



Lebanese American University Repository (LAUR)

Chapter

Publication metadata

Title: Adaptation technology and free-riding incentives in international environmental agreements

Author(s): Hassan Bencheckroun, Walid Marrouch, Amrita Ray Chaudhuri

Book title: Economics of international environmental agreements

Handle: <http://hdl.handle.net/10725/12135>

How to cite this post-print from LAUR:

Bencheckroun, H., Marrouch, W., & Chaudhuri, A. R. (2017). 11 Adaptation technology and free-riding incentives in international environmental agreements. In Economics of international environmental agreements: A critical approach. <http://hdl.handle.net/10725/12135>

© Year 2017

This Open Access post-print is licensed under a Creative Commons Attribution-Non Commercial-No Derivatives (CC-BY-NC-ND 4.0)



This paper is posted at LAU Repository

For more information, please contact: [archives@lau.edu.lb](mailto:archives@lau.edu.lb)

# Adaptation technology and free-riding incentives in international environmental agreements

Hassan Benchekroun\*, Walid Marrouch<sup>‡</sup> and Amrita Ray Chaudhuri<sup>†</sup>

## Abstract

A well established result in the literature on international environmental agreements (IEAs) is that when the gains from cooperation are large, the incentive of an individual country to free-ride and not participate in the IEA is also large. We show that a more efficient adaptation technology diminishes the incentive of individual countries to free-ride on a global agreement over emissions. Moreover, we show that this positive effect of an increase in adaptation's efficiency can also be accompanied by an increase in the gains from global cooperation over emissions. Thus, more efficient adaptation, rather than merely being a substitute for the failed attempts at negotiating an IEA, may actually foster international cooperation on mitigating emissions of GHGs.

**JEL Classifications:** Q54, Q59

**Keywords:** international environmental agreements, adaptation, transboundary pollution.

\*Department of Economics, CIREQ, McGill University, 855 Sherbrooke Ouest, Montreal, QC, Canada, H3A-2T7. E-mail: hassan.benchekroun@mcgill.ca

<sup>‡</sup>CORRESPONDING AUTHOR: Lebanese American University, School of Business, Department of Economics, Beirut, Lebanon, P.O. Box 13-5053 / F-15, Email: walid.marrouch@lau.edu.lb; CIRANO (Center for Interuniversity Research and Analysis on Organizations), 2020, University St., 25th Floor, Montreal, QC, Canada, H3A 2A5

<sup>†</sup>Department of Economics, The University of Winnipeg, 515 Portage Avenue, Winnipeg, MB, Canada R3B 2E9; E-mail: a.raychaudhuri@uwinnipeg.ca; Center, TILEC, Tilburg University, Warandelaan 2, P.O. Box 90153, 5000 LE Tilburg, The Netherlands

# 1 Introduction

The purpose of this paper is to investigate how increasing the efficiency of adaptation technology to avoid the damage from a transboundary pollutant affects individual countries' incentives to participate in international environmental agreements (IEAs) that mitigate emissions.

This question gains importance in light of the current policy debate surrounding climate change. Persistent failure plagues international negotiations at the UN Climate Conferences (for example, at recent UNFCCC COP Meetings at Copenhagen (2009), Cancun (2010) and Doha (2012)) such that binding commitments on emission targets remain elusive. At the same time, policy-makers are setting aside substantial funds for developing more efficient adaptive measures to safeguard against imminent damage from climate change.<sup>1</sup> Since 1980, the World Bank has approved more than 500 operations related to disaster management including those caused by climate change, amounting to more than US\$40 billion. There exist several adaptation funds run by the UNFCCC, World Bank and European Commission that have already contributed millions of dollars towards adaptation (see, for example, Le Goulven, 2008).

Our paper contributes to the vast literature on IEAs (for a survey, see Barrett, 2005 and Jørgensen et al., 2010). The general conclusion in this literature is that stable coalitions are small and that large coalitions are only stable when gains from cooperation are small (see, for example, Barrett, 1994; Rubio and Ulph, 2006; de Zeeuw, 2008). This paper sets up a game theoretic framework, which incorporates both adaptation and participation in a global agreement on emission reduction as strategies available to individual countries dealing with a global pollutant. We show that the existence of a more efficient adaptation technology reduces the incentive of a coalition member to free-ride and leave the grand coalition, that

---

<sup>1</sup>A recent article (November 27, 2010) in *The Economist*, entitled "How to live with climate change: It won't be stopped, but its effects can be made less bad", captures the ongoing developments as follows: "... in the wake of the Copenhagen summit, there is a growing acceptance that the effort to avert serious climate change has run out of steam... Acceptance, however, does not mean inaction. Since the beginning of time, creatures have adapted to changes in their environment..."

is, the coalition that includes all countries. Moreover, we show that this positive impact of increased efficiency of adaptation technology can be accompanied by an increase in the gains from cooperation over the control of emissions. This is a rather optimistic result about the impact of having more efficient adaptation when compared to the existing literature. Thus, more efficient adaptation, rather than merely being a substitute for the failed attempts at negotiating an IEA, as suggested, for example, in *The Economist* (November 2010) and other media outlets, may actually foster international cooperation on mitigating emissions of GHGs.<sup>2</sup>

We note that a recent strand in the literature focuses on international cooperation on R&D and/or development and adoption of 'breakthrough technologies' (e.g. Barrett, 2006; El-Sayed and Rubio, 2011; Hoel and de Zeeuw, 2010). In this paper, we abstract from the development and the adoption phase of the technology and focus on international cooperation on emissions.

In order to model adaptation, we follow closely the recent theoretical models of adaptation. The existing literature on adaptation can be broadly categorized into two streams. The first provides a description of the trade-off facing countries when deciding how to allocate resources between mitigation of emissions and adaptation (see for example Ingham et al. 2005; Tol, 2005; Tulkens and van Steenberghe, 2009). The second stream explicitly incorporates adaptation in integrated assessment models to analyze the interaction between mitigation and adaptation (see for example Bosello et al., 2011; De Bruin et al. 2009). Other integrated assessment models such as RICE (Nordhaus and Yang, 1996) implicitly capture adaptation by incorporating the costs of adaptation in the regional damage function. For a survey of the literature on the economics of adaptation, please refer to Agrawala et al. (2011).

---

<sup>2</sup>Indeed, it seems that countries are realizing the importance of including adaptation in international negotiations, given the "Cancun Adaptation Fund" that was established at the COP16 Meetings held at Cancun in December 2010. (UNFCCC Press Release, 11 December 2011, [http://unfccc.int/files/press/news\\_room/press\\_releases\\_and\\_advisories/application/pdf/pr\\_20101211\\_cop16\\_closing.pdf](http://unfccc.int/files/press/news_room/press_releases_and_advisories/application/pdf/pr_20101211_cop16_closing.pdf))

This paper is more closely related to a set of recent studies that model adaptation within a two-country framework and compare the non-cooperative and cooperative equilibria (Ebert and Welsch, 2012; Eisenack and Kähler, 2012; Zehaie, 2009). Whilst these papers examine how the presence of adaptation affects emission and welfare levels at the non-cooperative and cooperative equilibria, they do not examine how adaptation affects the incentives of an individual country to participate in a global effort to curb emissions. In this paper we investigate the relationship between adaptation technology and the likelihood of sustaining a self-enforcing international environmental agreement over emissions. We model an increase in the efficiency of adaptation technology as a reduction in the marginal cost of providing adaptive measures. We show that the existence of more efficient adaptation technologies reduces an individual country's incentive to free-ride on an international environmental agreement over pollution emissions.

We follow Ebert and Welsch (2012) and assume that countries undertake adaptation and emission decisions simultaneously, as, for example, in the case of liming of lakes to avoid acidification. As shown by Zehaie (2009), this model is equivalent to one in which adaptation occurs after emission decisions are undertaken, as, for example, in the case of flood evacuation programs. Thus, our results are applicable to those types of adaptive measures that are undertaken either simultaneously with emissions or ex-post.<sup>3</sup>

Ebert and Welsch (2012) find that when countries undertake adaptation and emissions simultaneously, adaptation alters the effective damage function such that the emission strategies of countries may be either strategic substitutes or strategic complements depending on the shape of the effective damage function. Eisenack and Kähler (2012) build on this result and show that, when one country acts as a Stackelberg leader and the follower has a positively sloped best response function, the leader has an incentive to unilaterally mitigate emissions

---

<sup>3</sup>Given the wide variety of adaptive measures, the existing literature is divided on the issue of the timing of adaptation in relation to mitigation. Zehaie (2009) shows that if adaptation is undertaken before mitigation, adaptation can be used strategically by each country to reduce its own mitigation efforts and increase that of others. Zehaie (2009) also shows that this strategic effect of adaptation disappears if either adaptation and mitigation are undertaken simultaneously or if adaptation occurs after mitigation.

leading to higher global welfare. By contrast, we study the free-riding incentives of individual countries from a given coalition where all coalition members jointly mitigate emissions. Moreover, to ensure that our results are not driven by specific non-standard assumptions about the properties of the effective damage function, we use the standard framework used in this literature where pollution damage is quadratic in emissions. We obtain emission strategies that are strategic substitutes, in line with the standard result in the literature on transboundary pollution games. This facilitates comparison of our results to the existing literature examining free-riding incentives from international environmental agreements.

A related paper by de Bruin et al. (2011) explicitly models coalition formation in the presence of adaptation. They present a calibrated model of climate change which provides numerical results about the stability of coalitions for two different levels of adaptation (the levels pertaining to the non-cooperative equilibrium and the cooperative equilibrium), assuming sequential decisions about adaptation and mitigation, and a damage function that is linear in emissions. In contrast, our aim is to determine the impact of a change in the equilibrium adaptation level following a change in adaptation technology. Furthermore, in our framework, decisions about adaptation and mitigation are simultaneous, the damage from pollution is strictly convex and the results are derived analytically.

We proceed as follows. Section 2 presents the model. Section 3 characterizes the equilibrium of the model. Section 4 presents the effects of increasing the efficiency of adaptation technology on countries' free-riding incentives. Section 5 provides concluding remarks.

## 2 The Model

Let  $N = \{1, \dots, n\}$  denote the set of all countries, with  $n \geq 3$ . The model consists of two stages.

*Stage 1:*

Each country decides whether to be a member of a given coalition structure within which

members set their emission levels cooperatively. Our objective is to examine how the free-riding incentive of an individual country is affected by increasing the efficiency of adaptation. We define the free-riding incentive to be the negative of the internal stability criterion, as used in d'Aspremont et al. (1983). The internal stability criterion is denoted by the welfare of an individual country from being a member of a given coalition less the welfare of the country if it were to unilaterally leave the coalition. This concept is extensively used in the IEA literature (see, for example, Barrett, 1994; Rubio and Ulph, 2006, and others). We use it in this paper to facilitate comparison of our results to the existing literature.<sup>4</sup>

*Stage 2:*

In stage 2, each country implements its emission and adaptation strategies simultaneously.

We assume that a by-product of production activities of each country is the emission of a global pollutant. Country  $i$  emits  $e_i \geq 0$  units of the pollutant with the aggregate emissions denoted by  $E = \sum_{i=1}^n e_i$ . Let  $B(e_i)$  represent the benefit to country  $i$  from its own emissions as follows:

$$B(e_i) \equiv e_i \left( \alpha - \beta \frac{e_i}{2} \right) \quad (1)$$

with  $\alpha > 0$  and  $\beta > 0$ . We have  $B'(e_i) > 0$  and  $B''(e_i) < 0$  for all  $e_i < \bar{e} \equiv \frac{\alpha}{\beta}$ .

Each country can spend resources on adaptation to avoid the damage from pollution. The level of adaptation chosen by country  $i$  is denoted by  $a_i$ .

Let  $D(E, a_i)$  represent the damage to country  $i$  from pollution as follows:

$$D(E, a_i) \equiv \frac{\omega}{2} E^2 - a_i E \quad (2)$$

with  $\omega > 0$ .

---

<sup>4</sup>We note that there exist other approaches to analyzing coalition stability. For example, Breton et al. (2010) model the dynamic aspect of coalition formation using an evolutionary process to determine which countries join and/or leave the coalition over time. Another approach examines the 'farsighted' stability criterion that allows for a more sophisticated behavior of players. In deciding whether to join or leave a coalition, a player considers the implication of her decision on other players's decision to leave or stay in a coalition (see, for example, Diamantoudi and Sartzetakis, 2002; Osmani and Tol, 2009; de Zeeuw, 2008).

The damage function, as given by (2), captures two features pertaining to climate change. First, the damage is strictly convex in global emissions. Second, the marginal damage from emissions,  $\frac{\partial D(E, a_i)}{\partial E} = \omega E - a_i$ , is decreasing in the level of adaptation.<sup>5</sup> We also have that  $\frac{\partial D(E, a_i)}{\partial a_i} = -E < 0$  for all  $E > 0$ , that is, pollution damage faced by country  $i$  is decreasing in the level of country  $i$ 's adaptation. Adaptation is, thus, modeled as a private good to each country while mitigation is a global public good. Therefore,  $a_i$  reduces the damage of country  $i$  only. This is in line with real examples of adaptive measures currently being undertaken by different countries, as mentioned earlier, such as flood evacuation schemes and construction of levees. From (2), it also follows that  $\frac{\partial^2 D(E, a_i)}{\partial E \partial a_i} = \frac{\partial^2 D(E, a_i)}{\partial a_i \partial E} = -1$ . This implies that the marginal benefit of adaptation is increasing in the global emission level,  $E$ .

Let  $C(a_i)$  represent the cost of adaptation of country  $i$  as follows:

$$C(a_i) \equiv \frac{c}{2} a_i^2 \quad (3)$$

where  $c > 0$ . Our modeling of adaptation in (2) and (3) is in line with Tulkens and van Steenberghe (2009) who consider the full cost minimization problem faced by countries in the presence of both mitigation and adaptation. The cost function, (3), reflects the fact that undertaking adaptive measures is indeed costly in reality (see Le Goulven, 2008).<sup>6</sup> We also assume that the cost of adaptation is strictly convex and increasing in  $a_i$ , in line with de Bruin et al. (2011) and Zehaie (2009). This reflects that some types of adaptation are associated with an increasing marginal cost. For example, starting with those stretches of the coast that are easiest to protect, elongating levees implies incurring increasing physical difficulties

---

<sup>5</sup>For the interior solution levels of emissions and abatement derived below, we have  $\frac{\partial D}{\partial E} \geq 0$ . Please refer to Assumption 1, as specified at the end of this section. As long as Assumption 1 is satisfied, it can be shown that  $\frac{\partial D}{\partial E} \geq 0$ .

<sup>6</sup>According to Le Goulven (2008), existing adaptation funds include the following. The UNFCCC pledged \$50 million through the SPA (Strategic Priority "Piloting an Operational Approach to Adaptation") in 2001. The UNFCCC pledged \$165 million through the LDCF (Least Developed Countries Fund) in 2001. The UNFCCC pledged \$65 million through the SCCF (Special Climate Change Fund) in 2001. Also in 2001, the Kyoto Protocol set up an Adaptation Fund which pledged \$160-950 million by 2012. In 2008, the World Bank's Pilot Program for Climate Resilience under the Strategic Climate Fund pledged \$500 million. In 2007, the European Commission pledged EUR 50 million under the Global Climate Change Alliance and the German Ministry of the Environment pledged EUR 60 million.

of protecting more irregularly shaped coastlines and opportunity costs of protection, such as destruction of landscape/beaches or economic costs of uprooting fishing villages.<sup>7</sup>

Social welfare of each country is assumed to be given by the following:

$$W(E, a_i) \equiv B(e_i) - D(E, a_i) - C(a_i) \quad (4)$$

where  $B(e_i)$ ,  $D(E, a_i)$  and  $C(a_i)$  are given by (1), (2), and (3) respectively.

In the non-cooperative case, the objective of country  $i$ 's government, with  $i = 1, \dots, n$ , is to simultaneously choose  $e_i$  and  $a_i$  that maximize its own welfare, taking as given the emissions and adaptation strategies of the other countries. That is,

$$\max_{e_i, a_i} W(E, a_i) \quad (5)$$

where  $W(E, a_i)$  is given by (4).<sup>8</sup>

In the fully cooperative case, the countries simultaneously choose  $e_i$  and  $a_i$  that maximize joint welfare. That is,

$$\max_{e_i, a_i} \sum_{i=1}^n W(E, a_i) \quad (6)$$

**Assumption 1:** *We have that  $\omega > \underline{\omega} \equiv \frac{1}{c}$ .*

Assumption 1 ensures that, in the non-cooperative equilibrium and the fully cooperative equilibrium, the marginal benefit to each country from its own emissions is non-negative, that is,  $e_i < \bar{e}$  such that  $B'(e_i) \geq 0$ . In the non-cooperative equilibrium, it can be shown that equilibrium emission of each country is given by  $e_{nc} \equiv \frac{\alpha}{\beta+n(\omega-\frac{1}{c})}$ . Assumption 1 ensures that  $e_{nc} \in (0, \bar{e})$ . In the fully cooperative equilibrium, it can be shown that equilibrium

---

<sup>7</sup>An alternative justification provided by Zehaie (2009) is the following: "Since self-protection decreases exposure to pollution but does not solve the problem of pollution, it is reasonable to assume that the opportunities to substitute self-protection for abatement gradually deteriorate when the level of self-protection increases."

<sup>8</sup>Given (2), the tradeoff facing each country when choosing its levels of emission,  $e_i$ , and adaptation,  $a_i$ , is similar to that in the literature on multiple pollutants in the context of climate change, where some pollutants such as CO<sub>2</sub> increase global warming and others such as SO<sub>2</sub> have a cooling effect (see, for example, Legras and Zaccour, 2011).

emission of each country is given by  $e_c \equiv \frac{\alpha}{\beta+n^2(\omega-\frac{1}{c})}$ . Assumption 1 ensures that  $e_c \in (0, \bar{e})$ . Moreover, as long as Assumption 1 is satisfied, it can be shown that  $\frac{\partial D}{\partial E} \geq 0$ , as mentioned in footnote 5.

### 3 The International Environmental Agreement

Consider the scenario where some of the countries decide to form an international environmental agreement. More specifically, let  $S \subset N$  denote the coalition of countries that sign an agreement over emissions and  $N \setminus S$  denote the set of countries who don't. We denote the size of coalition  $S$  by  $s$ .

We assume that the non-signatories and signatories simultaneously choose their emissions levels. The coalition members jointly play as a singleton in the game, taking as given the emission strategies of the  $(n - s)$  other players. Thus, the coalition maximizes the joint welfare of all its members. If we regard the coalition members jointly as a single player, the total number of players in the game is  $(n - s + 1)$ . Each of the non-signatories plays as a singleton in the game, taking as given the strategies of the  $(n - s)$  other players. Thus, each non-signatory maximizes its own welfare when choosing its emission level.

The objective of each non-signatory country  $i$ 's government, with  $i \in N \setminus S$ , is to simultaneously choose  $e_i$  and  $a_i$  that maximize its own welfare, taking as given the emissions and adaptation strategies of the coalition  $S$  and the other non-signatories. That is,

$$\max_{e_i, a_i} W \left( \left( e_i + \sum_{j \in N \setminus (S \cup \{i\})} e_j + \sum_{k \in S} e_k \right), a_i \right), \quad i \in N \setminus S. \quad (7)$$

This results in the following best response function of each non-signatory country:

$$e_i = \frac{\alpha - (\omega - \frac{1}{c}) \left( \sum_{j \in N \setminus (S \cup \{i\})} e_j + \sum_{k \in S} e_k \right)}{\beta + \omega - \frac{1}{c}}, \quad i \in N \setminus S. \quad (8)$$

The signatories' maximization problem is given by:

$$\max_{\{e_i\}_{i \in S}, \{a_i\}_{i \in S}} \sum_{i \in S} W \left( \left( e_i + \sum_{k \in S \setminus \{i\}} e_k + \sum_{j \in N \setminus S} e_j \right), a_i \right) \quad (9)$$

This results in the following best response function of each signatory country:

$$e_i = \frac{\alpha - s \left( \omega - \frac{1}{c} \right) \left( \sum_{j \in N \setminus S} e_j + \sum_{k \in S \setminus \{i\}} e_k \right)}{\beta + s \left( \omega - \frac{1}{c} \right)}, \quad i \in S. \quad (10)$$

For all countries, non-signatories and signatories, the adaptation strategies are given by

$$a_i = \frac{E}{c} \quad (11)$$

which implies that each country's adaptation level increases in total emissions.

By symmetry, let  $e_s$  denote the emission of a representative signatory, and  $e_{ns}$  denote the emissions generated by a representative non-signatory. The sum of the emissions of the signatory and non-signatory countries, that is global emissions, is given by  $E = se_s + (n - s)e_{ns}$ . The best response functions of each non-signatory and signatory country respectively can be written as:

$$e_{ns}(e_s) = \frac{\alpha - \left( \omega - \frac{1}{c} \right) se_s}{\beta + (n - s) \left( \omega - \frac{1}{c} \right)} \quad (12)$$

$$e_s(e_{ns}) = \frac{\alpha - s \left( \omega - \frac{1}{c} \right) (n - s) e_{ns}}{\beta + s^2 \left( \omega - \frac{1}{c} \right)} \quad (13)$$

The equilibrium emission levels of each non-signatory and signatory country respectively are given by:

$$e_{ns}^* = \frac{\beta + s(s - 1) \left( \omega - \frac{1}{c} \right) \alpha}{\beta + (n + s(s - 1)) \left( \omega - \frac{1}{c} \right) \beta} < \bar{e} \quad (14)$$

$$e_s^* = \frac{\beta - (n - s)(s - 1) \left( \omega - \frac{1}{c} \right) \alpha}{\beta + (n + s(s - 1)) \left( \omega - \frac{1}{c} \right) \beta} < \bar{e} \quad (15)$$

We note that  $e_{ns}^* > 0$ .

**Assumption 2:** We have that  $\omega < \bar{\omega} \equiv \underline{\omega} + \frac{\beta}{(n-s)(s-1)}$ .

Assumption 2 ensures that  $e_s^* > 0$ , as shown by (15).

The equilibrium level of total emissions is given by:

$$E^* = \frac{n\alpha}{\beta + (n + s(s-1))\left(\omega - \frac{1}{c}\right)} \quad (16)$$

We note that, under Assumption 1,  $E^*$  is always positive.

The equilibrium adaptation levels are given by:

$$a_s^* = a_{ns}^* = \frac{1}{c}E^* \quad (17)$$

where  $E^*$  is given by (16). Notice that in equilibrium, the non-signatory and signatory countries each choose the same level of adaptation. This is due to the fact that the effect of adaptation is purely local. Thus, the equilibrium level is the same for each country, regardless of whether the country is maximizing its individual welfare or the joint welfare of all signatories.

The welfare of each signatory country, at the equilibrium, is given by:

$$\begin{aligned} W_s^*(s) &\equiv e_s^* \left( \alpha - \beta \frac{e_s^*}{2} \right) - \left( \frac{\omega}{2} (E^*)^2 - a_s^* E^* \right) - \frac{c}{2} a_s^{*2} \\ &= \frac{\alpha^2 \left( \omega - \frac{1}{c} \right)^2 (s-1)(n-s)(s(1-s-n)-n) + \beta^2 - \beta(n(n-2) - 2s(s-1)) \left( \omega - \frac{1}{c} \right)}{2 \beta \left( \beta + (n-s+s^2) \left( \omega - \frac{1}{c} \right) \right)^2} \end{aligned} \quad (18)$$

The welfare of each non-signatory country, at the equilibrium, is given by:

$$\begin{aligned} W_{ns}^*(s) &\equiv e_{ns}^* \left( \alpha - \beta \frac{e_{ns}^*}{2} \right) - \left( \frac{\omega}{2} (E^*)^2 - a_{ns}^* E^* \right) - \frac{c}{2} a_{ns}^{*2} - F \\ &= \frac{\alpha^2 \left( \omega - \frac{1}{c} \right)^2 s(s-1)(2n-s+s^2) + \beta^2 - \beta(n(n-2) - 2s(s-1)) \left( \omega - \frac{1}{c} \right)}{2 \beta \left( \beta + (n-s+s^2) \left( \omega - \frac{1}{c} \right) \right)^2} - F \end{aligned} \quad (19)$$

In (19),  $F$  represents a fixed cost of not signing the IEA that is not related to environmental costs. For example, a non-signatory may face retaliation from the signatories in the

form of trade penalties such as border tax adjustments<sup>9</sup> (see Carraro and Marchiori, 2003, for a discussion of the literature on issue linkage in the context of IEAs) or a loss of reputation in the international political and economic forum (see Hoel and Schneider (1997); Rose and Spiegel, 2009).<sup>10</sup>

## 4 Free-riding and more efficient adaptation technologies

An increase in the efficiency of adaptation technology is equivalent to a decrease in the marginal cost of adaptation,  $c$ . In the following analysis, it is useful to define the following:

$$\Phi(s) \equiv W_s^*(s) - W_{ns}^*(s-1) - F$$

where  $W_s^*(s)$  represents the welfare of an individual country from participating in a coalition of size  $s$  and  $W_{ns}^*(s-1)$  represents the welfare of an individual country from leaving a coalition of size  $s$ .

Within our context, the incentive of a country to participate in a coalition of size  $s$  is given by:

$$\begin{aligned} W_s^*(s) - W_{ns}^*(s-1) &= \Phi(s) + F & (20) \\ &= \frac{n^2 \alpha^2 \left(\omega - \frac{1}{c}\right)^2 (s-1) \Psi}{2\beta \left(\left(\beta + (n-s+s^2)\left(\omega - \frac{1}{c}\right)\right)\left(\beta + \left(\omega - \frac{1}{c}\right)(-3s+n+s^2+2)\right)\right)^2} + F \end{aligned}$$

---

<sup>9</sup>The cost  $F$  is assumed independent of the coalition size  $s$ . This is done only for simplicity and is not crucial for the results of the paper.

<sup>10</sup>Rose and Spiegel (2009) apply the idea of reputation spillovers to the relationship between environmental interaction and international exchange. In a model of international asset exchange, they show that countries that participate more in IEAs also experience better economic outcomes, in both theory and practice.

where

$$\begin{aligned}\Psi \equiv & -(n - 3s + s^2)(n + s + ns - 2s^2 + s^3) \left(\omega - \frac{1}{c}\right)^2 \\ & - \beta^2 (s - 3) - 2\beta \left(\omega - \frac{1}{c}\right) (-n + 3s + ns - 4s^2 + s^3 - 2)\end{aligned}$$

Alternatively,  $-\Phi(s) - F$  can be interpreted as the incentive of an individual country, member of a coalition of size  $s$ , to free-ride and leave that coalition. We say that an increase in adaptation efficiency (i.e., a decrease of  $c$ ) reduces the incentive of a coalition member to leave a coalition of size  $s$  if  $\frac{\partial \Phi(s)}{\partial c} < 0$ . Let

$$\hat{F}(s) \equiv -\Phi(s) \text{ for all } s = 1, \dots, n. \quad (21)$$

Clearly if  $F$  is large enough, that is,  $F > \hat{F}(s)$ , then none of the members of a coalition of size  $s$  has an incentive to free-ride. Suppose  $F = \hat{F}(s)$ , a coalition member is just indifferent between staying in the coalition of size  $s$  or leaving the coalition. If  $\frac{\partial \hat{F}(s)}{\partial c} > 0$  then a marginal decrease in  $c$ , will result in the coalition member being strictly better off by staying in the coalition of size  $s$  than by leaving it.

In general, the existing literature shows that large coalitions can only be stable when the gains from the coalition are small. Our work is motivated by the question of whether more efficient adaptation can help reduce the incentives to free-ride from large coalitions. This is why our primary focus is on the analysis of the grand coalition and its internal stability. In this section, we, therefore, focus on the incentive of an individual country to free-ride and leave the grand coalition (i.e.  $s = n$ ) and determine how the presence of adaptation affects this free-riding incentive.

Note that Assumption 2 is satisfied for all finite values of  $\omega$  when  $s = n$ . We postpone the analysis of cases where  $s < n$ , to later in the section.

## 4.1 The grand coalition

We now study the impact of a change in  $c$  on  $\Phi(n)$ , for a given  $\omega$ . We note that, given  $F$ , the larger is  $\Phi$ , the smaller the incentive of a coalition member to free-ride and leave the coalition. In the following analysis, let  $X \equiv (\omega - \frac{1}{c})$ . Note that from Assumption 1 we have  $X > 0$ .

**Proposition 1:** *The incentive of a coalition member to free-ride and leave the grand coalition, i.e., the IEA that includes all countries, decreases when adaptation efficiency increases (i.e., when  $c$  decreases):  $\frac{\partial \Phi(n)}{\partial c} < 0$ .*

**Proof:** We have

$$\frac{\partial (\Phi(n))}{\partial X} = \frac{(z_3 X^3 + z_2 X^2 + z_1 X + z_0) (1-n) X n^2 \alpha^2}{(\beta + X n^2)^3 (2X + Xn + \beta + Xn(n-3))^3}$$

where  $z_1$ ,  $z_2$  and  $z_3$  are given by:

$$z_1 \equiv 3(n(n-1)(n-2) - 2)\beta^2 \tag{22}$$

$$z_2 \equiv (n(n-2)(3n(n(n-1)+2) - 4) - 4)\beta \tag{23}$$

$$z_3 \equiv n^2(n(n(n(n(n-3)+6) - 10) + 8) - 4). \tag{24}$$

From (22) – (24), it can be shown that  $z_1$ ,  $z_2$  and  $z_3$  are all positive for  $n \geq 3$  and therefore

$$\frac{\partial (\Phi(n))}{\partial X} < 0.$$

This, together with the fact that  $X$  is increasing in  $c$ , yields Proposition 1. ■

Given (21), a direct implication of Proposition 1 is that a decrease in  $c$  causes a decrease in  $\hat{F}(n)$ . That is, a more efficient adaptation technology enlarges the set of the fixed cost  $F$  under which the grand coalition is stable. Consider a decrease of  $c$  from some level  $c'$  to  $c'' < c'$  (resulting from an increase in efficiency of the adaptation technology), then there exists a range of  $F$ , such that the decrease in  $c$  stabilizes an otherwise unstable grand

coalition.

Why does more efficient adaptation technology reduce free-riding incentives? From (8) and (10), it follows that the more efficient is adaptation at reducing marginal damage from emissions, the flatter the best response function of each country in terms of emissions. That is, the lower is  $c$ , the less aggressive each country is in its emission strategy and, therefore, the lower the gap between the global emission levels under non-cooperation and under full cooperation, making it less costly to cooperate on emission strategies.

The best response functions are flatter the lower is  $c$  for the following reason. In the absence of adaptation, in response to an increase in other countries' emissions, the only option available to a given country is to decrease its own emissions. In the presence of adaptation, however, when other countries increase emissions, the given country may, instead of reducing its own emissions, decrease its own damage by increasing adaptation. The greater the efficiency of the adaptation technology, the greater the substitutability between mitigation and adaptation in response to changes in the level of others' emissions. This explains why the higher the efficiency of adaptation technology, the lower the free-riding incentives of individual countries in this transboundary pollution game.

Next, we study how the aggregate gains from cooperation change with  $c$ . Let  $G$  denote the gains from forming a coalition of size  $s$  as compared to the non-cooperative equilibrium. That is,

$$\begin{aligned} G &\equiv sW_s^* + (n-s)W_{ns}^* - n(W_{ns}^*|_{s=0}) \\ &= \frac{\Gamma(s-1)X^2n^2s\alpha^2}{2(\beta+(n-s+s^2)X)^2(\beta+nX)^2\beta} - sF \end{aligned} \quad (25)$$

with  $\Gamma = n(s-1)(s-n)X^2 + \beta n(2n-3s+s^2)X - \beta^2(s-2n+1)$ .

In the case of the grand coalition  $s = n$  we have the following.

**Proposition 2:** *There exists  $\bar{n} > 0$  such that for  $n > \bar{n}$ , we have that a marginal increase of adaptation efficiency results in an increase of the welfare gains from forming the grand*

coalition (i.e., of size  $s = n$ ) as compared to the non-cooperative equilibrium.

**Proof:** See Appendix A.

The value of  $\bar{n}$  depends on  $\beta, \omega$  and  $c$  and may well be smaller than 3. This happens, for example, when  $\beta$  is small enough. More precisely, we have the following corollary.

**Corollary:** Assume  $\beta \in (0, \bar{\beta})$  where  $\bar{\beta} \equiv \frac{6(\omega - \frac{1}{c})}{(\sqrt{\frac{11}{3}} + 1)}$ , then for all  $n \geq 3$ , a marginal increase in adaptation efficiency results in an increase in the gains from the formation of the grand coalition.

**Proof:** See Appendix B.

Why do the gains from cooperation increase with the efficiency of adaptation? Since the cost of adaptation is convex in the level of adaptation, failing to reach a cooperative equilibrium on emissions increases the cost to each individual country through this channel. This explains why the gains from cooperation increase as more adaptation is undertaken. This, together with the fact that more adaptation is undertaken in equilibrium the more efficient is adaptation, explains Proposition 2.

Propositions 1 and 2 give a rather optimistic message. An increase in the efficiency of adaptation technology can result in a decrease of individual countries' incentive to free-ride on a global agreement *and* an increase of the gains from a global agreement. This is at odds with the conventional wisdom in the existing literature on international environmental agreements that incentives to free-ride are small only when the gains from cooperation are negligible. See, for example, Barrett (1994) and Rubio and Ulph (2006) who show that large coalitions are stable only when the coalition-induced global welfare improvements relative to the non-cooperative outcome is small. While these papers use abatement effort as the choice variable of the countries, we use emission levels. As shown by Diamantoudi and Sartzetakis (2006), the model using abatement effort as the choice variable and that using emission levels, *ceteris paribus*, are equivalent. Hence, this difference in modeling technique is not the cause of the difference between our results and those of the Barrett (1994) and Rubio and Ulph (2006). However, a caveat is due. In our model, we use a Cournot approach where all

countries choose their emission strategies simultaneously while in Barrett (1994) and Rubio and Ulph (2006) the coalition acts as a Stackelberg leader. Moreover, we focus on studying the free-riding incentives from a given coalition rather than deriving the stable coalition size.

We also note that our optimistic result relies on our assumption that countries undertake adaptation and emissions simultaneously. This is because, under this assumption, countries cannot use adaptation strategically to reduce their own mitigation effort at the expense of others', as they do when adaptation decisions are undertaken prior to mitigation (see Zehaie, 2009). In those cases where adaptation decisions are undertaken prior to mitigation, more efficient abatement technologies may have a more pessimistic impact. As shown by Zehaie (2009) for the case of two countries, cooperation on abatement, after adaptation decision are undertaken, reduces environmental quality thereby reducing the gains from cooperation.

## 4.2 Subcoalitions

The main results of this paper, as given by Propositions 1 and 2, were shown for the grand coalition. Next, we examine the case of a coalition of countries of size  $s < n$ . It is possible to show that Proposition 1 extends to the case where  $s \in \{3, \dots, n\}$ . That is,

$$\frac{\partial(\Phi(s))}{\partial c} < 0 \text{ for all } n \geq 3 \text{ and } s \in \{3, \dots, n\}.$$

The approach to prove this result is similar to the case where  $s = n$ . To economize on space, we omit the details of the proof. The case where  $s = 2$  needs a special treatment which is provided later. For now, we proceed with analyzing the gains from cooperation for  $n \geq 3$  and  $s \in \{3, \dots, n\}$ .

The algebraic expressions for the impact of a change in adaptation efficiency on the gains from cooperation are too cumbersome to derive analytical results. We, therefore, proceed by fixing the number of countries  $n$  to 10 and consider coalitions of size  $s \in \{3, \dots, 9\}$ .

For notational convenience, let us define  $P(s, X)$  as given by:

$$\begin{aligned}
 P(s, X) \equiv & -\frac{X^3}{\beta^3} (80s^4 - 160s^3 + 2480s^2 - 400s + 16\,000) \\
 & + \frac{X^2}{\beta^2} (10s^4 - 20s^3 + 70s^2 - 660s + 600) \\
 & + \frac{X}{\beta} (30s^2 - 90s + 600) + 38 - 2s
 \end{aligned}$$

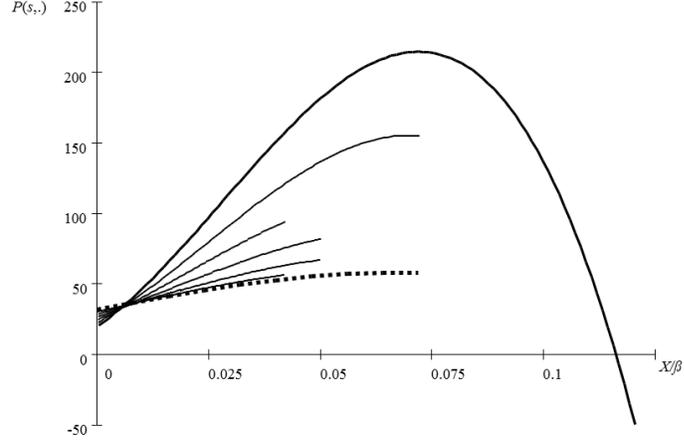
For  $n = 10$ , it can be shown that the sign of  $\frac{\partial G}{\partial c}$  is the opposite of the sign of  $P(s, X)$ . Moreover, for Assumption 2 to hold, we must have  $X < \bar{X} \equiv \frac{\beta}{(n-s)(s-1)}$  which in the case of  $n = 10$  becomes  $X < \frac{\beta}{(10-s)(s-1)}$ . Table 1 provides the value of the upper bound of  $\frac{X}{\beta}$ ,  $\frac{\bar{X}}{\beta}$ , for different values of  $s$  between 3 and 9.

**Table 1: Upper bound of  $X$  as  $s$  varies**

$s$	3	4	5	6	7	8	9
$\frac{\bar{X}}{\beta}$	$\frac{1}{14}$	$\frac{1}{24}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{1}{24}$	$\frac{1}{14}$	$\frac{1}{8}$

In Figure 1, we plot  $P(s, \cdot)$  for  $s \in \{3, \dots, 9\}$  with  $X < \bar{X}$ .

Chaudhuri Figure 1



1.pdf

Figure 1:  $P(s, \cdot)$  as a function of  $\frac{X}{\beta}$

The curve with the highest maximum corresponds to the plot of  $P(9, X)$  over its domain  $[0, \frac{1}{8}]$ . The dashed curve corresponds to the plot of  $P(3, X)$  over its domain  $[0, \frac{1}{14}]$ . The other curves correspond to the cases of  $s = 4, \dots, 8$ .

We can observe that for all  $s \in \{3, \dots, 8\}$  we have  $P(s, X) > 0$  for all  $X$  in the domain  $[0, \bar{X}]$  and therefore,  $\frac{\partial G}{\partial c} < 0$ , that is, an increase in adaptation efficiency increases the gains from the formation of a coalition. This is also true for  $s = 9$  when  $X$  does not exceed a certain threshold. A similar conclusion can be reached if we use other values of  $n$  instead of 10.

For completeness, we provide the results for a coalition member to leave a coalition of size  $s = 2$ . *The incentive of a coalition member to free-ride and leave the grand coalition, i.e., the IEA that includes all countries, decreases when adaptation efficiency increases (i.e., when  $c$  decreases):  $\frac{\partial \Phi(n)}{\partial c} < 0$ .*

**Proposition 3:** *For  $s = 2$ , there exists  $\hat{c}$  such that*

(i) for  $c > \hat{c}$ , the incentive of a coalition member to free-ride and leave the coalition decreases when adaptation efficiency increases (i.e., when  $c$  decreases):  $\frac{\partial \Phi(2)}{\partial c} < 0$ .

(ii) for  $c < \hat{c}$ , there exists  $\hat{n} > 2$  such that  $\frac{\partial \Phi(2)}{\partial c} > (<)0$  for all  $n < (>)\hat{n}$ .

**Proof:** See Appendix C.

For  $s = 2$ , the effect of increasing the efficiency of adaptation depends on the initial level of  $c$ . A marginal increase in the efficiency of adaptation technology reduces the incentive to free-ride when adaptive measures are relatively inefficient ( $c > \hat{c}$ ) or when the number of countries is large enough ( $n > \hat{n}$ ).

The approach to determine the sign of  $\frac{\partial G}{\partial c}$  for the case  $s \in \{3, \dots, 9\}$  can be repeated for  $s = 2$  and it yields  $\frac{\partial G}{\partial c} < 0$  for  $s = 2$ .

## 5 Concluding Remarks

According to The Economist (27 November 2010), "the green pressure groups and politicians who have driven the debate on climate change have often been loth to see attention paid to adaptation, on the ground that the more people thought about it, the less motivated they would be to push ahead with emissions reduction." We show that an increase in the efficiency of adaptation technology may result in a reduction of individual countries' incentives to free-ride on an IEA and an increase in the gains from forming the IEA. Therefore, the concern of environmentalists with adaptation is partially mitigated.

The incentives to free-ride on an IEA may decrease in the presence of adaptation. We show that the more efficient is adaptation at reducing marginal damage from emissions, the flatter the best response functions of each country in terms of emissions. This reduces the levels of global emissions in the non-cooperative equilibrium, making it less costly to cooperate on emission strategies. This is because, when other countries increase emissions, each individual country may, instead of reducing its own emissions, decrease its own damage by increasing adaptation. However, since the cost of adaptation is convex in the level of

adaptation, failing to reach a cooperative equilibrium on emissions increases the cost to each individual country through this channel. This explains why the gains from cooperation increase as more adaptation is undertaken.

In the current paper, we have made a few simplifying assumptions in order to illustrate the main insights as clearly as possible. Relaxing these may generate further insights, which is left for future work. For example, in this paper, we have analyzed the case of identical countries. In reality different regions are vulnerable to different degrees to the effects of climate change and will therefore undertake different amounts/types of adaptation, for example, Southern Europe is expected to be affected more than Northern Europe by climate change. Therefore, allowing for asymmetries across countries would be a relevant extension. Also, this paper assumes that the benefits and costs of adaptation and mitigation are contemporaneous, which may not be representative of certain adaptive measures. It would be useful to set up a dynamic model to understand the intertemporal tradeoffs arising from a relaxation of this assumption.

## References

- Agrawala, S., Bosello, F., Carraro, C., de Cian E., Lanzi, E., 2011. Adapting to Climate Change: Costs, Benefits, and Modelling Approaches. *International Review of Environmental and Resource Economics*, 5, 245-84.
- d'Aspremont, C. A., Jacquemin, J., Gabszewicz, J., Weymark, J. A., 1983. On the stability of collusive price leadership. *Canadian Journal of Economics* 16, 17-25.
- Barrett, S., 1994. Self-enforcing international environmental agreements. *Oxford Economic Papers*, 46, 878-94.
- Barrett, S., 2005. The Theory of International Environmental Agreements. In K.-G. Mäler and J. Vincent (eds.), *Handbook of Environmental Economics*, Vol. 3, Amsterdam: Elsevier, pp. 1457-1516.
- Barrett, S., 2006. Climate treaties and "breakthrough" technologies. *American Economic Review*, 96, 22-25.
- Bosello, F., Carraro, C., de Cian, E., 2011. Adaptation can help mitigation: an integrated approach to post-2012 climate policy. Working Papers 2011.69, Fondazione Eni Enrico Mattei.
- Breton, M., Sbragia, L., Zaccour, G., 2010. A dynamic model for international environmental agreements. *Environmental and Resource Economics*, 45, 25-48.
- de Bruin, K., Dellink, R., Tol, R., 2009. AD-DICE: An implementation of adaptation in the DICE model. *Climate Change*, 95, 63-81.
- de Bruin, K., Weikard, H.-P., Dellink, R., 2011. The role of proactive adaptation in international climate change mitigation agreements, CERE Working Paper 2011:9, Centre for Environmental and Resource Economics (CERE), Umeå University, Sweden.
- Carraro, C., Marchiori, C., 2003. Endogenous Strategic Issue Linkage in International Negotiations. Working Papers 2003.40, Fondazione Eni Enrico Mattei.
- Diamantoudi, E., Sartzetakis, E.S., 2002. International Environmental Agreements: the role of foresight. CORE Discussion Papers 2002061.

Diamantoudi, E., Sartzetakis, E.S., 2006. Stable International Environmental Agreements: An Analytical Approach. *Journal of Public Economic Theory* 8(2), 247-263.

Ebert, U., Welsch, H., 2012. Adaptation and mitigation in global pollution problems: Economic impacts of productivity, sensitivity, and adaptive capacity. *Environmental and Resource Economics* 52, 49-64.

Eisenack, K., Kähler, L., 2012. Unilateral emission reductions can lead to Pareto improvements when adaptation to damages is possible. Oldenburg Discussion Papers in Economics V-344-12.

El-Sayed, A., Rubio, S.J., 2011. Self-Enforcing International Environmental Agreements with R&D Spillovers. Mimeo, University of Valencia.

Hoel, M., Schneider, K., 1997. Incentives to participate in an international environmental agreement. *Environmental and Resource Economics* 9, 153-170.

Hoel, M., de Zeeuw, A., 2010. Can a focus on breakthrough technologies improve the performance of international environmental agreements? *Environmental and Resource Economics*, 47, 395-406.

Ingham, A., Ma, J., Ulph, A., 2005. Theory and practice of economic analysis of adaptation. Discussion paper, Tyndall Centre for Climate Research.

Jørgensen, S., Martín-Herrán, G., & Zaccour, G., 2010. Dynamic games in the economics and management of pollution. *Environmental modeling & assessment*, 15, 433-467.

Le Goulven, K., 2008. Financing Mechanisms for Adaptation. Commission on Climate Change and Development.

Legras, S., Zaccour, G., 2011. Temporal flexibility of permit trading when pollutants are correlated. *Automatica*, 47, 909-19.

Nordhaus, W., Yang, Z., 1996. A Regional Dynamic General-Equilibrium Model of Alternative Climate-Change Strategies. *American Economic Review*, 86, 741-765.

Osmani, D., Tol, R., 2009. Toward Farsightedly Stable International Environmental Agreements. *Journal of Public Economic Theory*, 11, 455-92.

Rose, A.K., Spiegel, M.M., 2009. Noneconomic Engagement and International Exchange: The Case of Environmental Treaties. *Journal of Money, Credit and Banking*, 41, 337-363.

Rubio, S.J., Ulph, A., 2006. Self-enforcing international environmental agreements revisited. *Oxford Economic Papers*, 58, 233-63.

Tol, R., 2005. Adaptation and mitigation: trade-offs in substance and methods. *Environmental Science and Policy*, 8, 572-578.

Tulkens, H., van Steenberghe, V., 2009. Mitigation, Adaptation, Suffering: In Search of the Right Mix in the Face of Climate Change, Working Papers 2009.79, Fondazione Eni Enrico Mattei.

de Zeeuw, A., 2008. Dynamic effects on the stability of international environmental agreements. *Journal of Environmental Economics and Management*, 55 (2), 163-74.

Zehaie, F., 2009. The timing and strategic role of self-protection. *Environmental and Resource Economics*, 44, 337-50.

## Appendices

### Appendix A: Proof of Proposition 2

For  $s = n$ , we have:

$$G = \frac{(n-1)^2 n^3 \alpha^2 X^2}{2(\beta + n^2 X)(\beta + nX)^2}$$

We have that

$$\frac{\partial G}{\partial X} = \frac{1}{2} \frac{X n^3 \alpha^2 (n-1)^2 n^3}{(X n^2 + \beta)^2 (\beta + X n)^3} (X - X_1)(X_2 - X)$$

where  $X_1 = -\frac{1}{2n}\beta \left( \sqrt{\frac{n+8}{n}} - 1 \right) < 0$  and  $X_2 = \frac{1}{2n}\beta \left( \sqrt{\frac{n+8}{n}} + 1 \right) > 0$ . Therefore, we have the following:

$$\frac{\partial G}{\partial X} > 0 \text{ iff } X < X_2$$

Since  $X \equiv \left( \omega - \frac{1}{c} \right)$  we have:

$$(i) \ X < X_2 \text{ iff } \omega - \frac{1}{c} < \frac{1}{2n}\beta \left( \sqrt{\frac{n+8}{n}} + 1 \right)$$

$$(ii) \ \frac{\partial G}{\partial c} = \frac{\partial G}{\partial X} \frac{\partial X}{\partial c} = \frac{1}{c^2} \frac{\partial G}{\partial X}$$

From (i) and (ii), it follows that:

$$\frac{\partial G}{\partial c} < 0 \text{ iff } \frac{\omega - \frac{1}{c}}{\beta} > \frac{1}{2n} \left( \sqrt{\frac{n+8}{n}} + 1 \right)$$

This along with the fact that  $L(n) \equiv \frac{1}{2n} \left( \sqrt{\frac{n+8}{n}} + 1 \right)$  is monotonically decreasing in  $n$  and  $\lim_{n \rightarrow \infty} L(n) = 0$  implies that there exists  $\bar{n}$  such that for any  $n > \bar{n}$  we have  $\frac{\omega - \frac{1}{c}}{\beta} > L(n)$  and therefore  $\frac{\partial G}{\partial c} < 0$ . ■

### Appendix B: Proof of Corollary

This follows from the fact that  $\frac{1}{2n} \left( \sqrt{\frac{n+8}{n}} + 1 \right)$  is monotonically decreasing in  $n$  and therefore if  $\frac{(\omega - \frac{1}{c})}{\beta} > \frac{1}{6} \left( \sqrt{\frac{11}{3}} + 1 \right)$  (or  $\beta < \bar{\beta}$ ) we necessarily have  $\frac{(\omega - \frac{1}{c})}{\beta} > \frac{1}{2n} \left( \sqrt{\frac{n+8}{n}} + 1 \right)$ . This, along with conditions (i) and (ii) in the proof of Proposition 2, gives  $\frac{\partial G}{\partial c} < 0$  for all  $n \geq 3$ . ■

### Appendix C: Proof of Proposition 3

We have

$$\frac{\partial \Phi(2)}{\partial X} = \frac{\Omega X n^2 \alpha^2}{(2X + Xn + \beta)^3 (Xn + \beta)^3}$$

where

$$\Omega \equiv X^3 (-5n^3 + 8n + 8) + X^2 \beta (-9n^2 + 12n + 16) + X \beta^2 (12 - 3n) + \beta^3$$

The sign of  $\frac{\partial \Phi(2)}{\partial X}$  is the same as that of  $\Omega$ . For convenience we use the notation  $\Omega(n)$  to specifically analyze  $\Omega$  as a function of  $n$ . We first note that

$$\Omega'(n) = X (12X\beta - 18Xn\beta + 8X^2 - 3\beta^2 - 15X^2n^2)$$

is strictly decreasing in  $n$ , implying the following:

$$\Omega'(n) < \Omega'(2) = - (24X\beta + 52X^2 + 3\beta^2) X < 0$$

Therefore, the function  $\Omega(n)$  is a strictly decreasing function of  $n$ . The evaluation of  $\Omega(2)$  gives the following:

$$\Omega(2) = (4X + \beta) (2X\beta - 4X^2 + \beta^2)$$

It can be shown that there exists a unique  $\hat{X} > 0$  such that  $\Omega(2) < 0$  for  $X > \hat{X}$ . Since  $\Omega'(n) < 0$ , we can state that there exists  $\hat{X} > 0$  such that  $\Omega(n) < 0$  for  $X > \hat{X}$  or  $\frac{\partial \Phi(2)}{\partial X} < 0$  for  $X > \hat{X}$ . This, along with the fact that  $\frac{\partial X}{\partial c} > 0$ , completes the proof of (i). When  $0 < X < \hat{X}$  we have  $\Omega(2) > 0$ . Moreover, from Assumption 2 we have  $X < \frac{\beta}{n-2}$  or  $n < \frac{\beta}{X} + 2$  with  $\Omega(\frac{\beta}{X} + 2) = -16(X + \beta)^3 < 0$ . This combined with  $\Omega'(n) < 0$  proves (ii). ■