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Delay-Tolerant Codes for the Decode-and-Forward Cooperation in Wireless Networks

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By

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Thesis Approval Form

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Abstract

With the great growth of wireless devices and communication, there is a huge demand for higher data rates. To respond to this growing demand, fourth generation (4G) wireless Local Area Networks (LANs) might adopt cooperation techniques where several collocated mobile terminals can cooperate with each other in order to enhance the quality of their corresponding communication links. One popular cooperation strategy is the Decode-and-Forward (DF) strategy. After a first phase where the source node broadcasts its message to the neighboring nodes (relays), these nodes will transmit their decoded versions of this message simultaneously to the destination. To conveniently encode the interfering data streams that are transmitted simultaneously from the different relays, Space-Time (ST) coding techniques must be applied. In this work, we propose a novel ST code that is adapted to the DF strategy with two relays. This code is tolerant to the delays that might occur between the different relays and it presents the main advantage of a reduced decoding complexity. In fact, only three data streams need to be decoded jointly rather than four streams as in the case of the existing solutions. An additional advantage is that the proposed code is totally real and, consequently, it is adapted to low cost carrier-less ultra-wideband transceivers that do not necessitate tracking the phase of the incoming signal.

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List of Abbreviations

BER: Bit Error Rate

dB: decibels

IR: Impulse Radio

MIMO: Multiple Input Multiple Output

ML: Maximum Likelihood

PAM: Pulse Amplitude Modulation

PCU: Per Channel Use

PPM: Pulse Position Modulation

QAM: Quadrature Amplitude Modulation

SER: Symbol Error Rate

SNR: Signal to noise ratio

STC: Space Time Code

UWB: Ultra Wideband

Introduction

Communication is sending and receiving data through a link. This is a simple definition that everybody can understand. Communication schemes are characterized by many factors, the most important factor is how the data is transfer from one location to another location, which is called the link in the telecommunication world. Wireless communication is transferring data without the use of physical links such as electrical wires or conductors. Also the distances between the source and destination differ. It can be short distance (a few meters) such as: television remote control. It can be a long distance (hundreds, thousands, and millions of kilometers) such as cellular communication, wireless networking, GPS, and satellite television.

Figure 1: A wireless device

The advantage of wireless communication is that it can permit services, on long distances or not reachable areas, which cannot be accessible by wires. It uses different types of energy such as radio frequency (RF), infrared light, laser light, visible light, etc.

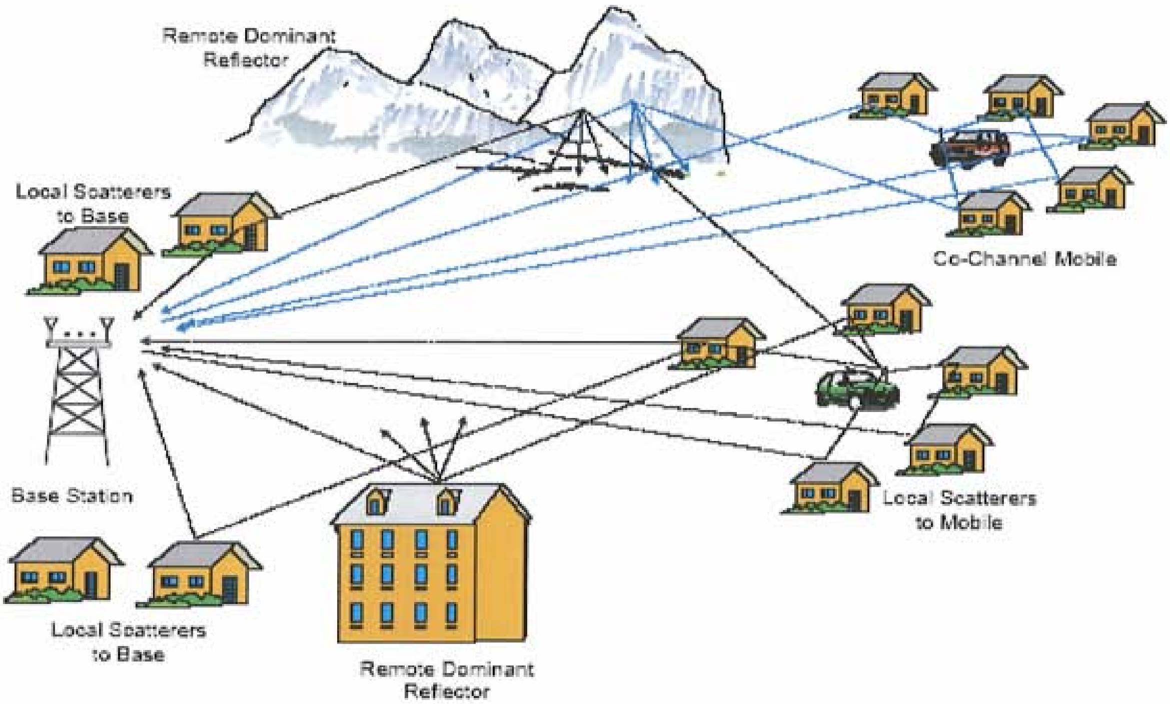

Figure 2: Wireless Communication area

Applications of wireless communication may involve point to point communication, multi point to point or point to multi point communication (Multiple input Multiple output MIMO) , and broadcasting.

A simple point to point communication can see as the figure below, a source terminal sends information (or data) to a destination terminal using a link or a channel.

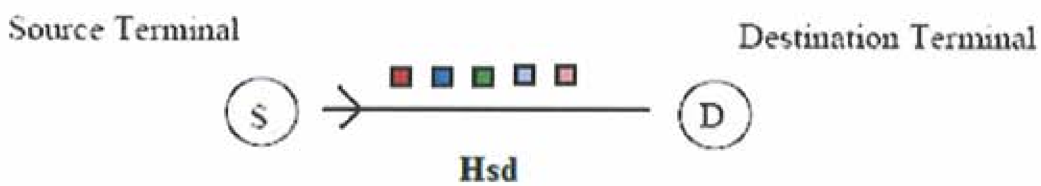

Figure 3: Point to point communication

The system may have multiple terminals (more that 2). These terminals can have one source that is broadcasting its message to the other entire terminals. So, the message or information is send from the source to all other destinations using different links or channels.

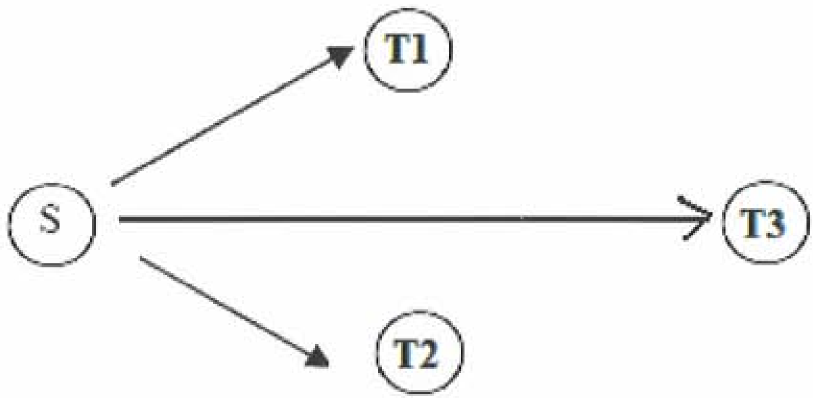

Figure 4: Broadcasting system

The third scheme of wireless communication can be viewed as a network communication. In such systems, we have more than one source and more than one destination. Each of the sources can send information to one or more destination and each distention can receive data from one or more sources.

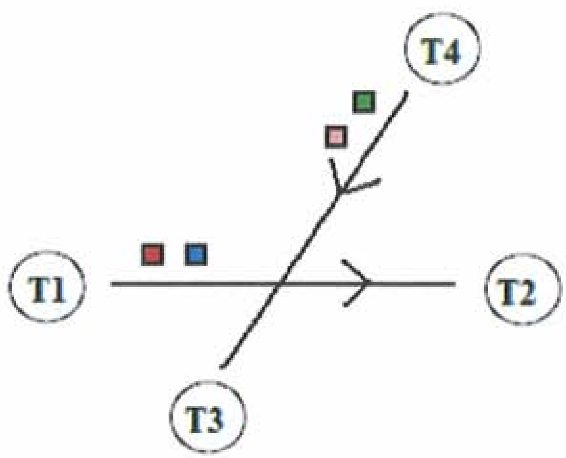

Figure 5: Network Communication

A wireless system has many phases or components between the source and the destination.

In figure below, point to point communication system is shown. First we have the binary source, in case of digital communication. Then these bits are modulated or changed using a certain scheme, as we will see later, in order to transmit over the channel. Then these modulated bits or signals are transmitted through the channel. On the other hand of the channel, there will be the demodulator to return the data as it was before sending and finally this data will be received to the destination.

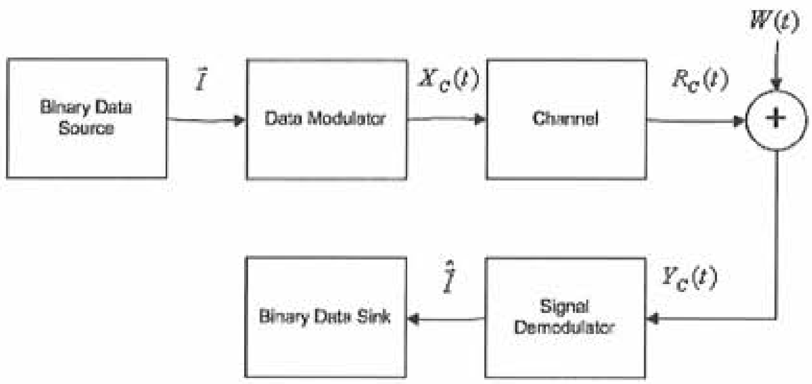

Figure 6: Simple point to point wireless communication system

In the next section, we will look Ultra Wideband (UWB) wireless systems.

IR-UWB Impulse Radio Ultra- Wideband systems

Ultra Wideband (UWB) systems are wireless systems which transmit signals across a much wider frequency spectrum than conventional wireless systems. According to a definition given by the US Federal Communications Commission (FCC), the radio signal of a UWB system has either a bandwidth of at least 20% of its center frequency, or a -10 dB bandwidth which exceeds 500 MHz. The UWB systems were originally used for radar, sensing, military communications and some niche applications [1].

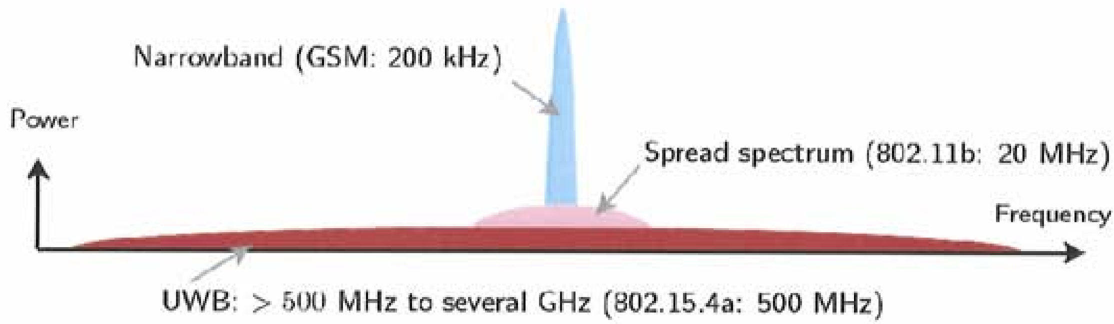

Figure 7 UWB signals [2]

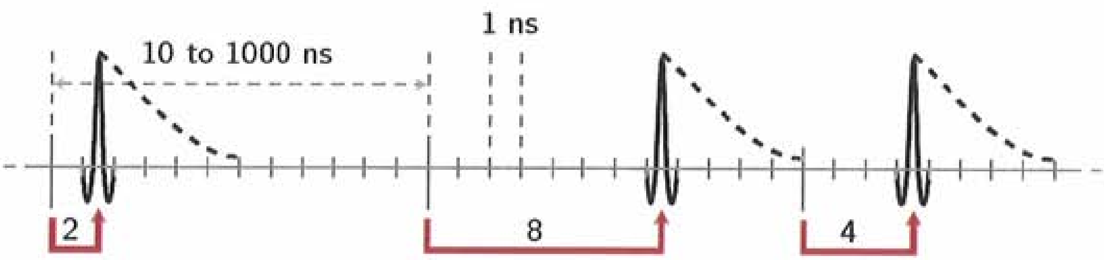

Figure 8 Impulse-radio UWB [2]

This technology uses short pulses between 1 and 1.5 nanoseconds along with a very low average power in the order of mille-watts. UWB can use time modulation with their waveforms. Using modulation of the signal in time domain, it is possible to represent the digital zero and the digital one using the following scheme. The digital one is when the pulse arrive earlier than expected and the digital zero is the when the pulse arrives later than expected.
The advantages of UWB are: low power consumption, low cost and simplicity.

Next, we will see different modulation techniques.

Modulation Techniques

There are many different modulation techniques used in wireless communication. Such of them are used for real data and the others are used with complex data or phase rotation. We will go throw two real modulation techniques: Pulse position modulation and Pulse Amplitude Modulation, and one complex modulation technique: Quadrature amplitude modulation.

Pulse Position Modulation

Pulse-position modulation PPM: is a type of signal modulation. M message bits are encoded then transmitted using a signal pulse in one of the 2^M possibilities of time slots. This procedure will be repeated every period of time T in seconds. Then the transmitted bit rate will be number of bits transmitted over the number of seconds it takes these bits to be transmitted. This will give R (bit rate) = M/T bits per second.

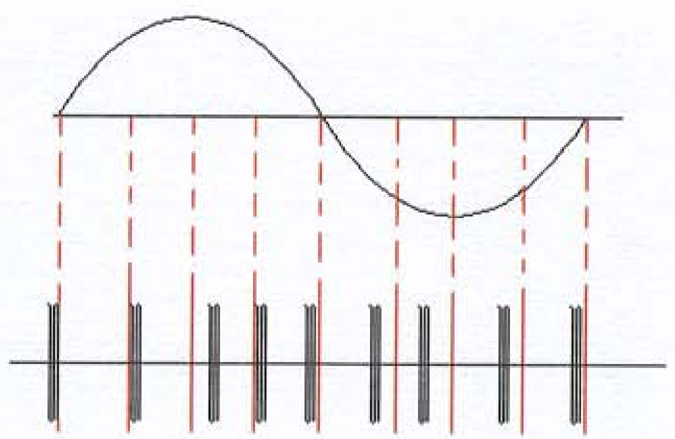

Figure 9: PPM Width and amplitude are constant. Its position is determined by the amplitude of the modulating signal. Pulse code modulation is measuring the amplitude at regular intervals and generating a binary number to represent this amplitude

Synchronization

The difficulty in implementing this scheme is that the receiver must be synchronized with a clock to see when each symbol will begin. For this reason, PPM coding and decoding is implemented differentially, which means that each pulse will be encoded according to the preceding one. On the other hand, the receiver will only measure the difference between the pulses. Also, it is possible to limit the propagation of errors to only adjacent symbols. So the error in measuring differential delay will not affect the whole sequence, but it will only affect two adjacent symbols.

Non-coherent Detection

The most important advantage in PPM is non- coherent detection. There is no need to use a Phase-locked loop PLL to track the phase of the carrier.

Pulse Amplitude Modulation

Pulse Amplitude Modulation, PAM, is another type of signal modulation. This technique takes the information and encodes it in the amplitude of signal pulses. For example, two bit modulator PAM-4 (2^2 levels) will take two bits at a time and map the signal amplitude to one the four possible levels. For example, the four levels will be [-3 -1 1 3].

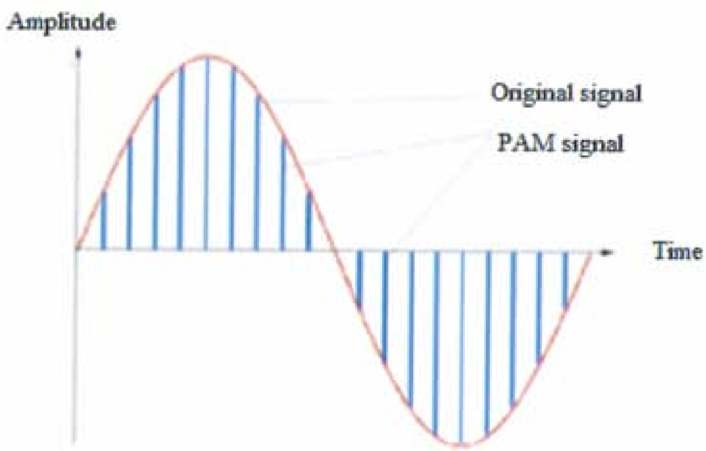

Figure 10: PAM Modulation

This technique is mainly use in baseband transmission of digital data. For telephone modes with bit rate of larger than 300 bits/s, they use quadrature amplitude modulation QAM.

Quadrature amplitude modulation

Quadrature amplitude modulation, QAM, is another modulation technique that takes two digital bits series or two analog messages and changes their amplitude using amplitude shift keying "ASK" for digital modulation or amplitude modulation "AM" for the analog case. The two phases of the two signals are usually out of phase by an angle of 90 degrees which are called quadrature carriers. The modulated wave, the resulting wave, is the sum of the two signals using both amplitudes shift keying and phase shift keying "PSK" for the digital case and amplitude modulation and phase modulation "PM" for the analog case.

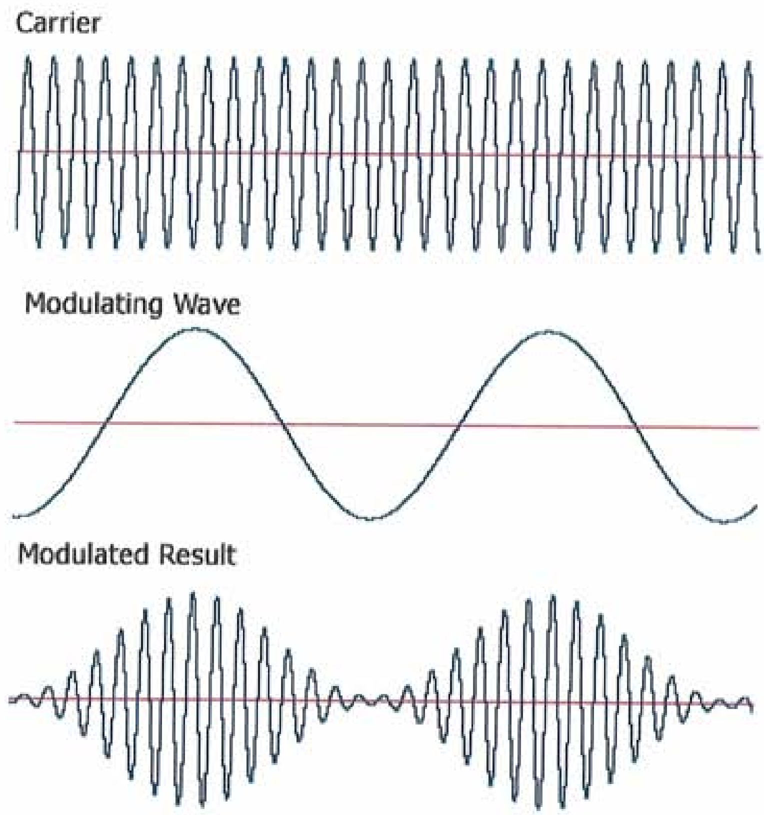

Figure 11: QAM, In AM, the amplitude of the carrier is varied by the incoming signal. The modulating wave is an analog signal.

In digital, at least there are two phases and two amplitudes are used. PSK principal is like QAM but it has constant amplitude.

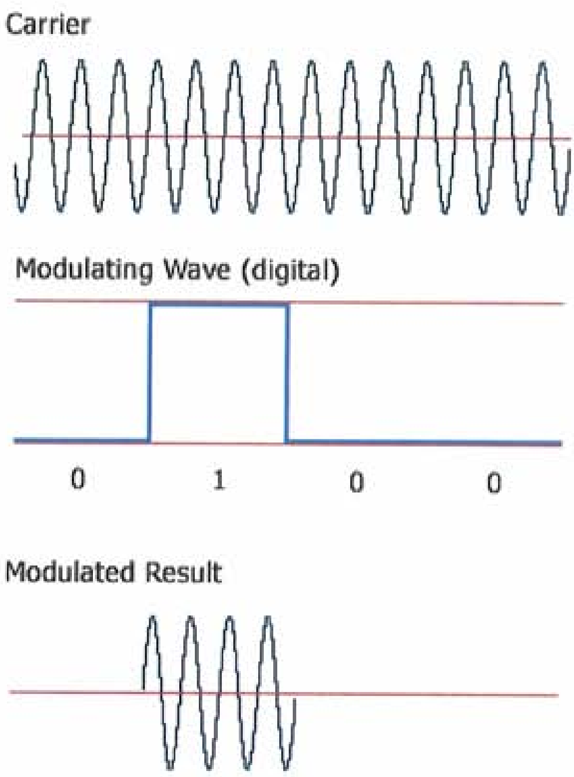

Figure 12: QAM: For digital signals, ASK uses two levels for 0 and 1.

The constellation is digital case is usually arranged in a square grid. There are equal horizontal and vertical spacing. But there are some cases that are not arranged in this mode such as cross-QAM. So, since digital uses binary presentation, the grid will be power of 2 (2, 4, 8, 16, 64, 128, 256…). We can transmit more bits per symbol by moving to higher order of QAM. But the points of constellation will be closer to each other and thus we have more noise on the signal and it will affect the bit error rate "BER". Therefore, we can transmit in higher order QAM more bits but less reliable.

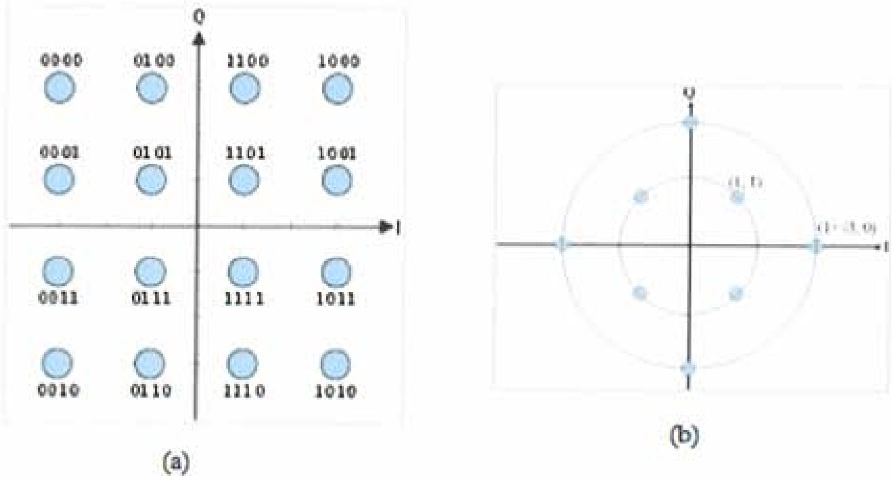

Figure 13: (a) Square 16 QAM constellation, (b) circular 8 QAM constellation

Rectangular or square QAM have advantage is that they can easily transmitted using two pulse amplitude modulation "PAM" over a quadrature carriers, and they can be easily demodulated. However, non- rectangular QAM constellation achieve better bit error rate, but they are hard to modulate and demodulate.

The next section will deal with MIMO systems.

Multiple Input and multiple output (MIMO)

MIMO, multiple-input and multiple-output, is the technology of using multiple antennas at both sides of a communication system. For wireless communication, MIMO uses different radio waveforms (two or more) at the source (transmitter) to carry inputs of a given channel. On the other hand, multiple antennas at the destination (receiver) are used as outputs to the radio channels, along with signal processing. The process will increase the throughput without adding any extra bandwidth or power. So this will lead to the following advantages:

1. Higher Spectral efficiency (more bits/second/HZ of bandwidth are sent)
2. Reliability link or special diversity

Channel Model

Consider a system that has M transmitter antennas with N receiver antennas. Then the channel can be represented by N × M. matrix channel. The received signal Y is equal to HX + w. X is M × 1 transmitted vector and W is N × 1 the noise matrix.

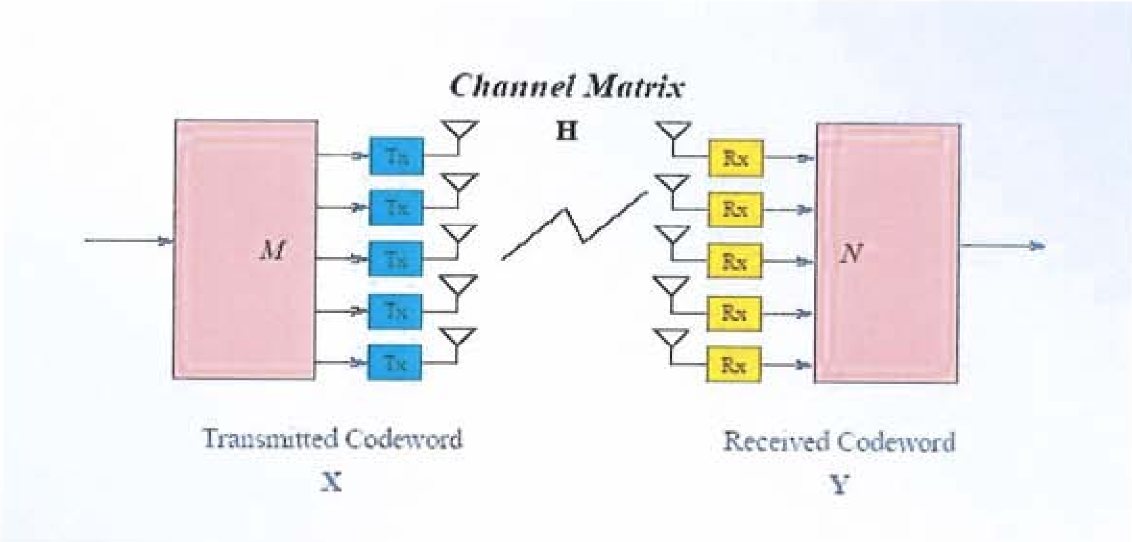

Figure 14: MIMO system

The next section will deal with Space time codes that are used with MIMO system in order to transmit in an efficient way.

Space Time Codes

Spaced time code, STC, is the method of using multiple antennas at the input, output, or both. It transmits multiple or redundant copies of data to the receiver in order to have a reliable decoding and reliable transmission through the physical link (channel). This redundancy will increase the probability of received data to be decoded correctly. STC combine the redundant copies of the received signal in order to get as much information as possible.

The STC uses the MIMO system in order to transmit from multiple relays to one or multiple receivers. Each transmitter sends the data at different time slot. For example of the figure below, we have the input matrix of the code that consists of three relays and three time slots. The yellow box indicates the information that is send from relay two at the third time slot and it is a function of the symbols used of the input set. S_i are the symbols used.

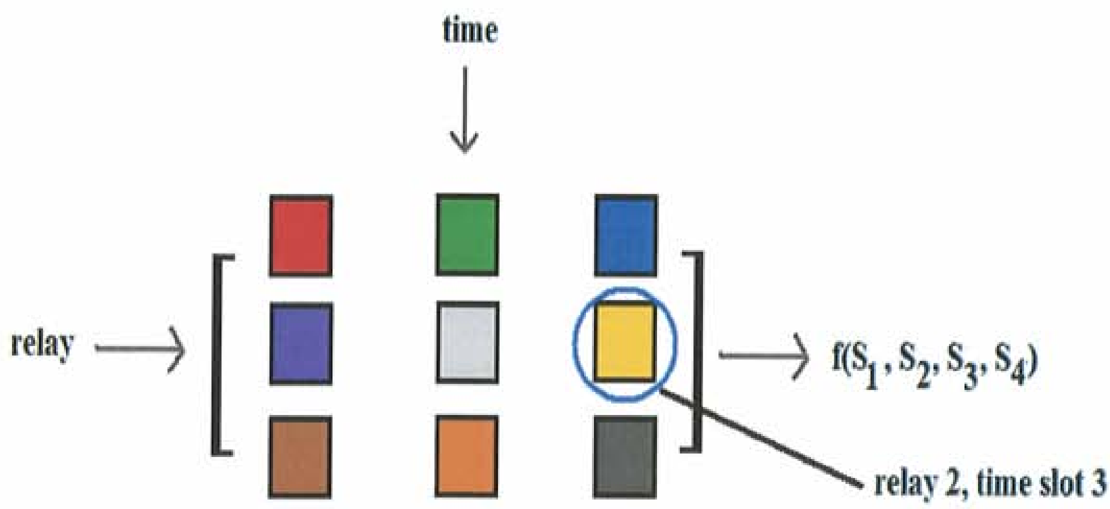

Figure 15: STC Matrix

Space time codes are divided into two major parts:

- The first one carries the code scheme over multiple antennas and multiple slot times. So, it will provide both coding gain and maximum diversity.
- The second one treats the data as a block and provides only diversity. But, its implementation is much simpler than the first one.

We can also divide the STC to more categories, coherent or non-coherent. In Coherent STC the receiver have some information about the channel using some training schemes or estimation schemes. For non-coherent, the receiver does not know the channel impairments but it know some statistical information of the channel. This process of receiving diverse copies of data is called diversity reception.

STC are usually represented by a matrix form. Each row represents a time slot and each column represent one antenna's transmission over time.
Next we will look at the different properties of space time codes.

Properties of Space Time Codes

Orthogonality and decoding complexity

STC are usually orthogonal. This means the code is designed in a way such that the vectors that represent any pairs of columns of the coding matrix are orthogonal. This will lead to a simple, linear, and optimal decoding at the receiver side.

We will present the Decoding complexity as M^a .

- M is the number of element in the constellation
- a is the number of elements that should be decoded jointly.

- ➢ Minimizing a => minimizing the decoding complexity.
- Decoding complexity can decrease if the input matrix of the code is orthogonal.

Code rate

Code rate or code information is the portion of the total amount of information that is useful, not redundant. It is a fractional amount. If the code rate is k/n. there are k bits that are useful or non-redundant and the total bits are n bits. Thus, there are n-k redundant bits.

If R is the bit rate of data signaling, then the useful bit rate is $\leq R \cdot k/n$

In Space time codes, the code rate can be seen as:

Code rate = k/T: it measures how many symbols per time slot transmitted on average.

Where, K is the number of encoding symbols and T is the total of time slots used.

Diversity

Usually the design of space time block codes is done using diversity criterion derived by Tarokh et al. this criterion can achieve maximum diversity.

Let the code word be:

$$\mathbf{c} = c_1^1 c_1^2 \ldots c_1^{n_T} c_2^1 c_2^2 \ldots c_2^{n_T} \ldots c_T^1 c_T^2 \ldots c_T^{n_T}$$

The decoded code words are:

$$\mathbf{e} = e_1^1 e_1^2 \ldots e_1^{n_T} e_2^1 e_2^2 \ldots e_2^{n_T} \ldots e_T^1 e_T^2 \ldots e_T^{n_T}.$$

Then the matrix:

$$\mathbf{B}(\mathbf{c}, \mathbf{e}) = \begin{bmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \cdots & e_T^1 - c_T^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \cdots & e_T^2 - c_T^2 \\ \vdots & \vdots & \ddots & \vdots \\ e_1^{n_T} - c_1^{n_T} & e_2^{n_T} - c_2^{n_T} & \cdots & e_T^{n_T} - c_T^{n_T} \end{bmatrix}$$

B has full rank for any pair of code-words c and e. This will give maximum diversity of order $n_I n_R$. If B(c, e) has minimum rank equal to b over any pair of code-words, then the diversity will be bn_R [3].

In the figure below, we can see the affection of diversity and coding gain advantages on the performance curve with respect to Signal to Noise Ratio (SNR).

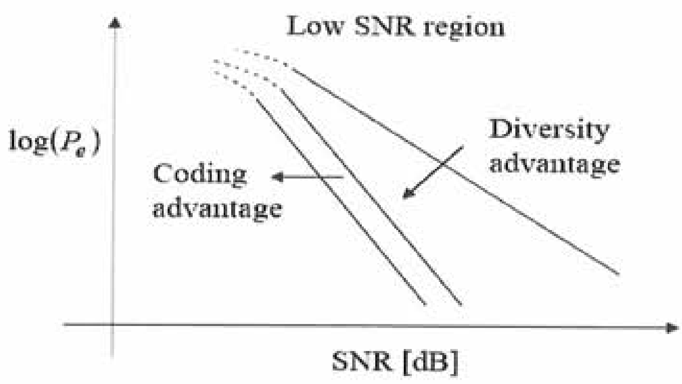

Figure 16: Coding gain and diversity advantages

Vanishing and non vanishing codes

Space-time block codes (STBC) with non-vanishing determinant (NVD) have attracted much attention lately. It have been shown that full rate STBC with NVD achieve the diversity [4].

There are two ways to construct STBC with NVD with full rate. The first way is to use the multi-layer structure. The second way is to use cyclic division algebra "CDA" structure [4].

In the next section, we will look at different STCs and look at their advantages and disadvantages.

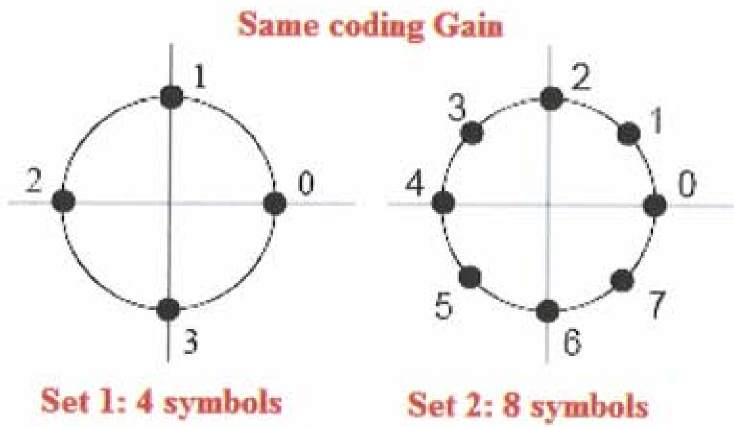

Figure 17: Non-vanishing coding gain

Real codes

In some systems, it is difficult to control the phase of pulses at high frequencies and large bandwidths. These systems must be totally-real.

In such systems, the shift occurs from using Quadrature amplitude modulation **(QAM)** (based on phase rotation) to either Pulse Position Modulation **(PPM)** or Pulse Amplitude Modulation **(PAM).**

Cooperative techniques

MIMO techniques and cooperative systems are merging as candidate solutions for enhancing the data rate, performance and communication distance. Neighboring terminals can cooperate with each other in order to benefit from spatial diversity. Decode-and-Forward (DF) protocol constitutes an interesting cooperative strategy.

Decode-and-Forward (DF) strategy

DF cooperation consists of two phases:

- During the first phase, the source node broadcasts its message to the neighboring nodes (relays),
- The second phase: these relays decode this message and transmit their encoded data streams versions simultaneously to the destination.

To conveniently encode the interfering data streams that are transmitted simultaneously from the different relays, ST coding techniques must be applied. During the second phase, each relay can simply transmit one row of a full rate ST codeword. We propose a novel ST code that is adapted to the DF strategy with two relays.

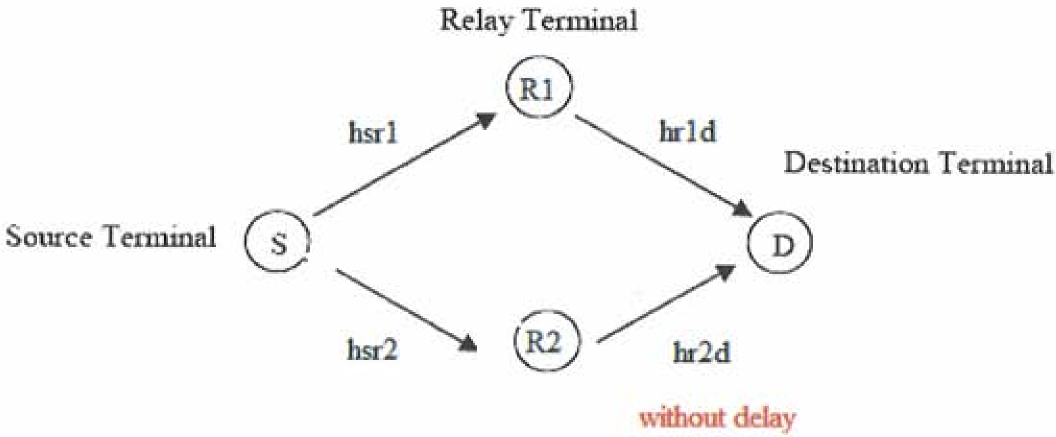

Figure 18: corporative technique without delay

The proposed code in [4], can be applied to DF systems, but the superiority of the proposed solution code is that it is delay-tolerant.

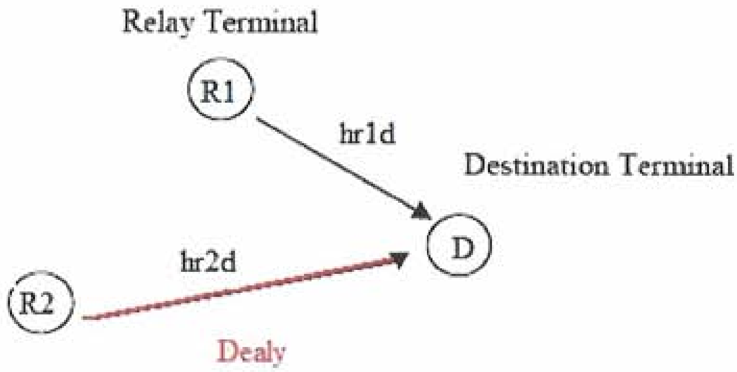

Figure 19: corporative technique with delay

A DF scheme is delay-tolerant if it keeps its diversity advantage even if the corresponding relays are asynchronous.

- Definition: A STC is delay-tolerant if all non-trivial code-words retain their full rank even when their columns are shifted.

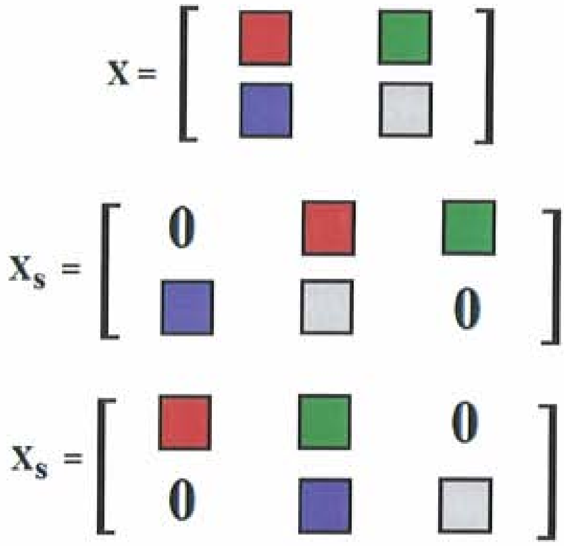

Figure 20: delay tolerant

The input matrix is X. if the first row or the second row is shifted and the rank of the shifted matrix is full, then the code is said to be delay tolerant.

Famous STC

We will look at different STCs in this section. Some of them are real codes and the others are not. We will consider all the properties and we will make a comparison between them according to these properties.

Alamouti- Matrix C code

In this section, we will look at Alamouti code presented in [6]. This code represent a simple two-branch transmit scheme. It uses two transmit antennas and one receiver antenna. It has the same order of diversity as in "maximum-ratio receiver combining "MRRC" that is composed of one transmitter and one receiver. This result is can be also generalized to obtain 2M diversity from M receivers. Also, this code is simple and does not need any extra bandwidth.

<u>"Two-Branch Transmit Diversity with two Receivers"</u>

In this scheme, we will consider the case of two transmitters and two receiver antennas.

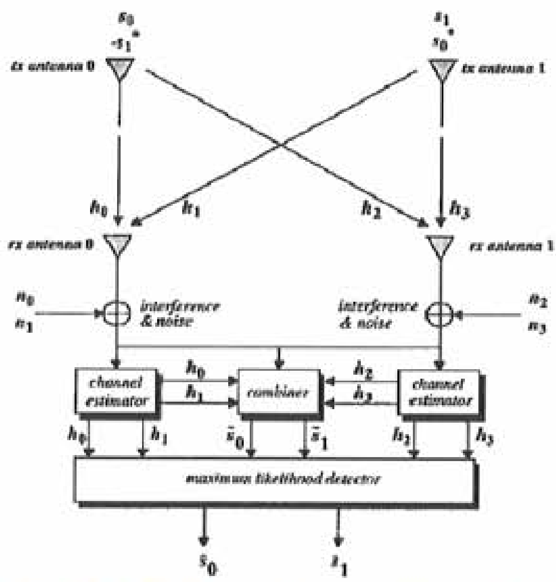

Figure 21: Two-Branch Transmit Diversity with two Receivers

	rx antenna 0	rx antenna 1
tx antenna 0	h_0	h_2
tx antenna 1	h_1	h_3

THE DEFINITION OF CHANNELS BETWEEN THE TRANSMIT AND RECEIVE ANTENNAS

	rx antenna 0	rx antenna 1
time t	r_0	r_2
time $t+T$	r_1	r_3

THE NOTATION FOR THE RECEIVED SIGNALS AT THE TWO RECEIVE ANTENNAS

Figure 22: Encoding scheme

From figure above and the two tables we can see that the received signals at the two antennas are:

$$
\begin{aligned}
r_0 &= h_0 s_0 + h_1 s_1 + n_0 \\
r_1 &= -h_0 s_1^* + h_1 s_0^* + n_1 \\
r_2 &= h_2 s_0 + h_3 s_1 + n_2 \\
r_3 &= -h_2 s_1^* + h_3 s_0^* + n_3
\end{aligned}
$$

n0, n1, n2, and n3 are complex random variables representing the noise.

In the figure above, the combiner builds the following two signals that are sent to the maximum likelihood detector:

$$
\begin{aligned}
\tilde{s}_0 &= h_0^* r_0 + h_1 r_1^* + h_2^* r_2 + h_3 r_3^* \\
\tilde{s}_1 &= h_1^* r_0 - h_0 r_1^* + h_3^* r_2 - h_2 r_3^* .
\end{aligned}
$$

By substituting the needed equation, we get:

$$\tilde{s}_0 = (\alpha_0^2+\alpha_1^2+\alpha_2^2+\alpha_3^2)s_0 + h_0^* n_0 + h_1 n_1^* + h_2^* n_2 + h_3 n_3^*$$
$$\tilde{s}_1 = (\alpha_0^2+\alpha_1^2+\alpha_2^2+\alpha_3^2)s_1 - h_0 n_1^* + h_1^* n_0 - h_2 n_3^* + h_3^* n_2.$$

These signals are sent to the maximum like-lihood detector:

Choose s_i iff
$$(\alpha_0^2+\alpha_1^2+\alpha_2^2+\alpha_3^2-1)|s_i|^2 + d^2(\tilde{s}_0, s_i) \le (\alpha_0^2+\alpha_1^2+\alpha_2^2+\alpha_3^2-1|s_k|^2 + d^2(\tilde{s}_0, s_k).$$

Choose s_i iff
$$d^2(\tilde{s}_0, s_i) \le d^2(\tilde{s}_0, s_k), \qquad \forall i \neq k.$$

Alamouti code for 2×2 MIMO (multiple input multiple output) systems:

$$X = \begin{bmatrix} S_1 & S_2 \\ -S_2{}^* & S_1{}^* \end{bmatrix}$$

Where S_1 and S_2 are the symbols to be sent and the star represents the conjugate of the symbols.

Alamouti code properties

1. Code rate:

   The code rate of the code is equal to r = k/T = 2/2 = 1. So, the code is half rate and not full rate.

2. The diversity of the code is given by the following equation:

$$d(M) = \min_{\substack{X,\hat{X} \in M \\ X \neq \hat{X}}} rank[(X-\hat{X})(X-\hat{X})^H]$$

M is the set of constellation used. X is the input matrix and X hat is the detected matrix. H is the transpose of the matrix.

The diversity of Alamouti code is equal to 2 (Full diversity).

3. The decoder complexity is equal to M (orthogonal code) as will have seen before.

4. Coding gain is presented by the minimum determinant of the input matrix:

$$\delta = \min_{\substack{X,\hat{X} \in M \\ X \neq \hat{X}}} det[(X-\hat{X})(X-\hat{X})^H]$$

   Coding gain = 2

5. Coding gain = 2 for all sets of constellation M (non-vanishing coding gain).

6. It is not a real code, since it used complex symbols (QAM constellation).

7. It is not a delay tolerant code because;

The original matrix is

$$X = \begin{bmatrix} S_1 & S_2 \\ -S_2{}^* & S_1{}^* \end{bmatrix}$$

The shifted matrix is

$$X_s = \begin{bmatrix} 0 & S_1 & S_2 \\ -S_2{}^* & S_1{}^* & 0 \end{bmatrix}$$

For example, S2 = 0 =>

$$X_s = \begin{bmatrix} 0 & S_1 & 0 \\ 0 & S_1{}^* & 0 \end{bmatrix}$$

The rank of this matrix is equal to 1. Therefore is not delay tolerant.

Golden code

We will look at the golden code presented in [5]. The golden code presented for 2×2 MIMO systems. This code has full rate and full diversity. The code is based on the golden number (1+√5)/2.

The construction satisfies the rank criterion for full diversity and maximizes the coding advantage.

Let K = Q(θ) "quadratic extension" of Q(i). Let the infinite code C∞ be defined as the following matrix.

$$\mathcal{C}_\infty = \left\{ \mathrm{X} = \begin{bmatrix} a - b\theta & c + d\theta \\ \gamma(c + d\bar{\theta}) & a + b\bar{\theta} \end{bmatrix} : a, b, c, d \in \mathbb{Z}[i] \right\}$$

C∞ is a linear code (X1+X2 ∈ C∞ for all X1, X2 ∈ C∞). The finite code C is obtained by limiting information symbols to a, b, c, d ∈ S ⊂ Z[i]. The signal constellation S is 2b-QAM (to ±1, ±3…) and b bits per symbol.

"The code C∞ is a discrete subset of a cyclic division algebra over Q(i), obtained by selecting γ _= $N_{K/Q(i)}$(x) for any x ∈ K."

Any codeword X ∈ C∞ the rank criterion is satisfied as det (X) ≠ 0.

The minimum determinant of C∞ is:

$$\delta_{\min}(\mathcal{C}_\infty) = \min_{\mathrm{X} \in \mathcal{C}_\infty, \mathrm{X} \neq 0} |\det(\mathrm{X})|^2$$

The minimum determinant of the finite code C is:

$$\begin{aligned} \delta_{\min}(\mathcal{C}) &\triangleq \min_{\mathrm{X}_1, \mathrm{X}_2 \in \mathcal{C}, \mathrm{X}_1 \neq \mathrm{X}_2} |\det(\mathrm{X}_1 - \mathrm{X}_2)|^2 \\ &= 4\delta_{\min}(\mathcal{C}_\infty) \end{aligned}$$

The algebraic code-words of the Golden code of the form:

$$\frac{1}{\sqrt{5}}\begin{bmatrix} \alpha(a+b\theta) & \alpha(c+d\theta) \\ i\bar{\alpha}(c+d\bar{\theta}) & \bar{\alpha}(a+b\bar{\theta}) \end{bmatrix} \qquad a,b,c,d \in \mathbb{Z}[i]$$

$\alpha = 1+i(1-\theta)$, $\theta = (1+\sqrt{5})/2$ and $\bar{\theta} = (1-\sqrt{5})/2$. This code has non-vanishing $\delta\min(C\infty)$ = 1/5, thus $\delta\min(C)$ = 1/5 for any size of the signal constellation [5].

Golden code properties

1. Code rate:

   The code rate of the code is equal to r = k/T = 4/2 = 2. So, the code is full.

2. The diversity is full.

3. The decoder complexity is equal to M^4. Therefore, there is no simplification in the detector and all the four symbols should be taken jointly at the same time.

4. Coding gain is equal to 3.2, which is a high coding gain and it gives this code a high advantage in its performance.

5. It has a Non-vanishing coding gain.

6. It is not a real code, since it used complex symbols (QAM constellation).

7. It is not a delay tolerant code; we can apply the same scheme to test for delay tolerant as we have done in the previous section.

S. Sezginer- Silver code

In this section, we will look at the code presented in [7]. This code is full rate, full diversity, and low complexity decoding. This leads to high performance full-rate in realistic systems.

For a general STC, the case of 2 ×2 transmission let the code book matrix be:

$$\mathbf{X} = \begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix}$$

The code optimization is generally concerned with pairwise error probability "PEP".

$P(X - \hat{X})$: is the probability of detecting $\hat{X}$ while X is transmitted.

Rank criterion: to achieve maximum diversity, the diversity gain should be maximized.

$$d(\mathcal{X}) = \min_{\substack{\mathbf{X},\hat{\mathbf{X}} \in \mathcal{X} \\ \mathbf{X} \neq \hat{\mathbf{X}}}} \operatorname{rank}[(\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^H]$$

If $(\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^H$ has a full rank for all code words, then the code is said to have full diversity.

Determinant gain: the coding gain is defined as:

$$\delta = \min_{\substack{\mathbf{X},\hat{\mathbf{X}} \in \mathcal{X} \\ \mathbf{X} \neq \hat{\mathbf{X}}}} \det[(\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^H].$$

This gain, for certain transmit power, should be maximized in order to achieve high performance.

The proposed code in [7] has the following code matrix form,

$$X = \begin{bmatrix} as_1 + bs_3 & -cs_2^* - ds_4^* \\ as_2 + bs_4 & cs_1^* + ds_3^* \end{bmatrix}$$

Four symbols are transmitted s1, s2, s3, and s4. (a ,b, c, and d) are complex-valued parameters. Star designates complex conjugate of the symbols.

Detection and simplification:

For the first receiver, we have two received signals that are due the first and the second time interval. The two received signals are:

$$r_1 = h_{11}(as_1 + bs_3) + h_{12}(as_2 + bs_4) + n_1$$
$$r_2 = h_{11}(-cs_2^* - ds_4^*) + h_{12}(cs_1^* + ds_3^*) + n_2.$$

Similarly for the second receiver, we have the following signals:

$$r_3 = h_{21}(as_1 + bs_3) + h_{22}(as_2 + bs_4) + n_3$$
$$r_4 = h_{21}(-cs_2^* - ds_4^*) + h_{22}(cs_1^* + ds_3^*) + n_4.$$

"In these equations, *hkl* designates the channel response between transmit antenna *l* and receive antenna *k*, and the *nk*, *k* = 1, . . . , 4, terms are circularly symmetric additive Gaussian noise terms with spectral density *N*0."[7]

"The maximum likelihood (ML) detector makes an exhaustive search over all possible values of the transmitted symbols and decides in favor of the quadruplet (*s*1, *s*2, *s*3, *s*4) which minimizes the Euclidean distance:" [7]

$$\begin{aligned} D(s_1, s_2, s_3, s_4) = \Big\{ & |r_1 - h_{11}(as_1 + bs_3) - h_{12}(as_2 + bs_4)|^2 \\ & + |r_2 + h_{11}(cs_2^* + ds_4^*) - h_{12}(cs_1^* + ds_3^*)|^2 \\ & + |r_3 - h_{21}(as_1 + bs_3) - h_{22}(as_2 + bs_4)|^2 \\ & + |r_4 + h_{21}(cs_2^* + ds_4^*) - h_{22}(cs_1^* + ds_3^*)|^2 \Big\} \end{aligned}$$

It is clearly that this equation is the search is done for M^4 symbols and M^4 -1 comparison. Suppose that for 64-QAM which will be very excessive. For this reason, the proposed code in [7] will design a minimized way for the ML detection that will take the system from M^4 to M^2.

First the following intermediate signals will be computed:

$$\begin{aligned} z_1 &= r_1 - b(h_{11}s_3 + h_{12}s_4) \\ z_2 &= r_2 - d(h_{12}s_3^* - h_{11}s_4^*) \\ z_3 &= r_3 - b(h_{21}s_3 + h_{22}s_4) \\ z_4 &= r_4 - d(h_{22}s_3^* - h_{21}s_4^*) \end{aligned}$$

For any given symbols (s3 and s4) and using the original received signals, the signals will be reduced to:

$$\begin{aligned} z_1 &= a(h_{11}s_1 + h_{12}s_2) + n_1 \\ z_2 &= c(h_{12}s_1^* - h_{11}s_2^*) + n_2 \\ z_3 &= a(h_{21}s_1 + h_{22}s_2) + n_3 \\ z_4 &= c(h_{22}s_1^* - h_{21}s_2^*) + n_4 \end{aligned}$$

Next, multiplying by factors of the channels, the signals will be as follows:

$$\begin{aligned} h_{11}^* z_1 &= a(|h_{11}|^2 s_1 + h_{11}^* h_{12} s_2) + h_{11}^* n_1 \\ h_{12} z_2^* &= c^*(|h_{12}|^2 s_1 - h_{11}^* h_{12} s_2) + h_{12} n_2^* \\ h_{21}^* z_3 &= a(|h_{21}|^2 s_1 + h_{21}^* h_{22} s_2) + h_{21}^* n_3 \\ h_{22} z_4^* &= c^*(|h_{22}|^2 s_1 - h_{21}^* h_{22} s_2) + h_{22} n_4^* \end{aligned}$$

From those signals y1 can be computed as the received signal at the first transmitter as:

$$\begin{aligned} y_1 &= (h_{11}^* z_1 + h_{21}^* z_3)/a + (h_{12} z_2^* + h_{22} z_4^*)/c^* \\ &= (|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2) s_1 + w_1 \end{aligned}$$

Where

$$w_1 = (h_{11}^* n_1 + h_{21}^* n_3)/a + (h_{12} n_2^* + h_{22} n_4^*)/c^*$$

We can note that this signal does not involve the symbols s2. So by sending this signal to the ML detector we can computer s1 for a given (s3, s4). This is true only when the symbols s1 and s2 are the same in the two columns.

Similarly, for s2, y2 is computes and detection complexity will be reduced to M^2.

$$\begin{aligned} y_2 &= (h_{12}^* z_1 + h_{22}^* z_3)/a - (h_{11} z_2^* + h_{21} z_4^*)/c^* \\ &= (|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2) s_2 + w_2 \end{aligned}$$

$$w_2 = (h_{12}^* n_1 + h_{22}^* n_3)/a - (h_{11} n_2^* + h_{21} n_4^*)/c^*$$

This detection is optimal when a= c, if we want to detect s_1 and s_2, and b=d if we want to reverse the order and detect s3 and s4.

Performance:

The *(a, b, c, d)* parameters should be optimized in order to obtain full-diversity STC with large coding gain. Also the transmit power should be optimized.
The condition is as follows:

$$\begin{aligned} |a|^2 + |b|^2 = 1 = |c|^2 + |d|^2 \\ |a|^2 + |c|^2 = 1 = |b|^2 + |d|^2 \end{aligned}$$

"The first condition ensures an equal transmit power at each symbol time, while the second condition ensures that equal total power is transmitted for each symbol. These equalities together with the constraint |a| = |c| for optimal detection lead immediately to the fact that all the design parameters should have the same magnitude, example: |a| = |b| = |c| = |d| = 1/√2." [7].

In order to get full diversity, we have to have the matrix in its full rank as defined previously.
Take *a* = *c* = 1/√2. "It can be shown that there exists a set of parameters *(b, d)* for which the resulting STC will have nonzero coding gain for any QAM constellation size."[7].

Silver code properties

1. Code rate:

   The code rate of the code is equal to r = k/T = 4/2 = 2. So, the code is full.

2. The diversity is full.

3. The decoder complexity is equal to M^2. Therefore, there is simplification in the detector and not all the four symbols should be taken jointly at the same time. Only two symbols should be taken jointly.

4. Coding gain is equal to 2, which is a good coding gain.

5. It has a Non-vanishing coding gain.

6. It is not a real code, since it used complex symbols (QAM constellation).

7. It is not a delay tolerant code; we can apply the same scheme to test for delay tolerant as we have done in the previous section.

First and second real codes

In some applications, it is much efficient to use totally real valued transition techniques. Real valued STC is suitable for IR-UWB communication system. "In fact, the last generation of complex-valued ST codes (namely, the perfect codes) cannot be associated with IR-UWB systems where the phase reconstitution at the receiver side is practically infeasible." [9]. So, we are going to study the possibilities of constructing real codes that uses PPM or PAM modulation techniques instead of using infinite fields (complex numbers). The proposed codes are totally real, achieve full diversity and full rate, and have non-vanishing coding gain.

Based on [8], STC constructed from the following equation with achieve full rate, full diversity, and non-vanishing determinants.

$$C = \sum_{i=0}^{n-1} \mathrm{diag}\left(\mathcal{M}\left[a_{in+1}, \ldots, a_{(i+1)n}\right]^T\right)\Omega^i$$

Where,

M is n×n matrix.

$$\Omega = \begin{bmatrix} \mathbf{0}_{(n-1)\times 1} & I_{n-1} \\ \gamma & \mathbf{0}_{1\times(n-1)} \end{bmatrix}$$

"For totally real code-words, transmitting a uniform average energy per antenna can be realized uniquely by $\gamma = \pm 1$."[8]

For 2×2 codes:

$$\mathcal{M} = \begin{bmatrix} \sqrt{\alpha} & 0 \\ 0 & \sqrt{\sigma(\alpha)} \end{bmatrix} \begin{bmatrix} 1 & \theta \\ 1 & \sigma(\theta) \end{bmatrix}$$

First Code

Construct a constellation by choosing a field which has units whose norms are equal to 1. Choosing the norm to be 1, we can find that $\gamma = 2$

X1 =

$$\begin{bmatrix} \sqrt{\alpha}(a_1 + a_2\theta) & \sqrt{\alpha}(a_3 + a_4\theta) \\ 2\sqrt{\alpha_1}(a_3 + a_4\theta_1) & \sqrt{\alpha_1}(a_1 + a_2\theta_1) \end{bmatrix}$$

$\alpha_1 = \sigma(\alpha)$, $\theta_1 = \sigma(\theta) = (1 - \sqrt{5})/2.$,and $\alpha = 1/(1 - \theta^2) = (3 - \theta)/5$

Second Code

Choosing $\gamma = -1$, K is n-dimensional vector space over Q.

K must be chosen to have no elements whose norm is -1.

The code is balanced because the transmitted power at each antenna is the same during the two symbol durations.

The code is X2 =

$$\frac{1}{2}\begin{bmatrix} a_1 + \sqrt{3}a_2 & a_3 + \sqrt{3}a_4 \\ -(a_3 - \sqrt{3}a_4) & a_1 - \sqrt{3}a_2 \end{bmatrix}$$

Where $\alpha 1$, $\alpha 2$, $\alpha 3$, and $\alpha 4$ are the symbols.

Studying the real codes

1st and 2nd code diversity

For the following two codes, the first thing that we are going to study is the coding gain and diversity of these codes.

Based on the diversity criterion proposed [7], to achieve maximum diversity, the diversity gain should be maximized.

$$d(\mathcal{X}) = \min_{\substack{\mathbf{X},\hat{\mathbf{X}} \in \mathcal{X} \\ \mathbf{X} \neq \hat{\mathbf{X}}}} \operatorname{rank}[(\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^H]$$

Referring to the equation, we want to get the rank of $(X - \hat{X})(X - \hat{X})^H$. Since we are using a 2×2 matrix, the maximum rank that we can achieve is 2. We need the rank difference matrix between the transmitted code and the detected code multiplied by the transpose of this matrix to be 2 in order to gain maximum diversity. For this, we calculate all the possible values of difference, but eliminating the cases where X $=\hat{X}$. Then using MATLAB, we write a program to go throw these differences and get the rank of each of the matrixes obtained and finally taking the minimum. We have done that for the two codes and the results show that both of these codes have full diversity equals to 2. Note that we have used the PAM set, for example 2-PAM = {-1 1} and 4-PAM = {-3 -1 1 3}.

<u>Basis vectors:</u>

For the first, note that any component of the matrix is composed from the combination of the two numbers $(1, \theta)$ .So, theses two numbers will be the basis of the vector space that we have. Same will apply to the second code with the two components $(1, \sqrt{3})$. So, if we take the determinant of any of these codes, we will have this vector will appear. For example it will be $a + \sqrt{3b}$, which will be zero if and only if a= b= 0. This is the case that we are eliminating, so the matrix will be full rank for any arbitrary vale taken.

1st and 2nd code: coding gain

Based on the determinant gain proposed in [7], the following equation of gain should be maximized.

$$\delta = \min_{\substack{\mathbf{X},\hat{\mathbf{X}} \in \mathcal{X} \\ \mathbf{X} \neq \hat{\mathbf{X}}}} \det[(\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^H].$$

This gain, for certain transmit power, should be maximized in order to achieve high performance.

For this we always have to transmit an average power which will not affect in increasing or decreasing this gain. Using the same procedure as in getting diversity, we will get the coding gain of the two codes. Results show that the coding gain of the first code and second code are $1.442 = 8/3\sqrt{5} = 1.19$ and 1.0 respectively. Compared to [7], we can see that the coding gain decreases in both cases. On the other hand, we have the advantage of maximum diversity and real valued code with lower complexity and cost.

It was know that the coding gain of any size set of constellation will be the same. So taking the set 2-PAM, 4-PAM, 8-PAM, ... will not affect the coding gain. This is very important characteristic of these codes which is called non-vanishing codes. So, we can increase the constellation to 64-PAM for example and use it in WiMAX systems and downlink systems that uses this kind of modulation.

Optimal Detector for the second real code

In this section, we will propose and optimal detector technique for the code presented in [8].

We will consider the case of 2×2 input matrix for both of the codes 1 and 2. Output channel H is also 2×2 matrix form that is generated randomly and has the following form:

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

The first code input matrix can be written as follows:

$$X = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{1+\theta^2}} & 0 \\ 0 & \frac{1}{\sqrt{1+\bar{\theta}^2}} \end{bmatrix} \begin{bmatrix} S_1 + \theta S_2 & \sqrt{2}(S_3 + \theta S_4) \\ \sqrt{2}(S_3 + \bar{\theta} S_4) & S_1 + \bar{\theta} S_2 \end{bmatrix}$$

$$\theta = \frac{1+\sqrt{5}}{2} \; and \; \bar{\theta} = \frac{1-\sqrt{5}}{2}$$

$$S_{1,}\, S_{2,} S_{3,} and\ S_{4,}\ are\ the\ symbols\ to\ be\ sent$$

Finally the system will have the following structure:

$$Y = HX + N$$

Y is 2 ×2 output matrix, and N is 2 ×2 matrix additive noise.

So, the input matrix will be send through the channels along with some noise, which is generated from the channel and equipments, to the Maximum Likelihood (ML) detector in order to decide on the symbols that are sent.

Here, we are assuming that our code know the channels. If we want to detect the output of these four symbols sent over the channel, we have to do the metric relation.
The ML will make a search over all possible values of the transmitted symbols. It will decide the group of symbols ($S_{1,}\ S_{2,}S_{3,}\ S_{4,}$) that minimize the Euclidean distance.
For the first code, we don't have a minimized detector since the build of the input matrix do not have terms that can be eliminated. There is no, for example, a negative column or row to be eliminated with the other row or column or even a component of the matrix. Thus, searching for the transmitted symbols will require a loop of order 4. The complexity is M^4, where M is the size of constellation.

For the second code, the input matrix is

$$X = \frac{1}{2}\begin{bmatrix} S_1 + \sqrt{3}S_2 & S_3 + \sqrt{3}S_4 \\ -(S_3 - \sqrt{3}S_4) & S_1 + \sqrt{3}S_2 \end{bmatrix}$$

In this code, we have some elements that help us in simplifying the detector. We will present here a minimized optimal detector with complexity of M^2, where M is the size of constellation.

The received signals will be as follows:
On the first receiver antenna, these are the two signals received during the first and the second time interval.

$$r_1 = \frac{1}{2}[h_{11}(S_1 + \sqrt{3}S_2) + h_{12}\left(-(S_3 - \sqrt{3}S_4)\right) + n_1]$$

$$r_2 = \frac{1}{2}[h_{11}(S_3 + \sqrt{3}S_4) + h_{12}(S_1 + \sqrt{3}S_2) + n_2]$$

Similarly, for the second receiver, the first and the second time interval signals are:

$$r_3 = \frac{1}{2}[h_{21}(S_1 + \sqrt{3}S_2) + h_{22}\left(-(S_3 - \sqrt{3}S_4)\right) + n_3]$$

$$r_4 = \frac{1}{2}[h_{21}(S_3 + \sqrt{3}S_4) + h_{22}(S_1 + \sqrt{3}S_2) + n_4]$$

In these equation hij corresponds to the response of the channel between the received antenna i and the transmitted antenna j. nk is the additive noise to the signals, where k = 1, 2, 3, and 4.

Now, the ML will search all the possible values of symbols and decide in the favor of minimum Euclidean distance D.

$$D = \left|r_1 - \frac{1}{2}[h_{11}(S_1 + \sqrt{3}S_2) + h_{12}\left(-(S_3 - \sqrt{3}S_4)\right)\right|^2$$
$$+ \left|r_2 - \frac{1}{2}[h_{11}(S_3 + \sqrt{3}S_4) + h_{12}(S_1 + \sqrt{3}S_2)\right|^2$$
$$+ \left|r_3 - \frac{1}{2}[h_{21}(S_1 + \sqrt{3}S_2) + h_{22}\left(-(S_3 - \sqrt{3}S_4)\right)\right|^2$$
$$+ \left|r_4 - \frac{1}{2}[h_{21}(S_3 + \sqrt{3}S_4) + h_{22}(S_1 + \sqrt{3}S_2)\right|^2$$

This criterion will need M^4 metrics and $M^4 - 1$ comparisons. For this, we are going to computer the following intermediate signals.

$$Z_1 = 2r_1 + h_{12}S_3 - h_{12}\sqrt{3}S_3$$
$$Z_2 = 2r_2 - h_{11}S_3 - h_{11}\sqrt{3}S_3$$
$$Z_3 = 2r_3 + h_{22}S_3 - h_{22}\sqrt{3}S_3$$
$$Z_4 = 2r_4 - h_{21}S_3 - h_{21}\sqrt{3}S_3$$

For a given symbol pair (S_3, S_4).These signal can be written as:

$$Z_1 = h_{11}(S_1 + \sqrt{3}S_2) + n_1$$
$$Z_2 = h_{12}(S_1 - \sqrt{3}S_2) + n_2$$
$$Z_3 = h_{21}(S_1 + \sqrt{3}S_2) + n_3$$
$$Z_4 = h_{22}(S_1 - \sqrt{3}S_2) + n_4$$

Now, computer the following signals:

$$h_{12}Z_1 = h_{11}h_{12}S_1 + h_{11}h_{12}\sqrt{3}S_2 + h_{12}n_1$$
$$h_{11}Z_2 = h_{11}h_{12}S_1 - h_{11}h_{12}\sqrt{3}S_2 + h_{11}n_2$$
$$h_{22}Z_3 = h_{21}h_{22}S_1 + h_{21}h_{22}\sqrt{3}S_2 + h_{22}n_3$$
$$h_{21}Z_4 = h_{21}h_{22}S_1 - h_{21}h_{22}\sqrt{3}S_2 + h_{21}n_4$$

From these signals, we can compute

$$y_1 = h_{12}Z_1 + h_{11}Z_2 + h_{22}Z_3 + h_{21}Z_4$$
$$= 2(h_{11}h_{12} + h_{21}h_{22})\, S_1 + \, w_1$$

Where $w_1 = h_{12}n_1 + h_{11}n_2 + h_{22}n_3 + h_{21}n_4$

We can see that $y_1$does not include terms of S_2. It can be clearly seen that it can be used for estimatingS_1. So, by sending this signal to the ML detection, condition on (S_3, S_4), we can estimateS_1.

In the same way, multiplying $(Z_1, Z_2, Z_3, and\ Z_4)$ by$(h_{12}, -h_{11}, h_{22}, and\ -h_{21})$ rspectively.

We can compute

$$y_2 = \sqrt{3}(h_{12}Z_1 - h_{11}Z_2 + h_{22}Z_3 - h_{21}Z_4)$$
$$= 2\sqrt{3}\,(2h_{11}h_{12} + h_{21}h_{22})\, S_2 + \, w_2$$

Where $w_1 = h_{12}n_1 + h_{11}n_2 + h_{22}n_3 + h_{21}n_4$

We can see that $y_2$does not include terms of S_1. It can be clearly seen that it can be used for estimatingS_2. So, by sending this signal to the ML detection, condition on (S_3, S_4), we can estimateS_2.

Now, instead of using D(s_1, s_2, s_3, s_4) , we can compute D($s_1{}^{ML}, s_2{}^{ML}, s_3, s_4$). Where $s_1{}^{ML}, s_2{}^{ML}$ are the detected symbols using the two signals $y_1$and y_2. It can also be in the second order, so we can compute $s_3{}^{ML}, s_4{}^{ML}$ and then get D($s_1, s_2, s_3{}^{ML}, s_4{}^{ML}$).

Thus, we are going from M^4 metrics to M^2 metrics. It will be very significant in we use 64-PAM.

First and second codes properties

1. Code rate:

   The code rate of both codes is equal to $r = k/T = 4/2 = 2$. So, the codes are full.

2. The diversity of both codes is also full.

3. The decoder complexity is equal to M^2 for the second code. While for the first one we don't have any minimizing scheme

4. Coding gain is equal to 1.19 for the first one and 1 for the second one. Here we have some decrease in the coding gain, but we have make advantages on other properties.

5. Both are Non-vanishing coding gain.

6. Both are real codes, they can be used with PAM constellation.

7. Both are not tolerant to delays.

The Proposed Code

We proposed a 2×2 ST code for PAM constellation. The code satisfies a large number of constraints:

- It is totally real, it has full rate and it achieves a full diversity order with non-vanishing coding gain.
- It is delay tolerant.
- Reduced decoding complexity (only three data streams need to be decoded jointly rather than four streams M^3).

This code can be applied to M'-PAM symbols.

$$\mathcal{A} \triangleq DA$$
$$= \frac{1}{\sqrt{3}} \begin{bmatrix} \frac{1}{\sqrt{1+\theta^2}} & 0 \\ 0 & \frac{1}{\sqrt{1+\bar{\theta}^2}} \end{bmatrix} \begin{bmatrix} a_1 + \theta a_2 + \sqrt{2} a_3 + \sqrt{2}\theta a_4 & -(a_1 + \theta a_2 - \sqrt{2} a_3 - \sqrt{2}\theta a_4) \\ a_1 + \theta_1 a_2 + \sqrt{2} a_3 + \sqrt{2}\theta_1 a_4 & a_1 + \theta_1 a_2 - \sqrt{2} a_3 - \sqrt{2}\theta_1 a_4 \end{bmatrix}$$

Denote by α1…α4 four M'-PAM symbols. θ is the golden code and θ1 is its conjugate.

$$\theta = \frac{1+\sqrt{5}}{2} \; and \; \theta_1 = \frac{1-\sqrt{5}}{2}$$

D is a diagonal matrix introduced for normalization purposes.

Proposition 1: the code is full diversity and achieve a non-vanishing coding gain of $\frac{8}{3\sqrt{5}}$ with M'-PAM constellation for all values of M'.

Proof: the code word $\mathcal{A}$ has the same rank of matrix A.

- Then the matrix can be written as:

$$A = UA_0V$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} a_1 + \theta S_2 & \sqrt{2}(a_3 + \theta a_4) \\ \sqrt{2}(a_3 + \theta_1 a_4) & a_1 + \theta_1 a_2 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$$

This implies that A can be obtained from A0 by linear combinations of its rows and its columns. These operations do not change the rank of the matrix.

A and A_0 have the same rank. Looking at A_0, it has the same structure of the code constructed from cyclic division algebras in [7].

The corresponding cyclic algebra is denoted by

$$\mathcal{D}(\mathbb{K}|\mathcal{Q}, \sigma, \gamma) where\ \gamma = 2$$

The second order cyclic field extension

$$\mathbb{K} = \mathcal{Q}(\theta)$$

It has a Galois group

$$Gal\ (\mathbb{K}|\mathcal{Q}) =< \sigma > with\ \sigma^2 = 1\ and\ \sigma(\theta) = \theta_1$$

Non-zero code-words A0 have a full rank if the algebra $\mathcal{D}$ is division algebra. This is true if not element in $\mathbb{K}$ whose algebraic norm is equal to γ =2 [4].

Using KANT software:

Ideal $2\mathcal{O}_{\mathbb{K}}$ is prime => 2 is a non-norm element. Thus the code proposed in full diverse.

Where, $\mathcal{O}_{\mathbb{K}}$ is the ring of integers of $\mathbb{K}$.

Non-zero determinant of A_0 is 1. => The coding gain over any set of constellation $\mathcal{Z}$ is equal to $\frac{2}{3\sqrt{d_k}}$ where $d_k = 5$ corresponds to absolute discriminate of $\mathbb{K}$.

Since PAM symbols ε to $2\mathbb{K}$, then the coding gain with M'-PAM constellation is equal to $\frac{2}{3\sqrt{d_k}}$.

Finally, since $\gamma = 2$, this implies that the coding gain is non-vanishing and it keeps the same value for all values of M'.

Proposition 2: tolerant to delays.

A can be written as follows:

$$A = \begin{bmatrix} x_1 & -x_3 \\ x_2 & x_4 \end{bmatrix} \quad \text{Equation (1)}$$

Where x1..x4 corresponds to the PAM information symbols a1…a4:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \frac{1}{\sqrt{3}}\mathcal{D}\begin{bmatrix} 1 & \theta & \sqrt{2} & \sqrt{2}\theta \\ 1 & \theta_1 & \sqrt{2} & \sqrt{2}\theta_1 \\ 1 & \theta & -\sqrt{2} & -\sqrt{2}\theta \\ 1 & \theta_1 & -\sqrt{2} & -\sqrt{2}\theta_1 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad \text{Equation (2)}$$

Where $\mathcal{D}$ is related to D in the original equation by

- $\mathcal{D} = I_2 \otimes D$

Where $\otimes$ stands for the Kronecker product and I_2 is 2×2 identity matrix.

Definition: A STC is delay-tolerant if all non-trivial code-words retain their full rank even when their columns are shifted.

$$A_s = \begin{bmatrix} 0 & x_1 & -x_3 \\ x_2 & x_4 & 0 \end{bmatrix} \quad \text{Equation (3)}$$

Therefore, non-trivial code-words A_s should be full rank.

Equation 2 shows that the x1…x4 ε $\mathbb{Q}(\theta,\square)$, where the golden number $\theta = \frac{1+\sqrt{5}}{2}$ is a degree-2 algebraic number that is a solution to the minimal polynomial

$$f_1(x) = x^2 - x - 1$$

$and\ \theta_1 = \frac{1-\sqrt{5}}{2}$ is its conjugate.

In the same way, $\emptyset = \sqrt{2}$ is a degree-2 algebraic number that is a solution to the minimal polynomial

$$f_2(x) = x^2 - 2$$

and $\emptyset_1 = -\sqrt{2}$ *is its conjucate.*

Since $f_1(x)$ is irreducible over $\mathbb{Q}(\square)$ and $f_2(x)$ is irreducible over $\mathbb{Q}(\theta)$, then $\mathbb{Q}(\theta,\square)$ is a degree-4 algebraic extension of $\mathbb{Q}$ implying that the set $\{1, \theta, \square, \square\theta\}$ [and all their conjugates] forms a basis over $\mathbb{Q}$.

=> Xi = 0 if and only if a1= … =a4=0 for i= 1… 4.

Consider the sub-matrix formed from the first two rows and the two columns of matrix A_s (eq.3).

$$A_{s(2\times2)} = \begin{bmatrix} 0 & x_1 \\ x_2 & x_4 \end{bmatrix}$$

The determinant of this matrix is equal to -x1x2. It is equal to zero if x1 = 0 or x2 = 0. Based on what proceed, "Xi = 0 if and only if a1= … =a4=0 for i= 1… 4", this shows that all non-trivial matrices A_s have a rank two.

Therefore, the code will remain full diverse even when one of its relays is delayed by one symbol duration with respect to the other one.

Proposition 3: It has an optimal detector

<u>Optimal Detector:</u>

Our system has the following form:

$$Y = HX + N$$

Where X is the input matrix, H is the channel, N is the noise, and Y is the output. All these variable are 2×2 matricies.

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} + \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}$$

Input matrix of the proposed code is:

$$\mathcal{A} \triangleq DA = \frac{1}{\sqrt{3}} \begin{bmatrix} \frac{1}{\sqrt{1+\theta^2}} & 0 \\ 0 & \frac{1}{\sqrt{1+\bar{\theta}^2}} \end{bmatrix} \begin{bmatrix} a_1 + \theta a_2 + \sqrt{2} a_3 + \sqrt{2}\theta a_4 & -(a_1 + \theta a_2 - \sqrt{2} a_3 - \sqrt{2}\theta a_4) \\ a_1 + \theta_1 a_2 + \sqrt{2} a_3 + \sqrt{2}\theta_1 a_4 & a_1 + \theta_1 a_2 - \sqrt{2} a_3 - \sqrt{2}\theta_1 a_4 \end{bmatrix}$$

$$\theta = \frac{1+\sqrt{5}}{2} \; and \; \bar{\theta} = \frac{1-\sqrt{5}}{2}$$

$a_{1,}\ a_{2,} a_{3,} and\ a_{4,}\ are\ the\ symbols\ to\ be\ sent$

So, the input matrix will be send through the channels along with some noise, which is generated from the channel and equipments, to the Maximum Likelihood (ML) detector in order to decide on the symbols that are sent.

Here, we are assuming that our code know the channels. If we want to detect the output of these four symbols sent over the channel, we have to do the metric relation.
The ML will make a search over all possible values of the transmitted symbols. It will decide the group of symbols ($a_{1,}\ a_{2,} a_{3,} and\ a_{4,}$) that minimize the Euclidean distance. Thus, searching for the transmitted symbols will require a loop of order 4. The complexity is M^4, where M is the size of constellation.

We have some elements that help us in simplifying the detector. We will present here a minimized optimal detector with complexity of M^3, where M is the size of constellation.

$$X = \begin{bmatrix} a_1 + \theta a_2 + \sqrt{2}a_3 + \sqrt{2}\theta a_4 & -(a_1 + \theta a_2 - \sqrt{2}a_3 - \sqrt{2}\theta a_4) \\ a_1 + \theta_1 a_2 + \sqrt{2}a_3 + \sqrt{2}\theta_1 a_4 & a_1 + \theta_1 a_2 - \sqrt{2}a_3 - \sqrt{2}\theta_1 a_4 \end{bmatrix}$$

The received signals will be as follows:

On the first receiver antenna, these are the two signals received during the first and the second time interval.

$$r_1 = [h_{11}(a_1 + \theta a_2 + \sqrt{2}a_3 + \sqrt{2}\theta a_4) + h_{12}(a_1 + \theta_1 a_2 + \sqrt{2}a_3 + \sqrt{2}\theta_1 a_4) + n_1]$$
$$r_2 = [h_{11}(-a_1 - \theta a_2 + \sqrt{2}a_3 + \sqrt{2}\theta a_4) + h_{12}(a_1 + \theta_1 a_2 - \sqrt{2}a_3 - \sqrt{2}\theta_1 a_4) + n_2]$$

Similarly, for the second receiver, the first and the second time interval signals are:

$$r_3 = [h_{21}(a_1 + \theta a_2 + \sqrt{2}a_3 + \sqrt{2}\theta a_4) + h_{22}(a_1 + \theta_1 a_2 + \sqrt{2}a_3 + \sqrt{2}\theta_1 a_4) + n_3]$$
$$r_4 = [h_{21}(-a_1 - \theta a_2 + \sqrt{2}a_3 + \sqrt{2}\theta a_4) + h_{22}(a_1 + \theta_1 a_2 - \sqrt{2}a_3 - \sqrt{2}\theta_1 a_4) + n_4]$$

In these equation hij corresponds to the response of the channel between the received antenna i and the transmitted antenna j. nk is the additive noise to the signals, where k = 1, 2, 3, and 4.

Now, the ML will search all the possible values of symbols and decide in the favor of minimum Euclidean distance D.

$$\begin{aligned} D = {} & \left|r_1 - h_{11}(a_1 + \theta a_2 + \sqrt{2}a_3 + \sqrt{2}\theta a_4) - h_{12}(a_1 + \theta_1 a_2 + \sqrt{2}a_3 + \sqrt{2}\theta_1 a_4)\right|^2 \\ & + \left|r_2 - h_{11}(-a_1 - \theta a_2 + \sqrt{2}a_3 + \sqrt{2}\theta a_4) - h_{12}(a_1 + \theta_1 a_2 - \sqrt{2}a_3 - \sqrt{2}\theta_1 a_4)\right|^2 \\ & + \left|r_3 - h_{21}(a_1 + \theta a_2 + \sqrt{2}a_3 + \sqrt{2}\theta a_4) - h_{22}(a_1 + \theta_1 a_2 + \sqrt{2}a_3 + \sqrt{2}\theta_1 a_4)\right|^2 \\ & + \left|r_4 - h_{21}(-a_1 - \theta a_2 + \sqrt{2}a_3 + \sqrt{2}\theta a_4) - h_{22}(a_1 + \theta_1 a_2 - \sqrt{2}a_3 - \sqrt{2}\theta_1 a_4)\right|^2 \end{aligned}$$

This criterion will need M^4 metrics and $M^4 - 1$ comparisons. For this, we are going to compute the following intermediate signals.

$$Z_1 = r_1 - h_{11}(\theta a_2 + \sqrt{2}a_3 + \sqrt{2}\theta a_4) - h_{12}(\theta_1 a_2 + \sqrt{2}a_3 + \sqrt{2}\theta_1 a_4)$$
$$Z_2 = r_2 - h_{11}(-\theta a_2 + \sqrt{2}a_3 + \sqrt{2}\theta a_4) - h_{12}(\theta_1 a_2 - \sqrt{2}a_3 - \sqrt{2}\theta_1 a_4)$$
$$Z_3 = r_3 - h_{21}(\theta a_2 + \sqrt{2}a_3 + \sqrt{2}\theta a_4) - h_{22}(\theta_1 a_2 + \sqrt{2}a_3 + \sqrt{2}\theta_1 a_4)$$
$$Z_4 = r_4 - h_{21}(-\theta a_2 + \sqrt{2}a_3 + \sqrt{2}\theta a_4) - h_{22}(\theta_1 a_2 - \sqrt{2}a_3 - \sqrt{2}\theta_1 a_4)$$

For a given symbol (a_2, a_3, a_4).These signal can be written as:

$$Z_1 = h_{11}a_1 + h_{12}a_1 + n_1$$
$$Z_2 = -h_{11}a_1 + h_{12}a_1 + n_2$$
$$Z_3 = h_{21}a_1 + h_{22}a_1 + n_3$$
$$Z_4 = -h_{21}a_1 + h_{22}a_1 + n_4$$

Now, compute the following signals:

$$h_{11}Z_1 = |h_{11}|^2 a_1 + h_{11}h_{12}a_1 + h_{11}n_1$$
$$h_{12}Z_2 = -h_{11}h_{12}a_1 + |h_{12}|^2 a_1 + h_{12}n_2$$
$$h_{21}Z_3 = |h_{21}|^2 a_1 + h_{21}h_{22}a_1 + h_{21}n_3$$
$$h_{22}Z_4 = -h_{21}h_{22}a_1 - |h_{22}|^2 a_1 + h_{22}n_4$$

From these signals, we can compute

$$y_1 = h_{11}Z_1 + h_{12}Z_2 + h_{21}Z_3 + h_{22}Z_4$$
$$= (|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{221}|^2)a_1 + w_1$$

Where $w_1 = h_{11}n_1 + h_{12}n_2 + h_{21}n_3 + h_{22}n_4$

We can see that y_1 include only terms of a_2. It can be clearly seen that it can be used for estimatinga_1. So, by sending this signal to the ML detection, condition on (a_2, a_3, a_4), we can estimate a_1.

Now, instead of using D(a_1, a_2, a_3, a_4) , we can compute D(${a_1}^{ML}, a_2, a_3, a_4$). Where ${a_1}^{ML}$ is the detected symbol using the signal y_1.
Thus, we are going from M^4 metrics to M^3 metrics. It will be very significant in we use 64-PAM and 128-PAM.

Proposed code properties

1. Code rate:

   The code rate is r = k/T = 4/2 = 2. So, the code is full.

2. The diversity is also full.

3. The decoder complexity is equal to M^3.

4. Coding gain is equal to 1.19 which is equal to the coding gain of the first real code presented before.

5. It has a Non-vanishing coding gain.

6. It is a real code; it can be used with PAM constellation.

7. It is tolerant to delays that might occur.

Performance

The performance of the two-relay cooperative with 2-PAM is shown in the figure below:

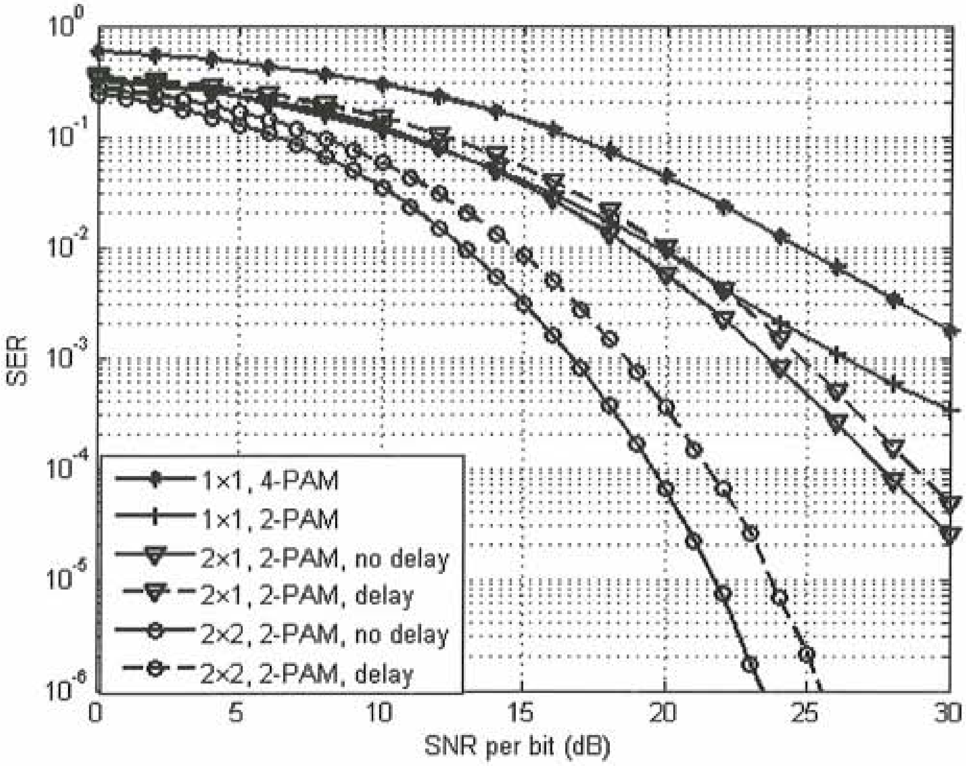

Figure 23: Code performance

The Symbol Error Rare (SER) under synchronous transmissions and under asynchronous transmissions are parallel to each other at high Signal to Noise Ratio (SNR)

=> This scheme is delay-tolerant.

Note that in the case of synchronous, the proposed code transmits at a rate of two PCU. While under asynchronous, it transmit at rate 4/3.Result shows the superiority of the proposed code with respect to 2-PAM (rate 1 bit PCU) non-cooperative and with respect to 4-PAM (rate 2 bit PCU) especially at high SNR.

Comparisons

Comparing the proposed code to well know code is given in the following figure:

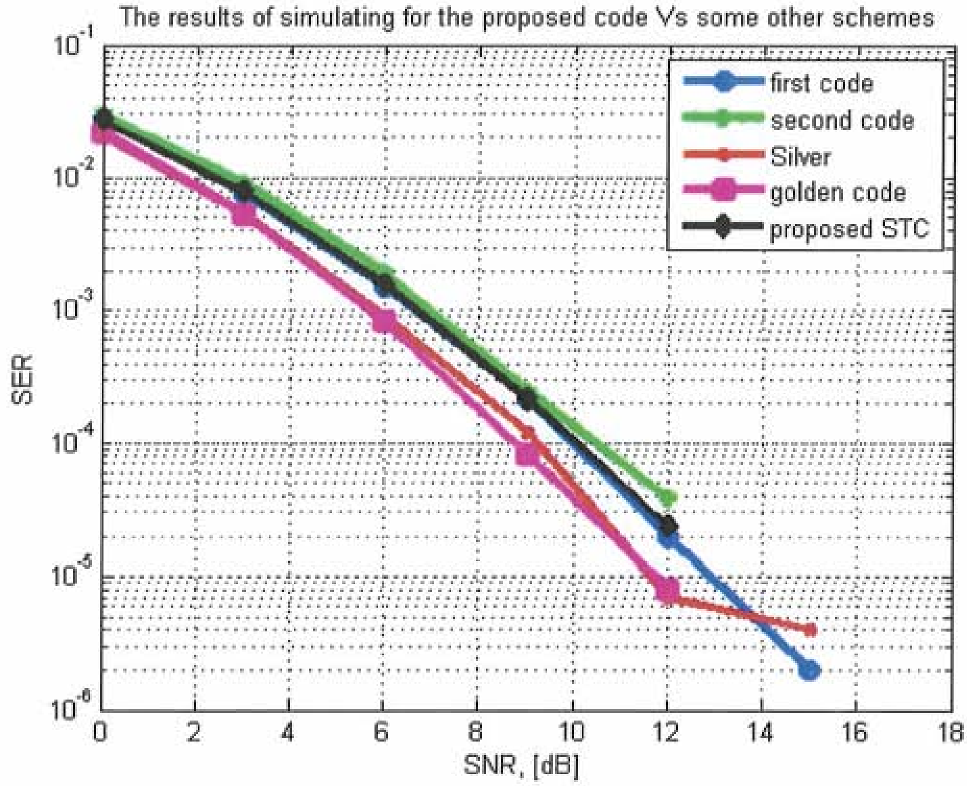

Figure 24: Comparisons 1

This figure describes the Symbol Error Rate (SER) Vs SNR using 2-PAM constellation. We have used the Matlab software on one thousand symbols for one thousand different channels in order to get high performance. In the simulations, the Symbol Error Rate (SER) was plotted with respect to the Signal to noise ratio (SNR).

We can see that the five codes go parallel to each other. Our code with have higher performance at high SNR (above 12 dB) and approximately equal at low SNR.

If we introduce delay to all the codes, the proposed code will show superiority over the other codes at all values of SNR. This is the power of our proposed code.

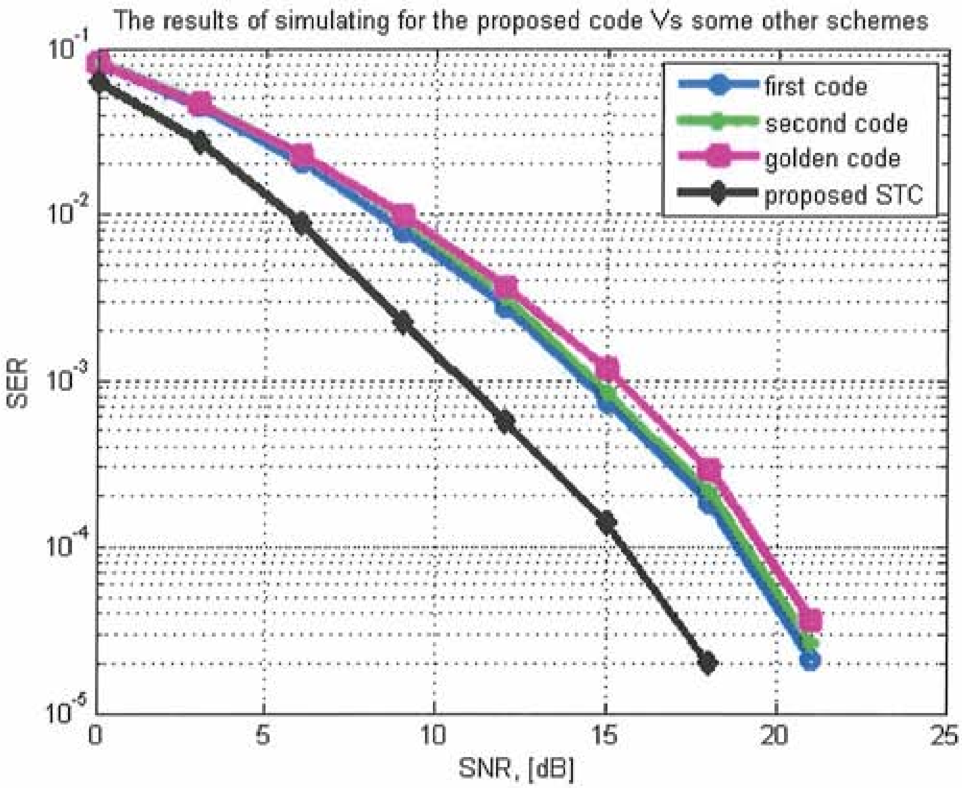

Figure 25: comparisons 2

Looking more in details about the different code, we can see in the table below that we cannot gain all the time. If we have gained in one or more properties, we will lose on the other side. The red color indicates that we have a high advantage compared to other codes. The blue color indicates that we have a medium advantage over the other code.

Code	Rate	Coding gain	Diversity	Non-vanishing	Decoder complexity	Delay tolerant	Real
Alamouti	Half	2	Full	√	M	X	X
Golden	Full	3.20	Full	√	M^4	X	X
Silver code	Full	2	Full	√	M^2	X	X
First code	Full	1.19	Full	√	M^4	X	√
Second code	Full	1	Full	√	M^2	X	√
Proposed Code	Full	1.19	Full	√	M^3	√	√

Table 1: comparing different codes by their properties.

We can see that all have full diversity and a non-vanishing coding gain. The most powerful code is the code that can compromise between all these properties. For our code, its coding gain is less than the golden and silver codes for example, but it has a main advantage; it is real code. Also, it does not have a linear decoder as Alamouti, but it is tolerant to delays. Moreover, it still has an optimal detector.

In the figure below, we can see how we gain and lose some advantages in different coding techniques.

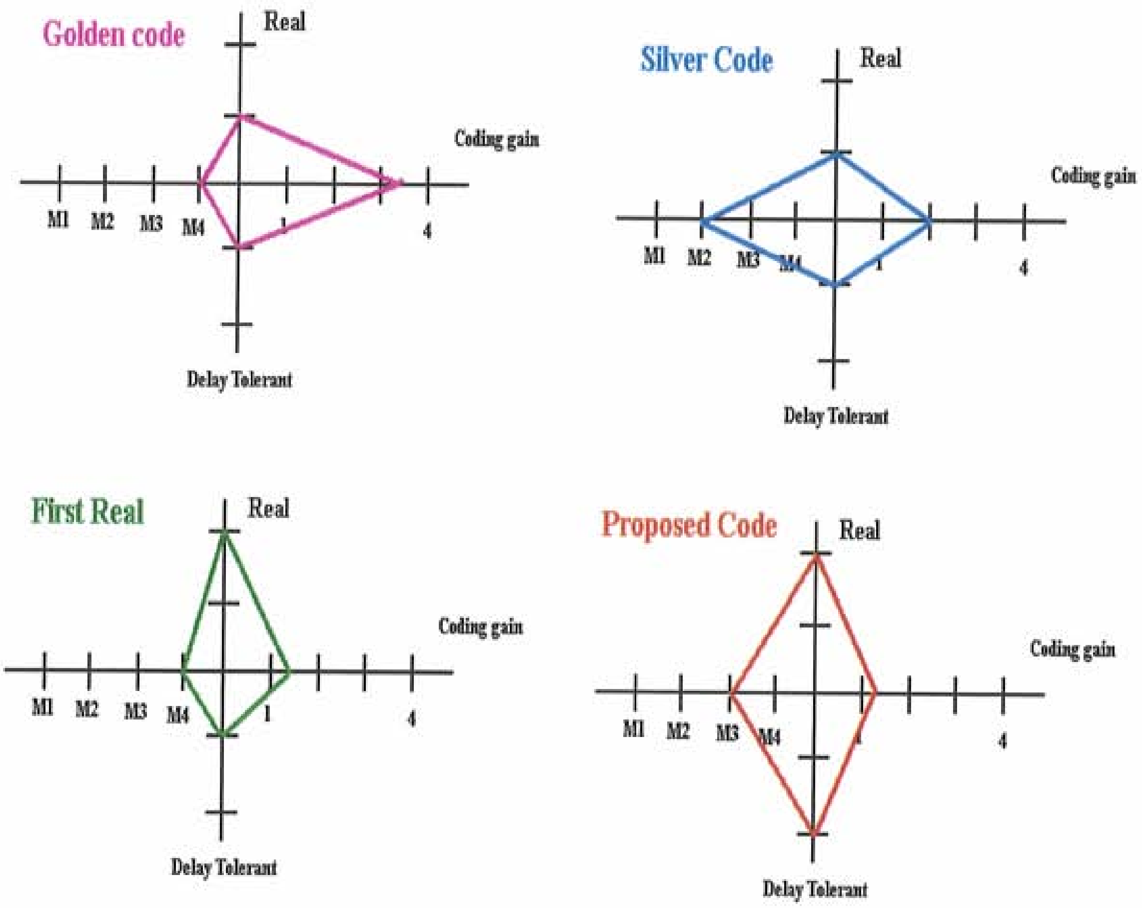

Figure 26: comparisons 3

For example, the coding gain of the golden code is the highest; this advantage will put more restrictions on the other properties. So, it cannot be real and tolerant for delay. The silver code is balanced between the medium coding gain and medium optimal detector. For our proposed code, we can see that is has the same coding gain of the first real code proposed and above that it is delay tolerant and have a minimal detector at the output. So, it has more advantages of the real codes proposed.

Conclusion

We presented novel transmission strategy for MIMO DF UWB systems using PAM. This code has the same constrained of STCs: full rate, full diversity, and non-vanishing coding gain. It addition to that it is totally real, have minimal decoding scheme, and tolerant to delays of one symbol duration. These characteristics have advantages in implementing such system such as: low cost (using real). It also can be used to solve the problem of delays in wireless communication

We have published this work in a paper called "Novel High-Rate Transmit Diversity Schemes for MIMO IR-UWB and Delay-Tolerant Decode-and-Forward IR-UWB Transmissions" and it was accepted by the International conference on Ultra Wideband (ICUWB).

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