LEBANESE AMERICAN UNIVERSITY

Developing and Piloting a DGS Based Unit for Overcoming Difficulties Faced by Grade-9 Lebanese Students while Learning Locus of Points

By
Noura Moh’d Hilal Nackouzi

A thesis
Submitted in partial fulfillment of the requirements for the degree of Master of Arts in Education/Emphasis Math Education

School of Arts and Sciences
August, 2013
Thesis Proposal Form

Name of Student: _Noura Nakkouzi_ I.D.#: _200401862_

Department: _Education_

On (dd/mm/yy) _May 11, 2012_, has presented a Thesis proposal entitled:

**Developing and piloting a DGS based unit for overcoming difficulties faced by grade 9 Lebanese students while learning “Locus of Points”**

in the presence of the Committee Members and Thesis Advisor:

Advisor  
Dr. Iman Osta  
(Name and Signature)

Committee Member  
Dr. Samer Habre  
(Name and Signature)

Committee Member  
Dr. Tamer Amin  
(Name and Signature)

Proposal Approved on (dd/mm/yy): _May 11, 2012_

Comments / Remarks / Conditions to proposal approval (if any):

Minor modifications that will be taken into consideration during the thesis work

Date: May 14, 2013  
Acknowledged by  
(Dean, School of Arts and Sciences)

cc:  
Dean  
Chair  
Advisor  
Student
LEBANESE AMERICAN UNIVERSITY  
School of Arts and Sciences - Beirut Campus

Thesis Defense Result Form

Name of student: Noura Nackouzi  I.D.: 200401862

Program / Department: Education

Date of thesis defense: August 27, 2013

Thesis title: Developing and piloting a DGS based unit for overcoming difficulties faced by grade 9 Lebanese students while learning Locus of Points

Result of Thesis defense:

☐ Thesis was successfully defended. Passing grade is granted

☒ Thesis is approved pending corrections. Passing grade to be granted upon review and approval by thesis Advisor

☐ Thesis is not approved. Grade NP is recorded

Committee Members:

Advisor: Dr. Iman Osta
(Name and Signature)

Committee Member: Dr. Samer Habre
(Name and Signature)

Committee Member: Dr. Tamer Amin
(Name and Signature)

Advisor’s report on completion of corrections (if any):

Completed

Changes Approved by Thesis Advisor: Dr. Iman Osta  Signature:

Date: __________________________

Acknowledged by __________________________
(Dean, School of Arts & Sciences)

Cc: Registrar, Dean, Chair, Advisor, Student
Thesis Approval Form

Student Name: Noura Nackouzi
I.D. #: 200401862

Thesis Title: Developing and piloting a DGS based unit for overcoming difficulties faced by grade 9 Lebanese students while learning Locus of Points

Program: Master of Arts in Education. Emphasis: Mathematics Education
Department: Education
School: School of Arts and Sciences

Approved by:
Thesis Advisor: Dr. Iman Osta
Signature:

Member: Dr. Samer Habre
Signature:

Member: Dr. Tamer Amin
Signature:

Date: August 27, 2013
THESIS COPYRIGHT RELEASE FORM

LEBANESE AMERICAN UNIVERSITY NON-EXCLUSIVE DISTRIBUTION LICENSE

By signing and submitting this license, you (the author(s) or copyright owner) grants to Lebanese American University (LAU) the non-exclusive right to reproduce, translate (as defined below), and/or distribute your submission (including the abstract) worldwide in print and electronic format and in any medium, including but not limited to audio or video. You agree that LAU may, without changing the content, translate the submission to any medium or format for the purpose of preservation. You also agree that LAU may keep more than one copy of this submission for purposes of security, backup and preservation. You represent that the submission is your original work, and that you have the right to grant the rights contained in this license. You also represent that your submission does not, to the best of your knowledge, infringe upon anyone’s copyright. If the submission contains material for which you do not hold copyright, you represent that you have obtained the unrestricted permission of the copyright owner to grant LAU the rights required by this license, and that such third-party owned material is clearly identified and acknowledged within the text or content of the submission. IF THE SUBMISSION IS BASED UPON WORK THAT HAS BEEN SPONSORED OR SUPPORTED BY AN AGENCY OR ORGANIZATION OTHER THAN LAU, YOU REPRESENT THAT YOU HAVE FULFILLED ANY RIGHT OF REVIEW OR OTHER OBLIGATIONS REQUIRED BY SUCH CONTRACT OR AGREEMENT. LAU will clearly identify your name(s) as the author(s) or owner(s) of the submission, and will not make any alteration, other than as allowed by this license, to your submission.

Name: Noura Nackouzi

Signature: 

Date: August, 2013
Plagiarism Policy Compliance Statement

I certify that I have read and understood LAU’s Plagiarism Policy. I understand that failure to comply with this Policy can lead to academic and disciplinary actions against me. This work is substantially my own, and to the extent that any part of this work is not my own I have indicated that by acknowledging its sources.

Name: Noura Nackouzi
Signature: 
Date: August, 2013
ACKNOWLEDGMENTS

This research would not have been possible without the help and assistance of many persons. First I would like to express my gratitude to Dr. Iman Osta for her continuous guidance, support, patience, care and encouragement throughout my Thesis work. I am also grateful to Dr. Samer Habre and Dr. Tamer Amin for their time and guidance.

Thanks go also to my caring mother. Thank you for everything you have done for me.

Finally, special thanks go also to my beloved husband for his patience, support, love, and blessings. Your trust in me made me able to complete this work.

With Love
Dedication

To my Mother, loving husband Sami and beloved son Salah
Abstract

The concept of “locus” is often considered confusing to students due to its overly abstract nature. The abstract approach that is used in solving problems involving locus of points in classrooms is probably one of the reasons learners face difficulties in acquiring and understanding “locus”. This paper presents a qualitative research study that is concerned with the use of a constructivist approach where the students have to explore the cases of locus of a point with the integration of Dynamic Geometry Software (DGS), namely Geometer’s Sketchpad, in the context of open geometry problems requiring conjecturing and proving. The purpose of this study is to explore the difficulties that grade-9 students face in learning locus and solving locus problems. It also aims to investigate whether using DGS can help improving the learning process and better preparing students for solving problems involving locus of points. A series of activities were developed and implemented over seven sessions that integrate the use of DGS, namely Geometer’s Sketchpad, to teach geometric locus of points. Participants are two classes of grade-9 students (over two academic years), and a group of three grade-9 math teachers, at a private school in South-Lebanon. The total number of students from two classes over two consecutive years is 41 including 18 girls and 23 boys. The students’ average mathematics scores in grade 8 for each class were considered to determine a base for comparison. The math average of the non-DGS class was found to be 82 out of 100 and that of the DGS class was 73 out of 100. The study involved several stages conducted over two academic years: semi-structured interviews with three grade-9 teachers, classroom observations of grade-9 classes where locus is taught without students’ use of DGS, open interviews with eight students about the difficulties that they face while learning Locus, classroom observation of a major grade-9 problem-solving session using DGS during the implementation of the unit, clinical interviews with selected pairs of students solving geometric problems using DGS, and paper-pencil test (the same one administered to both, the non-DGS and the DGS groups) to investigate whether the use of DGS enhances students’ understanding and if the abilities of finding locus of points, developed in a DGS environment, are transferable to a non-DGS environment. Data collected was analyzed according to a framework compiled by the researcher based on frameworks used in the literature and on a primary overview of students’ work. Results showed that although the two classes were of different levels of achievement, the DGS group performed higher in the paper-pencil test than the non-DGS class. The use of DGS positively impacted students’ ability to find and formulate...
conjectures about locus of points. It also gave the DGS group confidence in attempting proofs.

**Keywords:** Locus of points, Dynamic Geometry Software, Conjecturing, Proof difficulties, Lebanese middle school students, and Lebanese geometry curriculum.
# TABLE OF CONTENTS

| Cover Page | i |
| Thesis Proposal Form | ii |
| Thesis Defense Result Form | iii |
| Thesis Approval Form | iv |
| Thesis Copyright Release Form | v |
| Plagiarism Policy Compliance Statement | vi |
| Acknowledgments | vii |
| Dedication | viii |
| Abstract | ix |
| Table of Contents | xi |
| List of Tables | xiv |
| List of Figures | xv |

## Chapter

### I – Introduction

1.1 – Statement of the Problem
1.2 – Purpose of the Study
1.3 – Research Questions
1.4 – Definition of Terms
1.5 – Significance of the Study
1.6 – Organization of the Remainder of the Study

### II – Literature Review

2.1 – Solution process of locus problems
2.2 – DGS in mathematics teaching/learning
   2.2.1 – Conjecture-generation in a DGE
   2.2.2 – Dragging modalities in a DGE
2.3 – Proof
   2.3.1 – Difficulties that students face while proving
   2.3.2 – Students’ ways of justifying conjectures
   2.3.3 – Proof in DGS situations
2.4 – Teacher’s role
III – Method

3.1 – Participants 23
3.2 – Procedures
   3.2.1 – Interview with the teachers 23
   3.2.2 – Classroom Observations 24
   3.2.3 – Interview with the students 24
   3.2.4 – Paper-pencil test 25
   3.2.5 – Analysis of the “Locus” unit in students’ textbook 26
   3.2.6 – Development of the math teaching unit about locus 26
       3.2.6.1 – Time-line 27
       3.2.6.2 – Context 27
       3.2.6.3 – Material 27
       3.2.6.4 – Lab sessions 28
       3.2.6.5 – The general objectives of the instructional unit 28
       3.2.6.6 – Lab settings 30
       3.2.6.7 – Clinical interviews 30
   3.2.7 – Data collection instruments 31

IV – Findings

4.1 – “Locus” unit in students’ textbook 33
4.2 – Students’ answers to sheets 1 and 2 in the math unit 35
4.3 – Interview with the teachers 39
   4.3.1 – Teachers’ strategies of teaching mathematics, especially ‘locus’ 39
   4.3.2 – Teachers’ views of students’ major difficulties in learning locus 40
   4.3.3 – Teachers’ awareness of the benefits of using DGS in class 41
   4.3.4 – Teachers’ views of ways to enhance students’ proving abilities 43
   4.3.5 – Conclusion 43
4.4 – Interview with the students 44
   4.4.1 – Difficulties faced while learning “locus” 45
   4.4.2 – Ways of thinking to find the locus of a point 45
   4.4.3 – Difficulties faced when solving geometric problems about Locus 46
   4.4.4 – Conclusion 47
4.5 – Observation of the Non-DGS class 48
4.6 – Observation of classes with students’ use of DGS 49
4.7 – Framework for analyzing students’ conjectures and proofs 51
4.8 – Results 63
4.8.1 – Clinical interviews
  4.8.1.1 – Conjecture and proof categories 66
  4.8.1.2 – Ways DGS was used in making conjectures 79
  4.8.1.3 – Summary 80
4.8.2 – Paper-pencil test
  4.8.2.1 – Categories of conjectures used by students 85
  4.8.2.2 – Categories of proofs used by students 91
  4.8.2.3 – Qualitative analysis of some students’ tests 97

V – Discussions and Conclusions 102-110

  5.1 – Research Question 1 103
  5.2 – Research Question 2 105
  5.3 – Research Question 3 106
  5.4 – Research Question 4 107
  5.5 – Limitations of the study 108
  5.6 – Recommendations 109
  5.7 – Perspectives for further research 110

VI – References 111-114

VII – Appendices 115-157

Appendix A : Questions of semi-structured interview with Teacher(s) 116
Appendix B : Rubric for observation in class 117
Appendix C : Interview with the students 119
Appendix D : Paper-pencil test 120
Appendix E : “Locus” unit in students’ textbook 122
Appendix F : Instructional Unit on Geometric Locus 130


**LIST OF TABLES**

<table>
<thead>
<tr>
<th>Table</th>
<th>Table Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1</td>
<td>Instructional unit sessions in glance</td>
<td>28</td>
</tr>
<tr>
<td>Table 2</td>
<td>Case addressed in each part of sheets 1 and 2</td>
<td>36</td>
</tr>
<tr>
<td>Table 3</td>
<td>Outcomes of sheets 1 and 2</td>
<td>37</td>
</tr>
<tr>
<td>Table 4</td>
<td>Conjecture and proof categories</td>
<td>62</td>
</tr>
<tr>
<td>Table 5</td>
<td>Categories of conjectures produced in DGS before using any DGS feature</td>
<td>66</td>
</tr>
<tr>
<td>Table 6</td>
<td>Categories of conjectures produced in DGS after using DGS features</td>
<td>67</td>
</tr>
<tr>
<td>Table 7</td>
<td>Proof categories produced in DGS before using any DGS feature</td>
<td>68</td>
</tr>
<tr>
<td>Table 8</td>
<td>Proof categories produced in DGS after using DGS features</td>
<td>69</td>
</tr>
<tr>
<td>Table 9</td>
<td>Categories of conjectures of non-DGS class (23 students)</td>
<td>85</td>
</tr>
<tr>
<td>Table 10</td>
<td>Categories of conjectures of DGS class (18 students)</td>
<td>86</td>
</tr>
<tr>
<td>Table 11</td>
<td>Categories of conjectures of a 100-student non-DGS class</td>
<td>87</td>
</tr>
<tr>
<td>Table 12</td>
<td>Categories of conjectures of a 100-student DGS class</td>
<td>88</td>
</tr>
<tr>
<td>Table 13</td>
<td>Categories of proofs of non-DGS class (23 students)</td>
<td>92</td>
</tr>
<tr>
<td>Table 14</td>
<td>Categories of proofs of DGS class (18 students)</td>
<td>93</td>
</tr>
<tr>
<td>Table 15</td>
<td>Categories of proofs of a 100-student non-DGS class</td>
<td>94</td>
</tr>
<tr>
<td>Table 16</td>
<td>Categories of proofs of a 100-student DGS class</td>
<td>95</td>
</tr>
</tbody>
</table>
## LISTS OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Figure Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>Perpendicular bisector case</td>
<td>33</td>
</tr>
<tr>
<td>Figure 2</td>
<td>Example of MNL</td>
<td>52</td>
</tr>
<tr>
<td>Figure 3</td>
<td>Example of MFV</td>
<td>53</td>
</tr>
<tr>
<td>Figure 4</td>
<td>Example of IC category</td>
<td>53</td>
</tr>
<tr>
<td>Figure 5</td>
<td>Example of CIC</td>
<td>54</td>
</tr>
<tr>
<td>Figure 6</td>
<td>Example of CCNS</td>
<td>55</td>
</tr>
<tr>
<td>Figure 7</td>
<td>Example of CCED</td>
<td>56</td>
</tr>
<tr>
<td>Figure 8</td>
<td>Example of CPL</td>
<td>57</td>
</tr>
<tr>
<td>Figure 9</td>
<td>Example of NP category</td>
<td>57</td>
</tr>
<tr>
<td>Figure 10</td>
<td>Example of IPFV</td>
<td>58</td>
</tr>
<tr>
<td>Figure 11</td>
<td>Example of IPGM</td>
<td>59</td>
</tr>
<tr>
<td>Figure 12</td>
<td>Example of CNCP</td>
<td>60</td>
</tr>
<tr>
<td>Figure 13</td>
<td>Example of CPUS</td>
<td>61</td>
</tr>
<tr>
<td>Figure 14</td>
<td>Example of CCP</td>
<td>62</td>
</tr>
<tr>
<td>Figure 15</td>
<td>Question number 1 in the paper-pencil test</td>
<td>82</td>
</tr>
<tr>
<td>Figure 16</td>
<td>Question number 2 in the paper-pencil test</td>
<td>84</td>
</tr>
<tr>
<td>Figure 17</td>
<td>Solution of the test by the first student</td>
<td>98</td>
</tr>
<tr>
<td>Figure 18</td>
<td>Solution of part 1 by the second student</td>
<td>99</td>
</tr>
<tr>
<td>Figure 19</td>
<td>Solution of the test by the third student</td>
<td>100</td>
</tr>
<tr>
<td>Figure 20</td>
<td>Solution of the test by the fourth student</td>
<td>101</td>
</tr>
</tbody>
</table>
CHAPTER ONE

INTRODUCTION

Mathematics is considered a subject in which most students face difficulties, as it requires analytical thinking, logic, structuring, organization, and reasoning. It should be learned through real life problems, games, and explorations rather than the traditional way of lecturing and memorizing that might leave the learner uninterested in knowledge that s/he might find abstract. A constructivist-visual approach which encourages students to seek answers for themselves and to visualize the representations of the concepts is believed to attract students and make them actively involved learners rather than passive listeners.

Many studies (e.g. Dreyfus 1991; Olive & Leatham 2000; Borba & Villareal, 2005; Calder 2004; Palais, 1999) note that visualization through computer graphics facilitates understanding and conveys insight and knowledge through making it possible to examine features that were unapproachable without computers. However, according to many studies (e.g. Roschelle, Pea, Hoadley, Gordin and Means 2000; Sacristán et al. 2010), technology alone is not enough. Students learn most effectively when they are actively engaged and when they construct knowledge from their experience and interactions with peers and teachers, and when they participate in groups, in which they carry out more complex tasks than they could alone, discuss the task, and make thinking visible.
Technology software programs can help the learners in constructing their knowledge through visualization and exploration. The visualization helps in facilitating understanding the mathematical concepts through visualization of problems or mathematical processes in ways that were not possible before. Moreover, the explorative activities that result from dragging or moving particular objects within the representation can lead the learner, not only to detect and explore invariants or mathematical relations, but also to explore whether the relation is valid for a family of cases.

1.1 Statement of the Problem

The concept of “locus” is often considered confusing to students due to its overly abstract nature. The abstract approach that is used in solving problems involving locus of points in classrooms is probably one of the reasons learners face difficulties in acquiring and understanding “locus”. It is important that the learners first understand the meaning of the concept “locus”, visualize it, and explore it.

This paper presents a qualitative research study in a grade-9 class. The study is concerned with the use of a constructivist approach where the students have to explore the cases of locus of a point with the integration of Dynamic Geometry Software (DGS), namely Geometer’s Sketchpad, in the context of open geometry problems requiring conjecturing and proving.

1.2 Purpose of the Study

The purpose of this study is to explore the difficulties that students face in learning locus and solving locus problems. It also aims to investigate whether using
DGS can help improving the learning process and better preparing students for solving problems involving locus of points. A series of activities will be developed and implemented over seven sessions that integrate the use of DGS, namely Geometer’s Sketchpad, to teach geometric locus of points.

1.3 Research Questions

The research attempts to answer the following questions:

1. What are the major difficulties that students face in learning the concept of locus and in finding and proving locus of points?

2. How does DGS modify/change the class instructional environment (class interactions, teacher vs. student centeredness …) while teaching locus of points?

3. How does DGS support learning about locus of points?

4. In problem situations involving Locus, are the abilities developed in a DGS environment transferable to a non-DGS environment?

1.4 Definition of Terms

Locus: The word “locus” is a Latin word that means place. “When we use visual thinking in geometry to describe points that satisfy a set of conditions or the path of something moving according to a given set of instructions, the set of points or the path is called a locus of points” (Serra, 1997). For example, a locus of a point that is always at a fixed distance from a fixed point is a circle.
Invariant property: A property of a set of mathematical objects that remains unchanged when alterations of a certain type are applied to the objects.

Conjecture: A statement likely to be true based on available evidence, but which has not been formally proven.

1.5 Significance of the Study

Many studies have been conducted worldwide on visualization and conjecturing with DGS (e.g. Sacristán et al., 2010, Palais, 1999). Other studies investigated the conjectures found by students and proofs written using paper-pencil after hands-on exploration activities (e.g. Borba & Villareal, 2005, Zbiek, 1998, Hanna, 1989). However, although locus is considered a tough lesson to many students because it highly depends on analysis and imagination of a variable element which holds an invariant property and not all students visualize mentally without an external help, very few studies explored the difficulties that students might face while learning or applying knowledge about locus, and how DGS can help in improving the learning process and better preparing students for solving problems involving locus of points.

In Lebanon, few schools use DGS as a tool for teaching geometry. The use of such software is not considered in the texts of the Lebanese curriculum. The results of such a study might contribute to the body of the Lebanese literature related to teaching/learning mathematics through using technology and may bring remarkable results. Teachers can hopefully benefit from the potentials of DGS in geometry classes. The aim of the research is to explore the difficulties that students usually face in learning or applying knowledge about locus concept. It also focuses on the role that DGS can
play in helping students with improving their learning process and better preparing them for solving problems involving locus of points. Hence, it focuses on understanding students’ geometric reasoning in two different contexts (DGS and paper-pencil). It will highlight the benefits that a DGS can play in students’ geometrical reasoning. Also, it will provide teachers with a better understanding about the potentials of DGS in geometrical classes.

**1.6 Organization of the Remainder of the Study**

This study is organized in four more chapters. Chapter two includes a review of the literature related to concept of locus and characteristics of questions asked about locus, dynamic geometry software, conjectures in dynamic geometry software, and proof. It also includes a theoretical background on which this study is based. Chapter three includes information about the participants, the procedure, and the instruments used in this study. Chapter four provides the analysis of the data and the findings. Chapter five presents conclusions, comparison between the findings of this study and those found in the literature, recommendations and limitations.
CHAPTER TWO

LITERATURE REVIEW

The present section includes a review of the literature on the importance of DGS in geometry classes, the role of visualization and exploration in Dynamic Geometry Environments (DGE), and on the formulation of conjectures and proofs. It also considers literature concerning the importance of proof, difficulties that students face while proving, students’ ways of justifying conjectures, and teachers’ role and importance of appropriate intervention.

2.1 Solution process of locus problems

Problems in the chapter “locus” are usually referred to as “open problems” which are problems or questions stated in a form that does not reveal their solution, where the student has to find the invariant property, make a conjecture about it, and afterwards justify or prove it. More precisely open problems have been characterized in the following way:

*The statement is short, and does not suggest any particular solution method or the solution itself. It usually consists of a simple description of a configuration and a generic request for a statement about relationships between elements of the configuration or properties of the configuration. The questions are expressed in the form ‘which configuration does… assume when…?’, ‘which relationship can you find between…?’ ‘What kind of figure can… be transformed into?’ These requests are different from traditional closed expressions such as ‘prove that…’*, which present students with an already established result (Mogetta, Olivero and Jones, 1999, pp. 91–92).
The solution process of an open problem goes into two phases: a) a conjecturing phase, during which students engage in exploration of a figure including variable elements (moving points) and argumentation leading to the written formulation of a statement; and b) a proving phase, during which students attempt to prove their conjecture. As referenced in Morselli (2006), Garuti, Boero and Lemut (1998) stated that the production of a conjecture may give important hints for proof where students gradually work out their statements through an argumentative activity that intermingles with the justification of their choices. During the proving stage, students organize some of the produced arguments in a logical way.

Mogetta et al. (1999) mentioned that students undergo a process in order to solve an open problem. The process is: exploring the situation, making conjectures, validating conjectures and proving them. Arzarello, Andriano, Olivero and Robutti (1998) carried out an analysis of the performances of mathematics teachers in high school and at university on solving open geometric problems. The analysis ended up in a theoretical model which describes the way conjectures are produced and the transition from the conjecturing to the proving phase. The three main modalities of the theoretical model are: *ascending control* (Gallo, 1994) which relates to exploring a certain situation; *abduction (hypothesis)* (Magnani, 2001) in which the solver tries to choose the rule that suits a particular case, thus explorations are transformed into conjectures; and *descending control* (Gallo, 1994) where after producing a conjecture, the solver seeks for a confirmation. In this way, the solver refers to a theory in order to justify and validate the conjecture. The model shows the process of transition from exploring-conjecturing to proving. Abduction guides the transition, in that it is the moment in
which the conjectures produced are written in a logical form 'if…then'. This model suggests an essential continuity in the process exploring-conjecturing-validating proving, for experts. Thus, it points out a crucial continuity of thought which rules the successful transition from the conjecturing phase to the proving one through explorations and appropriate heuristics.

2.2 DGS in mathematics teaching/learning

According to Frank and Mariotti (2010), a DGS may become a potential bridge between the world of mathematics, which is a crystallized world where logical dependency is the hierarchical organizer, and the world of experience, where dragging can allow a real-time physical experience and dynamism. During the last years, an extensive amount of research concerning the processes of conjecture production and construction of proofs was connected with interactive learning environments and different software packages (Harel & Papert, 1990; Davis 1991), in particular, dynamic geometry software (Arzarello et al, 1998; Mariotti, 2000; Furinghetti, Morselli & Paola, 2005).

Furinghetti, Morselli and Paola (2005) conducted an experiment on 15 year-old students to test whether Cabri enlarges the scope of the exploration in a way that couldn’t be possible in a paper and pencil environment. Students worked in groups of three with each group working at one computer. The analysis of students’ work was based on video-tapes, protocols, and field notes. Results indicated that by using Cabri students dealt easily with geometric representation of the problem (the dynamic figure)
and analytic representation of the problem (the diagram) and had deep insight of “linear dependence”.

Mariotti (2000) conducted a long-term teaching experiment carried out in the 9th and 10th grades of a scientific high school to investigate how the students’ views of geometry change from being intuitive to being theoretical. Results showed that DGS supports this transition as it uses exploration and visualization.

There is a consensus among many educators regarding the positive role of visualization or graphic approaches in the facilitation of understanding in mathematics education (Dreyfus 1991; Olive & Leatham 2000; Borba & Villareal, 2005; Calder 2004). Dynamic representations of mathematical objects allow learners to visualize problems or mathematical processes in ways that were not possible before (Sacristán et al., 2010). For example, learners can view a process as it develops, rather than trying to analyze it from its fixed initial, partial or end results. As cited in Hanna (2000), Palais (1999) notes that visualization through computer graphics facilitates understanding and conveys insight and knowledge through making it possible to examine features that were unapproachable without computers. However, according to Malaty (2006), there are problems in the use of visualization in classroom as it offers a ready model that makes children jump to conclusions without leaving space for searching for a cause. In other words, the misusage of visualization misleads children and makes them jump to conclusions, which have not been justified.

According to Roschelle, Pea, Hoadley, Gordin and Means (2000) and Sacristán et al. (2010), using technology alone is not enough. Students learn most effectively when (1) learning through active engagement, in which students construct knowledge from
their experience and interactions with peers and teachers; and (2) learning through participation in groups, in which students carry out more complex tasks than they could alone, discuss the task, and make thinking visible. For example, Borba and Villereal (2005) and Zbiek (1998) believe that the creation and exploration of dynamic models can enhance students’ ability to create mathematical models in a reflective way. In the case of dynamic geometry, students can engage in explorative activities that result from dragging or moving particular objects within the representation: in such environment, the controlled movement of some elements within a geometric configuration can lead the learner, not only to detect and explore invariants or mathematical relations, but also to explore whether the relation is valid for a family of cases.

2.2.1 Conjecture-generation in a DGE

According to Frank and Mariotti (2009), students are able, in a DGE, to develop dynamic-conjecture where they can drag and observe changes in the figures and the provided measurements. However, in static environments such as in a paper-and-pencil situation, students build a static-conjecture where they are not allowed to deform the figures by dragging. Moreover, in a dynamic environment, the invariant geometrical properties of a construction, which lead to conjectures, can easily be grasped.

Talmon and Yerushalmy (2004) stated that, while drawing a geometric figure in DGE, there are relations between objects. These relations establish a hierarchy of dependences in which some elements depend on other more basic elements. This hierarchy is stated in terms such as parent-child relationships. For example, when a point is constructed on a given line, it can be dragged only on that line; in this case, dragging
does not affect other components of the construction. However, when a line is constructed through a given basic point, this point can be dragged freely in the plane, and the location of the line changes with it. An element that is built on, and related to a previous one is a child; the previous element is its parent, which creates parent-child relations representing the hierarchy of dependences between the elements of the construction.

2.2.2 Dragging modalities in a DGE

During the conjecture-generation in dynamic geometry software various dragging modalities are used by the solver. The dragging modalities are important affordances in the DGS context that help in dynamically exploring the figures and give the students new material to observe, which they do not have in paper and pencil situations. While exploring figures by moving them, the users are allowed to discover properties of geometric figures. This way dragging supports the production of conjectures, provides explanations of a conjecture or property, and consequently supports the role of proofs (Hanna, 1989). According to Hanna (2000), teachers have to utilize the enjoyment and excitement of the exploration to motivate students and thus to encourage them to supply a proof. According to Olivero (1999), students who are engaged in activities which include explorations and production of conjectures can organize a proof better than students who are presented with a recognized statement and asked to prove it.

The different ways of dragging points as a conjecture is elaborated and tested shift from “ascending control” to “descending control”. The four dragging modalities that are used by students for investigation are: 1) wandering/random dragging, that is when students randomly drag a basic point on the screen to look for interesting configurations
or regularities of the DGS-figure or to maintain a geometrical property of the figure (intentionally induced invariance); 2) \textit{maintaining dragging} or \textit{dummy locus dragging} that consists of trying to drag a basic point while maintaining some interesting property observed (invariant observed during dragging). It involves the user’s recognition of a particular configuration as interesting, and attempt to induce the particular property to become an invariant during dragging; 3) \textit{dragging with trace activated} in which students intend any form of dragging after the trace function has been activated on one or more objects of the figure; and 4) \textit{dragging test} that is a way used for testing a conjecture in which students move a figure through all its draggable points and observe that it keeps the considered property. It is used as a means of validating a conjecture (Frank & Mariotti, 2010).

\textbf{2.3 Proof}

The notion of proof is viewed as a central construct in mathematical thinking, and learning to understand and develop formal proofs is seen as an important aspect of students’ mathematical learning (Yackel & Hanna, 2003).

Mathematics educators consider eight roles that proof plays in mathematics (Bell, 1976; de Villers, 1999; Hanna & Jahnke, 1996). These roles are: to verify that a statement is true, to explain why a statement is true, to communicate mathematics to other mathematicians, to discover new mathematics, to incorporate well known facts into a new framework, to explore the meaning of a definition or the sequence of a theory, to construct an empirical theory, and to systematize results into a deductive system of definitions, axioms and theorems.
In school, teachers do not always seem to distinguish important roles of proof or to understand roles other than verification. Little instructional time is dedicated to proof construction and appreciation (Biza, Nardi & Zachariades, 2009, De Villiers, 2004). However, proof is much more than a sequence of logical steps; it is also a sequence of ideas and insights (Jaffe, 1997; Kleiner, 1991; Manin, 1998; Rota, 1997). Thus, proof for mathematicians involves interpretation, understanding, reasoning, and sense-making. From this perspective, chains of logical argument do not function as satisfactory proofs unless they serve explanatory and communicative functions for an interpreting individual. Hanna (2000) noted that mathematicians have to first convince themselves that a mathematical statement is true and then move to a formal proof. Thus, conjecturing with verification, exploration and explanation constitute the necessary elements that precede formal proof.

2.3.1 Difficulties that students face while proving

A study was conducted by Moore (1994) on post-secondary school students. It examined the cognitive difficulties that university students experience in learning to develop formal proofs. Moore suggested that one of the causes of difficulty in producing a formal proof is the fact that “students are unable, or unwilling, to generate and use their own examples” (p.251). Examples may be useful to check the validity of a property and to formulate and communicate the conjecture. Moore identified three major sources for the students’ difficulties with the transition to formal proof starting with conceptual understanding, moving to mathematical language and notation, and finally getting started on a proof.
Balacheff (1991) noted that, in instruction, the emphasis is on the written form of proof rather than on students’ reasoning. Thus, students do not gain an appreciation for the role of proof as a tool that allows a mathematician to establish validity of a statement and convey that validity to others. Balacheff also noted that students engage in behaviors and activities that lead them to act as practical persons rather than as theoreticians. They tend to frame their solutions in a format that is acceptable to peers and teachers. Their aim is then to produce a solution rather than to produce knowledge. According to Simpson (1995), teaching proof through logic has no connection with the existing mental structure of the students. Students have to be taught proof through reasoning which involves investigations and embodies heuristic arguments.

Dreyfus and Hadas (1996) believe that the cognitive difficulty posed by the nature of proof is that students fail to see a need for proof. They also fail to see the explanatory and convincing roles of proof.

2.3.2 Students’ ways of justifying conjectures

Many authors (e.g. Arzarello et al., 1998; Balacheff, 1988; Bell, 1976; Harel and Sowder, 1996; Sowder and Harel, 1998, Marrades and Gutiérrez, 2000) studied, analyzed and categorized the ways in which students produce justifications.

Bell (1976) identifies two categories of students’ justifications used in proof problems: empirical justifications which are characterized by the use of examples as elements of checking/conviction, and deductive justifications that are characterized by the use of deductive arguments to connect the premises with the conclusions.
Balacheff (1988) considers two categories of students’ justifications: *pragmatic* and *conceptual* justifications. *Pragmatic justifications* are based on the use of examples or on actions or showings. *Conceptual justifications* are based on abstract formulations of properties or relationships.

Harel and Sowder (1996) and Sowder and Harel (1998) identify three categories of justifications: *externally based, empirical, and analytical or theoretical justifications*. *Externally based justifications* are based on the power of an external source to students, like teacher or textbook or a knowledgeable person. *Empirical justifications* are based on examples or drawings. *Analytical or theoretical justifications* result in formal mathematical proofs.

Bell (1976), Balacheff (1988), and Harel and Sowder (1996) only described students’ justifications. Bell analyzed only the completeness of sets of examples used by students. Balacheff focused on students' reasons for selecting examples and on how they used them. Sowder and Harel differentiated justifications based only on visual or tactile perception and on the observation of mathematical properties. Based on the findings of Bell, Balacheff, and Harel and Sowder, Marrades and Gutiérrez (2000) describe students’ justifications and consider the process of production of such justifications. They define a scheme in which all of the student's activity - generation of a conjecture, devising a justification, and the resulting justification - is considered. They differentiate between two main categories: empirical and deductive justifications. The scheme is as follows:
- Empirical justifications are divided into a number of subcategories depending on ways students select the example. Each subcategory has different types depending on different ways students use the selected example.

- **Naive empiricism:** when the conjecture is justified by showing that it is true in one or several examples.

- **Crucial experiment:** when the conjecture is justified by showing that it is true in a specific, carefully selected, example. Students assume that the conjecture is always true if it is true in this example.
  - **Example-based:** when the justification shows only the existence of an example or the lack of counter-examples.
  - **Constructive:** in which the justification focuses on the way of getting the example.
  - **Analytical:** in which the justification is based on properties empirically observed in the example or in auxiliary elements.
  - **Intellectual:** when the justification is based on empirical observation of the example, but the justification mainly uses accepted properties or relationships among elements of the example. Intellectual justifications show some decontextualization (Balacheff, 1988), since they include deductive parts in addition to arguments based on the example.

- **Generic example:** when the justification is based on a specific example and the justification refers to abstract properties and elements of a family, but it is clearly based on the example.
- **Failed answer**: when students use empirical strategies to solve a proof problem but they do not succeed in elaborating a correct conjecture or they do state a correct conjecture but they do not succeed in providing any justification.

- Deductive justifications are divided into three subcategories depending on whether students use an example to help organize their justification or not.
  - **Thought experiment**: when students justify deductively and use an example to help organize the justification.
  - **Formal deduction**: when the justification is based on mental operations without the help of specific examples.
  - **Failed**: when students use deductive strategies to solve proof problems but they do not succeed in elaborating a correct conjecture or they elaborate a correct conjecture but they fail in providing a justification.

### 2.3.3 Proof in DGS situations

Hoyles and Jones (1998) state that many students consider proof to be an elusive concept. Innovative activities must be designed to enable students make links between empirical and deductive reasoning. DGS provide a model of Euclidean geometry which offers feedback through “dragging” as to whether constructions or theorems are “correct”. Thus, students are able to generate ample empirical evidence for geometric theorems. For example, if a figure of a rhombus and its diagonals are given and the students are asked to construct that figure on DGS, they do not only construct the figure but also explain why the shape is a rhombus and come up with all its properties. The dragging facility in DGS allows conjectures to be tested by focusing attention on the
relationships between the geometrical objects that have been constructed. Explaining why these geometrical facts are necessarily true involves constructing chains of reasoning that helps in a meaningful experience of proof.

Christou, Mousoulides, Pittalis and Pitta-Pantazi (2004) indicate that dragging facilities in DGE allow one to experiment geometrical objects and consequently infer properties or theorems. In their article “Proofs through Exploration in Dynamic Geometry Environments” three phases are used to analyze students’ strategies in solving two geometrical problems in DGE: (a) the phase before proof, (b) the proof phase, and (c) the phase of intellectual challenge of extending proof to similar problems. At the phase before proof, students explored the given problems through constructing the figures and using the dragging facilities of the software. This exploration led students to form their own conjectures by visualizing the changes. This phase is necessary for students to understand the problem based on their own intellectual efforts. At the proof phase, the students who successfully found the conjectures based on the exploration phase (phase before proof) were able to define and identify the geometrical properties and provided a deductive proof of the problem. During the last phase, students felt a strong desire for explaining their conjectures and understanding how one conclusion is a consequence of other familiar problems, results or theorems. Students viewed a deductive argument as an attempt for explanation rather than for verification.

As cited in Christou, Mousoulides, Pittalis and Pitta-Pantazi (2004), Hoyles and Healy (1999) indicate that exploration of geometrical concepts in a DGE motivate students to explain their empirical conjectures using formal proof. They find that DGS
help students to define and identify geometrical properties and the dependencies between them.

2.4 Teacher’s role

Often in the traditional mathematics classroom, we do not realize, or even we ignore or suppress, intuitive or spontaneous ideas. But the teacher can adjust an intended learning trajectory to include tasks that stimulate creative, intuitive thinking, or alternatively allow space for imaginative exploration of the pedagogical medium or mathematical thinking as it emerges from engagement with the activities.

By using Digital Technology (DT) tools, the teacher’s role may become more than that of a mediator, where the teacher not only guides students through their DT tasks, but also intervenes to promote learning. As cited in Sacristán et al (2010), many (e.g. Clements 2002) have found that the mathematical knowledge constructed in a DT environment can remain hidden or “situated” within the technological context, unless teachers help make that knowledge explicit. On the other hand, some researchers have also indicated that the affordances provided by DT-environments, when facilitated appropriately by the teacher, may lead students to explore powerful ideas in mathematics, to learn to pose problems, and to create explanations of their own. The teachers’ appropriate intervention during the development of DT sessions involves guiding the learners to validate mathematical results or relations that emerge when they formulate and explore a problem through the use of the tools. Furthermore, DT attributes, coupled with appropriate teacher intervention, can enable the learner to not only explore problems but to make links between different content areas that may otherwise have developed discretely. When students feel that there is no need for further
conviction/verification since they are certain of the correctness of the conjecture, it is important for teachers to challenge them by asking why they think a particular result is true. Students quickly admit that the “why” questions urge them to view deductive arguments as an attempt for explanation, rather than verification. Thus, the challenge of educators is to convey clearly to the students the interplay of deduction and experimentation and the relationship between mathematics and the real world (Hanna, 2000).
CHAPTER THREE

METHOD

The study is a qualitative research aiming at exploring the difficulties that grade-9 students face in learning locus and solving locus problems, as well as the ways using dynamic geometry software can help improving the learning process and better preparing students for solving problems involving locus of points. For that purpose, a DGS based geometry curriculum unit, using Geometer’s Sketchpad, is developed and implemented, which allows an investigation of students’ geometric thinking in such a context.

The research method encompasses nine stages to be conducted over two academic years:

First year:

- semi-structured interviews with three grade-9 math teachers
- observation of grade-9 classes where locus is taught without students’ use of DGS
- open interviews with eight students (four low achievers and four middle achievers) about the difficulties that they face while learning Locus
- paper-pencil test to investigate the difficulties that grade-9 students (here after referred to as non-DGS group) might face in finding and proving locus of points
- analysis of the unit about Geometric loci in a locally published grade-9 textbook to assess its approach and suggested activities, to identify possible sources of difficulty, and to plan for the development of the DGS unit

Second year:

- development of a math teaching unit about locus integrating the use of DGS based on the findings obtained in the first year
- observation of a major grade-9 problem-solving session using DGS during the implementation of the unit (this group of students will be referred to as the DGS group)
- clinical interviews with selected groups of students solving geometric problems using DGS
- paper-pencil test (the same one administered to the non-DGS group in the first year knowing that the papers of the non-DGS group were collected and no trace of the test was left to students) to investigate whether the use of DGS enhances students’ understanding and if the abilities of finding locus of points, developed in a DGS environment, are transferable to a non-DGS environment

It is important to mention that the second-year grade-9 class consists of a different group of students who were in grade 8 in the first year of the research.
To guarantee comparability of the two groups’ students’ level of achievement, students’ grades in mathematics over two semesters prior to observation (while students were in grade 8) are collected, as well as their average in mathematics in grade 8.

3.1 Participants

Participants are two groups of grade-9 students (over two academic years), and a group of three grade-9 math teachers, at a reputable private school in South-Lebanon. The total number of students from two classes over two consecutive years is 41 including 18 girls and 23 boys. The grade-9 class that participated the first year consists of 23 students, including 10 girls and 13 boys. The grade-9 class that participated the second year consists of 18 students, including 8 girls and 10 boys.

3.2 Procedures

3.2.1 Interview with the teachers

A semi-structured interview was conducted with three different teachers, 2 females and 1 male. One of the three teachers has been teaching for 3 years during which she taught grade 9 during two years. The second teacher has been teaching for 10 years and taught grade 9 during three years. As for the third teacher, he has been teaching grade 9 throughout his 18 years of teaching career. The purpose of the interview is to provide insight about teachers’ methods of teaching math, especially ‘locus’, teachers’ perceptions about students’ major difficulties in learning the Locus topic, and teachers’ views of ways to enhance proving abilities. Moreover, it investigates the teachers’ perceptions of, experience in, and actual use of technology while teaching.
Appendix A provides the protocol (questions) of the semi-structured interview. The interview is audio-taped, transcribed, analyzed and categorized based on themes related to the research questions.

### 3.2.2 Classroom Observations

Non-participant observations were conducted in a grade-9 class introducing locus for the first time by the teacher having 18 years of teaching experience. The observations were conducted over two sessions including the introduction of the topic of locus in the first one and solving geometric problems related to locus, which includes making conjectures and making proofs in the second session. During observation, detailed observation log tool was used to capture the obstacles that students might face. The classes were videotaped and a rubric (Appendix B) was filled based on the video tape of the class. The transcription of the videotapes and the analysis of the observation log and the rubric identify the difficulties that students face in dealing with the locus topic. The rubric was used as a checklist to give an idea about class instructional interactions (teacher-student interactions); it includes items that help observe the interactions between the teacher and the students.

### 3.2.3 Interview with the students

Appendix C provides the questions that were asked to the students during the interview. The interview aims to provide insights about the students’ thinking about Locus and the difficulties that they face while solving geometric problems involving locus. Four low achievers and four middle achievers were interviewed. The level of the
students was determined by the teacher based on their achievement in math during the current year.

The interview is audio-taped, transcribed, analyzed and then categorized.

3.2.4 Paper-pencil test

The test was administered to the two grade-9 student groups for the two consecutive years within a time frame of two months from instruction of the “locus” chapter. Students were informed about the test and told to revise. In the first year, the test was given to the class that was taught the chapter “Locus” without using DGS. In the following year, the same test was given to the class that learned “locus” with the use of DGS. The test was done in class with a maximum time of 25 minutes where talking and asking questions were not allowed. The tools that students can use were the following: solution paper, scratch paper where students have to write down their thinking, geometric set (compass, protractor, ruler, set square), pen and pencil. The test (Appendix D) was designed by the researcher and consists of one problem composed of two parts, both requiring finding locus of a point, thus involving making a conjecture and proving it. The test starts with instructions to encourage students to show their thinking (see Appendix D).

The students’ papers were collected. They are analyzed using a marking scheme that the teacher usually uses to correct a test including conjecture and proof. The marking scheme provides an idea about what the teacher expects from students when solving problems about locus of points, based on the assumption that most of the students write proofs according to the expectations of the teacher. Also, a qualitative set
of criteria is developed based on the literature to analyze students’ conjecturing processes, proving strategies and proof writing format.

3.2.5 Analysis of the “Locus” unit in students’ textbook (for a copy of textbook unit, refer to Appendix E)

The unit about Geometric Loci in grade-9 textbook is analyzed to investigate the approach used. The approach and the content of the unit are examined. As for the approach, the way the unit is presented is analyzed to check the method of introducing the concept of locus and whether it allows the students to construct their knowledge through observing, testing, analyzing… etc. Moreover, the method of teaching is inspected by the researcher to check whether the unit adopts the problem solving method advocated in the curriculum, that is starting from real life situations and stressing on the usefulness of mathematics in real life world. As for the content, further analysis was conducted to check whether the activities allow the students to understand the concept of locus and whether the exercises and problems contained in the unit are enough and whether the exercises give an opportunity for the students to use technology. A detailed analysis of the unit is presented in the chapter 4 on Findings.

3.2.6 Development of the math teaching unit about locus (for the unit plan, refer to Appendix F)

Based on the analysis of the unit in students’ textbook, the researcher developed an instructional unit for teaching Locus. The unit covers the objectives of grade 9 related to “Locus”. It emphasizes on: a) real life situations where students have to model situations in which locus is used in real life, b) students’ active involvement in the
exploration of the locus of a point, c) the use of technology namely DGS that is Geometer’s Sketchpad.

3.2.6.1 Time-line

The unit is developed based on seven sessions (same number of sessions, for the same unit, as the year before). In three of the sessions, DGS is used. The researcher provided the teacher with the lesson plans and activities that were discussed and agreed upon.

3.2.6.2 Context

The class teacher (same grade-9 teacher as the previous year) taught the class during the implementation of the unit based on the lesson plans that were provided by the researcher. The researcher attended the sessions to guarantee that the lesson plans were correctly implemented and to collect data. Three out of the seven sessions took place in a computer lab in which students had access to computers with Geometer’s Sketchpad installed. The other sessions were conducted in a regular classroom with a chalk board and an overhead projector.

3.2.6.3 Material

The activities to be conducted in the computer lab were prepared by the researcher. These activities allow the students to come up with the different cases of locus of a point. The same homework exercises that were given to the non-DGS grade-9 students in the first year were given to the DGS grade-9 students of the second year.
3.2.6.4 Lab sessions

Three sessions took place in the computer lab. These sessions were videotaped in order to compare students’ interaction with the teacher in the first-year and the second-year classes (without vs. with the use of DGS).

3.2.6.5 The general objectives of the instructional unit

The general objectives of the unit are:

At the end of this unit, students should be able to:

- Identify the geometric loci of points satisfying given properties (listed in Appendix F)

- Solve open geometric locus problems requiring conjecturing and proving.

The distribution of the instructional unit is according to Table 1 below that provides an outline of the sessions that were covered. For more detailed information about the procedure of each session refer to Appendix F.

Table 1

Instructional unit sessions in glance

<table>
<thead>
<tr>
<th>Session number</th>
<th>Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 1</td>
<td>Students solve two questions prepared by the researcher with an objective to distinguish between fixed and variable elements.</td>
</tr>
</tbody>
</table>
Teacher introduces two cases of locus of a point through problem situations. Together, the teacher and the students, will model the problem situations.

Using a computer and an overhead projector, the teacher will model the two cases of locus of a point mathematically.

Session 2  Correction of H.W. + Teacher introduces three cases of locus of a point using a computer and an overhead projector + C.W. two exercises from students’ book

Session 3  Correction of H.W. + Teacher introduces the last two cases of locus of a point through problem situations. Together, the teacher and the students, will model the problem situations.

Using a computer and an overhead projector, the teacher will model the two cases of locus of a point mathematically

C.W. one exercise from students’ book

Session 4  Correction of H.W. + Students solve activities 1 and 2 in the computer lab

Session 5  Correction of H.W. + Students solve activity 3
3.2.6.6 Lab settings

First, students were prepared for the use of Geometer’s Sketchpad. Due to time limitation, the researcher asked the computer teacher to train all the 18 students on Geometer’s Sketchpad in the computer sessions by asking them simple constructions, two weeks before starting the unit implementation. Second, students with similar levels of achievement were paired. Class teacher explained to the students the activity procedures and the researcher interviewed one pair of students at a time, according to the clinical interview technique (Ginsburg, 1981), to investigate their thinking processes.

3.2.6.7 Clinical interviews

During each of the three DGS-based sessions, clinical interviews with students were conducted. Two pairs of low achievers and two pairs of middle achievers were interviewed. In the first lab session two pairs of low achievers were interviewed. In the second lab session one pair of middle achievers was interviewed. In the third lab session one pair of middle achievers was interviewed. One of the low achievers’ pair was chosen to be interviewed based on their request; however, the other pairs were chosen
randomly. Students were asked questions about their way of thinking while solving the problem and were free to answer the way they want. The pair was asked questions that reveal students’ way of thinking but did not affect their thinking. Examples of questions are: “Can you tell me how you reached the solution?” “Why did you choose to do this?” “Why are you moving this point?” .. etc.

The interviews were audio taped and the students’ computer files were saved and kept for analysis. (Note: the interviewed students were asked to save their work every 5 minutes under different files’ names in order to follow their work and compare the audio taping results with their sequences of figure manipulation).

### 3.2.7 Data collection instruments

This study uses a qualitative method of collecting and analyzing data. Qualitative data is collected through semi-structured interviews with the teachers of grade 9, classroom observations of grade-9 classes where locus is taught without students’ use of DGS, open interviews with eight students about the difficulties that they face while learning Locus, classroom observation of a major grade-9 problem-solving session using DGS during the implementation of the unit and clinical interviews with selected pairs of students solving geometric problems using DGS. Also, qualitative data are collected through administering tests to the participants and comparing their problem solving strategies and their scores in order to determine the effectiveness of the employed approach.
CHAPTER FOUR

FINDINGS

The present chapter presents the analysis of the “Locus” unit in the textbook, and the analysis of data collected through each instrument: semi-structured interviews with three grade-9 teachers, observation of the non-DGS class, open interviews with eight students, observation of a major grade-9 problem-solving session using DGS during the implementation of the unit, clinical interviews with four pairs of students solving geometric problems using DGS, and paper-pencil test administered to the two grade-9 classes (DGS class and non-DGS class), over two consecutive years.

To explore the major difficulties that students face while learning the concept of locus and while finding and proving locus of points, the audio-taped data of the interviews with the three teachers and the interviews with the students are transcribed, analyzed and categorized. Afterwards, the data of the sessions observed in class (transcription of the videotapes, observation log, and rubric) are analyzed.

In order to discover the ways DGS helps improving the learning process and better preparing students for solving problems involving locus of points, the conjectures and proofs presented by the non-DGS students in the paper-pencil test are categorized according to a framework developed prior to the analysis and refined during the analysis. Those conjectures and proofs are compared with conjectures and proofs presented by the students in the same paper-pencil test, after using DGS. Also, the
audio-taped data of the clinical interviews with students while solving Geometer’s Sketchpad activities are analyzed, along with their computer files.

### 4.1 “Locus” unit in students’ textbook

Although locus as per the Lebanese books is mentioned as of grade 7 and further introductions are made in grade 8, most schools do not introduce this topic before grade 9 since they believe it is not an important subject due to the low weight it carries in the official exams. The unit about Locus in grade-9 textbook (Refer to Appendix E) introduces locus by stating six different typical cases of locus and providing an example for each. The following figure (Fig. 1) provides an example of introducing one of the six cases, the perpendicular bisector case:

![Equidistant points of extremities of a segment](image)

**Fig. 1 Perpendicular bisector case**

To introduce the case of perpendicular bisector, the textbook starts by stating the title “Equidistant points of extremities of a segment” under which there is a sentence that explains it: “A and B are two fixed points. If M is a variable point such that MA =
MB, then the geometric locus of M is the perpendicular bisector of the segment [AB].”

Under the explanation of such a case, the textbook provides the following question with its solution as an example: “E and F are two fixed points. Find the geometric locus of points O, variable centers of circle passing through E and F. We have OE = OF = radius. With E and F fixed. The geometric locus of O is then the perpendicular bisector of segment [EF].” It is noticed that the textbook does not introduce the cases through real life problem situations. In other words, the chapter does not adopt the method of teaching through problem solving, that is starting from real life situations and stressing on the usefulness of mathematics in real life world. It does not encourage students to model real life problem situations into geometrical configurations to reach a solution to a problem. Thus, although the concept of geometric loci is closely related to real life problem situations, the way the chapter is presented in the textbook is purely informative and theoretical. Moreover, there is no explanation of the words “fixed points” and “variable points”. However, understanding what is meant by these words and the cases when some points are variable and some points are fixed is an important perception to the understanding of the idea of locus of points. As for the approach, no discovery activities are presented in the unit. On the other hand, the exercises do not hold any opportunity to use technology as a tool for exploration or even visualization.

To conclude, the presentation of the chapter in the textbook needs modifications by introducing real life problem situations and applying discovery activities that would encourage students to discover all locus cases that are important in their grade level. In addition to that the terms “fixed points” and “variable points” need further explanation.
by adding a section for this purpose. In addition, technology could be integrated to benefit from the dynamic aspects for exploration, better presentation and elaboration.

4.2 Students’ answers to sheets 1 and 2 in the math unit (Appendix F)

Based on the analysis of the unit in students’ textbook and the analysis of the interviews with the students about the difficulties they faced while learning locus, an instructional unit to teach Locus was developed. Sheets 1 and 2 (in Appendix F) are part of this instructional unit. The DGS class worked on these two sheets. The rationale behind using these two sheets is that understanding what is meant by variable and fixed elements is necessary to the understanding of the notion of locus; these two sheets were solved in the first session of the unit. The objective of sheet 1 is to allow the students to explore/induce the conditions on their own and determine when they have fixed or variable elements. Before students started solving sheet 1, the researcher explained the meaning of a fixed and a variable element (fixed element is an element that has only one location, whereas the variable element is an element that may move and take different locations, usually on a specific geometric object, such as a circle, a straight line, etc.). The objective of sheet 2 is to check if the students understood the conditions for having fixed or variable elements following the discussion of their answers to sheet 1, conducted by the teacher with the whole class. The different parts in sheets 1 and 2 respectively address the same objectives and similar contents; however, sheet 1 was introduced prior to any explanation and sheet 2 followed the discussion of sheet 1. Following is a table (Table 2) presenting the case addressed in each part:
Table 2

Case addressed in each part of sheets 1 and 2

<table>
<thead>
<tr>
<th>Parts</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sheets 1 and 2</strong></td>
<td></td>
</tr>
<tr>
<td>part 1</td>
<td>The midpoint of a fixed segment is fixed</td>
</tr>
<tr>
<td>part 2</td>
<td>A segment with a variable endpoint is variable</td>
</tr>
<tr>
<td>part 3</td>
<td>A straight line drawn from a fixed point and perpendicular to a fixed straight line is fixed</td>
</tr>
<tr>
<td>part 4</td>
<td>A straight line drawn from a fixed point and parallel to a fixed straight line is fixed</td>
</tr>
<tr>
<td>part 5</td>
<td>A circle whose center is variable or whose radius is not constant is variable</td>
</tr>
<tr>
<td><strong>Sheet 1</strong></td>
<td></td>
</tr>
<tr>
<td>part 6</td>
<td>A straight line passing through two fixed points is fixed</td>
</tr>
<tr>
<td><strong>Sheet 2</strong></td>
<td></td>
</tr>
<tr>
<td>part 6</td>
<td>A circle with fixed center and constant radius is fixed</td>
</tr>
</tbody>
</table>
The following table (Table 3) provides a summary of the outcome of sheets 1 and 2.

**Table 3**

**Outcome of sheets 1 and 2**

<table>
<thead>
<tr>
<th>Student's number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sheet 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>figure</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sheet 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>part 1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sheet 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>part 2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sheet 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>part 2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sheet 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>part 2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sheet 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>part 3</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sheet 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>part 3</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sheet 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>part 4</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sheet 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>part 4</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sheet 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>part 5</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sheet 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>part 5</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sheet 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>part 6</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sheet 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>part 6</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

It is noted from the table above that more than half of the students did not solve sheet 1 completely or solved it completely with mistakes; however, after in-class explanation of sheet 1 they solved all of sheet 2 or most of it correctly.

The outcome of the above practice of sheet 1 was that 10 out of 18 students drew the figure and correctly answered all the six parts on their own.
The following is a summary of the number of students who did not answer parts of sheet 1 correctly:

- 2 out of 18 did not answer parts 1 and 2 correctly.
- 3 out of 18 did not answer part 3 correctly.
- 1 out of 18 did not answer part 4 correctly.
- 4 out of 18 did not answer part 5 correctly.
- 5 out of 18 did not answer part 6 correctly.

This outcome outlines the fact that the cases addressed in parts 1 and 2 are relatively easy for students, since only 2 out of 18 students did not answer these parts correctly. Those are the case of the midpoint of a fixed segment and the case of a segment with variable endpoint. The cases addressed in parts 5 and 6 (sheet 1) seem to be the most challenging to students. These cases are: the case of a circle with variable center or variable radius measure, and the case of a straight line passing through two fixed points.

After the students solved sheet 1, the teacher drew the figure on the board and tried to help the students come up on their own with the conditions for having fixed or variable elements. After that explanation, 14 out of 18 students solved the whole sheet 2 correctly.

The following is a summary of the number of students who did not answer parts of sheet 2 correctly:

- All the students answered parts 1, 3 and 4 correctly.
- 1 out of 18 did not answer part 2 correctly.
- 3 out of 18 did not answer part 5 correctly.
- 2 out of 18 did not answer part 6 correctly.

This outcome tells that a few students still have difficulty in the cases addressed in parts 5 and 6 (sheet 2). These are the case of a circle with variable center and the case of a circle with fixed center and constant radius.

One of the aims of this research is to show that the use of technology may allow students to visualize relationships of dependency between geometric objects, and to explore the conditions for variability of those objects.

### 4.3 Interview with the teachers

The interview is conducted with three different teachers named as Teacher 1, Teacher 2, and Teacher 3. The interview consists of 12 questions intended to uncover the following categories: teachers’ strategies of teaching mathematics, especially ‘locus’, teachers’ views of students’ major difficulties in learning locus, teachers’ awareness of the benefits of using DGS in class, and teachers’ views of ways to enhance proof abilities. The interview is scheduled for 30 minutes and audio-taped, transcribed, analyzed and then categorized.

#### 4.3.1 Teachers’ strategies of teaching mathematics, especially ‘locus’

When asked about their strategies of teaching mathematics, all three teachers said that they do not have enough time to do activities in class. According to the three teachers, the strategy of explanation of any mathematics lesson starts with prerequisites on which builds the new concept; a rule or formula is stated for the students to apply,
imitating the teacher’s demonstration. Teacher 1 said: “I give one example on the board to illustrate the way of solving then the students complete the remaining problems alone”. Teacher 3 said: “I start with easy problems that are direct application, of which algebra exercises are solved on the board by the students while geometry problems are discussed by the students and I write the proof on the board”. Teacher 2 said: “activities depend on years of experience (if I do not have much of experience I might not have found a nice activity that would make the objectives reach easily, in this case I might use an example to help students reach the objective)”.

When asked about the strategy that they use to introduce the locus topic, all three teachers revealed that they start with stating the definitions of the terms (fixed and variable points) and the different cases of locus of points. Teachers 1 and 3 said that they explain the terms _“variable”_ and _“fixed”_ points and then tell the students that a locus of a point can either be a straight line or a circle. Then a PowerPoint slide show of geometric diagrams prepared on Sketchpad is presented to students about examples on every case of locus. The students have to memorize these properties of a point that make it move on a certain path (either circle or straight line). To conclude, the classes of the three teachers are teacher-centered and leave no room for student-student discussions.

**4.3.2 Teachers’ views of students’ major difficulties in learning locus**

When asked about the special aspects of the locus topic that make it difficult, all three teachers said that it’s not a direct application; it needs analysis and higher-order thinking. Teachers 2 and 3 stated that the way a locus question is stated discourages students from solving it, only high-order thinking students or those who study and
memorize the cases well and who search a lot are able to solve locus questions. Teacher 2 said: “50% of the class can’t see the answer of geometric problems dealing with locus topic”. The three teachers agreed that the low percentage that the locus part carries of the overall grade makes the students believe that it would be a waste of time to study for it; students hear from others that it’s not an important chapter and it does not affect the grade in the official exams.

When asked about the difficulties that students face while solving geometric problems involving locus, teacher 1 said: “looking for the condition and sometimes discovering the fixed elements (students mix between fixed and variable elements)”.

Teachers 2 and 3 are not sure of the difficulties that their students face. According to them, the proof of locus problems is easy; however, not all the students know the conjecture because they either do not study much or they do not memorize the cases.

4.3.3 Teachers’ awareness of the benefits of using DGS in class

The three teachers are aware of the benefits of DGS in class. All three teachers agreed that there are difficulties while explaining locus but with the use of DGS these difficulties are remedied since DGS illustrates the answer graphically for the students. Teachers 1 and 3 actually use Sketchpad during the explanation of the lesson and sometimes during solving the exercises in which justifications are not clear as it helps clarify the geometric property. Teacher 2 uses Sketchpad to help students visualize the solution, and asks them to draw on their own. In her opinion, it is better for students to work on the computer but due to time constraints students only draw in the copybook since this is the situation in the official exams.
All teachers agreed on the benefit of using dynamic geometry software for teaching the locus concept. Teacher 1 said: “It’s very important. It will make students see the path”. Teacher 2 said: “Sure, it can help students visualize the answer while on paper they can’t see something is moving”. Teacher 3 said: “Sure, it saves time needed to draw many figures to conclude the locus. It makes explanation easier and clearer when the students see the color and the variable point moving”.

When asked whether using dynamic software convinces students about the conjecture, and about the software’s effect on the students’ conception of proof, teacher 3 said: “the proof of locus of a certain point is easy as it only asks students to indicate: the fixed points, if the length is constant or if the angle formed by the fixed points and the variable point is 90°, the locus, and the property”. Teacher 2 said: “sometimes when using the software students discover that the locus is a circle but they can’t find the center or the radius so they have to go back and try their own drawing”. Teacher 1 said: “writing proof is an easy work; one has to know the variable point and fixed elements and to notice the condition that will directly lead to knowing the locus and writing the proof”.

When asked whether DGS hinders the teaching of proof and whether the students feel that the proof isn’t necessary anymore, Teacher 1 said: “students have to write the proof; they have to write to explain”. Teacher 2 said: “According to the premise of a certain problem, students might visualize from the software that the locus is a circle unaware that the question might be asking about an arc”. Teacher 3 said: “DGS helps in finding the conjecture but not in proving. While using the DGS, the teacher has to coordinate between writing the proof and finding the conjecture.”
4.3.4 Teachers’ views of ways to enhance students’ proving abilities

When asked about the ways to encourage students to prove a statement that they are convinced is true, all teachers said that students have to write the proof. Teacher 1: “there is a way of proving that they have to follow while writing”. Teacher 2: “asking students to prove something would let them make sure that their figure is right. Proving would justify the solution to the reader”. Teacher 3: “Students have to get accustomed to writing a proof in geometry from grade 7”.

4.3.5 Conclusion

It is notable that the three classes are teacher-centered and leave no room for class discussions. All three teachers were teaching for the purpose of the official exams justifying that they did not have enough time to do in-class activities or to let the students work in the computer lab. In their opinion, students should only draw on the copybook since this is the situation in the official exams. When asked about the strategy that they use to introduce the locus topic, all three teachers revealed that they start with stating the definitions of the terms (fixed and variable points) and the different cases of locus of points. The teachers are not sure of the difficulties that their students face in the locus topic. When asked about the special aspects of the locus topic that makes it difficult, all three teachers said that it is not a direct application; it needs analysis and higher-order thinking. The three teachers added that the low percentage that the locus part carries of the overall grade makes the students believe that it would be a waste of time to study or practice for it; students hear from others that it is not an important chapter and it does not affect much the grade in the official exams. Although all three
teachers agreed that explaining locus the traditional way imposes a lot of difficulties that may be alleviated with the use of DGS as it provides a graphical solution, still they did not provide sessions in the computer lab for that purpose due to time constraints knowing that ninth graders already had a dedicated computer session per week. In that session, the math class teacher could have provided activity worksheets to the computer class teacher for students to solve and to explore since understanding the concept of locus related to many chapters in grades 10, 11 and 12 will enhance understanding and save time for the coming years. According to the three teachers, the proof of locus problems is easy. However, not all students can easily find the conjecture because they either do not study much or they do not memorize the cases. The teachers relate knowing the conjecture to the memorizing of the cases not to the understanding of each case or to students’ ability to visualize or to relate geometric objects. However, when the student understands the case and the situation of each case, the proof (analyzing the premises and reaching a conjecture) becomes easy. For the teachers, the proof of locus is a way that the students have to follow, that is, identifying the fixed and constant elements, finding the locus, and applying the property to write the proof.

4.4 Interview with the students

Open interviews were conducted with eight students (four low achievers and four middle achievers) in order to investigate the difficulties that they face while learning Locus. The interview with the students consists of 6 questions. The questions ask about the definition of the term ‘locus’, the difficulties faced while learning locus, the thinking processes while searching for the locus of a point, the difficulties faced while solving geometric problems about locus, the procedure suggested for helping a classmate or a
friend understand locus, and possible changes to be made by the class teacher to help students in better understanding locus. Four low achievers and four middle achievers were interviewed. The audio-taped interviews were transcribed, analyzed and then categorized into the following categories: difficulties faced while learning “locus”, ways of thinking to find a locus of a point, and difficulties faced when solving geometric problems about Locus.

4.4.1 Difficulties faced while learning “locus”

When asked about the difficulties faced while learning locus, two middle achievers stated that the difficulty is the imagination required while finding the locus of a certain point. Other students referred to the number of cases that they have to memorize. Thus, it can be concluded that the students’ perceptions of the nature and requirements of the Locus theme are limited to memorizing a number of cases; however, teaching should guide students to seek solutions, formulate conjectures, explore patterns not just memorize properties and procedures to solve exercises.

4.4.2 Ways of thinking to find the locus of a point

When asked about their thinking processes while searching for the locus of a point, one low achiever said: “I do not solve the question on locus since if I solve it or not, my grade won’t be affected much”. Thus, one of the reasons that reduces students’ motivation to understand locus is that the questions of Locus lesson carry low percentage in the official exams. The other students stated that they imitate the teachers’ technique in solving the question by marking the fixed and the variable points and plotting many possible locations of the variable point to find the path/locus of the
variable point. But all of them said that when the figure has too many fixed points and variable points they get confused.

When the interviewer asked the students to suggest a way for them to explain to a classmate/friend who does not understand locus well, one low achiever answered by: “I’ll try to understand the explanation of the book, I would explain it if I understood it otherwise I will not explain”. Another low achiever repeated the technique that he learned from his teacher that is to draw many possible locations of the variable points/geometric objects and explain how the variable point moves. However, when the interviewer asked him to explain to her, he answered by: “I do not know”. One middle achiever answered by: “if I tell him, I’ll make him more confused”. Another middle achiever answered by: “after explaining the writing of the proof, I’ll teach how to draw many possible locations of the variable points”. One other middle achiever answered by: “I’ll let him memorize all the cases” when the interviewer asked him: “automatically?” he answered: “the teacher asked us to memorize all the cases before letting us understand the meaning of the word locus”.

**4.4.3 Difficulties faced when solving geometric problems about Locus**

The interviews revealed that one of the reasons for the low achievers to face difficulty in dealing with geometric problems about locus might be that they do not understand what is meant by the question. When asked to state the meaning of the locus of a point, three out of the four low achievers immediately answered by “I do not know” and the fourth low achiever answered by “the figure that’s formed by a variable….!” And then she said: “I do not know”.
When asked about suggesting more techniques for the teacher to explain locus, one middle achiever student answered: “the teacher has to explain locus using more slides and has to present and ask us to memorize all the cases and not to let us focus only on four cases; and the definition of locus has to be explained first and then ask us to memorize the cases. Because when we started memorizing the cases, we did not know what we are memorizing”. Another middle achiever student answered: “he has to give us more time to understand it well, for example, the usage of the overhead projector was useful but the teacher started to give us complicated questions that confused me”. One low achiever answered: “the teacher has to explain more how to move the variable point; the Sketchpad automatically showed us the answer/the path of the variable point”. One middle achiever answered: “solve more exercises”. One low achiever answered: “I would like the teacher to explain more the four cases to me alone in the recess”. This student thinks that the cases of the locus of a point are only four cases since his teacher asked the whole class to stress on four cases that are the most common in the official exam questions. One middle achiever said that the way the teacher explained the lesson was a good way. Also, two low achievers suggested the same way of explanation (teacher solves some exercises using Sketchpad and students watch and try to visualize).

4.4.4 Conclusion

The interviews with the eight students revealed that their perceptions of the nature and requirements of the locus theme were limited to memorizing a number of cases and they did not care much to understand locus as it carries a low percentage in the official exams. Moreover, students faced difficulty in dealing with geometric problems about locus since they do not understand the meaning of the question. The notion of
locus of a point was still vague to them. They should have a definition that is stated and explained through real life examples in order to be understood before getting memorized.

4.5 Observation of the Non-DGS class

Two sessions were observed. The locus topic was introduced in the first, while the second session included solving geometric problems related to locus and involving making conjectures and proving. The classes were videotaped and a rubric (Appendix B) was filled based on watching and transcribing the video tape. The researcher took notes and kept an observation log throughout the two sessions. The transcription of the videotapes and the analysis of the observation log aimed to identify the difficulties that students face in dealing with the locus topic. The analysis of the rubric provides a picture of the class instructional interactions (teacher-student and student-student interactions).

After the transcription of the videotapes and the analysis of the observation log, the following points were noted:

- The class is teacher-centered with the teacher making all the decisions without involving the students in any discovery or exploration process. He starts the chapter by defining the word locus as: “the set of points that possess a certain property” and he presents all the conditions for having fixed or variable elements in a Power point slide.
The teacher uses Geometer’s Sketchpad to demonstrate the different locus fundamentals/cases. He explains the definition of the locus, the conditions for having fixed and variables elements, and all the 8 fundamentals/cases in one session.

The interaction between the teacher and the students occurs at the time when the teacher states a case and demonstrates a way for proving it whereas the students tend to imitate the teacher's explanation.

In solving problems involving locus, the teacher helps students in their thinking process to formulate a geometric proof by explaining the steps they should follow. He stresses on reading the premises correctly and accurately in order to raise attention to fixed and variable elements. In addition to that, he helps the students in analyzing the premises in order to reach and formulate a proof.

4.6 Observation of classes with students’ use of DGS

Three computer lab sessions were observed and videotaped. The researcher took notes and kept an observation log throughout these sessions. The transcription of the videotapes and the analysis of the observation log aimed to check out how the use of dynamic geometry software changes the class environment and to compare students’ interaction with the teacher between the non-DGS class and the DGS class.

After the transcription of the videotapes and the analysis of the observation log, the following points were noted:
- The DGS class showed motivation as students were socially active and productive. Thus the DGS provided a non-traditional way for students to learn and understand new mathematical concepts.

- The students were encouraged with conversations that expanded their communication about locus. When the students worked in pairs, they had the opportunity not only to imitate the work of others, but they were also involved with discussions that deepened their understanding of the locus topic.

- The interaction between the teacher and the students occurred at the time when a student wanted to check the writing format of the justification. The teacher would listen to students’ answers and justifications and try to convince the students why their answers were wrong, whenever applicable.

- After solving each activity sheet, the teacher discussed the solution with the students using Geometer’s Sketchpad. He raised their attention to fixed and variable elements. In addition to that, he helped the students in analyzing the premises in order to reach and formulate a proof.

In conclusion, the DGS sessions can be described as student centered as there was room for in-class interaction and the students were more motivated about the topic that seemed more interesting to them. Through these sessions they were also involved with discussions that deepened their understanding of the locus topic. The teacher communicated with the students about their answers and justifications and tried to convince them why their answers were wrong, whenever applicable. On the other hand, the Non-DGS sessions can be described as teacher centered due to minimal interaction
where the teacher states a case and demonstrates a way for proving it whereas the students tend to imitate the teacher's explanation without having a role or a clear understanding about the discovery or exploration process. Besides reaching a proof was easier with using DGS as the fixed and variable points were clear which helped the students in their analysis.

4.7 Framework for analyzing students’ conjectures and proofs

To qualitatively analyze students’ conjecturing processes, proving strategies and proof writing format produced in both, the DGS environment and the paper-pencil test, a framework was developed and adopted.

Note that: 1) for clarity reasons, the students’ handwritten answers given as example under each category were typed and provided after each figure, and 2) the scratches and the check marks on the scanned students’ handwritten answers were all done by the teacher while correcting the test.

The framework was compiled by the researcher based on frameworks used in the literature and on a primary overview of students’ work. Following are the categories in the framework, with examples illustrating each one. These examples relate to the problem included in the paper-pencil test. The problem is:

*Draw a circle (C) of fixed center O and a fixed diameter [MN]. [Nx) is the tangent at N to (C). D is a variable point on [Nx). [DM] cuts the circle at E.*

1) What is the geometric locus of I, the midpoint of [MD]?

2) Let K be the symmetric of N with respect to E.

What is the geometric locus of K?
i. **Framework for analyzing students’ conjectures:**

a. **No Conjecture (NC):** It is when student does not provide any conjecture.

b. **Invalid Conjecture based on Basic Misunderstandings (ICBM).** This category includes two subcategories: Invalid conjecture reflecting Misunderstanding of the Notion of Locus (MNL) and Invalid conjecture reflecting Misunderstanding of Fixed/Variable elements (MFV).

   b1. **Invalid conjecture reflecting Misunderstanding of the Notion of Locus (MNL):** It is when student provides a wrong conjecture based on misunderstanding of the notion of locus. The following figure (Fig. 2) is an example (Part 1 of the problem):

   ![Image of MNL](image124x259_to_525x315)

   **Fig. 2 Example of MNL**

   *The locus of I is a pt moving on the \( \perp \) bisector of \([MN]\).*

   Note: The student stated the locus, not in terms of the fixed geometric object, but in terms of a point moving on it.
b2. Invalid conjecture reflecting Misunderstanding of Fixed/Variable elements (MFV): It is when student provides an incorrect conjecture where the locus assumed is based on variable elements. The following figure (Fig. 3) is an example (Part 2 of the problem):

![Fig. 3 Example of MFV](image)

*The geometric locus of K is circle of center E diameter KN.*

Note: The student provided a conjecture based on a variable point E.

c. Incorrect Conjecture (IC): It is when student provides a wrong conjecture. The following figure (Fig. 4) is an example (Part 1 of the problem):

![Fig. 4 Example of IC category](image)

*The locus of I is a circle (O, 5cm).*

Note: In this part of the problem, the locus is either a straight line (OI) // (xy) or the perpendicular bisector of [MN].
d. Correct Conjecture (CC): It is when student provides a correct conjecture about the right locus. This category includes four subcategories: Correct Incomplete Conjecture (CIC), Correct Conjecture with No Signs (CCNS), Correct Conjecture Empirical Drawing (CCED), and Conjecture providing a Partial Locus (CPL). Following are the different subcategories:

d1. Correct Incomplete Conjecture (CIC): It is when student provides a correct incomplete conjecture. The following figure (Fig. 5) is an example (Part 2 of the problem):

![Example of CIC](image)

2) Let K be the symmetric of N with respect to E. What is the geometric locus of K?
- The geometric locus of K is a circle.

Fig. 5 Example of CIC

The geometric locus of K is circle.

Note: The student stopped her conjecture at “circle”, and missed saying that the locus is a circle of center M and radius [MN].

d2. Correct Conjecture with No Signs (CCNS): It is when student provides a correct conjecture but no signs appear on the figure to indicate how it was found.
Since students were not allowed to use scratch paper other than the test paper, the absence of codes or signs from the figure imply that finding the conjecture was based on reasoning rather than on empirical evidence. The following figure (Fig. 6) is an example (Part 1 of the problem):

**Given:** Draw a circle (C) of fixed center O and a fixed diameter [MN].

\((x, y)\) is the tangent to (C) at N. D is a variable point on \((x, y)\). [DM] cuts the circle at E.

1) What is the geometric locus of I, the midpoint of [MD]?

- The locus of I is a str. Line parallel to \((xy)\) drawn from pt. O

**Fig. 6 Example of CCNS**

_The locus of I is a str. Line parallel to (xy) drawn from pt. O_

Note: The figure shows exactly the geometric objects involved, with neither signs nor multiple cases. The conjecture being correct, we can safely assume that student relied on reasoning to find the conjecture.

d3. Correct Conjecture based on Empirical Drawing (CCED): It is when student provides a correct conjecture, and the figure shows many cases of the variable objects drawn, which implies that student tried to find the locus by empirically creating many points and by visually inducing the locus. The following figure (Fig. 7) is an example:
**Fig. 7 Example of CCED**

The locus of \( I \) is the st. line // \((xy)\)

The geometric locus of \( K \) is circle \((C')\) \((M, [MN])\)

Note: It is clear from the figure that student drew several cases of point \( D \), variable point on the tangent, which in turn led to several cases of points \( E, I \) and \( K \).

d4. Conjecture providing a Partial Locus (CPL): It is when student provides a conjecture providing a correct part of the locus. The following figure (Fig. 8) is an example (Part 1 of the problem):

\[
\text{Given: Draw a circle (C) of fixed center O and a fixed diameter [MN].}\]
\[
(x, y) \text{ is the tangent to (C) at N. D is a variable point on (x, y). [DM] cuts the circle at E.}\]

1) What is the geometric locus of \( I \), the midpoint of \([MD]\)?
   - The locus of \( I \) is the st. line // \((xy)\).

2) Let \( K \) be the symmetric of \( N \) with respect to \( E \). What is the geometric locus of \( K \)?
   - The geometric locus of \( K \) is circle \((C')\) \((M, [MN])\).
The locus of I is a str. Line (OI) // (xy) and from the same side of [MN]

Note: In this part of the problem, the correct locus is the straight line (OI). The student stated that the locus would be the part of (OI) that is from one side with respect to (MN).

ii. Framework for analyzing students’ proof:

Some categories of the framework developed by Marrades and Gutiérrez (2000) were used. Some other categories have been modified based on students’ work.

a. No Proof (NP): It is when student does not provide any proof and this would occur in both cases whether a correct or an incorrect conjecture was provided. The following figure (Fig. 9) is an example (Part 2 of the problem):

The geometric locus of K is a circle of radius 2R, center M
b. Incorrect Proof (IP): It is when student provides a wrong proof. This category includes two subcategories: Incorrect Proof reflecting confusion between Fixed/Variable (IPFV) and Incorrect Proof based on Geometric Misconceptions (IPGM). Following are the different subcategories:

b1. Incorrect Proof reflecting confusion between Fixed/Variable (IPFV): It is when student provides an incorrect proof because of confusion between fixed/variable elements. The following figure (Fig. 10) provides an example (Part 2 of the problem):

**Proof:** K symmetric of N with respect to E then E midpoint of [NK].

NE = EK (def. of midp.) then when K varies E stays fixed which would form a circle of center E and diameter KN.

then K is a pt on a circle of center E.

Note: The major error in this proof is the statement “E stays fixed”.

b2. Incorrect Proof based on Geometric Misconceptions (IPGM): It is when student provides a wrong proof because of geometric misconceptions related to definitions, properties, theorems, or because of wrong inferences. The following figure (Fig. 11) provides an example (Part 2 of the problem):
Fig. 11 Example of IPGM

Proof: $E$ midpt $[KN]$ (by central symmetry)
$M, O, N$ are fixed pts. $E$ varies on $(C)$, $D$ varies on $(xy)$
In $\Delta MKN$ we have: $O$ midpt $[MN]$ (proved)
$E$ midpt $[KN]$ (proved)
Then $(OE)/(KM)$ and $OE = KN/2 = R$ (mid segment theorem)
then $KN = 2R$ then the locus of $K$ is a circle $(N, 2R)$

Note: The major error in this proof is the student’s wrong inference while applying the mid-segment theorem. $O$ being the midpoint of $[MN]$ and $E$ being the midpoint of $[KN]$ then $OE = KM/2$ and not $OE = KN/2$.

c. Correct Proof (CP): It is when student is able to provide a correct justification for the correct conjecture. This category includes three subcategories: Correct Not Complete Proof (CNCP), Correct Proof adding Unnecessary Statements (CPUS) and Correct Complete Proof (CCP). Following are the different subcategories:

c1. Correct Not Complete Proof (CNCP): It is when student is able to provide a correct proof but missing some important steps or statements. The following figure (Fig. 12) provides an example (Part 2 of the problem):
The geometric locus of $K$ is a circle $(M, 2R)$.

Proof: in $\triangle MNK$ we have: $K$ symmetric of $N$ w.r.t. $E$ then $E$ is the midpt of $[KN]$

$O$ midpt $[MN]$ then $(OE) \parallel (MK)$ and $OE = \frac{1}{2}$ of it and $OE = R$ then $MK = 2R$.

Note: The student stopped her proof at $MK = 2R$, and missing stating that, since $M$ is a fixed point and $E$ is a variable point the locus of $K$ is a circle $(M, 2R)$

c2. Correct Proof adding Unnecessary Statements (CPUS): It is when student provides a correct proof but includes statements that do not contribute to the proof. The following figure (Fig. 13) provides an example (Part 1 of the problem):
Fig. 13 Example of CPUS

The locus of I is a st. line // (xy)

Proof: O midpoint [MN] (center is the midpoint of the diameter). M, O, N are fixed pts. D is a variable pt on (xy) and E varies on (C) and $\overline{DN}$ right (tangent) then $\Delta DNM$ is right (triangle having a right angle is right) I midpt [MD] then In $\Delta DNM$ (IO) // (xy) (midsegment theorem of a $\Delta$) then the locus of I is a st. line (IO) // (xy)

Note: The student provided a correct justification for a correct conjecture; however, he added unnecessary statements such as “$\overline{DN}$ right (tangent) then $\Delta DNM$ is right (triangle having a right angle is right)”. The fact that the triangle DNM is right does not contribute to the proof.

c3. Correct Complete Proof (CCP): It is when student is able to provide a correct complete proof for the correct conjecture. The following figure (Fig. 14) provides an example (Part 1 of the problem):
Fig. 14 Example of CCP

**Proof:** In $\triangle MDN$ we have:
- $O$ midpt of $[MN]$ (center of a circle is midpt of diameters)
- $I$ midpt of $[MD]$ (given)

Then $(OI) \parallel (DN)$ and $OI = DN/2$ (midsegment theorem of a $\Delta$

$O$ is a fixed point, $D$ is a variable point on $(xy)$

Then the locus of $I$ is a str. Line parallel to $(xy)$ passing through $O$ or drawn from $O$.

The following table (Table 4) summarizes the framework:

<table>
<thead>
<tr>
<th>Conjecture Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. No Conjecture (NC)</td>
</tr>
<tr>
<td>b. Invalid Conjecture based on Basic Misunderstandings (ICBM)</td>
</tr>
<tr>
<td>b1. Invalid conjecture reflecting Misunderstanding of the Notion of Locus (MNL)</td>
</tr>
<tr>
<td>b2. Invalid conjecture reflecting Misunderstanding of Fixed/Variable elements (MFV)</td>
</tr>
<tr>
<td>c. Incorrect Conjecture (IC)</td>
</tr>
<tr>
<td>d. Correct Conjecture (CC)</td>
</tr>
</tbody>
</table>
d1. Correct Incomplete Conjecture (CIC)

d2. Correct Conjecture with No Signs (CCNS)

d3. Correct Conjecture based on Empirical Drawing (CCED)

d4. Conjecture providing a Partial Locus (CPL)

Proof Categories

a. No Proof (NP)

b. Incorrect Proof (IP)

b1. Incorrect Proof reflecting confusion between Fixed/Variable (IPFV)

b2. Incorrect Proof based on Geometric Misconceptions (IPGM)

c. Correct Proof (CP)

c1. Correct Not Complete Proof (CNCP)

c2. Correct Proof adding Unnecessary Statements (CPUS)

c3. Correct Complete Proof (CCP)

4.8 Results

In this section, the clinical interviews are analyzed. In addition, some categories of the adopted framework are used to categorize the conjectures and the proofs produced by the clinically interviewed pairs when solving the DGS activity sheets. Moreover, the conjectures and the proofs of the paper-pencil test of the non-DGS class and of the DGS-class are categorized according to the adopted framework. Last, the section presents a
comparison between the non-DGS class and the DGS class in terms of the conjectures and the proofs produced by the students in the paper-pencil test.

4.8.1 Clinical interviews

This section presents the analysis of four clinical interviews conducted with four different groups of students with different levels of achievement (two pairs of low achievers -group 1 and group 2- and two pairs of middle achievers -group 3 and group 4). The aim of these clinical interviews is to explore the thinking strategies used by the selected students, by discovering and identifying their cognitive thinking for solving different Geometer’s Sketchpad activities involving locus problems. One of the two pairs of low achievers was chosen to be interviewed based on personal request; however, other pairs were chosen based on two reasons:

- They are homogeneous and belong to two levels of achievement (two pairs of low achievers and two pairs of middle achievers).

- They use fluently the functionalities of Geometer’s Sketchpad.

All along the interview, care was taken in order not to interfere in the solution process or suggest any solution path. The pairs were asked “how” and “why” they approach the problem in a way or another, such as: “Can you tell me how you reached the solution?” “Why did you choose to do this?” “Why are you moving this point?”

To explore the thinking strategies used by the selected students, the analysis of the transcribed data and Geometer’s Sketchpad files is undertaken according to three steps:
- The conjectures and proofs presented by the pairs are categorized according to some categories of the framework.

- The transcripts of the audio-tapes and the Sketchpad’s files are analyzed.

- The ways DGS was used in exploring and making conjectures are identified.

The interviews were audio-taped in order to keep track of the groups’ thinking-aloud discourse and their interactions with the interviewer. The students’ computer files were saved and kept for analysis. (Note: the interviewed students were asked to save their work every 5 minutes under different files’ names in order to follow their work and triangulate the audio taping results with their sequences of figure manipulation). Data are transcribed and Geometer’s Sketchpad’s files are analyzed. To note that all the groups were able to reach a correct justification for their conjectures.

In the first lab session, group 1 worked on activity sheet 1 and group 2 worked on activity sheet 2 (refer to Appendix F). The session for solving these two activities lasted around 45 minutes.

In the second session, group 3 worked in activity sheet 3 (refer to Appendix F). The session for solving this activity lasted around 25 minutes.

In the third computer lab session, group 4 was interviewed. This group did activity sheet 4 (refer to Appendix F). The session for solving this activity lasted around 40 minutes.

The following section represents an account of the results of clinical interviews’ analysis.
4.8.1.1 Conjecture and proof categories

To select some categories from the framework to categorize the conjectures and proofs presented by the student pairs, the researcher referred to the DGS activity sheets, transcripts of the audio-tapes and the Sketchpad’s files.

The following tables (Table 5 and Table 6) summarize the conjecture categories for each part of the four activity sheets.

Table 5

*Categories of conjectures produced in DGS before using any DGS feature*

<table>
<thead>
<tr>
<th>Group number</th>
<th>Conjecture Categories</th>
<th>Activity sheet number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Sheet1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sheet3 part 1</td>
</tr>
<tr>
<td>1</td>
<td>CCNS</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>IC</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>IC</td>
<td>CCNS</td>
</tr>
<tr>
<td>4</td>
<td>CCNS</td>
<td>CCNS</td>
</tr>
</tbody>
</table>

NC: No Conjecture is provided
IC: Incorrect Conjecture is provided
CCNS: Correct Conjecture with No Signs in the figure to indicate how it was found
CCED: Correct Conjecture based on Empirical Drawing of many cases
Table 6

Categories of conjectures produced in DGS after using DGS features

<table>
<thead>
<tr>
<th>Group number</th>
<th>Conjecture Categories</th>
<th>Activity sheet number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Sheet1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>part 1</td>
</tr>
<tr>
<td>1</td>
<td>CCED</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>CCED</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>CCED  CCNS</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>CCNS  CCNS  CCED</td>
<td></td>
</tr>
</tbody>
</table>

NC: No Conjecture is provided  
IC: Incorrect Conjecture is provided  
CCNS: Correct Conjecture with No Signs in the figure to indicate how it was found  
CCED: Correct Conjecture based on Empirical Drawing of many cases

In table 5, there were 1 “No Conjecture” (NC) case and 2 “Incorrect Conjecture” (IC) cases. These disappeared in table 6. Also, all conjectures in table 6 are under “Correct Conjecture” (CC) category. Before using any DGS feature, group 2 provided an incorrect conjecture (IC), group 3 provided an incorrect conjecture (IC) for part 1 of activity sheet 3 and provided correct conjectures with No Signs in the figure to indicate how they were found (CCNS) for part 2, and group 4 provided correct conjectures with no signs in the figure to indicate how they were found (CCNS) for parts 1 and 2 of activity sheet 4 and did not provide any conjecture for part 3. However, after using some DGS features, these groups were able to provide a correct conjecture based on empirical
drawing of many cases. As a result, DGS features helped the students to adjust their way of thinking.

Categories of proofs for each part of the activity sheets were collected and summarized in the following tables. (Table 7 and Table 8)

*Table 7*

*Proof categories produced in DGS before using any DGS feature*

<table>
<thead>
<tr>
<th>Group number</th>
<th>Proof Categories</th>
<th>Activity sheet number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Sheet1 Sheet2 Sheet3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sheet3 Sheet4 Sheet4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>part 1 part 2 part 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>part 2 part 2 part 3</td>
</tr>
<tr>
<td>1</td>
<td>NP</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>NP</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>IPFV CCP</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>CCP CCP NP</td>
<td></td>
</tr>
</tbody>
</table>

NP: No Proof category containing a conjecture with no justification.
IPFV: Incorrect Proof reflecting confusion between Fixed/Variable.
CNCP: Correct Not Complete Proof missing some important steps or statements.
CCP: Correct Complete Proof for the correct conjecture.
Table 8

Proof categories produced in DGS after using DGS features

<table>
<thead>
<tr>
<th>Group number</th>
<th>Proof Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CCP</td>
</tr>
<tr>
<td>2</td>
<td>CNCP</td>
</tr>
<tr>
<td>3</td>
<td>CNCP</td>
</tr>
<tr>
<td></td>
<td>CCP</td>
</tr>
<tr>
<td></td>
<td>CCP</td>
</tr>
<tr>
<td>4</td>
<td>CCP</td>
</tr>
</tbody>
</table>

NP: No Proof category containing a conjecture with no justification.
IPFV: Incorrect Proof reflecting confusion between Fixed/Variable.
CNCP: Correct Not Complete Proof missing some important steps or statements.
CCP: Correct Complete Proof for the correct conjecture.

In table 7, there were 3 “No Proof” (NP) cases and 1 “Incorrect Proof reflecting confusion between Fixed/Variable” (IPFV) case. These disappeared in table 8, representing proofs after DGS use. Also, all proofs in table 8 are under “Correct Proof” category. Before using DGS features, group 1 could not provide a justification. However, after using DGS features, this group was able to provide a correct conjecture based on empirical drawings and was able to provide a “Correct Complete Proof” (CCP). Also, at the beginning, group 2 provided an incorrect conjecture and could not provide any justification; however, after visualizing the correct conjecture, they were able to provide a “Correct Not Complete Proof missing some important steps or statements” (CNCP). Although the proof that was provided was not a detailed proof since it was missing important properties, this group was able to provide a correct proof.
Moreover, at the beginning, group 3 provided an incorrect proof reflecting confusion between fixed/variable elements (IPFV) for an incorrect conjecture in Part 1 of sheet 3. However, after using DGS features, the students of this group were able to notice the correct conjecture and they were able to provide a correct not complete proof missing some important statements (CNCP) (like properties). Group 4, for the third part, could not notice the conjecture until DGS features were used. DGS features helped this group to provide a “Correct Complete Proof” (CCP).

Activity sheet 1

Group 1 started drawing the figure of activity sheet 1 and dragging segment [CD] in order to recognize the path of the center O of the rectangle ABCD. After dragging segment [CD], students came up with a conjecture of the path of point O; however, they could not provide a justification for that conjecture.

Interviewer: What’s the path?

Student 1: [CD] when it moves it can become below [AB] or above [AB]. It is already above [AB]. Then when the center of the rectangle moves it will make a straight line which will cut [AB] into 2 equal parts. So O can become the midpoint of [AB] or the path of it can become the perpendicular bisector.

Interviewer: So what’s the path of the center of the rectangle as [CD] varies?

Student 2: perpendicular bisector of [AB].

Interviewer: Why?

Student 1: If we move [CD] down, it will be here. If we move [CD] up, it will be here. When we move [CD], the center will vary on a straight line. The point will become perpendicular bisector of [AB].
The students knew the conjecture but they were not able to provide a justification. Afterwards, when asked about the path of center O when point C varies, the students thought that this part is unrelated to the first part (the path of center O when C varies).

**Student 1:** Now, we need to find the locus of center O of the rectangle ABCD when C varies on (d). If C moves but D stays in the same position... Let’s say for example AC = 4cm but the rectangle ABCD is on the upper level of point D, if we lower point C to the lower level of point D but AC remains 4cm I think it can stay a rectangle, ABCD can stay a rectangle.

After the interviewer’s question: “How do you think this software can help you in finding the locus of center O?” student 2 immediately used tracing and animating without answering.

**Interviewer:** After tracing and animating what did you find the path of center O?

**Student 2:** perpendicular bisector of [AB]

**Interviewer:** How do we justify?

**Student 1:** since D varies as C varies it depends on C then since O is the center of the rectangle then OC = OD = OA = OB then the locus of O is the perpendicular bisector of [AB].

The students were able to provide a correct justification for a correct conjecture.

To summarize, at the beginning, group 1 provided a correct conjecture from random dragging facility. Thus a correct conjecture, with no signs in the figure to indicate how it was found, was provided. In other words, this group came up with the
correct conjecture, not from using the *dragging with trace activated* modality. For this conjecture this group did not provide a justification. However, after using the *dragging with trace activated* modality, this group was able to provide a correct conjecture based on empirical drawings and was able to provide a correct complete proof. Thus the *random dragging* facility helped the students to come up only with a correct conjecture and not with a correct justification. However, after using the *dragging with trace activated* facility, the students were able to visualize the conjecture and provide a correct justification.

*Activity sheet 2*

Group 2 started by drawing the figure. Before using any facility of Sketchpad, this group provided an incorrect conjecture and was not able to provide any justification.

*Student 1:* Point I will move on straight line (AM) then the path will be on straight line (AM).

*Interviewer:* So, what’s the path of point I?

*Student 1:* Point I will move on line (AM) so the path of it will be straight line (AM).

Afterwards, students had to check their answer by tracing point I and animating point M.

*Student 2:* Miss, it’s a circle!! Of center B and radius [BI]
Interviewer: You both said before that it varies on straight line (AM). Why do you think your answer went to be wrong?

Student 1: Miss, I thought that I must only move point I, I thought that everything else was fixed. Now since we animated point M and traced point I, it showed that it's a circle of center B and radius [BI].

Interviewer: Why do you think that the locus of point I is a circle?

Student 2: $BI = 2 OM = 2 OA = BA$.
$I$ varies on circle of center $B$ and radius $2 OA$.

To conclude, the *dragging with trace activated* facility helped the students to adjust their way of thinking. First, they provided an incorrect conjecture and could not provide any justification. After using the *dragging with trace activated* modality, the students were able to visualize the correct conjecture and were able to provide a proof. Although the proof that was provided was not a detailed proof since it was missing important properties (e.g. $BI = 2 OM$ because of mid-segment theorem), this group was able to provide a correct proof.

*Activity sheet 3*

Group 3 started with drawing the figure of activity sheet 3. When the students of this group were asked about the path of point M, they were sure that they were able to know it. However, when they were told that they must not use any feature of Sketchpad, they felt that the path is difficult to be known and expressed their difficulty.
Interviewer: Do you think you are able to know the path of point M such that OAMB is a parallelogram?

Student1: Yes

Interviewer: What do you think the path/locus is? Without moving any point

Student1: Without moving any point Miss?? It’s difficult (With student smiling)

Afterwards, students were able to provide a conjecture and a justification.

Student2: Since it is a chord, and this is perpendicular to it

Student1: This is a parallelogram, since this is equal to that then it’s a parallelogram, then the locus of M is the perpendicular bisector of [AB]. Miss this is a rhombus, perpendicular bisector of [AB].

Interviewer: So always MA = MB?

Student1: equidistant

However, as a response to the part in the activity sheet asking to trace point M and animate point B, the students noticed their mistake and they were able provide a correct proof for the correct conjecture.

Interviewer: Write it down, write what is the path of point M and justify. To check your answer trace point M and animate point B.

Student1: M varies on the perpendicular bisector of [AB] since AM = MB

Students: Aha, it’s a circle.

Interviewer: You have to tell me now why the path is circle and not perpendicular bisector.
Student1: Miss, when we said perpendicular bisector, we forgot that B is variable. So for sure not perpendicular bisector

Interviewer: So?

Students: Not always MA = MB.

Interviewer: So?

Student1: O and A fixed and OA = AM then locus is circle of center A and radius [OA].

The students knew how to reason in order to come up with a correct conjecture.

Interviewer: What did you conclude?

Student2: that we must always look at the fixed points not variable

Interviewer: and what else?

Student1: What else? (With student smiling)

Interviewer: Only we look at fixed points?

Student1: and see what is the relation with the variable point

Interviewer: Okay, then we need to see what is the condition.

Students: Yes

Interviewer: And here what was the condition?

Student1: OA = AM

Interviewer: Before you said AM = MB.

Student: But we need to relate with the fixed points.

When solving part 2 of activity sheet 3 students were able to visualize the answer without the help of the DGS. However, students used the dragging test modality to check their conjecture.

Teacher: Where does the center of the parallelogram move when B varies?
Student1: I midpoint since it is a point of intersection since this is a rhombus \( AI = IB = \frac{1}{2} OA \). Circle of center \( I \) and radius \( IA = R/2 \). Ah, Miss, it makes angle 90° with \([AO]\), circle of diameter \([AO]\).

Interviewer: Always it is 90°?

Student2: Yes, since we proved that it is rhombus so always diagonals are perpendicular so it is always 90°.

Student1: First, we proved \( AOBM \) is a rhombus then diagonals are perpendicular bisector then \( AIO \) is a right angle then the path of \( I \) is a circle of diameter \([AO]\).

In brief, while solving part 1 of activity sheet 3, before using any feature of DGS, group 3 provided an incorrect proof reflecting confusion between fixed/variable elements for an incorrect conjecture. However, after using the dragging test modality, the students of this group were able to notice the correct conjecture and they were able to provide a correct proof missing some important statements (like properties) “\( O \) and \( A \) fixed and \( OA = AM \) then locus is circle of center \( A \) and radius \([OA]\)”.

Activity sheet 4

Group 4 was able to come up with a correct justification for a correct conjecture for part 1 of activity sheet 4 without the use of any dynamic Sketchpad facility.

Interviewer: What is the locus of point \( J \)? Without moving any point.

Student1: Parallel to \((AM)\)

Interviewer: Why?

Student1: If two perpendiculars to the same straight line then they are parallel.

Interviewer: Which two straight lines are perpendicular to the same straight line?

Student1: \((AM)\) and \((OJ)\) are perpendicular to \((AO)\)

Interviewer: So what is the path/locus of point \( J \)?
Student2: Straight line perpendicular to (AB) at O.

Interviewer: Ok. If we want to write a proof for the first part, how do we write it?

Student2: (x'y') perpendicular to (AO) at A

Student1: J is the midpoint of [MB]

Student2: Can we say O is the midpoint of [AB] then mid-segment theorem?

Interviewer: What do you think?

Student1: Yes.

While solving part 2 of activity sheet 4, group 4 came up with a correct conjecture and correct justification although there was a mistake in their drawing.

Student 2: We drew a circle of center J and J is a variable then the circle is variable all the time. (AJ) cuts (D) at K. Hide circle (D). On what path does K move as M varies?

Interviewer: without tracing

Student 2: O midpoint of [AB], J midpoint of [AK] (by central symmetry) then (OJ)//(KB) then (KB) perpendicular to (AB) then the path of K is straight line perpendicular to (AB) at B. only.

Interviewer: (reading the question to the students) Trace K and animate M in order to check your answer. Animate M not J. Why this is the path of K? Is K symmetric of A with respect to J? Is J the midpoint?

Students: No there is something wrong in the figure!!


Students: Yes

Interviewer: Redraw it.

Student1: We drew circle of center J and kept on extending it to reach point A then we located point K.
Interviewer: No, this is not the way to draw circle knowing center and radius.

(Interviewer corrected the figure)

To solve part 3 of activity sheet 4, group 4 could not come up with any conjecture until Sketchpad’s dynamic facilities were used. After using tracing and animating, group 4 came up with a complete proof for a correct conjecture.

Interviewer: So on what path does N move as M varies? (Students answered the question after they used the tracing and animating facility)

So what do you think?

Student1: Circle of center B.

Interviewer: Why do you think?

Student1: \( \angle AIB \) is a right angle.

Student2: Inscribed angle facing diameter is right.

Student1: Opposite angles are equal .... (after 1 minute of thinking) .. \( BA = BN \)

Interviewer: Why they are equal?

Student1: In congruent triangles.

Interviewer: How?

Student1: Right triangles, common side \([BI]\) , and \( IM = IA \) then they are congruent triangles by SAS then corresponding elements.

Interviewer: So always \( BN = BA \)? And what else?

Student1: and they have same vertex

Interviewer: Do you mean a common endpoint B?

What about this point?

Student1: it is a fixed point

Interviewer: Then? What is the locus of point \( N \)?

Student1: The locus of \( N \) is a circle of radius \([BA]\) and center \( B \).
Briefly, group 4 provided a correct proof for a correct conjecture for the first two parts of activity sheet 4 without the use of DGS facilities. However, for the third part, this group could not notice the conjecture until the *dragging with trace activated* facility was used. After using the tracing facility, students of this group provided a correct proof. Thus Sketchpad’s *dragging with trace activated* facility helped them to provide a formulated proof.

4.8.1.2 Ways DGS was used in making conjectures

After analyzing Sketchpad’s files of the interviewed pairs and the audio-taped data, the researcher found out that in the four clinical interviews the dragging feature of DGS (mainly Geometer’s Sketchpad) helped students to see as many examples as possible that directed them to check their conjecture or to formulate a conjecture. The researcher was able to identify the ways students used the dynamic geometry software to make conjectures. To develop a dynamic-conjecture (where students drag figures and observe changes) the students used these three different modalities of dragging points that made them shift from “ascending control” to “descending control”:

- *Wandering/random dragging*, that is when students randomly drag a base point on the screen to look for interesting configurations or regularities of the DGS-figure or to maintain a geometrical property of the figure (intentionally induced invariance).

- *Dragging with trace activated* in which students intend any form of dragging after the trace function has been activated on one or more points of the figure.

- *Dragging test* is a way used for testing a conjecture in which students move a figure through all its draggable points and observe that it keeps the asked property.
4.8.1.3 Summary

Before using the *dragging with trace activated* facility, students of group 1 (low achiever students) found a correct conjecture but could not provide a justification. However, after using the *dragging with trace activated* modality, this group was able to provide a correct conjecture based on empirical drawings and was able to provide a correct complete proof. Thus after using the *dragging with trace activated* facility, the students were able to visualize the conjecture and provide a correct justification.

For group 2 (low achiever students), the *dragging with trace activated* facility helped the students to adjust their way of thinking. First, they provided an incorrect conjecture and could not provide any justification. After using the *dragging with trace activated* modality, the students were able to visualize the correct conjecture and were able to provide a correct not complete proof missing some important steps or statements (CNCP).

For group 3 (middle achiever students), while solving part 1 of activity sheet 3, before using any feature of DGS, group 3 provided an incorrect proof reflecting confusion between fixed/variable elements (IPFV) for an incorrect conjecture. However, after using the *dragging test* modality, the students of this group were able to notice the correct conjecture and they were able to provide a correct not complete proof missing some important statements (CNCP).

Group 4 (middle achiever students) provided a correct proof for a correct conjecture for the first two parts of activity sheet 4 without the use of DGS facilities. However, for the third part, this group could not notice the conjecture until the *dragging*
with trace activated facility was used. After using the tracing facility, students of this group provided a correct proof. Thus Sketchpad’s dragging with trace activated facility helped them to provide a formulated proof.

It is noted from the four clinical interviews that DGS helped all these students who were low and middle achievers to reason visually the conjecture, enhance their geometric reasoning and their abilities in proof writing, and to analyze the premises of the problem in order to reach and formulate a proof.

4.8.2 Paper-pencil test

The same paper-pencil test was administered to the two student groups, the first year to the non-DGS class and the second year to the DGS class, each time within a time frame of two months from instruction of the chapter “locus”. Students were informed ahead of time about the date of the test and told to revise.

The initial achievement levels of the two classes were determined by the average of students’ average grades in mathematics over the period of two semesters in grade 8. The two classes were of different levels of achievement. The math average of the non-DGS class was 82 out of 100 and that of the DGS class was 73 out of 100.

The test consists of one problem composed of two parts requiring both finding the locus of a point, thus involving making a conjecture and proving it. A scoring scheme based on a qualitative set of criteria developed from the literature was used to score students’ work on the test, including their conjecture and proof, as well as students’ conjecturing processes, proving strategies and proof writing format.
To discuss the content of the paper-pencil test, the first question asks the students to find the locus of a point and to provide a proof (Refer to Figure 15). The possible conjectures to this question are: the locus of point I is a straight line parallel to (xy) drawn from point O or the locus of point I is the perpendicular bisector of [MN].

Draw a circle (C) of fixed center O and a fixed diameter [MN]. (xy) is the tangent to (C) at N. D is a variable point on (xy). [DM] cuts the circle at E.

1) What is the geometric locus of I, the midpoint of [MD]?
   - The locus of I is ____________________________
   - Proof:

Fig. 15 Question number 1 in the paper-pencil test

The way to answer this part is as follows:

![Figure](image.png)
The locus of I is the perpendicular bisector of [MN].

Proof: M and N are fixed points. (xy) tangent to (C) at N then (xy) ⊥ (MN) at N (definition of tangent) then \( \overline{NM} \) is a right angle then \( \triangle DNM \) is right at N then \( IN = MD/2 \) (Median relative to the hypotenuse of a right triangle is equal to half of it). And I midpoint of [MD] then \( MI = MD/2 \). Then \( MI = IN \). So I belongs to the perpendicular bisector of the fixed segment [MN]. Then the locus of I is the perpendicular bisector of [MN].

OR

The locus of I is the perpendicular bisector of [MN].

Proof: \( \overline{DN} \perp (MN) \) at N (definition of tangent). I and O are midpoints of [DM] and [MN] respectively. Then \( \overline{OI} \parallel (DN) \) (mid-segment theorem of a triangle). Then \( \overline{OI} \perp (MN) \) at O (If two straight line are //, every perpendicular to one is perpendicular to the other). Then \( \overline{OI} \) perpendicular bisector of [MN]. M, N and O are fixed points. Then the locus of I is the straight line \( \perp \) bisector of [MN].

OR

The locus of I is a straight line parallel to (xy) drawn from point O.

Proof: In \( \triangle MDN \) we have: O midpoint of [MN] (center of a circle is midpoint of diameter) I midpoint of [MD] (given) then \( \overline{OI} \parallel (DN) \) and \( OI = DN/2 \) (mid-segment theorem of a triangle). O is a fixed point, D is a variable point on (xy) then the locus of I is a straight line parallel to (xy) drawn from O.

The second question in the paper-pencil test asks the students to find a locus of a point and to provide a proof (Refer to Figure 16). The correct conjecture is: the locus of K is a circle of center M and radius 2R.
2) Let K be the symmetric of N with respect to E. What is the geometric locus of K?

- The locus of K is _________________________
- Proof:

Fig. 16 Question number 2 in the paper-pencil test

The way to answer this part is as follows:

The locus of K is a circle (M, 2R)

Proof: In \( \triangle KMN \) we have: O midpoint of \([MN]\) and E midpoint of \([KN]\) (by central symmetry). Then \((OE) // (KM)\) and \(OE = KM/2 = R\) (mid-segment theorem of a triangle) (OE is a radius of a circle of radius equal to R) then \(KM = 2OE = 2R\). M, N and O are fixed points, E varies on circle \((C)\) then the locus of K is a circle \((M, 2R)\).
4.8.2.1 Categories of conjectures used by students

The following table (Table 9) summarizes the conjectures’ categories of the non-DGS class.

Table 9

*Categories of conjectures of non-DGS class (23 students)*

<table>
<thead>
<tr>
<th>Conjecture Categories</th>
<th>Frequency</th>
<th></th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part 1</td>
<td>Part 2</td>
<td></td>
</tr>
<tr>
<td>NC</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MNL</td>
<td>9</td>
<td>10</td>
<td>19</td>
</tr>
<tr>
<td>MFV</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>IC</td>
<td>3</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CIC</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>CCNS</td>
<td>7</td>
<td>6</td>
<td>13</td>
</tr>
<tr>
<td>CCED</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>CPL</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>28</td>
<td>29</td>
<td>57</td>
</tr>
</tbody>
</table>

NC: No Conjecture is provided
MNL: Invalid conjecture reflecting Misunderstanding of the Notion of Locus
MFV: Invalid conjecture reflecting Misunderstanding of Fixed/Variable elements
IC: Incorrect Conjecture is provided
CIC: Correct Incomplete Conjecture is provided
CCNS: Correct Conjecture with No Signs in the figure to indicate how it was found
CCED: Correct Conjecture based on Empirical Drawing of many cases
CPL: Correct Conjecture providing a Partial Locus
The following table (Table 10) summarizes the conjectures’ categories of the DGS class.

Table 10

*Categories of conjectures of DGS class (18 students)*

<table>
<thead>
<tr>
<th>Conjecture Categories</th>
<th>Frequency</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part 1</td>
<td>Part 2</td>
</tr>
<tr>
<td>NC</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>MNL</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>MFV</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>IC</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>CIC</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>CCNS</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>CCED</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>CPL</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>18</strong></td>
<td><strong>19</strong></td>
</tr>
</tbody>
</table>

NC: No Conjecture is provided  
MNL: Invalid conjecture reflecting Misunderstanding of the Notion of Locus  
MFV: Invalid conjecture reflecting Misunderstanding of Fixed/Variable elements  
IC: Incorrect Conjecture is provided  
CIC: Correct Incomplete Conjecture is provided  
CCNS: Correct Conjecture with No Signs in the figure to indicate how it was found  
CCED: Correct Conjecture based on Empirical Drawing of many cases  
CPL: Correct Conjecture providing a Partial Locus

Note: The two tables above (Table 9 and Table 10) represent the numbers of conjectures under each category. The same conjecture may have been classified under
more than one category, which means that the Total does not correspond to the total
number of conjectures, nor to the number of students.

In order to be able to compare the numbers in the two tables, and because of the
difference between the numbers of students in the two classes, it is essential to unify the
base of comparison, by considering two similar classes with 100 students each. So the
numbers in each table were respectively divided by the number of students in the class
and multiplied by 100.

The following table (Table 11) summarizes the conjectures’ categories of a 100-
student non-DGS class.

Table 11

*Categories of conjectures of a 100-student non-DGS class*

<table>
<thead>
<tr>
<th>Conjecture Categories</th>
<th>Frequency</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part 1</td>
<td>Part 2</td>
</tr>
<tr>
<td>NC</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MNL</td>
<td>39</td>
<td>43</td>
</tr>
<tr>
<td>MFV</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>IC</td>
<td>13</td>
<td>35</td>
</tr>
</tbody>
</table>

|                       |           |       | 143 cases |
| CIC                   | 26        | 0      | 26        |
| CCNS                  | 30        | 26     | 56        |
| CCED                  | 9         | 8      | 17        |
| CPL                   | 4         | 0      | 4         |

|                       |           |       | 103 cases |

NC: No Conjecture is provided
The following table (Table 12) summarizes the conjectures’ categories of a 100-student DGS class.

Table 12

Categories of conjectures of a 100-student DGS class

<table>
<thead>
<tr>
<th>Conjecture Categories</th>
<th>Frequency</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part 1</td>
<td>Part 2</td>
</tr>
<tr>
<td>NC</td>
<td>0</td>
<td>17</td>
</tr>
<tr>
<td>MNL</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>MFV</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>IC</td>
<td>28</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CIC</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>CCNS</td>
<td>28</td>
<td>11</td>
</tr>
<tr>
<td>CCED</td>
<td>17</td>
<td>11</td>
</tr>
<tr>
<td>CPL</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NC: No Conjecture is provided  
MNL: Invalid conjecture reflecting Misunderstanding of the Notion of Locus  
MFV: Invalid conjecture reflecting Misunderstanding of Fixed/Variable elements  
IC: Incorrect Conjecture is provided  
CIC: Correct Incomplete Conjecture is provided  
CCNS: Correct Conjecture with No Signs in the figure to indicate how it was found  
CCED: Correct Conjecture based on Empirical Drawing of many cases  
CPL: Correct Conjecture providing a Partial Locus
The tables above (Table 11 and Table 12) show that all students in the non-DGS class provided conjectures for parts 1 and 2 and all students in the DGS class provided a conjecture for part 1. However, there are 17 cases of “No Conjecture” (NC) for part 2 in the DGS class. After referring to the students’ tests, one can notice that the students who did not provide any conjecture for part 2 drew a wrong figure. In part 2, the question names K the symmetric of N with respect to E, thus makes E the midpoint of [KN]; however, these students made point N the midpoint of [KE].

The Invalid Conjecture (IC) category was divided into two subcategories. Table 11 shows that the number of cases under “invalid conjecture reflecting Misunderstanding of the Notion of Locus” (MNL) subcategory is 82; whereas, this number is reduced to 22 in the DGS class as shown in Table 12. This shows that the notion of locus is significantly clearer to the students in the DGS class. The “invalid conjecture reflecting Misunderstanding of Fixed/Variable elements” (MFV) subcategory of both classes (DGS and non-DGS classes) turned out to be empty in part 1; however, in part 2, the number of cases in the non-DGS class is 13 and that of the DGS class is 22. Half the number of cases under MFV subcategory in the DGS class was committed by high achiever students. It was noted from the way they proved their conjectures that they considered a particular figure. An example of an invalid conjecture reflecting misunderstanding of fixed/variable elements is “The locus of K is a circle of center N and radius 2EK”; the way the student proved this conjecture was: “E is midpoint of [KN] by central symmetry then KE = EN = KN/2. E is variable and N is fixed then the
locus of K is a circle of center N and radius 2EK”. This student considered that the radius “2EK” does not change.

Table 11 shows that the number of Incorrect Conjecture (IC) cases is 13 for part 1 and 35 for part 2. However, Table 12 shows that the number of Incorrect Conjecture (IC) cases is 28 for part 1 and 22 for part 2. Therefore, less incorrect conjectures were provided by the students in the DGS class in part 2 although the geometric locus case/fundamental of part 2 (circle with fixed center and constant radius) is considered by most teachers a more advanced level than that of part1 (perpendicular bisector).

It is noted from the tables above (Table 11 and Table 12) that the number of cases on invalid/incorrect conjectures in the non-DGS class was reduced in the DGS class (from 143 cases to 111 cases).

Table 11 shows that the number of Correct Incomplete Conjecture (CIC) cases is 26 for part 1 and 0 for part 2. Whereas, Table 12 shows that the number of Correct Incomplete Conjecture (CIC) cases is 11 for part 1 and 11 for part 2. In the DGS class, most of the students who answered by correct incomplete conjecture for part 2 drew many positions of the variable points in the figure and answered by “circle of center M” but they did not determine the radius.

Under “Correct Conjecture with No Signs in the figure to indicate how it was found” (CCNS) subcategory, Table 11 shows that the number of cases is 56; whereas in the DGS class the number of cases is 39 as it is shown in Table 12. This difference is due to high analytical thinking of the non-DGS class, being the higher-achiever class, based on students’ average scores in mathematics (82 versus 73).
As Table 11 shows that the number of Correct Conjecture based on Empirical Drawing of many cases (CCED) cases is 17; however, Table 12 shows that the number of cases is 28. It was noticed by the researcher that most of the students in the DGS class who provided CCED were clinically interviewed and provided incorrect conjecture with no proof before the use of any DGS facility. This indicates that DGS affected their geometric reasoning in order to provide a conjecture after observing many cases of the figure.

Under the “Correct Conjecture providing a Partial Locus” (CPL) subcategory, the total number of cases in the non-DGS class is 4; whereas, that of the DGS class is 6.

To conclude, one can notice that the number of invalid conjecture reflecting Misunderstanding of the Notion of Locus (MNL) cases is reduced in the DGS class; thus the conceptual understanding of the notion of locus is clearer to DGS class’s student. Moreover, the empirical drawing of many cases is improved in the DGS in a way that the number of cases in the DGS class under “Correct Conjecture based on Empirical Drawing of many cases” (CCED) category is higher than that in the non-DGS class.

### 4.8.2.2 Categories of proofs used by students

The following table (Table 13) summarizes the proofs’ categories of the non-DGS class.
Table 13

Categories of proofs of non-DGS class (23 students)

<table>
<thead>
<tr>
<th>Proof Categories</th>
<th>Frequency</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part 1</td>
<td>Part 2</td>
</tr>
<tr>
<td>NP</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>IPFV</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>IPGM</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>CNCP</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>CPUS</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CCP</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>23</strong></td>
<td><strong>24</strong></td>
</tr>
</tbody>
</table>

NP: No Proof category containing a conjecture with no justification
IPFV: Incorrect Proof reflecting confusion between Fixed/Variable.
IPGM: Incorrect Proof based on Geometric Misconceptions
CNCP: Correct Not Complete Proof missing some important steps or statements.
CPUS: Correct Proof is provided but includes statements that do not contribute to the proof
CCP: Correct Complete Proof for the correct conjecture

The following table (Table 14) summarizes the proofs’ categories of the DGS class.
Table 14

Categories of proofs of DGS class (18 students)

<table>
<thead>
<tr>
<th>Proof Categories</th>
<th>Frequency</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part 1</td>
<td>Part 2</td>
</tr>
<tr>
<td>NP</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>IPFV</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>IPGM</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>CNCP</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>CPUS</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>CCP</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>19</td>
<td>19</td>
</tr>
</tbody>
</table>

NP: No Proof category containing a conjecture with no justification
IPFV: Incorrect Proof reflecting confusion between Fixed/Variable.
IPGM: Incorrect Proof based on Geometric Misconceptions
CNCP: Correct Not Complete Proof missing some important steps or statements.
CPUS: Correct Proof is provided but includes statements that do not contribute to the proof
CCP: Correct Complete Proof for the correct conjecture

Note: The two tables above (Table 13 and Table 14) represent the numbers of proofs under each category. The same proof may have been classified under more than one category, which means that the Total does not correspond to the total number of proofs, nor to the number of students.

In order to be able to compare the numbers in the two tables, and because of the difference between the numbers of students in the two classes, it is essential to unify the base of comparison, by considering two similar classes with 100 students each. So the
numbers in each table were respectively divided by the number of students in the class and multiplied by 100.

The following table (Table 15) summarizes the proofs’ categories of a 100-student non-DGS class.

Table 15

*Categories of proofs of a 100-student non-DGS class*

<table>
<thead>
<tr>
<th>Proof Categories</th>
<th>Frequency</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part 1</td>
<td>Part 2</td>
</tr>
<tr>
<td>NP</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>IPFV</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>IPGM</td>
<td>13</td>
<td>35</td>
</tr>
<tr>
<td>CNCP</td>
<td>43</td>
<td>17</td>
</tr>
<tr>
<td>CPUS</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CCP</td>
<td>43</td>
<td>9</td>
</tr>
</tbody>
</table>

91 cases

112 cases

NP: No Proof category containing a conjecture with no justification
IPFV: Incorrect Proof reflecting confusion between Fixed/Variable.
IPGM: Incorrect Proof based on Geometric Misconceptions
CNCP: Correct Not Complete Proof missing some important steps or statements.
CPUS: Correct Proof is provided but includes statements that do not contribute to the proof
CCP: Correct Complete Proof for the correct conjecture
The following table (Table 16) summarizes the proofs’ categories of a 100-student DGS class.

*Table 16*

*Categories of proofs of a 100-student DGS class*

<table>
<thead>
<tr>
<th>Proof Categories</th>
<th>Frequency</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part 1</td>
<td>Part 2</td>
</tr>
<tr>
<td>NP</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>IPFV</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>IPGM</td>
<td>33</td>
<td>44</td>
</tr>
</tbody>
</table>

111 cases

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CNCP</td>
<td>22</td>
<td>6</td>
</tr>
<tr>
<td>CPUS</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>CCP</td>
<td>44</td>
<td>22</td>
</tr>
</tbody>
</table>

100 cases

NP: No Proof category containing a conjecture with no justification
IPFV: Incorrect Proof reflecting confusion between Fixed/Variable.
IPGM: Incorrect Proof based on Geometric Misconceptions
CNCP: Correct Not Complete Proof missing some important steps or statements.
CPUS: Correct Proof is provided but includes statements that do not contribute to the proof
CCP: Correct Complete Proof for the correct conjecture

The two tables above (Table 15 and Table 16) show that all students in both classes (non-DGS and DGS classes) provided a proof for part 1; however, for part 2 the number of No Proof (NP) cases in the non-DGS is 30 and in the DGS class is 17. This difference means that the DGS class had more confidence in trying to provide a proof despite their low achievement in math.
The incorrect proof category is divided into two subcategories. Under the “Incorrect Proof reflecting confusion between Fixed/Variable” (IPFV) subcategory, the number of cases in both classes is 0 for part 1. However, that for part 2 is 13 in the non-DGS class and 22 in the DGS class. This is because the DGS class provided more number of proofs than the non-DGS class. Under the “Incorrect Proof based on Geometric Misconceptions” (IPGM) subcategory, the number of cases in the non-DGS is 48 as it is shown in Table 15; whereas that in the DGS class is 78 as it is shown in Table 16. This difference is due to the low achievement of the DGS class in math thus more geometric misconceptions this class has.

The correct proof category is divided into three subcategories: Correct Not Complete Proof missing some important steps or statements (CNCP), Correct Proof that includes statements that do not contribute to the proof (CPUS), and Correct Complete Proof for the correct conjecture (CCP). The number of Correct Not Complete Proof missing some important steps or statement (CNCP) cases in the non-DGS class is 60, whereas in the DGS class is 28. Under “Correct Proof that includes statements that do not contribute to the proof” (CPUS) subcategory, 0 is the number of cases in the non-DGS class and 6 is that in the DGS class. Under the “Correct Complete Proof” (CCP) subcategory, 52 is the number of cases in the non-DGS class as it is shown in Table 15 and that in the DGS class is 66 as it is shown in Table 16. Some students in the DGS class who are under CCP subcategory were clinically interviewed and were considered as low achievers.

To conclude, one can notice that the DGS class had more confidence to provide a proof than the non-DGS class in a way that the number of No Proof (NP) cases is reduced from 30 in the non-DGS class to 11 in the DGS class. Also, more correct
complete proofs were provided by the DGS class. Most of the students who were under “Correct Complete Proof” (CCP) category were clinically interviewed and could not provide formulated proofs before the use of DGS facilities. The comparison of the proofs produced by students in paper-pencil test between the non-DGS class and the DGS class tells that the abilities developed in a DGS environment were transferable to a non-DGS environment in problem situations involving Locus.

4.8.2.3 Qualitative analysis of some students’ tests

In the DGS class, four students showed a significant improvement in their analysis and proof writing. These students were clinically interviewed and could not find either a conjecture or a proof before the use of DGS facilities. Three of them are considered to be low achievers and one of them is considered to be middle achiever.

The three low achievers showed a significant improvement in their proof writing format/style. This improvement was noticed by their teacher. Below is the analysis of the first student work:

His school grades and his teacher’s opinion about him indicate that this student is a very low achiever. In the paper pencil test he excelled and his grade was better than many students whom are considered to be high achievers. To analyze his test: in the first part he understands that the point I is a movable point on the perpendicular bisector of [MN] but when asked about the locus/path of point I he has to answer by: “the locus of I is the perpendicular bisector of [MN]”; whereas, he answered by: “the locus of I is a point moving on the perpendicular bisector of [MN]”. While proving his conjecture, it is
clear that he understands the concept but he is unable to prove it and his proof misses important steps like: (IO) is perpendicular to [MN] at its midpoint. In the second part, this student provided a correct complete conjecture and a correct proof that he missed to state which are the fixed and the variable elements. The following figure (Fig. 17) provides this student’s test:

![Solution of the test by the first student](image)

Fig. 17 Solution of the test by the first student
In analysis of the second student’s work:

This student is considered to be a low average student. In part 1 of the paper-pencil test she answered a correct complete conjecture and proof. The following figure (Fig. 18) provides her answer on part 1.

![Fig. 18 Solution of part 1 by the second student](image)

In analysis of the third student’s work:

This student is considered to be a low achiever. In the two parts of the paper-pencil test she wrote a correct complete conjecture and proof. Part 2 needs a high analysis level and many high achievers in the non-DGS class did not answer this part correctly. The following figure (Fig. 19) provides this student’s test:
In analysis of the fourth student’s work:

This student is considered to be a middle achiever student. He answered the two parts of the paper-pencil test correctly and he provided “Conjecture providing a Partial Locus” (CPL) and a correct proof. His way of analyzing the two parts was significant.
and clearly realized by his teacher. The following figure (Fig. 20) provides this student’s test:

Fig. 20 Solution of the test by the fourth student
CHAPTER FIVE

DISCUSSIONS AND CONCLUSIONS

The present study aimed at exploring the difficulties that students face in learning locus and solving locus problems, as well as the ways using dynamic geometry software can help improving the learning process and better preparing students for solving problems involving locus of points. In more specific words, the research attempted to answer the following questions:

- Research Question 1: What are the major difficulties that students face in learning the concept of locus and in finding and proving locus of points?
- Research Question 2: How does DGS modify/change the class instructional environment (class interactions, teacher vs. student centeredness …) while teaching locus of points?
- Research Question 3: How does DGS support learning about locus of points?
- Research Question 4: In problem situations involving Locus, are the abilities developed in a DGS environment transferable to a non-DGS environment?

The following section aims to answer the above questions based on the findings.
5.1 Research Question 1

What are the major difficulties that students face in learning the concept of locus and in finding and proving locus of points?

The analysis of the locus chapter in students’ textbook, the interviews with the three grade-9 teachers, the interviews with students and the observations of sessions in the non-DGS class provided an insight about the difficulties that students face while solving geometric questions involving locus.

The interviews with students revealed that the major difficulty they face while learning locus is not the imaginary part as much as the memorizing part of the fundamentals/cases of the locus of a point. Their perceptions of the nature and requirements of the locus theme were limited to memorizing a number of cases. Students were asked to memorize the cases that are useful for grade-9 official exams. This might be one of the reasons that students face difficulties while studying chapters that are related to locus in different grade levels later. Also, it can be concluded from the eight interviews that the notion of locus of a point was still vague to them. This fact was clearly noticed in the paper-pencil test of the non-DGS class, as the total number of cases under the “invalid conjecture reflecting Misunderstanding of the Notion of Locus” (MNL) subcategory was 82, whereas, this number was only 22 in the DGS class. This shows that the notion of locus is significantly clearer to the students in the DGS class. In the students’ textbook, there is no clear definition for the word “locus” and there is no explanation of the words “fixed points” and “variable points”; although, understanding what is meant by these words and the cases when some points are variable and when
some points are fixed is an important perception to the understanding of the idea of locus of points. Moreover, during the interviews the students stated that even though the teacher used DGS in class for visualization but the way the teacher adopted was complicated. According to them the teacher started with complicated problems and did not explain how to move the variable point so that DGS automatically shows the path of the variable point. This was clearly noticed in the paper-pencil test in a way that the total number of cases under “Correct Conjecture based on Empirical Drawing of many cases” (CCED) subcategory is 17 in the non-DGS class; however, that number is greater in the DGS class, namely 28. The fact that students face difficulties in learning the locus topic was clearly noticed during the observation of the non-DGS class. The analysis of the transcription of the videotapes along with the analysis of the observation logs identified that the teaching strategy is teacher-centered, with the teacher making all the decisions without involving the students in any discovery or exploration process. The teacher explained the definition of the locus, the conditions for having fixed and variables elements, and all the 8 fundamentals/cases, all in one session. This fact is consistent with Malaty (2006), who states that the misusage of visualization misleads children and makes them jump to conclusions, which have not been justified. Thus there are problems in the use of visualization in classroom as it offers a ready model that makes children jump to conclusions without leaving space for searching for a cause or a conviction. Some researchers (e.g. Clements 2002) have indicated that the mathematical knowledge can remain hidden or “situated” within the technological context, unless teachers help make that knowledge explicit. In addition to that, results of paper-pencil test clearly showed that the students had difficulty in proving locus problems. However, this fact is not noticed or recognized by the teachers, as for them, the proof of locus is an easy way
that the students have to follow, that is, identifying the fixed and constant elements, finding the locus, and applying the property to write the proof. The teachers are not aware of the difficulties that their students face in the locus topic.

**5.2 Research Question 2**

How does DGS modify/change the class instructional environment (class interactions, teacher vs. student centeredness …) while teaching locus of points?

In order to answer this question, comparisons of the observations of the sessions in the lab settings and in the class the year before, were conducted. The observation of the sessions in the lab setting revealed that DGS supports the teaching-learning process of ‘locus’ as it modified the class instructional environment from a teacher-centered class to a student-centered class. While examining the videotapes and the observation logs in the non-DGS class, it was evident that the action was revolving around the teacher as he did all the explanation without involving the students into enough problem solving or interaction. As for the DGS class the students were active and interacted with each other and with the teacher. When using DGS there was room for in-class interaction and the students were more motivated about the topic that seemed more interesting to them. DGS helped in changing the communication between the teacher and the students as they were involved with discussions that deepened their understanding of the locus topic. The teacher communicated with the students about their answers and justifications and tried to convince them why their answers were wrong, whenever applicable. This is a different scenario than when observing the non-DGS sessions that can be described as a teacher centered due to minimal interaction where the teacher states a case and
demonstrates a way for proving it whereas the students tend to be passive receivers of information and to imitate the teacher's explanation without having a role or a clear understanding about the discovery or exploration process. Thus DGS helped in changing the class instructional environment while teaching locus of points in such a clear and unique way.

**5.3 Research Question 3**

How does DGS support learning about locus of points?

The observation of the sessions in the lab setting compared with the observations done in the non-DGS class the year before, the clinical interviews with selected groups solving geometric problems using Geometer’s Sketchpad and the analysis of the results of the paper-pencil quiz after implementing the unit, provide a picture about how DGS supports the teaching/ learning of ‘locus’.

In the computer lab, the students were exploring the figures. They were working with excitement and asking questions about every difficulty. They were reasoning visually to construct a conjecture and trying to analyze the premises of the problem through dragging in order to reach and formulate a proof. This was not the case while observing the non-DGS group. The transcription of the videotapes and the analysis of the observation log revealed that the students were not involved in any discovery or exploration process. The interaction between the teacher and the students occurred only at the time when the teacher stated a locus case and demonstrated a way for proving it whereas the students tended to imitate the teacher's explanation.
The analysis of the clinical interviews revealed that when students used the different dragging modalities *wandering/random dragging, dragging with trace activated*, and *dragging test*, they developed a dynamic-conjecture where they can drag and observe changes in the figures and the provided measurements. However, in static environments such as in a paper-and-pencil situation, students built a static-conjecture where they were not allowed to modify the figures by dragging. Moreover, in a dynamic environment, the invariant geometrical properties of a construction, which lead to conjectures, were easily grasped (Frank & Mariotti, 2009). Thus, the students explored a certain situation, tried to identify the fixed and variable elements, as well as the variant and invariant properties, choose the rule that suits a particular case (where explorations were transformed into conjectures), and then they tried to justify and validate their conjectures. In other words, the dynamic-conjecture made them shift from exploring-conjecturing to proving. Thus DGS helped in improving the learning process and better prepared students for solving problems involving locus of points.

5.4 Research Question 4

In problem situations involving Locus, are the abilities developed in a DGS environment transferable to a non-DGS environment?

The analysis of the results of the paper-pencil test after implementing the unit and comparing students’ strategies to those of last year students of the non-DGS, showed that the abilities of finding locus of points that were developed in DGS environment were transferable to a non-DGS environment. One indicator is the fact that the students who faced difficulties when they were clinically interviewed in the computer lab,
excelled in the paper-pencil test. In other words, DGS enhanced students’ construction of a conjecture about the locus of points and strengthened the willingness and/or the ability to build a proof of a conjecture about the locus of points, in a way that in the paper-pencil test the number of No Proof (NP) cases, which was 30 in the non-DGS class, was only 11 in the DGS class. Also, more correct complete proofs were provided by the DGS class. Most of the students who were under “Correct Complete Proof” (CCP) category were clinically interviewed before and could not provide formulated proofs before the use of DGS facilities.

Before implementing the unit, the two classes were of different levels of achievement. The math average of the non-DGS class in grade 8 was 82 out of 100 and that of the DGS class was 73 out of 100. After implementing the unit, the DGS class performed slightly higher in the paper-pencil test than the Non-DGS class.

From the paper-pencil test results, one can notice that the number of invalid conjectures reflecting Misunderstanding of the Notion of Locus (MNL) cases was much smaller than in the DGS class; thus the conceptual understanding of the notion of locus is clearer to DGS class student. Moreover, the empirical drawing of many cases is improved in the DGS class, as the number of cases in the DGS class under “Correct Conjecture based on Empirical Drawing of many cases” (CCED) category is higher than that in the non-DGS class.

5.5 Limitations of the study

Though the study might have given insights on whether the DGS can help improve the learning process and better prepare students for solving problems involving
locus of a point, it has several limitations. First, the sample was small (41 students); therefore, the results that were obtained cannot be generalized to other populations. In addition, the study was done on two different groups for which the initial levels of achievement were different, which made it difficult to reach conclusive results in some aspects. Moreover, the time allotted for the study was short. The total number of taught DGS sessions was only three sessions. Also, due to the limitation in the number of computers, students solved the DGS-based worksheets in pairs. Furthermore, the interview questions addressed to the teachers should have been structured in a way that allows them to explain the difficulties that students face while learning locus in an indirect way as this might prevent them from providing the complete data thinking that there is something wrong with their teaching style. Moreover, while analyzing students’ work and developing the framework, it occurred to the researcher that an interesting case was overlooked, which is the case where the locus is only a part of geometric object. In the paper-pencil test there were no cases where the locus needed to be limited to only a part of a straight line or a circle. This is one of the limitations of this research.

5.6 Recommendations

The recommendations go to the teachers, the schools, and the curriculum developers.

Teachers should not rely completely on the explanation of the book but they are rather expected to introduce activities that would allow students to explore the information and be active learners.
The administration of the schools should provide teachers with workshops that point out on the importance of the activities and of the use of technology for visualization and exploration. In addition to that, they should dedicate a computer session for math activities once a week.

Math curriculum developers should develop books that integrate the use of technology in every chapter. They are also expected to change the way the books present the chapters so as the students would have the chance to explore the information.

5.7 Perspectives for further research

This study shows that using carefully designed DGS tasks enhance students’ geometric reasoning abilities. In order to be able to generalize the results of this study, further studies are needed to apply on this research or similar ones including selecting larger sample for having a better input in addition to structuring the interview with teachers in a different way to get better feedback without doubting their teaching style. Moreover, similar studies that investigate the effects of DGS need to be done for a longer period of time in which students will have the chance to spend more time working on DGS. Additionally, studies in the literature are about conjecturing but not about locus topic. More studies need to examine the effect of DGS on students’ learning the concept of locus topic as well as in finding and proving locus of points.
REFERENCES


APPENDIX A

Questions of semi-structured interview with Teacher(s)

1. How do you introduce Locus topic?
2. What are the common difficulties faced by students learning Locus?
3. What are the difficulties that you face while teaching Locus?
4. What is special about the Locus topic that makes it more difficult than other topics?
5. Do you use any DGS in your classes?
   - If no: Why not?
   - If yes: How do you use it? What for? Do you think it can assist you in geometry teaching?
   - More specifically, how do you use DGS for teaching locus? Please give me a few examples.

6. How do you start teaching any math lesson?

7. Do you present the information to students? Or do you let them discover it?
   - Do you start with problems that students are expected to solve?
   - When solving problems on the board, do you solve or you would ask students to solve?
   - How heavily do you rely on questioning students during your teaching? (Never, fairly, usually, or always?)
   - How do you proceed for solving problems involving locus in class?

8. What difficulties do students face when they have to solve geometric problems about Locus?

9. Do you agree or disagree with using dynamic geometry software for teaching locus? Why? Or why not?

10. If a teacher uses software that might visually convince students about the conjecture, what do you think will be its effect on the students’ conception of proof?

11. How do you encourage students to prove a statement they are convinced is true?

12. Do you think DGS can hinder the teaching of proof? Do you think students will feel that proof is not necessary anymore?
**APPENDIX B**

**Rubric for observation in class**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher asking students to give conjecture</td>
<td></td>
</tr>
<tr>
<td>Student giving a right conjecture</td>
<td></td>
</tr>
<tr>
<td>Student giving a wrong conjecture</td>
<td></td>
</tr>
<tr>
<td>Student not knowing what to answer when asked about conjecture</td>
<td></td>
</tr>
<tr>
<td>Teacher helping students to find the conjecture</td>
<td></td>
</tr>
<tr>
<td>Teacher giving the conjecture without asking students</td>
<td></td>
</tr>
<tr>
<td>Teacher giving the conjecture after wrong conjectures from students</td>
<td></td>
</tr>
<tr>
<td>Teacher asking another student to correct student’s conjecture</td>
<td></td>
</tr>
<tr>
<td>Teacher questioning student about his/her wrong conjecture</td>
<td></td>
</tr>
<tr>
<td>Teacher questioning student about his/her correct conjecture</td>
<td></td>
</tr>
<tr>
<td>Teacher asking about fixed and variable elements</td>
<td></td>
</tr>
<tr>
<td>Student knowing the fixed and the variable elements</td>
<td></td>
</tr>
<tr>
<td>Student not knowing the fixed and the variable elements</td>
<td></td>
</tr>
<tr>
<td>Student not knowing what to answer when asked about fixed and variable elements</td>
<td></td>
</tr>
<tr>
<td>Teacher helping students to find the fixed and the variable elements</td>
<td></td>
</tr>
<tr>
<td>Teacher giving the fixed and the variable elements without asking students</td>
<td></td>
</tr>
<tr>
<td>Teacher giving the fixed and the variable elements after wrong answers from students</td>
<td></td>
</tr>
<tr>
<td>Teacher asking another student to correct student’s answer about fixed and variable elements</td>
<td></td>
</tr>
<tr>
<td>Teacher questioning student about his/her wrong answer about fixed and variable elements</td>
<td></td>
</tr>
<tr>
<td>Event</td>
<td>Description</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Teacher questioning student about his/her correct answer about fixed and variable elements</td>
<td></td>
</tr>
<tr>
<td>Teacher asking about condition</td>
<td></td>
</tr>
<tr>
<td>Student knowing the right condition</td>
<td></td>
</tr>
<tr>
<td>Student not knowing the right condition</td>
<td></td>
</tr>
<tr>
<td>Student not knowing what to answer when asked about the condition</td>
<td></td>
</tr>
<tr>
<td>Teacher helping students to find the condition</td>
<td></td>
</tr>
<tr>
<td>Teacher giving the condition without asking students</td>
<td></td>
</tr>
<tr>
<td>Teacher giving the condition after wrong answers from students</td>
<td></td>
</tr>
<tr>
<td>Teacher asking another student to correct student’s answer about condition</td>
<td></td>
</tr>
<tr>
<td>Teacher questioning student about his/her wrong answer about condition</td>
<td></td>
</tr>
<tr>
<td>Teacher questioning student about his/her correct answer about condition</td>
<td></td>
</tr>
<tr>
<td>Teacher asking student to suggest a justification/proof</td>
<td></td>
</tr>
<tr>
<td>Student suggesting a correct justification/proof</td>
<td></td>
</tr>
<tr>
<td>Student suggesting a wrong justification/proof</td>
<td></td>
</tr>
<tr>
<td>Student not knowing what to answer when asked to suggest a justification/proof</td>
<td></td>
</tr>
<tr>
<td>Teacher helping students to find the justification/proof</td>
<td></td>
</tr>
<tr>
<td>Teacher suggesting the justification/proof</td>
<td></td>
</tr>
<tr>
<td>Teacher giving the justification/proof after wrong justifications/proofs from students</td>
<td></td>
</tr>
<tr>
<td>Teacher asking another student to correct student’s justification/proof</td>
<td></td>
</tr>
<tr>
<td>Teacher questioning student about his/her wrong justification</td>
<td></td>
</tr>
<tr>
<td>Teacher questioning student about his/her correct justification</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX C

Interview with the students

1. What is meant by a locus of a point?

2. Do you face difficulties while learning Locus? If yes, what is special about the Locus topic that makes it difficult?

3. How do you think/what do you do when you are asked to find a locus of a point?

4. Do you face difficulties when you have to solve geometric problems about Locus? If yes, what are they?

5. If your classmate/ friend asked you to help him/her understand locus, how would you do it? What is that you will focus on?

6. What do you think you need your teacher to do or to explain to you so that you understand locus?
APPENDIX D

Paper-pencil test

Grade Level: 9  Duration: 25 minutes
Section:     Name: ______________________________

Instructions:

• Use only the scratch paper while thinking about the problem.

• If you want to modify your method or start over, do not cross out or erase your previous work. Draw a line and start another method.

• When you feel that you have completely solved the problem, write the complete solution on the solution paper.
**Given:**

Draw a circle (C) of fixed center O and a fixed diameter [MN].

(xy) is the tangent to (C) at N. D is a variable point on (xy). [DM] cuts the circle at E.

1) What is the geometric locus of I, the midpoint of [MD]?
   - The locus of I is ____________________________.
   - **Proof:**

2) Let K be the symmetric of N with respect to E. What is the geometric locus of K?
   - The geometric locus of K is ____________________________.
   - **Proof:**

**GOOD WORK**
APPENDIX E

“Locus” unit in students’ textbook

SOME GEOMETRIC LOCUS

Objective

Know some easy geometric loci.

PLAN OF THE CHAPTER

1. Equidistant points of extremeties of a segment
2. Equidistant points of two sides of an angle
3. Points situated at a constant distance from a fixed point
4. Points varying on a circle of fixed diameter
5. Points varying on a line forming a constant angle with another fixed line
6. Points situated at a constant distance from a fixed line

EXERCISES AND PROBLEMS

209
Reminder

1. Equidistant points of extremities of a segment

A and B are two fixed points.

If M is a variable point such that \( MA = MB \), then the geometric locus of M is the perpendicular bisector of the segment \([AB]\).

Example

E and F are two fixed points.

Find the geometric locus of points O, variable centers of circles passing through E and F.

We have \( OE = OF = \text{radius} \), with E and F fixed.

The geometric locus of O is then the perpendicular bisector of segment \([EF]\).

2. Equidistant points of two sides of an angle

The semi-lines \([Ox]\) and \([Oy]\) are fixed.

If M is a variable point and is equal distant of \([Ox]\) and \([Oy]\), then the geometric locus of M is the bisector of angle \(xOy\).
### Example

The two semi-lines \([Ax]\) and \([Ay]\) are fixed and perpendicular.

\(M\) and \(N\) are two variable points respectively on \([Ax]\) and \([Ay]\) such that \(AM = AN\).

Find the geometric locus of \(D\) of the square \(MAND\).

We have \([Ax]\) and \([Ay]\) fixed, and \(MAND\) a square.

Since \(D\) is at equidistant of \([Ax]\) and \([Ay]\) \((DM = DN)\), then the geometric locus of \(D\) is the bisector \([AU]\) of the angle \(\angle AxAy\).

### Points situated at a constant distance from a fixed point

\(I\) is a fixed point and \(M\) is variable, such that \(IM\) is a constant \(k\) positive.

The geometric locus of \(M\) is the circle of center \(I\) and of radius \(k\).

### Example

The two points \(B\) and \(C\) are fixed with \(BC = 6\) cm and \(I\) the midpoint of \([BC]\).

Find the geometric locus of the variable points \(A\), such that \(ABC\) is rectangle in \(A\).

Since \(I\) fixed is the midpoint of the fixed segment \([BC]\).

\([AI]\) is the relative median to the hypotenuse \([BC]\) of the rectangle triangle \(ABC\), therefore \(IA = \frac{BC}{2}\); \(IA = 3\) cm.

The geometric locus of \(A\) is then the circle of center \(I\) and of a radius 3 cm.
4 Points varying on a circle of fixed diameter

The two points $A$ and $B$ are fixed.

If $M$ is a variable point such that the angle $\angle AMB = 90^\circ$, then the geometric locus of $M$ is the circle of diameter $[AB]$ excluding $A$ and $B$.

**Example**

$RIT$ is a triangle with $R$ and $I$ fixed and $T$ variable.

Find the geometric locus of point $H$ the foot of the height $[RH]$ of this triangle.

We have $R$ and $I$ two fixed points and $H$ is variable (because $T$ is variable) such that $\angle RHI = 90^\circ$.

The geometric locus of $H$ is then the circle of diameter $[RT]$.

5 Points varying on a line forming a constant angle with another fixed line

$A$ is fixed point of a fixed line $(D)$.

If $M$ is variable such that the line $(AM)$ make a constant angle $\alpha$ with the line $(D)$, then the geometric locus of $M$ is formed of the two lines $(D_1)$ and $(D_2)$ whose form respectively an angle $\alpha$ with $(D)$. 
**Example**

A is a fixed point of a fixed line \((d)\) and

\(B\) is a variable point of \((d)\).

Find the geometric locus of center \(O\)

of a square where \([AB]\) is one of its

sides.

\(B_1, B_2, B_3\) and \(B_4\) are possible positions

of \(B\). \(O_1, O_2, O_3\) and \(O_4\) are also

possible positions of \(O\).

The two lines \((AB)\) and \((AO)\) form an

angle of 45°.

The geometric locus of \(O\) is then formed

by the two lines \((\Delta_1)\) and \((\Delta_2)\) whose

form respectively with \((d)\) an angle

of 45°.

---

**6 Points situated at a constant distance from a fixed line**

\((D)\) is a fixed line.

If \(M\) is a variable point such that the distance of \(M\)

to \((D)\) is a constant \(k\) positive, then the geometric

locus of \(M\) is formed by the two lines \((D_1)\) and

\((D_2)\) whose are respectively parallel to \((D)\) and at

a distant \(k\) of \((D)\).

\(M_1\) and \(M_2\) are possible positions of \(M\).

---

**Example**

\((d)\) is a fixed line.

The vertices \(A\) and \(C\) and the center \(O\) of

rhombus \(ABCD\) vary on \((d)\). Find the

geometric locus of the vertices \(B\) and \(D\) if

\(OB\) is a positive constant \(k\).

We have \(OB = OD = k\) and \((BD)\) is perpendicular at \((d)\).

The geometric locus of \(B\) and \(D\) is formed by

the two lines \((\Delta_1)\) and \((\Delta_2)\) parallel at \((d)\) and

of a distant \(k\) of \((d)\).
Test your knowledge

1. The diagonal [AC] of a rhombus ABCD is fixed and the vertices B and D are variables. Determine the geometric locus of B and D.

2. The angle $Oy$ is fixed. The two points B and C are variables respectively on ($Ox$) and ($Oy$) such that $OB = OC$.
   1° What is the nature of triangle BOC?
   2° Find the geometric locus of M midpoint of [BC].

3. ABC is a triangle such that A and B are fixed and C is variable. If [AH] is a height-segment of this triangle, find the geometric locus of H.

4. C ($O ; R$) is a circle of a fixed diameter [AB].
   M is a variable point of this circle and I is the symmetric of A in terms of M.
   1° Show that (OM) and (BI) are two parallel lines.
   2° Find the geometric locus of I.

5. Consider (C) a fixed circle of center O and a radius R.
   [AB] is a variable diameter of (C) and M is a variable point distinct from A and B. Since I the midpoint of [AM], the lines (BI) and (MO) intersect in G. What is the geometric locus of G?

6. ABC is a triangle of vertex A variable and a fixed base [BC], such that the area of this triangle is $15 \text{ cm}^2$ and $BC = 6 \text{ cm}$.
   1° Calculate the length of the height [AH].
   2° Deduce the geometric locus of A.

7. The triangle ABC is fixed and M is a variable point of the segment [AB]. What is the geometric locus of I midpoint of [CM]?
   (Indication: take E, midpoint of [AC] and F midpoint of [BC].

8. (C) is a fixed circle of center O and a radius R.
   A is a fixed point of (C) and B is a variable point of (C).
   The point M is the fourth vertex of the parallelogram OAMB.
   1° What is the geometric locus of M?
   2° What is the geometric locus of the center I of the parallelogram OAMB?
9. Given \( \hat{A} \text{y} \) a fixed angle with \( \hat{A} \text{y} = 60^\circ \). \( M \) is a variable point of \( [Ax] \) and \( N \) is the symmetric of \( M \) with respect to \( [Ay] \). What is the geometric locus of \( N \) ?

10. \( \hat{O} \text{y} \) is a fixed angle and \( A \) is a variable point on \( (Ox) \). Draw from \( A \) the parallel at \( (Oy) \) and place the point \( B \) on this parallel such that \( OA = AB \). \( ([AB]) \) and \( (Oy) \) are on the same side in terms of \( (Ox) \). What is the geometric locus of \( B \) ?

11. \( \text{xy} \) is a fixed line and \( A \) is a fixed point not belonging to \( \text{xy} \). \([AH]\) is the perpendicular to \( \text{xy} \). \( H \) is on \( \text{xy} \), \( I \) is the midpoint of \([AH]\). A variable line passing through \( A \) cuts \( \text{xy} \) in \( B \). What is the geometric locus of point \( M \) midpoint of \([AB]\) ?

For seeking

12. \( O \) is a fixed point of a fixed line \( \text{xy} \). \( A \) is a variable point of \( \text{xy} \). On the perpendicular to \( \text{xy} \) at \( A \), place \( M \) such that \( OA = AM \).

1° What is the nature of the triangle \( AOM \) ?

2° What is the geometric locus of \( M \) ?

13. Given a circle \((C)\) of center \( O \) and of a radius 5 cm.

1° Find the geometric locus of center \( M \) of a variable circle of radius 2 cm, and tangent interiorly to \((C)\).

2° Find the geometric locus of center \( N \) of a variable circle of radius 2 cm, and tangent exteriorly to \((C)\).

14. \((C)\) is a circle of center \( O \) and of a radius \( R \).

\([AB]\) is a fixed diameter of \((C)\).

\( \hat{A}x \text{Ax} \) is tangent to \((C) \) in \( A \). \( M \) is a variable point on \([Ax] \). \([BM]\) cuts \((C) \) in \( I \). \( J \) is the midpoint of \([BM]\).

Find the geometric locus of:

1° \( J \).

2° \( K \) the symmetric of \( A \) with respect to \( J \).

3° \( N \) the symmetric of \( A \) with respect to \( I \).
15. Given $MNP$ isosceles triangle in $N$.

1° Determine the center $O$ of the circle $(C)$ circumscribed about triangle $MNP$. Draw $(C)$.

2° A variable chord drawn from $N$ cuts $(C)$ in $A$ such that $A$ is a point of the small arc $MN$. $B$ is the foot of the perpendicular from $M$ to $(NA)$. $(PA)$ and $(MB)$ intersect in $C$. Calculate the angles $\angle CAB$ and $\angle BAM$ in terms of the angles of the triangle $MNP$ and deduce that $\angle CAB = \angle BAM$.

3° Show that $C$ is the symmetric of $M$ in terms of $(NA)$ and deduce the line where the point $C$ varies.

16. On a semi-circle $(C)$ with center $O$ and of diameter $[AB]$, let $E$ be the midpoint of the arc $\overset{\frown}{AB}$. $C$ is a variable point of $(C)$. $H$ is the orthocenter of $C$ on $[AB]$. Place on $[OC]$ the point $D$ such that $CH = OD$.

1° Compare the two triangles $OCH$ and $DOE$.

2° Determine the geometric locus of $D$.

17. $A$ and $B$ are two fixed points of a line $(xy)$ and such that $AB = 6$ cm.

1° What is the geometric locus of the centers $O$ of the circles $(O)$ tangent to $(xy)$ at $A$?

2° What is the geometric locus of the midpoint of $[OB]$?

3° $(BE)$ is the second tangent to $(O)$ in $E$, $(AE)$ and $(BO)$ intersect in $K$. What is the geometric locus of $E$?

4° What is the geometric locus of $K$?

18. $C(O, R)$ and $C'(O', R')$ are two exterior circles tangent at $A$. Suppose that $R > R'$.

The line $(OO')$ cuts $C(O, R)$ in $B$ and $C'(O', R')$ in $C$. $[BP]$ is a chord of $C(O, R)$ and $[AM]$ is a chord of $C'(O', R')$ parallel to $[BP]$.

$(BF)$ and $(CM)$ intersect in $T$.

1° Determine the nature of the quadrilateral $AMTP$ and deduce that $PM = AT$.

2° What is the geometric locus of $T$?

3° What is the geometric locus of the midpoint of $[PT]$?

19. $[AB]$ is a diameter of a circle $C(O; 4$ cm$)$. $M$ is a variable point of $C(O; 4$ cm$)$. Since $(\Delta)$ and $(\Delta')$, the tangents to this circle respectively at $A$ and $B$. The tangent to this circle in $M$ cuts $(\Delta)$ in $E$ and $(\Delta')$ in $F$.

1° Show that the triangle $FOE$ is right at $O$.

2° Consider $AE = x$ and $BF = y$. Show that $xy = 16$.

3° $(MA)$ and $(OE)$ intersect in $I$; $(MB)$ and $(OF)$ intersect in $J$. Show that the length of $IJ$ is constant when $M$ varies on a circle.

4° $(IJ)$ cuts $(OM)$ in $K$. Determine the geometric locus of $K$. 

216
APPENDIX F

Instructional Unit on Geometric Locus

Setting:

The unit is developed based on the same number of sessions of the year before. Some sessions take place in a computer lab in which students have access to computers with Geometer’s Sketchpad installed. The other sessions are conducted in a regular classroom with a chalk board and an overhead projector.

Title of the unit: Some Geometric Loci

Grade level: Grade 9

Number of sessions: 7 sessions

General objectives of the unit: At the end of this unit, students should be able to:

1- Identify the geometric loci of points satisfying given properties (listed below).

2- Solve open geometric locus problems requiring conjecturing and proving.

The locus properties to be considered are:

- A variable point keeping a constant distance $r$
  from a fixed point $O$ moves on circle of center
  $O$ and radius $r$. 
- A variable point equidistant from endpoints of a given segment moves on the perpendicular bisector of this segment.

- A variable point equidistant from the two arms of an angle moves on the bisector of this angle.

- A variable point M such that a given fixed semi-straight line \([Ox]\) forms a constant angle \(m\) with \([OM]\), moves on a pair of semi-straight lines forming an angle \(m\) with \([Ox]\).

- A variable point M which always forms a right angle with the endpoints A and B of a given fixed segment \((\overline{AMB} = 90^\circ)\) moves on a circle of diameter \([AB]\).

- A variable point keeping a constant distance \(L\) from a given fixed straight line moves on two straight lines parallel to the given straight line and lying at a distance \(L\) from this given
straight line.

- A variable point keeping a distance k from two fixed parallel straight lines moves on a straight line parallel to the two fixed straight lines and located midway between them.

Session 1: (50 minutes)

*Specific objectives of the session:* At the end of this session, students should be able to:

- Distinguish between fixed and variable elements of a geometric figure
- Identify two types of locus that are related to the following properties:
  - A variable point keeping a constant distance \( r \) from a fixed point \( O \) moves on circle of center \( O \) and radius \( r \).
  - A variable point equidistant from endpoints of a given segment moves on the perpendicular bisector of this segment.
- Come up with a definition for the term locus of a point

*Setting:* Classroom

*Procedure:*

(1) Students solve a question with an objective to distinguish between fixed and variable elements. (Refer to sheet1 in this appendix)
(2) Teacher discusses the solution of the question with the students. After the discussion, students should be able to come up with the following statements:

- the midpoint of a fixed segment is fixed
- a segment with a variable endpoint is variable
- a straight line drawn from a fixed point and perpendicular to a fixed straight line is fixed
- a straight line drawn from a fixed point and parallel to a fixed straight line is fixed
- a circle whose center is variable or whose radius is not constant is variable
- a straight line passing through two fixed points is fixed

(3) Students solve another question about fixed and variable elements. (Refer to sheet 2 in this appendix)

(4) Teacher introduces two types of locus of a point that are related to the properties listed in the specific objectives of this session through problem situations. Together, the teacher and the students, will model the problem situations.

*Problem situation 1:*

You are standing in the middle of your classroom. The teacher asks Ahmad, your classmate, to stand 3 meters away from you. The teacher then asks Ola, Aya, Rachelle and Khodor to also stand 3 meters away from you. Can you start to picture what is happening? If all your classmates were to stand 3 meters away from you, what geometric shape or path would your classmates be forming?
Problem situation 2:

You are practicing Basketball (there are two baskets in the court). Where could you possibly stand so that at the same time your shooting distance to each basket will be exactly the same length?

(5) Using a computer and an overhead projector, the teacher will model the two types of locus of a point mathematically. (Refer to sheet 3 in this appendix “types 1 and 2”)

(6) Assigning homework: numbers 1 and 2. (Refer to sheet 5 in this appendix)

Session 2: (50 minutes)

Specific objective of the session:

At the end of the session, students should be able to find the locus of a point satisfying given properties:

- A variable point equidistant from the arms sides of an angle moves on the bisector of this angle
- A variable point M which always forms a right angle with the endpoints A and B of a given fixed segment (\( \overline{AB} = 90^\circ \)) moves a circle of diameter \([AB]\).
- A variable point M such that a given fixed semi-straight line \([Ox]\) forms a constant angle \(m\) with \([OM]\), moves on a pair of semi-straight lines forming an angle \(m\) with \([Ox]\).

Setting: Classroom
Procedure:

(1) Correction of homework in class on board.

(2) Teacher introduces three types of locus of a point that are related to the properties listed in the specific objectives of this session using a computer and an overhead projector. (Refer to sheet 3 in this appendix, “types 3, 4 and 5”)

(3) Class work: numbers 3 and 4. (Refer to sheet 5 in this appendix)

(4) Assigning homework: numbers 5 and 6. (Refer to sheet 5 in this appendix)

Session 3: (50 minutes)

Specific objective of the session:

At the end of the session, students should be able to find the locus of a point satisfying a given properties:

- A variable point keeping a distance k from two fixed parallel straight lines moves on a straight line parallel to the two fixed straight lines and located midway between them.

- A variable point keeping a constant distance $L$ from a given fixed straight line moves on two straight lines parallel to the given straight line and lying at a distance $L$ from this given straight line.

Setting: Classroom
Procedure:

(1) Correction of homework in class on board.

(2) Teacher introduces the last two types of locus of a point that are related to the properties listed in the specific objectives of this session through problem situations. Together, the teacher and the students, will model the problem situations.

Problem situation 1:

Your teacher has placed a strip of tape on the classroom floor which forms a straight line. The teacher gives each student a stick, 1 meter long, and asks that each student stand exactly 1 meter away from the line on the floor. Can you picture what will happen? If you, and all of your classmates, stand exactly 1 meter away from the line, describe where you and your classmates will be standing?

Problem situation 2:

During your morning jog, you run down an alley between two buildings which are parallel to one another and are 10 meters apart. Describe your path through the alley so that you are always the same distance from both buildings.

(3) Using a computer and an overhead projector, the teacher will model the two types of locus of a point mathematically. (Refer to sheet 3 in this appendix, “types 6 and 7”)

(4) Class work: number 7. (Refer to sheet 5 in this appendix)

(5) Assigning homework: number 8. (Refer to sheet 5 in this appendix)
Session 4: (50 minutes)

Specific objective of the session:

At the end of the session, students should be able to find locus of points satisfying different properties using Geometer’s Sketchpad.

Setting: Computer Lab

Procedure:

(1) In class: Correction of homework on board.

(2) In computer lab: Students solve activities 1 and 2 (Refer to sheet 4 in this appendix)

(3) Assigning homework: exercise 9 in sheet 5 in this appendix (same as activity 2) for students to solve it using paper-pencil without using a computer.

Lab settings: First, students are prepared for the use of Geometer’s Sketchpad. Due to time limitation, the researcher asks the computer teacher to train all the 20 students on Geometer’s Sketchpad in the computer sessions by asking them simple constructions before two weeks from starting the unit implementation. Second, students with similar levels of achievement are paired. Class teacher explains to the students the activity procedures and the researcher interviews one pair of students at a time, according to the clinical interview technique (Ginsburg, 1981), to assess their way of thinking.
Clinical interview: During each of the four DGS-based sessions, clinical interviews with students are conducted. Two pairs of low achievers and two pairs of middle achievers are interviewed. Two pairs of low achievers and two pairs of middle achievers were interviewed. In the first lab session two pairs of low achievers were interviewed. In the second lab session one pair of middle achievers was interviewed. In the third lab session one pair of middle achievers was interviewed. One of the low achievers’ pair was chosen to be interviewed based on their request; however, the other pairs were chosen randomly. Students were asked questions about their way of thinking while solving the problem and were free to answer the way they want. The pair was asked questions that reveal students’ way of thinking but do not affect their thinking. Examples of questions might be: “Can you tell me how you reached the solution?” “Why did you choose to do this?” “Why are you moving this point?” .. etc.

The interviews were audio taped and the students’ computer files were saved and kept for analysis. (Note: the interviewed students were asked to save their work every 5 minutes under different files’ names in order to compare the audio taping results with their work).

Session 5: (50 minutes)

Specific objective of the session:

At the end of the session, students should be able to find locus of points satisfying different properties using Geometer’s Sketchpad.

Setting: Computer Lab
Procedure:

(1) In class: Correction of homework on board.

(2) In computer lab: Students solve activity 3 (Refer to sheet 4 in this appendix)

(3) Assigning homework: exercise 10 in sheet 5 in this appendix (same as activity 3) for students to solve it using paper-pencil without using a computer.

Session 6: (50 minutes)

Specific objective of the session:

At the end of the session, students should be able to find locus of points satisfying different properties using Geometer’s Sketchpad.

Setting: Computer Lab

Procedure:

(1) In class: Correction of homework on board.

(2) In computer lab: Students solve activity 4 (Refer to sheet 4 in this appendix)

(3) Assigning homework: exercise 11 in sheet 5 in this appendix (same as activity 4) for students to solve it using paper-pencil without using a computer.

Session 7: (50 minutes)

Specific objective of the session:

At the end of the session, students should be able to solve high level problems about locus of a point.
Setting: Classroom

Procedure:

Application: Students solve assigned problems in class with their teacher using only paper and pencil. When they solve the problems, the teacher uses an overhead projector in order to demonstrate the problems and chooses a student to write the proof on the board.

Problem 1:

Given a straight line (D) and a point A outside (D). Points B and C are two variable points on (D). The heights drawn from B and C in triangle ABC intersect in H. What is the geometric locus of H?

Problem 2:

B is a fixed pint on a fixed circle of center O. A is a variable point on (C). S is midpoint of [AB].

1) Show that (OS) is the perpendicular bisector of [AB].

2) What is the geometric locus of S?
Given a circle $C(O, 4\text{cm})$ and a fixed point $B$ on circle $C$.

1. Mark point $M$, the midpoint of the radius $[OB]$.

   Point $M$ is (fixed / variable).

2. Locate a point $P$, not on $[OB]$, such that $OP = 2\text{cm}$.

   Are there many possibilities for point $P$? (yes/ no).

3. Draw $(d) \perp (OB)$ at $B$. $(d)$ is (fixed/ variable).

4. Draw $(r) \parallel (d)$ and passing through $O$. $(r)$ is (fixed/ variable).

5. Mark point $Z$ on $(d)$. Let point $Z$ be a moving point on $(d)$. Draw circle $\mathcal{N}$ of center $Z$ and radius $3\text{ cm}$. Circle $\mathcal{N}$ is (fixed/ variable).

6. $(r)$ cuts circle $C$ at $E$ and $F$. Straight line $(BE)$ is (fixed/ variable).
Given segment [AB] such that AB = 5 cm.

1. Mark point P, midpoint of [AB]. Point P is (variable/ fixed).

2. Mark a point C, not on (AB) such that BC = 3 cm.
   
   Are there many possibilities for point C? (yes/ no).

3. Draw (d) ⊥ (AB) at P. (d) is (variable/ fixed).

4. Draw (z) // (d) passing through A. (z) is (variable/ fixed).

5. Draw circle D (C, 2cm). Circle D is (variable/ fixed).

6. Draw circle T (B, AB). Circle T is (variable/ fixed).
Sheet 3

Type I of locus of a point that is related to the following property:

A variable point keeping a constant distance \( r \) from a fixed point \( A \) moves on circle of center \( A \) and radius \( r \).
Type 2 of locus of a point that is related to the following property:

A variable point equidistant from endpoints of a given segment moves on the perpendicular bisector of this segment.
Type 3 of locus of a point that is related to the following property:

A variable point equidistant from the arms sides of an angle moves on the bisector of this angle
Type 4 of locus of a point that is related to the following property:

A variable point M such that a given fixed semi-straight line [Ox) forms a constant angle \( m \) with [OM), moves on a pair of semi-straight lines forming an angle \( m \) with [Ox).
Type 5 of locus of a point that is related to the following property:

A variable point M which always forms a right angle with the endpoints A and B of a given fixed segment $\overline{AMB} = 90^\circ$ moves a circle of diameter $[AB]$. 
Type 6 of locus of a point that is related to the following property:

A variable point keeping a constant distance $L$ from a given fixed straight line moves on two straight lines parallel to the given straight line and lying at a distance $L$ from this given straight line.
Type 7 of locus of a point that is related to the following property:

A variable point keeping a distance $k$ from two fixed parallel straight lines moves on a straight line parallel to the two fixed straight lines and located midway between them.
Sheet 4

Geometer’s Sketchpad Activities

Name: ___________________
Date: ___________________
BE 9 (A/B)

Activity Sheet 1

Procedure:

1- Open a blank Geometer’s Sketchpad page.

2- Draw rectangle ABCD by following the steps below:

   a- Draw a segment [AB].

   b- Construct from point A straight line (d) ⊥ to [AB].

   c- Construct from point B straight line (k) ⊥ to [AB].

   d- Locate point C on (d).

   e- From point C construct straight line (n) || (AB).

   f- (n) intersects (k) at D.

3- Let O be the center of the rectangle ABCD.

4- Without moving any point, tell on what path does the center of the rectangle move when C varies? ________________________________

5- In order to check your answer trace point O and animate point C.

6- Considering that points A and B are given and fixed, indicate the variable and the fixed elements.

                                                                                                           
                                                                                                           

150
7- What is the locus of center O of the rectangle ABCD when point C varies on (d)? Why?

_________________________________________________________________

_________________________________________________________________

Activity Sheet 2

Procedure:

1- Open a blank Geometer’s Sketchpad page.

2- Draw segment [AB] of midpoint O.

3- Draw circle C (O, OA).

4- Mark a variable point M on (C).

5- Draw straight line (AM).

6- Find the distance from A to M.

7- Construct circle (L) of center M and radius equal to the distance from A to M.

8- Circle (L) and (AM) intersect at point I

   (I will be the symmetric of A with respect to M).

9- Hide circle (L)

10- How are the segments [OM] and [BI] related?

11- Without moving any point, tell on what path does I move?

12- In order to check your answer trace point I and animate point M.

_________________________________________________________________

_________________________________________________________________
Activity Sheet 3

Procedure:

1- Open a blank Geometer’s Sketchpad page.

2- Draw circle C (O, OA).

3- Plot B on circle (C) / B is a variable point.

4- Connect O and B by a segment.

5- From B construct straight line (d) // to [OA].

6- From A construct straight line (z) // to [OB].

7- (d) and (z) intersect at M.

8- Join A and M by a segment.

9- Join B and M by a segment.

(AOBM is a parallelogram)

10- Hide (d) and (z).

11- Without moving any point, tell on what path does M move as B varies.

____________________________________________________________

____________________________________________________________

12- To check your answer, trace M and animate B. Justify the answer.

____________________________________________________________

____________________________________________________________

____________________________________________________________

13- Let I be the center of the parallelogram AOBM.

14- Without moving any point, tell on what path does I move as B varies.

____________________________________________________________

____________________________________________________________

15- To check your answer, trace I and animate B. Justify the answer.

____________________________________________________________
Activity Sheet 4

Procedure:

1- Open a blank Geometer’s Sketchpad page.
2- Draw segment [AB] of midpoint O.
3- Draw circle \(C\) (O, OA).
4- From point A construct straight line (x’x) perpendicular to [AB].
   
   (x’x) is tangent to circle (\(C\)) at A
5- Plot M on [Ax).
6- (BM) cuts (\(C\)) in I.
7- Connect B and M by a segment.
8- Let J be the midpoint of [BM].
9- Without moving any point, on what path do you think does J move as M varies?
10- In order to check, trace J and animate M. Justify your answer.

________________________________________________________________________
________________________________________________________________________
11- Erase traces.
12- Draw a circle \(D\) (J, AJ).
13- Connect A and J by a straight line.
14- (AJ) cuts (\(D\)) at K.
15- Hide circle (\(D\)).

(K is symmetric to A with respect to J).
16- On what path does K move as M varies? Trace K and animate M.
Justify your answer.

17- Erase traces and hide K, J, (AJ), (BM), and circle (D).

18- Draw circle \(H\) (I, AI).

19- Connect A and I by a straight line

20- (AI) cuts circle (\(H\)) at N.

21- Hide circle (\(H\)).

\(N\) is symmetric to A with respect to I.

22- On what path does N move as M varies? Trace N and animate M.
Justify your answer.
Sheet 5

Book Exercises

Number 1

The diagonal [AC] of a rhombus ABCD is fixed and the vertices B and D are variables.

Determine the geometric locus of B and D.

Number 2

Triangle ABC is a right triangle at A (A is a fixed point) such that BC = 8cm.

What is the geometric locus of point I, the midpoint of [BC]?

Number 3

O is a fixed point of a fixed line (xy). A is a variable point of (xy). On the perpendicular to (xy) at A, place M such that OA = AM.

1) What is the nature of the triangle AOM?
2) What is the geometric locus of M?

Number 4

ABC is a triangle such that A and B are fixed and C is variable. If [AH] is a height-segment of this triangle, find the geometric locus of H.

Number 5

Given $\widehat{xAy}$ is a fixed angle with $\widehat{xAy} = 60^\circ$. M is a variable point of [Ax) and N is the symmetric of M with respect to [Ay).

What is the geometric locus of N?
**Number 6**

The angle $x\overrightarrow{O}y$ is fixed. The two points $B$ and $C$ are variables respectively on $[Ox)$ and $[Oy)$ such that $OB = OC$.

1) What is the nature of triangle $BOC$?

2) Find the geometric locus of $M$, the midpoint of $[BC]$.

**Number 7**

$(xy)$ is a fixed line and $A$ is a fixed point not belonging to $(xy)$. $[AH]$ is the perpendicular to $(xy)$, $H$ is on $(xy)$, $I$ is the midpoint of $[AH]$. A variable line passing through $A$ cuts $(xy)$ in $B$.

What is the geometric locus of point $M$, the midpoint of $[AB]$?

**Number 8**

$ABC$ is a triangle of vertex $A$ variable and a fixed base $[BC]$, such that the area of this triangle is $15 \text{ cm}^2$ and $BC = 6\text{cm}$.

1) Calculate the length of the height $[AH]$.

2) Deduce the geometric locus of $A$.

**Number 9**

$C (O, R)$ is a circle of a fixed diameter $[AB]$.

$M$ is a variable point of this circle and $I$ is the symmetric of $A$ in terms of $M$.

1) Show that $(OM)$ and $(BI)$ are two parallel lines.

2) Find the geometric locus of $I$. 

**Number 10**

(C) is a fixed circle of center O and a radius R.
A is a fixed point of (C) and B is a variable point of (C).
The point M is the fourth vertex of the parallelogram OAMB.
1) What is the geometric locus of M?
2) What is the geometric locus of the center I of the parallelogram OAMB?

**Number 11**

(C) is a circle of center O and of radius R.
[AB] is a fixed diameter of (C).
(x’Ax) is tangent to (C) in A. M is a variable point on [Ax). (BM) cuts (C) in I. J is the midpoint of [BM].
Find the geometric locus of:
1) J
2) K symmetric of A with respect to J.
3) N symmetric of A with respect to I.