LEBANESE AMERICAN UNIVERSITY

Investigating the Alignment of the Lebanese National Mathematics Tests with the Curriculum Foundations at the Secondary Level

By

DIMA MISBAH ITANI

A thesis

Submitted in partial fulfillment of the requirements

for the degree of Master of Arts in Education

School of Arts and Sciences

May 2022

LAU
الجامعة اللبنانية الأميركية
Lebanese American University

THESIS APPROVAL FORM

Student Name: Dima Itani I.D. #: 200501348

Thesis Title: Investigating the Alignment of the Lebanese National Mathematics Tests with the Curric

Program: Education

Department: Department of Social and Education Sciences

School: School of Arts and Sciences

The undersigned certify that they have examined the final electronic copy of this thesis and approved it in Partial Fulfillment of the requirements for the degree of:

Master of Art (MA) in the major of Education

Thesis Advisor's Name: Iman Osta

Signature: [redacted] Date: 19 / 05 / 2022
Day Month Year

Committee Member's Name: Hagop Yacoubian

Signature: [redacted] Date: 19 / 05 / 2022
Day Month Year

Committee Member's Name: Diane Nauffal

Signature: [redacted] Date: 19 / 05 / 2022
Day Month Year

LAU
الجامعة اللبنانية الأميركية
Lebanese American University

THESIS COPYRIGHT RELEASE FORM

Name: Dima Itani

Signature: [redacted]

Date: May, 2022

الجامعة اللبنانية الأميركية
Lebanese American University

PLAGIARISM POLICY COMPLIANCE STATEMENT

I certify that:

1. I have read and understood LAU's Plagiarism Policy.
2. I understand that failure to comply with this Policy can lead to academic and disciplinary actions against me.
3. This work is substantially my own, and to the extent that any part of this work is not my own I have indicated that by acknowledging its sources.

Name: Dima Itani

Signature: [redacted]

Date: May, 2022

DEDICATION

To my road companions

Mostafa and Hala

ACKNOWLEDGMENTS

This research would not have been possible without the help and assistance of many persons.

Many thanks go to my advisor Dr. Iman Osta for her great patience, continuous support, and guidance. I am also grateful to Dr. Hagop Yacoubian and Dr. Diane Nauffal for being members in my thesis committee and for monitoring my progress. Thanks also go to Dr, Mohamad Nasser for doing the same analysis of the test items and then meeting and discussing with me to agree on the unified analysis used in this study. Mr. Nasser is Lebanese and has a PhD in Group Representation Theory from Beirut Arab University. He is the educational manager of 11 educational institutions and centers in Lebanon and has 10 years of experience in teaching Mathematics at the middle, secondary, and university levels.

Finally, thanks to my family, my work supervisor - Mr. Khaled Al Shafii, and numerous friends who kept on supporting me during this long process.

Investigating the Alignment of the Lebanese National Mathematics Tests with the Curriculum Foundations at the Secondary Level

Dima Itani

ABSTRACT

The aim of the present paper is to investigate the alignment of the Lebanese mathematics national tests with the foundations of the 1997 reformed curriculum, for the "Literature and Humanities" (LH) and "Life Sciences" (LS) tracks of the secondary level. Qualitative and quantitative content analysis techniques were used. Different components of the curriculum foundations were analyzed qualitatively as well as the structure and content of ten model tests issued by MEHE and ECRD as annexes to the curriculum, and 16 national tests for each track. The model and the national tests were quantitatively analyzed using an analysis framework that crossed their respective cognitive domains and content objectives. The cognitive domains are those of the TIMSS international assessments. Correlations were calculated and interpreted, considering the math content domains and the cognitive domains and taking into account the existence of different model tests issued at different time periods, between different sets of the model and national tests for each track, specifically between: 1) all the tests items of each of the national tests and the model tests, 2) the test items of the national tests of the years 2011-2013, 2015-2016, 2017-2018, and 2019, and between each of them and the test items of their corresponding model tests, and 3) the test items of the two sessions (session 1 and session 2) of the national tests. The quantitative analysis showed an overall high correlation between the national tests and the model tests for each track (r=0.97 at each track). However, the qualitative analysis and the results of correlations of the remaining sets showed a notable high correspondence between the model tests issued in the recent years and the national tests previously administered, signifying that a tradition of past tests has developed in the national examination setting and eventually defined the curriculum. Results also revealed a steady structure of the national tests emphasizing the "knowing" and "applying" cognitive domains and overlooking the "reasoning" domain, which reflects weak alignment with the curriculum foundation.

Keywords: Curriculum Alignment, National Assessment, High Stakes Tests, Lebanon, Secondary Education, Mathematics.

Table of Contents

List of Tables

List of Figures

Chapter One

Introduction

1.1. Overview

We are living in an era characterized by continuous changes, developments, and challenges in the social, environmental, health, economic, and technological conditions. As a result, the need of societies and global labor market for highly-skilled workforce is increasing in view of overcoming the challenges, adapting to new changes, accompanying the rapid developments, and staying competent worldwide. Such need for highly-skilled individuals is also increasing as the ability to predict future jobs is becoming harder. Since education aims at developing one's knowledge and skills and focuses on the needs of society and changing world (Sowell, 2005), and since it is necessary to regularly update curricula to make them in consonance with these needs (Halai, 2008; Cachia et al., 2010), considerable efforts have been, and are still being, exerted worldwide for reforming curricula and integrating related goals.

Several meanings and views of the term "curriculum" exist in literature. Kelly (2004, p. 8) defines curriculum as "the totality of the experiences the pupil has as a result of the provision made", a definition that is in line with the actually implemented activities, based on provisions. On the other hand, curriculum is also seen as "a coherent series of aligned and interconnected learning events, which transform the content and structure of a discipline into an ordered series of learning experiences to communicate and define the parameters of learning for the learner" (Moye, 2019, pp. 2-3), a definition that is more in line with the curriculum plans and the intended activities in prospective. A distinction between three actualizations of curriculum was made by Van den Akker (2003) : (a) the intended curriculum: ideal - encompassing the philosophy and spirit of a curriculum - and formal - encompassing the intentions set by the curriculum documentation; (b) the implemented curriculum: perceived – depending on the interpretation of curriculum by teachers and educators - and operational – meaning the curriculum in action as taught and learned; and (c) the attained curriculum: experiential – demonstrated by learners' application of learning in authentic situations - and learned – based on the resulting learning outcomes. Alignment of a curriculum is sometimes defined by the consistency between these forms (Safa, 2013) and understanding the degree to which they work together to support a common goal (Martone & Sireci, 2009). Curriculum alignment is one of the major criteria for evaluating a curriculum (Safa, 2013). Being an important construct, a considerable body of research works have been carried out focusing on curriculum alignment.

Assessment is an important component of a curriculum. Assessment approaches affect school and teaching practices, culture, and learning outcomes (Osta, 2007; Cachia et al., 2010). According to Osta (2007), tests determine, for the educational community, the part of a subject that is valued and should be taught, as well as the way it should be taught. Cachia et al. (2010) describe assessment as being "both an enabler and a barrier for creative learning and innovative teaching" and concord with Osta's idea by stressing on the national assessment's role in guiding and implementing in practice any transformation in the curriculum's learning objectives.

Considering the importance of assessment and curriculum alignment, the present study aims to investigate the alignment of the Lebanese national mathematics tests with the curriculum foundations at the secondary level.

1.2. Context and Background

1.2.1. The Lebanese Educational System and Mathematics Curriculum

In Lebanon, the latest curriculum reform took place in 1997, after the end of the civil war. The Educational Center for Research and Development (ECRD), which is the academic arm of the Lebanese Ministry of Education and Higher Education (MEHE), undertook, starting 1995, developing and applying a reform plan, including curricula for all school disciplines, with new syllabi, textbooks, teacher's guides, evaluation guides and teacher training. The curricula then developed are still in effect till the date of preparing this paper.

In the 1997 reformed curriculum, the educational ladder consists of 2 stages: (1) Basic education which includes the elementary and intermediate levels and (2) Secondary education. The elementary level consists of 2 cycles: cycle 1 (grades 1, 2, and 3) and cycle 2 (grades 4, 5, and 6). The intermediate level consists of cycle 3 which includes grades 7, 8 and 9. By the end of grade 9 or cycle 3, students sit for the Brevet exam, a national official examination in various disciplines. These exams allow them to move to the secondary level in case they succeed. The secondary education consists of grades 10, 11, and 12. Grade 11 has two tracks, Sciences and Humanities, while Grade 12 has four tracks: Life Science (LS), General Sciences (GS), Literature and Humanities (LH) and Sociology-Economy (SE). By the end of grade 12, students sit for the end-of-school national official exams (Baccalaureate) whose result decides whether a student can get the General Secondary Certificate and may move to university education or not.

The Ministry of Education organizes, every year, two sessions of each of the two official exams (Brevet and Baccalaureate), one in June and the other in September. Students who fail the regular June session (Session 1) or would like to improve their results on Session 1 can sit for the exams in September (Session 2).

The 1997 reformed curriculum was implemented gradually across grade levels starting in the academic year 1998-1999 and was completely in effect, at all grade levels, in the academic year 2000-2001. By the end of that academic year, grade-9 and grade-12 students took the first national exams under the reformed curriculum.

According to the Lebanese laws, all public schools should abide by the national curriculum and follow the related books developed by ECRD. On the other hand, private schools have the freedom to adopt other curricula and series of books, provided that they cover the national curriculum's content. Especially in grades 9 and 12, the private schools follow more closely the national curriculum's teaching and assessment approaches, to achieve the best possible results on the national exams. This imposing power of the national exams is gained from the fact that their results also affect the reputation of schools and teachers. The better the students' achievement on the national exams, the better is the reputation of a school and the higher enrollment it can get.

The Mathematics curriculum has undergone, in the 1997 reform, considerable transformations. The general objectives of the math reformed curriculum, which are actually the objectives concerning the mathematical processes, are presented in the main one of the curriculum’s documents (ECRD, 1997a), and target higher-order thinking skills. There are five general objectives of the math curriculum, related to mathematical processes and supposed to set the foundations and spirit of the curriculum: developing mathematical reasoning, solving mathematical problems, establishing connections between mathematics and each of science and real life, communicating mathematically, and valuing mathematics. Teaching strategies and assessment guidelines that emphasize these skills are encouraged in the reformed curriculum, but no sufficient guidance is provided for their practical implementation.

In the same document, one can find the table of numbers of periods allocated for math per week and year for at each grade level. The scope and sequence for each cycle in the same document presents how the topics under each mathematical domain, such as geometry or numbers, are distributed over the three grade levels of each cycle, as well as the number of hours allocated to them in each grade level. The process objectives of the cycle are then listed for each, followed by the content domains’ objectives. The process objectives at all cycles are: mathematical reasoning, problem solving, and communication. The content domains for cycle 1 are: spacial, numerical and measurement. These domains are common to all cycles, but statistics is added to cycles 2 and 3 while statistics-and-probability and calculus are added to the secondary cycle. The secondary level presents separately the specific objectives and scope-and-sequence of each of the four sections: Literature and Humanities, Economics and Sociology, General Sciences, and Life Sciences. Contents of each grade level and their more specific objectives are detailed in a series of three other documents, with explanations about the recommended method(s) of their teaching and guiding comments to teachers (ECRD, 1997b, 1998 & 1999)

1.2.2. Studies on the Alignment of the Lebanese Curriculum

The alignment of the Lebanese Curriculum has been the interest of many researchers. Osta (2007) developed and piloted a framework for analyzing math tests that are not in the style of "objective tests" (multiple choice, matching, True or False, etc.). The analysis of such non-objective style tests, especially in mathematics that is a highly internally connected discipline, requires more complex techniques, which should take into consideration, not only the tested specific objectives in each test item, but also the pre-requisites on which they are built. Osta's study aimed at

> "developing and piloting a methodological framework to investigate the alignment between the Lebanese national (official) exam tests and the mathematics curriculum, at the middle school level, during the transitory period of a major curricular reform" (Osta, 2007, p. 172).

The researcher considered two sets of exam tests: 1) the three math model tests of the reformed curriculum and 2) eleven pre-reform official (national) tests used over six consecutive years. Analysis and comparison of the two sets of tests were made based on their respective cognitive level, and their math content within the curriculum, using the National Assessment of Educational Progress (NAEP) mathematical abilities, which are: Conceptual Understanding, Procedural Knowledge, and Problem Solving. The results showed a stable structure in the format and content of the tests. They also showed that the globality of the test items, over the six years, represent a "mini curriculum" that does not cover all the math content in the original curriculum. That mini curriculum was implicitly established, reinforced and practically adopted by all implicated parties through the years. Osta concluded that the pre-reform assessment culture may still be rooted and thus affecting official exams under the 1997 reformed curriculum. She also stipulated that the lack of alignment between the high-stake official tests and the contents of the curriculum affects the actualization of the curriculum, mainly demonstrated in the implemented curriculum. Teachers in this case perceive that the topics covered in the tests are the only important ones, and thus focus their teaching around them and end up teaching for the tests. As a result, Osta called for an assessment-led new reform or a reconsideration of the testing policies, contents, and formats in the reformed curriculum.

Sleiman (2012) used the framework developed by Osta (2007) to analyze the Lebanese 1997 reformed math curriculum of the “Literature and Humanities” track of secondary education and to study the alignment of the national math tests of this track with the reformed curriculum. Sleiman conducted:

- semi-structured interviews with two members of the national-tests developing committee,
- a content analysis of the following:
    - the general and specific objectives (ECRD, 1997a) of the reformed mathematics curriculum,
    - the details of contents (ECRD, 1999) of the grade 12 math curriculum, LH track,

    - the Evaluation Guide for Mathematics (ECRD, 2000) for the Secondary level consisting of the secondary cycle competencies,
    - a set of model tests illustrating the orientations for the official examination under the reformed curriculum for the grade 12, LH track (ECRD, 2000), and
    - a sample of 20 national math tests for the 2001 to 2010 LH track and consisting of two tests each year, one for session 1 and one for session 2,
- a comparison of the 2001-2005 tests and the 2006-2010 tests to investigate their evolution, and
- a comparison between the 2001-2010 session 1 and the session 2 tests to investigate their consistency and comparability.

For the above analyses, Sleiman classified the test items according to a two-entry matrix, the content domains and the TIMSS's three cognitive domains, "Knowing", "Applying" and "Reasoning". Results showed that the national tests are reasonably aligned, over the years 2001-2010, with the reformed curriculum in terms of the math domains tested, but they considerably lack alignment with the curriculum's general objectives and cycle's specific objectives (Sleiman, 2012). The tests focus most on the "knowing" and "applying" cognitive domains and neglect other more important general objectives: mathematical reasoning, problem solving, communication, and connections. The researcher recommended revising the test banks and the approach to developing math tests for the official exams, taking into consideration the neglected general objectives.

Safa (2013) also adopted Osta's framework (2007) to investigate the alignment between the grade 12 math national tests for the Life Science (LS) track and the Lebanese math curriculum. Safa analyzed the structure, content, and objectives of the curriculum along with four model tests of the reformed curriculum and 12 national math tests administered between 2001 and 2012 (six session-1 and six session-2). Safa conducted, as well, a comparison between the tests of the years 2001-2003 and tests of the years 2010-2012 to investigate their evolution. He also compared the first-session and second-session tests of the years 2001-2012 to investigate their consistency and comparability. The results of Safa's study showed a stable structure of the official tests. They confirmed Osta's view of the fact that they represent a "mini curriculum" by not targeting, over the years, all of the reformed curriculum content. Such a problem causes teachers to "teach to the test", not for real understanding of math or development of mathematical thinking. Moreover, the cognitive domains mostly targeted by the national tests were found to be "Knowing", then "Applying" and then, with a much lower emphasis, "Reasoning". Based on that study, Safa recommends a revision of the national tests in terms of content and design. Such revision has to take into consideration: including different types of questions in an increasing order of difficulty, targeting different levels of cognitive domains, including real life situations and non-routine reasoning questions, and allowing the use of a graphical calculator in solving certain questions.

Shatila (2014) investigated the alignment of the general objectives of the Math reformed curriculum with the math objectives of the intermediate level, the specific objectives of the intermediate grade levels, the national books of the intermediate level grades, and the math national Brevet tests by studying 18 national grade-9 tests under the reformed curriculum. The studied tests are nine first-session tests and nine second-session tests between the years 2001 and 2013. She also compared the national tests of years 2001-2003, 2006-2008, and 2011-2013 to study their evolution and compared as well all the first-session tests and all the second-session tests to investigate their consistency and comparability. The results of the study showed that the objectives of the intermediate-level cycle are aligned with the general objectives of the math curriculum. A lack of alignment was found, however, between the general objectives of the math curriculum and the specific ones, since the latter neglect several of the general objectives. As for the textbooks, they are more reflective of the specific objectives than the general ones. Grade 9 book is the least reflective of the general objectives while grade 8 is the most reflective of those objectives. Moreover, the reasoning and communication skills decrease in these books as the grade level gets higher. In contrast to the textbook, grade 9 national tests showed a slight level of alignment with two of the general objectives, problem solving and reasoning, which increased throughout the years, but they still neglected completely the other two out of the four general objectives. Shatila called for a revision of the curriculum, textbooks, and the national tests to align with the math general objectives, which set, in principle, the foundations of the discipline's curriculum.

1.2.3. Lebanon's Results on TIMSS Assessment for Grades 8 and 12

Countries assess their students' acquisition of knowledge and skills and evaluate their educational programs by participating in international exams and analyzing their students' performance compared to students' performance in other countries. Examples of these international exams are the Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS).

TIMSS exams take place every four years to assess and analyze the participating countries'grade-4 and grade-8 students' achievement in math and science. TIMSS Advanced exams assess students' achievement in mathematics and physics at grade 12. The reports on the results of TIMSS assessments and the research based on them are usually published by the TIMSS & PIRLS (Progress in International Reading Literacy Study) International Study Center (https://timssandpirls.bc.edu/).

Lebanon has participated in grade-8 TIMSS assessments in the years 2003, 2007, 2011, 2015, and 2019. The following data are extracted from the reports (Mullis et al., 2016; Mullis et al., 2020) regarding Lebanon's achievement in math.

Table 1 shows that, in 2003, 45 countries participated, among which eight were Arab countries. In 2007, 48 countries participated, among which twelve were Arab countries. In 2011, 42 countries participated, among which twelve were Arab countries. In 2015 and 2019, 39 countries participated, among which ten were Arab countries.

Table 1: *Participation of Countries in grade 8 TIMSS for the Years 2003, 2007, 2011, 2015 and 2019*

Year	Number of Participating Countries	Number of Arab Countries
2003	45	8
2007	48	12
2011	42	12
2015	39	10
2019	39	10

Table 2: *Lebanon's scores and ranks in grade 8 TIMSS for the Years 2003, 2007, 2011, 2015 and 2019*

Year	Lebanon's Average Score	Lebanon's rank / Number of Countries
2003	433	31 / 45
2007	449	28 / 48
2011	449	25 / 43
2015	442	27 / 39
2019	429	32 / 39

In all its participations, Lebanon's average performance was less than the international average test score that is 500, as shown in Table 2. The score was under average in all participations: 433 in 2003, 449 in 2007 and 2011, 442 in 2015, and 429 in 2019. The score was the highest in 2007 and 2011 and started decreasing to reach the lowest score of 429 in 2019. Lebanon ranked 31st of 45, 28th of 48, 25th of 43, 27th of 39, and 32nd of 39 in the years 2003, 2007, 2011, 2015, and 2019 respectively.

On the cognitive domains, Lebanon's scores were always higher on "Knowing" than on "Applying" and on "Reasoning", the latter being always the lowest score. Such results raise concerns regarding the achievement of the general objectives of the reformed Lebanese curriculum, especially in the recent years. This issue was previously discussed by Shatila (2014) who raised, based on the Brevet results of the years 2003, 2007, and 2011, a significant question: since reasoning is the first general objective of the math curriculum, how could students' scores in international assessments be the least among their scores on cognitive domains (Shatila, 2014)?

Moreover, Lebanon participated in TIMSS Advanced in 2008 and 2015. Only Lebanese students in the GS track of grade 12 usually participate in TIMSS Advanced. The math scores of grade 12 Lebanese students on those tests were better than those of grade-8 TIMSS math scores. According to the data retrieved from the reports of the TIMSS & PIRLS International Study Center, Lebanon ranked third among ten participating countries in 2008 and second among ten participating countries in 2015 with scores of 554 and 533 in the years 2008 and 2015 respectively (Mullis et al., 2009; Mullis at al., 2016).

On the cognitive domains, in both participations, Lebanon's highest scores were on the "Knowing" level, then on "Applying', and least on "Reasoning". The previous results may be a good indicator of Lebanese students' achievement at this grade level. However, these results should be analyzed by considering also the number of math instructional hours that students take over the academic year at this grade level, which, according to the same retrieved data, exceed the average number of instructional hours of other participating countries by 40 to 145 hours (Mullis et al., 2016). A better explanation for this good performance is perhaps what Squires (2012) and Schmidt et al. (2001) highlighted, referring to TIMSS results, maintaining that the more a class spends on a topic, the better achievement they'll have on that topic, especially if the curriculum and assessment are mostly based on drill and practice.

Sleiman (2012) and Safa (2013) studied the math national tests for LH and LS secondary level respectively, under the 1997 reformed curriculum, until the years 2010 and 2012 respectively. Several national examinations were held since then till the present time. This study intends to complete the picture for both tracks and continue the analyses to investigate whether the tests or their alignment with the curriculum's foundations are undergoing any changes over the years.

1.3. Purpose of the Study

This paper aims at studying the degree of alignment of the Lebanese math national tests for the grade-12 secondary LS and LH tracks with the Lebanese mathematics curriculum as reflected in the curriculum document (ECRD, 1997a). It is a continuation of the studies conducted, using the framework developed by Osta (2007), by Sleiman (2012) and Safa (2013) in terms of the alignment of the national examinations with the foundations of the curriculum, mainly the introduction of the curriculum (ECRD, 1997a), presenting the philosophy and general objectives. This study will continue investigating the alignment, starting from the year 2011 till the year 2019. Even though Safa's study included the 2011 and 2012 math tests for LS, this study will consider them as part of its analyzed documents, in order to have a common range of years for the LS and LH test analysis. No official examination occurred at the end of year 2020 because of the COVID19 pandemic.

The study also aims at investigating the evolution of the alignment for each track during the last nine years of implementation, by comparing the tests for the periods 2011-2013 and 2015-2016, 2017-2018, and 2019.

This paper will also investigate differences between sessions 1 and 2 for each track in terms of their alignment with the curriculum foundations, their consistency and comparability.

1.4. Research Questions

The research questions are as follows:

1. What is the extent of alignment of the Lebanese national math tests at the secondary level (for each of LS and LH tracks), over the years 2011-2019, with the national curriculum as reflected in the curriculum document (ECRD, 1997a)?
2. Is there any improvement in the alignment of the national math tests for each track, in the last nine years of implementation, compared to the extent of alignment in the previous years, as reflected in the two previous studies by Sleiman (2012) and Safa (2013)?
3. Are there differences between the tests in sessions 1 and 2, over the last nine years of implementation, for each of the Secondary LS and LH national math tests, in terms of content and cognitive domains addressed?

1.5. Rationale of the Study

The World is rapidly changing on all aspects. The need for highly-skilled individuals with high problem-solving and critical thinking abilities is becoming crucial. COVID 19 pandemic has brought a lot more challenges and changes to the World. Since mathematics is basic in the development of the learner's intellect (Haylock, 2018), good mathematics teaching and learning must be ensured to develop the needed skills and abilities in order to face the arising challenges.

Though apparently acceptable, the results of Lebanon's participations in TIMSS (math) for grade 12 were not satisfactory, if we take into consideration the number of instructional hours compared to other countries, on one hand, and the predominance of low-level knowing scores over the reasoning scores, on the other hand. These results, added to the results of Lebanon's participations in TIMSS (math) for grade 8, raise questions about the quality of the mathematics curriculum and its teaching in Lebanon. These results also raise questions on the curriculum alignment because research showed that curriculum alignment plays an important role in students' achievement. The more the alignment, the better the achievement becomes (Squires, 2012). It is very important to note that the results on TIMSS are given here a great importance since the philosophy of the Lebanese mathematics curriculum, as reflected by its general objectives, is well represented by TIMSS cognitive domains.

Additionally, research shows that high-stake national tests have negative effects on teaching and learning, because they promote teaching to the test. Among the factors that can yield positive effects instead, a close correspondence must exist between the intended curriculum and assessment (Popham, 1987, 2001; Hughes, 1989).

Moreover, the results of Osta's study (2007) reflect that a pre-reform assessment culture has been rooted in the assessment practices of the Lebanese educational community, and thus it is unlikely that exams can change enough to reflect the drastic changes that occurred in the curriculum, especially in its cognitive higher-order thinking foundations as reflected by the introduction and general goals of the curriculum. This also raises a question on the effectiveness of the 1997 curriculum because changes in curriculum and objectives are ineffective if assessment practices remain unchanged (Cachia et al., 2010).

However, a direct change is sometimes difficult to occur especially after 30 years of implementing the old curriculum. Consequently, much insight can be gained from studying the alignment after 10 years of implementation, in the years that followed the two studies by Sleiman (2012) and Safa (2013). Such study can provide insights about the evolution of the alignment of the national math tests with the curriculum foundations. “Alignment should be looked at over time and across instruments” (Webb, 1997, p. 11). The analysis of the results compared to literature will also provide insights on whether the national tests are having positive or negative effects on teaching and learning.

Moreover, the results will provide an idea about the impact of research results in the Lebanese educational community, by checking whether the committee responsible for developing the national exams is aware of, or has taken into consideration, the reviews of the national tests previously made by educators.

1.6. Significance of the Study

The results of the study will show policymakers and curriculum and assessment developers where they stand from the agreed-on goals (objectives), thus encouraging them to reflect on the current educational and assessment reality and to reform the assessment process and tools on the national level. Such study is especially due because an action of revising the curriculum as a whole is expected to occur soon. The results of this study will provide evidence based insights on the possible gaps and problems in the studied area, allowing policymakers and developers to make certain decisions and refinements to ensure a better alignment of the assessment framework with the aims and objectives of the curriculum to be set. This will in turn increase the effectiveness of the changes they are expected to make to the curriculum.

Moreover, the alignment research familiarizes teachers and educators with the concept and importance of alignment between assessment and curriculum foundations. Anderson (2002) gives four reasons why curriculum alignment should concern teachers: 1) what matters is what students actually learn as a result of their schooling experience, 2) when curriculum is properly aligned, teachers can understand the differences in the schooling effects on achievement, 3) when the alignment is poor, the effect of instruction is misjudged and underestimated, and 4) in order to make the educational accountability successful, schools should be held accountable- just as students- by showing that the learning experiences offered to students meet the set standards (Anderson, 2002). In fact, alignment has a great effect on teachers' practices in class. Their awareness of this fact urges them to improve their teaching practices by bringing all the curriculum components to work together to achieve the main goals reflected by the intended curriculum.

This research will contribute to the literature in the fields of education, mathematics education, assessment, curriculum development, etc. It is a longitudinal study that follows upon the curriculum alignment over a number of years of the reform implementation, then includes the other (previous) years by comparing the results to those of previous studies, in order to provide a more complete picture. Previous research works have studied alignment between assessments and curricula over a certain period of time, and not over the whole implementation years. Longitudinal studies are encouraged in alignment research since they demonstrate how alignment changes with time after implementing a reform, especially with the fact that "curriculum and policy are volatile and rarely mobilized as the creator/s intended" (Alfrey et al., 2017).

This study also contributes to advances in education in the conclusions and interpretations it yields since "by continuing to collaborate on alignment studies, educators, researchers, and policymakers contribute to the advances in education" (Case et al., 2004).

It is hoped that the results of this study will inform the efforts that the MEHE and ECRD are launching for reforming the curricula and assessment systems. It is also hoped that it will contribute to the body of research on the issues of relations between assessments and curriculum foundations, mostly between the intended and assessed curricula.

Chapter Two

Literature Review

In this chapter, a review of the literature related to my research is presented. The literature review provides a background for the current study, highlights the existing research, and presents the points of reference when discussing the results of the current study (Merriam, 2009). This chapter is divided into four main sections. The first section describes the main theories that framed Math Education in recent decades. The second section discusses some definitions of curriculum through literature. In the third section, assessment definitions, types and areas of importance, and the washback effects of high-stake tests are discussed. In the fourth section, the curriculum alignment definitions and value, methods, most used models, and the results of studies on curriculum alignment are presented.

2.1. Mathematics Education

2.1.1. Theories Affecting Mathematics Education

The nature of taught mathematics and ways it is learned have developed and changed through years. This change is due to many factors, among which the evolution of societal goals, the fast technological developments, and the theories of psychology and education that have always considerably influenced, and are still influencing, this field. Three main learning theories have largely influenced the field of math education. These theories are: behaviorism, cognitivism, and social constructivism.

Behaviorism, that describes learning as being based on stimulus-response mechanisms, views mathematics education as an accumulation of acquired associations and trained skills (Montilla, 2019; Verschaffel et al., 2015). According to Thorndike (1929), one of the behaviorist pioneers, the connections in these stimulus-response mechanisms are reinforced and strengthened as they are used. The perspective of this theory on mathematics education had and still has its great implications on this field. Its ideas still permeate into many practices. Some of these are the drill and practice exercises, training students on only one way for solving a mathematical problem, and keeping the same type and wording of questions.

Cognitivism, that views learning as pure individual mental process of knowledge and skill acquisition, conceives mathematical learning as changes in universal cognitive schemes and rules that define mathematical knowledge in individual learners. Theories under this perspective also have a great impact on practices in mathematics education. Performing analysis of the concepts and skills needed for certain mathematical tasks, looking at processes rather than learning outcomes, valuing conceptual understanding, paying great attention to the role of prior knowledge, and valuing problem solving besides procedural fluency are among the practices under this view of mathematics education (Verschaffel et al.,2015).

Social Constructivism, which views that development of knowledge is a socio-cognitive process, attributes a great importance to social processes and interactions in learning mathematics. Practices under this view include collaborative activities that include group work for solving a problem, research, projects, and debates. Activities of socio-cultural nature are also among these practices.

Many other theories in mathematics education emerged and continuously emerge on the basis or in connection to the three major theories above, affecting the practices carried out in this field. Despite the difference in views toward mathematics education, the importance of this field and its impact on the individual and social levels has not been doubted and continues to gain more attention as the need for higher-level math and STEM skills continues to gain more momentum and impacts the development of math curricula and teaching / learning materials.

2.1.2. The Importance of Mathematics Education

Mathematics education has its importance not only on the academic and cognitive development of students. The discussion of this importance occupied a great place in literature. As a result, great attention was given to mathematics curricula and math curricular reforms all over the world. The questions of *why we teach math, why we learn math, why is math education important*, and *what are the aims of math education* are all questions that lead to important debates and discussions. According to Francis Su, the former president of the Mathematical Association of America (MAA), the way we answer these "why" questions "strongly influences who we think should do mathematics and how we will teach it" (Larson, 2018).

Ernest (2010) describes three major categories of mathematics as reasons for teaching mathematics:

(1) Necessary mathematics, which is for the benefit of the employment, society and economics. This includes functional numeracy, practical-work related knowledge, and advanced specialist knowledge.

(2) Social and personal mathematics, which is related to personal, social, and cultural relevance. This includes mathematical problem posing and solving, mathematical confidence, and social empowerment through mathematics.

(3) Appreciation of mathematics as an element of culture, history, and society.

Haylock (2018) relates the reasons for teaching mathematics to its importance in many domains: (1) everyday life and society, (2) other subject areas, (3) learner's intellectual development, (4) child's enjoyment of learning, and (5) the body of human knowledge and culture.

The Lebanese national curriculum document (ECRD, 1997a) points out some of the above mentioned domains of importance. Math is a means to explore the world around us in various domains and to prepare capable individuals with decision-making and problem-solving skills necessary for their careers and their future (ECRD, 1997a). Hashmi et al. (2018) contend that math becomes essential, in this advanced era, to cope with the challenging world. Finally, NCTM (2018) asserts that mathematics is increasingly becoming essential to understand the world today and to engage in a democratic society.

Therefore, considering quality and coherence when designing math curricula is essential for achieving such aims at the social and individual levels.

2.2. Curriculum Definitions

Curriculum was defined in many different ways throughout literature. In Latin, "curriculum" refers to "currere", which means a "course or track to be followed" (Van Den Akker, 2004, p.2). Accordingly, Taba defines curriculum as a "plan for learning", while Tyler defines it as "all of the learning of students which is planned and directed by the school to attain its educational goals" (Scott, 2011). Similarly, Kelly (2004, p.8) defines curriculum as "the totality of the experiences the pupil has as a result of the provision made". Moye (2019) synthesized all the implicit concepts underlying the

Latin definition and came up with a new detailed definition for curriculum. Curriculum is "a coherent series of aligned and interconnected learning events, which transform the content and structure of a discipline into an ordered series of learning experiences to communicate and define the parameters of learning for the learner" (Moye, 2019, pp. 2-3).

Anderson (2002) states that curriculum includes aims and objectives, instructional activities, support materials, and assessment. Adirika (2020) adds to these definitions the component of evaluating the effectiveness of a curriculum in achieving its goals. Adirika (2020, p. 324) states that, in developing a curriculum, the following must be considered: "the selection of objectives, content, learning experiences as well as organizing and evaluating these experiences to determine the extent to which they are effective in achieving stated objectives".

In addition to Van Den Akker (2004; 2010) curriculum forms- intended, implemented, and attained, Robitaille et al. (1993), Valverde et al. (2002), and Schmidt et al. (1997) added the potentially implemented curriculum, which they regarded as the link between the intended and the implemented curricula. This form is represented by textbooks and other organized resource material.

On the other hand, Porter (2006) divides the curriculum into four components: the intended- a set of guidelines of what students are expected to know and be able to do, and it is captured in the content standards, enacted- the instruction or what is taught, assessed- student achievement tests, and learned- what is achieved. The definition of the intended curriculum according to Van Den Akker (2004; 2010) is broader than

Porter's definition (2006) since it includes the philosophy behind the curriculum and not only the content standards.

Miller and seller (1985) differentiate between two components of the curriculum: the explicit curriculum and implicit curriculum. They define the curriculum as "an explicitly and implicitly intentional set of interactions designed to facilitate learning and development to impose meaning on experience. The explicit intentions are expressed in the written curricula and in courses of study; the implicit intentions are found in the "hidden curriculum" by which we mean the rules and norms that underlie interactions in the school" (1985, pp. 3-4). Cornbleth (1984) defines the hidden curriculum as the learning that is not publicly stated in the statements of school's philosophy or curriculum documents, such as knowledge, beliefs, and social conduct. This curriculum is shaped by different elements some of which are teachers, society, and awareness. The hidden curriculum can affect students' achievement and beliefs positively or negatively, so understanding it by teachers is crucial (Alsubaie, 2015).

In this study, certain components of the explicitly intended math curriculum will be analyzed to investigate their alignment. These components are the foundations of the curriculum- the intended curriculum- and the national examination- the assessed curriculum.

2.3. Assessment

2.3.1. Definition and Value

Assessment is an important component of curriculum. A considerable body of research exists on assessment types, practices, and standards. According to Torkildsen and Erickson (2016), assessment is a dynamic and collaborative activity that is connected to planning, enacting and evaluating learning activities. On the other hand, Contino (2013) defines assessment as the use of tests and other practices to collect information that enables making inferences about students' learning and achievement of standards. Cachia, Ferrari, and Punie (2009) also point out to the importance of assessment in education, stating that it is an important component in the educational process as it allows the judgment and improvement of the quality of teaching and learning.

From these definitions, assessment can be used to provide useful information at different stages of the educational process and in different areas. Popham (2011) states some of these areas: 1) diagnosing areas of strengths and weaknesses of students, 2) tracking the improvement and progress of students, 3) assigning grades, and 4) evaluating the effectiveness of instruction. Based on recent uses of assessment, Popham adds to these areas three more areas which are: 1) influencing the public perception of the effectiveness of education, 2) evaluating teachers, and 3) making the instructional intentions of teachers clear. Influencing the public perception of the educational effectiveness is one of the results of international examinations. Based on the results of such examinations, countries evaluate their educational curricula. Moreover, as in the case of Lebanon, based on the results of the national examination, teachers and schools are being evaluated.

2.3.2 . Types of Assessment

Assessment practices take different forms based on the aims behind them. Some of these practices are made before or at the beginning of instruction. This is called diagnostic assessment. This type of assessment evaluates students' knowledge or prerequisites needed for introducing the new lesson. Another type of assessment is done during the lesson or course of study. This is formative assessment. This type is intended to enhance and enrich the learning process (Kibble, 2017; Broadbent et al., 2018). These two types provide non-judgmental feedback and aim to inform instruction by detecting students' understandings, misconceptions or gaps, and so they fall under a broader type of assessment called Assessment for Learning.

Another type of practices is applied at the end of a unit or instruction. This is summative assessment and is used to measure the outcome of student learning for grading purposes and to ensure that standards are met (Shute & Kim, 2014). This also falls under Assessment of Learning. Summative assessments "are high stakes for all concerned … in the sense that the data may be used to drive course improvement, to assess teaching effectiveness, and for program-level assessments such as accreditation" (Kibble, 2017, p. 110). The national examination at the Lebanese LS and LH tracks of grade 12 fall under this type of assessment as it aims at measuring if the standards are met and if students are eligible to move on to the university education.

The type and way the assessment is conducted has a great influence on students' learning and academic achievement (Black & William, 1998). Therefore, a great importance is given by curriculum developers, administrators, and teachers to the types of assessment to be adopted, the process of assessment development, and the analysis of the assessment's results.

2.3.3 . High Stakes Tests and Their Washback Effects

A test is considered high stakes if important decisions are to be made based on its results such as students' graduation or promotion to another grade level, teachers' certification, allocation of a certain fund to a school, etc. (Madaus, 1988). "Washback" refers to the effects of tests on teaching and learning. Alderson and Wall (1993, p. 117) define washback as when "teachers and learners do things they would not necessarily otherwise do, because of the test". At the macro level, washback is related to the effects that testing has with the society including policy makers, school administrators, etc. (Zhao et al., 2016), while at the micro level, it refers to the influences that testing has inside the classroom on instruction and learning (Chapman & Snyder, 2000). A great debate takes place in literature on whether high stake tests have positive or negative washback effects.

Bailey (1996) discusses the washback concept and combines the way it works, as described in the literature, into two categories. The first is washback on learners which includes practicing items whose format is similar to the test items, applying test-taking strategies, enrolling in test-preparation courses, etc. (Bailey, 1996). The second is washback to the programme, which includes curriculum developers, teachers, administrators, and so on. This includes the test influence on the what will be taught, the way of teaching, sequence, degree, and depth of teaching, attitudes to the content and method, etc. (Bailey, 1996). These washback effects can be positive or negative.

Madaus (1988) discusses the negative washback effects or consequences of high stakes tests in six principles. These principles are:

1) The power of tests to affect individuals, institutions, or instruction is based on how the importance of these tests is perceived. If individuals believe that high-importance decisions are made based on their results, then the effect of these tests is great on instruction and learning.

2) When a quantitative social indicator is extensively used for social decision-making, it will more likely corrupt the social processes that it is intended to monitor.
3) Teachers teach to the test when important decisions are made based on their results. This is because of the social pressure exerted on them to see if their students' results are satisfactory and because the importance associated to the test dictates that their instructional time focus on test preparation. Students will also be affected, so they adjust themselves to the tests just as their teachers emphasizing materials covered in the test only and ignoring the non-covered topics.
4) A tradition of past tests develops in high-stakes test setting. These past tests eventually define the curriculum.
5) The form of the test questions can narrow instruction and learning to the detriment of other skills. This is because teachers will adjust their instruction based on the format and form of the high-stake test questions.
6) When tests are given a great importance for the future of the students, then society will consider the tests as the major schooling goal.

Noble and Smith (1994) discuss additional negative washback on teachers. High-stake tests cause good teachers to quit teaching while decreasing the skills of those who do not quit and lower their professional self-images (Noble & Smith, 1994). This is because teachers under the mentioned social pressure will be teaching, without using their skills and creativity, the format, content, and strategies of the high-stake tests which might contradict their pedagogical and ethical views.

On the other hand, high-stake tests have positive washback. High-stake tests can improve instruction and enhance students' achievement (Okitowamba et al., 2018; Chapman & Snyder, 2000). High-stake tests can lead to the innovation of new teaching materials and methodologies that influence positively students' learning (Chapman & Snyder, 2000; Zhao, 2016).

However, to attain a positive washback, there should be a close correspondence between the test and the syllabus. Moreover, tests should be properly conceived and implemented (Popham, 1987; 2001). Bailey (1996) states the seven ways the high-stakes should be, as outlined by Hughes (1989), to achieve positive washback:

"1) Test the abilities whose development you want to encourage.

2) Sample widely and unpredictably.

3) Use direct testing.

4) Make testing criterion-referenced.

5) Base achievement tests on objectives.

6) Ensure [that the] test is known and understood by students and teachers.

7) Where necessary provide assistance to teachers." (Hughes, 1989, as cited in Bailey, 1996, p. 2).

2.4. Curriculum Alignment

2.4.1. Curriculum Alignment and its Value

Van Den Akker (2004; 2010) explains what each form of the curriculum forms- intended, implemented, and attained- represents. The intended curriculum represents the philosophy, objectives, and learning outcomes to be achieved. The implemented is what is being taught in classroom and how it is taught. As for the attained curriculum, it refers to what is experiences and learned by the students.

Alignment in general is an agreement of two categories or a match between them (Squires, 2012). According to Martone and Sireci (2009) and Webb (2007), alignment in education is the degree to which different components of the system (content standards, instruction, and testing) work together to support one common goal. In particular, alignment between curriculum foundations and assessment is the degree of agreement or congruence between these two components.

“Webb elaborated stating that the alignment between curriculum and assessment is the degree to which they guide learners to learn what they need to know” (Bhaw & Kriek, 2020, p. 2).

The value and importance of curriculum alignment have been discussed in the literature. According to Martone and Sireci (2009), the study of curriculum alignment:

1) gives students the opportunity to learn and demonstrate what they achieved,
2) helps policymakers and assessment developers, through knowing where they stand relative to agreed-on goals, do refinements in order to allow the curriculum, assessment, and instruction support each other in what is expected of students,
3) allows the public to understand how assessment does or does not support what is supposed to occur in classrooms and what changes needed to be done in components of educational systems.

Cachia et al. (2010) emphasize on the importance of curriculum alignment. To obtain the results sought from any changes performed in the curriculum and its objectives, assessment practices must change to reflect these changes, otherwise the changes will be ineffective. Squires (2012) relates curriculum alignment with students' achievement stating that alignment leads to better achievement.

In fact, the alignment of the curriculum in terms of its foundations and assessment is basic in any curriculum and educational system. Without such alignment, the curriculum objectives might not be achieved.

2.4.2. The Intended-Assessed Curriculum Alignment Methods

Because of the mentioned importance of the alignment between the intended and the assessed curriculum, efforts have been made to study this alignment in different contexts. Three methods or approaches exist for evaluating the alignment between the intended and assessed curriculum: sequential development, expert review, and document analysis (Web, 1997; Case et al., 2004). In studying the alignment of the mentioned curriculum components, more than one method can be used.

In the sequential development method, the standards which constitute the intended curriculum are developed first then used by test developers as a blueprint in terms of structure and content to create the assessment. This method follows a logical process since having developed the standards, they develop the knowledge of the criteria needed for assessment, and this makes it an advantage (Web, 1997; Case et al., 2004).

The expert review method analyzes the alignment between the two components of the curriculum when they are both already developed. It depends on opinions of experts knowledgeable about the intended curriculum and assessment development. In this method, a trained committee of trained specialists typically carries an item-by-item review of an assessment.

In the document analysis method, the intended curriculum and assessment documents are encoded for their content and structure. Then, the alignment between them is quantified and systematically compared. This method is used for complex alignment studies, and it can be used in studying the alignment of other curriculum components (Web, 1997; Case et al., 2004).

For the purpose of enabling more sophisticated alignment analyses, several alignment models have been developed using one or more of the mentioned methods.

2.4.3 . Models of Curriculum Alignment

The No Child Left Behind Act (NCLB) in 2001 mandated, in the USA, accountability assessments that are aligned with the state's content standards and required the states to provide evidence of the alignment from a study done by a third party (Case et al., 2004). The NCLB's requirements made the alignment analysis sophisticated. To enable this sophisticated process, different models have been developed using one or more of the above-mentioned methods. Three models are the most widespread: Webb's Alignment Model, Porter's Model or the Surveys of Enacted Curriculum (SEC) Model, and the Achieve Model.

1. Webb's Alignment Model (Webb, 2007): In this model, Webb's alignment criteria include: content focus- related to the development of student knowledge of a certain subject matter, articulation across grades and ages- related to the way the student's knowledge changes over time, equity and fairness- related to the diversity in students' population, pedagogical

implications- factor that affect students' learning, and system applicability which requires the alignment of standards (representing the intended curriculum) and assessment in realistically and credibly (Case et al., 2004). This model was then pared by Webb (1999) to evaluate the assessment "content focus". This content focus, according to Webb (1999), has four aspects: 1) categorical occurrence which indicates whether all standards are measured in the assessment items, 2) depth-of-knowledge (DOK) consistency which compares the complexity of knowledge required by standards and assessment and which is of four levels: recall, skill, strategic thinking, and extended thinking, 3) range of knowledge correspondence which indicates if both the assessment and standards have corresponding span (breadth) of knowledge, and 4) balance of representations which compares the emphasis given to objectives and topics in assessment and standards. These aspects are what constitute Webb's model criteria. These criteria are rated numerically to allow the objectivity of quantifying and reporting the results. To achieve alignment, an accepted level is necessary on each.

2. Porter's Model or The Survey of the Enacted Curriculum Model (Porter, 2002): In this model, the standards and assessments are categorized according to content and cognitive skills or demand. The cognitive demand is described based on categories specific to each subject area. A content-by-cognitive level matrix is used, and reviewers categorize the required curriculum component onto the matrix. The reviewers map the studied

curriculum components to this common framework and not to each other (Webb, 2007). After categorizing the curriculum components studied, the alignment can be quantified through the use of statistical tables using Porter's alignment index.

3. Achieve Model (Case et al., 2004; Webb, 2007): In this model, a group of experts reaches consensus on the degree of alignment of standards and assessment of a certain state. This model uses five criteria: 1) content centrality which compares the content of the test questions to the standards, 2) performance centrality which studies the correspondence between the cognitive demand of the questions and that of their corresponding standards, 3) challenge which tests whether a set of items reflects the proficiency level needed by the standards, 4) balance, 5) and range- these present evaluating the emphasis of topics in each of the assessment and standards (Case et al., 2004).

Several alignment studies were based on the above mentioned models especially in the states where the NCLB was in act. Other alignment studies were based on other frameworks developed to suit the curricula they are studying.

2.4.4. Studies on Curriculum Alignment

Curriculum alignment's importance captures the attention of researchers, curriculum developers, and policy makers especially when a curriculum reform is about to occur or after a reform is implemented. Several studies aimed at analyzing the alignment of different curriculum components.

Contino (2012) and Bhaw and Kreik (2020) studied the intended-assessed curriculum alignment. Contino (2012) studied the alignment of the New York's Earth Science curricula represented by the New York State Learning Standards for Mathematics, Science, and Technology and the Physical Setting/ Earth Science Common Core and the New York State Physical Setting/ Earth Science Regents Exams. The components were categorized into matrices using performance indicators and cognitive demands according to Bloom's Taxonomy and compared using Porter's Alignment Index. Findings showed that the Core focused on *understanding* and *applying* skills while the tests focused on *applying* followed by *understanding* and *remembering*. Bhaw and Kreik (2020) analyzed the alignment between grade 12 physics examination and the Curriculum and Assessment Policy Statement (CAPS) curriculum in South Africa. The Surveys of Enacted Curriculum method for document analysis and Bloom's taxonomy was used as a classification tool, and Porter's alignment index was used for alignment. Results showed a disjoint alignment represented by an index of 0.76 between the curriculum and exams.

Phaeton and Stears (2017) studied the intended-implemented curriculum alignment for the Zimbabwean A-level Biology curriculum through the lens of interpretation of the curriculum by teachers. Results show a misalignment represented by teachers' misinterpretation of the implemented curriculum. They interpreted it through examinations and didn't engage with the curriculum to understand the objectives of the implemented curriculum (Phaeton & Stears, 2017).

Hashmi et al. (2018) studied the alignment between the intended and potentially implemented curriculum- textbooks- at grade eight level in Punjab using content analysis. Results showed a misalignment between the learning outcomes and the curriculum content.

Yilmaz and Sunkur (2021) and Seitz (2017) studied the alignment among the intended, implemented, and assessed curricula. Yilmaz and Sunkur (2021) studied the alignment of the Life Science curriculum at grade 3 level in Turkey. Twenty-nine objective elements were sampled from the curriculum and 134 instructional activities and 90 assessment questions were analyzed relating to those cognitive elements by two researchers using Revised Bloom's Taxonomy Matrix. Results showed the complete alignment of 9 objectives, partial alignment of 17, and misalignment of 3 objectives. Seitz (2017) studied the alignment of the three curriculum forms in grade 9 mathematics curriculum in Canada according to content and cognitive domains. The researcher used the program of study for the content classification, Delphi method for cognitive classification, and classroom observation for the enacted curriculum identification. Results showed a high alignment between the components at the component level but a low alignment at the cognitive level.

Osta's framework was used, as mentioned before, by Sleiman (2012) and Safa (2013) to study the alignment of the Lebanese math curriculum foundations and the national tests for the LH and LS tracks of grade 12. However, Osta's framework was not only used to study the curriculum alignment. Shehayeb (2017) used Osta's framework in studying the alignment of the Lebanese national exams for the General Science track of grade 12 with the TIMSS Advanced framework in general, as well as with its items derived from the TIMSS Almanac in particular. The purpose of the study was to study the alignment in terms of mathematics content and cognitive domains. Since all exams are similar in structure, Shehayeb used one sample of the national tests in the study. The qualitative analysis of this sample was carried out according to Osta's framework, then Porter's alignment index (2002) was calculated. The results showed that the alignment index between the national test and TIMSS framework content and cognitive domains was greater than its alignment with TIMSS items in particular.

In short, the literature review includes four sections studying theories related to mathematics education, curriculum, and assessment.

The first section studies the theories affecting mathematics education. It describes how different theories, mainly Behaviorism, Cognitivism, and Social Constructivism, still affect practices in mathematics education. It then states the importance of mathematics education on different levels: everyday life, cognitive and intellectual development, society, different subjects, and in facing arising daily challenges.

The second section examines the definitions of the word "curriculum" in literature. It includes the forms and elements of curriculum as categorized by different scholars.

The third section includes theories on assessment. It defines assessment and explains its value. Types of assessment are then discussed. The section ends with high stakes tests and how their washback effects can be positive or negative on teachers, students, curriculum, curriculum developers, and society.

The final section addresses curriculum alignment. It starts with defining this term. Then the value of curriculum alignment is discussed. Some methods of the intended-assessed curriculum are then described followed by the most used models of curriculum alignment. This section ends with presenting several studies done on the alignment of different curriculum components.

This study is concerned with how certain theories still affect practices in mathematics education, the washback effects of high stakes tests in Lebanon, and the alignment between the intended and assessed curricula. The definition of the intended curriculum by Van Den Akker (2004, 2010) and the definition of Porter (2006) of the assessed curriculum component are adopted. Discussion of the results will be based on the previously mentioned ideas and on the importance of mathematics education and curriculum alignment on all levels.

Chapter Three

Research Method

In this chapter, the research method is discussed. The chapter is divided into four main sections: (1) research paradigm, (2) design and procedures, (3) framework for analyzing the tests, and (4), the validity and reliability controls of the study.

3.1. Research Paradigm

This study follows the post-positivist paradigm. Post-positivism balances both the positivist and interpretive approaches (Panhwar, 2017). While the positivist approach assumes that reality is objective and expressed in the observable statistical regularities of facts and behaviors, thus tends to study it using purely quantitative methods, the interpretive approach assumes reality is subjective and socially constructed and to understand it qualitative methods are used (Wildemuth, 1993). While a positivist approach reduces the studied reality and limits it by "controlling variables", post-positivism emphasizes a more complete understanding of the studied situation from multi-dimensional perspective. It promotes the use of both, qualitative and quantitative methods to explore different researchable facts. It values all findings as essential components for knowledge development (Fischer, 1998). In the present study, qualitative and quantitative methods are used to investigate the alignment of the Lebanese math curriculum foundations and assessment. The obtained results, including the quantitative ones, are then interpreted in light of the qualitative analysis, through pattern finding, trends of change, literature and the context being studied, including social and educational factors, in order to have a rather complete understanding of the studied issue.

3.2. Design and Procedures

This research is a longitudinal research. A longitudinal research is “a research emphasizing the study of change and containing at minimum three repeated observations (although more than three is better) on at least one of the substantive constructs of interest” (Ployhart & Vandenberg, 2010, p. 97). The change studied is the evolution of the alignment of the grade 12 national math tests with the Lebanese curriculum foundations over a period of nine years, as well as in comparison to the results of the two previous studies by Sleiman (2012) and Safa (2013), making up a total period of 19 years.

The method used is document content analysis. The Lebanese national math tests at each of the LS and LH tracks are analyzed, as done by Sleiman (2012) and Safa (2013), in terms of content and cognitive domains, compared to the Lebanese mathematics curriculum foundations represented by its objectives, contents, and model tests. These documents are analyzed qualitatively and quantitatively, following Osta’s framework (2007).

The documents analyzed are as follows:

1) The text of the mathematics official curriculum for the secondary school level as issued in 1997 by the MEHE and ECRD – Decree No: 10227 – (referenced as the Main Document in Appendix A), which includes the

curriculum's general and specific objectives, and the scope and sequence delineating the distribution of contents over the years of the secondary level.

2) Curriculum of mathematics – Decree No: 10227 – details of contents of the third year of each cycle, a document issued in May 1999 by MEHE and ECRD (referenced as ECRD (1999) under Three Details-of-Content Documents in Appendix A). It includes the detailed contents along with the corresponding objectives and comments (guidelines for teaching) for the third year of each cycle. The detailed contents of the LS and LH tracks are considered (Appendices B and C respectively).
3) Evaluation Guide for Mathematics for the Secondary Cycle, a document issued in October 2000 by the MEHE and ECRD (referenced as ECRD (2000) in Appendix A). It consists of two units. The first includes the competencies for each year of the secondary cycle. The second includes a set of criteria for the content and format of the official tests (see I in Appendix D), in addition to model tests for each of the four tracks in grade 12 and their corresponding "elements of solution and marking scheme". Another set of guidelines for each track was issued in 2017 (see II in Appendix D). These sets and the model tests for the LH and LS tracks (see samples in Appendices E and F) are considered in this study.
4) The document titled "Details and Results of the Workshops Carried out by ECRD as Part of the Curriculum Evaluation and Development Plan" (referenced in Appendix G).

5) Five sample tests for the LH and six samples for the LS track. Three samples for the LH track and four samples for the LS track were put to public in 2017 and two at each track were put to public in 2019. The model tests and the new sample tests are used as a reference that represents the philosophy of assessment in the reformed curriculum, while the official tests represent the practical implementation, through assessment, of that philosophy.

6) The national tests at grade12 LH track. Sixteen tests administered between 2011 and 2019 are considered (not 18, for the same reason as above). These tests include eight regular sessions (session 1) administered in June and eight second-session tests administered in September (see sample in Appendix H). Note that tests of the year 2014 were not put to public since during that year, around the period of the official exams, Lebanese teachers were on strike and the committee responsible for correcting and grading the national tests abstained. The national examinations were then aborted and students were given certificates of completion grade 12 with no grades (under the decision number 781/m/2014) This explains the fact that, over nine years, there were only 16 tests, for the two sessions, instead of 18.

7) The national tests at grade12 LS track. Sixteen tests administered between 2011 and 2019 are considered. These tests include eight regular sessions (session 1) administered in June and eight second-session tests administered in September. (see sample in Appendix I).

8) The documents containing the details of contents of the first and second year of each cycle (referenced as ECRD (1998) and ECRD (1999) under Three Details-of-Content Documents in Appendix A). These documents are used, when needed, as additional documents since the objectives of previous grade level(s) are implicitly assessed or included in certain questions in the national tests.

When analyzed quantitatively, the national tests are compared to the model tests using the model tests' analysis results of Sleiman (2012) and Safa (2013) for the model tests issued in the year 2000 and using descriptive correlational statistics- Pearson Product-Moment coefficient.

In the quantitative analysis, data are analyzed as follows:

1. The official national math tests for each track are analyzed and compared to the model tests quantitatively, in terms of their test items' percentages under the content and cognitive domains and using correlational statistics of these percentages.
2. The results of analysis of the national math tests for each track over the years 2011 to 2013, 2015 to 2016, 2017 to 2018, and 2019 are compiled and compared to the model tests and to each other, in order to check the evolution of the alignment of the tests under the reformed curriculum.
3. The session-1 official exams of the years 2011-2019 are analyzed and compared to those of session-2 in order to check their compatibility.

3.3. Framework for Analyzing Math Tests

The methodological framework used by Sleiman (2012) and Safa (2013), based on the framework developed by Osta (2007), is used in this study to investigate the alignment at grade 12 LS and LH tracks. Osta (2007) developed a mixed-method framework with both, qualitative and quantitative analyses of the tests. Analysis techniques are crafted to address the complexity of math tests that are not multiple-choice or True-False questions. Two categories of tests are analyzed: 1) the model tests provided as part of the curriculum documentation, and 2) the national math tests.

The quantitative analysis used two-entry analysis tables for analyzing the tests. The two entries are: content domains and cognitive domains of the test items. To each test corresponds a table where the test items are mapped. Each cell in a table includes the number of test items addressing the content domain and the cognitive domain whose intersection is that cell. The mapping techniques within this framework are explained in more details in section 3.3.3.2.

Osta (2007) mapped the test items of each test under these two categories according to their respective math content within the curriculum, and their cognitive level, using the National Assessment of Educational Progress (NAEP) mathematical abilities which are: Procedural Knowledge, Conceptual Understanding, and Problem Solving. The statistical tables were then used to find the Pearson correlation between the items of the two categories of tests.

Both Sleiman (2012) and Safa (2013) adopted this framework using the same technique of classifying the questions of each test (model test and national tests) within a two-entry statistical table (one entry being the math content strands and the other being the cognitive domain). However, both considered the cognitive domains:

knowing, applying, and reasoning of TIMSS Advanced 2008 instead of relating to the NAEP mathematical abilities. The reasons behind using TIMSS cognitive domains according to Safa (2013, p. 25) are two:

> "The first reason is that the TIMSS cognitive domains represent well the philosophy of the Lebanese reformed math curriculum delineated in the Introduction and general objectives and based on: critical thinking, use of math in everyday life, long life learning, and students constructing their own knowledge. The second reason is that Lebanon is one of the countries participating in TIMSS assessment, and adopting the TIMSS cognitive domains would shed light on the extent to which the national exams take into consideration the preparation of Lebanese students for TIMSS."

Osta's framework detailed in this section is adopted in analyzing the grade-12 tests and their alignment with the curriculum foundations in the present study. However, TIMSS Advanced 2015 Framework (Appendix J) will be used for mapping the test items to the cognitive domains.

3.3.1. Definition of Test Item

Sleiman (2012) and Safa (2013) adopted the definition of the "test item" in Osta's (2007) methodological framework. This definition is also adopted in the present study. It states:

> We define a "test item" as being any part of the test that requires a response from the student which entitles him/her to a part of the grade. A test item may take one of the two following forms:
> - A question that requires an answer. For example, "What is the nature of triangle ABC?"
> - An imperative sentence, such as "Calculate the coordinates of point I."
>
> In the case of many components required in one sentence, it is considered to stand for more than one test item. For example, "Plot the points A, B, C, and the straight line (D)" is counted for four items, because it stands for "Plot point A, plot point B, plot point C, and plot straight line (D)." (Osta, 2007, pp. 185-186)

This definition sets the basis of the simple statistics carried out and enables a clearer and more reliable classification of the test items.

3.3.2. Qualitative Analysis

The content of the grade-12 national math tests at each track are qualitatively analyzed based on Osta's framework followed. They are described in terms of their structure and content, through detecting patterns, similarities, differences and content covered.

The test items in the national tests of the LS track are classified according to the organizing content domains and topics under each domain in the curriculum documents. Similarly, the test items in the national tests of the LH track are classified according to the content domains and topics under each domain.

Then, description of the test items occurrence in the said topics under each track follows.

3.3.3. Quantitative Analysis

Statistical tables are used to analyze quantitatively the test items of the model and national tests. Test items are classified in these tables according to their corresponding curriculum contents and the cognitive domains, as classified by TIMSS, that they aim to assess. In order to systematically process the considerable amount of data, a coding system is used - the same coding system adopted by Sleiman (2012) and Safa (2013) following the coding criteria in the methodological framework by Osta (2007).

3.3.3.1. Coding

The official tests for the LS track are coded as LS131, LS132, LS151, LS152, and so on. The letters specify the track of these test. Then the year of administering the test and the number of session are represented by the first two numbers and last number respectively. For example, LS131 means the national test administered for the LS track in the year 2013 being the first session. The model tests are coded as LSM1, LSM2, and so on. The letters LS represent the track, the letter M refers to model, and the number represents the number of the model test. For the LS track 10 model tests exist. The model tests LSM1, LSM2, LSM3, and LSM4 were issued with the curriculum reform in the year 2000, the model tests LSM5, LSM6, LSM7 and LSM8 were issued in the academic year 2016-2017, and the model tests LSM9 and LSM10 were issued in the academic year 2018-2019.

Similar coding will be used for the national tests of the LH track. The coding will be LH111, LH112, LH121, LH122, and so on The letters specify the track of these test. Then the year of administering the test and the number of session are represented by the first two numbers and last number respectively. For example, LH152 means the national test administered for the LH track in the year 2015 being the second session. The model tests are coded as LHM1, LHM2, and so on. The letters LH represent the track, the letter M refers to model, and the number represent the number of the model test. For the LS track 10 model tests exist. The model tests LHM1, LHM2, LHM3, and LHM4 were issued with the curriculum reform in the year 2000, the model tests LHM5, LHM6, and LHM7 were issued in the academic year 2016-2017, and the model tests LHM8 and LHM9 were issued in the academic year 2018-2019.

The analysis tables of the model tests constructed by Sleiman (2012) for the LH track and Safa (2013) for the LS track are taken as they are for later comparison and correlation with the new tests over the years considered in this study.

This study adopts the same coding system of the details of contents of the national curricula of the LS and LH tracks as done by Safa (2013) and Sleiman (2012) respectively. In developing the coding systems of their studies, Safa and Sleiman adopted the coding system of the tracks LS and LH respectively, provided in the "Details of contents" curriculum document of the third year of every cycle (ECRD, 1999). The sub-objectives are coded using the Roman numbering i, ii, iii, etc. Appendices B and C represent the coding of the curriculum details of contents of the LS and LH grade-12 tracks respectively.

Since the math contents of the three secondary years are included to be assessed in the national exams at the end of the Grade 12 LS track, the items that are addressed in the national tests, associated with Grades 10 or 11 curriculum contents, are also coded (A, B… TT). In addition, they are classified per topics. Similar coding is made for the tests of the LH track. Appendix K represents the grade-10 and 11 objectives from each track.

3.3.3.2. Mapping of Test Items

Osta's (2007) technique in mapping the test items is adopted in this study.

Table 3 shows how the quantitative analysis will be carried out in terms of the corresponding curriculum content domains, as well as cognitive domains that they measure. Each test item will have a code under the "curriculum objectives" column and will be classified according to the cognitive domain(s) it measures. Each test item is assigned one tally (the number +1) in the relevant cell. If a test item addresses more than one content domain and/or more than cognitive domain, thus should be mapped in more than one cell, a fraction is assigned to each relevant cell, so that the fractions in those cells add up to 1. Therefore, the sum of the results of the cognitive domains must be equal to the total number of the test items. "The numerical point for each test item is split over the objectives and cognitive domains. That is,

1. If a test item covers two objectives x and y such that x corresponds to knowing and y corresponds to applying, then ½ is assigned to the cell representing x-knowing and another ½ is assigned to the cell representing y-applying.

2. If a test item can be solved in two methods, then half a point is assigned to each method. For example, if the first method covers objective x that corresponds to knowing and applying, and the second method requires objective y that corresponds to knowing, then ¼ for x-knowing, ¼ for x-applying, and ½ for y-knowing" (Sleiman, 2012).

Table 3: *Sample Quantitative Analysis for the Official national Exam*

Curriculum of Mathematics - Decree No 10227 - Date: 08 May 1997 Details of Contents / Objectives of Grade 12 - LS section	Mathematics Framework - TIMSS Advanced 2015 - Cognitive Domains			Math Official Exam - Grade 12 - LS Section - Year - Session
	Knowing	Applying	Reasoning	Test Items

1.2.1.1 1.2.1.1.i 1.2.1.2				
Total				

The resultant tables are 16 for each track in addition to the model tests' tables. The tables are summed in tables as follows:

1. The sum of model tests in each track used by Sleiman (2012) and Safa (2013) named as Mod in each track is used in the analysis. The sum of model tests under the LS track will be named ModLS and the sum of model tests under the LH track will be named Mod LH.
2. The tables of the model tests issued in the years 2017 and 2019 in the LH track are added in tables named ModLH5-7 (of the year 2017) and ModLH8-9 (of the year 2019). Similarly, the tables of the model tests issued in the years 2017 and 2019 in the LS track are added in tables named ModLS5-8 (of the year 2017) and ModLS9-10 (of the year 2019).
3. The tables of all the model tests under each track are added in tables named AllModLH and AllModLS.
4. The tables for the official exams of LH track are added in one table to be named OffExLH.

5. The tables for the official exams of LS track are added in one table to be named OffExLS.
6. The tables for the official LH exams of the years 2011-2013 are added in one table OffExLH11-13, of the years 2015-2016 are added in one table OffExLH15-16, of the years 2017-2018 are added in one table OffExLH17-18, and of the year 2019 are added in one table OffExLH19.
7. The tables for the official LS exams of the years 2011-2013 are added in one table to be named OffExLS11-13, of the years 2015-2016 are added in one table to be OffExLS15-16, of the years 2017-2018 are added in one table OffExLS17-18, and of the year 2019 are added in one table OffExLS19..
8. The tables for the official LH exams of the session-1 are added in two tables named OffExLH11 (for tests of the years 2011-2016), OffExLH12 (for tests of years 2017 to 2018), and table of LH191 is kept separate. The tables for the official exams of the session-2 are added similarly in tables named OffExLH21, OffExLH22, and LH192. The reason is that the comparison of each chuck with its corresponding model test is more valid and reasonable.
9. The tables for the official LS exams of the sessions 1 and 2 are added similarly under tables OffExLS11, OffExLS12, LS191, OffExLS21, OffExLS22, and LS192.

3.3.3.2. Correlations

Using Pearson Correlation, the obtained tables are compared as follows:

1. Tables AllMod and OffEx under each track are compared in order to analyze quantitatively the alignment between the official exams and the model tests.
2. Each of the tables OffExLH11-13 and OffExLH15-16 is compared to table Mod of LH track, while the table OffExLH17-18 is compared to the table ModLH5-7 and the table OffExLH19. Moreover, the tables OffExLH11-13, OffExLH15-16, OffExLH17-18, and OffExLH19 are compared to each other using correlation in order to quantitatively determine the compatibility between the official exams of the years 2011-2013, 2015-2016, 2017-2018, and 2019.
3. Each of the tables OffExLH11 and OffExLH21 is compared to table Mod using correlation. Each of OffExLH12 and OffExLH22 is compared to table ModLH5-7 using correlation, and Each of LH191 and LH192 is compared to ModLH8-9. The obtained correlations are compared in order to determine quantitatively if there are differences among the official exams in the first and second sessions. Moreover, each of the tables OffExLH11 and OffExLH21, the tables OffExLH12 and OffExLH22, and the tables LH191 and LH192 are compared to each other using correlation in order to quantitatively determine the compatibility between the official exams in the first and second sessions.
4. Similar comparisons in 2 and 3 will be carried out on the LS track exams.

3.3.4. Validity and Reliability of the Analysis

The validation of the analysis of the LS and LH grade-12 tests in mapping the test items in the most objective way possible is insured by having the same items mapped by another researcher/rater. A specialist in mathematics, especially in grade 12 level, was asked to map the test items classified in the tables above, regarding the TIMSS Advanced 2015 cognitive domains that they assess. Porter et al. (2008) emphasize the importance of the rater's background when establishing inter-rater reliability.

The specialist was asked to perform the same coding of a sample of the national tests after discussing the mapping techniques and being training on them for some questions. The inter-rater reliability using Cohen's Kappa Reliability Index (1960) is then calculated for a sample of 2 tests. The agreement was k=0.77. The results and difference in mapping was then resolved. For the differences that appeared in the classification of test items, the researcher and the math specialist discussed the classification and agreed on a unified way of analysis and unified criteria for classification of the test items. When a unified analysis could not be reached, they considered the average of the results reached for the corresponding test item. After discussion, the researchers continued mapping the remaining test items, and the inter-rater reliability of another 2 samples was then calculated, and a stronger agreement was obtained k=0.88.

Chapter Four

Findings

This chapter includes four parts: 1) the qualitative analysis of the Lebanese mathematics curriculum foundations - through analyzing the introduction and general objectives of the main curriculum document (ECRD, 1997a) and the Evaluation Guide (ECRD, 2000), 2) the analysis of the document “Details and Results of the Workshops Carried out by ECRD as Part of the Curriculum Evaluation and Development Plan” (referenced in Appendix G), 3) the analysis of the LH track model and national tests (sample model and national tests quantitative analyses are referenced in appendices L and M respectively) , and 4) the analysis of the LS track model and national tests (sample model and national tests quantitative analyses are referenced in appendices N and O respectively).

4.1. Analysis of the Math Curriculum’s Introduction, General Objectives, and The Evaluation Guide

The introduction and general objectives of the Lebanese mathematics curriculum is issued under the Curriculum Document (ECRD, 1997a), referenced as “Main Document” in Appendix A. The Evaluation Guide (ECRD, 2000), referenced in Appendix D) includes a section in Arabic language titled: General principles about the guidelines and the way of developing the official exam questions in mathematics for the general secondary school certificate (see Appendix D). This section is analyzed in the study.

4.1.1. Introduction

The introduction of the 1997 reformed math curriculum highlights the importance of mathematics on all levels (the level of the individual, society, and the World). It states that mathematics plays an important role in changing and developing societies, understanding the whole world, and developing logical, critical, and creative thinking skills. Mathematics helps in modeling precisely and quantitatively the description of reality. The introduction describes how the spirit and teaching of mathematics are reformed through three axes: (a*) Formulation of objectives,* which stresses the individual construction of knowledge. Students construct their knowledge through mental activities by having the opportunity to be immersed in real-life situations where inquiry is the starting point. Developing communication through mathematics as reading and interpreting mathematical texts, writing proofs, and explaining situations, graphs and tables are essential objectives in the reformed curriculum; (b) *Remodeling Contents*, which stresses the importance of eliminating the theoretical overuse and emphasizing the practicality of the topics given and introducing the use of calculator and computer technologies to raise generations capable of facing socio-economic challenges; and (c) *Method of Teaching,* which recommends starting from real life-situations, showing that math is not separated from everyday life and ensuring the accessibility of mathematics learning by all. These changes constitute an important shift in the mathematics curriculum on all levels.

4.1.2. General Objectives

The general objectives of the math curriculum are: (a) *Mathematical Reasoning* through training students to construct and evaluate arguments, doubt, abstract, synthesize, etc.; (b) *Solving Mathematical Problems* by using different mathematical strategies in addition to reading and interpreting real life situations; (c) *Relating Mathematics to the Surrounding Reality* by developing research skills, practicing scientific approaches, and understanding and valuing the role of mathematics in "technological, economic and cultural development"; (d) *Communicating Mathematically* orally and in writing in a variety of contexts; and (e) *Valuing Mathematics* by allowing students to experience the beauty and harmony of mathematical theories.

4.1.3. Evaluation Guide

The section under the Evaluation Guide, which includes the principles of developing mathematics questions in the secondary official examination (see Part I Appendix D), contains the bases for the test items' selection in terms of content and format. These criteria are for all grade 12 tracks. In terms of content, this section emphasizes following the philosophy of the curriculum and its general and specific objectives, having a balance in the three cognitive domains (knowledge, application, and reasoning), choosing competencies from different domains and including questions that integrate different competencies covering different curriculum topics, not following the same type of exams throughout the years by continuously including and excluding same topics and questions, and including different forms of questions such as open-ended questions, multiple choice questions, questions based on graphs, data, text, etc. A new guidelines document was published in 2017 containing similar guidelines (see Part II Appendix D). However, the new document adds one guideline which states that questions should not be limited to grade 12 content but should also include contents from grades 10 and 11. These guidelines are well aligned with the introduction and general objectives of the reformed curriculum.

4.2. Analysis of the Document "Details and Results of the Workshops Carried out by ECRD as Part of the Curriculum Evaluation and Development Plan"

A series of workshops was carried out by ECRD after the implementation of the reformed curriculum to reflect on it, evaluate it and recommend ways for improving it. Appendix G, represents the proceedings (minutes) of one of these workshops, which is dedicated to the mathematics subject at the secondary level. Representatives of different educational institutions, such as the Lebanese University, University of Kaslik, Secondary School for Girls Saida, Teacher's Association, Union of Orthodox Schools, and other schools, attended these workshops and discussions were made with the coordinator of these workshops. The following points are the main points related to the third year of secondary education discussed during the workshop, which took place in the year 2003 and which pinned out the existence of many problems at the curriculum's level.

The reformed curriculum was designed first without planning for evaluation. It was built based on behavioral objectives. General objectives were inspired from standards that prevailed at that time worldwide, then specific secondary objectives, content, and textbooks were established. A great confusion regarding what should be tested occurred when planning for evaluation started. Therefore, the concept of competence was adopted by ECRD and work started toward reforming the curriculum to align with the newly- adopted concept.

Different things were debatable during the workshops. Among these is the reason and importance behind mathematics education. Another is the source of problems, being the curriculum, insufficiency of teachers' guides, or students' low abilities. This lead to another debate on the role of the curriculum's developers and excluding from it proposing teachers' methods. Another debatable issue was whether to keep certain topics or remove them from the curriculum such as "propositional calculus", which is very important for students finishing the LH track and majoring in philosophy at the university level.

Problems related to content were among the discussed issues. Topics such as statistics were newly introduced to the secondary education causing great problems as they were also new to teachers, while others were included based on the request of other curriculum's developers (such as physics). Moreover, topics were weighed by specialists and teachers with great experience but from specific schools and backgrounds different from the general average norm of the country, and this showed that the time allocated to these topics was not sufficient in reality.

In short, these minutes reflect the major problems and gaps that may help in understanding the situation of assessment under the 1997 curriculum, and show indicators of an awareness by the educational community of the existence of problems in general, which need to be specified and characterized in more specific ways that would guide a new reform. Here exists the importance of this study.

4.3. Analysis of the LH Model and National Tests

This section includes the analysis of the LH model tests and national tests.

4.3.1. LH Track Content

The process objectives of the LH track of the secondary cycle are: mathematical reasoning, problem solving, and communication, while the content domain objectives are: spacial, numerical and algebraic, calculus, and statistics & probability. The contents of these domains are distributed over the three years of the secondary cycle but not necessarily all included in each.

Mathematics is assigned two sessions per week for the LH track of grade 12. These constitute 60 sessions per academic year. The content domains of this grade level and their corresponding topics and allotted time are shown in Table 4. Appendix C presents the details of the math contents of the LH track.

Table 4: *The Math Topics in the LH Track of the Third Secondary Year*

Code	Math Topics	Allocated Time
1	**ALGEBRA**	20 hours
1.1.	Foundations	10 hours
1.1.1.	→ Binary operations	
1.1.2.	→ Structure of group	
1.1.3.	→ Prepositional calculus	
1.2.	Equations & Inequalities	10 hours
1.2.1.	→ Situations- problems leading to the solutions of equations and inequalities	
2	**CALCULUS (NUMERICAL FUNCTIONS)**	25 hours
2.1.	Definitions & Representations	15 hours
2.1.1.	→ Simple rational functions	
2.1.2.	→ Graphical interpretation	
2.1.3.	→ Exponential growth and exponential function	
2.2.	Mathematical Models for Economics and Social Sciences	10 hours
2.2.1.	→ Simple interest, compound interest	
3	**STATISTICS AND PROBABILITY**	15 hours
3.1.	Statistics	10 hours
3.1.1.	→Measures of central tendency and measures of variability of a distribution of one (continuous or discrete) variable	
3.2.	Probability	5 hours
3.2.1.	→Conditional probability: definition, independence of two events	

These topics were reduced by a decision taken by MEHE and ECRD and after establishing the curriculum and issuing the books, because it was found during implementation that they were too heavy to be covered in the allocated time. The omitted topics include *binary operations and group structure* under the content domain *Algebra.* They also include *exponential growth and exponential function* under the content domain *Calculus.* The topic *propositional calculus* which is also under *Algebra* was added to these omitted topics in the academic year 2016-2017.

4.3.2. Qualitative Analysis of the Model Tests

Nine model tests for the LH track were issued throughout the years. Four model tests (LHM1, LHM2, LHM3, and LHM4) were issued in the year 2000 with the curriculum documents of the 1997 reformed curriculum in the evaluation guide (referenced in Appendix D). Three model tests (LHM5, LHM6, and LHM7) were issued in the year 2017, and two model tests (LHM8 and LHM9) were issued in the year 2019. In this part of the qualitative analysis, the change that happened in the model tests from the date the first model tests were issued to the year 2017 and then 2019 is analyzed. This gives insight about the developers' point of view of the importance of the curriculum's different topics.

The Evaluation Guide (referenced in Appendix D) contains the model tests issued in 2000 (Appendix E presents a sample model test: LHM2). The following analysis is extracted from Sleiman's (2012) analysis of the model tests.

The model test LHM4 is made up of two problems. One is a problem on *statistics* and the second is a problem on *exponential growth and function*, a topic included in the omitted topics. Therefore, it will not be included in this study as it doesn't totally represent the taught curriculum at this track (Sleiman, 2012) and might affect the results.

Table 1 in Appendix P displays the math topics covered by the model tests as well as the official tests studied in this research. Each of LHM1 and LHM2 is made up of three problems. Each problem is based on one of the three content domains of this track. On the other hand, LHM3 consists of two problems covering *Algebra* and *Calculus* only. Thus, LHM3 doesn't cover a considerable part of the curriculum (Sleiman, 2012).

Moreover, Table 1 in Appendix P shows that the *topic definitions and representations* occurs in each of the three model tests where the given is a graph of a rational function in LHM1 and LHM2, while being the algebraic expression of the rational function in LHM3. A problem on *equations / inequalities* is also found in the three model tests as problem on *equations* in LHM2 and LHM3 and as problem on *inequality* in LHM1. A problem on *probability* constitutes the third problem in LHM1 and LHM2. A problem on *propositional calculus* constitutes the third problem in LHM3. The *topics statistics* and *simple and compound interest* are not included in the mentioned model tests.

On the other hand, model tests issued in the year 2017 and the model tests issued in the year 2019 are all three-problem tests (refer to Table 1 in Appendix P). All these tests have a problem on *definitions and representations* and a problem on *probability*. The problems on definitions and representations are of different forms. In the model tests LHM5, LHM6, and LHM7, the given in *definitions and representations* is the function's table of variation, graph, and algebraic expression of the function respectively. In the model tests LHM8 and LHM9 the given is the algebraic expression and graph respectively. The first four model tests have each a part of a problem on *statistics* which is limited to completing the frequency table, which is part of the content in the previous secondary years. Four model tests LHM5, LHM6, LHM8, and LHM9 have each a problem on *equations*. The fifth test has a part of the *definitions and representations* problem tackling the *solving-an-equation* topic. As for the topic *inequalities*, it does not occur in any of these five tests. Test LHM7 is the only among these five model tests that includes a problem on *compound interest*. The topic *propositional calculus* is not included in any of these tests since it is among the suspended lessons starting from the year 2016.

Table 2 in Appendix P displays the grade points allocated to the math topics in the model and official tests for the LH track. In the model tests LHM1 and LHM3, the highest grade point is assigned to the *topic equalities/ inequalities* in LHM1 and LHM3. In LHM2, the highest grade point is assigned to *definitions and representations*. On the other hand, in LHM1 and LHM2, the *topic probability* is assigned the lowest grade. The *topics propositional calculus* and *functions* have same grade distribution in LHM3. On the other hand, the grade distribution in the model tests issued in 2017 and 2019 is the same to all. The *definitions and representations* problem has the highest grade point (10). The remaining two problems have 5 grade points each.

Looking at the length of each model test, it is clear how the model tests issued with the curriculum documents differ from the model tests issued in the years 2017 and 2019. The model tests LHM1, LHM2, and LHM3 have a total of 49 test items making an average of about 16 test item per test, while the model tests LHM5, LHM6, and LHM7 have a total of 90 test items making an average of about 30 test items per test, and the model tests LHM8 and LHM9 have a total of 66 test items making an average of about 33 test items per test.

The qualitative analysis of the model tests shows that the developers' view regarding the importance of certain topics has changed over time. The topic *propositional calculus* and *inequalities* under the topic *equalities and inequalities,* which appear in the model tests before 2017, are considered unimportant over time; therefore, the first was omitted in the year 2017, while the second was never addressed in any of the model tests issued in the years 2017 and 2019. On the other hand, the topic *simple and compound interest* which was never addressed in the model tests before 2017, was regarded as important in the model tests of the year 2017, but then it was neglected in the model tests of the years 2019. The topic *statistics* was also considered important again, so it was tackled in the model tests of the years 2017 and 2019 but at a basic level. Moreover, the A*lgebra* content domain which is represented by the topic *equalities and inequalities* does not seem to have a considerable importance since it can be replaced by the topic *simple and compound interest* which is under *Calculus* content domain.

The change in the model tests is investigated further in terms of content and cognitive domains in the Qualitative Analysis of the LH Model and National Tests' Test Items.

4.3.3. Qualitative Analysis of the National LH Math Tests

The sixteen national math tests (sessions 1 and 2) of the years 2011 to 2019 for the LH track are analyzed in this section. The official tests of year 2014 were not put in public since Lebanese teachers were on strike, and the committee responsible for correcting and grading the national tests abstained. Appendix H presents a sample national test-LH182 which is the national test of the 2nd session of the year 2018).

Table 1 in Appendix P displays the math topics covered by the national tests of the years starting from 2011 till 2019. Each official test consists of three problems covering the three content domains except for the official tests LH112 and LH122 which do not cover the domain *Algebra*.

According to table 1, all national tests include problems on the *topic definitions and representations*. *The topic probability* is also included in every national test, while the *topic propositional calculus* has no problems in any national test. The *topic equations* has test items in the national tests of every year, except for the official tests LH112 and LH122. No questions on *inequalities* are included in the official tests. *Compound interest* problems are just included in the official tests LH112 and LH122 instead of the *equation* problems. The official test LH192 includes a problem combining both *equations* and *compound interest* topics. The official tests LH111, LH112, LH131, LH152, LH162, LH172, and LH191 have, each, a problem combining both *statistics* and *probability*. However, the occurrence of the *statistic topic* here is just limited for basic knowledge on statistics taken in previous years.

In short, the topics *propositional calculus*, *inequalities* under the topic *equations and inequalities,* and *statistics* are never addressed although the number of sessions allocated to them according to the syllabus is not little, that is 10, a fraction of 10, and 10 hours respectively. Moreover, the topic *simple and compound interest* is addressed in only three out of 16 official tests although it also constitutes a good portion of the topics in the syllabus having allocated time of ten hours. These topics form around 50 to 60% of the topics but are rarely or never addressed.

Table 2 in Appendix P displays the grade points allocated to the math topics in the national tests for the LH track. The grades allocated the three content domains is constant in all the official tests. *Algebra* and *Statistics and Probability* domains are allocated 5 points each out of a total of 20. The highest grade points (half the total grade:10 points) are assigned to the problem on *definitions and representations* in each official test. The official tests LH121, LH131, LH151, LH181, LH182, LH191, and LH192 include a test item on *equations* (1.2 – Equations & Inequalities) in the problem of *definitions and representations* which caused the difference in the distribution in Table 2. Similarly, official test LH162 has a test item on equations in the problem on *probability* and the official test LH192 has a test item on compound interest in the problem on *equations*.

The qualitative analysis of the national math tests shows that the scale of importance given by the curriculum to most of the topics, as reflected by the content and number of hours allocated to each, differ from the scale of importance shown in the tests to these topics, as reflected by the test items occurrence and grade distribution. While the topic *definitions and representations* has an equal number of allocated hours as the domain *statistics and probability*, the first is assigned double the grade points assigned to the latter and a greater number of test items. On the other hand, the topic *simple and compound interest* rarely occurs in the national tests, unlike the topic *equations/inequalities* which is allocated the same number of hours and occurs in almost every national test, and the topic *probability* which is allocated half the number of hours and occurs in all national tests.

4.3.4. Qualitative Analysis of the LH Model and National Tests' Test Items

This section aims at analyzing qualitatively and comparing the model and national tests to check whether the 2017 issued model tests have reflected more the previous national tests or have impacted the subsequent national tests, and how. The test items of the model tests and national tests are studied. The analysis is based on the *topics* covered. Out of the six *topics* constituting the math curriculum at this track, the topics *propositional calculus* and *inequalities* occur only in the model tests.

Propositional Calculus (Under Algebra Domain)

Table 3 in Appendix P displays the occurrences of test items on the *topic propositional calculus* as well as the tests in which they appear. No test items under this *topic* are included in any of the national tests studied while they occur in one of the model tests. Thus, no alignment exits between the national tests and the model tests this *topic* until the year 2017, when this *topic* was added to the suspended topics under the LH track. This reflects that the curriculum, in this case, is modified to be based on assessment rather than modifying assessment to be aligned with the curriculum content.

Equations & Inequalities (Under Algebra Domain)

Table 4 in Appendix P displays the occurrences of test items on this topic as well as the tests in which they appear. The test items require implicitly three steps: identifying the unknown, translating problems which are given in word form into equations or inequalities, and solving the system. The cognitive abilities required for these steps are: knowing, reasoning, and knowing-and-applying respectively.

The following is a problem on *equations* from the model test LHM2.

The average monthly income of either an employee or a technician in a firm is 600,000 LP.
If we raise the wage of the employee by 10% and we reduce that of the technician by 10%, the average income becomes 590,000 LP.
What is the monthly income of each of them?

Figure 1: *Sample Problem 1 on Equations from the Model Test LHM2*

The following is a problem on *equations* from the model test LHM5.

A Shop sells phone books. At the beginning of the season, the prices of a phone and an IPAD together is 1 800 000 LL. At the end of the season, after a 30% decrease in the price of the phone and 25% increase in the price of the IPAD, the prices together become 1 920 000 LL.

1) Find the price of the phone and the price of the IPAD at the beginning of the season.

2) Find the price of the phone and the price of the IPAD at the end of the season.

3) Samir wants to buy 5 phones and 2 IPADs. Is it profitable for him to buy them at the beginning or at the end of the season? Justify your answer.

Figure 2: *Sample Problem 2 on Equations from the Model Test LHM2*

The model tests issued in 2017 and 2019 that include problems on equations have a close structure but different real life example, and some require higher cognitive skills than the questions of the model tests that were issued in the year 2000.

The following is a sample problem on *equations* retrieved from the national test LH162.

1. Solve the following system: $\begin{cases} 2x + y = 80 \\ 1.7x + 2.7y = 105 \end{cases}$
2. The price of two shirts and one belt is 80 thousand L.L.
   After a discount of 15% on the price of one shirt and a discount of 10% on the price of a belt, the price of two shirts and three belts becomes 105 thousand L.L.
   a. Show that this text is modeled by the system given in the first question.
   b. Find the original price of one shirt and the original price of a belt.
   c. Nadim bought four shirts and three belts after the discount. How much did he pay?

Figure 3: *Sample Problem 3 on Equations from Session-2 National Test of the Year 2016*

All the national tests that include a problem on *equations* include some test items on systems of equations as in test items 1, 2a, and 2b and some test items that require forming and solving an equation with one unknown or some calculations to find the price of several items after a discount as in part 2c in the above problem. The second part of test items also requires translating the situation into an equation and then solving or doing the calculation, and this also requires the same cognitive abilities required by the steps on the system of equations part. The part on system of equations differs in structure among the tests. In some tests, the system of equations is given, then a situation is given and students are required to show that it models the previously given system. In other tests, the situation is directly given and students have to translate it to a system and solve. In both structures, the same abilities are required and the same objectives are assessed.

The test items on *equations* in the model tests of the years 2017 and 2019 and all the national tests are more variant in structure, situations, and number than the previous model tests. The problems in both the model and national tests have real life contexts. We conclude that the model tests and the national tests partially align and match under *equations* before the year 2017 and are well aligned after the year 2017.

Inequalities, on the other hand, does not appear in the national tests to be compared to test items corresponding to it in the model tests issued in the year 2000.

Thus, the national tests are not aligned with the model tests under *inequalities* before the year 2017.

In fact, this reflects how the model tests are changing based on assessment and to align more with the curriculum contents and objectives.

Definitions and representations (Under Calculus Domain)

Table 5 in Appendix P displays the test items on *definitions and representations* and the tests in which they appear. This topic is included in all national exams being assigned half the test's grade points which makes it considered as a very important topic. The cognitive domains required by the test items under this topic vary between: *knowing*, *applying* or both, and sometimes *reasoning*.

The following is a sample problem on definitions and representations retrieved from the model test LHM3. This example is made up of two questions.

> Let f be a function defined on]0,∞[by $f(x) = x - 1 + \frac{1}{x}$. We call (C) its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$ (1 unit = 1 cm).
>
> 1) Prove that the lines x = 0 and y = x – 1 are asymptotes of (C).
>
> 2) Study the variations of f and sketch (C).

Figure 4: *Sample Problem 1 on Definitions and Representations from the Model Tests LHM3*

These questions require: finding the limits to answer the first part, finding the derivative, solving f'(x) = 0, (d) studying the variation, and sketching the graph to answer the second part (Sleiman, 2012).

The following is sample problem on definitions and representations topics retrieved from the model tests LHM8.

Let f be the function defined over $\mathbb{R}$ as $f(x) = \frac{4}{x^2 + 2x + 2}$.

Denote by (C) its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$.

1- a) Determine $\lim_{x \to +\infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.

Deduce that (C) has an asymptote.

b) For all x in $\mathbb{R}$, prove that (C) is above its asymptote.

c) Determine the coordinates of A and B, the meeting points of (C) and the line with equation $y = 2$; $(x_A < 0)$.

2- a) Show that $f'(x) = \frac{-8(x+1)}{(x^2 + 2x + 2)^2}$, then set up the table of variations of f.

b) Calculate $f(-3)$ and $f(1)$, then draw (C).

3- S is the vertex of (C).

a) Prove that (SA) is tangent to (C) at A.

b) Solve graphically $f(x) < 2$.

c) Write an equation of (T), the tangent at B to (C).

Verify that (T) is passing through S.

4- Let g be the function defined as $g(x) = ax + \frac{b}{x-1}$ (C') is the representative curve of g in the same system as that of (C).

Calculate a and b so that (C') is tangent at B to (C).

5- In what follows, let $a = -4$ and $b = -2$.

a) Determine the domain of definition of g.

b) Determine the asymptotes for (C').

Figure 5: *Sample Problem 2 on Definitions and Representations from the Model Tests LHM8*

The model tests issued in 2017 and 2019 have similar structure, length, and a wide variety of test items under this topic. There are big differences between sample problems 1 and 2 in terms of specific objectives covered as well as the cognitive domains required. Sample problem 2 is more comprehensive of the topic than sample problem 1.

Questions under the *Definitions and representations* topic in the national tests are extended in some parts in a somehow guided way where each test item asks about a step in the solution. For example, instead of being asked to prove an asymptote, students are asked to find the limits then find the asymptote. The following is a sample problem on definitions and representations retrieved from the national test LH182. It also starts with the algebraic form of a rational function as in sample problem 2 but has several specific, step-by-step questions.

Let f be the function defined on the interval $I =]-1;+\infty[$ as $f(x) = \frac{x^2+3}{x+1}$ and let (C) be its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Show that $f(x) = x - 1 + \frac{4}{x+1}$.
2) a. Determine $\lim_{\substack{x\to -1 \\ x>-1}} f(x)$ and deduce an equation of an asymptote to (C).

   b. Determine $\lim_{x\to+\infty} f(x)$.

   c. Prove that the line (d) with equation $y = x - 1$ is an asymptote to (C).
3) a. Verify that $f'(x) = \frac{(x-1)(x+3)}{(x+1)^2}$.

   b. Copy and complete the following table of variations of f.

x	-1	1	$+\infty$
f'(x)		0	
f(x)			

4) a. Calculate the coordinates of the points of intersection of (C) and the line with equation $y = 3$.

   b. Find an equation of the tangent to (C) at its point with abscissa 0.

   c. Draw the curve (C) and its two asymptotes.
5) Solve graphically: $2 < f(x) \leq 3$.

Figure 6: *Sample Problem 3 on Definitions and Representations from the National Test LH182*

From samples 1 and 3, we conclude that the test items under the topic *definitions and representations* in both the model tests issued in the year 2000 and national tests have in some parts similar content but different structure. However, the questions in the national tests are more elaborated and varied. From samples 2 and 3, it is noticed that the questions under the *definitions and representations* problem in the national tests are similar, under this topic, to the model tests issued in 2017 and 2019. However, the model tests issued in 2017 and 2019 have more test items giving all the possible forms a certain question might have. Moreover, the questions in both the model and national tests do not describe real-life situations but are purely abstract. National tests LH122, LH131, LH151, LH162, LH181, LH182, and LH191 are similar to the model tests of the years 2017 and 2019, however, they include test items on a different topic- *equations*. We conclude that the national and model tests are also partially aligned under the topic *definitions and representations* before the year 2017, but have a better alignment starting from the year 2017 which shows that the revision of model tests is being made based on assessment and not on a rational revision of the curriculum and its goals and objectives.

Simple Interest, Compound Interest (Under *Calculus Domain)*

. Table 6 in Appendix P presents the tests items under this *topic* and the tests where they appear. This *topic* appears in the national tests in only three tests where the test items are mainly about calculating the compound interest. The cognitive abilities required for these test items are mainly knowing and applying.

Only one objective out of 6 objectives under this *topic* is addressed in the national tests. The remaining five are never addressed.

The following is a sample interest problem retrieved from the national test LH112.

Fadi deposited, in a bank, for a period of 5 years, a capital of 10 000 000 L.L at an annual interest rate of 10%. The interests are compounded quarterly.

1) a. Calculate the future value of this capital at the end of the fifth year.
   b. Calculate the total interest.

2) By the end of the fifth year, Fadi withdraws 35% of the total interest of his account. What is the remaining amount in this account?

Figure 7: *Sample Problem 1 on Simple and Compound Interest from the National Test LH112*

Test items under this *topic* are only included in the national tests and the model tests of the years 2017 and 2019. The model tests issued in 2000 include no test items under this *topic* to compare with the test items of the national tests. Therefore, the model and national tests are not aligned under the *topic simple and compound interest* before the year 2017.

The following is the interest problem retrieved from the national test LHM7.

Rami deposited, for a period of 4 years, a sum of 10 000 000 LL in a savings account at an annual interest rate of 5 %. The interests are compounded quarterly.

1) Calculate the future value of this sum.
2) Calculate the total interest.

Figure 8: *Sample Problem 2 on Simple and Compound Interest from the Model Test LHM7*

This problem is very similar in content and structure to the problems in the national tests under the topic *simple and compound interest*. However, although it has less parts, these parts tackle the same ideas tackled by the national tests under the topic. If we compare the national tests of the years 2017 and 2018, none has a question on this topic. Therefore, a lack of alignment exists under this topic between the model tests LHM5, LHM6, and LHM7 and the national tests LH171, LH172, LH181, and LH182 representing both the same period of time. If we consider the model tests of the year 2019, they do not include problems under this topic, while the national test LH192 includes a part on *simple and compound interest* under the *equations* problem. Therefore, there is no alignment between the national tests and the model tests of the year 2019.

In fact, the *topic simple and compound interest* is not considered as an important topic since even when it is tackled, it appears in one or two very simple test items which are routine questions.

Statistics (Under *Statistics and Probability Domain*)

Table 7 in Appendix P displays the tests items under this topic as well as the tests they appear in. The cognitive abilities required for these test items are mainly knowing. This topic is never addressed in the model tests issued with the curriculum documents. However, it occurs in 4 out of the model tests of the years 2017 and 2019, but limited to completing the table which is an objective covered in the previous grade levels. This topic is present in half the national tests, and its occurrence is also limited to the mentioned objective and sometimes to finding the average of the given data. Therefore, a lack of alignment exists under this topic between the national tests and the model tests issued with the curriculum documents, but a partial alignment exists between the national tests and the model tests of the years 2017 and 2019.

Probability (Under *Statistics and Probability Domain*)

Table 8 in Appendix P displays the tests items under this topic as well as the tests they appear in. Its problems include test items to find the probability of events using basic rules of probability and the rules of conditional probability. The cognitive abilities required for these test items are mainly knowing and applying.

The following is a sample probability problem retrieved from the model test LHM2

The students of a secondary school are distributed according to the following table:

	Boys	Girls
External	650	850
Half-internal	550	450

We randomly pick up one student.
Compute the probability that this student is external given that he is a boy.

Figure 9: *Sample Problem 1 on Probability from the Model Test LHM2*The following is a sample probability problem retrieved from the model test LHM7.

In a survey about the best social media used, 250 persons were asked and the following data were collected.

	Facebook	Twitter	What's App.
Men	70		
Women			50

We know that:

- 40% of the persons are females
- 20% of the men prefer "Twitter"
- The number of men and women who prefer "What's App." is the same.
- 34% of the women prefer "Facebook"

1) Copy and complete the above table.
2) One person is chosen and interviewed.
   a- Calculate the probability of choosing a boy.
   b- Calculate the probability of choosing a woman who prefers "Facebook".
   c- Calculate the probability of choosing a person who prefers "Twitter".
3) One girl is chosen randomly. What is the probability of being a person who prefers "What's App."?
4) Two persons are chosen randomly and successively without replacement and interviewed. What is the probability of being boys who prefer "Twitter"?

Figure 10: *Sample Problem 2 on Probability from the Model Test LHM7*

The problems on *probability* in the model tests of the years 2017 and 2019 have similar structure and similarly tackle all the specific objectives under this *topic*. Samples one and two show the big difference between the model tests issued in the year 2000 and the model tests of the years 2017 and 2019 in terms of content coverage and diversity of test items.

The following is an example of the probability problems retrieved from the model test LH181.

A survey is done on a population formed of 40 men and 60 women about their usage of three kinds of soaps A, B and C. The results are shown in the following table:

	Soap A	Soap B	Soap C
Men	20	5	15
Women	15	20	25

A person is randomly selected from this population and interviewed :

Consider the following events :

A : « The interviewed person uses soap A »

B : « The interviewed person uses soap B »

M : « The interviewed person is a man ».

1) Calculate the following probabilities:

   $P(M)$; $P(A \cap M)$; $P(A / M)$; $P(B \cup M)$ and $P(\overline{B})$.

2) The interviewed person doesn't use soap A. Calculate the probability that this person is a man.

Figure 11: *Sample Problem 3 on Probability from the National Test LH181*

Thus, we conclude from the previous three samples that the test items under *probability* in the model tests of the years 2017 and 2019 and national exams are more numerous than those in the model tests issued in the year 2000, yet the problems are similar in structure.

The qualitative analysis in this section shows that the *topics inequalities* and *propositional calculus* were never addressed in the national tests and the model tests of the years 2017 and 2019, but were addressed in the model tests issued in the year 2000. The *topic propositional calculus* was not addressed in the model tests and national tests starting from 2017 because it was omitted from the required topics for this track. The *topic simple and compound interest* didn't appear in the model tests issued in the year

2000 and the model tests of the year 2019, and was rarely addressed in both the model tests of the year 2017 and the national tests. Moreover, the topic *equations, definitions and representations, and probability* are over-emphasized and include a variety of test items in the model tests of the years 2017 and 2019 and national tests compared to the model tests issued in the year 2000. The *topic statistics* was never addressed in the model tests before the year 2017, but occurred in most of the model tests of the years 2017 and 2019 and in half of the national tests. However, its occurrence was limited to a couple of basic questions which are taken in the previous years.

Moreover, many specific objectives were not tackled in both the model tests and the national tests. Many of these objectives are under the *topics simple and compound interest* and *statistics*. All the specific objectives under *statistics* for grade 12 were never addressed. Some of the specific objectives under the topic *equations* were never addressed. Thus, model tests are obviously modified based on assessment excluding the topics and specific objectives that never occur in the previous national tests and maintaining a low occurrence of certain topics, instead of basing the revision on the curriculum's objectives.

A steady structure and content exist in all the national tests throughout the years. Three problems on the *topic definitions and representations, equations, and probability* are present in almost all of the tests. The *topic equations* is rarely replaced with the *topic simple and compound interest*, but when it was replaced, it appears as a part or two in one of the other two problems on *probability* and *definitions and representations*. The occurrence of the *simple and compound interest topic* is limited to session-2 of the national tests. The *topic propositional calculus* which was neglected in many national tests was added to the suspended *topics* later.

In fact, three topics are considered essential in the national test: *definitions and representations, probability*, and *equations*. The topic *simple and compound interest* is rarely addressed, and *statistics* of grade 12 and *inequality* are completely neglected.

LH official tests, as analyzed qualitatively, help to some extent in the implementation of the reform of mathematics education as mentioned in the mathematics curriculum's introduction and which emphasizes the nonseparation of mathematics from real-life but eliminating the theoretical overuse and including the use of technologies. Test items under the *topic definitions and representations* which has the most test items in the national tests occur in a purely abstract context separated from any life application which contradicts the spirit of the reform. As for the use of calculators and technological tools, the use of calculators is limited to simple calculations which might lead students to performing lesser mental calculations by over-depending on them.

The process objectives of the LH track, as presented in the curriculum document, emphasize mathematical reasoning, solving mathematical problems, and communicating mathematically (ECRD, 1997a). As for communication, the subskills under communication in the process objectives of the LH track under the main curriculum document (Appendix A) are limited to: 1) getting the formulas and relations out of a mathematical text and 2) doing the work with precision. According to these subskills, and as shown in the qualitative analysis of the LH national tests, this objective is reflected to a good extent in the official tests.

4.3.5. Quantitative Analysis of the LH National Tests

The percentages of test items under the content and cognitive domains of the model tests and national tests are presented in tables, compared, and analyzed in this section. The Pearson Product-Moment correlation coefficients between the data in the resultant tables are then presented, interpreted and discussed.

4.3.5.1 – Overall Alignment Between the Model Tests and the National Tests

According to the test item definition adopted in this study, there exist 205 test items in 8 model tests and 417 test items in 16 national tests. For the sake of having a unified base for comparison, the data in the Table AllModLH, that presents the quantitative data of the nine model tests, and Table OffExLH, that presents the quantitative data of the sixteen national tests, were converted to percentages. Table 1 in Appendix Q displays the resultant percentages of the test items in all the model and national tests distributed over the cognitive domains and *topics* they tackle.

According to Table 1, the model and national tests, compared to each other in terms of their test items' percentages, cover in a balanced way most of the math curriculum topics. The topic *definitions and representations* is assigned around half of the test items (50.73%) in the model tests and (52.76%) in the national tests. *Equations* and *probability* are two *topics* assigned almost equal amounts of test items in the national tests (17.75% and 17.98 % respectively), while they are assigned 18.05% and 15.12% respectively in the model tests. *Statistics* comes in the third place having 10.07% of the test items in the national tests and 11.71% in the model tests. The *topic simple and compound interest* has the lowest percentage with very close values in the national (1.44%) and the model tests (1.46%). The topic propositional calculus has a greater percentage in the model tests (2.93%) than in the national tests (0%) since it was never addressed in the latter and then omitted in the year 2017 from the topics required for grade 12 LH track.

Table 1 in Appendix Q shows an imbalance between the cognitive domains in each the model and national tests. About half of the test items in both the model (50.85%) and the national tests (51.57%) require the cognitive domain *knowing*. Next required cognitive domain is *applying* being required by 33.45% of the test items in the model tests and 38.81% in the national tests. R*easoning* is required by the least percentage of test items in both the model tests (15.69%) and the national tests (9.62%) with an obvious discrepancy. Both the model tests, representing the curriculum, and the national tests emphasize *knowing* over *applying* and *reasoning*. The curriculum emphasizes *reasoning* much more than the national tests do, as *Mathematical Reasoning* is stated in the curriculum as the first of the general objectives, as well as the first of the specific objectives of each cycle; while the national tests emphasize *applying* more than the curriculum does.

Correlations were made, using Pearson Product-Moment coefficient by Microsoft Excel, between the calculated percentages by correlating data in Table 1 of Appendix Q. Correlations were not made between the specific objectives in Tables AllModLH and OffExLH since it is hard to have a good correspondence under every specific objective and cognitive domain. Thus, to have more valid results, an overall correlation was calculated between all cells of the model tests and all cells of the national tests, correlations under each math domain between the model and national tests were calculated, and correlations under each cognitive domain between the model and national tests were calculated.

Table 5 presents the correlation between all the national tests of the years 2011 to 2019 and the model tests for the LH track.

Table 5: *Correlations Between the National Tests of the Years 2011 to 2019 and the Model Tests for LH Track*

	Overall Correlation	In terms of Cognitive Domains			In terms of Math Contents		
		Knowing	Applying	Reasoning	Algebra	Calculus	Statistic & Probability
NT & MT	0.97	0.99	0.99	0.85	0.93	0.974	0.94

NT & MT : Correlation between the national tests of the years 2011-2019 (NT) and the model tests (MT)

According to Table 5, the overall correlation between the model tests and the national tests is very high (r=0.97). This is because the percentages are very close under both the math *domains* and the cognitive domains.

The correlation between the national tests and model tests in terms of the cognitive domains is similarly calculated between the numbers in the columns of each domain of Table 1 in Appendix Q and presented in Table 5. The correlation, refer to Table 5, between the national tests and model tests in terms of *knowing* is 0.99, *applying* is 0.99, and *reasoning* is 0.85.

Correlations between the model and national tests in terms of the math domains *Algebra, Calculus,* and *Statistics and Probability* were similarly calculated by finding Pearson Product-Moment coefficient between the data in the rows of Table 1 in

Appendix Q. The correlation in terms of *Algebra* is 0.93, *Calculus* is 0.974, and *Statistics and Probability* is 0.94 between the national tests and model tests.

The correlations under the cognitive domains and the math domains are very high. This is because the percentages under each math content and cognitive domain are very close. Discrepancy exists in the percentages under *reasoning* especially in the topic *propositional calculus* and *definitions and representations* causing the correlation under this cognitive domain not perfect. *Reasoning* under these topics is more emphasized in the model tests than the national tests.

In short, according to the analysis presented in this section, the national tests are well aligned with the model tests under all content domains. Alignment is almost perfect at the cognitive domains *knowing* and *applying*, but not at *reasoning* which is the first general and specific objective of the math curriculum for the LH tack. Moreover, the national tests give great importance to certain topics while ignoring some others.

4.3.5.2 – Alignment Between the Model Tests and the National Tests over the years 2011-2013, 2015-2016, 2017-2018, and 2019 and its Evolution

4.3.5.2.1 – Alignment Between the Model Tests and the National Tests

More accurate results are obtained when comparisons and correlations are made between the national tests and their corresponding model tests. Therefore, to study the alignment in a more valid way, comparison and correlations between the model tests and the national tests over the years 2011-2013, 2015-2016, 2017-2018, and 2019 are made in this section. Comparison is made between the model tests (LHM1, LHM2, and LHM3) issued with the curriculum documents (in 2000) and the national tests of the years 2011-2013 and 2015-2016, between the model tests (LHM5, LHM6, and LHM7) issued in the year 2017 and the national tests of the years 2017-2018, and between the model tests (LHM8 and LHM9) issued in the year 2019 and the national tests of this year are made. This comparison is in terms of content domains and cognitive domains to see how the content and cognitive domains coverage is in each set, then correlations are calculated to check alignment of each set.

According to the test item definition adopted in this study, there exist 49 test items in the three model tests issued in the year 2000, 90 test items in the three model tests issued in the year 2017, 66 test items in the two model tests issued in the year 2019, 140 test items in six national tests (2011-2013), 104 test items in four national tests (2015-2016), 113 test items in four national tests (2017-2018), and 60 test items in four national tests of the year. For the sake of having a unified base for comparison, data in Tables Mod, ModLH5-7, ModLH8-9, NewModLH, OffExLH11-13, OffExLH15-16, OffExLH17-18, and OffExLH19 were changed to percentages. Tables 2, 3, and 4 in Appendix Q display the resultant percentages of the test items distributed over the cognitive domains and *topics* they tackle. Table 2 presents the percentages of test items in the model tests issued in the year 2000 and the national tests of the years 2011-2013 and 2015-2016. Table 3 presents the percentages of test items in the model tests issued in the year 2017 and the national tests of the years 2017-2018, and Table 4 presents the percentages of test items in the model tests issued in the year 20019 and the national tests of the same year.

Content Domains

When comparing the model tests issued with the curriculum documents to the national tests of the years 2011-2013 and the national tests of the years 2015-2016, it is obvious in table 2 in Appendix Q that the model tests and the corresponding national tests do not tackle in a balanced way the content domains of the math curriculum. The percentages of the test items in the model tests are distributed over four out of six topics, while they are distributed in the national tests of the years 2011-2013 over five topics and of the years 2015-2016 over four topics. More than half of the test items are assigned to the topic *definitions and representations* (57.14 %) in the model tests, (53.93 %) in the national exams of the years 2011-2013, and (50.48 %) in the national exams of the years 2015-2016.

The *topic definitions and representations* has the highest percentage of test items in all tests. The *topic equations and inequalities* has the second highest percentage in the old model tests (26.53%) and the national tests of the years 2015-2016 (23.56%), while it has the third highest percentage in the national tests of the years 2011-2013 (12.5%) after the *topic probability* (17.86%).

Considerable discrepancies are obvious between the percentages of the test items in these model tests, and each of the national tests of the years 2011-2013 and the years 2015-2016 under the topics *propositional calculus* (12.24%, 0%, and 0% respectively), *statistics* (0%, 12.14%, and 9.62% respectively), and *probability* (4.08%, 17.86%, and 16.35% respectively). Considerable discrepancy is also obvious under the topic *equations* between the national tests of the years 2011-2013 (12.5%) and each of the model tests (26.53%) and the national tests of the years 2015-2016 (23.56%). As for the topic *simple and compound interest*, although there are no test items on this topic in the model tests and the national tests of the years 2015-2016, its percentage in the national tests 2011-2013 is very low (3.57%).

When comparing the model tests of the year 2017 to the national tests of the years 2017-2018, it is obvious in Table 3 in Appendix Q that the model tests and the corresponding national tests cover in a balanced way some of the content domains of the math curriculum. While the model tests and the national tests of the years 2017-2018 cover the five *topics* of the math, the national tests cover only four *topics*. The *topic propositional calculus* had become among the suspended lessons for those years. The *topic definitions and representations* has the highest percentages of test items in the model tests issued in 2017 (46.67%) and the national tests of the years 2017-2018 (53.53%) constituting around half of the test items. Then comes *probability* with 16.67% of the test items of the model tests and 19.47% of the test items of the national tests of the years 2017-2018. *Equations* follows with a discrepancy between its occurrence in the model tests and that in the national tests of the years 2017-2018 (13.33% and 19.03% respectively). A considerable discrepancy is obvious under *statistics* with a percentage of 20 in the model tests and of 7.97 of the test items in the national tests of the years 2017-2018. Lastly comes *simple and compound interest* with a small percentage (3.34 %) in the model tests and with no test items in the national tests of the years 2017-2018.

Comparing the model tests of the year 2019 to their corresponding national tests of the year 2019, it is obvious in Table 4 of Appendix Q that the model tests and their corresponding national tests assess in a more balanced way all of the *topics* of the math curriculum. The model tests cover four *topics*, while the national tests of the years 2019 cover all the five topics of the math curriculum. The topic propositional calculus is among the suspended lessons for this year. The topic *definitions and representations* has the highest percentages of test items in the new model tests (51.51%) and the national tests of the years 2019 (52.49%) constituting around half of the test items. Then comes *probability* with 21.21% of the test items of the model tests and 18.33% of the test items of the national tests of the year 2019. *Equations* follows with percentages of 18.16 and 17.51 in the model tests and the national tests of the year 2019 respectively. *Statistics* has a percentage of 9.09 in the model tests and of 18.33 of the test items in the national tests of the year 2019. Lastly comes *simple and compound interest* with a very small percentage (1.66%) in the national tests and with no occurrence in the model tests of the year 2019.

In short, the qualitative analysis of the model and national tests over the mentioned sets of years shows considerable discrepancies in test item percentages in the national tests of each of the years 2011-2013 and 2015-2016 and the model tests issued in the year 2000 under the *content domains* mainly *Algebra* and *Statistics and Probability*. Discrepancy exists in the percentages of national tests of the years 2017-2019 and the model tests of the year 2017 mainly under the *domain Statistics and Probability*. The national tests and the model tests of the year 2019 have close percentages under all content domains. A noticeable change exists in the percentages of test items under all the *math topics* of the model tests of the year 2019 being closer to the percentages of the national tests of the previous years. This intersects with the results of the qualitative analysis of the LH track tests: the modifications made to the curriculum were made to be aligned with assessment and not based on a revision of the curriculum and its objectives.

Cognitive Domains

As to the cognitive domains, Tables 2, 3, and 4 in Appendix Q show that through the four periods, the model tests and their corresponding national tests have the following close percentages under the cognitive domain *knowing* ranging between 50% and 51.13%.

However, discrepancies occur under the domains *applying* and *reasoning* over the periods 2011-2013, 2015-2016, and the year 2019 being the highest in the model tests and national tests of the year 2019. The following are the respective percentages under the domain *applying*:

- 32.99% for the model tests issued in 2000 and 41.61% for the national tests of the years 2011-2013
- 32.99% for the model tests issued in 2000 and 36.06% for the national tests of the years 2015-2016
- 34.63% for the model tests issued in 2017 and 35.84 % for the national tests of the years 2017-2018
- 32.2% for the model tests issued in 2019 and 42.2 % for the national tests of the year 2019.

The following are the respective percentages under the domain *reasoning*:

- 17% for the model tests issued in 2000 and 10.83% for the national tests of the years 2011-2013

- 17% for the model tests issued in 2000 and 9.455% for the national tests of the years 2015-2016
- 14.26% for the model tests issued in 2017 and 10.62 % for the national tests of the years 2017-2018
- 16.64% for the model tests issued in 2019 and 5.14 % for the national tests of the year 2019.

In short, the curriculum, as demonstrated in the model tests, and the national tests emphasize the cognitive domain *knowing* over *applying* and *reasoning*. However, more attention is given by all the model tests to the domain *reasoning* than the national tests over the mentioned periods of time. It is also noted that the national tests of the year 2019 has the lowest percentage under the domain *reasoning*.

Correlations

In addition to percentages, correlations were calculated between the respective numbers in Tables 2, 3, and 4 in Appendix Q. Correlations were calculated between the model tests issued in 2000 and the national tests of the years 2011-2013, the model tests issued with the curriculum documents and the national tests of the years 2015-2016, the model tests issued in 2017 and the national tests of the years 2017-2018, and the model tests issued in 2019 and the national tests of the year 2019. Table 6 presents these correlations.

Table 6: *Correlations Between the National tests of the Years 2011-2019 and the Model Tests for Grade 12 LH Track*

	Overall Correlation	**In terms of Cognitive Domains**			**In terms of Math Contents**		
		Knowing	**Applying**	**Reasoning**	**Algebra**	**Calculus**	**Statistics &Probability**
NT11-13 &MT1	0.695	0.867	0.706	-0.084	0.375	0.842	0.634
NT15-16 &MT1	0.817	0.912	0.855	-0.064	0.484	0.943	0.549
NT17-18 &MT2	0.85	0.72	0.98	0.91	0.69	0.98	0.51
NT19 &MT3	0.9	0.98	0.96	0.24	0.99	0.98	0.99

MT1: model tests issued in the year 2000
MT2: model tests issued in the year 2017
MT3: model tests issued in the year 2019
NT11-13: national tests of the years 2011-2013
NT15-16: national tests of the years 2015-2016
NT17-18: national tests of the years 2017-2018
NT19: national tests of the year 2019

According to Table 6, the correlation is r = 0.695 between the model tests issued with the curriculum documents and the national tests of the years 2011-2013. This correlation is mainly because of the imbalance of content coverage between the two sets of the topics *propositional calculus, simple and compound interest* and *statistics* and the discrepancy in the percentages of test items under *probability*. The overall correlation is r=0.817 between the model tests of the year 2000 and the national tests of the years 2015-2016. The overall correlation is higher between the model tests issued in 2017 and the national tests of the years 2017-2018 (0.87) and higher between the model tests issued in 2019 and the national tests of the year 2019 (0.9).

Model tests issued in the year 2000 and the national tests of the years 2011-2013 are aligned under the cognitive domains *knowing* and *applying* having high correlations, but not under *reasoning* where the correlation is -0.084. As for the content domains. The model and national tests under this period are well aligned under *Calculus* and *Statistics and Probability*, but not under *Algebra* under which a low correlation exists (0.375).

Model tests issued in the year 2000 and the national tests of the years 2015-2016 are aligned under the cognitive domains *knowing* and *applying* having high correlations, but not under *reasoning* where the correlation is -0.064. As for the content domains. The model and national tests under this period are also well aligned under *Calculus*, but not under *Algebra* under which a low correlation exists (0.484) and *Statistics and Probability* where the correlation is average (0.549).

Model tests issued in the year 2017 and the national tests of the years 2017-2018 are very well aligned under all cognitive domains and content domains (correlation ranging between 0.69 and 0.98) except *Statistics and Probability* content domain where the correlation is average (0.51).

Model tests issued in the year 2019 and the national tests of the year 2019 are very well aligned under all cognitive domains and content domains (correlation ranging between 0.98 and 0.99) except for *reasoning* cognitive domain where the correlation is very low (0.24).

4.3.5.2.2 – Evolution of the National Tests of the LH track Over the Years

In this section, comparison between the national tests of the years 2011-2013, 2015-2016, 2017-2018, and 2019 is in terms of content domains and cognitive domains to see the evolution of the national test is over time, then correlations are calculated to check their alignment.

Content Domains

Comparing the national tests of the years 2011-2013, 2015-2016, 2017-2018, and 2019 in terms of math topics, it is obvious, according to tables 2, 3, and 4 in the appendix Q, that the topic *definitions and representations* has close percentages among the tests of the four periods (53.93%, 50.48%, 53.53%, 52.49). The topics probability and statistics have also close percentages among the tests of the three periods with 17.86%, 16.35, 19.47% and 18.33% for *probability* and 12.14%, 9.615%, 7.97% and 10% for *statistics*. Discrepancies are found in the percentages of the two topics *equations* (12.5%, 23.56%, 19.03%, and 17.51%) and *simple and compound interest* (3.57%, 0%, 0%, and 1.66%). While the topic *equations* is being more emphasized, the topic *simple and compound interest* is being more neglected with the years.

Cognitive Domains

As to the cognitive domains, Tables 2, 3, and 4 in Appendix Q show that the national tests of the years 2011-2013, 2015-2016, 2017-2018, and 2019 have close percentages under the cognitive domain *knowing* (47.56%, 54.49%, 53.54 %, and 52.22 % respectively).

Close percentages also exist under the cognitive domain *applying* 41.61% for the national tests of the years 2011-2013, 36.06% for the national tests of the years 2015-2016, 35.84 % for the national tests of the years 2017-2018, and 42.2 % for the national tests of the year 2019. Percentages under reasoning, on the other hand, are very close between the national tests of all periods except for the tests of the year 2019 (10.83%, 9.455%, 10.62%, and 5.14% respectively).

In short, the national tests cover most of the content domains similarly. The percentages under all cognitive domains are also very close except under the domain *reasoning* where a noticeable decline in the percentage under this domain occurs in the national tests of the year 2019.

Correlations

Correlations were made between the national tests of the years 2011-2013 and the years 2015-2016, the national tests of the years 2011-2013 and the years 2017-2018, the national tests of the years 2011-2013 and the year 2019, the national tests of the years 2015-2016 and the years 2017-2018, the national tests of the years 2015-2016 and the year 2019, the national tests of the years 2017-2018 and the year 2019. When taking the correlations between the national tests, the *topic propositional calculus* had been excluded. The fact is that this *topic* was never addressed in any national test before the year 2017, and it was omitted since the year 2017. Therefore, keeping it in the national tests of the years before 2017 and omitting it in the tests of the years 2017 till 2019 naturally resulted in tables which are not of equal size and structure to be correlated. Moreover, keeping it will affect slightly the correlation results. To solve this problem, we had to remove that *topic* from Tables 2, 3, and 4 in Appendix Q when doing the correlations in this section.

Table 7: *Correlations Between the National Exams of the Years 2011-2019 and the Model Tests for Grade 12 LH Track*

	Overall Correlation	In terms of Cognitive Domains			In terms of Math Contents		
		K	A	R	Alg.	Calc.	S.P
OT11-13& OT15-16	0.945	0.957	0.967	0.873	0.974	0.972	0.972
OT11-13&OT17-18	0.9	0.9	1	1	1	0.9	0.8
OT11-13& OT19	0.97	0.98	0.99	0.49	0.85	0.97	0.99
OT15-16&OT17-18	0.97	0.97	0.98	0.93	0.98	0.99	0.85
OT15-16&OT19	0.97	0.99	0.99	0.87	1	0.98	0.98
OT17-18&OT19	0.95	0.96	0.99	0.66	0.99	0.96	0.86

K: Knowing
A: Applying
R: Reasoning
Alg.: Algebra
Calc.: Calculus (Numerical Functions)
S.P.: Statistics & Probability
MT1: model tests LHM1, LHM2, and LHM3
MT2: model tests LHM5, LHM6, and LHM7
MT3: model tests LHM8 and LHM9
OT11-13: official tests of the years 2011-2013
OT15-16: official tests of the years 2015-2016
OT17-18: official tests of the years 2017-2018
OT19: official tests of the year 2019

The overall correlations between the official tests of the years 2011-2013 and the national tests of the years 2015-2016, the national tests of the years 2011-2013 and the years 2017-2018, the national tests of the years 2011-2013 and the year 2019, the national tests of the years 2015-2016 and the years 2017-2018, the national tests of the years 2015-2016 and the year 2019, the national tests of the years 2017-2018 and the year 2019 are very high (0.945, 0.9, 0.97, 0.97, 0.97, and 0.95 respectively) which shows that the national tests of the years 2011-2013, 2015-2016, 2017-2018 and 2019 are consistent with each other.

In terms of cognitive domains, refer to table 7, the correlations between the national tests of the years 2011-2013 and the years 2015-2016, the national tests of the years 2011-2013 and the years 2017-2018, the national tests of the years 2011-2013 and the year 2019, the national tests of the years 2015-2016 and the years 2017-2018, the national tests of the years 2015-2016 and the year 2019, the national tests of the years 2017-2018 and the year 2019. are high positive under the cognitive domain *knowing* (ranging between 0.957 and 0.99) and *applying* (ranging between 0.967 and 1). These correlations reflect the consistency between the tests under each set under the cognitive domains *knowing* and *applying*. On the other hand, correlations under the domain *reasoning* are also high between all the tests of the studied sets except between the tests 2011-2013 and 2019 (0.49).

In terms of content domains, refer to table 7, the correlations between the national tests are very high under all domains *Algebra, Calculus,* and *Statistics and Probability*.

In fact, the correlations between the national tests show that the tests did not change over the years. They are steady in terms of content domains and cognitive domains coverage. This is despite the considerable change that happened in the model tests between the year 2000 and 2017. This in turn emphasizes the conclusion of the qualitative analysis of the LH track model and national tests that assumes that the model tests representing the curriculum were modified to align with the assessment and not based on a revision of the curriculum's objectives.

4.3.5.3 – Correlations Between the Model Tests and the Official Tests of Sessions 1 and 2

To study if there is any difference in the alignment between the curriculum and session-1 and session-2 official tests, comparison is made in this section between the model tests issued with the curriculum documents and the corresponding official session-1 and session-2 tests (of the years 2011-2013 and 2015-2016), between the model tests issued in 2017 and the corresponding official session-1 and session-2 tests (of the years 2017-2018), and between the model tests of the year 2019 and their corresponding official tests of the year 2019. Comparison is also made between sessions 1 and 2 official tests of the years 2011-2013 and 2015-2016, between sessions 1 and 2 official tests of the years 2017-2018, and between session 1 and 2 official tests of the year 2019 to see how consistent are the official tests of sessions 1 and 2.

According to the definition of a test item (Osta, 2017) adopted in this study, there are 49 test items in the model tests (LHM1. LHM2, and LHM3), 90 test items in the model tests (LHM5, LHM6, and LHM7), 66 test items in the model tests LHM8 and LHM9, 114 test items in five session-1 official tests of the years 2011-2016, 130 test items in five session-2 official tests of the years 2011-2016, 58 test items in four session-1 official tests of the years 2017-2018, 55 test items in four session-2 official tests of the years 2017-2018, 35 test items in session-1 official test of the year 2019, and 25 test items in session-2 official test of the year 2019. Tables Mod, ModLH5-7, ModLH8-9, OffExLH11, OffExLH21, OffExLH12, OffExLH122, LH191, and LH192 were converted to percentages to have a unified base for comparison.

Table 5 in Appendix Q presents the distribution in percentages of the test items in the model tests issued in the year 2000, session-1 official tests of the years 2011-2016, and session-2 official tests of the years 2011-2016 to their corresponding cognitive domains and math topics they address. Table 6 in Appendix Q presents the distribution in percentages of the test items in the model tests of the year 2017, session-1 official tests of the years 2017-2018, and session-2 official tests of the years 2017-2018 to their corresponding cognitive domains and math topics they address. Table 7 in Appendix Q presents the distribution in percentages of the test items in the model tests of the year 2019, session-1 official tests of the year 2019, and session-2 official tests of the year 2019 to their corresponding cognitive domains and math topics they address. The data in these tables are extracted from the Tables Mod, ModLH5-7, ModLH8-9, OffExLH11, OffExLH21, OffExLH12, OffExLH122, LH191, and LH192.

Model Tests Compared to Each of Sessions 1 and 2 of the National Tests

When comparing the model tests issued in the year 2000 to session-1 official tests of the years 2011-2016, and session-2 official tests of the years 2011-2016, it is obvious in table 5 in Appendix Q that the model tests and the corresponding official tests do not assess in a balanced way the different topics of the math curriculum. The percentages of the test items in the model tests are distributed over four out of six topics, while they are distributed in session-1 official tests of the years 2011-2016 over four topics, one which is different than the model tests, and in session-2 official tests over five topics. More than half of the test items are assigned to the topic *rational functions* (57.14 %) in the model tests, (51.31 %) in session-1 official exams of the years 2011-2016, and (53.47 %) in session-2 official exams of the years 2011-2016.

The topic *rational functions* has the highest percentage of test items in all tests. The topic *equations and inequalities* has the second highest percentage in the old model tests (26.53%) and session-1 official exams of the years 2011-2016 (21.49%), while it has the fourth highest percentage in session-2 official exams of the years 2011-2016 (13.46%) after the topic *probability* (15.38%) and *statistics* (13.85%).

Considerable discrepancies are obvious between the percentages of the test items in the model tests, and each of session-1 official exams of the years 2011-2016, and (53.47 %) in session-2 official exams of the years 2011-2016 under the topics *propositional calculus* (12.24%, 0%, and 0% respectively), *statistics* (0%, 7.9%, and 13.85% respectively), and *probability* (4.08%, 19.29%, and 15.38% respectively). Huge discrepancy is also obvious under the topic *equations* between session-2 official exams of the years 2011-2016 (13.46%) and each of the model tests (26.53%) and session-1 official exams of the years 2011-2016 (21.49%). As for the topic *simple and compound interest*, it only appears in session-2 official exams of the years 2011-2016. However, its percentage is very low (3.84%).

When comparing the model tests issued in the year 2017 to session 1 and official tests of the years 2017-2018, it is obvious in table 5 in Appendix Q that the model tests and session-1 official tests do not assess in a balanced way most of the topics of the math curriculum. The model tests cover all the five topics of the math curriculum, while session-1 official tests cover only 3 topics by not covering the topics *statistic and simple and compound interest*. On the other hand, the model tests cover all the five topics of the math curriculum and session-2 official tests of the years 2017-2018 cover four topics of the math. The topic propositional calculus is among the suspended lessons for these years. The topic *rational functions* has the highest percentages of test items in the model tests (46.67%), session-1 official tests of the years 2017-2018 (58.62%), and session-2 official tests of the years 2017-2018 (48.19%) constituting around half of the test items.

Then comes *probability* with 16.67% of the test items of these model tests, 18.97% of session-1 official tests of the years 2017-2018, and 20% of session-2 official tests of the years 2017-2018. Discrepancies exist under the topic *equations* having percentages of 13.33, 22.42 and 15.45 in the model tests, session-1, and session-2 official tests of the years 2017-2018 respectively. *Statistics* which has no test items in session-1 official tests, has somehow close percentages in the model tests and session-2 official tests of the years 2017-2018 (16.67% and 20% respectively). Lastly comes simple and compound interest with very small percentages 3.34 % in the model tests and no test items in session-1 and session-2 official tests of the years 2017-2018.

Comparing the model tests of the year 2019 and the official tests of the same year, it is obvious, according to table 6 in appendix Q, that each of the model tests and session-1 and session-2 official tests cover five topics of the math curriculum. While the model tests and session-1 official tests do not cover the topic *simple and compound interest*, this topic is included in session-2 official tests but the topic *statistics* is excluded in the tests of this session. The topics *rational functions*, *probability*, and *equations* are the topics of highest occurrence in the model tests, session-1 tests, and session-2 tests, while *statistics* which has a high percentage (17.14%) in session-1 official tests, is totally ignored in both the model tests and session-2 of the official tests of this year. The topic *simple and compound interest* has a very low percentage (4%) in session-2 but never occurred in the model tests or session-1 official tests of the year 2019.

In short, inconsistency exists at the content domain level between the model tests of the year 2000 and each of national tests sessions 1 and 2 of the year 2011-2016 under the *Algebra* and *Statistics and Probability* content domains. This inconsistency is also seen under the same content domains between the model tests issued in 2017 and session-1 national tests of the years 2017-2018, and under mainly the topic statistics between the model tests issued in 2019 and session-2 national test of the years 2019.

As to the cognitive domains, Tables 5 and 6 in Appendix Q show that very close percentages of test items exist among the model tests, session-1 official tests and session-2 official tests of each of the years 2011-2016, 2017-2018, and 2019 under the cognitive domain *knowing* ranging between 46% and 55.7%. There is also a balance under the domain *applying* between the model tests and session-1 and 2 official tests of the years 2011-2016 and of the years 2017-2018 having percentages ranging between 32.99% and 39.36%. A discrepancy exists under *applying* when comparing each of session-1 and 2 official tests of the year 2019 to their corresponding model test (40.95%, 45%, and 32.2% respectively). On the other hand, the model tests issued in the year 2000, the model tests of the year 2017, and the model tests of the year 2019 have more test item percentages under the domain *reasoning* than their corresponding official tests (17%, 14.26% and 16.64% for the mentioned model tests respectively) and less test items percentages under the domain *applying* than the official tests.

Session-1 National Tests Compared to Session-2 National Tests

Comparing sessions 1 and 2 of the official tests of the years 2011-2013, 2015-2016, 2017-2018, and 2019 in terms of math topics, it is obvious, according to tables 5 and 6 in the appendix Q, that the topic *rational functions* and *probability* has close percentages among all the tests. The topic *simple and compound interest* appears only in sessions 2. Discrepancies are found in the percentages of the topic statistics between session-1 and session-2 official tests with no specific pattern in its occurrence among sessions 1 and 2.

As to the cognitive domains, close percentages exist under the cognitive domain *applying* then knowing between session-1 and 2 official tests under each set of years. On the other hand, the domain *reasoning* is not consistent between sessions 1 and 2 of the official tests. Official tests of the years 2011-2016 have more percentage under *reasoning* in session 1 than session 2, while in the years 2017-2018 and 2019, session-2 tests have more percentage under *reasoning* than session-1 official tests.

The curriculum, as demonstrated in the model tests, and session-1 and 2 official tests emphasize the cognitive domain *knowing* over *applying* and *reasoning*. However, more attention is given by the model tests to the domain *reasoning* than the official tests over the mentioned periods of time.

Correlations

In addition to percentages, correlations were calculated between the respective numbers in Tables 5 and 6 in Appendix Q. Correlations were made between the model tests issued with the curriculum documents and the corresponding official session-1 and session-2 tests (of the years 2011-2016), between the model tests issued in 2017and the corresponding official session-1 and session-2 tests (of the years 2017-2018), and between the model tests of the year 2019 and their corresponding official tests (year 2019). Correlations were calculated between sessions 1 and 2 official tests of each of the years 2011-2016, 2017-2018, and 2019.

According to Table 8, the correlations between the model tests issued with the curriculum and session-1 official tests of years 2011-2016, the model tests and session-2 official tests of years 2011-2016, the model tests and session-1 official tests of years 2017-2018, the model tests and session-2 official tests of years 2017-2018, the model tests and session-1 official test of year 2019, and the model tests and session-2 official tests of the year 2019 are (0.755, 0.738, 0.775, 0.923, 0.856, and 0.869 respectively. This shows that the tests under each compared set are consistent. However, this consistency increases between model tests issued in 2017 and 2019 with their corresponding sessions 1 and 2 official tests.

Table 8: *Correlations Between Sessions 1 and 2 of the Official Exams of the Years 2011-2016, 2017-2018, and 2019, and the Model Tests for Grade 12 LH Track*

	Overall Correlation	In terms of Cognitive Domains			In terms of Math Contents		
		K	A	R	Alg.	Calc.	S.P
OT11-16(1)&MT1	0.755	0.916	0.778	-0.083	0.444	0.875	0.819
OT11-16(2)&MT1	0.738	0.852	0.754	-0.068	0.441	0.889	0.388
OT17-18(1)&MT2	0.775	0.622	0.958	0.915	0.616	0.963	0.214
OT17-18(2)&MT2	0.923	0.842	0.998	0.833	0.798	0.991	0.748
OT19(1)&MT3	0.856	0.9	0.933	0.051	0.892	0.869	0.811
OT19(2)&MT3	0.869	0.883	0.99	0.395	0.98	0.893	0.75
OT11-16(1)& OT11-16(2)	0.931	0.918	0.969	0.931	1	0.958	0.914
OT17-18(1)& OT17-19(2)	0.911	0.884	0.964	0.847	0.967	0.982	0.645
OT19(1)& OT19(2)	0.837	0.633	0.962	0.936	0.784	0.994	0.237

K: Knowing
A: Applying
R: Reasoning
Alg.: Algebra
Calc.: Calculus (Numerical Functions)
S.P.: Statistics & Probability
MT1: model tests issued with the curriculum documents
MT2: model tests issued in the year 2017
MT3: model tests issued in the year 2019
OT11-16(1): session-1 official tests of the years 2011-2016
OT11-16(2): session-2 official tests of the years 2011-2016
OT17-18(1): session-1 official tests of the years 2017-2018
OT17-18(2): session-2 official tests of the years 2017-2018
OT19(1): session-1 official tests of the year 2019
OT19(2): session-2 official tests of the year 2019

In terms of cognitive domains, refer to Table 8, the correlations under the domains *knowing* and *applying* between the model tests and sessions 1 and 2 national tests over all the periods are very high reflecting an alignment ranging between good and high between the model tests and each of sessions-1 and 2 national tests in general. This is also the case under the domain *reasoning* between the model tests of the year 2017 and each of sessions 1 and 2 national tests. However, the correlations under reasoning between the model tests issued in the year 2000 and session1- and session-2 national tests of the years 2011-2016 are very low negative (-0.083 and -0.068) and between the model tests of the year 2019 and each of sessions 1 and 2 of the year 2019 are also very low (0.051 and 0.391 respectively). This shows that no alignment exists under reasoning between the model tests and session-1 and 2 national tests of the years 2011-2016 and 2019.

In terms of math domain *Algebra*, refer to Table 8, the correlation between the model tests issued in the year 2000 and sessions 1 and 2 of the national tests of the years 2011-2016 is very low (0.444 and 0.441% respectively). This is because under this math domain, the topic *propositional calculus* is only addressed in the old model tests, while the national tests cover only the other topic *equations*. Correlations under *Algebra* is high between the model tests and sessions 1 and 2 of the years 2017-2018 and 2019 is very high reflecting a very good alignment between them. Correlations under *Calculus* is very high between all sets which reflects a very good alignment between the model tests and sessions 1 and 2 national tests under *Calculus*. On the other hand, correlation under *Statistics and Probability* between the model tests issued in 2000 and session-2 national test of the years 2011-2016 is 0.388%. Similarly, the correlation between sessions 1 of the official tests of each of the years 2017-218 is very low (0.214). This shows that the tests under the mentioned sets are not aligned under *statistics and probability*.

In short, alignment is good between the model tests and national tests sessions 1 and 2 over *knowing* and *applying* cognitive domains. Problems exist at the *reasoning* domain between the model tests and national tests sessions 1 and 2 of the years 2011-

2016 and between the model tests and session 2 national tests of the year 2019. Alignment is very good under the content *domains* for all sets of years studied except under *Algebra* between the model tests and sessions 1 and 2 national tests of the year 2011-2016 and under *probability and statistics* between sessions 2 and model tests of the years 2011-2016 and 2017-2018.

Comparing sessions 1 and 2 national tests, overall correlations between sessions 1 and 2 national tests of years 2011-2016, sessions 1 and 2 national tests of years 2017-2018, and sessions 1 and 2 national tests of the year 2019 are 0.931, 0.911 0.837 respectively. This shows that sessions 1 and 2 national tests are very well aligned. More specifically, the correlations under all cognitive domains between sessions 1 and 2 are high positive ranging between 0.633 and 0.964, so sessions 1 and 2 are well aligned under all cognitive domains. As to the content domains, correlations are also very high positive between sessions 1 and 2 national tests ranging from 0.784 and 1 except for sessions 1 and 2 of the year 2019 under Statistics and Probability (0.237).

In conclusion, the quantitative analysis of the LH model and national tests shows that the alignment between the model and national tests of this track increases over time. However, the alignment between the national tests themselves is almost stable over all the years. This added to the noticeable change in the percentages of test items under all the *math topics* of the model tests of the year 2019 becoming closer to the percentages of the national tests of the previous years reflects that the modifications made to the curriculum were made to be aligned with assessment and not based on a revision of the curriculum and its objectives.

As to the cognitive domains, the process objectives of the LH track, as presented in the curriculum document, emphasize mathematical reasoning, solving mathematical problems, and communicating mathematically (ECRD, 1997a). *Mathematical reasoning*, as defined by TIMSS Advanced 2015 Assessment Framework (Appendix J), involves analyzing, synthesizing, and generalizing to solve problems, and justifying through mathematical arguments or proofs. This objective is reflected in the LH official tests in a very low percentage (9.62% of the total test items). Solving mathematical problems range between *applying* and *reasoning* based on the complexity and familiarity of the problems. Problems under *applying* typically reflect standard types of problems that are familiar to students (Mullis & Martin, 2014), while they are more complex requiring logical and systematic thinking under *reasoning*. This objective is reflected in the official tests in 9.62% of the test items under *reasoning* and 38.8% under *applying*, so only routine problems that require direct application of knowledge and procedures are emphasized in the official tests for this objective.

4.4. Analysis of the LS Model and National Tests

This section includes the analysis of the LS track model tests and national official tests.

4.4.1. LS Track Content

The process objectives of the LS track of the secondary cycle are: mathematical reasoning, problem solving, and communication, while the domain objectives are: spacial, numerical and algebraic, calculus, and statistics & probability. The content of these domains are distributed over the three years of the LS secondary cycle and not necessarily all included in each.

Mathematics is assigned two sessions per week for the LS track of grade 12. These constitute 150 sessions per academic year. Table 9 shows the five domains of this grade level with their main content and the allocated time for each (refer to Appendix B for the details of contents of the LS track).

Table 9: *The Math Topics in the LS Track of the Third Secondary Year*

Code	Math Topics	Allocated Time
1	**ALGEBRA**	35 hours
1.1.	Foundations	8 hours
1.1.1.	→ Binary operations	
1.1.2.	→ Structure of group	
1.2.	Literal and numerical calculations	10 hours
1.2.1.	→ Combinations: definition, notation, binomial formula, Pascal's Triangle	
1.3.	Equations & Inequalities	7 hours
1.3.1.	→ System of linear equations (m×n): definition, Elementary operations on the rows, Gauss Method	
1.4	Numbers	10 hours
1.4.1.	→ Module and argument of a complex number, properties	
1.4.2.	→ Trigonometric and exponential forms of a complex number	
1.4.3.	→ Geometric interpretation of addition and multiplication of complex numbers and the passing to the conjugate	
1.4.4.	→ De Moivre's formula, applications	
2	**GEOMETRY**	15 hours
2.1.	Classical study	
2.1.1.	→ Components of the vector product. Mixed product	
2.1.2.	→ Equation of a plane and of a straight line in space	
2.1.3.	→ Orthogonality of two straight lines, of a straight line and a plane; perpendicular planes	
2.1.4.	→ Parallelism of straight lines and of planes	
2.1.5.	→ Distance from a point to a plane, to a straight line	
3	**CALCULUS**	65 hours
3.1.	Definitions and Representations	25 hours
3.1.1.	→ Inverse functions	
3.1.2.	→ Inverse trigonometric functions	
3.1.3.	→ Natural (Naperian) logarithmic functions	
3.1.4.	→ Exponential functions	

3.2.	Continuity and derivation	15 hours
3.2.1.	→ Image of a closed interval by a continuous function	
3.2.2.	→ Derivative of composite functions	
3.2.3.	→ Derivative of an inverse function	
3.2.4.	→ Second derivative, successive derivatives	
3.2.5.	→ L'Hopital's rule	
3.3.	Integration	15 hours
3.3.1.	→ Integral: definitions, properties	
3.3.2.	→ Rules of integration	
3.3.3.	→ Application of the integral calculations	
3.4.	Differential equations	10 hours
3.4.1.	→ Definition	
3.4.2.	→ Equations in separable variables	
3.4.3.	→ Linear first order equations with constant coefficients.	
3.4.4.	→ Linear second order equations with constant coefficients	
4	**TRIGONOMETRY**	5 hours
4.1	Circular functions	
4.1.1.	→ Study of the circular functions of the form acos(bx+c) and asin(bx+c)	
5	**STATISTICS AND PROBABILITY**	30 hours
5.1.	Statistics	10 hours
5.1.1.	→ Measure of central tendency and measures of variability of a distribution of one (continuous or discrete) variable	
5.2	Probability	20 hours
5.2.1.	→ Conditional probability: definition, independence of two events	
5.2.2.	→ Formula for all probabilities	
5.2.3.	→ Random real variable, law of associated probability, distribution function, characteristics	
5.2.4.	Bernoulli variable	
5.2.5.	Binomial law	

These topics were reduced by a decision taken by MEHE and ECRD and after establishing the curriculum and issuing the books, because it was found during implementation that they were too heavy to be covered in the allocated time. The omitted *topics* include: "*binary operations, structure of group, system of linear equations (m × n): definition, elementary operations on the rows, gauss method, inverse* *trigonometric functions, logarithmic function to the base a, successive derivatives, Bernoulli variable*, *binomial law"*. The *topics circular functions* and *statistics* were added to these omitted *topics* in the academic year 2018-2019.

4.4.2. Qualitative Analysis of the LS Model Tests

Ten model tests for the LS track were issued throughout the years. Four model tests (LSM1, LSM2, LSM3, and LSM4) were issued in the year 2000 with the curriculum documents of the 1997 reformed curriculum in the evaluation guide (referenced in Appendix D). Four model tests (LSM5, LSM6, LSM7, and LSM8) were issued in the year 2017, and two model tests (LSM9 and LSM10) were issued in the year 2019. Refer to Document II- Appendix F which presents a sample model test: LSM5. In this part of the qualitative analysis, the change that happened in the model tests of the LS track from the date the first model tests were issued to the year 2017 and then 2019 is analyzed. This gives insight about the developers' point of view of the importance of the curriculum's different topics.

The Evaluation Guide (referenced in Appendix D) contains the model tests issued in 2000 (refer to Document I- Appendix F which presents a sample model test: LSM1). The following analysis is extracted from Safa's (2012) analysis of the model tests.

The math topics in the model tests for the LS track are presented in Table 1 in Appendix R. LSM1, LSM3, and LSM4 are made of three parts each, while LSM2 consists of four parts (Safa, 2012).

LSM1 has problems covering the domains: *Algebra, Calculus*, and *Geometry*. However, these problems are not comprehensive of all the topics under the mentioned domains. The model test LSM2 is made up of four parts based on the domains: *Algebra, Calculus*, and *Statistics and Probability*: two parts covering Algebra, one covering *Statistics and Probability*, and the last covering *Calculus*. LSM2 also does not cover all the *topics* under the *domains* tackled. LSM3 *involves* three parts: one on *Algebra*, one on *Statistics and Probability*, one part on *Calculus and Algebra*. No test items occur on Geometry. It is noted that the part on *Algebra* includes test item linked in content to the domain *Statistics and Probability*, and the test items on *numbers* are integrated to the test items on *Calculus*. LSM4 is made up of three parts based on the domains *Statistics and Probability, Algebra*, and *Calculus*. The topics covered in these model tests issued in the year 2000 *are numbers, geometry, literal and numerical calculations, definitions and representations, continuity and differentiation, integration, differentiation, statistics* and *probability* with different levels of occurrence in each model test. The occurrence of the *topic statistics* is always limited to objectives from grade 11, while the occurrence of the *topic literal and numerical calculations* include test items that tackle this *topic* directly not being parts of integrated with other *topics*.

Table 1 in Appendix R presents the math topics covered in the model tests issued in the years 2017 and 2019. These model tests are all four-problem tests covering the *domains*: *Algebra, Geometry, Calculus,* and *Statistics and Probability*. The problems on *Algebra* in these model tests cover the *topic numbers*. The *topic literal and numerical calculations* occurs only in the tests LSM5, LSM7, and LSM9 under the problems on *Probability and Statistics* included indirectly on test items on *probability*. The problems on *Calculus* in the tests LSM5, LSM6, and LSM8 are limited to the topics *definitions and representations* and *continuity and differentiation*, while the topic *integration* occurs in the model tests LSM7, LSM9, and LSM10 and the topic differential equations occurs only in the model test LSM7. The problems on *Probability and Statistics* in all these model tests do not include any test item in the topic *statistics*. The topic *circular functions* does not appear in any of the model tests before the year of its omission from the required lessons for this track, 2018.

Table 2 in Appendix R displays the grade points allocated to the math topics in the model and official tests for the LS track. The parts on *Calculus* occurred on all the model tests and are allocated the highest grades that range from 7 to 9 grades, having a fixed grade point (8 points) in all the model tests issued in 2017 and 2019. The parts *on Algebra*, *Geometry*, and *Statistics and probability* are allocated similar grades that range from 4 to 6 grades in all the model tests but having a fixed grade point (4 points) in all the model tests issued in 2017 and 2019.

The qualitative analysis of the model tests shows that the developers' view regarding the importance of some topics has slightly changed over time. The topic *statistics* which appear in the model tests before 2018, although appearing tackling objectives of grade 11, is considered unimportant over time; therefore, it was omitted in the year 2018. Moreover, the topic *literal and numerical calculations,* which has test items on the *topic* covering different objectives, occurs at the basic level under the *probability* problems as means to calculate the *probability* of certain events. On the other hand, t*he topic circular functions* which has no occurrence in any model tests, was omitted from the required lessons in the year 2018.

The change in the model tests is investigated further in terms of content and cognitive domains in the Qualitative Analysis of the LS Model and Official Tests' Test Items section.

4.4.3. Qualitative Analysis of the LS Official Tests

The sixteen national math tests (sessions 1 and 2) of the years 2011 to 2019 for the LS track are analyzed in this section. The official tests of year 2014 were not put in public since Lebanese teachers were on strike, and the committee responsible for correcting and grading the national tests abstained. Appendix I presents a sample national test-LS131 which is the official test of the 1st session of the year 2013).

Table 1 in Appendix R displays the math topics covered by the official tests of the years starting from 2011 till 2019. Each official test consists of four problems covering the four content domains of this track.

According to table 1, all official tests contain test items on the topics *literal and numerical calculations, numbers, definition and presentations, continuity and differentiation, integration*, and *probability* except for the official test LS131 which doesn't have test items on the topic *literal and numerical calculations* and official tests LS151, LS161, LS162, and LS192 which do not have test items on *integration*. All the test items on the topic *literal and numerical calculations* are basically integrated in the *probability* topic and not direct questions on this topic. The official test LS192 is the only test that has test items on differential equations. The topic *circular functions* is never included in any official test before 2018 when it was excluded from the required lessons.

Table 2 in Appendix R displays the grade points allocated to the math topics in the national tests for the LS track. The parts on *Calculus* are allocated a fixed number of grades in all official tests (8 grade points) presenting the highest grade. The parts on *Algebra, Geometry,* and *Statistics and probability* are allocated similar grades (4 grade points) in all the official tests.

The qualitative analysis of the national math tests shows that the scale of importance given by the curriculum to most of the topics, as reflected by the content and number of hours allocated to each, differ from the scale of importance shown in the tests to these topics, as reflected by the test items occurrence and grade distribution. While the topics *literal and numerical calculations* and *numbers* have an equal number of allocated hours per year (10 hours) (refer to Table 9 in the section 4.4.1), the first does not always appear in official tests, but when it does, it appears as means of calculating some test items under *probability*. On the other hand, the *topic numbers* always appears in official tests as a problem with several test items. Moreover, the *topic differential equations,* although allocated a considerable number of hours per year (10 hours), rarely occurs in the official tests. Similarly, the *topic statistics* which is also allocated 10 hours has no occurrence in any official test before being omitted in the year 2018.

4.4.4. Qualitative Analysis of the LS Model and Official Tests' Test Items

This section aims at analyzing qualitatively and comparing the model and official tests to check whether the 2017 issued model tests have reflected more the previous official tests or have impacted the subsequent national tests, and how. The test items of the model tests and official tests are studied. The analysis will be based on the topics covered (refer to Table 9 in the section 4.4.1). Out of the topics constituting the math curriculum at this track, the topic *statistics* occurs only in the model tests. Analysis of the old model tests and their examples are retrieved from Safa's study (2013).

Literal and numerical calculations

Table 3 in Appendix R displays the occurrences of test items on the *topic literal and numerical calculations* as well as the tests in which they appear.

Test items on *literal and numerical calculations* require mainly finding the number $^{n}C_{p}$ of all the combinations of p elements of a set of n elements. Test items under this topic occur in the model tests LSM1, LSM3, LSM5, LSM7, and LSM9 where the test items go under the cognitive domain *knowing*. The test items in LSM3, LSM5, and LSM7 are integrated within questions on probability to be used in calculating the probability of an event, but this is not the case for the model test LSM1 where the test items are direct. The following is the part on *literal and numerical calculations* retrieved from the model test LSM1.

> In a computer club of a school, there are four boys, numbered from 1 to 4, and five girls, numbered from 1 to 5. The manager of the club wishes to form a committee of three members.
> 1) How many committees of boys can be formed? Deduce the possible number of committees having at least one girl.
> 2) How many committees having only one boy and a member numbered 2 can be formed?

Figure 12: *Sample Problem 1 on Literal and Numerical Calculations from the Model Test LSM1*

The following is a part on this *topic* from the model test LSM5.

U1 and U 2 are two boxes so that:
U1 contains 10 balls: 6 red and 4 black.
U2 contains 10 balls: 5 red and 5 black.
A die numbered 1 through 6 is rolled. If this die shows 1 or 2, then two balls are randomly selected at a time from the box U1. Otherwise, two balls are randomly selected one after another with replacement from the box U2.
Consider the following events:
U1: The selected box is U1.
U2: The selected box is U2.
R: The selected balls are red.
1) Calculate P(R/U1).

Figure 13: *Sample Problem 2 on Literal and Numerical Calculations from the Model Test LSM5*

The topic on *literal and numerical calculations* occurs in all official tests except for LS131. The occurrence of this topic in all test item is integrated within questions on probability. All the test items under this topic in Table 3 in Appendix R go under the cognitive domain *knowing*. The test items in the official tests are all similar to the test item of the model test LSM5 mentioned sample 2.

It can be noted from what is mentioned that the official tests are almost aligned with the model tests under this topic. However, six objectives under this topic were never addressed in both the model and the official tests.

Therefore, the model tests issued in 2017 and 2019 did not take in to consideration the revision of the curriculum’s objectives under this topic to have the assessment more aligned with the curriculum. Instead, the norm of having this topic in its simplest forms and objectives and which was developed over the years in the official tests, became a part of the curriculum as represented in the model tests.

Numbers

Table 4 in Appendix R displays the occurrences of test items on the *topic numbers* as well as the tests in which they appear. The topic *numbers* involves the study of complex numbers. Most test items require moving from one form to another of a complex number (algebraic, trigonometric, and exponential), calculating and using the properties of each of the modulus and argument of a complex number in finding relations and solving geometric problems, and using De Moivre's formula.

The following is the part under *numbers* retrieved from the model test LSM4.

1) Solve in C the equation $z^2 + \sqrt{3}\, z + 1 = 0$ (E)
We call z_1 and z_2 the roots of the equation (E), the root z_1 is the one that has a positive imaginary part.
2) In the orthonormal plane let A_1 and A_2 be the points representing z_1 and z_2 respectively. Let A be the point representing $z_A = i$.
a- Prove that the points A_1, A_2 and A are on a circle for which you should determine the center and the radius.
b- Calculate $|z_A - z_1|$. Deduce the type of triangle OAA_1.
c- Specify the type of the quadrilateral OAA_1A_2.

Figure 14: *Sample Problem 1 on Numbers from the Model Test LSM4*

The following is the part under *numbers* retrieved from the model test LSM9.

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points $M(Z)$, $M'(Z')$, $I(1+2i)$, and $E(5)$. The complex numbers Z and Z' are so that: $Z' = 2iZ + 5$.

1- a) If Z is pure imaginary, prove that Z' is real.
   b) If $Z' = 5i\sqrt{3}$, write Z in exponential form.

2- a) Prove that $Z_{\overrightarrow{IM'}} = 2iZ_{\overrightarrow{IM}}$.
   b) Express IM' in terms of IM and show that $(\overrightarrow{IM},\overrightarrow{IM'}) = \frac{\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$.
   c) Deduce that if M moves on the line (Δ) with equation $(x = 1)$, then M' moves on a line whose equation is to be determined.

3- Let $Z = x + iy$ and $Z' = x' + iy'$ where $x, y, x', and\ y'$ are real numbers.
   a) Express x' and y' in terms of y and x.
   b) If $x + 2y = 5$, prove that (MM') is parallel to $(y'y)$. Then use the result $(\overrightarrow{IM},\overrightarrow{IM'}) = \frac{\pi}{2} + 2k\pi$ to construct M' when $x + 2y = 5$.
   c) If M' moves on the circle (C') with center E and radius 2, prove that M moves on the circle (C) with center O and radius 1.

Figure 15: *Sample Problem 2 on Numbers from the Model Test LSM9*

It is noticed from the samples 1 and 2 and table 4 Appendix R that the model tests issued in 2017 and 2019 are more comprehensive of the *topic* and include more complex problems on this *topic* than the previous model tests. The model test LSM4 includes only one test item on *numbers*, while the model test LSM1 doesn't include any test item on *numbers*. On the other hand, all the other model tests include a whole problem on *numbers* with several test items. More diverse test items are included in the model tests of the years 2017 and 2019. Moreover, test items under the topic *numbers* differ in form and structure in the model tests issued in the years 2017 and 2019. The model test LSM6 is a True/False problem-type, while LSM10 is a multiple-choice problem-type. Problems on *numbers* in the remaining model tests are short-answer and open-ended problem-types.

On the other hand, according to table 4 Appendix R, test items under this *topic* in the official tests are diverse. Many test items keep occurring almost in all official tests, but these test items are based on the different conditions and situations given. Multiple-choice problems occur only in official tests LS112 and LS122, while problems of other official tests are of short-answer and open-ended type.

The following is the part under the topic numbers retrieved from the official test LS162.

In the complex plane referred to an orthonormal system $(O; \vec{u}, \vec{v})$, consider the points A(1), M(z) and M'(z') so that: $z' = (1-i)z + i$ with $z \neq 1$.

1) a- Verify that $z' - 1 = (1-i)(z-1)$.

b- Verify that $AM' = AM\sqrt{2}$. Deduce that if M moves on the circle with center A and radius $\sqrt{2}$, then M' moves on a circle (C) whose center and radius should be determined.

c- Prove that: $(\vec{u}; \overrightarrow{AM'}) = -\frac{\pi}{4} + (\vec{u}; \overrightarrow{AM}) + 2k\pi$ with $k \in \mathbb{Z}$.

d- Compare $|z' - z|$ and $|z-1|$, then prove that the triangle AMM' is right isosceles.

2) Let $z = x + iy$ and $z' = x' + iy'$ where x, y, x' and y' are real numbers.

a- Express x' and y' in terms of x and y.

b- Verify that if M' moves on a line (D) with equation $y = x$, then M moves on a line (Δ) to be determined.

Figure 16: *Sample Problem 3 on Numbers from the Official Test LS162*

It is noticed that the official tests are more aligned with the model tests of the years 2017 and 2019 than with the previous model tests. The content is almost similar in all the tests. However, both the model tests of the years 2017 and 2019 and the official tests contain more diverse test items that require high order thinking skills and are more comprehensive of the objectives of this topic. Writing in exponential form and calculating and using the properties of argument are emphasized in the official tests and model tests issued in 2017 and 2018 but ignored in the previous model test, while the objective on writing in trigonometric form does not occur in official tests, and representing a complex number graphically occurs only in the model test LSM9.

The model tests of the years 2017 and 2019 have almost same test items as all the official tests, while they differ from the model tests issued in the year 2000. This reflects that the model tests, representing the curriculum, were modified to be aligned with the assessment.

Classical Study (Geometry)

Classical Study is a topic classified under Geometry. It involves the use of the knowledge of plane geometry and space geometry, analytical geometry, and visualizing geometric elements and sketching 3D drawings, to find equations of straight lines and planes in the space and to study their relative positions. Table 5 in Appendix R displays the occurrences of test items on the *topic classical study* as well as the tests in which they appear. The tests items under *classical study* occur with no specific pattern in the model and official tests.

The test items on *classical geometry* occurred only in the model tests LSM1 and LSM3. In both tests, no figure was shown. The following is the part under the topic *classical study* retrieved from the model test LSM3.

The space has the orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$.

Consider the planes (P) and (Q) of equations:

$(P) \quad : 2x + 2y - z + 5 = 0$

$(Q) \quad : 2x + y + 6z - 8 = 0$

1) Prove that (P) and (Q) are orthogonal. *2) Deduce the distance from the point A (2, 1, 4) to the line (D), intersection of the two planes (P) and (Q).*

3) Give an equation of the line (D).

4) Use the value found in question 2) to calculate the coordinates of the point H, orthogonal projection of the point A on the line (D).

Figure 17: *Sample Problem 1 on Classical Geometry from the Model Test LSM3*

On the other hand, the model tests of the years 2017 and 2019 have similar structure and content under this topic. They all have more questions on this topic which are comprehensive of the topic and more diverse. Moreover, no figures are shown in any of them. The following is the part under the topic *classical study* retrieved from the model test LSM8.

In the space of an orthonormal system $(O ; \vec{i}, \vec{j}, \vec{k})$, consider the plane (P) with equation : $x + y + z - 1 = 0$, and the line (d) with parametric equations $\begin{cases} x = -t - 1 \\ y = t + 5 \\ z = 3t + 9 \end{cases} (t \in \mathbb{R})$,

Let H (1, 1, -1) be a point on (P).

1) Determine A, the common point between (d) and (P).

2) Let (Δ) be the line passing through H and perpendicular to the plane (P).

a- Write a system of parametric equations of (Δ).

b- Verify that E (2,2,0) is the intersection point between (Δ) and (d).

c- Calculate the angle formed by (d) and (P).

3) Let (Q) be the plane passing through O and the point F (2,1,0) and perpendicular to(P).

a- Write an equation of the plane (Q).

b- Let M(x,y,z) be a variable point on (Q).

Prove that the volume of the tetrahedron MEAH is constant.

c- Deduce that the two planes (Q) and (EAH) are parallel

Figure 18: *Sample Problem 2 on Classical Geometry from the Model Test LSM8*

As for the official tests, test items under *classical study* are of similar content and structure to those of the model tests. Only one test LS192 contains a figure. No pattern in the occurrence of test items is obvious. The test items are very diverse and differ clearly between the tests. The following is the part under the topic *classical study* retrieved from the official test LS151.

The space is referred to a direct orthonormal system $(O ; \vec{i}, \vec{j}, \vec{k})$.

Consider the plane (P) with equation $x-2y+2z-6=0$ and the two lines (d) and (d') defined as:

$(d):\begin{cases} x=m+1 \\ y=2m+1 \\ z=2m+2 \end{cases}$ and $(d'):\begin{cases} x=2t \\ y=5t-3 \\ z=4t \end{cases}$ (m and t are real parameters)

1) Find the coordinates of A, the intersection point of line (d) and plane (P).

2) Verify that A is on line (d'), and that (d') is contained in plane (P).

3) a- Write an equation of plane (Q) determined by the lines (d) and (d').

b- Show that the two planes (P) and (Q) are perpendicular.

4) Let $B(1;1;2)$ be a point on (d).

Calculate the distance from point B to line (d').

Figure 19: *Sample Problem 3 on Classical Geometry from the Official Test LS151*

Definitions & Representations

One problem under *Calculus* occurs in each model and official test. It is the most important topic as it is assigned the highest grade points. This problem involves all the topics under this *domain* in an integrated form. *Definition and representations* is one of these topics. It involves the study of exponential and trigonometric functions in terms of: Domain, variation, limits and asymptotes, graphical representations, derivative and primitive, composite functions and inverse functions (Safa, 2013). Table 6 Appendix R displays the test items on *definition and representations* and the tests in which they appear.

This *topic* occurs in all the model tests in a similar structure where no tables or graphs are presented. It is noted that the model tests of the years 2017 and 2019 have more varied test items and include composite functions which have no test items in previous old model tests nor the official tests. It is also noted that the model tests of the years 2017 and 2019 contain all the possible test items that might occur under this topic. Therefore, they are very comprehensive of the topic and include test items that are challenging requiring a good level of reasoning.

The following is a question retrieved from the model test LSM3 under the *topic Definitions & Representations*.

f is the function defined on]0; + ∞ [by $f(x) = \frac{1}{2}x - 1 + \frac{x}{\ln x}$.

1) *Study the limits of f at 0 and at + ∞. Prove that the line (Δ):* $y = \frac{1}{2}x - 1$ *is an oblique asymptote of the graph (C) of f. Specify the relative positions of (C) and (Δ).*

2) *a- Calculate f'(x) and then f''(x). Deduce the variations of f.*
b- Calculate $f'(e^{1.5})$ *and deduce the sign of f'.*
c- Make the table of variations of f.

Figure 20: *Sample Problem 1 on Definitions & Representations from the Model Test LSM3*

The following is a question retrieved from the model test LSM10 under the *topic Definitions & Representations*.

Consider the function f defined over $\mathbb{R}$ as: $f(x) = (2 - x)e^x + x - 2$, and denote by (C) its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$.

1- a) Determine $\lim\limits_{x \to +\infty} f(x)$ and calculate $f(2.5)$.
   b) Determine $\lim\limits_{x \to -\infty} f(x)$ and prove that the straight line (d) with equation $y = x - 2$ is an asymptote to (C).
   c) Study the relative positions of (C) & (d).

2- a) Verify that $f'(x) = g(x)$, then set up the table of variations of f.
   b) Show that $f(\alpha) = \frac{(\alpha-2)^2}{\alpha-1}$.

Figure 21: *Sample Problem 2 on Definitions & Representations from the Model Test LSM10*

The following is a question from the Calculus problem retrieved from the official test LS152 under the *topic Definitions & Representations.*

A- Let g be the function defined on $]0;+\infty[$ as $g(x) = x^3 - 1 + 2\ln x$.
1) Determine $\lim\limits_{x \to 0} g(x)$ and $\lim\limits_{x \to +\infty} g(x)$.
2) Calculate $g'(x)$ then set up the table of variations of g.
3) Calculate $g(1)$ then deduce the sign of $g(x)$ according to the value of x.

Figure 22: *Sample Problem 3 on Definitions & Representations from the Model Test LS152*

The official tests vary a little in structure in three tests LS172, LS182 and LS191 where in a part of the problem a graph is given. The content is similar between the official and the model tests. Moreover, although similar forms of test items might frequently occur, they vary between the three cognitive domains: *knowing, applying* and *reasoning* as the function and context differ.

Continuity and Differentiation

The objectives of this *topic* are regarded as part of the *topic differentiation*. They include studying: Image of a closed interval by a continuous function, derivative of composite and inverse functions, successive derivatives, and L'Hopital's rule. Table 7 in Appendix R displays the test items on this topic and the tests where they occur. All model and official tests include test items either targeting this topic directly or indirectly. Similar content and structure exist under test items of this topic.

The following is a question retrieved from the model test LSM1 under this topic.

Let f be the function defined by $f(x) = lnx - mx$ where m is a non-zero real number. Let C_m be the graph of f in an orthonormal system.
1) For which values of m, the function f is strictly monotone increasing?
2) For which values of m, C_m has a maximum or a minimum?

Figure 23: *Sample Problem 1 on Differentiation from the Model Test LSM1*

The following is part of a problem under *Calculus* retrieved from the model test LSM7 targeting this topic.

In what follows, suppose that $f(x) = x\,e^{-x} + x$, and f is defined over R.

3) **a-** Verify that $f'(x) = g(x)$ and set up the table of variations of f.
b- Discuss according to x the concavity of (C).
c- Determine the point E on (C) where the tangent (T) is parallel to (d).

Figure 24: *Sample Problem 2 on Differentiation from the Model Test LSM7*

The official tests have test items that are similar to the test items of the model tests issued in the years 2017 and 2019. In fact, this *topic* becomes more limited in the official tests and the model tests of the years 2017 and 2019 to test items that routine questions. Therefore, questions that might appear to require *reasoning* under this *topic*, when being routine, they require *applying* instead.

Integration

Integration which is also a topic under *Calculus* domain, includes different methods of calculating integrals to find the primitive of a function to calculate areas and volumes. This topic occurs in the model tests and the official tests as part(s) under the problem on *Calculus*. Table 8 in Appendix R displays the test items on *integration* and the tests they appear in.

The model tests issued in the year 2000 address the topic *integration* in the tests LSM1, LSM2, and LSM4. The questions are direct and address limited objectives.

The following is a question under this *topic* retrieved from the model test LSM1.

Let f be a function defined by $f(x)=\ln x - mx$ where m is a non-zero real number. Let C_m be the graph of f in an orthonormal system

- *Calculate the area of the domain limited by C_1, the lines $y = -x$, $y = 1$ and $x = e$.*

Figure 25: *Sample Problem 1 on Integration from the Model Test LSM1*

The model tests of the years 2017 and 2019 address the *topic integration* in the tests LSM7, LSM9, and LSM10 in a similar way as means to calculate the area of the region bounded by the curve and given lines.

The following is a question retrieved from the model test LSM10 under the topic integration.

Consider the function f defined over $\mathbb{R}$ as: $f(x) = (2 - x)e^x + x - 2$, and denote by (C) its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$.

5- Denote by A the area bounded by (C), (d), (yy') and the line with equation $x = \alpha$. Show that $A = \frac{6-4\alpha}{\alpha-1}$ units of area.

Figure 26: *Sample Problem 2 on Integration from the Model Test LSM10*

Integration occurs in all the official tests except for LS151, LS161, LS162, and LS192. Test items similar to the ones occurring in the model tests occur in the official tests under this *topic*. However, some official tests emphasize this *topic* more than other test items by including more test items.

The following is a question tackling this *topic* retrieved from the official test LS152.

4) Let α be a real number greater than 1. Denote by $A(\alpha)$ the area of the region bounded by (C), (d) and the two lines with equations $x = 1$ and $x = \alpha$.

a- Verify that $\int \frac{\ln x}{x^2} dx = \frac{-1 - \ln x}{x} + k$, where k is a real number.

b- Express $A(\alpha)$ in terms of α.

c- Using the graphic, show that $A(\alpha) < \frac{(\alpha - 1)^2}{2}$.

Figure 27: *Sample Problem 3 on Integration from the Official Test LS152*

Three objectives under *integration* were never addressed in both the model tests and the official tests. This reflects the model tests were not modified based on a revision of the curriculum's objectives to have a better alignment, but kept tackling, in the same forms used, the content covered by the previous national tests.

Differential Equations

Table 9 in Appendix R presents the test items on *differential equations*, another *topic* under *Calculus*, and the tests where they appear. The model test LSM7 is the only test that tackles this topic.

The following is the part on the topic *differential equations* retrieved from the model test LSM7.

Consider the differential equation (E): $y'' + 2y' + y = x + 2$.

1) **a-** Verify that $u = x$ is a particular solution for (E).
b- Let $y = z + u$; Form the differential equation (E')satisfied by z and solve this equation .
c- Deduce the general solution $y = f(x)$ for the equation (E).

d-Denote by (C) the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.
Determine f so that (C) is tangent at O to the line $y = 2x$.

Figure 28: *Sample Problem 1 on Differential Equations from the Model Test LSM7*

On the other hand, the topic *differential equations* appears only in one official test LS192.

The following is the part on the topic *differential equations* retrieved from the test LS192 from the problem on Calculus.

Consider the differential equation $(E): y' - y = -2x$.
Let $y = z + 2x + 2$.
1) Form the differential equation (E') satisfied by z.
2) Solve (E') and deduce the particular solution of (E) satisfying $y(0) = 0$.

Figure 29: *Sample Problem 2 on Differential Equations from the Official Test LS192*

This shows that no alignment exists between the model tests and the official tests under this topic. This topic, based on its rare appearance in both the model and official tests which does not cover all the objectives, seems to be unimportant.

Statistics

The topic *statistics* is under *Statistics and Probability*. It does not occur in any official test or model test of the years 2017 and 2019. It only appears in the model test LSM4. Table 10 in Appendix R presents the test items on *statistics*.

Probability

Table 11 in Appendix R displays the objectives under this topic occurring in the model and official tests. Probability involves the study of probability of events focused on conditional probability and dependent events. The study of real random variables and distribution functions is also included under this topic.

Probability occurs in the model tests LSM2 and LSM3 with a focus on binomial distribution which is among the lessons which were omitted directly after the curriculum reform. On the other hand, this topic has a problem in every new model test and official test with a focus on the concepts *conditional probability* and *determining probability distribution of X*.

The following is a sample probability problems retrieved from the model test LSM2.

In a factory where we make shirts, we notice that:
4% of the shirts have colour defect (called "defect C"),
2% of the shirts have defect in size (called "defect T")
The existence of one defect in a shirt is independent from the existence or not of the other defect.

1) a- Determine the probability that a shirt has the two defects C and T.
b- Determine the probability that a shirt has, at least, one defect.

2) Let X be the random variable representing the number of shirts having at least one defect in a set of 80 shirts.
a- What is the law of X?
b- What is the probability that each shirt of the set has at least one defect?
c- What is the mathematical expectation of X? Give an interpretation of the value you found.

Figure 30: *Sample Problem 1 on Probability from the Model Test LSM2*

The following is a sample probability problem retrieved from the model test LSM5.

U_1 and U_2 are two boxes so that :
U_1 contains 10 balls : 6 red and 4 black .
U_2 contains 10 balls: 5 red and 5 black .
A die numbered 1 through 6 is rolled .
. If this die shows 1 or 2 , then two balls are randomly selected at a time from the box U_1 .
.Otherwise , two balls are randomly selected one after another with replacement from the box U_2
Consider the following events :
U_1 :''The selected box is U_1.''
U_2 :''The selected box is U_2.''
R :''The selected balls are red ''.
1) calculate $P(R\ /U_1), P(R \cap U_1)$
2) verify that $P(R)=\frac{5}{18}$.
3) The two balls selected are red , calculate the probability that they come from U_1 .
4) Let X be the random variable that is equal to the number of the red balls selected .

Figure 31: *Sample Problem 2 on Probability from the Model Test LSM5*

The following is a sample probability problem retrieved from the official test LS182.

II- (4 points)
An urn U contains **six** balls: **four** red balls and **two** blue balls.
A bag S contains **five** bills: **one** 50 000 LL bill, **two** 20 000 LL bills and **two** 10 000 LL bills.

Part A

One ball is randomly drawn from U

- If this ball is red, then **two** bills are drawn successively without replacement at random from S.
- If this ball is blue, then **three** bills are drawn simultaneously at random from S.

Consider the following events:

R: " the drawn ball is red ".

A: " the sum of the values of the bills drawn is 70 000 LL ".

1) Calculate the probabilities $P(R)$, $P\left(A/R\right)$ then verify that $P(A \cap R) = \frac{2}{15}$.
2) Calculate $P(A \cap \overline{R})$. Deduce $P(A)$.

Part B

In this part, **two** bills are drawn successively with replacement at random from the bag S.
Designate by X, the random variable that is equal to the sum of the values of the two drawn bills.

1) Determine the six possible values of X.
2) Show that $P(X = 70\ 000) = \frac{4}{25}$.
3) Calculate $P(X < 70\ 000)$.

Figure 32: *Sample Problem 3 on Differential Equations from the Official Test LS182*

No pattern is obvious in the occurrence of test items in the official tests. It is noticed that the tests are aligned with the model tests of the years 2017 and 2019 under *probability* but not with the previous model tests.

In conclusion, the qualitative analysis shows certain topics were neglected or never addressed in the model tests and the official tests. The topic *circular functions* was never addressed in the model tests and the official tests before it was cancelled in 2018. Similarly, the topic *statistics* appeared in one of the model tests that were issued with the curriculum but not in any model test of the year 2017 or official test before being cancelled in 2018. On the other hand, *differential equations* which was ignored in most of the model tests and the official tests throughout the years, appears back in one of the model tests of the year 2017 and in the official test LS192. Several objectives were never addressed in the official and model tests. These objectives are under *integration, literal and numerical calculations,* and *probability*. Moreover, all *topics* occur in a purely abstract context except for probability which always occurs in real-life contexts.

It is also clear how the model tests of the years 2017 and 2019 were designed to be more aligned with all the previous national tests. This reflects that the modification was not based on a rational revision of the curriculum taking into consideration its objectives and content details.

In fact, the qualitative analysis of the official tests of the LS track shows that the national examination of this track does not reflect the major points of the reform as mentioned in the introduction of the mathematics curriculum: stressing the individual construction of knowledge by giving students the opportunity to be immersed in real-life situations where inquiry is their starting point (b) stresses the importance of eliminating the theoretical overuse, emphasizing the practicality of the topics given, and the use of calculator and computer technologies; and (c) recommending starting from real-life situations which shows that math is not separated from everyday life. The test items of most of the topics of the official tests of this track are purely abstract and theoretical, do not relate to real-life or other scientific subjects, and do not require the use of any technological tool except the calculator for basic calculations. The only topic that appears in a real-life context is *probability*. This contradicts the three major points mentioned.

On the other hand, the process objectives of the LS track, as presented in the curriculum document, emphasize mathematical reasoning, solving mathematical problems, and communicating mathematically (ECRD, 1997a). The qualitative analysis shows that communication through mathematics which includes, as stated in the introduction of the curriculum, reading and interpreting texts, writing demonstration, and explaining situations, graphs and tables, for the LS track, is also partially reflected in the official tests of this track since questions are mostly direct and theoretical.

4.4.5. Quantitative Analysis of the LS National Tests

The percentages of test items under the content and cognitive domains of the model tests and national tests for the LS track are presented in tables, compared, and analyzed in this section. The Pearson Product-Moment correlation coefficients between the data in the resultant tables are then presented, interpreted and discussed.

4.4.5.1 – Overall Correlation Between the Model Tests and the Official Tests

According to the definition of test item adopted in this study, there are 324 test items in 10 model tests and 556 test items in 16 official tests. For the sake of having a unified base for comparison, the data in the Table AllModLS, that presents the quantitative data of the ten model tests, and Table OffExLS, that presents the quantitative data of the sixteen national tests, were converted to percentages.

Table 1 in Appendix S displays the resultant percentages of the test items in all the model and national tests distributed over the cognitive domains and *topics* they tackle.

According to the table, the model and official tests assess in a balanced way most of the topics of the curriculum. The topic *definitions and representations* is assigned the highest number of the test items in both the model and the official tests (41.36% and 35.58% respectively), and this reflects the high importance given to this topic in both the model and official tests. The topic *classical study (geometry)* has the second highest number of test items in both the model and official tests (17.67% and 19.6% respectively). Then the topic *probability* follows with 14.05% of the model tests' test items and 16.74% of the official tests' test items and the topic *numbers* with a very close percentage to *probability*'s percentages in each of the model and official tests (15.66% and 16.28% respectively). The remaining topics have low percentages of the test items in each of the model and official tests:

- *continuity and differentiation* (5.87% and 5.28% respectively)
- *integration* (1.86% and 4.39% respectively)
- *literal and numerical calculations* (1.7% and 2.14% respectively)
- *differentiation* (0.93% and 0.54% respectively)
- *statistics* (0.92% and 0 respectively)

Discrepancy is obvious mainly in the percentages under the topic *integration* which is emphasized more in the official tests than the model tests. On the other hand, the topic *statistics* is never addressed in the official tests although it was added to the suspended lessons in the last two years (2018-2019), while it has a very low percentage of test items in the model tests.

Considering the math domains, *Algebra* which consists of the *topics literal and numerical calculations* and *numbers* constitutes 17.36% and 18.42% of the test items of the model and official test respectively. *Geometry* constitutes 17.67% and 19.16% of the test items of the model and official test respectively. *Calculus* which consists of the *topics definitions and representations*, *continuity and differentiation*, *integration*, and *differential equations* constitutes 50.02% of the test items in the model tests and 45.69% of the official tests' test items. The domain *Probability and Statistics* which consists of the *topics statistics* and *probability* constitutes 14.97% and 16.74% of the test items in the model and official tests respectively.

Table 1 in Appendix S presents the percentages of test items addressing the cognitive domains *knowing*, *applying*, and *reasoning* in the model and official tests. According to table 1, the model and official tests address each cognitive domain with very close percentages of test items. While the cognitive domain *knowing* has the highest percentage of test items in the model tests (43.66%) followed by *applying* (40.49%), it has the second highest percentage in the official tests (40.26%) after *applying* (43.34%). On the other hand, the domain *reasoning* has a low percentage of test items in each the model (15.87%) and the official tests (16.41%) compared to the other two domains.

Comparing the model tests and official tests to both math *topics* and cognitive domains, close percentages of test items appear under each. However, it is noted that under the *topic probability,* the domain knowing is emphasized over *applying* and *reasoning* (7.10%, 3.94%, and 3.01% respectively), while in the official tests, the domains *knowing* and *applying* have almost equal percentages (6.69% and 6.77% respectively) over *reasoning* (3.28%) reflecting a discrepancy in the percentages under the domain *applying* under this topic. Moreover, the topic *literal and numerical* *calculations'* test items are distributed over *knowing* and *reasoning* in the model tests, while they tackle only *knowing* in the official tests. On the other hand, the topics *definitions and representations*, *continuity and differentiation*, and *integration* under the domain *Calculus* are the only *topics* where the test items require the domain *applying* more that the domains *knowing* and *reasoning*. Lastly, the topic *differential equations*, its rare presence in both the model and official tests is limited to the domains *knowing* and *applying*.

Correlations were made, using Pearson Product-Moment coefficient by Microsoft Excel, between the calculated percentages by correlating data in Table 1 of Appendix S. An overall correlation was calculated between all cells of the model tests and all cells of the official tests, correlations under each math domain between the model and official tests were calculated, and correlations under each cognitive domain between the model and official tests were calculated.

Table 10 presents the correlation between all the official exams of the years 2011- 2019, and the model tests for the LS track at grade 12.

Table 10 *Correlations Between the Official Tests of the Years 2011-2019 and the Model Tests for Grade 12 LS Track*

	Overall Correlation	**In terms of Cognitive Domains**			**In terms of Math Contents**			
		K	**A**	**R**	**Alge.**	**Geom,**	**Cal.**	**S&P**
OT & MT	0.97	0.99	0.97	0.95	0.93	0.85	0.99	0.92

K: Knowing
A: Applying
R: Reasoning
Alge.: Algebra
Cal. : Calculus
Geom.: Geometry
S&P : Statistics & Probability
OT & MT : Official tests and model tests

According to Table 10, the overall correlation between the model tests and the national tests is very high (r=0.97). This shows that the model tests and the official tests are very consistent in general. This high correlation is the result of having very close percentages of test items under almost every topic and cognitive domain.

The correlation between the national tests and model tests in terms of the cognitive domains is similarly calculated between the numbers in the columns of each domain of Table 1 in Appendix S and presented in Table 6. The correlation, refer to Table 6, between the national tests and model tests in terms of *knowing* is 0.99, *applying* is 0.97, and *reasoning* is 0.95. This high correlation shows the consistency of the model and official tests under each cognitive domain. This great alignment is due to the correspondence in the percentages of *topics* covered under each cognitive domain as presented in table 1 Appendix S.

Correlations between the math domains were similarly calculated by finding Pearson Product-Moment coefficient between the data in the rows of Table 1 in Appendix S. High positive correlations, at the math *domains,* are also shown in Table 10. *Algebra* has a correlation of 0.93 between the model tests and the official tests. This is due to the close percentages of each topic under each domain as shown in Table 1 Appendix S. Moreover, the domain *Geometry* has a correlation of 0.85 between the model and official tests which reflects also a high consistency between the model and official tests under this domain. The domain *Calculus* has a correlation of 0.99. The high correlation reflects the high consistency between the model tests and official tests under this domain and its topics. Similarly, the correlation under the domain *Probability and Statistics* is very high positive (0.92). This reflects the consistency between the model tests and official tests under this domain. Although the topic statistics, under this domain, doesn't occur in the official tests, unlike the model tests, its occurrence in the model tests is negligible (0.92) which didn't affect the consistency under the whole domain which is reflected clearly in the close percentages under the topic probability in Table 1 Appendix S.

In short, results show an overall very high correlation between the model tests and official tests under all math topics and cognitive domains.

4.4.5.2 – Correlations Between the Model Tests and the Official Tests over the years 2011-2013, 2015-2016, 2017-2018, and 2019

4.4.5.2.1 – Alignment Between the Model Tests and the National Tests Over the Years

More valid results, are obtained when comparisons and correlations are made between the national tests and their corresponding model tests. Therefore, comparison and correlations between the model tests and the national tests over the years 2011-2013, 2015-2016, 2017-2018 and 2019 are made in this section. Comparison is made between the model tests (LSM1, LSM2, and LSM3) issued with the curriculum documents (in 2000) and the national tests of the years 2011-2013 and 2015-2016, between the model tests (LSM5, LSM6, LSM7, and LSM8) issued in the year 2017 and the national tests of the years 2017-2018, and between the model tests (LSM8 and LSM9) issued in the year 2019 and the national tests of this year are made.

There are 77 test items in four model tests issued in the year 2000, 161 test items in the four model tests issued in the year 2017, model tests issued in the year 2017, 84 test items in the two model tests issued in the year 2019, 202 test items in six official tests (2011-2013), 146 test items in four official tests (2015-2016), 137 test items in four official tests (2017-2018), and 71 test items in the two official tests of the year 2019. Tables Mod, NewModLS, OffExLS11-13, OffExLS15-16, OffExLS17-18, and OffExLS19 were converted to percentages to have a unified base for comparison. Tables 2, 3, and 4 in Appendix S display the resultant percentages of the test items distributed over the cognitive domains and *topics* they tackle for the LS track. Table 2 presents the percentages of test items in the model tests issued in the year 2000 and the national tests of the years 2011-2013 and 2015-2016. Table 3 presents the percentages of test items in the model tests issued in the year 2017 and the national tests of the years 2017-2018, and Table 4 presents the percentages of test items in the model tests issued in the year 20019 and the national tests of the same year.

Content Domains

According to tables 2, 3, and 4, the topic *definitions and representations* has the highest percentage of test items in all the studied sets of tests ranging between 30% and 44%. The topics *classical study, numbers,* and *probability* come next with very close percentages of test items. The topics *continuity and differentiation, integration, differential equations, literal and numerical calculations,* and *statistics* are assigned small percentages of test items in all the sets of tests studied.

When comparing the model tests issued in the year 2000 to the official tests of the years 2011-2013, it is obvious in table 2 in Appendix S that all the topics are assessed in a somehow imbalanced way in the model and official tests except the topics *literal and numerical calculations* (1.95% and1.196% respectively) and *number*s (11.37% and 12.87% respectively). The *topics definitions and representations* and *continuity and differentiation* have more test item percentages in the model tests (42.68% and10.38% respectively) than the official tests (35.52% and 6.4% respectively). Discrepancies also occur under the topics *integration* and *statistics*. While the first is emphasized more in the official tests than the model tests (6.38% and 3.91% respectively), the latter occurs in the model tests only but with low percentage (3.9%). It is noticed that *differential equations* doesn't occur in any of the model tests and official tests under this set. Test items on *probability* are more in the official tests than the model tests (16.13% and 11.04% respectively).

Discrepancies are obvious between the percentages of the test items in these model tests, and of the official tests of the years 2015-2016 mainly under the topics *numbers* (11.37% and 21.61% respectively), continuity and differentiation (10.38%, and 4.11% respectively), and *probability* (11.04%, and 14.56% respectively). *Statistics* also has no test items in the official tests of the years 2015-2016, while *differential equations* doesn't occur in both the model and official tests.

When comparing the model tests of the year 2017 to the official tests of the years 2017-2018, it is obvious in table 3 in Appendix S that the model tests and the corresponding official tests assess in a balanced way most of the topics of the math curriculum: *numbers, classical study,* and *continuity and differentiation*. Discrepancies are obvious between the model tests and the official tests mainly in the percentages of the topics *literal and numerical calculations* (0.62% and 3.65% respectively), *definitions and representations* (44.1% and 34.32% respectively*)*, and *integration* (0.62 and 4.38% respectively). The topic *statistics* is added to the suspended lessons starting from the year 2017. It is also noted that the topic *differential equations* is covered in the model tests of the year 2017 with very low percentage (1.86%) but not in the official tests. Therefore, official tests are not aligned with the model tests in terms of content coverage of this topic.

Comparing the model tests of the year 2019 to their corresponding official tests of the year 2019, it is obvious in table 4 of Appendix S that the model tests and their corresponding official tests are very well aligned under the Algebra topics (*literal and numerical calculations* and *numbers*). Some differences exist in the percentages of the test items under the remaining topics which make them less aligned under these topics. The topic *differential equations* is just covered in the official tests but not the model tests which makes them unaligned under this topic.

In short, the qualitative analysis of the model and national tests over the mentioned sets shows that the national tests over the years are becoming better aligned with their respective model tests in terms of content domain coverage.

Cognitive Domains

As to the cognitive domains, the cognitive domain *knowing* (refer to Table 2 appendix S) has very close percentages in the model and official tests of the year 2011-2013 (39.18 % and 38.93% respectively). On the other hand, discrepancies occur under *reasoning* which is emphasized more in the official tests that the model tests (19.32% and15.81% respectively) and *applying* which is emphasized more in the model tests having about half of the test items (45.03%) than the official tests (41.78%). Moreover, it is noted that the topic *numbers* in the model tests have a great percentage under the domain *reasoning* (6.28% out of 11.37%) while having close percentages over the three cognitive domains in the official tests. Therefore, the model tests and the official tests of the yeas 2011-2013 are well aligned only under the topics of the *Algebra* domain and under *knowing*. This alignment decreases under other topics and cognitive domains.

The model tests and the official tests of the year 2015-2016 (refer to Table 2 appendix S) have very close percentages under all the cognitive *domains knowing, applying,* and *reasoning* (39.18%, 45.03%, and 15.81% respectively for the model tests and 41.13%, 45.79%, and 13.2% respectively for the model tests). Therefore, the model tests and the official tests are well aligned under the *topics classical study* and *definitions and representations* only and under all the cognitive domains.

On the other hand, the model tests issued in 2017 and the official tests of the year 2017-2019 have close percentages under all the cognitive domains *knowing, applying,* and *reasoning* with a more emphasis on *reasoning* in the official tests than model tests (17.98% and 15.68% respectively) and a more emphasis on *knowing* in the model tests than official tests for these years (43.47% and 40.23% respectively).

On the other hand, discrepancies exist under the cognitive domains mainly under *knowing* and *applying* in the model tests and national tests of the year 2019. While the model tests emphasize *knowing* over *applying* (49.09% and 36.6% respectively), the official tests emphasize *applying* over *knowing* (45.66% and 42.49% respectively).

These discrepancies are because of the differences in the percentages of each of the topics *numbers* and *classical study* under these two cognitive domains. The official tests have more test items under *applying* covering these two topics, while the model tests have more test items under *knowing* under these topics.

In short, the national tests of the years 2015-2016 and 2017-2018 are more aligned with their corresponding model tests in terms of cognitive domains than the national tests of the years 2011-2013 and 2019. Correspondence of percentages under *knowing* and *applying* is the greatest with a more emphasis on applying by the national tests. Discrepancy occurs mainly under *reasoning* being more emphasized in the model tests over all the years. The curriculum, as demonstrated in the model tests, emphasizes the cognitive domain *knowing* over *applying* and *reasoning*, while the official tests emphasize *applying* over *knowing* and *reasoning*. However, more attention is given by all the model tests to the domain *reasoning* than the official tests over the mentioned periods of time.

Correlations

In addition to percentages, correlations were calculated between the respective numbers in Tables 2, 3, and 4 in Appendix S. Correlations were calculated between the model tests issued in the year 2000 and the official tests of the yeas 2011-2013, the model tests issued with the curriculum documents and the official tests of the years 2015-2016, the model tests issued in 2017 and the official tests of the years 2017-2018, and the model tests issued in 2019 and the official tests of the year 2019.

According to Table 11, the overall correlation is very high positive (ranging between 0.82 and 0.96) between the tests of all the sets correlated. This shows that the tests of each set have an overall good alignment under each set.

Table 11: *Correlations Between the Official Tests and the Model Tests Over the Years 2011-2013, 2015-2016, 2017-2018, and 2019 for Grade 12 LS Track*

	Overall Correlation	**In terms of Cognitive Domains**			**In terms of Math Contents**			
		Knowing	**Applying**	**Reasoning**	**Alg.**	**Geo.**	**Calc.**	**S.P.**
OT11-13&MT1	0.92	0.9	0.96	0.77	0.79	0.98	0.98	0.8
OT15-16&MT1	0.88	0.7	0.95	0.63	0.36	0.99	0.98	0.6
OT17-18&MT2	0.96	0.93	0.94	0.66	0.84	0.98	0.95	0.8
OT19&MT3	0.89	0.89	0.9	0.85	0.66	0.36	0.97	0.95

Alg.: Algebra
Geo.: Geometry
Calc. : Calculus (Numerical Functions)
S.P. : Statistics & Probability
MT1: model tests LSM1, LSM2, LSM3 and LSM4
MT2: model tests LSM5, LSM6, LSM7, and LSM8
MT3: model tests LSM9 and LSM10
OT11-13: official tests of the years 2011-2013
OT15-16: official tests of the years 2015-2016
OT17-18: official tests of the years 2017-2018
OT19: official tests of the year 2019

Model tests issued in the year 2000 and the national tests of the years 2011-2013 are well aligned under all the cognitive domains and content domains having correlations ranging between 0.77 and 0.98.

Model tests issued in the year 2000 and the national tests of the years 2015-2016 are somehow well-aligned under the cognitive domains *knowing* and *reasoning* having correlations of 0.7 and 0.66 respectively, and very well-aligned under *applying* having a very high correlation (r=0.98). As for the content domains, the model and national tests under this period are very well aligned under *Geometry* and *Calculus* and somehow under *Statistics and Probability*(r=0.6), but not under *Algebra* under which a low correlation exists (0.36).

Model tests issued in the year 2017 and the national tests of the years 2017-2018 are well aligned under all the cognitive domains and content domains having correlations ranging between 0.66 and 0.98 with the lowest correlation under *reasoning*.

Model tests issued in the year 2019 and the national tests of the year 2019 are very well aligned under all cognitive domains and all content domains (correlation ranging between 0.66 and 0.97) except for *Geometry* content domain where the correlation is very low (0.36).

4.4.5.2.2 – Evolution of the National Tests of the LS track Over the Years

In this section, comparison between the national tests of the years 2011-2013, 2015-2016, 2017-2018, and 2019 is in terms of content domains and cognitive domains to see the evolution of the national test is over time, then correlations are calculated to check their alignment.

Content Domains

Comparing the official tests of the years 2011-2013, 2015-2016, 2017-2018, and 2019 in terms of math topics, it is obvious, according to tables 2, 3, and 4 in the appendix S, that the official tests of the years 2011-2013 and 2017-2018 have close percentages under the math topics *literal and numerical calculations*, *classical study, continuity and differentiation,* and *probability* and under *definitions and representations* with the official tests of the years 2015-2016. The topic *differential equations* is never addressed in the tests of these three sets of years. The topics numbers and classical studies have close percentages of test items in the official tests of the years 2015-2016 and the year 2019.

Cognitive Domains

The official tests of the years 2011-2013, 2015-2016, 2017-2018, and 2019 have very close percentages under the domain *knowing* ranging between 38.9% and 42.5%. Tests of the years 2011-2013 and those of the years 2017-2018 have close percentages under the domains *applying* and *reasoning* too (41.77% and 19.32% respectively for official tests of the years 2011-2013 and 41.89% and 17.98% respectively for official tests of the years 2017-2018). The official tests of the years 2015-2016 and the year 2019 have very close percentages under all cognitive domains (41.13% and 42.49% respectively under *knowing*, 45.76% and 45.67% respectively under *applying*, and 13.2% and 11.86% respectively under *reasoning*.

In short, the national tests over the years have very close percentages under the cognitive domains. It is noted that more emphasis is given over years to *applying* on the expense of *reasoning*.

Correlations

In addition to percentages, correlations were calculated between the respective numbers in Tables 2, 3, and 4 in Appendix S. Correlations were calculated between the official tests of the years 2011-2013 and the years 2015-2016, the official tests of the years 2011-2013 and the years 2017-2018, the official tests of the years 2011-2013 and the year 2019, the official tests of the years 2015-2016 and the years 2017-2018, the official tests of the years 2015-2016 and the year 2019, the official tests of the years 2017-2018 and the year 2019 of the LS track. When taking the correlations between the official tests, the topic *statistics* is excluded. The fact is that this *topic* was never addressed in any national test before the year 2018, and it was omitted since the year 2018. Therefore, keeping it in the national tests of the years before 2018 and omitting it in the tests of the years 2018 till 2019 naturally resulted in tables which are not of equal size and structure to be correlated. Moreover, keeping it will affect slightly the correlation results. To solve this problem, we had to remove that *topic* from Tables 2, 3, and 4 in Appendix S when doing the correlations in this section.

Table 12

Correlations Between the Official Exams of the Years 2011-2019 and the Model Tests for Grade 12 LS Track

	Overall Correlation	In terms of Cognitive Domains			In terms of Math Contents			
		Knowing	Applying	Reasoning	Alg.	Geo.	Calc.	S.P.
OT11-13& OT15-16	0.92	0.78	0.97	0.9	0.78	0.99	0.99	0.44
OT11-13&OT17-18	0.93	0.91	0.93	0.87	0.83	0.8	0.98	-0.28
OT11-13& OT19	0.89	0.9	0.9	0.91	0.91	0.82	0.94	0.84
OT15-16&OT17-18	0.92	0.85	0.98	0.7	0.89	0.75	0.96	0.74
OT15-16&OT19	0.87	0.77	0.93	0.84	0.81	0.86	0.93	0.86
OT17-18&OT19	0.88	0.8	0.94	0.74	0.89	0.31	0.96	0.3

Alg.: Algebra
Geo.: Geometry
Calc. : Calculus (Numerical Functions)
S.P. : Statistics & Probability
OT11-13: official tests of the years 2011-2013
OT15-16: official tests of the years 2015-2016
OT17-18: official tests of the years 2017-2018
OT19: official tests of the year 2019

The official tests of the years 2011-2013, 2015-2016, 2017-2018, and 2019 are consistent with each other under all the cognitive domains, refer to Table 12, having high correlations ranging between 0.7 and 0.98 under these cognitive domains.

In terms of math domains, refer to table 12, high positive correlations (ranging between 0.75 and 0.99) exist between the tests of all the sets studied under the content domains *Algebra*, *Geometry*, and *Calculus* except for the national tests of the years 2017-2018 and 2019 under Geometry (r=0.31). This reflects that all the official tests are aligned with each other under these content domains. Low correlations exist between the model official tests of the years 2011-2013 and the years 2015-2016 (r=0.44), and between official tests of the years 2017-2018 and the year 2019 (r=0.3), and a negative correlation between official tests of the years 2011-2013 and the year 2017-2018 (r=-0.28) under the domain *Probability and Statistics*. Therefore, a lack of alignment exists between the mentioned sets of the official tests under this domain. The official tests 2011-2013 and 2019, 2015-2016 and 2017-2018, and 2015-2016 and 2019, however, have a good alignment under this domain represented by their high positive correlations (0.84, 0.74, and 0.86 respectively).

In fact, the correlations between the national tests show that the tests did not change over the years. They are steady in terms of content domains and cognitive domains coverage.

4.4.5.3 – Correlations Between the Model Tests and the Official Tests of Sessions 1 and 2

To study if there is any difference in the alignment between the curriculum and session-1 and session-2 official tests, comparison is made in this section between the model tests issued with the curriculum documents and the corresponding official session-1 and session-2 tests (of the years 2011-2013 and 2015-2016), between the model tests issued in 2017 and the corresponding official session-1 and session-2 tests (of the years 2017-2018), and between the model tests of the year 2019 and their corresponding official tests of the year 2019 for the LS track. Comparison is also made between sessions 1 and 2 official tests of the years 2011-2013 and 2015-2016, between sessions 1 and 2 official tests of the years 2017-2018, and between session 1 and 2 official tests of the year 2019 to see how consistent the official tests of sessions 1 and 2 are.

There are 77 test items in four model tests issued with the curriculum document, 161 test items in the four model tests LSM5, LSM6, LSM7, and LSM8 model tests issued in the year 2017, 84 test items in the two model tests issued in the year 2019 (LSM9 and LSM10), 170 test items in five session-1 official tests of the years 2011-2016, 178 test items in five session-2 official tests of the years 2011-2016, 74 test items in four session-1 official tests of the years 2017-2018, 63 test items in four session-2 official tests of the years 2017-2018, 33 test items in session-1 official test of the year

2019, and 38 test items in session-2 official test of the year 2019. Tables ModLS, ModLS5-8, ModLS9-10, OffExLS11, OffExLS21, OffExLS12, OffExLS122, LS191, and LS192 were converted to percentages to have a unified base for comparison.

Table 5 in Appendix S presents the distribution in percentages of the test items in the model tests (LSM1, LSM2, LSM3, and LSM4), session-1 official tests of the years 2011-2016, and session-2 official tests of the years 2011-2016 to their corresponding cognitive domains and math topics they address. Table 6 in Appendix S presents the distribution in percentages of the test items in the model tests of the year 2017, session-1 official tests of the years 2017-2018, and session-2 official tests of the years 2017-2018 to their corresponding cognitive domains and math topics they address. Table 7 in Appendix S presents the distribution in percentages of the test items in the model tests of the year 2019, session-1 official tests of the year 2019, and session-2 official tests of the year 2019 to their corresponding cognitive domains and math topics they address. The data in these tables are extracted from the Tables ModLS, ModLS5-8, ModLS9-10, OffExLS11, OffExLS21, OffExLS12, OffExLS122, LS191, and LS192.

Model Tests Compared to Each of Sessions 1 and 2 of the National Tests

When comparing the model tests issued with the curriculum documents to session-1 official tests of the years 2011-2016, and session-2 official tests of the years 2011-2016, it is obvious in table 5 in Appendix S that the model tests and the corresponding official tests do not assess in a balanced way the different topics of the math curriculum. The percentages of the test items in the model tests are distributed over eight out of nine topics, while they are distributed in session-1 official tests and session-2 official tests of the years 2011-2016 over seven topics. It is noted that no test items tackled the topic differential equations in both the model and the official tests, while statistics is only addressed in the model tests. The topic *definitions and representations* has the highest percentage of test items in all tests (42.86% in the model tests, 35.3% in session-1 official tests, and 38.34% in session-2 official tests). There is a small difference in the percentages of this topic between the model tests and each of session-1 and 2 official tests. The topic *classical study* has the second highest percentage in the model tests issued with the curriculum documents (14.61%), session-1 official tests of the years 2011-2016 (20.23%), and session-2 official tests of the years 2011-2016 (19.1%) with obvious discrepancies between the model tests and each of session-1 and session-2 official tests under this topic. Similarly, the topics *numbers* and *probability*, which have the third highest percentage of test items in the model tests, have significant differences in the percentages between the model tests and each of session-1 and session-2 official tests of the year 2011-2016 (11.37%, 17.94%, and 15.17% respectively for numbers and 11.04%, 16.82%, and 14.19% respectively). A huge discrepancy exists under the topic *continuity and differentiation* between the model tests and each of session-1 and session-2 official tests taking 10.38% in the model tests, 4.71% in session-1 official tests, and 5.82% in session-2 official tests.

When comparing the model tests issued in the year 2017 to each of session-1and session-2 official tests of the years 2017-2018, it is obvious in table 6 in Appendix S that the model tests compared to each of session-1 and session-2 official tests do not assess in a balanced way some of the topics of the math curriculum. While the model tests have some test items on *differential equations*, both session-1 and session-2 official tests do not cover this topic. The *topics numbers, classical study, definitions and* *representations, continuity and differentiation*, and *probability* have very close percentages of test items in the model tests (16.77%, 18.01%, 44.1%, 4.34%, and 13.66% respectively) and session-1 official tests of the years 2017-2018 (17.57%, 18.92%, 37.84%, 4.06%, and 14.53% respectively), while there are obvious discrepancies under the topics *literal and numerical calculations, numbers, definitions and representations*, *continuity and differentiation, integration,* and *probability* between the model tests (0.62%, 16.77%, 44.1%, 4.34%, 0.62%, and 13.66% respectively) and session-2 official tests (4.37%, 11.12%, 30.15%, 9.53%, 4.76%, and 21.03% respectively). This shows a misalignment of the model tests with session-2 official tests of the year 2017-2018 under these topics.

Comparing the model tests of the year 2019 and the official tests of the same year, it is obvious, according to table 7 in appendix S, that there is a good alignment between the model tests and each of session-1 and session-2 under most of the math topics. The topic *differential equations* occurs only in session-2 official tests of the year 2019 without appearing in the model tests and official session-1 tests. Moreover, the topic integration has a good percentage in the official tests 2019 (9.09%), while it is almost negligible in the model tests (2.37%) and with no occurrence under session-2 official tests of the year 2019.

As to the cognitive domains, Tables 5, 6, and 7 in Appendix S show that close percentages of test items exist among the model tests, session-1 official tests and session-2 official tests of the years 2011-2016, 2017-2018, and 2019 under all the cognitive domains. It is notable that session-2 of the years 2011-2016 has a higher percentage under *reasoning* than the model tests and session-1 of the same years

(17.84%, 15.81%, and 15.59% respectively). A balance in the percentages under the cognitive domains *knowing*, *applying* and *reasoning* also exists between the model tests (43.47%, 40.83%, and 15.68% respectively) and session-1 official tests of the years 2017-2018 (46.17%, 39.77%, and 14.08% respectively). A discrepancy exists under this domain when comparing each of the model tests and session-1 official tests with session-2 official tests of the year 2017-2018 (the latter has the percentages 33.2%, 44.32%, and 22.49% respectively). It is notable that session-2 official tests of the years 2017-2018 have the highest percentage under the domain *reasoning* among all studied sets of tests (22.49%). Discrepancies exist under the domain *knowing* when comparing the model tests of the year 2019 to each of sessions 1 and 2 official tests of the same year (49.09%, 42.93%, and 42.1% respectively), under *applying* between the mentioned tests (36.6%, 42.17%, and 48.73% respectively), and under *reasoning* when comparing between each of the model tests and session-1 official test of the year 2019 and session-2 2019 (14.28%, 15.9%, and 9.21% respectively).

In short, discrepancies occur between the model tests and sessions 1 and 2 national tests mainly under the topic *numbers*, *continuity and differentiation*, and *probability*. Correspondence under all the cognitive domains is better between the model tests of the 2017 and session-1 national tests of the years 2017-2018 than between the model tests and session-2 national tests of the same period, and between the model tests of the 2019 and session-1 national tests of the year 2019 than between the model tests and session-2 national tests of the same period. Close percentages of test items exist among the model tests, session-1 official tests and session-2 official tests of the years 2011-2016 under all cognitive domains.

Session-1 National Tests Compared to Session-2 National Tests

Comparing sessions 1 and 2 of the official tests of the years 2011-2013, 2015-2016, 2017-2018, and 2019 in terms of math topics, it is obvious, according to tables 5, 6, and 7 in Appendix S, that sessions 1 and 2 of the years 2011-2016 are well aligned having very close percentages under each topic. However, this is not the case between sessions 1 and 2 of each of the years 2017-2018 and 2019 because discrepancies exist in the percentages of test items under different *topics*.

In terms of cognitive domains, according to tables 5, 6, and 7 of Appendix S, sessions 1 and 2 of the years 2011-2016 have good correspondence under *applying* (42.43% and 44.37% respectively) and *reasoning* (15.59% and 17.84% respectively) but with discrepancies under *knowing* (41.93% and 37.79% respectively). Sessions 1 and 2 of the years 2017-2018 have a somehow good correspondence under *applying* (39.77% and 44.32% respectively) but with discrepancies under *knowing* (46.17% and 33.2% respectively) and *reasoning* (14.08% and 22.49% respectively). It is notable that reasoning in sessions 2 of the years 2011-2016 and 2017-2018 emphasize *reasoning* more than sessions 1 of the same years. Sessions 1 and 2 of the year 2019 have a good correspondence under *knowing* (42.93% and 42.1% respectively) but with discrepancies under *applying* (42.17% and 48.73% respectively) and *reasoning* (15.9% and 9.21% respectively). The lowest percentage under *reasoning* exists in session-2 of the year 2019.

Correlations

In addition to percentages, correlations were calculated between the respective numbers in Tables 5 and 6 in Appendix S. Correlations were made between the model tests issued with the curriculum documents and the corresponding official session-1 and session-2 tests (of the years 2011-2016), between the model tests issued in 2017and the corresponding official session-1 and session-2 tests (of the years 2017-2018), and between the model tests of the year 2019 and their corresponding official tests (year 2019).

According to Table 13, the correlations between the model tests issued with the curriculum and session-1 official tests of years 2011-2016, the model tests and session-2 official tests of years 2011-2016, the model tests and session-1 official tests of years 2017-2018, the model tests and session-2 official tests of years 2017-2018, the model tests and session-1 official test of year 2019, and the model tests and session-2 official tests of the year 2019 are 0.915, 0.906, 0.903, 0.815, 0.88, and 0.77 respectively. This shows that the tests under each compared set are consistent. However, this consistency is greater between model tests issued in the year 2000 and with their corresponding sessions 1 and 2 official tests then between model tests issued in 2017 and their corresponding official tests. This consistency decreases between the model tests of the year 2019 and its corresponding official tests.

Table 12: *Correlations Between Sessions 1 and 2 of the Official Exams of the Years 2011-2016, 2017-2018, and 2019, and the Model Tests for Grade 12 LS Track*

	Overall Correlation	**In terms of Cognitive Domains**			**In terms of Math Contents**			
		K	**A**	**R**	**Alg.**	**Geo.**	**Calc.**	**S.P**
OT11-16(1)&MT1	0.915	0.79	0.97	0.86	0.62	0.99	0.99	0.81

OT11-16(2)&MT1	0.906	0.94	0.99	0.7	0.48	0.98	0.97	0.63
OT17-18(1)&MT2	0.903	0.93	0.97	0.58	0.97	0.99	0.92	-0.64
OT17-18(2)&MT2	0.815	0.76	0.88	0.57	0.33	0.94	0.94	0.15
OT19(1)&MT3	0.88	0.89	0.93	0.61	0.83	0.64	0.95	-0.38
OT19(2)&MT3	0.77	0.71	0.79	0.66	0.37	0.2	0.93	0.94
OT11-16(1)& OT11-16(2)	0.96	0.94	0.99	0.7	0.88	1	1	0.89
OT17-18(1)& OT17-19(2)	0.83	0.77	0.96	0.65	0.48	0.97	0.9	0.67
OT19(1)& OT19(2)	0.7	0.6	0.82	0.01	0.41	0.88	0.87	-0.06

K: Knowing
A: Applying
R: Reasoning
Alg.: Algebra
Calc.: Calculus (Numerical Functions)
S.P.: Statistics & Probability
MT1: model tests issued with the curriculum documents
MT2: model tests issued in the year 2017
MT3: model tests issued in the year 2019
OT11-16(1): session-1 official tests of the years 2011-2016
OT11-16(2): session-2 official tests of the years 2011-2016
OT17-18(1): session-1 official tests of the years 2017-2018
OT17-18(2): session-2 official tests of the years 2017-2018
OT19(1): session-1 official tests of the year 2019
OT19(2): session-2 official tests of the year 2019

In terms of cognitive domains, refer to table 12, the correlations between the model tests and their corresponding sessions 1 and 2 over all the years are high positive under all cognitive domain (ranging between 0.7 and 0.9) except for *reasoning* when correlating the model tests to each of session-1 and session-2 official tests of the year 2019, having average correlations of 0.58 and 0.57 respectively.

In terms of math domain *algebra*, refer to table 12, an obvious low correlation (ranging between 0.33 and 0.48) exists between session-2 official tests of each year and their corresponding model tests. This is because under this math domain, percentages of test items vary a lot between session-2 tests and the model tests in terms of math domain and both math topic and cognitive domain. Therefore, there is consistency between sessions-1 official tests of the years 2011-2016, 2017-2018 and 2019 and their corresponding model tests under the math domain *Algebra* but not among the tests of the remaining sets.

In terms of math domains *Geometry* and *Calculus*, very high correlations exist between the tests of each sets ranging between 0.87 and 1 except for the tests of the set: model tests of the year 2019 and session-1 official test and model tests of the year 2019 and session-2 official test of the same year (r=0.64 and r=0.2 respectively) under *Geometry*. These high correlations reflect a great consistency between the mentioned sets, while the average and low correlations reflect an average consistency and a low consistency between the model tests of the year 2019 and session-1 test of the same year and between the model tests and session-2 test of the same year respectively.

In terms of math domain *statistics and probability*, refer to table 12, negative and very low correlations exist between the model tests issued in 2017 and each of session-1 and session-2 official tests of the year 2017, and the model tests of the year 2019 and session-1 of the year 2019 (-0.64, 0.15, and -0.38). This reflects an inconsistency between each of the mentioned set. This is due to discrepancies of the total percentages of test items under this topic or to discrepancies of the percentages under the cognitive domains under this math domain. On the other hand, the correlations between the tests of the remaining sets range between 0.63 and 0.94 reflecting a consistency ranging between good to high consistency between the sets under the domain *Statistics and Probability*.

Correlations were calculated between sessions 1 and 2 official tests of each of the years 2011-2016, 2017-2018, and 2019. Correlations between sessions 1 and 2 official tests of years 2011-2016, sessions 1 and 2 official tests of years 2017-2018, and sessions 1 and 2 of the year 2019 are 0.96, 0.83 0.7 respectively.

Correlations in term of cognitive domains, refer to table 12, range between good (r=0.6) to high positive (r=0.99) between sessions 1 and 2 national tests under all cognitive domains except under reasoning between session-1 and session-2 official tests of the year 2019 having a very low correlation (r=0.01).

In terms of math domain *algebra*, refer to table 12, obvious low correlations exist between sessions 1 and 2 official tests of the years 2017-2018 and the year 2019 (r=0.48 and 0.41 respectively). High positive correlation exist under Algebra between sessions 1 and 2 of the years 2011-2016 (r=0.88). In terms of math domains Geometry and Calculus, very high correlations exist between sessions 1 and 2 of all the years ranging between 0.87 and 1. These high correlations reflect a great consistency between sessions 1 and 2 of the national tests. In terms of math domain *Statistics and Probability*, refer to table 12, negative and very low correlations exist between each of session-1 and session-2 official tests of the year 2017 and sessions 1 and 2 of the year 2019 (0.67 and -0.06 respectively). This reflects an inconsistency between each of the mentioned set. This is due to discrepancies of the total percentages of test items under this topic or to discrepancies of the percentages under the cognitive domains under this math domain. On the other hand, the correlation sessions 1 and 2 of the years 2011-2016 is high positive (r= 0.89) reflecting a high consistency between sessions 1 and 2 of the year 2011-2016 under the domain *Statistics and Probability*.

In short, the quantitative analysis of the official tests of the LS track show that the objective *reasoning* has a percentage (16.41%) of the test items which is low and does not reflect the general and specific objectives of mathematics at this track. Solving mathematical problems, which is included in both cognitive domains *applying,* which is required by 43.34% of the tests items of the official tests for the LS track, and *reasoning* which is required by 16.41% of the tests items of the official tests for the same track, is emphasized ranging from simple mathematical problems to some complex non-routine problems. It is important to note at this level that since *applying* involves solving routine problems and *reasoning* consists of solving non-routine problems, many problems that normally require *reasoning*, when seen repeated over several tests are considered as routine questions that students are used to solve and thus classified under *applying* rather than *reasoning*. Moreover, a considerable change is obvious in the model tests over the years emphasizing *knowing* over *applying*, while a steady content and cognitive structure is obvious in the national tests over the years.

Chapter Five

Conclusions and Recommendations

This chapter includes six parts: 1) introduction, 2) discussion, 3) conclusions, 4) recommendations, 5) limitations, 6) and recommendations for future studies.

5.1. Introduction

The purpose of this study was to examine the alignment between the foundations of the Lebanese mathematics intended curriculum, as represented by the curriculum documentation, and national assessment, as represented by the national math tests for the LH and the LS tracks of grade 12 (assessed curriculum) between the years 2011 and 2019. The study also investigated the alignment's evolution throughout the implementation years of the reformed curriculum starting from the year 2001 taking into consideration the results of Sleiman's study (2012) and Safa's study (2013) on the alignment of the mentioned curriculum components during the previous years for the LH and LS tracks respectively. The study also studied if there exist any differences between session-1 and session-2 of the official tests for both tracks, by investigating their alignment with the 1997 reformed curriculum.

Content analysis techniques were used in the study for both the LH and LS tracks. The qualitative section studied the structure and content of each of the curriculum, model tests, and national official tests. The qualitative analysis of the tests took into consideration the topics and the test items. The model tests and official tests were analyzed quantitatively using double-entry statistical tables. Test items, as identified by Osta (2007), for each test were analyzed in these tables as to their corresponding objectives in the math curriculum and to the cognitive domains they address, based on TIMSS Advanced 2015 framework. These tables were then compared using Pearson Product-Moment coefficient to check the degree of alignment between the model tests and official tests.

The Lebanese official tests for the LH track, as shown in the analysis, are characterized by the following:

- All the official tests have the same structure. Each is made up of three parts with constant grade distribution each year. Each part covers a domain: *Algebra* (17.75%), *Calculus* (54.2%), and *Statistics* and *Probability* (28.05%).
- *Inequalities* under the *topic equalities and inequalities* and the *topic statistics* of grade 12 are never addressed in the official tests. *Statistics* in the official tests is only limited to very basic concepts taken in previous year.
- The topic *simple and compound interest* is addressed in session-2 only of the official tests. Its occurrence is rare (1.44% of the official tests) and limited to one objective out of five objectives.
- Propositional calculus is never addressed in the official tests. It was added to the omitted tests starting from the year 2016.
- The problems on the topics *equations* and *statistics and probability* are always presented in the real-life context with examples that are different each time but requiring applying what is learned under this *topic*. On the other hand, the problems on *equations* are always related to money and discounts.

- *Definitions and representations topic* is considered to be the most important topic since its test items cover 52.76% of the official tests with the highest assigned grade points (10 points out of 20). The problem on *definitions and representations* is never presented in the context of real-life situations. The given varies between three types: a graph, a table of variation, and a function. Many test items are of similar content but vary in structure as the given varies. Other different test items appear under this topic each time.
- The official tests put a great emphasis on the cognitive domain *knowing* (51.57% of the test items in the official tests), then *applying* gets 38.81%. Official tests almost neglect the domain *reasoning* (9.62%). The test items in the official tests are mostly routine questions previously seen in class. They don't challenge students' critical thinking and mathematical reasoning. What does this inform us about the connection of these tests to the foundations of the curriculum which emphasize mainly the *reasoning* cognitive domain?

On the other hand, according to the results of the analysis, the Lebanese official tests for the LS track are characterized by the following:

- All the tests have the same structure. Each is made up of four parts with constant grade distribution every year. Each part covers a domain: *Algebra* (18.42% of the test items), *Geometry* (19.16% of the test items), *Calculus* (45.69% of the test items), and *Statistics and Probability* (16.74% of the test items).
- The cognitive domains are emphasized differently in the national tests. The cognitive domain *applying* is the most emphasized (43.34% of the test

items). Then the domain *knowing* (40.26% of the test items) follows and *reasoning* (16.41% of the test items).

- The domain *Trigonometry* has no occurrence in any test before being omitted in the year 2018. The topic *statistics* also has no test items in the official tests before being omitted in the year 2018. Some objectives under different topics never occurred such as *distribution function* under the topic *probability*, *Pascal's Triangle* under the topic *literal and numerical calculations*, and *second order equations* under the topic *differential equations*.
- The domain *Calculus* which occurs in all tests have the highest percentage of test items and highest allocated grade points (8 points out of 20 each year). This domain contains four *topics*, but in every test two of these four topics always occur *definitions and representations* and *continuity and differentiation* with a great emphasis on the first (35.48% of the test items) compared to the second (5.28% of the test items). On the other hand, the *topic integration* seldom occurs having the same forms of test items (4.39% of the test items), and the *topic differential equations* rarely occurs (0.54% of the test items). *Differential equations* is tackled in only one test (session-2 of the year 2019) with basic questions that are limited to *knowing* and *applying* cognitive domains.
- The *topics numbers*, *classical study*, and *probability* occur without an obvious pattern in the national tests. On the other hand, the *topics* under the domain *Calculus* have similar repetitive form but of different functions and situations.

- A greater emphasis on *reasoning* appears in session-2 official tests of the years 2011-2016 and 2017-2018 than sessions-1 of the same periods.
- Three domains are tackled in the national tests in a purely abstract form (*Algebra, Calculus,* and *Geometry*), while *Probability* is always presented in a real-life context.
- The given in the national tests is rarely presented as a graph or table.

From all the above, what do the results tell us about the tests? The assessment system in the Lebanese curriculum? The problems from which the curriculum and assessment suffer?

The results of this study are in agreement with the results of the research done by Osta (2007), Sleiman (2012), and Safa (2013) regarding the characteristics of the official tests. The official tests have a steady structure and cover a narrowed part of the curriculum, a “mini curriculum” as named by Osta (2007).

The negative effects of high-stakes tests have been extensively discussed in literature. Since the results of the Lebanese official tests are the only criterion that determines the student’s eligibility of graduating from school and moving to the university in Lebanon, students will adjust themselves to the tests by only focusing on the materials covered in the exams (Madaus, 1988) and practicing items similar in content and format to the official tests (Bailey, 1996), which might result in not acquiring the understanding, skills and knowledge that students are supposed to acquire at this grade level.

High stakes tests also have their effects on the program itself, including teachers and curriculum developers. Teachers will teach to the test emphasizing only materials that are covered in the tests, because of the social pressure exerted on them (Madaus, 1988). Schools and parents want students' results to be satisfactory as these results determine the students' future and the schools' reputation. Moreover, a tradition of "past tests" develops in high-stakes test setting, and eventually these past tests define the curriculum (Madaus, 1988). This is obvious for the LH track in the model tests, which were released in the years 2017 and 2019. Both, the qualitative and quantitative analyses of the study show a great alignment (ranging between 0.9 and 0.97) between these late model tests and the corresponding official tests (of the years 2017-2018 and 2019) and between the official tests of the years 2011 to 2016 and the official tests of the years 2017-2018 and 2019. These two results show that the model tests had undergone modifications, while the national tests remained stable in structure, content, and cognitive levels required, and that the modification made to the model tests, representing the curriculum, seem to be made to align with the assessment instead of being based on a revision of the curriculum in terms of objectives and content.

This in turn raises concerns regarding the negative impact of the steady and low-level thinking education students are getting over this long period of time on them as individuals and on the whole society taking into consideration the importance of mathematics education on the intellectual development, other sciences, and society.

Moreover, on the website, it is clear that the model tests issued in the years 2017 and 2019 are prepared by different schools and authors with many years of experience of teaching students for the official tests. This raises the following concerns:

- These tests are being prepared by consumers. These authors are teachers who are teaching their students questions similar to the model tests which raises a question about the credibility of the national tests' results.

- Teachers preparing the tests are teachers who have been preparing the national tests years before the reform of the curriculum. Therefore, how much would they, their mentality, and assessment strategies have changed to keep pace with the drastic changes that the reform has brought to the mathematics curriculum?

The results of this longitudinal study contributes to the literature in a very important idea. The study showed that the model tests changed to align with the assessment while the opposite should happen. The assessment has to contribute to the development of the curriculum, while in the case of this study the assessment led to changing the curriculum in a way that didn't end in achieving its goals. This would not have been clear and obvious if the study was not carried in the way it was carried and over this period of time. The comparison of the model tests and the official tests and studying their alignment and the comparison and correlations done between the official tests over years showed this important idea and ended in the conclusion that a tradition of past tests has developed in high-stakes test setting and eventually defined the curriculum.

5.2. Discussion

The findings of the study will be discussed based on three main aspects: 1) the foundations of the Lebanese mathematics curriculum represented by its introduction, general objectives, and the Evaluation Guide (Appendix A), 2) the document "Details and Results of the Workshops Carried out by ECRD as Part of the Curriculum Evaluation and Development Plan" (Appendix G), and 3) the socio-political situation in Lebanon during the years of the implementation of the reformed curriculum. The discussion based on what's mentioned is important since the educational system in any country can't be separated from its context, and this is basic to have a better reading of the findings of this study and to better understand the reasons behind them.

5.2.1 Foundation of the Lebanese Mathematics Curriculum

The introduction of the math curriculum, as mentioned in the main curriculum document (referenced in Appendix A), states the role of mathematics and describes how the spirit and teaching of mathematics were reformed stressing the individual construction of knowledge through mental activities, the importance of eliminating the theoretical overuse, the use of calculator and computer technologies, and starting from real-life situations which shows that math is not separated from everyday life and ensuring the accessibility of mathematics learning by all.

The LH official tests, as analyzed qualitatively, help to some extent in the implementation of the reform of mathematics education as mentioned in the mathematics curriculum's introduction. About half of the *topics* are given in real-life context in the tests (*equations* and *statistics and probability*). On the other hand, the qualitative analysis of the official tests of the LS track shows that the national examination of this track does not reflect the major points of the reform as mentioned in the introduction of the mathematics curriculum. Test items of most of the topics of the official tests of this track are purely abstract, not relating to real-life, except for the *topic probability*. Moreover, national tests of both tracks do not require the use of any technological tool except the calculator for basic calculations. These major points of the introduction are very important in the LS track since students graduating from it are eligible to major in STEM majors (Science, Technology, Engineering, and Mathematics). Linking the topics learnt to real-life applications and understanding their usefulness constitute an important element that fosters students' interest in STEM subjects and majors (Baran et al., 2016; Chittum et al., 2017).

The general objectives of the reformed mathematics curriculum as mentioned in the main curriculum document (referenced in Appendix A) are: mathematical reasoning, solving mathematical problems, relating mathematics to the surrounding reality, communicating mathematically, and valuing mathematics (ECRD, 1997a). *Mathematical reasoning* is reflected in the LH official tests in a very low percentage (9.62% of the total test items). As for solving mathematical problems, only routine problems that require direct application of knowledge and procedures are emphasized in the official tests for this objective. As for communication, this objective is reflected to a good extent in the official tests based on if compared to the subskills required by the process objectives of the LH track. However, these subskills do not reflect completely this objective as mentioned in the introduction and general objectives and do not reflect what mathematical communication truly is. Sumarmo et al. (2012) give a more extended list for mathematical communication's characteristics. It includes: "(a) constructing real objects, figures and diagrams into mathematical ideas; (b) explaining mathematical ideas, situations, and relationships by oral and written expressions, or by means of real objects, pictures, figures, and algebra; (c) explaining daily events in mathematical symbol languages; (d) listening, discussing, and writing upon mathematics, comprehensive reading upon mathematical presentations; (e) explaining and drafting questions upon learnt mathematical materials" (Sumarmo et al., 2015, p. 351).

On the other hand, the quantitative analysis of the official tests of the LS track show that the objective *mathematical reasoning* has a percentage (16.41%) of the test items which is low and does not reflect the general and specific objectives of mathematics at this track. Solving mathematical problems, which is included in both cognitive domains *applying* and *reasoning* is emphasized ranging from simple mathematical problems to some complex non-routine problems. Moreover, only probability problems are related to reality, while no problems relate to other sciences. Communication, on the other hand, is also partially reflected in the official tests of this track since questions are mostly direct and theoretical.

Moreover, the stable structure and repetitive type of questions in the tests, as shown in the qualitative analysis, and the focus on *knowing* and *applying* rather than *reasoning* cognitive domains, as shown in the quantitative, reflect and promote the behaviorist approach, rather than the constructivist approach announced in the introduction and general objectives. This reflects that this approach still has its great implications on mathematics education and its practices in the 1997 reformed curriculum in Lebanon in terms of drill and practice questions and having same type and wording of questions.

The section "General principles about the guidelines and the way of developing the national test questions in mathematics for the general secondary school certificate" (Appendix D) under the Evaluation Guide contains the bases for the question selection in terms of content and format. These criteria are for all grade 12 tracks. The national tests should:

1. "Abide by the general and specific objectives": As mentioned earlier in this section, the official tests of the LH track require the communication skills mentioned in the specific objectives, while these skills are not emphasized in the official tests of the LS track. The official tests also require problem-solving skills but limited to routine problems for the LH track, and not including real-life problems or being integrated with other sciences for the LS track. However, reasoning is tackled in low percentages in the tests of both the LH and LS tracks.
2. "Have a balance among the basic three levels of knowledge: acquisition, application, and analysis". These levels can be associated to TIMSS cognitive domains: *knowing*, *applying,* and *reasoning.* Table 1 in Appendix Q shows an imbalance of these three levels in the LH track. *Knowing* has the main emphasis (51.57% of the test items), the *applying* (38.81% of the test items) and *reasoning* (9.62% of the test items). Similarly, an imbalance exists between these three cognitive domain in the official tests for the LS track. At this track, 40.26% of the test items require *knowing*, 43.34% require *applying*, and 16.41% require *reasoning*.
3. "Consider competences from all the domains and should include questions that test the competences tackling different parts of the curriculum". Each of the official tests has three parts covering the three domains for the LH track- *Algebra*, *Calculus*, and *Statistics and Probability*- except for 2 tests out of 16

tests that do not cover the domain *Algebra*. Some topics of the curriculum under each domain are neglected: *inequalities, simple and compound interest*, and *statistics*. For the LS track, the domain *Trigonometry* and the *topic statistics* never occur in the official tests before being omitted. Covering all competencies under all domains is not possible knowing that some objectives are always neglected while many others rarely appear in the official tests.

4. "Not follow a specific pattern, neglect any part of the curriculum, or consider continuously a specific topic". The LH and LS official tests do not respect this criterion since many topics are never tackled while others are always adopted. Moreover, the official tests of this track follow a steady structure with the same grade distribution.
5. "Must ask diverse types of questions (open-ended questions, short response, multiple-choice questions) and questions based on text, diagram, graph, etc". Most of the test items in the official tests of the LH track are open-ended. Multiple choice questions never occur. However, the problems on rational functions differ in representation each time between a graph, a table, or an explicit function form. Similarly, almost all of the LS national tests are of open-ended questions or short response. Two out of sixteen official tests have different forms of questions under the topic *numbers*; one is multiple choice and the other is true or false question. Graphs also appear in only two tests, while the remaining tests have problems based on text only.

6. “Be clearly communicated to escape multiple interpretations”. The official tests for the LH and LS tracks are well written and clearly communicated.
7. “Not be limited to grade 12 concepts, but include items that tackle concepts or objectives of grade 10 and 11”. This criterion is respected in the official tests of the LH track. Appendix K part II lists the curriculum content of grades 9, 10, and 11 of the LH track that are addressed in the official tests. Probability concepts taken in grade 11 are always tackled in the tests. Statistics test items are limited to concepts taken in previous years. Many concepts in the topic rational functions are taken in previous grade levels. The official tests for the LS track also respect this criterion as many concepts under all topics relate to concepts taken in previous years. Appendix K part I lists the curriculum content of grades 9, 10, and 11 of the LS track that are addressed in the official tests.

5.2.2 Details and Results of the Workshops Carried out by ECRD as Part of the Curriculum Evaluation and Development Plan

Several points were the topics of discussions in the workshops carried out by the ECRD and presented under the document “Details and Results of the Workshops Carried out by ECRD as Part of the Curriculum Evaluation and Development Plan”. These points are very important in understanding the results of this study.

Above all, it is clear that the curriculum development was not carried out based on a thorough planning of the whole curriculum forms or components altogether. The intended, implemented, and potentially implemented curricula were planned and put into application/effect first, then planning the evaluation or assessment process was thought of. Moreover, the discussion reflects an unclear understanding of the aims behind the mathematics curriculum and education. This is a crucial point because the way questions on the reasons and aims of mathematics education are answered strongly influences how math will be taught and who should do mathematics (Larson, 2018). This also raises questions on whether the introduction and the general objectives, in which the importance of mathematics on both the individual and social levels is described and in which clear objectives and essential skills needed to be acquired are stated, are the actual philosophy adopted in this reformed curriculum, or if they were only crafted in an appealing way just to be an image and not to be adopted.

Moreover, according to the document, curriculum planning and development seem to be carried out not only in separate phases, but also by different committees that seemed to work independently. This in turn reflects a distorted view of the curriculum by the curriculum developers and a possible absence of the concept "curriculum alignment" to them.

Another important issue can be concluded from this document. Developing the mathematics curriculum was based on specialists and experts who have great experience, but whose backgrounds do not reflect the average norm of the country. As a result, crafting the curriculum didn't take into consideration the level, needs, and abilities of the average students in the country. This contradicts the fact that they are designing a national curriculum which must take into consideration students' nature and needs all over the country. This is, as discussed in the document, the reason behind thinking that the allocated time for mathematics is enough to cover all the topics planned first, and then omitting many of them shortly after the curriculum reform in both the LH and the LS tracks.

In the document, it is not clear how reducing the lessons occurred, especially that some were directly omitted directly after the reform. This leads us to a very important question. Was the reduction of lessons which used to happen over years based on a series of global evaluation of the curriculum, or was it made randomly just because of not having time to cover them?

Moreover, it is obvious that examination in the Lebanese mathematics curriculum is controlling the curriculum. It was decided that the whole curriculum should be reformed again to align with the new evaluation system adopted. Beloe Report (SSEC, 1960) states that examination systems can control and govern the kind of curriculum adopted. Moreover, this confirms one of the negative effects high stake tests have which is "becoming the major schooling goal" as stated by Madaus (1988).

In addition, despite this fragile curriculum development process, the blame is on students' abilities in the existing problems and not on the program. However, to make educational accountability successful, curriculum alignment must be ensured, and thus curriculum developers and schools be held accountable- just as students- by showing that what is offered to students meet the standards set (Anderson, 2002).

For the LH track, it is clear that some of the topics which were always neglected in the official tests, namely *statistics* and *propositional calculus*, were possibly neglected for the following main reasons. The first is that *statistics* was new to teachers thus caused problems, but it is essential as stated in the curriculum introduction. Therefore, this topic remained in the curriculum without being tackled in the official tests until a decision was made to omit it in the year 2017. As for the topic *propositional calculus*, the debate over this topic might be the main reason behind keeping it but not tackling it. Some refused to remove it because it is essential for the university stage later on, while others saw it hard for students of LH track. This topic was kept for years then was added to the omitted topics.

For the LS track, the *topic statistics* might be never occurring for the same mentioned reason for this *topic* in the LH curriculum. However, according to the document, several *topics* were asked to be included by other subjects' developers (as the physics developers) in the mathematics curricula despite the mathematics curriculum developers' belief that they are not important. This might be the reason behind including several topics but limiting them to very low percentages of test items. The *topics trigonometry*, *integration*, and *differential equations* are known to be of high importance in physics and engineering majors. Therefore, having very low percentages in the official tests might be for this mentioned reason.

Finally, lowering the cognitive level required by the test items in the LH national tests, especially the reasoning domain, might be because of the low ability students of this track have as described in the document.

5.2.3 The Socio-Political Situation in Lebanon

Lebanon has passed through periods characterized by unstable political and security situations. During the years of the implementation of the reformed curriculum and since the year 2005, the assassinations of many political figures occurred. Since the year 2005 till the year of this study, the political and security situation in Lebanon has not reached stability. Many governments continuously came to power, then either quitted or were forced to quit by the public. This created an unstable situation on all aspects because when a government quits or does not get the minimum number of votes to reach power, it becomes a care-taking government. The care-taking government has jurisdictions which are very limited not exceeding issues that are considered very critical. As a result, decisions which are limited to ministers are barely taken and when taken, they are taken after months or years. Moreover, many important decisions in Lebanon are taken based on the agreement between all political parties. A decision such as the curriculum reform is considered a big decision that requires such agreement.

The curriculum as noticed in the documents of Appendix G had been going through revisions and evaluations. However, no change has been noticed in the assessment or the curriculum foundation and content till the year 2017. The reason behind this might be because of the mentioned chaotic political situation in the country which can also be noticed also in what happened in the year 2014 when the committee responsible for correcting the tests refrained from correcting the tests because they were not given their rights. New model tests were put to public in the academic year 2016-2017, some topics were omitted, and new guidelines for assessment were published as an attempt to improve the educational system. Other model tests were also issued in the academic year 2018-2019 with more topics being omitted.

5.3. Conclusions

This paper aimed to answer the following research questions: (1) Are the Lebanese official math exams at the secondary level (for each of LS and LH tracks), over the years 2011-2019, aligned with the national curriculum as reflected in the curriculum document (ECRD, 1997a)? (2) Is there any improvement in the alignment of the official math exams for each track, in the last years of implementation, compared to the extent of alignment that resulted from the two previous studies? (3) Are there differences between the tests in sessions 1 and 2, over the last years of implementation, for each of the Secondary LS and LH tests, in terms of content and cognitive domains addressed?

The three research questions will be discussed based on the results of this study.

5.3.1. Research Question 1

Are the Lebanese official math exams at the secondary level (for each of LS and LH tracks), over the years 2011-2019, aligned with the national curriculum as reflected in the curriculum document (ECRD, 1997a)?

To answer this question for the LH track, the quantitative analysis (refer to Table 5 chapter 4) shows that there is a very high correlation (r = 0.967) between the official tests and the model tests when considering the math domains and cognitive abilities. Moreover, the percentages presented in Table 1 Appendix Q show that the model tests and the official tests emphasize the *knowing* domain over *applying* and *reasoning*. One reason for this high correlation might be that the total number of test items of the model tests is 205, forty-nine of this total number corresponding to the model tests issued in the year 2000, while the remaining corresponding to the model tests issued in 2017 and 2019 which are highly aligned with the official tests of the years 2017-2019 and are very similar in content and cognitive domains as the national tests of the previous years.

Another possible reason for this high alignment might be the idea that a tradition of past tests develops in high-stakes test setting eventually defining the curriculum (Madaus, 1988). The high correspondence is obvious between the model tests issued in 2017 and 2019 and all the official tests and not only the corresponding official tests of the years 2017-2019. To test this hypothesis, correlations between the tests of the year 2019 were correlated with the tests of the years 2015-2016 and the years 2017-2018. Results are presented in Table 13.

Table 13: *Correlations Between the Official Tests of the Years 2015-2016 and 2017-2018 and the Model Tests of the year 2019 for Grade 12 LH Track*

	Overall Correlation	**In terms of Cognitive Domains**			**In terms of Math Contents**		
		Knowing	**Applying**	**Reasoning**	**Algebra**	**Calculus**	**Statistics &Probability**
OT15-16 & MT	0.949	1	0.96	0.72	0.98	0.95	0.97
OT17-18&MT	0.95	0.97	0.98	0.85	0.95	0.96	0.90

OT15-16: official tests of the years 2015-2016
OT17-18: official tests of the years 2017-2018
MT: model tests of the year 2019

According to Table 13, very high overall correlations and very high correlations under all the cognitive and math domain exist between the model tests of the year 2019 and the official tests of the years 2015-2016 and 2017-2018. This shows that the above hypothesis might be true for the LH track to a great extent based on the results presented in table 13.

To answer this question for the LS track, the quantitative analysis (refer to Table 10 chapter 4) shows that there is a very high correlation (r = 0.94) between the official tests and the model tests when considering the math domains and cognitive abilities. Moreover, the percentages presented in Table 1 Appendix G show that the model tests and the official tests emphasize the knowing domain over applying and reasoning. The correlations under each the cognitive domains and the content domains are also very high.

The reason of this high correlation might be also that the total number of test items of the model tests is 324. Seventy-seven of this total number correspond to the model tests issued with the curriculum documents, while the remaining correspond to the model tests issued in 2017 and 2019 which are highly aligned with the official tests of the years 2017-2019 and very similar in content and cognitive domains as the national tests of the previous years. Similar to the LH testes, another possible reason for this high alignment for the LS track might be idea that a tradition of past tests develops in high-stakes test setting eventually defining the curriculum (Madaus, 1988). To test this hypothesis, correlations between the tests of the year 2019 were correlated with the tests of the years 2015-2016 and the years 2017-2018. Results are presented in Table 14.

Table 14: *Correlations Between the Official Tests of the Years 2015-2016 and 2017-2018 and the Model Tests of the year 2019 for Grade 12 LS Track*

	Overall Correlation	**In terms of Cognitive Domains**			**In terms of Math Contents**			
		K	**A**	**R**	**Alg.**	**Geo.**	**Calc.**	**S.P.**
OT15-16 & MT	0.94	0.95	0.99	0.75	0.97	0.8	0.8	0.4
OT17-18&MT	0.92	0.89	0.96	0.94	0.76	1	1	-0.33

K: Knowing
A: Applying
R: Reasoning
Alg.: Algebra
Geo,: Geometry
Calc. : Calculus (Numerical Functions)
S.P. : Statistics & Probability
OT15-16: official tests of the years 2015-2016
OT17-18: official tests of the years 2017-2018

MT: model tests of the year 2019

According to Table 14, very high positive overall correlations and very high correlations under all the cognitive and math domain exist between the model tests of the year 2019 and the official tests of the years 2015-2016 and 2017-2018 except under the domain *Statistics and Probability*. This shows that the above hypothesis might be true to a good extent for the LS track based on the results presented in Table 14.

This reflects that a "testing culture" have been developed and rooted over the years. This intersects with the results of Osta's study's (2007) that suggest "that the still rooted assessment culture is affecting the new official exams and consequently every other component of the curriculum" (Osta. 2007; p. 197).

5.3.2. Research Question 2

Is there any improvement in the alignment of the official math tests for each track, in the last years of implementation, compared to the extent of alignment that resulted from the two previous studies?

The correlations between the model tests and the official tests for the LH track are presented in Table 15. Correlations of the years 2001-2010 are taken from Sleiman's study (2012). The correlations in Table 15 reflect the alignment of the tests in a better way since each group of official tests is correlated with its corresponding model tests.

Table 15: *Correlations Between the Official Tests of each of the Years 2001-2005, 2006-2010, 2011-2013, 2015-2016, 2017-2018, and 2019 and the Model Tests for Grade 12 LH Track*

	2001-2005	**2006-2010**	**2011-2013**	**2015-2016**	**2017-2018**	**2019**
OT & MT	0.80	0.80	0.695	0.817	0.85	0.9

OT & MT : Correlation between the official tests of the years 2011-2019 (OT) and the model tests (MT)

According to Table 15, the correlation is the same between the tests of the years 2001-2005 and the tests of the years 2006-2010 (r=0.8) although according to Table 7 Appendix Q there are some changing aspects of the evolution of these tests under the *topics propositional calculus* and *simple and compound interests*. These topics are more neglected in the official tests of the years 2006-2010. However, the correlation decreases to r=0.695 for the official tests of the years 2011-2013. This is because of the discrepancies between percentages of the model tests and the official tests of these years under the *topics propositional calculus*, *simple and compound interests*, and *statistics*. *Propositional calculus* has no test items in the official tests of these years, while *simple and compound interests* and *statistics* have no test items in the model tests issued with the curriculum documents. Moreover, the *topic equations* is less emphasized in the official tests of these years than in the model tests (12.5% and 26.53% respectively). The correlation increases in the official tests of the years 2015-2016 (r=0.817). The *simple and compound interest* is neglected in the tests of these years, while the topic *equations* is more emphasized causing a better correlation with the model tests under the domain *Algebra.* The correlation increases to r=0.85 between the model tests of the year 2017 and the official tests of the years 2017-2018, and r=0.9 for tests of the year 2019. The occurrence of every *topic* has almost the same percentage in the model tests and the official tests of the years 2017-2018 and 2019. Similarly, the percentages under the cognitive domains *knowing* and *applying* are very close. The discrepancy is only under the domain *reasoning* being more emphasized in the model tests issued in the years

2017 and 2019 than the official tests, making the correlation not perfect between the model tests and the official tests of these years.

The correlations between the model tests and the official tests for the LS track are presented in Table 16. Correlations of the years 2001-2003 and 2009-2010 are taken from Safa's study (2013). However, to avoid an overlap between the periods of correlations, new correlations were made between the tests of the years 2009-2010 using Safa's results since Safa (2013) calculated the correlation over the periods 2001-2003 and 2009-2012.

Table 16: *Correlations Between the Official Tests of each of the Years 2001-2003, 2009-2010, 2011-2013, 2015-2016, 2017-2018, and 2019 and the Model Tests for Grade 12 LS Track*

	2001-2003	**2009-2010**	**2011-2013**	**2015-2016**	**2017-2018**	**2019**
OT & MT	0.80	0.92	0.92	0.88	0.96	0.89

OT & MT : Correlation between the official tests of the years 2011-2019 (OT) and the model tests (MT)

According to Table 16, the correlation between the model tests and the official tests of the years 2001-2003 is 0.8. This correlation remarkably increases in the year 2009-2010 (r=0.92). This increase is due to emphasizing *applying* at the expense of *knowing* in the tests of this period making them better aligned with the official tests. Same correlation is obtained for the tests of the years 2011-2013, while it decreases to 0.88 in the tests of the years 2015-2016. This decrease is mainly due to the discrepancies between the model tests and the official tests of this year under the *topics numbers* (11.37% and 21.6% of the test items respectively) *and continuity and differentiation*

(10.38% and 4.11% respectively). Moreover, *statistics* has no test items in the official tests of this year unlike the model tests. The correlation increases to 0.96 in the years 2017-2018 because of the close correspondence between the percentages under all cognitive and content domains reflecting a high alignment between the model and the official tests of these years. The correlation decreases again between the tests of the years 2019 due to the discrepancies of the cognitive domains percentages under all math domains.

5.3.3. Research Question 3

Are there differences between the tests in sessions 1 and 2, over the last years of implementation, for each of the Secondary LS and LH tests, in terms of content and cognitive domains addressed?

Table 8 in chapter 4 represents the correlations between the model tests of the LH track and sessions 1 and 2 of the official tests of the LH track during the last years of implementation (2011-2019). In general, the session-1 and session-2 official exams are very consistent and aligned among each other. This is shown in the high correlations r = 0.931 between the session-1 and session-2 official exams of the years 2011-2016, r = 0.911 between the session-1 and session-2 official exams of the years 2017-2018, and r = 0.937 between the session-1 and session-2 official exams of the year 2019.

The correlation between the model tests and the official tests is very high under all cognitive domains and under all math domains reflecting a very good alignment between sessions 1 and 2 national tests.

Moreover, the *topic statistics* is more emphasized in session 2 of the official tests, while the *topic simple and compound interest* occurs only in session 2 of the official tests (refer to Tables 4 and 5 in Appendix Q).

On the other hand, Table 12 in chapter 4 represents the correlations between the model tests of the LS track and sessions 1 and 2 of the official tests of the LS track during the last years of implementation (2011-2019). In general, session-1 and session-2 official tests are consistent and aligned among each other. However, this consistency decreases over the years. This is shown in the high correlations r = 0.96 between the session-1 and session-2 official exams of the years 2011-2016, r = 0.83 between the session-1 and session-2 official exams of the years 2017-2018, and r = 0.7 between the session-1 and session-2 official exams of the year 2019.

The correlation between the model tests and the official tests is very high under all cognitive domains and under all math domains in the first years, but decreases mainly in the year 2019 where discrepancies occur under the *Algebra* (0.41) and Statistics and Probability (-0.06) content domains and the cognitive domain *reasoning* (0.01).

5.4. Recommendations

The findings of this study reveal that the official tests have a stable structure and target a narrow part of the curriculum. This leads teachers to teach to the tests and students to concentrate on the materials covered by the tests (Madaus, 1988). Moreover, the tests emphasize *knowing* over *applying* and *reasoning* cognitive domains. The tests do not address the main concerns of the curriculum presented in the curriculum's introduction and general objectives: mathematical reasoning, solving non-routine problems, communication, and connections.

However, the analysis of these findings shows that problems in the alignment between the official tests and the curriculum foundation might be due to a great extent to the curriculum development process of the reformed curriculum which occurred in an inconsistent and coherent way.

Therefore, it is recommended that the new reform starts by reconsidering the testing policies and contents based on the curriculum foundations and not the opposite. It is important that several terms made clear and defined well when developing the new curriculum. The most important of these terms is "curriculum alignment" and its importance. Curriculum forms: intended, implemented, assessed, and attained must be taken into consideration when developing the new curriculum. A thorough planning of all these forms should take place. The definition and role of assessment in the curriculum and educational process should be taken into consideration to avoid the negative effects of the high stake national examination and to avoid making examination the basic goal behind education. To attain a positive washback of the tests, there should be a close correspondence between the test and the syllabus, and tests should be properly conceived and implemented (Popham, 1987; 2001).

Moreover, it is recommended that the reduction of lessons in the curriculum, when it happens, should be based on a periodic global evaluation of the curriculum and not randomly. Many lessons are important to the university education, to the development of certain skills which are important for the individual intellect and future, and in daily life. Therefore, assessing the lessons and curriculum is very important to consider when reducing the curriculum.

In addition, it is recommended to include experts from different institutions (at the level of schools and universities) and backgrounds in the curriculum planning process. Teachers should also be included in this process since they play a basic role in the educational process.

Available curriculum alignment models, frameworks, and studies can be made use of when planning for any new curriculum to ensure the alignment of all forms of the curriculum.

After reforming the curriculum, it is recommended to carry out professional development on assessment and strategies followed for teachers, committee members who prepare the assessment, and correctors of the official tests.

On the official tests level, it is recommended that the official tests take into consideration including the neglected general objectives: reasoning, problem solving of non-routine problems, and relating mathematics to the surrounding society and sciences. Presenting problems in a real-life context is highly recommended for the LS track. Moreover, including the use of technology and a more complex use of scientific calculators is recommended in both tracks. Including questions of different types and forms which trigger students' higher order cognitive skills are also recommended. Another very important point should be considered. Solving problems can go under *applying* and *reasoning*. Solving problems which are routine problems done or seen before, even if they apparently require reasoning, go under *applying.* Therefore, it is recommended that questions in the national tests do not take the same form and structure each time. Including real-life situations might help a lot in varying the form of the questions. Moreover, to attain a positive washback of the national tests, it is recommended that these tests test the abilities whose development is required to be encouraged and to be sampled widely and unpredictably (Bailey, 1996).

5.5. Limitations of the Study

This paper has two limitations. The first limitation is that no issued model tests were found after the model tests issued with the reform until the year 2017. However, according to the press, schools and several institutions complained when model tests of the year 2017 were issued stating that this not more than telling students the questions of the tests. The researcher concluded that these might be the only issued model tests besides the ones issued with the reformed curriculum in the year 2000.

Another limitation is that the model tests which were issued with the reformed curriculum have a low number of test items compared to the official tests and the new model tests which might have affected the results of the study.

5.6. Recommended Future Research

The following are recommendations for future research:

1. A study on how the “curriculum alignment” concept as conceived among the curriculum developers, administrators, and teachers.
2. A study on the perceptions of grade 12 teachers towards the national examination and its positive or negative effects on their instruction.
3. A study on the philosophy of the national examination as perceived by test developers and test correctors and comparing them to the documents named

"Specialized Reports For the Results of the Official Examination" found under the ECRD website.

4. A study on the alignment between the intended curriculum and the implemented curriculum (perceived –the interpretation of curriculum by teachers- and operational –the curriculum in action as taught and learned) at each of grade 12 tracks.

References

Adirika, B. (2020). Examining models of curriculum development and processes: Implications for African educational heritage and review. *Social Science and Humanities Journal, 2017* (6), 324-342.

Alderson, J. & Wall, D. (1993). Does washback exist*? Applied Linguistics, 14*, 115-129. http://dx.doi.org/10.1093/applin/14.2.115

Alfrey, L., O'Connor, J., & Jeanes, R. (2017). Teachers as policy actors: co-creating and enacting critical inquiry in secondary health and physical education. *Physical Education and Sport Pedagogy, 22*(2), 107–120. https://doi.org/10.1080/17408989.2015.1123237

Anderson, L. (2002). Curricular alignment: A re-examination. *Theory into Practice, 4,* 255-261.

Alsubaei, A. (2015). Hidden curriculum as one of current issue of curriculum. *Journal of Education and Practice, 6*(33), 125-128.

Bailey, K. (1996) Working for washback: A review of the washback concept in language testing, *Language Testing, 13*(3), 257–279.

Baran, E., Bilici, S., Mesutoglu, C., & Ocak, C. (2016). Moving STEM beyond schools: students' perceptions about an out-of-School STEM education program. *International Journal of Education in Mathematics, Science and Technology, 4(1),* 9-19.

Bhaw, N. & Kriek, J. (2020). The alignment of the grade 12 physics examinations with the CAPS curriculum: (November 2014-March 2018). *South African Journal of Education, 40*(1), 1-9.

Black, P., and Wiliam, D. (1998). Assessment and Classroom Learning, *Assessment in Education: Principles, Policy & Practice, 5* (1), 7–74.

Broadbent, J., Panadero, E., & Boud, D. (2018). Implementing summative assessment with a formative flavor: a case study in a large class. *Assessment and Evaluation in Higher Education, 43*(2), 307-322.

Cachia, R., Ferrari, A., Ala-Mutka, K. & Punie, Y. (2009) *Creative Learning and Innovative Teaching: Final Report on the Study on Creativity and Innovation in Education in EU Member States*. Luxembourg: Publications Office of the European Union.

Cachia, R., Ferrari, A., Ala-Mutka, K. & Punie, Y. (2010) *Creative Learning and Innovative Teaching: Final Report on the Study on Creativity and Innovation* *in Education in EU Member States.* Luxembourg: Publications Office of the European Union.

Case, B., Jorgensen, M., & Zucker, S. (2004). *Alignment in educational assessment*. New Jersey: Pearson Education.

Chapman, D. & Snyder, C. (2000). Can high stakes national testing improve instruction: Reexamining conventional wisdom. *International Journal of Educational Development, 20*, 457-474. http://dx.doi.org/10.1016/S0738-0593(00)00020-1

Chittum, J., Jones, B., Akalin, S., & Schram, A. (2017). The effects of an afterschool STEM program on students' motivation and engagement. *International Journal of STEM Education, 4*(11), 1-16.

Contino, J. (2013). A case study of the alignment between curriculum and assessment in the New York State Earth Science standards-based system. *Journal of Science Education and Technology, 22*(1), 62-72.

Cornbleth, C. (1984). Beyond hidden curriculum. *Journal of Curriculum Studies 16*(1):29–36.

ECRD (1997a). Mathematics curricula. In *General Education curricula and their objectives. Decree n^o 10227* (pp. 287-327). Lebanon: Ministry of National Education, Youth and Sports & National Center of Educational Research and Development.

ECRD (1997b). *Curriculum of Mathematics. Decree n^o 10227. Details of the contents of the first year of each cycle.* Lebanon: Ministry of Education and Higher Education (MEHE) & Educational Center for Research and Development (ECRD).

ECRD (1998). *Curriculum of Mathematics. Decree n^o 10227. Details of the contents of the second year of each cycle.* Lebanon: Ministry of Education and Higher Education (MEHE) & Educational Center for Research and Development (ECRD).

ECRD (1999). *Curriculum of Mathematics. Decree n^o 10227. Details of the contents of the third year of each cycle.* Lebanon: Ministry of Education and Higher Education (MEHE) & Educational Center for Research and Development (ECRD).

ECRD (2000). *Curriculum of Mathematics. Decree n^o 10227. Evaluation Guide. Mathematics Secondary Cycle.* Lebanon: Ministry of Education and Higher Education (MEHE) & Educational Center for Research and Development (ECRD).

Ernest, P. (2010). Why teach mathematics? *Professional Educator, 9*(2), 44-47.

Fisher, F. (1998). Policy inquiry in a post-positivist perspective, *Policy Studies Journal*, *26*(1), 129-146.

Halai, N. (2008). Curriculum reform in science education in Pakistan. In R. Coll & N. Taylor (Eds.), *Science education in context: An international examination of the influence of context on science curricula development and implementation* (pp.115-129). Sense Publishers.

Hashmi, A., Hussain, T., & Shoaib, A. (2018). Alignment between mathematics curriculum and textbook of grade VIII in Punjab. *Bulletin of Education and Research*, *40*(1), 57-76.

Haylock, D. (2018). *Student workbook mathematics explained for primary teachers. (6th ed.).* SAGE Publications Ltd.

Hughes, A. 1989: *Testing for language teachers*. Cambridge: Cambridge University Press.

Kelly, A., (2004). *The curriculum: theory and practice* (5th ed.). SAGE Publications.

Kibble, J. (2017). Best practices in summative assessment. *Advances in Psychology Education*, *14*, 110-119.

Larson, M. (2018). Why Teach Mathematics. NCTM. Retrieved from:

https://my.nctm.org/blogs/matthew-larson/2018/02/21/why-teach-mathematics

Madaus, G. F. (1988). The distortion of teaching and testing: High-stakes testing and instruction. *Peabody Journal of Education, 65*, 29-46.

Martone, A., & Sireci, S. (2009). Evaluating Alignment between Curriculum, Assessment, and Instruction. *Review of Educational Research*, 79(4), 1332-1361.

Retrieved: http://www.jstor.org/stable/40469099

Miller, J., & Seller, W. (1985). *Curriculum: Perspectives and Practice*. New York: Longman.

Montilla, J.(2019). *Behaviorism: its implication to mathematics*.

Retrieved from:

https://www.researchgate.net/publication/338149249_BEHAVIOURISM_ITS_IMPLICATION_TO_MATHEMATICS_EDUCATION/citation/download

Moye, J.N. (2019), A Shared Definition of Curriculum Design, *Learning Differentiated Curriculum Design in Higher Education*, Emerald Publishing Limited, pp. 1-18. https://doi.org/10.1108/978-1-83867-114-320191004

Mullis, I.V.S., Martin, M.O., Robitaille, D.F., & Foy, P. (2009). *TIMSS advanced 2008 international report: Findings from IEA's study of achievement in advanced mathematics and physics in the final year of secondary school*. Chestnut Hill, MA: Boston College. Retrieved from

http://timssandpirls.bc.edu/timss_advanced/ir.html

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS Advanced 2015 International Results in Advanced Mathematics and Physics*. Retrieved from Boston College, TIMSS & PIRLS International Study Center.

website: http://timssandpirls.bc.edu/timss2015/international-results/

Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020). *TIMSS 2019 International Results in Mathematics and Science*. Retrieved from Boston College, TIMSS & PIRLS International Study Center.

website: https://timssandpirls.bc.edu/timss2019/international-results/

National Council of Teachers of Mathematics (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: Author.

Noble, A. & Smith, M. (1994). Measurement-driven reform: Research on policy, practice, repercussion. *CRESST/Arizona State University,* 1-28.

Okitowamba, O., Julie, C., & Mbekwa, M. (2018). The effects of examination-driven teaching on mathematics achievement in Grade 10 school-based high-stakes examinations. *Pythagoras, 39*(1), 1-10.

Osta, I. (2007). Developing and piloting a framework for studying the alignment of mathematics examinations with the curriculum: The case of Lebanon. *Journal of Educational Research and Evaluation, 13*, 171-198.

Panhwar, A., Ansari, A., & Shah, A. (2017). Post-positivism: An effective paradigm for social and educational research. *International Research Journal Arts & Humanities (IRJAH), 45*, 253-260.

Phaeton, M. & Stears, M. (2017). Exploring the alignment of the intended and implemented curriculum through teachers' interpretation: A Case study of A-Level biology practical work. *Eurasia Journal of Mathematics, Science and Technology Education, 13*, 723-740.

Ployhart, R. & Vandenberg, R. (2010). Longitudinal Research: The theory, design, and analysis of change. *Journal of Management*, 36(1), 94–120. https://doi.org/10.1177/0149206309352110

Popham, W. (1987). The merits of measurement-driven instruction. *Phi Delta Kappan, 68(9)*, 697-682.

Popham, W. (2001) Teaching to the test? *Educational Leadership, 58*(6), 16–20.

Popham, W. (2011). *Classroom assessment: What teachers need to know*. Boston, MA: Pearson.

Porter A. (2002). Measuring the content of instruction: uses in research and practice. *Edutional Research, 31*(7): 3–14.

Porter, A. C. (2006). Curriculum assessment. In J., Green, G., Camilli, P., Elmore, & P., Elmore, (Eds.), *Handbook of complementary methods in education research* (pp. 141-159). Taylor & Francis Group.

Retrieved from:

https://ebookcentral.proquest.com/lib/lau-ebooks/detail.action?docID=446575

Porter, A., Polikoff, M., Zeidner, T., & Smithson, J. (2008) The quality of content analyses of state students' achievement tests and content standards. *Educational Measurement Issues Practice, 27*(4): 2-14.

Robitaille, D., Schmidt, W., Raizen, S., McKnight, C., Britton, E., & Nicol, C. (1993). *Curriculum frameworks for mathematics and science* (Vol. TIMSS Monograph No.1). Vancouver: Pacific Educational Press.

Safa, W. (2013). *Evaluating the alignment between a mathematics curriculum and the national tests: The case of Lebanon secondary national exams for the life science section* (Master's Thesis under the supervision of I. Osta). Lebanese American University, Beirut.

Schmidt, W., McKnight, C., Valverde, G., Houang, R., & Wiley, D. (1997). *Many visions, many aims: A cross-national investigation of curricular intentions in school mathematics* (Vol. 1). Dordrecht: Kluwer.

Schmidt, W., McKnight, C. Houang, R., Wang, H., Wiley, D., Cogan, L., & Wolfe, R. (2001). *Why schools matter: A cross-national comparison of curriculum and learning*. San Francisco: Jossey-Bass.

Scott, H. (2011). *Relevancy of classic curriculum theories of Ralph Tyler and Hilda Taba*. Retrieved from:

www.hwscott.net/educational-technology/relevancy-of-classic-curriculum-theories-of-ralph-tyler-and-hilda-taba

Secondary Schools Examinations Council (1960) *Secondary School Examinations other than the GCE* (the Beloe Report), London: HMSO.

Retrieved from:

http://www.educationengland.org.uk/documents/beloe/beloe.html

Seitz, P. (2017). Curriculum alignment among the intended, enacted and assessed curricula for grade 9 mathematics. *Journal of the Canadian Association for Curriculum Studies (JCACS), 15*(1), 72-94.

Shatila D. (2014). *A study of the coherence of the Lebanese math curriculum at the intermediate level*. (Master's Thesis under the supervision of I. Osta). Lebanese American University, Beirut.

Shehayeb, S. (2017). TIMSS and National Examinations. *Academia.edu.* Retrieved from https://www.academia.edu/35693407/TIMSS_and_National_Examinations_Sanaa_Shehayeb_Article_2017_pdf

Shute, V. J. & Kim, Y. J. 2014. "Formative and Stealth Assessment." *Handbook of Research on Educational Communications and Technology*, 311–321. New York: Springer. 10.1007/978-1-4614-3185-5

Sleiman, L. H. (2012). *A study of the alignment between the Lebanese secondary-level national math exams for the literature and humanities track and the reformed math curriculum*. (Master's Thesis under the supervision of I. Osta). Retrieved from:

https://ecommons.lau.edu.lb:8443/xmlui/handle/10725/1460

Sowell, E. J. (2005). *Curriculum: An integrative introduction* (3rd ed.). Upper Saddle River, New Jersey: Merrill Prentice Hall.

Squires, D. (2012). Curriculum alignment research suggests that alignment can improve student achievement. *The Clearing House, 85*(4), 129-135.

Sumarmo, U., Koswara, U., & Kusumah, Y. (2012). Meningkatkan kemampuan penalaran dan komunikasi matematis siswa SMA melalui Pendekatan Konteksual Berbantuan Program Autograph. Universitas Pendidikan Indonesia. *Jurnal Educationist, 4*(2).

Sumarmo, U., Kleden, M., & Kusumah, Y. (2015). Analysis of enhancement of mathematical communication competency upon students of mathematics education study program through metacognitive learning. *International Journal of Education and Research, 3*(9), 349-358.

Thorndike, E. (1929). *The psychology of arithmetic*. The Macmillan Company. Retrieved from:

http://www.gutenberg.org/files/39300/39300-h/39300-h.htm#GENERAL_INTRODUCTION

Torkildsen, L. & Erickson, G. (2016). “If they’d written more…”- on students’ perceptions of assessment and assessment practices. *Education Inquiry, 7*(2), 137-157.

Valverde, G., Bianchi, L., Wolfe, R., Schmidt, W., & Houang, R. T. (2002). *According to the Book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks.* Dordrecht: Kluwer Academic Publishers.

Van den Akker, J. (2003). The science curriculum: between ideals and outcomes. In B. J. Fraser & K. G. Tobin (Eds.), *International Handbook of Science Education* (Vol. 1, pp. 421–449). Dordrecht: Kluwer Academic Publishers.

Van den Akker J. (2004) Curriculum Perspectives: An introduction. In: *Curriculum Landscapes and Trends*. Springer, Dordrecht.

Van den Akker, J. (2010). Curriculum perspectives: an introduction. In J. van den Akker, W. Kuiper & U. Hameyer (Eds.), *Curriculum landscapes and trends* (pp. 1–10). London: Kluwer Academic.

Verschaffel, L., Depaepe, F., & De Corte, E. (2015). Mathematics Education. In *International Encyclopedia of the Social & Behavioral Sciences.* Elsevier.

Webb, N. (1997). *Research monograph No. 6. Criteria for alignment of expectations and assessments in mathematics and science education.* Washington, DC: Council of Chief State School Officers.

Webb, N. L. (1999). *Research monograph No. 18. Alignment of science and mathematics standards and assessments in four states.* Washington, DC: Council of Chief State School Officers.

Webb, N. (2007). Issues related to judging the alignment of curriculum standards and assessments. *Applied Measurement in Education, 20*(1), 7-25.

Wildemuth, B. (1993). Post-positivist research: Two examples pf methodological pluralism. *Symposium on Qualitative Research: Theory, Methods, and Applications, 63*(4), 450-468.

Yilmaz, F., & Sunkur, M. (2021). A curriculum alignment analysis: A sample of life sciences course curriculum (2018) for 3rd-grade students. *Journal of Qualitative Research in Education, 27*, 279-297.

Zhao, M., Mu, B., & Lu, C. (2016). Teaching to the test: Approaches to teaching in senior secondary schools in the context of curriculum reform in china. *Creative Education, 7*(1), 32-43.

APPENDICES

APPENDIX A

The References of the Curriculum Documents

<u>Main Document</u>

ECRD (1997a). Mathematics curricula. In *General Education curricula and their objectives. Decree nº 10227* (pp. 287-327). Lebanon: Ministry of National Education, Youth and Sports & National Center of Educational Research and Development.

<u>Three *Details-of-Content* Documents</u>

ECRD (1997b). *Curriculum of Mathematics. Decree nº 10227. Details of the contents of the first year of each cycle.* Lebanon: Ministry of Education and Higher Education (MEHE) & Educational Center for Research and Development (ECRD).

ECRD (1998). *Curriculum of Mathematics. Decree nº 10227. Details of the contents of the second year of each cycle.* Lebanon: Ministry of Education and Higher Education (MEHE) & Educational Center for Research and Development (ECRD).

ECRD (1999). *Curriculum of Mathematics. Decree nº 10227. Details of the contents of the third year of each cycle.* Lebanon: Ministry of Education and Higher Education (MEHE) & Educational Center for Research and Development (ECRD).

ECRD (2000). *Evaluation Guide - Secondary.* Lebanon: Ministry of Education and Higher Education (MEHE) & Educational Center for Research and Development (ECRD).

APPENDIX B

Coding the Details of Contents of the Lebanese Reformed Math Curriculum

For the LS track at the Secondary School Level

Retrieved from:

Ministry of National Education, Youth and Sports & National Center of Educational Research and Development (1997). *Curriculum of Mathematics. Decree n^{o} 10227. Details of the contents of the third year of each cycle.* Lebanon: Ministry of National Education, Youth and Sports & National Center of Educational Research and Development.

Codes	*Math Curriculum for the LH track at the Secondary School Level*
1	**ALGEBRA**
1.1.	Foundations
1.1.1.	Binary operations
1.1.1.1.	Identify a binary operation.
1.1.1.1.i.	→ Identify a binary operation on a set E as a rule which associates to every pair $(x,y) \in E\times E$ an element $z \in E$.
1.1.1.2.	Recognize the properties of a binary operation.
1.1.1.2.i.	→ Identify an associative binary operation.
1.1.1.2.ii.	→ Identify a commutative binary operation.
1.1.1.3.	Recognize certain particular elements.
1.1.1.3.i.	→ Identify a neutral element (an identity element) for a binary operation.
1.1.1.3.ii.	→ Identify the symmetric element of an element for a binary operation.
1.1.2.	Structure of group
1.1.2.1.	Define a group and give examples of groups
1.1.2.1.i.	→ Identify an Abelian group
1.1.2.1.ii.	→ Identify a group.
1.2	Literal and numerical calculations
1.2.1	Combinations: definition, notation, binomial formula
1.2.1.1	Identify a combination of elements of a finite set
1.2.1.1.i.	→ Identify a combination of p elements of a set of n elements ($p \leq n$)as a part of this set formed of p elements
1.2.1.2	Calculate the number of combinations of p elements of a set of n elements ($p \leq n$)
1.2.1.2.i.	→ Determine, in simple cases, all the combinations of p elements of a set of n elements ($p \leq n$)

1.2.1.3. Construct the Pascal's triangle

1.2.1.4 Know and use the binomial formula

1.2.1.4.i. → Know and use the formula giving the number nCp of all combinations of p elemenets of a set of n elemenets ($p \leq n$)

1.2.1.4.ii. → Model situations by combinations

1.2.1.4.iii. → Know and use the binomial formula for expanding (a+b)^n

1.2.1.4.iv. → Know and use the formula nCp = (n-1)Cp + (n-1) C (p-1)

1.3.	Equations & Inequalities
1.3.1.	System of linear equations (m×n): definition, elementary operations on therows, Gauss' method

1.3.1.1. Identify a linear system (m×n)

1.3.1.2. Reduce a linear system (m×n) by successive applications of elementary operations

1.3.1.2.i. → Apply an elementary operation on the equations of a linear system and know that it transforms it into an equivalent system

1.3.1.3. Solve a linear system (m×n) by the Gauss method

1.3.1.3.i. → Recognize a solution of a linear system

1.3.1.3.ii. → Classify the linear systems into impossible systems, indeterminate systems, and *determinate* systems.

1.3.1.3.iii. → Recognize an impossible reduced linear system

1.3.1.3.iv. → Recognize a reduced linear system possessing a unique solution

1.3.1.3.v. → Recognize a reduced linear system possessing an infinity of solutions and identify in this case the rank and the unknowns of the system

1.3.1.3.vi. → Solve a reduced linear system

1.4	Numbers
1.4.1.	Modulus and argument of a complex number. Properties

1.4.1.1. Calculate and Interpret geometrically the modulus (absolute value) and argument (amplitude) of a complex number

1.4.1.1.i. → Calculate the modulus of a complex number written in an algebraic form

1.4.1.1.ii. → Interpret geometrically the modulus of a complex number

1.4.1.1.iii. → Calculate the argument of a non-zero complex number written in an algebraic form

1.4.1.1.iv. → Interpret geometrically the argument of a non-zero complex number

1.4.1.2 Know and use the formulas relative to the modulus and argument of a complex number.

1.4.1.2.i. → Know and use the following properties relative to the modulus of a complex numbers: mod(z)≥ 0 , mod(z) is a real number , [mod(z) = 0] ..

1.4.1.2.ii. → Know and use the following properties relative to the argument of a non-zero complex numbers: arg(-z)= π + arg(z) (2π).....

1.4.2.	Trigonometric and exponential form of a complex number

1.4.2.1. Write a complex number in the trigonometric form

1.4.2.1.i. → Write a non-zero complex number z, given in algebraic form, in the trigonometric form $z = r(\cos\theta + i\sin\theta)$ where r, θ are real numbers, r>0

2.1.1.2.i.	→ Recognize the mixed product of three vectors
2.1.1.2.ii.	→ Determine the analytic expression of the mixed product ina direct orthonormal system
2.1.1.2.iii	→ Use the mixed product to calculate the volume of a parallelepiped and hat of a tetrahedron.
2.1.1.2.iv	→ Know that the mixed product of three vectors is zero if , and only if, these vectors are coplanar.
2.1.2.	**Equation of a plane and a stright line in space**
2.1.2.1.	Determine the cartesian equation of a plane and a line defined by geometric elements in an orthonormal system.
2.1.2.1.i.	→ Recognize the equation ux + vx+ wz + r = 0 as that of a plane perpendicular to the non-zero vector V (u, v, w)
2.1.2.1.ii.	→ Determine an equation of the plane passing through a given point and perpendicular to a non-zero vector.
2.1.2.1.iii.	→ Determine an equation of a plane passing through three non-collinear points
2.1.2.1.iv.	→ Determine an equation of a plane passing through a given point and parallel to two non-parallel given directions.
2.1.2.1.v.	→ Know that the line of non-zero direction vector V(a, b, c) and passing through a point A (x0, y0, z0) is the set of points M(x,y,z) verifying the system of parametric equations: x =at+x0 , y=bt+y0 , ,z = ct + z0 where t is a real parameter
2.1.2.1.vi.	→ Determine a system of parametric equations of a line passing through two given points
Additional	Show that a given point lies in a plane
Additional	Show that a line passes through a given point
Additional	Show that a line lies in a plane
Additional	Determine an equation of plane passing through a point and a line
Additional	Determine an equation of plane passing through 2 points and perpendicular to a plane
Additional	Determine an equation of plane containing 2 lines
Additional	Determine an equation of plane passing through a point and parallel to a plane
2.1.3.	**Orthogonality of two straight lines, of a straight line and a plane; perpendicular planes**
2.1.3.1	Characterize the orthogonality of two lines, of a line and a plane and of wo planes, knowing their equations, in an orthonormal system
2.1.3.1.i.	→ Know that two lines of respective direction vectors V(a,b,c) and V'(a',b',c') are orthogonal if, and only if, aa'+bb'+cc' = 0
2.1.3.1.ii.	→ Know that a line of a direction vector V and a plane of normal vector V' are orthogonal if, and only if, V and V' are collinear.
2.1.3.1.iii.	→ Know that two planes of respective normal vectors V(u,v,w) and V'(u', v', w') are orthogonal if, and only if , uu'+vv'+ww' = 0
2.1.4.	**Parallelism of straight lines and planes**
2.1.4.1	Study the relative positions of two planes, two lines and of a plane and a line, knowing their equations, in an orthonormal system.

2.1.4.1.i.	→ Know that two lines of respective direction vectors V and V' are parallel (or confounded) if, and only if, V and V' are collinear.
2.1.4.1.ii.	→ Know that a line of a direction vector V and a plane of normal vector V' are parallel if, and only if,, V and V' are orthogonal
2.1.4.1.iii.	→ Know that two planes of respective normal vectors V and V' are parallel (or confounded) if, and only if, V and V' are collinear
2.1.4.1.iv.	→ Determine the system of parametric equations of the line of intersection of two secant planes
2.1.4.1.v.	→ Determine the intersection of two secant lines.
2.1.4.1.vi.	→ Determine the intersection of a line and a plane
Additional	Prove two lines are skew
2.1.5.	**Distance from a point to a plane, to a straight line.**
2.1.5.1.	Determine the distance from a point to a plane and the distance from a point o a line in an orthonormal system.
2.1.5.1.i.	→ Kniow and use the relation d = abs(ux0 + vy0 + wz0 + r)/sqr (u^2 + v^2 +w^2) expressing d from a point A(x0,y0,z0) to the plane of equation ux+vy+wz+r = 0
2.1.5.1.ii.	→ Calculate the distance from a point to a plane
Additional	Calculate the distance between two lines
3	**CALCULUS (NUMERICAL FUNCTIONS)**
3.1.	**Definitions & Representations**
3.1.1.	**Inverse functions**
3.1.1.1.	Determine the composite functions of two given functions.
3.1.1.1.i.	→ Recognize and calculate the composite function of two functions
3.1.1.2.	Characterize the functions having an inverse function.
3.1.1.2.i.	→ Recognize the reciprocal function f^(-1) of a continuous and strictly monotonous function f
3.1.1.2.ii.	→ Know that the reciprocal function f^(-1) of f exists only if f is continuous and strictly monotone
3.1.1.3.	Compare graphically the graphs of a function and its inverse
3.1.1.3.i.	→ Determine the domain of definition of a reciprocal function
3.1.1.3.ii.	→ Know that a function and its reciprocal have the same sense of
3.1.1.3.iii.	→ Calculate, if possible, the explicit expression of the reciprocal function.
3.1.1.3.iv.	→ Know that the graphs of a function and its reciprocal are symmetric each other with respect to the first bisector of the orthonormal system
3.1.2.	**Inverse trigonometric functions**
3.1.2.1.	Study the functions Arcsin, Arccos and Arctan
3.1.2.1.i.	→ Recognize the inverse function of the sine function over [-π /2, π /2] and represent it graphically.
3.1.2.1.ii.	→ Recognize the inverse function of the cosine function over [0, π[and present it graphically.
3.1.2.1.iii.	→ Recognize the inverse function of the stangent function over]-π /2, π

	/2[and represent it graphically.
3.1.3.	**Natural (Naperian) logarithmic function. Logarithmic function to the base a**
3.1.3.1.	Study and represent graphically the natural logarithmic function ln.
3.1.3.1.i.	→ Recognize the domain of definition, variation and graph of the natural logarithmic funtion.
3.1.3.1.ii.	→ Know and use the properties of the natural logarithmic function: a and b are two strictly positive real numbers. $Ln(ab) = lna + lnb$ $ln(a/b)=lna-lnb$, $lnsqrt\ a = (1/2)lna$
3.1.3.1.iii.	→ Characterize the number e
3.1.3.1.iv.	→ Recognize the following limits: lim lnx ($x \rightarrow 0+$) , lim lnx ($x \rightarrow +\infty$), lim lnx/x ($x \rightarrow +\infty$), limxlnx ($x \rightarrow 0+$), limln(1+x)/x ($x \rightarrow 0$)
3.1.3.2.	Differentiate functions of the form ln(u) and calculate the primitives of functions of the form u/u' where u is a function
3.1.3.2.i.	→ Recognize the derivative of lnu where u is a function of x and a primitive of u'/u with $u \neq 0$.
3.1.3.3.	Know the relation which links the function ln to the logarithmic function to base a ($a>0$ and $a \neq 1$) and deduce the properties of the latter.
3.1.3.3.i.	→ Know that loga(x) = lnx/lna with $a>0$ and $a \neq 1$
3.1.3.3.ii.	→ Know that the function log a is strictly increasing for $a > 1$ and strictly decreasing for $1>a>0$.
3.1.3.3.iii.	→ Solve equations and inequalities that include the logarithmic function
3.1.3.	**Exponential functions**
3.1.3.1.	Study and represent graphically the exponential function to base e
3.1.3.1.i.	→ Recognize the domain of definition, variation and the representative curve of the exponential function to base e
3.1.3.1.ii.	→ Know and use the properties of the exponential function to base e : $e^{(x+y)} = e^x + e^y$, $e^{(x-y)} = e^x / e^y$ $(e^x)^y = e^{(xy)}$
3.1.3.1.iii.	→ Recognize the following limits: lim e^x ($x \rightarrow -\infty$), lim e^x ($x \rightarrow +\infty$), lim(e^x)/x ($x \rightarrow +\infty$), lim abs(x)e^x ($x \rightarrow -\infty$), lim (e^x -1)/x ($x \rightarrow 0$)
3.1.3.1.iv.	→ Recognize the derivative of the function e^u and a primitive of u(e^u) where u is a function of x.
	→ Know that $a^b = e^{blna}$ where $a>0$ and $a \neq 1$.
3.1.3.2.	Study and represent graphically the exponential function to base a
3.1.3.2.i.	→ Recognize the domain of definition, variation and representative curve of the function a^x
3.1.3.3.	Study the power function $x \rightarrow x^{(x \rightarrow -\infty)}$
3.1.3.3.i.	→ Know that the power function $x \rightarrow x^\alpha$, where α is a real number is only defined if $x>0$
3.1.3.3.ii.	→ Recognize the variation and the representative curve of the power function.
3.1.3.3.iii.	→ Recognize the following limits: lim lnx/ x^α ($x \rightarrow +\infty$), lim x^αlnx ($x \rightarrow 0+$), lim e^x/x^α ($x \rightarrow +\infty$), lim abs(x)^αe^x ($x \rightarrow -\infty$) ($\alpha > 0$)
3.1.3.4.	Compare the increases of the functions ln , e^x, and x^α
3.1.3.4.i.	→ Solve equations and inequalities that including logarithmic and

exponential functions.

3.2.	Continuity and differentiation
3.2.1.	Image of a closed inteval by a continuous function
3.2.1.1.	Characterize the image of a closed interval by a continuous function
3.2.1.1.i.	→ Know that the image of an interval by a contiuous function is an interval of the same nature
3.2.1.1.ii.	→ Know the fact that a continuous function on a closed interval reaches a maximum and a minimum on this interval and that it takes every intermediate value between the two extremes (theorem of intermediate values)
3.2.1.2.	Locate a root for a continuous function on a closed interval and justify the existence of this root.
3.2.1.2.i.	→ Know that if a function f is continuous and strictly monotonous on an interval I, it defines a bijection of I on f(I)
3.2.1.2.ii.	→ Know that if a function f is continuous on the interval [a,b] with f(a).f(b)≤0, it possesses at least one root in [a,b]
3.2.1.2.iii.	→ Know that if a function f is continuous and strictly monotonous on an interval [a,b] with f(a)f(b)≤0, it possesses one only root in [a,b].
3.2.2.	Derivatives of composite functions
3.2.2.1.	Differentiate a composite function.
3.2.2.1.i.	→ Recognize and calculate the derivative of a composite function at a point.
3.2.2.1.ii.	→ Recognize and calculate the derivative of a composite function of two functions on an interval.
3.2.3.	Derivatives of an inverse function
3.2.3.1.	Differentiate an inverse function
3.2.3.1.i.	→ Use the formula [f^(-1)]' (y0) = 1/ f'(x0), with y0 = f(x0)
3.2.3.1.ii.	→ Recognize the derivative of an inverse function on an interval.
3.2.4.	Second derivative, successive derivatives.
3.2.4.1.	Calculate the second derivative and the successive derivatives of a function.
3.2.4.1.i.	→ Calculate the second derivative of a function at a point and on an interval
3.2.4.1.ii.	→ Calculate the successive derivatives of a function at a point and on an interval.
Additional	Prove a point to be a point of inflection
Additional	Find the point of inflection
3.2.5.	L'Hopital's rule
3.2.5.1.	Use L'Hopital's rule when finding limits
3.2.5.1.i.	→ Use L'Hopital's rule to calculate limits
3.3.	Integration
3.3.1.	Integral: definition, properties
3.3.1.1.	Define the integral of a function f continuous on an interval [a,b]
3.3.1.1.i.	→ Recognize the integral of a continuous function f on the closed

APPENDIX C

Coding the Details of Contents of the Lebanese Reformed Math Curriculum

For the LH track at the Secondary School Level

Retrieved from:

Ministry of National Education, Youth and Sports & National Center of Educational Research and Development (1997). *Curriculum of Mathematics. Decree n^{o} 10227. Details of the contents of the third year of each cycle.* Lebanon: Ministry of National Education, Youth and Sports & National Center of Educational Research and Development.

Codes	*Math Curriculum for the LH track at the Secondary School Level*
1	ALGEBRA
1.1.	Foundations
1.1.1.	Binary operations
1.1.1.1.	Identify a binary operation.
1.1.1.1.i.	→ Identify a binary operation on a set *E* as a rule which associates to every pair (x,y) ϵ *E*×*E* an element *z* ϵ *E*.
1.1.1.2.	Recognize the properties of a binary operation.
1.1.1.2.i.	→ Identify an associative binary operation.
1.1.1.2.ii.	→ Identify a commutative binary operation.
1.1.1.3.	Recognize certain particular elements.
1.1.1.3.i.	→ Identify a neutral element (an identity element) for a binary operation.
1.1.1.3.ii.	→ Identify the symmetric element of an element for a binary operation.
1.1.2.	Structure of group
1.1.2.1.	Define a group.
1.1.2.1.i.	→ Clarify the structure of the set of integers provided by addition.

1.1.2.1.ii.	→ Identify a group as being a set provided by a binary operation which verifies certain properties.
1.1.3.	Propositional calculus
1.1.3.1.	Identify a proposition.
1.1.3.1.i.	→ Identify a proposition as being a declarative phrase.
1.1.3.1.ii.	→ Identify a tautology as being the proposition that is always true.
1.1.3.2.	Recognize and use the basic logical operators.
1.1.3.2.i.	→ Identify the negation of a proposition.
1.1.3.2.ii.	→ Identify the conjunction of a proposition.
1.1.3.2.iii.	→ Identify the disjunction of a proposition.
1.1.3.2.iv.	→ Identify the implication as being the proposition $(\neg P) \vee Q$.
1.1.3.2.v.	→ Identify the equivalence as being the proposition $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$.
1.1.3.3.	Use the table of truth.
1.1.3.3.i.	→ Fill the table of truth of a proposition.
1.2. 1.2.1.	Equations & Inequalities Situations- problems leading to the solutions of equations and inequalities
1.2.1.1.	Analyze a problem and put it in equations and/or inequalities.
1.2.1.1.i.	→ Choose the unknown or the unknowns.
1.2.1.1.ii.	→ Write the equations, systems of equations, inequalities or systems of inequalities which must verify the unknowns.
1.2.1.2.	Clarify the constraints on solutions imposed by the studied situation.
1.2.1.3.	Solve the equations and/or the inequalities and verify the validity of the solutions found.
1.2.1.3.i.	→ Solve the equations and/or inequalities.
1.2.1.3.ii.	→ Asses the relevance of the solutions.
2 2.1. 2.1.1.	CALCULUS (NUMERICAL FUNCTIONS) Definitions & Representations Simple rational functions
2.1.1.1.	Study and represent graphically simple rational functions.
2.1.1.1.i.	→ Recognize a rational function as being a function of the form $x \rightarrow f(x) = P(x) / Q(x)$ where P and Q are polynomials.
2.1.1.1.ii.	→ Determine the domain of definition of a rational function.
2.1.1.1.iii.	→ Determine the parity of a rational function and exploit it.
2.1.1.1.iv.	→ Study the sense of variation of a rational function.

Codes	*Math Curriculum for the LH track at the Secondary School Level*
2.1.1.1.v.	→ Calculate the limits at the neighborhood of the domain of definition of a rational function.
2.1.1.1.vi.	→ Find the vertical, horizontal asymptotes.
2.1.1.1.vii.	→ Interpret the limits graphically.
2.1.1.1.iix.	→ Find that a given line is an asymptote.
2.1.1.1.ix.	→ Represent graphically a rational function.
2.1.1.1.x.	→ Solve graphically an equation of the form P(x) / Q(x) = m where *m* is a real number.
2.1.2.	**Graphical interpretation**
2.1.2.1.	Interpret a graph and grasp the essential information that are presented.
2.1.2.2.	Use the representative curve of a function to:
2.1.2.2.i.	→ Find from a graph the domain of definition of the function corresponding to this graph.
2.1.2.2.ii.	→ Determine the intervals of increase (resp. of decrease) of the correspondent function.
2.1.2.2.iii.	→ Determine graphically the extrema and characterize them.
2.1.2.2.iv.	→ Determine graphically the points of discontinuity.
2.1.2.2.v.	→ Clarify the limits if they exist.
2.1.2.2.vi.	→ Graphically locate the value of $f(x)$ for a given x.
2.1.2.2.vii.	→ Graphically locate the value of x for a given $f(x)$.
2.1.2.2.iix.	→ Solve graphically inequalities of the form: $f(x) \geq m$ (resp. $\leq$) for a given real value of m.
2.1.2.2.ix.	→ Compare f and g on a given interval where g is a reference function for a given x.
2.1.3.	**Exponential growth and exponential function**
2.1.3.1.	Calculate a^x for a real positive number a in the two cases $a > 1$ and $0 < a < 1$.
2.1.3.2.	Know and use the properties: $a^x . a^y = a^{x+y}$ $(a^x)^y = a^{x.y}$
2.1.3.3.i.	→ Represent graphically, point by point the function: $x \rightarrow a^x$ for a given real positive number a.
2.1.3.3.ii.	→ Read graphically the variation of the function: $x \rightarrow a^x$ according to a.
.1.3.3.iii.	→ Compare graphically the two functions: $x \rightarrow x^n$ where n is a positive integer and $x \rightarrow a^x$ where a is a positive real number.

Codes	*Math Curriculum for the LH track at the Secondary School Level*
2.2. 2.2.1.	Mathematical Models for Economics and Social Sciences Simple interest, compound interest
2.2.1.1.	Calculate the simple interest or the compound interest returned by acapital placed at a given rate for a given duration.
2.2.1.2.	Find an element among the four elements concerned by the calculationof interest knowing the other three.
2.2.1.2.i.	→ Know the terminology: capital, simple interest, compound interest, interest rate, period of placement, actual value, acquired value.
2.2.1.2.ii.	→ Know and apply the relation linking the capital, rate, duration and interest.
2.2.1.2.iii.	→ Know and apply the formula linking the acquired value, capital, interest rate and duration.
2.2.1.2.iv.	→ Know and use the formulas of annuity.
3 3.1. 3.1.1.	STATISTICS AND PROBABILITY Statistics Measures of central tendency and measures of variability of a distribution of one (continuous or discrete) variable
3.1.1.1.	Calculate the measures of central tendency and measures of variabilityand know how to interpret them.
3.1.1.1.i.	→ Recognize the median class.
3.1.1.1.ii.	→ Recognize the modal class(es).
3.1.1.1.iii.	→ Identify and calculate analytically and graphically (if it can be done) the median and the mode(s).
3.1.1.1.iv.	→ Identify and determine the range.
3.1.1.1.v.	→ Identify and calculate the mean, mean deviation, variance and standard deviation.
3.1.1.1.vi.	→ Compare and interpret two distributions of the same mean and ofdifferent standard deviations.
3.2. 3.2.1.	Probability Conditional probability: definition, independence of two events
3.2.1.1.	Define and calculate the probability of an event *A*, knowing that anevent *B* is achieved.
3.2.1.1.i.	→ Calculate $P_B(A)$ *by the formula* $P_B(A) = P(A/B) = P(A \cap B) / P(B)$.

3.2.1.1.ii. → Calculate $P(A \cap B)$ by the formula: $P(A \cap B) = P(A/B) \times P(B) = P(B/A) \times P(A)$ where *A* and *B* are two non- impossible events.

3.2.1.2. Define two independent events:

3.2.1.2.i. → Recognize two independent events *A* and *B* by the fact that $P(A/B) = P(A)$.

APPENDIX D

General Principles about the Guidelines and the Way of Developing the Official Exam Questions in Mathematics for the General Secondary School Certificate

Part I:

Retrieved from:

Ministry of Education and Higher Education & Educational Center for Research and Development (2000). *Evaluation Guide. Mathematics Secondary Cycle.* Lebanon: Ministry of National Education, Youth and Sports & National Center of Educational Research and Development.

مبادئ عامة حول أصول وطريقة وضع أسئلة الامتحانات الرسمية في الرياضيات للشهادة الثانوية العامة

تهدف مسابقة الرياضيات في الامتحانات الرسمية إلى قياس مدى اكتساب التلاميذ للكفايات العائدة لهذه المرحلة (راجع لوائح

الكفايات لمادة الرياضيات العائدة لصفوف الثالث ثانوي بفروعها الأربعة.)

الأسس المتبعة لاختيار الأسئلة

في المضمون

ينبغي أن تراعي أسئلة الرياضيات الأسس التالية:

- التقيّد بأهداف المادة (العامة والخاصة) وذلك من خلال احترام نظام التقييم الجديد وفلسفته (دليل المعلم للتقييم).
- التوازن بين مستويات المعرفة الأساسية الثلاثة (الاكتساب ــ التطبيق ــ التحليل).
- اختيار الكفايات من كافة المجالات وتضمين الاختبار أسئلة تقيّم كفايات متدامجة تغطي عدة مواضيع من المنهاج.
- الابتعاد عن نمط معين للاختبار، وذلك من خلال عدم إهمال أي جزء من المنهاج بشكل دائم (بمعنى ألا يُستبعد بشكل دائم موضوع ما من أسئلة الاختبار)، وكذلك عدم اعتماد حتمية وجود موضوع ما في كافة الاختبارات.

- العناية بصياغة الأسئلة ووضوحها منعاً[3] لكل التباس.
- تنوع أشكال الأسئلة: أسئلة مغلقة أو مفتوحة (تتطلب اتخاذ قرار من قبل المرشح)، أسئلة الاختيارات المتعددة، أسئلة .
- مبنية على مستند (نصّ – جدول – بيانات – رسومات هندسية أو تحليلية – الخ.) أو غير ذلك.

في الشكل

- يتكون اختبار الرياضيات من عدة مسائل إلزامية (ليس هناك شرط على عدد المسائل).
- تأتي الأسئلة في كراس (على الأقل أربع صفحات A3) مطوية.
- ينبغي أن يكون الاختبار سهل القراءة لجهة اختيار نوع الخط (Font) وحجمه، والمسافات بين الأسطر والهوامش العامة أو الداخلية.
- ترقم المسائل بالترقيم الروماني (I, II, III, etc.). ترقم الأسئلة للمسألة الواحدة بالأرقام العربية (1, 2, 3, etc.). وترقم الأسئلة الفرعية بالأحرف اللاتينية (a, b, c, etc.).
- تذكر علامة كل مسألة من المسائل الواردة في الاختبار دون تحديد العلامة لكل سؤال في المسألة الواحدة.
- تخصص الصفحة الأولى من كرّاس أسئلة الاختبار لتوصيف الاختبار وتتضمن بعض الإرشادات العامة (أنظر التفصيل لاحقا).

تتضمن الصفحة الأولى المعلومات التالية:

- الكتابة الرسمية (الجمهورية اللبنانية – وزارة التربية .. الخ.)
- اسم الشهادة الرسمي.
- المادة.
- اللغة.
- عدد المسائل .
- مدة الاختبار.
- تعداد الأدوات اللازمة (أدوات الرسم الهندسي – آلة حاسبة غير قابلة للبرمجة أو لاختزان المعلومات أو لرسم
- البيانات – الخ).
- إرشادات عامة للمرشحين : قراءة كافة الأسئلة قبل البدء بالإجابة – اختيار الترتيب الذي يلائم المرشح في كتابة الحلول – الاعتناء بالخط لجهة الوضوح والترتيب وتجنب التشطيب قدر الإمكان – الخ.

Part II:

Retrieved from:

https://www.crdp.org/guideline-official-exams

توصيف مواد الامتحانات الرسمية

القرار رقم 142 /م/2017 تاريخ 16 شباط 2017: يمكنكم معاينة الملحق المصحح لتوصيف مسابقات الامتحانات الرسمية بناء على قرار معالي وزير التربية والتعليم العالي الأستاذ مروان حمادة رقم 142 /م/2017 تاريخ 16 شباط 2017، المتعلق بتصحيح الملحقين المرفقين بالقرار رقم 631/م/2016 تاريخ 2016/9/3 المتعلق بتوصيف مواد الامتحانات الرسمية للشهادتين المتوسطة والثانوية العامة بفروعها الأربعة،

القرار رقم 631/م/2016 تاريخ 2016/9/3: أصدر الوزير بو صعب قرارا آخر يتعلق بتوصيف المسابقات وكلّف المركز التربوي القيام بهذه المهمّة، وعقدت لهذه الغاية اجتماعات عمل بين جميع المعنيين والمتخصصين في المركز والقطاعين التربويين الرسمي والخاص، واستند العاملون في ذلك، إلى آلية عمل جديدة قائمة على بدء التنسيق بين مواد اللغات في ما بينها، وبين مواد العلوم في ما بينها، وبين مواد الإجتماعيات في ما بينها، وذلك بهدف توحيد اللغة بين المواد المتشابهة، ووضع عدد أكبر من الأسئلة الصغيرة، والسعي باستمرار إلى تغطية أكبر قسم من المنهج، على أن يواكب هذا التوجه إعطاء فرصة أكبر للإجابة، آخذين في الإعتبار مراعاة مجالات المعارف والتفكير المنطقي والتواصل.

وركزت اللجان في وضع أسئلة متسلسلة ضمن السؤال الواحد، على أن تكون هذه الأسئلة نابعة من واقع المتعلم، وأن تعتمد على التحليل، وأن يتم وضعها انطلاقا من اعتماد معايير محددة في توزيع العلامة عند تصحيح المسابقة.

واعطى الوزير توجيهاته إلى المديرية العامة للتربية لتوزيع هذين القرارين على المدارس كافة، وأن يتم وضع القرار والتعميم وملحقاتهما على الموقع الإلكتروني للمركز التربوي: www.crdp.org

ملاحظة: يمكنكم تحميل جدول توزيع العلامات في مواد الامتحانات الرسمية للشهادتين المتوسطة والثانوية العامة على الرابط التالي: جدول توزيع العلامات في مواد الامتحانات الرسمية للشهادتين المتوسطة والثانوية العامة

د- فرع الآداب والإنسانيّات:

تهدف مسابقة الرّياضيّات في الشّهادة الثّانويّة العامّة – فرع الآداب والإنسانيّات – إلى قياس مدى اكتساب التّلميذ للكفايات في المرحلة الثّانويّة (راجع لوائح الكفايات العائدة لمادّة الرّياضيّات للصّفّ الثّالث الثّانوي – فرع الآداب والإنسانيّات).

الأسس المتّبعة لاختيار الأسئلة:

في المضمون:

ينبغي أن تراعي أسئلة الرّياضيّات الأسس التّالية:

١- التّقيّد بأهداف المادّة (العامّة والخاصّة) وذلك من خلال احترام نظام التّقييم الجديد وفلسفته.

٢- التّوازن بين مستويات المعرفة الأساسيّة الثّلاثة (الاكتساب – التّطبيق – التّحليل).

٣- اختيار الكفايات من المجالات كافّة.

Processus de calcul – Résolutions de problèmes et communication.

وتضمين المسابقة أسئلة تقيّم كفايات متداخلة تغطّي عدّة مواضيع من المنهج.

٤- عدم إهمال أيّ جزء من المنهج بشكل دائم وعدم اعتماد حتميّة وجود موضوع ما في المسابقات كافّة.

٥- تنويع أشكال الأسئلة: أسئلة مغلقة أو مفتوحة (تتطلّب اتخاذ قرار من قبل المرشّح)، أسئلة تتضمّن اختيارًا من متعدّد، أسئلة مبنيّة على مستندات (نصوص – جداول – بيانات – رسومات هندسيّة أو تحليليّة إلخ...) أو غير ذلك.

٦- عدم التّقيّد بطرح الأسئلة من الصّفّ الثّالث ثانويّ فقط بل من الصّفّ الثّاني والثّالث ثانويّ.

في الشّكل

١- تتكوّن المسابقة من عدّة مسائل إلزاميّة (يمكن للتّلميذ التّعامل معها بالتّرتيب الّذي يراه مناسبًا).

٢- تُذكر علامة كلّ مسألة من المسائل الواردة في المسابقة دون تحديد العلامة في كلّ سؤال في المسألة الواحدة.

٣- عدد صفحات المسابقة أربع صفحات كحد أقصى.

٤- تُرقّم المسائل بالتّرقيم الرّوماني (I,II,III,...)، تُرقّم أسئلة المسألة الواحدة بالأرقام العربيّة (١،٢،٣،...) وتُرقّم الأسئلة الفرعيّة بالأحرف اللاتينيّة (a,b,c,...).

ب- فرع علوم الحياة

تهدف مسابقة الرّياضيّات في الشّهادة الثّانويّة العامّة – فرع علوم الحياة – إلى قياس مدى اكتساب التّلميذ للكفايات في المرحلة الثّانويّة (راجع لوائح الكفايات العائدة لمادّة الرّياضيّات للصّفّ الثّالث الثّانوي – فرع علوم الحياة).

الأسس المتّبعة لاختيار الأسئلة:

في المضمون:

ينبغي أن تراعي أسئلة الرّياضيّات الأسس التّالية:

١- التّقيّد بأهداف المادّة (العامّة والخاصّة) وذلك من خلال احترام نظام التّقييم الجديد وفلسفته.

٢- التّوازن بين مستويات المعرفة الأساسيّة الثّلاثة (الاكتساب – التّطبيق – التّحليل) .

٣- اختيار الكفايات من المجالات كافّة.

Processus de calcul – Fonctions numériques (Analyse) – Résolutions de problèmes et communication.

وتضمين المسابقة أسئلة تقييم كفايات متدامجة تغطّي عدّة مواضيع من المنهج.

٤- عدم إهمال أي جزء من المنهج بشكل دائم وعدم اعتماد حتميّة وجود موضوع ما في المسابقات كافّة.

٥- تنويع أشكال الأسئلة: أسئلة مغلقة أو مفتوحة (تتطلّب اتخاذ قرار من قبل المرشّح)، أسئلة تتضمّن اختيارًا من متعدّد، أسئلة مبنيّة على مستندات (نصوص – جداول – بيانات – رسومات هندسيّة أو تحليليّة إلخ...) أو غير ذلك.

٦- عدم التّقيّد بطرح الأسئلة من الصّفّ الثّالث ثانويّ فقط بل من الصّفّ الثّاني والثّالث ثانويّ.

في الشّكل

١- تتكوّن المسابقة من عدّة مسائل إلزاميّة (يمكن للتّلميذ التّعامل معها بالتّرتيب الّذي يراه مناسبًا).

٢- تُذكر علامة كلّ مسألة من المسائل الواردة في المسابقة دون تحديد العلامة في كلّ سؤال في المسألة الواحدة.

٣- عدد صفحات المسابقة أربع صفحات كحدّ أقصى .

٤- تُرقّم المسائل بالتّرقيم الرّوماني (I,II,III,...)، تُرقّم أسئلة المسألة الواحدة بالأرقام العربيّة (١,٢,٣,...) وتُرقّم الأسئلة الفرعيّة بالأحرف اللاتينيّة (a,b,c,...).

ملاحظة: يُطلب إلى المرشّح حيازة أدوات الرّسم الهندسيّ وآلة حاسبة غير قابلة للبرمجة أو تخزين المعلومات أو رسم البيانات.

APPENDIX E

Model Tests LH track

Model Test 2 (LHM2)

Retrieved from:

Ministry of Education and Higher Education & Educational Center for Research and Development (2000). Evaluation Guide. Mathematics Secondary Cycle. Lebanon: Ministry of National Education, Youth and Sports & National Center of Educational Research and Development.

الجمهورية اللبنانية
وزارة التربية والتعليم العالي

الشهادة الثانوية العامة

فرع الآداب والإنسانيات

اختبار الرياضيات (فرنسي)

(نموذج ٢)

السنة : ....
عدد الأسئلة : ثلاثة
مدة الاختبار : ساعة

إرشادات عامة:

- يجب أن يكون مع المرشح : أدوات الرسم الهندسي - آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يجب أن يستخدم المرشح قلم حبر (سائل أو ناشف) أزرق أو أسود بشكل عام. ويحق للمرشح استخدام أقلام ملونة أو قلم رصاص للرسم أو للإيضاح.
- يستحسن أن يقرأ المرشح كافة أسئلة الاختبار قبل البدء بالإجابة.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في الاختبار).
- إن لجان التصحيح تولي أهمية خاصة للخط (لجهة الوضوح) والترتيب، لذلك ينصح المرشح بالكتابة بشكل واضح والترتيب قدر الإمكان، مع تجنب التشطيب.

I. **(3 points)**

The students of a secondary school are distributed according to the following table:

	Boys	Girls
External	650	850
Half-internal	550	450

We randomly pick up one student.

Compute the probability that this student is external given that he is a boy.

II **(6 points)**

The average monthly income of either an employee or a technician in a firm is 600 000 LP.

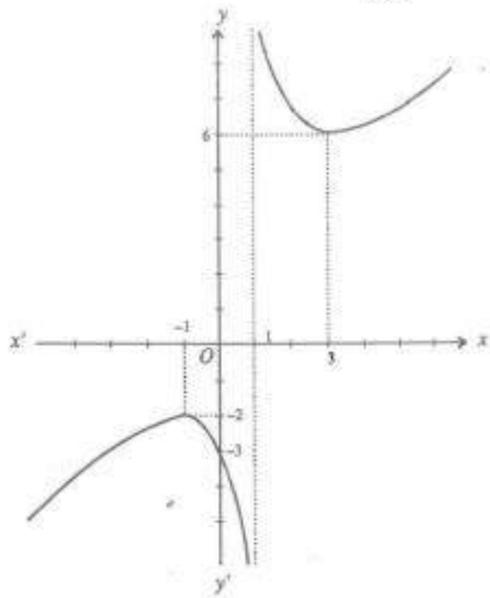
If we raise the wage of the employee by 10% and we reduce that of the technician by 10%, the average income becomes 590 000 LP.

What is the monthly income of each of them?

III. **(11 points)**

The following curve (*C*) represents the function f defined by $f(x) = ax + b + \frac{4}{x+c}$ where a, b, c are real numbers. By reading the graph :

1) Calculate c.
2) Determine $f(3)$ and $f(-1)$. Deduce the values of a and b.
3) Give the table of variations of f and specify the limits of $f(x)$ at the bounds of its domain of definition.
4) Prove that the line $y = x + 1$ is an asymptote of (*C*).
5) Solve the equations $f(x) = -3$ and $f(x) = 7$.
6) Find the set of values of x satisfying: $-3 \le f(x) \le 7$.



Elements of solutions and marking scheme

Question	Short answers	Note
I	$P(E/G)=\dfrac{P(E\cap G)}{P(G)}$ (2) Calculation (*1*).	2 *1*
II	Translation into a system $\begin{cases} x+y=1200000 \\ 1{,}1x+0{,}9y=1180000 \end{cases}$ (*4*) Solution of the system $x=500\,000$L.L. $y=700\,000$ L.L. (2)	*6*
III.1	$c=-1$	*1*
III.2	$a=1,\ b=1$	2
III.3	Readings from the representative curve.	*4*
III.4	Limit of $f(x)-(x+1)=0$	*1*
III.5	$x=0$ or $x=-3$; $x=2$ or $x=5$	*1*
III.6	$-3\le x\le 0$ or $2\le x\le 5$	2

Model Test 6 (LHM6)

Retrieved from:

https://www.crdp.org/official-examples-samples?term_node_tid_depth=82

المركز التربوي للبحوث والإنماء	الهيئة الأكاديمية المشتركة قسم : الرياضيات	المادة: الرياضيات الشهادة: الثانوية العامة ـ فرع الآداب والإنسانيات نموذج رقم ـ٢ـ المدة : ساعة واحدة

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ارشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (5 points)

The following table shows the results of a random sample of 50 students at a certain high school classified according to gender and age.

Gender \ Age	[14,16[	[16,18[	[18,20]	Total
Boys		**8**		
Girls	**3**	**10**		**25**
Total			**14**	**50**

1) Complete the missing values in the given table.
2) One student is selected at random from the 50-students sample. Calculate the probability of selecting:
    a) A girl whose age is below 16 years.
    b) Either a girl or a student whose age is 18 years or above.
    c) A boy knowing that he is older than 18 years.
3) Two differents students are selected one after another. What is the probability that the students are of different genders?

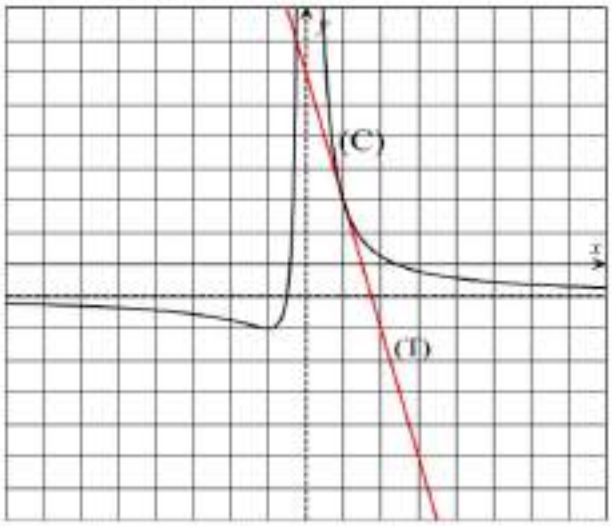
II- (5 points)

A bookshop offers a 20 % discount on its articles. The sum of original prices of a pen and a copybook is four times the price of the pen with the discount. The sum of prices of the pen and the copybook after discount is 16000 LBP.

1) Calculate the original price of the pen and that of the copybook.
2) Deduce the price of each item after the discount.
3) Rima benefits from the discount and buys 2 pens and 3 copybooks.
    How much does she pay?

III- (10 points)

Given the function f defined over its domain D and let (C) be its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$. (See figure)



Using the curve, answer to the questions 1. to 6.

1) Determine the domain of definition D of f
2) Find the limits: $\lim\limits_{x\to 0^-} f(x)$, $\lim\limits_{x\to 0^+} f(x)$, $\lim\limits_{x\to -\infty} f(x)$, and $\lim\limits_{x\to +\infty} f(x)$. Give a geometric interpretation of the obtained results.
3) Copy and complete the following using the symbols <, >, or =
   a) $f'(-1)$........0
   b) $f'(2)$........0
   c) $f(-1)$.......-2
   d) $f(-4)$......$f(-3)$
4) Determine the sign of $f(x)$ over $]-\infty,-1]$
5) Find an equation of the tangent (T) to (C) at x = 1. Deduce the value of $f'(1)$.
6) Set up the table of variations of f .
7) In what follows, assume that: $f(x)=\dfrac{-x^2+2x+1}{x^2}$.
   a) Solve for x, $-x^2+2x+1=0$. Deduce the points of intersection between (C) and the x-axis
   b) Verify that $f'(x)=\dfrac{-2x(x+1)}{x^4}$.

المادة: الرياضيات الشهادة: الثانوية العامة ـ فرع الآداب والإنسانيات نموذج رقم -٢- المدّة : ساعة واحدة	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المركز التربوي للبحوث والإنماء

أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

Solution

Question 1

1) Table (1pt)

Gender/Age	[14,16[	[16,18[	[18,20]	Total
Boys (B)	15	8	2	25
Girls (F)	3	10	12	25
Total	18	18	14	50

2) Probability (1pt+1pt+1pt)

a) $\frac{3}{50}$

b) $P(G \text{ or } age \geq 18) = \frac{25}{50} + \frac{14}{50} - \frac{12}{50} = \frac{27}{50}$

c) $P(B / Age \geq 18) = \frac{2}{14} = \frac{1}{7}$

3) $P(BG \text{ or } GB) = \frac{25}{50} \bullet \frac{25}{49} + \frac{25}{50} \bullet \frac{25}{49} = \frac{25}{49}$ (1pt)

Question 2

1) x : original price of a pen
y : original price of a copybook (1pt)

From the given we get the following system

$$\begin{cases} x + y = 4(0.8x) \\ 0.8x + 0.8y = 16000 \end{cases}$$ (2pt)

$x = 6250 LBP$

$y = 13750 LBP$

2) $0.8x = 5000$ LBP
$0.8y = 11000$ LBP (1pt)

3) Rima paid a sum equals to: $2 \times 5000 + 3 \times 11000 = 43000 LBP$ (1pt)

Question 3

1) $\left]-\infty,0\right[\cup\left]0,+\infty\right[$ (0.5pt)

2) $\lim_{x\to 0^-} f(x) = +\infty$, $\lim_{x\to 0^+} f(x) = +\infty$, $\lim_{x\to -\infty} f(x) = -1$ and $\lim_{x\to +\infty} f(x) = -1$ (2pts)

$x = 0$ V.A.
$y = -1$ HA (1pt)

3) Complete

1) $f'(-1) = 0$

2) $f'(2) < 0$

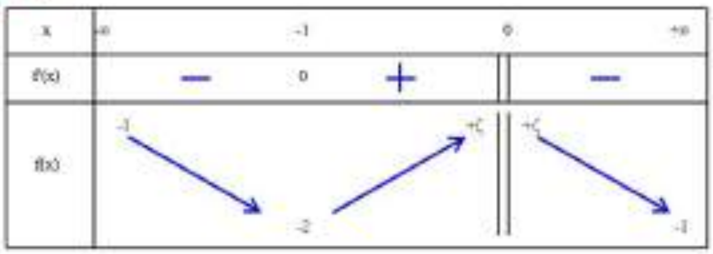
3) $f(-1) = -2$

4) f(-4) > f(-3) (1.5pt)

4) $f(x) < 0$ over $\left]-\infty,-1\right]$ (0.5pt)

5) Tangent passes through (1,2) and (0,6); $y=-4x+6$ then f ' (1) = - 4 (1pt)

6) (1.5pts)



7)

1) $x = 1 \pm \sqrt{2}$ then $(1+\sqrt{2}\ ;\ 0)$ and $(1-\sqrt{2}\ ;\ 0)$ are the two points of intersection of (C) and (x'x) (1pt)

2) $f'(x) = \dfrac{-2x(x+1)}{x^4}$ (1pt)

APPENDIX F

Model Tests LS track

Model Test 1 (LSM1)

Retrieved from:

Ministry of Education and Higher Education & Educational Center for Research and Development (2000). Evaluation Guide. Mathematics Secondary Cycle. Lebanon: Ministry of National Education, Youth and Sports & National Center of Educational Research and Development.

الجمهورية اللبنانية
وزارة التربية والتعليم العالي

الشهادة الثانوية العامة

فرع علوم الحياة

اختبار الرياضيات (انكليزي)

(نموذج ١)

السنة	: ....
عدد الأسئلة	: ثلاثة
مدة الاختبار	: ساعتان

<u>إرشادات عامة:</u>

- يجب أن يكون مع المرشح : أدوات الرسم الهندسي - آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يجب أن يستخدم المرشح قلم حبر (سائل أو ناشف) أزرق أو أسود بشكل عام. ويحق للمرشح استخدام أقلام ملونة أو قلم رصاص للرسم أو للايضاح.
- يستحسن أن يقرأ المرشح كافة أسئلة الاختبار قبل البدء بالإجابة.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في الاختبار).
- إن لجان التصحيح تولي أهمية خاصة للخط (لجهة الوضوح) والترتيب، لذلك ينصح المرشح بالكتابة

I. **(5 points)**

In a computer club of a school, there are four boys, numbered from 1 to 4, and five girls numbered from 1 to 5. The manager of the club wishes to form a committee of three members.

1) How many committees of boys can be formed?
   Deduce the possible number of committees having at least one girl.
2) How many committees having only one boy, and a member numbered 2 can be formed?

II. **(9 points)**

Let f be the function defined by $f(x) = \ln x - mx$ where m is a non zero real number. Let C_m be the graph of f in an orthonormal system.

1) Construct on the same sketch the graphs C_1 and C_{-1}.
2) Calculate the area of the domain limited by C_1, the lines $y = -x$, $x = 1$ and $x = e$.
3) For which values of m, the function f is strictly monotone increasing?
4) For which values of m, C_m has a maximum or a minimum?
5) In this question, we suppose that $m > 0$.
   a- Study, according to the values of m, the sign of $-1 - \ln m$.
   b- Use the variations of f to discuss, according to m, the number of solutions of the equation $f(x) = 0$.
6) Find the coordinates of a point of C_m at which the tangent to C_m contains the origin.

III. **(6 points)**

In the orthonormal space $(O;\vec{i},\vec{j},\vec{k})$, we consider the points $A(1,0,0)$, $B(1,1,1)$, $C(2,3,0)$ and $D(2,0,3)$.

1) Verify that $ABCD$ is a tetrahedron and calculate its volume.
2) Prove that (AB) is orthogonal to (CD).
3) Find the equation of the plane (ADC) and the coordinates of the point H orthogonal projection of B on the plane (ADC).
4) I is the midpoint of $[CD]$. Prove analytically that A, H and I are collinear and give a geometric interpretation.

Elements of solutions and marking scheme

Question	Short answers	Note
I.1	Number of committees with no girls : $C_4^3 = 4$ (*1*). Number of committees of at least one girl : $C_9^3 - C_4^3 = 80$ (*1*).	*2*
I.2	We expect the student to distinguish between two cases : Case 1. The number of committees having the boy numbered 2 is $C_4^2 = 6$ (the girl numbered 2 is rejected); Case 2. The number of committees having the girl numbered 2 is $C_3^1 \times C_4^1 = 12$ (we must choose one boy, other than the number 2, and one girl among the remaining four). Total number of committees: 18.	*3*
II.1	Argument based on the calculation of the derivative function: $m < 0$.	*1*
II.2	Expected answer: $m > 0$.	*1*
II.3	a- Expected answer: $m < 1/e$. (*1*) b- Table of variations that shows the extreme values : $\left(\frac{1}{m}, -1 - \ln m\right)$ (*1*). Conclusion: a unique solution for $m = 1/e$, two solutions for $m < 1/e$, and no solutions for $m > 1/e$ (*1*).	*3*
II.4	Expected answer: $(e, 1 - me)$.	*1*
II.5	Construction of the representative curves of the two functions.	*2*
II.6	Calculation of an integral.	*1*
III.1	The student is expected to show that the four points are not on the same plane (*1*). Calculation of the volume: it is equal to 1 (*1*).	*2*
III.2	Simple calculation of scalar product.	*1*
III.3	Determination of the equation of a plane containing three points. Expected equation : $3x - y - z - 3 = 0$ (*1*). Expected answer for H: $H\left(\frac{8}{11}, \frac{12}{11}, \frac{12}{11}\right)$ (*1*).	*2*
III.4	Simple argument of geometric orthogonality and the use of Pythagores allow to prove that ADC is isosceles and to conclude that the three points are on the same line.	*1*

Model Test 5 (LSM5)

Retrieved from:

https://www.crdp.org/official-examples-samples?term_node_tid_depth=82

المادة: الرياضيات الشهادة: الثانوية العامة الفرع: علوم الحياة نموذج رقم -١- المدة : ساعتان	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المركز التربوي للبحوث والإنماء

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (4 points)

In the space referred to an orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the points E(2 ; 2 ; 0) and F(0 ;0 ;-2), the plane (P) with equation x+y+z – 1=0 and the line (d) with parametric equations
$\begin{cases} x = -t - 1 \\ y = t + 5 \ (t \in \mathbb{R}). \\ z = 3t + 9 \end{cases}$

Denote by H the orthogonal projection of E on (P)

1)
   a- Verify that E is a point on (d).
   b- Determine the coordinates of A ,the intersection point of (d) and (P).

1)
   a-Verify that F is the symmetric of E with respect to (P).
   b-Write a system of parametric equations of the line (Δ) bisector of the angle EAF .

2) Let (Q) be the plane containing F and parallel to (P) and K the intersection point of (d) and the plane (Q).
   a) Write an equation of the plane (Q).
   b) Verify that A is the midpoint of [EK].

II- (4points)

U_1 and U_2 are two boxes so that :
   U_1 contains 10 balls : 6 red and 4 black .
   U_2 contains 10 balls: 5 red and 5 black .
A die numbered 1 through 6 is rolled .
. If this die shows 1 or 2 , then two balls are randomly selected at a time from the box U_1 .
.Otherwise , two balls are randomly selected one after another with replacement from the box U_2
Consider the following events :
U_1 :"The selected box is U_1."
U_2 :"The selected box is U_2."
R :"The selected balls are red ".

1) calculate $P(R/U_1), P(R\cap U_1)$

2) verify that $P(R)=\frac{5}{18}$.

3) The two balls selected are red , calculate the probability that they come from U_1 .

4) Let X be the random variable that is equal to the number of the red balls selected .

a) verify that $P(X=1)= \frac{23}{45}$.

b) Determine the probability distribution of X

III- (4points)

The complex plane is referred to an orthonormal system $(O;\vec{u},\vec{v})$.

Denote by A, Band C the points with respective affixes $z_A = 2-3i$, $z_B = i$ et $z_C = 6-i$.

1) Calculate $\frac{z_B - z_A}{z_C - z_A}$. Deduce the nature of the triangle ABC.

For each point M with affix distinct from i , we associate the point M' with affix :

$z' = \frac{i(z-2+3i)}{z-i}$.

2) If $z= 1-i$, determine the exponential form of z'.

3)a- If $z'= 2i$, find the algebraic form of z .(Denote by E the image point of z obtained).

b- Verify that E is a point on the line (AB).

4)Prove that if M moves on the perpendicular bissetor of [AB] then M' moves on a circle with center O and a radius to be detremined .

IV- (8points)

Consider the function defined over $\mathbb{R}$ by : $f(x)=\ln(e^{2x} - e^{x} + 1) - 1$. (C) is the representative curve of f in an orthonormal system $(O; \vec{i},\vec{j})$.

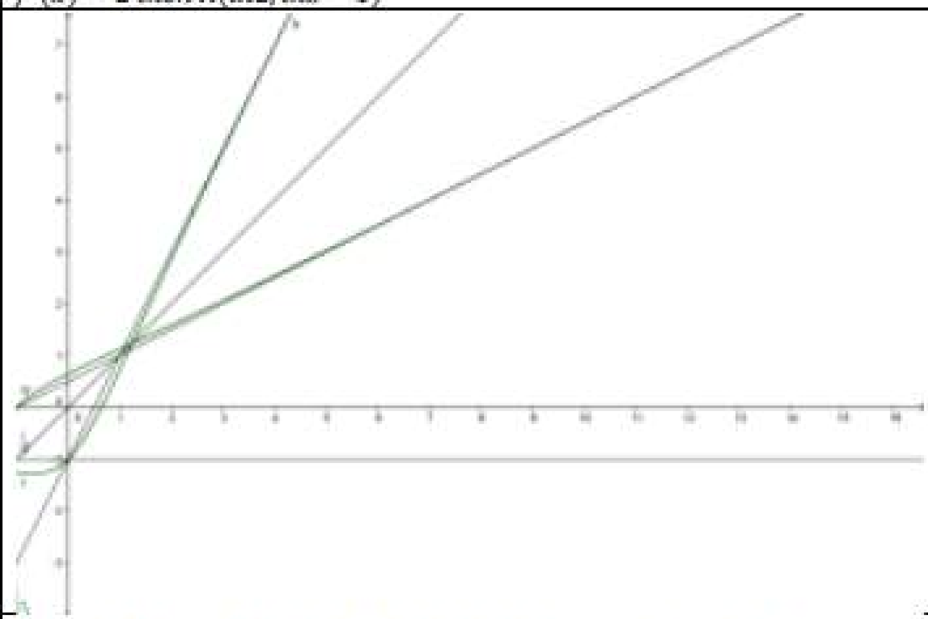
1) Determine the limit of f at $-\infty$ and deduce an asymptote to (C).
2) a.Show that the line (D) with equation $y=2x-1$ is an asymptote to (C).
   b.Discuss according to x , the relative position of (C) and (D).
3) Calculate $f'(x)$ and set up the table of variations of f .
4) Determine the coordinates of A ,where the tangent to (C) is parallel to (D).
5) Draw (D) and (C).
6) a) For $x \geq 0$,prove that f has an inverse function g whose domain of definition should be determined .
7) Let (G) be the representative curve of g and (D') its asymptote. Draw (G) and (D') in the same system as that of (C) .
8) Suppose that the area of the region bounded by (C) , (x'Ox),(y'Oy) is A .
   Calculate , in terms of A , the area of the region bounded by (G) ,its asymptote and the y-axis .

QI		Notes
1.a	E is a point on (d) for t=–3	0,5
1.b	A(3 ; 1 ; –3)	0,5
2.a	$\overrightarrow{EF}(-2,-2,-2) \Rightarrow (EF) \perp (p)$ Let H(1,1,-1) be the midpoint of [EF] and verify that H is on (P).	1
2.b	$(AH):\begin{cases} x=-2m+3 \\ y=1 \\ z=2m-3 \end{cases}$ the perpendicular bisector of [EF]	0.5
3.a	(Q): x+y+z+2=0	0.5
3.b	$K(4,0,-6)=(d)\cap(Q)$ and A is the midpoint of [EK].	1

QII		Notes
1	$P\left(R/U_1\right)=\frac{C_6^2}{C_{10}^2}=\frac{1}{3}$ $P(R\cap U_1)=P\left(R/U_1\right)\times P(U_1)=\frac{1}{9}$	0,5
2	$P(R)=P(R\cap U_1)+P(R\cap U_2)=\frac{1}{9}+\frac{5}{10}\times\frac{5}{10}\times\frac{2}{3}=\frac{5}{18}$	1
3	$p\left(U_1/R\right)=\frac{P(R\cap U_1)}{P(R)}=\frac{2}{5}$	0,5
4	$P(X=1)=\left(\frac{6\times 4}{C_{10}^2}\right)\times\frac{1}{3}+2\left(\frac{5}{10}\times\frac{5}{10}\times\frac{2}{3}\right)=\frac{23}{45}$	1
5	see table below p(X=0)=1-P(X=1)-P(X=2)	1

$X=x_i$	0	1	2
$p(X=x_i)$	$\frac{19}{90}$	$\frac{23}{45}$	$\frac{5}{18}$

QIII		Notes
1	ABC is a right isosceles triangle.	1
2	$z'=e^{\frac{-\pi}{2}i}$	0,5
3.a	$z_E=-2+5i$	0,5
3.b	$\frac{z_A-z_E}{z_B-z_E}=2$ then A,E and B are collinear .	0,5
4.a	$\lvert z'\rvert=\frac{\lvert i\rvert\lvert z-z_A\rvert}{\lvert z-z_B\rvert}$, then $OM'=\frac{AM}{BM}$	0,5
4.b	OM'=1, then M' is on the circle with center O and radius 1	1

QIV		Notes
1	$\lim_{x\to-\infty} f(x) = -1$ then y=−1 is a horizontal asymptote .	0.5
2.a	$\lim_{x\to+\infty} (f(x) - 2x + 1) = 0$ then y=2x − 1 is on O.Asymptote .	1
2.b	si x<0 then (C) is above (D) si x>0 then (C) is below (D) si x=0 (C) intersects (D)	1
3	$f'(x) = \dfrac{e^x(2e^x - 1)}{e^{2x} - e^x + 1}$	0,5
3	x: $-\infty$ … $-\ln 2$ … $+\infty$ f'(x): − … 0 … + f(x): ↘ … ↗	0,5
4	$f'(x) = 2$ *then* $A(\ln 2; \ln 3 - 1)$	1
5		1
6.a	For x ≥0 , f defined ,continuous and strictly increasing then f has an inverse function g and $D_g = [-1;+\infty[$	0,5
6.b	On the figure.	1
7	Because of the symetry with respect to y=x then Area = A – (area of the region bounded by (D') and the coordinates axes). Then Area = A – area of the triangle bounded by the coordinates axes = A- 0.25.	1

APPENDIX G

Details and Results of the Workshops Carried out by ECRD as Part of the Curriculum Evaluation and Development Plan

Retrieved from:

https://www.crdp.org/node/2897

3ème journée: le 17/12/2003

1ère séance

Sujet du débat d'aujourd'hui : le programme du **Cycle Secondaire**.

Dr MELHEM : Nous sommes réunis pour débattre ensemble de vos remarques ou critiques à l'encontre du programme actuel, et comment nous pouvons l'améliorer. Et pour vous montrer aussi dans quelle direction nous nous orientons actuellement en vue de le modifier. Je voudrais parler des principaux problèmes auxquels nous nous sommes heurtés, vous et nous, au niveau du Secondaire. Voici, à notre sens quelques-uns de ces problèmes (vous pourrez, bien sûr, en signaler d'autres et proposer des solutions pour y remédier) :

1- D'abord, quand nous avons conçu ces programmes, nous y avons introduit de nouveaux sujets telles les statistiques ou la programmation linéaire.

Tout cela a posé des problèmes à beaucoup d'enseignants qui n'y étaient pas préparés, ne l'avaient pas appris dans leur formation universitaire, ou l'avaient oublié. Il y a eu une période de démarrage assez pénible et problématique.

2- Le système éducatif, tel que les autorités concernées l'ont décidé, impose en 1ère année secondaire un **tronc commun**. Le but était de laisser à l'élève toute latitude de choisir son orientation après 15 ans, considérant qu'une orientation précoce donne lieu, souvent, à de mauvais choix et à des regrets et changements qui perturbent le cursus des élèves.

Pour nous, chargés de la conception des programmes de Mathématiques, cela nous a posé un problème :

- faut-il faciliter les sujets pour les rendre accessibles à tous ?
- ou faut-il les faire plus difficiles à l'intention de ceux qui iront vers les branches scientifiques (alors même que les Maths sont une matière exigible de tous) ?

Autre souci : nous ne voulions pas que des jeunes quittent l'école à cause des Mathématiques.

Nous avons choisi une solution médiane :

- Présenter des matières assez fortes, un peu difficiles par rapport aux littéraires, mais dans l'espoir que les futurs scientifiques soient préparés, sachent un peu ce qui les attend. Et de façon aussi à ce que les littéraires se disent : " Heureusement on y a échappé… C'est trop difficile pour moi… ".

Le problème n'est pas dans les Maths, mais dans l'estimation que fait l'élève de ce qu'il peut et de ce vers quoi il veut aller.

Si cette 1ère année reste un tronc commun, que pouvons-nous, faire ? Vous pouvez me dire : divisez cette 1ère année en 2 sections. Mais là, la décision ne dépend de nous. Et en attendant une quelconque révision du système, nous devons faire avec.

3- Nous avons réparti le programme sur les années du Secondaire de telle sorte que, quel que soit le sujet sur lequel l'élève est interrogé en 3e année (terminale), il l'ait vu au cours de cette année. Mais il l'aura aussi vu avant puisque le sujet devra être bâti sur des choses qu'il a apprises auparavant.

Cette progression graduelle correspond à une philosophie de l'éducation que nous avons adoptée. On n'apprend pas d'une fois, en une année, tout ce qu'on doit savoir sur un sujet. On progresse "en spirale". Ces passages successifs sur un sujet, avec oublis et rappels, à des niveaux de plus en plus avancés, donnent à l'élève le temps nécessaire pour l'assimiler, ou le digérer.

Sinon, nous devrions reprendre tout le programme et le répartir par sujets : tout sur la fonction doit être terminé en tant de semaines ou de mois ; tout sur l'équation, en classe de seconde, tout sur la fonction en classe de première, tout sur l'intégration en terminale... Et les questions de l'examen porteront sur ce contenu / bloc.

Ceci ne trouve l'approbation de personne, ni sur le plan pédagogique, ni du point de vue psychologique. L'expérience a prouvé que c'est un choix nocif.

Bien entendu, le système adopté actuellement comporte quelques inconvénients. Des problèmes se posent, pour lesquels il faut que nous trouvions des solutions. Il y des matières qu'on considère inutiles, d'autres qu'on voudrait rajouter... Les physiciens se plaignent toujours de lacunes mathématiques chez les élèves (exemple : "le vecteur", utilisé par eux pour représenter la force, alors que nous entendons, nous, autre chose par le vecteur).

4- Enfin, pour la structure même du programme, nous avons un autre problème à affronter.

Le programme actuel a été conçu sans prévision, dès le départ, de l'évaluation. Il a été bâti sur des objectifs, que l'on appelle comportementaux, chaque pas effectué par l'apprenant étant prévu et planifié dans le détail (comme vous le voyez à la lecture du programme).

Bien entendu, il y a des objectifs généraux, qui, en principe, doivent se retrouver entièrement dans les objectifs secondaires (ou partiels). Cette structure, pensée de manière très saine et cohérente, ne s'est pourtant pas réalisée à 100 % dans l'application.

Arrivés au stade de l'évaluation, nous nous sommes interrogés : devons-nous juste préparer l'élève à acquérir des comportements définis ? C'est-à-dire que je lui donne $\sqrt{14}$ et je lui dis d'en faire la réduction ; ou une équation et je lui dis de la résoudre ? Le but n'est-il pas plutôt d'obtenir, à partir de ces compétences, un résultat effectif, dans la pratique. Je lui apprends à résoudre une équation pour qu'il puisse résoudre un problème à une inconnue... Je lui apprends les proportions pour qu'il s'en serve en Physique ou en Chimie...

Les compétences sont-elles un but en elles-mêmes ? Dois-je me limiter à ces compétences simples ? Si je le faisais, je dénaturerais l'esprit même du programme. Il reste cependant que les objectifs secondaires n'ont pas toujours pris en considération l'acquisition précise des compétences.

Pour combler ces lacunes, tout en restant dans l'esprit du programme, nous avons abordé le problème de l'évaluation sur la base des compétences. Et là nous avons rencontré diverses écoles d'évaluation, diverses opinions et conceptions... Au Liban, jusqu'en 1998, il n'existait pas de spécialistes dans ce domaine et cela n'avait pas cours. Ailleurs, divers courants existent ; c'est normal, c'est la loi du développement. Nous avons fait venir des experts de divers pays. Les avis étaient chaque fois différents.

Nous nous sommes donc réunis, au CRDP pour, avant tout, **définir la compétence**.

La compétence est, selon nous, un grand objectif qui s'applique à des cas, à des sujets donnés précis, ou à des situations définies, soit en Mathématiques ou dans d'autres sciences, soit dans le domaine du savoir en général ou dans les activités de la vie courante, et dont la méthode de résolution est précise.

Il y a donc : un objectif – une situation – un contenu. C'est ce que nous avons adopté au CRDP. Et nous faisons une tentative d'application aux programmes actuels, dont nous essayons de garder les grands objectifs et la structure générale.

Dans notre précédente réunion concernant le programme du Complémentaire, nous vous avons donné des exemples sur quelques compétences ainsi envisagées.

D'autres exemples vont suivre (sur le Secondaire) que mes collègues vont vous exposer.

Maintenant, je demande votre avis pour la suite. Voulez-vous que nous passions directement à l'exposé sur les compétences ? Ou voulez-vous discuter les problèmes qui ont été soulevés ? Ou bien exposer et commenter les diverses correspondances que j'ai reçues au sujet des programmes ? Je vous écoute...

Dr Hussein ZEINEDDINE :

1- Les objectifs généraux du programme du Secondaire sont bien définis. Mais quand on en vient aux objectifs spécifiques, on découvre qu'en 10ème et 11ème année, il y a des objectifs généraux qui n'ont pas d'objectifs secondaires (ou spécifiques) qui leur correspondent. Pourquoi ? Est-ce voulu ? Est-ce qu'on a voulu réserver les objectifs spécifiques aux seules 4 sections de la 3ème année (terminale) ? Nous aimerions comprendre.

2- Pour la 11ème année (bac.), beaucoup de problèmes viennent de ce que le programme est trop long et dense. Il reste cependant que, pour la section littéraire, le programme est insuffisant, au dessous du niveau requis, pour les élèves qui s'orienteront vers l'économie, alors qu'il est trop lourd pour ceux qui choisiront le littéraire. Peut-être faut-il donc revoir le problème du tronc commun.

Dr MELHEM : Pour ce qui est de la densité des matières, c'est un problème de contenu qui, donc, nous concerne et je vous promets qu'il sera examiné. Pour ce qui est du tronc commun et des sections (2 ou 3 en 11ème année), cela ne se décide pas à notre niveau. Mais il serait bon d'attirer l'attention des décideurs sur ce problème. Une question : après "l'allègement" que nous avons opéré sur le programme de 1ère scientifique, y a-t-il encore surcharge, selon vous ?

– Plusieurs réponses ; oui, trop lourd encore.

Saria MAJZOUB (Secondaire pour filles–Saida) : il n'y a pas continuité entre le bac littéraire et l'économie, et cela au niveau du programme.

– Quelques intervenants attirent l'attention sur la sélection des élèves : certains ont un 7 en Maths et réussissent l'examen, et on les retrouve en bac scientifique. Il faut trouver une solution à cette situation.

Dr Leila NASR (Université de Kaslik) : J'aimerais savoir si la section littéraire est conçue de façon à ce l'élève s'oriente vers l'économie.

Dr MELHEM : Disons que ça lui est permis. Mais cela crée une situation sûrement inconfortable.

Dr Leila NASR : Certains de nos collègues ont réclamé un allègement plus important du programme de la 1ère scientifique. J'aimerais vous signaler que, moi qui enseigne à l'université, je vois arriver des étudiants qui ne savent presque rien en Algèbre, parce que l'allègement actuel n'a porté que sur l'Algèbre. Nous sommes obligés de leur faire refaire tout le programme d'Algèbre. Aussi, prière, si vous allégez encore ce programme (ce pour quoi je suis d'accord avec vous), que cet allègement ne porte pas encore sur l'Algèbre…

Dr. Ibrahim AL-HAJJ : Je veux rappeler qu'à la base, le minimum exigible pour passer de la 1ère scientifique à l'ES en terminale était une moyenne de $\frac{12}{20}$ en Mathématiques. Cette note / limite a été supprimée après que le programme et le manuel furent achevés. De là viennent les problèmes auxquels vous avez à faire face aujourd'hui.

Dr Moufid SKAF : Nous enseignons tous et corrigeons tous aux examens officiels. Mme Saria Majzoub dit que celui qui a fait le littéraire ne peut pas monter en ES. Pouvez-vous me dire ce qui, précisément, lui manque dans ce programme. Moi, je considère que c'est un problème d'élèves et non un problème de programme. Les élèves ont 5 heures de Maths en seconde (tronc commun) et 4 heures en première. Ça me paraît tout à fait suffisant, en tant que bagage mathématique, pour montrer en ES. Mais le problème c'est le niveau et la capacité des élèves. Vous savez tous que 30 % de ces élèves trop faibles ont $\frac{1}{2}/_{20}$ ou zéro en Maths. Mais ceux-là, tout leur est difficile : quel que soit le programme et quelle que soit la question que vous leur posez, ils auront ces notes-là. Ceux-là, sont perdus dans la seconde à tronc commun ; il leur faudrait un autre accès, plus facile, à cette matière. Nous devons attirer l'attention des décideurs sur cette seconde à tronc commun. Je crois que nous en sommes tous d'accord. Même le programme de LH est difficile pour ces élèves si leur problème n'est pas résolu dès la seconde. Pour ce qui est du programme, j'ai devant moi celui de ES et je n'y vois aucun thème qui ne soit préparé déjà soit en seconde, soit en première même littéraire. D'ailleurs les élèves capables en Maths ne ressentent aucun manque quand ils passent en ES. Où sont les lacunes dans le programme ? Si vous m'en donnez un exemple précis, un sujet qui manquerait à l'élève en ES, nous pourrons en discuter tout de suite.

Saria MAJZOUB : J'ai donné deux exemples :

- pour les fonctions inversées : les élèves qui ont un $\frac{10}{20}$ en bac littéraire ont le droit de faire Economie, mais ils n'arrivent pas à suivre sur ce sujet, ça leur est difficile ;
- pour les logarithmes : les élèves nous posent problème quand ils arrivent en Economie, nous sommes obligés d'y consacrer plus de temps que prévu (parce que les élèves ont eu une moyenne qui leur a permis l'accès à l'Economie, mais que leur analyse des questions est inefficace).

Dr Moufid SKAF : Vous confirmez ce que j'ai dit : le problème ne se pose pas au niveau du programme, mais au niveau de la préparation des élèves…

Nada MAKKÉ (Secondaire Al–Kawthar–Mabarrat) : Je suis d'accord avec M. Skaf : c'est un problème d'élèves plus que de programme. Nous avons principalement augmenté les heures consacrées à ces sujets en seconde.

Nehmé MAKSOUD (Amicale des Enseignants) : J'ai trois remarques à faire :

1- Je ne suis pas d'accord sur la forme de cette réunion ; le débat va-t-il continuer comme ça ou y a-t-il des points définis à l'ordre du jour ?
2- Pour ce qui est de l'Economie, pour moi, le problème réside surtout dans le manuel du CRDP (moi, j'enseigne à l'aide d'un autre manuel) qui est trop fort. Son niveau est plus haut que le SV. Il faut des profs très costauds pour s'en sortir. Mais le problème n'est pas dans le programme.
3- 80 % des élèves en Economie viennent du littéraire. J'ai dans ma classe 3 ou 4 élèves du scientifique et 35 à 36 du littéraire. La solution que nous avons adoptée pour ces élèves trop faibles venant du littéraire, c'est de faire une session d'été, d'un mois entier… car il ne faut pas oublier que nous commençons l'année le 13 ou 14 septembre. Voilà encore une solution des écoles privées.

Dr MELHEM : Je vous prie de noter, encore une fois, que cette réunion est réservée aux programmes et non aux manuels. Je vous remercie de vos remarques sur le manuel, qui pourraient nous servir plus tard, mais je vous en prie, tenons-nous au sujet d'aujourd'hui : le programme du secondaire.

Ghassan ANTOUN (Union des Ecoles Orthodoxes) : J'ai une proposition que le CRDP pourrait peut-être examiner. Ne serait-il pas possible d'adopter le modèle du programme français pour les Mathématiques, en créant, dans la classe littéraire, un programme "Spécialités" destiné à ceux qui veulent s'orienter vers l'Economie ?

Dr MELHEM : La proposition de M. Antoun est intéressante ; il y a peut-être là une voie à chercher. Beaucoup de choses intéressantes ont été proposées.

Ahmad DANKAR : Pour ma part, j'insiste sur la nécessité de ne pas permettre à n'importe qui d'aller en Economie. Comme l'a souligné le Dr Hajj, il faut rétablir un barème, un plafond précis en Maths. Peut-être un conseil des professeurs, dans les écoles, peut se charger d'orienter les élèves, de conseiller sans imposer…

Dr MELHEM : Ce n'est malheureusement pas dans nos prérogatives. Mais peut-être votre vœu, exprimé ici, sera-t-il entendu…

La parole est maintenant donnée à notre collaboratrice M^elle^ Yolla Farès qui va vous présenter un échantillon de notre travail sur les "compétences" qui constitue la base de notre plan de perfectionnement du programme pour l'avenir.

Yolla Farès fait un exposé, avec projection de fiches portant sur un exemple précis en statistiques. (Voir le Document N° 3 ci-joint).

L'exposé est suivi de quelques éclaircissements donnés par le Dr Melhem et de quelques remarques et objections soulevées dans l'assistance, et portant essentiellement sur le choix de l'exemple, tiré des statistiques (matière que certains assurent même ne pas traiter, vu le manque de temps). On aurait préféré un exemple plus probant et tiré d'une matière plus intéressante.
(Réponse du Dr Melhem : après tout ce n'est qu'un exemple pris au hasard ; on n'en est qu'aux premiers pas ; on vous promet bientôt d'autres spécimens qu'on vous enverra pour avoir votre avis). En somme, il rassure les inquiets ou les récalcitrants à toute innovation. Nombre de participants se montrent d'ailleurs favorables à cette nouvelle option et la défendent).

En conclusion, le Dr Melhem résume l'essentiel et insiste sur l'importance du "verbe actif" dans la désignation des compétences à acquérir et affirme que c'est sur ce point que devra porter l'essentiel des modifications à apporter au programme. De toute façon, on n'en est qu'aux premiers pas dans ce chantier d'avenir. Nous avons voulu vous présenter ici une description formelle de ce qu'on appelle "compétence". Nous espérons, dans l'avenir, pouvoir faire un programme selon cette méthode. Si certains d'entre vous ou si vos écoles voulaient bien nous faire des propositions (sous le titre : **quelles compétences ?**), nous vous en serions reconnaissants.

3ème journée: le 17/12/2003

2ème séance

Dr MELHEM : Avant de reprendre nos discussions (où il est impossible de satisfaire tout le monde !), j'ai une communication d'ordre administratif à vous faire. Nous avions opéré, dans les années précédentes, un allégement du programme de Mathématiques, qui a été suivi d'un second allégement. Tous ces allégements ont encore cours aujourd'hui, bien que le décret officiel eût mentionné qu'ils seraient applicables jusqu'en 2002. Nous allons donc requérir officiellement des autorités une décision ministérielle qui entérine et prolonge leur validité jusqu'à la parution des nouveaux programmes actuellement en chantier.

Moufid SKAF : C'est le dernier allégement, celui de 2001 – 2002, qui est le définitif. On lui a juste adjoint un petit corollaire pour la 2ème année scientifique.

Dr Hicham BANNOUT demande à revoir la fiche de " compétence " vue tout à l'heure. (On projette la 1ère fiche). " J'ai deux remarques :

1- Par rapport aux programmes actuels, un pas en avant a été fait dans la bonne direction : évolution et amélioration. Pour ce qui du contenu et des objectifs, la méthode de travail dans cette réunion consistait à demander leur avis aux enseignants sur le contenu : trop ? assez ? trop peu ?...
2- Ce que vous proposez maintenant comme modification, le système des " compétences " n'était-il pas déjà dans les programmes ? Peut-être pas tout à fait systématiquement, mais il y avait déjà des objectifs et des compétences à faire acquérir. Ma question est : dans quelle mesure cela a été pratiquement utile aux enseignants dans leur méthode d'enseignement et dans la réalisation de leurs objectifs ?

Ma proposition est : travaillons surtout, dans cette nouvelle avancée pour faire évoluer les programmes, sur la définition exacte des notions de " compétence ", " savoir " et " savoir-faire ", sur la définition des objectifs à réaliser dans chaque classe concernant chacune de ces compétences. Un point c'est tout ! Sur le terrain, centrer plutôt sur ce que doit faire le prof en classe. Se contenter de :

compétence + savoir + savoir-faire

Tout le reste, avec cet excès de détails, ne fera qu'embrouiller les enseignants et ne les aidera pas en classe. A quoi ça sert de leur dire : " Ça c'est le savoir-faire, ça c'est le critère, ça c'est l'indicateur... ".

Dr MELHEM : Vous caricaturez : personne n'aura à dire en classe : " Voici les indicateurs " !

Dr Hicham BANNOUT : Votre grand objectif, " Construire les mathématiques ", c'est le plus important, et aussi le plus actuel. Il faut seulement bien faire comprendre aux enseignants ce qu'ils doivent faire en classe, comment procéder. Beaucoup ne savent pas mettre en application ce " construire les mathématiques ".

Dr MELHEM : Nous sommes, nous, les concepteurs du programme. Voulez-vous que nous donnions aussi des méthodes d'enseignement ? Nous pensions que c'était le rôle de la Faculté de Pédagogie…

Dr Maroun BARAKAT (Université Libanaise) : Ce dont on parle dans cette nouvelle conception, c'est d'une " évaluation formative ", pour que le professeur évalue son propre enseignement (et non l'élève). Il doit travailler sur lui-même ; la fiche peut l'aider, mais elle ne suffit pas.

Moufid SKAF : Il m'a semblé que ce qui a été présenté dans ce tableau (dans l'exposé) n'est rien d'autre que ce qui est dans le programme lui-même, mais exprimé autrement, peut-être juste plus précis. Aucun des indicateurs mentionnés n'est autre chose que l'un des objectifs spécifiques définis dans le programme (ou dans les commentaires de ces objectifs). Jusqu'à la terminologie qui est la même. En fin de compte, j'ai l'impression que le programme va rester le même (avec quelques ajouts ou quelques allégements), mais écrit d'une autre manière. Nous rendons le programme responsable de tous les problèmes. Il y a peut-être quelques problèmes, mais ils sont dûs, non pas au programme, mais plutôt :

- soit à une insuffisance de la formation,
- soit à des carences dans les manuels qui seraient à réviser,
- soit, surtout, à une mauvaise application de la méthode active pas les enseignants.

Beaucoup d'entre nous n'ont pas tout à fait compris cette attitude à avoir à l'égard des élèves, ou ne savent pas bien comment la mettre en pratique. Mais il ne faut pas nier qu'un grand changement, qu'un grand pas en avant a été fait. Aux examens, nous avons constaté, par exemple, un meilleur niveau des questions. C'est un véritable saut qualitatif. Dans toutes nos écoles, nous l'avons senti. En dépit, bien sûr, de quelques lacunes ou insuffisances…

Dans ce qui est proposé aujourd'hui, je ne vois rien de si révolutionnaire. Sur le plan du contenu, les choses resteront les mêmes, mais présentées sous une autre forme. Espérons que les manuels seront aussi ambitieux que le programme. Et souhaitons que nous, la Faculté de Pédagogie ou les autres instances chargées de la formation, nous puissions faire parvenir ce nouveau message aux enseignants. S'il y en a parmi vous qui peuvent apporter leur témoignage sur ce sujet, qu'ils le fassent.

Jules ADOUANE (Collège Elysée-Kaslik) : Je pense également que le programme n'est pas le problème. C'est la méthode des profs qui est en question : beaucoup n'appliquent pas la méthode active. Si l'on a l'honnêteté de faire son autocritique, on découvrira que le " cours magistral " se pratique encore. On fait un effort les premières semaines puis, peu à peu, on revient à ses réflexes antérieurs... Moi je donne une 2ème année d'Agriculture à Kaslik ; nous recevons des élèves qui ont une bonne formation ; mais le problème est que le niveau varie d'une école à l'autre, souvent d'un prof à l'autre... C'est donc surtout la méthode d'enseignement qui pose problème.

Ahmad DANKAR : Quand nous avons construit le programme, nous n'avions pas en tête le mot " compétences ". On parlait alors d'objectifs généraux, puis d'objectifs particuliers pour chaque cycle et pour chaque classe. On a établi le contenu, puis sont venus les détails du programme et des livres. Quand une équipe a voulu ensuite établir une méthode d'évaluation, des experts étrangers (notamment français) sont venus, dont M. Colomb. Il nous a présenté la notion de compétence. Nous nous y sommes mis, mais en fait, nous étions tous un peu perdus. Je souhaiterais, dans ce qui va être fait maintenant (et c'est peut-être ce qui se fait au CRDP), que la notion de " compétences " soit définie, que les compétences exigibles soient clairement exprimées et de façon simplifiée et que sur leur base, on reconstruire le contenu du programme et sa nouvelle rédaction.

Dr MELHEM : En rebâtissant le programme, peut-être conserverons-nous les sujets, ou les changerons-nous seulement quand c'est nécessaire. Pour ce qui est de la formation, le CRDP a essayé d'établir un programme de formation avec les moyens du bord. Nous ne pouvions pas, avec ce dont nous disposions, faire plus. Quand à nos intentions futures, elles peuvent se résumer ainsi :

- un bon programme
- de bons livres
- une bonne formation.

Ghassan ANTOUN : J'ai deux questions :

1- Nous avons eu aujourd'hui un aperçu plus précis de la notion de compétence, à travers un exemple tiré du programme de statistiques. Peut-être pourriez-vous mettre au point d'autres exemples (2 ou 3) sur d'autres sujets et nous les faire distribuer pour nous permettre de mieux les étudier. De cette façon, si vous nous demandez notre avis par la suite, nous serons mieux préparés à participer à la réflexion commune.

2- Je n'ai pas encore trouvé de réponse claire à la problématique qui a été posée ici :
   - faut-il enseigner les " mathématiques utilitaires " en vue d'une utilisation postérieure ?
   - ou faut-il, dans une certaine mesure, enseigner les maths pour les maths ?

Dr MELHEM : Ce sujet n'a pas été tranché à travers le monde. Personne, ici, n'a confirmé qu'une option définitive a été prise. C'est d'ailleurs là un grand problème qui accompagne l'histoire des Mathématiques depuis les origines.

Bouchr MOUSSA (Centre pédagogique Omar Al-Moukhtar) : Vous avez souligné que beaucoup d'enseignants n'ont pas reçu de formation. Mais le fait est que, même formés, quand ils reviennent en classe, ils reviennent souvent à leurs pratiques traditionnelles et à leurs vieilles habitudes. Et cela parce que les problèmes véritables sont :

- les programmes trop longs : nous donnons 7 heures hebdomadaires pour y arriver…
- et les classes surpeuplées.

Dr MELHEM : Pour ce qui est des programmes, vous dites qu'ils sont très lourds. Même après les allégements ? Même en appliquant la méthode active, qui permet, comme nous l'a dit Dr Moufid Skaf, d'aller plus vite ? Quand nous avons fait les programmes, des spécialistes et des enseignants de grande expérience ont tout soupesé pour que ces programmes puissent être terminés (avec les allégements) en moins de 25 semaines. Nous ne pouvions les contredire. Plus tard, avec la pratique, sont apparues beaucoup de protestations. Peut-être avait-on pris les décisions en fonction d'écoles, de milieux spécifiques, différents de la norme moyenne générale du pays. Nous nous consultons et nous vous consultons tous à ce sujet et nous allons tâcher d'y remédier. Mais jusqu'à quel point peut-on alléger ? Je crains que plus tard on se plaigne de ce que le programme est trop léger ! Déjà les professeurs d'université nous le reprochent… Certains nous diront : " Nous terminons en février ! " Il faut savoir trouver un juste milieu.

Pour ce qui est des compétences, nous en sommes encore aux premiers pas. Considérez sous cet angle l'exemple que nous vous avons donné…

- De nombreux participants ont exprimé le souhait de recevoir, par écrit, quelques fiches supplémentaires sur les " compétences " pour les aider à mieux cerner la question.
- Certains reviennent sur la nécessité de mieux répartir les matières sur les trois années du secondaire. On donne des exemples (l'équation du 2^{d} degré passée en 1ère alors que l'élève la comprenait bien en seconde) de la surcharge du programme du bac.

Moufid SKAF : C'est possible d'étudier la répartition pour alléger un peu la 2ème année. Mais pour l'équation du 2^{d} degré, si l'élève la comprenait en seconde c'est parce qu'il ne faisait rien d'autre…

Dr MELHEM : Il faudra sûrement le faire, sur la base de vos propositions. J'en ai reçu beaucoup. Nous nous appuierons sur vos remarques (comme nous l'avons fait à la suite de la réunion de l'année dernière avec M. JOST). L'évaluation est faite par des institutions. Mais avec vous, nous découvrons des réalités précises sur le terrain. Vous nous êtes très utiles. Et je vous remercie tous pour l'effort de réflexion que vous fournissez avec nous. Beaucoup de points soulevés par vous ont été enregistrés. Vous les retrouverez dans un rapport sur cet atelier de travail.

Aujourd'hui, nous sommes sur le seuil de ce chantier de révision des programmes sur la base des " compétences ". Je vous demande par exemple d'y participer en nous proposant des **titres** relatifs aux compétences qui vous paraissent exigibles, de par votre expérience sur le terrain. Pas des détails, mais juste des titres. A vous de nous aider. Si vous nous laissez travailler seuls, après vous allez vous plaindre et nous faire un tas de reproches…

Pour les remaniements du contenu : ajouter ou retrancher des choses, mieux adapter la répartition par années, etc., j'ai gardé toutes vos propositions. Et j'attends d'autres propositions, que vous pourrez m'envoyer si vous en avez que nous n'avons pu aborder ici.

Je laisse la conclusion au Dr. Hajj.

Dr Ibrahim AL-HAJJ : Si nous voulons essayer de remodeler les objectifs secondaires selon la nouvelle méthode exposée aujourd'hui, il est très important de savoir ce que l'élève doit savoir à la fin de chaque étape :

- que doit-il savoir ?
- que doit-il savoir faire ?

Un détail à souligner : la matrice, en Algèbre linéaire, est très utile en Economie.

Dr MELHEM : Mais il faudra d'abord définir la compétence pour laquelle la matrice est utile.

Mohamed MOUSSAOUI (Secondaire de Ghobeyri) pose des questions sur des contenus trop lourds ou trop ambitieux dans les manuels (Ex : la " logique mathématique" en LH. "Même à l'Université on ne peut pas la comprendre ! ").

Dr MELHEM : Encore cette confusion entre le programme et le manuel… Pour nous, c'est un manuel parmi les autres. Beaucoup de problèmes administratifs ont pesé sur la rédaction de ces manuels… Mais ce n'est pas là notre propos aujourd'hui.

Dr Leila NASR : En conclusion :

- Je suis, comme certains d'entre vous, contre l'allègement sans cohérence. On en paie le prix à l'Université.
- Je suis pour la " logique mathématique " en LH : les philosophes, depuis toujours, ont été des logiciens.

Fin de séance. On se promet de se retrouver sans tarder dans d'autres rencontres.

APPENDIX H

LH Session-2 Official Test of the year 2018 LH182

Retrieved from:

https://www.crdp.org/en/official-exams-corrections-lebanon?sessionurl=3084&term_node_tid_depth=83

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسمية	امتحانات الشهادة الثانوية العامة فرع الآداب والإنسانيات	دورة العام ٢٠١٨ الاستثنائية الثلاثاء ٣١ تموز ٢٠١٨
عدد المسائل: ثلاث	مسابقة في مادة الرياضيات المدة: ساعة	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (5 points)

In a certain store, all the pants are sold at the same price and all the shirts are sold at the same price.
Diala bought 3 pants and 4 shirts for 240 000 LL.
Joudi bought 2 pants and 2 shirts for 140 000 LL.

1) Calculate the price of one pant and that of one shirt.
2) The store proposes two offers for Diala if she buys 5 shirts and 5 pants:
   - **Offer 1**
     10% discount on the price of each pant and 30% discount on the price of each shirt.
   - **Offer 2**
     A reduction of 60 000 LL on the total amount.

   Which one of the two offers is better for Diala? Justify your answer.

II- (5 points)

80 tourists are travelling on a boat to visit a certain island. These tourists are distributed as shown in the table below:

Age in years	[16 ; 24[	[24 ; 32[	[32 ; 40[	[40 ; 48[	[48 ; 56]
Europeans	6	7	12	3	8
Asians	3	13	11	12	5

1) Determine the average age of the European tourists on this boat.
2) The captain of this boat chose randomly one person from these tourists to be the guest of honor.
   Consider the following events:
   E: " the chosen tourist is European ".
   A: " the chosen tourist is Asian ".
   Y: " the chosen tourist is less than 32 years old ".

   a. Verify that the probability of Y is $\frac{29}{80}$.

   b. Calculate the following probabilities:

   $P(E)$, $P\left(Y/A\right)$, $P(Y \cap A)$, $P(Y \cup E)$ and $P\left(A/Y\right)$.

III- (10 points)

Let f be the function defined on the interval $I =]-1;+\infty[$ as $f(x) = \frac{x^2+3}{x+1}$ and let (C) be its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Show that $f(x) = x - 1 + \frac{4}{x+1}$.

2) a. Determine $\lim_{\substack{x \to -1 \\ x > -1}} f(x)$ and deduce an equation of an asymptote to (C).

   b. Determine $\lim_{x \to +\infty} f(x)$.

   c. Prove that the line (d) with equation $y = x - 1$ is an asymptote to (C).

3) a. Verify that $f'(x) = \frac{(x-1)(x+3)}{(x+1)^2}$.

   b. Copy and complete the following table of variations of f.

x	-1		1		$+\infty$
f '(x)			0		
f(x)					

4) a. Calculate the coordinates of the points of intersection of (C) and the line with equation $y = 3$.

   b. Find an equation of the tangent to (C) at its point with abscissa 0.

   c. Draw the curve (C) and its two asymptotes.

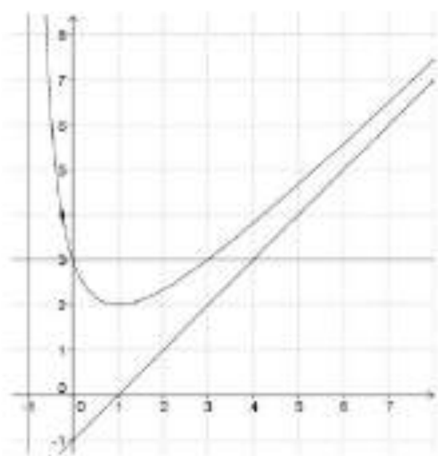
5) Solve graphically: $2 < f(x) \leq 3$.

أسس التصحيح مادة الرياضيات

QI	Answers	Mark
1	Let x be the price of a pant and y the price of a shirt $\begin{cases} 3x+4y=240000 \\ 2x+2y=140000 \end{cases}$ x = 40000 LL and y =30 000 LL	2
2	**Offer 1**: after the discount, the price of a pant is : $40000 \times 0.9 = 36000$ LL. And the price of a shirt is : $30000 \times 0.7 = 21000$ L.L. $5 \times 36000 + 5 \times 21000 = 285000$ LL **Offer 2** : $5 \times 40000 + 5 \times 30000 - 60000 = 290000$ LL Offer 1 is better for Diala.	3

QII	Answers	Mark
1	The average age of the Europeans is: $\frac{20\times6+7\times28+12\times36+3\times44+8\times52}{36} = 36$ years	1
2.a	$P(Y) = \frac{9+20}{80} = \frac{29}{80}$	1
2.b	$P(E) = \frac{36}{80}$; $P(Y/A) = \frac{10}{20}$; $P(Y \cap A) = \frac{6}{80}$, $P(Y \cup E) = P(Y) + P(E) - P(Y \cap E) = \frac{52}{80}$ $P(A/\overline{Y}) = \frac{28}{51}$	3

QIII	Answers	Mark
1	$x-1+\frac{4}{x+1} = \frac{(x-1)(x+1)+4}{x+1} = \frac{x^2+3}{x+1}$	1
2.a	$\lim\limits_{\substack{x \to -1 \\ x > -1}} f(x) = \frac{4}{0^+} = +\infty$ x= -1 vertical asymptote of (C)	1
2.b	$\lim\limits_{x \to +\infty} f(x) = +\infty$	1
2.c	$\lim\limits_{x \to +\infty} [f(x) - (x-1)] = \lim\limits_{x \to +\infty} \left[\frac{4}{x+1}\right] = 0$, $y = x - 1$ is an asymptote of (C)	1
3.a	$f'(x) = \frac{2x(x+1)-(1)(x^2+1)}{(x+1)^2} = \frac{x^2+2x-3}{(x+1)^2} = \frac{(x-1)(x+3)}{(x+1)^2}$	1

QIII	Answers	Mark
3.b	x: -1 … 1 … $+\infty$ f'(x): ‖ $-$ 0 $+$ f(x): ‖ $+\infty$ ↘ 2 ↗ $+\infty$	1
4.a	$\frac{x^2+3}{x+1}=3$, $x^2-3x=0$, then $x=0$ or $x=3$ then the points of intersection are $(0,3)$ and $(3,3)$	1
4.b	$y-f(0)=f'(0)(x)$ $y-3=-3x, y=-3x+3$	1
4.c		1
5.	$x\in[0;1[\cup]1,3]$	1

APPENDIX I

LS Session-1 Official Test of the year 2013- LS131

Retrieved from:

https://www.crdp.org/en/official-exams-corrections-lebanon?sessionurl=3084&term_node_tid_depth=83

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة فرع علوم الحياة	دورة سنة 2013 العادية
الاثنين 1 تموز 2013 عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I-(4points)

In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the points:

A (4; 2; 0), B (2; 3; 1) and C (2; 2; 2).

1) Prove that triangle ABC is right at B.

2) Show that an equation of the plane (P) determined by the three points A, B and C is $x + y + z - 6 = 0$.

3) Let (Q) be the plane passing through A and perpendicular to (AB).
   a- Determine an equation of (Q).
   b- Denote by (D) the line of intersection of (P) and (Q), show that (D) is parallel to (BC).

4) Let H(5;3;1) be a point in (Q).
   a- Show that A is the orthogonal projection of H on (P).
   b- Calculate the volume of the tetrahedron HABC.

II-(4points)

A music store sells classical and modern musical albums only.
The customers of this store are surveyed and the results are as follows:

- 20% of these customers bought each a classical album.
- Out of those who bought a classical album, 70% bought a modern album.
- 22% of the customers bought each a modern album.

A customer of the store is interviewed at random. Consider the following events:
C: «the interviewed customer bought a classical album »
M: «the interviewed customer bought a modern album ».

1) Calculate the probability $P(C\cap M)$ and verify that $P(C\cap \overline{M}) = 0.06$.

2) Prove that $P(\overline{C}\cap \overline{M}) = 0.72$.

3) Calculate the probability that the customer bought at least one album.

4) Knowing that the customer didn't buy a modern album, calculate the probability that he bought a classical album.

5) The classical album is sold for 30 000LL and the modern one is sold for 20 000LL.
   Let X be the random variable that is equal to the sum paid by a customer.
   a- Justify that the possible values of X are: 0, 20 000,30 000 and 50 000. Then, determine the probability distribution of X.
   b- During the month of June, 300 customers visited this music store. Estimate the revenue of this store during that month.

III(4 points)

In the plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A, B and C with respective affixes $z_A = i$, $z_B = 3-2i$ and $z_C = 1$.

1) Prove that the points A, B and C are collinear.

2) Consider the complex number $w = z_C - z_A$.

Write w in exponential form and deduce that w^{20} is a real negative number.

3) Let M be a point in the plane with affix z.

a- Give a geometric interpretation to $|z-i|$ and $|z-1|$.

b- Suppose that $|z-i| = |z-1|$; show that the point M moves on a line to be determined.

c- Prove that if $(z-i)\times(\bar{z}+i) = 16$, then the point M moves a circle whose center and radius to be determined.

IV-(8points)

Consider the function f defined on $\mathbb{R}$ as $f(x) = 3 - \dfrac{4}{e^{2x}+1}$.

Let (C) be its representative curve in an orthonormal system (**unit 2 cm**).

1) Calculate $\lim\limits_{x\to-\infty} f(x)$, $\lim\limits_{x\to+\infty} f(x)$ and deduce the asymptotes to (C).

2) Prove that f is strictly increasing over $\mathbb{R}$ and set up its table of variations.

3) The curve (C) has a point of inflection W with abscissa 0. Write an equation of (T), the tangent to (C) at the point W.

4) a- Calculate the abscissa of the point of intersection of (C) with the x-axis.

b- Draw (T) and (C).

5) a- Verify that $f(x) = -1 + \dfrac{4e^{2x}}{e^{2x}+1}$ and deduce an antiderivative F of f.

b- Calculate, in cm^2, the area of the region bounded by the curve (C), the x- axis, the y-axis and the line with equation x = ln2.

6) The function f has over $\mathbb{R}$ an inverse function g. Denote by (G) the representative curve of g.

a- Specify the domain of definition of g.

b- Show that (G) has a point of inflection J whose coordinates to be determined.

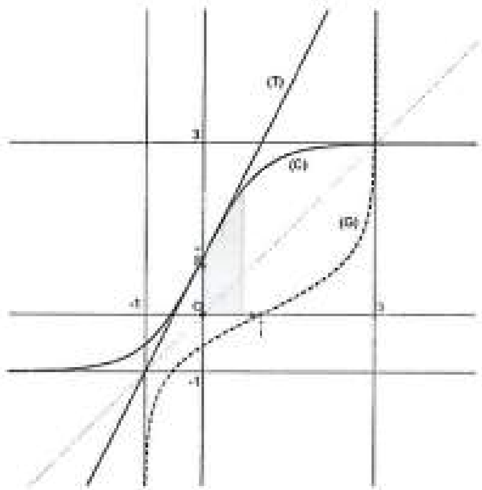
c- Draw (G) in the same system as (C).

d- Determine g(x) in terms of x.

Q1	Answers	M
1	$\overrightarrow{AB}(-2\,;1\,;1)$, $\overrightarrow{BC}(0\,;-1\,;1)$; $\overrightarrow{AB}\cdot\overrightarrow{BC}=0$, hence triangle ABC is right at B.	0.5
2	$x_A+y_A+z_A-6=0$, then A is in (P); $x_B+y_B+z_B-6=0$, then B belongs to (P) and $x_C+y_C+z_C-6=0$, then C is in (P) Therefore (P) : $x+y+z-6=0$. **Or** $\overrightarrow{AM}.(\overrightarrow{AB}\wedge\overrightarrow{AC})=0$ with M(x; y; z) any point in (P).	0.5
3. a	For any point M (x, y, z) in (Q); $\overrightarrow{AM}.\overrightarrow{AB}=0$; (Q): $-2x+y+2z+6=0$.	0.5
3. b	A directing vector of (D) is $\vec{V}=\vec{n}_P\wedge\vec{n}_Q$; hence $\vec{V}(0;-3;3)$, et $\overrightarrow{BC}(0;-1;1)$ and B $\notin$ (Q), so B $\notin$ (D). Thus, (D) is parallel to (BC). **Or** : Since (BC) is perpendicular to (AB) and (AB) is perpendicular to (D) in A, (BC) and (D) being coplanar in (P) and perpendicular to the same line (AB), are parallel.	1
4. a	$A\in(P)$, $\overrightarrow{AH}(1;1;1)$ and $\vec{n}_P(1;1;1)$ hence (AH) is perpendicular to (P).	1
4. b	The volume of tetrahedron HABC is equal to $V=\frac{1}{3}$ HA × area of triangle ABC $=\frac{1}{6}\times BA\times BC\times\sqrt{3}=1u^3$. **Or** $V=\frac{\left\|\overrightarrow{AH}.(\overrightarrow{AB}\wedge\overrightarrow{AC})\right\|}{6}=\frac{6}{6}=1\ u^3$.	0.5

<table>
<tr><th>Q 2</th><th>Answers</th><th>M</th></tr>
<tr><td>1</td><td>$P(C\cap M)=P(C)\times P(M/C)=0.14$. $P(C\cap\overline{M})=P(C)\times P(\overline{M}/C)=0.06$.</td><td>1</td></tr>
<tr><td>2</td><td>$P(C\cap\overline{M})+P(\overline{C}\cap\overline{M})=P(\overline{M})=1-P(M)$ then $P(\overline{C}\cap\overline{M})=0.78-0.06=0.72$.</td><td>0.5</td></tr>
<tr><td>3</td><td>P (at least an album) $=1-P(\overline{C}\cap\overline{M})=0.28$.</td><td>0.5</td></tr>
<tr><td>4</td><td>$P(C/\overline{M})=\frac{P(C\cap\overline{M})}{P(\overline{M})}=\frac{0.06}{0.78}=\frac{1}{13}$.</td><td>0.5</td></tr>
<tr><td>5a</td><td>The four possible values are : 0 (the costumer did not buy anything), 20 000 (the costumer bought a modern album), 30 000 (the costumer bought a classical album), 50 000 (the costumer bought two albums).
<table>
<tr><td>x_i</td><td>0</td><td>20 000</td><td>30 000</td><td>50 000</td></tr>
<tr><td>P_i</td><td>0.72</td><td>0.08</td><td>0.06</td><td>0.14</td></tr>
</table></td><td>1</td></tr>
<tr><td>5b</td><td>$E(X)=\sum P_iX_i=0\times0.72+20\,000\times0.08+30\,000\times0.06+50\,000\times0.14=10\,400$ L.L.
$R=E(X)\times300=10\,400\times300=3\,120\,000$ LL.</td><td>0.5</td></tr>
</table>

Q3	Answers	M
1	$z_A-z_B=-3+3i$ and $z_A-z_C=-1+i$. $z_A-z_B=3(z_A-z_C)$ and $z_{\overrightarrow{BA}}$ $_{\overrightarrow{CA}}$; par suite $\overrightarrow{BA}=3\overrightarrow{CA}$ and the three points A, B and C are collinear.	0.5
2	$w=z_{\overrightarrow{AC}}=1-i=\sqrt{2}e^{-i\frac{\pi}{4}}$, $w^{20}=\left(\sqrt{2}\right)^{20}e^{-5i\pi}=-\left(\sqrt{2}\right)^{20}$ which is real negative.	1
3. a	$\|z-i\|=\|z_M-z_A\|=AM$; $\|z-1\|=\|z_M-z_C\|=CM$.	0.5
3. b	If z_M verifies $\|z-i\|=\|z-1\|$, so MA = MC; and the point M varies on the perpendicular bisector of segment [AC].	1
3. c	If z_M verify $(z-i)\times(\overline{z}+i)=16 \Leftrightarrow (z_M-z_A)\times(\overline{z_M-z_A})=16 \Leftrightarrow \|z_M-z_A\|\times\|\overline{z_M-z_A}\|=16$ $\Leftrightarrow \|z_M-z_A\|\times\|z_M-z_A\|=16$. Hence $AM^2=16$; therefore the point M belongs to the circle with center A and radius 4.	1

Q$_s$	Answers	M
1	$\lim\limits_{x\to-\infty} f(x) = 3-4=-1$ and $\lim\limits_{x\to+\infty} f(x)=3$. Hence (C) has two asymptotes with equations y = 3 and y = − 1.	1
2	$f'(x)=\dfrac{8e^{2x}}{(e^{2x}+1)^2}>0$; f is strictly increasing over IR. x: $-\infty$ … $+\infty$ f '(x): + f(x): −1 ↗ 3	1
3	Slope of (T) = f '(0) = 2, and (T) passes through the point W (0 ; 1), then the equation of (T) is : y = 2 x + 1.	0.5
4. a	$f(x)=0 \Leftrightarrow 3=\dfrac{4}{e^{2x}+1} \Leftrightarrow e^{2x}=\dfrac{1}{3} \Leftrightarrow x=-\dfrac{\ln 3}{2}$.	0.5
4.b		1
5-a	$f(x)=3-\dfrac{4}{e^{2x}+1}=\dfrac{3e^{2x}-1}{e^{2x}+1}$ et $1+\dfrac{4e^{2x}}{e^{2x}+1}=\dfrac{3e^{2x}-1}{e^{2x}+1}$. $F(x)=\int f(x)\,dx=\int\left(-1+\dfrac{4e^{2x}}{e^{2x}+1}\right)dx=-x+2\int\dfrac{2e^{2x}}{e^{2x}+1}dx=-x+2\ln\left(e^{2x}+1\right)+c$.	1
5-b	A = 4A' cm^2. $A'=\int_0^{\ln 2} f(x)\,dx=\left[-x+2\ln\left(e^{2x}+1\right)\right]_0^{\ln 2}=2\ln 5-3\ln 2=\ln\left(\dfrac{25}{8}\right)$. Thus, $A=4\ln\left(\dfrac{25}{8}\right)$ cm^2.	0.5
6. a	Dom (g) =]−1; 3[.	0.5
6. b	W (0,1) is a point of inflection of (C), then the symmetric of W with respect to the line with equation y = x is the point J (1 ; 0), which is the point of inflection of (G).	0.5
6. c	(G) is the symmetric of ©with respect to the line with equation y = x.	0.5
6. d	$y=g(x) \leftrightarrows x=f(y) \leftrightarrows x=3-\dfrac{4}{e^{2y}+1} \leftrightarrows \dfrac{4}{e^{2y}+1}=3-x \leftrightarrows e^{2y}+1=\dfrac{4}{3-x} \leftrightarrows$ $e^{2y}=\dfrac{4}{3-x}-1=\dfrac{1+x}{3-x}$. Thus, $2y=\ln\left(\dfrac{1+x}{3-x}\right)$; $y=g(x)=\dfrac{1}{2}\ln\left(\dfrac{1+x}{3-x}\right)$.	1

APPENDIX J

TIMSS Advanced 2015 – Mathematics Cognitive Domains

Retrieved from:

Mullis, I. & Martin, M. (Eds.) (2014). *TIMSS Advanced 2015 Assessment Frameworks.* Retrieved from Boston College, TIMSS & PIRLS International Study Center website:

http://timssandpirls.bc.edu/timss2015-advanced/frameworks.html

TIMSS Advanced
Mathematics Cognitive Domains

The mathematics cognitive dimension consists of three domains based on what thinking processes students are expected to use when confronting the mathematics items developed for the TIMSS Advanced 2015 assessment. The first domain, knowing, addresses the students' ability to recall and recognize facts, procedures, and concepts necessary for a solid foundation in mathematics. The second domain, applying, focuses on using this knowledge to model and implement strategies to solve problems. The third domain, reasoning, includes analyzing, synthesizing, generalizing, and justifying through mathematical arguments or proofs. The situations requiring reasoning often are unfamiliar or complex.

While there is some hierarchy across the three cognitive domains (from knowing to applying to reasoning), each domain contains items representing a full range of difficulty. The following sections further describe the thinking skills and behaviors defining the cognitive domains. The general descriptions are followed by lists of specific behaviors to be elicited by items that are aligned with each domain.

Each content domain includes items developed to address each of the three cognitive domains. Accordingly, the algebra, calculus, and geometry domains include knowing, applying, and reasoning items.

Knowing

Knowing refers to students' knowledge of mathematical facts, concepts, and procedures. Mathematical facts and procedures form the foundation for mathematical thought.

Recall	Recall definitions, terminology, notation, mathematical conventions, number properties, and geometric properties.
Recognize	Recognize entities that are mathematically equivalent (e.g., different representations of the same function).
Compute	Carry out algorithmic procedures (e.g., determining derivatives of polynomial functions, and solving a simple equation).
Retrieve	Retrieve information from graphs, tables, texts, or other sources.

Applying

The applying domain involves the application of mathematics in a range of contexts. In this domain, students need to apply mathematical knowledge of facts, skills, and procedures or understanding of mathematical concepts to create representations and solve problems. The problems in this domain typically reflect standard types of problems expected to be familiar to students. Problems may be set in real-life situations, or may be purely mathematical in nature involving, for example, numeric or algebraic expressions, functions, equations, or geometric figures.

Determine	Determine efficient and appropriate methods, strategies, or tools for solving problems for which there are commonly used methods of solution.
Represent/Model	Generate an equation or diagram that models problem situations and generate equivalent representations for a given mathematical entity, or set of information.
Implement	Implement strategies and operations to solve problems in familiar mathematical concepts and procedures.

Reasoning

Reasoning mathematically involves logical, systematic thinking. Problems requiring reasoning may do so in different ways, because of the novelty of the context or the complexity of the situation, the number of decisions and steps, and may draw on knowledge and understanding from different areas of mathematics. Reasoning involves formulating conjectures, making logical deductions based on specific assumptions and rules, and justifying results.

Analyze	Identify the elements of a problem and determine the information, procedures, and strategies necessary to solve the problem.
Integrate/ Synthesize	Link different elements of knowledge, related representations, and procedures to solve problems.
Evaluate	Determine the appropriateness of alternative strategies and solutions.
Draw Conclusions	Make valid inferences on the basis of information and evidence
Generalize	Make statements that represent relationships in more general and more widely applicable terms.
Justify	Provide mathematical arguments to support a strategy, solution, or a statement.

APPENDIX K

The curriculum content of Grade 9, 10, or 11 that is associated with the items that were addressed in the official exam tests for the LS and LH tracks

Part I: For the LS Track

A. Arrangements and permutations: Calculate n!

B. Arrangements and permutations: Know and use the formulas that give the number of arrangements and number of permutations

C. Polynomials, equations and inequalities of degree 2: Determine if a quadratic equation with real coefficients has real roots.

D. Polynomials, equations and inequalities of degree 2: Find the roots of a quadratic equation with real coefficients if they exist.

E. Complex numbers: Identify the real part and the imaginary part of a complex number.

F. Complex numbers: Determine the set of points that satisfy a given condition.

G. Complex numbers: Represent geometrically a complex number.

H. Complex numbers: Know and use the fact that the image of z and its conjugate are symmetric with respect to the real axis.

I. Complex numbers: Calculate the conjugate of a complex number and use its properties.

J. Complex numbers: Solve a quadratic equation with real coefficients and a negative discriminant.

K. Complex numbers: Characterize two equal complex numbers.

L. Complex numbers: Know the fact that the function from the set of points. p(x,y) to C which assigns p(x,y) to z=x +iy is a bijection.

M. Vectorial study: Find the coordinates of the midpoint of a segment.

N. Vectorial study: Know and use that the relations X(AB) = X(A)-X(B).

O. Geometry: Calculate the angle between vectors (using dot product).

P. Geometry: Prove ABC is right (Given 3 points).

Q. Geometry: Prove ABC is isosceles (use distance Formula).

R. Geometry: Deduce circle is tangent to line.

S. Geometry: Deduce/prove nature of a quad.

T. Geometry: Know and use the properties of vector product.

U. Geometry: Prove E sym of B wrt W.

V. Geometry: Prove 3 points collinear.

W. Geometry: Prove w center of circumscribed circle.

X. Functions: Deduce V and/or H asymptotes using limits.

Y. Functions: Sketch an asymptote.

Z. Functions: Verify that a given line is an asymptote.

AA. Functions: Calculate coordinate of intersection of graph and asymptote/tangent.

BB. Functions: Study relative positions of C and asymptote/tangent.

CC. Functions: determine center of symmetry (by proving odd).

DD. Functions: Prove a point is a center of sym.

EE. Functions: discuss the number of roots f(x) = m.

FF. Functions: interpret f'(0) graphically.

GG. Continuity and differentiation: Know that the derivative is the slope of tangent and know the equation of the tangent to a graph at a point.

HH. Continuity and differentiation: Find m so that f is strictly monotonic.

II. Continuity and differentiation: Find m so that C has an extremum.

JJ. Continuity and differentiation: Justify f is increasing using a given graph of f'(x).

KK. Continuity and differentiation: Justify f is increasing using a given table of f'(x).

LL. Continuity and differentiation: Study sign of f(x) using a table of variation of h(x).

MM. Continuity and differentiation: study sign of f'(x) given table of variations of f'(x).

NN. Continuity and differentiation: Find h'(x) (where h(x) =xf(x).

OO. Antiderivative: Identify the antiderivative as the inverse operation of differentiation.

PP. Statistics: Draw I.C.F polygon.

QQ. Probability: Calculate the probability of an event using the basic properties of probability.

RR. Probability: Find P(A∩B) using formula when independent.

SS. Probability: Know that, for two events A and B, P(A or B) = P(A) + P(B) - P(A and B) .

TT. Probability: Know that if A and Ā are complementary events then: P(A) + P(Ā) = 1

Part II: For the LH track

A. *Lines:* Draw a line defined by its equation. (retrieved from the details of contents forgrade 9)

B. *Lines:* Write the equation of a line parallel to the y-axis. (retrieved from the detailsof contents for grade 9)

C. *Lines:* Find the equation of a line passing through two distinct points. (retrievedfrom the details of contents for grade 9)

D. *Functions:* Calculate f(2) given the algebraic expression f(x).

E. *Functions:* Write the equation of the tangent to the graph of the function at the point(a, f(a)). (retrieved from the details of contents for grade 11H)

F. *Functions:* Calculate the derivative and determine its sign. (retrieved from thedetails of contents for grade 11H)

G. *Functions:* Graphically, determine if $f'(2)$ is > 0, < 0, or $= 0$.

H. *Functions:* Determine analytically the points of intersection of curve and line.

I. *Functions:* Given table of variation, solve inequalities of the form: $f(x) \geq m$ (resp. $\leq$)

for a given real value of m.

J. *Functions*: Given f(x) in terms of a, b, c, or other unknowns. find the unknown(s)

K. *Functions*: Verify that the equation of line passing through 2 or 3 given point is

L. *Functions*: Compare $f'(1)$ and $f'(2)$ given table of variation

M. *Functions*: Show line is tangent to a curve.

N. *Functions*: Find the intersection of 2 lines.

O. *Functions*: Given table of variation, find f'(4).

P. *Functions*: Given table of variation, solve f'(x) $\geq$ m (resp. $\leq$, or =) for a given real value of m.

Q. *Functions*: Given f(x) in 1 form, prove it can be written in another form

R. *Interest:* Which choice is more profitable?

S. *Interest:* Find earned interest (new amount - old amount)

T. *Statistics:* Complete table of frequency.

U. *Statistics:* Find the average of statistical data.

V. *Probability:* Calculate the probability of an event using the basic properties ofprobability. (retrieved from the details of contents for grade 11H)

W. *Probability:* Know that, for two events A and B, P(A or B) = P(A) + P(B) - P(Aand B). (retrieved from the details of contents for grade 11H)

X. *Probability:* Know that if A and Ā are complementary events then: P(A) + P(Ā) = 1.

APPENDIX L

Quantitative Analysis of the LH Model Tests

Model Test 2 LHM2

Retrieved from:

Sleiman, L. H. (2012). *A study of the alignment between the Lebanese secondary-level national math exams for the literature and humanities track and the reformed math curriculum*.

Code of the Details of Contents of the LH track at Grade 12	Mathematics Framework - TIMSS Advanced 2008 - Cognitive Domains			Mathematics Model Test 2 (LHM2) Test items
	Knowing	Applying	Reasoning	
1.2.1.1.i.	2/3			II.i. , II.ii.
1.2.1.1.ii.			2/3	II.i. , II.ii.
1.2.1.3.i.	1/3	1/3		II.i. , II.ii.
2.1.1.1.iv.		1		III.3.i.
2.1.1.1.v.	4			III.3.ii. , III.3.iii. , III.3.iv. , III.3.v.
2.1.1.1.iix.	1			III.4.
J		2	1	III.1. , III.2.iii. , III.2.iv.
2.1.2.2.vi.	2			III.2.i. , III.2.ii.
2.1.2.2.vii.	2			III.5.i. , III.5.ii.
2.1.2.2.iix.		1		III.6.
3.2.1.1.	1/2	½		I.
Total	**10 1/2**	**4 5/6**	**1 2/3**	**17**

J refers to the test items addressed in the model tests and the official exams that relate to the curriculum content studied at grade levels preceding Grade 12 LH track.

J: Given f(x) in terms of a, b, c find a, b, c

Model Test 6 LHM6

Code of the Details of Contents of Grade 12 - LH	Mathematics Framework - TIMSS Advanced 2008 - Cognitive Domains			Math Model Test - Grade 12 - LH Section - Year 2017 – 2 LHM6
	Knowing	Applying	Reasoning	Test items
1				
1.2.1.1.i.	2/3			II1i-II1ii
1.2.1.1.ii.			2 1/6	II1i-II1ii-II2i-II2ii-II3
1.2.1.3.i.	1 5/6	1 1/3		II1i-II1ii-II2i-II2ii-II3-III7ai
2				
2.1.1.1.ii.	1			III1
2.1.1.1.vi.			1	III2v2
2.1.2.2.ii.	1 1/2	1	½	III3d-III4-III6
2.1.2.2.v.	4		1	III2i-III2ii-III2iii-III2iv-III2v1
2.1.2.2.vi.	1			III3c
E	½	½		III5i
F	1		1	III5ii-III7b
G	1		1	III3a-III3b
H	½		½	III7aii
3				
T	6			I1i-I1ii-I1iii-I1iv-I1v-I1vi
3.2.				
3.2.1.1.i.	½	½		I2c
3.2.1.2.i.			1	I3
V	1			I2a
W	½	½		I2b
	21	3 5/6	8 1/6	33

E, F, G, T, U, V and W refer to test items addressed in the model tests and the official tests that relate to the curriculum content studied at grade levels preceding Grade 12 LH track.
E: Write the equation of the tangent to the graph of the function at the point (a, f(a)).
F: Calculate the derivative and determine its sign.
G: Graphically, determine if f '(2) is > 0 , < 0 , or = 0.
T: Complete table of frequency
U: Find the average of statistical data.
V: Calculate the probability of an event using the basic properties of probability.
W: Know that, for two events A and B, P(A or B) = P(A) + P(B) - P(A and B).

APPENDIX M

Quantitative Analysis of the Official Test LH182 for the LH track

Code of the Details of Contents of Grade 12 - LH	Mathematics Framework - TIMSS Advanced 2008 - Cognitive Domains			Math Official Test - Grade 12 - LH Section - Year 2018 – 2 LH182
	Knowing	Applying	Reasoning	Test Items
1.2.				
1.2.1.1.i.	2/3			I1i-I1ii
1.2.1.1.ii.			1 1/6	I1i-I1ii-I2ii
1.2.1.3.i.	1 5/6	5/6		I1i-I1ii-I2i-I2ii-III4a
2				
2.1.1.1.iv.		1		III3b
2.1.1.1.v.	1	1		III2ai-III2b
2.1.1.1.vi.	1			III2aii
2.1.1.1.iix.	½	1/2		III2c
2.1.1.1.ix.		1		III4ci
2.1.2.				
2.1.2.2.iix.		1		III5
A		1		III4ciii
B	1			III4cii
E		1		III4b
F	½	1/2		III3a
H	½			III4a
Q	1			III1
3				
U	½	1/2		II1
3.2.				
3.2.1.1.i.	½	1 1/2		II2bii-II2bv
V	2	1		II2a-II2bi-II2biii
W	½	1/2		II2biv
	11 ½	11 1/3	1 1/6	24

A, B, E, F, H, Q, U, V and W refer to test items addressed in the model tests and the official tests that relate to the curriculum content studied at grade levels preceding Grade 12 LH track.
A: Draw a line defined by its equation.
B: Write the equation of a line parallel to the y-axis.
E: Write the equation of the tangent to the graph of the function at the point (a, f(a)).
F: Calculate the derivative and determine its sign.
H: Determine analytically the points of intersection of curve and line.
Q: Given f(x) in 1 form, prove it can be written in another form
U: Find the average of statistical data.
V: Calculate the probability of an event using the basic properties of probability.
W: Know that, for two events A and B, P(A or B) = P(A) + P(B) - P(A and B).

APPENDIX N

Quantitative Analysis of the Model Tests LS track

Model Test 1 LHM1

Retrieved from:

Safa, W. (2013). *Evaluating the alignment between a mathematics curriculum and the national tests: The case of Lebanon secondary national exams for the life science section* (Master's Thesis under the supervision of I. Osta). Lebanese American University, Beirut.

Curriculum of Mathematics - Decree No 10227 - Date: 08 May 1997 Details of Contents / Objectives of Grade 12 - LS section	Mathematics Framework - TIMSS Advanced 2008 - Cognitive Domains			Math - Model Test 1 - Grade 12 - LS Section
	Knowing	Applying	Reasoning	Test items
1.2.1.2	1		2	I1-I2
2.1.1.2.iii.	1/2	1/2		III1
2.1.1.2.iv.	1/3	1/3	1/3	III1
2.1.2.1.	1/4	1/4		III3
2.1.2.1.iii.	1/2	1/2		III3
2.1.3.1.i.	1/4	1/2		III2
2.1.4.1.vi.	1/4	1/4		III3
Grade 11	1/4			III2
Grade 11S	1/2	1/2		III4
3.1.3.1.i.		2		II1
3.1.3.3.iii.	1/3	1/3	1/3	II5a
Grade 11S	1/3	1/3	1/3	II5b
Grade 11S			1	II6
Grade 11S	1/3	1/3	1/3	II3
Grade 11S	1/3	1/3	1/3	II4
3.3.3.1.i.	1/3	1/3	1/3	II2
Total	5 1/2	6 1/2	5	17

Model Test 5 LHM5

Code of the Details of Contents of Grade 12 - LS	Mathematics Framework - TIMSS Advanced 2008 - Cognitive Domains			Math - Model Test 5 - Grade 12 - LS Section
	Knowing	Applying	Reasoning	Test items
1				
1.2.1.2	½			II1i
1.4.1.1.ii.		½		III1ii
1.4.1.1.iii.		½		III1ii
1.4.1.2		½	1/2	III2
1.4.1.2.iii.	1	¼	3/4	III1i-III3b
1.4.1.2.iv.		¼	1/4	III4
Grade 11S		¼	1/4	III4
Grade 11S	1			III3a
2				
2.1.2.1.vi.			1	I2b
Additional	1			I1a
2.1.4.1.ii.		¼	1/4	I2a
2.1.4.1.iii.	½	½		I3a
2.1.4.1.vi.	¾	1	1/4	I1b-I2a-I3b
Grade 9	½			I3b
3				
3.1.1.2.ii.	1			IV6a
3.1.1.3.i.		1		IV6b
3.1.1.3.iv.		1	1	IV7a-IV8
3.1.3.1.i.		3		IV3ii-IV5ii-IV7b
3.1.3.1.iv.			1	IV1i
3.1.3.2.i.		1		IV3i
Grade 11S	1			IV1ii
Grade 11S		1		IV5i
Grade 11S			1	IV2a
Grade 11S	1/3	1/3	1/3	IV2b
5				
Grade 11S	½	½		IV4
5.2.1.1.i.	½	½		II3
5.2.1.1.ii.	½	½		II1ii
5.2.2.1.	1			II2
5.2.3.1.iii.		1	1	II4a-II4b
grade 11 S	½			II1i
Total	**10 4/7**	**13 5/6**	**7 4/7**	**32**

APPENDIX O

Quantitative Analysis of the National Test LS131 for the LS track

Code of the Details of Contents of Grade 12 – LS	Mathematics Framework - TIMSS Advanced 2008 - Cognitive Domains			Math – National Test 2013 session 1 - Grade 12 - LS Section
	Knowing	Applying	Reasoning	Test Items
1				
1.4.1.1.ii.	1			III3a
1.4.1.2.	½	1/2		III2i
1.4.1.3.		1/2	½	III2ii
1.4.1.2.iii.		1 1/2		III1-III3c
Grade 11S	¼	1/4	¾	III3b-III3c
Grade 11S			¼	III3c
2				
2.1.1.2.iii.	½	1/2		I4b
2.1.2.1.ii.	½	1/2		I3a
2.1.2.1.iii.	½	1/2		I2
2.1.3.1.i.		1		I1
2.1.3.1.ii.		1		I4a
2.1.4.1.i.		1/4	¾	I3b
3				
3.1.1.3.i.	1			IV6a
3.1.1.3.iii.		1		IV6d
3.1.1.3.iv.		1		IV6c
3.1.3.1.i.		2		IV2ii-IV4bii
3.1.3.1.ii.		1		IV5ai
3.1.3.1.iii.	1	1		IV1i-IV1ii
3.1.3.1.iv.	1/3	1/3	1/3	IV2i
Grade 11S	½	1/2		IV1iii
Grade 11S		1		IV14bi
Grade 11S		1		IV4a
3.2.4.1.i.		1/2	½	IV6bi
Additional	1			IV6bii
Grade 11S	½	1/2		IV3
3.3.2.1.ii.		1		IV5aii
3.3.3.1.i.	½	1/2		IV5b
5				
5.2.1.1.i.	1/3	1/3	1/3	II4
5.2.1.1.ii.	2			II1i-II1ii
5.2.2.1.	¼		¾	II2
5.2.3.1.ii.		1		II5aii
5.2.3.1.iii.			1	II5aii
5.2.3.3.i.	½	1/2		II5b
grade 11 S	½		½	II3
Total	11 2/3	19 2/3	5 2/3	37

APPENDIX P

Qualitative Analysis of the LH Model Tests and Official Tests

Table 1

Occurrences of Test Items on Different Math Topics in the Model Tests and Official Tests of the LH Track at Grade 12

Math Topics	Model Tests								Official Exams of the LH Track at Grade 12															
									2011		2012		2013		2015		2016		2017		2018		2019	
	LHM1	LHM2	LHM3	LHM5	LHM6	LHM7	LHM8	LHM7	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
1. ALGEBRA																								
1.1.3 Propositional Calculus			X																					
1.2 Equations and Inequalities	X	X	X	X	X	X	X	X	X		X		X	X	X	X	X	X	X	X	X	X	X	X
2. CALCULUS (Numberical Functions)																								
2.1. Definitions and Representations of Rational Functions	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
2.2.1 Simple interest, Compound Interest						X				X		X												X
3. STATISTICS AND PROBABILITY																								
3.1. Statistics				X	X	X	X		X	X			X			X		X		X		X	X	
3.2. Probability	X	X		X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

Table 2

Distribution of Grades by Math Topics in the Model Tests and Official Tests of the LH Track at Grade 12

Math Topics	Model Tests								Official Exams of the LH Track at Grade 12															
									2011		2012		2013		2015		2016		2017		2018		2019	
	LHM1	LHM2	LHM3	LHM5	LHM6	LHM7	LHM8	LHM7	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
1. ALGEBRA																								
1.1.3 Propositional Calculus			6																					
1.2 Equations and Inequalitie	9	6	8	5	5	1	5	5	5		5.5		5.5	5	5.5	5	5	6	5	5	5.5	5.5	5.5	4.5
2. CALCULUS (Numberical Functions)																								
2.1. Definitions and Representations of Rational Functions	7	11	6	10	10	9	10	10	10	10	9.5	10	9.5	10	9.5	10	10	10	10	10	9.5	9.5	9.5	9.5
2.2.1 Simple interest, Compound Interest						5				5		5												1
3. STATISTICS AND PROBABILITY																								
3.1. Statistics				1	1	2	1		1.5	1			1			1				2		1	1.5	
3.2. Probability	4	3		4	4	3	4	5	3.5	4	5	5	4	5	5	4	5	4	5	3	5	4	3.5	5

Table 3

Occurrences of Test Items on the Math Topic "Propositional Calculus" in the Model Tests and Official Tests of the LH Track at Grade 12

Test Items on Propositional Calculus	Model Tests									Official Exams of the LH Track at Grade 12															
										2011		2012		2013		2015		2016		2017		2018		2019	
	LHM1	LHM2	LHM3		LHM5	LHM6	LHM7	LHM8	LHM7	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
-general problem on Logic			X																						

Table 4

Occurrences of Test Items on the Math Topic "Equations and Inequalities" in the Model Tests and Official Tests of the LH Track at Grade 12

Test Items on Equations and Inequalities	Model Tests									Official Exams of the LH Track at Grade 12															
										2011		2012		2013		2015		2016		2017		2018		2019	
	LHM1	LHM2	LHM3		LHM5	LHM6	LHM7	LHM8	LHM7	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
-equations		X	X		X	X	X	X	X	X		X		X	X	X	X	X	X	X	X	X	X	X	X
-inequalities	X																								

Table 5

Occurrences of Test Items on the Math Topic "Definitions and Representations" in the Model Tests and Official Tests of the LH Track at Grade 12

Test Items on Definitions and Representations	Model Tests								Official Exams of the LH Track at Grade 12															
									2011		2012		2013		2015		2016		2017		2018		2019	
	LHM1	LHM2	LHM3	LHM5	LHM6	LHM7	LHM8	LHM7	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
-domain of definition				X	X		X												X	X				
-find f(1)		X			X		X			X					X				X	X			X	
-determine f '(1)					X			X		X						X			X	X				
-compare f(2) and f(3)					X					X	X		X		X		X		X					
-compare f '(0) and f '(1)/ determine using graph f '(2) < > = 0					X					X			X	X	X		X			X				
-prove that I(2, 3) is center of symmetry									X							X		X						
-write equation of tangent line at point A/prove				X	X	X	X	X		X			X	X	X	X	X		X		X	X		
-determine the intersection of f(x) and a line					X		X		X		X	X	X		X							X	X	X
-find lim f(x) as x ->1+	X	X			X	X	X		X			X		X	X	X		X	X	X	X	X	X	X
-deduce asymptote/write equation			X		X	X	X	X	X			X				X		X	X	X	X	X	X	X
-find lim f(x) as x-> ∞	X	X		X	X	X	X		X			X		X	X	X		X	X		X	X	X	X
-prove y= k or y= 2x+1 asymptote/ write equation		X	X	X		X	X	X	X		X	X	X			X	X	X	X		X	X	X	X
-given f(x) = ... verify f(x) is also = ...								X														X	X	
-solve f(x)=g(x) algebraically		X				X															X			
-given f(x) in terms of a, b, c. (or another unknown to be found) Find a. b. c		X		X			X	X		X	X		X	X					X	X	X			X
-calculate f '(x)														X	X								X	
-find sign of f '(x) by calculation																							X	
-verify f '(x)=				X	X	X	X	X	X			X					X	X	X		X	X		X
-verify f ' (x) > 0																								

-complete table of variation					X									X									X	X		X
-set table of variation	X	X	X				X	X	X		X	X				X	X	X		X		X			X	
-draw line					X		X				X		X	X				X	X	X	X		X	X	X	X
-draw the graph of f			X		X		X	X			X		X	X	X			X	X	X	X		X	X	X	X
-write equation of vertical line given graph												X														
-write equation of oblique line given graph/points												X				X	X									
-verify the equation of straight line passing through 2 given points									X		X															
-determine using table f ' (2) < > = 0																					X					
-determine # of solution of f(x)=3 (table)													X		X											X
-determine # of solution of f(x)=0 (graph)																X			X			X				
-solve graphically f(x) ≤ 1	X	X						X	X		X	X		X		X						X	X	X	X	X
-solve graphically f ' (x)= 0																										
-solve graphically f '(x) > 0												X						X								
-given table of variation, solve f(x) < 0																		X			X					
-solve graphically f(x)< any line																										
-verify OMxON=0.5f(3)											X															
-prove that a line passes through 3 points.																				X						
-determine sign of f(d) at specific interval using the graph						X																				
-prove algebraicly that the function is above its asymptote								X																		
-solve graphically inequality f(x)≥g(x)													X							X						

Table 6

Occurrences of Test Items on the Math Topic "Simple and Compound Interest" in the Model Tests and Official Tests of the LH Track at Grade 12

Test Items on Simple and Compound Interest	Model Tests									Official Exams of the LH Track at Grade 12															
										2011		2012		2013		2015		2016		2017		2018		2019	
	LHM1	LHM2	LHM3		LHM5	LHM6	LHM7	LHM8	LHM7	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
-find new amount using compound interest							X				X		X												X
-compare which choice is more profitable													X												
-find earned interest (new - old amount)							X				X														

Table 7

Occurrences of Test Items on the Math Topic "Statistics" in the Model Tests and Official Tests of the LH Track at Grade 12

Test Items on Statistics	Model Tests									Official Exams of the LH Track at Grade 12															
										2011		2012		2013		2015		2016		2017		2018		2019	
	LHM1	LHM2	LHM3		LHM5	LHM6	LHM7	LHM8	LHM7	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
-complete table					X	X	X	X		X	X						X		X		X			X	
-find the average of statistical data														X							X		X		

Table 8

Occurrences of Test Items on the Math Topic "Probability" in the Model Tests and Official Tests of the LH Track at Grade 12

Test Items on Probability	Model Tests								Official Exams of the LH Track at Grade 12															
									2011		2012		2013		2015		2016		2017		2018		2019	
	LHM1	LHM2	LHM3	LHM5	LHM6	LHM7	LHM8	LHM7	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
-P(A)				X	X	X	X	X	X	X	X	X	X		X	X	X		X	X	X	X	X	X
-P(A and B)				X		X	X							X	X	X	X	X		X	X	X	X	X
-P(A or B)	X			X	X		X	X	X	X								X	X			X	X	X
-P(A/B)		X		X	X	X	X	X		X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
-P(… < > …)					X			X												X		X		
-P(A then B) with no replacement (knowing that)				X	X	X	X	X	X	X	X		X	X			X		X					X

APPENDIX Q

Quantitative Analysis of the Model Tests and Official Tests for the LH Track

Table 1

Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests and the Official Tests (2011-2019) for the LH Track of Grade 12 – Extracted from Table AllModLH and Table OffExLH.

The Topics of the Math Curriculum of the LH Track at Grade 12	Sum of Model Tests				Sum of Official Tests			
	K %	A %	R%	Total	K %	A %	R %	Total
1.1.3. Propositional Calculus	0	0	2.93	2.93	0	0	0	0
1.2. Equations & Inequalities	8.17	5.90	3.98	18.05	8.51	4.78	4.46	17.75
2.1. Definitions and Representations	26.10	17.07	7.56	50.73	25.44	22.52	4.8	52.76
2.2.1. Simple & Compound Interest	1.22	0.24	0	1.46	0.96	0.48	0	1.44
3.1. Statistics	10	1.71	0	11.71	8.63	1.08	0.36	10.07
3.2. Probability	5.37	8.54	1.22	15.13	8.03	9.95	0	17.98
Total	50.8537	33.4545	15.6894	≈100	51.57	38.81	9.62	≈100

K = Knowing
A = Applying
R = Reasoning
The sum of Totals is approximately equal to 100 because the percentages are rounded.

Table 2

Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests issued with the curriculum documents and the Official Tests 2011-2013 and 2015-2016 for the LH Track of Grade 12 – Extracted from Table ModLH, Table OffExLH11-13 and Table OffExLH15-16.

The Topics of the Math Curriculum of the LH Track at Grade 12	Sum of Model Tests LHM1, LHM2, and LHM3				Sum of 2011-2013 Official Tests				Sum of 2015-2016 Official Tests			
	K %	A %	R%	Total	K %	A %	R %	Total	K %	A %	R %	Total
1.1.3. Propositional Calculus	0	0	12.24	12.24	0	0	0	0	0	0	0	0
1.2. Equations & Inequalities	10.2	13.61	2.72	26.53	5.42	3.16	3.93	12.5	11.14	6.97	5.45	23.56
2.1. Definitions and Representations	38.78	16.32	2.04	57.14	22.14	24.88	6.91	53.93	25.8	20.67	4.01	50.48
2.2.1. Simple & Compound Interest	0	0	0	0	2.5	1.071	0	3.571	0	0	0	0
3.1. Statistics	0	0	0	0	10.36	1.79	0	12.14	9.62	0	0	9.615
3.2. Probability	1.02	3.06	0	4.08	7.143	10.71	0	17.86	7.93	8.41	0	16.35
Total	50	32.99	17	≈100	47.56	41.61	10.83	≈100 □	54.49	36.06	9.455	≈100

K = Knowing
A = Applying
R = Reasoning
The sum of Totals is approximately equal to 100 because the percentages are rounded.

Table 3

Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests issued in 2017 and the Official Tests 2017-2018 for the LH Track of Grade 12 – Extracted from Table ModLH5-7 and Table OffExLH17-18.

The Topics of the Math Curriculum for the LH Track at Grade 12	Sum of Model Tests (LHM5, LHM6, and LHM7)					Sum of 2017-2018 Official Tests			
	K %	A %	R %	Total		K %	A %	R %	Total
1.1.3. Propositional Calculus	0	0	0	0		0	0	0	0
1.2. Equations & Inequalities	5.56	2.96	4.81	13.33		10.18	4.57	4.28	19.03
2.1. Definitions and Representations	21.11	17.78	7.78	46.67		28.98	19.54	5.01	53.53
2.2.1. Simple & Compound Interest	2.78	0.56	0	3.34		0	0	0	0
3.1. Statistics	16.11	3.89	0	20		4.87	1.77	1.33	7.97
3.2. Probability	5.56	9.44	1.67	16.67		9.51	9.96	0.00	19.47
Total	51.12	34.63	14.26	□	□	53.54	35.84	10.62	≈100

K = Knowing
A = Applying
R = Reasoning
The sum of Totals is approximately equal to 100 because the percentages are rounded.

Table 4

Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests issued in 2019 and the Official Tests of the year 2019 for the LH Track of Grade 12 – Extracted from Table ModLH8-9 *and Table OffExLH17-18.*

The Topics of the Math Curriculum of the LH Track at Grade 12	Sum of Model Tests (LHM8 and LHM9)					Sum of 2019 Official Tests			
	K %	A %	R %	Total		K %	A %	R %	Total
1.1.3. Propositional Calculus	0	0	0	0		0	0	0	0
1.2. Equations & Inequalities	10.23	4.17	3.76	18.16		8.06	5.14	4.31	17.51
2.1. Definitions and Representations	23.48	16.67	11.36	51.51		25.83	25.83	0.83	52.49
2.2.1. Simple & Compound Interest	0	0	0	0		0.83	0.83	0	1.66
3.1. Statistics	9.09	0	0	9.09		10	0	0	10
3.2. Probability	8.33	11.36	1.52	21.21		7.50	10.83	0.00	18.33
Total	51.13	32.2	16.64	□	□	52.22	42.63	5.14	≈100

K = Knowing
A = Applying
R = Reasoning
The sum of Totals is approximately equal to 100 because the percentages are rounded.

Table 5

Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests issued in the year 2000 and Sessions 1 and 2 Official Tests of the years 2011-2016 for the LH Track of Grade 12– Extracted from Table Mod, Table OffExLH11, and OffExLH21

The Topics of the Math Curriculum of the LH Track at Grade 12	Sum of Old Model Tests LHM1, LHM2, and LHM3				Sum of Session-1 Official Tests of the years 2011-2016				Sum of Session-2 Official Tests of the years 2011-2016			
	K %	A %	R%	Total	K %	A %	R %	Total	K %	A %	R %	Total
1.1.3. Propositional Calculus	0	0	12.24	12.24	0	0	0	0	0	0	0	0
1.2. Equations & Inequalities	10.2	13.61	2.72	26.53	9.8	5.99	5.7	21.49	6.15	3.72	3.59	13.46
2.1. Definitions and Representations	38.78	16.32	2.04	57.14	21.93	20.61	8.77	51.31	25.26	25.26	2.95	53.47
2.2.1. Simple & Compound Interest	0	0	0	0	0	0	0	0	2.69	1.15	0	3.84
3.1. Statistics	0	0	0	0	6.58	1.32	0	7.9	13.08	0.77	0	13.85
3.2. Probability	1.02	3.06	0	4.08	8.11	11.18	0	19.29	6.92	8.46	0	15.38
Total	50	32.99	17	≈100	46.42	39.1	14.47	≈100	54.1	39.36	6.54	≈100

K = Knowing
A = Applying
R = Reasoning
The sum of Totals is approximately equal to 100 because the percentages are rounded.

Table 6

Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests issued in the year 2017 and Sessions 1 and 2 Official Tests of the years 2017-2018 for the LH Track of Grade 12– Extracted from Table ModLH5-7, Table OffExLH12, and Table OffExLH22

The Topics of the Math Curriculum of the LH Track at Grade 12	Sum of Model Tests (LHM5, LHM6, and LHM7)				Sum of Session-1 Official Tests 2017-2018				Sum of Session-2 Official Tests 2017-2018			
	K %	A %	R %	Total	K %	A %	R %	Total	K %	A %	R %	Total
1.1.3. Propositional Calculus	0	0	0	0	0	0	0	0	0	0	0	0
1.2. Equations & Inequalities	5.56	2.96	4.81	13.33	12.07	5.75	4.6	22.42	8.18	3.33	3.94	15.45
2.1. Definitions and Representations	21.11	17.78	7.78	46.67	34.05	20.26	4.31	58.62	23.64	18.79	5.76	48.19
2.2.1. Simple & Compound Interest	2.78	0.56	0	3.34	0	0	0	0	0	0	0	0
3.1. Statistics	16.11	3.89	0	20	0	0	0	0	10	3.64	2.73	16.37
3.2. Probability	5.56	9.44	1.67	16.67	8.19	10.78	0	18.97	10.91	9.09	0	20
Total	51.12	34.63	14.26	≈100	54.31	36.79	8.91	≈100	52.73	34.85	12.43	≈100

K = Knowing
A = Applying
R = Reasoning
The sum of Totals is approximately equal to 100 because the percentages are rounded.

Table 7

Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests issued in the year 2019 and Sessions 1 and 2 Official Tests of the year 2019 for the LH Track of Grade 12– Extracted from Table ModLH8-9, Table LH191, and Table LH192

The Topics of the Math Curriculum of the LH Track at Grade 12	Sum of Model Tests (LHM8 and LHM9)				Sum of Session-1 Official Tests 2019				Sum of Session-2 Official Tests 2019			
	K %	A %	R %	Total	K %	A %	R %	Total	K %	A %	R %	Total
1.1.3. Propositional Calculus	0	0	0	0	0	0	0	0	0	0	0	0
1.2. Equations & Inequalities	10.23	4.17	3.76	18.16	7.14	5.24	3.33	15.71	9.33	5	5.67	20
2.1. Definitions and Representations	23.48	16.67	11.36	51.51	25.71	27.14	0	52.85	26	24	2	52
2.2.1. Simple & Compound Interest	0	0	0	0	0	0	0	0	2	2	0	4
3.1. Statistics	9.09	0	0	9.09	17.14	0	0	17.14	0	0	0	0
3.2. Probability	8.33	11.36	1.52	21.21	5.71	8.57	0	14.28	10	14	0	24
Total	51.13	32.2	16.64	≈100	55.7	40.95	3.33	≈100	47.33	45	7.67	≈100

K = Knowing
A = Applying
R = Reasoning
The sum of Totals is approximately equal to 100 because the percentages are rounded.

APPENDIX R

Qualitative Analysis of the LS Model Tests and Official Tests

Table 1

Occurrences of Test Items on Different Math Topics in the Model Tests and Official Tests for the LS Track of Grade 12

Math Topics	Model Tests											Official Tests of the LS Track at Grade 12															
												2011		2012		2013		2015		2016		2017		2018		2019	
	LSM1	LSM2	LSM3	LSM4		LSM5	LSM6	LSM7	LSM8	LSM9	LSM10	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
1. ALGEBRA																											
1.2. Literal and numerical calculations	X		X			X		X		X		X	X	X	X		X	X	X	X	X	X	X	X	X	X	X
1.4. Numbers		X	X	X		X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
2. GEOMETRY																											
2.1. Classical study	X		X			X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
3. CALCULUS (NUMERICAL FUNCTIONS)																											
3.1. Definitions & Representations	X	X	X	X		X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
3.2. Continuity and differentation			X	X		X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
3.3. Integration	X	X		X				X		X	X	X	X	X	X	X	X		X			X	X	X	X	X	
3.4. Differential equations								X																			X
4. TRIGONOMETRY																											
4.1 Circular functions																											
5. STATISTICS AND PROBABILITY																											
5.1 Statistics				X																							
5.2 Probability		X	X			X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

Table 2

Distribution of Grades by Math Topics in the Model Tests and Official Tests for the LS Track of Grade 12

Math Topics	Model Tests										Official Tests of the LS Track at Grade 12															
											2011		2012		2013		2015		2016		2017		2018		2019	
	LSM1	LSM2	LSM3	LSM4	LSM5	LSM6	LSM7	LSM8	LSM9	LSM10	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
1. ALGEBRA																										
1.2. Literal and numerical calculations	5				0.5		0.5		1		2	3	1	2		1.5	0.5	1.5	1.25	0.5	1.5	1	2.5	1.25	1	
1.4. Numbers		6	4	5	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
2. GEOMETRY																										
2.1. Classical study	6		5		4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
3. CALCULUS																										
3.1. Definitions &																										
3.2. Continuity and differentation	9	9	7	10	8	8	8	8	8	8	8	8	8	8	8	8	8		8	8	8	8	8	8	8	8
3.3. Integration																										
3.4. Differential equations																										
4. TRIGONOMETRY																										
4.1 Circular functions																										
5. STATISTICS AND PROBABILITY																										
5.1 Statistics				5																						
5.2 Probability		6	5		3.5	4	3.5	4	3	4	2	1	3	2	4	2.5	3.5	2.5	2.75	3.5	2.5	3	1.5	2.75	3	4

Table 3

Occurrences of Test Items on the Math Topic "Literal and Numerical Calculations" in the Model Tests and Official Tests for the LS Track of Grade 12

Objectives of the test items on Literal and numerical calculations	Model Tests											Official Tests of the LS Track at Grade 12															
												2011		2012		2013		2015		2016		2017		2018		2019	
	LSM1	LSM2	LSM3	LSM4		LSM5	LSM6	LSM7	LSM8	LSM9	LSM10	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
No of combinations	x					x		x		x		x	x	x	x		x	x	x	x	x	x	x	x	x	x	x
No of combinations (specifications: at least...)	x											x	x		x		x										
Arrangements and permutations			x																								

Table 4

Occurrences of Test Items on the Math Topic "Number" in the Model Tests and Official Tests for the LS Track of Grade 12

Objectives of the test items on Numbers	Model Tests												Official Tests of the LS Track at Grade 12															
	LSM1	LSM2	LSM3	LSM4		LSM5	LSM6	LSM7	LSM8	LSM9	LSM10		2011		2012		2013		2015		2016		2017		2018		2019	
													session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
Linearize		x																										
write in trigo form		x					x																					
write in exp form						x	x		x	x				x	x	x	x	x		x			x	x	x	x	x	x
write in alg form (from exp or trigo?)								x																				x
Identify the real part of a complex number		x																										
Interpret geometrically the product zz'		x																										
Calculate the argument of z		x						x	x	x	x					x					x	x		x				
Calculate the modulus of z				x		x	x	x	x					x	x	x						x	x	x	x			x
Interpret geometrically the argument of z (prove collinear)		x				x											x											
Interpret geometrically the argument of z (u, OA)										x																		
Interpret geometrically the modulus of z								x	x						x		x		x									
Determine the set of points that satisfy a given condition			x	x		x	x		x	x	x		x		x	x	x	x	x	x	x	x	x	x	x	x	x	x
Know that AB = abs (z(b)-z(a))				x							x			x		x			x	x	x	x	x	x		x		x
Deduce or prove the type of triangle		x		x		x		x					x	x				x				x					x	
Prove parallelogram								x																		x		
Know and use the properties of modulus							x		x		x		x		x						x				x	x	x	x
Know and use the properties of an argument								x														x					x	x
Represent geometrically a complex number										x																		
express x' and y' interms of x and y									x	x			x					x		x	x	x	x		x		x	
Recognize pure real		x																	x									
Recognize pure imaginary										x										x							x	
Calculate cojugate of a complex number and use properties											x			x												x		
Solve a quadratic equation with complex roots				x																								
Perform operations on complex numbers						x		x	x	x									x	x	x	x	x	x	x	x		
show that 2 lines are perpendicular																					x							

Table 5

Occurrences of Test Items on the Math Topic "Classical Study" in the Model Tests and Official Tests for the LS Track of Grade 12

Objectives of the test items on Geometry	Model Tests											Official Tests of the LS Track at Grade 12															
	LSM1	LSM2	LSM3	LSM4		LSM5	LSM6	LSM7	LSM8	LSM9	LSM10	2011		2012		2013		2015		2016		2017		2018		2019	
												session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
Show that a line lies/not in plane						x		x		x	x	x			x			x					x	x	x	x	
Show that a point belong/belong to plane									x			x		x	x		x							x			
Show that a point belongs/not to a line						x	x		x		x		x				x	x				x	x				
Show that A sym A' wrt plane						x											x										
Use dot product										x																	
Calculate the distance from a point to a line												x			x			x									
Calculate the distance from a point to a plane												x															
Calculate the dist from a A to the line of intersection			x																								
Prove E is orth. Proj. of point on a line																					x						
Prove E is orth. Proj. of point on a plane														x		x				x		x				x	
Find the orth proj. of a point on a plane	x																										x
Find the orth proj. of a point on a line			x				x					x							x						x		
Find eq. of a plane (passing A and perp to line)																x											
Find eq. of plane (contains line and a point)												x															
Find eq. of plane (2 points perp to a plane)									x										x				x	x			
Find eq. of plane (containing two lines)																		x				x					
Find eq. of a plane (3 pts)	x													x												x	x
Find eq. of plane (A and parallel to a plane)						x																					
Prove an expression is an eq. of plane (3 pts)															x	x				x							

Prove an expression is an eq. of plane (1pt and a line)							x	x																			
Prove an expression is an eq. of plane (1pt parallel 2line)													x														
Prove two planes perpendicular			x					x		x					x			x			x					x	
Prove two planes parallel									x																		
Prove two planes intersect															x												
Find the line of intersection of two planes										x									x				x				
Find the line of intersection of two planes given A			x																								
Prove a given line is inter. of two planes																					x						
Find equation of a line(A and perp. to plane...)									x		x						x								x		
Find equation of a line(2 pts)													x	x			x				x						
Find equation of line tangent to a circle													x											x			
Prove two lines intersect at a given point									x	x									x						x		
Prove line perp. to a plane														x													
Determine m so that line perp. To plane												x															
Prove line parallel. to a plane							x										x										
Prove two lines are perp.	x																						x				
Prove two lines are parallel											x					x						x					
Prove two lines are skew							x			x															x		
Prove point equidistant from two lines													x														
Find E intersection of line and plane										x								x									
Determine the bisector of an angle (given one point)						x																x					
Determine coordinates of pt E (E on line) AE= 5																					x						
Verify that ABCD is a tetrahedron	x													x													
Calculate volume of tetrahedron	x						x							x		x				x							x
Calculate the area of triangle ABC							x													x		x					
Calculate the area of quad ABCE								x						x													

Calculate the area of triangle ABC							x													x		x					
Calculate the area of quad ABCE								x						x													
Prove ABC is right (Given 3 pts)																x											
Prove ABC is equilateral (Given 3 pts)																											x
Prove triangle is semi equilateral using dot product										x																	
Deduce the dist.from A to a plane knowing volume													x														
Deduce circle is tangent to line								x													x						
Find coordinates of tangency pt. between (C) and (d)								x																			
Deduce/prove nature of a quad															x												
Know and use the properties of vector product														x													
Prove area(volume, distance)is indep. Of….									x		x																
Prove E sym of B wrt W								x																			
Find the coordinates of the midpt of a segment/verify						x																					
Find cosα of angle formed between line and plane									x										x								
Find coord. of pts of intersection of a line and a circle																						x				x	
Find coordinates of a point (Given area of triangle)																							x				
Find coordinates of a point (Given volume of											x													x			
Find coordinates of a point (Given isosceles triangle)																				x				x			
Find center of circle tangent to a plane											x																
Prove w center of circumscribed circle							x																				x

Table 6

Occurrences of Test Items on the Math Topic "Definitions and Representations" in the Model Tests and Official Tests for the LS Track of Grade 12

Objectives of the test items on Definitions & Representations	Model Tests										Official Tests of the LS Track at Grade 12															
											2011		2012		2013		2015		2016		2017		2018		2019	
	LSM1	LSM2	LSM3	LSM4	LSM5	LSM6	LSM7	LSM8	LSM9	LSM10	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
Construct graph C1 (Given Cm)	x																									
Study according to x the sign of f(x)	x							x		x		x														
discuss using variations number of solutions	x															x						x				
Study variation of function (ln and exp)					x	x																				
Study variation of logarithmic function (base e)			x	x				x				x		x		x		x							x	
Study variation of exp function		x					x		x	x	x		x		x	x	x		x		x	x				x
sketch graph of exp Function		x					x		x		x		x		x		x		x	x	x	x	x			x
sketch graph of logarithmic function (base e)			x	x				x				x		x		x		x						x		
sketch graph of function (ln and exp)		x			x	x																				
Prove f admits an inverse fct f^(-1)		x		x	x	x		x	x					x			x		x	x			x		x	x
determine the explicit expression of f^(-1)		x							x						x											
verify an expression to be the f^(-1)							x	x		x				x									x			
Find domain of definition of f^(-1)		x		x	x	x			x				x	x	x					x			x		x	x
Graph f^(-1)		x		x	x	x		x					x	x					x	x			x		x	
Study variation of f^(-1)				x																					x	
Know and use that f(x) and f^(-1) are sym									x		x				x											
Find limit of log function of base e			x					x				x		x		x		x						x	x	
Find limit of exp function							x		x	x	x	x	x		x		x		x	x	x	x	x			x
Find limit of fct (exp and lnx)					x	x																				
Deduce V and/or H asymptotes using limits					x	x			x				x	x	x	x	x	x	x		x			x		
Verify that a given line is an asymptote		x	x		x	x	x	x		x	x						x	x		x		x		x		x
Find coorinates of inter. Of graph and asymptote/ line						x									x				x				x		x	

Study relative positions of C and asymptote/tangent			x			x	x	x	x		x		x					x	x	x		x	x	x	x	x		x
give table of variation without deriving		x																				x						
Find f'(x) of log. Function of base e			x											x		x				x								
Find f'(x) of exponential Function										x	x		x		x							x		x				
Find f'(x) of fuction (lnx and exp)						x																						
Prove f'(x) positive (f is inc)from expression of f'(x)													x															
Verify an expression to be the f'(x)							x											x	x		x		x		x		x	x
Deduce variation of f'(x) from expression of f''(x)			x																	x				x		x		
Sketch an asymptote			x	x		x		x	x				x		x		x		x	x	x	x	x	x	x	x		x
Plot points																		x										
deduce the sign of a function (f^(-1), f'(x)from table				x																								
Calculate f(0)				x					x		x				x				x	x	x	x	x	x	x			x
Calculate f'(0)				x																								
determine center of symmetry (by proving odd)										x																		
Prove a point is a center of sym.													x															
prove f(x)can be written as = …											x		x	x			x											
discuss the number of roots f(x)=m																										x		
Solve graphically inequality							x																					
Verify the domain of definition of f is..																							X					

Table 7

Occurrences of Test Items on the Math Topic "Continuity and Differentiation" in the Model Tests and Official Tests for the LS Track of Grade 12

Objectives of the test items on Continuity and differentation	Model Tests											Official Tests of the LS Track at Grade 12															
												2011		2012		2013		2015		2016		2017		2018		2019	
	LSM1	LSM2	LSM3	LSM4		LSM5	LSM6	LSM7	LSM8	LSM9	LSM10	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
Find m so that f is st monotone	x																										
Find m so that C has an extremum	x																										
Find f''(x) of log function			x																								
Find limit using L'Hopital's rule													x														
Justify f is increasing using a given table of f'(x)								x	x												x						
Study sign of f using a table of variation																		x	x								
Prove C has a point of inflection using graph of f'(x)																									x		
Prove C has a point of inflection using table of f'(x)																					x						
Prove C has a point of inflection by calculating f''(x)																						x					
Find the coordinates of C at which tangent…	x					x		x	x								x		x		x	x			x		
Calculate a and b such that C is tangent to a line ..									x																		
determine equation of tangent at a point				x			x			x						x					x			x		x	x
Determine f such that © is tangent at O to a line								x																			
Study according to x the concavity								x																			
Verify that a line is the tangent at a point										x																	
Find the point of inflection/ show											x										x	x			x		
Calculate the derivative of f^(-1)									x			x															
deduce/find slope(eq) of tangent using f^(-1)												x															
Prove f(x)=0 has a (unique) root in [a,b]											x		x	x													
Find h'(x) (where h(x) =xf(x)															x												

Table 8

Occurrences of Test Items on the Math Topic "Integration" in the Model Tests and Official Tests for the LS Track of Grade 12

Objectives of the test items on Integration	Model Tests											Official Tests of the LS Track at Grade 12															
												2011		2012		2013		2015		2016		2017		2018		2019	
	LSM1	LSM2	LSM3	LSM4		LSM5	LSM6	LSM7	LSM8	LSM9	LSM10	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
Calculate area under a curve	x			x									x	x	x	x						x	x	x			
Calculate area between fct and asymptote												x													x	x	
Calculate the area using graph of primitive										x	x																
deduce the area between fct and asym. (from integral)								x									x		x								
Calculate a definite integral		x											x						x							x	
Calculate an indefinite integral													x		x		x		x			x					
Know and use the properties of integrals													x			x								x			
Know and use the fundamental theorem of integration													x														
use by parts to find a definite integral								x			x														x	x	
Calculate a, b, c so that F is an antiderivative of f														x													

Table 9

Occurrences of Test Items on the Math Topic "Differential Equations" in the Model Tests and Official Tests for the LS Track of Grade 12

Objectives of the test items on Differential equations	Model Tests											Official Tests of the LS Track at Grade 12															
												2011		2012		2013		2015		2016		2017		2018		2019	
	LSM1	LSM2	LSM3	LSM4		LSM5	LSM6	LSM7	LSM8	LSM9	LSM10	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
Solve a linear second order diff equation								x																			
Verify a particular solution of 2nd order diff. eq								x																			
Write a diff equa satisfied by z								x																			x
deduce general sol. of (E) form (E')								x																			x

Table 10

Occurrences of Test Items on the Math Topic "Statistics" in the Model Tests and Official Tests for the LS Track of Grade 12

Objectives of the test items on Statistics	Model Tests											Official Tests of the LS Track at Grade 12															
												2011		2012		2013		2015		2016		2017		2018		2019	
	LSM1	LSM2	LSM3	LSM4		LSM5	LSM6	LSM7	LSM8	LSM9	LSM10	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
Organize the data in classes of amplitude 10				x																							
Calculate the median				x																							
Interpret the median				x																							
Calculate the mean given classes and freq.				x																							
Calculate the st. d. given classes and freq.				x																							

Table 11

Occurrences of Test Items on the Math Topic “Probability” in the Model Tests and Official Tests for the LS Track of Grade 12

Objectives of the test items on Probability	Model Tests											Official Tests of the LS Track at Grade 12															
												2011		2012		2013		2015		2016		2017		2018		2019	
	LSM1	LSM2	LSM3	LSM4		LSM5	LSM6	LSM7	LSM8	LSM9	LSM10	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2	session 1	session 2
P(event), 1 is chosen at a time (and)																			x	x							
P(event), 1 is chosen at a time (or)									x																		
P(A or B) (1-p(A)) at least when ind.		x									x	x												x			x
P(event), more than 1 at a time (or)													x		x						x	x	x		x		
P(event), more than 1 at a time (and)													x		x		x		x	x	x	x	x				
P(A/B)											x	x	x														
P(A/B) using formula						x	x	x		x		x				x	x	x	x	x	x	x	x	x	x	x	x
Find P(A∩B) using formula when						x				x		x			x	x		x		x	x	x			x	x	x
Find P(A∩B) using formula when		x												x													
P(A∩B bar)												x															
P(A∩B bar) using formula							x	x							x	x					x	x	x		x	x	
Total probability						x	x	x	x	x		x			x			x		x							
Find values of X							x	x	x				x			x								x	x		
Determine the probability distribution of						x	x	x	x	x	x	x	x	x		x	x		x					x	x		
Find /prove P(X >, <, =)		x	x			x		x		x	x			x			x							x	x	x	
Find E(X)									x		x					x											
interpret E(X) /Use E(X) to estimate…		x							x		x																
identify binomial dist		x	x								x																
find E(X) when X binomial		x	x																								
p(event) basic properties of probability		x	x			x	x	x	x	x	x	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x

APPENDIX S

Quantitative Analysis of the LS Model Tests and Official Tests

Table 1

Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests and the Official Tests (2011-2019) for the LS Track of Grade 12 – Extracted from Table AllModLS and Table OffExLS.

The Topics of the Math Curriculum for the LS Track of Grade 12	Sum of Model Tests				Sum of Official Tests			
	K %	A %	R%	Total	K %	A %	R %	Total
1.2. Literal and numerical calculations	1.08	0	0.62	1.7	2.14	0	0	2.14
1.4. Numbers	7.92	4.76	2.98	15.66	6.37	6.33	3.58	16.28
2.1. Classical study	9.44	5.04	3.19	17.67	8.41	7.33	3.42	19.16
3.1. Definitions & Representations	13.79	23.2	4.37	41.36	13.19	18.86	3.43	35.48
3.2. Continuity and differentiation	2.37	2.37	1.13	5.87	1.55	1.91	1.82	5.28
3.3. Integration	0.57	0.72	0.57	1.86	1.55	1.96	0.88	4.39
3.4. Differential equations	0.62	0.31	0	0.93	0.36	0.18	0.00	0.54
5.1. Statistics	0.77	0.15	0	0.92	0	0	0	0
5.2. Probability	7.1	3.94	3.01	14.05	6.69	6.77	3.28	16.74
Total	43.66	40.49	15.87	≈100	40.26	43.34	16.41	≈100

K = Knowing
A = Applying
R = Reasoning
The sum of Totals is approximately equal to 100 because the percentages are rounded.

Table 2

Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests issued in the year 2000 and the Official Tests 2011-2013 and 2015-2016 of the LS Track of Grade 12 – Extracted from Table ModLS, Table OffExLS11-13 and Table OffExLS15-16.

The Topics of the Math Curriculum of the LS Track of Grade 12	Sum of Model Tests LSM1, LSM2, LSM3, and LSM4				Sum of 2011-2013 Official Tests				Sum of 2015-2016 Official Tests			
	K %	A %	R%	Total	K %	A %	R %	Total	K %	A %	R %	Total
1.2. Literal and numerical calculations	1.95	0	0	1.95	1.196	0	0	1.196	2.57	0	0	2.57
1.4. Numbers	2.38	2.71	6.28	11.37	3.795	4.785	4.29	12.87	11.1	7.14	3.37	21.61
2.1. Classical study	5.84	6.17	2.60	14.61	9.12	8.5	4.17	21.79	6.96	6.79	3.03	16.78
a. Definitions & Representations	13.64	27.9	1.3	42.86	12.58	19.27	3.67	35.52	12.39	22.38	4	38.77
a. Continuity and differentiation	4.11	3.46	2.81	10.38	2.13	1.88	2.13	6.14	1.94	1.6	0.57	4.11
3.3. Integration	1.52	1.52	0.87	3.91	2.21	2.7	1.47	6.38	0.69	0.86	0.17	1.72
3.4. Differential equations	0	0	0	0	0	0	0	0	0	0	0	0
5.1. Statistics	3.25	0.65	0	3.9	0	0	0	0	0	0	0	0
5.2. Probability	6.49	2.6	1.95	11.04	7.9	4.64	3.589	16.129	5.48	7.02	2.06	14.56
Total	39.18	45.03	15.81	≈100	38.931	41.775	19.319	≈100	41.13	45.79	13.2	≈100

K = Knowing
A = Applying
R = Reasoning
The sum of Totals is approximately equal to 100 because the percentages are rounded.

Table 3

Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests issued in 2017 and the Official Tests 2017-2018 of the LS Track of Grade 12 – Extracted from Table ModLS5-8 and Table OffExLS17-18.

The Topics of the Math Curriculum of the LS Track of Grade 12	Sum of Model Tests LSM5, LSM6, LSM7, and LSM8				Sum of 2017-2018 Official Tests			
	K %	A %	R%	Total	K %	A %	R %	Total
1.2. Literal and numerical calculations	0.62	0	0	0.62	3.65	0	0	3.65
1.4. Numbers	9.06	5.95	1.76	16.77	5.6	6.33	2.68	14.61
2.1. Classical study	10.61	4.09	3.31	18.01	10.3	5.38	3.35	19.03
3.1. Definitions & Representations	13.56	23.5	7.04	44.1	14.23	16.79	3.3	34.32
3.2. Continuity and differentiation	2.17	2.17	0	4.34	0.73	2.56	3.3	6.59
3.3. Integration	0	0.31	0.31	0.62	1.34	2.07	0.97	4.38
3.4. Differential equations	1.24	0.62	0	1.86	0	0	0	0
5.1. Statistics	0	0	0	0	0	0	0	0
5.2. Probability	6.21	4.19	3.26	13.66	4.38	8.76	4.38	17.52
Total	43.47	40.83	15.68	≈100	40.23	41.89	17.98	≈100

K = Knowing
A = Applying
R = Reasoning
The sum of Totals is approximately equal to 100 because the percentages are rounded.

Table 4

Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests issued in 2019 and the Official Tests of the year 2019 of the LS Track of Grade 12 – Extracted from Table ModLS9-10 and Table OffExLS17-18.

The Topics of the Math Curriculum of the LS Track of Grade 12	Sum of Model Tests LSM9 and LSM10				Sum of 2019 Official Exams			
	K %	A %	R%	Total	K %	A %	R %	Total
1.2. Literal and numerical calculations	1.19	0	0	1.19	1.06	0	0	1.06
1.4. Numbers	11.01	4.46	2.38	17.85	5.52	9.04	3.76	18.32
2.1. Classical study	10.71	5.95	3.57	20.23	5.75	8.92	2.23	16.9
3.1. Definitions & Representations	14.68	18.85	2.18	35.71	14.55	14.55	1.88	30.98
3.2. Continuity and differentiation	1.19	1.79	1.79	4.77	0.7	1.41	0.7	2.81
3.3. Integration	0.79	0.79	0.79	2.37	1.88	1.88	0.47	4.23
3.4. Differential equations	0	0	0	0	2.82	1.408	0	4.228
5.1. Statistics	0	0	0	0	0	0	0	0
5.2. Probability	9.52	4.76	3.57	17.85	10.21	8.45	2.82	21.48
Total	49.09	36.6	14.28	≈100	42.49	45.658	11.86	≈100

K = Knowing
A = Applying
R = Reasoning
The sum of Totals is approximately equal to 100 because the percentages are rounded.

Table 5

Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests issued with the curriculum documents and the and the Session-1 and Session-2 Official Tests of the years 2011-2016 of the LS Track of Grade 12 – Extracted from Table ModLS, Table OffExLS11, and OffExLS21

The Topics of the Math Curriculum of the LS Track of Grade 12	Sum of Model Tests				Sum of Session-1 Official Tests of the years 2011-2016				Sum of Session-2 Official Tests of the years 2011-2016			
	K %	A %	R%	Total	K %	A %	R %	Total	K %	A %	R %	Total
1.2. Literal and numerical calculations	1.95	0	0	1.95	2.01	0	0	2.01	1.55	0	0	1.55
1.4. Numbers	2.38	2.71	6.28	11.37	8.14	4.9	4.9	17.94	5.62	6.6	2.95	15.17
2.1. Classical study	5.84	6.17	2.60	14.61	8.91	8.53	2.79	20.23	7.49	7.07	4.54	19.1
3.1. Definitions & Representations	13.64	27.92	1.3	42.86	11.77	21.18	2.35	35.3	13.2	19.94	5.2	38.34
3.2. Continuity and differentiation	4.11	3.46	2.81	10.38	1.86	1.57	1.28	4.71	2.22	1.94	1.66	5.82
3.3. Integration	1.52	1.52	0.87	3.91	0.882	1.47	0.59	2.942	2.23	2.36	1.24	5.83
3.4. Differential equations	0	0	0	0	0	0	0	0	0	0	0	0
5.1. Statistics	3.25	0.65	0	3.9	0	0	0	0	0	0	0	0
5.2. Probability	6.49	2.6	1.95	11.04	8.36	4.78	3.68	16.82	5.48	6.46	2.25	14.19
Total	39.18	45.03	15.81	≈100	41.932	42.43	15.59	≈100	37.79	44.37	17.84	≈100

K = Knowing
A = Applying
R = Reasoning
The sum of Totals is approximately equal to 100 because the percentages are rounded.

Table 6

Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests issued in the year 2017 and the and the Session-1 and Session-2 Official Tests of the years 2017-2018 of the LS Track of Grade 12 – Extracted from Table ModLS, Table OffExLS21, and OffExLS22

The Topics of the Math Curriculum of the LS Track at Grade 12	Sum of Model Tests				Sum of Session-1 Official Tests of the years 2017-2018				Sum of Session-2 Official Tests of the years 2017-2018			
	K %	A %	R%	Total	K %	A %	R %	Total	K %	A %	R %	Total
1.2. Literal and numerical calculations	0.62	0	0	0.62	3.04	0	0	3.04	4.37	0	0	4.37
1.4. Numbers	9.06	5.95	1.76	16.77	9.01	6.31	2.25	17.57	1.59	6.35	3.18	11.12
2.1. Classical study	10.61	4.09	3.31	18.01	11.15	4.73	3.04	18.92	9.26	6.09	3.7	19.05
3.1. Definitions & Representations	13.56	23.5	7.04	44.1	18.24	17.57	2.03	37.84	9.52	15.87	4.76	30.15
3.2. Continuity and differentiation	2.17	2.17	0	4.34	0.68	2.03	1.35	4.06	0.79	3.18	5.56	9.53
3.3. Integration	0	0.31	0.31	0.62	1.35	2.03	0.68	4.06	1.32	2.12	1.32	4.76
3.4. Differential equations	1.24	0.62	0	1.86	0	0	0	0	0	0	0	0
5.1. Statistics	0	0	0	0	0	0	0	0	0	0	0	0
5.2. Probability	6.21	4.19	3.26	13.66	2.7	7.1	4.73	14.53	6.35	10.71	3.97	21.03
Total	43.47	40.83	15.68	≈100	46.17	39.77	14.08	≈100	33.2	44.32	22.49	≈100

K = Knowing
A = Applying
R = Reasoning
The sum of Totals is approximately equal to 100 because the percentages are rounded.

Table 7

Distribution of Percentages of Test Items by Math Topics and Cognitive Domains in the Model Tests of the year 2017 and the and the Session-1 and Session-2 Official Tests of the year 2019 for the LS Track of Grade 12– Extracted from Table ModLS9-10, Table LS191, and Table LS192

The Topics of the Math Curriculum of the LS Track of Grade 12	Sum of Model Tests				Session-1 Official Test of the year 2019				Session-2 Official Test of the year 2019			
	K %	A %	R%	Total	K %	A %	R %	Total	K %	A %	R %	Total
1.2. Literal and numerical calculations	1.19	0	0	1.19	2.27	0	0	2.27	0	0	0	0
1.4. Numbers	11.01	4.46	2.38	17.85	8.59	5.56	7.07	21.22	2.85	12.06	0.88	15.79
2.1. Classical study	10.71	5.95	3.57	20.23	6.06	6.82	2.27	15.15	5.48	10.75	2.19	18.42
3.1. Definitions & Representations	14.68	18.85	2.18	35.71	15.15	15.15	0	30.3	14.04	14.04	3.51	31.59
3.2. Continuity and Differentiation	1.19	1.79	1.79	4.77	1.52	1.51	0	3.03	0	1.32	1.32	2.64
3.3. Integration	0.79	0.79	0.79	2.37	4.04	4.04	1.01	9.09	0	0	0	0
3.4. Differential equations	0	0	0	0	0	0	0	0	5.26	2.63	0	7.89
5.1. Statistics	0	0	0	0	0	0	0	0	0	0	0	0
5.2. Probability	9.52	4.76	3.57	17.85	5.3	9.09	5.55	19.94	14.47	7.93	1.31	23.71
Total	49.09	36.6	14.28	≈100	42.93	42.17	15.9	≈100	42.1	48.73	9.21	≈100

K = Knowing
A = Applying
R = Reasoning
The sum of Totals is approximately equal to 100 because the percentages are rounded.