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Influence of Realistic Lubricant Density-Pressure Dependence on the Stiffness of Elastohydrodynamic Lubricated Contacts

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Abstract

The analysis of the dynamic response of elastohydrodynamic lubricated contacts has often invoked a universal law for lubricant density-pressure dependence, even though the densities of many lubricants exhibit a substantial deviation from this widely adopted law. The current work investigates the influence of real lubricant density-pressure behavior on the stiffness of elastohydrodynamic lubricated contacts. It is shown that accounting for the real lubricant density-pressure dependence is crucial for an accurate estimation of the oil film stiffness, under steady-state considerations. The influence on the overall stiffness of the contact is found to be negligible though. Finally, an analytical correction procedure is provided, allowing a correction of oil film stiffness predictions that are based on the universal law for lubricant density-pressure dependence (or any other unrealistic law), to account for the real lubricant density-pressure response.

Keywords: Elastohydrodynamic Lubrication; Contact Stiffness; Lubricant Properties; Steady-State Response.

1. Introduction

Lubrication is often employed in machinery as a means of separating contacting surfaces, using an intermediate medium; the lubricant. The main purpose is to prevent direct contact between surface asperities, leading to reduced wear and friction. The lubricant also serves a secondary purpose; that of cooling the contact. In fact, a substantial amount of heat is generated within any contact and part of that heat is carried away from the lubricated contact by the lubricant. Elastohydrodynamic lubrication (EHL) is a fluid-film lubrication regime subject to high contact loads, such that the hydrodynamic pressure built-up within the lubricant film is high enough to induce elastic deformation of the contacting surfaces.

Understanding an elastohydrodynamic (EHD) contact’s response to variations in external applied loads has always been of interest to the multi-body dynamic analysis of its corresponding mechanical component. This response can be broken down to a superposition of the response of the solids and that of the lubricant film, placed in series. Under steady-state conditions, these individual responses are usually modelled using springs, whereas under transient conditions, parallel spring-dashpot systems are employed. The response of the solids only depends on their mechanical properties, whereas that of the lubricant film depends on the thermo-physical properties of the lubricant (density, viscosity, etc.) as well as the operating conditions (surface speeds, external applied load, etc.).
Over the years, the development of efficient numerical techniques and computational tools has allowed detailed numerical investigations of the stiffness and damping behaviors of EHL conjunctions. Wijnant et al. [1] were the pioneers on that front, analyzing the effects of structural vibrations on lubricant film thickness in circular EHL contacts. Nonato and Cavalca [2][3] investigated the non-linear response of EHL point contacts subject to vibrations. The vibrational behavior of EHL line contacts was also the subject of several numerical investigations, such as Qin et al. [4], Zhang et al. [5] and Tsuha et al. [6]. Wiegert et al. [7] presented a simplified semi-analytical approach for the analysis of the non-linear vibration behavior of EHL line contacts. All the aforementioned works assume a Newtonian lubricant behavior. The influence of lubricant non-Newtonian behavior was incorporated into the vibrational analysis of EHL line contacts by Zhou et al. [8] [9] [10]. All the works mentioned thus far invoke a universal law for lubricant density-pressure dependence proposed by Dowson and Higginson [11], whereby the lubricant dimensionless density $\bar{\rho} = \rho/\rho_0$ (where $\rho_0$ corresponds to the density at ambient pressure) is assumed to vary with pressure $p$ according to:

$$\bar{\rho} = 1 + \frac{0.6 \times 10^{-9} p}{1 + 1.7 \times 10^{-9} p}$$

However, from as far back as 1953, the report of the ASME Research Committee on Lubrication [12] revealed that the density-pressure dependence of many lubricants exhibits substantial deviations from the widely adopted Dowson and Higginson relation. For instance, figure 1 shows the dimensionless density-pressure dependence of the most compressible lubricant considered in the report (a silicone oil, dubbed “ASME 55”) and the least compressible one (a heavy naphthenic mineral oil, dubbed “ASME 38”). The Dowson and Higginson density-pressure dependence is also shown for comparison.

![Figure 1: Density-pressure dependence of ASME 38 and ASME 55 lubricants and its comparison to the widely adopted Dowson & Higginson relationship](image-url)
Differences in density-pressure responses of lubricants are not without consequence on the lubrication performance of EHL contacts. Venner and Bos [13] showed that lubricant film thickness at the center of the contact is inversely proportional to its density at the same location, under isothermal conditions. Habchi and Bair [14] later showed that this observation also holds under thermal conditions. Thus, accurately accounting for density-pressure dependence of lubricants is crucial for the prediction of the performance of these contacts. The current work investigates the influence of real lubricant density-pressure dependence on the stiffness of EHL contacts. Out of simplicity, smooth line contacts are considered under isothermal Newtonian considerations. Given that the focus here is on contact stiffness only, steady-state operation is assumed.

2. Governing Equations

Line contacts take place between infinitely long rotating cylindrical bodies of radii \( R_1 \) and \( R_2 \), with their surface velocities denoted by \( u_1 \) and \( u_2 \), pressed against each other by an external applied load per unit length \( F \). The infinite nature of the contact in one of the space directions means that the dimensions of both the hydrodynamic and elastic parts of the problem are reduced by one order. The hydrodynamic part governs the lubricant flow within the EHL conjunction, while the elastic part governs the elastic deformation of the solid bodies. The geometry of a line contact can be reduced to that of an equivalent contact between an elastic cylinder of equivalent radius \( R = R_1 R_2 / (R_1 + R_2) \) and a rigid flat plane, as shown in figure 2 (a). This is done by defining the modulus of elasticity \( E \) and Poisson coefficient \( \nu \) of the cylinder as a function of those of the two solids \( (E_1, \nu_1) \) and \( (E_2, \nu_2) \) as follows [15]:

\[
E = \frac{1}{\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}} \quad \text{and} \quad \nu = 0 \tag{2}
\]

This way, the elastic cylinder would accommodate the entire elastic deformation of the two solids. The computational domain of the EHL model is shown in figure 2 (b). The 1D contact domain \( \Omega_c \) (over which the hydrodynamic part is applied) is located on the surface of the 2D square solid domain \( \Omega \) (over which the elastic part is applied), between \( X_{\text{inlet}} = -4.5 \) and \( X_{\text{outlet}} = 1.5 \). The dimensions of the latter should be large enough compared to the former, such that a half-space configuration is attained [16]. All equations provided in this section are written in dimensionless form, using Hertzian / Dry contact parameters [17]:

**Hertzian Contact Half-Width:** \( a_h = \sqrt{\frac{4FR}{\pi E}} \)

**Hertzian Contact Pressure:** \( p_h = \frac{2F}{\pi a_h} \tag{3} \)
The lubricant flow within the contact is governed by Reynolds [18] equation, which is derived from the Navier-Stokes equations by applying the thin-film simplifying assumptions. It is applied to the 1D contact domain $\Omega_c$ (with the inlet located on the left side and the outlet on the right side) and it gives access to the dimensionless pressure distribution $P$ over the contact domain. The dimensionless Reynolds equation for steady-state line contacts is given by:

$$\frac{\partial}{\partial X} \left( \bar{e} \frac{\partial P}{\partial X} \right) + \frac{\partial (\bar{\rho} H)}{\partial X} = 0$$

Where: $$\bar{e} = \frac{\bar{\rho} H^3}{\bar{\mu} \lambda} \quad \text{with} \quad \lambda = \frac{12 u_m \mu_0 R^2}{a_h^3 p_0}$$

where $u_m = (u_1 + u_2)/2$ is the mean entrainment speed, $\bar{\mu} = \mu / \mu_0$ is the lubricant dimensionless viscosity (where $\mu_0$ corresponds to the viscosity at ambient pressure) and $H = hR/a_h^2$ is the dimensionless lubricant film thickness (with $h$ being the dimensional film thickness), defined as:

$$H = H_0 + \frac{X^2}{2} - W$$

The first term on the right-hand-side of equation (5) corresponds to the rigid-body separation (the distance between the undeformed solid surfaces), while the middle term represents the undeformed geometry of the lubricant gap and the last term corresponds to the normal elastic displacement of the contact surface $\Omega_c$, under the influence of the hydrodynamic pressure.
generated within the lubricant film. The displacement field is obtained by applying the linear elasticity equations to the solid domain $\Omega$, under a plane strain configuration (given the infinite nature of the solid domain in the $Y$-direction). Using the reduced material properties defined in equation (2), the dimensionless plane strain linear elasticity equations are given by (after simplification [15]):

$$
\begin{align}
-\frac{\partial^2 U}{\partial X^2} - \frac{\partial}{\partial Z} \left[ \frac{1}{2} \left( \frac{\partial U}{\partial Z} + \frac{\partial W}{\partial X} \right) \right] &= 0 \\
-\frac{\partial}{\partial X} \left[ \frac{1}{2} \left( \frac{\partial U}{\partial Z} + \frac{\partial W}{\partial X} \right) \right] - \frac{\partial^2 W}{\partial Z^2} &= 0
\end{align}
$$

(6)

where $U$ and $W$ are the $X$ and $Z$ components of the dimensionless elastic displacement field. These equations are solved while applying $P$ as a normal pressure load and zero tangential load over the contact domain $\Omega_c$, zero displacement over $\partial \Omega_b$ and zero normal and tangential loads over the remaining boundaries of $\Omega$. As for Reynolds equation, a zero-pressure boundary condition is applied over the inlet and outlet boundaries of $\Omega_c$. In addition, a penalty formulation is employed to handle the non-physical negative pressures that arise from the solution of Reynolds equation, as detailed in [19]. Finally, the value of the rigid body separation term (the scalar $H_0$) is determined by applying an equilibrium of forces to the contact, which results in the dimensionless load balance equation:

$$
\int_{\Omega_c} P \, dX = \frac{\pi}{2}
$$

Equation (7) balances the hydrodynamic pressure generated within the lubricant film against the external applied contact load per unit length $F$. It is added to the system of equations formed by the Reynolds and linear elasticity equations, while introducing $H_0$ as an additional scalar unknown. Reynolds equation (4), the linear elasticity equations (6) and the load balance equation (7) form a highly non-linear system of equations that fully describes the lubrication performance of a smooth isothermal Newtonian line contact. The solution of Reynolds equation provides the dimensionless pressure distribution $P$ within the lubricant film, while the solution of the linear elasticity equations provides the elastic displacement field $(U, W)$ of the solids and the load balance equation allows determining $H_0$. All equations are solved simultaneously, using the full-system finite element approach [20]. The Static Condensation with Splitting (SCS) model order reduction technique [15] is applied to speed-up the calculations. For more details, the reader is referred to [20].

3. Contact Stiffnesses

From a multi-body dynamics perspective, under steady-state considerations, an EHL contact is viewed as a 1-DOF system whose stiffness is composed of the stiffness of the elastic solids $k_c$
and that of the fluid film $k_f$, placed in series, as shown in figure 3. Let $h_c = H(X = 0) \times a_z^2 / R$ be the lubricant film thickness at the center of the contact $(X = x = 0)$ and $\delta = -W(X = 0) \times a_z^2 / R$ be the composite elastic deformation of the solids at the same location. Note that, according to equation (5): $h_c = h_0 + \delta$. The dimensional rigid body separation $h_0 = H_0 \times a_z^2 / R$ is usually negative (as is the case in figure 3, for instance), except for very lightly loaded contacts. This means that, without their deformation, the solids would interpenetrate under the influence of the external applied load.

![Figure 3: Stiffness of an EHL contact](image)

Then, it ensues that the stiffnesses $k_c$ and $k_f$ of the elastic solids and lubricant film are given by:

$$k_c = \frac{\partial F}{\partial \delta} \quad \text{and} \quad k_f = -\frac{\partial F}{\partial h_c} \quad (8)$$

And the overall contact stiffness $k = k_c k_f / (k_c + k_f)$ can be expressed as:

$$k = -\frac{\partial F}{\partial h_0} \quad (9)$$

This is because the overall contact stiffness is determined from the variation of the distance between the center of the elastic cylinder and the rigid flat plane $(h_0 + R$; with $R$ being constant), subject to a variation in the external applied load per unit length $F$. Note that a negative sign was added to the definitions of the oil film stiffness $k_f$ and the overall contact stiffness $k$ in equations (8) and (9), respectively. Otherwise, negative stiffness values would be obtained, since both $h_c$
and $h_0$ decrease when $F$ is increased. This was not necessary though for the definition of the elastic solid stiffness $k_e$, since $\delta$ increases with $F$.

4. Results and Discussion

To isolate the influence of density-pressure dependence on EHL contact stiffnesses, three hypothetical lubricants are considered, all with the same viscosity-pressure dependence, but with different density-pressure responses. The employed viscosity-pressure relation is that proposed by Roelands [21]:

$$\mu = \mu_0 \exp \left( \left( \ln \left( \mu_0 \right) + 9.67 \right) \left[ -1 + \left( 1 + 5.1 \times 10^{-9} p \right)^{Z_0} \right] \right)$$

Where:

$$Z_0 = \frac{\alpha}{5.1 \times 10^{-9} \ln \left( \mu_0 + 9.67 \right)}$$

The values for the ambient-pressure viscosity and viscosity-pressure coefficient are: $\mu_0 = 10 \text{ mPa} \cdot \text{s}$ and $\alpha = 20 \text{ GPa}^{-1}$, respectively, for all three fluids. As for the density-pressure dependence of the three fluids, three different responses are considered: that of Dowson & Higginson provided in equation (1) and those of ASME 38 and ASME 55. All three responses were shown in figure 1. For the ASME 38 and ASME 55 responses, they were fitted in [14] to the Tait [22] equation of state, given by:

$$\rho = \frac{\rho_0}{1 - \frac{1}{1 + K'_0} \ln \left[ 1 + \frac{p}{K_0} \left( 1 + K'_0 \right) \right]}$$

with $K'_0 = 9.650$ and $K_0 = 1.8364 \text{ GPa}$ for ASME 38, while $K'_0 = 9.735$ and $K_0 = 0.8376 \text{ GPa}$ for ASME 55. Note that the value of the ambient-pressure density $\rho_0$ is not needed as it cancels out within the two terms of equation (4).

Throughout this section, steel-steel contacts are considered ($E_1 = E_2 = 210 \text{ GPa}$ and $\nu_1 = \nu_2 = 0.3$) with $R = 15 \text{ mm}$, while the values of the external applied load per unit length $F$ and the mean entrainment speed $u_m$ were varied to cover sufficiently wide ranges of the Moes [23] dimensionless load parameter $M$ and material properties parameter $L$, defined as:

$$M = \frac{F}{2 \pi R} \left( \frac{\mu_0 u_m}{E R} \right)^{-1/2} \quad \text{and} \quad L = 2 \alpha E \left( \frac{\mu_0 u_m}{E R} \right)^{1/4}$$

In fact, the following ranges of $M$ and $L$ were considered: $5 \leq M \leq 1000$ and $1 \leq L \leq 20$, such that contact stiffnesses are evaluated and analyzed over a wide range of operating conditions.
These stiffnesses are evaluated from finite difference approximations of their corresponding derivatives, provided in equations (8) and (9).

4.1. Film Thicknesses

The central film thicknesses \( h_c \) are reported on a log-log scale in figure 4, as a function of \( M \) for different values of \( L \) (1, 2, 5, 10, 15 and 20), with the direction of increasing \( L \) values indicated by a dashed arrow. Figure 4 (a) shows the central film thicknesses for ASME 38, while those of ASME 55 are reported in figure 4 (b). In both figures, the central film thicknesses obtained with Dowson & Higginson are shown for comparison.

![Figure 4: Comparison of central film thicknesses for Dowson & Higginson with those for: a) ASME 38 and b) ASME 55, as a function of \( M \), for different values of \( L \) (1, 2, 5, 10, 15 and 20)](image)

It is clear from figure 4 (a) that, for a given \( L \), the use of the widely adopted Dowson & Higginson relation for density-pressure dependence leads to an overestimation of central film thickness at high values of \( M \), compared to ASME 38. This is because, for a given \( L \), high \( M \) values are associated with high external applied loads per unit length \( F \) and thus, high contact pressures. However, remember that the density of ASME 38 exceeds that of Dowson & Higginson at high pressures (above \( \approx 0.85 \) GPa), as was shown in figure 1. But, also remember that, according to Venner and Bos [13] or Habchi and Bair [14], central film thickness is inversely proportional to lubricant density at the contact center. At low contact pressures (low \( M \) and low \( L \) values), this trend is inverted and central film thicknesses for Dowson & Higginson are slightly underestimated, compared to ASME 38. On the other hand, figure 4 (b) reveals that Dowson & Higginson film thicknesses are higher than those of ASME 55, irrespective of \( M \) and \( L \) and that the deviation increases with both \( M \) for a given \( L \), and \( L \) for a given \( M \). In other words, the deviation increases with increasing external applied loads and thus, contact pressures. This is because ASME 55 density exceeds that of Dowson & Higginson for any pressure, as can be seen in figure 1, and the difference increases with pressure. The maximum relative errors in central film thickness...
predictions for Dowson & Higginson are 28.0% and 41.1%, with respect to ASME 38 and ASME 55, respectively. This sheds light on the importance of an accurate representation of lubricant density-pressure dependence, for an adequate prediction of central film thickness in EHL contacts, as was highlighted by Venner and Bos [13] or Habchi and Bair [14].

4.2. Stiffnesses

This section examines the influence of lubricant density-pressure dependence on contact stiffnesses i.e. the stiffnesses $k_e$ and $k_f$ of the elastic solids and lubricant film and the overall contact stiffness $k$. Given that for line contacts, the external applied load is given per unit contact length (in $N/m$), stiffnesses will be in $N/m^2$ or $Pa$.

Starting with the stiffness of the elastic solids $k_e$, it is derived from variations of the composite elastic deformation of the solids $\delta$ at the contact center with $M$, for different values of $L$ (1, 2, 5, 10, 15 and 20)

![Figure 5: Variations of the composite elastic deformation of the solids $\delta$ at the contact center with $M$, for different values of $L$ (1, 2, 5, 10, 15 and 20)](image)

$\delta$ decreases linearly with $M$, which means that $k_e$ is constant. This is not surprising, since $\delta$ is proportional to $a_b^2/R$. 

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, with the coefficient of proportionality depending on the size of the computational domain of the solids [20]. For the size adopted in this work (60×60, in dimensionless terms), \( \delta \approx 2.5 a_h^2 / R \). Replacing \( a_h \) by its expression provided in equation (3), one gets after simplification:

\[ \delta \approx 10 F / \pi E. \]

As such, \( k_e = \frac{\partial F}{\partial \delta} = \frac{\pi E}{10} \), which only depends on the mechanical properties of the solids, as expected, through the equivalent modulus of elasticity \( E \). Given that steel solids are considered throughout this work, then \( E \approx 115 \text{ GPa} \) and the stiffness of the elastic solids \( k_e \approx 36 \text{ GPa} \).

Next, the oil film stiffness \( k_f \) is examined. It is determined from variations in central film thickness \( h_c \) with contact external applied load per unit length \( F \), as detailed in equation (8). These were reported in figure 4, as a function of \( M \) for different values of \( L \). The corresponding oil film stiffnesses are shown in figure 7, with the direction of increasing \( L \) values indicated by a dashed arrow. Figure 7 (a) shows the oil film stiffnesses for ASME 38, while those of ASME 55 are reported in figure 7 (b). In both figures, the oil film stiffnesses obtained with Dowson & Higginson are shown for comparison. First, figure 7 reveals that oil film stiffness generally increases with increasing \( M \) for a given \( L \), but it increases with decreasing \( L \) for a given \( M \). This is because, under such conditions, lubricant films become thinner and thus, stiffer. Note that the oil film stiffness \( k_f \) is orders of magnitude higher than the elastic solid stiffness \( k_e \approx 36 \text{ GPa} \). Like film thicknesses and for the same reason, oil film stiffnesses for Dowson & Higginson increasingly deviate from those of ASME 38 or ASME 55 with increasing \( M \), for a given \( L \). This is not surprising, since the oil film stiffness is directly derived from its thickness, as depicted in equation (8). The maximum relative errors in oil film stiffness for Dowson & Higginson are 25.6%.

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**Figure 6:** Variations of the composite elastic deformation of the solids \( \delta \) at the contact center with \( F \), for different values of \( L \) (1, 2, 5, 10, 15 and 20)
and 22.0%, with respect to ASME 38 and ASME 55, respectively. This highlights the importance of an accurate representation of lubricant density-pressure dependence, for an adequate prediction of oil film stiffness in EHL contacts.

![Figure 7](image1.png)

**Figure 7:** Comparison of oil film stiffnesses for Dowson & Higginson with those for: a) ASME 38 and b) ASME 55, as a function of $M$, for different values of $L$ (1, 2, 5, 10, 15 and 20)

![Figure 8](image2.png)

**Figure 8:** Comparison of overall contact stiffnesses for Dowson & Higginson with those for: a) ASME 38 and b) ASME 55, as a function of $M$, for different values of $L$ (1, 2, 5, 10, 15 and 20)

The influence of the lubricant density-pressure response on the overall contact stiffness $k = k_c k_f / \left( k_c + k_f \right)$ remains limited though, as can be seen in figure 8. In fact, the overall contact stiffness $k$ for the different considered cases is reported in figure 8, as a function of $M$ for the different values of $L$ (1, 2, 5, 10, 15 and 20), with the direction of increasing $L$ values indicated by a dashed arrow. Figure 8 (a) shows the overall contact stiffnesses for ASME 38, while those of ASME 55 are reported in figure 8 (b). In both figures, the overall contact stiffnesses obtained with
Dowson & Higginson are shown for comparison. Clearly, the influence of the density-pressure response of the lubricant on the overall contact stiffness is rather limited. At high $M$, it is even nonexistent since under these conditions, $k_f$ is orders of magnitude higher than $k_e$ and the overall contact stiffness $k \approx k_e$. At low $M$ values on the other hand, $k_f$ approaches $k_e$ and some influence could be observed. This influence remains limited though, with the maximum relative errors in overall contact stiffness for Dowson & Higginson being equal to 0.33% and 0.37%, with respect to ASME 38 and ASME 55, respectively.

4.3. Fluid Film Stiffness Correction

It was shown in section 4.2 that EHL fluid film stiffness is significantly influenced by the lubricant density-pressure response and that, relative errors in its prediction exceeding 25% may be incurred from an inadequate representation of that response. In this section, an analytical correction methodology is proposed for correcting EHL fluid film stiffnesses that are based on universal laws for lubricant density-pressure dependence, to account for the real response of the fluid. The methodology is based on EHL film thickness correction for density-pressure dependence as proposed by Venner and Bos [13] or Habchi and Bair [14]. Both works suggest that the ratio of the central EHL film thicknesses for a correct density-pressure response to an incorrect one $h_{c,\text{correct}}/h_{c,\text{incorrect}}$ equals the inverse of the corresponding lubricant dimensionless densities $\rho_{\text{incorrect}}/\rho_{\text{correct}}$ at the contact center, where the pressure is assumed to be equal to the Hertzian contact pressure $p_h$. That is, central film thickness correction may be formulated as follows:

$$h_{c,\text{correct}} = h_{c,\text{incorrect}} \times \frac{\bar{\rho}_{\text{incorrect}}}{\bar{\rho}_{\text{correct}}} \bigg|_{p=p_h}$$

(13)

Remark: A slightly improved correction procedure with respect to equation (13) may be obtained by evaluating the ratio of lubricant densities at the contact center, using the actual pressure at that location. The latter may slightly differ from $p_h$, especially for lightly loaded contacts. This would require however a precise knowledge of the actual central contact pressure. The latter can only be determined through a full numerical resolution of the problem though, which defies the purpose of having a simple analytical correction methodology.

The correction procedure delineated in equation (13) can be used for instance to correct the Dowson & Higginson central film thickness predictions reported in figure 4, to account for the real density-pressure response of ASME 38 and ASME 55. The correction reduces the maximum relative errors in central film thickness predictions from 28.0% to 0.98% for ASME 38 and from 41.1% to 2.32% for ASME 55.
Corrected oil film stiffnesses can now be deduced from the corrected central film thicknesses, using equation (8). The corrected oil film stiffness results are shown in figure 9. Clearly, the corrected Dowson & Higginson oil film stiffnesses are in much better agreement with the predictions for both ASME 38 and ASME 55. In fact, the maximum relative errors are reduced from 25.6% to 1.74% for ASME 38 and from 22.0% to 3.09% for ASME 55.

Alternatively, corrected oil film stiffnesses could have been obtained as follows:
Equation (14) can be used to correct the Dowson & Higginson oil film stiffnesses by replacing \( h_{c,incorrect} \) and \( k_{f,incorrect} \) by the corresponding central film thickness and oil film stiffness numerical predictions, respectively. This would result in the same corrected oil film stiffnesses as shown in figure 9. But more importantly, equation (14) could be used to derive a fully analytical framework for estimating fluid film stiffnesses. Such a framework would require analytical expressions for \( h_{c,incorrect} \) and \( k_{f,incorrect} \), that are based on the universal law for density-pressure dependence (or any other unrealistic law). For the former, many film thickness formulae exist in the literature that could be used while for the latter, none could be found. The following regression formula was derived to fit the Dowson & Higginson oil film stiffnesses shown in figure 7 or figure 9:

\[
k_{f,incorrect} = a M^b \text{ (GPa)}
\]

with: \( a = 349.1 L^{-0.221} \text{ (GPa)} \) and \( b = \frac{4.65 L^{1.77} + 16.17}{3.69 L^{1.8} + 13.71} \) (15)

The curve fitted analytical estimation of \( k_{f,incorrect} \) using equation (15) fits the numerical data well, with a maximum relative error of 3.43%. As for central film thicknesses, analytical formulae exist that are often based on the assumption of a Dowson & Higginson density-pressure relation or even worse, that of incompressibility. For instance, a widely adopted central film thickness formula for EHL line contacts has been proposed by Moes [23], that assumes incompressible lubricants \( (\overline{\rho} = 1) \). It is given by:

\[
h_{c,Moes} = \sqrt{\frac{\mu_0 u_m R}{E}} \left\{ \left[ \frac{3}{M^3} \right] + \left[ \frac{2.62105}{M^{1/5}} \right] + \left[ \frac{1.28666L^{2/3}}{M^{1/8}} \right] \right\} \left[ \left[ 7 + 8 \exp(-1.74737M^{4/5}) \right] \right]^{1/2}
\]

with: \( s = \frac{1}{5} \left[ 7 + 8 \exp(-1.74737M^{4/5}) \right] \) (16)
This incompressible central film thickness formula could be adjusted to account for a Dowson & Higginson density-pressure dependence, as suggested by equation (13), so that an incorrect central film thickness $h_{c,\text{incorrect}}$ is obtained, to be used within equation (14):

$$h_{c,\text{incorrect}} = h_{c,\text{Moes}} \times \frac{1}{\rho_{D\&H} \big|_{p=p_h}}$$

(17)

The analytical estimation of $h_{c,\text{incorrect}}$ using equation (17) fits the numerical data well, with a maximum relative error of 4.45%. Equation (14) may now be used to correct the analytical Dowson & Higginson oil film stiffness predictions, to account for the real density-pressure response of ASME 38 and ASME 55. This is done by replacing $k_{f,\text{incorrect}}$ and $h_{c,\text{incorrect}}$ by their analytical expressions, provided in equations (15) and (17), respectively, while $\bar{\rho}_{\text{incorrect}}$ and $\bar{\rho}_{\text{correct}}$ at the contact center are replaced by the Dowson & Higginson and Tait values, respectively, as provided in equations (1) and (11). The analytical oil film stiffness results and the proposed correction procedure are delineated in figure 10. Clearly, the corrected analytical oil film stiffnesses are in much better agreement with the numerical predictions for both ASME 38 and ASME 55, compared to the uncorrected ones. In fact, the maximum relative errors are reduced from 28.6% to 2.15% for ASME 38 and from 23.5% to 4.75% for ASME 55.

5. Conclusion

The analysis of the dynamic response of elastohydrodynamic lubricated contacts has often been based on a universal law for lubricant density-pressure dependence; the Dowson & Higginson relation. Yet, it has been known for a long time now that the densities of many lubricants exhibit a substantial deviation from this widely adopted law and that this is not without consequence on the performance of EHL contacts. The current work investigates the influence of real lubricant density-pressure behavior on the stiffness of elastohydrodynamic lubricated contacts. For this, two typical density-pressure responses are considered; that of a highly compressible silicone oil and that of a heavy naphthenic mineral oil with a relatively low compressibility. The corresponding contact stiffnesses are examined and compared to those obtained using the Dowson & Higginson relation; all other parameters being equal. It is shown that accounting for the real lubricant density-pressure dependence is crucial for an accurate estimation of the oil film stiffness, under steady-state considerations. In fact, the use of the universal law for density-pressure dependence may result in errors in fluid film stiffness predictions exceeding 25%. The influence on the overall stiffness of the contact is found to be negligible though. Finally, an analytical correction procedure is provided, allowing a correction of oil film stiffness numerical predictions that are based on the universal law for lubricant density-pressure dependence (or any other unrealistic law), to account for the real lubricant density-pressure response. This correction procedure was shown to reduce the maximum relative errors in fluid film stiffness numerical predictions to less than 3%. The
correction procedure was also applied within a fully analytical framework for oil film stiffness predictions, reducing the maximum relative errors from about 30% to less than 5%.

Nomenclature

\( \alpha \): Lubricant pressure-viscosity coefficient (Pa\(^{-1}\))
\( \delta \): Composite elastic deformation of the contacting solids at the contact center (m)
\( \mu \): Lubricant viscosity (Pa.s)
\( \mu_0 \): Lubricant viscosity at ambient pressure (Pa.s)
\( \bar{\mu} \): Dimensionless lubricant viscosity
\( \nu \): Equivalent Poisson coefficient
\( \nu_1, \nu_2 \): Poisson coefficients of the contacting solids
\( \Omega \): Two-dimensional solid domain
\( \Omega_c \): One-dimensional contact domain
\( \partial \Omega_b \): Solid domain zero-displacement boundary
\( \rho \): Lubricant density (kg/m\(^3\))
\( \bar{\rho} \): Dimensionless lubricant density
\( \bar{\rho}_{\text{correct}} \): Correct dimensionless lubricant density
\( \bar{\rho}_{\text{incorrect}} \): Incorrect dimensionless lubricant density
\( \bar{\rho}_{\text{D&H}} \): Dowson & Higginson dimensionless lubricant density
\( \rho_0 \): Lubricant density at ambient pressure (kg/m\(^3\))
\( a_h \): Hertzian contact half-width (m)
\( E \): Equivalent Young’s modulus of elasticity (Pa)
\( E_1, E_2 \): Young’s moduli of elasticity of the contacting solids (Pa)
\( F \): Contact external applied load per unit length (N/m)
\( h \): Lubricant film thickness (m)
\( h_0 \): Rigid body separation (m)
\( h_c \): Lubricant film thickness at contact center (m)
\( h_{c,\text{correct}} \): Corrected lubricant film thickness at contact center (m)
\( h_{c,\text{incorrect}} \): Uncorrected lubricant film thickness at contact center (m)
\( h_{c,\text{Moes}} \): Moes lubricant film thickness at contact center (m)
\( H \): Dimensionless lubricant film thickness
\( H_0 \): Dimensionless rigid body separation
\( k \): Overall contact stiffness (Pa)
\( k_e \): Elastic solid stiffness (Pa)
\( k_f \) : Fluid film stiffness (Pa)
\( K_0 \) : Lubricant isothermal bulk modulus at ambient pressure (Pa)
\( K'_0 \) : Pressure rate of change of lubricant isothermal bulk modulus at ambient pressure
\( L \) : Moes dimensionless material properties parameter
\( M \) : Moes dimensionless load parameter
\( p \) : Pressure (Pa)
\( p_h \) : Hertzian contact pressure (Pa)
\( P \) : Dimensionless pressure
\( R \) : Radius of equivalent cylinder (m)
\( R_1, R_2 \) : Radii of the contacting solids / cylinders (m)
\( u, w \) : x and z-components of the solid elastic displacement field (m)
\( U, W \) : Dimensionless x and z-components of the solid elastic displacement field
\( u_1, u_2 \) : Surface velocities of the contacting solids (m/s)
\( u_m \) : Contact mean entrainment speed (m/s)
\( x, y, z \) : Space coordinates (m)
\( X, Y, Z \) : Dimensionless space coordinates

**Dimensionless Parameters**

\[
X = \frac{x}{a_h}, \quad Z = \frac{z}{a_h}, \quad H = \frac{hR}{a_h^2}, \quad U = \frac{uR}{a_h^2}, \quad W = \frac{wR}{a_h^2}, \quad P = \frac{p}{p_h}, \quad \bar{p} = \frac{p}{\rho_0}, \quad \bar{\mu} = \frac{\mu}{\mu_0}
\]

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