assumed to be demanded continuously. El Kassar et al developed two mathematical models to demonstrate the two cases possible. They obtained an optimal production quantity and provided numerical examples to better demonstrate their models.

El Kassar, Salemeah and Bitar (2012 c.) expanded an economic production model that not only integrates the effects of imperfect quality items of raw material, but also the time value of money. They developed a mathematical model and attained the optimal production quantity by maximizing the profit function. The solution determined is illustrated through numerical examples.

1.3 Objective of the Study

The objective of this research is to evaluate the models developed lately by El-Kassar et al. (2012 a. & b.) and to extend the model to the case where the percentage of imperfect quality items is a random variable having a known probability density

function. Based on examining the different parameters that affect the optimal solution, the deterministic models will be evaluated. The probabilistic models are developed. The theoretical and the computational solutions are compared.

The following will constitute the rest of the thesis. In chapter two, the classical EOQ and EPQ Models are considered, whereby we describe those models and find their optimal solutions. We then look at the effects of raw material on the EPQ model. In chapter three, we study the EPQ model that uses raw materials of imperfect quality items and we refer to other studies regarding this issue. In particular, we describe the models presented by El-Kassar et al. (2012, a). In chapter four, we present the EPQ of El-Kassar et al. (2012, b) that uses both perfect and imperfect quality items in the production process. The finished product is of two quality types. Two cases are considered depending on which of the two types of the finished product is sold out first.

In chapter five, a model presented in chapter three is evaluated and compared to previous models. Numerical examples for finding the optimal solution are provided. Microsoft Excel programs for finding the optimal solution are developed and illustrated. The evaluation of the models is conducted by varying the different parameters and the generated output is analysed. We show that the model is sensitive to changes in the percentage of imperfect quality items of raw material. Hence, the need of developing a probabilistic model is demonstrated.

In chapter six, we extend the models of chapter three to the case where the percentage of imperfect quality items of raw material is a random variable having a

known probability density function. This approach is more realistic. A mathematical model for this approach is developed. The optimal solution is obtained using a closed form formula. Numerical examples illustrating the model are provided. The results of this chapter are accepted to be presented at the IABE-2012 Las Vegas- Annual Conference and the submitted paper is accepted to be published in the *Journal of Academy of Business and Economics*, Vol. 12, No. 3, see Mikdashi, El-Kassar, and Joudi (2012).

In chapter seven, we give an overall assessment for the models presented as well as conclusions and recommendations. We also give suggestions for future works.

Chapter Two

The Classical EOQ & EPQ Models

In this chapter, we present the classical EOQ and EPQ models and give their mathematical models along with the optimal solutions. Also, we present a model that considers the impact of raw material on the EPQ model. To better illustrate those models, numerical examples will be employed.

2.1 The Optimal Solution of the EOQ Model

Being one of the simplest models, the classical EOQ model presumes that the quantity obtained is exactly equal to the quantity requested, so there are no lost items. As buyers receive their requested orders, they should pay suppliers in full. To construct the mathematical model, we define the following variables:

- D as the demand per unit time
- C as the unit cost of purchasing
- K as the cost of making orders, and it is expressed in dollars
- y as the number of raw materials ordered per production cycle
- y^* as the optimal ordering quantity
- T as the length of the inventory cycle in unit time
- h as the holding cost of 1 unit of the product per unit time

In our EOQ models, we will presume that the demand is known and fixed, replacement is immediate, and lead time is zero. It can be shown that the total inventory can be expressed as:

$$TCU(y) = CD + \frac{KD}{y} + \frac{hy}{2}. \quad (2.1)$$

The optimal order quantity is

$$y^* = \sqrt{\frac{2KD}{C}}, \quad (2.2)$$

and the optimal cycle length as

$$T^* = \frac{y^*}{D}. \quad (2.3)$$

The classical EOQ model is used to determine the order quantity (y) which balances the cost of ordering with the costs of holding h so that overall costs are minimized.

The inventory level is shown in figure 2.1.

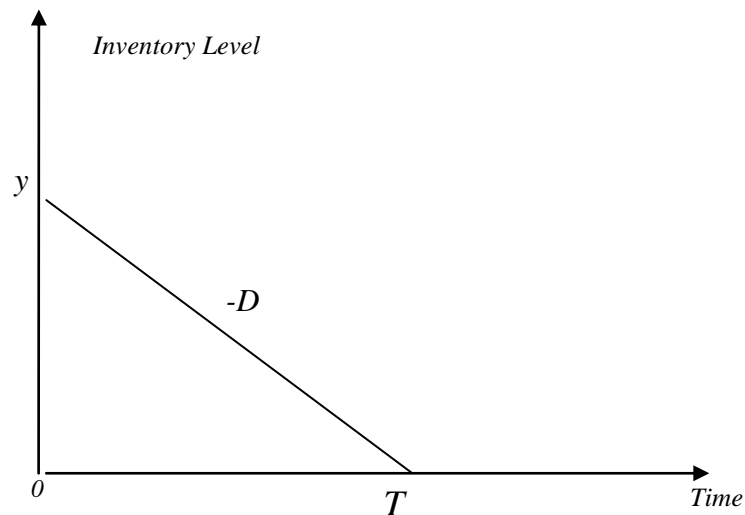


Figure 2.1: A typical cycle for the classical EOQ model

To illustrate, we consider the case where the daily demand for a product is 100 units. It costs \$ 100 to place an order and \$0.02 to keep one item in stock for one day. The unit purchasing cost is \$ 0.5. Then Optimal Order Quantity is:

$$Q^* = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2(100)(100)}{0.02}} = 1000 \text{ units.}$$

The *Optimal Cycle Length* is:

$$T^* = \frac{Q^*}{D} = \frac{1000}{100} = 10 \text{ days.}$$

The *Optimal Total Cost* per day is:

$$\begin{aligned} TC(Q^*) &= CD + \frac{KD}{Q^*} + \frac{HQ^*}{2} \\ &= 0.5(100) + \frac{(100)(100)}{1000} + \frac{(0.02)(1000)}{2} = \$70. \end{aligned}$$

2.2 The Optimal Solution of the EPQ Model

The EPQ model was established early in the 20th century. This model differs from EOQ models in its supposition that ordered items ordered are delivered gradually throughout the production process. In the classical EPQ model, the replenishment of the ordered quantity is not instantaneous because the delivery of items is distributed over a certain time period, which causes the gradual accumulation of inventory.

Under this model, the production process progresses during a restricted time period throughout the inventory cycle; yet consumption occurs during the whole cycle. As a result, the rate at which building up of inventory occurs is equal to the difference between the consumption and production rates of the items.

The parameters of the EPQ model are:

- D as the demand per unit time
- P as the production rate per unit time
- C as the unit cost of production
- K as the production run set up cost
- h as the holding cost

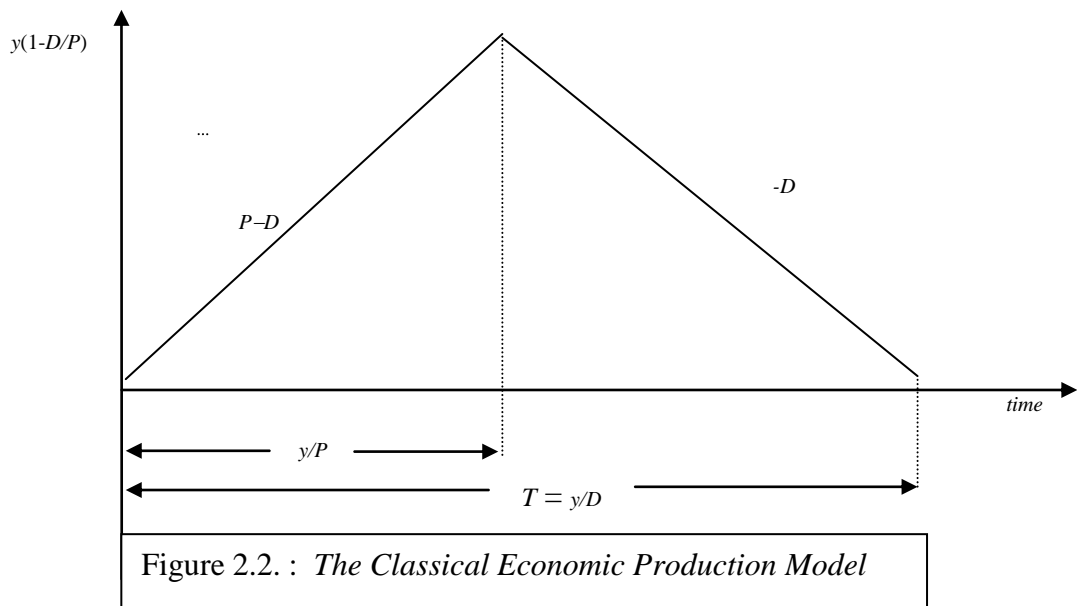
The maximum inventory is expressed as $y\left(1 - \frac{D}{P}\right)$. This maximum level is reached at the end of the production period. The length of the production is denoted as y/P , see figure 2.2.

The total inventory cost per unit time function is:

$$TCU(y) = CD + \frac{KD}{y} + h\left(1 - \frac{D}{P}\right)\frac{y}{2}. \quad (2.4)$$

Solving for y , we find out that the optimal lot size y^* is:

$$y^* = \sqrt{\frac{2KD}{h(1 - D/P)}}. \quad (2.5)$$



To illustrate, we will consider the case where the daily demand rate for a product is 100 units and the daily production rate is 300 units. The daily per unit holding cost of the product in inventory is \$ 0.02 and the ordering cost per order is assumed to be \$150. Each item is purchased at \$ 5 per unit.

The above given could be summarized as follows:

$D = 100$ units per day.

$P = 300$ units per day.

$h = \$ 0.02$ per unit per day.

$K = \$ 150$ per order.

$C = \$ 5$ per unit.

The ***Optimal Order Quantity*** is:

$$Q^* = \sqrt{\frac{2KD}{h(1-\frac{D}{P})}} = \sqrt{\frac{2(150)(100)}{(0.02)(1-\frac{100}{300})}} = 1500 \text{ units.}$$

The *Optimal Cycle Length* is equal to:

$$T^* = \frac{Q^*}{D} = \frac{1500}{100} = 15 \text{ Days.}$$

The *Optimal Total Cost* per day is equivalent to:

$$\begin{aligned} TC(Q^*) &= CD + \frac{KD}{Q^*} + \frac{Q^*}{2} \left(1 - \frac{D}{P}\right)h \\ &= 5100 + \frac{150(100)}{1500} + \left(\frac{1500}{2}\right) \left(1 - \frac{100}{300}\right)(0.02) = \$520. \end{aligned}$$

The *Optimal Maximum Inventory Level* is:

$$Q^* \left(1 - \frac{D}{P}\right) = (1500) \left(1 - \frac{100}{300}\right) = 1000 \text{ Units}$$

The *Optimal Production Period Length* is:

$$\frac{Q^*}{P} = \frac{1500}{300} = 5 \text{ days.}$$

2.3 Effects of Raw Material on the EPQ Model

The EOQ and EPQ models are generally used to find out the optimal ordering quantity that sets the total cost of inventory at a minimum. The least cost will be

attained when the purchase/manufacturing expenses and the ordered cost are equivalent to the holding cost. The two models differ in the delivery mode. In the EOQ model, delivery of the ordered goods occurs all at once, while in the EPQ, the items are delivered gradually within a definite time interval. One of the shortcomings of the EPQ model is that it does not take into account the various costs of the raw material used in the production process. Salameh & El-Kassar (2007) incorporated the holding cost of raw material into the classical EPQ model. This model combines the EOQ and EPQ models. In the following, we develop a model similar to that of Salameh & El-Kassar (2007). Define the following variables:

- D as demand per unit time
- C_r as the unit cost of purchasing raw materials
- C_p as the unit cost of producing one finished item
- y as the number of raw materials ordered per production cycle
- y^* as the optimal ordering quantity
- T as the length of the inventory cycle in unit time
- y as the number of units of raw material items ordered per production cycle
- K_p as the production run set up cost
- K_s as the fixed cost of placing raw material orders from suppliers
- h_r as the cost of holding one unit of raw material per unit time
- h_p as the holding cost due to production per unit time

The total inventory cost per unit time function, $TCU(y)$, is:

$$TCU(y) = \frac{(K_s + K_p)D}{y} + (C_s + C_p)D + \frac{h_r D}{2 P} y + \frac{(h_r + h_p) y}{2 P} (P - D). \quad (2.6)$$

The optimal order quantity is:

$$y^* = \sqrt{\frac{2(K_s + K_p)PD}{(P - D)(h_r + h_p) + Dh_r}}. \quad (2.7)$$

Consider the situation where the daily demand for a certain product is 5 items. The production rate is 10 items per day. The setup cost is \$183 and the cost of ordering is \$100 and. The purchasing cost is \$5 per unit, the production cost is \$10 per unit, the daily per unit holding cost of raw material is \$0.01, and the daily per unit production holding cost is \$0.01. Subsequently, the optimal quantity is $y^* = 376$ and the optimal length of inventory cycle will be $T = 75.2$ days and the minimum total inventory cost is $TCU(y^*) = \$82.52$. The graph of the TCU function generated by the mathematical software package Mathematics is given in figure 2.3.

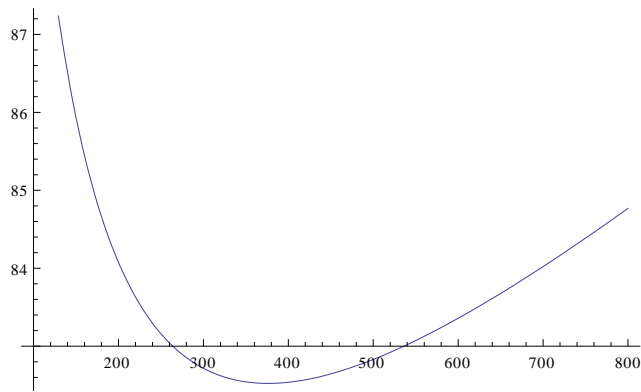


Figure 2.3: Behaviour of the TCU Function

Chapter Three

EPQ Model with Imperfect Quality Raw Material

In this chapter, the effect of imperfect quality items of raw materials is taken into account by depicting two EPQ models. The results of El-Kassar et al. (2012, a) are presented. Numerical examples are provided for better demonstration.

3.1 Imperfect Quality Items are Sold at a Lowered Price

Consider a production process whereby the raw material acquired from the supplier is partially of imperfect quality. Items of the raw material are received instantaneously as the inventory cycle initiates. After that, items within the raw material that are of imperfect quality are detected by performing a 100% screening process. Two different scenarios are considered, whereby the optimal solutions along with the total profit function are given. The models shown are clarified by numerical examples. In the first scenario, the imperfect quality items of the raw material are kept till the end of the screening period, when they are sold at a lowered price.

In this chapter, consider the following variables:

- D as the demand of finished product per unit time
- P as the rate of production (whereby $P > D$)
- y as the number of raw materials ordered per production cycle

- q as the proportion of raw material items that are of imperfect quality
- $1-q$ as the percentage of perfect quality items of raw material
- x as the screening rate for imperfect quality items of raw material ($x > P$)
- K_p as the set up cost for production
- K_s as the ordering cost from supplier
- h_r as the raw material holding cost per unit per unit time
- h_p as the production holding cost per unit per unit time
- C_r as the unit cost of raw material
- C_p as the unit production cost
- C_s as the unit cost screening cost
- S as the unit selling price of finished perfect product
- S_r as the discounted unit selling price of imperfect quality items
- T as the inventory cycle length
- t_s as the screening period
- t_p as the production period

To start, we formulate a model to describe the first scenario, whereby raw materials that are of imperfect quality items are sold at a reduced price towards the end of the screening period. Production takes place at a rate P , where P is greater than the rate of demand D for the final product. As the production cycle starts, raw material order of size y is requested and obtained. A proportion q of imperfect quality items is included within those raw materials supplied. The rest of the materials are of perfect quality, $y(1-q)$, and are consumed in producing the end product. Hence, the length of the production period is:

$$t_1 = \frac{y(1-q)}{p}, \quad (3.1)$$

and the length of inventory cycle is:

$$T = \frac{y(1-q)}{D}. \quad (3.2)$$

As the production cycle is initiated, a 100% screening procedure for distinguishing the imperfect quality items is performed at a screening rate x , whereby $x > P$. The length of the production period is

$$t_s = \frac{y}{x}. \quad (3.3)$$

During the screening period, the raw material items that are of perfect are used normally in the production process. Thus, the inventory level of raw material is exhausted at a rate P until the end of screening period. As the screening process is brought to a halt, the number of perfect and imperfect quality items of raw material will be:

$$y - pt_s = y - p \frac{y}{x} = y(1 - \frac{P}{x}). \quad (3.4)$$

As for the imperfect quality items at that time, they will be sold at a cut-price price S_r , which is less than the raw material unit purchasing cost, C_r . consequently, the inventory level of raw materials decreases from $y - pt_s$ by an amount qy . The total number of items of raw material left is

$$y - pt_s - qy = y(1 - \frac{P}{x}) - qy = y(1 - \frac{P}{x} - q). \quad (3.5)$$

The raw material inventory level drops at a rate P until the end of production period where it reaches zero. The inventory level of the finished items is shown in figure 3.1.

During Manufacturing, finished items are produced at a rate P and part of these items is sold at a rate D . Therefore, an inventory of finished items is accumulating throughout the production period at a rate $P-D$ until a maximum level of y_{max} is reached. The inventory level of the finished items is shown in figure 3.2. From earlier, the maximum level of finished items is:

$$y_{\max} = t_p(P - D) = \frac{y}{p}(1 - q)(P - D) = y(1 - q)\left(1 - \frac{D}{P}\right). \quad (3.6)$$

Since we aim to obtain the optimal order quantity, we start by finding the total cost per inventory cycle which is the sum of the cost of purchasing, cost of production cost of ordering raw material, setup cost of production, holding cost of raw material and holding cost of finished product. Except for the holding costs, the components are as follows:

$$\text{Purchasing cost and screening of raw material} = (C_r + C_s)y \quad (3.7)$$

$$\text{Production cost} = C_p y (1 - q) \quad (3.8)$$

$$\text{Ordering cost of raw material} = K_s \quad (3.9)$$

$$\text{Set up cost} = K_p \quad (3.10)$$

$$\text{Holding cost of raw material} = y^2 \left(\frac{(1 - q)^2}{2P} + \frac{q}{x} \right) \times h_r \quad (3.11)$$

$$\text{Holding cost for finished items} = \frac{y}{2}(1 - q) \left(1 - \frac{D}{P}\right) (h_p + h_r)T \quad (3.12)$$

The total inventory cost per cycle function $TC(y)$ will be calculated as follows:

$$TC(y) = (C_r + C_s)y + C_p y(1-q) + K_s + K_p + y^2 \left(\frac{(1-q)^2}{2P} + \frac{q}{x} \right) h_r \quad (3.13)$$

$$+ \frac{y}{2}(1-q) \left(1 - \frac{D}{p}\right) (h_p + h_r) T.$$

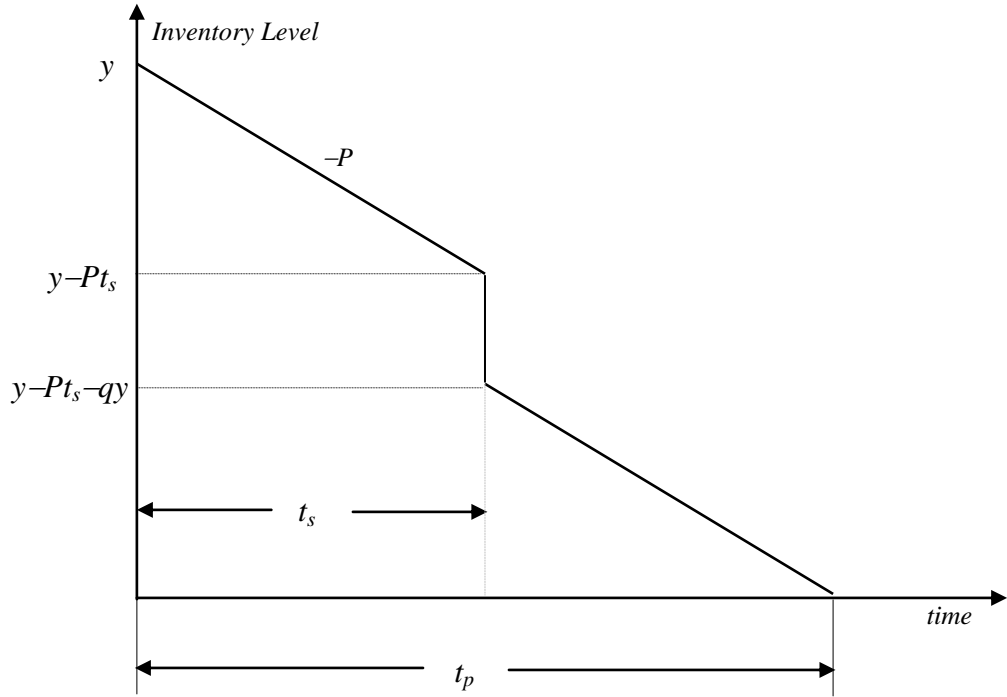
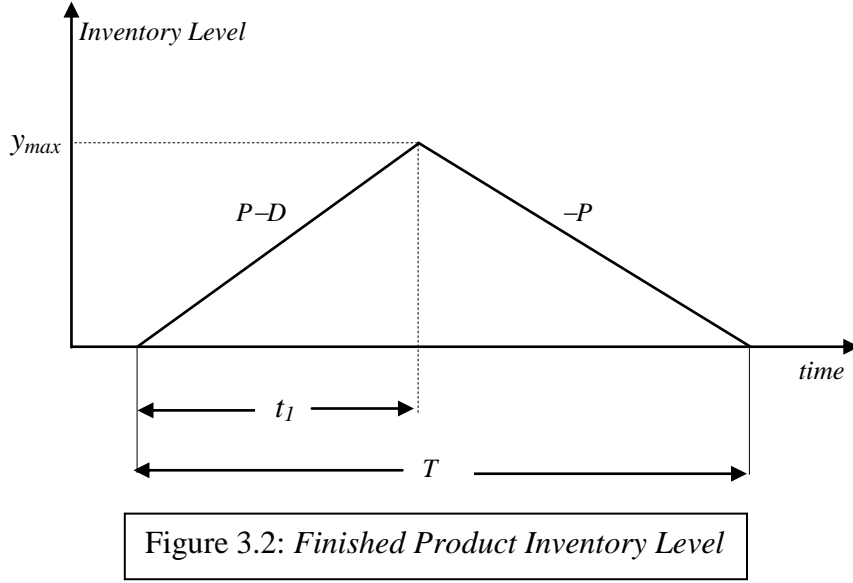


Figure 3.1: Raw Material Inventory Level, Imperfect Items Sold at a Discount

The total inventory cost per unit time function $TCU(y)$ is:

$$TCU(y) = (C_r + C_s) \frac{D}{1-q} + C_p D + (K_s + K_p) \frac{D}{y(1-q)} \quad (3.14)$$

$$+ y D \left(\frac{(1-q)}{2P} + \frac{q}{(1-q)x} \right) h_r + \frac{y}{2}(1-q) \left(1 - \frac{D}{p}\right) (h_p + h_r).$$



Now, the total revenue function $TR(y)$ will be:

$$TR(y) = Sy(1 - q) + S_r q y. \quad (3.15)$$

From the above, find the total revenue per unit time:

$$TRU(y) = SD + S_r q D \frac{1}{(1 - q)}. \quad (3.16)$$

The optimal order quantity is:

$$y^* = \sqrt{\frac{2(K_s + K_p)D}{(h_p + h_r)\left(1 - \frac{D}{P}\right)(1 - q)^2 + Dh_r\left(\frac{(1 - q)^2}{P} + \frac{2q}{x}\right)}}. \quad (3.17)$$

We note that it is easy to show that the EOQ is unique.

3.2 Imperfect Quality Items are Returned to Supplier

Now we develop the mathematical model for the second scenario. In this case, the screened imperfect quality items of raw material are kept on hold to be returned to

the supplier at the end of the inventory cycle when the next order is received. The optimal order quantity is obtained by calculating the total cost per cycle, which is identical to that of the first scenario except for the cost of holding the raw material.

The area under the curve representing the inventory level in figure 3.3 is:

$$\text{Area} = qyT + \frac{y(1-q)}{2}t_1 \quad (3.18)$$

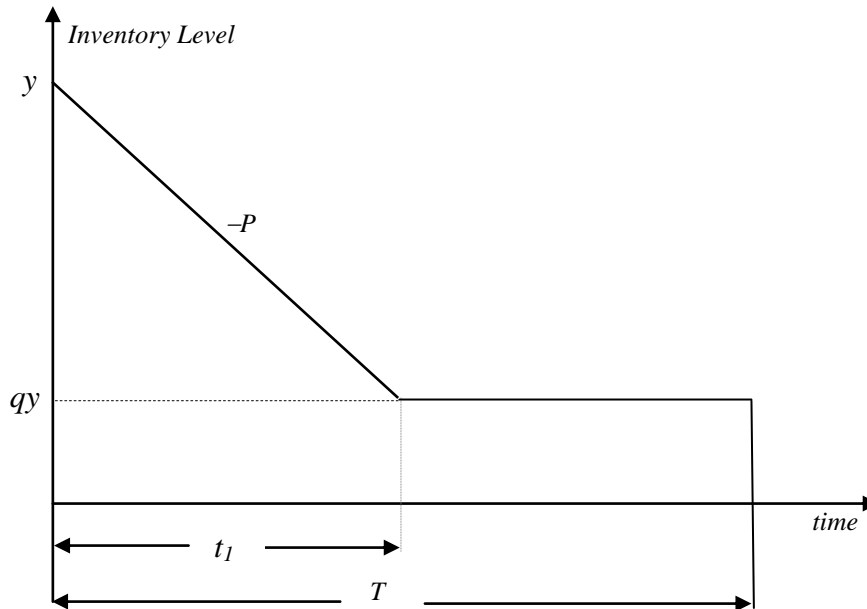


Figure 3.3: Raw Material Inventory Level, Imperfect Items Returned to Supplier

Dividing (3.18) by T , we get:

$$\text{Average inventory} = qy + \frac{D}{2P}y(1-q). \quad (3.19)$$

Multiplying (3.19) by h_r and T , we obtain:

$$\text{Holding cost of raw material} = y \left(q + \frac{D}{2P}(1-q) \right) h_r T. \quad (3.20)$$

From the above, we obtain the TCU function for the second scenario as:

$$\begin{aligned}
TCU(y) &= (C_r + C_s + C_p) \frac{D}{1-q} + (K_s + K_p) \frac{D}{y(1-q)} \\
&\quad + y \left(q + \frac{D}{2P} (1-q) \right) h_r + \frac{y}{2} (1-q) \left(1 - \frac{D}{P} \right) (h_p + h_r).
\end{aligned} \tag{3.21}$$

Since the imperfect quality items of raw material are returned to the supplier, the total revenue function is:

$$TR(y) = Sy(1-q) + C_r qy \tag{3.22}$$

Dividing (3.22) by T and subtracting (3.21), we have:

$$\begin{aligned}
TPU(y) &= SD - C_s \frac{D}{1-q} - (C_r + C_p) D - (K_s + K_p) \frac{D}{y(1-q)} \\
&\quad - y \left(q + \frac{D}{2P} (1-q) \right) h_r - \frac{y}{2} (1-q) \left(1 - \frac{D}{P} \right) (h_p + h_r).
\end{aligned} \tag{3.23}$$

We finally obtain the economic order quantity:

$$y^* = \sqrt{\frac{2(K_s + K_p)D}{(h_p + h_r) \left(1 - \frac{D}{P} \right) (1-q)^2 + h_r \left(\frac{D}{P} (1-q)^2 + 2q(1-q) \right)}}. \tag{3.24}$$

3.3 Numerical Examples

Consider a production process where the demand rate for an item is 5 units per day and the daily production rate is 10 units. The raw material used in production is ordered from the supplier where 30% of the items received are found defective. The daily screening rate for imperfect quality items of the raw material is 20. The cost of ordering the raw material is \$100 and the setup cost is \$183. The cost of holding raw material is \$0.01 per unit per day while the daily cost due to holding production is \$0.02 per unit. Hence, the holding cost of one unit of the finished product is \$0.03 per day. The purchasing cost of one item of raw material is \$5, the screening cost per unit is \$0.5, and the production cost is \$10 per unit. The selling price is \$25 per unit. The imperfect quality items screened may be sold at the end of screening period at a discounted price of \$3, or may be kept in stock and returned to the supplier when the next order arrives. The two scenarios are compared to establish the optimal order policy.

The parameters of the problem are: $D = 5$, $P = 10$, $q = 0.3$, $x = 20$, $K_s = 100$, $K_p = 183$, $h_p = 0.02$, $h_r = 0.01$, $C_r = 5$, $C_s = 0.5$, $C_p = 10$, $S = 25$, and $S_r = 3$.

Evaluating the policy where the imperfect quality items are sold at a discount, the optimal order quantity is obtained as $y^* = 500.4 \approx 500$. The optimal number of items manufactured during a production cycle is $y^*(1-q) = 350$ units. The length of inventory cycle is $T^* = \frac{y^*(1-q)}{D} = 70$ days, the production period is

$t_p^* = \frac{y^*(1-q)}{P} = 35$ days, and the screening period is $t_s^* = \frac{y^*}{x} = 25$ days. The total inventory cost per day is \$93.79, the total revenue per day is \$131.43, and the maximum total profit per day is \$37.64.

As for the case where the imperfect quality items are returned, the optimal order quantity $y^* = 449.6 \approx 450$. The optimal number of items produced during a production cycle is 315 units. The production period is 31.5 days and the inventory period length is 63 days. The total cost per day is \$94.71, the total revenue per day is \$134.71, and the total profit per day is \$40.00

From the above analysis, the optimal operating policy is to return the imperfect items to the supplier when the next order arrives.

Chapter Four

EPQ Model with Imperfect Quality Items of Raw

Material and Finished Product

The purpose of this chapter is to present the EPQ of El-Kassar et al. (2012 b) who included the perfect and imperfect quality items in the production process. The end result will be finished products of two quality types. Two cases are considered depending on which of the two types of the finished product is sold out first.

4.1 Using Imperfect Quality Items of Raw Material in Production

In the model we will discuss, the raw material is acquired from a supplier at the beginning of the inventory cycle. A percentage of imperfect quality items is found within the raw material. A 100% screening process for detecting the imperfect quality items of raw material is conducted at a rate greater than the production rate. The perfect and imperfect quality items screened are used in the production process. The perfect quality items of raw material used in production yield perfect quality finished items. On the other hand, when the imperfect quality items of raw material are used, the finished items produced are of imperfect quality. Both types of finished products are assumed to have continuous demand. Two cases are considered depending on which of the two types of the finished product is sold out first. For each case, the mathematical model is developed and the optimal order quantity is determined.

To develop this model, we use the following notations:

- D_p as the demand rate of perfect quality items of raw material
- D_i as the demand rate of imperfect quality items of raw material
- P as the production rate ($P < x$)
- x as the screening rate for imperfect quality items of raw material
- y as the raw material ordered size
- q as the percentage of imperfect quality items of raw material
- $1-q$ as the percentage of perfect quality items of raw material
- y_{imax} as the maximum inventory level of imperfect finished items
- y_{pmax} as the maximum inventory level of perfect finished items
- K_s as the ordering cost from supplier
- K_p as the set up cost for production
- h_p as the production holding cost per unit per unit time
- h_r as the raw material holding cost per unit per unit time
- C_r as the unit cost of raw material
- C_s as the unit screening cost
- S_p as the unit selling price of perfect quality items
- S_i as the discounted unit selling price of imperfect quality items
- S_d as the unit selling discounted price of imperfect quality items
- T as the inventory cycle length
- T_i as the inventory cycle length of imperfect quality items
- T_p as the inventory cycle length of perfect quality items
- t_s as the screening period
- t_{1i} as the production period for imperfect quality finished items

- t_{1p} as the production period for imperfect quality items

Consider a production model where the production rate is P . The raw material used in the production process is ordered at the beginning of the production period from a supplier. An order of size y of raw material is acquired at the beginning of the production cycle. An identified proportion q of imperfect quality items is present inside each batch of raw material received. Raw materials that are of perfect and imperfect quality are used in production. The perfect quality items of raw material screened, $y(1-q)$ units, are used to produce perfect quality finished product.

The remaining imperfect quality items, yq , yield imperfect quality finished product. The demand rates for the perfect and imperfect quality items produced are D_p and D_i , respectively.

Assume the following to develop this model:

- The demand rates, D_p and D_i , are known and constant.
- The percentage of imperfect quality items of raw material q is known and constant.
- The production rates for perfect and imperfect quality finished items are assumed to be larger than their corresponding demand rates.
- Raw material is ordered and received instantaneously at the beginning of the production period.
- One item of raw material is needed to produce one item of the finished product.

The order of size y of raw material contains qy imperfect quality items. To produce the finished product of imperfect quality, these items are processed at a rate P . Therefore, the production period length for the imperfect quality finished items is:

$$t_{1i} = \frac{qy}{P}. \quad (4.1)$$

Similarly, the $(1-q)y$ perfect quality items of raw material are also processed at the rate P so that the length of the production period for perfect quality finished items is:

$$t_{1p} = \frac{(1-q)y}{P}. \quad (4.2)$$

The finished items produced are used to meet the demand. Since the demand rates are D_p and D_i , for perfect and imperfect quality finished items, respectively, we have that the length of the inventory cycle for the imperfect quality finished items is:

$$T_i = \frac{qy}{D_i}, \quad (4.3)$$

and that of perfect quality is:

$$T_p = \frac{(1-q)y}{D_p}. \quad (4.4)$$

The larger value among T_i and T_p determines the combined inventory cycle length T for both types of finished items. As the production period approaches the end, inventory decreases at a rate of D until items of either type are out of stock. That is, $T = \max\{T_i, T_p\}$. Two cases are considered: $T_i \leq T_p$, and $T_i > T_p$. In the first case,

using (4.3) and (4.4), we have that the condition $T_i \leq T_p$ is equivalent to $\frac{D_i}{D_p} \geq \frac{q}{1-q}$.

Similarly, the condition for the second case is $\frac{D_p}{D_i} \geq \frac{1-q}{q}$.

Suppose that $T_i < T_P$. Then, $T = T_P$ and the levels of inventory for raw materials and the finished product are revealed in figures 4.1, 4.2, 4.3 and 4.4. At the beginning of the production cycle, a 100% screening process to identify the imperfect quality items is conducted at a screening rate x , where $x > P$. Throughout the screening period, the perfect and imperfect quality items of raw material are used in the production process. The behaviour of the inventory level of perfect quality items of raw material is shown in figure 4.1.

At the beginning of the inventory period, this level starts at $(1-q)y$ and is depleted at a rate P until the end of this production period where it reaches zero. See figure 4.1. Similarly, the inventory level of imperfect quality items of raw material starts at qy and decreases at a rate P , see figure 4.2.

During the production period, finished items of perfect quality are produced at a rate P . Part of these items is sold at a rate D_P . Therefore, an inventory of perfect quality finished items is accumulating throughout the production period at a rate $P - D_P$ until a maximum level of y_{pmax} is reached, see figure 4.3. From (4.2), this maximum level is:

$$y_{pmax} = (1-q) \left(1 - \frac{D_P}{P} \right) y. \quad (4.5)$$

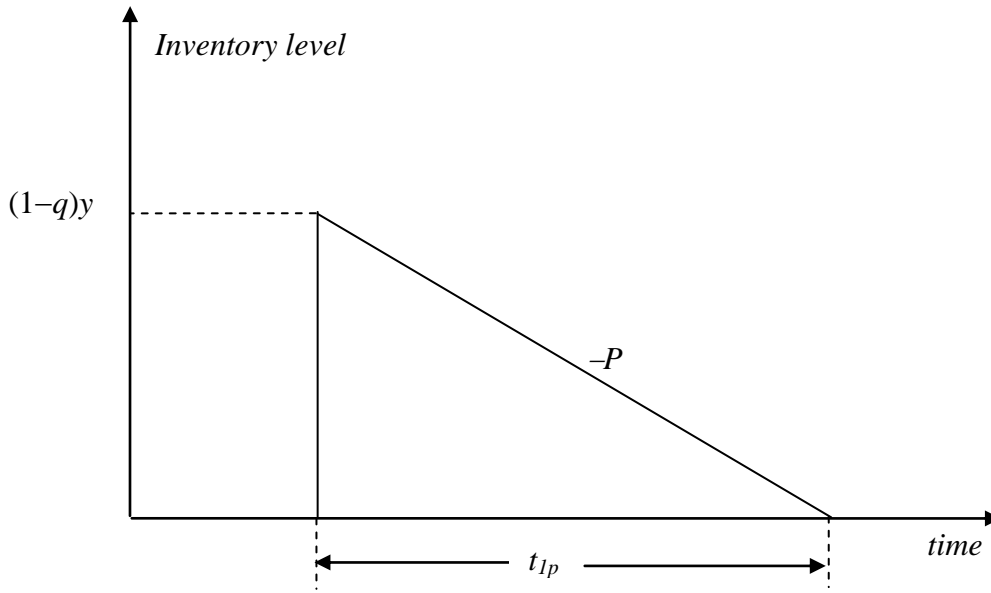


Figure 4.1: *Perfect items of raw material*

A similar argument gives that the maximum level of imperfect finished items is:

$$y_{i \max} = qy \left(1 - \frac{D_i}{P} \right). \quad (4.6)$$

To find the optimal order quantity, we start by calculating the total cost per inventory cycle, which is the sum of purchasing cost, production cost, ordering cost of perfect and imperfect raw material, setup cost of production, raw material holding cost and finished product holding cost. Except for the holding costs, the components are as follows:

$$\text{Purchasing and screening cost of raw material} = (C_r + C_s)y, \quad (4.7)$$

$$\text{Production cost} = C_p y, \quad (4.8)$$

$$\text{Ordering cost of raw material} = K_r \quad (4.9)$$

$$\text{Setup cost} = K_p. \quad (4.10)$$

The holding cost of perfect quality items of raw materials is the product of the average inventory, cycle length, and the holding cost per unit per unit time h_r . The area under the curve representing the inventory level of perfect quality raw material, see figure 4.1, is given by

$$\text{Area} = \frac{(1-q)y t_{1p}}{2}. \quad (4.11)$$

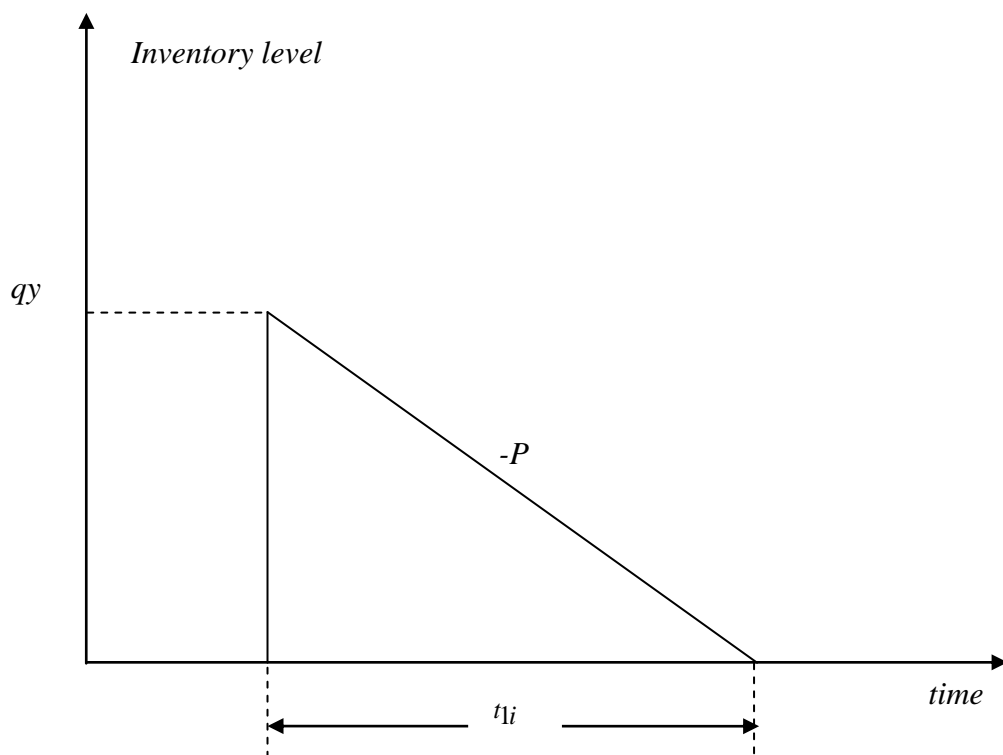


Figure 4.2: Imperfect items of raw materials

From (4.2), the area in (4.11) becomes:

$$\text{Area} = \frac{(1-q)^2 y^2}{2P}. \quad (4.12)$$

Using (4.12), the holding cost of perfect raw material becomes:

$$\text{Holding cost of perfect raw material} = \frac{(1-q)^2 y^2}{2P} h_r. \quad (4.13)$$

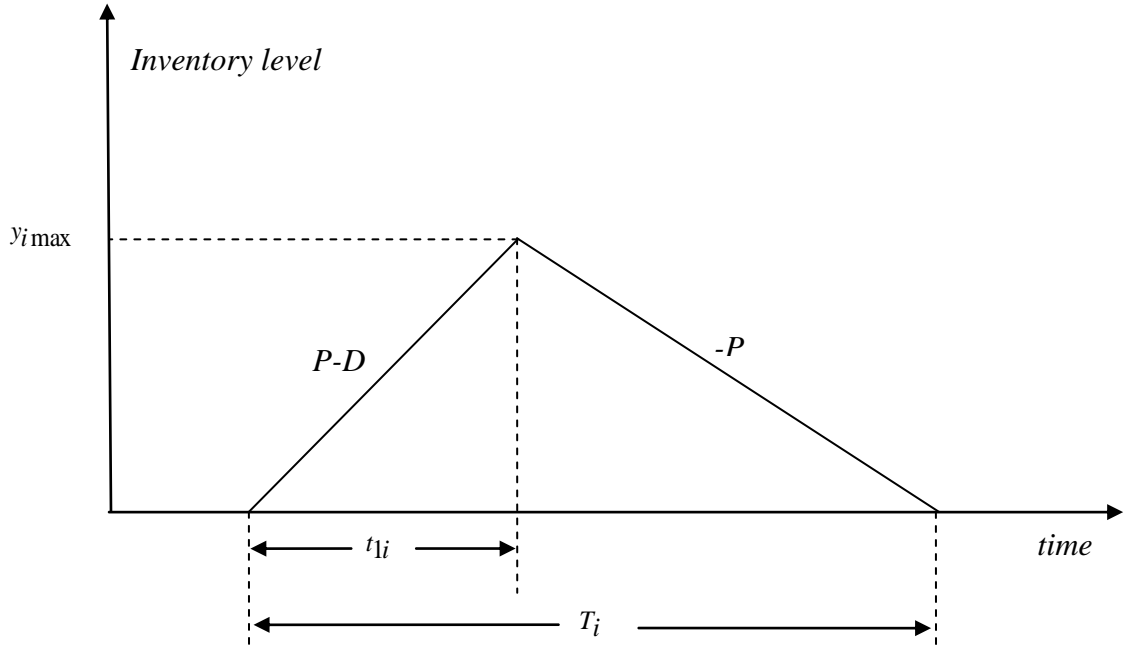


Figure 4.3: Finished Imperfect Product Inventory Level

Similarly, the holding cost of imperfect raw material, see figure 4. 2, is:

$$\text{Holding cost of imperfect raw material} = \frac{q^2 y^2}{2P} h_r. \quad (4.14)$$

The holding cost for perfect quality finished items is the product of the average inventory, cycle length and the sum of the two holding cost per unit per unit time, h_r and h_p . Since the area under the curve representing the inventory level of perfect finished items is $(T_p y_{p \max})/2$, we have:

$$\text{Average inventory} = \frac{y_{p \max}}{2} T_p \frac{1}{T} = \frac{y_{p \max}}{2}. \quad (4.15)$$

We obtain holding cost for perfect finished items:

$$\frac{y^2(1-q)^2}{2D_p} \left(1 - \frac{D_p}{P}\right) (h_p + h_r). \quad (4.16)$$

Similarly, by using (4.3) and (4.6), we obtain holding cost for imperfect finished items:

$$\frac{q^2 y^2}{2D_i} \left(1 - \frac{D_i}{P}\right) (h_p + h_r). \quad (4.17)$$

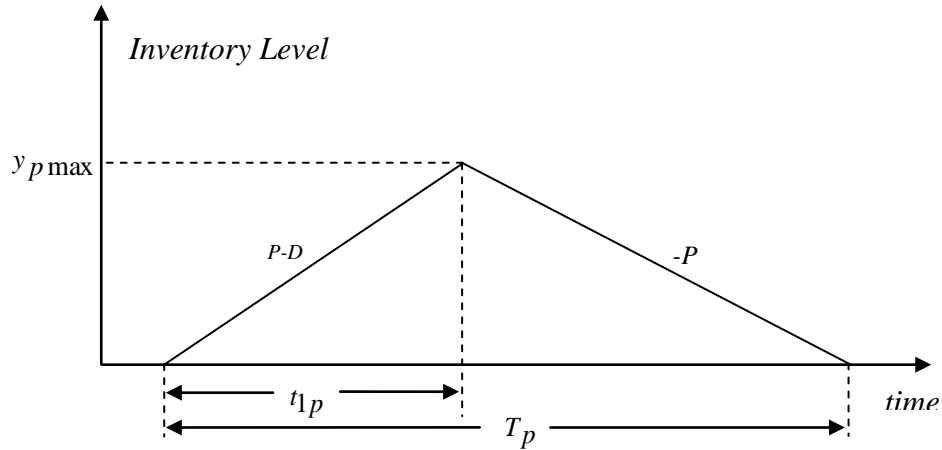


Figure 4. 4: Perfect Quality Finished Product

Now, the total inventory cost per cycle function $TC(y)$ is:

$$\begin{aligned} TC(y) = & (C_r + C_s)y + C_p y + K_r + K_p + \frac{y^2}{2P} (q^2 + (1-q)^2) h_r \\ & + \frac{q^2 y^2}{2D_i} \left(1 - \frac{D_i}{P}\right) (h_p + h_r) + \frac{(1-q)^2 y^2}{2D_p} \left(1 - \frac{D_p}{P}\right) (h_p + h_r). \end{aligned} \quad (4.18)$$

The total inventory cost per unit time function $TCU(y)$ is obtained by dividing (4.18)

by the inventory cycle length $T = T_p = \frac{(1-q)y}{D_p}$. Hence:

$$TCU(y) = \frac{(C_r + C_s + C_p)D_p}{1-q} + \frac{(K_r + K_p)D_p}{(1-q)} \frac{1}{y} + \frac{q^2 + (1-q)^2}{2(1-q)P} D_p h_r y$$

$$+ \left[\frac{q^2}{2(1-q)} \left(1 - \frac{D_i}{P}\right) \frac{D_p}{D_i} + \frac{(1-q)}{2} \left(1 - \frac{D_p}{P}\right) \right] (h_p + h_r) y. \quad (4.19)$$

Now, the total revenue function $TR(y)$ is the sum of the sales revenue of the finished product and the discounted sales of the imperfect quality items. That is:

$$TR(y) = S_p(1-q)y + S_i q y. \quad (4.20)$$

Dividing (4.20) by T , we have that the total revenue per unit time is:

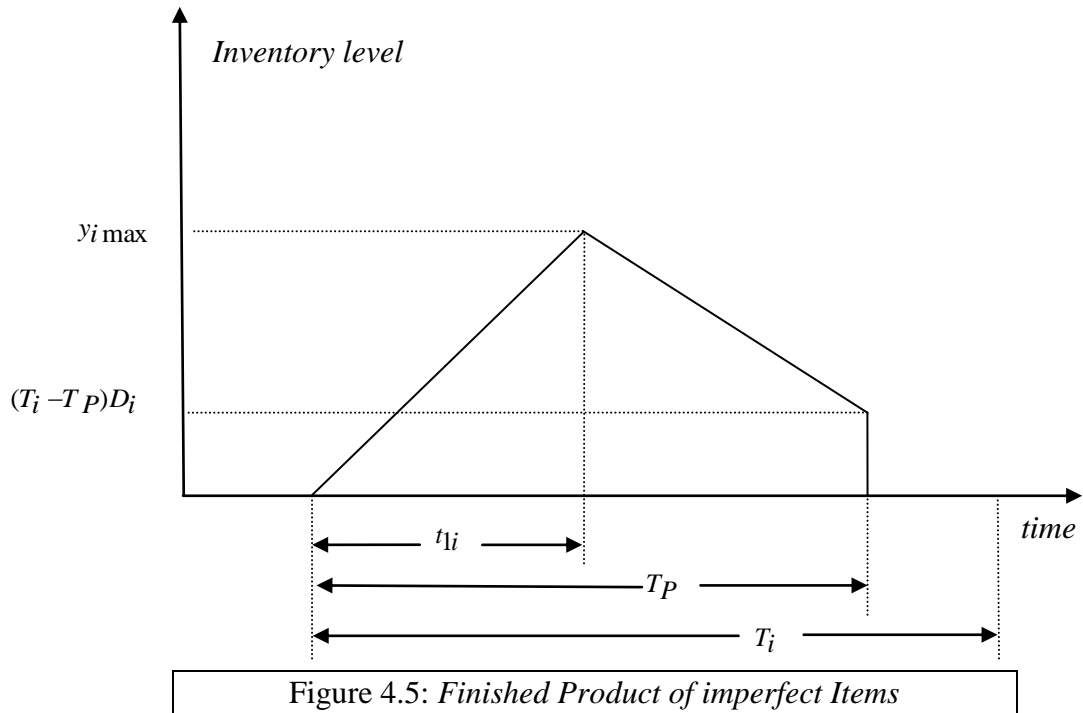
$$TRU(y) = S_p D_p + S_i D_p \frac{q}{1-q}. \quad (4.21)$$

The optimal order quantity is:

$$y^* = \sqrt{\frac{2P(K_r + K_p)D_p}{\left(q^2 + (1-q)^2\right)D_p h_r + (h_p + h_r) \left(q^2(P - D_i) \frac{D_p}{D_i} + (1-q)^2(P - D_p)\right)}}.$$

4.2 Cycle Length of Perfect Quality Finished Items is Greater

Now we develop the mathematical model for the case when $T_i > T_p$ so that $T = \max(T_i, T_p) = T_i$. It is assumed here that all the imperfect quality finished items remaining will be sold in one batch at a low cut-price price at time T_p . figure 4.5 shows the behaviour of the inventory level of raw material:



To find the optimal order quantity, we calculate the total cost per cycle which is identical to that of the previous case except for the holding cost of finished product of imperfect items. The area under the curve representing the inventory level in figure 4.5 is:

$$\text{Area} = \frac{-q^2 y^2}{2P} + \frac{q(1-q)y^2}{D_p} - \frac{(1-q)^2 y^2}{2(D_p)^2} D_i. \quad (4.26)$$

Dividing (4.26) by T , we get the average inventory. Then multiplying the average inventory by $(h_r + h_p)$ and T , we obtain the holding cost for finished imperfect items as:

$$\left\{ -\frac{q^2}{P} + \frac{2q(1-q)}{D_p} - \frac{(1-q)^2 D_i}{(D_p)^2} \right\} \frac{y^2}{2} (h_r + h_p). \quad (4.27)$$

Replacing the holding cost for finished imperfect items term in (4.17) by (4.27), we obtain:

$$\begin{aligned} TC(y) = & (C_r + C_s + C_p)y + K_r + K_p + \frac{y^2}{2P} (q^2 + (1-q)^2) h_r \\ & + \left[\frac{(1-q)^2}{D_p} \left(1 - \frac{D_p}{P}\right) - \frac{q^2}{P} + \frac{2q(1-q)}{D_p} - \frac{(1-q)^2 D_i}{(D_p)^2} \right] \frac{y^2 (h_p + h_r)}{2}. \end{aligned} \quad (4.28)$$

We obtain the $TCU(y)$ function for the second case as:

$$\begin{aligned} TCU(y) = & (C_r + C_s + C_p) \frac{D_i}{q} + \frac{(K_r + K_p) D_i}{q} \frac{1}{y} + \frac{D_i (q^2 + (1-q)^2) h_r}{2Pq} y \\ & + \left[\frac{(1-q)^2}{qD_p} \left(1 - \frac{D_p}{P}\right) - \frac{q}{P} + \frac{2(1-q)}{D_p} - \frac{(1-q)^2 D_i}{q(D_p)^2} \right] \frac{D_i (h_p + h_r)}{2} y. \end{aligned} \quad (4.29)$$

Now, the total revenue function $TR(y)$ is:

$$TR(y) = S_p(1-q)y + S_i(yq - (T_i - TP)D_i) + S_d(T_i - TP)D_i. \quad (4.30)$$

Dividing (4.30) by $T = T_i = \frac{qy}{D_i}$, we have that the total revenue per unit time is:

$$TRU(y) = \frac{S_p(1-q)}{q} D_i + (S_d - S_i) \left(1 - \frac{(1-q)^2 D_i^2}{qD_p} \right). \quad (4.31)$$

Maximizing the total profit per unit time function in a manner similar to that of the first case, we get the economic order quantity as:

$$y^* = \sqrt{\frac{2P(K_r + K_p)}{\left((q^2 + (1-q)^2) h_r + (h_p + h_r) \left[-q^2 + \frac{2Pq(1-q)}{D_p} + \frac{(1-q)^2}{D_p} \left(P - D_p - \frac{PD_i}{D_p} \right) \right] \right)}}. \quad (4.32)$$

Chapter Five

Evaluation of the EPQ Models with Imperfect Quality Items of Raw Material

The purpose of this chapter is to evaluate the EPQ models presented in chapters three and four. Spread sheet applications using Microsoft Excel and computer programs using Mathematica are developed to conduct the evaluation. The models are evaluated by analysing the output.

5.1 Spread Sheet Applications

For each model developed in chapters three and four, spread sheet applications using Microsoft Excel were developed. Given a set of values for the parameters of a model, the output is the optimal ordering policy. In the following we demonstrate the application for the model of section 3.2.

	A
1	
2	Input
3	
4	Proportion of Imperfect Quality Items of
5	Demand Rate (Units/days)
6	Production Rate (Units/days)
7	Screening Rate (Units/Day)
8	
9	Ordering Cost
10	Setup Cost
11	Unit Cost of Raw Material
12	Production Cost per Unit
13	Screening Cost Per Unit
14	Raw Material Holding Cost
15	Finished Product Holding Cost/Unit/Day
16	

The output is as follows.

		Model 1		EPQ		EPQ Raw Material	
	Output (Optimal)	Per Cycle	Per Day	Per Cycle	Per Day	Per Cycle	Per Day
3	Order Quantity of Raw Material	500		499		380	
4	Number of items produced	350		349		266	
5	Length of the inventory cycle	70		70		53	
6	Length of the production period	35		35		27	
7	Length of the screening period	25		25		19	
8	Maximum inventory	175		175		133	
9	Ordering cost	\$100.00	\$1.43	\$100.00	\$1.43	\$100.00	\$1.43
10	Setup cost	\$183.00	\$2.61	\$183.00	\$2.62	\$183.00	\$2.62
11	Purchasing cost of raw material	\$2,502.21	\$35.71	\$2,494.89	\$35.71	\$1,899.92	\$27.20
12	Production cost	\$3,503.10	\$50.00	\$3,492.85	\$50.00	\$2,659.89	\$38.08
13	Screening Cost	\$250.22	\$3.57	\$249.49	\$3.56	\$189.99	\$2.71
14	Raw material holding cost	\$98.92	\$1.41	\$98.35	\$1.41	\$57.03	\$0.82
15	Holding cost for items produced	\$122.72	\$1.75	\$122.00	\$1.75	\$70.75	\$1.01
16	Total cost	\$6,760.17	\$96.49	\$6,740.58	\$96.49	\$5,160.58	\$73.87
17	Total Sales	\$18,074.47	\$257.98	\$18,021.61	\$257.98	\$13,723.88	\$196.46
18	Total profit	\$11,314.30	\$161.49	\$11,281.03	\$161.49	\$8,563.30	\$122.58

Note that the results of the model of section 3.2 are compared with those of the classical EPQ model and the EPQ with raw material.

The following illustrate the calculation of the optimal order quantity.

Output (Optimal)				
Order Quantity of Raw Material	=SQRT((2*\$B\$6*\$B\$5*((\$B\$9+\$B\$10))/(((\$B\$15+\$B\$14)*(\$B\$6-\$B\$5)*(1-\$B\$4)^2+\$B\$14*\$B\$5*((1-\$B\$4)^2+2*\$B\$6*\$B\$4/\$B\$7))))			
Number of items produced				
Length of the inventory cycle				
Length of the production period	SQRT(number) 35		35	27
Length of the screening period			25	19
Maximum inventory		175	175	133

Production cost	\$3,503.10	\$50.00	\$3,492
Screening Cost	\$250.22	\$3.57	\$249
Raw material holding cost	\$98.92	\$1.41	\$98
Holding cost for items produced	\$122.72	\$1.75	\$122
Total cost	\$6,760.17	\$96.49	\$6,740
Total Sales		\$257.98	\$18,021
Total profit	=E17-E16	\$161.49	\$11,281

The following illustrate how the above application can be used to evaluate the model by varying one of the parameters.

24									
25	Proportion of Imperfect Quality Items(q)	EPQ Model1	TPU	EPQ Classic	TPU	EPQ Raw Maerial	TPU		
26	0.05	393	\$166.75	368	\$166.67	280	\$126.44		
27	0.1	412	\$165.95	388	\$165.88	296	\$125.85		
28	0.15	431	\$165.04	411	\$164.99	313	\$125.15		
29	0.2	453	\$164.02	437	\$163.98	332	\$124.43		
30	0.25	476	\$162.85	466	\$162.82	355	\$123.57		
31	0.3	500	\$161.49	499	\$161.49	380	\$122.58		
32	0.4	555	\$158.04	582	\$158.08	443	\$120.06		
33	0.5	614	\$153.10	699	\$153.13	532	\$116.44		
34									
35									

Demand Rate (Units/days)	Prod. Rate, & Screening Rate	EPQ Model1	TPU	EPQ Classic	TPU	EPQ Raw Maerial	TPU		
5	10 & 20	500	\$161.49	499	\$161.49	380	\$122.58		
7	10 & 20	631	\$227.87	762	\$227.85	474	\$141.35		
10	15 & 30	746	\$327.42	864	\$327.44	561	\$212.21		
20	30 & 60	1055	\$660.68	1222	\$660.71	794	\$428.46		
40	50 & 100	1564	\$1,329.95	2231	\$1,329.10	1166	\$694.15		
50	100 & 200	1583	\$1,664.15	1578	\$1,664.14	1202	\$1,266.04		
100	500 & 1000	2060	\$3,340.37	1764	\$3,339.07	1585	\$2,998.05		

5.2 Evaluation and Comparison

In the following e illustrate how the models can be evaluated by changing one of the parameters of the model. We start by analyzing the model of section 3.2 (refer to as Mod1). The optimal solution and optimal TCU values of the model are compared with those of the classical model of section 2.2 and the model of section 2.3 with perfect quality raw material.

Varying the proportion of imperfect quality items of raw material between 0 and 0.5, we obtain the following results.

q	y*			TCU		
	Mod1	Classical	Raw Material	Mod1	Classical	Raw Material
0	376	428	376	\$164.98	\$164.91	\$164.98
0.05	393	428	376	\$164.27	\$164.24	\$164.26
0.1	412	428	376	\$163.47	\$163.47	\$163.44
0.15	431	428	376	\$162.58	\$162.58	\$162.51
0.2	453	428	376	\$161.56	\$161.55	\$161.43
0.25	476	428	376	\$160.40	\$160.36	\$160.18
0.3	500	428	376	\$159.06	\$158.96	\$158.73
0.35	527	428	376	\$157.50	\$157.32	\$157.03
0.4	555	428	376	\$155.66	\$155.37	\$155.01
0.45	584	428	376	\$153.46	\$153.03	\$152.60
0.5	614	428	376	\$150.79	\$150.18	\$149.66

Examining the results in the above table, we observe the following:

- In the absence of imperfect quality items of raw material, i.e., when $q = 0$, the model of section 3.2 and the perfect quality raw material model are identical. Otherwise, the model of section 3.2 must be used since it results in the highest TCU value.
- As the percentage of imperfect quality items of raw material increases, the optimal order quantity increases when the model of section 3.2 is used, while it remains constant if the two other models are used.
- As the percentage of imperfect quality items becomes higher, the difference between the optimal TCU value of model of section 3.2 and the two other models increase.

The large variation in the optimal order quantity shows that the model is sensitive to variations in the percentage of imperfect quality items of raw material. In real life

situations, this percentage is usually unknown and can be considered as a random variable with a certain probability distribution. Hence, this demonstrates the importance of extending the model to a probabilistic one. This will be done in the next chapter.

Next we consider the effects of changes in the holding cost of raw material.

h_r	y^*			TCU		
	Mod1	Classical	Raw Material	Mod1	Classical	Raw Material
0	760	428	532	\$161.82	\$160.92	\$161.48
0.01	500	428	376	\$159.06	\$158.96	\$158.73
0.02	400	428	307	\$157.03	\$157.01	\$156.68
0.03	343	428	266	\$155.34	\$155.05	\$154.96
0.04	305	428	238	\$153.87	\$153.10	\$153.46
0.05	277	428	217	\$152.54	\$151.14	\$152.11
0.06	256	428	201	\$151.33	\$149.19	\$150.87
0.07	239	428	188	\$150.20	\$147.23	\$149.72
0.08	225	428	177	\$149.14	\$145.28	\$148.64
0.09	213	428	168	\$148.14	\$143.32	\$147.62
0.1	203	428	160	\$147.19	\$141.36	\$146.65
0.11	194	428	154	\$146.29	\$139.41	\$145.72
0.12	186	428	148	\$145.42	\$137.45	\$144.83
0.13	179	428	142	\$144.59	\$135.50	\$143.98
0.14	173	428	137	\$143.78	\$133.54	\$143.15
0.15	167	428	133	\$143.00	\$131.59	\$142.36
0.16	162	428	129	\$142.25	\$129.63	\$141.59
0.17	158	428	125	\$141.52	\$127.68	\$140.84
0.18	154	428	122	\$140.81	\$125.72	\$140.11
0.19	150	428	119	\$140.11	\$123.76	\$139.40
0.2	146	428	116	\$139.44	\$121.81	\$138.71
0.21	143	428	113	\$138.78	\$119.85	\$138.03
0.22	139	428	111	\$138.13	\$117.90	\$137.38
0.23	136	428	109	\$137.50	\$115.94	\$136.73
0.24	134	428	106	\$136.89	\$113.99	\$136.10
0.25	131	428	104	\$136.28	\$112.03	\$135.48
0.26	129	428	102	\$135.69	\$110.07	\$134.87
0.27	126	428	101	\$135.11	\$108.12	\$134.28
0.28	124	428	99	\$134.54	\$106.16	\$133.69
0.29	122	428	97	\$133.97	\$104.21	\$133.11
0.3	120	428	96	\$133.42	\$102.25	\$132.55

The above results show that the classical model is inappropriate in the presence of imperfect quality items of raw material. The model of section 3.2 is sensitive to the holding cost and always yields the best results. The optimal order quantity for this model is always larger.

Next we consider variations in the holding cost due to production.

hp	y*			TCU		
	Mod1	Classical	Raw Material	Mod1	Classical	Raw Material
0.01	565	605	434	\$159.99	\$159.98	\$159.74
0.02	500	428	376	\$159.06	\$158.96	\$158.73
0.03	454	349	336	\$158.23	\$157.93	\$157.83
0.04	418	302	307	\$157.47	\$156.96	\$157.01
0.05	390	271	284	\$156.76	\$156.07	\$156.25
0.06	366	247	266	\$156.10	\$155.24	\$155.53
0.07	347	229	251	\$155.48	\$154.46	\$154.86
0.08	330	214	238	\$154.89	\$153.72	\$154.23
0.09	315	202	227	\$154.32	\$153.02	\$153.62
0.1	303	191	217	\$153.78	\$152.35	\$153.04
0.11	291	182	209	\$153.26	\$151.72	\$152.48
0.12	281	175	201	\$152.76	\$151.10	\$151.95
0.13	272	168	194	\$152.28	\$150.51	\$151.43
0.14	264	162	188	\$151.81	\$149.94	\$150.93
0.15	256	156	182	\$151.36	\$149.39	\$150.44
0.16	249	151	177	\$150.91	\$148.85	\$149.97
0.17	243	147	173	\$150.48	\$148.33	\$149.51
0.18	237	143	168	\$150.06	\$147.82	\$149.06
0.19	231	139	164	\$149.66	\$147.33	\$148.62
0.2	226	135	160	\$149.26	\$146.85	\$148.19
0.21	221	132	157	\$148.86	\$146.38	\$147.77
0.22	217	129	154	\$148.48	\$145.92	\$147.37
0.23	212	126	150	\$148.11	\$145.46	\$146.96
0.24	208	123	148	\$147.74	\$145.02	\$146.57
0.25	205	121	145	\$147.38	\$144.59	\$146.19
0.26	201	119	142	\$147.02	\$144.16	\$145.81
0.27	198	116	140	\$146.67	\$143.75	\$145.43
0.28	194	114	137	\$146.33	\$143.34	\$145.07
0.29	191	112	135	\$145.99	\$142.93	\$144.71
0.3	188	110	133	\$145.66	\$142.54	\$144.35

All three models reveal similar behaviour in the presence of imperfect quality items of raw material. All three models are less sensitive to this holding cost. The model of section 3.2 yields the best results and its optimal order quantity for this model is always larger.

The following table shows the effects of changes in the production quantity.

P	y*			TCU		
	Mod1	Classical	Raw Material	Mod1	Classical	Raw Material
6	594	741	461	\$160.33	\$160.16	\$160.11
7	555	566	424	\$159.85	\$159.85	\$159.59
8	530	494	402	\$159.51	\$159.50	\$159.22
9	513	454	387	\$159.26	\$159.20	\$158.95
10	500	428	376	\$159.06	\$158.96	\$158.73
11	491	410	368	\$158.91	\$158.77	\$158.56
12	483	396	361	\$158.78	\$158.61	\$158.42
13	477	386	356	\$158.67	\$158.48	\$158.30
14	472	377	352	\$158.58	\$158.36	\$158.21
15	468	370	348	\$158.50	\$158.26	\$158.12
16	464	365	345	\$158.43	\$158.18	\$158.05
17	461	360	343	\$158.37	\$158.10	\$157.98
18	458	356	340	\$158.32	\$158.04	\$157.93
19	456	352	338	\$158.27	\$157.98	\$157.88
20	454	349	336	\$158.23	\$157.93	\$157.83

The results show that the model of section 3.2 produces different optimal order quantity as the production rate increases and should be used in this case. On the other hand, when the production rate is smaller and close to the demand rate, the classical yields a large optimal order quantity.

Next, the effect of the screening rate is analysed. The table below shows the different result as we increase the screening rate. The classical model and the model assuming perfect quality raw material do not reflect the changes in the screening rate. Once

again, the model of section 3 should especially when the screening rate becomes larger.

x	y*			TCU		
	Mod1	Classical	Raw Material	Mod1	Classical	Raw Material
10	470	428	376	\$158.54	\$158.51	\$158.33
11	475	428	376	\$158.64	\$158.59	\$158.40
12	480	428	376	\$158.71	\$158.66	\$158.46
13	483	428	376	\$158.78	\$158.72	\$158.52
14	487	428	376	\$158.84	\$158.77	\$158.56
15	490	428	376	\$158.89	\$158.81	\$158.60
16	492	428	376	\$158.93	\$158.85	\$158.63
17	495	428	376	\$158.97	\$158.88	\$158.66
18	497	428	376	\$159.00	\$158.91	\$158.69
19	499	428	376	\$159.04	\$158.94	\$158.71
20	500	428	376	\$159.06	\$158.96	\$158.73
21	502	428	376	\$159.09	\$158.99	\$158.75
22	503	428	376	\$159.11	\$159.01	\$158.77
23	505	428	376	\$159.13	\$159.02	\$158.79
24	506	428	376	\$159.15	\$159.04	\$158.80
25	507	428	376	\$159.17	\$159.06	\$158.81
26	508	428	376	\$159.19	\$159.07	\$158.83
27	509	428	376	\$159.20	\$159.08	\$158.84
28	510	428	376	\$159.22	\$159.10	\$158.85
29	511	428	376	\$159.23	\$159.11	\$158.86
30	512	428	376	\$159.25	\$159.12	\$158.87

Chapter Six

Probabilistic EPQ Model with Imperfect Quality

Items of Raw Material

In this chapter, we extend the models discussed in chapter three to the case where the percentage of defective items q as a random variable having a known probability density function $f(q)$. The mathematical models for the two scenarios presented in sections 3.2 and 3.3 are modified and expressions for the optimal solutions in terms of the expected value and the standard deviation of q are obtained. Numerical examples illustrating the new models are given. The results of this chapter are accepted to be presented at the IABE-2012 Las Vegas- Annual Conference and the submitted paper is accepted to be published in the *Journal of Academy of Business and Economics*, Vol. 12, No. 3 (Mikdashi, El-Kassar, and Joudi, 2012).

5.1 Probabilistic EOQ Models

One of the disadvantages of the previous discussed models is the assumption that the percentage quantity of imperfect quality items of raw materials is known. A more realistic approach is to consider the percentage of defective items q as a random variable having a known probability density function $f(q)$. In their work in the year 2000, Salameh and Jaber extended the EOQ model to the case where items received from the supplier contain imperfect quality items. This extension of the classical EOQ model assumed the following:

- The demand is deterministic
- The replenishment is immediate
- The percentage of imperfect quality items is probabilistic
- Imperfect quality items are isolated from perfect ones by a 100% screening
- Items of imperfect quality are sold at a discount as a single batch.

To illustrate this inventory model, let y be the lot size, D the demand rate, K the ordering cost, C the unit purchasing cost, q the percentage of imperfect quality items, C_s the unit screening cost, t_s the screening time, S the unit selling price, S_r the discounted unit selling price for imperfect quality items, and $T = \frac{(1-q)y}{D}$ is the cycle time.

The total cost in an inventory cycle is

$$TC(y) = K + (C + C_s)y + h \left(\frac{y(1-q)T}{2} + \frac{qy^2}{x} \right). \quad (5.1)$$

The total profit per cycle is

$$TP(y) = Syq + S_r yq - \left[K + (C + C_s)y + h \left(\frac{y(1-q)T}{2} + \frac{qy^2}{x} \right) \right]. \quad (5.2)$$

The expected per unit time profit function is

$$E(TPU(y)) = E \left[\frac{TP(y)}{T} \right] = D \left(S - S_r + \frac{hy}{x} \right) + D \left(S_r - \frac{hy}{x} - C - C_s - \frac{K}{y} \right) E \left(\frac{1}{1-q} \right) - \frac{hy(1-E(q))}{2}. \quad (5.3)$$

The optimal order quantity y^* that maximizes the equation (5.3) is

$$y^* = \sqrt{\frac{2KDE\left(\frac{1}{1-q}\right)}{h(1-E(q)) - \frac{2D\left(1-E\left(\frac{1}{1-q}\right)\right)}{x}}}. \quad (5.4)$$

This work triggered many extensions to both EOQ and EPQ models to include probabilistic imperfect quality items. A survey of such articles is found in Khan et al. (2011). However, all previous extensions of the EPQ model considered imperfect quality finished items. The purposed of this chapter is to extend the models of the first scenarios of chapter three to include discussed earlier where the imperfect quality items of raw material are of probabilistic nature.

5.2 Probabilistic EPQ Models with Imperfect Quality Raw Material

The following notations were used:

- D as the demand per unit time
- P as the production rate
- x as the screening rate for imperfect quality items
- C_r as the cost of raw materials
- C_p as the unit production cost
- C_s as the screening cost per unit
- K_s as the fixed cost of placing an order of raw material from suppliers
- K_p as the setup cost of a production run as the ordering quantity

- y as the raw material order size
- T as the length of inventory cycle in unit time
- h_r as the cost of holding one unit of raw material per unit time
- h_p as the holding cost due to production per unit per unit time
- S as the selling price of one unit of finished product
- S_r as the discounted selling price of one unit of imperfect quality item
- q as the proportion of raw material items that are of imperfect quality
- $1 - q$ as the proportion of raw material items that are of perfect quality
- μ as the expected proportion of imperfect quality items of raw material
- σ as the standard deviation of q

Consider the first model where the imperfect quality items of raw material of raw material are sold at a discounted price at the end of the screening period. From see equation (3.13), the total profit function is

$$\begin{aligned}
 TP(y) = & Sy(1-q) + S_r qy - (C_r + C_s)y - C_p y(1-q) - (K_s + K_p) - \\
 & - y^2 \left[\frac{(1-q)^2}{2P} + \frac{q}{x} \right] h_r - \frac{y^2}{2D} (1-q)^2 \left(1 - \frac{D}{p}\right) (h_p + h_r).
 \end{aligned} \tag{5.5}$$

The expected total profit per inventory cycle is

$$\begin{aligned}
 E(TP(y)) = & SyE(1-q) + S_r E(q)y - (C_r + C_s)y - C_p yE(1-q) \\
 & - (K_s + K_p) - y^2 \left[\frac{E((1-q)^2)}{2P} + \frac{E(q)}{x} \right] h_r \\
 & - \frac{y^2}{2D} E((1-q)^2) \left(1 - \frac{D}{p}\right) (h_p + h_r).
 \end{aligned} \tag{5.6}$$

Using the following facts regarding any probability density function $f(q)$ and its expectations

$$\int_{-\infty}^{\infty} f(q) dq = 1,$$

$$\int_{-\infty}^{\infty} qf(q) dq = E(q),$$

$$\int_{-\infty}^{\infty} q^2 f(q) dq = \mu^2 + \sigma^2,$$

equation (5.6) becomes

$$\begin{aligned} E(TP(y)) &= Sy(1-\mu) + S_r \mu y - (C_r + C_s)y - C_p y(1-\mu) \\ &\quad - (K_s + K_p) - y^2 \left[\frac{(1-\mu)^2 + \sigma^2}{2P} + \frac{\mu}{x} \right] h_r \\ &\quad - \frac{y^2}{2D} ((1-\mu)^2 + \sigma^2) \left(1 - \frac{D}{p}\right) (h_p + h_r). \end{aligned} \tag{5.7}$$

The renewal reward theorem can be used to approximated the expected total profit per unit time function can be by

$$\begin{aligned} E(TPU(y)) &= \frac{E(TP(y))}{E(T)} \\ &= \frac{D}{y(1-\mu)} \left\{ Sy(1-\mu) + S_r \mu y - (C_r + C_s)y - C_p y(1-\mu) \right. \\ &\quad - (K_s + K_p) - y^2 \left[\frac{(1-\mu)^2 + \sigma^2}{2P} + \frac{\mu}{x} \right] h_r \\ &\quad \left. - \frac{y^2}{2D} ((1-\mu)^2 + \sigma^2) \left(1 - \frac{D}{p}\right) (h_p + h_r) \right\}. \end{aligned} \tag{5.8}$$

Simplifying equation (5.8), we have

$$\begin{aligned}
E(TPU(y)) = & DS + \frac{DS_r\mu}{1-\mu} - \frac{D(C_r + C_s)}{1-\mu} - DC_p \\
& - \frac{D(K_s + K_p)}{y(1-\mu)} - \frac{D}{1-\mu} y \left[\frac{(1-\mu)^2 + \sigma^2}{2P} + \frac{\mu}{x} \right] h_r \\
& - \frac{y}{2(1-\mu)} ((1-\mu)^2 + \sigma^2) \left(1 - \frac{D}{p}\right) (h_p + h_r).
\end{aligned} \tag{5.9}$$

Differentiating the expression of (5.9) with respect to y we get

$$\begin{aligned}
\frac{d}{dy} E(TPU(y)) = & \frac{D(K_s + K_p)}{y^2(1-\mu)} - \frac{D}{1-\mu} \left[\frac{(1-\mu)^2 + \sigma^2}{2P} + \frac{\mu}{x} \right] h_r \\
& - \frac{1}{2(1-\mu)} ((1-\mu)^2 + \sigma^2) \left(1 - \frac{D}{p}\right) (h_p + h_r).
\end{aligned} \tag{5.10}$$

Setting the derivative in (5.10) equal to zero and solving for y , we obtain the economic order quantity

$$y^* = \sqrt{\frac{2D(K_s + K_p)}{((1-\mu)^2 + \sigma^2) \left(1 - \frac{D}{p}\right) (h_p + h_r) + D \left[\frac{(1-\mu)^2 + \sigma^2}{P} + \frac{2\mu}{x} \right] h_r}}. \tag{5.11}$$

Note that when the percentage of imperfect quality items of raw material q is deterministic, we have that $\mu = q$, $\sigma = 0$, and equation (5.11) reduces to the same expression in (3.17).

Now consider the case where the imperfect quality items are returned to the supplier when the next order arrives. From (3.23), the total profit per cycle function is

$$\begin{aligned}
TP(y) = & Sy(1-q) - (C_r + C_p)y(1-q) - C_s y - (K_s + K_p) - \\
& - y^2 \left(\frac{q(1-q)}{D} + \frac{(1-q)^2}{2P} \right) h_r - \frac{y^2}{2D} (1-q)^2 \left(1 - \frac{D}{P}\right) (h_p + h_r).
\end{aligned} \tag{5.12}$$

Therefore, the expected total profit per cycle function is:

$$\begin{aligned}
E(TP(y)) = & Sy(1-\mu) - (C_r + C_p)y(1-\mu) - C_s y - (K_s + K_p) - \\
& - y^2 \left(\frac{\mu - (\sigma^2 + \mu^2)}{D} + \frac{(\sigma^2 + (1-\mu)^2)}{2P} \right) h_r \\
& - \frac{y^2}{2D} (\sigma^2 + (1-\mu)^2) \left(1 - \frac{D}{P}\right) (h_p + h_r).
\end{aligned} \tag{5.13}$$

Therefore, the expected total profit per unit time function is:

$$\begin{aligned}
E(TPU(y)) = & \frac{E(TP(y))}{E(T)} = SD - (C_r + C_p) - \frac{C_s D}{1-\mu} - \frac{(K_s + K_p)D}{(1-\mu)y} - \\
& - y \left(\frac{\mu - (\sigma^2 + \mu^2)}{1-\mu} + \frac{D(\sigma^2 + (1-\mu)^2)}{2P(1-\mu)} \right) h_r \\
& - \frac{y}{2(1-\mu)} (\sigma^2 + (1-\mu)^2) \left(1 - \frac{D}{P}\right) (h_p + h_r).
\end{aligned} \tag{5.14}$$

The economic order quantity

$$y^* = \sqrt{\frac{2(K_s + K_p)D}{(\sigma^2 + (1-\mu)^2) \left(1 - \frac{D}{P}\right) (h_p + h_r) + \left(2(\mu - (\sigma^2 + \mu^2)) + \frac{D(\sigma^2 + (1-\mu)^2)}{P} \right) h_r}}.$$

5.3 Numerical Examples

Consider a production process where the daily demand rate for an item is 5 and the production rate is 10 units per day. Out of the entire order of raw material ordered from a supplier to be used in production, the proportion of the imperfect quality items is a random variable uniformly distributed over the interval [26%, 34%]. Screening for imperfect quality items of the raw material is performed at a rate of 20 items per day and at a cost of \$0.5 per unit. The ordering cost for the raw material is \$100 and the setup cost is \$183. The holding cost of raw material is \$0.01 per unit per day while the holding cost due to production is \$0.02 per unit per day. Hence, the holding cost of one unit of the finished product is \$0.03 per day. The purchasing cost of one item of raw material is \$5 and the unit production cost is \$10. The selling price is \$25 per unit. The imperfect quality items screened may be sold at the end of screening period at a discounted price of \$3, or may be kept in stock and returned to the supplier when the next order arrives. To determine the optimal order policy, the problem's parameters are: $D = 5$, $P = 10$, $x = 20$, $K_s = 100$, $K_p = 183$, $h_p = 0.02$, $h_r = 0.01$, $C_r = 5$, $C_p = 10$, $C_s = 0.5$, $S = 25$, and $S_r = 3$.

Since q is uniformly distributed over $[a, b] = [0.26, 0.34]$, we have that

$$\mu = E(q) = \int_{-\infty}^{\infty} qf(q) dq = \int_a^b q \frac{1}{b-a} dq = \frac{a+b}{2} = 0.3,$$

$$\sigma^2 = \int_{-\infty}^{\infty} q^2 f(q) dq - \mu^2 = \int_a^b q^2 \frac{1}{b-a} dq - \left(\frac{a+b}{2}\right)^2 = \frac{(b-a)^2}{12} = 0.000533,$$

and

$$\sigma^2 + (1-\mu)^2 = 0.490533.$$

Evaluating the policy where the imperfect quality items are sold at a discounted price, the optimal order quantity is obtained from a previous equation as $y^* = 500.074 \approx 500$. The expected number of items produced during a production cycle is $y^*(1-q) = 350$ units. The expected length of inventory cycle is $T^* = \frac{y^*(1-q)}{D} = 70$ days, the production period is $t_p^* = \frac{y^*(1-q)}{P} = 35$ days, and the screening period is $t_s^* = \frac{y^*}{x} = 25$ days. The expected total profit per day is \$159.06.

Regarding the case where the imperfect quality items are returned, the optimal order quantity is $y^* = 449.6 \approx 450$. The optimal number of items produced during a production cycle is 315 units. The production period is 31.5 days and the inventory period length is 63 days. The total cost per day is \$94.71, the total revenue per day is \$134.71, and the total profit per day is \$40.00.

Chapter Seven

CONCLUSION

In this thesis several extensions of the classic economic production quantity (EPQ) model were presented. The extensions considered the case where the raw material used in the production process contain imperfect quality items. The mathematical formulation of the classical and the extended models along with explicit expressions for the optimal order quantity were presented. Numerical examples were given to illustrate how the optimal policy can be determined. The extended economic production models with imperfect quality items of raw material were evaluated and extended to the case where the percentage of imperfect quality items is a random variable having a known probability density function. The deterministic models will be evaluated by examining the effects of the parameters on the optimal solution. The probabilistic models were simulated using various density functions for the percentage of imperfect quality items to compare the theoretical and the computational solutions.

In chapter one, the classical Economic Order Quantity Model and Economic Production Quantity Model were introduced. The presence of imperfect quality items of raw materials was discussed. Several studies related to this topic were surveyed in a literature review.

The classical EOQ and EPQ models were presented in chapter two and their mathematical models along with the optimal solutions were given. A model that incorporates the effects of raw material on the EPQ model was also considered. Numerical examples were employed.

In chapter three, two EPQ models that take into account the effects of imperfect quality items of raw materials were presented. Numerical examples were provided for better demonstration.

An EPQ that uses both perfect and imperfect quality items in the production process was presented in chapter four. The finished product is of two quality types. Two cases were considered depending on which of the two types of the finished product is sold out first.

In chapter five the EPQ model presented in chapters three is evaluated. Spreadsheet applications using Microsoft Excel and computer programs using Mathematica were developed to conduct the evaluation. The model is evaluated by analysing the output.

Two new models are developed in chapter six. The models discussed in chapter three are extended to the case where the percentage of defective items q as a random variable having a known probability density function $f(q)$. The mathematical models for the two scenarios presented in sections 3.2 and 3.3 are modified and expressions for the optimal solutions in terms of the expected value and the standard deviation of q are obtained. Numerical examples illustrating the new models are given.

The analysis demonstrated that the models should be employed to compute the optimal lot size and the total inventory cost whenever a two-stage, multistage, multistage multi-cycle production processes are considered. In particular, we discussed the use of the new model which is necessary when the production process uses raw material that contains imperfect quality items.

This thesis did not take into consideration the time value of money. If the time value of money is integrated within this new model, it will become much more appealing. Moreover, credit facilities and delay in payment were also not mentioned in our study. One of the essential assumptions of the models presented is that each inventory cycle contains one production run and each production run consists of one stage. Other restrictions may be the supposition that demand rate is constant, and the items produced are non-deteriorating and of perfect quality.

For future work, we suggest adding the concepts of time value of money and credit facility for these models. In another direction, we can explore the effect of machine breakdown on the new model. We can also study variable demand, deterioration and multi-stage and multi-cycle production processes.

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