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Avoiding Disruption: Optimal Supplier Selection in Two-Tier Supply Chain Production Model

By

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Dedication

I would like to dedicate this thesis especially to my mother who taught me that it is never too late to learn and who was and will always be my backbone. To my beloved father, may his soul rest in peace, who would be proud to see me graduating. My husband who never stopped encouraging me throughout the several years I spent learning and finally to my lovely children who suffered a lot of my absence.

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Avoiding Disruption: Optimal Supplier Selection in Two-Tier Supply Chain Production Model

Leyla Takkouch

ABSTRACT

During the COVID-19 pandemic, supply chain managers were faced with unprecedented challenges, risks, and disruptions of supplies. Firms were redesigning their strategies to ensure the continuity, sustainability and resilience of their operations. This necessitated the development of new approaches to supply chain management by considering factors that may hinder or disrupt their operations. The purpose of this thesis is to consider a two-tier supply chain with a retailer that acquires a certain product from two available suppliers, a primary unreliable one and a secondary reliable but more expensive supplier. Three mathematical models are formulated. The first model extends the Economic Order Quantity (EOQ) model to the case of dual suppliers. The second model allows for shortages while taking into account the unreliable nature of the primary supplier. The last model is an EOQ model with dual suppliers and price discount. In each case, the optimal solution is derived and numerical examples are presented to illustrate the calculations. Sensitivity analyses are provided to assess the robustness of the models.

Keywords: Inventory models, Economic Order Quantity, Dual supplier, Price discount, shortages.

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Chapter One

Introduction

This chapter encompasses an introduction consisting of three parts. The background of the topic is first presented. Then, the objectives of the thesis are highlighted. Finally, the organization of the thesis is given.

1.1 Background

In the age of COVID-19, supply chain managers are faced with the problem of disruptions of supplies in the form of goods, raw materials, and components required for the operation (De & Mahata, 2020; Saha et al., 2020). With such unprecedented global supply chain challenges and risks, firms and manufacturers are redesigning their strategies to ensure the continuity, sustainability, and resilience of their operations. This necessitates the development of new approaches to supply chain management by considering factors that may hinder or disrupt their operations. In particular, supply chain mangers must revisit the modeling approach to their operations to optimize the selection of suppliers of goods, raw materials and/or components. The stochastic nature of quality of acquired items, lead-time, and risk of disruption of delivery, as well as the environmental impact of transportation require the consideration and evaluation of multiple suppliers of raw materials/components (El-Khalil & El-Kassar, 2018; Kaur et al., 2020; Yassine, 2020; Yassine & Singh, 2020).

Andriolo et al. (2014) presented a review of articles about the classical lot size model, one of the earliest and most central models in the field of production planning and inventory control. They discussed the evolution of the classical model that was first introduced by Harris in 1913. They highlighted why different researchers have examined numerous models that extend and generalize the classical model.

Sahaa et al. (2020) studied an inventory model from the perspective of risks related to fractional interruption taking into account a combined facility location. Their findings indicate that consequences disruption in the chain include customers refusing backorder agreements and increase in overall cost among all members of the supply chain. However, they indicate that cost could significantly be reduced by producing and keeping safety stock.

A research conducted by Sahaa et al. (2020) studied a joint facility location and inventory model from the viewpoint of partial disruption. They studied the effect of products that can be substituted and the decisions of customers accepting or rejecting backorders by considering multinomial logit model and a nonlinear integer model. In addition, they considered a particle swarm optimization model. Those models increased the efficiency of the study. The research showed that the cost of supply chain is an essential factor in chain disruption with cost saving from producing a substitute for raw materials.

In addition to costs and their effect on the supply chain, environmental care and sustainability is a major field of study. Manufacturing firms need to work on sustainable operations that aim to reduce the negative impact of manufacturing on the environment and enhancing the firm's image and performance. A study conducted by Yassine (2018) examined the relationship between the used raw materials (perfect and imperfect) and transportation emission tax. This study resulted in a model that calculated the production cost. Yassine and Singh (2020) investigated a production model from a supply chain perspective. The effects of HRM performance and environmental features were incorporated. The research showed the best practices for maximizing the supply chain profit by carefully selecting suppliers. However, they ignored the probability of disruption in the reception goods and raw material items/components received from the various suppliers.

1.2 Statement of the Problem

The objective of this thesis is to formulate inventory models for a two-tier supply chain consisting of acquiring a certain product from two available suppliers, a primary unreliable one and a secondary reliable but more expensive supplier. Three mathematical models are formulated. The first model extends the Economic Order Quantity (EOQ) model to the case of dual suppliers. The second model allows for shortages while taking into account the unreliable nature of the primary supplier. The last model is an EOQ model with dual suppliers and price discount.

The formulated mathematical models enable the determination of the policy from each of the two suppliers that safeguard against possible disruption in deliveries. In each case, the optimal solution is derived and numerical examples are presented to illustrate the calculations. Sensitivity analyses are provided to assess the robustness of the models.

1.3 The Organization Of The Thesis

The rest of the study is structured in the following manner. A literature review is provided in chapter two and the classical models are explained. In chapter three, an EOQ model with dual supplier is developed and numerical examples are provided. In chapter four, the model presented in chapter three is modified to allow for shortages. The mathematical model is developed and an expression for the optimal solution is derived. In addition, a numerical example is presented. Chapter five has a purpose to modify the models presented in chapters three and four to allow for price discount offered by the unreliable supplier. Procedures are proposed for finding the optimal solution in each case. In addition, numerical examples illustrate the procedures presented. Chapter six comprises of the conclusion, discussion, limitations and implications.

1.4 Research Questions

This study discusses the following research questions that will be addressed throughout the thesis:

RQ1: What procurement strategies can be adopted to overcome the disruption challenge especially in the aftermath of Covid 19?

RQ2: How can such strategies be mathematically formulated to describe the real life situation and derive the optimal ordering policies?

Chapter Two

Literature Review and Classical Models

This chapter includes a literature review. The definitions of the classical models used in the paper are also explained and demonstrated with some examples.

2.1 Background

Supply chain disruptions are not new in the business field: they have been present for a long time since the supply chain was first applied in business. Therefore, the question is why is there an interest in this topic nowadays? The answer is there are many reasons. The most important reason is the Covid-19 problem, which caused an international scarcity of resources in the supply chain; in addition to other disruptions due to climate change problems such as global warming and other.

Second, many firms have concluded that just-in time technique is excellent under normal global economic situations. In this sense, many researches attracted their attention to supply chain disruptions. Any mistake in the supply chain can cause drastic economic crisis. For example, the Covid-19 lockdown has caused a lot of devastating effect on the global economy. Many firms have shut down especially due to the lock-down specifically the hospitality industry sector.

Many previous examples can shed light on this fact. For example, in 1988 strikes at General Motors yielded to problems in the supply chain, (Simison, 1998). In another study, Hendricks and Singhal (2003, 2005a, 2005b) have studied supply chain vulnerability and problems resulting from this issue. Lately in 1990s, Wall Street Journal and the Dow Jones News Service have nearly collapsed. Moreover the same scenario is being witnessed today where the stock prices of oil due to Covid-19 restrictions and lock down has decreased the price of stocks related to oil dramatically. These companies have shown steep fall in the oil stocks. In addition many companies' growth rate has declined and hence the supply chain is affected in this sense.

These articles will emphasize Operations Research/ Management Science (OR/MS) methods and other models to tackle supply disruptions. It will also evaluate these disruptions and implement various strategies to minimize them.

To begin with, we need to identify the difference between yield uncertainty and disruptions. The former is defined, as the stock handed to any supplier is a random variable that depends on the quantity. However, disruption in this context means the variable is obvious and can be estimated.

Researchers dwell on using specific terminologies to make things clear and more objective when talking about supply disruptions. They used also supply risk management while discussing unreachable facilities and scarcity of resources and production problems, (Tang2006a).To calculate the probability of supply disruptions, the researchers took into account the use of a network reliability that is able to function even during disruptions.(Shooman, 2002). Moreover, there are many models of disruptions discussed in different contexts. All models follow a certain mode. When the supply chain is normal, the models use the "up" state and when there are disruptions the models use “down” state irrespective of the time interval. A model has been identified as the two-state Markov process. Another model mentioned is the Bernoulli where the disruption length is specified. The Erlang-k model uses unspecified distribution. In addition, other researchers use the Markovian model and vary its disruption probabilities (Snyder & Tomlin, 2008). Song and Zipkin (1996) modified the Markov model to adjust disruption in using random lead-time supply uncertainty.

In general, supply chain variables can be categorized into two parts: 1, the strategy and 2, the evaluation.

Evaluation of disruptions is difficult to attain by calculation, so the simulation model will be the suitable tool for this purpose.

Schmitt and Singh (2012) identified that feedback is a powerful tool, which is more important than the disruption itself. According to Schmitt and Singh (2009), the quality of the service chain is as important as the supply itself. Overall risk level changes. In this aspect, Snyder and Shen (2006) claimed that Demand Uncertainty (DU) and Supply Uncertainty (SU) during disruptions follow indirect strategies .They even claimed that it is better to adhere to minimize disruptions and it is called the “risk-diversification effect.” However, in DU it is more advisable to follow a centralized design where pooling of inventory will have protection against disruption (Snyder and Shen 2006).

Snyder and Tomlin (2008) talk about how inventory systems play a major role in time of disruptions. They consider a system with reliable and another with unreliable resource. They have acknowledged that advanced information about the disruption affected the firm" threat level". They proclaimed that a threat advisory system will enhance earnings and inventory levels.

In this perspective, forecasting information about disruptions must be implemented in sourcing and managerial decisions, (Tomlin 2009b).

For this purpose, many businesses have studied inventory systems subject to disruptions. A paper discussed the case on single-stage continuous-review models and single-stage periodic-review models. In addition, models focused on the time and the quantity in ordering the inventory. One view claims that to order many quantities is subject to problems with disruptions than for those without (Greenbelt et al. (1992b); while Snyder (2014) suggests that extra inventory provides a buffer on inventory systems with a control policy. The business firms revealed that the best policy of ordering is the basic Economic Order Quantity (EOQ) model. The best was to achieve no lost inventory, which is identified by Parlar, and Berkin by (EOQD) where the demand is deterministic.

Other researchers claim that the best model is the Zero Inventory Ordering (ZIO) where the order of supply is placed when the inventory level is zero.

In this perspective, Kalpakam and Sapna (1997) and others mentioned that ignoring disruptions will affect the firms and cause opportunity cost. If the stock is zero and there is high demand for the supply, then the firm will lose their clients. For example, the Brexit in Britain has cost disruptions in supply where companies and trucks stayed for many days locked. If companies did not have enough supplies they would lose a lot in their operating system and the stock prices will soar high. In this view, Moinzadeh and Aggarwal(1997), Liu and Parlar (1997) , Mohebbi (2003,2004) ,and Mohebbiand Hao (2006, 2008) came to a conclusion that ignoring disruptions will yield to high costs and expenses on the company side.

Moreover, we can see other inventory mitigating strategies through sourcing flexibility where companies order the stock from routine suppliers in addition to referring to other new resources to minimize disruption.

Federgruen and Yang (2009) considered a model in which the unit costs is dependent on the supplier and fixed costs were assumed to be zero. The results were related to the expected total yield attributed to all suppliers and this is referred to as risk diversification. They claimed that using dual supply chain is very useful when looking for the expected effective supply; however, using risk concentration, which means using a reliable supply source, works well in the assembly system. Other contingent resources in times of disruption where some firms use financial incentive is an option, (Tang, 2006b). This approach makes a retailer who has few suppliers increase others to enter the market to order minimum quantities and to share market information with them.

This policy has positive effects in diversification of supply and in increasing of competition among suppliers. Tang (2006b) suggests another mitigation strategies used by buyers to improve the supply chain when using promotional strategies to deviate the demand of certain products. Moreover, it is vital to include inventory management into facility location decisions. It is crucial to locate facilities to serve customers efficiently when disruptions occur.

According to a study conducted by Yano and Lee (1995) have concluded that yield effectiveness is more reliable when dealing with single supplier system since the quality of the product is more certain. In this aspect, Gerchak and Parlar (1990) confirmed this result when they investigated dual sourcing yield uncertainty using the EOQ (Economic order Quantity) as a criterion.

On the other side, an opposite view is highly preferred for dual sourcing concerning supply disruption. In this aspect, many firms encourage secondary and multiple suppliers to enhance customer service and expected profits .Moreover, Anupindi and Akella (1993) address that allocating resources from different suppliers will impact the inventory policy of the buyer.

In this literature, Gurnani et al. (2000) simultaneously determine ordering and production decisions for a two different suppliers and concluded that businesses need to involve risk managerial decisions in the cost of the resources.

Supply Chain:

Literature Review being afraid of supply shortages to be at risk, business enterprises may choose to allocate its raw materials from different sources. This concern opt many companies to reduce their marginal costs incurred to their budget and to their business partners (Babish, 2006).In this sense, in order to minimize any risk whether from suppliers or human capital, companies should have various channels as well as different time ordering deliveries from their suppliers. The orders could range from the slowest supply chain to the fastest one. This action would reduce risk deadlines to customers as well as to the stock level of inventory shortages.

In this respect, (Yassine & Singh, 2020) have discussed in their paper the consequences of environmental aspects on the supply chain costs. They have investigated the extra cost on production related to carbon emission, quality of raw material, and total quality management. They have identified these factors to be very well considered in the cost of the supply chain (Yassine, 2018).

Not only would the shortages of supply result in negative consequences on the suppliers, but it will extend to the shareholder’s value and to the equity of the business (Wagner and Bode, 2011). These results have been announced due to a study conducted in Germany. The study revealed that more than 760 firms’ strategic decision-making was based on positive correlations on previous supply and demand supply chain business. This frequent risk factor directed the operational strategy of the supply chain source.

Another perspective to all supply chain factors mentioned above, a major factor should be taken into account which is sustainability (Giannakis & Papadopoulos, 2016).

This study acknowledges that sustainability should be involved into the supply chain costs.

Nowadays many business take into consideration the integrated part of sustainability in their supply chain process to reduce risk fees in their costs.

Moreover, risk management is a vital domain to be considered in the supply chain in addition to other fields of study such as economics, finance, insurance, and healthcare.

Risk management led to various risk models and methods used for descriptive supply chain to reduce any risk costs (Heckman ad Nickel, 2014).

In this aspect, Lawrence et al. (2016) review the Operations Research/Management Science (OR/MS) literature on six factors that are vulnerable to supply chain disruptions and problems. The factors involved are: strategic decisions; sourcing decisions; contracts and incentives; inventory; and facility location could increase the frequency of severe disruption if not clear strategies are considered for each factor.

In planning alleviative strategies related to probability tables (Chopra, S., Reinhardt, G., Mohan, U., 2007) mentioned that separating each factor of disruption with its respective risk probability can limit the impact of the supply default. Moreover, the researchers suggested to order stocks from a reliable source even if it is more expensive than to have any material from an unpredicted and cheaper supplier.

According to (Giri, 2011) sourcing from different suppliers has many advantages in supply management. Many companies are motivated to reduce supply risks, because single sourcing exposes companies to have interruptions .In the presence of demand uncertainty, multiple sourcing can improve customer service and reduce safety stocks. Nowadays, with an increasing importance of global supply chains, sourcing from a domestic (or onshore) supplier could guarantee the availability of products and improve responsiveness to customers. This direction may cause to have cheap resources but could be unreliable. In this sense, looking for overseas alternative is considered an alternative option for secure supplies if the resource is a reliable one.

De & Mahata (2020) case study revealed a model for default stock. They discussed that default risk is one primary factor in the supply chain default. The default risk of the customer and the fuzziness could be reduced due the stock and credit period in their case study for having better management decision analyses.

(Saha, Apurba & Paul, Ananna & Azeem, Abdullahil & Paul, Sanjoy,2020) discussed partial-disruption risk in relation to an inventory model considering customers' preference and the role of other alternative products and the role of back off offers .

Kebing Chen and Tiaojun Xiao (2015) developed a supply chain game model which would result in a dual supply chain with respect to production and stock inventory. The researchers find out that the products outsourced and the disruptive risk are inversely proportional. Moreover, various sourcing strategies can lead to a higher level of stock due to various vendors at hand. This supply channel perspective would result into a dominant manufacturer and a dominant retailer that would consequently reduce disruption risks.

2.2 The EOQ Model

The three letters EOQ refer to the Economic Order Quantity. (EOQ) is the perfect order quantity a company should purchase to minimize inventory costs such as holding costs, shortage costs, and order costs. The goal of the EOQ formula is to identify the optimal number of product units to order.

The objective of the EOQ model is:

To calculate Q in order for the total inventory cost to be minimized.

The optimal order quantity is denoted by $Q*$.

2.3 EOQ Model With Price Discount:

Suppose that the supplier is offering a price discount. The discount schedule has several price levels each of which is determined by a range for the quantity purchased.

The parameters of the EOQ model are:

D = Demand Rate (units/unit time)

Ci = Level i Purchasing Cost per Unit ($/unit)

C0 = Ordering cost ($)

H = Annual Holding Cost Rate (%)

C_h=Holding cost ($/unit/year) = H.Ci

Procedure 2.1

To find the optimal order quantity:

Step 1. For each level *i*, find the optimal order quantity

$$Q_i^* = \sqrt{\frac{2C_0\, D}{C_h}} = \sqrt{\frac{2C_0\, D}{HC_i}} \tag{2.1}$$

Step 2. Adjust Q_i^* according to:

If Q_i^* is within the quantity range for level *i*, keep Q_i^*.

If Q_i^* is more than the largest quantity of its range, ignore level *i*.

If Q_i^* is less than the smallest quantity of its range, set Q_i^* equal to the smallest quantity of its range.

Step 3. For each level *i*, use the adjusted values of Q_i^* to calculate

$$TCU\,(Q_i^*) = C_i D + \frac{C_0\, D}{Q_i^*} + C_h \frac{Q_i^*}{2} \tag{2.2}$$

Step 4. Select the Q_i^* that results in a minimum value of $TCU(Q_i^*)$.

The demand rate for a certain item is 10000 units per year. It costs $1,000 to place an order and $100 to purchase a single item. The holding cost rate is 20%.

Find the optimal order quantity if the supplier is offering a price discount according to the following schedule:

Level	**Quantity**	**Price per Unit**
1	1 to 499	$100
2	500 to 1999	90
3	2000 to 9999	85
4	10000 or more	80

The parameters of the problem are:

D = 10000 units/year

C_1 = $100 /unit, C2 = $90 /unit,

C_3 = $85 /unit, C4 = $80 /unit

C_0 = $1,000

H = 20%

C_h = 20% ($100) = $20/unit/year (level 1)

C_h = 20% ($90) = $18/unit/year (level 2)

C_h = 20% ($85) = $17/unit/year (level 3)

C_h = 20% ($80) = $16/unit/year (level 4)

Step 1. For each level I find the optimal order quantity

Level 1: $Q_1^* = \sqrt{\frac{2C_0\, D}{C_h}} = \sqrt{\frac{2(1000)\,(10000)}{20}}$ = 1000 units

Level 2: $Q_2^* = \sqrt{\frac{2C_0\, D}{C_h}} = \sqrt{\frac{2(1000)\,(10000)}{18}}$ = 1054 units

Level 3: $Q_3^* = \sqrt{\frac{2C_0\, D}{C_h}} = \sqrt{\frac{2(1000)\,(10000)}{17}}$ = 1085 units

Level 4: $Q_4^* = \sqrt{\frac{2C_0\, D}{C_h}} = \sqrt{\frac{2(1000)\,(10000)}{16}}$ = 1118 units

Step 2. Adjusting Q_i^*:

Level 1: $Q_1^* = 1000$ units is more than the largest quantity in its range (1- 499), **ignore this** level.

Level 2: $Q_2^* = 1{,}054$ units is within its range (500-1,999), **keep** $Q_2^* = 1{,}054$ units.

Level 3: $Q_3^* = 1{,}085$ units is less the smallest quantity in its range (2,000- 9,999), **set** $Q_3^* =$ 2,000 units.

Level 4: $Q_4^* = 1{,}118$ units is less the smallest quantity in its range (10,000 or more), **set** $Q_4^* =$ 10,000 units.

Step 3. Calculate *TCU*

The Total Inventory Cost is:

$$TCU:(Q_i^*) = C_i D + \frac{C_0\, D}{Q_i^*} + C_h \frac{Q_i^*}{2}: \tag{2.3}$$

Level 1: Level ignored.

Level 2: $TCU\ (Q_2^*) = 90(10000) + \frac{(1000)(10000)}{1054} + 18\left(\frac{1054}{2}\right)$

$TCU\ (Q_2^*) = \$918{,}973.67$

Level 3: $TCU\ (Q_3^*) = 85(10000) + \frac{(1000)(10000)}{2000} + 17\left(\frac{2000}{2}\right)$

$TCU\ (Q_3^*) = \$872{,}000$

Level 4: $TCU\ (Q_4^*) = 80(10000) + \frac{(1000)(10000)}{10000} + 16\left(\frac{10000}{2}\right)$

$TCU\ (Q_3^*) = \$881{,}000$

<u>Step 4</u>. The optimal order quantity is Q^* = 2,000 units. The minimum total inventory cost is $872,000 per year.

Chapter Three

An EOQ Model with Dual Supplier

The EOQ model is the perfect order quantity a company should purchase tominimize inventory costs. This chapter will focus on the model with dual supplier.

3.1 Problem Statement

In this chapter, we consider an EOQ model for a product that can be obtained from supplied two suppliers, one is unreliable and cheaper and another is reliable but more expensive. The underlying problem scenario follows the model proposed by Giri (2010) in which the delivery quantity from the unreliable supplier is assumed to follow a probability distribution independent of the order quantity. However, in the proposed model, the order received from the unreliable supplier follows a discrete distribution.

3.2 Assumptions and Notation

The notations used in this chapter are as follows:

D = Demand Rate (units/unit time)

S = Amount Ordered Per Inventory Cycle (units)

W = Purchasing Cost per Unit from Supplier 1 ($/unit)

C_1 = Ordering Cost from Supplier 1 ($)

R = Amount Reserved from Supplier 2 (units)

r = Reserve Cost per Unit from Supplier 2 ($/unit)

V = Purchasing Cost per Unit from Supplier 2 ($/unit)

C_2 = Ordering Cost from Supplier 2 if an Order is Placed ($)

C_u = Shortage Penalty Cost per Unit ($/unit)

C_h = Holding Cost Per Unit per Unit Time ($/unit/unit time)

π = Percentage of Order Received from Supplier 1

μ = Expected Value of π

X = Amount Received from Supplier 1 (random variable unit)

$P(X)$ = Probability Distribution of X

T = Inventory Cycle Length (unit time)

$TC(S, R)$ = Total Inventory Cost per Cycle

$TCU(S, R)$ = Total Inventory Cost per Unit Time

3.3 Mathematical model

To determine the optimal order quantity S^* and the reserved quantity R, we construct the TC function representing the total inventory cost per cycle. This function involved the purchasing cost from supplier 1, which is the product of the purchasing cost per unit from supplier 1 multiplied by the amount received from the supplier. Hence, the purchasing cost from supplier 1 is given by

$$\text{Purchasing Cost Form Supplier 1} = WX. \tag{3.1}$$

The TC function also includes the ordering cost from supplier 1, which is given by

$$\text{Ordering Cost from Supplier 1} = C_1. \tag{3.2}$$

The holding cost for the unit received during the inventory cycle is identical to the EOQ model. Therefore, the holding cost is given by

$$\text{Holding Cost} = C_h\,(S/2)T. \tag{3.3}$$

Note that the inventory cycle length is given by

$$T = S/D. \tag{3.4}$$

Since the amount X received from supplier 1 is the product of S by the proportion π of the order received from supplier 1, we have

$$X = S\pi. \tag{3.5}$$

The expected amount received from supplier 1 is

$$E[X] = S\,E[\pi] = S\mu. \tag{3.6}$$

The purchasing cost from supplier 2 is the product of the unit purchasing cost and the amount received from supplier 2, which is

$$\text{Purchasing Cost from Supplier 2} = V(S - X). \tag{3.7}$$

The ordering cost from supplier 2 is C_2 multiplied by the random variable Y defined by

$$Y = \begin{cases} 0 & if \quad X = S \\ 1 & if \quad X \neq S \end{cases}. \tag{3.8}$$

Hence,

$$\text{Ordering Cost from Supplier 2} = C_2\,Y. \tag{3.9}$$

Finally, the reservation cost of unit from supplier 2 is the product of the amount reserved from supplier 2 times the reserve cost per unit. That is,

$$\text{Reservation Cost from Supplier 2} = r\,R\,. \tag{3.10}$$

Assuming the total cost per inventory cycle function is obtained by adding the costs in Equations (3.1)-(3.3), (3.7), (3.9) and (3.10). Thus,

$$TC(S) = WX + C_1 + C_h\left(\frac{S}{2}\right)T + V(S - X) + C_2Y + rR. \tag{3.11}$$

In the following, it is assumed that either the entire order S or a fixed proportion of the order will be received from Supplier 1 so that that the random variable π takes on two values 100% and a fixed value $\pi_1 < 1$. In this case, the reserved amount is the difference between S and X. Therefore, in this case, the reserved amount from Supplier 2 is

$$R = S - S\,\pi_1 = S\,(1 - \pi_1), \tag{3.12}$$

and the total cost per inventory cycle given by Equation (3.11) becomes

$$TC(S) = WX + C_1 + C_h\left(\frac{S}{2}\right)T + V(S - X) + C_2Y + rS(1 - \pi_1). \tag{3.13}$$

In chapters 4 and 5, the general case will be dealt with.

Let *ETC*(*S*) be the expected total cost per inventory cycle function. From Equation (3.13), the *ETC*(*S*) is

$$ETC(S) = E[TC(S)] = WE[X] + \ C_1 + C_h\left(\frac{S}{2}\right)T + V(S - E[X]) + C_2E[Y] + rS(1 - \pi_1). \tag{3.14}$$

Let *P* denote the expected value of the random variable *Y*. Then,

$$P = E[Y]. \tag{3.15}$$

From Equations (3.5) and (3.15) and using the fact that μ is the expected value of π, the *ETC*(*S*) = *E*[*TC*(*S*)] given in Equation (3.13) becomes

$$ETC(S) = WS\mu + \ C_1 + C_h\left(\frac{S}{2}\right)T + V(S - S\mu) + C_2P + rS(1 - \pi_1). \tag{3.16}$$

Next, the expected total cost per unit time function *ETCU*(*S*) is obtained calculating the total cost per unit time function *TCU*(*S*) using

$$TCU(S) = \frac{TC(S)}{T}, \tag{3.17}$$

and taking its expected value.

Using the Renewal Reward theorem, see for instance (Giri, B. C. (2011), the expected total cost per unit time function is

$$ETCU(S) = \frac{ETC(S)}{T} = \frac{D}{S}\left(WS\mu + \ C_1 + C_h\left(\frac{S}{2}\right)T + VS(1-\mu) + C_2\ P + rS(1-\pi_1)\right). \tag{3.18}$$

Rearranging and grouping the terms of Equation (3.18), the *ETCU*(*S*) function becomes

$$ETCU(S) = \frac{D}{S}\big(WS\mu + C_1 + VS(1-\mu) + C_2\ P + rS(1-\pi_1)\big) + C_h\left(\frac{S}{2}\right). \tag{3.19}$$

To find the optimal order quantity *S**, the derivative of *ETCU* is calculated, equal to zero, and solved for *S*. The Derivative of *ETCU* is

$$\frac{dETCU}{dS} = -\frac{D}{S^2}\ C_1 - \frac{D}{S^2}C_2\ P + C_h\left(\frac{1}{2}\right). \tag{3.20}$$

Setting the Derivative equal to Zero,

$$-\frac{D}{S^2}\ C_1 - \frac{D}{S^2}C_2\ P + C_h\left(\frac{1}{2}\right) = 0. \tag{3.21}$$

Rearranging the terms of equation (3.21), we have

$$C_h\left(\frac{1}{2}\right) = \frac{D}{S^2}\ C_1 + \frac{D}{S^2}C_2\ P. \tag{3.22}$$

Solving Equation (3.22) for S, the optimal order quantity is

$$S^* = \sqrt{\frac{2D(\ C_1 + C_2\ P)}{Ch}}. \tag{3.23}$$

It is worth noting that the second derivative of *ETCU*

$$\frac{d^2\ ETCU}{dS^2} = \frac{D}{S^3}(\ C_1 + C_2\ P). \tag{3.24}$$

The second derivative of ETCU given in Equation (3.24) is always positive so that the optimal order quantity $S*$ is unique.

By determining an expression given by a closed form formula for the optimal order quantity and showing that the second derivative is always positive, the existence and uniqueness of the optimal solution has been demonstrated. In the next section, a numerical example is presented and the determination of the optimal solution is demonstrated.

3.4 Numerical Example

Consider the case that an item is purchased from two suppliers. Supplier 1 is the unreliable supplier but cheaper and supplier 2 is reliable and more expensive. An order of size S request from supplier will be received entirely with a probability of 80% or in part of 90% with a probability of 20%. The demand rate for the item is 100 units per day. The purchasing cost from supplier 1 is $5 per unit and ordering cost is $800.

The holding cost is $0.1 per unit per day. To assure the reception of the order quantity per cycle, an amount of size R (to be determined) is reserved for supplier 2 at a cost of $0.5/unit. The ordering cost from supplier 2 is $1000 and the purchasing cost is $7 per unit.

The parameters of the problem are:

D = 100 units/day

S = 1000 units

W = \$ 5 /units

C_1 = \$ 800

r =\$ 0.5 /unit

V =\$ 7/unit

C_2 = \$1000 (if ordered)

Ch = \$ 0.1/unit/day

CU = \$2/unit

X = Amount Received from Supplier 1

π = Percentage of Order Received from Supplier 1

$X = S\,\pi$ = 1000 * 90/100 = 900

R = S (1-π_1) = 1000 (1- 0.9) = 100

The percentage of order received from supplier 1 is represented as below:

π	(100%)	90%
$P(\pi)$	0.8	0.2

The amount received from supplier 1 is represented as below:

X	$100\%*S$	$90\%*S$
$P(X)$	0.8	0.2

The amount received from supplier 2 if ordered depends on the random variable Y defined wise probability distribution is:

Y	0	1
$P(Y)$	0.8	0.2

The expected value μ of the random variable π is

$$\mu = \Sigma\ \pi P\ (\pi) = 1(0.8) + 0.9(0.2) = 0.98$$

Also, the probability P of ordering from supplier 2 is

$$P = P(Y = 1) = P\ (X \neq S) = 0.2$$

The total inventory cost is received according to the equation (3.19)

$$\text{ETCU(S)} = \frac{100}{1000} * (5 * 1000 * 0.98 + 800 + 7 * 1000 * (1 - 0.98) + 1000 * 0.2 + 0.5 * 1000 * (1 - 0.9)) + 0.1\ (\frac{1000}{2})*10$$

ETCU(S) = $ 659

To find the optimal order quantity S, we use Equation (3.23) to obtain

$$S^* = \sqrt{\frac{2*100*(800+1000*0.2)}{0.1}} = 1414.21 \text{ units}$$

The optimal cycle length is:

$$T = \frac{S^*}{D} = \frac{1414.21}{100} = 14.14 \text{ days}$$

The minimum total cost per day is

E(X) = 1000*0.98 = 980 units

Depending to the equation (3.18),

$$ETCU(S) = \frac{100}{1414} * (5 * 1414 * 0.98 + 800 + 0.1\left(\frac{1414}{2}\right) * 14.14 + 7 * 1414(1-0.98) + 1000 * 0.2 + 0.5 * 1414(1-0.9)) = \$\ 650.224.$$

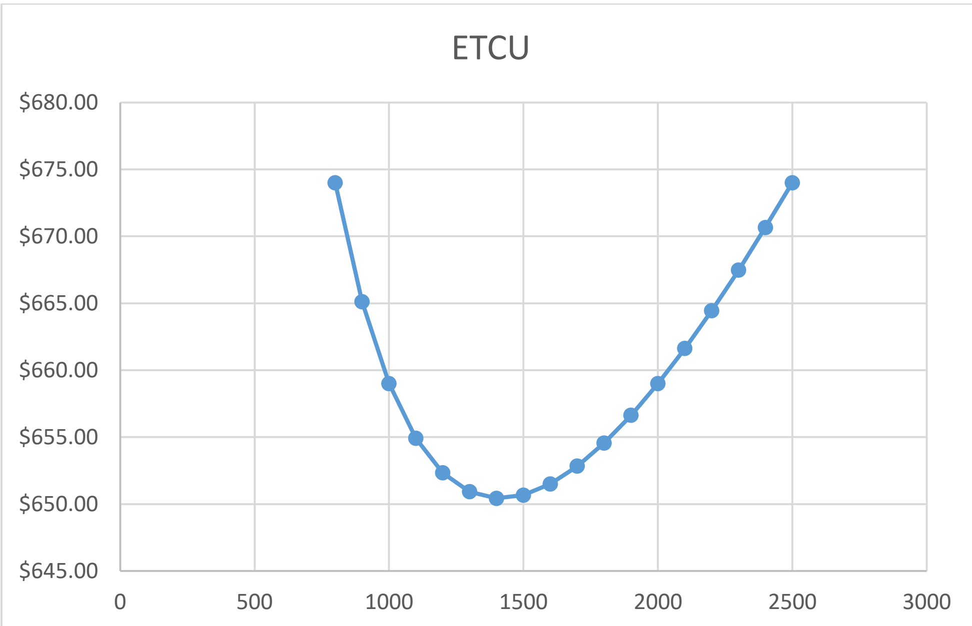
The optimal ordering policy is to order 1414.21 units every 14 days whenever the inventory reaches 0. The minimum total inventory cost is $ 650.224 per day.The expected order received from supplier 1 is 1385.72, the holding cost is 70.7, the Purchasing Cost is $490 and the Ordering cost is $56.58. The Reserved Amount from supplier 2 is 141.4, the Amount purchased is 28.28, the Purchasing cost is $14.00, the reserved cost is $5.00 and the Ordering cost is $14.14

Table 1: Cost Component for S = 1400

In the table below, we did a breakdown for the cost components by using the increment Of S = 100 where S ranges from 800 to 2000. The minimum value occurred around S = 1400 the total cost ETCU is around $ 650.43

Order Size	Cycle length	Reciprocal of S/D	Holding cost	Order Received	Purchasing Cost	Ordering cost	Reserved Amount	Amount purchased	Purchasing cost	Reserved cost	Ordering cost	Total cost
S	S/D	D/S	Ch(S/2)	Sµ	(D/S)SWµ	(D/S)C1	R	S(1-µ)	(D/S)VS(1-µ)	(D/S)rR	(D/S)C2P	ETCU
800	8	0.13	40	784	$490.00	$100.00	80	16	$14.00	$5.00	$25.00	$674.00
900	9	0.11	45	882	$490.00	$88.89	90	18	$14.00	$5.00	$22.22	$665.11
1000	10	0.10	50	980	$490.00	$80.00	100	20	$14.00	$5.00	$20.00	$659.00
1100	11	0.09	55	1078	$490.00	$72.73	110	22	$14.00	$5.00	$18.18	$654.91
1200	12	0.08	60	1176	$490.00	$66.67	120	24	$14.00	$5.00	$16.67	$652.33
1300	13	0.08	65	1274	$490.00	$61.54	130	26	$14.00	$5.00	$15.38	$650.92
1400	**14**	**0.07**	**70**	**1372**	**$490.00**	**$57.14**	**140**	**28**	**$14.00**	**$5.00**	**$14.29**	**$650.43**
1500	15	0.07	75	1470	$490.00	$53.33	150	30	$14.00	$5.00	$13.33	$650.67
1600	16	0.06	80	1568	$490.00	$50.00	160	32	$14.00	$5.00	$12.50	$651.50
1700	17	0.06	85	1666	$490.00	$47.06	170	34	$14.00	$5.00	$11.76	$652.82
1800	18	0.06	90	1764	$490.00	$44.44	180	36	$14.00	$5.00	$11.11	$654.56
1900	19	0.05	95	1862	$490.00	$42.11	190	38	$14.00	$5.00	$10.53	$656.63
2000	20	0.05	100	1960	$490.00	$40.00	200	40	$14.00	$5.00	$10.00	$659.00
2100	21	0.05	105	2058	$490.00	$38.10	210	42	$14.00	$5.00	$9.52	$661.62
2200	22	0.05	110	2156	$490.00	$36.36	220	44	$14.00	$5.00	$9.09	$664.45
2300	23	0.04	115	2254	$490.00	$34.78	230	46	$14.00	$5.00	$8.70	$667.48
2400	24	0.04	120	2352	$490.00	$33.33	240	48	$14.00	$5.00	$8.33	$670.67
2500	25	0.04	125	2450	$490.00	$32.00	250	50	$14.00	$5.00	$8.00	$674.00

The graph for the function is given in the figure below where optimal value for S is around 1400 and the total cost ECU is around $ 650 .



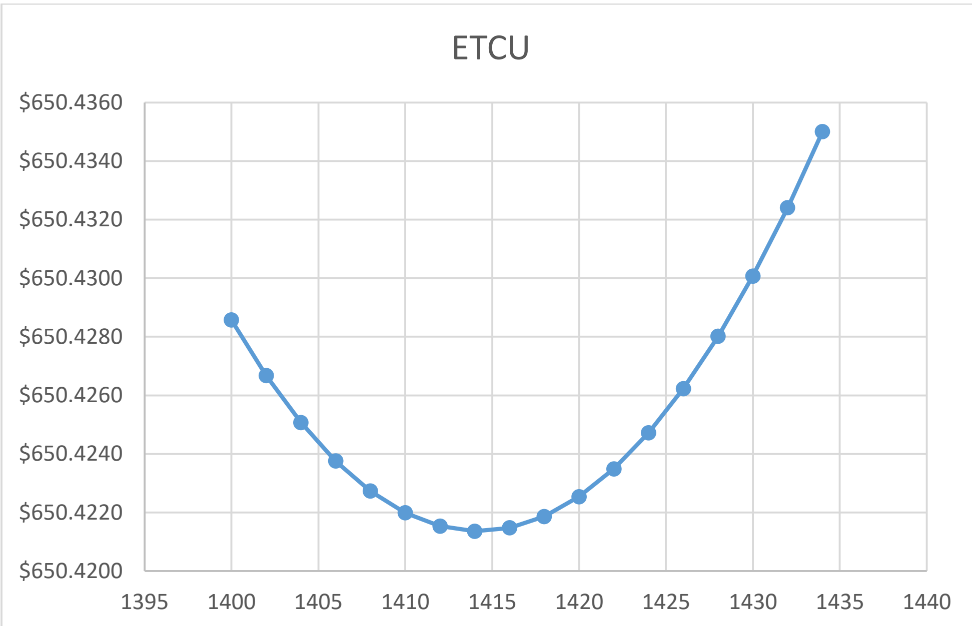
Graph 1: ETCU for S = 1400

Table 2: Cost Component for S = 1414

To obtain the exact value, in the table below we did a breakdown for the cost Components by using the increment of S = 2 where S ranges from 1400 to 1434. The table revealed the optimal value S = 1414 and the total cost ETCU is $ 650.42

				Supplier1			Supplier2					
Order Size	Cycle length	Reciprocal of S/D	Holding cost	Order Received	Purchasing Cost	Ordering cost	Reserved Amount	Amount purchased	Purchasing cost	Reserved cost	Ordering cost	Total cost
S	S/D	D/S	Ch(S/2)	Sμ	(D/S)SWμ	(D/S)C1	R	S(1-μ)	(D/S)VS(1-μ)	(D/S)Rr	(D/S)C2P	ETCU
1400	14	0.07	70	1372	$490.00	$57.14	140	28	$14.00	$5.00	$14.29	$650.4286
1402	14.02	0.07	70.1	1373.96	$490.00	$57.06	140.2	28.04	$14.00	$5.00	$14.27	$650.4267
1404	14.04	0.07	70.2	1375.92	$490.00	$56.98	140.4	28.08	$14.00	$5.00	$14.25	$650.4251
1406	14.06	0.07	70.3	1377.88	$490.00	$56.90	140.6	28.12	$14.00	$5.00	$14.22	$650.4238
1408	14.08	0.07	70.4	1379.84	$490.00	$56.82	140.8	28.16	$14.00	$5.00	$14.20	$650.4227
1410	14.1	0.07	70.5	1381.8	$490.00	$56.74	141	28.2	$14.00	$5.00	$14.18	$650.4220
1412	14.12	0.07	70.6	1383.76	$490.00	$56.66	141.2	28.24	$14.00	$5.00	$14.16	$650.4215
1414	**14.14**	**0.07**	**70.7**	**1385.72**	**$490.00**	**$56.58**	**141.4**	**28.28**	**$14.00**	**$5.00**	**$14.14**	**$650.4214**
1416	14.16	0.07	70.8	1387.68	$490.00	$56.50	141.6	28.32	$14.00	$5.00	$14.12	$650.4215
1418	14.18	0.07	70.9	1389.64	$490.00	$56.42	141.8	28.36	$14.00	$5.00	$14.10	$650.4219
1420	14.2	0.07	71	1391.6	$490.00	$56.34	142	28.4	$14.00	$5.00	$14.08	$650.4225
1422	14.22	0.07	71.1	1393.56	$490.00	$56.26	142.2	28.44	$14.00	$5.00	$14.06	$650.4235
1424	14.24	0.07	71.2	1395.52	$490.00	$56.18	142.4	28.48	$14.00	$5.00	$14.04	$650.4247
1426	14.26	0.07	71.3	1397.48	$490.00	$56.10	142.6	28.52	$14.00	$5.00	$14.03	$650.4262
1428	14.28	0.07	71.4	1399.44	$490.00	$56.02	142.8	28.56	$14.00	$5.00	$14.01	$650.4280
1430	14.3	0.07	71.5	1401.4	$490.00	$55.94	143	28.6	$14.00	$5.00	$13.99	$650.4301
1432	14.32	0.07	71.6	1403.36	$490.00	$55.87	143.2	28.64	$14.00	$5.00	$13.97	$650.4324
1434	14.34	0.07	71.7	1405.32	$490.00	$55.79	143.4	28.68	$14.00	$5.00	$13.95	$650.4350

The graph for the function is given in the figure below where optimal value for S is 1414 and the total cost ECU is $ 650.42



Graph 2 : ETCU for S = 1414

The numerical analysis verified the result.

Table 3: Sensitivity Analysis:

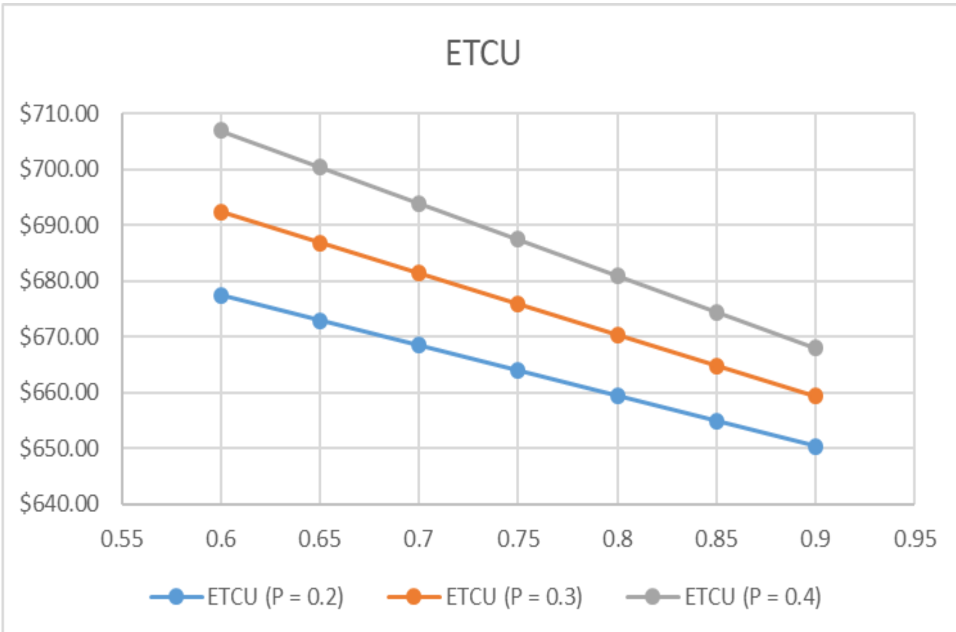
Sensitivity analysis was conducted to determine the effect of changes in the value of π and its corresponding probability for the optimal order size and the total cost function.

Each time the percentage of the order received from supplier 1 decreases, the TCU function increases.

The more supplier 1 is unreliable the higher the cost will be. The same is repeated from 0.2 to 0.3 or 0.4

Again we notice that the higher the probability of not receiving the entire order the higher the total cost will be.

						Supplier1			Supplier2					
			Order Size	Cycle length	Holding cost	Order Recived	Purchasing Cost	Ordering cost	Reserved Amount	Amount purchased	Purchaing cost	Reserved cost	Ordering cost	Total cost
π1	P	μ	S*	S/D	Ch(S/2)	Sμ	(D/S)SWμ	(D/S)C1	R	S(1-μ)	(D/S)VS(1-μ)	(D/S)rR	(D/S)C2P	ETCU
0.9	0.2	0.98	1414	14.14	$70.71	1386	$490.00	$56.57	141	28	$14.00	$5.00	$14.14	$650.42
0.85	0.2	0.97	1414	14.14	$70.71	1372	$485.00	$56.57	212	42	$21.00	$7.50	$14.14	$654.92
0.8	0.2	0.96	1414	14.14	$70.71	1358	$480.00	$56.57	283	57	$28.00	$10.00	$14.14	$659.42
0.75	0.2	0.95	1414	14.14	$70.71	1344	$475.00	$56.57	354	71	$35.00	$12.50	$14.14	$663.92
0.7	0.2	0.94	1414	14.14	$70.71	1329	$470.00	$56.57	424	85	$42.00	$15.00	$14.14	$668.42
0.65	0.2	0.93	1414	14.14	$70.71	1315	$465.00	$56.57	495	99	$49.00	$17.50	$14.14	$672.92
0.6	0.2	0.92	1414	14.14	$70.71	1301	$460.00	$56.57	566	113	$56.00	$20.00	$14.14	$677.42
0.9	0.3	0.97	1483	14.83	$74.16	1439	$485.00	$53.94	148	44	$21.00	$5.00	$20.23	$659.32
0.85	0.3	0.955	1483	14.83	$74.16	1416	$477.50	$53.94	222	67	$31.50	$7.50	$20.23	$664.82
0.8	0.3	0.94	1483	14.83	$74.16	1394	$470.00	$53.94	297	89	$42.00	$10.00	$20.23	$670.32
0.75	0.3	0.925	1483	14.83	$74.16	1372	$462.50	$53.94	371	111	$52.50	$12.50	$20.23	$675.82
0.7	0.3	0.91	1483	14.83	$74.16	1350	$455.00	$53.94	445	133	$63.00	$15.00	$20.23	$681.32
0.65	0.3	0.895	1483	14.83	$74.16	1327	$447.50	$53.94	519	156	$73.50	$17.50	$20.23	$686.82
0.6	0.3	0.88	1483	14.83	$74.16	1305	$440.00	$53.94	593	178	$84.00	$20.00	$20.23	$692.32
0.9	0.4	0.96	1549	15.49	$77.46	1487	$480.00	$51.64	155	62	$28.00	$5.00	$25.82	$667.92
0.85	0.4	0.94	1549	15.49	$77.46	1456	$470.00	$51.64	232	93	$42.00	$7.50	$25.82	$674.42
0.8	0.4	0.92	1549	15.49	$77.46	1425	$460.00	$51.64	310	124	$56.00	$10.00	$25.82	$680.92
0.75	0.4	0.9	1549	15.49	$77.46	1394	$450.00	$51.64	387	155	$70.00	$12.50	$25.82	$687.42
0.7	0.4	0.88	1549	15.49	$77.46	1363	$440.00	$51.64	465	186	$84.00	$15.00	$25.82	$693.92
0.65	0.4	0.86	1549	15.49	$77.46	1332	$430.00	$51.64	542	217	$98.00	$17.50	$25.82	$700.42
0.6	0.4	0.84	1549	15.49	$77.46	1301	$420.00	$51.64	620	248	$112.00	$20.00	$25.82	$706.92



Graph 3 : results of the sensitivity analysis representing ETCU function.

Chapter Four

Model with Shortage

The purpose of this chapter is to modify the model presented in chapter 3 to allow for shortages. The mathematical model is developed and an expression for the optimal solution is derived. In addition, a numerical example is presented.

4.1 Problem Statement

Let π be the portion of the order received from supplier 1. It is assumed that π is a discrete random variable with K values, where $K \geq 2$. Let $\pi_1, \pi_2, \ldots, \pi_K$ be the possible values π can take. Supposed that $\pi_1 > \pi_2 > \ldots > \pi_K$ and let $\pi_1 = 100\%$. Then, $P(\pi = \pi_1) = P(\pi_1)$ represents the probability that the amount received from supplier 1 is exactly the amount S ordered. The expected value of the proportion of the order received is

$$\mu = \sum_{i=1}^{K} \pi_i \, P(\pi_i). \tag{4.1}$$

The reserved quantity for supplier 2 is set equal to correspond to the maximum quantity that can be received from supplier 1 that is not equal to the total order S. Then

$$R = S - \pi_2 S = S\,(1 - \pi_2), \tag{4.2}$$

where π_2 is the highest percentage not equal to 100%. Let $X = \pi S$ be the amount received from supplier 1. Then,

$$E(X) = S\,E(\pi) = \mu S \tag{4.3}$$

Define the random variable Y to represent whether or not an order from amount received from supplier 1 is less than the total amount S. That is,

$$Y = \begin{cases} 0 & if \quad X = S \\ 1 & if \quad X \neq S \end{cases}. \tag{4.4}$$

Note that a value of Y equals to 1 indicates that an order from supplier 2 would be purchased. Hence,

$$E(Y) = P(X \neq S) = 1 - P(\pi_1) = P, \tag{4.5}$$

where P is probability that $\pi \neq 100\%$.

In this inventory situation, shortages may occur during an inventory cycle. Note that shortages will not occur if the amount received from Supplier 1 is either $X = \pi_1 S = S$ or $X = \pi_2 S$. In the first case, no items are ordered from Supplier 2. As for the second case, the difference between the amount ordered S and the amount received $X = \pi_2 S$ is exactly the reserved R. In this case, R units are ordered from Supplier 2 making the total amount received equal to $X + R = \pi_2 S + (S - \pi_2 S) = S$. Accordingly, shortages will occur when the amount received $X = \pi S$ is less than $\pi_2 S$. In this case, R units are ordered from Supplier 2 making the total amount received equal to $X + R = \pi S + (S - \pi_2 S)$ which is less than S. Thus, the amount short is the difference between the amount ordered and amount received. That is, when $\pi < \pi_2$, the quantity short during a cycle Q is

$$Q = S - (X + R) = S - (\pi S + (S - \pi_2 S)) = \pi_2 S - \pi S = S(\pi_2 - \pi) \tag{4.6}$$

In general, the quantity short is given by

$$Q = \begin{cases} 0 & if \quad \pi \geq \pi_2 \\ S(\pi_2 - \pi) & if \quad \pi < \pi_2 \end{cases}. \tag{4.7}$$

The expected value of quantity short is

$$E(Q) = S\left(\sum_{i=3}^{K} (\pi_2 - \pi_i)\, P(\pi_i)\right) = S\mu_Q. \tag{4.8}$$

Since $\pi_1 > \pi_2 > \ldots > \pi_K$, $\sum_{i=3}^{K} (\pi_2 - \pi_i)\, P(\pi_i) > 0$ and $E(Q)$ is positive.

4.2 Optimal Solution

The various costs involved are as follows:

Purchasing Cost from Supplier 1 = $W X$ (4.6)

Ordering Cost from Supplier 1 = C_1 (4.7)

Purchasing Cost from Supplier 2 = $Y.\ V.\ (S - \pi_2 S)$ (4.8)

Ordering Cost from Supplier 2 = $Y.C_2$ (4.9)

Reservation Cost = $R\ r = r\ S\ (1 - \pi_2)$ (4.10)

Holding Cost = $(1-Y).\frac{Ch}{2}.\ S.\ T + Y.\frac{Ch}{2}\ (\pi.S + S - \pi_2 S).\ T$ (4.11)

Shortage Cost = $C_u\ Q$ (4.12)

Note that the inventory cycle length is given by

T = S/D.

The Total Cost per Cycle Function is

$$TC(S) = WX + C_1 + (1-Y)C_h\left(\frac{S}{2}\right)T + YC_h\left(\frac{S}{2}\right)(\pi + 1 - \pi_2)T + YVS(1 - \pi_2) + YC_2 + rS(1 - \pi_2) + C_u\,Q. \qquad (4.13)$$

The expected total cost per cycle function is

$$ETC(S) = W\mu S + C_1 + (1-P)C_h\left(\frac{S}{2}\right)T + PC_h\left(\frac{S}{2}\right)(\mu + 1 - \pi_2)T + PVS(1 - \pi_2) + PC_2 + rS(1 - \pi_2) + S\mu_Q. \qquad (4.14)$$

Using the Renewal Reward theorem and the fact that *T* = *S/D*, the expected total cost per unit time function is

$$ETCU(S) = \frac{ETC(S)}{T} = W\mu D + C_1\left(\frac{D}{S}\right) + (1-P)C_h\left(\frac{S}{2}\right) + PC_h\left(\frac{S}{2}\right)(\mu + 1 - \pi_2) + PVD(1 - \pi_2) + PC_2\left(\frac{D}{S}\right) + rD(1 - \pi_2) + Cu\,D\mu_Q. \qquad (4.15)$$

To find the optimal order quantity, we first differentiate (4.15) to obtain

$$\frac{dETCU}{dS} = -\frac{C_1 D}{S^2} + (1 - P)\frac{Ch}{2} + P.\frac{Ch}{2}(\mu + 1 - \pi_2) - \frac{PC_2 D}{S^2} \qquad (4.16)$$

Setting the derivative equal to zero,

$$0 = -\frac{C_1 D}{S^2} + (1 - P)\frac{Ch}{2} + P.\frac{Ch}{2}(\mu + 1 - \pi_2) - \frac{PC_2 D}{S^2}$$

and solving for the order size S, we obtain

$$S^* = \sqrt{\frac{(C_1 + PC_2)D}{(1-P\,)\frac{Ch}{2} + P.\frac{Ch}{2}\,(\,\mu\, + 1\, -\, \pi_2\,\,)}} \qquad (4.17)$$

4.3 Numerical Example

Suppose the parameters of the problem are:

D = 100 units/day

W = \$ 5 /units

C_1 = \$ 800

r =\$ 0.5 /unit

V = \$ 7/unit

C_2 = \$1000 (if ordered)

Ch = \$ 0.1/unit/day

C_U = \$2/unit

X = Amount Received From Supplier 1

π_1 = Percentage Of Order Received From Supplier 1

π_2 = First value in the probability distribution of π not corresponding to 100%

The Percentage of Order Received from Supplier 1 is represented as below:

	π_1	π_2	π_3
Π	100%	90%	80%
P(π)	0.7	0.2	0.1

The Amount Received from Supplier 1 is represented as below:

X	100%**S*	90%**S*	80%**S*
P(X)	0.7 *S*	0.2 *S*	0.1 *S*

The Amount Received from Supplier 2 if ordered depends on the random variable Y defined wise probability distribution is:

Y	0	1
P(Y)	0.7	0.3

The expected value μ of the random variable π is

$$\mu = \Sigma \pi P(\pi) = 1(0.7) + 0.9(0.2) + 0.8(0.1) = 0.96$$

and the value μ_Q in the expression for the expected quantity short given in Eq. (4.8) is

$$\mu_Q = \pi_3 * P(\pi_3) = 0.8 * 0.1 = 0.08$$

The probability *P* of ordering from supplier 2 is

$P = P(Y = 1) = P\ (X \neq S) = 0.3$

Now consider the optimal order quantity obtained by Eq. (4.15). The optimal order quantity is

$$S^* = \sqrt{\frac{(C_1+PC_2)D}{(1-P\ )\frac{Ch}{2}+P.\frac{Ch}{2}\,(\ \mu+1-\pi_2\ )}} = \sqrt{\frac{(800+0.3*1000)100}{(1-0.3\ )\frac{0.1}{2}+0.3.\frac{0.1}{2}\,(\ 0.96+1-0.9\ )}} = 1470.07$$

Hence, the optimal cycle length is

$T = S/D = 1470.07/100 = 14.7$ days,

the optimal reservation quantity is

$R^* = S^* - \pi_2\, S^* = 1470 - 0.9*1470 = 147$ units,

and expected number of units short is

$E(Q) = S\mu_Q = 1470(0.08) = 117.6$ units,

and the expected total cost per day function is

$ETCU(147) = \$682.90$.

Chapter Five

EOQ Model with Dual Supplier and Price Discount

The purpose of this chapter is to modify the models presented in chapter 3 to allow for price discount offered by the unreliable supplier. The procedure is proposed for finding the optimal solution with and without shortage. In addition, numerical examples are presented.

5.1 EOQ Model with Price Discount and No Shortages

Consider the dual supplier models presented in Chapters 3 and 4. Suppose that the unreliable supplier (Supplier 1) is offering a price discount. The discount schedule has several price levels each of which is determined by a range for the quantity purchased.

<u>The Parameters of the EOQ Model are:</u>

D = Demand Rate (units/unit time)

Wi = Level i Purchasing Cost Per Unit from Supplier 1 ($/unit)

C_1 = Ordering Cost from Supplier 1 ($)

H = Annual Holding Cost Rate (%)

C_h =Holding Cost ($/unit/year) = H.Wi

V_2 = Purchasing Cost Per Unit from Supplier 2 ($/unit)

S = Amount Ordered Per Cycle (units)

R = Amount Reserved from Supplier 2 (units)

r = Reserve Cost Per Unit from Supplier 2 ($/unit)

C_2 = Ordering Cost from Supplier 2 if an order is placed ($)

π = Percentage of Order Received from Supplier 1

μ = Expected Value Of π

π_1 = Highest percentage of π not equal to 100%

X = Amount Received from Supplier 1 (random variable)

$P(X)$ = Probability Distribution of X

T = Cycle Length, $T = S/D$ (unit time)

$TC(S)$ = Total Inventory Cost per Cycle

$TCU(S)$ = Total Inventory Cost per Unit Time, $TCU(S) = TC(S)/T$

The procedure presented in Chapter 3 for finding the optimal order quantity for an EOQ model with price discount is modified for the case of dual suppliers.

Procedure 5.1

Step 1. For each level *i*, find the optimal order quantity using Eq. (3.23)

$$S^* = \sqrt{\frac{2D(\ C_1 + C_2\ P)}{HW_i}}$$

Step 2. Adjust S_i^* according to:

If S_i^* is within the quantity range for level *i*, keep S_i^*.

If S_i^* is more than the largest quantity of its range, ignore level *i*.

If S_i^* is less than the smallest quantity of its range, set S_i^* equal to the smallest quantity of its range.

Step 3. For each level *i*, use the adjusted values of S_i^* to calculate *TCU* (S_i^*,R^*) by modifying Eq. (3.19)

$$ETCU(S_i^*) = \frac{D}{S_i^*}\left(W_i S_i^* \mu + C_1 + VS_i^*(1-\mu) + C_2\ P + rS_i^*(1-\pi_1)\right) + HW_i\left(\frac{S_i^*}{2}\right).$$

Step 4. Select the S_i^* that results in a minimum value of $ETCU(S_i^*)$.

In the following, a numerical example is provided to illustrate the determination of the optimal solution using procedure 5.1.

5.2 A Numerical Example for an EOQ model with price discount

The demand rate for a certain item is 10000 units per year. It costs $1,000 to place an order from supplier 1 and $1,000 to purchase a single item. The holding cost rate is 20%. Items can be ordered from two suppliers. Supplier 1 is unreliable and delivers the order or part of the order according to the random variable π representing of the percentage of the order delivered. The probability distribution for π is:

The Percentage of Order Received from Supplier 1 is represented as below:

π	100%	90%
$P(\pi)$	0.8	0.2

The aim is to find the optimal order quantity if the supplier is offering a price discount according to the following schedule:

Level	**Quantity**	**Price per Unit**
1	1 to 1999	$5
2	2000 to 9999	4
3	10000 or more	3.5

The parameters of the problem are:

D = 10000 units/year

C_1 = \$1,000

C_2 = \$1,000

W_1 = \$5 /unit,

W_2 = \$4 /unit,

W_3= \$3.5 /unit

H = 20%

C_h = 20% (\$5) = \$ 1/unit/year (level 1)

C_h = 20% (\$4) = \$ 0.8/unit/year (level 2)

C_h = 20% (\$3.5) = \$ 0.7/unit/year (level 3)

V_2 =\$ 7/unit

r =\$ 0.5 /unit

C_U = \$2/unit

X = Amount Received from Supplier 1

The Amount Received from Supplier 1 is represented as below:

X	100%**S*	90%**S*
P(X)	0.8 *S*	0.2 *S*

The Amount Received from Supplier 2 if ordered depends on the random variable Y defined wise probability distribution is:

Y	0	1
P(Y)	0.8	0.2

The Expected value μ of the random variable π is

$$\mu = \Sigma \pi P(\pi) = 1(0.8) + 0.9(0.2) = 0.98$$

Also, the Probability P of Ordering from Supplier 2 is

$$P = P(Y = 1) = P(X \neq S) = 0.2,$$

and $\pi_1 = 0.9$.

Step 1. For each level I find the optimal order quantity

Level 1: $S_1^* = \sqrt{\frac{2D(\,C_1+C_2\,P)}{HW_1}} = \sqrt{\frac{2*10000*(1000+1000*0.2)}{0.2*5}} = 4898.97 \approx 4899$ units

Level 2: $S_2^* = \sqrt{\frac{2D(\,C_1+C_2\,P)}{HW_2}} = \sqrt{\frac{2*10000*(1000+1000*0.2)}{0.2*4}} = 5477.22 \approx 5477$ units

Level 3: $S_3^* = \sqrt{\frac{2D(\,C_1+C_2\,P)}{HW_3}} = \sqrt{\frac{2*10000*(1000+1000*0.2)}{0.2*3.5}} = 5855.4 \approx 5855$ units

Step 2. Adjusting the values of S_i^*,

Level 1: Since S_1^*= 4899 units is outside its range and higher than the largest value in the range, ignore this level. (1 to 1999)

Level 2: Since $S_2^* = 5477$ units is within its range, we keep this level (2000 to 9999).

Level 3: Since S_3^*= 5855 units is outside the range and less than the smallest value in its range, it is adjusted to S_3^* = 10,000 units.

Step 3. For each level *i*, the adjusted values of S_i^* are used to calculate *ETCU* (S_i^*) based on Eq. (4.15)

Level 1: Level 1 is ignored.

Level 2: Using S_2^* = 5477 units

$$ETCU(S) = \frac{D}{S_2^*}\big(W_i\mu S_2^* + C_1 + VS_2^*(1-\mu) + C_2\,P + r(S_2^*1 - \pi_1)\big) + HW_i\left(\frac{S_2^*}{2}\right).$$

Purchasing Cost For Supplier 1 = $\frac{D}{S_2^*} W_2 S_2^*$ μ $= \frac{10000}{5477}$ * 4 * 0.98 *5477 = $ 39 200

Ordering Cost For Supplier 1 = $\frac{C_1 D}{S_2^*} = \frac{1000*10000}{5477}$ = $ 1825.81

Holding Cost for Supplier 2 = $HW_2 \left(\frac{S_2^*}{2}\right)$ = 0.2* 4 * ($\frac{5477}{2}$) = $ 2190.8

Purchasing Cost For Supplier 2 = $\frac{D}{S_2^*}(VS_2^*(1-\mu))$ = $\frac{10000}{5477}$ (7 * 5477 (1- 0.98)) = $ 1400

Ordering Cost For Supplier 2 = $\frac{D}{S_2^*} C_2\ P$ = $\frac{10000}{5477}$ * 1000 * 0.2 = $ 365.16

Reservation Cost For supplier 2 = $\frac{D}{S_2^*}(rS_2^*(1-\pi_1)$ = $\frac{10000}{5477}$ (0.5 * 5477 (1 - 0.9)) = $500

$ETCU(S) = \frac{D}{S_2^*}\left(W_2 S_2^* \mu + C_1 + VS_2^*(1-\mu) + C_2\ P + rS_2^*(1-\pi_1)\right) + HW_2\left(\frac{S_2^*}{2}\right)$

= $\frac{10000}{5477}$ (4 * 5477 * 0.98 + 1000 + 7 * 5477 (1 – 0.98) + 1000 * 0.2 + 0.5 * 5477 (1 -0.9)) + 0.2*

4 * ($\frac{5477}{2}$) = $ 45 481.37

Level 3: Using S_3^*= 10000 units

the optimal reservation quantity is

R*= S* − π_1 S* = 10000 – 0.9*10000 = 1000 units.

$$ETCU(S_3^*) = \frac{D}{S_3^*}\left(W_i S_3^* \mu + C_1 + VS_3^*(1-\mu) + C_2\ P + r(S_3^* 1 - \pi_1)\right) + HW_i\left(\frac{S_3^*}{2}\right).$$

Purchasing Cost For Supplier 1 $= \frac{D}{S_3^*} W_3 S_3^* \mu = \frac{10000}{10000}$ * 3.5 * 0.98* 10000 = $ 34300

Ordering Cost For Supplier 1 = $\frac{C_1 D}{S_3^*} = \frac{1000*10000}{10000}$ = $ 1000

Holding Cost For Supplier 2 = $HW_3 \left(\frac{S_3^*}{2}\right)$ = 0.2 * 3.5 *($\frac{10000}{2}$) = $ 3500

Purchasing Cost For Supplier 2 = $\frac{D}{S_3^*}(VS_3^*(1-\mu)) = \frac{10000}{10000}$ (7 * 10000 (1- 0.98)) = $ 1400

Ordering Cost For Supplier 2 = $\frac{D}{S_3^*} C_2 P = \frac{10000}{10000}$ * 1000 * 0.2 = $ 200

Reservation Cost For supplier 2 $= \frac{D}{S_3^*}(rS_3^*(1-\pi_1) = \frac{10000}{10000}$ (0.5 * 10000 (1 - 0.9)) = 500$

$$ETCU(S) = \frac{D}{S_3^*}\left(W_3 S_3^* \mu + C_1 + VS_3^*(1-\mu) + C_2 P + rS_3^*(1-\pi_1)\right) + HW_3\left(\frac{S_3^*}{2}\right)$$

$= \frac{10000}{10000}$ (3.5 * 10000 * 0.98 +1000 + 10000 * 7 (1 – 0.98) + 1000 * 0.2 + 0.5 * 10000 (1 – 0.9)) + 0.2 * 3.5 ($\frac{10000}{2}$) = $ 40 900

Step 4. Level 3 resulted in the minimum expected total cost. The optimal order quantity is $S_3^* =$ 10000.

Since $X = \pi S^*$ is the amount received from supplier 1, the expected value of X is

Since $Y = \begin{cases} 0 & if \quad X = S \\ 1 & if \quad X \neq S \end{cases}$,

$$E(Y) = P\ (X \neq S) = P = 0.2$$

The optimal cycle is T = S*/D = $\frac{10000}{1000}$ = 10 days

According to the proposed procedure, the optimal policy is to order 10000 units, every 10 days. This policy calls for reserving 1000 units, to purchase $34,300 a cost for supplier 1, to order 1000 units a cost for supplier 1, to hold a cost of $ 3500 for supplier 2, to purchase a cost of $ 1400 for supplier 2, to order a cost of $ 200 for supplier 2, to reserve a cost of $ 500 for supplier 2.

5.3 EOQ Model with Price Discount and Shortages

Consider the dual supplier model presented in Chapter 4. Suppose that the unreliable supplier (Supplier 1) is offering a price discount. The discount schedule has several price levels each of which is determined by a range for the quantity purchased.

The Parameters of the EOQ Model are:

D = Demand Rate (units/unit time)

W_i = Level i Purchasing Cost Per Unit from Supplier 1 ($/unit)

C_1 = Ordering Cost from Supplier 1 ($)

H = Annual Holding Cost Rate (%)

C_h =Holding Cost ($/unit/year) = H.Wi

C_u = Shortage Cost ($/unit)

V_2 = Purchasing Cost Per Unit from Supplier 2 ($/unit)

S = Amount Ordered Per Cycle (units)

R = Amount Reserved from Supplier 2 (units)

r = Reserve Cost Per Unit from Supplier 2 ($/unit)

C_2 = Ordering Cost from Supplier 2 if an order is placed ($)

π = Percentage of Order Received from Supplier 1

μ = Expected Value Of π

π_1 = Highest percentage of π not equal to 100%

X = Amount Received from Supplier 1 (random variable)

$P(X)$ = Probability Distribution of X

Q = the quantity short during a cycle

T = Cycle Length, $T = S/D$ (unit time)

$TC(S)$ = Total Inventory Cost per Cycle

$TCU(S)$ = Total Inventory Cost per Unit Time, $TCU(S) = TC(S)/T$

The procedure presented in Chapter 4 for finding the optimal order quantity for an EOQ model with price discount is modified for the case of dual suppliers.

Procedure 5.2

Step 1. For each level *i*, find the optimal order quantity using Eq. (4.17)

$$S^* = \sqrt{\frac{(C_1 + PC_2)D}{(1-P)\frac{HW_i}{2} + P.\frac{HW_i}{2}(\mu + 1 - \pi_2)}}$$

Step 2. Adjust S_i^* according to:

If S_i^* is within the quantity range for level *i*, keep S_i^*.

If S_i^* is more than the largest quantity of its range, ignore level *i*.

If S_i^* is less than the smallest quantity of its range, set S_i^* equal to the smallest quantity of its range.

Step 3. For each level *i*, use the adjusted values of S_i^* to calculate $TCU(S_i^*, R^*)$ by modifying Eq. (4.15)

$$ETCU(S_i^*) = \frac{ETC(S_i^*)}{T} = W\mu D + C_1\left(\frac{D}{S_i^*}\right) + (1-P)C_h\left(\frac{S_i^*}{2}\right) + PC_h\left(\frac{S_i^*}{2}\right)(\mu + 1 - \pi_2) +$$

$$PVD(1-\pi_2) + PC_2\left(\frac{D}{S_i^*}\right) + rD(1-\pi_2) + Cu\, D\mu_Q.$$

Step 4. Select the S_i^* that results in a minimum value of $ETCU(S_i^*)$.

In the following, a numerical example is provided to illustrate the determination of the optimal solution using procedure 5.2.

5.4 A Numerical Example for an EOQ model with Discount Price and Shortage

The aim is to find the optimal order quantity if the supplier is offering a price discount according to the following schedule:

Level	**Quantity**	**Price per Unit**
1	1 to 4999	$5
2	5000 to 9999	4.5
3	10000 or more	3.5

The parameters of the problem are:

D = 10000 units/year

C_1 = $1,000

C_2 = $1,000

W_1 = $5 /unit,

W_2 = $4 /unit,

W_3 = $3.5 /unit

H = 20%

C_h = 20% (\$5) = \$ 1/unit/year (level 1)

C_h = 20% (\$4) = \$ 0.8/unit/year (level 2)

C_h = 20% (\$3.5) = \$ 0.7/unit/year (level 3)

V_2 =\$ 7/unit

r =\$ 0.5 /unit

C_U = \$2/unit

X = Amount Received from Supplier 1

The percentage of order received from supplier 1 is represented as below:

π	100%	90%	80%
$P(\pi)$	0.7	0.2	0.1

The amount received from supplier 1 is represented as below:

X	100%*S	90%*S	80%*S
$P(X)$	0.7	0.2	0.1

The amount received from supplier 2 if ordered depends on the random variable Y defined wise probability distribution is:

Y	0	1
P(Y)	0.7	0.3

The expected value μ of the random variable π is

$$\mu = \Sigma\ \pi\ P(\pi) = 1(0.7) + 0.9(0.2) + 0.8(0.1) = 0.96$$

In addition, the probability P of ordering from supplier 2 is

$$P = P(Y = 1) = P(X \neq S) = 0.3$$

Suppose that S is 1000 units. Since $\pi 1 = 90\% = 0.9$, the reservation quantity is

$$R = S - \pi_1 S = 1000 - 0.9 * 1000 = 100 \text{ units.}$$

$$\mu Q = \pi_3 * P(\pi_3) = 0.8 * 0.1 = 0.08$$

Note that shortages may occur in this case. Since $X = \pi S$ is the amount received from supplier 1, the expected value of X is

$$E(X) = S\ E(\pi) = \mu S \quad = 0.96 * 1000 = 960.$$

Since $Y = \begin{cases} 0 & if \quad X = S \\ 1 & if \quad X \neq S \end{cases}$,

E and π_1 = 0.9.

Step 1. For each level I find the optimal order quantity

Level 1: $S_1^* = \sqrt{\frac{(C1+PC2)D}{(1-P)\frac{HW1}{2}+P.\frac{HW1}{2}(\mu+1-\pi 2)}} = \sqrt{\frac{(1000+03*1000)*10000}{(1-0.3)\frac{1}{2}+0.3*\frac{1}{2}(0.96+1-0.9)}} = \sqrt{\frac{13000000}{0.509}}$ = 5053.73 ≈ 5054 units

Level 2: $S_2^* = \sqrt{\frac{(C1+PC2)D}{(1-P)\frac{HW2}{2}+P.\frac{HW2}{2}(\mu+1-\pi 2)}} = \sqrt{\frac{(1000+0.3*1000)*10000}{(1-0.3)0.4+0.3*0.4(0.96+1-0.9)}} = \sqrt{\frac{13000000}{0.4072}}$ = 5650.25 ≈ 5650 units

Level 3: $S_3^* = \sqrt{\frac{(C1+PC2)D}{(1-P)\frac{HW3}{2}+P.\frac{HW3}{2}(\mu+1-\pi 2)}} = \sqrt{\frac{(1000+0.3*1000)*10000}{(1-0.3)0.35+0.3*0.35(0.96+1-0.9)}} = \sqrt{\frac{13000000}{0.3563}}$ = 6040.37 ≈ 6040 units

(Y) = P (X≠S) =P = 0.3.

Step 2. Adjusting the values of S_i^*,

Level 1: Since $S_1^*= 5054$ units is outside its range and higher than the largest value in the range, ignore this level. (1 to 4999)

Level 2: Since $S_2^* = 5650$ units is within its range, we keep this level (5000 to 9999).

Level 3: Since S_3^*= 6040 units is outside the range and less than the smallest value in its range, it is adjusted to S_3^* = 10,000 units.

Step 3. For each level *i*, the adjusted values of S_i^* are used to calculate $ETCU(S_i^*)$ based on Eq. (4.15)

Level 1: Level 1 is ignored.

Level 2: Using the adjusted S_2^* = 5650.25 units

$$ETCU(S_2^*) = \frac{ETC(S_2^*)}{T} = W_2\ \mu D + C_1\left(\frac{D}{S_2^*}\right) + (1-P)\frac{HW_2}{2}\left(\frac{S_2^*}{2}\right) + P\frac{HW_2}{2}\left(\frac{S_2^*}{2}\right)(\ \mu\ + 1 - \pi_2\ ) +$$

$$PV_2D(1-\pi_2) + PC_2\left(\frac{D}{S_2^*}\right) + rD(1-\pi_2) + Cu\ D\mu_Q = \$\ 47201.6 \text{ per year}$$

Level 3: Using the adjusted S_3^*= 10000 units

$$ETCU(S_3^*) = \frac{ETC(S_3^*)}{T} = W_3\ \mu D + C_1\left(\frac{D}{S_3^*}\right) + (1-P)\frac{HW_3}{2}\left(\frac{S_3^*}{2}\right) + P\frac{HW_3}{2}\left(\frac{S_3^*}{2}\right)(\ \mu\ + 1 - \pi_2\ ) +$$

$$PV_2D(1-\pi_2) + PC_2\left(\frac{D}{S_3^*}\right) + rD(1-\pi_2) + Cu\ D\mu_Q = \$\ 48922.3 \text{ per year.}$$

Chapter six

Conclusion, Discussion, Limitations and Implications

This chapter provides a summary of the results and highlights the importance of findings. In addition, recommendations for future research are provided.

6.1 Conclusion and Discussion

The purpose was to extend the EOQ model of an unreliable supplier using several mathematical models. These were formulated and the case of optimal solution was derived. Chapter one encompasses on an introduction consisting of three parts. The background of the topic is first presented. Then, the objectives of the thesis are highlighted. Finally, the organization of the thesis is given. Chapter two includes a literature review about supply chain disruptions and how researchers derived solutions .The definitions of the classical models used in the paper are also explained and demonstrated with some examples. In chapter three, we considered an EOQ model for a product that can be obtained from supplied two suppliers, one is unreliable and cheaper and another is reliable but more expensive. The order received from the unreliable supplier follows a discrete distribution. In chapter four we modified the model presented in chapter three to allow for shortages. The mathematical model is developed and an expression for the optimal solution is derived. In addition, a numerical example is presented. In chapter five we modified the models presented in chapter three to allow for price discount offered by the unreliable supplier. The procedure is proposed for finding the optimal solution with and without shortage. In addition, numerical examples are presented

Moreover, programs on excel worksheet were developed to do the calculation.

6.2 Implications, Limitations, and Future research

During the COVID-19 pandemic, supply chain managers were faced with unprecedented challenges, risks, and disruptions of supplies. Firms were redesigning their strategies to ensure the continuity, sustainability and resilience of their operations. This necessitated the development of new approaches to supply chain management by considering factors that may hinder or disrupt their operations. The mathematical model presented above can be used by managers to anticipate unreliable the delivery of orders and shortage. Managers can use this model in their real life to obtain the optimal solution in their daily operations.

The models developed in this thesis have certain limitations. Only costs were used to formulate the mathematical model; however, profits were ignored. Other limitation includes the assumptions that the proportion of the order received are discrete random variables. We suggest including the profit component. We consider the model with the profit not only the cost and continuous random variable of the proportion of the order delivered. Finally, the time value can be incorporated into the model presented in this case.

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