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# Optimal Supercharge Scheduling of Electric Vehicles: Centralized vs. Decentralized Methods

Ribal Atallah, Chadi Assi, Wissam Fawaz, Mosaddek Hossain Kamal Tushar, Maurice Khabbaz

**Abstract**—The contemporary problem of scheduling the recharge operations of electric vehicles (EVs) has gained a lot of research attention. This is particularly true given the governmental and industrial confidence in a bright future for EVs accompanied with the widespread installation of an enormous number of charging stations across the world. As such, this paper addresses the delay-optimal scheduling of charging EVs at several Charging Stations (CSs) each with multiple charging outlets. At first, a centralized optimization framework is formulated using an Integer Linear Problem (ILP) that accounts for the delayed arrival of EVs to CSs and the randomness in the requested recharge time interval. Simulation results showed the efficacy of the ILP model when compared to naive as well as sophisticated scheduling heuristics. Next, motivated by the scalability issues of the ILP model, this paper then proposes a distributed game-theoretical approach where each EV communicates its selected CS and iterates on modifying its strategy until all EVs converge to selecting an appropriate CS that minimizes their waiting times for receiving services. The distributed game-theoretical approach recorded promising results especially when compared to the well-known Shortest Job First scheduling algorithm. Further, unlike the other approaches which normally are centralized and suited for offline scheduling, the game-based method is suited for online scheduling since it is played at anytime a batch of EVs requests charging services. The running time of the game is remarkably small and outperforms all other heuristics and its convergence to Nash equilibrium is guaranteed after only small number of iterations.

**Index Terms**—Linear Optimization, Electric Vehicles, Scheduling, Game Theory

## I. INTRODUCTION

### A. Overview:

According to statistics recently published by the International Energy Agency (IEA) [1], the transport sector currently accounts for about 23% of total energy-related CO<sub>2</sub> emissions. This is expected to increase by nearly 50% in 2030 and more than 80% by 2050, especially in the absence of appropriate measures. Globally, resources have been devoted to Electric Vehicles (EVs) development since the energy crises of the 1970s; however, in that time, only small markets have developed. Presently, a renewed political push for EV deployment is linked to climate change abatement and energy conservation [2]. EVs appear to have a bright future, since they promise, among other benefits, greenhouse gas emission reduction and independence from fossil fuels. Several national governments have set ambitious targets for EV deployment, particularly the U.S. and Canada [3]. Also, many authorities joined forces with the electric automotive industry and introduced strong incentives to promote EVs, which include *i*) exemption from import and value added tax, *b*) free toll roads, *c*) free parking spots and *d*) privileged access to bus lanes. However, the

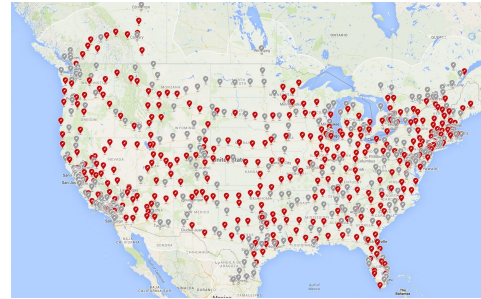


Fig. 1: The Expansion of Tesla Supercharging Sites

inconvenience of EVs recharging as well as their limited range are two main factors that have been delaying the EVs' wide market penetration. In fact, the process of charging/recharging an EV's battery to capacity requires a significantly larger amount of time (i.e. in the order of hours) than the conventional refuelling of an Internal Combustion Engine (ICE)-driven vehicle's reservoir, [4]. At this point, several major factors limit the widespread evolution of the EV industry, including: *a*) range anxiety which is the fear that a vehicle has insufficient range to reach its destination, *b*) lack of state-sponsored signs for EV charging stations hence creating the delusion that there is not enough charging facilities, and *c*) limited availability of fast chargers along major highways. Recent reports are highlighting the market growth in both EV batteries and stationary energy storage as the confluence of regulatory policy, technological capabilities, and business models continue to drive interest forward. The introduction of mass-market battery EVs with ranges of 150 to 200-plus miles has significant implications for the EV market. During the next few years, significant growth is further expected, particularly in the North American EV market. That growth will be driven by sales of the Tesla S and X Models, the second-generation Volt, and by the introduction of the Chevrolet Bolt 200-mile range battery electric vehicle. Given the growing popularity of EVs, it is anticipated that future garages (such as the parking lots for office buildings or business districts) will provide fast EV charging services. In point of fact, and as illustrated in Figure 1, EV industrial leaders, and particularly Tesla Motors, have already started expanding their supercharging services as they consider this step the key to their future plans. This expansion is however accompanied by a critical challenge presented by the increasing number of Tesla users requesting charging services especially during peak hours. Now, the EV research studies have reached a consensus that EV charging should be controlled to avoid distribution congestion and higher peak-to-average ratios [5]. The increasing

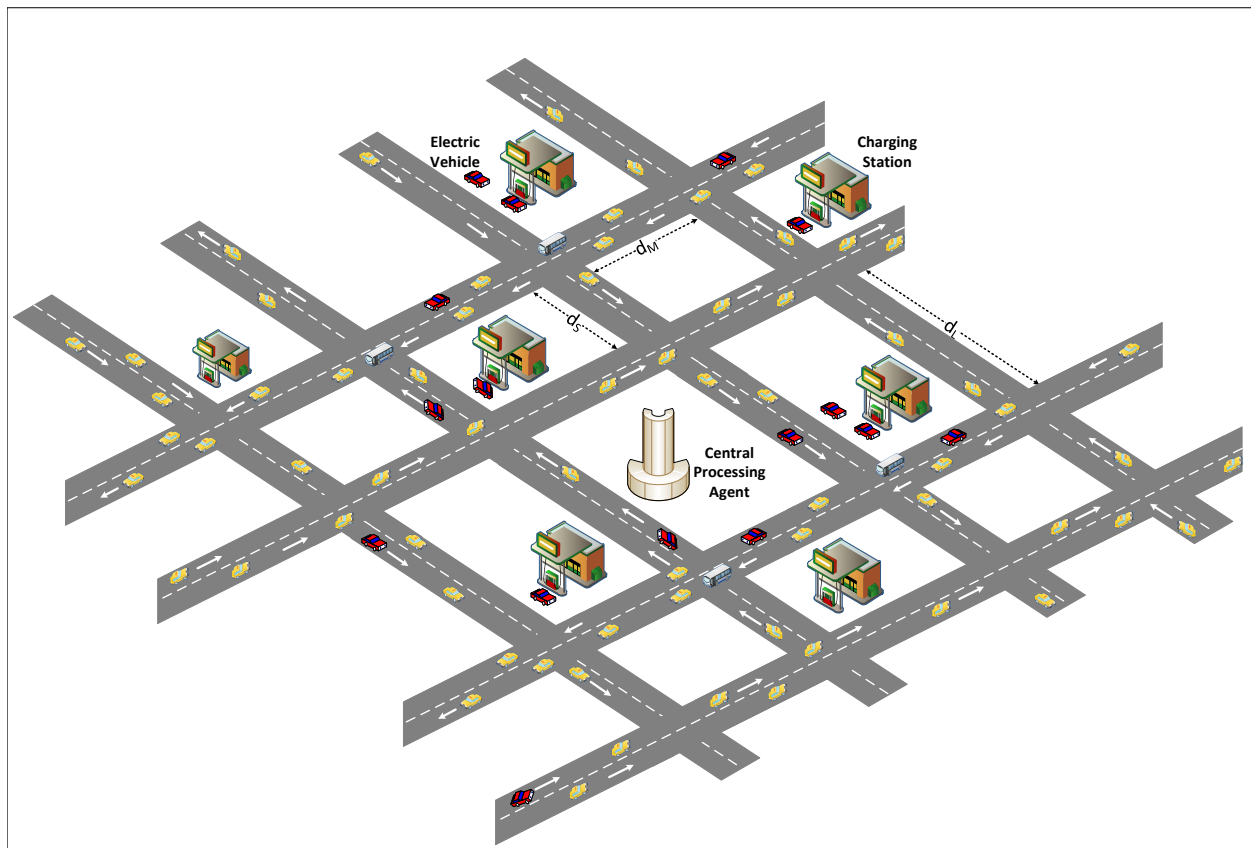


Fig. 2: Scheduling EV Recharge

need to satisfy growing EV charging demands gives rise to a contemporary challenging problem of scheduling EVs on charging stations. In fact, a proper EV charging schedule, especially for urgent charging stations supporting fast EV charging, can enhance the utilization of charging facilities, increase their expected revenues and increase the customers' satisfaction; hence, promoting the EV industry. Now, without a proper coordinated charging technique, either the EV will experience long waiting times at the charging station or the latter may be forced to decline the service requests. These negative effects may severely affect the customer satisfaction and result in revenue losses for the charging stations. To mitigate these negative effects, it is crucial to design efficient admission control and scheduling algorithms for EV charging stations. As such, this work addresses the problem of EV scheduling on charging stations that provide fast charging in a unidirectional grid-to-vehicle (G2V) power transfer scenario. Note that, an EV can also discharge its battery to the grid for reasonable incentives [6], [7]. However, this is outside the scope of this current work. Ideally, as illustrated in Figure 2, neighbouring charging stations are connected to a central intelligent agent which is aware of all current and future EV locations, destinations and charging demands, and thus, plans the scheduling of EVs on designated CSs accordingly. In point of fact, the full realization of the next generation wireless communications leverages smart city solutions built on 5G infrastructure [8], hence, allowing for connected EVs to communicate with each other as well as with a backend agent

with negligible delays for the purpose of generating significant EV scheduling efficiencies.

The primary objective of this work is to allocate CSs to EVs requesting charging services such that the maximum EV waiting time at the charging station is minimal. For this purpose, an optimization framework is first proposed that serves to realize an optimal offline scheduling policy using Integer Linear Programming (ILP). The latter model is then compared with simple as well as complex scheduling heuristics. Due to the exponential-time complexity of the optimal allocation method, *i.e.*, *ILP model*, as well as the unavailability of knowledge about future EV recharge requests, this work then establishes a distributed game-theoretical approach where each EV is aware of the status of each neighbouring CS and hence, strategically chooses at which CS to recharge in such a way that minimizes the total EV waiting times. This work analyses the various properties of the resulting game, in particular, the existence of a generalized Nash equilibrium is shown and also, the EV selection behaviour is discussed. As such, it becomes clear that coordinated charging is an effective EV charging plan which serves to improve the overall system energy utilization and avoid overloading on EV charging stations. In fact, the proposed game-theoretical approach enables the system to adapt to time-varying conditions such as arrival of new EVs with recharging requests.

## B. Novel Contributions:

The main contributions of this work are summarized as follow:

- 1) In an attempt to address the emerging problem of allocating EVs on CSs, this work examines and accounts for the road topology, the vehicular mobility model, and the EVs' required State-of-Charge (SoC) at their respective destinations. With the objective of minimizing the maximum EV waiting time at a CS, this paper first formulates a mixed ILP model that meets the EVs' requirements. The optimal solution of the underlying scheduling problem assigns EVs to CSs that fulfill their charging requests in the least amount of waiting time. The results obtained by solving the ILP model serve as an optimal benchmark for comparison purposes given the ILP's time complexity, especially with large input instances.
- 2) This work conducts a performance analysis of the formulated ILP model. For this purpose, this work then proposes and implements multiple simple as well as complex heuristics that allocate EVs to corresponding CSs. In particular, random scheduling, closest to source/destination, shortest/longest processing time algorithms were implemented and their recorded results were compared with the ILP model. By using this approach, observations about the EVs' average and maximum waiting times as well as the time complexity to realize a scheduling plan are recorded.
- 3) In order to overcome the limitations of a centralized scheduling algorithm (particularly the time to solve the ILP), the interaction between the EVs is modelled as a non-cooperative pure strategy game. The structure of the game is common knowledge of all the EVs, but each EV knows only its own utility (revenue) function. The Bayes Nash equilibria of this game establish the strategies (in terms of expected utilities) that the EVs should adopt while competing for the CSs. In a non-cooperative game, all players (*i.e.*, EVs) are assumed to be rational and each EV strategy is known to all other EVs, hence the Nash equilibrium is realized.
- 4) Using extensive simulations, this work finally discusses the performances of the presented scheduling algorithms. Based on the obtained results, this paper highlights the efficacy of the game-theoretic model for realizing successful scheduling plans for recharging EVs.

## C. Paper Organization:

The remainder of this paper is structured as follows. Section II presents a brief overview of the related work. A description of the system model is presented in Section III. Section IV lays out, in details, two centralized offline ILP models. Section V presents the distributed game-theoretical approach. The simulation setup and the proposed scheduling heuristics used for comparison purposes are presented in Section VI. The performances of the two ILP models as well as the game model are examined and compared to the proposed scheduling heuristics in Section VII. Finally, concluding remarks are presented in Section VIII.

## II. RELATED WORK

In [5], a comprehensive survey of economy-driven schemes for EV charging was provided. The authors highlighted important questions that should be addressed when scheduling the charging of EVs, including: *a)* how charging stations are specifying their prices, *b)* what the mobility models of EVs are and *c)* the direction of power flow. The authors of [5] also differentiated between the static and dynamic scheduling approaches, where, in the former, scheduling decisions are based on a snapshot of the system regardless of impacts of future variations, and in the latter, the uncertainty of future events should be taken into account.

In [9], the authors shed the light over an inherent uncertainty that governs the charging behaviour and demands of EVs in a certain regional transmission network or local distribution network. Such uncertainty originates from different random factors such as the number of EVs being charged, these EVs voltage and current levels as well as their power battery start/end and capacity, the charging time duration and so forth. Given this uncertainty, the authors of [9] conducted Probabilistic Power Flow-based (PPF) analysis in order to capture and quantify the impact of EV charging on the power grid.

In addition to the above, the work of [10] and [11] shed the light on the power system overloading problem especially at the distribution system level resulting from the widespread adoption of EVs in the transportation system and their consequent burdening charging loads. This problem becomes even more crucial when fast charging is demanded. This is especially true since this latter required much higher power than regular charging. To this end, in an attempt to work around power system overload as well as to improve energy utilization while avoiding additional deployment costs, efficient load management and distribution strategies are required. In particular, the work of [10] and [11] revolved around coordinated EV charging schemes throughout which EVs receive energy from charging stations through grid-to-vehicle (G2V) transfers. Nevertheless, G2V is strictly confined to the technical limitations of the power system and may not be able to cope with elevated charging demands. At this point, Vehicle-to-Vehicle (V2V) energy swapping presents itself as a promising solution, [12]. Explicitly, V2V energy swapping takes place through direct energy transfer among EVs at an aggregator that is connected to the grid and controlled by the grid operator for the purpose of offloading heavy power demands, [13]. This allows for improving the EV charging efficiency with minor infrastructure modifications.

The authors of [14] proposed a centralized EV recharge scheduling system for smart parking lots. Their study considered two types of EV arriving patterns, being regular and irregular. Regular EV arrivals are associated with daily commuters that follow the same journey pattern, whereas, irregular EV arrivals are associated with arbitrary new guests requesting a parking spot to recharge their EVs. The authors formulated optimization problems that either maximize the revenue of the parking lot owner or maximize the percentage of EVs with a fulfilled recharge request. The reported results

in [14] were compared to First-Come-First-Served (FCFS) and Earliest-Deadline-First (EDF) scheduling algorithms. In [15], the authors studied scheduling the charging of multiple EVs aiming to maximize the profit of the charging station under time-of-use (TOU) pricing. The authors developed a multi-charger framework considering both the customers' and charging station's interests. The authors of [16] identified the critical challenges and research problems associated with the high EV mobility, vehicle range anxiety, and power systems overload. Then, the authors investigated innovating charging and discharging potentials for mobile EVs based on real-time information collections (via VANETS and/or cellular networks). The authors of [17] considered delay-optimal charging scheduling of the electric vehicles EVs at a charging station with multiple charge points where the uncertainty of the arrival of the EVs, the intermittence of the renewable energy, and the variation of the grid power price are taken into account and described as independent Markov processes. The work in [18] studied the charging policies in smart microgrids with EVs and renewable energy sources. Knowing the states of the renewable energy sources and the number of charging EVs, an optimal charging policy was obtained to maximize the energy utilization.

Major concerns for operators of Fast Charging Stations (FCS) are highlighted in [19]; those being: a) profitable operation of the FCS and b) the FCS's high Quality of Service (QoS) to the arriving Electric Vehicles (EVs) in terms of throughput and waiting time. Then, the authors shed the light over the important fact that during constant current charging, whenever an EV's battery reaches its threshold value, the charging mode switches to constant voltage charging, which, in turn, exhibits an exponential current decrease as a function of the EV's battery State-of-Charge (SoC). Such a phenomenon leads to a significant increase in the charging time whenever higher SoCs are desired and, hence, lower revenues for FCS operators as a consequence of the increased EV service time during which the EV charges less energy. In this light, the authors of [19] derived an SoC dependent charging power function and formulated an accurate relation between the requested charging energy and its required charging time. The authors adopted a strategy of limiting the requested SoC by an EV in order to increase the revenue of the FCS and accordingly, they computed an optimal SoC request limit that maximized the FCS revenue.

### III. SYSTEM MODEL

This paper considers the scenario illustrated in Figure 2, where a number of 15 EVs, present within a certain geographical area, wish to recharge their batteries such that, upon arriving to their respective destinations, their batteries are above a desired State-of-Charge (SoC). As depicted in Figure 2, the 15 EVs (illustrated as red vehicles) have entered the considered network from any one of the 8 entry points and shall exit the network from any one of 8 exit points. The considered network embraces 7 charging stations each located at a road intersection, and an EV may be scheduled to charge on any suitable CS knowing that its current SOC

allows it to reach the chosen CS. Note that the location of the CSs does not affect the problem formulation nor the reported results in this paper since changing the location of the charging station is an input parameter for any scheduling method. The considered traffic network is composed of three different segment types of long length,  $d_L$ , medium length,  $d_M$ , and short length,  $d_S$ . Now, since multiple potential options are viable to recharge a user's EV, CSs may quickly become unmanageable if handled improperly. Hence, we advocate that EV users are encouraged to seek assistance in order to find the *best* charging station, which provides the fastest service. As a result, a centralized infrastructure management system would certainly allow for better utilization of the available CSs. Now, for an EV to transfer its current status (*i.e.*, current SoC, required SoC at destination, range and battery capacity) to the centralized scheduling agent, there are two supporting communication frameworks. Indeed, as mentioned earlier, as the wireless communication technology cruises towards its 5<sup>th</sup> generation, smart cities will soon support delay-minimal communications among vehicles and between vehicles and infrastructure devices. On the other hand, EVs may exploit VANET platforms that are exclusively designed for information exchange among highly mobile vehicles and infrastructure units in a multi-hop fashion, where the required real-time information can be delivered efficiently via short-range V2V and V2I communications.

An EV issues a recharge request once its battery SoC drops to a certain threshold. At this point, the EV user forwards his/her EV's current characteristics being: 1) the EV's SoC, 2) the EV's destination, 3) the EV's battery capacity and finally 4) the desired SoC at the destination<sup>1</sup>. The central processing agent defines time windows for receiving the charging requests from the EVs before optimizing the charging schedule. The EVs which miss the current time slot have to wait for the next time slot to submit their requests. Now, once the centralized agent, knowing the current statuses of all neighbouring CSs, collects all charging requests from the EVs and accordingly, schedules their charging process in such a way that minimizes the maximum waiting time an EV spends at a CS. Note that, it is assumed herein that all EVs requesting a charging service are known to the scheduling discipline, and no EV is allowed to start charging at a CS without being scheduled.<sup>2</sup>

Now, the scheduling agent should account for two important constraints when scheduling an EV on a CS: a) an EV should have enough residual battery power to reach to the CS it is scheduled on and b) an EV should recharge its battery enough to reach its destination with a SoC above a requested level.

It is worthwhile noting that, this work does not consider the monetary cost/value charged by the CSs for charging EVs. In fact, this work assumes identical pricing at all CSs and hence concerned with the actual demand, supply and utilization of energy by EVs and how different scheduling algorithms for

<sup>1</sup>EV users may or may not have access to a charging outlet at their final destination whether it is their work or home. Therefore, the desired destination SoC differs from one EV to the other.

<sup>2</sup>It is true that the event of an EV arriving to a CS without an assigned charging time is indeed a likely one, however it is outside the scope of this current work and is being accounted for in the extension of this current problem.

charging of EVs on nearby CSs affect the trip performance of these EVs. The cost optimization arising from different pricing at the charging stations is however considered as future work.

The next section presents a mathematical formulation for this problem with the objective to realize a scheduling policy that minimizes the EV waiting times at charging outlets.

#### IV. CENTRALIZED OFFLINE PROBLEM FORMULATION

In this section, it is assumed that a centralized intelligent agent is already aware of all current and future EV recharging requests. The formulation of the centralized offline optimization model serves for the realization of an optimal scheduling policy.

Consider the scenario illustrated in Figure 2. Assume that, over a time period of length  $T$ , a total of  $I$  EVs request to recharge on one of the available  $J$  neighbouring charging stations in the network. Assume that each CS  $j$  has a number of  $L_j$  charging outlets that can be used. In this optimization framework, assume that the time axis is divided into  $N$  slots of length  $\tau$  each. Let  $x_{ij}$  be a binary decision variable defined as follow:

$$x_{ij} = \begin{cases} 1, & \text{if vehicle } i \text{ is assigned to CS } j \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where  $1 \leq i \leq I$  and  $1 \leq j \leq J$ .

Let  $x_{ij}^t$  be a binary decision variable defined as follow:

$$x_{ij}^t = \begin{cases} 1, & \text{if } i \text{ is charging at } j \text{ during } t \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

where  $1 \leq t \leq N$ . Note that  $x_{ij}$  decides whether a vehicle is assigned to a charging station. A vehicle, however, may queue behind other vehicles before it actually is served.

Assume that the charging rate of the considered charging stations is constant, and denoted herein as  $\zeta$ . Let  $P_{ij}$  be the amount of power vehicle  $i$  has to recharge at charging station  $j$ . Also, let  $\Delta_{ij}$  be the time required for vehicle  $i$  to recharge  $P_{ij}$  at charging station  $j$ . Let  $y_{ij}^t$  be a binary decision variable defined as follow:

$$y_{ij}^t = \begin{cases} 1, & \text{if } i \text{ starts charging at } j \text{ during } t \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

Let  $\text{SOC}_i$  represent vehicle  $i$ 's State of Charge (SOC) at the time it submits its recharging request.  $\text{SOC}_i$  is EV  $i$ 's percentage of available battery power. Let  $E_i = \text{SOC}_i \times C_i$  be the current amount of energy (kWh) stored in the EV  $i$ 's battery at the time when  $i$  submits its recharging request, where  $C_i$  is EV  $i$ 's battery capacity. Also, assume that EV  $i$  consumes an amount of energy  $\text{LEC}_{ij} = \text{SOC}_{ij} \times C_i$  (LEC stands for Lost Energy until Charge) in order to reach CS  $j$ , where  $\text{SOC}_{ij}$  is  $i$ 's consumed battery percentage to arrive at charging station  $j$ .

Let  $\text{TC}_i$  be the percentage SOC target required by vehicle  $i$  at its destination, and let  $\text{LED}_{ij}$  (LED stands for Lost Energy to Destination) denote vehicle  $i$ 's consumed power from Charging station  $j$  to  $i$ 's destination. Finally, let  $t_{ij}$  be the time at which vehicle  $i$  arrives to station  $j$ , and  $z_{ij}$  be the

time at which vehicle  $i$  starts charging at CS  $j$  ( $z_{ij} \geq t_{ij}$ ). The objective of this optimization problem is to minimize the time spent by EVs waiting at charging stations. As such, the problem formulation is given next:

$$\text{Min} \sum_j (z_{ij} - t_{ij} + \Delta_{ij}) x_{ij} \quad \forall i \in I$$

subject to

$$\sum_j x_{ij} = 1 \quad \forall i \in I \quad (4)$$

$$\sum_i x_{ij}^t \leq L_j \quad \forall j \in J, t \in T \quad (5)$$

$$\sum_t x_{ij}^t \zeta \geq P_{ij} x_{ij} \quad \forall i \in I, j \in J \quad (6)$$

$$x_{ij} \geq x_{ij}^t \quad \forall i \in I, j \in J, t \in T \quad (7)$$

$$x_{ij} \leq \sum_t x_{ij}^t \quad \forall i \in I, j \in J \quad (8)$$

$$\sum_j \sum_t y_{ij}^t = 1 \quad \forall i \in I \quad (9)$$

$$\sum_t y_{ij}^t \leq x_{ij} \quad \forall i \in I, j \in J \quad (10)$$

$$\sum_t x_{ij}^t = \Delta_{ij} x_{ij} \quad \forall i \in I, j \in J \quad (11)$$

$$x_{ij}^{t'} \geq y_{ij}^t \quad \forall i \in I, j \in J, t \in T, t' \in T, \text{ and } t \leq t' \leq t + \Delta_{ij} \quad (12)$$

$$y_{ij}^t = 0 \quad \forall i \in I, j \in J, t \in T, \text{ where } t \leq t_{ij} \quad (13)$$

$$\text{TC}_i \times C_i \geq \text{SOC}_i \times C_i -$$

$$\sum_j (\text{LEC}_{ij} - P_{ij} + \text{LED}_{ij} \times C_i) x_{ij} \quad \forall i \in I \quad (14)$$

$$z_{ij} = \sum_t y_{ij}^t \times t \quad \forall i \in I, j \in J \quad (15)$$

Equation (4) forces a vehicle  $i$  to be assigned to a single CS. Equation (5) allows for a single vehicle to charge at CS  $j$  at a time. Equation (6) makes sure that EV  $i$  acquires its requested recharge power at CS  $j$ . Equations (7) and (8) does not allow an EV to recharge on any CS if it was not assigned to it. Equation (9) makes sure that an EV starts recharging only once. Equation (10) makes sure that the EV only starts its recharging process at the designated CS. Equations (11) and (12) forces EV  $i$  to charge on CS  $j$  for a consecutive time of  $\Delta_{ij}$ . Equation (13) does not allow an EV  $i$  to start recharging on CS  $j$  before it actually arrives to it. Equation (14) makes sure that an EV's SOC at destination is larger than a pre-defined threshold taking into account the power lost during the commute and the power added in the charging process. Finally Equation (15) identifies the time slot during which vehicle  $i$  starts charging at CS  $j$ . Since this ILP model's objective is to minimize the sum of EV waiting times at CSs, it will be denoted as Min-Sum ILP (MS-ILP) throughout this paper.

It is worthwhile noting that the installation of a charging station is subject to the code of conduct and agreement signed by both the station owner and the power supplier or utility

company, which utilizes methods for voltage stability such as voltage regulation and VAR compensator [20] in order to avoid voltage fluctuation and instability of the distribution network. As such, the voltage requirements are transparent to the charging algorithm and hence the above ILP model omits any constraint related to the voltage in the distribution network.

Now, another interesting objective for the ILP could be minimizing the maximum EV waiting time at a charging station. The resolution of the min max problem helps mitigating the maximum waiting time and frustration of unfortunate EVs that arrive to CSs at peak hours. The Min-Max ILP model will be denoted by MM-ILP in the sequel. Let  $\Gamma$  be the maximum EV waiting time at a CS. It is given by:

$$\Gamma \geq \sum_j (z_{ij} - t_{ij} + \Delta_{ij}) x_{ij} \quad \forall i \in I \quad (16)$$

The objective function of the MM-ILP becomes:

$$\text{Min } \Gamma \quad (17)$$

We should note that in both of the above ILP models, some constraints may contain non linear terms (namely, a product of a binary variable and a real value, e.g., in (14)), whose linearization is straight forward and omitted for brevity. Now, the ILP model's most limiting disadvantage is its time required to solve large-scale problems, which is a fact illustrated in Figure 5 in Section VII. Consequently, this paper opts to solve the scheduling of EV charging more efficiently, where the problem is decomposed into several subproblems, particularly, one problem per vehicle, and the solution approach is a game theoretic one, as explained in the next section.

## V. GAME THEORETIC APPROACH

The distributed non-cooperative game starts as each EV  $i$  selects one of the CSs it can reach given its current SOC. The EVs (*i.e.*, players in this game) will autonomously choose their strategies which result in the least service time given the latest network characteristics. Each EV will then communicate its selected strategy to the other players in this game. Recall from Section III that each connected EV shall exploit the 5G infrastructure in order to communicate its selected strategy with negligible latency. Once an EV becomes aware of the remaining players' strategies, it then modifies its strategy based on this most recent game status, and thereafter, the EV re-communicates its updated strategy. EVs will repeatedly update and communicate their strategies until the game reaches its Nash equilibrium state, where neither of the EVs can further choose a better strategy than its current strategy. Note that, once all players (*i.e.*, the EVs requesting charging services) establish their strategy that is realized by the Nash equilibrium of the game, it becomes impractical to replay the game due to the arrival of a new EV that is requesting a charging service. However, the newly arriving vehicle may join the game in the next scheduling process.

Let  $M_i$  be the set of CSs which can be reached by EV  $i$  given its location and current SOC,  $M_i \subseteq J$ . And let (vector)

$S_i$  be the EV  $i$ 's strategy (set of actions), which is given by:

$$S_i = \{x_{ij} | \forall j \in M_i\} \quad (18)$$

where, as defined in Section IV,  $x_{ij}$  is a binary variable which becomes 1 if EV  $i$  decides to recharge its battery on CS  $j$ . As such,  $x_{ij}$  becomes EV  $i$ 's pure strategy herein. Note that, in order for a CS  $j$  to be considered in set  $M_i$ , EV  $i$  has to reach  $j$  given its current SOC, thus satisfying the following condition:

$$SOC_i \times C_i - LEC_{ij} \geq 0; \forall j \in M_i \quad (19)$$

Now, an EV  $i$  chooses a strategy by selecting a CS  $j$  to receive its recharging service (see Fig. 3) at ( $\sum_j x_{ij} = 1$ ), and hence sets its binary decision variable  $x_{ij}$  accordingly while making sure the following condition is satisfied:

$$TC_i \times C_i \geq SOC_i \times C_i - \sum_j (LEC_{ij} - P_{ij} + LED_{ij} \times C_i) x_{ij} \quad (20)$$

Equation (20) forces an EV  $i$  to choose a CS  $j$  from  $M_i$  which provides a charging service that allows EV  $i$  to reach its final destination with the required target SOC.

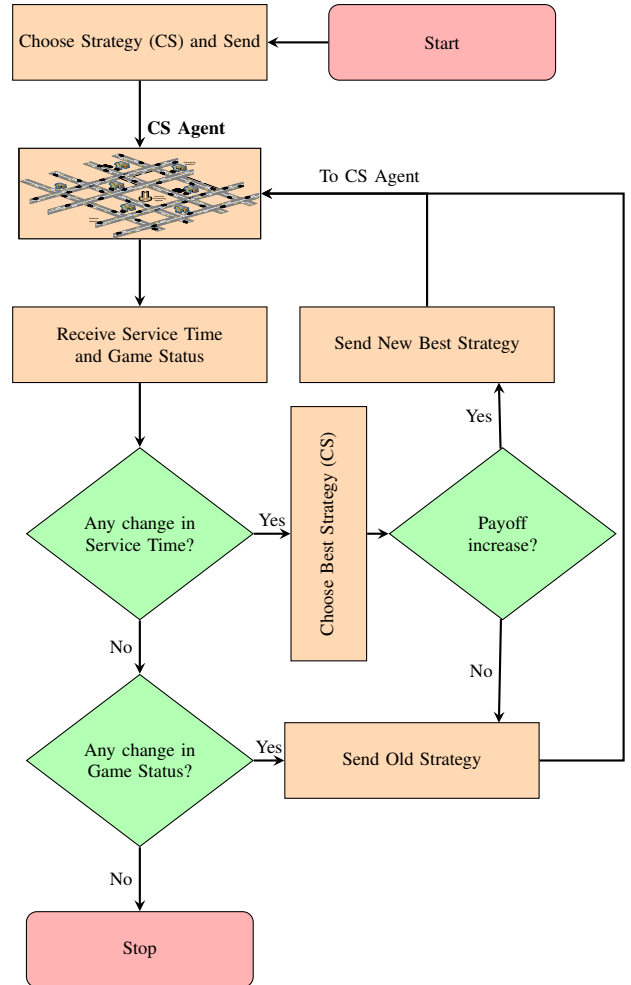


Fig. 3: Flowchart for EV Charging Game

For an EV  $i$  to select an appropriate strategy, it forwards

to CS  $j$ ,  $j \in M_i$ , its tentative arrival time  $t_{ij}$  as well as the expected amount of consumed energy along the way to  $j$ ,  $LEC_{ij}$ . Each CS  $j$  then gathers  $|A_j|$  EV requests, where  $A_j$  is the set of all EVs that can reach CS  $j$ , and sends to each EV  $i \in A_j$  its service time (including wait time) on  $j$ . Denote by  $\gamma_{ij}$  the computed service time of EV  $i$  at CS  $j$ ; then:

$$\gamma_{ij} = w_{ij} + \Delta_{ij}, \forall i, \forall j \in M_i \quad (21)$$

where  $w_{ij}$  and  $\Delta_{ij}$  are respectively the waiting time and charging duration of EV  $i$  at CS  $j$ .

Each CS  $j$  now maintains an ordered list of arriving times  $t_{ij}$  and requested service duration  $\Delta_{ij}$  for all  $i \in A_j$ .

$$\{(t_{ij}, \Delta_{ij}, i) | \forall i \in A_j\} \quad (22)$$

The CS  $j$  orders the EVs according to the following:

$$(t_{lj}, P_{lj}, l) < (t_{kj}, P_{kj}, k) \quad (23)$$

where  $l \neq k$ ,  $l \in A_j$ ,  $k \in A_j$ . The EVs  $l$  and  $k$  are ordered according to Equation (23) based on the following set of rules:

$$t_{lj} < t_{kj} \quad (24)$$

or

$$P_{lj} < P_{kj}, \text{ when } t_{lj} = t_{kj} \quad (25)$$

or

$$l < k, \text{ when } t_{lj} = t_{kj} \text{ and } P_{lj} = P_{kj} \quad (26)$$

Upon receiving the request from the EV  $i$ , CS  $j$  determines the position  $p$  of EV  $i$  (CS agent in Fig. 3) in its ordered list and hence evaluates  $i$ 's total service time according to the following equation:

$$\gamma_{ij} = (t_{p-1j} + \Delta_{p-1j} - t_{pj})\delta_{pj} + \Delta_{ij} \quad (27)$$

where  $w_{ij} = (t_{p-1j} + \Delta_{p-1j} - t_{pj})\delta_{pj}$ . Note that  $\delta_{pj} = 0$  when  $(t_{p-1j} + \Delta_{p-1j} - t_{pj}) \leq 0$ , and otherwise,  $\delta_{pj} = 1$ .

During iteration  $r$  of the game, all CSs  $j \in J$  respond to the EVs with their associated service times; then, each EV  $i$  selects, during that particular iteration, the best strategy (see Fig.3) that chooses the CS that results in the minimum service time:

$$x_{ij}^{*r} = \arg \min_{x_{ij}} \sum_{j \in M_i} \gamma_{ij}^r \times x_{ij}^r \quad (28)$$

Hence, the maximum payoff of EV  $i$  of an instance (iteration  $r$ ) of the non-cooperative game can be defined as:

$$\sigma_i^r(x_{ij}^{*r}, x_{-ij}^r) = B - \min_{x_{ij}} \sum_{j \in M_i} \gamma_{ij}^r \times x_{ij}^r \quad (29)$$

where  $B$  is a constant, and  $x_{ij}^*$  is the best strategy for EV  $i$  with respect to the strategies of all other EVs  $x_{-ij}$ . The game iterates until all participating EVs converge to selecting their best strategy (see Fig. 3). This is known as the Nash equilibrium state of the game, where the optimal payoff of EV  $i$  is:

$$\sigma_i^*(x_{ij}^*, x_{-ij}^*) = B - \min_{x_{ij}} \sum_{j \in M_i} \gamma_{ij}^* \times x_{ij} \quad \forall i \in I \quad (30)$$

where  $x_{-ij}^*$  is the optimal strategies of all players other than

$i$  and  $\sigma_i^*(x_{ij}^*, x_{-ij}^*) \geq \sigma_i(x_{ij}^*, x_{-ij}) \quad \forall i \in I$ .

**Lemma 1.** *The EV scheduling game with a finite number strategies converges to a Nash Equilibrium in a finite number of iterations.*

*Proof:* In the above EV scheduling game, the action of each EV is  $x_{ij}$ , which is finite since  $x_{ij} \in \{0, 1\}$ . The number of available stations (which translate to feasible actions/strategies) is also finite. In other words, the strategy set of each EV is finite and non-empty i.e.,  $S_i = \{x_{ij}, \forall j \in M_i\}$ . Hence, it can be shown that the number of plays or games is also finite since, in each iteration, an EV chooses a single charging station  $j$  such that  $\sigma(x_{ij}, x_{-ij}^*) \geq \sigma(x_{ij'}, x_{-ij}^*)$ , where  $j' \neq j$ . Furthermore, An EV will not change its strategy unless the payoff (30) improves with a new strategy. At each iteration, the strategy of all other players are known, therefore an EV can find the best CS, which leads to the optimal payoff ((30)). Consequently, the game converges to a Nash equilibrium in a finite number of iterations. ■

## VI. SIMULATION SETUP AND HEURISTICS

This section presents the simulation setup needed to resolve the ILP models presented earlier using the network topology illustrated in Figure 2. Furthermore, the scheduling heuristics whose results are compared to the ILP and game theoretic approaches are also laid out in this section. As illustrated in Figure 2, J=7 charging stations are considered. Moreover, the following distance values are used for the short, medium, and long distances portrayed in the figure: a)  $d_S = 1$  Km, b)  $d_M = 2$  Km, and c)  $d_L = 3$  Km. A vehicle  $i$  enters the considered traffic network through one of the 8 entry points illustrated in Figure 2 with a random initial state of charge  $SOC_i$  that is in the range of 40% to 80% of the vehicle's battery capacity, which is assumed to be 80 KWh. During its presence within any roadway segment, a vehicle may decide to initiate a charging request. A vehicle must have enough power to reach the selected charging station assuming that a power loss of 0.43 kWh/km is considered. At a charging station  $j$ , the charging rate is 16 KW per hour. The individual vehicle speeds are independent and identically distributed random variables with values in the range of  $[V_{min}; V_{max}]$ . Particularly, vehicles' speeds are generated from a truncated Normal distribution [21] and assumed to be constant for the entire duration of the navigation of an arriving vehicle to the assigned charging station. Realistic mobility traces were obtained via SUMO [22] with the objective of evaluating the impact of the different traffic theoretic parameters on the vehicles' individual speeds. It is important to note in this regard that vehicles entry and exit points are generated randomly within the investigated topology. Last but not least, a time slot length of 5 minutes is used.

Armed with these different system parameter values, the ILP models were solved using CPLEX, a state-of-the-art commercial solver. It is important to note that the resolution of the ILP models via CPLEX is preceded by a pre-processing step and followed by a post-processing one. On the one hand, the aim of the pre-processing step is to determine the



values of the input parameters underlying the formulated ILP models. On the other hand, the post-processing step aimed at processing the output decision variables resulting from the ILP resolution step in such a way so as to calculate both the maximum station waiting time as well as the average station waiting time.

Notice that the above-described problem is similar to the well-known problem of scheduling jobs on parallel machines [23]. In fact, the latter problem is an instance of this current problem once the time required for an EV to reach any CS becomes zero. Now, since scheduling jobs on parallel machines is an NP-hard problem [24], therefore, our problem is also NP-hard. Therefore, this paper proposes a number of heuristic strategies that schedule the service of arriving EVs on one of the existing charging stations. The results obtained from solving the ILP models are compared to the following heuristic approaches:

- Closest to Source (CTS) heuristic: assigns an arriving vehicle to the charging station that happens to be the closest to a vehicle's entry point.
- Closest to Destination (CTD) heuristic: assigns an arriving vehicle to the charging station that is found to be the nearest to a vehicle's exit point.
- Random Station Selection (RSS) heuristic: dispatches an arriving vehicle to a randomly selected charging station.
- Vehicle with Shortest Service Time First (VSSTF): creates sorted lists of vehicles per station, where vehicles are arranged in an ascending order of their service times. Then, a round robin scheduling of vehicle service is performed among the stations whereby the vehicle with the shortest service time is assigned to the charging station in each round.
- Vehicle with Longest Service Time First (VLSTF): operates similarly to VSSTF but unlike VSSTF assigns the vehicle with the longest service time to the charging station in each iteration of the round robin scheduling approach.

Given that the CTS, CTD, and RSS heuristics are straightforward and self-descriptive, a pseudo-code description of the VSSTF heuristic is presented in Algorithm 1 below. It is important to note that the VLSTF algorithm operates similarly to VSSTF with the only difference being that the vehicle with the longest service time is selected.

Generating mobility traces using SUMO, solving the ILP using CPLEX, and simulating the whole scenario under the above-described heuristics were performed on an Intel Core i7-4790 3.6 GHz CPU powered machine with 16.0 GB of RAM. It is important to note the following with respect to the results. Each data point represents the average of multiple runs with different initial conditions in order to achieve the highest degree of accuracy in terms of the reported values.

## VII. RESULTS AND DISCUSSIONS

### A. ILP Results and Discussion

The results of the ILP models as well as those resulting from the heuristic scheduling algorithms are reported in Figure 4. In particular, Figure 4(a) plots the maximum waiting time as

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### Algorithm 1 Vehicle with Shortest Service Time First (VSSTF) Heuristic

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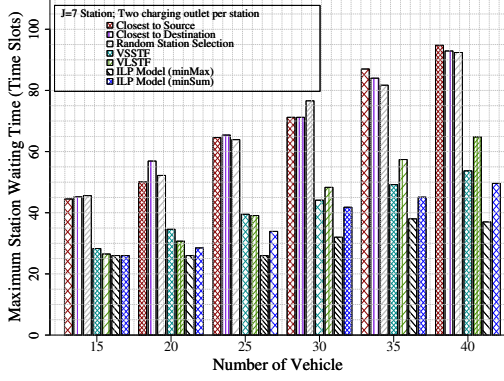
function VSSTF( $I, J$ )
  for  $j \in J$  do  $\triangleright$  populate sorted list of vehicles per station
    Create empty priority list  $L_j$   $\triangleright$  vehicles sorted by  $\Delta_{ij}$ 
    for  $i \in I$  do
      if ( $i$  can reach  $j$ ) then
        Calculate service time  $\Delta_{ij}$  of  $i$  at  $j$ 
        Insert  $i$  into  $L_j$ 
      end if
    end for
  end for
  repeat  $\triangleright$  assign vehicles to stations
    for  $j \in J$  do
      if ( $L_j$  is not empty) then
         $i \leftarrow$  remove vehicle with smallest  $\Delta_{ij}$  from  $L_j$ 
        Assign  $i$  to  $j$ 
        for  $k \in J$  such that  $k \neq j$  do
          Remove  $i$  from  $L_k$ 
        end for
      end if
    end for
  until All  $L_{ij}$  lists are empty
end function

```

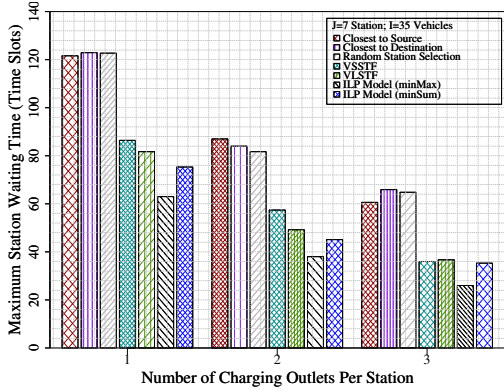
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a function of the number of vehicles whose services need to be scheduled. This performance metric indicates the worst service experience an EV driver may witness, which could be indicative of the provided quality of service. Indeed, our *min-max* model is designed to improve this overall experience, by forcing the max waiting time to be as low as it can possibly be. Our results (shown in the figures) clearly reveal the ability of the MM-ILP model to minimize this maximum waiting time for the vehicles as compared to the heuristic algorithms. One also observes based on the results that the proposed heuristic algorithms can be classified into two categories, namely naive and good ones. While CTS, CTD, and RSS belong to the naive category, the VSSTF and VLSTF heuristics can be categorized as being good algorithms. This classification is driven by the fact that VSSTF and VLSTF consistently and for different values of  $I$  achieve better performance in terms of the maximum waiting in comparison to the CTS, CTD and RSS heuristics. For instance, it can be seen from Figure 4(a) that when the number of vehicles is 40, the MM-ILP model reduces the max waiting time by more than 20 times slots (100 minutes) over the best heuristic method (i.e., a reduction of 33% in max waiting time is obtained for this instance). Finally, one might be surprised by the fact that CTS and CTD have a worse performance when compared to the RSS. Hence, note that it is observed that for larger number of vehicles, CTS and CTD have the tendency to group together a larger number of vehicles at the same station, especially those vehicles whose sources/destinations are close to the same charging station. This explains the relatively higher maximum waiting time resulting from CTS and CTD as compared to RSS, which inherently distribute arriving vehicles evenly among charging stations reducing thus the size of vehicle clusters per station, and hence the observed maximum waiting time.

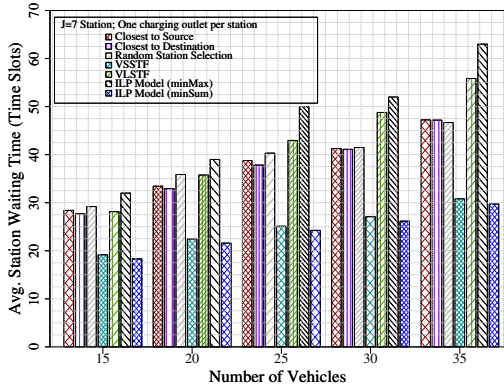
Figure 4(b) studies the impact of the number of charging outlets per station on the resulting maximum waiting time.



(a) Max WT versus N



(b) Max WT versus L



(c) Avg WT versus N

Fig. 4: Performance Evaluation of the ILP Models

TABLE I: MM-ILP CPU Run Time for different values of I and L

Number of Vehicles	L = 1	L = 2	L = 3
15 vehicles	0.0070 hours	0.0017 hours	0.0003 hours
20 vehicles	0.5150 hours	0.0038 hours	0.0005 hours
25 vehicles	1.9938 hours	0.0123 hours	0.0008 hours
30 vehicles	10.1962 hours	0.0375 hours	0.0092 hours
35 vehicles	12.5628 hours	0.0817 hours	0.0096 hours

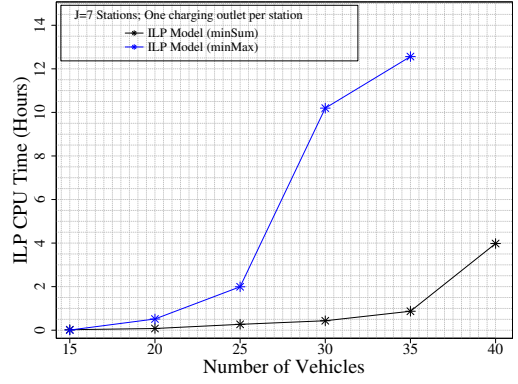


Fig. 5: CPU Run Time

Indeed similar conclusion can be drawn, further underlining the superiority of the MM-ILP model when it comes to improving the overall quality of service (i.e., reducing the maximum EV waiting time). The good heuristics VSSTF and VLSTF produce comparable results (i.e., the gain of the MM-ILP becomes less pronounced as we increase the number of charging outlets per station) which are still better than the values observed in the case of the naive heuristics algorithms, namely, CTS, CTD, and RSS. It is worthwhile noting in this respect that the maximum waiting time decreases considerably as the number of charging outlets per station increases. This is expected as increasing the rate of departures from charging stations has the effect of reducing the amount of time a vehicle waits at a station until it starts receiving its charging service.

Now, although the maximum waiting time is improved (by the MM-ILP method), this may be at the expense of increasing the average waiting times of EVs. Figure 4(c) shows the average waiting time for different number of EVs. As expected, first, the average waiting time is an increasing function of the number of vehicles. According to the reported results, it is clear that the VSSTF heuristic algorithm yields the shortest average station waiting time as compared to the rest of the scheduling strategies, except for the MS-ILP. VSSTF is found even to be able to outperform the MM-ILP; this can be justified by the fact that even though the MM-ILP aims at minimizing the maximum station waiting time, this does not guarantee a control on the average station waiting time. It is clear that, since the objective of the MS-ILP is to optimize the average EV waiting time, it records the best results for that performance metric. Here, it is important to mention that, depending on the application and its requirements, one may choose to minimize the maximum waiting time using the MM-ILP model or opt to minimizing the average EV waiting time using the MS-ILP model. In this paper, both models were resolved for completeness.

Figure 5 provides insight into the scalability of the ILP-based solutions by plotting the ILP running time as a function of the number of vehicles. The running time increases impractically fast with the number of vehicles. Obviously, a centralized agent utilizing any of the ILP models to perform the scheduling will fail to cater to the vehicles requirements within reasonable time frames. Note that the case MM-ILP

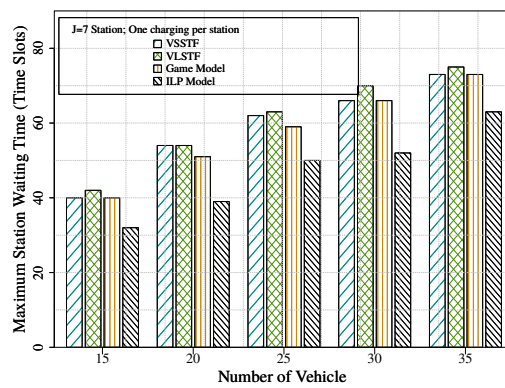
with 40 vehicles is not included in the figure as the MM-ILP model was stopped after 24 hours of runtime. Response time values that are in this order of magnitude are clearly not practical in the context of the considered problem. This is particularly true since vehicles expect the agent to respond to their requests in a quasi-real time fashion. This clearly demonstrates the limitation of the ILP-based solution owing to its inability to scale well for large number of vehicles. It is in this context that solutions like the game-theoretic one introduced in the next section becomes necessary. Note that, the CPU runtime for MS-ILP is much less than that of MM-ILP, particularly for a small number of vehicles since the time horizon is much wider for MM-ILP resulting in an increase in terms of the number of time-related constraints and as such requiring long periods of time for its resolution.

Finally, Table I illustrates the evolution of the MM-ILP's running time for different values of the number of vehicles and number of charging outlets. These results assert the fact that the running time of the ILP is greatly improved when the number of charging outlets per station increases. This is justified by the fact that the maximum waiting time decreases in this case and as a result, the number of instances of the time-related constraints, namely Equations (5), (7), (12), and (13), would significantly decrease.

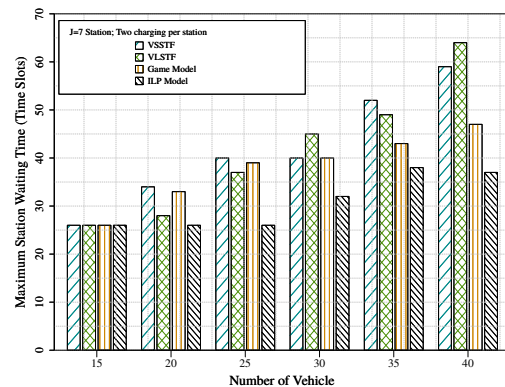
### B. Game Theoretic Results

This section is dedicated to present and discuss the reported results of scheduling EVs on charging stations in terms of average waiting time and maximum waiting time. Furthermore, the number of iterations required for the game to converge to Nash equilibrium as well as the CPU run time are also laid out in this section. We consider two systems to study, a small one (as shown in Figure 2) and another larger one which consists of replicating the previous scenario 4 times. The number of available CSs becomes 28, and the number of EVs requesting charging services is varied between 500 and 1000EVs. The performance metrics of interests are the maximum waiting time, the average waiting times and the CPU run time. As discussed earlier, the maximum waiting time is measured after running the different methods and measuring the largest experienced waiting time (this serves as a good metric to compare the proposed heuristics and game methods with the ILP), which can indicate the level or quality of experience of EVs' charging.

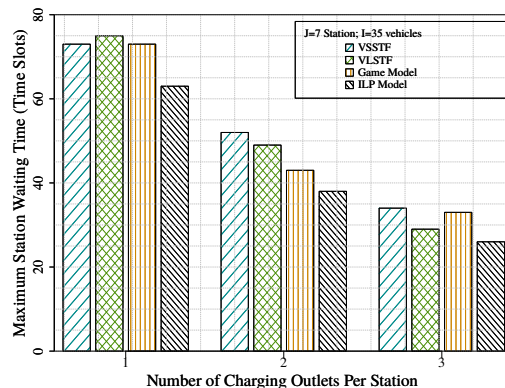
Figure 6 plots the maximum waiting time experienced at a charging station by the EVs (considering the system depicted in Figure 2), as we vary the number of vehicles and charging outlets per station. The reported results under the game theoretic scheduling technique are compared to the ILP models as well as VSSTF and VLSFT. Clearly, the maximum EV waiting time at a CS increases with the number of EVs seeking charging services. It is also clear from Figures 6(a) and 6(b) that increasing the number of charging outlets from  $L = 1$  to  $L = 2$  significantly decreases the maximum waiting times by 30 – 40%. Figure 6 also demonstrates the superiority of the MM-ILP model in terms of reducing the EV maximum waiting time and hence improving the quality



(a) Maximum WT with  $L = 1$



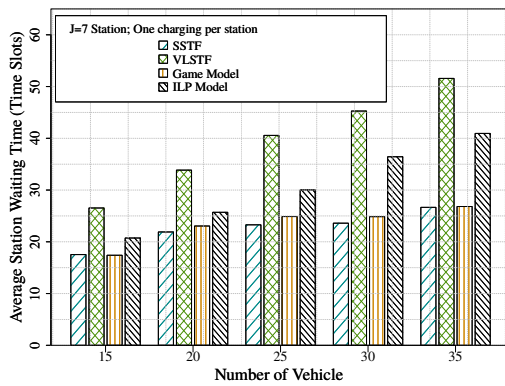
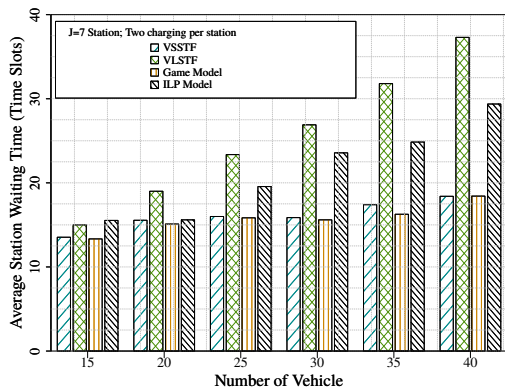
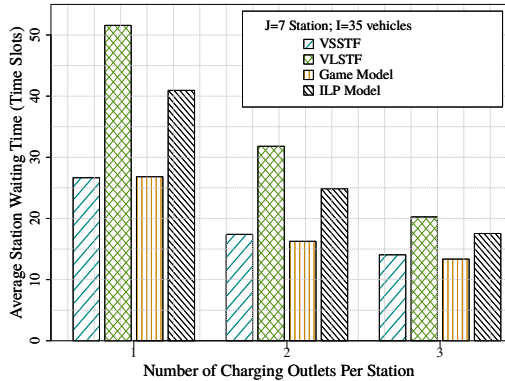
(b) Maximum WT with  $L = 2$



(c) Maximum WT

Fig. 6: Maximum Waiting Time: Game vs. ILP

of experience. This is indeed due to the objective of the MM-ILP model. Alternatively, it is promising to note that the game theoretic approach records sub-optimal results in terms of maximum waiting times and, most of the time, outperforms the good heuristics (particularly as the number of EVs increase). Note that, in the game model, each player (*i.e.*, EV) seeks to minimize its own waiting time, rather than the worst waiting time in the system. It should be recalled here that the game method is a distributed approach, where each EV optimizes its payoff (*i.e.*, wait time); further, the game approach can be employed as an online scheduling where at anytime an EV decides to get service, gets to play with other

(a) Average WT with  $L = 1$ (b) Average WT with  $L = 2$ 

(c) Average WT

Fig. 7: Average Waiting Time: Game vs. ILP

EVs requesting service at the same time in order to decide their best strategy. The ILP models however, as well as the heuristics, are centralized and expected to be run periodically at the agent upon receiving a batch of service requests from EVs.

Figure 7 plots the average EV waiting time and compares the results collected from the game theoretic approach with both the ILP models as well as with the two heuristics (VSSTF and VLSTF). It is clear from Figures 7(a) through 7(c) that, independent from the scheduling algorithm, the average time an EV spends in a CS increases as the total number of EVs requesting recharging services increases. Furthermore,

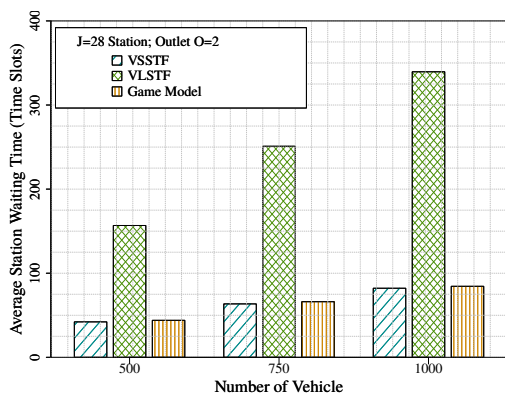
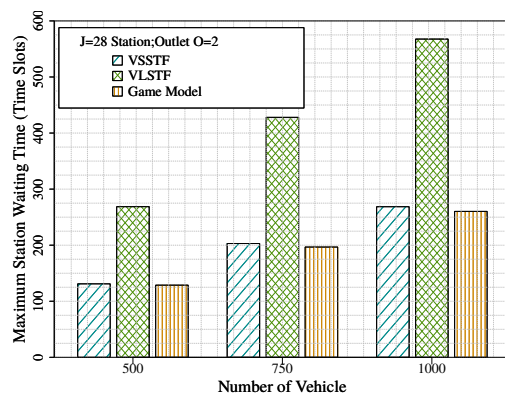
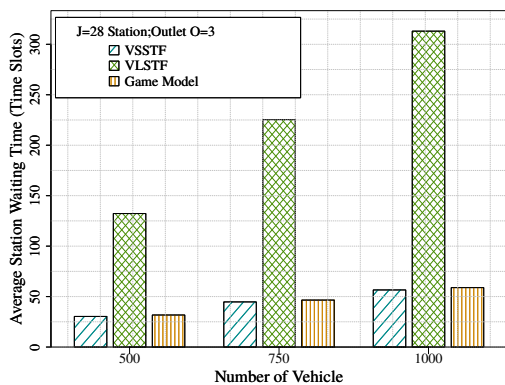
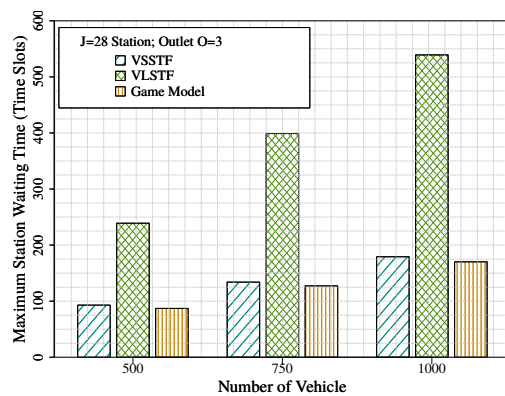
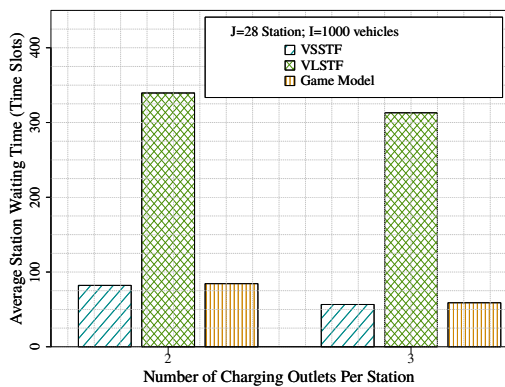
the availability of more charging outlets per charging station helps decrease the EV average waiting time. Figure 7 shows that the game model outperforms its centralized MM-ILP counterpart in terms of the average EV waiting time; e.g., an average wait time lower by 15 time slots is shown when 35 vehicles are simulated in both the ILP and the game method (recall that the objective of the MM-ILP model was to minimize the maximum EV waiting time). Compared to the optimal average waiting time realized by the MS-ILP model, the game theoretic approach records sub-optimal results. Another observation worthy of mentioning here is that the heuristics perform remarkably well in terms of average wait times, especially VSSTF, showing comparable results to those obtained using the game and the MS-ILP. Here, VSSTF follows a shortest job first policy for scheduling EVs and indeed this discipline is known for its simplicity and because it minimizes the average amount of time each EV has to wait until its execution is complete [25]. Nonetheless, as shown in Figure 6, the VSSTF discipline may be a source of unfairness as it exhibits the worst maximum wait times (affecting the quality of experience of EVs).

Now, recall from Figure 5 that the time required to solve the ILP model increases uncontrollably as the number of vehicles continues to increase. As such, it becomes computationally intensive and eventually infeasible to solve any ILP model and schedule the charging of EV vehicles when their number exceeds, e.g., say a hundred. On the other hand, as mentioned earlier, the game model is solved in a distributed manner where each EV individually modifies its strategy until the game converges to a Nash equilibrium. Figures 8 and 9 present the results reported from solving the game model for a very large number of EVs requesting charging services in a larger system.

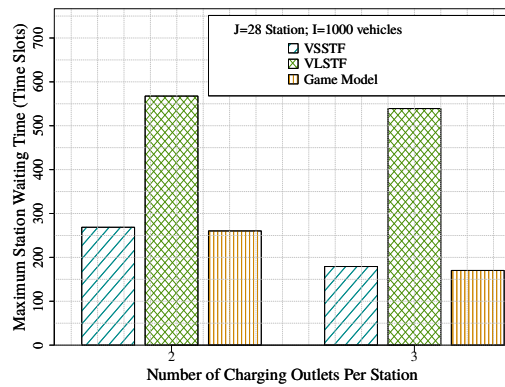
Figure 8 plots the average EV waiting time under the game model and compares the results with the two heuristics VSSTF and VLSTF. Figures 8(a)-8(c) reassure the significance of multiple outlets per CS to decrease the average waiting times especially when the total number of scheduled EVs increases. Furthermore, Figures 8(a)-8(c) show that the average EV waiting time under the game theoretic approach is almost identical to that under VSSTF. This further demonstrates the ability of the game model to achieve lowest average waiting times especially when compared to a shortest job first algorithm which is widely known to record the least average waiting time [25].

Figure 9 plots the maximum EV waiting time under the game model as well as the VSSTF and VLSTF heuristics. Figures 9(a)-9(c) clearly show that the game model significantly outperforms VLSTF in terms of the maximum EV waiting time. Also, the game model scheduling algorithm records a smaller maximum EV waiting time than that under VSSTF. This promising result highlights the competence of the distributed game scheduling approach especially given its comparable results with centralized scheduling heuristics.

Figure 10 depicts the required number of iterations for the game model to converge to Nash equilibrium when 1000 EVs are requesting charging services on 28 CSs with 3 outlets per CS. It is clear from Figure 10 that after 10 iterations

(a) Average WT with  $L = 2$ (a) Maximum WT with  $L = 2$ (b) Average WT with  $L = 3$ (b) Maximum WT with  $L = 3$ 

(c) Average WT



(c) Maximum WT

Fig. 8: Average Waiting Time: Distributed Game Approach

Fig. 9: Maximum Waiting Time: Distributed Game Approach

of the game, Nash equilibrium has been reached. This is an important indicator of the fast convergence of the distributed pure strategy game model. Finally, Figure 11 plots the CPU run time required to schedule between 500 and 1000 EVs on 28 CSs with 3 outlets per CS. The distributed nature of the game theoretic approach allows each EV (player in the game) to compute its payoff and modify its strategy simultaneously, which is not possible under a centralized algorithm. As such, as clearly shown in Figure 11, the CPU run time under the game theoretic approach is slightly affected by the number of EVs, whereas, the CPU run time increases dramatically under the two centralized heuristic algorithms, namely VSSTF and

VLSTF. This result motivates the adoption of a distributed algorithm, particularly a game theoretic approach, to schedule the charging service of a large number of EVs. Figure 11 is a clear evidence that the VSSTF and VLSTF heuristics have approximately the same average running time. This is due to the fact that these two algorithms differ solely in terms of the selection process from the priority list maintained per station (see Algorithm 1). In particular, while VSSTF repeatedly selects from the priority list the vehicle with the shortest service time, the VLSTF repeatedly gets from the priority list the vehicle with the longest running time. This results in the same running time for the selection process, and hence,

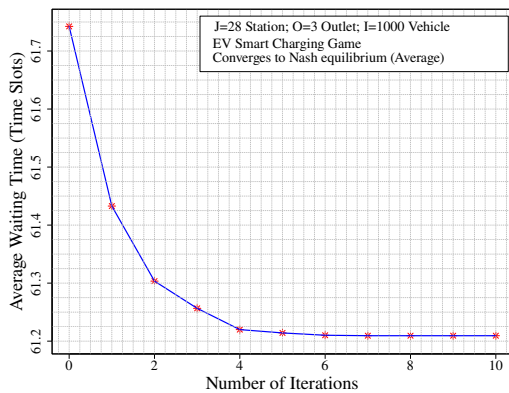


Fig. 10: Game convergence to Nash Eq.

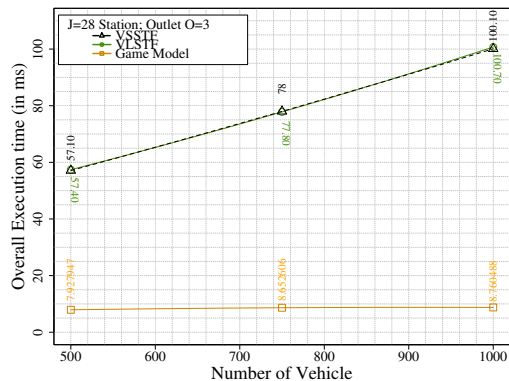


Fig. 11: CPU Run Time

the same running time for the overall algorithms. The slight running time difference observed in Figure 11 between VSSTF and VLSTF is within the simulation's confidence interval.

Number of Vehicles	L=1	L=2	L=3
15 Vehicles	4.70787 ms	4.63073 ms	3.7770642 ms
20 Vehicles	3.766845 ms	2.8293675 ms	2.8450275 ms
25 Vehicles	4.6698032 ms	3.6999676 ms	3.43251 ms
30 Vehicles	3.7325605 ms	3.7402898 ms	4.6100632 ms
35 Vehicles	3.7285909 ms	3.7448452 ms	3.6882075 ms

TABLE II: Game Theoretic Approach CPU Run time for different values of I and L

Case	EV 500 (iteration)	EV 750 (iteration)	EV 1000 (iteration)
1	9.20012 (10)	8.22954 (9)	6.93883(7)
2	8.23923 (9)	9.32271 (10)	9.18581 (10)
3	9.25103 (10)	7.18268 (8)	10.2325(11)
4	7.2444 (8)	9.26022 (10)	7.97883 (8)
5	11.3443 (12)	8.27376 (9)	9.23738 (10)
6	6.18713(7)	9.22441 (10)	8.17854 (9)
7	6.17314 (7)	8.29878 (9)	11.0478 (12)
8	7.17016 (8)	8.24979 (9)	10.2548 (10)
9	8.25618 (9)	9.27801 (10)	6.23513 (7)
10	6.21378 (7)	9.20616 (10)	8.31526 (9)

TABLE III: Examples of CPU Run Time (ms) and Game Iterations for large number of EVs (3 outlets per CS)

Table II illustrates the CPU runtime required to schedule a small number of EVs on  $J$  charging outlets per CS. Moreover, Table III presents the CPU runtime as well as the required number of iterations for the game to converge and schedule a

large number of EVs on CSs. Tables II and III further prove that the quick realization of a scheduling policy using the game theoretic approach is independent of the number of EVs and outlets per CS, which is an expected result due to the distributed nature of the scheduling algorithm.

## VIII. CONCLUSION

The widespread installation of supercharging services for EVs necessitates the realization of proper scheduling policies to avoid extended queues at CSs and elevated EV waiting times. This paper established centralized as well as distributed scheduling techniques that organize the charging operation of EVs at different CSs. The centralized linear optimization model recorded the optimal results for the maximum EV waiting time when compared with several other scheduling heuristics. However, given that problem turned out to be NP-hard, and its scalability is a limiting factor. Hence, this paper then formulated a distributed game theoretic approach that overcomes the scalability issue of the ILP model and records promising results.

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