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A Production Model with Continuous Demand for Imperfect Finished Items Resulting from the Quality of Raw Material

By

Mohammad Nasr El Dine

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Student Name: Mohammad Nasr El Dine I.D. #: 201404447

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The undersigned certify that they have examined the final electronic copy of this thesis and approved it in Partial Fulfillment of the requirements for the degree of:

Master of Business Administration in the major of Business

Thesis Advisor's Name: Abdul-Nasser Kassar

Signature [redacted] Date: 15/08 /2020
Day Month Year

Committee Member's Name: Manal Yunis

Signature: Digitally signed by Manal Yunis Date: 15/08 /2020
Day Month Year

Committee Member's Name: Raed El-Khali

Signature: [redacted] Date: 15/08 /2020
Day Month Year

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And say, "My Lord, increase me in knowledge."

-Holy Quran [20:114]-

First and foremost, my profound gratitude to Allah (SWT), the Most Beneficent, the Most Merciful.

I dedicate this work to my parents who have always supported me and taught me that everything can be accomplished once you set your mind to it and invest your heart in it.

My mom, thank you for teaching me that even the sky isn't the limit. Thank you for all your encouragement and mid-night snacks! Thank you for waking up so early all those years to make sure we comfortably started each day. Thank you for always inspiring me to give my very best at everything I do. None of this would have been possible without your prayers.

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A Production Model with Continuous Demand for Imperfect Finished Items Resulting from the Quality of Raw Material

Mohammad Nasr El Dine

ABSTRACT

This thesis aims to investigate the production process having different qualities of a single type of raw material which is utilized in the production process of the finished good. From preceding studies, researchers usually discarded the idea of including imperfect quality items in production. This thesis, however, regards the case in which the two types of raw material, the perfect and the imperfect qualities items, are not discarded but rather used in to produce the finished product. This consequently results in the production of two types of finished product, perfect and imperfect qualities, respectively. That being said, it is also assumed that the two types of finished products have continuous. It must be noted that this modeling approach still has not been researched. The development of two models that are subject to the inventory cycle length of each type of raw material is done in this thesis.
For that, and to better illustrate how optimal production quantity is determined, numerical examples are provided. In this thesis, we present theoretical implications with recommendations being provided in the end.

Keywords: Economic Order Quantity, Economic production Quantity, Inventory, Quality, Perfect Quality, Imperfect Quality, Raw Material.

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EOQ: Economic Ordering Quantity (model)

EPQ: Economic Production Quantity (model)

Chapter One

Scope of the Study

This chapter includes an introduction, which ends with highlights of the research procedure, and a literature review that presents a brief history of this study, as well as the current trends of this field.

1.1 Introduction

Inventory management and production control are the most important functions in a business model, having the main objective of managing materials used in production and manufacturing. The reason these two functions are of that much importance consists of the fact that they direct and control the level of inventory and help organizations avoid shortages and excess inventory, hence keep a balance while still being able to satisfy the demand of consumers.

Globally speaking, demands are growing rapidly, and thus organizations must have the ability to quickly respond to the fluctuating demand of the costumers.

The two most used inventory control techniques are the standard models. First, the economic order quantity, its most basic, while the second, the standard economic production quantity, deals with the production of the inventory items. The identification of optimal quantities of raw materials to order is done by the economic production quantity and the economic order quantity for manufacturing and purchasing processes. According to the standard Economic Production Quantity model, the manufacturing process is perfect with no failures, in other words, everything produced is of perfect quality. This, however, is inconsistent with real-life production situations (Pal et al, 2016), in which flawed items are produced as a result of having defective raw material or an error in the processes. Our thesis agrees with such a viewpoint, and asserts that the production policy put by the classical Economic Production Quantity model is inaccurate and does not take into consideration real-life situations and what might happen during production.

Aiming to include real-life conditions, several researchers addressed this matter through extending the standard Economic Order Quantity and Economic Production Quantity models by relaxing their underlying assumptions, such as quality, the value of money, and even credit facilities. This examination course was started by Salameh and Jaber (2000), who built up an Economic Production Quantity model that meant the flawed things conveyed by a provider with a known distribution function. Right after, Hayek and Salameh (2001) proposed an Economic Production Quantity model where imperfect quality items were taken into account. Lately, a sum of research has considered Economic Production Quantity / Economic Order Quantity models with imperfect quality raw materials as being part of the production process. A study by Salameh and El-Kassar (2007), considered the Economic Production Quantity model taking into account the raw material utilized to produce the finished product. El-Kassar (2009) gave an Economic Order Quantity model quality in which there is an ongoing demand for the two types of products received from a supplier, items of perfect quality, and items of imperfect quality.
Moving on, several current studies have regarded the quality factor of raw material used in production and its effect on the process. For instance, Yassine (2016) considered an assembly model that uses several components whereby each type contains some perfect and some imperfect components. Yassine and AlSagheer (2017) determined the optimal solution for a production model with shortages and raw materials. Yassine et al. (2018) introduced a bundling problem with quality. Yassine and El-Rabih (2019) incorporated the probabilistic lead times for the raw materials used in production. Yassine (2018) proposed a sustainable production model with quality. Partnerships have been participating in environmentally friendly exercises that improve execution (El-Kassar and Singh, 2019; Singh et al., 2019, El-Khalil and El-Kassar, 2018; El-Khalil and El-Kassar, 2016). These studies concluded that such practices lead to favorable results. For example, ElGammal et al., (2018) showed that these practices can lead to a more significant level of governance, El-Kassar et al. (2017) suggested they can generate ideal employee mentality and conduct. Additionally, the utilization of data, reporting skills, and development can result in enhancing their competitive level (Singh and El-Kassar, 2019; Yunis et al.,

2017; Yunis et al., 2018). As of late, these variables have been consolidated into the old-style Economic Production Quantity model (Lamba et al., 2019; Yassine, 2018). In this thesis, we present an Economic Production Quantity model that accounts for the usage of a type of items of raw material that are of two different qualities. In production, we use both types of raw material that are of perfect and imperfect qualities, hence resulting in both qualities of products. As we said earlier, we assumed that the demand for both the quality of finished products is continuous. Two models, within every two cases, relying on the inventory cycle length of the two types of the finished items, are presented. Add to that, the perfect quality raw material, consequently, the perfect quality finished products have the same random variable qj having a known probability density function $f_j(q_j)$.

The first model regards the producer as the sole decision-maker, with two cases respectively. In this model, products of different qualities (perfect and imperfect) are randomly produced, yet for the first case, perfect quality products are more, and hence imperfect quality is sold out first. The second case has imperfect quality products to be more, and thus perfect quality is sold out first. For that, the left-over products of imperfect quality are sold once at a lower price.

The second model considers collaboration between the producer and the supplier of raw material in making the decision. Both cases of the first model were used in the second (i.e. first case perfect are produced more, and in the second case imperfect are produced more).

For each case of each model, the problem is modeled mathematically with the optimal solution achieved by maximizing the function of the total profit. In the first model, the manufacturer's profit function is maximized, on the other hand, a maximization of the supply chain function is done in the second model. The results are to be compared with the problem. Moreover, to be able to investigate the effects of changes in the various parameters on the optimal solution, sensitivity analysis is conducted, with computations performed using Microsoft Excel. Numerical examples are given to illustrate the model.

1.2 Literature Review

The body of literature shows that the two basic inventory models are an active area of research that has been increasingly growing. This could be attributed to the fact that with globalization and increasing competition, business organizations have considered the importance of supply chain management. The supply chain model, a chain of suppliers, manufacturers, distributors, and customers, is considered by managers to obtain optimal levels of raw materials and goods production (Pal et al, 2016).

The history of the economic order quantity dates back to 1913 when F.W. Harris laid the foundation for the present-day inventory models. In 1934, Wilson developed a statistical strategy to obtain order points where the demand rate is considered constant. Moreover, the famous assumption regarding the basic Economic Order Quantity model is that everything produced is of perfect quality, is invalid due to various causes which include errors in production processes, issues within the facilities, or even problems that occur during transportation.

For that, a decent amount of effort has been made to break through the constraining assumptions of the Economic Order Quantity Model, by creating lot-sizing models when quality has imperfections. This issue was first encountered by Karlin in 1958, who studied the assumptions that are associated with inventory cost. Porteus then investigated the effect of defective items on the standard Economic Order Quantity model in 1986. During the same year, Rosenblatt and Lee suggested production to be in smaller lots in the case of imperfections. Besides, they considered that the flawed items are altered immediately and that the time difference between in-control and out-of-control states of a process depends on an exponential distribution. A year later, Lee and Rossenblatt (1987) wanted to detect the change to an out-of-control state, so they desired that process inspection to be done during the production run. Later in 1988, Gerchak et al studied a production problem with a single-period, having a production process being under variable yield and uncertain demand. Furthermore, the single-period model was then relaxed into an *n*-period model. The

Economic Order Quantity model presented by Cheng in 1991, had the demand depend on the cost of production per unit and the processes of imperfect production. The formulation of the inventory model is a geometric program that was designed to deduce optimal solutions. The model which describes the learning effects in the production process was modeled by Urban. Urban's model (1992) represented a process' rate of defection as a function of the running length. For imperfect production processes, Ben-Daya presented multiple-stage lot-sizing models in 1999. In 2000, Salameh and Jaber came up with an Economic Order Quantity model in which the resulting products of imperfect quality are sold under reduced pricing. In 2003, Chan et al proposed an Economic Production Quantity model having imperfect products being are either rejected, reworked, or sold. In the same year, Chiu (2003) presented a more generalized model, considering a production function with a random defect rate. Another direction was set by Salameh & El-Kassar (2007) and El-Kassar (2009) to deal with imperfect items and defective finished products. They formulated a production model, where raw material will be used, and in which the demand is continuous for both perfect and imperfect quality products. Then in 2005, Rezaei expanded the model of Salameh and Jaber with the assumption that cycle shortages that result from imperfect quality items are to be completely backordered starting each cycle. As an addition to the work of Salameh and Jaber, Papachristos and Konstantaras (2006) took into consideration that the defective items are sold at the end of the interval of replenishment rather than at the end of the screening process.

Wahab and Jaber (2010) came up with an Economic Order Quantity model that taking into consideration the quality of the items. In their model, the demand is assumed known and constant, and the holding cost is governed by a linear function of the item's quality. In 2012, El-Kassar et al studied an Economic Production Quantity model in which the raw material received from the supplier includes a fraction of defective quality. Detecting the imperfect quality items is done with a complete screening process, and two scenarios are considered: (1) defective raw material items being sold at a reduced price, and (2) defective items are sent to the supplier at the termination of each inventory cycle.

On the other hand, Sarkar (2012) studied an Economic Order Quantity model with the assumption that defective products occur within every cycle under a progressive payment scheme and dependent demand. Accordingly, the better the item quality is, the higher would be the holding cost. Guchhait et al (2013) proposed a production-inventory model that incorporates an imperfect production process, imperfect items, and a time-variant holding cost. The authors considered different demand functions that are either material or time-dependent, with both production rate and the percentage of defected products considered decision variables. In 2014, the option of repairing defective products instead of buying new ones was proposed by Jaber et al. as a new variant for Salameh and Jaber's model. After that, Taleizadeh et al. went even further in 2016 and developed an Economic Order Quantity model where the defective items are sent to repair stores. Not only that but also they assumed that shortages are partially backordered which is a continuation of the model put forth by Jaber et al. (2014). More recently, Kundu and Chakrabarti (2015) observed a production model of imperfect quality, which presents the random production of defective items, with the assumption that defective items are reworked. The objective was to minimize the total cost of the inventory-production system. Moreover, Tripathi and Singh (2015) proposed stock-dependent demand models, and Yadav and Swami (2018) developed an Economic Production Quantity model for defective items assuming a time-variant demand rate and incompletely backlogged shortages reworked. Sharifi et al (2015) introduced an Economic Order Quantity model for imperfect quality with the assumption of partial backordering shortages as well as errors resulting from the screening process. In the year 2017, Pal and Mahapatra studied distinct ways of dealing with a defective product. In the most recent years, Economic Order Quantity modeling took an interesting course towards sustainability, when Kazemi et al. (2019) studied an Economic Order Quantity model of imperfect quality considering carbon emissions.

Chapter Two

Decision by Producer

In this chapter we assess the production model and find the optimal ordering cost that grants the producer the maximum total profit, taking into consideration that the decision is solely made by the producer. In these models, we are assuming that we are receiving the perfect and imperfect quality of raw material and this demand for raw material is continuous. We then undergo production using perfect and imperfect raw material, producing perfect and imperfect finished products respectively, with different demand rates and salvage values. The results of this chapter were published in the proceedings of the ICORES 2020 conference (El-Kassar, Yunis, & Nasr El Dine, 2020).

2.1 Mathematical Model

Take into consideration a case where the received raw material from the supplier are of different qualities, perfect and imperfect, a production process that uses both types is investigated. Thus, this results in finished items that are also of two types of quality, perfect and imperfect, respectively. Moreover, the assumption that a continuous demand for items of the finished product of both qualities is imposed.

2.1.1 Notation

The parameters used in all cases are found below:

- Q: Units ordered each cycle
- Q^*: Units ordered each cycle (optimal)
- D: Rate of demand
- D_p: Rate of demand for finished products (perfect quality)
- D_i: Rate of demand for finished products (imperfect quality)
- C: Unit cost of purchase
- C_{sup}: Unit cost of purchase (supplier)

- C_p: Production cost per unit
- K_o: Raw material ordering cost
- K_{sup}: Ordering cost (supplier)
- K_s: Production set-up cost
- C_s: Cost of screening per unit
- C_{hr}: Raw material holding cost
- C_{hf}: Finished product holding cost
- C_{hs}: Raw material holding cost (Supplier)
- q: Portion of raw material that is of perfect quality
- S_p: Price of products that are of perfect quality
- S_i: Price of products that are of imperfect quality
- S_d: Price of products that are of imperfect quality (discounted)
- T: Length of the inventory cycle
- T_p: Length of the inventory cycle (perfect)
- T_i: Length of the inventory cycle (imperfect)
- T_s: Raw material screening duration
- T_{pr}: Length of the production cycle
- X: Rate of raw material screening
- P: Total rate of production
- P_p: Rate of production (perfect)
- P_i: Rate of production (imperfect)
- N: Production cycles executed by the producer with respect to supplier

2.1.2 Formulation of Model 1: $T_p > T_i$

The following model was developed to find the optimal number of units ordered to be produced. This number, denoted by Q*, maximizes the value of the function representing the total profit during one unit of time.

Figure 1 illustrates the levels of raw material and finished products' inventories of both qualities perfect and imperfect. A random variable denoted by 'q', is the fraction of raw material that is perfect with an identified probabilistic distribution and an expected value denoted by E[q].

Screening is done to find the percentage of perfect quality material ‘q’, for that, the perfect quality raw material used in production is ‘qQ’. The remaining raw material items, which are of imperfect quality, are used in the production process at a value of ‘(1-q)Q’. An assumption of linearity is imposed on the relationship among the rates of production of the two types of qualities of the items produced. The combined production period and the inventory cycle for both the perfect and imperfect finished items is the following:

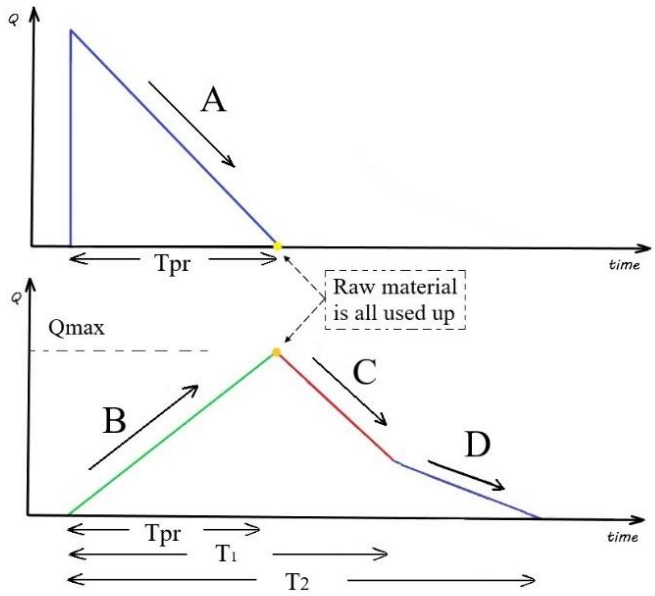

Figure 1. Inventory Levels: Raw Material (top) and Finished Items (bottom) for Tp ≥ Ti

The letters A, B, C, and D represent:

A. The raw material is being used in production
B. Production while satisfying demand for perfect and imperfect quality products
C. Satisfying demand for perfect and imperfect quality products
D. Satisfying demand for perfect quality products

As seen in Figure 1, the raw material inventory level and the finished goods inventory level are depicted. Items of raw material received at the start of the production run are screening resulting in a percentage of perfect quality raw material (q). Since the production rates of the two types of finished items are assumed to follow a linear association, the finished products of perfect quality are proportional constants.

TC(Q) which is the total cost per production cycle includes several costs, of which we mention:

- Manufacturing set-up cost
- Raw material ordering cost
- Manufacturing cost,
- Raw material purchasing cost
- Cost of holding items of raw material
- Cost of holding manufactured products
- Cost of raw material screening.

The parameters used in the production period are the following:

$$T_{pr}=\frac{Q}{P} \qquad (1)$$

$$T_p=\frac{qQ}{D_p} \qquad (2)$$

$$T_i=\frac{(1-q)Q}{D_i} \qquad (3)$$

Let T_1 = min{T_p, T_i}and let T_2 = max{T_p, T_i}. Assuming that $T_p > T_i$, we have that $T_1 = T_i$ and $T_2 = T_p$. Therefore,

$TC(Q) = K_o + K_s + CQ + C_pQ + C_sQ + C_{hr} \times$ [Area under top curve of Figure 1] + $C_{hf} \times$ [Area under bottom curve of Figure 1]

To begin with, we should first get the area underneath the upper curve of Figure 1. One must note that the level of inventory is depleted as we produce at a rate of production ‘P’. Hence, the processing of raw material, the slope of A, occurs at a rate of '-P'.

The area under the top curve of Figure 1 is given by:

$$A_{triangle}=\frac{1}{2}T_{pr}Q$$

When finding the area under the curve of the bottom part of Figure 1, we notice that final products are manufactured at a rate of ‘P’ and sold, or utilized at the demand rate, given by $D = D_p + D_i$, during the production period. There is also an assumption that $P > D$ to avoid shortages. Henceforth, inventory of manufactured products is added up at a degree of ‘$P – (D_p+D_i)$’ until we achieve the inventory’s maximum level Q_{max} that is attained whenever the production process terminates. Therefore, we note that the slope of B is ‘$P – (D_p+D_i)$’ and $Q_{max} = T_{pr}[P – (D_p+D_i)]$

The area under the bottom curve of Figure 1 is given by:

Next, the area below the inventory level of the finished items, Figure 1, is calculated using three parts as shown in Figure 1 (green, red, and blue).

i. First, we determine the area under the ***green curve*** as follows.

$$A_{triangle}= \frac{b\times h}{2}, \text{ where}$$

$b = T_{pr,}$ $h = Qmax$, with $P– (D_p+D_i)$ being the hypotenuse’s slope. Hence,

$$Q = [P – (D_p+D_i)]t + 0 \text{ at } t = T_{pr},$$

$$Q = Qmax = T_{pr}[P – (D_p+D_i)].$$

Thus,

$$A_{triangle}= \frac{1}{2} Tpr^2 [P – (D_p+D_i)]$$

ii. Seconds, we find the area underneath the ***red curve***. It is to be noted that the inventory of manufactured products (finished) is used up at a rate of $–D$ which is $– (D_p+D_i)$.

$$A_{trapezoid} = \frac{(b_1+b_2)\times h}{2}$$

$$b_1 = T_{pr}[P – (D_p+D_i)]$$

$$h = T_1 – T_{pr}$$

With the slope of the hypotenuse $C = -(D_p+D_i)$ consequently the equation would be: $Q = -(D_p+D_i)t + Q_{o.}$ At $t=T_{pr}$, $Q = Qmax = T_{pr}[P – (D_p+D_i)]$. Hence, $T_{pr}[P – (D_p+D_i)] = -(D_p+D_i) T_{pr} + Q_o$

Thus $Q_o = T_{pr}P$, and hence $Q = – (D_p+D_i) t + T_{pr}P$. When $t=T_1$, $Q = -(D_p+D_i) T_1 + T_{pr}P$. Thus,

$$A_{trapezoid} = \frac{1}{2} \{ T_{pr}[P – (D_p+D_i)] + T_{pr}P – (D_p+D_i) T_1\}\{ T_1 – T_{pr} \}$$

iii. Third, we find the area underneath the ***blue curve***. At this stage, the only remaining products are of perfect quality. These items are used up at a rate of $–D_p$. which is the slope.

$A_{triangle} = \frac{1}{2}\,[T_{pr}P - (D_p+D_i)\,T_1\,] \times [\,T_2 - T_1]$

Therefore,

$$TC(Q) = K_o + K_s + CQ + C_pQ + C_sQ + \frac{1}{2}C_{hr} \times (T_{pr}Q) + \frac{1}{2}C_{hf} \times \Big[T_{pr}^2\,(P - (D_p + D_i)) + \big(T_{pr}[P - (D_p + D_i)] + T_{pr}P - T_1\,(D_p + D_i)\big)(\,T_1 - T_{pr}\,) + \big(T_{pr}P - T_1\,(D_p + D_i)\big)(\,T_2 - T_1)\Big] \quad (4)$$

Substituting for $T_{pr} = \frac{Q}{P}$, $T_1 = \min\{T_p, T_i\}$, and $T_2 = \max\{T_p, T_i\}$ and assuming that $T_p > T_i$, we have $T_p = \frac{qQ}{D_p}$, and $T_i = \frac{(1-q)Q}{D_i}$. Hence, $T_1 = T_i$ and $T_2 = T_p$. Then, the TC(Q) function becomes:

$$TC(Q) = K_o + K_s + CQ + C_pQ + C_sQ + \frac{1}{2}C_{hr} \times \left(\frac{Q^2}{P}\right) + \frac{1}{2}C_{hf} \times \left[\left(\frac{Q}{P}\right)^2\,\big(P - (D_p + D_i)\big) + \left(\left(\frac{Q}{P}\right)\,[P - (D_p + D_i)] + Q - \left(\frac{(1\text{-}q)Q}{D_i}\right)\,(D_p + D_i)\right)\left(\frac{(1\text{-}q)Q}{D_i} - \frac{Q}{P}\right) + \left(Q - \left(\frac{(1\text{-}q)Q}{D_i}\right)\,(D_p + D_i)\right)\left(\frac{qQ}{D_p} - \frac{(1\text{-}q)Q}{D_i}\right)\right] \quad (5)$$

Simplifying we get:

$$TC(Q) = K_o + K_s + CQ + C_pQ + C_sQ + \frac{1}{2}C_{hr} \times \left(\frac{Q^2}{P}\right) + \frac{1}{2}C_{hf} \times \left(\frac{Q^2}{D_i} - \frac{qQ^2}{D_i} - \frac{Q^2}{P} + \frac{qQ^2}{D_p} - \frac{(D_p + D_i)qQ^2}{D_pD_i} + \frac{(D_p + D_i)q^2Q^2}{D_pD_i}\right) \quad (6)$$

TO find the total profit we have:

$TR(Q) = S_pqQ + S_i(1-q)Q$

$$TP(Q) = S_pqQ + S_i(1-q)Q - \left[K_o + K_s + CQ + C_pQ + C_sQ + \frac{1}{2}\,C_{hr} \times \left(\frac{Q^2}{P}\right) + \frac{1}{2}\,C_{hf} \times \left(\frac{Q^2}{D_i} - \frac{qQ^2}{D_p} - \frac{Q^2}{P} + \frac{qQ^2}{D_p} - \frac{(D_p + D_i)qQ^2}{D_pD_i} + \frac{(D_p + D_i)q^2Q^2}{D_pD_i}\right)\right] \quad (7)$$

TP(Q)'s expected value is:

$$E[TP(Q)] = S_pQ \cdot E[q] + S_iQ \cdot E[1-q] - K_o - K_s - CQ - C_pQ - C_sQ - \frac{1}{2} C_{hr} \times \left(\frac{Q^2}{P}\right) - \frac{1}{2} C_{hf} \times \left(\frac{Q^2}{D_i} - \frac{Q^2}{P}\right) - \frac{1}{2} C_{hf} \times \left(\frac{Q^2}{D_p} - \frac{Q^2}{D_i} - \frac{(D_p+D_i)Q^2}{D_pD_i}\right) \cdot E[q] - \frac{1}{2} C_{hf} \times \left(\frac{(D_p+D_i)Q^2}{D_pD_i}\right) \cdot E[q^2]$$

From the theorem of renewal reward, we have $E[TPU(Q)] = \frac{E[TP(Q)]}{E[T]}$, where $T = T_p = \frac{qQ}{D_p}$. Thus, for practical purposes of presentation, let:

$$a \equiv S_pQ \cdot E[q] + S_iQ \cdot E[1-q] - K_o - K_s - CQ - C_pQ - C_sQ - \frac{1}{2} C_{hr} \times \left(\frac{Q^2}{P}\right)$$

$$b \equiv \frac{1}{2} C_{hf} \times \left(\frac{Q^2}{D_i} - \frac{Q^2}{P}\right) + \frac{1}{2} C_{hf} \times \left(\frac{Q^2}{D_p} - \frac{Q^2}{D_i} - \frac{(D_p+D_i)Q^2}{D_pD_i}\right) \times E[q]$$

$$c \equiv \frac{1}{2} C_{hf} \times \left(\frac{(D_p+D_i)Q^2}{D_pD_i}\right) \cdot E[q^2]$$

$$d \equiv \frac{Q}{D_p} \cdot E[q]$$

Therefore, $E[TPU(Q)] = \frac{a-b-c}{d}$

Upon solving for Q we get:

$$Q^* = \sqrt{\frac{K_o+K_s}{\frac{1}{2} C_{hr} \times \left(\frac{1}{P}\right) + \frac{1}{2} C_{hf} \times \left(\frac{1}{D_i} - \frac{1}{P}\right) + \frac{1}{2} C_{hf} \times \left(\frac{1}{D_p} - \frac{1}{D_i} - \frac{(D_p+D_i)}{D_pD_i}\right) \cdot E[q] + \frac{1}{2} C_{hf} \times \left(\frac{(D_p+D_i)}{D_pD_i}\right) \cdot E[q^2]}} \quad (8)$$

Uniqueness:

$$\frac{d^2TPU(Q)}{dQ^2} = -\frac{\left(K_o+K_s+\frac{1}{2}\times C_{hr}\times Q^2\right)}{\frac{q\times Q^3}{D_p}} < 0$$ for all $Q > 0$, hence it is unique

2.1.2.1 Numerical Example

To better explain this model, let us consider a case where a carpenter manufactures and sells wooden closets to retailers. The carpenter manufactures single door closets using a wooden board he orders from a supplier. It is predicted that the range [70%→ 90%] is the percentage of perfect quality boards. However, after screening ($0.03 per item) is done, we come to realize that a portion of the boards have a height of 180cm and others have a height of 160cm. We thus depict the 180cm boards as perfect and 160cm boards as imperfect. Each day, the carpenter can manufacture per day 300 closets that are of perfect quality (i.e. 180cm), and 100 closets that are of imperfect quality (i.e. 160cm), with \$10 being the production cost. The wood board is purchased at \$4 from the supplier, and the cost of placing an order is \$1,000, and the production set-up cost is \$250.

Every 180cm closet is sold at 450\$ and the 160cm is sold at 300\$. The cost of holding the wooden boards per unit per day is 0.01\$. The cost of holding closets per unit per day is 0.02\$. With the demand being as follows:

- 100 units of 180cm closets per day
- 50 units of 160cm closets per day

How many wooden boards should the carpenter order, and what would be his total profit?

$$E[q] = \mu = \frac{0.7+0.9}{2} = 0.8$$

$$E[q^2] = Var(q) + (E[q])^2 = 0.00367+0.64= 0.64367$$

$$Q^* = \sqrt{\frac{K_o+K_s}{\frac{1}{2} C_{hr} \times \left(\frac{1}{P}\right) + \frac{1}{2} C_{hf} \times \left\{ \left(\frac{1}{D_i} - \frac{1}{P}\right) + \left(\frac{1}{D_p} - \frac{1}{D_i} - \frac{(D_p+D_i)}{D_p D_i}\right) E[q] + \left(\frac{(D_p+D_i)}{D_p D_i}\right) E[q^2] \right\}}}$$

$$Q^* = \sqrt{\frac{1,250}{\frac{1}{2}(0.01)\times\left(\frac{1}{400}\right) + \frac{1}{2}(0.02) \times \left(\frac{1}{50} - \frac{1}{400}\right) + \frac{1}{2}(0.02) \times \left(\frac{1}{100} - \frac{1}{50} - \frac{(150)}{5000}\right) \times (0.8) + \frac{1}{2}(0.02) \times \left(\frac{(150)}{5000}\right) \times (0.64367)}}$$

$$Q^* = 4,541.6 \approx 4,542 \text{ units}$$

And the total profit per cycle is:

$$TP(Q) = S_p qQ + S_i(1-q)Q - \left[K_o + K_s + CQ + C_p Q + C_s Q + \frac{1}{2} C_{hr} \times \left(\frac{Q^2}{P}\right) + \frac{1}{2} C_{hf} \times \left(\frac{Q^2}{D_i} - \frac{qQ^2}{D_p} - \frac{Q^2}{P} + \frac{qQ^2}{D_p} - \frac{(D_p+D_i)qQ^2}{D_p D_i} + \frac{(D_p+D_i)q^2 Q^2}{D_p D_i}\right)\right]$$

$$TP(Q^*) = 450(0.75)(4542) + 300(0.25)(4542) - \left[1250 + 4(4542) + 10(4542) + (0.03)(4542) + \frac{1}{2}(0.01) \times (51574.41) + \frac{1}{2}(0.02) \times (244978.4)\right]$$

$= 1{,}805{,}893.1\$$

$$\text{TPU}\left(Q^*\right) = \frac{\text{TP}(Q^*)}{T} = \frac{\text{TP}(Q^*)}{\frac{qQ}{D_p}} = 53{,}013.2\$ \text{ per cycle.}$$

2.1.3 Formulation of Model 2: $T_p \leq T_i$

In this case, it is assumed that we have more imperfect than perfect products, in other words, the imperfect cycle is longer than the perfect products' cycle. For that, it is logical that the perfect products are sold out first and what is left are imperfect products and are sold in a single batch under a lower price of S_d, with $S_p > S_i > S_d$.

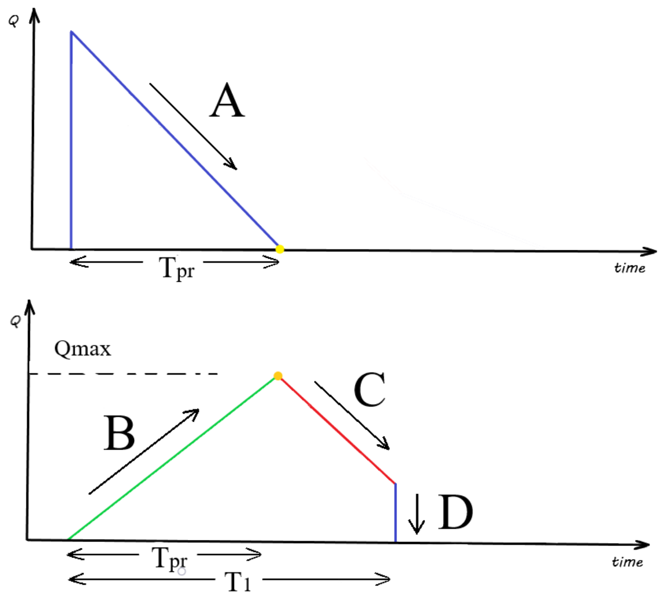

Figure 2. Inventory Levels: Raw Material (top) and Finished Items (bottom) for $T_p \leq T_i$

The letters A, B, C, and D represent:

A. The raw material is being used in production
B. Production while satisfying demand for perfect and imperfect quality products
C. Satisfying demand for perfect and imperfect quality products
D. Satisfying demand of imperfect quality products

In this case, $T_1 = \min\{T_p, T_i\} = T_p = \frac{(1-q)Q}{D_i}$. Similar steps followed in the previous case, yet with Figure 2, results in:

$$TC(Q) = K_o + K_s + CQ + C_pQ + C_sQ + \frac{1}{2} C_{hr} \times \left(\frac{Q^2}{P}\right) + \frac{1}{2} C_{hf} \times \left[\left(\frac{Q}{P}\right)^2 \left(P - (D_p + D_i)\right) + \left(\left(\frac{Q}{P}\right) [P - (D_p + D_i)] + Q - \left(\frac{(1\text{-}q)Q}{D_i}\right) (D_p + D_i)\right) \left(\frac{(1\text{-}q)Q}{D_i} - \frac{Q}{P}\right)\right] \quad (9)$$

The outstanding imperfect quality products are to be sold after T_1 in one batch under a selling price S_d. Hence, the amount of remaining finished products is $T_{pr}P - (D_p+D_i)$ T_1. Thus:

$$TR(Q) = S_pqQ + S_i(qQ)\left(\frac{D_p}{D_i}\right) + S_d\left[Q - \left(\frac{qQ}{D_p}\right) (D_p + D_i)\right] \quad (10)$$

That being said: $TP(Q) = TR(Q) - TC(Q)$

Hence TP(Q) is:

$$TP(Q) = S_pqQ + S_i(qQ)\left(\frac{D_p}{D_i}\right) + S_d\left[Q - \left(\frac{qQ}{D_p}\right) (D_p + D_i)\right] - K_o - K_s - CQ - C_pQ - C_sQ - \frac{1}{2} C_{hr} \times \left(\frac{Q^2}{P}\right) - \frac{1}{2} C_{hf} \times \left[\left(\frac{Q}{P}\right)^2 \left(P - (D_p + D_i)\right) + \left(\left(\frac{Q}{P}\right) [P - (D_p + D_i)] + Q - \left(\frac{(1\text{-}q)Q}{D_i}\right) (D_p + D_i)\right) \left(\frac{(1\text{-}q)Q}{D_i} - \frac{Q}{P}\right)\right] \quad (11)$$

The expected total profit is

$$E[TP(Q)] = QE(q) + S_i\left(\frac{QD_p}{D_i}\right)E(q) + S_dQ - Sd\left(\frac{Q}{D_p}\right) (D_p + D_i)\, E(q) - K_o - K_s - CQ - C_pQ - C_sQ - \frac{1}{2} C_{hr} \times \left(\frac{Q^2}{P}\right) - \frac{1}{2} C_{hf} \times \left\{\left[\frac{2Q^2}{D_i} - \frac{Q^2D}{D_i^2} - \frac{Q^2}{P}\right] + \left[\frac{2Q^2D}{D_i^2} - \frac{2Q^2}{D_i}\right] E(q) - \left[\frac{Q^2D}{D_i^2}\right] E(q^2)\right\} \quad (12)$$

Using $E[TPU(Q)] = \frac{E[TP(Q)]}{E[T]}$ with $T = T_p = \frac{qQ}{D_p}$, we have

For practical purposes of presentation, let:

$$a \equiv S_pQE(q) + S_i\left(\frac{QD_p}{D_i}\right) E(q) + S_dQ - S_d\left(\frac{Q}{D_p}\right) \left(D_p + D_i\right)E(q)$$

$$b \equiv Ko + Ks + CQ + CpQ + CsQ$$

$c \equiv \frac{1}{2}\, C_{hr} \times \left(\frac{Q^2}{P}\right)$

$d \equiv \frac{1}{2}\, C_{hf}\, Q^2 \times \left\{\left[\frac{2}{D_i} - \frac{D}{D_i^2} - \frac{1}{P}\right] + \left[\frac{2D}{D_i^2} - \frac{2}{D_i}\right] E(q) - \left[\frac{D}{D_i^2}\right] E(q^2)\right\}$

$e \equiv \frac{Q}{D_p} \cdot E[q]$

$$E[TPU(Q)] = \frac{a\text{-}b\text{-}c\text{-}d}{e}$$

The optimal order quantity is

$$Q^* = \sqrt{\frac{K_o+K_s}{\frac{1}{2} C_{hr} \times \left(\frac{1}{P}\right) + \frac{1}{2} C_{hf} \times \left\{\left(\frac{2}{D_i} - \frac{(D_i+D_p)}{D_i^2} - \frac{1}{P}\right) + \left(\frac{2(D_i+D_p)}{D_i^2} - \frac{2}{D_i}\right) \cdot E[q] - \left(\frac{(D_i+D_p)}{D_i^2}\right) \cdot E[q^2]\right\}}} \tag{13}$$

Uniqueness:

$\frac{d^2TPU(Q)}{dQ^2} = -\frac{2(K_o+K_s)}{\frac{q\times Q^3}{D_p}} < 0$ for all $Q > 0$, hence it is unique

2.1.3.1 Numerical Example

Recall the previous carpenter example, however, here we suppose that the range [60% → 80%] is the percentage of perfect quality boards, and now the carpenter is manufacturing tables. The tables of 3cm are considered perfect and 2cm denoted as imperfect. Each day the carpenter can produce 200 tables that are of perfect quality (i.e. 3cm thick), and 150 tables that are of imperfect quality (i.e. 2cm thick). The cost of production is $5 per table. The cost of purchasing a wooden board is 1$, and the cost of placing an order is 400$, with $100 being the production set-up cost. The tables that have a thickness of 3cm are sold at 30$ per piece and the 2cm thick tables are sold at 20$ per piece. The daily cost of holding the wooden boards per unit is 0.01$, and the daily cost of holding tables per unit is 0.015$. We come to realize that the 3cm tables got sold out before the 2cm tables. For that, and since 2cm are left, we will sell them at a reduced price of $15. The daily demand for tables is the following:

- 100 units for the tables of thickness 3cm
- 50 units for the tables of thickness 2cm

$E(q) = \mu = \frac{0.6+0.8}{2} = 0.7$

$E(q^2) = Var(q) + (E[q])^2 = 0.00367+0.49 = 0.49367$

$$Q^* = \sqrt{\frac{K_o+K_s}{\frac{1}{2}C_{hr} \times \left(\frac{1}{P}\right) + \frac{1}{2}C_{hf} \times \left\{\left(\frac{2}{D_i} - \frac{(D_i+D_p)}{D_i^2} - \frac{1}{P}\right) + \left(\frac{2(D_i+D_p)}{D_i^2} - \frac{2}{D_i}\right) \cdot E[q] - \left(\frac{(D_i+D_p)}{D_i^2}\right) \cdot E[q^2]\right\}}}$$

$$Q^* = \sqrt{\frac{500}{\frac{1}{2}(0.01) \times \left(\frac{1}{350}\right) + \frac{1}{2}(0.015) \times \left\{\left(\frac{2}{50} - \frac{150}{2500} - \frac{1}{350}\right) + \left(\frac{300}{2500} - \frac{2}{50}\right)(0.7) - \left(\frac{150}{2500}\right)(0.49367)\right\}}}$$

$Q^* = 3{,}504.7 \approx 3{,}505$ units

And the total profit per cycle is:

$$TP(Q) = S_p qQ + S_i(qQ)\left(\frac{D_p}{D_i}\right) + S_d\left[Q - \left(\frac{qQ}{D_p}\right)(D_p+D_i)\right] - K_o - K_s - CQ - C_pQ - C_sQ - \frac{1}{2}C_{hr} \times \left(\frac{Q^2}{P}\right) - \frac{1}{2}C_{hf} \times \left[\left(\frac{Q}{P}\right)^2 \left(P - (D_p+D_i)\right) + \left(\left(\frac{Q}{P}\right)[P - (D_p+D_i)] + Q - \left(\frac{(1-q)Q}{D_i}\right)(D_p+D_i)\right)\left(\frac{(1-q)Q}{D_i} - \frac{Q}{P}\right)\right]$$

$$TP(Q^*) = 30(0.62)(3{,}505) + 20(0.62)(3{,}505)\left(\frac{100}{50}\right) + 15\left[3{,}505 - \left(\frac{0.62 \times 3{,}505}{100}\right)(150)\right] - 400 - 100 - (3{,}505) - 5 \times (3{,}505) - 0.02 \times (3{,}505) - \frac{1}{2} \times 0.01 \times \left(\frac{3{,}505^2}{350}\right) - \frac{1}{2} \times 0.015 \times \left[\left(\frac{3{,}505}{350}\right)^2 (350 - (150)) + \left(\left(\frac{3{,}505}{350}\right)[350 - (150)] + 3{,}505 - \left(\frac{(0.38)(3{,}505)}{50}\right)(150)\right)\left(\frac{(0.38)(3{,}505)}{50} - \frac{3{,}505}{350}\right)\right]$$

$TP(Q^*) = 7{,}967{,}357.6\$$

$TPU(Q^*) = \frac{TP(Q^*)}{T} = \frac{TP(Q^*)}{\frac{qQ}{D_p}} = 366{,}636\$$ per cycle.

Chapter Three

Supply Chain

In this chapter, the decision making is a collaboration between the supplier of raw material and the producer of finished goods. This is done to maximize the total supply chain profit. Moreover, in this model, a completed cycle by the supplier is equivalent to 'N' cycles completed by the producer. Recall that the supplier provides the producer with 'Q'. Hence the inventory level of the supplier for this model is:

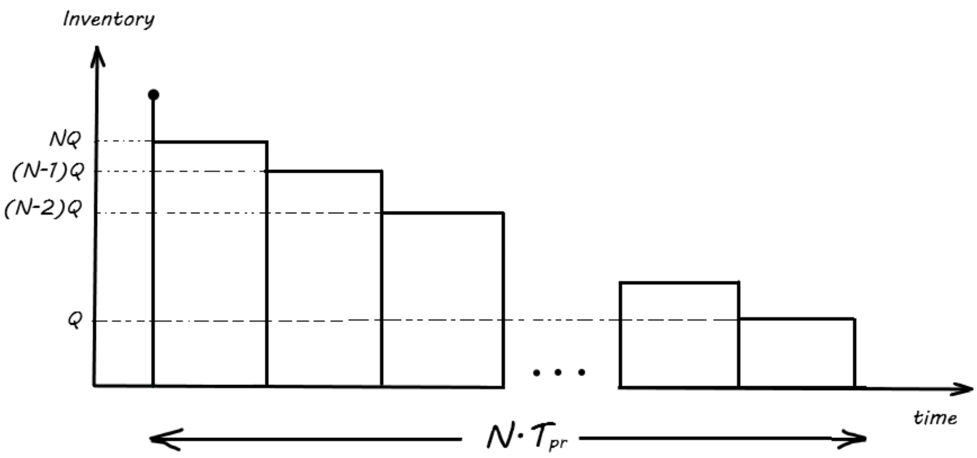

Figure 3. Supplier Inventory Level

3.1 Mathematical Model

The total revenue is actually what the supplier earned by selling the producer an amount of 'Q'.

TR(Q) = CQ

Total cost is the purchasing cost, ordering cost, and the holding cost.

$TC(Q) = C_{sup}NQ + K_{sup} + C_{hs}\times$[Area under curve of Figure 3]

Since the producer is ordering the value of 'Q', for that, the supplier's inventory is decreasing by the same amount. Hence it's an arithmetic sequence which is: $T_{pr}Q\sum_{i=1}^{N-i}(N-i) = T_{pr}Q\frac{N(N-1)}{2}$, and we have that $T_{pr}=\frac{Q}{P}$, thus the total cost becomes:

$TC(Q) = C_{sup}NQ + K_{sup} + C_{hs}\frac{Q^2}{P}\ \frac{N(N-1)}{2}$,

Hence the total profit $TP(Q) = TR(Q) - TC(Q)$

$= CQ - C_{sup}NQ - K_{sup} - C_{hs}\frac{Q^2}{P}\ \frac{N(N-1)}{2}$

$$TPU(Q) = \frac{TR(Q) - TC(Q)}{NT_{pr}} = \frac{CQ - C_{sup}NQ - K_{sup} - C_{hs}\frac{Q^2}{P}\frac{N(N-1)}{2}}{\frac{NQ}{P}} \qquad (14)$$

Since all the variables are not random, thus the expected value $E[TPU(Q)] = \frac{E[TP(Q)]}{E[T]}$ $= TPU(Q)$

As for the total profit of the supply chain per unit time:

$TPU(Q)_{supply\ chain} = TPU(Q)_{producer} + TPU(Q)_{supplier}$

Consequently, $E[TPU(Q)_{supply\ chain}] = E[TPU(Q)_{producer}] + E[TPU(Q)_{supplier}]$

3.1.1 Formulation of Model 1: $T_p > T_i$

For practical purposes of presentation, let:

$$a \equiv S_pQ \cdot E[q] + S_iQ \cdot E[1-q] - K_o - K_s - CQ - C_pQ - C_sQ - \frac{1}{2}C_{hr} \times \left(\frac{Q^2}{P}\right)$$

$$b \equiv \frac{1}{2}C_{hf} \times \left(\frac{Q^2}{D_i} - \frac{Q^2}{P}\right) + \frac{1}{2}C_{hf} \times \left(\frac{Q^2}{D_p} - \frac{Q^2}{D_i} - \frac{(D_p+D_i)Q^2}{D_pD_i}\right) \times E[q]$$

$$c \equiv \frac{1}{2}C_{hf} \times \left(\frac{(D_p+D_i)Q^2}{D_pD_i}\right) \cdot E[q^2]$$

$$d \equiv \frac{Q}{D_p} \cdot E[q]$$

$$E[TPU(Q)_{\text{supply chain}}] = \frac{a\text{-}b\text{-}c}{d} + \frac{CQ - C_{sup}NQ - K_{sup} - C_{hs}\frac{Q^2}{P}\frac{N(N-1)}{2}}{\frac{NQ}{P}}$$

Setting the above expression equal zero; solve for Q and get Q*:

$$Q^*_{\text{supply chain}} = \sqrt{\frac{\frac{K_oD_p}{E[q]}+\frac{K_sD_p}{E[q]}+\frac{K_{sup}P}{N}}{\frac{1}{2}\left\{\frac{C_{hr}\times D_p}{P\times E[q]}+C_{hf}\times\left[\frac{D_p}{D_i\times E[q]}-\frac{D_p}{P\times E[q]}+1+\frac{D_p+D_i}{D_i}\left(\frac{E[q^2]}{E[q]}-1\right)-\frac{D_p}{D_i}\right]+C_{hs}\times(N-1)\right\}}} \quad (15)$$

3.1.2 Formulation of Model 2: $T_p \leq T_i$

For practical purposes of presentation, let:

$$a \equiv S_pQE(q) + S_i\left(\frac{QD_p}{D_i}\right)E(q) + S_dQ - S_d\left(\frac{Q}{D_p}\right)(D_p + D_i)E(q)$$

$$b \equiv Ko + Ks + CQ + CpQ + CsQ$$

$$c \equiv \frac{1}{2}C_{hr} \times \left(\frac{Q^2}{P}\right)$$

$$d \equiv \frac{1}{2}C_{hf}Q^2 \times \left\{\left[\frac{2}{D_i}-\frac{D}{D_i^2}-\frac{1}{P}\right]+\left[\frac{2D}{D_i^2}-\frac{2}{D_i}\right]E(q)-\left[\frac{D}{D_i^2}\right]E(q^2)\right\}$$

$$e \equiv \frac{Q}{D_p} \cdot E[q]$$

$$E[TPU(Q)_{\text{supply chain}}] = \frac{a\text{-}b\text{-}c\text{-}d}{e} + \frac{CQ - C_{sup}NQ - K_{sup} - C_{hs}\frac{Q^2}{P}\frac{N(N-1)}{2}}{\frac{NQ}{P}}$$

Setting the derivative of the above expression equal to zero and solving for Q, we get:

$$Q^*_{\text{supply chain}} = \sqrt{\frac{\frac{K_o\times D_p}{E[q]}+\frac{K_s\times D_p}{E[q]}+\frac{K_{sup}\times P}{N}}{\frac{1}{2}\left\{\frac{C_{hr}\times D_p}{P\times E[q]}+C_{hs}(N-1)+C_{hf}\left[\frac{D_p}{E[q]}\times\left(\frac{2}{D_i}-\frac{(D_p+D_i)}{D_i^2}-\frac{1}{P}\right)-\frac{(D_p+D_i)\times D_p\times E[q^2]}{D_i^2\times E[q]}+D_p\times\left(\frac{2\times(D_p+D_i)}{D_i^2}-\frac{2}{D_i}\right)\right]\right\}}}$$

Uniqueness:

$$\frac{d^2TPU(Q)}{dQ^2} = -\frac{2\times(K_{sup})}{\frac{N\times Q^3}{D_p}} < 0 \text{ for all } Q > 0\text{, hence it is unique}$$

3.1.3 Numerical Example

Recall the numerical example of section 2.1.2.1, now the supplier is undergoing 2 cycles each time the producer completes a single cycle. The supplier acquired the boards for $1 each. It costs the supplier $500 to order the raw material for retail, and the holding cost is 0.01$ per unit per day. Repeat the problem and find $Q^*_{\text{supply chain}}$, and then find the total cost per unit time of the supply chain.

$Q^*_{\text{supply chain}} \approx 2{,}353$ units and the total cost per unit time of the supply chain is $TPU(Q)_{\text{supply chain}} = \frac{TP(Q^*)_{\text{producer}}}{T_p} + \frac{TP(Q^*)_{\text{supplier}}}{NT_{pr}} = \$53{,}363.05$

Chapter Four

Sensitivity Analysis

To be able to study how some variables affect the optimal number of units per order ‘Q*’, we undergo sensitivity analysis using excel, by varying a single variable at a time while holding others constant. The change in values for the variable is done linearly. For both models, we assumed the first case respectively (i.e. Tp > Ti).

4.1 Decision by Producer

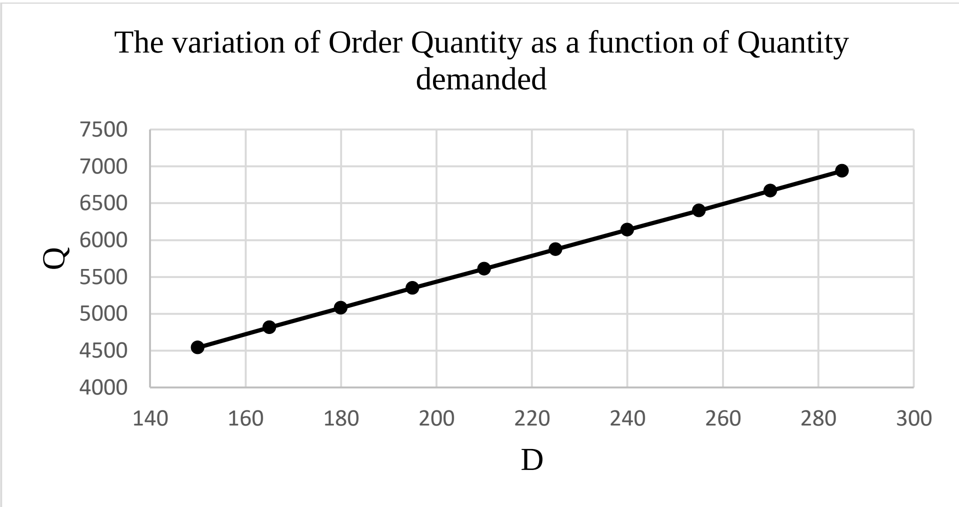

Figure 4. The variation of Order Quantity as a function of quantity demanded

Interpretation: As seen in Figure 4, as the quantity demanded increases, the quantity ordered by the producer increases linearly. The results are acceptable since a greater demand triggers a higher order for raw material.

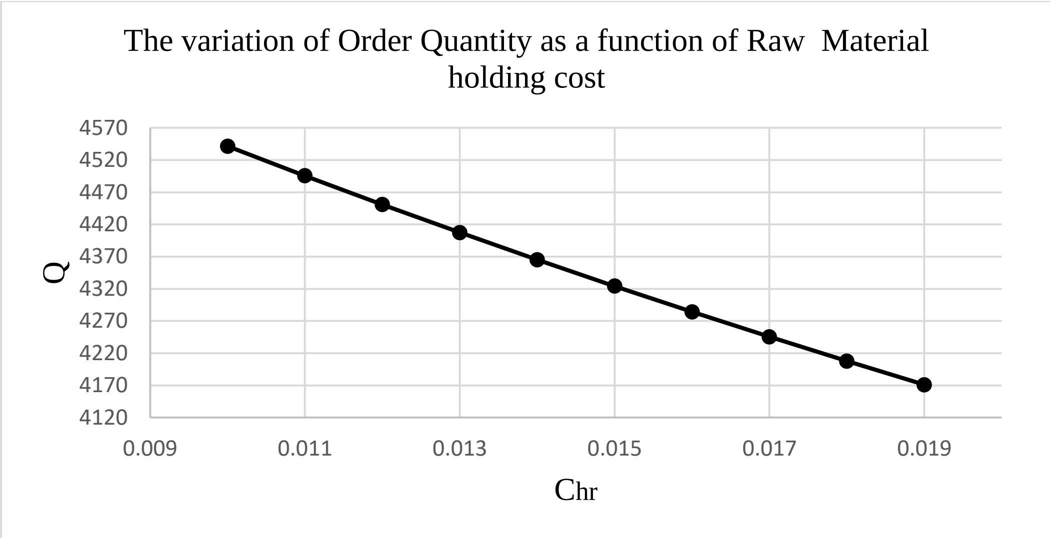

Figure 5. The variation of Order Quantity as a function of Raw Material holding cost

Interpretation: As seen in Figure 5, as the holding cost for raw material increases, the quantity ordered by the producer decreases linearly. The results are acceptable since as the cost of holding inventory increases, the manufacturer would decrease his inventory levels, consequently his order of raw material.

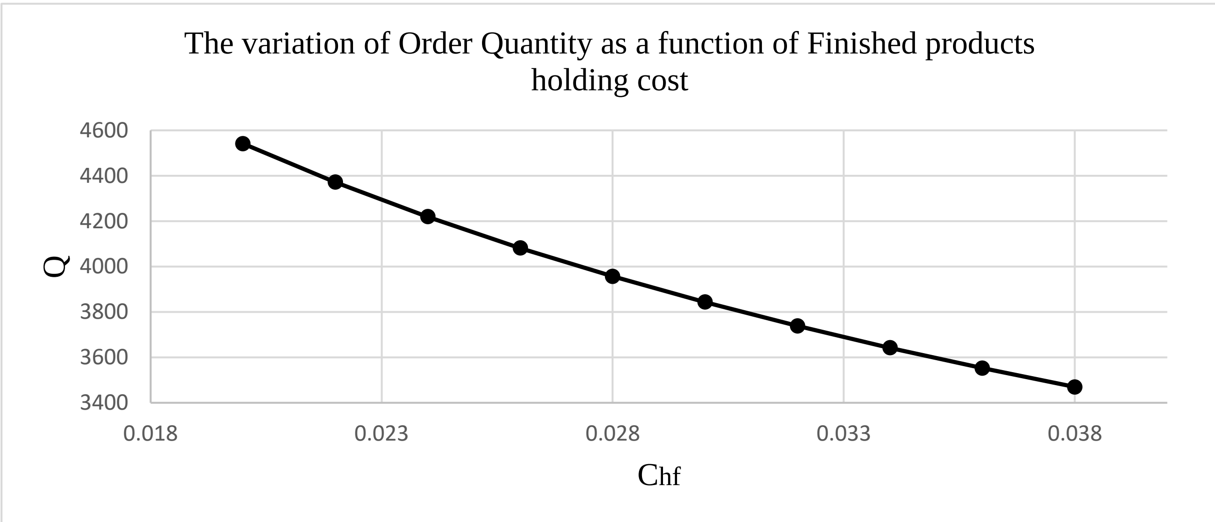

Figure 6. Variation of Order Quantity as a function of finished products holding cost

Interpretation: As seen in Figure 6, as the holding cost for finished products increases, the quantity ordered by the producer decreases. The results are acceptable since as the cost of holding inventory increases, the manufacturer would decrease his inventory levels, consequently his order of raw material.

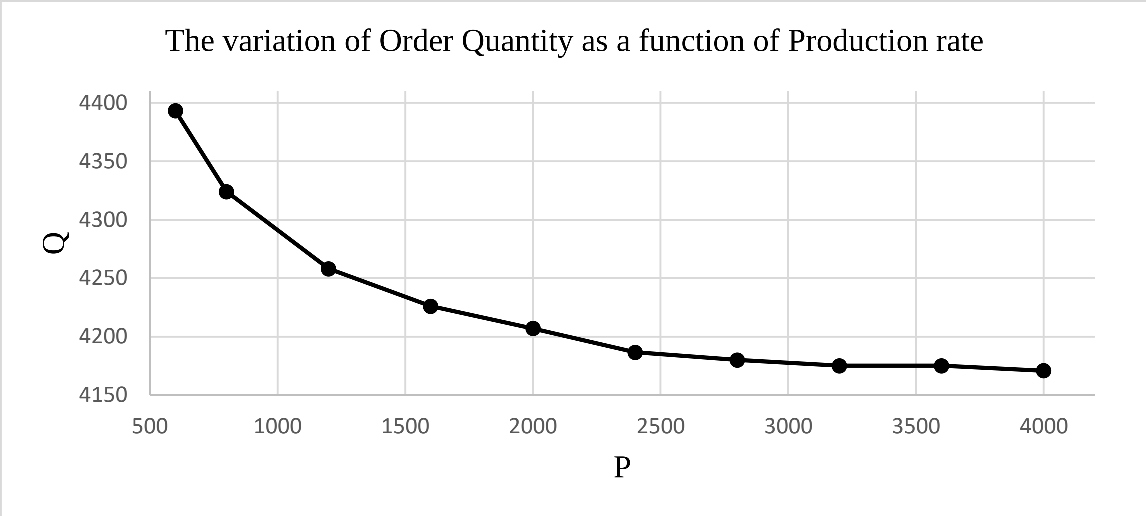

Figure 7. The variation of Order Quantity as a function of Production rate

Interpretation: As seen in Figure 7, as the production rate increases, the quantity ordered by the producer decreases negatively until it almost reaches a plateau. The results have shown debatable and encourage further investigation and deeper research. However, this can be due to factors of micro-production which is out of the scope of this thesis.

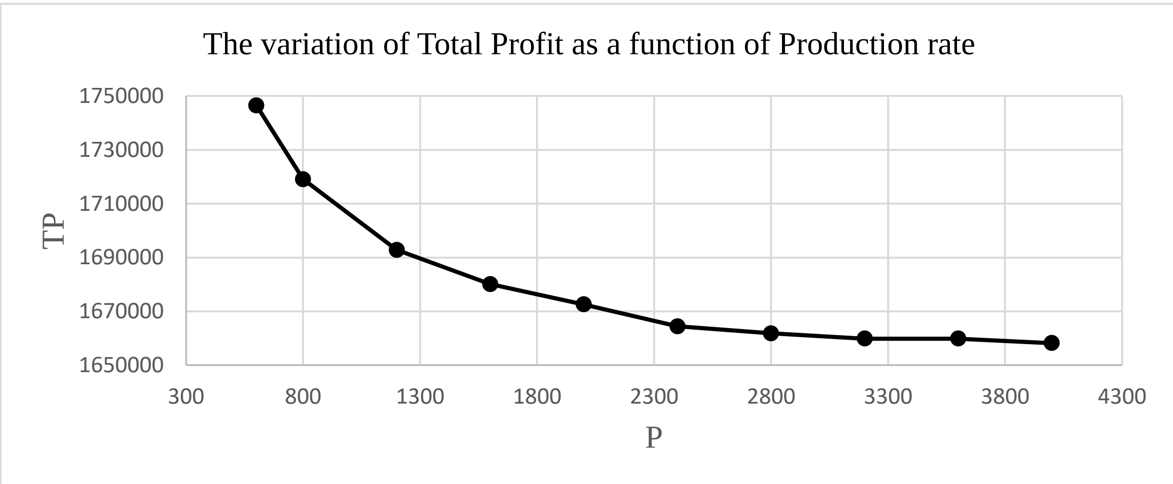

Figure 8. The variation of Total Profit as a function of Production rate

Interpretation: As seen in Figure 8, as the production rate increases, the producer's total profit decreases negatively until it almost reaches a plateau. The results have shown debatable and encourage further investigation and deeper research. However, this can be due to factors of micro-production which is out of the scope of this thesis

4.2 Collaboration between Producer and Supplier

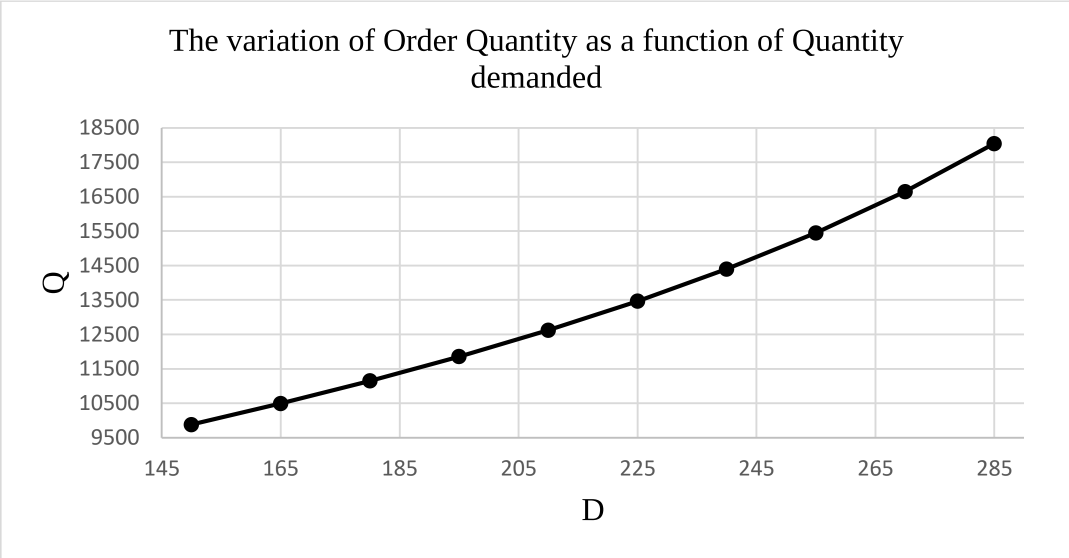

Figure 9. The variation of Order Quantity as a function of quantity demanded

Interpretation: As seen in Figure 9, as the quantity demanded increases, the quantity ordered by the producer from the supplier increases. The results are acceptable since a greater demand triggers a higher order for raw material.

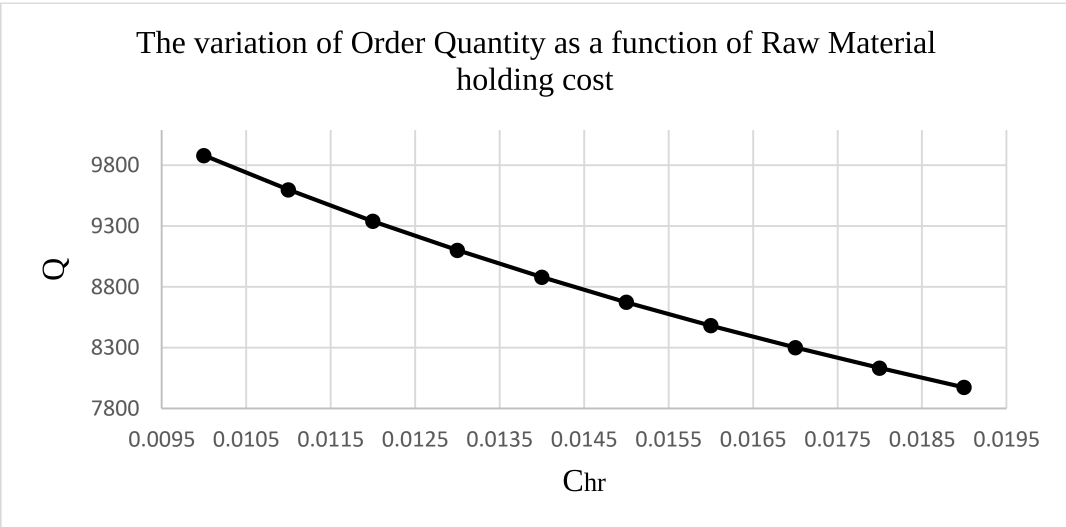

Figure 10. The variation of Order Quantity as a function of Raw Material holding cost

Interpretation: As seen in Figure 10, as the producer's raw material holding cost increases, the quantity ordered by the producer from the supplier decreases. The results are acceptable since as the cost of holding inventory increases, the manufacturer would decrease his inventory levels, consequently his order of raw material.

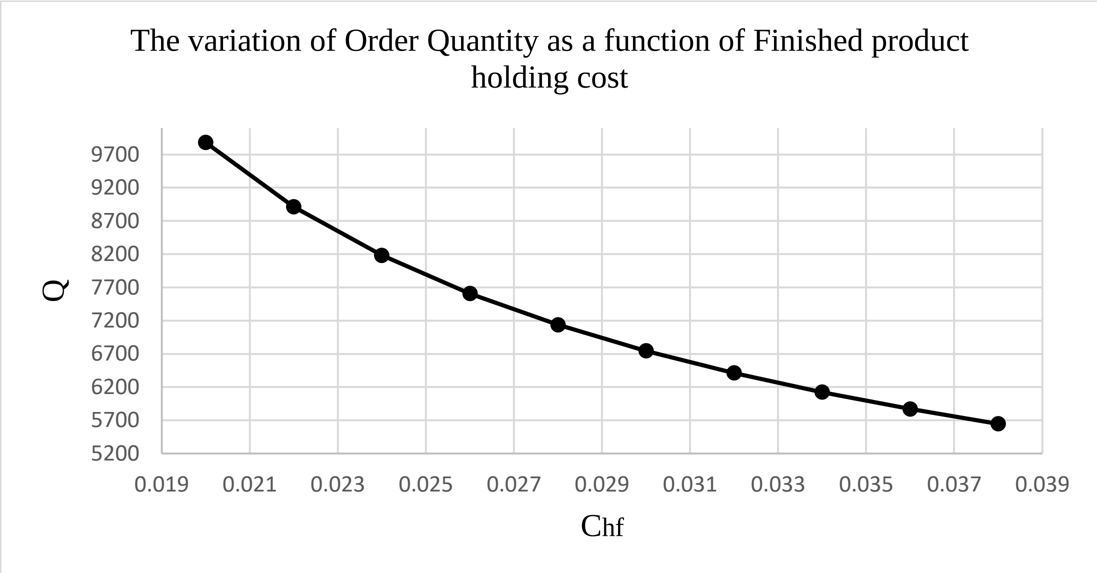

Figure 11. The variation of Order Quantity as a function of finished product holding cost

Interpretation: As seen in Figure 11, as the producer's finished product holding cost increases, the quantity ordered by the producer from the supplier decreases. The results are acceptable since as the cost of holding inventory increases, the manufacturer would decrease his inventory levels, consequently his order of raw material.

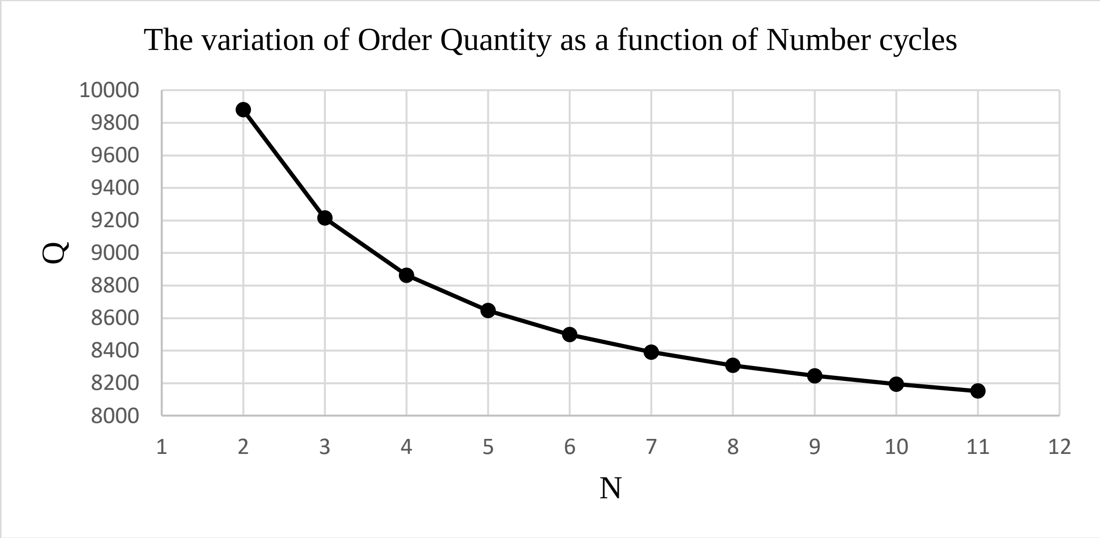

Figure 12. The variation of Order Quantity as a function of Number cycles

Interpretation: As seen in Figure 12, as the producer's number of cycles completed relative to each cycle completed by the supplier increases, the quantity ordered by the producer from the supplier negatively decreases. The more cycles the producer completes relatively to the supplier means that the producer is frequently ordering, in other words, the producer is ordering little orders but placing these orders many times. Hence, the result of a decreasing quantity ordered as the number of cycles completed increases is acceptable.

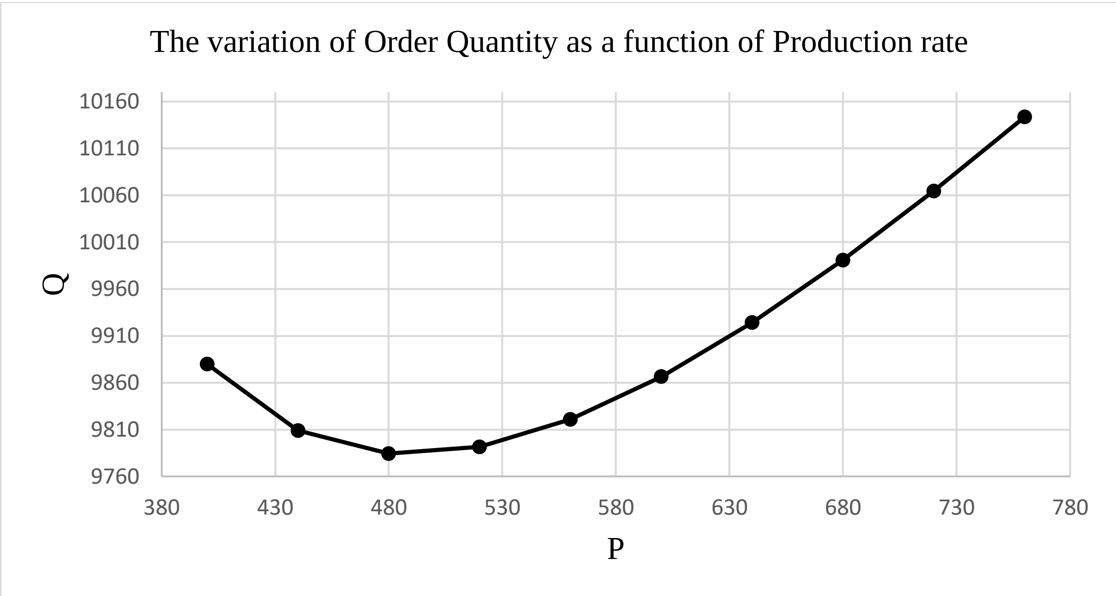

Figure 13. The variation of Order Quantity as a function of Production rate

Interpretation: As seen in Figure 13, as the producer's production rate increases, the quantity ordered by the producer from the supplier negatively decreases, reaches a minimum, and then increases. The results have shown debatable and encourage further investigation and deeper research. However, this can be due to factors of micro-production which is out of the scope of this thesis

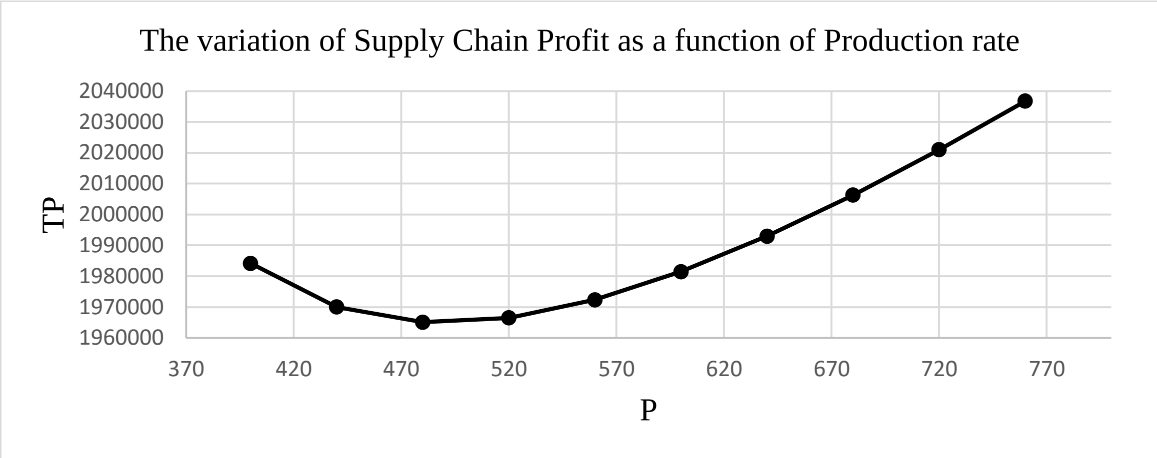

Figure 14. The variation of Supply Chain Profit as a function of Production rate

Interpretation: As seen in Figure 14, as the producer's production rate increases, the supply chain total profit negatively decreases, reaches a minimum, and then increases. The results have shown debatable and encourage further investigation and deeper research. However, this can be due to factors of micro-production which is out of the scope of this thesis

Chapter Five

Conclusion and Recommendations

A mathematical model was created to incorporate two types of finished products under the Economic Production Quantity model. The demand for the two types of finished goods (perfect and imperfect) is assumed to be continuous. For limitations and recommendations, I suggest the following:

The holding cost inconsistency is an important necessity, presenting a real-life suggestion that is applicable in real-life situations. Furthermore, for some special cases, the cost of holding inventory positively increases with storage periods. This is because bigger storages might require more sophisticated, hence costly storage equipment and conditions (for instance: food, pharmaceutical products…etc.).

The models were confirmed with the help of numerical examples. To better demonstrate real-life applications, a case study must be implemented. The model must also take include the holding cost, holding time, stock type, demand for good-quality items, and demand for defective items. That being said, the models must include things like pricing decisions, as well as factors that demand is affected by, for each type respectively. For future models, we should consider multiple objectives instead of a single objective only like adding the impact of the time value of money to maximizing the total profit or minimizing the total cost.

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