LEBANESE AMERICAN UNIVERSITY

INVESTIGATING DIFFICULTIES FACED BY GRADE 1 TO 6 STUDENTS WHILE LEARNING GEOMETRY BASED ON THE LEBANESE CURRICULUM’S OBJECTIVES AND CONTENT

By

ARAZ MINAS KEUROGHLIAN

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Name of Student: Anaz Keuroghlian  I.D.#: 200903109

Department: Education

On (dd/mm/yyyy) 19/12/2011, has presented a Thesis proposal entitled:

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in the presence of the Committee Members and Thesis Advisor:

Advisor: Iman Osta
(Name and Signature)

Committee Member: Mona Majdalani
(Name and Signature)

Committee Member: Tamer Amin
(Name and Signature)

Proposal Approved on (dd/mm/yyyy) 19/12/2011

Comments / Remarks / Conditions to proposal approval (if any):

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Date: 20/12/2011 Acknowledged by [Signature] 21/12/2011
(Dean, School of Arts and Sciences)

cc: Dean
Chair
Advisor
Student
Thesis Defense Result Form

Name of Student: Araz Keuroghlian
I.D. #: 200903109

Program / Department: Education – Emphasis Math Education

Date of thesis defense: January 29, 2013

Thesis title: Investigating Difficulties Faced by Grade 1 to 6 Students While Learning Geometry Based on the Lebanese Curriculum’s Objectives and Content

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☐ Thesis is not approved. Grade NP is recorded

Committee Members:

Advisor: Dr. Iman Osta, PhD, Mathematics Education

Committee Member: Dr. Mona Majdalani, PhD, Curriculum and Instruction

Committee Member: Dr. Tamer Amin, PhD, Science Education

Advisor’s report on completion of corrections (if any):

Corrections have been completed according to the Committee’s recommendations

Changes Approved by Thesis Advisor: __________________ Signature: ________________
Date: 5/2/13

Acknowledged by: __________________

cc: Registrar, Dean, Chair, Advisor, Student
Thesis approval Form

Student Name: Araz Keuroghlian  I.D. #: 200903109

Thesis Title: Investigating Difficulties Faced by Grade 1 to 6 Students While Learning Geometry Based on the Lebanese Curriculum’s Objectives and Content

Program: MA in Education, Emphasis Math Education

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School: School of Arts and Sciences

Approved by:

Advisor: Dr. Iman Osta, PhD, Mathematics Education

Committee Member: Dr. Mona Majdalani, PhD, Curriculum and Instruction

Committee Member: Dr. Tamer Amin, PhD, Science Education

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Investigating Difficulties Faced By Grade 1 To 6 Students
While Learning Geometry
Based on the Lebanese Curriculum’s Objectives and Content

Araz Minas Keuroghlian

Abstract

Geometry is an important aspect of one’s everyday life. As such it is crucial for young learners to study, understand and apply geometry during their academic years. Yet, time and again, young learners complain that geometry is difficult. This research investigates difficulties faced by grade 1 to 6 students while learning geometry based on the Lebanese curriculum’s objectives and content. The study spans two academic years during which the geometric knowledge and skills of students from grade 1 to 6 are investigated through a series of tests, observations and interviews. The first test took place at the end of the first academic year. The test consisted of knowledge and application level exercises based on the objectives provided by the Lebanese geometry curriculum. The second test takes place at the beginning of the following academic year. It is parallel and equivalent to the first test, content wise. The aim is to study how maturity influences some skills, and how much students can retain after some time has passed. The third test is based on performance and objective based tasks which also took place at the beginning of the second academic year, but after the second test. It involves problem solving, as well as procedural exercises. During the third test students are closely observed and interviewed while detailed notes are taken about their behaviors and responses. As a result of this study, it becomes obvious that some of the students’ major difficulties with understanding and applying geometry, as well as some of the sources for such difficulties are based on their inability to conceptualize, recall terminology, recall properties, recall procedures, recognize, coordinate their visual motor skills, and use geometric tools. Yet since these difficulties are mostly based on knowing (the first cognitive domain of TIMSS), a deeper look shows that the problem starts with the poor distribution of objectives across the years, their inconsistency with the cognitive development of the students at each grade level, and the insufficient time allocated to geometry in the elementary classes.

Keywords: National Curriculum, Geometry, Learning Difficulties, Elementary School Education, Lebanon.
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Abbreviations
Listed in this section are all the acronyms used in this thesis alongside their definitions.

ECRD: Educational Center for Research and Development
FIL: Final Intensity Level
IL: Intensity Level
K-12: Kindergarten to Twelfth Grade
MKO: More Knowledgeable Other
NCTM: National Council of Teachers Of Mathematics
OPV: Objective Percentage Value
PCK: Pedagogical Content Knowledge
SPV: Student Percentage Value
TIMSS: Trends in International Mathematics and Science Study
VAK: Visual, Auditory, and Kinesthetic
VAKT: Visual, Auditory, Kinesthetic, and Tactile
ZPD: Zone of Proximal Development
CHAPTER ONE

INTRODUCTION

1.1 - Overview

Mathematics is useful in everyday life of mankind. Different situations require the use of different mathematical concepts. Such is the case for geometry. As it is directly related to our environment, learning and using it become important, if not essential.

Geometry is the study of shapes and space. Since these are already available in mostly everyone’s life even before starting to understand number concepts, it is assumed that students would easily understand and grasp geometrical knowledge and develop geometrical skills. Instead, teachers often hear statements such as: “We are quite good in arithmetic, but we just don’t understand geometry”. It is surprising that students should have difficulty in a domain that is tightly related to the environment in which they live. Awareness of shapes and their properties, relationships between lines, patterns, and other components of geometry, play a large role in daily life. For instance, students can locate objects in space. They can also navigate through paths when moving from one location to another, such as when going from the classroom to the playground, and so on. These are naturally occurring events and they, in turn, follow the basic principles of geometry (Cawley, Foley, & Hayes, 2009).

This being said, it is startling to realize that although one can find a multitude of published studies about arithmetic, literature that discusses elementary-level geometry is scarce. Hence, there is a need to investigate the difficulties that young learners face
when they try to tackle geometric tasks. In order to highlight these difficulties, elementary level students are assessed for their knowledge and skills in geometry through multiple paper-pencil testing, performance tasks and oral communication.

Furthermore, the following points are reasons that make the study of geometry essential enough to ascertain its proper development throughout the school years.

- Geometry and other contents of mathematics are connected and interrelated
- Geometry is part of the universal intellectual heritage
- Geometry is by excellence the field of development and application of inductive and deductive reasoning and logic
- Geometry is the knowledge base for many future specialty domains, such as architecture, engineering, plastic arts, design…

Therefore, dealing with geometry cannot rely plainly on rote learning or completion of tasks, but rather, the construction of mental schemas and well developed skills for their application. Learning geometry goes beyond memorizing facts, definitions and properties. It involves understanding, visualizing (figures, patterns, movement), applying, and reasoning. In order to do so, one must connect the tasks that students face with the definitions, properties and procedures that have been learnt (Cawley, Foley, & Hayes, 2009).

The standards issued by the National Council of Teachers of Mathematics (NCTM) also insist on the importance of building connections among different mathematical topics, as mathematics is not separated into an arbitrary set of rules and skills (National Council of Teachers of Mathematics, 2000). Through geometry, learners are offered opportunities to develop their reasoning, critical thinking, as well as
their communication processes, language comprehension, and cognitive performance (Cawley, Foley, & Hayes, 2009). As a result, through the proper study of geometry, one can learn to be critical while dealing with everyday life problems or situations, while taking part in intellectual activities (Cawley, Foley, & Hayes, 2009).

1.2 - The National Council of Teachers of Mathematics (NCTM)

Standards

The National Council of Teachers of Mathematics Standards (2000) provide recommendations regarding the curriculum, teaching style and technique, and assessment for K-12 grade levels. The standards are based on what is valued by the society and work force. They state priorities and goals as well as what students should know and what they should be able to do (Hiebert, 1999). Furthermore, the Standards provide support for mathematics teachers to assist them in ensuring proper learning outcomes for all students. No longer is what students learn that is the main focus, but rather how they learn it that plays a critical role.

The NCTM Curriculum and Evaluation Standards (1989) advocates the Van Hiele’s theory (with the hierarchy of its levels) in order to ensure the development of geometric thought. Students initially start by naming shapes, then stating properties, and in a later stage, making relations amongst the shapes, and thus making simple deductions (Mason, 1998).

Then, in 2000, the NCTM Principles and Standards for School Mathematics were published. It discusses five processes that endorse students’ evolution in a mathematical community (Gavin, Casa, Adelson, Carroll, Sheffield & Spinelli, 2007).
As stated by NCTM (n.d.), the five processes are as follows:

*Problem solving* is where students are given the opportunity to construct new mathematical knowledge. The problems must have different contexts and must permit students to apply and adapt different strategies while solving.

*Reasoning and proof* is where students are given the opportunity to realize that reasoning and proof are fundamental. Students should investigate and make mathematical conjectures, as well as evaluate and develop mathematical proofs, while they use different types of mathematical reasoning and proof.

*Communication* is where students learn how to organize their mathematical thoughts through clear and coherent communication with their classmates and teacher. As such, students get to evaluate and analyze other students’ and the teacher’s mathematical thoughts and strategies, as well as learn the correct terminologies that are required to convey mathematical ideas correctly.

*Connection* is where students are urged to recognize and connect different mathematical ideas. Students must understand how they interconnect and together generate a coherent whole. These connections are important for students to be able to apply mathematics in different contexts, even those outside of mathematics.

*Representation* is where students are urged to use and create representations in order to organize, and communicate their mathematical ideas. Students should learn how to select, translate among, and apply mathematical representations while solving problems.

In accord with NCTM, the TIMSS framework also promotes better understanding and learning of mathematics in a more elaborate and precise manner.
1.3 - Trends in International Mathematics and Science Study (TIMSS)

Framework

TIMSS is a large-scale test for assessing fourth and eighth grade students’ achievement in mathematics and science at the international level. TIMSS has been collecting data every four years since 1995. Each four years is considered a cycle (National Center for Education Statistics, n.d.). Research based on TIMSS data is crucial as it benefits the field of mathematics education by highlighting what and how students learn.

TIMMS routinely gathers background information concerning the content of instruction, its quality and quantity. For instance, TIMSS 2007, the fourth cycle of internationally comparative studies, collected information about curriculum coverage, teacher preparation, and the availability of resources and technology (IEA, 2010).

This data was dedicated to the improvement of the teaching and learning of mathematics and science for learners around the world.

The TIMSS 2007 framework for mathematics is organized around two dimensions:

1- Content: It is the dimension whereby the subject matter within mathematics is specified (arithmetic, algebra, geometry…)

2- Cognitive domain: It is the dimension whereby the cognitive domains are specified (knowing, applying, and reasoning).
More specifically, Mullis, et al. (2005), delineates the TIMSS 2007 mental behaviors related to each cognitive ability as follows:

The first of the three cognitive domains is *knowing*. It covers facts, procedures and concepts that students must have learned. The behaviors that are related to *knowing* are *recalling*, *recognizing*, *computing*, *retrieving*, *measuring* and *classifying/ordering*.

The second domain is *applying*. It focuses on students’ ability to apply their knowledge as well as their conceptual understanding in order to *solve routine problems*. The behaviors that are related to *applying* are *selecting*, *representing*, *modeling*, *implementing* and *solving routine problems*.

The third cognitive domain is *reasoning*. It encompasses unfamiliar conditions while solving multi-step problems in elaborate contexts (Mullis, et al., 2005). The behaviors that are related to *reasoning* are *analyzing*, *generalizing*, *synthesizing/integrating*, *justifying* and *solving non-routine problems*.

The present study will henceforth follow the TIMSS framework. The reasons for this selection are as follows:

1- The cognitive levels can be applied at every elementary grade level.

2- The objectives in the Lebanese geometry curriculum can be categorized under the three cognitive domains.

3- Lebanon has already taken part in TIMSS testing on an international level, whereby the test results are based on the cognitive levels, and statistics are already available in comparison to other countries around the world.

It is noteworthy that Lebanon has participated in the TIMSS cycles, for the years of 2003, 2007 and 2011, at the level of the eighth grade (NCESb, n.d.).
The TIMSS scale average score is 500 as scores range from 0 to 1000. The results for the years of 2003 and 2007 show that Lebanese students’ average scores are lower than those of the scale average score. They are also lower than the US students’ average score, but higher than those of the Arab countries. In 2003, Chinese Taipei scored highest with 585, the US scored 504, Lebanon scored 433, and Jordan (the closest Arab country to the Lebanese score) scored 424.

In 2007, the highest score was 598, by students from Chinese Taipei. US had a score of 508, Lebanon had a score of 449, and the closest Arab country to Lebanon, being Jordan, had a score of 427. Lebanon ranked 28\textsuperscript{th} in the world (out of 48 participating countries), 19\textsuperscript{th} after the US, and first amongst the Arab countries (Martin, Mullis, & Foy, 2008).
Although all countries showed improvement in 2007, as compared to 2003, the results were quite close. These numbers, obtained by comparing data from the numbers provided by NCES, also show that Lebanon had the highest rate of improvement over the years.

In regards to the cognitive domains, in 2003, Lebanon scored 447 for knowing, 426 for applying, and 410 for reasoning, all of which showed a relatively good rate of improvement in 2007. Lebanon scored 464 for knowing, 448 for applying, and 429 for reasoning (Martin, Mullis, & Foy, 2008). Regardless, still there is an obvious need for improvement.
Chart 2

Comparing Lebanon’s 2003 and 2007 scores across the cognitive domains, based on numbers obtained from Martin, Mullis, and Foy (2008)

Furthermore, Lebanon again showed improvement from 2003 to 2007, and scored 454 in number theory, 465 in algebra, 462 in geometry, and 407 in data and chance. Yet again it is clear that the scores lag way below the scale average.
While studying the TIMSS results provided by NCES, one notices that even though grades are below the scale average when it comes to “Lebanese” mathematics in general, the lowest being the score for the domain of data and chance, from a more global perspective, one can also notice that in some domains, Lebanese scores are relatively more acceptable than in other domains. For example, in geometry, Lebanon ranks in the 24th place out of the participating 48 countries, 7th after the US, and 1st amongst the Arab countries (Martin, Mullis, & Foy, 2008).
As a result, after having viewed the assessment results provided by TIMSS and NCES, it is clear that, although there is much improvement from the 2003 cycle to the 2007 cycle, there is still a lot of room for growth. Lebanon’s scores are below the scale average, and as such, this fact points out the need for academic improvement in all areas of mathematics, geometry included.

Results also show that Lebanese students face difficulty with the three cognitive domains highlighted by TIMSS (see charts 2 and 3).

It is noteworthy to mention that although this framework applies to all fields of education, it particularly plays an important role in geometry learning as well.

1.4 - Statement of the problem

Teaching experience and observation of learners’ behaviors make it apparent that learners find difficulty when trying to deal with geometry. Yet, another more valid indicator of students facing difficulty with geometry is the TIMSS results. On average, students were only able to answer 46.2% of the geometry test items correctly (Martin, Mullis, & Foy, 2008).

Together, the teaching experience and test results conducted on an international level, show that Lebanese students are having difficulties with the learning and using of geometry. Furthermore, the possible sources of the difficulties are numerous. Learners might have difficulty using the geometry tool set appropriately; such is the case when using the set-square, the compass, the protractor. Another difficulty that learners face is in understanding what is required from a written task, and then relating their understanding to the knowledge that has been previously acquired. This is directly related, but not limited, to learners’ ability to understand and to use vocabulary and other
communication skills to their advantage. According to Cawley, Foley, and Hayes (2009), students solving geometry tasks are expected and encouraged to use the proper vocabulary of geometry.

Yet a more analytic look will show some other possible sources for the difficulties that many students face in the formal learning of geometry. The difficulties may be due to the amount of geometry that the textbooks include. Aside from being insufficient, these amounts are also far less than those dedicated to other components of mathematics, and the number of periods assigned in the curriculum (Educational Center for Research and Development [ECRD], 1997) to geometry is also far less than that assigned to other components. This means that not much emphasis is put on geometry.

Some other reasons for students facing difficulty would include the lack of textbooks and curriculum alignment with the many well developed and recognized stages of cognitive development.

1.5 - Purpose of the study

The purpose of this study is three-fold.

The first purpose is to study the scope and sequence of the Lebanese curriculum for geometry, and to check if the objectives are set based on proper pedagogical foundations.

The second is to investigate some of the elementary level students’ major difficulties while learning geometry by conducting an analysis of elementary students’ procedures and thinking as they try to perform geometric tasks and solve geometric problems.
The third purpose is to investigate whether these difficulties increase or decrease across the six elementary grade levels.

In order to conduct an analytic study students will be tested based on objectives that are set by the Lebanese curriculum for geometry (ECRD, 1997). Additionally, the Framework of Trends in International Mathematics and Science Study (Mullis, et al., 2005) will be adopted especially for its three cognitive domains: knowing, applying and reasoning, and the mathematical abilities that fall under each domain.

The tools used to interpret the difficulties throughout this study are:

1- The geometry section in the Lebanese Curriculum, for grades 1 to 6 of Basic Education (ECRD, 1997), as well as its pedagogical and developmental foundations.

2- The difficulties as stated in the literature review.

1.6 - Research Questions

Throughout this study, the following questions will be examined.

1- How are the objectives and content of the Lebanese curriculum for geometry (ECRD, 1997) at the elementary level (grades 1 to 6, age 6 to 12) distributed among the three cognitive domains of TIMSS?

2- What are the major difficulties that students face at each grade level while applying their acquired geometric knowledge as they perform geometry tasks and solve geometric problems?

3- How do the major difficulties that students face while applying their acquired geometric knowledge evolve across the grade levels?
4- Are the major difficulties faced by students learning geometry subject to change over time? How do retention and/or maturation change students’ geometric abilities?

5- Does changing the testing style from paper-pencil to performance-based change the intensity or the type of difficulties that students face while applying their acquired geometric knowledge?

1.7 - Significance and rationale of the study

Through this study, a few key points will be investigated in regards to difficulties that elementary students face as they try to learn, understand and use geometry. Some of the sources of these difficulties may be as follows:

- Objectives and content of the Lebanese curriculum may be distributed among the three cognitive domains in an unorganized or inappropriate manner.

- Textbooks used may lack alignment with the curriculum goals or with the common stages of development.

- Time or emphasis attributed to geometry may not be sufficient.

Thus, by attending to these problems and potential others that become evident from the findings, the study will provide curriculum designers with a picture of the gaps that need to be addressed. It will also provide textbook publishers objective evidence of consistency or inconsistency in regards to the curriculum as well as to students’ cognitive development. More importantly, teachers will become better aware of the factors that create obstacles to students when performing geometric tasks. As such, teachers can make the appropriate adjustments, ultimately allowing young learners a
better opportunity to smoothly advance in their geometric knowledge and abilities throughout the years.
CHAPTER TWO
LITERATURE REVIEW

The following section presents a preliminary account of the major themes discussed in the literature that are related to the purpose of the study.

2.1 - Cognitive theories of learning

Cognitive theories help represent and better understand the psychological and educational factors that influence students’ thinking and learning. Educators are constantly concerned about factors and circumstances that influence the gradual evolution of students’ performances, knowledge structures, and conceptions.

Understanding and comprehension is an educator’s main concern. However, students’ understanding highly depends on their prior knowledge, and the progress from their prior knowledge to the new information is made.

Thus, cognitive theories of learning are referred to in order to analyze the source to some difficulties students face while learning geometry.

The first of the many cognitive theories of learning is that of Van Hiele’s. It heavily applies to the learning of geometry and describes the development of geometrical abilities.

Van Hieles’ theory is based on the five levels through which students reason while dealing with geometry. The levels as stated by Mason (1998) are as follows:

Level 1, Visualization involves the recognition of figures by comparing them to other figures that students have already encountered. At this stage, students do not perceive properties; they merely rely on perception, void of reasoning.
Level 2, *Analysis* involves seeing figures and realizing some of its properties. Students may recognize and state properties of the given geometric figures; however the relationships between the properties are not realized yet. At this level, when students describe objects, they will list all the apparent properties without realizing which properties are sufficient for the description of the object.

Level 3, *Abstraction* involves students perceiving relationships between properties and between figures. At this level, students can create meaningful definitions and give informal arguments to justify their reasoning. Logical implications and class inclusions, such as squares being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood.

Level 4, *Deduction* involves understanding the importance of having axioms and definitions, the construction of proofs, and knowing the role different conditions play on justifying their responses. However, this level involves the construction of proofs typically found at the level of high school.

Level 5, *Rigor* involves deduction and comparing different mathematical systems. At this level, students may rely on indirect proofs and/or proofs by contrapositive. No longer is Euclidean the only system they are exposed to. Again, this level is typical for high school students.

Mason (1998), goes on to state that the progress students make from one level to the next depends on their educational experiences that either facilitate or obstruct the advancement within levels. According to Van Hieles, students’ progress is based on the accessible *information*. Educators must identify what their students already know, and what else they need to know in order to have students better prepared for the new lesson.
This is followed by *guided orientation*, where students are given the opportunities to explore the contents of the lesson and its objectives by following carefully constructed tasks that may involve folding, tracing, cutting, measuring, or constructing at the elementary level. The educator in turn ensures that students get multiple and various encounters of the important concepts of the lesson. Once this phase is covered, *explication* takes place. Students discuss what they learned in their own words. This is when the educator introduces the relevant terminologies and allows students *free orientation*. Here students apply what they learnt while solving problems as well as some open-ended tasks. And lastly, *integration* allows students to summarize and integrate the main objectives of the lesson. As a result, students develop a new network of relations among the objectives, and thus developing their mental schemas.

It is important to note that students cannot advance among the levels without mastering the previous level (Mason, 1998).

Although Van Hiele’s framework seems most appropriate for this study, it will not be followed since at the elementary grades, not all levels of the framework can be satisfied.

Other frameworks and theories, such as those established by Jean Piaget and Lev Vygotsky, must also be taken into consideration as they are considered to be the most influential developmental psychologists to date.

According to Piaget (1997) learning and development is ensured by external factors and factors influencing *maturation*, valid *experience*, *social transmission*, along with *equilibrium*. According to him the prior factors do not reach the desired outcomes if there is no equilibrium, no assimilation.
Maturation relies on the development of the nervous system and prepares students for the phase that involves experience. Experience with objects of the physical realm helps develop students’ cognitive structure. However, students still cannot relate factors of different contexts on their own without teacher guidance. At this stage, students’ knowledge progresses when they are given experiences based on situations provoking logical-mathematical thinking. For instance, a student may arrange pebbles in many ways, but no matter the different arrangements, the number of pebbles does not vary. This experience teaches students that ordering does not influence the sum of objects. This in itself is a logical-mathematical deduction.

Social transmission is the phase where students express their thoughts using language and representations that are at the level of their understanding, either with their peers or teachers. It is through classroom discussion that students realize different points of view and better understand the situations they face. Through this phase, active students who face opposing views or new ideas, deal with uncertainty in their mind with respect to the knowledge they have compiled. To compensate, the mind assimilates the information, self-regulates, and finally the student finds equilibrium. Therefore, students need stimulations that permit this evolution from experiences to equilibrium. However, it is important for the students to have the mental structure and knowledge to be able to react and respond to the stimulations. Students can only learn by being able to connect the new information to their already existing, though simpler, structures of knowledge (Piaget, 1997).

Furthermore, Vygotsky, being one of the founders of constructivism, parallels Piaget’s theory, and discussed children’s cognitive development. He discusses three
major themes, the first of which is \textit{social interaction}. Through this interaction, students are exposed to knowledge in different contexts, bringing us to the next theme being the \textit{More Knowledgeable Other} (MKO). Educators are considered as MKOs as they are able to think at higher levels than their young learners. MKOs guide and expose students through particular tasks, procedures and concepts. These tasks may promote cognitive disequilibrium, and as a result promote assimilation. The third theme is that of the \textit{Zone of Proximal Development} (ZPD). Educators must be aware of the experiences they provide their students with just as much as they are aware of their students’ abilities and limitations. Students’ ZPD depends on their ability to complete tasks at a slightly higher level than their own level of thinking, under the guidance of an MKO or peer collaboration until students are able to think and solve at this higher level independently. According to Vygotsky, for learning to take place, students must solve tasks at this zone (Social Development Theory (Vygotsky), 2011). This is because, if the tasks are too easy, students get bored, while if the tasks are too difficult, students get frustrated and are discouraged from their exploration and will be unable to construct new knowledge correctly.

\textbf{2.2 - Geometry as an Object of Learning}

Geometry, coming from the Greek words \textit{geo}-earth and \textit{metron}-measure is a field in mathematics that studies geometric objects such as points and lines (Bogomolny, n.d.).

According to Jaber and Daher (2010), geometry is a subject that deals with shapes. The shapes are directly connected to our natural environment. Learning geometry makes students more aware of the need to justify claims in various settings,
and students are permitted to develop their exploration, discovery and research skills. As a result, students recognize geometry as an achievement, appreciate it, and thus become more flexible in their professional fields.

However, sometimes learning is not achieved without causing some misconceptions in the minds of students. The reason for this is that objects of geometry are abstractions contrived by convention, rather than being of the tangible sort, readily available in the physical realm. In Euclidean geometry for instance, a point has no dimensions and a line has length, but no width. Students need to realize that by drawing a dot or a straight line, one is merely communicating geometric thought by fostering pictorial representations (Bogomolny, n.d.). Regardless, using diagrams such as the dot or other diagrams to represent mathematical objects and notions allows students to make sense of abstract objects, bring them “down to earth” and thus, manipulate them (Osorio, n.d.).

2.3 - Research on geometry teaching and learning

When students find it difficult to understand a concept, they start to rely on rote memorization. However, unlike understanding, rote memorization limits students’ ability to use the concept in different contexts and be critical thinkers. Students eventually forget memorized material and thus, will not be able to apply it unfamiliar situation (Mason, 1998).

For learning based on understanding to occur, students should focus on, perceive and assimilate various types of information (Gantasala & Gantasala, 2009). Capraro and Capraro (2006) report that, according to Piaget, in order for students to progress from their initial understanding, they must make meaningful connections between their
personal knowledge and the task at hand. In turn, the task must be carefully developed in order to promote the connections. It is the educator’s role to ensure this.

Educators must first realize what students know, and what level of thinking they follow. This eliminates further difficulties that students might face. For instance, when a teacher uses a square, one student may only visualize its physical appearance, while others will recall a square’s properties, a third student might be able to state what properties required to prove having a square, and so on… All these are examples of different levels of thought with respect to a single word, the square, and an educator must make sure that students are at a given level of thought before they can handle the slightly higher levels (Mason, 1998).

Note that, at each level, students interpret information and express their thoughts differently. Active classroom discourse leads to uncovering information, explicating new knowledge, and integration. Students thus, reorganize their mental schemas as ideas become clearer to them (Mason, 1998).

Never the less, learning geometry involves not only verbal processes as previously mentioned, but also, visual reasoning. Piaget, and others alike, suggest that, when learning mathematics, visualization should include processes whereby students construct and transform both, their visual and mental imagery, as well as their inscriptions of the object in a spatial nature (Cannizzaro & Menghini, 2006).

Similarly, other learning theories point out other important factors to take into consideration while learning and teaching geometry. For instance, the Dunn and Dunn’s VAK model for learning geometry bases the theory not only on Visual, but also the Auditory, and Kinesthetic as sensory receivers. It is sometimes known as VAKT as it
also includes *Tactile* learning. Though one sensory receiver might be more dominant than the others, all four receivers are important for students’ learning. This model bases its premise on students’ relatively fixed characteristics and traits that inevitably affect their performance and achievements (Gantasala & Gantasala, 2009).

As cited by Gantasala and Gantasala (2009), Honey and Mumford (1992) discuss the *active experimentation* by students and their *reflective observations* in order to attain *abstract conceptualization*, as well as *concrete experience* with what they are trying to learn.

Presmeg (as cited in Pitta-Pantazi & Christou, 2009), discusses the distinction amongst verbalizers and imagers. Though it is not viewed as a cognitive style, it is however a thinking mode that varies with students’ individual differences.

Another key note when teaching/learning geometry is that students should be taught in a way whereby they are slowly led towards *inclusive* definitions. Definitions should include the *minimum properties required* to describe a given object, the *descriptive* definition, and the *prescriptive* or constructive definition (Cannizzaro & Menghini, 2006).

Correspondingly, students’ intuition must be stimulated. Students need to manipulate objects, and find patterns in order to advance their mathematical/ geometric thinking. As they do so, teachers should pay attention to students’ verbalization of characteristics and properties that they perceive. Dubinsky (as cited in Cannizzaro & Menghini, 2006) states that some students may need scaffolding and repetition in order to internalize the information.
All these need to be considered when teaching and learning geometry; but that is not all. Research also discusses other key aspects that need to be tackled when teaching and learning geometry. Skemp (as cited in Cannizzaro & Menghini, 2006), explores the relationship among concepts, mental models, and conceptual structures. Van Hiele (1986) further discusses the ability of students to move from perception to descriptions and definitions of geometric figures and then further on to even higher levels in Euclidean geometry. The Van Hieles’ theory expresses the importance of orienting students through these different stages, starting from the symbol, to the significant signal, and then to the definition.

According to the Van Hieles’, symbols are at the level of perception/visualization. This is when students condense all the properties of the studied geometrical figure. Signals are at the level of description/analysis and exploration. This is best described as the translation of perceptions into descriptions without involving explicit linguistic abilities. At this time, one significant property emerges, whereby students prepare, and are able to anticipate other properties. Definition/descriptive definition is when students begin to logically observe the different relations and order them. The Van Hieles (1958) considers this as the essence of geometry.

Furthermore, Capraro and Capraro (2006) state the constructivist view on learning, whereby students build their knowledge by integrating new, personalized experiences with their prior knowledge. Students’ experiences should include interaction with the teacher, their classmates, and the environment surrounding them. This is because, Capraro and Capraro (2006), like Piaget, point out that substantive learning only occurs when children’s mathematical curiosity is stimulated, particularly by
teachers. Students need to be given the opportunity to experience an assortment of interactions that would create internal and external conflict, or even surprise them. It is through classroom interaction, discussion, and purposeful questioning by the teacher, that meaningful connections occur. Another factor that educators must take into consideration is the importance of mathematics vocabulary as well as students’ ability to demonstrate their knowledge (Capraro & Capraro, 2006).

In addition, selected mathematics textbooks should help students solidify concepts such that they reflect the correct and complete mathematical idea. Finding literature that helps create cognitive dissonance in order to tackle students’ misconceptions also helps reduce obstacles students may face while learning (Capraro & Capraro, 2006).

2.4 - Difficulties faced by elementary students while solving geometry tasks

Research that examines the difficulties students face when solving geometry tasks, show various obstacles that students face. Among these obstacles are understanding and using geometry due to language barriers and communication skills, inability to use geometric tools precisely, misconceptions, inaccurate measurements, lack in preliminary content knowledge, and/or their inability to relate or connect what is being taught to their everyday lives and interests.

Nonetheless, geometry is part of the curriculum. It allows learners to reflect on and interpret their physical environment as well as the space around them. To be able to do so is an ability on its own. According to Panoura, Gagatsis and Lemonides (2007), this ability consists of different aspects. Spatial ability consists of image generation,
storage of information, retrieval of relevant information, and transformation. Spatial ability also consists of three dimensions; visualization, orientation and relations. The human mind also needs to be able to manipulate images, rotate them mentally and state coordinates from different perspectives.

Spatial visualization is the ability to mentally manipulate a given visual stimulus. This ability involves not only recognizing, but also retaining and recalling changes as parts of the figure are moved around. On the other hand, spatial orientation is the ability to realize that changing the orientation of a certain visual stimulus does not change its main properties. Rather, a mental rotation of the figure is sufficient (Unal, Jakubowski & Corey, 2009, & Osorio, n.d.).

Note that, research highlights the significance of spatial ability as a factor influencing conceptual understanding. As a result, solving geometry based activities is highly dependent on this ability. Instructions should be adapted such that they are not of a higher level of understanding than that of the students’. The insufficient alignment prevents students’ growth and understanding based on the Van Hieles levels (Unal, Jakubowski & Corey, 2009).

Moreover, researchers have found other varying aspects that play important roles in the mastering of geometry. For instance, according to Jaber and Daher (2010), when it comes to geometry, the general skills a student needs are linguistic skills, cognitive skills and their corresponding procedures. Meanwhile, Giudicea et al. (2000) clearly state the importance of some cognitive domains that need to be considered, which are as follows:

Visual scanning is the first domain to consider. It consists of tasks that include sequentially counting randomly placed items.
Visual perceptual ability consists of tasks whereby students estimate length, orientation, and the relative positions of points.

Representational abilities consist of tasks whereby students represent mentally some spatial relationship after recognizing complex shapes, identifying hidden geometric figures from complex patterns, and mentally assembling parts.

Visual motor coordination consists, for example, of tasks whereby students trace lines such that they are parallel and in between two other parallel lines.

Executive (grapho-motor) ability consists of tasks whereby students copy different models (dot patterns, geometric figures…).

These abilities are important for the organization of geometric knowledge and its construction. They allow students to develop their drawing abilities (lines and angles of relative size, representation of relationships…). It is noteworthy to mention that other reasons for drawing disabilities in students can be related to their inability to use a pencil efficiently or poor hand-eye coordination.

Deficiency in the prior mentioned domains may also lead to students’ inability to relate individual parts of a drawing to obtain a unified whole. This is known as developmental constructional apraxia.

Chawla (2009) states that apraxia is a behavioral neurology syndrome. Students with apraxia have difficulty in using tools, performing coordinated movements, and manipulating objects. As such, students are unlikely to be able to perform tasks as well as their corresponding peers. Apraxia isn’t caused by weakness or poor comprehension, but rather the inability to know how to perform the required skilled movements.
Therefore, the more the geometry problems involve complex figures, the more these students rely on intuition, and what they expect the correct answer to be.

Intuition is yet another problematic area to consider in the role it plays in students’ learning and responses. Tsamir (2003) stated Stavy and Tirosh’s intuitive rule theory. Students’ intuition may account for some of their incorrect responses to mathematical tasks. This is because students’ responses are immediate, and self-evident to them. Making claims based on visual intuition void of any formal justification may also lead to mistakes. For instance, some students often confuse a tilted square to be a diamond. Students rely on what they see, or believe the answer should be. However, it is important to note that students’ immediate and confident responses largely depend on their individualized conceptions that may be faulty or incomplete. Furthermore, some students may even try to justify their intuitive claims; however, their justifications are based on informal explanations and theories, as well as misconceptions. As such, students end up with invalid, incomplete and incorrect answers.

Thus Gal, Lin and Ying (2009) clearly state, the importance of realizing students’ perceptual difficulties. Understanding such figures requires perceptual abilities, just as much as it requires geometrical knowledge.

Teachers often do not realize the role that perceptual difficulty plays. Researchers state that teachers must become more aware of the strengths and difficulties associated with visual processing (Pitta-Pantazi & Christou, 2009).

This brings about another problem. How qualified is the teacher in order to realize these problematic issues? What is the teachers’ knowledge of and experience with geometry?
Mathematics teachers, who cannot successfully engage their students and give them appropriate geometry instruction, deny their students the ‘opportunity to learn’. Previous studies show that this occurs when teachers’ individualized geometric knowledge and spatial ability are found of limited capacity. Hence, the teacher will not be able to make the necessary adjustments to help students advance appropriately. Thus, it is clear that teachers greatly influence students’ learning of geometry based on the instructional decisions they make during the critical moments when in class (Unal, Jakubowski & Corey, 2009).

Inadequate instructions may also result in students who also face difficulties in conceptualizing and grasping related geometrical concepts (Gal, Lin & Ying, 2009). This means that some students might not have fully understood and learnt concepts, theories, properties related to geometry due to pseudo-conceptual/analytical behavior (superficial connections of terminologies and processes) whereby students’ mental processes lack conceptual behavior (Gal, Lin & Ying, 2009). Such is the case of students stating that all squares are rectangles, but without really understanding why. They do not relate the basic properties, nor make the right connections in their mental schema. Such a pseudo-conceptual/analytical behavior can be detected by asking students what they would alter in a square in order to get a rectangle. If students make any alterations, such as elongating a pair of opposite sides, or changing the angle of intersection by the diagonals, this would show that they have not fully understood what is meant by all squares are rectangles.

Note that, sources for lack of conceptual understanding are students’ misconceptions of properties, which in turn, may be due to students’ lack of prerequisite
knowledge, practice and/or interaction. Which in turn means, teachers have not been following the steps stated by the likes of Van Hieles, Piaget and Vygotsky, and that are required for a student to learn efficiently. Furthermore, when students make mistakes, although teachers do correct them, and explain how to do it right, what they often neglect to do or are incapable of doing is try to investigate the source of the mistake, explain and justify why the previous response is incorrect (Gal, Lin, & Ying, 2009).

Gal and Linchevski (as cited in Gal, Lin & Ying, 2009), add that conceptual difficulties happen when students learn properties as a group or a package without properly understanding why the properties hold true. As a result, these students think that the properties are bound to each other, whereby they all coexist, but only together. It’s an “all” or “nothing at all” outlook. This outlook is void of understanding that one property contributes to another and so on.

Gutiérrez (as cited in Pitta-Pantazi & Christou, 2009), suggests that reasons for complication may lie in differentiating between different concepts with similar names.

The same problem arises when discussing a single concept having different names, or various definitions. Such is the case of multifaceted concepts. Multifaceted concepts are terminologies that have various definitions, based on what is required from each situation. For instance, defining an angle varies, depending on the desired emphasis. Some define an angle as “a pair of rays with a common end point”, but this definition does not incorporate the aspect of rotation with respect to angles (Mitchelmore & White, 2000).

It is such incomplete definitions that may inhibit students from building connections within different aspects of geometry (Mitchelmore & White, 2000).
Keiser (2004) thus discusses Tall and Vinner’s (1981) proposal underlying the need for concept image and definition, as well as compartmentalizing and organizing the relevant properties (Concept image describes the cognitive structure associated with concepts. It includes mental pictures, properties, processes…while the concept definition describes the concept in word form). Thus, to prevent incomplete structures of concepts, students should be allowed multiple encounters under different contexts, so as to broaden their concept image and relate their corresponding definitions. It is equally important for students to also be able to compartmentalize and organize the different components of a task when needed.

On the same note, students’ inability to visualize, describe, use, manipulate, decompose, and/or recall properties of geometric shapes highlights incomplete conceptual understanding. Students need to recognize patterns, combine shapes, use perception-bound strategies as well as mental images. As stated above, it becomes even more evident that students need to be able to organize and classify all the different components so they may progress in knowledge development (Clements, Wilson & Sarama, 2004).

However, putting conceptual understanding aside, we previously mentioned that teachers may be at times unable to adapt their level of thinking and teaching to the level of their students’ knowledge and understanding. Yet, this problem is also available in the school textbook. Even curriculum designers at times fail to adapt their instruction according to the different students’ needs and/or level of cognitive development. For instance, visualizers may fail in school mathematics due to the mismatch between the
predominant verbal teaching methods and assessment, and the students’ preferred learning style (Gal, Lin, & Ying, 2009).

Also mentioned before is the language and communication barrier. As students have preferred learning styles, they also have a preferred language, a language that they understand the most. Such is the case with foreign students, where they have been raised communicating, learning, and thinking in one language and are then expected to learn mathematics in a different language that is somewhat foreign to them.

Language is used during teacher-student interaction. This poses a problem for students whose mother tongue varies from the one used in class. While studying geometry, specialized terminologies are used, which may vary in definition from those used during every day communication. Although this is may be difficult to master by all students, it is more so for foreign learners since native students solely focus on understanding of the specialized language, while the foreigners have to first grasp the instructional language and afterwards the geometry language (Barwell, 2008).

Here it is important to note that although research shows that, many ESL (English as a Second Language) students may quickly develop the required skills to communicate in English, yet, research also shows that it takes years for learners to master the required “academic” English (Barwell, 2008). Research shows that the language used in textbooks and instruction requires a strong familiarity with terms, grammar, styles of presentation, as well as arguments that are different than informal talk. So, the used academic language during geometry class may cause difficulty for second-language learners. Geometry requires the use of specific terminology, grammar, syntax, symbols… Even though, the words used are based on the language of
instruction, however, the descriptions of the terms used are more specific and narrower in scope. Therefore, the ability to understand geometry in a second language is highly influenced by various language skills (Cuevas, 1984). Geometry learners need to build relationships amongst the numbers, categories, geometric forms, variables and so on. These relationships, abstract by nature, are expressed and interpreted linguistically. This is further proof that learning geometry highly depends on language. For students who have not yet mastered the language followed in the classroom, the challenge is quite significant (Barwell, 2008).

Research shows that students achieve better when the instructor uses the student's native language. The studies conducted on Hispanics show that after tests were translated, the students attained higher scores on the Spanish version than they did when solving the English version (Cuevas, 1984).

Henceforth, since every student is unique, and has a personalized understanding of the surrounding, this means that each student may be inflicted by many obstacles while learning and using geometry. As such, each student may face different types of difficulties which must be highlighted in order to prevent such obstacles in the future, or minimize the effects.

2.5 - Research on geometry curricula

Parkinson and Redmond (as cited in Pitta-Pantazi & Christou, 2009), state that in the past, research made evident that individual differences play a significant role in the learning of geometry. These differences directly influence the manner in which students acquire and process information. As such, Pitta-Pantazi and Christou (2009) state that geometry could be presented in various forms, of which are the visual, wholistic and
dynamic formats. Thus, it is important to identify how learners perceive geometry, whether they are wholist and visual thinkers or more analytic and verbal thinkers.

As stated by Gantasala and Gantasala (2009), the curriculum and teaching approach should be consistent with the students’ learning style preferences. In doing so, students are given an opportunity to improve their academic level and have a better attitude towards learning. Thus, education must be designed and developed around teaching methods that use pedagogy that complement the students’ learning styles.

Furthermore, Schwartz (2008) stated that learning and progress doesn’t occur naturally, but rather as a result of direct and appropriate experiences. Unfortunately, studies show that these experiences haven’t been added into the curriculum until very recently, and that the experiences added at the elementary and intermediate levels are of a low level. On the other hand, once at the secondary level, students are expected to jump to higher levels of thinking, reasoning and proof. This unreasonable expectation is yet another reason for students facing obstacles with geometry.

When curriculum designers, and educators alike, approach learning, they should consider the students, and a flexible pedagogy that allows case analysis, team projects, lectures, and games that provide the appropriate experiences. The curriculum should be designed to include projects, coaching, mentoring, and creating students’ awareness of their complementary learning strategies. It should also include activity based learning that helps foster flexible learning strategies. In turn, teachers should provide the students with solid support which enables them to reflect on the what, how and why of learning. In addition, curriculum should be designed such that students are exposed to many
learning environments, helping them develop coping skills and behaviors (Gantasala & Gantasala, 2009).

In order to impart the necessary knowledge, skills and competencies, especially for the level of the corporate world, teaching strategies should take into consideration students’ learning styles (Gantasala & Gantasala, 2009), and all that is required in content knowledge and experience that will permit the gradual advancement throughout the academic years.
CHAPTER THREE

METHOD

As mentioned in the introduction, this study will uncover difficulties that elementary students face while learning geometry. The geometry program that will be investigated is based on the Lebanese curriculum’s objectives and content. For this purpose, the study involves a mixed-method research involving both qualitative and quantitative analyses, which in turn is based on several techniques of data collection and processing.

3.1 - Participants:

The study takes place over the course of two academic years. The participants from the first year sum up to 44 elementary students (grades 1 to 6, age 6 to 12) of a Lebanese/Armenian school (Originally there were 45 students; however a student from the 6th grade decided not to participate). However, four students changed schools by the second academic year (one of which was the student who didn’t wish to participate in the first round of testing), and the number of students tested at the second stage of the study was reduced to 41 students.

The school is located in Beirut, Lebanon. Not only do the students come from different socioeconomic backgrounds, they also have different nationalities and cultural backgrounds.

The school has about 120 students ranging between grades K to 12. All teachers have bachelor’s degrees and a teaching diploma; they also have a minimum of two-year
teaching experience. Moreover, the school usually shows relatively good results during the Brevet (end of grade 9) and Bacc. II (end of grade 12) national exams.

It is already apparent that the number of students at this school is limited. Table 1 represents the number of students at each elementary grade level during the first academic year. Table 2 represents the number of students at each elementary grade level during the second academic year. Both tables are also subdivided by gender.

Table 1

*Number of Students at Each Grade Level of Academic Year 1, with Breakdown by Gender.*

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Number of students</th>
<th>Number of girls</th>
<th>Number of boys</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>44</strong></td>
<td><strong>26</strong></td>
<td><strong>18</strong></td>
</tr>
</tbody>
</table>

Table 2

*Number of Students at Each Grade Level of Academic Year 2, with Breakdown by Gender.*

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Number of students</th>
<th>Number of girls</th>
<th>Number of boys</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>41</strong></td>
<td><strong>24</strong></td>
<td><strong>17</strong></td>
</tr>
</tbody>
</table>
Note that the grade levels in table 1 range from grade 1 to 6 while in table 2 they range from grade 2 to 7. This is because students who were in grade 1 during the first testing, had been promoted to grade 2 during the second testing. Similarly, the students who were in grade 6 in table 1, were promoted to grade 7 in table 2.

Moreover, even though the participants are relatively few in number, which does not allow the generalization of findings, it still benefits this study by allowing the opportunity for studying students’ responses on an individual basis, which in turn promotes the in-depth analysis of students’ thinking.

3.2 - Procedures:

The investigation in this study is based on several major components.

3.2.1. Curriculum analysis

This component is the study and analysis of the Lebanese geometry curriculum’s objectives and content (ECRD, 1997). It provides an insight to the development and sequencing of the geometry curriculum throughout the elementary years and the way objectives are distributed among the three cognitive domains for each grade level.

Thus, during this phase, objectives of the geometry section for grades 1 to 6 are classified amongst the three cognitive domains as specified by TIMSS (See Appendix I). Furthermore, this analysis allows the researcher to create and organize the research test items accurately.

3.2.2. Testing 1

The second component consists of a first set of paper-pencil testing. It addresses most objectives of the Lebanese curriculum’s content. Testing was
conducted in class, during a single period, consisting of 45 minutes. It was conducted during the last month of the first academic year.

Test items are based on the first two cognitive domains of TIMSS framework, *knowing* and *applying*. They include solving tasks and applying knowledge, matching, multiple choice questions, labeling, filling in the blanks, and *true or false* questions (for examples, see Appendix II).

Each test includes a number of test items that correspond to the objectives for the respective grade level, as stated by the Lebanese curriculum for geometry. However, because of the complex and cumulative nature of mathematical tasks, some objectives may encompass other simpler objectives. This means that each task or test item is also based on the use of underlying objectives required to complete the task accurately.

The tests will be studied mainly to analyze students’ individual answers, to identify the mistakes made, and the questions that could not be answered. This will uncover a picture of students’ knowledge, as well as the geometric knowledge and application skills that they have not mastered, according to the objectives stated by the curriculum.

### 3.2.3. Testing 2

The third component is a second paper-pencil testing. It is a retest that took place after the summer vacation, as soon as students returned to school the following academic year. The retest parallels the original test in content and structure. The reason for this retest is to analyze how much students have
retained from the previous year, after a three-month vacation, as well as to check if maturation helps with certain geometric skills, such as using geometric tools.

3.2.4. **Comparison between tests one and two**

In this phase, the aim was to investigate the way time has influenced students’ responses based on each objective, and to highlight what teachers and educators alike should take into consideration while teaching certain objectives.

3.2.5. **Linking mistakes to difficulties as stated in the literature review**

In this phase the aim was to compile the mistakes that students made during testing and to connect them to the major difficulties faced while dealing with geometry as provided by the literature review (see difficulties with description and examples in section 4.3- Difficulties discussed in the literature review).

The importance of this component is to pinpoint and organize some factors that may cause students to face difficulties and obstacles during their learning process of learning and applying geometry.

3.2.6. **Performance and objective based tasks**

This component is based on further testing. During this stage, students were asked to perform / execute geometric tasks assessing objectives that were not covered during the paper-pencil testing, either because they go beyond the cognitive abilities “knowing” and “applying” (see Appendix II), or because they require the researcher’s observation of students’ performance.
This component is subdivided into various categories.

a- The first is solving a set of problems.

b- The second involves refined motor skills (such as constructing nets and using geometry tools).

c- The third category involves oral communication with selective interviewing. (A student is interviewed when his/her answer on the worksheet isn’t clear and the researcher needs to better understand what the student is thinking.)

In this stage, students used their geometry skills as they solved problems. The students were monitored closely and were followed-up on their line of thought, and whether there are any skills on which they may seem to have difficulty. If for any reason, students’ responses were not clear, students were interviewed on an individual basis, to explain their thinking orally, or through drawing, or through any other form of representation and/or communication.

It is during this stage that students were asked to further explain their responses, state what strategies they used in order to proceed through the tasks, what difficulties they faced, and so on.

Note that the researcher monitored the students’ progress through the tasks and problems. As students were being monitored, the researcher took notes, tape-recorded any dialogue, as each student expressed his/her thoughts out loud, and videotaped the tasks that require students to use refined motor skills. Students were also asked to take notes of their own thoughts, on their
worksheets. Students were constantly urged and encouraged to justify and explain their solutions.

The first reason for this assessment technique was to study students’ thinking skills. As students solved geometry problems, and expressed themselves, the researcher tape recorded, and took notes, in order to review certain instances that were originally overlooked, thus allowing a deeper reflection of the students’ reactions. Furthermore, the answers allowed further study and analysis of students’ thinking, allowing the uncovering of misconceptions, or other factors causing weakness when it comes to geometry based on the three cognitive domains.

By combining both assessment techniques, students were tested for the outcomes of their learning of the geometry curriculum’s learning outcomes as desired by the curriculum. Thus, students’ test responses were seen as indicators of whether the learning outcomes have been met.

3.2.7. Analyzing mistakes to understand students’ difficulties

In this stage, the researcher compares, categorizes and organizes mistakes made by students under main headings. The reason for this is to link the mistakes to the findings in the literature review. This in turn, allows to understand why students have difficulty with certain objectives, and the foundations for those difficulties. Although the first tool for analyzing students’ difficulties is based on the information provided by the literature review, the second tool is the researcher’s analysis of the Lebanese geometry curriculum.
This analysis showed that the scope, sequence and language of the curriculum are not up to par. Some difficulties are thus also related to the complications provided by the curriculum itself (see Appendix III).

3.3 - Testing environment and test designs:

The testing environment plays an important role while students are tested. As such, students were always tested in their classrooms, where they are already accustomed to paper-pencil testing. Students were also constantly reminded that they had nothing to fear, as test results will have no consequence on their grades. Furthermore, if students did not know how to deal with a certain task, they could move on to the next, as the order in which they solve is not important.

The paper-pencil tests were designed by the researcher, and were reviewed by a specialist in the domain. The test items are direct applications of the demands of each objective in the Lebanese geometry curriculum. This means that each test item included in the tests corresponds to one of the objectives as stated by the curriculum, according to the grade level (for examples, see Appendix II).

Furthermore, during the last stage of testing, although some of the test items were direct applications of what the researcher understood as to the demands of each objective in the Lebanese geometry curriculum, problem solving was also required. For this aspect, the test designer resorted to problems available in the Lebanese textbooks as blueprints on which new but similar problems were designed.

Lastly, a few objectives were ignored, as they may have been too general or ambiguous to test as intended.
3.4 - **Validity and reliability**

It is important to note that throughout the research process, a specialist in mathematics education was consulted. She consistently monitored the progress of the researcher’s work, and reviewed and validated the distribution of the objectives over the three cognitive domains as defined by TIMSS. The specialist also validated the test items beforehand so that the items be consistent with the learning outcomes of the objectives as set by the curriculum. Furthermore, she also confirmed the connections made between students’ mistakes, and the difficulties based on the framework and tools adopted for the analysis. The specialist’s reflections and suggestions were followed in order to keep the reliability and validity throughout the research.

3.5 - **Instruments:**

Different types of instruments are relied on to back the findings of this research.

3.5.1. **Description of instruments**

- Review of related literature and theoretical frameworks regarding the cognitive development of children’s geometrical thinking and skills and development of a framework for analysis.

- Review of the geometry addressed by the Lebanese Curriculum Paper-pencil testing in June and October that are based on geometry tasks at the TIMSS framework levels of knowing and applying.

Performance and objective based tasks that involve solving geometrical problems requiring manipulating, recalling procedures and properties, relying on conceptual understanding, and using various skills such as visual motor skills
3.5.2. Purpose of instruments

- The literature provides an insight on some major difficulties that students face while learning and dealing with geometry. In the literature review, difficulties discussed with explanations are adequate descriptions that allow the researcher to translate the mistakes students make into difficulties.

- The Lebanese Curriculum for geometry is studied for two purposes. The first purpose is to set test items based on the objectives provided by the curriculum. The second purpose is to check if there is proper scope and sequence regarding the objectives as students are promoted from one grade level to the next.

- Two paper-pencil tests take place. First of all, these tests allow the researcher to check the students’ knowing and applying skills of objectives stated in the curriculum. However, more importantly, the reason for the repetition of the same test is based solely to analyze how time influences students’ knowledge and applying skills. Due to the passage of time, some objectives may show increase or decrease in difficulty rates, while other objectives may show no change at all.

- The performance and objective based tasks serves multiple purposes. The first is to analyze students’ abilities regarding performance tasks. The second is to analyze students’ abilities to solve routine problems. The third purpose is to re-test certain objectives that were tested during the paper-pencil testing. The reason for this is so that the researcher can have a better understanding of the difficulties students faced. To ensure this, students were encouraged to explain their thinking skills as they solved the test items.
To conclude, the research conducted throughout this study is based on a mixed method analysis. It investigates why difficulties are faced with some objectives, and why they have not been grasped as students’ responses to the test items are analyzed.

The research starts with a qualitative study of the mistakes made by students and converts these mistakes to difficulties as discussed in the literature review.

This phase is followed by a quantitative study whereby the number of students and number of objectives per grade level are taken into consideration in order to calculate the intensity of each difficulty.

Together, this mixed method of study not only highlights the difficulties that students face, but also which difficulties are to be considered major difficulties, and which one are minor in comparison to other difficulties.

As such, there triangulation of findings between the literature review, and the two different types of testing, provide a clear view of the major difficulties the tested students face.

Furthermore, the analysis of the geometry section of the Lebanese geometry curriculum as stated for the elementary level shows whether there is proper scope and sequencing to the stated objectives, and whether the curriculum takes into consideration the cognitive domains as stated by TIMSS in order to provide students with a well-rounded experience in the learning of geometry at the elementary level.
CHAPTER FOUR

FINDINGS

This section discusses how the collected data is organized, and how the procedures followed ensure proper analysis of the results obtained, which in turn will help answer this study’s research questions.

4.1 - Level of the Objectives Based on the TIMSS Cognitive Domains

It is important to point out at what level the objectives of the Lebanese curriculum of geometry at the elementary grades are set. This permits a better understanding of what students are expected to know, what they are expected to apply, and how much importance is given to reasoning and critical thinking. The distribution of the objectives according to the cognitive domains is available in Appendix I. Table 3 presents the percentage given to each cognitive domain per grade level.

Table 3

*The Percentage Given to Each Cognitive Domain per Grade Level*

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Number of hours</th>
<th>Number of objectives</th>
<th>% for knowing</th>
<th>% for applying</th>
<th>% for reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>20</td>
<td>70.00</td>
<td>30.00</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>15</td>
<td>53.33</td>
<td>46.67</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>26</td>
<td>65.385</td>
<td>34.615</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>27</td>
<td>66.67</td>
<td>33.33</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>20</td>
<td>75.00</td>
<td>25.00</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>56</td>
<td>55.36</td>
<td>44.64</td>
<td>0</td>
</tr>
</tbody>
</table>

From table 3, we notice the following: The first issue of concern is that there is no room for reasoning (0%). On the other hand, *knowing* is highly valued (ranging from 53.33% to 75.00%). The uneven distribution between *knowing* and *applying* (ranging...
from 25.00% to 46.67%) shows that the curriculum at the elementary level is based on knowing, and drill and practice rather than on developing well rounded thinking skills. It is worth to note that the highest percentage for applying is still less than the lowest percentage for knowing. This is problematic as the set objectives are of the same level of abstraction. Many objectives that are at the knowing level, require applying to ensure that the students would properly assimilate the objectives. Yet more disturbing is the opposite, whereby certain objectives require direct applying without the consideration of prior knowledge.

Moreover, as mentioned in the introduction of this manuscript, Lebanon scored 464 for knowing, 448 for applying, and 429 for reasoning (Martin, Mullis, & Foy, 2008). This too can be explained by looking at the distribution of the objectives.

Since most of the objectives are stated under the knowing domain followed by a decrease in the number of objectives with applying, and nil with reasoning, it comes as no surprise that the students were able to answer more test items that required knowing than test items that required applying or reasoning.

Yet another issue to highlight is the drop in the number of objectives addressed in grades 2 and 5 (from 20 to 15 objectives for grade 2, and from 27 to 20 objectives for grade 5), and then the leap in the number of objectives from grade 5 to 6 (from 20 to 56 objectives). Instead of a gradual increase in the number of objectives, there is a drastic jump. The reason for this inconsistency relies on how the curriculum was originally set.

This shows that the curriculum lacks proper distribution of objectives.

Lastly, in grade 6, 25 hours are dedicated to the teaching, learning, understanding, and manipulating of 56 objectives. Although 20 to 25 hours may be
sufficient in the previous grades since the number of objectives is not as many, it is clear that in the sixth grade more time should be dedicated to the learning of geometry.

4.2 – Linking Mistakes to Difficulties Discussed in the Literature

Tests 1 and 2 target students’ responses towards the objectives, based on knowledge and application exercises. On the other hand, the performance and objective based tasks target the performance and problem solving skills of the students. It is during this latter phase that it was possible to observe students’ performances, as well as complete a systematic interview of students when necessary. Through probing the students and selective interviewing and observation, uncovering the sources for difficulties becomes clearer. Together, the three tests with the students’ responses give a general insight as to what types of difficulties students face according to each objective.

After students completed their tests, responses were analyzed for mistakes, or difficulties faced with respect to each objective.

4.3 - Difficulties discussed in the literature review

In this section, the difficulties that have been mentioned in the literature review are categorized under major headings. For some of those difficulties, examples taken from the actual participants’ work are provided. The difficulties explained in this section are classified in a table format in Appendix IV. It is the main tool allowing the translation and attribution of students’ mistakes to difficulties.

Difficulties in: Conceptualizing:

Conceptual misunderstanding creates major obstacles for the student. This section is divided into many smaller entities.
1- Pseudo-conceptual /analytical behavior (superficial connections of terminologies and processes).

2- Lack of comprehension of multifaceted concepts (a single concept having different names, or various definitions based on the different situations that may arise).

3- Incomplete structure of a concept (includes mental pictures, properties, and processes).

4- Misconceptions (faulty understanding of a concept).

(Gal, Lin & Ying, 2009; Pitta-Pantazi & Christou, 2009; Mitchelmore & White, 2000; Keiser, 2004; Capraro & Capraro, 2006).

Example of incomplete structure of a concept: Some students did not realize the importance of the perpendicular line as the shortest distance between a point and a slanting line; instead, students in the fourth grade drew the horizontal segment from the point to the line which, for them, represented the distance between the parallel lines.

Difficulties with the cognitive level:

Knowing: According to Mullis, et al. (2005), the first of the three cognitive domains is knowing. It covers facts, procedures and concepts that students must have learned. The behaviors that are related to knowing are
recalling, recognizing, computing, retrieving, measuring and
classifying/ordering.

Examples:

Recalling Property: Some students forgot the number of faces/ vertices/
edges of solid figures by the time of the second testing.

![Image of a cube with labeled faces, vertices, and edges]

Recalling Procedure: By the second phase of testing, some students
forgot that in order to draw a line parallel to another line, they needed to draw the
perpendicular to a perpendicular to the given line.

![Image of drawing a line parallel to another line]

Recognizing: Some students did not recognize the slanting line as a
straight line.

![Image of a line with a slanting line nearby]
Computing: Some students made mistakes while placing the midpoint of a given segment because they were unable to correctly divide the length of the segment by 2.

Measuring: Students had a hard time using the protractor in order to measure the angles correctly.

Classifying/ordering: Students were unable to classify and group solid figures or plane figures accurately because they chose to classify them according to their heights instead of the type of solid or plane figure. Also, although students classified some triangles as triangles, and some rhombi as rhombi, they also grouped one large triangle with one large rhombus, saying that they did not belong with the previous two groups.
Applying: The second domain is *applying*. It focuses on students’ ability to apply their knowledge as well as their conceptual understanding in order to solve *routine problems*. The behaviors that are related to *applying* are *selecting*, *representing*, *modeling*, *implementing* and *solving routine problems*.

Examples:

Selecting: Some students were unable to select the correct nets in order to build a given model.

Implementing: Some students were able to solve the geometric task but they disregarded the given conditions. For instance when told to move 3 steps left, students moved left, but the number of steps they took varied.

Reasoning: The third cognitive domain is *reasoning*. It encompasses unfamiliar conditions while solving multi-step problems in elaborate contexts (Mullis, et al., 2005). The behaviors that are related to *reasoning* are *analyzing*, *generalizing*, *synthesizing/integrating*, *justifying* and *solving non-routine problems*. (See appendix I for elaboration).
Difficulties in: Understanding/ thinking/ using geometry due to: Social / Cultural differences:

Students’ backgrounds, cultures and level of maturity influence students’ ability to learn and use geometry.

1- Effects of culture: This is apparent during problem solving and reasoning.

Asking students to solve a problem involving how to score in a baseball game, when students do not know the rules of the game, is an example.

2- Effects of limited exposure: educational experiences either facilitate or obstruct students’ progress (Piaget, 1997; Mason, 1998).

Difficulties with guessing based on perception:

According to Tsamir (2003), intuition may be another reason for students’ facing difficulty. Students base their answers on what they know, see or expect. If what students know, see and/or expect is incomplete, misconceived and/or wrong, then the students’ responses that are based on that knowledge will be faulty.

Example: Difficulties with guessing based on perception:

A sixth grade student still mistook the slanting square for a rhombus.
Difficulties in: Understanding/ thinking/ using geometry due to: Language:

Throughout the years, researchers have mentioned the role that language plays in the understanding, learning and using of geometry skills in the classroom. It is often through classroom discussion that the teacher and students communicate their thoughts. This in turn allows the teacher to pinpoint some misconceptions, while students are introduced to new viewpoints and ways of thinking. Thus, if students do not understand the language properly, their understanding of the concepts will also be limited (Piaget, 1997; Barwell, 2008).

Example: Difficulties in: Understanding/ thinking/ using geometry due to: Language: Some students did not understand the question. For instance, when asked to place the midpoint of a segment, a couple of students answered “yes”.

![Mark the midpoint of RS](image)

Difficulties in: Recalling Terminology and Definitions:

Students should be introduced to the correct terminology, and in return, they should be able to use the terms correctly. The reason for this is simple. Many terms used in geometry are readily available in our everyday language; however, when a term is used in geometry, it is used specifically to point out certain properties, and thus its definition is narrower in scope than what is used in every day conversations. Not knowing these terms and their importance hinders students’ ability to relate what they are learning with other concepts that they may know, thus resulting in an incomplete mental schema and limited
content knowledge (Cawley, Foley, & Hayes, 2009; Cuevas, 1984; Barwell, 2008; Cannizzaro&Menghini, 2006; Jaber&Daher, 2010).

Example: Recalling Terminology: Students often answered correctly, but using Armenian words instead of the corresponding English terminology.

Difficulties with: Visual Reasoning:

Learning geometry involves visual reasoning. When learning mathematics, visualization includes both the visual and mental imagery.

1- Perception/Visualization: the translation of perceptions into descriptions without involving explicit linguistic abilities

2- Spatial ability consisting of image generation, storage of information, retrieval of relevant information, and transformation. Spatial ability also consists of three dimensions; visualization, orientation and relations. The human mind needs to be able to manipulate images
3- Example: Difficulties with:

Spatial ability: Some students could not differentiate between their left and their right.

4- **Visual scanning**: sequentially counting / identifying randomly placed items

Example: Visual scanning: This difficulty becomes apparent when students are asked to complete a figure to make it identical to a given figure. The mistake committed here is that students managed to draw most of the figure to be identical, but missed joining one side, or ignored to place a minor detail that was available in the original figure.

5- **Representational abilities**: The ability of mentally representing some spatial relationships of complex shapes by identifying hidden geometric figures, and mentally assembling parts.

6- **Visual motor coordination**: for example, students’ ability to trace a line parallel, and in between, two other parallel lines

Example: Visual motor coordination: When asked to draw a straight line perpendicular to a given straight line and passing through a given point,
some students had been able to draw the perpendicular line, but about a millimeter away from the given point.

7- Difficulties regarding their executive (grapho-motor) ability: students cannot copy different models (dot patterns, geometric figures…).


Example: Executive (grapho-motor) ability: Some students were not able to reproduce simple figures on grids because they drew sides longer or shorter than the given sides.
4.4 - Steps followed for analysis

During this phase a few steps were followed before reaching the final results. The steps are as follows:

4.4.1. Correcting the tests and taking notes when necessary

This step involved more than just checking if an answer is wrong or right. It involved taking notes of the type of mistake made. For instance in grade 1, when asked to describe the position of an object in space, students answered correctly but in Armenian instead of English. This shows that the main problem is the academic language used, rather than the actual understanding or knowing of concepts. Thus this objective is categorized under “difficulties in understanding/thinking/ using geometry due to language”.

4.4.2. Highlighting the major difficulties and linking the mistakes to these difficulties.

This involved a qualitative study. The major difficulties were tabulated in Appendix IV, and explained in section “4.3 - Difficulties discussed in the literature review”. In section 4.3, examples are also given as to how mistakes are translated and attributed to difficulties. For instance, in the previous example, answering in Armenian is categorized as a language problem.

4.4.3. Organizing and tabulating the objectives based on the number of mistakes made under the corresponding difficulties

This step starts by stating the major difficulties for each grade level. Then under each difficulty are the objectives with which students faced that difficulty. Furthermore, next to each objective is the ratio of students who faced that same
difficulty while solving the task, first during test 1, then test 2, and lastly during the performance and objective based tasks, if applicable (see Appendix V). Once all the information was organized, the numbers were tabulated whereby averages and comparisons may become easier to conduct.

Example:

Difficulties in: Recalling: Terminology

- 1.1.3. Recognize the interior, the exterior, and the boundary of a simple \( 3/7; 3/7 \)
- 1.1.4. Use the terms: interior, exterior, open, closed \( 7/7 \)

We notice that each objective is preceded by a numerical code and followed by ratios. The first two numbers of the numerical code preceding an objective represent the content domain of geometry to which the objective belongs, as used in the geometry curriculum text, while the last number of the numerical code represents the number of the objective within that content domain. In this case, 1.1. means the domain Location. Furthermore, ‘Recognize the interior, the exterior, and the boundary of a simple‘ is the third objective in this content domain.

As for the ratios, we notice slight variations in them. The bold ratio represents the ratio of students who faced difficulty with recalling terminologies during test 1 (coded as T1), the underlined ratio is for those who faced the same difficulty during test 2 (T2), and the regular font ratio represents the number of students who faced difficulties with recalling terminologies in the performance and objective based tasks (T3).
Once this step was completed, numbers were converted using the Student Percentage Value (SPV) and represented in table format.

\[
SPV = \frac{\text{# of students who made mistakes} \times 100}{\text{total number of students in that grade level}}
\]

Table 4

Distribution of Objectives under Difficulties Based on Numbers and Percentages for each Grade Level

<table>
<thead>
<tr>
<th>G1 Diff.</th>
<th>Obj #</th>
<th>T 1</th>
<th>T 2</th>
<th>T 3</th>
<th>SPV1</th>
<th>SPV2</th>
<th>SPV3</th>
<th>V1-V2</th>
<th>A(V1,V2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term.</td>
<td>1.1.3.</td>
<td>3</td>
<td>3</td>
<td></td>
<td>42.86</td>
<td>42.86</td>
<td>0.00</td>
<td>42.86</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.1.1.</td>
<td>3</td>
<td>2</td>
<td>7</td>
<td>42.86</td>
<td>28.57</td>
<td>100.00</td>
<td>14.29</td>
<td>35.71</td>
</tr>
<tr>
<td></td>
<td>3.2.1.</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>14.29</td>
<td>0.00</td>
<td>71.43</td>
<td>14.29</td>
<td>7.14</td>
</tr>
</tbody>
</table>

Legend:

G1 Diff: Difficulties that appear while analyzing test results for grade 1
Obj #: Objective code(1.x.y.= Location, 2.x.y.= Solid figures, 3.x.y.= Plane figures)
T1: Number of students who had a difficulty while solving test 1
T2: Number of students who had a difficulty while solving test 2
T3: Number of students who had a difficulty while solving the performance and objective based tasks
SPV1: Student percentage value for test 1
SPV2: Student percentage value for test 2
SPV3: Student percentage value for the performance and objective based tasks
V1 – V2: The difference between SPV1 and SPV2
A(V1,V2): The average of SPV1 and SPV2

Note: Table 4 is extracted from Appendix VI.

Table 4 is extracted from Appendix VI whereby, the first column shows the difficulty (difficulty in recalling terminology). The second column shows the code of the objective (1.1.3. is the third objective in location, domain…). The third, fourth and fifth columns show the numbers of students who had difficulty with recalling the terminology in test 1 (T1), test 2 (T2) and the performance and objective based tasks (T3). Note that not all cells in these columns are filled. This is because many objectives tested in T1 and T2 were not retested in T3. Similarly some objectives could not be tested during T1 and T2.
Furthermore, the columns SPV1, SPV2 and SPV3 represent the percentage of the students who made mistakes. The reason for this is that the number of students varies between the different grade levels. Thus there was a need to calculate the Student Percentage Value (SPV).

In grade 1, there are 7 students. This means that the percentage value of 3 students out of 7 is 42.86%.

Note that the percentages are rounded to the nearest hundredth.

The column V1-V2 shows the retention or the maturation rate based on the percentage according to which time has altered students’ answers, negatively or positively. Test1 took place at the end of the first academic year, while test2 took place at the beginning of the following year, after three months of summer vacation. Therefore it is expected that students may have forgotten some concepts, procedures, skills, properties… but they may also have matured with, for example, visual-motor related skills. As such when looking at column V1-V2 one may get one of three results.

Result 1 < 0: Students forgot certain requirements to solve a test item.
Result 2 = 0: Students’ knowledge has not changed after the 3 months
Result 3 > 0: Students’ knowledge has improved after the 3 months.

The column A(V1,V2) represents the average of difficulties faced by students during T1 and T2.

\[ A(V1,V2) = \frac{[SPV1 + SPV2]}{2}. \]
The numbers obtained in this column are mainly used in the necessary comparisons, be it between the same difficulties, the same grade level, or across the different grade levels as well.

4.4.4. Quantitative analysis based on number of mistakes per objective and number of students

This stage consists of the manipulation, evaluation and comparisons the percentages and averages obtained once the test results were tabulated. This in turn provides a clearer view of the major issues.

However, in order to compare results obtained in table 4 among the different difficulties and across the different grade levels, another indicator was calculated for each difficulty per grade level, which is the Objective Percentage Value (OPV). The reason for this is the varying number of objectives across the grade levels.

\[ OPV = \frac{\# \text{ of objectives where difficulty appeared} \times 100}{\text{total } \# \text{ of objectives in the grade level}} \]

Table 5 shows, in columns G. 1 to G. 6, the numbers of objectives that are categorized under each difficulty for each grade level. However, since each grade level has a different number of objectives to satisfy, the Objective Percentage Value of each objective, per grade level, varies. The OPVs for the grade levels are presented in the last six columns.
Table 5

Objective Percentage Value (OPV) per grade level and difficulty.

<table>
<thead>
<tr>
<th>Difficulty</th>
<th>G. 1</th>
<th>G. 2</th>
<th>G. 3</th>
<th>G. 4</th>
<th>G. 5</th>
<th>O. 1</th>
<th>O. 2</th>
<th>O. 3</th>
<th>O. 4</th>
<th>O. 5</th>
<th>O. 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptual</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>6</td>
<td>27</td>
<td>5.00</td>
<td>26.67</td>
<td>26.92</td>
<td>25.93</td>
<td>31.58</td>
<td>49.09</td>
</tr>
<tr>
<td>Language</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>15.00</td>
<td>6.67</td>
<td>15.38</td>
<td>3.70</td>
<td>5.26</td>
</tr>
<tr>
<td>Terminology</td>
<td>7</td>
<td>6</td>
<td>10</td>
<td>2</td>
<td>1</td>
<td>6</td>
<td>35.00</td>
<td>40.00</td>
<td>38.46</td>
<td>7.41</td>
<td>5.26</td>
</tr>
<tr>
<td>Property</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>17</td>
<td>10.00</td>
<td>20.00</td>
<td>23.08</td>
<td>11.11</td>
<td>15.79</td>
</tr>
<tr>
<td>Procedure</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>12</td>
<td>1</td>
<td>11</td>
<td>5.00</td>
<td>26.67</td>
<td>11.54</td>
<td>44.44</td>
<td>21.05</td>
</tr>
<tr>
<td>Vis. Scan.</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>15.00</td>
<td>6.67</td>
<td>19.23</td>
<td>14.81</td>
<td>21.05</td>
</tr>
<tr>
<td>Recognizing</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>25.00</td>
<td>20.00</td>
<td>15.38</td>
<td>3.70</td>
<td>10.53</td>
</tr>
<tr>
<td>Selecting</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>11.11</td>
<td>0.00</td>
</tr>
<tr>
<td>Classify</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Represent</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.00</td>
<td>6.67</td>
<td>7.69</td>
<td>0.00</td>
<td>5.26</td>
</tr>
<tr>
<td>Tools (draw)</td>
<td>1</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>5.00</td>
<td>40.00</td>
<td>23.08</td>
<td>14.81</td>
<td>10.53</td>
</tr>
<tr>
<td>Measuring</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>0.00</td>
<td>20.00</td>
<td>3.85</td>
<td>0.00</td>
<td>15.79</td>
</tr>
<tr>
<td>Vis. M. Coor.</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>10.00</td>
<td>26.67</td>
<td>30.77</td>
<td>7.41</td>
<td>5.26</td>
</tr>
<tr>
<td>Executive</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>5.00</td>
<td>6.67</td>
<td>3.85</td>
<td>7.41</td>
<td>5.26</td>
</tr>
<tr>
<td>Spatial</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>20.00</td>
<td>6.67</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Implement</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>8</td>
<td>20.00</td>
<td>33.33</td>
<td>11.54</td>
<td>7.41</td>
<td>15.79</td>
</tr>
<tr>
<td>Guess (Perc)</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.00</td>
<td>6.67</td>
<td>3.85</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Computing</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>3.85</td>
<td>0.00</td>
<td>26.32</td>
</tr>
<tr>
<td># of Obj</td>
<td>20</td>
<td>15</td>
<td>26</td>
<td>27</td>
<td>19</td>
<td>55</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Legend:
- Vis. Scan.: Difficulty in visual scanning
- Vis. M. Coor.: Difficulties with visual motor coordination
- Guess (Perc): Difficulties with guessing based on perception
- # of Obj: Total number of objectives per grade level
- G.: Grade
- O.: Objective percentage value
In grade 2, we have a total of 15 objectives; however, in grade 4 we have a total of 27 objectives, and so on. This difference amongst grade levels causes the difference in each difficulty’s OPV per grade level. As such, the difference in the number of objectives per grade level changes the value of the ‘1’ in language that is stated under each of G. 2, G. 4 and G. 5 to ‘6.67 %’, ‘3.70%’ and ‘5.26%’, respectively.

This explains the variations in the Objective Percentage Value columns that have been stated as O. 2, O. 4 and O. 5 (OPV for grade 2, OPV for grade 4, and OPV for grade 5 respectively).

However, it is not enough to state the OPV to judge the real intensity of each difficulty. Aside from the presence of a number of objectives, we also need to consider the number of students who had difficulty with the particular objective at each grade level. For this reason, both the SPV and OPV combined together give a more accurate value to the intensity of the difficulty which may be later compared across the various difficulties in a single grade level as well as across the varying grade levels.

Take for instance the column for V1-V2 from table 4 across the various grade levels. In order for proper comparison to take place amongst the years there is a need to be able to compare them based on common ground which is obtained by the combination of both SPV and OPV.

This is why the values of V1-V2 obtained from table 4 that are based on SPV are then converted using OPV. It is then that the results obtained by table 4
can be accurately compared. With SPV and OPV together, an indicator of the intensity % of each difficulty is calculated as follows:

\[
\text{Intensity } \% = \frac{\text{SPV} \times \text{OPV}}{100}
\]

The intensity % is calculated for all items in table 4 and the tables of Appendix VI, in order to analyze and compare the obtained results (such as averages obtained from SPV3...) amongst difficulties in each of the six grade levels as well as in order to better understand the progress of difficulties across the six grade levels.

4.5 - Analysis of results

Results in appendix VI are based on SPV alone. However, in order to continue the analysis, these results are converted to intensity % using OPV and are used in the analysis. The intensity % allow the proper comparison among the different aspects (type of difficulty, different grade levels...).

4.5.1. Major Difficulties Based on Average Intensity %

This section discusses the major difficulties that students faced while learning and using geometry based on the difficulties’ intensity % of T1, T2 and T3.

Table 6 highlights a few key factors. Not only does it show the intensity % of the difficulties for each grade level, but also how the difficulty intensifies or diminishes as students progress throughout the academic years.
For instance in grade 1, the difficulty having the highest intensity % is that of recalling terminologies with 19.00%. In contrast, difficulty in executing or reproducing a simple figure has the lowest intensity % of 0.18%.

On the other hand, difficulties regarding selecting, guessing based on perception, measuring, representing and computing show no intensity %. This is because they were either not tested, or did not arise during one or more of the testing phases.

Table 6

*Averages between the Various Tests Based on Difficulty for Each Grade Level*

<table>
<thead>
<tr>
<th>Difficulty</th>
<th>G. 1</th>
<th>G. 2</th>
<th>G. 3</th>
<th>G. 4</th>
<th>G. 5</th>
<th>G. 6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I_{(1,2,3)})%</td>
<td>(I_{(1,2,3)})%</td>
<td>(I_{(1,2,3)})%</td>
<td>(I_{(1,2,3)})%</td>
<td>(I_{(1,2,3)})%</td>
<td>(I_{(1,2,3)})%</td>
</tr>
<tr>
<td>Concept</td>
<td>0.54</td>
<td>16.08</td>
<td>6.23</td>
<td>14.05</td>
<td>18.05</td>
<td>21.57</td>
</tr>
<tr>
<td>Property</td>
<td>1.43</td>
<td>14.80</td>
<td>3.31</td>
<td>10.03</td>
<td>10.53</td>
<td>14.07</td>
</tr>
<tr>
<td>Procedure</td>
<td>2.50</td>
<td>13.53</td>
<td>1.28</td>
<td>8.64</td>
<td>2.26</td>
<td>10.40</td>
</tr>
<tr>
<td>Language</td>
<td>3.57</td>
<td>3.35</td>
<td>3.00</td>
<td>0.16</td>
<td>0.57</td>
<td>1.45</td>
</tr>
<tr>
<td>Term</td>
<td>19.00</td>
<td>18.00</td>
<td>10.79</td>
<td>2.40</td>
<td>0.19</td>
<td>5.90</td>
</tr>
<tr>
<td>Vis.Scan.</td>
<td>8.04</td>
<td>0.54</td>
<td>1.93</td>
<td>1.24</td>
<td>1.32</td>
<td></td>
</tr>
<tr>
<td>Recognize</td>
<td>14.65</td>
<td>5.39</td>
<td>0.86</td>
<td>0.70</td>
<td>1.13</td>
<td></td>
</tr>
<tr>
<td>Selecting</td>
<td>1.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Classify</td>
<td>4.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Guess(Perc)</td>
<td></td>
<td>3.34</td>
<td>0.43</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tools(draw)</td>
<td>1.07</td>
<td>5.59</td>
<td>9.75</td>
<td>2.39</td>
<td>0.75</td>
<td>1.82</td>
</tr>
<tr>
<td>Measuring</td>
<td>9.72</td>
<td>0.11</td>
<td>0.01</td>
<td>0.57</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>Vis.M.Coor.</td>
<td>1.79</td>
<td>4.17</td>
<td>13.73</td>
<td>0.47</td>
<td>0.19</td>
<td>1.45</td>
</tr>
<tr>
<td>Executive</td>
<td>0.18</td>
<td>0.35</td>
<td>0.22</td>
<td>0.70</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>Implement</td>
<td>10.00</td>
<td>3.04</td>
<td>0.43</td>
<td>0.70</td>
<td>1.13</td>
<td>3.06</td>
</tr>
<tr>
<td>Represent</td>
<td>0.17</td>
<td>0.32</td>
<td>0.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spatial</td>
<td>5.36</td>
<td>0.91</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computing</td>
<td></td>
<td></td>
<td></td>
<td>0.22</td>
<td>1.32</td>
<td></td>
</tr>
</tbody>
</table>

Legend:
G. : Grade
\(I_{(1,2,3)}\)%: Average intensity % from T1, T2 and T3 = \([I(1,2)+I3]/2\)
Term: Difficulty in recalling terminologies
Vis. Scan.: Difficulties with visual scanning
Furthermore, if we follow the intensity % for each difficulty across the years, different deductions can also be made. By looking at the difficulty in conceptualizing one not only notices that the intensity % increased drastically from grade 1 to grade 6 (from 0.54% to 21.57%) but also that the increase in itself is inconsistent. From grade 1 to grade 2, the intensity % increases drastically, but then decreases again during grade 3 (0.54% increases to 16.08% and then decreases again to 6.23%), and so on.

However, before, studying the changes in intensity % across the grade levels, it is important to pinpoint the major difficulties for each grade level. As such, the intensity % for each difficulty at each grade level is charted in Appendix X. This way, major and minor difficulties can be visualized better.

After charting results obtained from table 6 (see Appendix X) for each grade level, the major difficulties obtained are thus tabulated in table 7.
Table 7
*Distribution of the Major Difficulties Based on Grade Level*

<table>
<thead>
<tr>
<th></th>
<th>G. 1</th>
<th>G. 2</th>
<th>G. 3</th>
<th>G. 4</th>
<th>G. 5</th>
<th>G. 6</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>Concept</td>
<td>Concept</td>
<td>Concept</td>
<td>Concept</td>
<td>Concept</td>
<td>Concept</td>
<td>5/6</td>
</tr>
<tr>
<td>Property</td>
<td>Term</td>
<td>Term</td>
<td>Property</td>
<td>Property</td>
<td>Property</td>
<td>Property</td>
<td>4/6</td>
</tr>
<tr>
<td>Procedure</td>
<td></td>
<td></td>
<td>Procedure</td>
<td>Procedure</td>
<td>Procedure</td>
<td>3/6</td>
<td></td>
</tr>
<tr>
<td>Recognize</td>
<td></td>
<td></td>
<td>Recognize</td>
<td>T(draw)</td>
<td>T(draw)</td>
<td>2/6</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Vis.M.C.</td>
<td>Vis.M.C.</td>
<td>2/6</td>
<td></td>
</tr>
<tr>
<td>Implement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td></td>
</tr>
<tr>
<td>Vis. Scan.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td></td>
</tr>
<tr>
<td>Spatial</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td></td>
</tr>
<tr>
<td>Classify</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td></td>
</tr>
</tbody>
</table>

**Measure**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/6</td>
</tr>
</tbody>
</table>

**Legend:**
- **G.:** Grade
- **Term:** Difficulty in recalling terminologies
- **Vis. Scan.:** Difficulties with visual scanning
- **T (draw):** Difficulties with using geometric tools while drawing
- **Vis.M.C.:** Difficulties with visual motor coordination
- **Spatial:** Difficulty with the spatial ability (distinguishing between left and right)
- **Implement:** Difficulty with implementing

<table>
<thead>
<tr>
<th>Ratio = number of times the difficulty appeared / 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Note: The denominator in the ratio is 6 because testing was conducted on 6 grade levels.</td>
</tr>
<tr>
<td>Note: Difficulties with intensity % &lt; 4 are considered minor, and are not presented in table 7.</td>
</tr>
</tbody>
</table>

Table 7 shows that for grade 1, some of the major difficulties are recalling terminologies, recognizing, implementing, visually scanning, distinguishing their left and right based on spatial abilities, and classifying. Similarly, it shows the major difficulties for the remaining grade levels. However, table 7 does not show the intensity % for each difficulty. This appears in table 8.

Table 7 also shows the number of times the same difficulty appeared to have a high intensity %. Difficulty with conceptualizing appeared 5 times through grades 2 to 5, yet spatial difficulty only appeared once with grade 1 and so on.
The number of times a difficulty appears throughout the grade levels is considered an important factor to consider because for instance there are no objectives testing students’ spatial abilities starting from grade 3. Thus the ratio plays a critical role in finding the \textit{Intensity Level} (IL).

\[ IL = (\text{sum of intensity } \% \text{ for each difficulty}) \times (\text{ratio of the difficulty}) \]

Thus, table 8 not only represents the difficulties with their corresponding intensity \% but also the total intensity level for each difficulty.

Table 8 shows the major difficulties faced by students throughout the grades. \textit{Conceptualizing} is a major difficulty with the highest IL of 63.32. Then there is a noticeable decrease in the IL of the difficulties to a little over half its value. At this stage, the difficulties are in \textit{recalling terminologies and properties} which still have relatively high IL of 35.79 and 32.95 respectively. After another drastic decrease in IL, the difficulties that still show a considerable IL are \textit{recalling procedures, recognizing, visual motor coordination}, and \textit{using geometric tools to draw} [5.11; 6.68].

On the other hand, the difficulties with IL less than 2.00 (difficulties with \textit{implementing, measuring, visual scanning, spatial abilities, and classifying and ordering}) are considered minor and insignificant in comparison to the ILs of the other difficulties.

In general, with the exception of \textit{visual motor coordination}, the stated major difficulties are categorized under the \textit{knowing} section of TIMSS. Those difficulties are: \textit{conceptualizing, recalling terminology, recalling properties, recalling procedures, recognizing, and using geometric tools to draw}. 
Table 8

*Intensity of the Major Difficulties Based on Grade Level*

<table>
<thead>
<tr>
<th>Difficulty</th>
<th>G. 1</th>
<th>G. 2</th>
<th>G. 3</th>
<th>G. 4</th>
<th>G. 5</th>
<th>G. 6</th>
<th>IL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept</td>
<td>16.08</td>
<td>6.23</td>
<td>14.05</td>
<td>18.05</td>
<td>21.57</td>
<td>63.32</td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>19.00</td>
<td>18.00</td>
<td>10.79</td>
<td></td>
<td>5.90</td>
<td>35.79</td>
<td></td>
</tr>
<tr>
<td>Property</td>
<td>14.80</td>
<td>10.03</td>
<td>10.53</td>
<td>14.07</td>
<td>32.95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Procedure</td>
<td>13.53</td>
<td>8.64</td>
<td></td>
<td>10.40</td>
<td>16.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recognize</td>
<td>14.65</td>
<td>5.39</td>
<td></td>
<td></td>
<td>6.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T(draw)</td>
<td>5.59</td>
<td>9.75</td>
<td></td>
<td></td>
<td>5.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vis.M.C.</td>
<td>4.17</td>
<td>13.73</td>
<td></td>
<td></td>
<td>5.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Implement</td>
<td>10.00</td>
<td></td>
<td></td>
<td></td>
<td>1.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vis. Scan.</td>
<td>8.04</td>
<td></td>
<td></td>
<td></td>
<td>1.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spatial</td>
<td>5.36</td>
<td></td>
<td></td>
<td></td>
<td>0.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Classify</td>
<td>4.83</td>
<td></td>
<td></td>
<td></td>
<td>0.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measure</td>
<td>9.72</td>
<td></td>
<td></td>
<td></td>
<td>1.62</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Legend:
G.: Grade
IL: Intensity level = (Sum of intensity % on each row) x (ratio)
Term: Difficulty in recalling terminologies
Vis. Scan.: Difficulties with visual scanning
T (draw): Difficulties with using geometric tools while drawing
Vis.M.C.: Difficulties with visual motor coordination
Spatial: Difficulty with the spatial ability (distinguishing between left and right)
Implement: Difficulty with implementing

4.5.2. Study of Difficulties across the Grade Levels

This section analyzes how the intensity % of the difficulties increase or decrease as we progress from one grade level to the next.

Vygotsky, being one of the founders of constructivism, studied children’s cognitive development. He discusses the Zone of Proximal Development (ZPD). It calls for the awareness of the experiences provided to students based on their abilities and limitations. According to Vygotsky, students’ ZPD depends on their ability to complete tasks at a slightly higher level than their own level of thinking, with guidance and peer collaboration until students are able to think and solve at this higher level independently. According to Vygotsky, for learning
to take place, students must solve tasks at this zone (Social Development Theory (Vygotsky), 2011). Having said that, some difficulty regarding the objectives is expected, but objectives must be well studied because if the tasks are too difficult, students get frustrated and are unable to construct new knowledge correctly, and eventually learn geometry.

Thus, for smooth and gradual progress among the grade levels, students are expected to face some difficulty but of relatively low intensity %. However, a look at table 6 shows that the intensity % of the difficulties are not consistent with Vygotsky’s theory about ZPD. Instead, not only do some difficulties have relatively high percentages, but they also fluctuate by initially increasing, and then decreasing again, and vice versa. For instance, a look at the difficulty with recalling procedures, one notices that at grade 1, the difficulty intensity % is 2.50% and increases to 13.53% in grade 2. This instant increase in the difficulty’s intensity % shows that objectives regarding procedural skills are not at the second grade students’ ZPD. Then at grade 3, the intensity % decreases again to a low level of 1.28%.

This drastic decrease in the difficulty’s intensity % shows that some objectives that were beyond grade 2 students’ abilities may be introduced in grade 3 instead.

Similarly, the intensity % increases from 1.28% to 8.64% as students are promoted to the fourth grade. This again shows that the objectives in grade 4 are inconsistent with respect to the constructivist point of view and cognitive development, especially since the intensity % decreases again in grade 5. This in
turn means that some objectives in grade 4 can be introduced in grade 5 instead, and so on.

Furthermore, this fluctuation in intensity % does not only appear with the difficulty in recalling procedures, but in a lot of other difficulties.

Another factor to consider is that with some difficulties, the intensity % increases. For instance, when regarding difficulties in conceptualizing, if a concept is not well developed to begin with, building new knowledge on those concepts will result in an increase in the intensity %. This is because, once a difficulty starts to appear, remediating becomes much harder, if the problems are left unresolved for a long period of time (Chard, et al., 2008).

Therefore, it comes as no surprise that although in grade 1, difficulty with conceptualizing starts at 0.54% and eventually increases extraordinarily to 21.57% by the time students reach the 6th grade.

This not only means that there is an urgent need to redistribute the objectives across the grade levels, to minimize the difficulties across the years, but there’s also a need to monitor the difficulties and try to minimize them from the beginning instead of leaving them unresolved for years to come.

4.5.3. Different Aspects Comparison between Intensities of V1 and V2 vs. V3

What this section entails are a few key factors. Since the testing type changes, from paper-pencil testing, where students are expected to write down and apply their knowledge to performance and objective based tasks, so do the intensity and types of difficulties. Therefore, table 9 analyzes the difference resulting from the change of testing style and tested abilities.
Table 9

*Evaluating Average Intensities of V1 and V2, versus Intensity of V3*

<table>
<thead>
<tr>
<th>Difficulty</th>
<th>G. 1 I3</th>
<th>G. 2 I3</th>
<th>I_{(1,2-3)}</th>
<th>G. 1 I3</th>
<th>G. 2 I3</th>
<th>I_{(1,2-3)}</th>
<th>G. 3 I3</th>
<th>G. 4 I3</th>
<th>I_{(1,2-3)}</th>
<th>G. 5 I3</th>
<th>G. 6 I3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept</td>
<td>17.78</td>
<td>-3.41</td>
<td>0.50</td>
<td>-6.48</td>
<td>-18.05</td>
<td>-5.96</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Property</td>
<td>20.00</td>
<td>-10.41</td>
<td>-2.16</td>
<td>-6.01</td>
<td>-0.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Procedure</td>
<td>5.00</td>
<td>17.78</td>
<td>-8.50</td>
<td>17.28</td>
<td>3.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Language</td>
<td>7.14</td>
<td>6.70</td>
<td>-4.27</td>
<td></td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>28.00</td>
<td>22.22</td>
<td>-18.00</td>
<td>-8.44</td>
<td>-10.48</td>
<td>-3.49</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vis.Scan.</td>
<td>15.00</td>
<td></td>
<td></td>
<td>-13.93</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recognize</td>
<td>25.00</td>
<td>-20.71</td>
<td></td>
<td></td>
<td>-0.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Classify</td>
<td>7.86</td>
<td></td>
<td></td>
<td>-6.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Guess(Perc)</td>
<td></td>
<td>6.67</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tools(draw)</td>
<td>4.44</td>
<td></td>
<td>2.30</td>
<td>-6.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measuring</td>
<td>13.33</td>
<td></td>
<td>-7.22</td>
<td>1.13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vis.M.Coor.</td>
<td>1.43</td>
<td>5.93</td>
<td>0.71</td>
<td>-3.52</td>
<td>-15.84</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Executive</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Implement.</td>
<td>18.57</td>
<td></td>
<td>-17.14</td>
<td></td>
<td></td>
<td>-3.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Represent.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spatial</td>
<td>8.57</td>
<td></td>
<td>-6.43</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Legend:
- Vis. Scan.: Difficulty in visual scanning
- Vis. M. Coor.: Difficulties with visual motor coordination
- Guess (Perc): Difficulties with guessing based on perception
- Tools (draw): Difficulties with using geometric tools while drawing
- G.: Grade
- I3: Intensity of difficulty during T3

From G. 1 to G. 6, only G. 1 I3 and G. 2 I3 have been selected from the other grades because only in grades 1 and 2 did students show difficulties during T3, that were hidden in T1 and T2.

I_{(1,2-3)}: The difference between the average intensity of T1 and T2 versus T3.

Note that table 9 is obtained by combining information provided by appendix VIII.

In table 9 there are two factors to consider.

The first factor is the section that includes columns G. 1 I3 and G. 2 I3. In these columns some numbers are underlined (5.00 in procedure, 7.14 and 6.70 in language, and 6.67 in Guess (Perc)). These numbers are important because they show difficulties that did not appear during T1 and T2, where tasks were paper-
pencil based, as opposed to T3 which was based on performance and communication. It is during T3 that grade 1 students’ inability to recall procedures and their inability to understand and communicate in the academic language appeared. Throughout T3, students kept communicating with the researcher in Armenian asking for help and explanation on what is required of them during the testing process. Similarly, language was found to be an issue for the second graders. However, they also appeared to be relying on perception more than actually remembering properties. What this means is that students may have guessed some of the test items correctly in T1 and T2 based on perception instead of justifiable thinking.

The second factor is the section which includes the $I_{(1,2,3)}$ columns. These columns show how different types of testing require students to use different abilities and skills. With the use of different skills and abilities to solve geometry tasks, exercises and problems, one can expect to get a better view of the major difficulties students may face in a more global manner.

Therefore, to better visualize the major difficulties resulting from changing the testing style, the intensity % obtained from table 9 were charted in appendix IX taking each grade level separately.

Table 10 shows the major difficulties based on grade level after changing testing style.
Table 10

Major Difficulties after Changing Testing Style Based on Grade Level

<table>
<thead>
<tr>
<th>G. 1</th>
<th>G. 2</th>
<th>G. 3</th>
<th>G. 4</th>
<th>G. 5</th>
<th>G. 6</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term.</td>
<td>Term.</td>
<td>Term.</td>
<td>Term.</td>
<td>Term.</td>
<td>Impl.</td>
<td>4/6</td>
</tr>
<tr>
<td>Concept</td>
<td>Concept</td>
<td>Concept</td>
<td>Concept</td>
<td>Concept</td>
<td>3/6</td>
<td></td>
</tr>
<tr>
<td>Property</td>
<td>Property</td>
<td>Property</td>
<td>Property</td>
<td>Property</td>
<td>2/6</td>
<td></td>
</tr>
<tr>
<td>Impl.</td>
<td>Vis. M.C.</td>
<td>Vis. M.C.</td>
<td></td>
<td></td>
<td></td>
<td>2/6</td>
</tr>
<tr>
<td>Recognize</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>Vis. Scan.</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>Spatial</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>Classify</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>Procedures</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>Measuring</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>T (draw)</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>Language</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
</tr>
</tbody>
</table>

Legend:
Term.: Difficulties in recalling terminology
Impl.: Difficulties with implementing
Vis. Scan.: Difficulty in visual scanning
Vis. M. C.: Difficulties with visual motor coordination
Guess (Perc): Difficulties with guessing based on perception
T (draw): Difficulties with using geometric tools while drawing
G.: Grade

Table 10 shows the major difficulties that students face in each grade level, after changing testing type from paper-pencil testing to performance and objective based tasks. For instance, in grade 1, the major difficulties are related to students’ ability to recall terminology, implement, recognize, visually scanning, distinguish between their left and right based on their spatial ability, and classify and order.

On the other hand, one notices that some of the major difficulties appear during the course of other grade levels as well. As such, difficulties with conceptualizing and recalling terminology appear with high intensity % in 4 out of the 6 grade levels, while difficulties regarding recognizing, visual scanning,
spatial abilities, classifying and ordering, recalling procedures, measuring, using geometric tools while drawing, and understanding and using academic language appear only once throughout the 6 grades.

However, to better highlight the major difficulties that arise due to the change in the type of the testing questionnaire, difficulties in table 10 are replaced with their corresponding intensity % in table 11. As such, table 11 allows the calculation of Intensity Levels (IL) of each difficulty.

Table 11 shows that IL for the difficulties vary from 0.86 to 57.79. This range highlights the major difficulties and those that are considered minor. Based on the intensity levels, the major difficulties that appear after changing the type of testing are: conceptualizing (with the highest IL of 57.79), recalling terminology (with the second highest IL of 49.26), and recalling properties (with another high IL of 22.32).

Although the ILs for the previous three difficulties are relatively much higher than the remaining difficulties, visual motor coordination, implementing, and recognizing also show considerable IL results (9.19, 7.74, and 4.17 respectively).

Moreover, even though some difficulties may have high intensity % in some grade levels, in general they do are not considered since their IL are considerably small (less than 3.00). Such is the case with difficulties regarding: visual scanning, spatial abilities, classifying and ordering, recalling procedures, measuring, using geometric tools while drawing, and using and understanding the academic language.
Table 11

Intensity Level of Major Difficulties after Changing Testing Style Based on Grade Level

<table>
<thead>
<tr>
<th>Difficulty</th>
<th>G. 1</th>
<th>G. 2</th>
<th>G. 3</th>
<th>G. 4</th>
<th>G. 5</th>
<th>G. 6</th>
<th>R.</th>
<th>IL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term.</td>
<td>28.00</td>
<td>22.22</td>
<td>16.03</td>
<td></td>
<td></td>
<td></td>
<td>7.64</td>
<td>49.26</td>
</tr>
<tr>
<td>Concept</td>
<td>17.78</td>
<td></td>
<td>17.29</td>
<td>27.07</td>
<td>24.55</td>
<td></td>
<td>4/6</td>
<td>57.79</td>
</tr>
<tr>
<td>Properties</td>
<td>20.00</td>
<td></td>
<td>11.11</td>
<td>13.53</td>
<td></td>
<td></td>
<td>3/6</td>
<td>22.32</td>
</tr>
<tr>
<td>Implement</td>
<td>18.57</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.66</td>
<td>2/6</td>
<td>7.74</td>
</tr>
<tr>
<td>Vis. M. Coor.</td>
<td>5.93</td>
<td>21.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2/6</td>
<td>9.19</td>
</tr>
<tr>
<td>Recognize</td>
<td>25.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td>4.17</td>
</tr>
<tr>
<td>Visual Scanning</td>
<td>15.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td>2.50</td>
</tr>
<tr>
<td>Spatial</td>
<td>8.57</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td>1.43</td>
</tr>
<tr>
<td>Classify</td>
<td>7.86</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td>1.31</td>
</tr>
<tr>
<td>Procedures</td>
<td>17.78</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td>2.96</td>
</tr>
<tr>
<td>Measuring</td>
<td>13.33</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td>2.22</td>
</tr>
<tr>
<td>Tools (Draw)</td>
<td>12.82</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td>2.14</td>
</tr>
<tr>
<td>Language</td>
<td>5.13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Legend:
Term.: Difficulty in recalling terminology
Vis. M. Coor.: Difficulties with visual motor coordination
Tools (draw): Difficulties with using geometric tools while drawing
G.: Grade
R.: Total ratio

Therefore, again the majority of the difficulties is based on the knowing section of TIMSS cognitive domain and can be addressed in class. Implementing however, is from the applying section.

4.5.4. Chronological Comparison between Tests One and Two

This section consists of an analysis of the progress/decrease of students’ knowledge, skills and understanding of geometry after allowing the passage of time. This information is provided by analyzing the intensity % of SPV1-SPV2 as presented in table 12.
Table 12

**Difficulties across the Six Grade Levels, Based on the Intensity % of SPV1-SPV2**

<table>
<thead>
<tr>
<th>Difficulties</th>
<th>G.1</th>
<th>G.2</th>
<th>G.3</th>
<th>G.4</th>
<th>G.5</th>
<th>G.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept</td>
<td>-2.14</td>
<td>-12.74</td>
<td>1.99</td>
<td>3.09</td>
<td>-4.51</td>
<td>-6.31</td>
</tr>
<tr>
<td>Property</td>
<td>-2.86</td>
<td>-4.52</td>
<td>-2.99</td>
<td>3.09</td>
<td>-3.01</td>
<td>-7.73</td>
</tr>
<tr>
<td>Measuring</td>
<td>-5.56</td>
<td>0.43</td>
<td>-2.26</td>
<td>-1.09</td>
<td>-1.99</td>
<td>-4.51</td>
</tr>
<tr>
<td>Vis. Scan.</td>
<td>2.14</td>
<td>-0.82</td>
<td>-4.27</td>
<td>-2.47</td>
<td>-3.76</td>
<td>-2.26</td>
</tr>
<tr>
<td>Terminology</td>
<td>-2.00</td>
<td>-11.56</td>
<td>2.57</td>
<td>0.15</td>
<td>-0.75</td>
<td>1.31</td>
</tr>
<tr>
<td>Computing</td>
<td>-0.86</td>
<td>-2.26</td>
<td>-1.54</td>
<td>1.50</td>
<td>2.86</td>
<td>4.15</td>
</tr>
<tr>
<td>Selecting</td>
<td>1.43</td>
<td>-9.56</td>
<td>2.56</td>
<td>-2.78</td>
<td>1.50</td>
<td>-1.46</td>
</tr>
<tr>
<td>Recognizing</td>
<td>2.86</td>
<td>-4.15</td>
<td>-0.86</td>
<td>0.93</td>
<td>-4.51</td>
<td>1.33</td>
</tr>
<tr>
<td>Implement</td>
<td>2.77</td>
<td>1.71</td>
<td>-6.17</td>
<td>-6.01</td>
<td>0.93</td>
<td>0.75</td>
</tr>
<tr>
<td>Procedure</td>
<td>1.43</td>
<td>1.85</td>
<td>-2.05</td>
<td>-1.85</td>
<td>-0.25</td>
<td>0.00</td>
</tr>
<tr>
<td>Vis. M. Coor.</td>
<td>1.43</td>
<td>1.85</td>
<td>-2.05</td>
<td>-1.85</td>
<td>-0.25</td>
<td>0.00</td>
</tr>
<tr>
<td>Language</td>
<td>0.57</td>
<td>0.62</td>
<td>-2.25</td>
<td>0.97</td>
<td>-0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Executive</td>
<td>0.71</td>
<td>-0.07</td>
<td>0.00</td>
<td>0.93</td>
<td>-0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Represent</td>
<td>0.67</td>
<td>-0.43</td>
<td>0.75</td>
<td>0.86</td>
<td>0.00</td>
<td>0.73</td>
</tr>
<tr>
<td>Guess (Perc)</td>
<td>4.29</td>
<td>-2.30</td>
<td>-1.54</td>
<td>1.50</td>
<td>2.86</td>
<td>4.15</td>
</tr>
<tr>
<td>Spatial</td>
<td>2.14</td>
<td>10.52</td>
<td>-2.05</td>
<td>4.01</td>
<td>0.00</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Legend:
- Vis. Scan.: Difficulty in visual scanning
- Vis. M. Coor.: Difficulties with visual motor coordination
- Guess (Perc): Difficulties with guessing based on perception
- Tools (draw): Difficulty in using geometric tools while drawing
- G.: Grade

In table 12, the intensity % vary. Where, the intensity % > 0, it implies that after the passage of time, students have less difficulty with some objectives. Where the intensity % = 0, it implies that after the passage of time, students have not improved or forgotten how to solve certain objectives. Where the intensity % < 0, it implies that students have difficulty in retaining the required knowledge and abilities in order to solve certain objectives.
Yet, in order to better understand and visualize table 12, each grade level is taken separately, and represented in chart formation in Appendix VII. The results obtained from Appendix VII show the major difficulties that arise after the passage of time for each grade level and are tabulated in table 13.

Table 13 shows the major difficulties per grade level.

Table 13

*Grouping of Major Difficulties Based on the Intensity % of V1-V2*

<table>
<thead>
<tr>
<th>Grade 1</th>
<th>Grade 2</th>
<th>Grade 3</th>
<th>Grade 4</th>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Total Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property Concept</td>
<td>Property Concept</td>
<td>Property Concept</td>
<td>Property Concept</td>
<td>Property Concept</td>
<td>Property Concept</td>
<td>5/6</td>
</tr>
<tr>
<td>Term.</td>
<td>Property Concept</td>
<td>Vis.Scan.</td>
<td>Term.</td>
<td>Vis.Scan.</td>
<td>Vis.Scan.</td>
<td>4/6</td>
</tr>
<tr>
<td>Recognize</td>
<td>Vis.Scan.</td>
<td>Recognize</td>
<td>Measure</td>
<td>Measure</td>
<td>Measure</td>
<td>3/6</td>
</tr>
<tr>
<td>Tools (draw)</td>
<td>Selecting</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
</tr>
</tbody>
</table>

Legend:
Term.: Difficulty in recalling terminology
Vis.Scan.: Difficulty in visual scanning
Vis. M. Coor.: Difficulties with visual motor coordination
Tools (draw): Difficulties with using geometric tools while drawing

As before, table 13 shows two things. First of all it shows the major difficulties per grade level, and the second is the number of times a difficulty appears across the various grade levels.

Correspondingly, the difficulties in table 13 are replaced with their intensity % to calculate the IL in table 14.
Table 14

_Intensity of the Major Difficulties per Grade Level_

<table>
<thead>
<tr>
<th>Difficulty</th>
<th>G. 1</th>
<th>G. 2</th>
<th>G. 3</th>
<th>G. 4</th>
<th>G. 5</th>
<th>G. 6</th>
<th>Total Ratio</th>
<th>IL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Properties</td>
<td>-2.86</td>
<td>-4.52</td>
<td>-2.99</td>
<td>-3.01</td>
<td>-7.73</td>
<td></td>
<td>5/6</td>
<td>-17.59</td>
</tr>
<tr>
<td>Concepts</td>
<td>-2.14</td>
<td>-12.74</td>
<td>-4.51</td>
<td>-6.31</td>
<td></td>
<td></td>
<td>4/6</td>
<td>-17.13</td>
</tr>
<tr>
<td>Vis. Scan.</td>
<td></td>
<td>-4.27</td>
<td>-2.47</td>
<td>-3.76</td>
<td></td>
<td></td>
<td>3/6</td>
<td>-5.25</td>
</tr>
<tr>
<td>Terminology</td>
<td>-2.00</td>
<td>-11.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2/6</td>
<td>-4.52</td>
</tr>
<tr>
<td>Recognizing</td>
<td></td>
<td>-9.56</td>
<td>-2.78</td>
<td></td>
<td></td>
<td></td>
<td>2/6</td>
<td>-4.11</td>
</tr>
<tr>
<td>Measuring</td>
<td></td>
<td>-5.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2/6</td>
<td>-2.22</td>
</tr>
<tr>
<td>Implementing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-4.51</td>
<td>-1.46</td>
<td>2/6</td>
<td>-1.99</td>
</tr>
<tr>
<td>Vis. M. Coor.</td>
<td>-2.05</td>
<td>-1.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2/6</td>
<td>-1.30</td>
</tr>
<tr>
<td>Procedures</td>
<td></td>
<td>-6.17</td>
<td>-6.01</td>
<td></td>
<td></td>
<td></td>
<td>2/6</td>
<td>-4.06</td>
</tr>
<tr>
<td>Selecting</td>
<td></td>
<td>-1.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td>-0.26</td>
</tr>
<tr>
<td>Tools (draw)</td>
<td></td>
<td>-2.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/6</td>
<td>-0.34</td>
</tr>
</tbody>
</table>

Legend:
Vis. Scan.: Difficulty in visual scanning
Vis. M. Coor.: Difficulties with visual motor coordination
Tools (draw): Difficulties with using geometric tools while drawing
G.: Grade

In order to compare the IL results from table 14, from now on the absolute values are taken and discussed. Having said that, again |IL| results show a wide range of [0.26; 17.59]. As such, from the |IL|, one notices that some of the difficulties are more imposing than others. The two major difficulties with highest absolute value of IL are difficulties in recalling properties (17.59) and conceptualizing (17.13). There also are difficulties ranging between [4.00; 6.00]. These difficulties (in order) are: _visual scanning, recalling terminology, recognizing, and recalling procedure._

Other minor difficulties that range between [0.00; 3.00] (in order) are: _measuring, implementing, visual motor coordination, using geometric tools while_
drawing, and selecting (Since this study aims to find the major difficulties, these minor difficulties are considered irrelevant).

It is important to realize however, that the difficulties having an $|IL|>4.00$, with the exception of visual scanning, are based on the knowing cognitive domain of TIMSS (see Appendix IV).

4.5.5. Difficulties based on the content domains of geometry

This section analyzes the major difficulties for each of the content domains of geometry. The domains are: Location (L), Solid figures (S), Plane figures (P) and Transformations (T).

In order to find the intensity level of the difficulties for each content domain, a few steps were followed: For each grade level, each objective within each difficulty was categorized depending on which content domain to which the objective belongs. Then the averages as presented in Appendix VI were converted to their corresponding intensity % and tabulated in appendix XI. Furthermore, the results in Appendix XI were chunked based on the cognitive domains, and new averages were found. These averages are represented in charts in order to give a clear view of the major difficulties that the students face with respect to each content domain of geometry (see Appendix XI).

Table 15 thus shows the major difficulties faced in each content domain.

Again as before, table 15 shows two things. First of all it shows the major difficulties per grade level, and the second is the number of times a difficulty appears across the various grade levels.
Table 15

*Distribution of Major Difficulties Based on the Content Domains of Geometry*

<table>
<thead>
<tr>
<th>Location</th>
<th>Solid Figures</th>
<th>Plane Figures</th>
<th>Transformations</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term.</td>
<td>Term.</td>
<td>Term.</td>
<td>Term.</td>
<td>4/4</td>
</tr>
<tr>
<td>Procedure</td>
<td>Procedure</td>
<td>Procedure</td>
<td>Procedure</td>
<td>4/4</td>
</tr>
<tr>
<td>Properties</td>
<td>Properties</td>
<td>Properties</td>
<td>Properties</td>
<td>4/4</td>
</tr>
<tr>
<td>T(draw)</td>
<td>T(draw)</td>
<td>T(draw)</td>
<td></td>
<td>3/4</td>
</tr>
<tr>
<td>Recognize</td>
<td>Recognize</td>
<td></td>
<td></td>
<td>2/4</td>
</tr>
<tr>
<td>Vis.M.C.</td>
<td>Vis.M.C.</td>
<td></td>
<td></td>
<td>2/4</td>
</tr>
<tr>
<td>Implement</td>
<td></td>
<td></td>
<td></td>
<td>1/4</td>
</tr>
<tr>
<td>Spatial</td>
<td></td>
<td></td>
<td></td>
<td>1/4</td>
</tr>
<tr>
<td>Vis. Scan.</td>
<td></td>
<td></td>
<td></td>
<td>1/4</td>
</tr>
<tr>
<td>Classify</td>
<td></td>
<td></td>
<td></td>
<td>1/4</td>
</tr>
<tr>
<td>Select</td>
<td></td>
<td></td>
<td></td>
<td>1/4</td>
</tr>
<tr>
<td>G. (Perc)</td>
<td></td>
<td></td>
<td></td>
<td>1/4</td>
</tr>
<tr>
<td>Measure</td>
<td></td>
<td></td>
<td></td>
<td>1/4</td>
</tr>
</tbody>
</table>

Legend:
- Term.: Difficulty in recalling terminology
- Vis. M. C.: Difficulties with visual motor coordination
- Guess (Perc): Difficulties with guessing based on perception
- T (draw): Difficulties with using geometric tools while drawing

Correspondingly, the difficulties in table 15 are replaced with their intensity % in order to calculate the IL for each difficulty in table 16.

In table 16, the difficulties are replaced with their corresponding intensities that are available in appendix XI.

Again, IL results show a wide range of [1.04; 58.28]. As such, from the IL, one notices that some of the difficulties are more imposing than others.

The IL in table 16 show that the major difficulties faced by students based on the cognitive domain of geometry (in order) are: recalling terminology(58.28), conceptualizing(53.94), recalling properties(45.06), and recalling procedures(37.20).
Table 16

*The Intensity of Major Difficulties Based on the Content Domains of Geometry*

<table>
<thead>
<tr>
<th>Difficulty</th>
<th>L</th>
<th>S</th>
<th>P</th>
<th>T</th>
<th>IL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terms</td>
<td>12.46</td>
<td>18.33</td>
<td>7.29</td>
<td>20.20</td>
<td>58.28</td>
</tr>
<tr>
<td>Concepts</td>
<td>10.49</td>
<td>13.59</td>
<td>13.41</td>
<td>16.45</td>
<td>53.94</td>
</tr>
<tr>
<td>Procedure</td>
<td>8.43</td>
<td>8.00</td>
<td>9.30</td>
<td>11.47</td>
<td>37.20</td>
</tr>
<tr>
<td>Properties</td>
<td>7.73</td>
<td>6.27</td>
<td>10.05</td>
<td>21.01</td>
<td>45.06</td>
</tr>
<tr>
<td>T(draw)</td>
<td>6.41</td>
<td>4.86</td>
<td>8.11</td>
<td></td>
<td>14.54</td>
</tr>
<tr>
<td>Recognize</td>
<td>5.36</td>
<td>7.92</td>
<td></td>
<td></td>
<td>6.64</td>
</tr>
<tr>
<td>Vis.M.C.</td>
<td>4.15</td>
<td>16.84</td>
<td></td>
<td></td>
<td>10.50</td>
</tr>
<tr>
<td>Implement</td>
<td>6.10</td>
<td></td>
<td></td>
<td></td>
<td>1.53</td>
</tr>
<tr>
<td>Spatial</td>
<td>4.18</td>
<td></td>
<td></td>
<td></td>
<td>1.05</td>
</tr>
<tr>
<td>Vis. Scan.</td>
<td></td>
<td>6.47</td>
<td></td>
<td></td>
<td>1.62</td>
</tr>
<tr>
<td>Classify</td>
<td>6.43</td>
<td></td>
<td></td>
<td></td>
<td>1.61</td>
</tr>
<tr>
<td>Select</td>
<td>4.17</td>
<td></td>
<td></td>
<td></td>
<td>1.04</td>
</tr>
<tr>
<td>Guess (Perc)</td>
<td></td>
<td>6.67</td>
<td></td>
<td></td>
<td>1.67</td>
</tr>
<tr>
<td>Measure</td>
<td>5.20</td>
<td></td>
<td></td>
<td></td>
<td>1.30</td>
</tr>
</tbody>
</table>

Legend:
- Term.: Difficulty in recalling terminology
- Vis. M. C.: Difficulties with visual motor coordination
- Guess (Perc): Difficulties with guessing based on perception
- T (draw): Difficulties with using geometric tools while drawing
- L: Location
- S: Solid figures
- P: Plane figures
- T: Transformations
- IL: Intensity level

In addition, even though using geometric tools to draw (14.54), visual motor coordination (10.50), and recognizing (6.64) have much lower ILs than the previous difficulties, these difficulties still have a considerable intensity level, and cannot be ignored.

On the other hand, the IL for implementing, spatial abilities, visual scanning, classifying and ordering, selecting, guessing based on perception and
measuring are considered too minimal to consider. This is because their IL ranges from [1.05; 1.67].

Thus, aside from the visual motor coordination, all the difficulties are based on knowing, the first cognitive domain of TIMSS.

**4.6- Conclusion to analysis**

In order to get a global understanding of the major difficulties that students face, intensities obtained in section 4.5 are tabulated in table 17.

Table 17

*Intensities of Major Difficulties on a Global Scale*

<table>
<thead>
<tr>
<th>Difficulty</th>
<th>4.5.1.</th>
<th>4.5.3.</th>
<th>4.5.4.</th>
<th>4.5.5.</th>
<th>FIL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptualizing</td>
<td>63.32</td>
<td>57.79</td>
<td>17.13</td>
<td>53.94</td>
<td>192.18</td>
</tr>
<tr>
<td>Recalling Terminology</td>
<td>35.79</td>
<td>49.26</td>
<td>4.52</td>
<td>58.28</td>
<td>147.85</td>
</tr>
<tr>
<td>Recalling Properties</td>
<td>32.95</td>
<td>22.32</td>
<td>17.59</td>
<td>45.06</td>
<td>117.92</td>
</tr>
<tr>
<td>Recalling Procedures</td>
<td>16.29</td>
<td>4.06</td>
<td>37.20</td>
<td>43.16</td>
<td></td>
</tr>
<tr>
<td>Recognizing</td>
<td>6.68</td>
<td>4.17</td>
<td>4.11</td>
<td>6.64</td>
<td>21.60</td>
</tr>
<tr>
<td>Using geometric tools to draw</td>
<td>5.11</td>
<td>14.54</td>
<td>9.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Visual Motor Coordination</td>
<td>5.97</td>
<td>9.19</td>
<td>10.50</td>
<td>19.25</td>
<td></td>
</tr>
<tr>
<td>Visual Scanning</td>
<td>5.25</td>
<td></td>
<td>1.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Implement</td>
<td>7.74</td>
<td></td>
<td></td>
<td>1.94</td>
<td></td>
</tr>
</tbody>
</table>

Legend:

4.5.1.: section 4.5.1. from thesis: Comparison of major difficulties across the years
4.5.3.: section 4.5.3. from thesis: Different Aspects Comparison between Intensities of V1 and V2 vs. V3
4.5.4.: section 4.5.4. from thesis: Chronological Comparison between Tests One and Two
4.5.5.: section 4.5.5. from thesis: Comparison of difficulties based on the content domains of geometry
FIL: Final Intensity level

The major difficulties based on the intensity levels found in table 17 (in order) are: *conceptualizing (192.18), recalling terminology (147.85), and recalling properties*
Then FIL results show a drastic drop regarding difficulties with: recalling procedures (43.16), recognizing (21.60), visual motor coordination (19.25), and using geometric tools (9.83).

On the other hand, the Final Intensity Level (FIL) for visual scanning and implementing, which are 1.31 and 1.94 respectively, are too minimal to consider.
CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 – Introduction

Learning and using geometry is essential, not only for students during geometry class, but also in our everyday lives.

If the 2007 geometry results obtained by TIMSS are considered, one notices that students from Lebanon, on average, were only able to solve 46.20% of the tested items correctly (Martin, Mullis, & Foy, 2008).

This means that students were unable to answer more than half the questionnaire accurately. Furthermore, out of the 48 participating countries, Lebanon ranked 24th (Martin, Mullis, & Foy, 2008).

These numbers alone highlight the fact that students from Lebanon have difficulties regarding geometry, and thus, this research aimed to pinpoint some of the major difficulties that the students of the elementary grades face while recalling, retrieving and applying their acquired geometric knowledge as well as reasoning geometrically and performing geometry tasks.

5.2 – Conclusions

The first research question aims to find out if the objectives and content of the geometry Lebanese curriculum (ECRD, 1997) for the elementary level (grades 1 to 6) are well distributed among the three cognitive domains of TIMSS. However, by looking at table 3, we notice that the distribution is not well balanced. Much more importance is given to knowing than the other cognitive domains, but that is not all. The more
bothersome factor is the fact that there are no objectives aiming for higher thinking levels such as reasoning. This promotes superficial learning of procedures through drill and practice activities.

Unfortunately, a more in-depth look at the curriculum shows that, not enough importance is given to geometry in general.

Table 18

*Average Time for Learning and Practicing Geometry Objectives per Grade Level*

<table>
<thead>
<tr>
<th>Grade</th>
<th>Number of objectives</th>
<th>Number of hours for geometry</th>
<th>Time dedicated per objective in hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>25</td>
<td>1.25</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>20</td>
<td>1.33</td>
</tr>
<tr>
<td>3</td>
<td>26</td>
<td>20</td>
<td>0.77</td>
</tr>
<tr>
<td>4</td>
<td>27</td>
<td>20</td>
<td>0.74</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>25</td>
<td>1.25</td>
</tr>
<tr>
<td>6</td>
<td>56</td>
<td>25</td>
<td>0.45</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>0.97</td>
</tr>
</tbody>
</table>

What table 18 shows is that the most time average for an objective is dedicated in grade 2 (1.33 hours), and the least dedicated is in grade 6 (0.45 hours). Yet in average, 0.97 hours (58.2 minutes) are dedicated to explaining, understanding, learning, practicing and assessing each objective.

As stated by Gantasala and Gantasala (2009), when curriculum designers, and educators alike, approach learning, they should consider the students, and a flexible pedagogy that allows case analysis, team projects, lectures, and games that provide the appropriate experiences. The curriculum should be designed to include projects, coaching, mentoring, and creating students’ awareness of their complementary learning strategies. It should also include activity based learning that helps foster flexible learning
strategies. In addition, curriculum should be designed such that students are exposed to many learning environments, helping them develop coping skills and behaviors. Yet with the time frame dedicated to geometry, it becomes impossible for the teacher to follow. Therefore, it is no wonder that some of the students’ major difficulties lie in recalling properties, procedures and terminology, as well as conceptualizing.

Yet, another in-depth look shows that not only is the distribution of the objectives in the three cognitive domains inconsistent, and the time dedicated for each objective insufficient, but the distribution of objectives across the grade levels are also inconsistent. From table 6 we notice the constant rise and fall in the intensity % of each difficulty as students progress from one grade level to the next.

When the difficulty intensity percentage increases drastically from one grade level to the next, this shows that the students did not have enough prior knowledge to cope with the new objectives, or that the set objectives are not aligned with the many well developed and recognized stages of cognitive development. Moreover, as stated by Gantasala and Gantasala (2009), the curriculum and teaching approach should be consistent with the students’ cognitive development not only to give students an opportunity to improve their academic level, but also to promote a better attitude towards learning.

The second research question tackles the major difficulties that students face while applying their acquired geometric knowledge as they perform geometry tasks and solve problems.

To answer this research question, a research was conducted on elementary students of a Lebanese school having different socioeconomic backgrounds. The
research results show that the major difficulties students have are based on conceptualizing, and recalling terminology, properties, and procedures.

These difficulties are based on the first cognitive domain of TIMSS which is *knowing*. There in is the problem. Students who lack prior knowledge, will face even more difficulty in learning and using geometry in the future. According to the constructivist view on learning, students can only build their knowledge by integrating new, personalized experiences with their prior knowledge (Capraro & Capraro, 2006).

As such, educators and teachers alike must tackle these major difficulties. Educators must monitor students’ knowledge of concepts as they me be incomplete, and based on misconceptions. Gal, Lin and Ying (2009) even state that some students might not fully understand and learn concepts due to pseudo-conceptual/analytical behavior (superficial connections of terminologies and processes) whereby the students’ mental processes lack the necessary conceptual behavior. Therefore, educators must be attentive to such behaviors.

Almost as concerning is the fact that students have difficulty recalling terminologies, and properties. This too may be because students did not understand the terminologies or properties, and thus relied on rote memorization which results in the inability to recall. As stated by Mason (1998), when students rely on rote memorization, students’ understanding becomes limited. Furthermore, students eventually forget memorized material and thus, will not be able to apply it unfamiliar situation.

Nevertheless, rote memorization does not only influence the ability to recall terminologies and properties, but also procedures. When students are unable to
understand and visualize the logic behind procedures, then it is no wonder that they do not recall the procedures themselves.

Furthermore, if students do not know concepts, properties, and terminology, then they would not be able to state what the figure set in front of them is. Students thus become unable to recognize those figures.

Moreover, students learn how to use geometric tools starting from a very early age. At first they learn to draw lines using a ruler, then how to measure segments using that ruler. But then they get introduced to a new tool, the set-square. Students learn how to draw right angles and how to verify that an angle is right. Next, students learn how to use the compass for drawing circles and bisectors, as well as comparing distances. And finally they are introduced to the protractor in order to draw and measure angles. This constant introduction of a new tool, or a new way to use that tool, may be why students have difficulty with the tools. At grade 6, students no longer have difficulty in using a ruler, but are hesitant when it comes to using the protractor.

Thus, the more experience students have in using geometric tools, the less they will face difficulties when it comes to drawing.

This in turn will improve students’ visual motor skills. In the first grade, students still cannot hold the ruler steady in order to draw a straight line. On the other hand, in the sixth grade, students have difficulty in placing the protractor correctly on the first side in order to draw the second based on a given measurement, or they manage to fix the side, but then the vertex of the angle is no longer under the center of the protractor, and so on.
In conclusion, it becomes clear that each difficulty may be a reason for the presence of other difficulties, and as such, educators must encourage classroom discussion, and must provide students with extra exercises that demand different skills and abilities that may show just how much students know and what they know. In addition, it is through classroom discussion that students realize different points of view and better understand the situations they face. Through this phase, active students who face opposing views or new ideas, deal with disturbances in their mind with respect to the knowledge they have compiled. To compensate, the mind assimilates the information, self-regulates, and finally the student finds equilibrium (Piaget, 1997). This assimilation is crucial for the smooth and gradual progress of the student since this progress depends on their educational experiences that may either facilitate or obstruct the advancement within levels (Mason, 1998).

Therefore, educators must focus on preparing early instruction plans that would help avoid these difficulties, especially since, once a difficulty starts to appear, remediating becomes much harder, if the problems are left unresolved for a long period of time (Chard, et al., 2008).

5.3 – Recommendations

The findings of this study clearly show that students do face different types of difficulties, and that the difficulties vary in intensity among the different grade levels. As such, recommendations may be made at different levels.
5.3.1. Recommendations at the school level

Since the students who took part in this study are limited in number, the results obtained cannot be generalized. However, the findings may still be beneficial since the students come from different backgrounds, be it socio-economic and/or racial.

Alleviating the difficulties faced by students would demand an increase in time allocated for geometry to allow the adequate amount of exercises and experiences under various contexts, and that would in turn ensure a better construction of concepts, and better learning and understanding of terminology, properties and procedures so as to prevent difficulties that arise due to the inability to retain knowledge.

Another way to eliminate difficulties arising from the inability to recall terminology, properties and procedures is to have review sessions. During these sessions, classroom discussion should be promoted whereby students communicate what they know with their classmates, with the guidance of the teacher. The teacher in turn monitors what students know, if there are misconceptions that need to be addressed, and provides scenarios that would allow students to create better connections among the terminology, properties and procedures, in order to have a well developed understanding of concepts.

Furthermore, since findings not only show the major difficulties across the grade levels in general, but also the major difficulties faced at each grade level separately, and each content domain separately, teachers should be brought to awareness of these difficulties, and must try to minimize the difficulties.
starting from the beginning of the academic year, and throughout the academic years, as new objectives are introduced. Also, at the beginning of each academic year, students must be tested to diagnose their current knowledge and retention of learned concepts, in order to ensure a smoother transition between the grade levels, between the academic cycles, and thus attain better academic results in geometry.

5.3.2. Recommendations for geometry teachers

Considering that the time allocated to the learning of geometry during school hours is limited, teachers should find ways to maximize the amount of substantial learning that takes place in the classroom by providing students with exercises that are well developed and can target the different facets of each objective, and may equate the application of what they learn under different situations. Similarly, adjusting teaching techniques, using supplementary materials, and conducting hands-on activities will improve students’ understanding, learning, and using of geometry.

In order to do so, teachers should be better prepared, and kept up to date with the latest trends of geometry by reading the latest researches and their findings, taking part in workshops, and communicating with other teachers to understand what techniques other used that may be useful when addressing certain difficulties.

Lastly, teachers should also communicate with other teachers across the different grade levels to get aware of the knowledge as well as the application and reasoning skills of their students at the beginning of each academic year.
This will give the teachers a better understanding of what the students know, what difficulties they may have, and thus, make the transition from one grade level to the next as smooth as possible for the students.

**5.3.3. Recommendations for curriculum designers**

Since the major difficulties are due to insufficient exposure to terminology, properties, procedures, which in turn influence students’ ability to conceptualize the objectives, more time must be allocated for the learning and using of geometry in the classroom. On average, teachers have less than an hour to introduce a new objective to the students, and within that time frame, students are permitted only a limited amount of exposure, and thus reasoning and critical thinking about the objective become unattainable. Hence, there is a need for more time so that students manage to learn each objective properly. As such, it is important for curriculum designers to take into consideration the importance of geometry, and to come up with a better organized time plan. Extra time should not only be dedicated to the teaching of the objectives, but also to introduce students to the concepts in multiple settings that require the use of different skills while solving for the same objective. This will give students a better understanding, and the required experience and exposure in order to satisfy the objective’s outcome.

Yet, time is not the only concern. The distribution of the objectives across the grade levels and the three cognitive domains as stated by TIMSS is not well developed either. This can be observed by taking into consideration the
distribution of the objectives under the three cognitive domains, and how the
difficulty intensities vary from one year to the next.

As such, curriculum designers should also re-evaluate the set objectives,
and redistribute them in order to make the curriculum more coherent with respect
to the cognitive development theories that are based on strong pedagogical
foundations.

5.4– Limitations to the Study

- Purposive convenience sampling: The researcher is the elementary mathematics
teacher at the school that these students attend. Thus, participants are selected
based on easy access and availability to assessment.

- Limited number of students, non–generalizability: Due to the limited number of
students attending this school, findings resulting from student assessment scores
and testing may not be generalized. They will only provide deep insight in
students’ geometric thinking and difficulties.

- Inconsistency in difficulty intensities: Since each grade level has a different
academic standing than the other grade levels, and since the students in each grade level
may vary in ethnicity and preferred language for learning, the intensities obtained may
not be consistent. A better consistency would have been obtained had the students had
the same ethnicity and academic standing across the six grade levels.

5.5 – Recommended Future Research

This research was conducted on students from grades 1 to 6. However, since
some classes have better academic standings than others, comparing across grade levels
would not be absolutely accurate, as other factors may come into place. Furthermore, the
fact that students are not all of the same race or ethnicity, their language and thinking
skills vary. As such, to better understand the difficulties, and observe its progression
throughout the years, it would be better to select students of specific academic standings,
and follow their evolution throughout the academic years.

Another concern to note is related to the type of difficulties. In section 4.1, it is
stated that objectives related to knowing have the highest percentage, ranging from
53.33% to 75.00%. On the other hand, applying ranges from 25.00% to 46.67%, and an
even 0% for reasoning.

Since knowing has the highest number of objectives, then it comes as no surprise
that the major difficulties lie within the knowing cognitive domain.

Thus a separate study should be conducted that solely monitors students’
applying and reasoning skills.

Furthermore, it would be also interesting to study how the difficulties vary
amongst the gender.

Yet more importantly, research should be conducted on the objectives
themselves with respect to the cognitive development for each grade level, in order to
redistribute the objectives across the years in a more suitable manner.
References


Appendix I

Distribution of Objectives across the TIMSS Cognitive Domains

The following section represents the distribution of objectives based on grade level and the three cognitive domains as defined by Mullis, et al. (2005). These domains are: knowing, applying and reasoning.

Knowing

Knowing is considered to be the relevant knowledge that has been understood by students and that will allow them to recall a wide range of concepts. This in turn will prepare the students and allow them to engage in more complex mathematical thinking and understanding.

If for some reason however, students do not have the relevant knowledge, then these students will be unable to make the necessary connections between more recent knowledge of concepts, thus hindering their ability to think mathematically.

The behaviors covered by this cognitive domain are:

1. **Recall** definitions; terms; properties; and notation
2. **Recognize** geometric objects, shapes, expressions, and different orientations of regular geometric figures.
3. **Compute** routine procedures.
4. **Retrieve** information from graphs, or tables; read simple scales.
5. **Measure** by using geometric tools appropriately.
6. **Classify/Order** objects, shapes, according to their common properties;

Applying

Problem solving is considered essential in the teaching of geometry, and hence is prominently featured in the second cognitive domain of applying. In this stage, students apply their mathematical knowledge of concepts, skills, facts and procedures while problem solving and creating representations of their ideas

Note that, at this stage, the problems have routine settings. This means that the problems have been introduced during classroom exercises, and are mostly designed to allow students to practice the specific methods or skills required to solve similar problems. Routine problems merely allow students to select and apply their knowledge

The behaviors covered by this cognitive domain are:
1. **Select** efficient/appropriate methods or strategies for solving problems.
2. **Represent** information in tables, charts, graphs and generate representations.
3. **Model** by generating appropriate models, such as diagrams while solving routine problems.
4. **Implement** by following instructions to draw figures and/or shapes.
5. **Solve Routine Problems** that are similar to those encountered in class while using learnt properties.

**Reasoning**

Reasoning involves the ability to think logically and systematically by using intuition and inductive reasoning as students analyze patterns in order to solve non-routine problems (that are unfamiliar to students). The need for reasoning may be because of the context of the problem is new or more complex than the more routine problems solved in class. The problems also may involve several steps that may require knowledge from the various areas of mathematics. Reasoning also involves making logical deductions as students make assumptions as well as conjectures that are based on certain facts and rules.

The behaviors covered by this cognitive domain are:

1. **Analyze** involves the ability to describe relationships between various objects in a geometrical situation; decompose figures and visualize their transformation while solving a problem; and make correct inferences based on the given information.
2. **Generalize** by extending the result obtained during problem solving and restating those results in a more general manner.
3. **Synthesize/Integrate** by combining various geometrical procedures to obtain certain results, that in turn will produce other results. It involves making connections between different learnt facts and representations, and linking different mathematical ideas.
4. **Justify** by providing justifications to show that a statement is truthful or a falsity by referring to other mathematical properties.
5. **Solve Non-routine Problems** involves solving problems that students have not encountered before.

**GRADE 1: (25hrs)**

**KNOWING**

**Location:** Domain
- Recognize an open domain, a closed domain
- Drawing open or closed domains
- Recognize the interior, the exterior, and the boundary of a simple domain
- Use the terms: interior, exterior, open, closed

**Location:** Position in space
- Recognize a position on a curve or in a plane
- Locate a point (an object) between two others on a curve

**Solid Figures:** Rectangle Prism. Cube. Sphere. Cylinder. Cone
- Sorting and classifying solids according to their shapes by using their names

**Plane Figures:** Lines
- Recognize a straight line
- Drawing a line by connecting a number of given dots
- Drawing a straight line with the ruler
- Reproduce a simple figure on a grid paper

**Plane Figures:** Square. Rectangle. Triangle. Disc
- Recognize geometric figures in a given drawing
- Classify geometric figures according to their shapes

**Transformations:** Axis of symmetry
- Recognize if a given axis is an axis of symmetry of a figure

**APPLYING**

**Location:** Displacement
- Moving in space by following given instructions
- Moving in plane by following given instructions
- Describe a position or a displacement by using the appropriate vocabulary

**Location:** Position in space
- Describe a position or a displacement by using appropriate vocabulary

**Plane Figures:** Square. Rectangle. Triangle. Disc

- Verify the congruence of two figures by tracing or cutting

**Transformations:** Axis of symmetry

Verify if an axis is an axis of symmetry of a figure by tracing, cutting and folding

**REASONING**

None

**GRADE 2: (20hrs)**

**KNOWING**

**Location and References:** Location of a point:

- Determine the position of a place or a knot on a grid using a code.
- Locate a point on a line with respect to other points on the line.

**Solid Bodies:** Description of solids:

- Distinguish between vertices, edges and faces of solids.
- Recognize edges of solids from given objects or their images.
- Recognize the faces of solids from given objects or their images.

**Plane Figures:** Description of plane figures:

- Know the relation between the length of a side of a square and a rectangle.
- Know the number of sides and vertices of a triangle, square or rectangle.
- Distinguish a square from a rectangle by coinciding their sides

**APPLYING**

**Location and References:** Location of a point:

- Locate a point on a curve or on a grid, starting with specific conditions.

**Plane Figures:** Segment of a line:

- Using a ruler, draw a segment limited by two points.
- Draw a segment of a given length, knowing one end point.
- Draw a segment of a given length on a line, knowing one endpoint.

**Plane Figures:** Description of plane figures:

- Construct a square from given elements.

**Transformation:** Figures having an axis of symmetry:
- Find an axis of symmetry of a given figure by folding.

**Transformation:** Axis of symmetry:

Complete, by symmetry on a grid, the drawing of a given figure with an axis of symmetry.

**REASONING**

None

**GRADE 3: (20hrs)**

**KNOWING**

**Location:** Midpoint of a segment:
- Use the terms “midpoint” and “half” correctly.
- Check with a ruler that the given point is the midpoint of a given segment.
- Place the midpoint of a given segment.

**Location:** Perpendicular lines:
- Verify that two lines are perpendicular.
- Draw at a point on a line a perpendicular to this line.

**Solid Figures:** Construction of a cube and a rectangular prism:
- Recognize the pattern of a cube from patterns given by tracing and cutting.
- Recognize the pattern of a parallelepiped from patterns given by tracing and cutting.
- Know that the square has 6 superposable square faces.
- Count the number of faces, vertices, sides of a cube from an image.
- Count the number of faces, vertices, sides of a parallelepiped from an image.

**Plane figures:** Right angles:
- Recognize a right angle.
- Verify if an angle is right using a set-square or equivalent.
- Draw a right angle

**Plane figures:** Rectangle, Square:
- Identify a rectangle by its four right angles.
- Identify a square by its four right angles and the superposition of its sides.
- Reproduce a figure on different grids.
Transformation: Axial Symmetry:
- Use terms: symmetry, symmetric figures, superposable

APPLYING

Solid Figures: Construction of a cube and a rectangular prism:
- Construct a cube from a given pattern.
- Construct a parallelepiped from a given pattern.

Plane figures: Rectangle, Square:
- Construct rectangles and squares.
- Complete a square when knowing a side.
- Complete a rectangle when knowing two consecutive sides.
- Complete a figure to make it identical to a given figure.

Transformation: Axial Symmetry:
- Verify that two given figures are symmetrical with respect to a line.
- Draw, on a grid, a symmetric figure of a given figure with respect to a given axis.
- Distinguish between superposable figures by symmetry and figures that are superposable without being symmetrical.

REASONING
None

GRADE 4: (20hrs)

KNOWING

Location: Distance from point to a line:
- Drawing the distance from a given point to a given straight line.
- Finding on a given drawing the distance from a point to a given straight line.

Location: Point on a square grid:
- Coding the points, the cases, of a grid.
- Situating a point, of a given code, on a grid.

Plane figures: Intersecting straight lines. Parallel straight lines:
- Distinguish two intersecting straight lines, two parallel straight lines.
- Identifying parallel straight lines in a figure.
- Drawing two parallel straight lines on a grid.
- Drawing a straight line parallel to a given straight line.

**Plane figures:** Classifying quadrilaterals according to their sides.
- Using the terms: rhombus, parallelogram, and trapezoid.
- Completing the drawing of a rhombus where we know two consecutive sides.
- Completing a parallelogram where we know two consecutive sides.
- Classifying quadrilaterals according to the parallelism of sides.
- Classifying quadrilaterals according to the congruence of sides, their parallelism and their orthogonality.

**Plane figures:** Circle. Disc.
- Using the instruments of geometry to continue a border.
- Using the terms: circle, center and radius.
- Drawing a circle of center and of a given radius.
- Using the compass for comparing lengths.
- Using the compass to carry over distances.

**APPLYING**

**Location:** Distance from point to a line:
- Situating a point (or several) at a given distance from a given straight line.

**Solid Figures:** Building Models:
- Building models starting from patterns

**Plane figures:** Intersecting straight lines. Parallel straight lines:
- Drawing a straight line perpendicular to a given straight line and passing by a given point.

**Plane figures:** Circle. Disc.
- Reproducing a given triangle or a given particular quadrilateral by using the ruler, the compass and the square.

**Transformation:** Drawing the symmetric of a figure with respect to an axis:
- Constructing with the help of the square and ruler, the symmetric of a triangle.
- Constructing with the help of the square and ruler, the symmetric of a particular quadrilateral.
- Constructing with the help of the square and ruler, the symmetric of a simple figure.
- Finding the axes of symmetry of the rhombus.
- Verifying that the corresponding parts of symmetric figures are congruent.

**REASONING**

None

**GRADE 5: (25hrs)**

**KNOWING**

**Localization and references:** Distance of two parallel lines:
- Recognize the distance between two parallel lines.
- Measure the distance between two parallel lines.

**Solid Bodies:** Development of solids:
- Recognize the bases of a cylinder and their superposition.
- Recognize the pattern of a cylinder.
- Recognize different patterns of the same solid and construct them by folding and cutting.

**Plane Figures:** Angles:
- Recognize the sides and the vertex of an angle
- Use correctly the notion of an angle

**Plane Figures:** Diagonals of a polygon:
- Recognize the diagonals in a polygon
- Draw the diagonals of a polygon.

**Plane Figures:** Classification of quadrilaterals according to diagonals:
- Know the properties of diagonals of special quadrilaterals.
- Recognize a quadrilateral from its diagonals.

**Plane Figures:** Diameter of a circle:
- Draw the diameter of a given circle, where the center is already located.
- Draw a circle knowing its diameter.

**Transformation:** Homothetic figures:
- Transpose the figure of a grid to another grid homothetic to the first
- Verify that two homothetic figures are similar.*

**APPLYING**

**Localization and references:** Distance between parallel lines:
- Use the invariance of a distance between two parallel lines to draw a parallel line to a given line from a given distance.

**Plane Figures:** Diameter of a circle:
- Use the relation Diameter = 2 x Radius.
- Draw a circle knowing the length of its diameter

**Transformation:** Homothetic figures:
- Enlarge a figure in a simple ratio.
- Reduce a figure in a simple ratio.

**REASONING**

None

**GRADE 6: (25hrs)**

**KNOWING**

**Location:** Relative position of 2 straight lines in a plane:
- Knowing that two lines in the plane can be concurrent, parallel or coincident.
- Recognize three concurrent lines.
- Recognize that two lines parallel to the same line are parallel.

**Location:** Relative position of a straight lines and a circle:
- Recognize the circle as being the figure formed by the points situated at an equal distance from a given point called center.
- Distinguish between arc and chord
- Determine the position of a point with respect to a circle.
- Determine the position of a line with respect to a circle as a function of the distance from the center to this line.
- Use the terms: tangent and secant.
- Determine the distance from the center of a given circle with respect to a given line.
- Draw the tangent to the circle at one of its points.
Solid Figures: Patterns of solids:
- Distinguish between the intersection of a sphere with a plane and that of a ball with a plane.**

Plane Figures: Adjacent and vertically opposite angles:
- Recognize two adjacent angles.
- Define and recognize two vertical angles.
- Construct a vertical angle to a given angle.

Plane Figures: Bisector of an angle:
- Recognize the bisector as an axis of symmetry of an angle.
- Construct the bisector of an angle with a protractor.
- Construct the bisector of an angle with a compass.

Plane Figures: Perpendicular bisector of a segment:
- Recognize the perpendicular bisector of a segment as its axis of symmetry.
- Draw the perpendicular bisector with the set-square and the ruler.
- Draw the perpendicular bisector of a segment with a ruler and compasses.
- Find the midpoint of a segment with the compasses and the ruler.

Plane Figures: Triangles:
- Draw the three angle bisectors in a triangle and know that they are concurrent.
- Draw the three perpendicular bisectors in a triangle and know that they are concurrent.
- Draw the three heights of a triangle and know that they are concurrent.
- Draw the three medians of a triangle and know that they are concurrent.
- Bring an angle with the compass.
- Identify the right, isosceles and equilateral triangles, by the sides and the angles.
- Use the vocabulary: principal vertex and base (in an isosceles triangle), hypotenuse and sides of the right angle (in a right triangle).
- Know that the sum of angles in a triangle is $180^\circ$.
- Determine the center of a circle passing through three non-collinear points.

Transformations: Central symmetry:
- Recognize the superposition of two symmetric figures with respect to a given point.

**APPLYING**

**Location:** Relative position of 2 straight lines in a plane:
- Use the uniqueness of the perpendicular taken from a point on a line to determine the distance from this point to the line.
- Use the uniqueness of the perpendicular taken from a point on a line to verify that three points are collinear.
- Use the uniqueness of the parallel taken from a point on a line to verify that three points are collinear.
- Use the following property: if two lines are parallel, every line perpendicular to one is perpendicular to the other.
- Apply the fact that two lines that are perpendicular to the same line are parallel.

**Solid Figures:** Patterns of solids:
- Construct different patterns of a solid.

**Plane Figures:** Adjacent and vertically opposite angles:
- Construct an angle adjacent to a given angle responding to given conditions.
- Use the equality of two vertical angles.

**Plane Figures:** Bisector of an angle:
- Use the following property: the bisector of an angle divides it into two equal angles.

**Plane Figures:** Perpendicular bisector of a segment:
- Use the fact that every point on the perpendicular bisector is at an equal distance from its extremities.

**Transformations:** Central symmetry:
- Draw a figure symmetrical to a given figure with respect to a given point.

**Transformations:** Study of figures from their elements of symmetry (apply in problem format):
- The perpendicular bisector is an axis of symmetry.
- The bisector is an axis of symmetry of an angle.
- The diameter is an axis of symmetry of a circle.
- The isosceles triangle has an axis of symmetry.
- The trapezoid has an axis of symmetry.
- The rectangle has the perpendicular bisectors of the sides as axes of symmetry.
- The diagonals of a rhombus are its axes of symmetry.
- The square has four axes of symmetry.
- The midpoint of a segment is its center of symmetry.
- The center of a circle is its center of symmetry.
- The point where the diagonals of a parallelogram meet is its center of symmetry.
- Draw the symmetric of a figure with respect to an axis of symmetry or a center of symmetry.
- Apply, in problems, the fact that two symmetric figures are superposable.
- Characterize an isosceles triangle and an equilateral triangle by the number of their axes of symmetry.

**REASONING**

None

The following section states the distribution of the objectives based on time allotted to each grade level, as well as the percentage of each cognitive domain the curriculum emphasizes on.

**Grade 1:**
20 objectives in 25 hours
Number of knowing objectives: 14 = 70%
Number of applying objectives: 6 =30%
Number of reasoning objectives: 0 = 0%

**Grade 2:**
15 objectives in 20 hours
Number of knowing objectives: 8 about 53.33%
Number of applying objectives: 7 about 46.67%
Number of reasoning objectives: 0 = 0%
Grade 3:
26 objectives in 20 hours
Number of knowing objectives: 17 about 65.385%
Number of applying objectives: 9 about 34.615%
Number of reasoning objectives: 0 = 0%

Grade 4:
27 objectives in 20 hours
Number of knowing objectives: 18 about 66.67%
Number of applying objectives: 9 about 33.33%
Number of reasoning objectives: 0 = 0%

Grade 5:
20 objectives in 25 hours
Number of knowing objectives: 15 = 75%
Number of applying objectives: 5 = 25%
Number of reasoning objectives: 0 = 0%

Grade 6:
56 objectives in 25 hours
Number of knowing objectives: 31 about 55.36 %
Number of applying objectives: 25 about 44.64%
Number of reasoning objectives: 0 = 0 %

*This objective was not tested since students are not yet familiar with the term or concept for similitude. Students are introduced to it in the 9th grade.

**This objective was not tested since the researcher was unable to answer the objective even after researching about it in the text books used by the students.
Appendix II

Sample Test Items Based on the Different Grade Levels and Content Domains within Geometry

In this section, test items from test 1, test 2, and the performance and objective based tasks are presented along with the objectives related to each test item.

I- A matching exercise for grade 1 (Location):

3- Match the arrows with the correct position words

![Diagram showing matching of arrows to boundary, exterior, and interior of a domain]

The objective being tested for this test item is: “1.1.3. Recognize the interior, the exterior, and the boundary of a simple domain”.

II- A multiple choice exercise for grade 2 (Plane Figures):

7- Circle the correct statements.
   a- A square has only 2 sides equal.
   b- A rectangle has only 3 sides equal.
   c- A square has only 3 sides equal.
   d- A rectangle has its opposite sides equal.
   e- A square has its opposite sides equal.
   f- A rectangle does not have any of its sides equal to another.
   g- A square has all sides equal.
   h- A rectangle has all sides equal.

The objective being tested for this test item is: “3.2.1. Know the relation between the length of a side of a square and a rectangle”

III- A fill-in-the-blanks exercise for grade 3 (Solid):

5- How many faces, vertices and edges do you see?
   - Faces: ..........................
   - Vertices: ..........................
   - Edges: ..........................

The objective being tested for this test item is: “2.1.2. Count the number of faces, vertices, sides of a cube from an image”
IV- A drawing exercise for grade 4 (Transformations):

15- Draw the symmetric of the following figures, with respect to (S), the axis of symmetry.

The objectives being tested for this test item is: “4.1.3/4/5 Construct with the help of the square and ruler, the symmetric of a (Triangle/ Particular quadrilateral/ Simple figure)”

V- A grade 5 exercise (Plane figures):

10- Fill in the table with ticks when applicable.

<table>
<thead>
<tr>
<th>Quadrilateral</th>
<th>Diagonals have equal length</th>
<th>Diagonals are perpendicular</th>
<th>Diagonals bisect each other</th>
<th>Diagonals are axes of symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trapezoid</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parallelogram</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rhombus</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kite</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangle</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The objective being tested for this test item is: “3.3.1. Know the properties of diagonals of special quadrilaterals (Superposition/ Orthogonality/ Same midpoint/ Axes of symmetry)”
VI-A labeling exercise for grade 6 (Plane figures):

26- Label the following:

The objective being tested for this test item is: “3.4.7. Use the vocabulary: principal vertex and base (in an isosceles triangle), hypotenuse and sides of the right angle (in a right triangle)”.

VII- A problem exercise for grade 6 (Transformation):

4- Draw two intersecting lines. (d) and (d'). They intersect at O.
Place A on (d).
Draw (p), the perpendicular bisector of [OA].
(p) cuts (d') at B.
What is the nature of triangle AOB?
What does (p) represent for triangle AOB?

The objective being tested for this test item is: “4.2.4. The isosceles triangle has an axis of symmetry”.
Appendix III

Flaws Found in the Lebanese Geometry Curriculum

This section includes a review of the objectives available in the curriculum, and the difficulties that may arise because of them.

Henceforth, the researcher discusses some of the issues that may be reason for students’ difficulties.

1- Redundant repetition of objectives:

Example:

During grade 2, the following objectives appear:

- Determine the position of a place or a knot on a grid using a code.
- Locate a point on a curve or on a grid, starting with specific conditions.

Then these objectives are skipped during grade 3, and reappear in grade 4.

- Coding the points, the cases, of a grid.
- Situating a point, of a given code, on a grid.

2- Applying concepts that students do not have prior knowledge of:

Examples:

a- In grade 2, students are asked to construct squares from given elements. Although students know how to draw segments of given lengths in order to draw the sides of a square, they have yet to be introduced to right angles, which takes place in the third grade.

b- In grade 6 students are asked to use the uniqueness of parallel lines and/or perpendicular lines to prove that points are collinear. Note that there are no previous objectives that demand students to know what collinear points are.

c- In grade 6, students are asked to draw the three medians of a triangle; however, there are no previous objectives that might have introduced what a median is to the students.

d- In grade 6, students are expected to draw a figure symmetrical to a given figure with respect to a given point. However, students have not encountered previous objectives that initially allow them to recognize
what it means to be symmetric with respect to a point. As such, drawing it becomes quite difficult.

3- Poor translation of objectives making them ambiguous and unclear:

   Example:
   In grade 4, students are asked to build models starting from patterns. However, while reviewing the objectives, pattern sometimes means net of a solid figure. As such, within this objective lies the problem. It is unclear whether students are supposed to construct solid bodies from their nets, or if they are supposed to use solid bodies to continue an already existing pattern. Yet another way to understand this objective would be that students are supposed to use solid bodies to reproduce an already existing pattern.

4- Faulty objectives:

   Example:
   There is an objective stating that a trapezoid has an axis of symmetry. This objective is inaccurate. Regular trapezoids do not have axes of symmetry. It is only the isosceles trapezoid that permits one.
Appendix IV

Major Difficulties Organized in Table Format

In this section, the difficulties are subcategorized under 4 main headings. Since the framework utilized in the tests is based on the three cognitive domains of TIMSS, the difficulties are thus categorized under the three cognitive domains, knowing, applying and reasoning. However, since some difficulties arise due to social, neurological or other factors, there is also a column titled others.

Note that these difficulties are discussed vividly in the literature review and then categorized and summarized in Appendix III.

The table starts by presenting the behaviors related to each of the cognitive domains (rows one to six). The reason why they are stated is because students may have difficulty with these behaviors along with the other difficulties discussed in the literature review.

The following points are the difficulties that students face while solving/ thinking/ using geometry.

Table IV.1

<table>
<thead>
<tr>
<th>Knowing</th>
<th>Applying</th>
<th>Reasoning</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Difficulties in: Recalling</td>
<td>Difficulties in: Selecting</td>
<td>Difficulties in: Analyzing</td>
</tr>
<tr>
<td></td>
<td>- Procedures</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Skills</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Terminology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Difficulties in: Recognizing</td>
<td>Difficulties in: Representing</td>
<td>Difficulties in: Generalizing</td>
</tr>
<tr>
<td>3</td>
<td>Difficulties in: Computing</td>
<td>Difficulties in: Modeling</td>
<td>Difficulties in: Synthesizing/Integrating</td>
</tr>
<tr>
<td>4</td>
<td>Difficulties in: Measuring</td>
<td>Difficulties in: Implementing</td>
<td>Difficulties in: Justifying</td>
</tr>
<tr>
<td>5</td>
<td>Difficulties in: Classifying/Ordering</td>
<td>Difficulties in: Solving Routine Problems</td>
<td>Difficulties in: Solving Non-routine Problems</td>
</tr>
</tbody>
</table>
|   | - Lack of experience/exposure  
|   | - Lack of knowledge  
|   | - Lack of experience/exposure  
|   | - Inability to combine various concepts and skills  
|   | - Lack of knowledge and application skills  
| 6 | Difficulties in retrieving  
| 7 | Difficulties in: Conceptualizing  
|   | Difficulties in: Understanding/thinking/using geometry due to:  
|   | - Apraxia  
|   | - Social/Cultural difference  
|   | - Language  
| 8 | Difficulties in: Using geometric tools to draw  
|   | Difficulties regarding: Visual scanning  
| 9 | Difficulties regarding: Visual Motor Coordination  
| 10 | Difficulties in: Perception  
| 11 | Difficulties with: Spatial ability  
|   | - Knowing left/right  
|   | Difficulties with: Visual Reasoning  
|   | Difficulties with: Spatial ability:  
|   | - Estimating distances  
| 12 | Difficulties in: Executive ability  
| 13 | Difficulties with guessing based on perception  
|   |  

Appendix V

Distribution of Objectives With Respect to Difficulties

In this section, a few key factors have been highlighted. First of all, the main difficulties faced by students of each grade level have been stated. Then under each difficulty are objectives. The objectives are stated only when students appear to face the same difficulty under which the objective is found. In front of each objective are three numbers separated with dots. The first two numbers are the numbers with which the objective is categorized in ECRD (1997) and the third number is the number of the objective in that category (1.x.y. = Location, 2.x.y. = Solid figures, 3.x.y. = Plane figures, 4.x.y. = Transformations). Furthermore, next to each objective is the ratio of students who faced that particular difficulty with that particular objective. However, next to each objective we have one, two or three ratios. The reason for this is that the particular objective may be tested in more than one test. Henceforth, the ratios written in bold are the results obtained from test 1, those underlined are obtained from test 2, and the regular ones are results obtained from the performance and objective based tasks.

Note that some objectives that were tested in test 1 and test 2 have been repeated in the performance and objective based tasks with the aim of following up on the procedure and thinking skills with which the students answered the questions in a more meticulous manner. However, not all objectives that were tested in the performance based tasks were tested during tests 1 and 2, and vice versa.

Lastly, when an objective appears with only a bold ratio, or only an underlined ratio, this means that during the other test, students did not face the same difficulty.

GRADE 1: (7 students)

Difficulties in: Conceptualizing
- 3.1.1. Recognize a straight line 3/7

Difficulties in: Understanding/ thinking/ using geometry due to: Language
- 1.2.1. Moving in space by following given instructions 1/7
- 1.3.3. Describe a position or a displacement by using appropriate vocabulary 6/7
- 3.2.3. Verify the congruence of two figures by tracing or cutting 3/7
Difficulties in: Recalling: Terminology
- 1.1.3. Recognize the interior, the exterior, and the boundary of a simple 3/7
- 1.1.4. Use the terms: interior, exterior, open, closed 7/7
- 1.3.3. Describe a position or a displacement by using appropriate vocabulary 1/7
- 2.1.1. Sorting and classifying solids according to their shapes by using their names 3/7 2/7 7/7
- 3.2.1. Classify geometric figures according to their shapes 1/7 5/7
- 3.2.2. Recognize geometric figures in a given drawing 1/7 1/7
- 4.1.2. Verify if an axis is an axis of symmetry of a figure by tracing, cutting and folding 2/7

Difficulties in: Recalling: Properties
- 2.1.1. Sorting and classifying solids according to their shapes by using their names 2/7 3/7
- 3.1.1. Recognize a straight line 3/7

Difficulties in: Recalling: Procedures
- 3.2.3. Verify the congruence of two figures by tracing or cutting 7/7

Difficulties regarding: Visual scanning
- 2.1.1. Sorting and classifying solids according to their shapes by using their names 7/7
- 3.2.1. Classify geometric figures according to their shapes 1/7
- 3.2.2. Recognize geometric figures in a given drawing 1/7

Difficulties in: Recognizing
- 1.3.2. Recognize a position on a curve or in a plane 2/7 1/7
- 2.1.1. Sorting and classifying solids according to their shapes by using their names 3/7 1/7 7/7
- 3.1.1. Recognize a straight line 3/7
- 3.2.1. Classify geometric figures according to their shapes 1/7
- 3.2.2. Recognize geometric figures in a given drawing 1/7
Difficulties in: Classifying/Ordering
- 2.1.1. Sorting and classifying solids according to their shapes by using their names 3/7 1/7 7/7
- 3.2.1. Classify geometric figures according to their shapes 1/7 4/7

Difficulties in: Using geometric tools to draw
- 3.1.4. Reproducing a simple figure on a grid paper 4/7 2/7

Difficulties regarding: Visual Motor Coordination
- 3.1.4. Reproducing a simple figure on a grid paper 2/7 1/7
- 4.1.2. Verify if an axis is an axis of symmetry of a figure by tracing, cutting and folding 1/7

Difficulties in: Executive ability
- 3.1.4. Reproducing a simple figure on a grid paper 1/7

Difficulties with: Spatial ability: Knowing left/right
- 1.2.1. Moving in space by following given instructions 2/7
- 1.2.2. Moving in plane by following given instructions 1/7
- 1.2.3. Describe a position or a displacement by using the appropriate vocabulary 2/7
- 1.3.3. Describe a position or a displacement by using appropriate vocabulary 4/7

Difficulties in: Implementing
- 1.2.1. Moving in space by following given instructions 6/7
- 1.2.2. Moving in plane by following given instructions 1/7
- 1.2.3. Describe a position or a displacement by using the appropriate vocabulary 1/7
- 1.3.3. Describe a position or a displacement by using appropriate vocabulary 6/7

GRADE 2: (10 students for test 1, 9 students for test 2)

Difficulties in: Conceptualizing
- 2.1.1. Distinguish between vertices, edges and faces of solids. 3/10 7/9
- 2.1.2. Recognize edges of solids from given objects or their images. 3/10
  7/9
- 2.1.3. Recognize the faces of solids from given objects or their images. 3/10
  7/9
- 4.1.1. Find an axis of symmetry of a given figure by folding. 6/9

Difficulties in: Understanding/ thinking/ using geometry due to: Language
- 3.2.1. Know the relation between the length of a side of a square and a rectangle. 0/10 0/9 9/9

Difficulties in: Recalling: Terminology
- 2.1.1. Distinguish between vertices, edges and faces of solids. 3/10 7/9
- 2.1.2. Recognize edges of solids from given objects or their images. 3/10 7/9
- 2.1.3. Recognize the faces of solids from given objects or their images. 3/10 7/9
- 3.1.3. Draw a segment of a given length, knowing one end point. 0/10 0/9 4/9
- 3.2.2. Know the number of sides and vertices of a triangle, square or rectangle. 1/10 1/9
- 4.1.1. Find an axis of symmetry of a given figure by folding 6/9

Difficulties in: Recalling: Properties
- 3.2.1. Know the relation between the length of a side of a square and a rectangle. 6/10 9/9 9/9
- 3.2.2. Know the number of sides and vertices of a triangle, square or rectangle. 1/10 6/9
- 3.2.4. Construct a square from given elements. 4/10 1/9

Difficulties in: Recalling: Procedures
- 1.1.2. Determine the position of a place or a knot on a grid using a code. 5/10 2/9
- 3.2.4. Construct a square from given elements. 7/10 5/9
- 4.1.1. Find an axis of symmetry of a given figure by folding. 6/9
- 4.1.2. Complete, by symmetry on a grid, the drawing of a given figure with an axis of symmetry. 1/9

Difficulties regarding: Visual scanning
- 4.1.2. Complete, by symmetry on a grid, the drawing of a given figure with an axis of symmetry. 1/10 2/9

Difficulties in: Recognizing
- 2.1.1. Distinguish between vertices, edges and faces of solids. 3/10 7/9
- 2.1.2. Recognize edges of solids from given objects or their images. 3/10
  7/9
- 2.1.3. Recognize the faces of solids from given objects or their images. 3/10
  7/9

Difficulties in: Representing
- 1.1.1. Locate a point on a line with respect to other points on the line. 1/10

Difficulties in: Using geometric tools to draw
- 3.1.1. Using a ruler, draw a segment limited by two points. 1/10
- 3.1.2. Draw a segment of a given length on a line, knowing one endpoint. 1/10
- 3.1.3. Draw a segment of a given length, knowing one end point. 1/10 1/9
  1/9
- 3.2.4. Construct a square from given elements. 8/10
- 4.1.1. Find an axis of symmetry of a given figure by folding. 3/10
- 4.1.2. Complete, by symmetry on a grid, the drawing of a given figure with an axis of symmetry. 4/10 1/9

Difficulties in: Measuring
- 3.1.2. Draw a segment of a given length on a line, knowing one endpoint. 3/10
  7/9
- 3.1.3. Draw a segment of a given length, knowing one end point. 1/10 5/9
  6/9
- 3.2.4. Construct a square from given elements. 1/10

Difficulties regarding: Visual Motor Coordination

Plane Figures: Description of plane figures:
- 3.1.2. Draw a segment of a given length on a line, knowing one endpoint. 2/10
   1/9
- 3.1.3. Draw a segment of a given length, knowing one end point. 0/10 0/9
   2/9
- 3.2.4. Construct a square from given elements. 3/10
- 4.1.2. Complete, by symmetry on a grid, the drawing of a given figure with an
  axis of symmetry. 1/9

Difficulties in: Executive ability
- 4.1.2. Complete, by symmetry on a grid, the drawing of a given figure with an
  axis of symmetry. 1/10 1/9

Difficulties with: Spatial ability: Knowing left/right
- 1.1.3. Locate a point on a curve or on a grid, starting with specific conditions.
  1/10 4/9

Difficulties in: Implementing
- 1.1.1. Locate a point on a line with respect to other points on the line. 1/10
- 1.1.2. Determine the position of a place or a knot on a grid using a code. 2/10
  1/9
- 1.1.3. Locate a point on a curve or on a grid, starting with specific
  conditions. 1/10 8/9
- 3.2.4. Construct a square from given elements. 1/10
- 4.1.2. Complete, by symmetry on a grid, the drawing of a given figure with an
  axis of symmetry. 1/10 2/9

Difficulties with guessing based on perception
- 3.2.3. Distinguish a square from a rectangle by coinciding their sides 9/9

GRADE 3: (9 students in all)

Difficulties in: Conceptualizing
- 1.2.1. Verify that two lines are perpendicular. 2/9
- 3.1.1. Recognize a right angle. 1/9
- 3.1.3. Draw a right angle 1/9
- 3.2.3. Construct rectangles and squares. 3/9
- 3.2.4. Complete a square when knowing a side. 3/9
- 4.1.2. Draw, on a grid, a symmetric figure of a given figure with respect to a given axis. 5/9
- 4.1.3. Distinguish between superposable figures by symmetry and figures that are superposable without being symmetrical. 7/9 6/9

Difficulties in: Understanding/ thinking/ using geometry due to: Language
- 1.1.1. Check with a ruler that the given point is the midpoint of a given segment. 1/9
- 1.1.2. Place the midpoint of a given segment. 1/9
- 1.1.3. Use the terms “midpoint” and “half” correctly. 1/9
- 1.2.1. Verify that two lines are perpendicular. 3/9

Difficulties in: Recalling: Terminology
- 1.1.3. Use the terms “midpoint” and “half” correctly. 1/9
- 1.2.1. Verify that two lines are perpendicular. 3/9
- 2.1.2. Count the number of faces, vertices, sides of a cube from an image. 2/9
- 2.1.5. Count the number of faces, vertices, sides of a parallelepiped from an image. 2/9
- 3.1.3. Draw a right angle 1/9
- 3.2.1. Identify a rectangle by its four right angles. 1/9
- 3.2.2. Identify a square by its four right angles and the superposition of its sides. 2/9
- 3.2.3. Construct rectangles and squares 1/9
- 4.1.3. Distinguish between superposable figures by symmetry and figures that are superposable without being symmetrical. 7/9
- 4.1.4. Use terms: symmetry, symmetric figures, superposable 9/9

Difficulties in: Recalling: Properties
- 2.1.2. Count the number of faces, vertices, sides of a cube from an image. 1/9 3/9
- 2.1.3. Recognize the pattern of a cube from patterns given by tracing and cutting. 1/9 1/9
- 2.1.5. Count the number of faces, vertices, sides of a parallelepiped from an image. 2/9 4/9

- 2.1.6. Recognize the pattern of a parallelepiped from patterns given by tracing and cutting. 1/9 4/9

- 2.1.7. Know that the square has 6 superposable square faces. 1/9

- 4.1.3. Distinguish between superposable figures by symmetry and figures that are superposable without being symmetrical. 7/9 6/9

**Difficulties in: Recalling: Procedures**

- 3.2.3. Construct rectangles and squares. 3/9

- 3.2.4. Complete a square when knowing a side. 3/9

- 4.1.2. Draw, on a grid, a symmetric figure of a given figure with respect to a given axis. 2/9 4/9

**Difficulties regarding: Visual scanning**

- 2.1.2. Count the number of faces, vertices, sides of a cube from an image. 1/9 3/9

- 2.1.5. Count the number of faces, vertices, sides of a parallelepiped from an image. 2/9 4/9

- 3.2.6. Reproduce a figure on different grids. 1/9 1/9

- 3.2.7. Complete a figure to make it identical to a given figure. 3/9

- 4.1.2. Draw, on a grid, a symmetric figure of a given figure with respect to a given axis. 3/9

**Difficulties in: Recognizing**

- 2.1.2. Count the number of faces, vertices, sides of a cube from an image. 1/9

- 2.1.3. Recognize the pattern of a cube from patterns given by tracing and cutting. 1/9

- 2.1.5. Count the number of faces, vertices, sides of a parallelepiped from an image. 2/9

- 2.1.6. Recognize the pattern of a parallelepiped from patterns given by tracing and cutting. 4/9

**Difficulties in: Representing**
1.1.2. Place the midpoint of a given segment. 1/9 1/9
1.2.2. Draw at a point on a line a perpendicular to this line. 1/9

Difficulties in: Using geometric tools to draw
- 1.2.2. Draw at a point on a line a perpendicular to this line. 1/9 4/9
- 3.1.2. Verify if an angle is right using a set-square or equivalent. 5/9
- 3.1.3. Draw a right angle 2/9 2/9
- 3.2.3. Construct rectangles and squares. 3/9 4/9
- 3.2.4. Complete a square when knowing a side. 3/9 3/9
- 3.2.5. Complete a rectangle when knowing two consecutive sides. 2/9 2/9

Difficulties in: Measuring
- 3.2.4. Complete a square when knowing a side. 1/9

Difficulties regarding: Visual Motor Coordination
- 1.2.2. Draw at a point on a line a perpendicular to this line. 2/9 2/9
- 2.1.1. Construct a cube from a given pattern. 9/9
- 2.1.4. Construct a parallelepiped from a given pattern. 9/9
- 3.2.3. Construct rectangles and squares. 1/9
- 3.2.6. Reproduce a figure on different grids. 2/9 1/9
- 3.2.7. Complete a figure to make it identical to a given figure. 2/9 3/9
- 4.1.1. Verify that two given figures are symmetrical with respect to a line. 1/9
- 4.1.2. Draw, on a grid, a symmetric figure of a given figure with respect to a given axis. 1/9 3/9

Difficulties in: Executive ability
- 3.2.3. Construct rectangles and squares. 1/9 1/9

Difficulties in: Implementing
- 1.2.2. Draw at a point on a line a perpendicular to this line. 1/9 1/9
- 3.2.4. Complete a square when knowing a side. 1/9
- 3.2.5. Complete a rectangle when knowing two consecutive sides. 1/9

Difficulties with guessing based on perception
- 1.1.1. Check with a ruler that the given point is the midpoint of a given segment. 3/9 1/9
Difficulties in: Computing

- 1.1.2. Place the midpoint of a given segment. 2/9

GRADE 4: (6 students for test 1, 4 students for test 2)

Difficulties in: Conceptualizing

- 1.1.3. Situating a point (or several) at a given distance from a given straight line. 2/4 0/4
- 3.2.1. Classifying quadrilaterals according to the parallelism of sides. 6/6 4/4 4/4
- 3.2.2. Classifying quadrilaterals according to the congruence of sides, their parallelism and their orthogonality. 6/6 4/4 4/4
- 3.2.5. Using the terms: rhombus, parallelogram, and trapezoid. 5/6
- 4.1.3. Constructing with the help of the square and ruler, the symmetric of a triangle. 1/6
- 4.1.4. Constructing with the help of the square and ruler, the symmetric of a particular quadrilateral. 1/6
- 4.1.5. Constructing with the help of the square and ruler, the symmetric of a simple figure. 1/6

Difficulties in: Understanding/ thinking/ using geometry due to: Language

- 4.1.1. Finding the axes of symmetry of the rhombus. 1/6

Difficulties in: Recalling: Terminology

- 3.2.5. Using the terms: rhombus, parallelogram, and trapezoid. 5/6 2/4
- 3.3.6. Using the terms: circle, center and radius. 3/6 3/4

Difficulties in: Recalling: Properties

- 3.2.1. Classifying quadrilaterals according to the parallelism of sides. 6/6 4/4 4/4
- 3.2.2. Classifying quadrilaterals according to the congruence of sides, their parallelism and their orthogonality. 6/6 4/4 4/4
- 3.2.5. Using the terms: rhombus, parallelogram, and trapezoid. 5/6

Difficulties in: Recalling: Procedures

- 1.1.1. Drawing the distance from a given point to a given straight line. 3/6 2/4
- 1.1.2. Finding on a given drawing the distance from a point to a given straight line. 3/6 2/4
- 1.1.3. Situating a point (or several) at a given distance from a given straight line. \(2/6 \ 2/4 \ 0/4\)
- 3.1.3. Drawing a straight line perpendicular to a given straight line and passing by a given point. \(2/4\)
- 3.1.5. Drawing a straight line parallel to a given straight line. \(2/4\)
- 3.2.3. Completing the drawing of a rhombus where we know two consecutive sides. \(3/6 \ 1/4\)
- 3.2.4. Completing a parallelogram where we know two consecutive sides. \(3/6\)
- 3.3.4. Reproducing a given triangle or a given particular quadrilateral by using the ruler, the compass and the square. \(1/4\)
- 4.1.2. Verifying that the corresponding parts of symmetric figures are congruent. \(1/4\)
- 4.1.3. Constructing with the help of the square and ruler, the symmetric of a triangle. \(3/6 \ 3/4\)
- 4.1.4. Constructing with the help of the square and ruler, the symmetric of a particular quadrilateral. \(3/6 \ 3/4\)
- 4.1.5. Constructing with the help of the square and ruler, the symmetric of a simple figure. \(3/6 \ 3/4\)

**Difficulties regarding: Visual scanning**
- 2.1.1. Building models starting from patterns \(3/4\)
- 3.1.1. Distinguish two intersecting straight lines, two parallel straight lines. \(1/6\)
- 3.1.2. Identifying parallel straight lines in a figure. \(1/6\)
- 3.3.5. Using the instruments of geometry to continue a border. \(1/4\)

**Difficulties in: Recognizing**
- 2.1.1. Building models starting from patterns \(3/4\)

**Difficulties in: Selecting**
- 2.1.1. Building models starting from patterns \(3/4\)
- 3.1.1. Distinguish two intersecting straight lines, two parallel straight lines. \(1/6\)
- 3.1.2. Identifying parallel straight lines in a figure. \(1/6\)

**Difficulties in: Using geometric tools to draw**
- 3.2.3. Completing the drawing of a rhombus where we know two consecutive sides. 3/6
- 3.2.4. Completing a parallelogram where we know two consecutive sides. 3/6
- 3.3.4. Reproducing a given triangle or a given particular quadrilateral by using the ruler, the compass and the square. 3/6
- 3.3.5. Using the instruments of geometry to continue a border. 2/6 3/4

**Difficulties regarding: Visual Motor Coordination**
- 3.1.3. Drawing a straight line perpendicular to a given straight line and passing by a given point. 1/4
- 3.2.3. Completing the drawing of a rhombus where we know two consecutive sides. 1/4

**Difficulties in: Executive ability**
- 3.3.4. Reproducing a given triangle or a given particular quadrilateral by using the ruler, the compass and the square. 3/6
- 3.3.5. Using the instruments of geometry to continue a border. 1/4

**Difficulties in: Implementing**
- 3.3.4. Reproducing a given triangle or a given particular quadrilateral by using the ruler, the compass and the square. 3/6
- 3.3.5. Using the instruments of geometry to continue a border. 1/4

**Grade 5: (7 students in all)**

**Difficulties in: Conceptualizing**
- 2.1.3. Recognize the pattern of a cylinder. 3/7
- 3.1.1. Recognize the sides and the vertex of an angle 1/7 1/7
- 3.2.2. Draw the diagonals of a polygon. 1/7
- 3.3.1. Know the properties of diagonals of special quadrilaterals. 7/7 7/7 6/7
- 3.3.2. Recognize a quadrilateral from its diagonals. 1/7 2/7
- 3.4.3. Draw a circle knowing the length of its diameter 1/7

**Difficulties in: Understanding/ thinking/ using geometry due to: Language**
- 3.1.2. Use correctly the notion of an angle 3/7

**Difficulties in: Recalling: Terminology**
- 3.4.3. Draw a circle knowing the length of its diameter 1/7

**Difficulties in: Recalling: Properties**
- 2.1.3. Recognize the pattern of a cylinder. 3/7
- 3.3.1. Know the properties of diagonals of special quadrilaterals. 7/7 7/7 6/7
- 3.3.2. Recognize a quadrilateral from its diagonals. 1/7 2/7

**Difficulties in: Recalling: Procedures**
- 1.1.2. Measure the distance between two parallel lines. 1/7 4/7
- 4.1.1. Transpose the figure of a grid to another grid homothetic to the first 2/7
- 4.1.2. Enlarge a figure in a simple ratio. 2/7
- 4.1.3. Reduce a figure in a simple ratio. 1/7 2/7

**Difficulties regarding: Visual scanning**
- 3.2.2. Draw the diagonals of a polygon. 1/7 2/7
- 4.1.1. Transpose the figure of a grid to another grid homothetic to the first 2/7
- 4.1.2. Enlarge a figure in a simple ratio. 1/7
- 4.1.3. Reduce a figure in a simple ratio. 1/7

**Difficulties in: Recognizing**
- 2.1.1. Recognize different patterns of the same solid and construct them by folding and cutting. 1/7
- 3.1.1. Recognize the sides and the vertex of an angle 1/7

**Difficulties in: Representing**
- 3.1.2. Use correctly the notion of an angle 1/7

**Difficulties in: Using geometric tools to draw**
- 3.4.3. Draw a circle knowing the length of its diameter 1/7
- 3.4.4. Draw a circle knowing its diameter. 2/7 1/7

**Difficulties in: Measuring**
- 1.1.2. Measure the distance between two parallel lines. 1/7
- 1.1.3. Use the invariance of a distance between two parallel lines to draw a parallel line to a given line from a given distance. 1/7 0/7
- 3.4.4. Draw a circle knowing its diameter. 1/7

**Difficulties regarding: Visual Motor Coordination**
- 4.1.2. Enlarge a figure in a simple ratio. 1/7

**Difficulties in: Executive ability**

- 4.1.1. Transpose the figure of a grid to another grid homothetic to the first 2/7

**Difficulties in: Implementing**

- 4.1.1. Transpose the figure of a grid to another grid homothetic to the first 2/7
- 4.1.2. Enlarge a figure in a simple ratio. 3/7
- 4.1.3. Reduce a figure in a simple ratio. 1/7

**Difficulties in: Computing**

- 2.1.2. Recognize the bases of a cylinder and their superposition. 1/7
- 3.4.2. Use the relation Diameter = 2 x Radius. 1/7
- 3.4.4. Draw a circle knowing its diameter. 1/7
- 4.1.2. Enlarge a figure in a simple ratio. 3/7
- 4.1.3. Reduce a figure in a simple ratio. 1/7

**GRADE 6: (5 students in all)**

**Difficulties in: Conceptualizing**

- 1.1.2. Use the uniqueness of the perpendicular taken from a point on a line to verify that three points are collinear. 1/5 2/5
- 1.1.3. Use the uniqueness of the parallel taken from a point on a line to verify that three points are collinear. 1/5 4/5
- 1.1.7. Knowing that two lines in the plane can be concurrent, parallel or coincident. 5/5 4/5 0/0
- 1.2.1. Recognize the circle as being the figure formed by the points situated at an equal distance from a given point called center. 1/5 4/5
- 1.2.2. Distinguish between arc and chord. 5/5 5/5 3/5
- 1.2.3. Determine the position of a point with respect to a circle. 1/5
- 1.2.5. Determine the position of a line with respect to a circle as a function of the distance from the center to this line. 1/5 2/5
- 2.1.1. Construct different patterns of a solid. 2/5
- 3.1.1. Recognize two adjacent angles. 2/5 2/5
- 3.1.2. Define and recognize two vertical angles. 3/5 2/5
- 3.1.3. Construct an angle adjacent to a given angle responding to given conditions. 1/5
- 3.1.4. Construct a vertical angle to a given angle. 1/5 2/5
- 3.1.5. Use the equality of two vertical angles. 1/5 1/5
- 3.3.2. Use the fact that every point on the perpendicular bisector is at an equal distance from its extremities. 1/5
- 3.4.4. Draw the three medians of a triangle and know that they are concurrent. 1/5
- 3.4.6. Identify the right, isosceles and equilateral triangles, by the sides and the angles. 1/5 2/5
- 4.1.2. Recognize the superposition of two symmetric figures with respect to a given point. 3/5
- 4.2.1. The perpendicular bisector is an axis of symmetry. 2/5
- 4.2.2. The bisector is an axis of symmetry of an angle. 1/5
- 4.2.3. The diameter is an axis of symmetry of a circle. 4/5
- 4.2.5. The trapezoid has an axis of symmetry. 4/5
- 4.2.6. The rectangle has the perpendicular bisectors of the sides as axes of symmetry. 4/5
- 4.2.8. The square has four axes of symmetry. 4/5
- 4.2.9. The midpoint of a segment is its center of symmetry. 2/5
- 4.2.10. The center of a circle is its center of symmetry. 5/5
- 4.2.11. The point where the diagonals of a parallelogram meet is its center of symmetry. 3/5
- 4.2.13. Apply, in problems, the fact that two symmetric figures are superposable. 2/5

Difficulties in: Understanding/ thinking/ using geometry due to: Language
- 1.1.8. Recognize three concurrent lines. 2/5
- 1.2.3. Determine the position of a point with respect to a circle. 1/5 2/5
- 3.3.2. Use the fact that every point on the perpendicular bisector is at an equal distance from its extremities. 1/5
- 4.2.9. The midpoint of a segment is its center of symmetry. 1/5

**Difficulties in: Recalling: Terminology**

- 1.1.7. Knowing that two lines in the plane can be concurrent, parallel or coincident. 0/5 0/5 2/5
- 1.2.7. Use the terms: tangent and secant. 5/5 5/5
- 3.1.2. Define and recognize two vertical angles. 3/5
- 3.1.4. Construct a vertical angle to a given angle. 1/5
- 3.4.7. Use the vocabulary: principal vertex and base (in an isosceles triangle), hypotenuse and sides of the right angle (in a right triangle). 2/5 5/5
- 4.2.10. The center of a circle is its center of symmetry. 5/5

**Difficulties in: Recalling: Properties**

- 1.1.2. Use the uniqueness of the perpendicular taken from a point on a line to verify that three points are collinear. 1/5
- 1.1.3. Use the uniqueness of the parallel taken from a point on a line to verify that three points are collinear. 1/5 4/5
- 1.1.4. Apply the fact that two lines that are perpendicular to the same line are parallel. 2/5
- 1.2.1. Recognize the circle as being the figure formed by the points situated at an equal distance from a given point called center. 4/5
- 2.1.1. Construct different patterns of a solid. 2/5
- 3.3.2. Use the fact that every point on the perpendicular bisector is at an equal distance from its extremities. 1/5
- 3.4.2. Draw the three perpendicular bisectors in a triangle and know that they are concurrent. 1/5
- 3.4.6. Identify the right, isosceles and equilateral triangles, by the sides and the angles. 1/5 2/5
- 4.2.4. The isosceles triangle has an axis of symmetry. 5/5 5/5 2/5
- 4.2.14. Characterize an isosceles triangle and an equilateral triangle by the number of their axes of symmetry. 5/5 5/5
- 4.2.1. The perpendicular bisector is an axis of symmetry. 5/5
4.2.5. The trapezoid has an axis of symmetry. 4/5
4.2.6. The rectangle has the perpendicular bisectors of the sides as axes of symmetry. 1/5
4.2.8. The square has four axes of symmetry. 1/5
4.2.9. The midpoint of a segment is its center of symmetry. 2/5
4.2.11. The point where the diagonals of a parallelogram meet is its center of symmetry. 3/5
4.2.13. Apply, in problems, the fact that two symmetric figures are superposable. 2/5

Difficulties in: Recalling: Procedures
1.2.4. Determine the distance from the center of a given circle with respect to a given line. 2/5
1.2.6. Draw the tangent to the circle at one of its points. 2/5
2.1.1. Construct different patterns of a solid. 2/5
3.2.2. Construct the bisector of an angle with a protractor. 2/5
3.4.1. Draw the three angle bisectors in a triangle and know that they are concurrent. 1/5
3.4.2. Draw the three perpendicular bisectors in a triangle and know that they are concurrent. 2/5
3.4.4. Draw the three medians of a triangle and know that they are concurrent. 4/5
3.4.5. Determine the center of a circle passing through three non-collinear points. 5/5 5/5
3.4.9. Bring an angle with the compass. 2/5
4.1.1. Draw a figure symmetrical to a given figure with respect to a given point. 5/5 5/5
4.2.12. Draw the symmetric of a figure with respect to an axis of symmetry or a center of symmetry. 5/5 5/5

Difficulties in: Using geometric tools to draw
3.2.2. Construct the bisector of an angle with a protractor. 2/5
Difficulties in: Measuring

- 1.1.2. Use the uniqueness of the perpendicular taken from a point on a line to determine the distance from this point to the line. 1/5 2/5
- 1.2.4. Determine the distance from the center of a given circle with respect to a given line. 2/5

Difficulties regarding: Visual Motor Coordination

- 1.2.6. Draw the tangent to the circle at one of its points. 2/5
- 2.1.1. Construct different patterns of a solid. 2/5
- 3.1.3. Construct an angle adjacent to a given angle responding to given conditions. 2/5
- 4.2.7. The diagonals of a rhombus are its axes of symmetry. 1/5

Difficulties in: Implementing

- 1.2.6. Draw the tangent to the circle at one of its points. 1/5
- 3.1.3. Construct an angle adjacent to a given angle responding to given conditions. 1/5 1/5
- 3.3.1. Draw the perpendicular bisector with the set-square and the ruler. 1/5
- 3.4.4. Draw the three heights of a triangle and know that they are concurrent. 1/5
- 3.4.9. Bring an angle with the compass. 1/5
- 4.2.4. The isosceles triangle has an axis of symmetry. 0/5 0/5 1/5
- 4.2.6. The rectangle has the perpendicular bisectors of the sides as axes of symmetry. 4/5
- 4.2.7. The diagonals of a rhombus are its axes of symmetry. 1/5
Appendix VI

Difficulties versus Number of Students Who Made Mistakes per Objective

This section shows the distribution of the difficulties for each grade level in the first column. The second column shows the objectives’ codes. Columns T1 to T3 show the number of students who faced the corresponding difficulty based on the objective during the different phases of testing. Columns $SPV_1$ to $SPV_3$ show the number of $T_1$, $T_2$, and $T_3$, based on the Students Percentage Value (SPV) whereby

$$ SPV = \frac{\text{# of students who made mistakes} \times 100}{\text{total number of students in that grade level}} $$

The column $V_1-V_2$ shows how time influences students’ knowledge whereby $V_1-V_2 = SPV_1-SPV_2$.

The column $A(V_1,V_2)$ shows the average of SPV1 and SPV2. $A(V_1,V_2) = \frac{SPV_1+SPV_2}{2}$.

$A(V_1,V_2),CD$ is the average of the combined $A(V_1,V_2)$ based on the content domain in geometry (be it location, solid figures, and so on..). What this means is that if there are 3 objectives based on location, then the sum is obtained for $A(V_1,V_2)$ for these objectives, and then the sum is divided by 3. ($1.x.y. = Location$, $2.x.y. = Solid$ $Figures$, $3.x.y. = Plane$ $Figures$ $and$ $4.x.y. = Transformation$).

Lastly, $AV_3,CD$ is the average of the combined SPV3 based on the content domain in geometry.
### Table VI.1

**Difficulties versus Number of Students Who Made Mistakes per Objective in Grade 1**

<table>
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<tr>
<th>Grade 1 Difficulties</th>
<th>Obj #</th>
<th>T 1</th>
<th>T 2</th>
<th>T 3</th>
<th>SPV1</th>
<th>SPV2</th>
<th>SPV3</th>
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Legend:

Obj #: Objective code based on the numbering in the Lebanese geometry curriculum
T1: Number of students having difficulty in test 1
T2: Number of students having difficulty in test 2
T3: Number of students having difficulty in test 3
SPV1: Student percentage value for T1
SPV2: Student percentage value for T2
SPV3: Student percentage value for T3
V1-V2: SPV1 – SPV2
A(V1,V2): Average of SPV1 and SPV2 = (SPV1 + SPV2) ÷ 2
A(V1,V2),CD: Average of SPV1 and SPV2 based on the cognitive domains of geometry
AV3,CD: Average of SPV3 based on the cognitive domains of geometry
Avg.: Averages
Vis. Scan.: Difficulties with visual scanning
Tools (draw): Difficulties with using geometric tools while drawing
Vis.M.Coor.: Difficulties with visual motor coordination
Executive: Difficulties with the ability to execute or reproduce a figure
Spatial: Difficulty with the spatial ability (distinguishing between left and right)
Implement: Difficulty with implementing
Table VI.2  
*Difficulties versus Number of Students Who Made Mistakes per Objective in Grade 2*

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Legend:

Obj #: Objective code based on the numbering in the Lebanese geometry curriculum
T1: Number of students having difficulty in test 1
T2: Number of students having difficulty in test 2
T3: Number of students having difficulty in test 3
SPV1: Student percentage value for T1
SPV2: Student percentage value for T2
SPV3: Student percentage value for T3
V1-V2: SPV1 – SPV2
A(V1,V2): Average of SPV1 and SPV2 = (SPV1 + SPV2) ÷ 2
A(V1,V2),CD: Average of SPV1 and SPV2 based on the cognitive domains of geometry
AV3,CD: Average of SPV3 based on the cognitive domains of geometry
Avg.: Averages
Represent: Difficulty with representing
Vis. Scan.: Difficulties with visual scanning
Tools (draw): Difficulties with using geometric tools while drawing
Vis.M.Coor.: Difficulties with visual motor coordination
Executive: Difficulties with the ability to execute or reproduce a figure
Spatial: Difficulty with the spatial ability (distinguishing between left and right)
Implement: Difficulty with implementing
Guess (perc): Difficulties with guessing based on perception
Table VI.3

*Difficulties versus Number of Students Who Made Mistakes per Objective in Grade 3*

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| 2.1.4 | 9 | 100.00 | 100.00 |

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| 3.2.6. | 2 | 1 | 22.22 | 16.67 |

| 3.2.7. | 2 | 3 | 22.22 | 33.33 | -11.11 | 27.78 |
|        |   |   | 11.11 | 11.11 | -11.11 | 27.78 |
|        |   |   | 22.22 | 33.33 | -11.11 | 27.78 |
|        |   |   | 11.11 | 11.11 | -11.11 | 16.67 |
| Avg    |   |   | 11.11 | 11.11 | -11.11 | 16.67 |

| 4.1.1. | 1 | 3 | 11.11 | 33.33 | -22.22 | 22.22 |
| 4.1.2. | 1 | 3 | 11.11 | 33.33 | -22.22 | 22.22 |
| Avg    |   |   | 15.56 | 22.22 | 70.37 | -6.67 |

| Executive | 3.2.3. | 1 | 1 | 11.11 | 11.11 | 0.00 | 11.11 |
| Implement. | 3.2.4. | 0 | 1 | 0.00 | 11.11 | -11.11 | 5.56 |
|           | 3.2.5. | 0 | 1 | 0.00 | 11.11 | -11.11 | 5.56 |
|           | Avg    |   | 3.70 | 11.11 | -7.41 | 7.41 |

| 1.1.1. | 3 | 1 | 33.33 | 11.11 | 22.22 | 22.22 |
| 1.1.2. | 0 | 2 | 0.00 | 22.22 | -22.22 | 11.11 |

Legend:
Obj #: Objective code based on the numbering in the Lebanese geometry curriculum
T1: Number of students having difficulty in test 1
T2: Number of students having difficulty in test 2
T3: Number of students having difficulty in test 3
SPV1: Student percentage value for T1
SPV2: Student percentage value for T2
SPV3: Student percentage value for T3
V1-V2: SPV1 – SPV2
A(V1,V2): Average of SPV1 and SPV2 = (SPV1+ SPV2) ÷ 2
A(V1,V2),CD: Average of SPV1 and SPV2 based on the cognitive domains of geometry
AV3,CD: Average of SPV3 based on the cognitive domains of geometry
Avg.: Averages
Represent: Difficulty with representing
Vis. Scan.: Difficulties with visual scanning
Tools (draw): Difficulties with using geometric tools while drawing
Vis.M.Coor.: Difficulties with visual motor coordination
Executive: Difficulties with the ability to execute or reproduce a figure
Spatial: Difficulty with the spatial ability (distinguishing between left and right)
Implement: Difficulty with implementing
Guess (perc): Difficulties with guessing based on perception
Table VI.4

Difficulties versus Number of Students Who Made Mistakes per Objective in Grade 4

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<th>SPV3</th>
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Legend:
Obj #: Objective code based on the numbering in the Lebanese geometry curriculum
T1: Number of students having difficulty in test 1
T2: Number of students having difficulty in test 2
T3: Number of students having difficulty in test 3
SPV1: Student percentage value for T1
SPV2: Student percentage value for T2
SPV3: Student percentage value for T3
V1-V2: SPV1 – SPV2
A(V1,V2): Average of SPV1 and SPV2 = (SPV1 + SPV2) ÷ 2
A(V1,V2),CD: Average of SPV1 and SPV2 based on the cognitive domains of geometry
AV3,CD: Average of SPV3 based on the cognitive domains of geometry
Avg.: Averages
Vis. Scan.: Difficulties with visual scanning
Tools (draw): Difficulties with using geometric tools while drawing
Vis.M.Coor.: Difficulties with visual motor coordination
Executive: Difficulties with the ability to execute or reproduce a figure
Implement: Difficulty with implementing
Table VI.5

Difficulties versus Number of Students Who Made Mistakes per Objective in Grade 5

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AV3,CD: Average of SPV3 based on the cognitive domains of geometry
Avg.: Averages
Represent: Difficulty with representing
Vis. Scan,: Difficulties with visual scanning
Tools (draw): Difficulties with using geometric tools while drawing
Vis.M.Coor.: Difficulties with visual motor coordination
Executive: Difficulties with the ability to execute or reproduce a figure
Implement: Difficulty with implementing


Table VI.6

Difficulties versus Number of Students Who Made Mistakes per Objective in Grade 6

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| 1.2.4. | 0 | 2 | 0.00 | 40.00 | -40.00 | 20.00 | 20.00 |
| Avg | | | | | | | |

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| 2.1.1. | 2 | 0 | 40.00 | 40.00 | 40.00 | 20.00 | 20.00 |
| 3.1.3. | 2 | 0 | 40.00 | 0.00 | 40.00 | 20.00 | 20.00 |
| 4.2.7. | 1 | 0 | 20.00 | 20.00 | 20.00 | 20.00 | 20.00 |
| Avg | | | | | | | |

<p>| Implement. | 1.2.6. | 0 | 1 | 0.00 | 20.00 | -20.00 | 10.00 | 20.00 |
| 3.1.3. | 1 | 1 | 20.00 | 20.00 | 0.00 | 20.00 | 20.00 |
| 3.3.1. | 0 | 1 | 0.00 | 20.00 | -20.00 | 10.00 | 20.00 |</p>
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<th>T2</th>
<th>T3</th>
<th>SPV1</th>
<th>SPV2</th>
<th>SPV3</th>
<th>V1-V2</th>
<th>A(V1,V2),CD</th>
<th>AV3,CD</th>
<th>Avg.</th>
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<td>-10.00</td>
<td>10.00</td>
<td></td>
</tr>
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</table>

Legend:
Obj #: Objective code based on the numbering in the Lebanese geometry curriculum
T1: Number of students having difficulty in test 1
T2: Number of students having difficulty in test 2
T3: Number of students having difficulty in test 3
SPV1: Student percentage value for T1
SPV2: Student percentage value for T2
SPV3: Student percentage value for T3
V1-V2: SPV1 – SPV2
A(V1,V2): Average of SPV1 and SPV2 = (SPV1+ SPV2) ÷ 2
A(V1,V2),CD: Average of SPV1 and SPV2 based on the cognitive domains of geometry
AV3,CD: Average of SPV3 based on the cognitive domains of geometry
Avg.: Averages
Tools (draw): Difficulties with using geometric tools while drawing
Vis.M.Coor.: Difficulties with visual motor coordination
Executive: Difficulties with the ability to execute or reproduce a figure
Spatial: Difficulty with the spatial ability (distinguishing between left and right)
Implement: Difficulty with implementing
Appendix VII

Difference between Tests 1 and 2 in Chart Format for Each Grade Level

Chart VII.1

Change of students’ responses after laps of time for Grade 1

For grade 1, students show high levels of forgetfulness with respect to properties and terminology. But they also show that their original conceptual understanding was at a superficial level, and as such, with time, they were unable to refer to what they had learnt.

On the other hand, students’ visual scanning, recognizing, implementing, classifying and ordering skills have improved along with their visual motor coordination, ability to use geometric tools, and differentiating between their left and right. This improvement may be due to maturation, as well as increased exposure and experience.
The improvement however, in the students’ executive skills is considered minimal.

Chart VII.2

*Change of students’ responses after laps of time for Grade 2*

For grade 2, students show high levels of forgetfulness with respect to properties and terminology. But they also show that their original conceptual understanding was at a superficial level, and as such, with time, they were unable to refer to what they had learnt. Other difficulties students faced were with measuring, recognizing, implementing (paying attention to conditions) and distinguishing between their left and right. This too shows that superficial learning and rote memorization has taken place, as such, students were unable to apply what they have learnt.

On the other hand, students’ ability to recall procedures have improved along with their visual motor coordination and drawing while using geometric tools. This improvement may be due to maturation, as well as increased exposure and experience.
There is minimal change in students’ abilities regarding visual scanning and executing as well as representing.

Chart VII.3

Change of students’ responses after laps of time for Grade 3

For grade 3, students show high levels of forgetfulness with respect to properties. This may be the result of rote memorization. Other difficulties students faced were with their visual scanning, visual motor coordination as well as their ability to use geometric tools to draw. Note that having difficulty in using a tool may also be a reason for visual motor coordination. Also, it is in grade 3 that students are introduced to the set-square, as such the problem may be due to insufficient experience in using the tool itself.

On the other hand, students showed an improvement in their conceptual understanding which may be a result of maturation, which in turn improved their ability to recognize geometric figures in a plane. Furthermore, students’ ability to recall terminology and procedure had improved. This may be due to increased exposure.
There is minimal change in students’ abilities regarding measuring, computing, implementing, executing, and representing. Students also showed a slight improvement with their language skills and were not as likely to rely on their perception while judging the figures on the plane. The later is important to note as it maturation in geometric thought.

Chart VII.4

*Change of students’ responses after laps of time for Grade 4*

For grade 4, students show high levels of forgetfulness with respect to recalling procedures. This may be the result of insufficient experience. Other difficulties students faced were with their visual motor coordination. This may be the result of students’ inability to focus, or faults in their perception. Other difficulties (related to perception) that students of grade 4 faced were in visual scanning, selecting and recognizing.

On the other hand, students showed an improvement in their conceptual understanding, which may be a result of maturation. Students also showed an improvement in their ability to recall properties. This may be due to increased exposure.
Increased exposure may also be the reason for the improvement in students’ ability to draw while using geometric tools.

There is minimal change in students’ abilities to recall terminology, implement and execute. There is also a slight improvement in understanding the language used during testing.

Chart VII.5

*Change of students’ responses after laps of time for Grade 5*

The fifth grade students’ results are extremely worrisome. Students’ ability to recognize has improved, as well as their ability to represent (which is too minimal to consider).

On the other hand, students show high levels of forgetfulness with respect to properties and procedure. They also show that their original conceptual understanding was at a superficial level, and as such, with time, they were unable to refer to what they had learnt. Other difficulties students faced were with measuring, recognizing,
implementing (paying attention to conditions). This too shows that superficial learning and rote memorization has taken place, as such, students were unable to apply what they have learnt. Students also showed difficulties related to perception such as visual scanning and visual motor coordination (also too minimal to consider).

Note that at this grade level there is also considerable difficulties considering computation and understanding the language used in the tests. Although language and computation is not something taught in geometry class, they still pose as difficulties in the learning and using of geometry.

Lastly, along with difficulties regarding representing and visual motor coordination that show minimal change is students’ abilities to recall terminology and reproduce/execute simple drawings. There is also absolutely no change in students’ abilities to draw using the geometric tools.

Chart VII.6

*Change of students’ responses after laps of time for Grade 6*

For grade 6, students show high levels of forgetfulness with respect to properties. They also show that their original conceptual understanding was at a superficial level,
and as such, with time, they were unable to refer to what they had learnt. Students also showed difficulty in implementing and measuring. This is because at the sixth grade students are introduced to a new tool for measuring and students have not had enough experience in using the tool.

On the other hand, students’ ability to recall terminology and procedures improved.

Other minor improvements were recorded in understanding the language used in the test, and students’ ability to draw while using geometric tools.
Appendix VIII

Distribution of the intensity % based on grade level and difficulty

The results presented in the tables below are obtained by converting results of appendix VI based on the Objective Percentage Value, OPV where

\[
OPV = \frac{\text{# of objectives in difficulty} \times 100}{\text{total # of objectives based on the grade level}}
\]

Table VIII.1 shows the average intensities obtained from tests 1 and 2 together based on grade level and difficulty, as well as the performance and objective based tasks.

In table VIII.1, results in columns I(1,2)% are obtained by finding the average intensity % of T1 and T2 whereby the intensity % = SPV% x OPV% ÷ 100, and the average intensity % = (intensity % from T1 + intensity % from T2)/2.

Results in column I3% are the intensity % obtained from T3.

Furthermore, there are numbers that have been underlined in columns I3%. This is because some difficulties per grade level may have been hidden or not tested for during T1 and T2.

Results obtained from table VIII.1 are used in Table VIII.2 to show the change that occurs in the types of difficulties students face when the testing style changes.

By solving for I(1,2-3)% = I(1,2)% - I3%, the researcher extricate the major difficulties that appear due to the change in testing styles when applicable. Thus, I(1,2-3)% may have three types of values.

1- I(1,2-3)% > 0 implies that the intensity of the difficulty diminishes after changing testing style. For instance in grade 1, I(1,2-3)% has gone up 5.00%

2- I(1,2-3)% = 0 implies that the intensity of the difficulty does not change after changing testing style. Such is the case with the visual motor skills in grade 6.

3- I(1,2-3)% < 0 implies that the intensity of the difficulty increases after changing testing style. For instance in grade 2, I(1,2-3)% = -3.41, which means that more difficulty was faced in T3
### Table VIII.1

*Average intensity % Based on Grade Level and Difficulty*

<table>
<thead>
<tr>
<th>Difficulty</th>
<th>G. 1</th>
<th>G. 2</th>
<th>G. 3</th>
<th>G. 4</th>
<th>G. 5</th>
<th>G. 6</th>
<th>G. 1</th>
<th>G. 2</th>
<th>G. 3</th>
<th>G. 4</th>
<th>G. 5</th>
<th>G. 6</th>
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<tbody>
<tr>
<td>I(1,2)%</td>
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<td>14.37</td>
<td>6.48</td>
<td>10.81</td>
<td>9.02</td>
<td>18.59</td>
<td>17.78</td>
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<td>17.29</td>
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**Legend:**
- **I(1,2)%:** Average intensity % obtained from T1 and T2
- **I3%:** Average intensity % obtained from T3
- **G.:** Grade
- **Term:** Difficulty in recalling terminology
- **Vis. Scan.:** Difficulties with visual scanning
G(Perc): Difficulties with guessing based on perception
T (draw): Difficulties with using geometric tools while drawing
Vis.M.Coor.: Difficulties with visual motor coordination
Executive: Difficulties with the ability to execute or reproduce a figure
Spatial: Difficulty with the spatial ability (distinguishing between left and right)
Implement: Difficulty with implementing
Represent: Difficulty with representing

Table VIII.2

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Difference in intensity % of T1 and T2 versus T3 Based on Grade Level and Difficulty
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<td>I(1,2-3)%: Average intensity % obtained from I(1,2)% - I3%</td>
</tr>
<tr>
<td>G.: Grade</td>
</tr>
<tr>
<td>Term: Difficulty in recalling terminology</td>
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<tr>
<td>Vis. Scan.: Difficulties with visual scanning</td>
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<td>G(Perc): Difficulties with guessing based on perception</td>
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<tr>
<td>T (draw): Difficulties with using geometric tools while drawing</td>
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<td>Vis.M.Coor.: Difficulties with visual motor coordination</td>
</tr>
<tr>
<td>Executive: Difficulties with the ability to execute or reproduce a figure</td>
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<tr>
<td>Spatial: Difficulty with the spatial ability (distinguishing between left and right)</td>
</tr>
<tr>
<td>Implement: Difficulty with implementing</td>
</tr>
<tr>
<td>Represent: Difficulty with representing</td>
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</table>

<table>
<thead>
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</tr>
</thead>
<tbody>
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Appendix IX

Tests 1 and 2 Intensities versus Performance and Objective Based Intensities in Charted Format

In this section, the intensities obtained from the performance and objective based tasks are subtracted from the average intensities of results obtained from tests 1 and 2 appendix VIII are converted into chart format based on the grade level. The charts help visualize the major difficulties that came forth from the performance and objective based tasks.

Chart IX.1

First Grade Results for Tests 1 and 2 versus Performance and Objective Based Tasks

![Chart IX.1]

Legend:
Term: Difficulty in recalling terminology
Vis. Scan.: Difficulties with visual scanning
Classify: Difficulties with classifying and recognizing
Implement: Difficulty with implementing
Vis.M.Coor.: Difficulties with visual motor coordination
Spatial: Difficulty with the spatial ability (distinguishing between left and right)

After changing the type of questionnaire, the main difficulties that became noticeable in grade 1 (in order) are: recognizing, recalling terminology, implementing, visual scanning, distinguishing between their left and right, and classifying and ordering.

Note that aside from the fact that the difficulty concerning visual motor coordination shows some improvement, it is almost none existent.
Chart IX.2

*Second Grade Results for Tests 1 and 2 versus Performance and Objective Based Tasks*

After changing the type of questionnaire, the main that became noticeable in grade 2 (in order) are: *recalling properties, recalling procedures, recalling terminology, measuring, visual motor coordination*, and *conceptualizing*.

Note that students’ ability to draw using geometric tools has improved.
After changing the type of questionnaire, the main difficulties that became noticeable in grade 3 (in order) are: visual motor coordination, recalling terminology, using geometric tools to draw, and language.

Note that aside from the fact that the difficulty concerning conceptualizing shows some improvement, it is almost none existent.
After changing the type of questionnaire, the main difficulties that became noticeable in grade 4 (in order) are: conceptualizing and recalling properties. Note that there is also a noticeable improvement in recalling procedures.

After changing the type of questionnaire, the main difficulties that became noticeable in grade 5 (in order) are: conceptualizing and recalling properties.
Note that although there is some difficulty concerning recognizing, but it is too minimal to consider. Similarly, even though there is improvement with measuring, it too is almost none existent.

Chart IX.6

*Sixth Grade Results for Tests 1 and 2 versus Performance and Objective Based Tasks*

![Chart IX.6](chart.png)

Legend:
- Term: Difficulty in recalling terminology
- Implement: Difficulty with implementing
- Vis.M.Coor.: Difficulties with visual motor coordination

After changing the type of questionnaire, the main difficulties that became noticeable in grade 6 (in order) are: conceptualizing, recalling terminology, and implementing. Again, there is noticeable improvement in recalling procedures.

Note that there is some difficulty with recalling properties, however, it is too minimal to consider. On the other hand the intensity % of each, having difficulty regarding using and understanding the academic language, as well visual motor coordination, is nil.
Appendix X
Difficulties per Grade Level in Charted Format

Chart X.1
First Grade’s Major Difficulties in Charted Format

The difficulties are of different intensity % for the first grade. The major difficulties are those difficulties that have intensity % > 4.00. Yet even though six out of thirteen difficulties are greater than 4.00%, they too need to be considered separately as their intensity % may vary drastically. Thus, the major difficulties (in order) are: recalling terminology, recognizing, implementing, visual scanning, distinguishing between left and right, and classifying and ordering.

Note that difficulties concerning conceptualizing and executing are almost none existent.
Chart X.2

Second Grade’s Major Difficulties in Charted Format

The difficulties are of different intensity % for the second grade as well. The major difficulties are those difficulties that have intensity % > 4.00. Yet even though eight out of fifteen difficulties are greater than 4.00%, they too need to be considered separately as their intensity % may vary drastically. Thus, the major difficulties (in order) are: recalling terminology, conceptualizing, recalling properties, recalling procedures, measuring, using geometric tools to draw, recognizing, and visual motor coordination.

Note that difficulties in visual scanning, executing and representing are almost none existent.
In the third grade, there are only four out of fifteen difficulties that are considered major difficulties (having intensity % > 4.00), and they too need to be considered separately as their intensity % may vary drastically. Thus, the major difficulties (in order) are: visual motor coordination, recalling terminology, using geometric tools to draw, and conceptualizing.

On the other hand, difficulties such as guessing based on perception, measuring, executing, implementing, representing and computing are almost none existent.
In the fourth grade, there are only three out of twelve difficulties that are considered major difficulties (having intensity % > 4.00), and they too need to be considered separately as their intensity % may vary drastically. Thus, the major difficulties (in order) are: conceptualizing, recalling properties, and recalling procedures.

On the other hand, the difficulty in using and understanding the academic language is almost none existent.
In the fifth grade, there are only two out of fourteen difficulties that are considered major difficulties (having intensity % > 4.00), and they too need to be considered separately as their intensity % may vary drastically. Thus, the major difficulties (in order) are: conceptualizing and recalling properties.

On the other hand, difficulties such as recalling terminology, visual motor coordination, executing and representing are almost none existent.
In the fifth grade, there are only four out of nine difficulties that are considered major difficulties (having intensity % > 4.00), and they too need to be considered separately as their intensity % may vary drastically. Thus, the major difficulties (in order) are: conceptualizing, recalling properties, recalling procedures, and recalling terminology.

On the other hand, the difficulty in measuring is almost none existent.
Appendix XI  

Comparing the Major Difficulties Based on the Content Domain of Geometry

The tables in appendix XI are based on grouping the difficulties based on the content domain of geometry (Location, Solid figures, Plane figures, Transformations). The way the average intensity % are calculated are by summing up the intensity % for each difficulty, and then dividing it by the number of times the difficulty appeared throughout the various grade levels, as well as the different types of testing.

For instance, in table XI.1, the average intensity % for conceptualizing is: \( \frac{6.48 + 25.25 + 5.98 + 0.00 + 14.73}{5} \).

It is through the average intensity levels that the major difficulties for each content domain can be pinpointed. Any intensity under 4.00% is not considered as a major difficulty. In table XI.1 for instance, out of the fourteen difficulties faced in location, the first cognitive domain of geometry, one can notice that the average intensity % varies from 0.43% with computing to 12.56% with recalling terminology.

Due to the wide variation in the range, one can deduce some difficulties are more prominent than others. Thus the main difficulties (in order) are: recalling terminology, conceptualizing (10.49), recalling procedure (8.43), recalling properties (7.73). These difficulties are then followed by other difficulties that are not as persistent as the previous, but still have considerable average intensity %. Those difficulties (in order) are: using the geometric tools to draw, implementing, recognizing, visual motor coordination, and distinguishing between left and right. Their average intensity % is between 4 and 6%.

Note that difficulties in language, representing, measuring, guessing based on perception, and computing are ignored as their average intensity % is less than 4.00 %
### Table XI.1

**Distribution of Averages Based on Location**

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<tr>
<th>Difficulty</th>
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<th>G. 3</th>
<th>G. 4</th>
<th>G. 5</th>
<th>G. 6</th>
<th>G. 1</th>
<th>G. 2</th>
<th>G. 3</th>
<th>G. 4</th>
<th>G. 5</th>
<th>G. 6</th>
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<td>I(1,2)%</td>
<td>I(1,2)%</td>
<td>I(1,2)%</td>
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Legend:
- G.: Grade
- I(1,2)%: Average intensity % from T1 and T2
- I3%: Intensity % from T3
- Avg. Average
- L: Location
- Term: Difficulty in recalling terminology
- Represent: Difficulty with representing
- T (draw): Difficulties with using geometric tools while drawing
- Vis.M.C.: Difficulties with visual motor coordination
Spatial: Difficulty with the spatial ability (distinguishing between left and right)
Implement: Difficulty with implementing
G (perc): Difficulties with guessing based on perception

Table XI.2

Distribution of Averages Based on Solid Figures

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<th>G. 4</th>
<th>G. 5</th>
<th>G. 6</th>
<th>G. 1</th>
<th>G. 2</th>
<th>G. 3</th>
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<th>G. 5</th>
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</tbody>
</table>

Legend:
G.: Grade
I(1,2)%: Average intensity % from T1 and T2
I3%: Intensity % from T3
Avg. Average
S: Solid figures
Term: Difficulty in recalling terminology
Vis. Scan.: Difficulties with visual scanning
Vis.M.C.: Difficulties with visual motor coordination
In table XI.2, out of the ten difficulties faced in solid figures, the second cognitive domain of geometry, one can notice that the average intensity % varies from 1.88% with computing again to 18.33% with recalling terminology.

Due to the wide variation in the range, one can deduce some difficulties are more prominent than others. Thus the main difficulties (in order) are: recalling terminology (18.33%), visual motor coordination (16.84%), conceptualizing (13.59%), recalling procedure (8.00%), and recognizing (7.92). These difficulties are followed by other difficulties whose average intensity % lie between 4.00% and 7.00% such as: visual scanning, classifying and ordering, recalling properties, and selecting.

Note that the difficulty with computing is ignored as its average intensity % is less than 4.00 %

In table XI.3, out of the seventeen difficulties faced in plane figures, the third cognitive domain of geometry, one can notice that the average intensity % varies from 0.38% with representing to 13.41% with conceptualizing.

Due to the wide variation in the range, one can deduce some difficulties are more prominent than others. Thus the main difficulties (in order) are: conceptualizing (13.41%), recalling properties (10.05%), recalling procedure (9.30%), and recalling terminology (7.29%). These difficulties are followed by other difficulties whose average intensity % lie between 4.00% and 7.00% such as: guessing (based on perception), measuring, and using geometric tools to draw.

Note that difficulties with language, visual scanning, recognizing, classifying and ordering, representing, visual motor coordination, executing, implementing and computing are ignored as their average intensity % is less than 4.00 %
Table XI.3

*Distribution of Averages Based on Plane Figures*

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<th>G. 5</th>
<th>G. 6</th>
<th>G. 1</th>
<th>G. 2</th>
<th>G. 3</th>
<th>G. 4</th>
<th>G. 5</th>
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<th>Avg</th>
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<td>I(1,2)%</td>
<td>I(1,2)%</td>
<td>I(1,2)%</td>
<td>I(1,2)%</td>
<td>I(1,2)%</td>
<td>I(1,2)%</td>
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Legend:
G.: Grade
I(1,2)%: Average intensity % from T1 and T2
I3%: Intensity % from T3
Avg. Average
P: Plane figures
Term: Difficulty in recalling terminology  
Vis. Scan.: Difficulties with visual scanning  
Represent: Difficulty with representing  
T (draw): Difficulties with using geometric tools while drawing  
Vis.M.C.: Difficulties with visual motor coordination  
Executive: Difficulties with the ability to execute or reproduce a figure  
Implement: Difficulty with implementing  
G (perc): Difficulties with guessing based on perception

Table XI.4

Distribution of Averages Based on Transformation

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<th>G. 3 I(1,2)%</th>
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<th>G. 5 I(1,2)%</th>
<th>G. 6 I(1,2)%</th>
<th>G. 1 I3%</th>
<th>G. 2 I3%</th>
<th>G. 3 I3%</th>
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Legend:
G.: Grade
I(1,2)%: Average intensity % from T1 and T2
I3%: Intensity % from T3
In table XI.4, out of the eleven difficulties faced in transformations, the fourth cognitive domain of geometry, one can notice that the average intensity % varies from 0.78% with executing to 21.01% with recalling properties.

Due to the wide variation in the range, one can deduce some difficulties are more prominent than others. Thus the main difficulties (in order) are: recalling properties (21.01%), recalling terminology (20.20%), conceptualizing (16.45%), recalling procedure (11.47%), and using geometric tools to draw (8.11%).

Note that difficulties with language, visual scanning, visual motor coordination, executing, implementing and computing are ignored as their average intensity % is less than 4.00 %.

However, there is one more thing that can be concluded form the tables in appendix XI. At the bottom right hand corner of each table is the total average intensity % for difficulties based on the content domains of geometry.

Students have most difficulty with solid figures (8.99%) directly followed by transformations (8.23%). However, for location (5.11%) and plane figures (4.44%), the total average intensity % for difficulties based on the content domains of geometry slightly decreases. Thus students have most difficulty with solid figures, and the least difficulty with plane figures.