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Implementing Dynamic Geometry Software-Based Constructivist Approach (DGS-CA) in Teaching Thales' Theorem

Impact on Students' Achievement, Problem Solving Skills and Motivation

By

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Love

To my beloved parents and fiancé Essam

**Implementing Dynamic Geometry Software-Based Constructivist Approach (DGS-CA) in Teaching Thales' Theorem**

**Impact on Students' Achievement, Problem Solving Skills and Motivation**

**Mirna Abi Saab**

## ABSTRACT

Teaching mathematics is no longer the act of teaching already-made mathematics, but rather starting from real-life problem situations, modeling, exploring, analyzing and hypothesizing. This study develops and pilots an instructional unit on Thales' Theorem that applies the guidelines of constructivist approach, counts on problem solving and reasoning skills (inductive and deductive), and integrates the use of Dynamic Geometry Software (DGS), namely Cabri Geometry, in teaching and learning. The study, comparing control and experimental groups, aims to investigate the effect of a DGS-based Constructivist Approach (referred to as DGS-CA) on students' motivation, geometric problem solving abilities, and learning achievement. The participants are grade 9 students learning under the Lebanese Mathematics Curriculum in two private Lebanese schools. The total number of participants is 32 students, 24 boys and 8 girls. The study used qualitative and quantitative methods to examine the effect of DGS-CA; an interview to investigate teaching methods of the control group's teacher, a questionnaire to test for motivation, and two tests to examine the effect on achievement and problem solving skills. Results showed that DGS-CA had a positive effect on students' motivation to learn mathematics and on students' mathematical achievement, however this effect was non-significant. On the other hand, DGS-CA had a significant effect on students' geometry problem solving skills, more specifically on their proving skills.

Key words: DGS, Cabri Geometry, Thales' Theorem, Achievement, Motivation, Problem Solving.

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# CHAPTER 1

# INTRODUCTION

Mathematics is believed to be one of the difficult subjects to learn as it requires both intuitive and analytic reasoning. Traditional mathematics teaching methods place a strong emphasis on lecturing and textual information. As a result, the number of students interested in mathematics is relatively limited. It is becoming increasingly important to motivate students and present materials in a way that maximizes learning, enjoyment and satisfaction. A visual, constructivist approach which adopts the representations of concepts as well as their practical significance, is believed to attract students to the field.

The Lebanese Mathematics Curriculum (LMC) considers mathematics to be a useful science that should offer students the necessary knowledge "to understand and explore the real world in all domains: physical, chemical, biological, social, psychological, computer…etc" (ECRD, 1997, p. 288). Thus, teaching mathematics is no longer the act of teaching already made mathematics, but rather starting from real-life situations, modeling, exploring, analyzing and hypothesizing. In its "General Objectives", the LMC states that: "students will be given the chance to observe, analyze, abstract, doubt, foresee, conjecture, generalize, synthesize, interpret and demonstrate" (ECRD, 1997, p. 289).

The "Principles and Standards for School Mathematics" of the National Council of Teachers of Mathematics (NCTM) state that students should learn mathematics with conceptual understanding. This is because the memorization of facts and procedures does not necessarily lead to students' awareness of how and when to use their knowledge. Besides, the NCTM (2000) urges math educators to provide means that enable students to use mathematics in everyday life and in the workplace. One of the ways towards reaching those means is teaching/learning through connecting to the real world to develop students' ability to use knowledge in other contexts and improve students' mathematical problem solving skills. Moreover, mathematics should make sense to students. Reasoning and proof should be a permeating part of classroom discussions. "Reasoning and proof enable students to abstract and codify their observation" (NCTM, 2000, p.344).

Technology software programs, on the other hand, can play an important role in giving educators the opportunity to create learning situations that are constructivist, visually attractive, and motivating. Computers provide a wide range of opportunities for teaching and learning mathematics. Software programs facilitate organization of thoughts, help in analyzing situations, and computing efficiently and accurately. Thus, computers support investigation by students, enhance decision making, reflection, reasoning, and problem solving (NCTM, 2000). Malabar and Poutney (2000, p. 3) state: "The constructivist use of technology allows the opportunity to change the nature of the material to be taught and learnt from routine-based to discovery-based activities".

### 1.1 Statement of the Problem

Experience and observation show that when Thales Property is taught the traditional way, students face difficulties acquiring it. First, they are unable to recognize the proportionality of segments and tend to believe that the segments cut by two parallel lines are equal. Moreover, students seem unable to manipulate proportional relationships in order to deduce other relationships between measures, which might be due to the lack of proportional reasoning or algebraic abilities.

This study is concerned with the use of a constructivist approach based on problem solving and reasoning (inductive and deductive) in teaching and learning Thales' Property in grade 9 with the integration of Dynamic Geometry Software (DGS), namely Cabri Geometry.

### 1.2 Purpose of the Study

The purpose of this study is to develop and pilot an instructional unit emphasizing problem solving and reasoning standards in teaching/learning Thales' Property. It also aims to investigate the effect of a DGS-based Constructivist Approach (hereafter referred to as DGS-CA) on students' motivation, geometric problem solving abilities, and learning achievement. A series of activities are developed and implemented over 14 teaching sessions that incorporate the use of DGS, namely Cabri Geometry, to teach "Thales Property". It is important to note that a constructivist approach to learning is applied in an explorative and interactive format.

## 1.3 Research Questions

This study addresses the following research questions:

1. How is Thales Property addressed in the Lebanese Mathematics Curriculum and in the national textbook?

Note: "Building Up Mathematics" is the Lebanese national textbook.

2. What is the effect of the DGS-CA on students' overall learning of Thales Property as reflected by their academic achievement (or the results of a test)?
3. What is the effect of the DGS-CA on students' geometric problem solving strategies?
4. What is the effect of the DGS-CA on students' motivation toward mathematics in general and particularly geometry?

## 1.4 Definition of Terms

Problem solving: "Problem solving means engaging in a task for which the solution is not known in advance" (NCTM, 2000).

The following criteria, adopted from NCTM (1989), will be adopted for assessing the evolution of students' geometric problem solving abilities:

- Draw a figure

- Select and apply learned properties in the right context to deduce other properties or relationships
- Gather, organize and select appropriate data to solve a geometric problem
- Make and formulate conjectures by inducing previously observed relationships
- Construct a whole and consistent proof

Motivation: “The spontaneous drive, force, or incentive, which partly determines the direction and strength of the response of an individual to a given situation; it arises out of the internal state of the individual” (Harold, 2003).

**1.5 Significance of the Study**

The National Council of Teachers of Mathematics (NCTM, 2000) has suggested that mathematical proficiency required for the 21$^{st}$ century develops through conceptual understanding and through the appropriate use of technology. Moreover, in today's evolving world, leaders expect students to enter the workplace with a wide range of communication, math, and technology skills. For this reason teachers cannot disregard current technologies. They are urged to properly implement those technologies for the purpose of better understanding and better preparating for the workplace.

Moreover, we are witnessing a trend towards constructivism in the teaching approaches and methods. Students should be given the chance to construct, explore, manipulate and interact with knowledge to build their understanding. Dynamic software serves the intention of teaching through constructivism in a technological environment.

Therefore, the results of this study may be useful for many teachers and educators who are willing to make their instruction more constructivist by incorporating technology into their math classroom.

Besides, Thales Property is considered to be a tough topic for several students. It depends heavily on proportions and proportional thinking and students seem anxious to flexibly deal with proportions. The study, through piloting the developed unit, fills the gap of students' weaknesses in dealing with proportionality.

Since this study uses problem solving, reasoning and technology in the implementation of an instructional unit, and since, to my knowledge, there is no study in Lebanon that investigates the teaching of Thales Property, the results of this study will draw teachers' attention to:

- The necessity of accounting for the role of problem solving and reasoning in learning/ teaching mathematics, not only as a context for applying mathematical concepts but also as a context for learning mathematical concepts.
- The necessity to use the national mathematics textbook with a critical eye and supplement it with materials to work towards achieving the general curriculum objectives
- The necessity of integrating real life situations and activities that help students reason and solve problems.
- The benefits of using DGS in teaching/learning geometry.

Moreover, this study will contribute to the body of Lebanese literature related to teaching/learning mathematics through using technology at the high school level.

### 1.6 Hypothesis on Achievement

Using DGS-CA affects students' achievement in mathematics.

### 1.7 Hypothesis on Problem Solving Abilities

Using DGS-CA affects students' geometrical problem solving abilities.

### 1.8 Hypothesis on Motivation

Using DGS-CA affects students' motivation toward mathematics.

### 1.9 Null hypothesis

DGS-CA has no effect on students' achievement, problem solving abilities, or motivation toward mathematics.

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 Thales Theorem

Thales' theorem is named after the Greek philosopher *Thales of Miletus.* In fact, according to Wikipedia, textbooks call two theorems *Thales' Theorem*. The first theorem states that: if A, B and C are three points on a circle and the segment BC is a diameter of that circle, then triangle ABC is right at A.

The second theorem, also called the *intercept* theorem, relates the ratios of various line segments that are created when intercepted by parallel lines. *Intercept* theorem in a triangle is equivalent to the theorem about ratios in similar triangles when these have a common vertex and parallel base. In this research we are concerned with studying the teaching / learning of the second theorem that is also named the *intercept* theorem.

Thales theorem is one of the key chapters in studying plane geometry in grade 9 of the third cycle of the LMC. Studying Thales' Theorem is vital as it reviews and emphasizes proportionality concepts that are studied in previous intermediate grades and serves as an accommodating introduction to the chapter on similar triangles. The applicability of Thales Theorem also appears later in grade 10 of the LMC in studying vectors. Besides the practicality of Thales theorem such as for measuring the heights of high objects (pyramids, trees…etc), measuring ship-to-shore distance...etc, a deep understanding of this concept is considered important for the success of students in geometry in grade 9 and later years. However, experience and practice indicate that understanding Thales Theorem seems to be difficult for students. Changes in instructional approaches and integrating technology is investigated in the following reserach to help in teaching/learning this theorem. The following review of the lierature presents the theoretical foundation on which this study is based. It begins with defining constructivist approach (active learning and scaffolding) and highlights the role of technology (specifically DGS) and how it connects to constructivism in teaching/learning of mathematics. Next, it presents a review of research on problem-solving and reasoning in learning/ teaching math and their connection to technology.

## 2.2 Theoretical Framework (Constructivist Approach)

The present research adopts the constructivist approach as a framework. The design of the instructional unit implemented in this study follows constructivist guidelines and the National Council of Teachers of Mathematics (NCTM, 2000) recommendations. Furthermore, the instructional unit is developed in a way that realizes the general objectives of the LMC and integrates technology into instruction.

Traditionally, teaching mathematics was, in most classes, the act of transmitting knowledge, procedures and formulas to passive observers. Learning was a process of a repetitive activity in which students used to imitate newly provided procedures by the teacher in the tests (Jong Suk, 2005). However, by the beginning of the twentieth century a group of scholars such as Piaget, Dewey, Bruner, Vygotski and many others developed views about more active approaches to learning –commonly known as constructivist approaches. Constructivist learning approaches assume “that people are purposive and knowledge-seeking individuals” (Gage & Berliner, 1998, p. 255). Thus, knowledge is constructed through engaging in active interactions with the physical environment; i.e. learning by doing. Students must learn with activities, with hands-on learning situations and with opportunities to experiment and manipulate the objects of the world (Hein, 1991). The learning process, in a constructivist environment, involves “students’ communication of questions, intuitions, conjectures, reasons, explanations and ideas” (Sheppard, 2008, p. 51). Besides, learning should engage students in critical thinking and inquiry and in thoughtful and open-ended questions that enable them to transform new information and use it in new contexts. Thus, instruction in a constructivist environment aims to activate students’ thoughts enabling them to elaborate and test new ideas and information.

Constructivist teachers are not knowledge dispensers of the class. Teachers only guide students through asking leading questions, accept or refute answers in a nonjudgmental way, and know when to intervene. Jong Suk adds, “…teachers may invite transformations but may neither mandate nor prevent them” (Jong, 2005, p.10). Yager (1991) defined the characteristics of constructivist teachers as those who engage students in learning tasks, promote student ideas and questions, allow and recognize students' own ideas, call for students’ presentation of their ideas, encourage students to challenge the ideas of others, and modify their instructional strategies to meet students’ needs.

On the other hand, Vygotsky, a well-known *social constructivist* theorist, emphasizes the social organization of instruction known as *Social Constructivism Theory*. From a social constructivist viewpoint, learning is not an individual activity but an accomplishment that takes place through class discussions and interactions with peers and/or with the teacher. This cooperation and interaction serve as *scaffold*. Instructional scaffolding is providing means that support the learning process by which peers or adults mediate the leaner's attempt to acquire new information (Wood, Bruner, & Ross, 1976). These supports are temporary and need to be gradually removed as students develop. Cazden defines a scaffold as "a temporary framework for construction in progress" (Cazden, 1983, p. 6). To better understand the role of scaffolding, Vygotsky identifies two areas of student learning: the Zone of Actual Development (ZAD) and the Zone of Proximal Development (ZPD). The ZAD is the set of tasks a learner is capable of doing independently i.e. without support. The ZPD is defined as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under support, or in collaboration with more capable peers" (Morris, 2008, p. 1). In other words, it is that region where the student needs a "support" in order to understand and apply learned materials. Thus, children interacting with each other and discussing with the teacher are able to solve a certain range of problems that they alone cannot solve. Furthermore, students in communication situations may develop awareness of their own misconceptions or the misconceptions of others, they learn to be` clear and convincing, challenge others' ideas and thoughts, and sharpen their thinking (NCTM, 2000).

## 2.3 Technology and DGS in Mathematics Teaching / Learning

Teaching / learning of mathematics has for long centuries been dominated by the use of pen and paper. However, traditional approaches to mathematics learning and teaching are being challenged by technology advancements. The infinite opportunities that computers provide have brought new tools and approaches to teaching and learning mathematics. Pierce and Ball state: “an extensive range of sophisticated technology is now generally available to teachers” (Pierce & Ball, 2009, p. 299).

What kind of technology integration is wanted? Are teachers expected to use technology only to keep record of their grades or to present a lesson on a Microsoft PowerPoint?

Muir (2007) explains that technology can be used in two ways: to replicate actions that were completed before the use of technology (like Microsoft Word to prepare worksheets, Microsoft Excel to keep record of grades, PowerPoint presentations…etc.) or to permit the performance of actions for the actual process of teaching/learning which were not possible, or were too time consuming, before technology use. The first way leads only to a more organized, easy-to-access and easy-to-modify instructional materials. But, the second level leads to “innovation in teaching and learning” (Muir, 2007, p. 1), and a shift to constructivist teaching methods and more cooperative learning. Thus, teachers are urged to include technology in their instructional approaches to enhance pupils' learning and not only to present lessons passively or to deliver drill-and-practice questions. The use of technology should facilitate and encourage the constructive learning process. Oldknow (2004) came up with six ways in which technology could assist students' learning of mathematics which are: learning from feedback, observing patterns, seeing connections, working with dynamic images, exploring data, and teaching about computers. Therefore, the activities designed to be performed on the computer should involve students in reasoning processes of constructing, realizing patterns, and making reliable conjectures.

In geometry, Dynamic Geometry Software (DGS) provides a range of tools for constructing geometric objects such as points, segments, lines, circles…etc. The tools available in the software also include classical constructions (such as midpoint, perpendicular, parallel, perpendicular bisector…etc.) as well as transformations (such as reflection, rotation, translation… etc.). Besides, it is important to mention the 'dynamic' aspect of the program, in which the user is able to drag defined objects, such as points, around on the screen and watch for invariants while the figure moves and changes accordingly. DGS has become one of the most commonly used software programs in schools and colleges all over the world (Jones, 2005). This is because the features of DGS are believed to add more meaning and sense to geometrical activities. Cabri is one of the first developed DGS tools. Using Cabri, students construct geometric figures, explore their figures and then induce mathematical properties. A geometrical figure or an equation on the Cabri screen becomes an object to manipulate and interact with rather than a static drawing or expression. The practicality of DGS programs has attracted several researchers to look in depth at how students interrelate with the software. For example, Arzarello and colleagues observed two processes that students demonstrate when using the drag mode. They call them: *ascending* and *descending* processes. Ascending processes, observed when students freely discover a situation, arise when students are looking for regularities, invariants… etc. These are shifts from drawings to theory. Descending processes are shifts from theory to drawings, and involve students making conjectures, checking specific geometric properties…etc. (Arzarello, Olivero, Paola, & Robutti, 2002). Jones maintains that “these movements in the use of dragging are believed to reveal cognitive shifts from the perceptual level to the theoretical level” (Jones, 2005, p.28). Thus, with dragging, students are able to see the invariants that usually hold a theoretical reason. Students, after dragging experiences, are able to discover properties and conclude conjectures which is believed to enhance their understanding and conceptualization (Holzl, 1996).

In addition, research highlights that the act of constructing diagrams on the use of DGS plays an important role in the learning process. Jones explains how students construct objects and realize their relationships when using DGS. Tasks completed using DGS may require different plans than those completed with paper and pencil (Jones, 2001). For example constructing a parallelogram on DGS requires a strategy that contains sophisticated and dependent set of steps (like point on object, parallel to this line passing through this point,…etc), while constructing it on paper seems much easier. Thus, constructing on DGS needs more “didactic efforts” to provoke learners to focus on the significant mathematical relationships (Jones, 2001). Laborde draws attention to an important difference between a *drawing* and a *figure:* "drawing refers to the material entity while figure refers to a theoretical object" (Laborde, 1993, p.49). Constructing on DGS will yield to a *figure* and not a *drawing* because the steps required by the learner to accomplish the mission need to be based on a theoretical background. Furthermore, the dragging feature of the software validates the correctness of the figure (Healy & Hoyles, 2001). The criteria of validation do not depend on the appearance of the result (like that on the paper) of the construction because this appearance can be changed using the dragging feature. However, what validates the result is the figure holding its properties and remaining consistent with the geometrical theory even if dragged. Furthermore, the sequenced and organized set of steps that learners should pass through to construct an “unmessable” figure helps them build a foundation for their deductive reasoning (Jones, 2001). The communication with those tools grants students a deeper conceptual knowledge of geometrical properties (Hollebrands, Laborde & Straβer 2008).

### 2.4 How Technology Connects to Constructivism and vice/versa.

The NCTM mentions that technology is an essential and highly recommended tool in the teaching of mathematics (NTCM, 2000). However, literature suggests that the effect of integrating technology depends heavily on the way it is used (Sheehan & Nillas, 2010). The integration of technology into instruction is supposed to accompany a constructivist approach as technology is believed to give learners the chance to explore, manipulate, observe, and appreciate data or figures. Almeqdadi (2005) and Funkhouser (2002) found that integrating technology with a constructivist teaching pedagogy leads to more constructivist, student-centered teaching approach. Galbraith, in another study on the effects of technology on learning mathematics, found that it enables students to collaboratively communicate, conjecture, rationalize, and generalize findings (Galbraith, 2006). On the other hand, a number of studies showed that teaching mathematics with the help of technology enables less-capable students to get involved in exploration processes (Ruthven & Hennessey, 2002). The dynamic nature of the software aids the process of *doing mathematics,* as the constructivist theorists express. The features such as measuring, moving, dragging…etc. can enhance the exploration process. Therefore, DGS, such as Cabri, can play an important role in giving educators the opportunity to create learning situations that are explorative, visually attractive, and motivating. Furthermore, Malabar and Pountney (2002) add that DGS features promote 'what if' situations for students to investigate. This is because the software serves as instant feedback tool. It allows learners to inspect the result of any question that comes to their minds while solving a certain problem.

Furthermore, "Technology software programs provide visualization of seemingly abstract mathematical ideas" (Sheehan & Nillas, 2010, p. 1). Technology software programs aid the formation of visual images which fosters students' success in mathematics. Habre (2001) supposes that "students who naturally use images in their thinking can easily make sense of mathematical tasks, while students who are not good visualisers often do not" .Thus, DGS, by providing those images, helps students better abstract their concepts. Malabar and Pountney contend that "knowledge is built up from personal experiences, and making these experiences more dynamic will assist in the development of cognitive structures" (Malabar & Pountney, 2002, p.2). A study by Vincent reports that students working with DGS formed valid proofs because the activities they went through on the software offered a “verification of the truth of their conjectures and an explanation as to why the linkage works the way it does” (Vincent, 2005, p. 108). Vincent concludes that the software helped students to achieve, understand and appreciate geometric proofs.

Social interaction and discussions are also suggested to adjoin to the use of technology. Malabar and Poutney (2002) state that graphic representations together with social interactions, expand learners’ knowledge and familiarize them with abstract concepts. Besides, Scher (2005) notes that the use of technology software together with a meaningful class discussion may lead to a deep conceptual understanding.

### 2.5 Problem Solving and Reasoning

Extensive research and curriculum guiding principles emphasize the use of problem solving and reasoning in teaching and learning mathematics (NCTM, 1989, 2000; LMC, 1997). NCTM claims that “problem solving must be the focus of school mathematics" (NCTM, 1989, p.3). However, the calls for integrating problem solving do not only indicate teaching for problem solving, but teaching through problem solving; That is teaching strategies that guide students to seek solutions, formulate conjectures , explore patterns not just memorize procedures and formulas to solve exercises (Schoenfeld, 1992). Therefore, problem solving coaches the reasoning skills of students. Students learning under a problem solving approach are continually encouraged to come up with generalizations about rules and concepts. They are constantly thinking, analyzing, realizing patterns, structure, or regularities. Moreover, students who reason do not accept information unless it satisfies their logic. Reasonable students become critical thinkers and tend to question whether patterns that they see are accidental or occur for a reason. Hence they make observations and furthermore try to prove their conjectures (NCTM, 2000).

Learning mathematics by means of problem solving and reasoning becomes *empowering* as Schoenfeld (1992) states. Problem solvers are empowered by important reasoning skills that help them understand and interpret information in their daily life, enhance their logic, and accordingly taking reasonable decisions. Experiencing reasoning skills will aid learners to view mathematics as a subject that makes sense. Taplin maintains that problem solving is a vehicle for students to construct, evaluate, refine their own mathematical theories, and utilize them in real life (Taplin, 1988).

Teachers, in a problem solving approach, introduce and present concepts through real life problems. The skills needed to solve problems are in such a situation developed by both students and teachers. The problem solving approach, as Taplin (1988) describes, promotes interactions between students, peers and teachers in their attempts to explain, interpret, and formulate a solution process. Research in cognitive psychology showed that learning occurs when the learner is engaged in some sort of cognitive restructuring or elaboration (Kramarski & Mevarech, 2003). Thus, activities must be structured to maximize the chances for each student to be engaged in questioning, explanation, elaboration, and other forms of communication and reasoning through which students express their ideas and can give and receive feedback. Relating concepts to problem solving contexts is believed to increase motivation and to give problem solving special value for students. Taplin adds that approaching mathematics through problem solving can create a context which "simulates real life and therefore justifies the mathematics rather than treating it as an end in itself" (Taplin, 1988, p. 3).

## 2.6 How Technology Connects to Problem Solving and Reasoning

Math educators have been calling for problem solving and reasoning as a major target in teaching mathematics. However, many teachers find it hard to provide students with experiences that guarantee the foundation of the reasoning skills of pupils. Considerable research has reported that even after considerable effort in teaching for problem solving and reasoning, many students are unable to differentiate the different types of mathematical reasoning such as observation, explanation, forming arguments, and justification…etc. (Hanna & Jahnke, 1996; Dreyfus, 1999, as cited in Jones, 2001). Furthermore, teachers often tend to concentrate on verification procedures and fail to recognize or omit exploration processes. This might have been the case because no tools were available to enhance exploration processes that link applications to theoretical knowledge.

In geometry, DGS can provide students with direct experiences in geometrical theories. Learners with DGS connect geometrical construction and exploration to deductive reasoning of geometrical theories (Jones, 2000). Many studies found that DGS environment helped students to improve their justification processes from experimental to deductive justifications (Healy & Hoyles, 2001; Jones, 2000; Mariotti, 2000, 2001; Marrades &

Gutierrez, 2000 as cited in Jurdak & Nakhal, 2008). A study done by Jones (2000) to examine how students reason under DGS suggests that Cabri inspired students' thinking and motivated them to develop justifications. Jones suggests in his research that the "students' explanations can evolve from imprecise, everyday expressions, through reasoning that is overtly mediated by the software environment, to mathematical explanations of the geometric situation that transcend the particular tool being used" (Jones, 2008, p. 30). This final stage grants a basis for deductive reasoning. Thus the software serves as a mediator to make students gain reasoning skills and practice them infinitely.

Using Cabri, learners construct figures, drag their figures, realize patterns, and look for justifications of their realizations to make conjectures. Thus, Cabri serves as a way where students can experiment and deal with abstract theories more flexibly. Furthermore, DGS allows the progress from procedures and experimentation to observation and the formation of concepts. Pea (1987) believes that DGS are not only amplifiers of students' capabilities but rather cognitive reorganizers. A study done by Jurdak and Nakhal (2008) investigating the impact of Cabri on reasoning, showed a positive effect on learners' level of reasoning. It was claimed that "Cabri helped learners connect empirical and theoretical aspects of geometry" (Jurdak & Nakhal, 2008, p.68).

### 2.7 Conclusion

In conclusion, teachers and students should view mathematics in general and geometry in specific as a problem solving activity. Teachers should create classroom activities that actively engage students in problem solving and reasoning. Teachers are urged to integrate technology as it enhances a constructivist approach and serves as a foundation for geometric reasoning. Furthermore, a free classroom culture where teachers continuously ask questions, engage and challenge students, and encourage them to reason is recommended.

# CHAPTER 3

# METHOD

The study is an action research whereby an instructional unit on Thales' Theorem is developed and piloted by the researcher. The unit integrates the use of DGS and applies a constructivist approach to learning. A pretest-posttest experimental design is used to investigate the effect of DGS-CA on students' learning and motivation. A causal comparative method is used to compare the differences between students' learning when DGS-CA unit is implemented (experimental class) and students' learning in the usual computer-free lecturing approach (control class).

## 3.1 Participants

The participants are the grade 9 students in two private Lebanese schools-hereafter referred to as school 1 and school 2-located in Mount-Lebanon. The schools are considered to be two of the top schools in the near region, based on the students' results in the official exams. The schools provide education from nursery to grade 12. Students are mostly from the town where the schools are located and belong to average socio-economic class. The implemented curriculum and books are based on the National Lebanese Curriculum.

The study is conducted with two groups of participants: an experimental grade 9 class (from school 1) and a control grade 9 class (from school 2). The researcher conducting this study is the teacher of the experimental class. The DGS-CA unit is implemented in the experimental class that consists of 19 students. However, only 16 of the students (12 boys and 4 girls), whose distribution of results forms a bell curve, are selected for analysis to form the experimental group. The same number (16) and gender composition (12 boys and 4 girls) is chosen from the control class to form the control group. The selection of the control group participants is based on two criteria: the results of the diagnostic test conducted prior to any instruction (or pre-test) and the average grade of the students in mathematics in the previous year that is in grade 8. Thus the total number of student participants is 32. Their age ranges between 13 and 15 years.

### 3.2 Procedures

Procedures that are followed to accomplish this research:

1. An interview is conducted with the teacher of the control group to take a close look at her teaching style.
2. A questionnaire to test for students' motivation is prepared and administered to the experimental group before and after the implementation of DGS-CA. the questionnaire is also administered for the control group after their completion of the Thales' Theorem unit.
3. Before any instruction, a diagnostic test is administered to both groups to determine the base-line level of students' mathematics achievement, critical thinking, and geometry problem solving skills.

4. The general objectives of the unit on “Thales’ Theorem” in the national textbook are reviewed and compared with those of the Lebanese math curriculum (ECRD, 1997), to study the consistency between curriculum and textbook.

5. An instructional unit is developed on Thales’ Theorem. The progress and the implementation of the instructional unit attribute a special emphasis to: students’ construction of their own knowledge, problem solving as a context for learning, reasoning and problem solving, connections to real life situations, technology software as a tool for learning.

6. Implementation of the unit in the experimental class. The experimental and control class were instructed by their teacher, each for a period of 15 sessions lasting for 50 minutes each. The researcher taught the DGS-CA unit in her own class, whereas the control group was instructed by their own teacher. The two teachers agreed on some details such as: objectives, timeline, quizzes and tests. However, no discussion about instruction methods was shared by the two teachers.

7. After the implementation, data is collected using a unit test (post-test) to check for improvement and development in the process of learning. The unit test was administered to both classes for final evaluation in session 15 (final session).

**3.2.1 Interview.** (Refer to Appendix A)

The purpose of the interview is to investigate the teaching style of the control group’s teacher. The interview consists of 7 questions. The questions asked are intended to uncover the way the teacher introduces a lesson, her use of learning aids, kinds of questions asked, use of technology…etc. The interview is scheduled for 30 minutes. It is tape recorded, transcribed and then analyzed.

**3.2.2 Questionnaire.** (Refer to Appendix B)

The intention of the questionnaire is to answer the research question of whether DGS-CA affects students' motivation. The questionnaire is developed by the researcher and then piloted through running it by 15 students in the grade 8 class of school 1 to check for any mistakes or ambiguous items. After minor changes introduced as a result of the piloting, the questionnaire is administered before and after the implementation of the experimental unit to grade 9 students (the experimental class) to compare the differences. It is also administered to the control group after their completion of Thales' Theorem unit. The questionnaire is a Likert scale type one; in which students selected, for every item of the questionnaire, one of the following choices: strongly agree, agree, disagree, or strongly disagree. Some examples of the statements are:  math is interesting, geometry is enjoyable, geometry problems are meaningless…etc. To highlight the role of Cabri Geometry software, some items asked students if they wish to rotate a geometrical figure, to measure the parts of a figure…etc. The questionnaire is made of 25 questions and students took 15 minutes to complete it.

**3.2.3 Diagnostic test.** (Refer to Appendix C)

Before teaching the unit on “Thales’ Theorem”, both teachers administered the diagnostic test in both classes (the experimental and the control class). In the diagnostic test, students have to solve geometric problems that cover the following: geometric properties of figures (parallelograms and rhombuses), midpoint theorem, proportional relations and ratios of segments. The purpose of this diagnostic test is to investigate students’ geometrical problem solving skills and their ability to recognize and use proportionality between segments. Students sat for the diagnostic test, which lasted for 50 minutes, without any previous preparation. Teachers did not answer any questions that might hint students to reach any solution.

**3.2.4 Analysis of Thales’ Theorem unit in ECRD textbook.** (For a copy of textbook unit, Refer to Appendix D)

The unit about Thales’ Theorem in grade 9 textbook *Building up Mathematics* of the Lebanese mathematics curriculum (Chalak et al., 2000) is analyzed to check for the consistency with the general objectives of the curriculum (ECRD, 1997). The purpose, approach and content of the unit are examined. Regarding the purpose of learning mathematics, the activities through which the unit introduces the concept are analyzed to check whether they allow students to construct their knowledge and to undertake reasoning procedures such as observing, testing, hypothesizing…etc. As for the approach, the researcher examined whether the unit’s arrangement follows a problem solving approach to learning; that is starting from real life situations and highlighting the usefulness of mathematics in everyday life.  Besides, the researcher inspected if the unit permits students to practice the “scientific approach and critical thinking procedures” (ECRD, 1997, p. 282) that the LMC recommends. Further analysis was conducted to check whether the unit holds an opportunity to use technology as a tool for learning. In the content analysis, an investigation was conducted of whether the unit covers all the objectives on *Thales' Theorem* stated in LMC and whether the content of the unit is sufficient to solve the exercises and problems contained in it. A detailed analysis of the unit in national textbook is presented in the chapter on *Data Analysis*.

**3.2.5 Development of the DGS-CA instructional unit.** (For the unit plan, Refer to Appendix E)

After the analysis of the unit in national textbook, the researcher developed an instructional unit to teach Thales' Theorem. The unit covers the objectives of the grade 9 mathematics curriculum related to Thales' Theorem. The instructional unit emphasizes: a) students' active involvement in the process of constructing their knowledge, b) the improvement of problem solving and reasoning skills of students, c) the use of problem solving as a context for learning as well as for applying the concepts d) the use of technology namely Dynamic Geometry software that is Cabri Geometry.

***3.2.5.1 Time-Line.*** The duration of the DGS-CA unit is 15 teaching sessions each lasting for 50 minutes. In the LMC, the allocated time for the section *Plane Figures* is 20 sessions (ECRD, 1997, p. 295). Those 20 sessions are distributed to cover 2 chapters: 1) Thales' Theorem, and 2) Similar Triangles. Since the DGS-CA unit works on improving the proportional reasoning of the students and it insures the acquisition of Thales' theorem, students will not require more than 5 sessions to cover the chapter on “Similar Triangles”. Thus, the time of the instructional unit (15 sessions) is the same as that allocated for it by the LMC. Note also that the same time is allocated for the control group.

***3.2.5.2 Material.*** While the national textbook is the main source for the control class, DGS-CA unit materials were developed by the researcher to be used in the experimental class. The researcher prepared a problem to introduce the theorem by connecting it to history and real life needs (see p.108). Also Cabri activity sheets, quizzes, and a unit test were developed. Only, exercises and problems used for classwork and homework were taken from the LMC textbook.

***3.2.5.3 Context.*** Some of the 15 sessions are planned to take place in a computer lab in which each student has access to a computer with Cabri Geometry software installed. The other sessions are conducted in a regular classroom with a chalk board.

***3.2.5.4 Content of the unit and its objectives*****.** The general Objectives of the DGS-CA unit are:

At the end of this lesson, students should be able to:

1. Apply the Thales’ Theorem in a triangle
2. Apply the converse of Thales’ Theorem in a triangle
3. Construct the fourth proportional geometrically
4. Enlarge or reduce a figure knowing the scale factor

5. Apply Thales' Theorem to geometric cases involving parallel lines cut by 2 intersecting lines

6. Recognize the significance of Thales' theorem in solving real-life problems and apply it

Note that the first four objectives are the same as those set by the LMC for the unit on Thales' Theorem. However, objectives 5 and 6 are not mentioned in the LMC objectives and are added by the researcher.

***3.2.5.5 The distribution of DGS-CA unit sessions.*** The distribution of teaching/learning tasks over the 15 sessions is according to Table 1 which provides the main duties each session will cover. For more detailed information refer to Appendix E.

***3.2.5.6 Approach and design of the DGS-CA unit.*** It is believed that students who actively engage in activities during the learning process are more likely to recall information (Bruner, 1961). Mayer (2004) reiterates that learners should be behaviorally and cognitively active during the learning process. Learners should be invited to search for meanings, observe, find relations, and make conjectures. Besides, Vygotsky believes that "learners should constantly be challenged with tasks that refer to skills and knowledge just beyond their current level of mastery" but close enough to be able to accomplish the task (Vygotsky, 1978). Thus, engaging students in situations that require them not only to apply what is learned but also to gather, organize, and search for new information is supportive in their learning process. Vygotsky adds that "knowledge is constructed in a social context" (Vygotsky, 1978). Accordingly, sharing and discussing individuals' thoughts and beliefs with a group results in understanding what they couldn't achieve on their own. Thus, the level of *potential development* as determined through problem-solving under teacher guidance or in group of more competent peers is helpful. The DGS-CA instructional unit is planned based on a constructivist learning approach according to Vygotsky's views, while integrating Cabri Geometry software.

*3.2.5.6.1 Four-phase approach*. The teaching/learning to achieve every objective in the DGS-CA unit is accomplished using a four-phase approach: 1) *Discovery* phase, 2) *Making a Conjecture* phase, 3) *Teacher-Class Discussions* phase, and 4) *Application* phase.

*Table 1*

*DGS-CA sessions in glance*

| **Session number** | **Topics and skills covered** |
|---|---|
| Session 1 | Thales' in a triangle and Activity Sheet 1 |
| Sessions 2 and 3 | Thales' in an extended triangle and Converse of Thales and Activity Sheet 2 |
| Session 4 | Drop Quiz |
| Session 5 and 6 | Correction of drop quiz + illustration of some consequences proportions to Thales' |
| Session 7 | Correction of H.W. and explain some tips for h. w. on Cabri and Activity Sheet 3 |
| Session 8 | Thales' in any problem situation including parallel lines |
| Sessions 9 and 10 | Correction of H.W. and explain reduction and enlargement |
| Session 11 | Quiz2 and Thales in real life applications |
| Session 12 and 13 | Correction of H.W. and correction of Quiz2 |
| Session 14 | Correction of Extra Exercises H.W. |

| Session 15 | Unit Test |
|---|---|

1) *Discovery phase.* Since the DGS-CA unit follows a constructivist approach to learning with integration of technology, properties are introduced to students through Cabri-based activity sheets (Refer to Activity sheets 1, 2 and 3 in Appendix E). The activity sheets allow students to freely interact with the software. While solving the activity sheet, learners have the chance to construct, measure, calculate, drag, ... figures. During the discovery process the teacher does not interfere with students' work. The teacher's role is to help students whenever they face difficulties in using Cabri Geometry, or to interact with students through a questioning sequence to prompt their thinking. .

*2) Making a conjecture phase.* As students solve the activity sheets, they are continuously asked to drag, observe, notice, compare, generalize, conjecture and reason. The activity sheets also urge students to animate and tabulate the results that seem constant. Thus, the leading questions of the sheets along with the unique features of Cabri help students notice invariant properties and make conjectures based on previously observed relationships.

*3) Teacher-class discussion phase.* In this phase, students would have developed the relations on their own and are ready to discuss with the teacher. Discussions are oriented by the teacher due to time limitations but students are given the chance to answer the questions of the activity. Thus, the student-teacher communication is shaped in a way that students themselves would develop the theorem and formulate the properties.

*4) Application phase.* In the application phase, students have to apply what they have just learned either in the form of classwork or homework. The application usually is a set of direct-application exercises to practice the property learned.

*3.2.5.6.2 Questioning.* Questioning is emphasized by the teacher during the implementation of the whole DGS-CA instructional unit. The teacher ensures a secure environment in the classroom so that students feel free to ask or answer questions. Questions are used to aid the learning process through triggering students' curiosity. Sometimes questions are used to help students organize their thoughts, build up their reasoning and draw out conclusions. At other times questions are used for formative assessment purposes to help the teacher get feedback on students' learning.

*3.2.5.6. 3 Homework.* At the end of every session, the teacher gives a homework assignment that relates directly to what was discussed during the session. The main function of the homework exercises is to apply what have been learned. The homework exercises and problems are assigned from the students' textbook *Building up Mathematics* (ECRD, 2000). At the beginning of every session, the teacher checks the homework and invites students to solve on the board. (Refer to Appendix E, homework part of every session).

*3.2.5.6. 4 Technology integration.* Technology is integrated in the teaching/learning of the DGS-CA unit. In 5 sessions, each student has access to a computer with Cabri Geometry installed. Students have to individually solve the activity sheets 1 and 2 on the computer. They are given the chance to play around (drag, calculate, rotate …etc.) the figures they construct. Through solving the sheets, students are expected to appreciate the special characteristics of the software such as the *tabulation* and *animation* features that help them make observations about the invariants and come up with conjectures. After students finish their activity sheets, the teacher illustrates the solution of the activity sheets on the LCD screen and conducts a question-answer session to discusses students' work. Students also have the chance to work with technology at home as they solve a homework activity based on Cabri Geometry (Refer to Appendix E, Activity sheet 3).

*3.2.5.6.5 Introduction through problem solving***.** During the first session, the teacher presents a problem situation to the students in order to introduce Thales' Theorem (Refer to Problem Statement under the paragraph *Activities*). Together, the teacher and the students model the problem mathematically. Afterwards, students conjecture the Thales' theorem formula on their own due to their involvement in the experimental situations on Cabri software.

*3.2.5.6.6 Modeling.* The students try to model the problem situation and represent it in mathematical terms. Through discussions with the teacher, they represent the problem with a triangle, and a segment that cuts two sides of the triangle and is parallel to the third (Refer to Fig. 2). Afterwards, the teacher asks the students to start with Activity Sheet 1 (Refer to appendix E, Activity Sheet 1) in order to conjecture a property that helps them find the height of the pyramid and thus solve the problem.

***3.2.5.7 Activities.*** There are three activities developed in the DGS-CA instructional unit.

3.2.5.7.1 *Activity sheet 1.* In Activity sheet 1, students have to construct the triangle that represents the problem situation using Cabri Geometry (Refer to Fig. 3). In Fig. 3, [BC] represents the height of the pyramid, [MN] Osiris's height, [BA] the shadow of the pyramid and [MA] Osiris's shadow. The activity asks students to calculate the ratios of the segments, tabulate them, and then animate the figure, observe and compare to make a conjecture. Afterwards, the teacher takes (AM/AB= AN/AC ) as the generalization given by the students, no matter how the shape of the figure changes (they pass from a right triangle to a scalene triangle). The teacher then discusses, with the students, the reason that keeps the ratios equal no matter how the triangle is modified by dragging one of its vertices. Students realize that the activity starts by constructing a parallel line that leads to Thales' Theorem: "Any line parallel to one side of a triangle divides the other two sides proportionally". Moreover, students will recognize other properties of Thales' theorem as being AM/AB= AN/AC = MN/BC.

*Problem Statement (*The problem statement through which the chapter on Thales' Theorem is introduced)

*Osiris wanted to find the height of a pyramid. However in his time (2,000 years BC) there were no tools available to measure the height of the pyramid. So Osiris thought that the shadow of the pyramid might be useful to help him find out the height of the pyramid. He stood in front of the pyramid such that his shadow overlaps the pyramid's shadow and both shadows (his and pyramid's) end at the same place (Refer to Fig. 1). His shadow was 2 meters. The pyramid's shadow was 6 meters. Knowing that Osiris is 1.8 meters tall, how do you think he was able to find the height of the pyramid?? How high was it??*

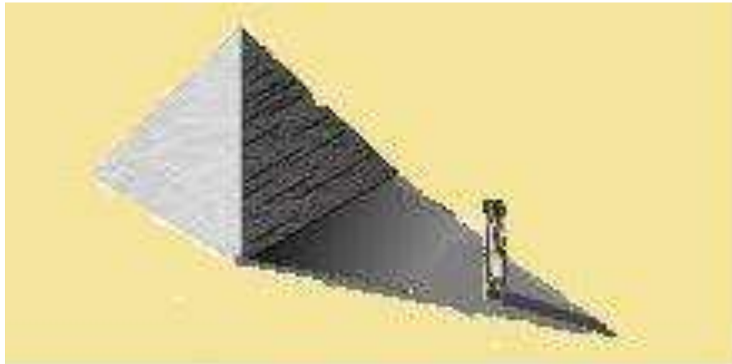


Figure 1. Osiris standing in front of the pyramid with his shadow overlapping with the pyramid's shadow such that both shadows (his and pyramid's) end at the same place.

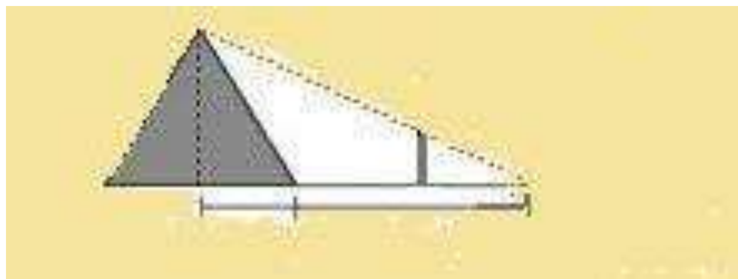


Figure 2. The triangle cut by parallel line that represents the real life problem.

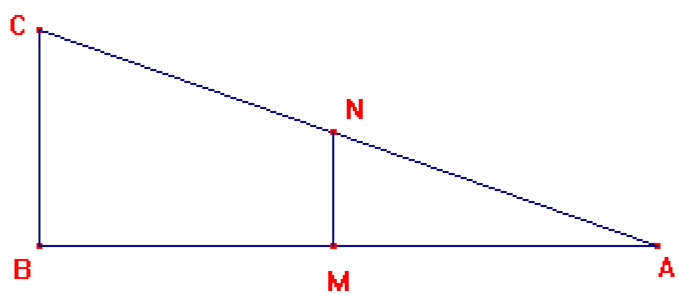
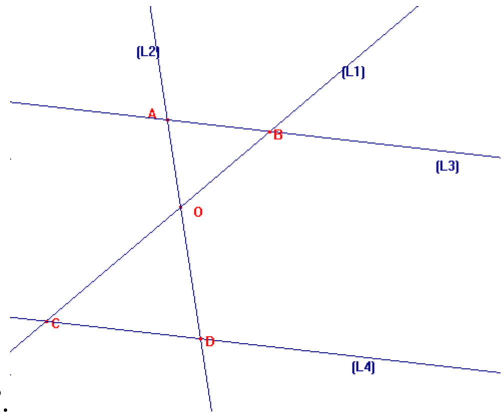



Figure 3. The triangle constructed on Cabri software to solve Activity sheet 1.

3.2.5.7.2 *Activity sheet 2* (sessions 2 and 3). Students apply Thales' Theorem in the extension of triangles. Students in this session have to solve Activity sheet 2 (Refer to Appendix E, Activity Sheet 2) on Cabri Geometry. They construct figure 4 and through answering the questions of the activity sheet they conclude: “Any line parallel to one side of a triangle divides the other two or their extensions




proportionally”.

Figure 4. The figure constructed on Cabri software to solve Activity sheet 1.

3.2.5.7.3 *Activity Sheet 3 (session 8).* The activity asks students to construct figure 5 using Cabri. Students make a conjecture that AC/CE=BD/DF. They recognize Thales in any geometry problem where lines intersect parallel lines (including trapezoids) from their homework assignment. The teacher takes the generalizations given by the students that is AC/CD=BD/DF, no matter how the shape of the figure is changing. The teacher asks the students why they think this is the case, and what is the main reason for the ratios to remain equal. Students should realize that they started with parallel lines and come up with: when the lines are parallel, the lines intersecting them will always be cut proportionally. And that’s why: AC/CD=BD/DF. Finally, the teacher emphasizes Thales’ Theorem on the board and states it clearly, just in case a student fails to generalize it during class discussion.

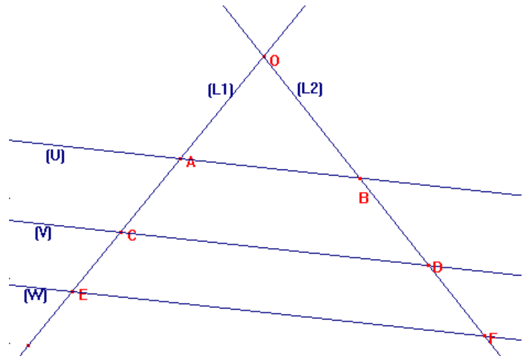



Figure 5. The figure constructed on Cabri software to solve Activity sheet 3

**3.2.5.8** ***Connecting Thales' Theorem and proportionality Rules.*** In sessions 5 and 6 students recognize different forms of Thales' Theorem and apply them. The teacher uses an overhead projector to illustrate some proportions that relate to Thales' Theorem and that the chapter in the national book doesn't mention. The teacher opens the file for figure 3 (refer to fig. 3) and illustrates using the drag mode to verify the following proportions: AM/MB = AN/NC and MB/AB = NC/AC.

**3.2.6 Unit Test.**

The purpose of the unit test (Refer to Appendix E) is to investigate students' acquisition of knowledge on the subject of Thales' Theorem. The results will be used to answer the research questions of whether DGS-CA enhances students' problem solving skills and overall learning of Thales' Theorem. Note that the unit test is paper-pencil based. It consists of problems ranging from direct application to problem solving and reasoning (deduction).

### 3.2.7 Data collection instruments

This study uses both qualitative and quantitative methods of collecting and analyzing data. Qualitative data are collected through an interview with the teacher of the control group to take a close look at the teaching method followed, and quantitative data through a questionnaire administered to the participants in the control group after they have learned Thales' Theorem, and to the participants of the experimental group before and after piloting the unit, in order to determine the effect of piloting the unit with a DGS-CA on motivation. Qualitative and quantitative data are collected through administering tests to the participants and comparing their problem solving strategies and their scores in order to determine the effectiveness of the employed approach.

# CHAPTER 4

# DATA ANALYSIS

The present chapter starts with a revision of the content and structure of the unit on Thales' Theorem in the national textbook and its objectives are compared with those of the Lebanese math curriculum (ECRD, 1997).It provides next an analysis of the interview with the teacher of the control group, in order to deepen our understanding about her instructional method. Afterwards, it reports the quantitative analysis of the attitude questionnaire to check whether DGS-CA affected students' motivation towards mathematics and geometry. Finally, quantitative and qualitative analyses of the tests' results and problem solving skills development are presented.

## 4.1 Revision of Thales' Theorem Unit in ECRD Textbook

The LMC states in its general objectives that: "mathematics constitute as an activity of the mind" (ECRD, 1997, p. 288) and that the learning of mathematics is a mental activity that develops the formation of students' mathematical reasoning and logic. The LMC recommends that learning mathematics happens through students' involvement in the construction of knowledge. Starting with real life situations, students should learn mathematical concepts through raising questions, laying down problems, discussing and making conjectures. On the other hand, the objectives of the LMC advise the demonstration of the practical usefulness of the mathematical concepts, since there is "…no divorce between mathematics and everyday life” (ECRD, 1997, p. 288). The LMC also proposes the use of technology as a tool for learning.

#### 4.1.1 Introducing the theorem.

The unit about Thales’ Theorem in grade 9 textbook (Refer to Appendix D) titled: “Building up Mathematics” of the Lebanese mathematics curriculum (Chalak et al. 2000) introduces the theorem through plane geometry figures. The unit does not introduce the topic through real life problem settings. The chapter, however, mentions some real life problem applications of Thales’ theorem in the problems section at the end of the chapter. In other words, the chapter does not adopt the method of teaching through problem solving. It does not allow students to model real life problem situations into geometrical configurations to reach a solution to a problem, which would lead to construct their knowledge.

#### 4.1.2 Activities.

The activity presented in the unit, through which the theorem is introduced, asks students to measure segments of straight lines cut by parallel line in a triangle and to calculate their ratios in order to conclude Thales’ theorem (Refer to Appendix D, p.57). The activity does not allow students to recognize ratios of segments cut by parallel lines since it does not consider more than one case. Since students are unable to observe, realize and make a conjecture, the activity will indirectly lead the teacher to state the theorem. Thus, it does not account for the role of problem solving and reasoning as a context for learning mathematical concepts. Consequently, the unit disregards building on students' problem solving skills such as making and formulating conjectures based on previously observed relationships and applying deductive reasoning, drawing logical conclusions,…etc.

#### 4.1.3 Content.

The unit does not mention all the needed proportional relations between segments in Thales' Theorem such as: AM/MB = AN/NC, MB/AB = NC/AC…etc (Refer to Fig. 1 in chapter 3). However, their usage appears later in the *exercises* section of the unit (Refer to Appendix D, p. 61). The unit focuses on the applicability of Thales' Theorem in a triangle and disregards its applicability in any geometry problem involving segments and parallel lines such as trapezoids. Moreover, in a paper -pencil setting, unlike a technology setting, students are unable to recognize or verify other proportional relations that they wish to prove or disprove for themselves. On the other hand, the unit does not help students that are unable to manipulate proportional quantities in order to deduce other relationships between measures.

### 4.2 Analysis of the Interview

The purpose of the interview is to investigate the teaching style of the control group's teacher. The interview consists of seven questions that reveal the way the teacher introduces a lesson, her use of learning aids, kinds of questions she asks, her use of technology…etc.

The analysis of the interview revealed that the teacher of the control class determines the content and makes all the decisions in her class. Most of the times, the teacher does not involve the students in any discovery or exploration process. As she states: “I begin the chapter by a figure such as a triangle, graph, drawing…etc. that the chapter is about.” When asked about how to introduce a topic, the teacher answers: “I start explaining a chapter by giving some examples on the board like some figures; in geometry I draw a figure for example and try to explain the concept or the property”. Thus, the teacher tends to state the property or the theorem and students tend to imitate what the teacher explains. The teacher does not seem to use learning aids during instruction, such as manipulatives, activities, games…etc. She has never used technology as a learning tool in any of her previous classes, and group work is rarely applied.

Interaction between the teacher and the students of the control class takes place through her use of questioning techniques. The teacher asks questions to check for understanding or leading questions to help students solve the problem at hand. Sometimes, the teacher asks tricky questions to check for deep understanding of the concepts.

When asked how to enhance the reasoning and problem solving skills of the students, the teacher immediately related the issue to geometry teaching. She answered that she helps students in their thinking process to formulate a geometry poof by explaining the steps they should follow. As she states: “… as a first step they have to read correctly and accurately; as a second step, they have to write the given and think how they can use it to relate to the question asked”. The teacher also stated that from time to time she gives her students some extra higher-level problems. Thus, it can be concluded that the teacher guides the reasoning skills of the students only through geometry chapters (through teaching how to conduct a geometric proof) and that she does not teach for reasoning skills as an approach of teaching/learning mathematics in general. Besides, the control group teacher does not introduce chapters through problem solving settings. However, problem solving is only used as an application.

### 4.3 Quantitative Analysis of the Questionnaire

To analyze the results of the questionnaire, the items were divided into 3 themes. The first theme (hereafter referred to as Theme_1) contains the items that determine students' attitude towards the mathematics subject in general, the second theme (hereafter referred to as Theme_2) contains the items that determine students' attitude towards geometry in specific, and the third theme (hereafter referred to as Theme_3) contains the items that inspect students' needs of some manipulations that can be facilitated through the use of features of Cabri software. Two comparisons are conducted. First *between-group analyses,* that is comparing the three themes between post control and post experimental. Second, *within-group analysis,* that is comparing the three themes between pre-experimental and post-experimental.

### 4.3.1 Comparison between Post-Control and Post-Experimental Group Analysis of Questionnaire

#### 4.3.1.1 *Assumptions of MANOVA (Multivariate Analysis Of Variance)*.

*Dependent variable:* Use of technology

*Independent variables:* attitude towards math, attitude toward geometry, and manipulations needed.

*Independence of Scores:* The scores are statistically independent; the participants did not influence each other.

*Random Sampling:* The data was collected at random and measured at interval level.

*Unequal Sample Sizes and Missing Data:* SPSS FREQUENCIES was run for the Dependent variables (hereafter referred to as DVs). Results revealed the absence of any missing values. The distribution of the sample sizes can be viewed in the Table 5. Note that the data presented an almost equal cell sizes.

*Outliers:* The inspection for outliers revealed their absence.

*Linearity:* Linearity is assumed since there are only two levels within each DV.

*Multivariate Normality*: We assume that the DVs (collectively) have multivariate normality within groups. In order to test this assumption, the univariate normality assumption within each DV was measured using the Kolmogorov-Smirnov[a] (hereafter referred to as K-S) test of normality (Refer to Table 2). Results revealed that all DVs have a normal distribution within each group except for Theme_2 in the control group (D (19) = .25, p<.05. Consequently, multivariate normality cannot be assumed in Theme_2. However, this violation does not threaten the validity of the MANOVA results, since our data has almost equal sample sizes in the cells and there are far more cases than DVs in the smallest cell (Tabachnick & Fidell, 2007).

*Table 2*

*Test of Normality in Post-Control and Post-Experimental Questionnaires*

| | | Kolmogorov-Smirnov[a] | | |
|---|---|---|---|---|
| | Group | Statistic | Df | Sig. |
| Theme_1 | Control | .137 | 19 | .200[*] |
| | Experimental | .135 | 18 | .200[*] |
| Theme_2 | Control | .248 | 19 | .003 |
| | Experimental | .199 | 18 | .057 |
| Theme_3 | Control | .154 | 19 | .200[*] |
| | Experimental | .191 | 18 | .082 |

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

*Homogeneity of Covariance Matrices*: In MANOVA, we assume that in each group the variance is roughly equal (homogeneity of variance assumed), and also the correlation between two DVs is the same across all groups. This assumption is measured by testing whether the population variance-covariance matrices of different groups in the analysis are equal. Since Box's test is significant, F (6, 8798.39) = 4.23, p<.05, homogeneity of covariance matrices cannot be assumed. However, since we have almost equal cell sizes and Pillai's Trace Test is used, the results of MANOVA would be robust to this violation (Tabachnick & Fidell, 2007).

#### *4.3.1.2 Results.*

MANOVA analysis was conducted to examine the effect of the DGS-CA approach on students' motivation toward mathematics and geometry (Theme_1 and Theme_2). Pillai's Trace Test revealed significant multivariate test statistic ($V= .92$, $F(3, 33) = 4.531$, $p<.05$, $\eta^2 = .29$), suggesting that the DGS-CA had some main effect on motivation.

Subsequent ANOVA analyses (simple contrasts) were conducted to pinpoint the exact effect of DGS-CA on motivation. The assumption of homogeneity of variance was tested using Levene's Test of Equality of Error Variances. This assumption was met in Theme_1 and Theme _3 ($F(1, 35) = .61$, $p>.05$ and $F(1, 35) = .94$, $p>.05$ respectively). The assumption was violated in Theme_2 ($F(1, 35) = 10.98$, $p<.05$).

Separate univariate ANOVA tests revealed a significant effect of the DGS-CA for Theme_1 and Theme_2 ($F(1, 35) = 7.37$, $p<.05$, $\eta^2= .17$ and $F(1, 35) = 7.95$, $p<.05$, $\eta^2 =19$ respectively. However, no significant difference in Theme_3 ( $F(1, 35) = 3.05$, $p>.05$).

For Theme_1, the results of the simple contrast test showed that the experimental group scored significantly higher level of motivation towards mathematics ($M = 3.43$, $SD= .33$) than the control group ($M= 3.12$, $SD= .38$), $t(35) = -2.71$, $p<.05$, $r = .06$. For Theme_2, the experimental group scored significantly higher level of motivation toward geometry (M= 3.23, SD= .30) than the control group (M= 3.02, SD= .15), t (35) = -2.77, p<.05, r = .06. However, there are no significant differences between the experimental and control groups in their needs for the features of Cabri software t (35) = -.20, p>.05 (Refer to Table 3).

*Table 3*
*Descriptive Statistics in Post-Control and Post-Experimental Questionnaires*

| | Group | Mean | Std. Deviation | N |
|---|---|---|---|---|
| Theme_1 | Control | 3.1158 | .37751 | 19 |
| | Experimental | 3.4333 | .33077 | 18 |
| | Total | 3.2703 | .38576 | 37 |
| Theme_2 | Control | 3.0158 | .15005 | 19 |
| | Experimental | 3.2333 | .29902 | 18 |
| | Total | 3.1216 | .25618 | 37 |
| Theme_3 | Control | 2.9789 | .37650 | 19 |
| | Experimental | 3.1806 | .32137 | 18 |
| | Total | 3.0770 | .36067 | 37 |

***4.3.1.3 Summary.***

The experimental group showed higher level of motivation toward mathematics in general and geometry in specific than control group. However, both groups showed the same level of needs for Cabri features.

### 4.3.2 Comparison *within Pre-Experimental and Post-Experimental Group Analysis of Questionnaire.*

#### 4.3.2.1 *Assumptions of MANOVA (Multivariate Analysis of Variance)*.

*Independence of Scores:* The scores are statistically independent; the participants did not influence each other.

*Random Sampling:* The data were collected at random and measured at interval level. *Sample Sizes and Missing Data:* SPSS FREQUENCIES was run for the DVs. Results revealed the absence of any missing values. The distribution of the sample sizes can be viewed in Table 4. Note that the data presented almost equal cell sizes.

*Outliers:* The inspection for outliers revealed their absence.

*Linearity:* Linearity is assumed since there are only two levels within each DV.

*Multivariate Normality:* We assume that the DVs (collectively) have multivariate normality within groups. The normality assumption within each DV was measured using the K-S test of normality. Results revealed that the pre-experimental data violated the normality assumption on Theme_1 and Theme_2 ($D(19) = .20$, $p<.05$ and $D(19) = .27$, $p<.05$ respectively). Moreover, the post-experimental data violated the assumption of normality on Theme_3 ($D(18) = .50$, $p<.05$) (Refer to Table 4). Consequently, multivariate normality cannot be assumed. However, this violation does not threaten the validity of the

MANOVA results, since the data has almost equal sample sizes in the cells and there are far more cases than DVs in the smallest cell (Tabachnick & Fidell, 2007).

*Homogeneity of Covariance Matrices:* In MANOVA, it is assumed that in each group the variance is roughly equal (homogeneity of variance assumed), and also the correlation between two dependent variables is the same across all groups. This assumption is measured by testing whether the population variance-covariance matrices of different groups in the analysis are equal. Since Box's test is significant, $F(6, 8798.39) = 11.03$, $p<.05$, the homogeneity of covariance matrices is not assumed. However, since we have almost equal cell sizes and Pillai's Trace Test will be used, the results of MANOVA would be robust to this violation (Tabachnick & Fidell, 2007).

*Table 4*

*Test of Normality in Pre-Experimental and Post-Experimental Questionnaires*

| | | Kolmogorov-Smirnov[a] | | |
|---|---|---|---|---|
| | Group | Statistic | Df | Sig. |
| Theme_1 | Experimental PRE | .204 | 19 | .036 |
| | Experimental POST | .135 | 18 | .200* |
| Theme_2 | Experimental PRE | .270 | 19 | .001 |
| | Experimental POST | .199 | 18 | .057 |
| Theme_3 | Experimental PRE | .186 | 19 | .082 |
| | Experimental POST | .501 | 18 | .000 |

a. Lillief Significance Correction

*. This is a lower bound of the true significance.

#### 4.3.2.2 *Results.*

The pre- and post-experimental data showed similar levels of motivation on Theme_1 (M= 3.26, SD = .31 and M = 3.43, SD = .33 respectively). The same similarity pattern was noticed in Theme_2 in the pre- and post-experimental data (M = 3.06, SD = .50, and M = 3.23, SD = .30). However, the post-experimental data showed more need and appreciation for the features of Cabri (Theme_3) (M = 4.24, SD = 4.54) than the pre-experimental group (M = 2.90, SD = .40) (Refer to Table 5).

To find out if this difference is significant, a MANOVA was conducted on the data set. Pillai's Trace Test revealed a non-significant multivariate test statistic (V= .10, F (3, 33) =1.21, p>.05), suggesting that the attitudes of students did not differ in the post and pre conditions.

Subsequent ANOVA analyses (simple contrasts) were conducted to further examine some possible effect of the DGS-CA on motivation (though if Pillai's test is non-significant, no tests should be followed).

The assumption of Homogeneity of variance was tested using Levene's Test of Equality of Error Variances. This assumption was met in the three themes (F (1, 35) = .80, p>.05, F (1, 35) = 3.13, p>.05, and F (1, 35) = 3.42, p>.05 respectively).

Separate univariate ANOVA tests revealed higher but non-significant effects between pre- and post-motivation for the three Themes (for Theme_1: F (1, 35) = 2.79, p>.05; Theme_2: F (1, 35) = 1.66, p>.05, and Theme_3: F (1, 35) = 1, 64, p>.05).

*Table 5*
*Descriptive Statistics in Pre-Experimental and Post-Experimental Questionnaires*

| | Group | Mean | Std. Deviation | N |
|---|---|---|---|---|
| Theme_1 | Experimental Pre | 3.2579 | .30789 | 19 |
| | Experimental Post | 3.4333 | .33077 | 18 |
| | Total | 3.3432 | .32706 | 37 |
| Theme_2 | Experimental Pre | 3.0579 | .49924 | 19 |
| | Experimental Post | 3.2333 | .29902 | 18 |
| | Total | 3.1432 | .41802 | 37 |
| Theme_3 | Experimental Pre | 2.9053 | .40204 | 19 |
| | Experimental Post | 4.2444 | 4.54238 | 18 |
| | Total | 3.5568 | 3.20698 | 37 |

**4.3.2.3 *Summary.***

Experimental group students showed higher but non-significant difference between pre- and post-motivation toward mathematics and geometry. Moreover, they showed the same level of needs of features of Cabri.

## 4.4 Quantitative Analysis of the Tests

In the attempts to investigate the effect of DGS-CA on students' achievement and development of problem-solving skills, this section presents the quantitative analysis of the tests. It starts by explaining the way the experimental and control group were formed. Next it clarifies the method that the researcher followed to carry out the comparison of achievement and problem-solving skills between the two groups. Finally, it presents the statistical results of the comparisons made.

### 4.4.1 Formation of Experimental and Control Group.

The diagnostic tests of both classes (the experimental and control) were administered and collected by the researcher. The researcher corrected the diagnostic tests of both classes according to an answer key previously developed (Refer to Appendix G). In order to increase the validity of the scores, the final mathematics grade of previous year (grade eight) of each student (in both control and experimental group) is averaged with his/her grade on the diagnostic test. From the experimental class, 16 students, whose average grades are distributed close to a bell shape curve, are chosen to form the experimental group. Similarly, 16 students from the control class, whose average grades match those of the experimental group, are chosen to from the control group.

### 4.4.2 Comparison of the Results.

The comparison between the control and the experimental groups is made to study the effect of DGS-CA on the overall learning of Thales' Theorem and on problem solving strategies. Thus, every student will have 2 kinds of scores: 1) achievement score to study the effect of the DGS-CA on students' overall learning, and 2) problem-solving score to study the effect of the DGS-CA on students' geometric problem solving abilities. The achievement score is determined based on the usual rules of a regular test scoring. As for the problem solving score, the researcher first distributed the questions of both the diagnostic and unit test over the problem-solving skills (Refer to Tables 6 and 7) and then the problem-solving score is determined by using a rubric that evaluates each students' problem-solving ability using a scale from 1 to 4 (Refer to Table 8).

#### 4.4.2.1 ***Comparison between Control and Experimental Groups as pertains to Achievement.***

To closely test for achievement results, a comparison was made between pre achievement scores of experimental and control groups and between post achievement scores of control and experimental groups.

##### 4.4.2.1.1 *Comparison between achievement of the control and the experimental groups on the pre-test.*

4.4.2.1.1.1 <u>*Assumptions.*</u>

The assumptions of an independent t-test were examined.

*Data Measured at the Interval Level:* Data was measured on an interval level.

*Independent Scores:* The scores are independent because they came from different people, and there were no influences from one on another.

*Normally Distributed Data*: The K-S test for normality revealed that the scores in both conditions are normally distributed.

Control: $D(16) = .156, p > .05$

Experimental: $D(16) = .165, p > .05$

*Homogeneity of Variance*: The homogeneity of variance assumption was met, $F(1, 30) = 1.53, p > .05$.

4.4.2.1.1.2 *Means and Error Bar.*

The mean score for the control group was 9.31 with a Standard deviation of 3.42, indicating that the participants varied in their achievement scores. The mean score for the experimental group was 8.53 with a standard deviation of 4.33 also indicating that the participants varied in their achievement scores. The error bar with 95% confidence interval is presented in Fig. 6.

A mean difference of 1.19 was noticed between the two groups; indicating that the control group scored higher than the experimental group. In order to examine if this mean difference is significant, an independent t test was conducted on the data.

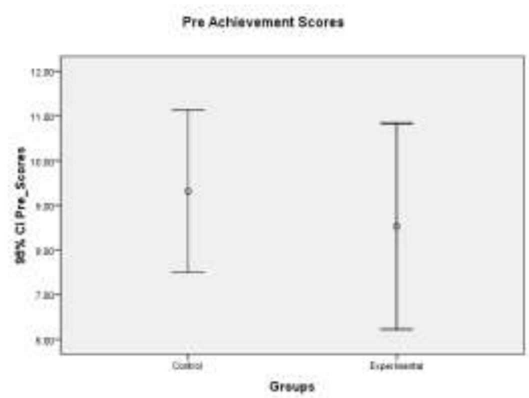



Figure 6: Error Bar of the Pre Scores for Achievement

The fact that the confidence intervals are overlapping means that the two groups are plausibly from the same population, meaning that there are no significant differences between the groups.

*Table 6*
*Distribution of the questions of the diagnostic Test (Pre-test) on the problem solving skills:*

| | Draw a figure | Selects and applies learned properties in the right context to deduce other properties or relationships | Gathers, organizes and selects appropriate data to solve a geometric problem | Makes and formulates conjectures based on previously observed relationships | Constructs a whole and consistent proof |
|---|---|---|---|---|---|
| | 1.b.i | 1.b.iii | 1.b.iii | 1.b.ii | 1.b.iii |
| | | | 2.a.i | 2.a.ii | |
| | | 2.a.iii | 2.a.iii | 2.a.iii | |
| <u>Diagnostic</u> | | 2.b.iii | 2.b.i | 2.b.ii | |
| | | 2.c.i | 2.b.iii | 2.b.iii | |
| | | | 2.c.i | 2.c.ii | |
| | | 3.a.ii | | | |
| | | 3.b.ii | | 3.a.i | 3.a.ii |
| | | 3.c.ii | 3.f.ii | 3.b.i | 3.b.ii |
| | | 3.d.ii | | 3.c.i | 3.c.ii |
| | | 3.e | | 3.d.i | 3.d.ii |
| | | 3.f.i | | | |
| | | 3.f.ii | | | |

*Table 7*
*Distribution of the questions of the Unit Test(post-test) on the problem solving skills:*

| | Draw a figure | Selects and applies learned properties in the right context to deduce other properties or relationships | Gathers, organizes and selects appropriate data to solve a geometric problem | Makes and formulates conjectures based on previously observed relationships | Constructs a whole and consistent proof |
|---|---|---|---|---|---|
| | 1.a | 1.b | 1.e | 1.f.i | 1.f.ii |
| | | 1.c | 1.d | | |
| | | 1.d | | | |
| | | 1.e | | | |
| UnitTest | | 1.f.ii | | | |
| | | 2.a | 2.c | | |
| | | 2.b | | | |
| | | 2.c | | | |
| | 3.b | 3.a.ii | 3.c | 3.a.i | 3.a.ii |
| | | 3.c | 3.e.ii | 3.e.i | 3.e.ii |
| | | 3.d | 3.f.ii | 3.f.i | 3.f.ii |
| | | 3.f.ii | | | |
| | | 3.e.ii | | | |

*Table 8*
*Problem Solving Skills-Rubric*

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Skill 1: Draw a figure | Not found | Wrong figure | Half correct or incomplete figure | Correct, complete and neat figure |
| Skill 2: Select and apply learned properties in the right context to deduce other properties or relationships | Not found | Select but fail to apply learned properties to deduce other relationships | Select and apply learned properties and deduce other relationships without stating the theorem behind | Select and apply learned properties and deduce other relationships stating the theorem behind in a complete and neat writing |
| Skill 3: Gather, organize and select appropriate data to solve a geometric problem | Not found | Gather and organize data but fail to solve | Gather, organize and correctly solve without justification | Gather, organize and correctly solve with justification |
| Skill 4: Make and formulate conjectures based on previously observed relationships | Not found | Wrong conjecture | Partly correct conjecture | Correct Conjecture |
| Skill 5: Construct a whole and consistent proof | Not found | Wrong reasoning | Correct proof but illogical sequenced reasoning | Correct, complete and logically sequenced reasoning |

Note: The error bar displays the mean and the 95% confidence interval of the mean of each group. 95% confidence level means that 95 of those 100 confidence intervals would contain the value of the mean.

On average, participants in the control condition scored higher in their achievement exam (M= 9.31, SE= .85), than their colleagues in the experimental condition (M= 8.33, SE=1.08). This difference was not significant t (30) = .57, p>.05.

4.4.2.1.1.3 *Summary of T-test result.*

Control group participants scored higher in their achievement on the pre-test than their colleagues in the experimental group. However, this difference in scores was not significant.

*Table 9*
*Independent T-Test of Pre Control and Pre Experimental Group Achievement*

| | | Levene's Test for Equality of Variances | | t-test for Equality of Means | | | |
|---|---|---|---|---|---|---|---|
| | | F | Sig. | t | df | Sig. (2-tailed) | Mean Difference |
| Pre-Scores | Equal variances assumed | 1.532 | .225 | .567 | 30 | .575 | .78125 |
| | Equal variances not assumed | | | .567 | 28.482 | .575 | .78125 |

4.4.2.1.2 *Comparison between achievement of the control and the experimental groups on the post-test.*

4.4.2.1.2.1 *Assumptions.*

The assumptions of an independent t-test were examined.

*Data Measured at the Interval Level*: Data was measured on an interval level.

*Independent Scores*: The scores are independent because they came from different people, and there were no influences from one on another.

*Normally Distributed Data*: The K-S test for normality revealed that the scores in both conditions are normally distributed.

Control: D (16) = .173, p>.05

Experimental: D (16) = .182, p>.05

*Homogeneity of Variance*: The homogeneity of variance assumption was not met, F (1, 30) = 6.29, p<.05).

4.4.2.1.2.2 *Means and Error Bar.*

The mean score for the control group is 12.39 with a Standard deviation of 2.17, indicating that the participants varied in their achievement scores. The mean score for the experimental group was 13.75 with a standard deviation of 3.43 also indicating that the participants varied in their achievement scores. The error bar with 95% confidence interval is presented in the figure below (Refer to Figure 7).

A mean difference of 1.36 was noticed between the two groups; indicating that the DGS-CA had some effect on participants' achievement scores. In order to examine if this difference is significant an in dependent t-test was conducted on the data.

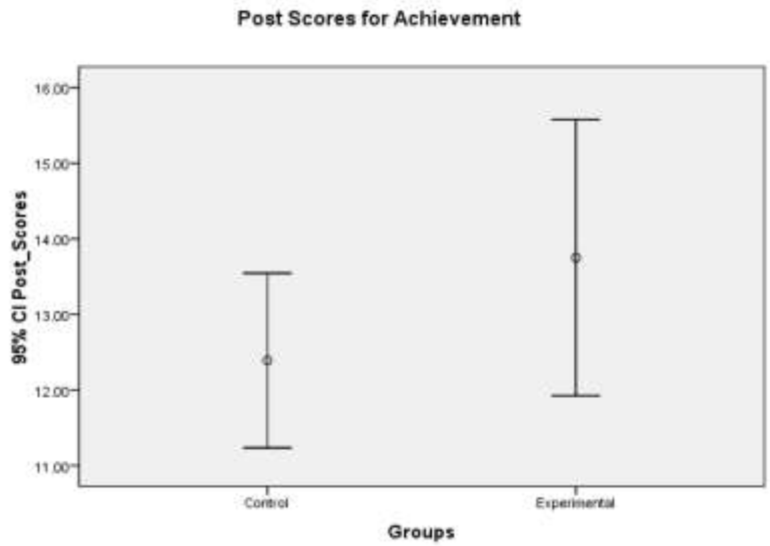



Figure 7: Error Bar of the Post Scores for Achievement

The fact that the confidence intervals are overlapping means that the two groups are plausibly from the same population, meaning that there are no significant differences between the groups.

Note: The error bar displays the mean and the 95% confidence interval of the mean of each group. 95% confidence level means that 95 of those 100 confidence intervals would contain the value of the mean.

On average, participants in the experimental group scored higher in their achievement exam (M= 13.75, SE= .86), than their colleagues in the control group (M= 12.39, SE= .54). However, this difference was not significant t (30) = -1.34, $p>.05$; it represents a small effect, r = .25 (Refer to Table 10).

4.4.2.1.2.3 *Summary of T-test Result.*

Experimental group participants scored higher in their achievement on the post-test than their colleagues in the control group. However, this difference in scores was not significant. This non-significance pinpoints that the DGS-CA did not have any significant effect on the achievement scores of the students.

*Table 10*
*Independent T-Test of Post Control and Post Experimental Group Achievement*

| | | Levene's Test for Equality of Variances | | t-test for Equality of Means | | | | |
|---|---|---|---|---|---|---|---|---|
| | | F | Sig. | t | Df | Sig. (2-tailed) | Mean Difference | Std. Error Difference |
| Post Scores | Equal variances assumed | 6.292 | .018 | -1.341 | 30 | .190 | -1.35938 | 1.01396 |
| | Equal variances not assumed | | | -1.341 | 25.363 | .192 | -1.35938 | 1.01396 |

**4.4.2.2 *Comparison between Control and Experimental groups as pertains to Problem-Solving Skills.***

4.4.2.2.1 *Assumptions.*

*Independence of Scores:* The scores are statistically independent; the participants did not influence each other.

*Random Sampling:* The data was collected at random and measured at interval level.

*Unequal Sample Sizes and Missing Data:* SPSS FREQUENCIES was run for the DVs. Results revealed the absence of any missing values. The distribution of the sample sizes can be viewed in the table 11. To note that the data presented almost equal cell sizes.

*Linearity:* Linearity is assumed since there are only two levels within each DV.

*Outliers:* The inspection for outliers revealed their absence.

*Multivariate Normality:* We assume that the DVs (five problem-solving skills) have multivariate normality within groups. The normality assumption within each DV was measured using the Kolmogorov-Smirnov test of normality. Results revealed that all DVs had a normal distribution within each group (Refer to Table 11).

*Homogeneity of Covariance Matrices*: In MANOVA, we assume that in each group the variance is roughly equal (homogeneity of variance assumed), and also the correlation between two dependent variables is the same across all groups. This assumption is measured by testing whether the population variance-covariance matrices of different groups in the analysis are equal. Since Box's test is non-significant ($F$ (3, 162000) = 1.24, $p > .05$) the homogeneity of covariance matrices is assumed.

4.4.2.2.2 *Results.*

MANOVA analysis was conducted to examine the significance of the DGS-CA on problem-solving skills. Pillai's Trace Test revealed significant multivariate test statistic ($V = .35$, $F$ (2, 29) = 7.68, $p<.05$, $\eta^2 = .34$), suggesting that DGS-CA had some effect on the average problem-solving skills.

*Table 11*

*Tests of Normality in Control and Experimental Groups' Problem-Solving skills*

| | | Kolmogorov-Smirnov[a] | | |
|---|---|---|---|---|
| | Group | Statistic | Df | Sig. |
| Pre PS score | Control | .189 | 16 | .128 |
| | Experimental | .156 | 16 | .200* |
| Post PS score | Control | .148 | 16 | .200* |
| | Experimental | .153 | 16 | .200* |

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

Subsequent ANOVA analyses (simple contrasts) were conducted to pinpoint the exact effect of the DGS-CA. The assumption of Homogeneity of variance was tested using Levene's Test of Equality of Error Variances. This assumption was met in the pre and post averaged problem-solving skills ($F (1, 30) = 3.54$, $p>.05$ and $F(1, 30)= .64$, $p>.05$ respectively).

Separate univariate ANOVA revealed non-significant difference in the problem solving level between the control and experimental groups on the pre-test ($F (1, 30) = .07$, $p>.05$). However, the univariate ANOVA tests revealed a significant effect of the DGS-CA on the problem-solving level between the control and experimental groups on the post-test ($F (1, 30) = 12.06$, $p<.05$, $\eta^2 = .29$).

For pre problem-solving level, the experimental group scored very similar (M= 2.31, SD= .54) to the control group (M= 2.28, SD= .38), and the mean difference was not significant, $t (30) = -.043$, $p>.05$. The results of the simple contrast showed that the experimental group scored higher (M = 3.20, SD= .33) than the control group (M= 2.73, SD= .42) in post problem-solving level, t (30) =- 3.475, p<.05, r = .0.08 (Refer to Table 12).

4.4.2.2.3 *Summary.*

Results showed a non-significant difference in the problem solving level between the control and experimental groups on the pre-test. However, a significant effect of the DGS-CA on the problem-solving level between the control and experimental groups on the post-test was revealed.

*Table 12*
*Descriptive Statistics in Control and Experimental Problem Solving skills*

| | Group | Mean | Std. Deviation | N |
|---|---|---|---|---|
| Pre PS score | Control | 2.2752 | .38561 | 16 |
| | Experimental | 2.3187 | .53998 | 16 |
| | Total | 2.2970 | .46209 | 32 |
| Post PS score | Control | 2.7396 | .41754 | 16 |
| | Experimental | 3.2044 | .33497 | 16 |
| | Total | 2.9720 | .44090 | 32 |

**4.4.2.3 *Comparison between control and experimental groups as pertains to problem-solving skills 4 and 5.***

4.4.2.3.1 *Assumptions.*

*Independence of Scores:* The scores are statistically independent; the participants did not influence each other.

*Random Sampling:* The data were collected at random and measured at interval level.

*Unequal Sample Sizes and Missing Data:* SPSS FREQUENCIES was run for the DVs. Results revealed the absence of any missing values. The distribution of the sample sizes can be viewed in the D table 14. To note that the data presented almost equal cell sizes.

*Outliers:* The inspection for outliers revealed their absence.

*Linearity:* Linearity is assumed since there are only two levels within each DV.

*Multivariate Normality:* We assume that the DVs (collectively) have multivariate normality within groups. The normality assumption within each DV was measured using the K-S test of normality. Results revealed that problem-solving skills 4 and 5 in the pre-test have a normal distribution within each group (D (16) = .11, p>.05, D (16) = .17, p>.05, D (16) = .17, p>.05, and D (16) = .11, p>.05 respectively). Moreover, the assumption was also met for post problem-solving skill 5 for the experimental condition (D (16) = .17, p>.05). The normality assumption was violated for Post problem-solving skill 4 in both conditions and Post problem-solving skill 5 in the control condition (D (16) = .24, p<.05, D (16) = .24, p<.05, and D (16) = .27, p<.05) (Refer to Table 13). Consequently, multivariate normality cannot be assumed. However, this violation does not threaten the validity of the MANOVA results, since the data has almost equal sample sizes in the cells and there are far more cases than DVs in the smallest cell (Tabachnick & Fidell, 2007).

*Homogeneity of Covariance Matrices:* In MANOVA, we assume that in each group the variance is roughly equal (homogeneity of variance assumed), and also the correlation between two dependent variables is the same across all groups. This assumption is measured by testing whether the population variance-covariance matrices of different groups in the analysis are equal. Since Box's test is non-significant (F (9, 4302.79) = .88, p>.05), homogeneity of covariance matrices is assumed.

*Table 13*

*Tests of Normality of Problem-Solving Skills 4 and 5 between Control and Experimental Groups*

| | | Kolmogorov-Smirnov[a] | | |
|---|---|---|---|---|
| | Group | Statistic | Df | Sig. |
| Post PS skill 4 | Control | .244 | 16 | .012 |
| | Experimental | .238 | 16 | .016 |
| Post PS skill 5 | Control | .269 | 16 | .003 |
| | Experimental | .166 | 16 | .200* |
| Pre PS skill 4 | Control | .114 | 16 | .200* |
| | Experimental | .166 | 16 | .200* |
| Pre PS skill 5 | Control | .168 | 16 | .200* |
| | Experimental | .117 | 16 | .200* |

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

4.4.2.3.2 *Results.*

MANOVA analysis was conducted to examine the significance of the DGS-CA on problem-solving skills 4 and 5.

Pillai's Trace Test revealed a non-significant multivariate test statistic ($V= .23$, $F$ (4, 27) = 2.05, $p>.05$), suggesting that the experimental manipulation had no main effect on performance of problem-solving skills 4 and 5.

Subsequent ANOVA analyses (simple contrasts) were conducted to pinpoint possible specific effects of the experimental manipulation on the post and pre problem-solving skills 4 and 5. The assumption of Homogeneity of variance was tested using Levene's Test of Equality of Error Variances. This assumption was met in Post problem-solving skills 4 and 5 ($F$ (1, 30) = 1.88, $p>.05$ and $F$ (1, 30) = .50, p>.05k respectively). Moreover, the assumption was also met in Post problem-solving skill 5 ($F$ (1, 30) = 4.12, $p>.05$). In pre problem-solving skill 4, the assumption of homogeneity of variance was violated, $F$ (1, 30) = 5.72, $p>.05$.

Separate univariate ANOVA tests revealed a non-significant difference between pre control and pre experimental on problem-solving skills 4 and 5 ($F$ (1,30) = .07, $p>.05$ and $F(1,30) = .00$, $P>.05$, respectively). Besides, univariate ANOVA revealed a non-significant effect on post skill 4 ($F$ (1, 30) = 1.96, $p>.05$. However, univariate ANOVA revealed a significant effect of the experimental manipulations only for post problem-solving skill 5 ($F$ (1, 30) = 8.72, $p<.05$, $\eta^2 = .23$). The results of the simple contrast showed that the experimental group scored higher ($M = 2.81$, $SD=.60$) than the control group ($M= 2.18$, $SD=.60$) on post problem-solving skill 5, $t$ (30) = 2.95, $p<.05$, $r = .07$.

4.4.2.3.3 *Summary*.

Results revealed that the control and experimental groups showed a non-significant difference on problem-solving skills 4 and 5 in pre-test. In post-test, DGS-CA had a non-significant effect on problem-solving skill 4 (that is conjecturing) and a significant effect on problem-solving skill 5 (that is proving).

*Table 14*
*Descriptive Statistics of Problem-Solving Skills 4 and 5 between Control and Experimental groups*

| | Group | Mean | Std. Deviation | N |
|---|---|---|---|---|
| Post PS skill4 | Control | 2.9375 | .66771 | 16 |
| | Experimental | 3.2344 | .52017 | 16 |
| | Total | 3.0859 | .60778 | 32 |
| Post PS skill5 | Control | 2.1875 | .59512 | 16 |
| | Experimental | 2.8125 | .60208 | 16 |
| | Total | 2.5000 | .66901 | 32 |
| Pre PS skill4 | Control | 2.4544 | .49602 | 16 |
| | Experimental | 2.5106 | .72364 | 16 |
| | Total | 2.4825 | .61094 | 32 |
| Pre PS skill5 | Control | 2.2250 | .50000 | 16 |
| | Experimental | 2.2250 | .80291 | 16 |
| | Total | 2.2250 | .65795 | 32 |

### 4.4.2.4 ***Comparison of Pre and Post Experimental Group as pertains to Problem-Solving Skills 4 and 5.***

4.4.2.4.1 *Assumptions.*

*Independence of Scores:* The scores are statistically independent; the participants did not influence each other.

*Random Sampling:* The data were collected at random and measured at interval level.

*Unequal Sample Sizes and Missing Data:* SPSS FREQUENCIES was run for the DVs. Results revealed the absence of any missing values. The distribution of the sample sizes can be viewed in the table 16. To note that the data presented almost equal cell sizes.

*Linearity:* Linearity is assumed since there are only two levels within each DV.

*Multivariate Normality:* We assume that the DVs (collectively) have multivariate normality within groups. The normality assumption within each DV was measured using the K-S test of normality. Results revealed that Pre and Post PROBLEM-SOLVING skill 4 and Pre PROBLEM-SOLVING skill 5 have a normal distribution ($D(32) = .11$, $p>.05$, $D(32) = .15$, $p>.05$, $D(16) = .17$, $p>.05$, and $D(32) = .14$, $p>.05$ respectively) (Refer to Table 15). The assumption was violated for post PROBLEM-SOLVING skill 5 ($D(32) = .16$, $p<.05$). The multivariate normality hence cannot be assumed. However, this violation does not threaten the validity of the MANOVA results, since our data has almost equal sample sizes in the cells and there are far more cases than DVs in the smallest cell (Tabachnick & Fidell, 2007).

*The Assumption of Sphericity:* Since the factors have only two levels each, the assumption of sphericity is no longer needed for the analysis. The assumption of sphericity can be linked to the assumption of homogeneity of variance in between-group ANOVA. It is similar to compound symmetry which holds true when both variances across conditions are equal and the co variances between conditions are equal.

*Table 15*
*Tests of Normality of Problem Solving Skills 4 and 5 within Pre- and Post-Experimental Group*

| | Kolmogorov-Smirnov[a] | | |
|---|---|---|---|
| | Statistic | Df | Sig. |
| Pre PS skill 4 | .114 | 32 | .200[*] |
| Pre PS skill 5 | .141 | 32 | .106 |
| Post PS skill 4 | .152 | 32 | .057 |
| Post PS skill 5 | .156 | 32 | .045 |

a. Lilliefors Significance Correction

4.4.2.4.2 *Results.*

The results of the repeated measure ANOVA for the within subject variables revealed a main effect of the manipulation of Pre-Post for problem-solving skill 4 ($F$ (1, 31) = 35.48, $p<.05$, $\eta^2 = .54$ and PROBLEM-SOLVING skill 5 ($F$ (1, 31) =12.29, $p<.05$, $\eta^2 = .29$).

Students scored significantly higher in post problem-solving skill 4 (M = 3.09, SD = .61), compared to pre problem-solving skill 4 (M = 2.48, SD = .61). Students scored significantly higher on post problem-solving skill 5 (M= 2.50, SD= .70) than the pre problem-solving skill 5 (M= 2.23, SD = .66).

4.4.2.4.3 *Summary.*

Students scored significantly higher in problem-solving skills 4 and 5 in post-test compared to their scores in pre-test. Thus, DGS-CA had a significant effect on students' conjecturing and proving problem-solving abilities.

*Table 16*

*Descriptive Statistics of Problem-Solving Skills 4 and 5 within Pre and Post Experimental Group*

| | Mean | Std. Deviation | N |
|---|---|---|---|
| Pre PS skill 4 | 2.4825 | .61094 | 32 |
| Post PS skill 4 | 3.0859 | .60778 | 32 |
| Pre PS skill 5 | 2.2250 | .65795 | 32 |
| Post PS skill 5 | 2.5000 | .66901 | 32 |

## 4.5 Qualitative Analysis of Tests

In the following a qualitative analysis of the tests of the experimental group is presented.

### 4.5.1 Construction and DGS.

The first question of the diagnostic test administrated prior to any integration of technology asked students to construct a Figure (Refer to Figure 8). The construction of this figure requires students to think about the theoretical properties of a parallelogram before performing any construction. Furthermore, after recognizing the properties of a parallelogram, students have to draw a triangle given the measures of its three sides. Thus, upon making the construction, students have to go through a process of geometrical theory-based dependent steps, a process that might be complex in a paper-pencil setting. Upon constructing with a paper and pencil, students tend to “cheat” and not apply the theoretical properties that a figure holds. For example, and from personal observations, students draw an isosceles triangle by drawing a horizontal line and finding its midpoint then going upwards remaining above the midpoint to find the third vertex. They do not realize that the property behind this “cheating” procedure is the definition of a perpendicular bisector, and thus any point on the perpendicular bisector is equidistant from both extremities. Another example is that upon drawing a parallelogram, students draw two horizontal straight lines that are not on the same level (one is moved 1 square to the left or right) and then they join those horizontal lines with two oblique lines thus a parallelogram is formed. As a result, students usually avoid thinking about geometric procedures upon constructing figures and they heavily rely on tricks or cheats that they follow and the figure works. This might be the reason that prevented almost all the experimental group from drawing a complete and correct figure. One out of 16 students drew a correct figure (that is 6.25% of the class). It is suggested that if students were used to construct geometrical figures on DGS or any software that requires them to think more about the figure's theoretical properties, they most probably would have succeeded in building this figure. This analysis is concluded built on the belief that using DGS requires students to go through a set of theoretical dependent steps (Jones, 2001; Laborde, 1993).

#### 4.5.2 Some Participants.

To qualitatively analyze the change or the development of students' learning and problem solving skills, three participants who showed a significant improvement in their overall achievement score and their problem solving skills score are selected and their work is deeply analyzed.

a. List all the properties of a rhombus (include sides', angles', and diagonals' properties.
b. Given a parallelogram ABCD where AB = 7 cm, BD = 8 cm, and AC = 12 cm.

i. Draw a figure.

ii. Is ABCD a rhombus? Justify your answer.

Figure 8 Question number 1 in the Pre-Test

**4.5.2.1** ***Participant 1.*** Participant 1 tests are chosen to be analyzed because of two reasons. First, an increase of 89.5% in the overall achievement grade mark (from 9.5 to 18) between pre and post test is realized. Second, comparisons between pre and post test indicates a development in the reasoning skill “realizing patterns to seek solutions”.

In question 2 of the pretest (Refer to Figure 9), part c is the only part in the problem that requires students to stop dealing with the given numbers and measures and shift to observe, recognize equal ratios, and figure out a ratio of segments that is equal to the given ratio. Participant 1 achieved a full score on the previous parts of the problem, but failed to solve part c. Note that her solution of parts a and b of this problem is presented in a neat and complete way where no evidence of confusion or misconception was observed (Refer to Figure 10).

However, in the analysis of question 2 of the participant’s post test and on a similar question that requires the skill of “realizing equal ratios” a progress is observed (Refer to Figure 11). This progress towards realizing patterns (such as equal ratios) is attributed to the use of DGS. This is because in the activity sheets that students solved on DGS, there is emphasis on realizing patterns, tabulating them, and then using them to make conjectures (Refer to Appendix E, Activity Sheets). Thus, students are more aware of looking at the problem from a perspective of realizing invariants that usually hold a theoretical reason.

Consider a scalene triangle ABC where M, N and P are the respective midpoints of the sides [AB], [BC] and [AC]. Let O be the meeting point of the three medians (centroid). Given AO=4cm, AN=6cm, BO=3cm, and BP=4.5.

**a**.

i. Find the ratios $\frac{AO}{ON}$ and $\frac{BO}{OP}$

ii. What do you notice? Write a proportion

iii. Find AO in terms of ON and BO in terms of OP

**b.**

i. Find the ratios $\frac{AO}{AN}$ and $\frac{BO}{BP}$

ii. What do you notice? Write a proportion

iii. Find AO in terms of AN and BO in terms of BP

**c.** Using the relations in parts a. and b.,

i. what are the two segments whose ratio is equal to $\frac{ON}{AN}$

ii. Conjecture a property of the centroid

Figure 9. Question number 2 of Pre-Test

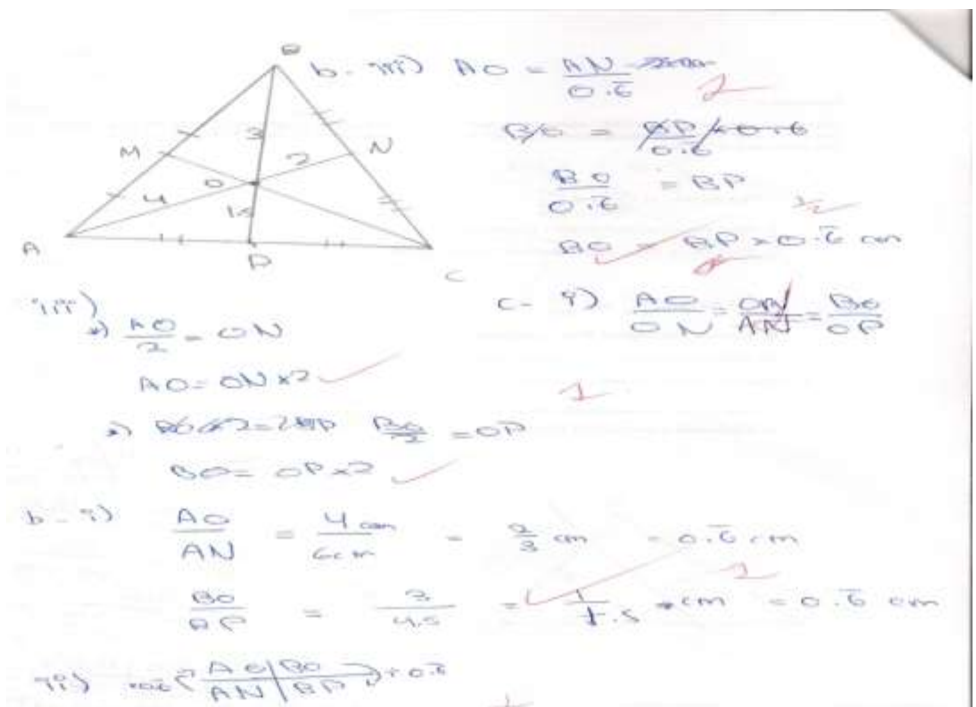


Figure 10. Solution of number 2 by Participant 1 in Pre-Test

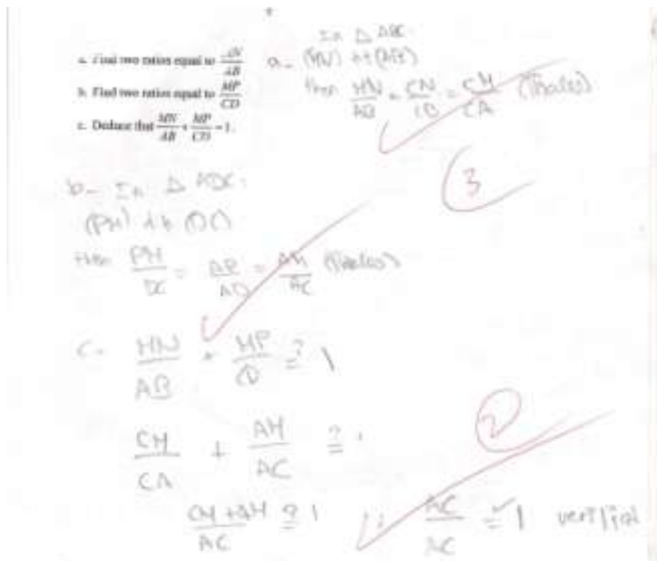


Figure 11. Solution of number 2 by Participant 1 in Posttest

4.5.2.2 ***Participant 2.*** Participant 2 tests are chosen to be analyzed for two reasons. First, an increase of 157% in the overall achievement grade mark (from 7 to 18)) between pre and post test took place. Second, the participant shows a development in the ability to manipulate and deal with proportional relations.

In the analysis of question 2 in pre-test (Refer to Figure 12), the participant correctly solved the parts that require a substitution of numbers to get the values of ratios (that are parts: a (i. ii); b (i, ii)) and failed to solve the parts that require manipulation of equivalent ratios to get the measure of a segment in terms of another segment (such as parts: a (iii) and b (iii)) (Refer to figure 12). Moreover, the participant showed the ability to realize relations that require a rather high level of problem solving ability such as part c of the same problem. Thus, this failure in parts a (iii) and b (iii) cannot be attributed to his reasoning level or achievement level.

However, in the post test the participant achieved a great success in dealing with or manipulating proportional relations (Refer to Figure 13). This success reflects a full and deep understanding of proportional relations that can be attributed to DGS-CA implementation. This is because the unit through its plan and its DGS-CA based activities focuses on building solid proportional reasoning skills of students. Note that prior to the administration of the diagnostic test, all participants had learned the chapter on "proportionality" and this manipulation cannot be due to lack of knowledge on the proportionality topic.

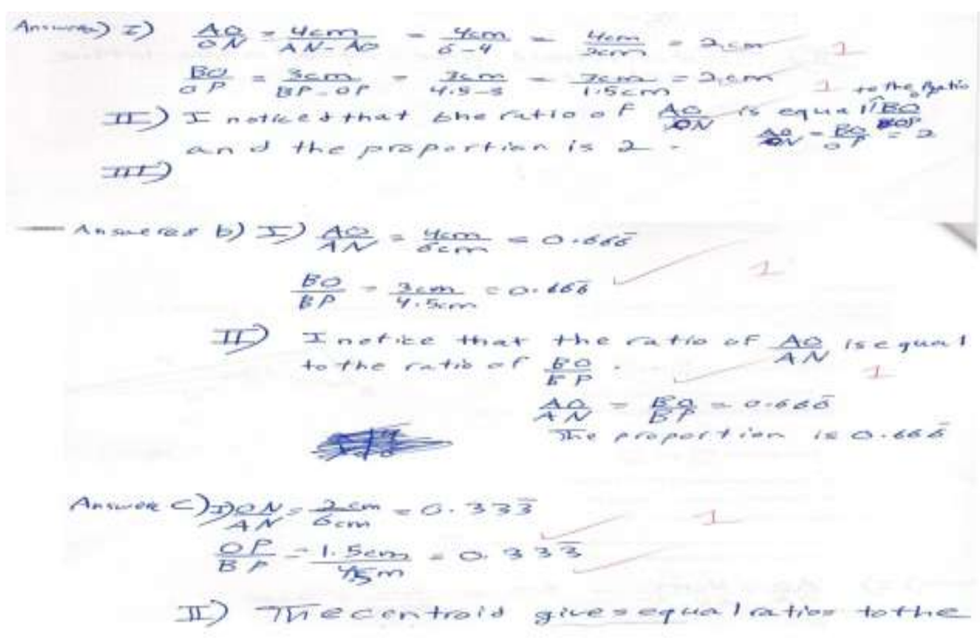


Figure 12 Solution of Question 2 by Participant 2 in Pre-Test

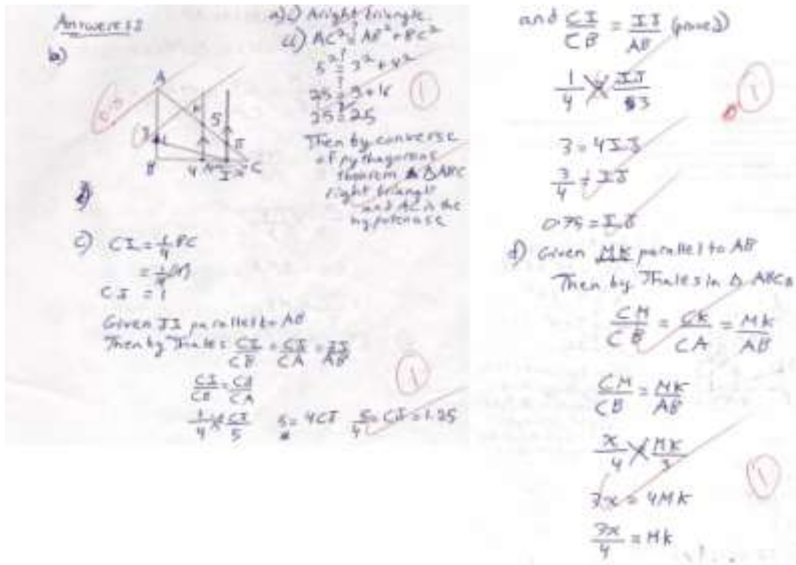


Figure 13 Solution of Question 2 by Participant 2 in Post-Test

# CHAPTER 5

# DISCUSSIONS AND CONCLUSION

The present study aimed to investigate the effect of DGS-CA on Lebanese students' achievement and on their problem solving skills development in geometry, as well as on their motivation toward mathematics. More specifically, the research aimed to answer the following questions:

- Research Question 1: How is Thales Property addressed in the Lebanese mathematics curriculum and in the national textbook "Building up Mathematics"?
- Research Question 2: What is the effect of the DGS-CA on students' overall learning of Thales Property as reflected by their academic achievement (or the results of a test)?
- Research Question 3: What is the effect of the DGS-CA on students' geometric problem solving strategies?
- Research Question 4: What is the effect of the DGS-CA on students' motivation toward mathematics in general and particularly geometry?

Based on the data analysis and on a synthesis of results from various instruments, the following section attempts to answer the above questions.

## 5.1 Research Question 1

How is Thales Property addressed in the Lebanese mathematics curriculum and in the national textbook “Building up Mathematics”?

The unit on Thales’ Theorem in national textbook was analyzed in terms of its approach, presentation, and content. It is realized that the presentation of the chapter does not put in practice the method of problem-solving-based approach and thus it does not allow students to model real life problem situations into geometrical ones. Moreover, the activities presented in the textbook chapter do not account for the role of problem solving and reasoning as a context for learning mathematical concepts. The unit disregards developing students’ problem solving skills such as making and formulating conjectures based on previously observed relationships and applying deductive reasoning, drawing logical conclusions…etc. As a result, the chapter does not reflect the general objectives of the LMC where students are supposed to construct their knowledge under reasoning procedures. As for the content, the unit focuses on the applicability of Thales’ Theorem in a triangle and disregards its applicability in any geometry problem where secants cut parallel lines such as in trapezoids. It can be concluded that the content of the chapter does not cover the requirements of the exercises and problems presented in that chapter.

By implementing the unit developed for the purpose of this research, the researcher tried to bridge this gap between the general objectives of the curriculum advocating problem-solving-based approaches and the textbook.

## 5.2 Research Question 2

What is the effect of the DGS-CA on students' overall learning of Thales Property as reflected by their academic achievement (or the results of a test)?

In order to answer this question, comparisons of achievement scores prior and post to the implementation of DGS-CA were made. The analysis of students' learning of Thales' theorem as reflected by their achievement scores revealed that prior to any implementation, the control and the experimental group differed in their mean achievement score in favor of the control group (control group's mean was higher than that of experimental group); however, this difference was not significant. In the analysis of their achievement scores post to the implementation of DGS-CA, the control and the experimental group differed in their mean achievement score in favor of the experimental group, but this difference was not significant either. Thus, it can be concluded that even though the post scores were non-significantly different, DGS-CA has actually positively affected the learning of Thales' Theorem. This is because the results prior to DGS-CA and post to it shifted in favor of the experimental group. Moreover, a qualitative analysis of the experimental group students' tests revealed that their strategies and proportional manipulation skills improved from pre to post as explained in chapter 4. Though a relatively short time was spent on DGS, students' level of understanding proportional relations apparently increased according to the researcher. It is believed that DGS-CA aided their understanding. The results of a study similar to this have stated that when students are primary users of technology, they engage in learning and attaining higher levels of understanding (Sheehan & Nillas, 2010). It can also be interpreted that this achievement was not significant due to many reasons which include the relatively short time that students directly worked with DGS. Longer periods of usage of DGS might have affected students' achievement more notably. Funkhouser and Almeqdadi (2002) found that the use of geometric software after a long period of time leads to increased student achievement. In another study, the technology-constructivist-based group outperformed the traditional group in academic achievement, yet the study was performed over a period of 9 weeks that equate 40 hours of instruction (Jong, 2005). Hence, using the software over a longer period of time would produce more ample differences in the results.

### 5.3 Research Question 3

What is the effect of the DGS-CA on students' geometric problem-solving strategies?

The comparisons that were made to examine the effect of the DGS-CA on problem solving skills revealed that there was non-significant difference in the average problem-solving skills of groups in the pre-test. However, a significant difference in the average problem-solving skills appeared in the post-test in favor of the experimental group. As a result, DGS-CA had a main effect on students' problem-solving skills. The integration of technology in an approach that emphasizes students' construction of knowledge, reasoning procedures, continuous interaction and discussion of thoughts, assisted in the development of students' problem-solving skills. The results of this study consent with many studies that applied a similar approach (Malabar & Pountney, 2002; Jurdak & Nakhal, 2008; Jones, 2005).

Further comparisons revealed that the control and the experimental group had a non-significant difference in problem-solving skills 4 and 5 in the pretest; meaning that they showed similar levels of conjecturing and proving prior to DGS-CA. However, the experimental group scored a higher but non-significant score on problem-solving skill 4 and a significantly higher average on problem-solving skill 5. That means, students showed improvement in the skills of conjecturing and proofing. The fact that the development of problem-solving skill 4, that is making conjectures, was not significant might have resulted from the fact that the tests were pencil-paper based and students did not have the chance to build their conjectures using the special features of Cabri software. If tests were Cabri-based, it is predicted that students would have shown a better improvement in conjecturing skills.

On the other hand, Marrades and Guttierrez (2000) argue that it will take students several years to progress from reasoning experimentally to reasoning formally. Another study pointed that students need a considerable amount of time working with DGS in order to reach recognized deductive justifications skills (Miyazaki & Yumoto, 2009). This brings back to the relatively short period of time the students used DGS-based conjecturing, which is not enough for them to transfer this ability to paper-pencil settings.

However, in the present study, students' proving ability developed significantly, which agrees with many studies that analyzed the effect of DGS on proving abilities of students. Moreover, further comparison was made on problem-solving skills 4 and 5 within pre and post experimental which revealed a significant effect of DGS-CA on the development of these skills. After the implementation of DGS-CA, experimental group showed a significant improvement in both problem-solving skills of conjecturing and proving compared to their own pre condition. This result agrees with Vincent's (2005) study which demonstrated that the software helps students achieve better understanding of and appreciation for geometric proof enabling them to conduct better proofs. Besides, Malabar and Pountney (2002) have found that technology-constructivist-approach broadens student's skills base. Results of another study that compared the level of reasoning in proofs produced by grade eight students show a positive effect with students instructed in a Cabri learning environment (Jurdak & Nakhal, 2008). Jones also argued that dynamic geometry software facilitates and enhances proof and proving (2005).

**5.4 Research Question 4:**

What is the effect of the DGS-CA on students' motivation toward mathematics in general and particularly geometry?

Comparison between post control and post experimental groups revealed a significant positive effect of DGS-CA on students' motivation towards mathematics and towards geometry (Themes 1 and 2). However, the experimental group's students showed a higher but non-significant difference in their needs for the features of Cabri. An explanation would probably be that they have seen those needs realized through the actual use of DGS.

In further analysis comparing pre experimental and post experimental motivation, students after DGS-CA implementation showed a higher but non-significant increase in the level of motivation towards mathematics subject in general and towards geometry in specific (themes 1 and 2). This non-significance can be due to the short time lag between pre and post. Moreover, the experimental group showed a higher need and appreciation for the features of Cabri after DGS-CA implementation but this increase was also non-significant. This result concurs with other studies' suggestions that technology use might not lead to significantly positive attitudes toward math as a subject (Funkhouser, 2002; Hull & Brovey, 2004).

Even though statistical results did not show any significant changes in motivation, the researcher realized that with DGS, mathematics has changed in the eyes of students. This was concluded from the personal interaction that took place between the researcher and her students and the record that she kept of their comments which explained that mathematics has become an interesting discipline to discover and investigate. The students appreciated this way of doing mathematics using Cabri. They enjoyed experimenting perfectly drawn figures and found them helpful.

## 5.5 Conclusion

As technology continues to advance and become available and affordable, it is increasingly incorporated by teachers and increasingly becoming a subject of interest for researchers. This study demonstrates the use of Cabri software in a constructivist approach where the activities are designed in a way that stimulates students' reasoning and problem solving skills. Thirty-two Lebanese grade 9 students were selected for this study and were divided in two groups (control and experimental). The experimental group studied Thales' theorem under DGS-CA while the control group studied the same chapter in a traditional approach. The total hours for treatment were 15 sessions (50 minutes each). The instruments used in this research were an interview with the teacher of the control group, questionnaire for students' motivation towards mathematics, and tests (pre-test and post-test). It can be concluded that DGS-CA enhances students' geometric achievement but insignificantly; they were better able to manipulate and deal with proportional relations. Besides, the students' problem solving skills were considerably positively affected after the implementation of DGS-CA, specifically their ability to prove and write justifications for their conjectures. It cannot be said that DGS-CA increased students' motivation toward mathematics or geometry.

### 5.6 Limitations

The study has several limitations. First, the results obtained cannot be generalized because the sample is small (32 students) and conveniently selected, hence not representative of Lebanese students learning under LMC. In addition, the time allotted for the study is short. The total number of taught DGS-CA sessions are 15 of which 3 sessions are assigned to work on Cabri software. Also, not every student had the chance to individually work on a computer; sometimes students shared the same computer and took turns to apply the activities. A final limitation is that students used Cabri software for the first time in this study and they took time to gain familiarity with the software which might have affected them focusing on the problem at hand.

### 5.7 Perspectives for Further Research

To be able to generalize the results of this study, further research is needed to apply this research or similar ones with a bigger and representative sample. Moreover, similar studies that investigate the effects of DGS-CA need to be done for a longer period of time in which students will have the chance to spend more time working on DGS. Additionally, studies should be conducted to investigate the effect of DGS-CA where tests are made on the software and are not only paper-and-pencil based.

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**Appendix A**

**Interview Questions**:

1. How do you introduce a mathematical topic/chapter? What do you start with? How do you develop the concept?
2. What kinds of learning aids do you use during instruction?
3. What kinds of questions you ask your students? How heavily do you rely on questioning students during your teaching?
4. Have you ever taught math or illustrated a topic through technological software?
5. Do you apply group work from time to time? How frequently?
6. How do you enhance reasoning and problem solving skills of your students?
7. Tell us more about your instructional style. How do you teach?

## Appendix B

## Questionnaire for Motivation

**Directions:** For each statement, circle the letter that shows how closely you agree or disagree with each statement. SD (Strongly Disagree), D (Disagree), A (Agree), and SA (Strongly Agree).

| | | | | |
|---|---|---|---|---|
| 1. Mathematics is enjoyable and interesting to me. | ○ SD | ○ D | ○ A | ○ SA |
| 2. I am interested and willing to acquire further knowledge of mathematics. | ○ SD | ○ D | ○ A | ○ SA |
| 3. I enjoy going beyond the assigned work and trying to solve new problems in math. | ○ SD | ○ D | ○ A | ○ SA |
| 4. Mathematics is boring because it leaves no room for personal opinion. | ○ SD | ○ D | ○ A | ○ SA |
| 5. Geometry problems are very exciting. | ○ SD | ○ D | ○ A | ○ SA |
| 6. Mathematics makes me feel confused. | ○ SD | ○ D | ○ A | ○ SA |
| 7. I usually enjoy geometry classes. | ○ SD | ○ D | ○ A | ○ SA |

| | | | | |
|---|---|---|---|---|
| 8. I wish I can rotate in my mind a geometrical figure to better see the solution of a geometry problem | ○ SD | ○ D | ○ A | ○ SA |
| 9. I have never liked math classes. | ○ SD | ○ D | ○ A | ○ SA |
| 10. Having to solve geometry problems makes me nervous. | ○ SD | ○ D | ○ A | ○ SA |
| 11. Math helps develop people's logic and teaches them how to think. | ○ SD | ○ D | ○ A | ○ SA |
| 12. There is nothing creative about geometry, it is just about memorizing formulas. | ○ SD | ○ D | ○ A | ○ SA |
| 13. Geometry problems are meaningless. | ○ SD | ○ D | ○ A | ○ SA |
| 14. It is very difficult for me to see geometrical properties in a figure. | ○ SD | ○ D | ○ A | ○ SA |
| 15. Mathematics allows me to develop good logic. | ○ SD | ○ D | ○ A | ○ SA |
| 16. At many times, I apply geometry to solve real life problems or to do things outside the classroom | ○ SD | ○ D | ○ A | ○ SA |
| 17. I feel that I am creating something interesting when I am solving a geometry problem. | ○ SD | ○ D | ○ A | ○ SA |
| 18. I enjoy observing relations between different | ○ SD | ○ D | ○ A | |

| | | | | |
|---|---|---|---|---|
| parts of a geometry problem. | | | | ○ SA |
| 19. Math contributes to other sciences and fields of knowledge. | ○ SD | ○ D | ○ A | ○ SA |
| 20. Learning mathematics is constantly discovering something new. | ○ SD | ○ D | ○ A | ○ SA |
| 21. When confronted with a geometry problem, I cannot move parts of the figure in my mind. | ○ SD | ○ D | ○ A | ○ SA |
| 22. The geometry chapters in my math book are unattractive. | ○ SD | ○ D | ○ A | ○ SA |
| 23. I wish I can move a geometrical figure to solve a locus question. | ○ SD | ○ D | ○ A | ○ SA |
| 24. When I am solving a geometry problem, I prefer to construct the figure from scratch rather than working on a ready-made figure | ○ SD | ○ D | ○ A | ○ SA |
| 25. While working on a geometric proof, I wish I can measure different parts of a geometrical figure (segments, angles…etc.) | ○ SD | ○ D | ○ A | ○ SA |

# Appendix C

## Diagnostic Test

## (50 minutes)

**Question 1:**

**a.** List all the properties of a rhombus (include sides', angles', and diagonals' properties).

**b.** Given a parallelogram ABCD where AB = 7 cm, BD = 8 cm, and AC = 12 cm.

i. Draw a figure.

ii. Is ABCD a rhombus? Justify your answer.

**Question 2:**

Consider a scalene triangle ABC where M, N and P are the respective midpoints of the sides [AB], [BC] and [AC]. Let O be the meeting point of the three medians (centroid). Given AO=4cm, AN=6cm, BO=3cm, and BP=4.5.

**a**.

i. Find the ratios $\frac{AO}{ON}$ and $\frac{BO}{OP}$

ii. What do you notice? Write a proportion

iii. Find AO in terms of ON and BO in terms of OP

**b.**

i. Find the ratios $\frac{AO}{AN}$ and $\frac{BO}{BP}$

ii. What do you notice? Write a proportion

iii. Find AO in terms of AN and BO in terms of BP

**c.** Using the relations in parts a. and b.,

i. what are the two segments whose ratio is equal to $\frac{ON}{AN}$

ii. Conjecture a property of the centroid

**Question 3:**

ABCD is a parallelogram with center O. P and S are the symmetric points of B and D, respectively, with respect to C.

**a.** What is the nature of BSPD? Justify.

**b.** What is the nature of ACSB? Justify.

**c.** What is the relative position of (AC) and (DP)? Justify.

**d.** Let (AC) meet (PS) at E. What is the position of E with respect to [PS]?Justify.

**e.** Find CE in terms of DP.

**f.** Let X be the midpoint of [CS] and Y be the midpoint of [SE].

i. Find XY in terms of CE.

ii. Deduce XY in terms of DP.

## Appendix D

## Instructional Unit on Thales' Theorem

# 5 Thales' property

### introduction

**Thales of Miletus** (624-547 B.C.) is a Greek geometer, physicist and astronomer. It is believed that he was the first astronomer to predict the eclipse of the sun.

His name is well-known, being associated with a famous theorem in geometry, the celebrated theorem of Thales to be stated shortly. This theorem was first proved 250 years before being declared in book VI of the "Elements" of **Euclid**.

Legend has it that Thales amazed the pharaoh in determining the height of the largest pyramid **Chepos** by means of its shadow.

*At the beginning of this chapter, I am able to:*

- recognize a proportion;
- calculate the fourth term of a proportion;
- calculate the rate of a proportion knowing two proportional quantities.

*At the end of this chapter, I will be able to:*

- use Thales' theorem and its converse in a triangle;
- construct the fourth proportional geometrically;
- Enlarge or reduce a geometric figure in a given ratio.

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## Recall Activity

Activity

### *The web of the spider geometer!..*

In the following figure, what is the length of each of the segments $[AB]$, $[EF]$ and $[GH]$?

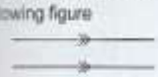

Explain your work.

## Preparatory Activities

Activity 1

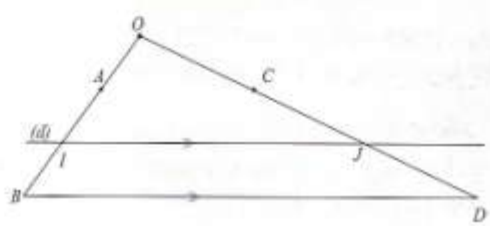
### *P, for parallel or for proportional?*

**Notation:**
Use the arrows as in the following figure



To indicate two parallel lines.

1) Using a graduated ruler, measure the segments to calculate the ratios $\frac{OA}{OB}$ and $\frac{OC}{OD}$, What is the relative position of $(AC)$ with respect to $(BD)$?

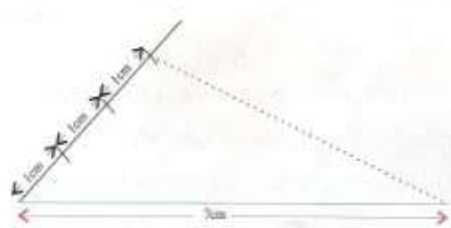



2) If $(d)$ is the line parallel to $(BD)$ from the point $I$ of $[OB]$ such that $OI = 2$ cm, can you figure out the length of $OJ$?

Activity 2

### *To cut a third by a third!..*

To divide the blue segment into three equal parts, Samia drew the black segment and divided it into three segments, 1cm each. Can you complete her work?

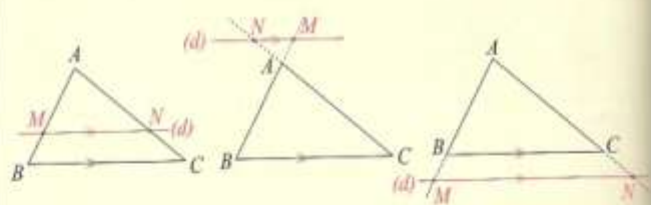


## Text

### I. Thales' theorem

$ABC$ is a triangle, $(d)$ is a line parallel to $(BC)$ cutting $(AB)$ in $M$ and $(AC)$ in $N$. We have:

$$\frac{AM}{AB}=\frac{AN}{AC}$$

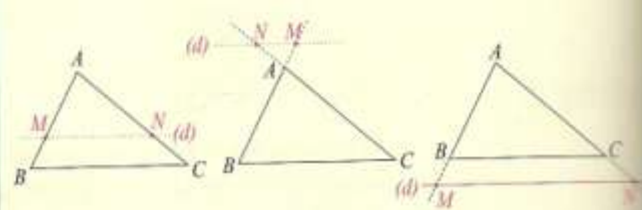



**Hence the line $(d)$ cuts the lines $(AB)$ and $(AC)$ proportionally.**

**PROPERTY 1** Any line parallel to one side of a triangle divides the other two (or their extensions) proportionally.

### II. The converse of Thales' theorem

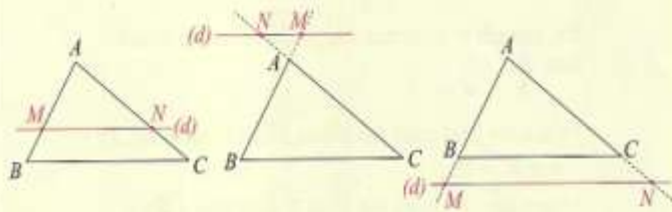
$M$ is a point of $(AB)$ and $N$ is a point of $(AC)$ such that $\frac{AM}{AB}=\frac{AN}{AC}$.
The line $(MN)$ is parallel to $(BC)$ .




**PROPERTY 2** Any line dividing the two sides of a triangle (or their extensions) proportionally is parallel to the third side .

## III. Other properties of Thales' theorem:

If a line $(d)$ is parallel to the side $[BC]$ of triangle $ABC$, then we have the following relations:



$$\frac{AM}{AB}=\frac{AN}{AC}=\frac{MN}{BC} \quad \text{and} \quad \frac{MA}{MB}=\frac{NA}{NC}.$$

## Focus

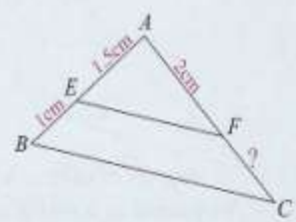
**Thales' theorem and its converse:**

- A line parallel to one side of a triangle divides the other two proportionally.
- Any line dividing the two sides of a triangle proportionally is parallel to the third side.

**Where to use Thales'Theorem and its converse?**

**1) To calculate the length of a segment:**

*Example*




In the adjacent figure, Thales' Theorem allows us to calculate the length $AC$.

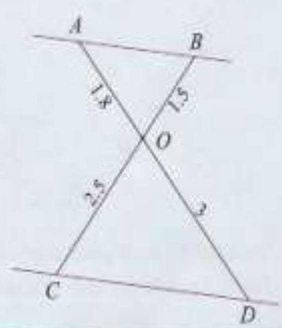
In fact, according to Thales we have:

$\frac{AE}{AB}=\frac{AF}{AC}$, which gives $\frac{1.5}{2.5}=\frac{2}{AC}$ so $AC=\frac{5}{1.5}$;

Consequently, $FC=AC-AF=\frac{5}{1.5}-2=\frac{4}{3}$.

**2) To prove two lines are parallel:**

*Example*




In the adjacent figure we have:

$\frac{OA}{OD}=\frac{1.8}{3}=0.6$ and $\frac{OB}{OC}=\frac{1.5}{2.5}=0.6$.

So, $\frac{OA}{OD}=\frac{OB}{OC}$ and consequently, according to the converse of Thales' theorem, $(AB)$ is parallel to $(CD)$.

### 3) To construct the fourth proportional:

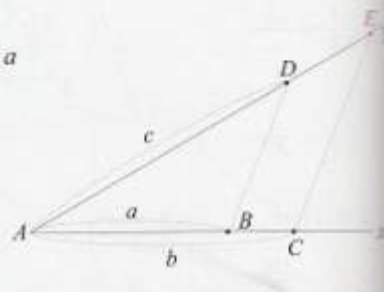

*Example*

To construct a segment whose length is the fourth proportional of the lengths $a$, $b$ and $c$.

It's enough to construct a segment of length $d$ such that: $\frac{a}{b} = \frac{c}{d}$.

* On a ray $[Ax)$ mark the points $B$ and $C$ such that $AB = a$ and $AC = b$.
* On a ray $[Ay)$ mark the point $D$ such that $AD = c$.
* Join $B$ and $D$, then through $C$ draw the parallel to $BD$ cutting $[Ay)$ in $E$.

  According to Thales' theorem we can assert that $AE = d$.



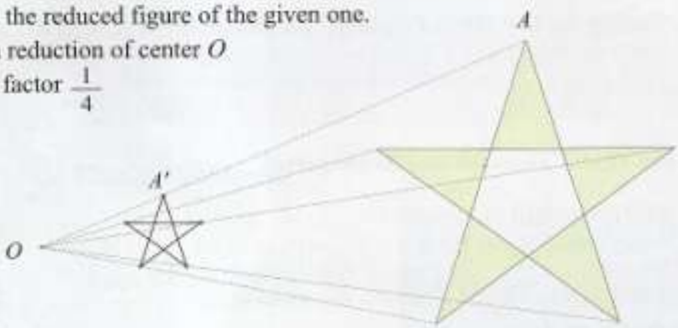

### 4) To reduce a figure in a given ratio:

In the following figure, join the five vertices of the figure to a point $O$. On $[OA]$ mark the point $A'$ such that $\frac{OA'}{OA} = \frac{1}{4}$.

Using $A'$ we draw segments parallel to the different segments of the figure.

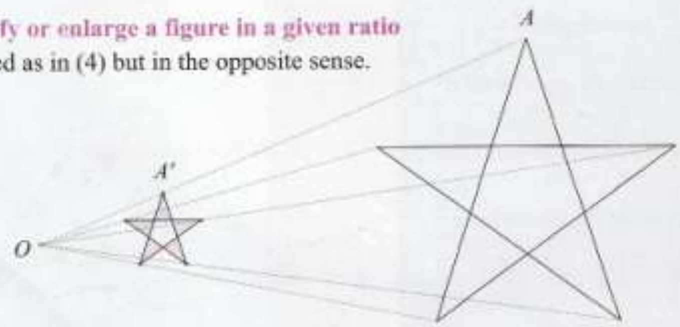
We obtain the reduced figure of the given one.

We have a reduction of center $O$ and scale factor $\frac{1}{4}$




### 5) To magnify or enlarge a figure in a given ratio

We proceed as in (4) but in the opposite sense.




We have an enlargement of center $O$ and scale factor 4.

Note: An enlargement or a reduction is a dilation

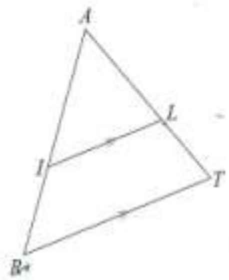
# Exercises

**1.** Complete :

$\frac{AI}{IR} = \frac{\ldots\ldots}{LT}$

$\frac{AI}{AR} = \frac{AL}{\ldots\ldots}$

$\frac{AI}{AR} = \frac{\ldots\ldots}{RT}$

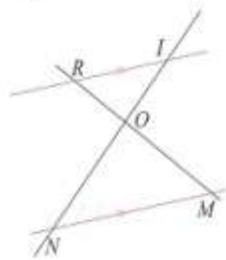



**2.** Complete :

$\frac{OI}{ON} = \frac{OR}{\ldots\ldots}$

$\frac{OI}{OR} = \frac{ON}{\ldots\ldots}$

$OR \times \ldots\ldots = OI \times OM.$

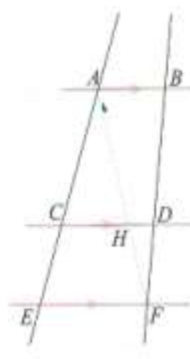



**3.** Complete :

$\frac{AC}{CE} = \frac{AH}{\ldots\ldots} = \frac{BD}{\ldots\ldots}$

$\frac{BD}{BF} = \frac{\ldots\ldots}{AF} = \frac{AC}{\ldots\ldots}$

$\frac{AE}{AC} = \frac{BF}{\ldots\ldots}$




**4.** Knowing that $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, show that :

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f}.$$

Hint : Let $k = \frac{a}{b}$, then $a = kb$ ... ).

**5.** In the following figure $(MN)$ is parallel to $(XY)$. Determine which of the following ratios is true:

a) $\frac{OM}{MX} = \frac{ON}{NY}$ ;

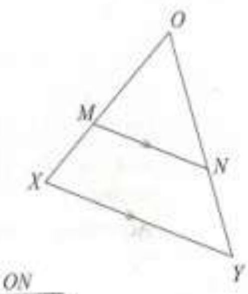
b) $\frac{OX}{MX} = \frac{OY}{NY}$ ;

c) $\frac{OM}{ON} = \frac{OX}{OY}$ ;

d) $\frac{OX}{OM} = \frac{OY}{ON}$ ;

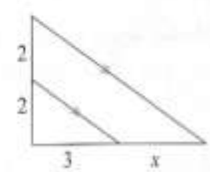
e) $\frac{ON}{OM} = \frac{MX}{NY}$ ;

f) $\frac{OM}{OX - OM} = \frac{ON}{OY - NY}$ .

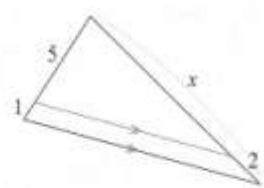



**6.** Determine the value of $x$ in each of the following cases :

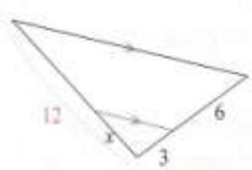
a)



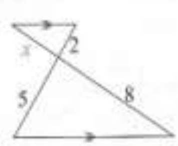

b)



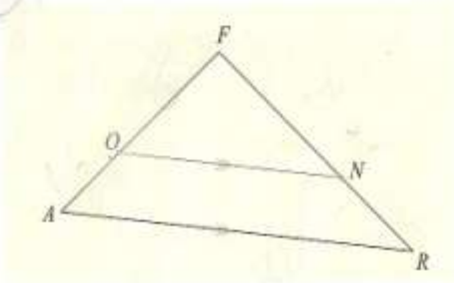

c)




d)



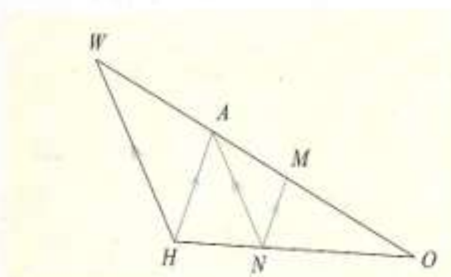
**7.** In triangle $FAR$ of the following figure we have $(ON)$ parallel to $(AR)$.




In each of the following cases, fill in the empty blanks in the following table:

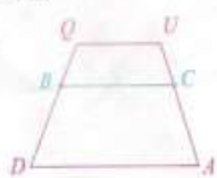
| | $FO$ | [illegible] | $FA$ | $FN$ | $NR$ | $FR$ |
|---|---|---|---|---|---|---|
| a) | 8 | | 12 | | 3 | |
| b) | 6 | 3 | | 12 | | |
| c) | | | 15 | 10 | 4 | |
| d) | | 9 | 36 | | | 48 |

**8.** In the following figure we have $(WH)$ and $(AN)$ are parallel, and so are $(AH)$ and $(MN)$.




Show that : $\dfrac{MO}{AM} = \dfrac{AO}{AW}$.

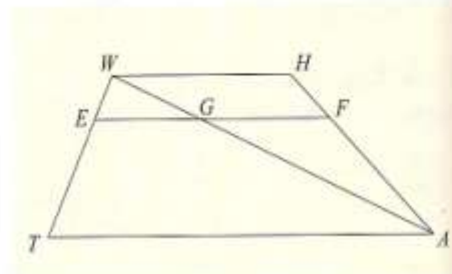
**9.** The nonparallel sides $[QD]$ and $[UA]$ of a trapezoid $QUAD$ are cut by a line parallel to the bases in $B$ and $C$ respectively.




Show that $\dfrac{QB}{BD} = \dfrac{UC}{CA}$.

Find a ratio equal to $\dfrac{BC}{DA}$.

**10.** $WHAT$ is any trapezoid. A straight line parallel to the bases cuts $[WT]$, $[WA]$, and $[HA]$ at $E$, $G$, and $F$ respectively.

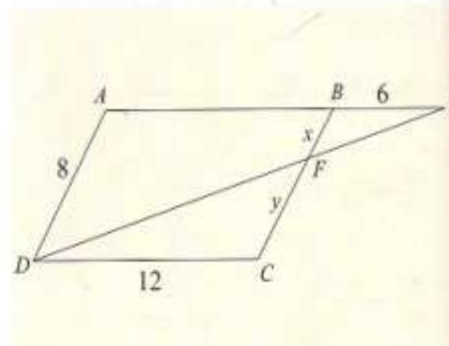



a) Calculate $WE$ and $ET$ such that $WT = 30$ and $2FA = 3HF$.

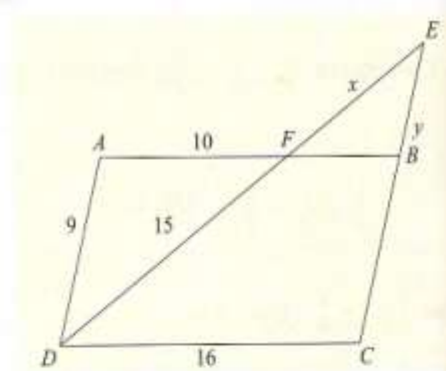
b) Calculate $HF$, $FA$, $WE$ and $ET$ such that $HA = 8$, $WT = 12$ and $GA = 2WG$.

**11.** In each of the following figures, $ABCD$ is a parallelogram. Calculate $x$ and $y$.

a)




b)

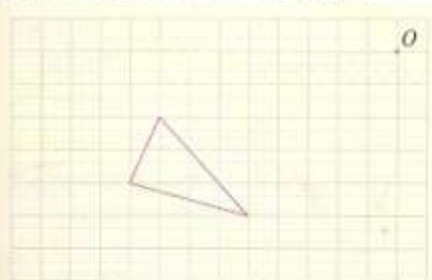


**12.** Given two points $O$ and $A$, 7 cm apart.

a) Construct the point $M$ of $[OA]$ such that $\frac{OM}{OA} = \frac{1}{3}$.

b) Construct point $N$ de $[OA)$ tel que $\frac{ON}{OA} = 3$.

**13.** a ) Using checked squares on your copybook reproduce the following figure.




b) Construct a reduction of $ABC$ with center $O$ and scale factor $\frac{3}{4}$ .

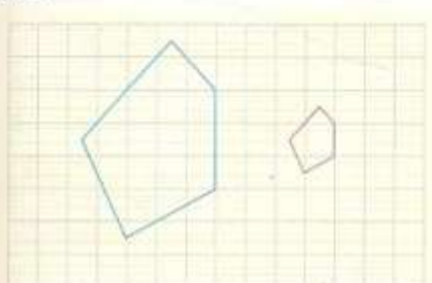
c) Reproduce the same figure and enlarge $ABC$ with $O$ as a center and scale factor 2.

**14.** a) Given a circle $C(O\,;4)$.

b) $M$ is a point of $C$. Construct the point $N$ of $[OM]$ such that $OM = 3ON$.

c) Using a center of your choice, construct an enlargement of the figure in which the image of $[ON]$ has a length of 2. What is the scal factor of this enlargement?

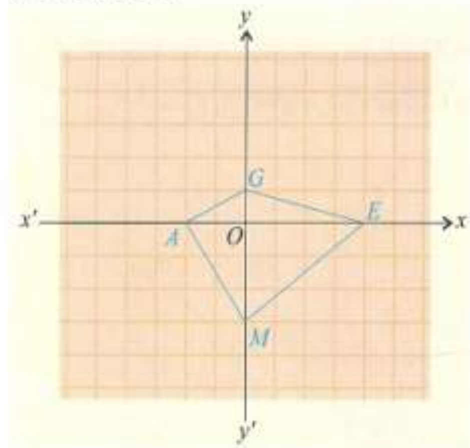
**15.** The red figure is a reduction of the blue figure.



a) Using the checked squares on your copybook reproduce the figure above.

b) Determine the center and the scale factor of this reduction.

**16.** a) Reproduce the figure below in a system of axes $x'Ox$, $y'Oy$.




b) Determine the coordinates of the points $G$, $A$, $M$ and $E$.

c) Construct the quadrilateral $FIND$ by enlarging $GAME$ with center $O$ and scale factor 1.5 .

d) Determine the coordinates of the points $F$, $I$, $N$ and $D$.

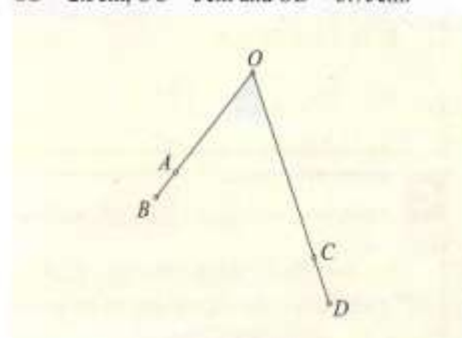
e) Construct $PO'ST$ by reducing $GAME$ with $O$ as a center and a scale factor of 0.5.

f) Determine the coordinates of the points $P$, $O'$, $S$ and $T$.

g) Applying Pythagoras' theorem, calculate the lengths of the sides of $GAME$.

h) Deduce the lengths of the sides of $FIND$ and of $PO'ST$.

**17.** In the following figure, we have $OA = 2$cm, $OB = 2.5$cm, $OC = 3$cm and $OD = 3.75$cm.

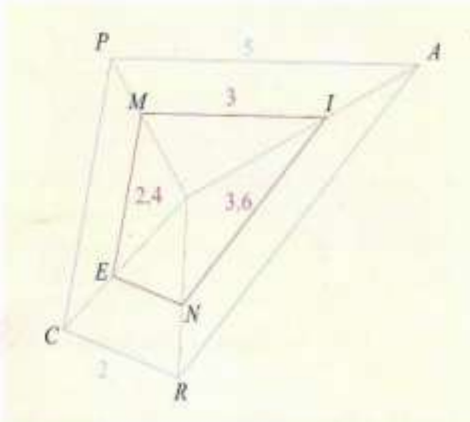



Show that $(AC)$ and $(BD)$ are parallel.

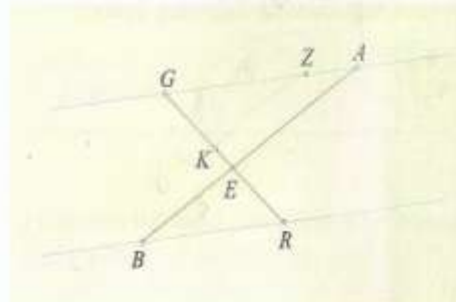
**18.** In the following figure *PARC* is the enlargement with center *O* of *MINE*.

a) What is the scale factor of this enlargement?

b) Calculate the lengths *AR*, *EN* and *PC*.




**19.** In the following figure the lines (*AG*) and (*RB*) are parallel. (*AB*) and (*RG*) intersect in *E*. Take the centimeter as a unit of length. Given that $BE = 3$; $AE = 5$; $AG = 10$ and $EG = 8$. (*The dimensions may not be represented correctly on the diagram*).




a) Calculate the distances *RB* and *RE* (justify).

b) Given $GK = 6.4$ and $GZ = 8$. Show that (*ZK*) and (*AE*) are parallel.

## Self-evaluation

**A**

*ABC* is any triangle, *D* is a point of [*AB*] and *E* a point of [*AC*]. Tell whether each of the following statements is true or false:

a) If $\frac{DB}{AD} = \frac{EC}{AC}$, then (*DE*) is parallel to (*BC*).

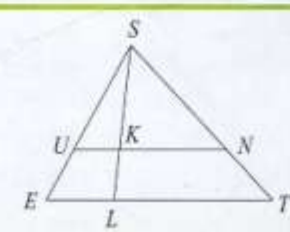
b) If $\frac{AB}{AD} = \frac{AC}{AE}$, then (*DE*) is parallel to (*BC*).

c) If (*DE*) is parallel to (*BC*), then $\frac{AD}{AE} = \frac{AB}{AC}$.

**B**

*SET* is a triangle, *L* is a point of [*ET*], and *U* a point of [*SE*]; the parallel through *U* to *ET* cuts *SL* in *K* and *ST* in *N*.

Show that $\frac{UK}{KN} = \frac{EL}{LT}$ .



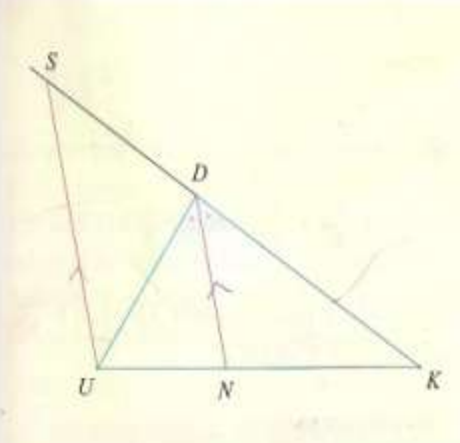

**C**

a) In a system of axes $x'Ox$, $y'Oy$, plot the points $D(-1\ ;\ 1)$; $E(2\ ;\ 2)$; $U(3\ ;\ -1)$ and $X(-3\ ;\ -2)$.

b) Construct *CINQ* by reducing *DEUX* with scale factor 0.5 and center O.
Determine the coordinates of its vertices.

## Problems

**1.** $DUK$ is any triangle. The bisector of $\hat{D}$ cuts $(UK)$ in $N$. The parallel to $(DN)$ drawn through $U$ cuts $(DK)$ in $S$.

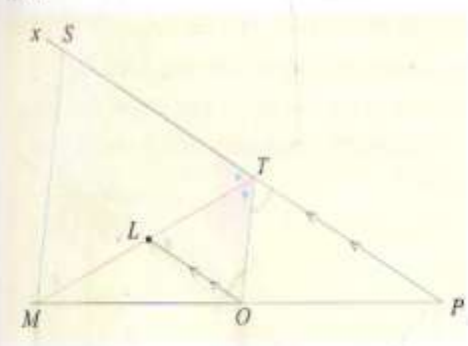



a) What is the nature of triangle $SUD$? Justify your answer.

b) Show that $\frac{NU}{NK} = \frac{DU}{DK}$.

c) Calculate $UN$ and $NK$, knowing that $DK = 15$, $UK = 18$ and $DU = 10$.

**2.** $TOP$ is any triangle, $[Tx)$ is the ray obtained by producing $[TP]$. The bisector of $\widehat{OTx}$ cuts $(PO)$ in $M$. The parallel from $M$ to $(TO)$ cuts $(Tx)$ in $S$.

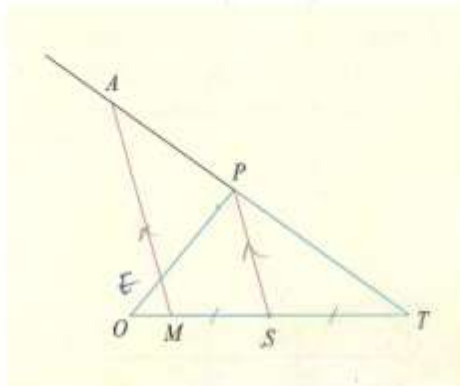



a) What is the nature of $MST$? Justify your answer.

b) Show that $\frac{OM}{SM} = \frac{OP}{TP}$.

Use $[OL]$ to show that $\frac{MO}{MP} = \frac{TO}{TP}$

**3.** $POT$ is a triangle, $S$ is the midpoint of $[OT]$, $M$ is a point of $[OS]$. The parallel from $M$ to $(PS)$ meets $(TP)$ in $A$ and $(OP)$ in $E$.

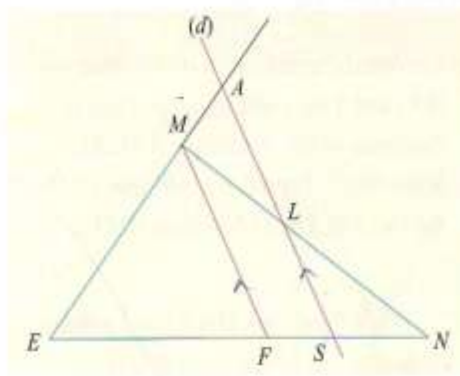



a) Calculate $EM$ in terms of $PS$, $OM$ and $OS$.

b) Calculate $AM$ in terms of $PS$, $TM$ and $TS$.

c) Deduce that $ME + MA = 2PS$.

**4.** $MEN$ is any triangle. $(d)$ is a line which intersects $(EM)$ in $A$, $(MN)$ in $L$ and $(EN)$ in $S$. The parallel through $M$ to $(d)$ cuts $(EN)$ in $F$.

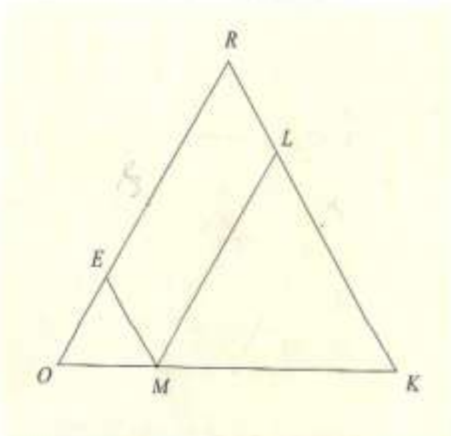



a) Show that $\frac{LN}{LM} \times \frac{SF}{SN} = 1$.

b) Show that $\frac{SE}{SN} \times \frac{AM}{AE} = \frac{SF}{SN}$.

c) Deduce that $\frac{SE}{SN} \times \frac{LN}{LM} \times \frac{AM}{AE} = 1$.

**5.** $ROK$ is an isosceles triangle of vertex $R$. $M$ is a point of $[OK]$. The parallel through $M$ to $(OR)$ intersects $(RK)$ in $L$, and the parallel through $M$ to $[RK]$ intersects $[RO]$ in $E$.




a) Draw the figure.

b) What is the nature of the quadrilateral $MERL$? Justify your answer.

c) Prove that both triangles $OME$ and $LMK$ are isosceles.

d) Show that $\frac{MO}{MK} = \frac{RL}{RE}$.

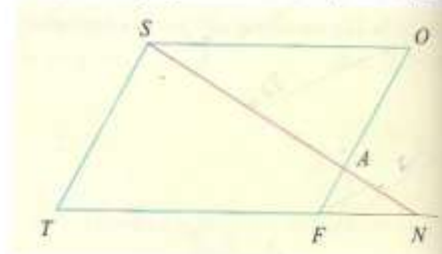
e) Let $I$ be the midpoint of $[OR]$, $J$ that of $[RK]$ and $S$ the point of intersection of the diagonals of the quadrilateral $MERL$. Show that $I$, $J$ and $S$ are collinear. Deduce the locus of $S$ as $M$ describes $[OK]$.

**6.** $(D)$ is a fixed line, $O$ is a fixed point outside $(D)$, and $M$ is a point of $(D)$.

a) Construct a point $N$ such that $ON = 3OM$ and $O$, $N$ and $M$ are collinear.

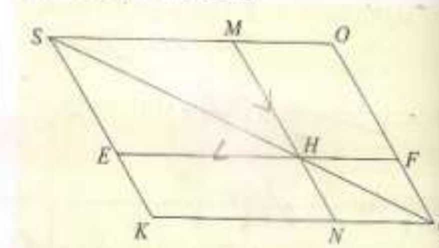
b) Determine the locus of $N$ as $M$ moves on $(D)$.

**7.** $SOFT$ is a parallelogram, $A$ is a point of $[OF]$ and $(SA)$ meets $(TF)$ in $N$. $M$ is the midpoint of $[SA]$ and $E$ is the midpoint of $[AN$



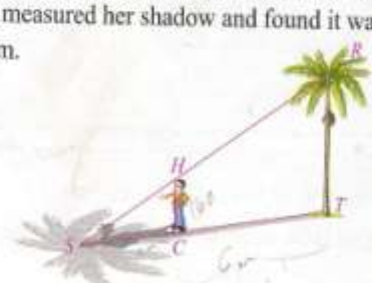

Show that $\frac{AF}{AO} = \frac{FN}{FT}$ and $(OM) \parallel (FE)$.

**8.** $SOCK$ is a parallelogram. Through a variable point $H$ of $[SC]$



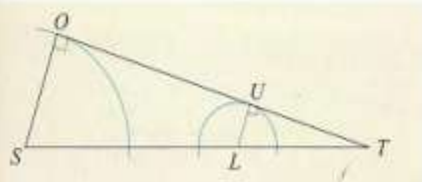

draw the parallel to $[OS]$ which cuts $[SK]$ in $E$ and $[OC]$ in $F$. The parallel to $[OC]$ cuts $[OS]$ in $M$ and $[KC]$ in $N$. Show that:
$\frac{SM}{MO} = \frac{SE}{EK}$.
Find the locus of $I$ midpoint of $[HM]$.

**9.** Rima wanted to find the height of a paln tree using its shadow. She measured the shadow of the tree and found it was 6m long She measured her shadow and found it was 2.4 m.




If Rima was 160 cm tall, how high was the palm tree?

**10.** A person was observing the eclipse of the sun. Suppose this diagram represents this situation.



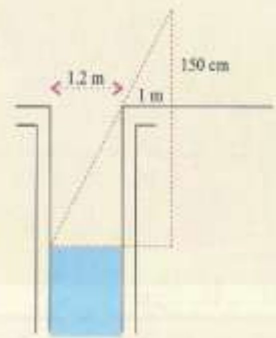

The observer is at $T$. The points $S$ (center of the sun), $L$ (center of the moon), and $T$ are collinear. The radius of the sun $SO$ is 695 000 km. The radius of the moon measures 1736 km, and the distance $TS$ is 150 million km.
Calculate the distance $TL$ (rounded to km).

*Brevet 1997*

**11.** To measure the depth of the level of water in a well, Euclid proposed a technique that we will describe it in the following example:

A girl whose eye is 150 cm above the ground, approaches a well until she sees the level of water in the well.

She measures the distance from this position to the side of the well and finds it to be 1m.She measures the diameter of the well, and finds it to be 1.2m.




How can she calculate the distance from the ground to the level of water?

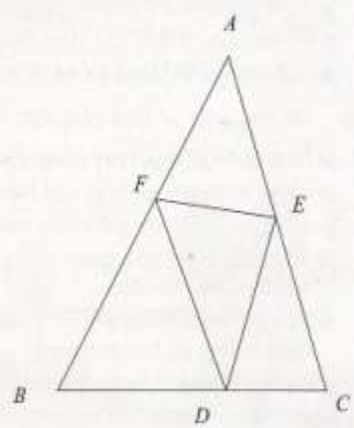
## Just For Fun

### *When a triangle encloses another!..*

$ABC$ is a triangle.

$D$ is a point on $[BC]$, $E$ is a point on $[AC]$, and $F$ is a point on $[AB]$.

For what positions of $D$, $E$ and $F$ are the sides of triangle $DEF$ respectively parallel to those of $ABC$?



## Appendix E

## Instructional Unit on Thales' Theorem

**Setting:**

This sequence of lessons is to take place in a computer lab for some session, in which each student has an access to independent computer with Cabri software installed and in a normal class with a board in other sessions. Students will have to follow the instructions of the teacher; solve, apply, discuss, interact…etc. when necessary.

**Title of the Unit:** Thales' Theorem

**Grade Level:** Grade 9

**Number of sessions:** 15 sessions

**Prerequisites:**

- Recognize a proportion
- Write a proportion knowing the values involved
- Knowing the proportional quantities, calculate the rate of proportionality
- Calculate the fourth term of a proportion
- Use Cabri software features

**General Objectives of the unit following DGS-BCA approach:** At the end of this lesson, students should be able to:

7. Apply the Thales' Theorem in a triangle and in extended triangles

8. Apply the converse of Thales' Theorem in a triangle

9. Construct the fourth proportional

10. Enlarge or reduce a figure knowing the scale factor

11. Apply Thales' Theorem in any context where parallel lines cut by 2 intersecting lines exist

12. Recognize the significance of Thales' theorem in solving real-life problems

**Note** that the first three objectives are the same as those of the LMC objectives. However, the fourth and the fifth objective are not mentioned in the LMC objectives.

Table of sessions:

*Table of sessions*

| Session number | **Topics and skills covered** |
|---|---|
| Sessions 1 | Thales' in a triangle |
| Sessions 2 and 3 | Thales' in an extended triangle and Converse of Thales |
| Session 4 | Drop Quiz |
| Session 5 and 6 | Correction of drop quiz + illustration of some consequences proportions to Thales' |
| Session 7 | Correction of H.W. and explain some tips for h. w. on Cabri |
| Session 8 | Thales' in any problem situation including parallel lines (including trapezoids) |
| Sessions 9 and 10 | Correction of H.W. and explain reduction and enlargement |
| Session 11 | Quiz2 and Thales in real life applications |
| Session 12 and 13 | Correction of H.W. and correction of Quiz2 |
| Session 14 | Correction of Extra Exercises H.W. |
| Session 15 | Test |

Note that in all the sessions, the specific objectives are those of the DGS-BCA and not those of the LMC

## Session 1: (50 minutes)

*Specific objectives of the session*: At the end of this session, students should be able to:

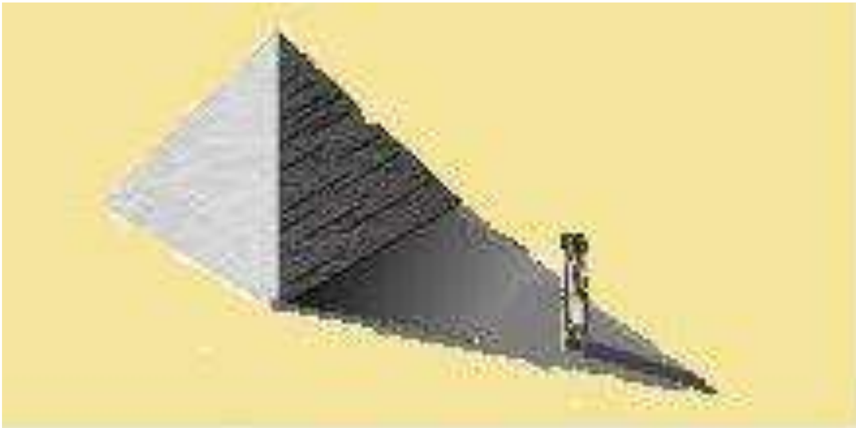
- conjecture Thales' theorem formula by their own involvement in experimental situations on Cabri software
- apply Thales' Theorem in a Triangle

*Procedure outline:*

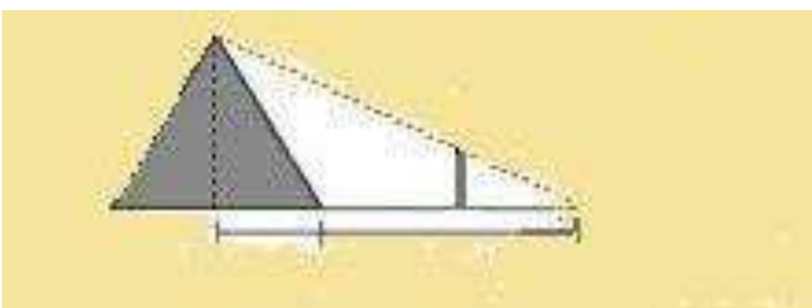
1. Teacher will present a problem situation to the students. Together, the teacher and the students, will model the problem mathematically. (10 minutes)

*Problem statement*:

Osiris wanted to find the height of a pyramid. However in his time (2,000 years BC) there were no tools available to measure the height of the pyramid. So Osiris thought that the shadow of the pyramid might be useful to help him find out the height of the pyramid. He stood in front of the pyramid such that his shadow overlaps the pyramid shadow and both shadows (his and pyramid's) end at the same place. His shadow was 2 meters. He measured the pyramid shadow that was 6 meters. Knowing that Isis is 1.8 meters tall, how do you think Osiris was able to find the height of the pyramid?? How much was it??



The teacher and the students will try to model the problem situation and represent it in mathematical terms. Through discussions, they will represent the problem with a triangle and a segment cutting 2 sides of the triangle and parallel to the third. Afterwards, the teacher will ask the students to start with Activity Sheet 1 in order to conjecture a property that will help them find the height of the pyramid and thus solve the problem.



2. Start with the Activity Sheet1 (Refer to Activity Sheet 1) (20 minutes)
3. Teacher and class discussion (10 minutes)
4. Application

5. Assigning homework

*Teacher and class discussion:*

Teacher will take the generalizations given by the students that is AM/AB=AN/AC, no matter how the shape of the figure is changing. Teacher will ask the students why they think this is the case. Thus, what is the main reason that kept the ratios equal? Students should realize that we started by constructing a parallel line and thus will come up with Thales' Theorem: "Any line parallel to one side of a triangle divides the other two proportionally". Also, students will recognize other properties of Thales' theorem (AM/AB=AN/AC =MN/BC).

*Application:* Refer to Appendix D, p.61 solve number 6 (parts a, b, c)

*Home -Assignment*: Refer to Appendix D, p. 62 solve number 7

## Session 2 and 3: (100 minutes)

*Specific Objectives:* At the end of this session, student should be able to

- apply Thales' Theorem in extended triangles
- apply the converse of Thales' Theorem

*Procedure outline:*

1. Correction of the homework (30 minutes)
2. Start with Activity Sheet 2 (Refer to Activity sheet 2) (40 minutes)

3. Teacher and class discussion (10 minutes)

4. Application 2 and correcting it (20 minutes)

5. Assigning homework

*Teacher and class discussion*:

Teacher takes answers from the students and together come up Defining Thales' Property in the extension of a triangle case.

Teacher emphasize that the definition of Thales' property becomes: "Any line parallel to one side of a triangle divides the other two, or their extensions, proportionally". Teacher will also explain the converse of Thales' theorem and will demonstrate it on the overhead (using fig.1 and 2)

*Application 2:* Refer to Appendix D, p. 61 number 2 and number 6(part d)

*Home Assignment:* Refer to Appendix D, p. 62 number 11 and p. 63 number 17

**Session 4 (50 minutes)**

- Correction of homework (30 minutes)
- Drop Quiz (Refer Quiz 1) to check for understanding (20 minutes)

## Session 5 and 6 (100 minutes)

*Specific objectives:* At the end of this session, student should be able to

- Recognize different forms of Thales' Theorem proportions and apply them.

*Procedure outline:*

1. Correction of quiz 1 and discussing mistakes (30 minutes)
2. Teacher then will use an overhead projector to illustrate some consequence proportions to Thales' Theorem. The teacher will open the Cabri-page of the triangle in Fig. 1 (Refer to Activity sheet 1) and illustrate using the drag mode to verify the following proportions: AM/MB = AN/NC and MB/AB = NC/AC

After that, teacher will try with the class to verify these proportions algebraically by using the proportions rules they already know. Thus, students will see the verification of these proportions algebraically and on the figure. (50 minutes)

3. Application: Refer to Appendix D, p. 61numbers 1, 3 and 5 ( 20 minutes)
4. Homework: Appendix D, p. 62 number 8 and p. 64 number 19

## Session 7: (50 minutes)

*Specific objectives:* At the end of this session, students should be able to

- Solve problems including Thales' theorem

*Procedure:*

1. Correction of the homework exercises (40 minutes)

2. Teacher will assign Activity Sheet 3 as homework on Cabri software (Refer to Activity Sheet 3)

## Session 8: (50 minutes)

*Specific objective:* At the end of this session, students should be able to

- Recognize Thales in any problem situation where lines intersect parallel lines (including trapezoids).

*Procedure:*

1. Teacher will pass and check the soft copy of the assignment and will discuss the results students came up with the class.(5 minutes)
2. *Teacher and student discussion:* (20 minutes)

Teacher will take the generalizations given by the students that is AC/CD=BD/DF, no matter how the shape of the figure is changing. Teacher will ask the students why they think this is the case. Thus, what is the main reason that kept the ratios equal? Students should realize that we started with parallel lines and thus will come up with that when the lines are parallel, the lines intersecting them will always cut them proportionally. And that's why: AC/CD=BD/DF. Finally, the teacher will emphasize Thales' Theorem on the board and will state it clearly, in case one of the students fail to generalize or got lost in the class discussion.

3. *Application*: students will solve alone as a classwork: (15minutes) Refer to Appendix D, p. 62 numbers 9 and 10

4. Teacher will correct the class work application (10 minutes)

5. Teacher will assign *homework:* Refer to Appendix D, p. 65 problems 1 and 2 and 3

## Session 9 and 10: (100 minutes)

*Specific objectives:* At the end of this session, student should be able to

- Solve high level problems containing Thales' Theorem
- Reduce or enlarge a figure knowing the scale

*Procedure:*

1. Checking the H.W.
2. Correction of H.W. on the board (60 minutes)
3. Class Work Refer to Appendix D, p. 66 problems 5 and 8 and correcting them (40 minutes)

Note: in number 8, teacher will allow students to find the locus using the software.

4. Assigning a quiz for the second day.

## Session 11: (50 minutes)

*Specific objectives:*

- Recognize the applicability of Thales' in real life

*Procedure:*

1. Quiz. Refer to Quiz 2 (30 minutes)
2. Classwork: solve problem 9 Refer to Appendix D, p.66
3. Teacher will assign homework In Appendix D, p. 67 number 11

## Session 12and 13: (100 minutes)

*Specific objectives:* At the end of this session, student should be able to

- Solve high level problems containing Thales' Theorem

*Procedure:*

1. Correction of H.W. (20 minutes)
2. Correction of Quiz 2 (30 minutes)
3. Solve Extra Exercises ( 30 minutes)
4. Correcting them on the board (20 minutes)
5. Assign the rest H.W.

**Session 14 (50 minutes)**

Correction of H.W

**Session 15 (50 minutes):** Unit Test Refer to Appendix F

*Activity Sheet 1:*

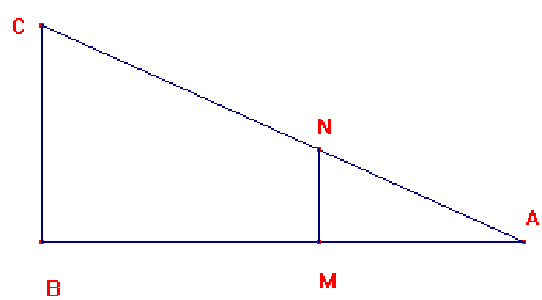
1. Open a blank Cabri page on the computer in front of you

2. Draw a Triangle ABC

3. Join a segment from A to B. Plot a point M on [AB] (using point-on-object option).

From M, draw a line parallel to the base (BC) intersecting [AC] at N.

4. Join a segment from M to N and hide the line so that your figure looks like this:




(Fig. 1)

5. Drag any vertex of the triangle to check if the figure holds the property of parallelism.

6. Calculate the measures of the following segments:

AM, AB, AN, AC

7. Find using the calculator feature of Cabri the following ratios:

AM/AB, AN/AC

8. What do you notice about the value of the ratios obtained in part 7?

9. Drag the point M and tell whether the observation can be generalized.

10. Now, tabulate the measures you get in parts 6. And 7.

11. Animate that point M and observe the changes in the measures in the table.

12. Analyze the table and generalize what you have constantly observed.

13. Come up with a generalization relating a parallel line to a base of a triangle and the corresponding ratios of the other two sides of the triangle.

14. Compare and discuss your conclusion with your classmates.

15. Now it is time for a discussion with the teacher.

16. Find the measures MN, BC, and MN/BC.

17. Add them to your table and repeat the steps in parts 11 and 12.

18. Conclude the third ratio that equates the other two ratios.

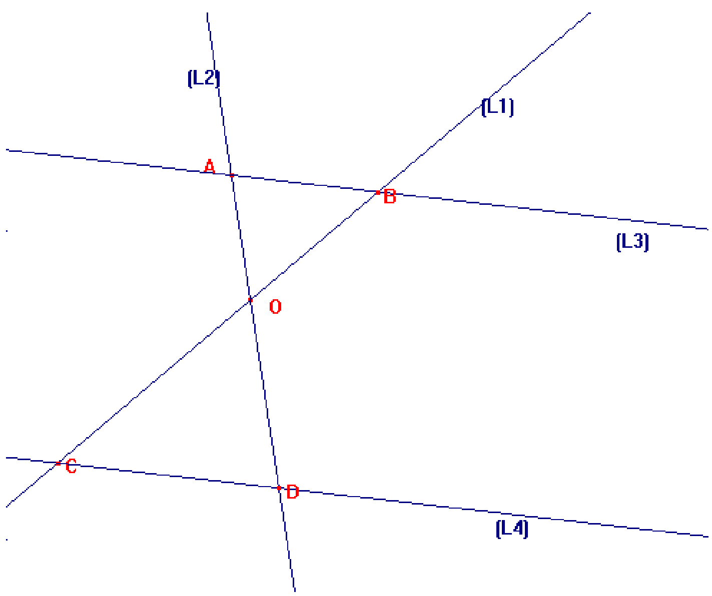
*Activity Sheet 2:*

1. Open a blank Cabri page

2. Construct 2 lines (L1) and (L2) intersecting at point 0.

3. Plot a point A on (L2) (using point-on-object option) and construct line (L3) passing through A and intersecting (L1) at point B.

4. Plot a point C on (L1) (using point-on-object option) and construct line (L4) passing through C and parallel to (L3) and intersecting (L2) at point D.




(Fig. 2)

As a usual step, drag the figure by dragging (L1) or (L2) to check if its parallel property persists.

5. Calculate the measures of the following segments:

AO, OD, BO, OC, AB, DC.

6. Find using the calculator feature of Cabri the following ratios:

AO/OD, BO/OC, AB/DC

7. What do you notice about the value of the ratios obtained in part 7. ?

8. Drag one of the lines and tell whether this observation can be generalized.

9. Now, tabulate the measures you get in parts 6. And 7.

10. Animate the figure by dragging point A and observe the table.

11. Analyze the table and conclude. Discuss with the teacher.

*Activity Sheet 3:*

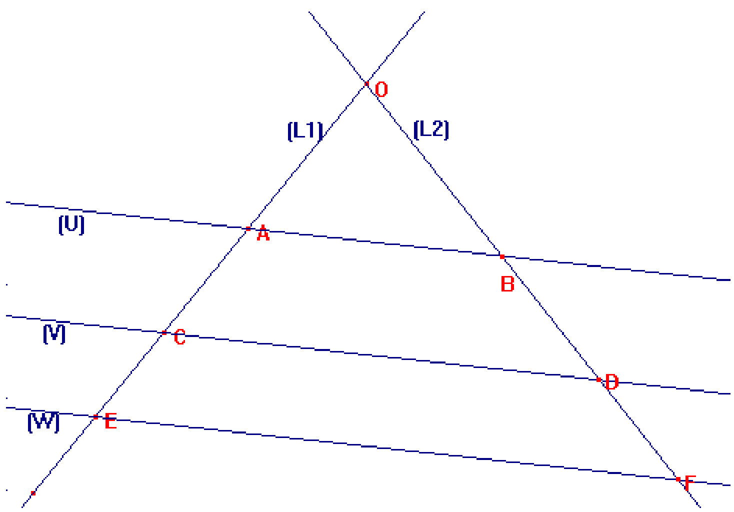
1. Open a blank Cabri page.

2. Draw a line (L1) then plot a point O on (L1) (using point-on-object option) and through O draw a line (L2).

3. Take a point A on (L1) distinct from O (using point-on-object option) and draw a line (U) passing through A and intersecting (L2) at point B.

4. Take a point C on (L1) distinct from O and A (using point-on-object option) and draw a line (V) passing through C and parallel to (U) intersecting (L2) at point D.

5. Take a point E on (L1) distinct from O, A and C (using point-on-object option) and draw a line (W) passing through E and parallel to (U) intersecting (L2) at point F.




(Fig. 3)

6. Calculate the measure of the following segments:

AC; CE; BD; DF

7. Calculate the following ratios:

AC/CE; BD/DF

8. Drag any of the points A, C or E and analyze what happens to your measures and ratios? Did the two ratios change in their value?

9. Now, tabulate the measures you got in numbers 6. And 7.

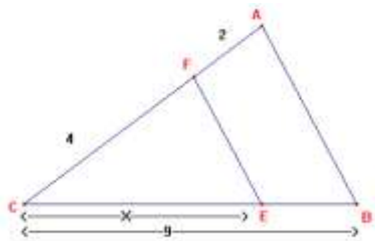
10. Animate the figure by dragging point A, C or E and observe the table

11. What do you notice? Any generalization? Discuss with the teacher.

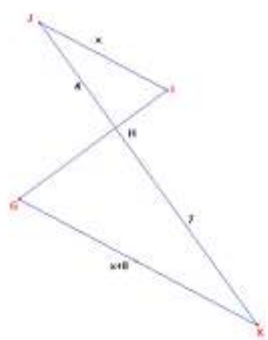
**Quiz1 (10 minutes)**

I. Calculate x in each of the following:

a. Given (FE) parallel to (AB). CE= x, CB=9, CF=4, FA=2.




b. Given (JI) parallel to (GK). JI= x, JH=4, HK=7, GK= x+8



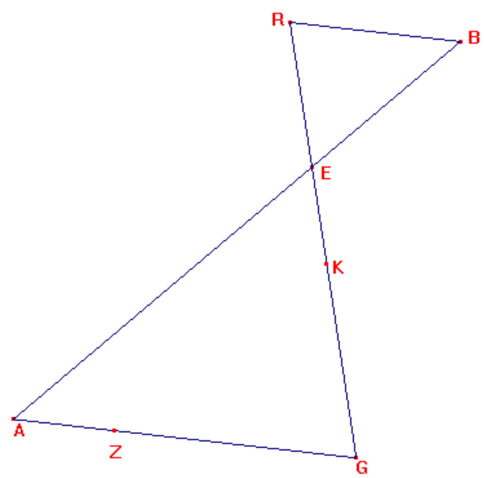
II. In the following figure: AN=6, NB=7, NF=5, FD=2, CE=4 and NC=14

a. What is the relative position of (EF) and (DC)? Justify

b. What is the relative position of (DC) and (AB)? Justify

**Quiz 2 (30minutes)**

I. In the following figure, lines (AG) and (RB) are parallel. Given BE=4; AE=8; AG=7 and EG=6




a. Calculate RB and RE. Justify.

b. Given GK= 4 and GZ=5.

i. Are (ZK) and (AE) parallel?

ii. Justify.

II. In the following figure, given a parallelogram ABCD such that JI=6, DI=8, JK= x

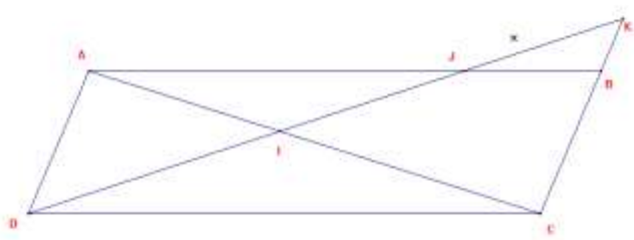
a. List the properties of the sides of a parallelogram

b. Find the value of the ratio $\frac{AJ}{DC}$

c. Deduce that $AJ = \frac{3}{4} AB$

d. Using part c. deduce the value of the ratio $\frac{JB}{AB}$

f. Find the value of x.



## Appendix F

## Unit Test

**I**. Given a parallelogram ABCD of center O. A parallel to the diagonal (AC) cuts [AB] in E and [BC] in F. Through E and F, two parallel lines to (BD) are drawn and cut [AD] in H and [CD] in G.

a. Draw a figure

b. Compare the ratios $\frac{AH}{AD} \quad and \quad \frac{AE}{AB}$

c. Compare the ratios $\frac{CG}{CD} \quad and \quad \frac{CF}{CB}$

d. Compare the ratios $\frac{AE}{AB} \quad and \quad \frac{CF}{CB}$

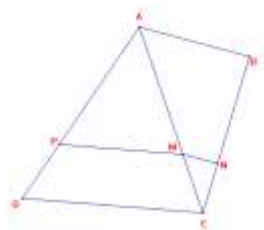
e. Deduce that $\frac{AH}{AD} = \frac{CG}{CD}$

f.

i. What is the relative position of (GH) and (AC)?

ii. Justify

**II.** Given (DC) parallel to (PM) and (AB) parallel (MN).



a. Find two ratios equal to $\frac{MN}{AB}$

b. Find two ratios equal to $\frac{MP}{CD}$

c. Deduce that $\frac{MN}{AB} + \frac{MP}{CD} = 1$.

**III**. Given a triangle ABC such that: AB=3, BC=4 and AC=5.

a.

i. What is nature of triangle ABC?

ii. Justify

b. Draw a figure

c. On [BC], draw point I so that CI=1/4 CB. The parallel to (AB) passing through I cuts (AC) at J. Calculate CJ and IJ

d. On the segment [CB], consider now the point M such that CM=x. the parallel drawn through M to (AB) cuts (AC) at K. Calculate MK in terms of x.

e. On [BA], draw the point L such that BL=0.75

i. what is the relative position of (LI) and (AC)

ii. Justify.

f.

i. What is the nature of AJIL

ii. Justify

## Appendix G

## Answer Keys

**Answer-Key of Diagnostic Test (/20)**

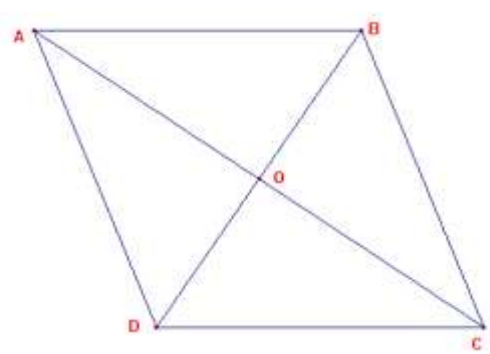
Question 1

a. Properties of a rhombus: (1 pt)

- All sides equal
- Opposite sides parallel
- Opposite angles equal
- Diagonals are perpendicular bisectors of each other
- Diagonals are angle bisectors

b.

i. (0.5 pt)



ii. No (0.5 pt)

iii. (1 pt)

In triangle AOB:

BO= BD/2= 4 cm (diagonals of a parallelogram bisect each other)

AO=AC/2=6 cm (diagonals of a parallelogram bisect each other)

AB=7 cm (given)

$AB^2?=AO^2+OB^2$

49?=36+16

$49\neq50$

Then, triangle AOB is not right at O (by converse of Pythagoras Theorem)

So, diagonals are not perpendicular and ABCD is not a Rhombus.

Question 2

a.

i. AO/ON=2 and BO/OP=2 (1 pt)

ii. They are equal ratios. AO/ON BO/OP (1 pt)

iii. AO=2ON and BO=2OP(1 pt)

b.

i. AO/AN=0.6 and BO/BP=0.6 (1 pt)

ii. They are equal. AO/AN= BO/BP (1 pt)

iii. AO=0.6AN and BO=0.6BP (1 pt)

c.

i. ON/AN=OM/CM=OP/BP (1 pt)

ii. Centroid cuts the medians in a triangle proportionally. The centroid is located on the median such that it is far from the vertex 2/3 and from the base 1/3. (1 pt)

Question 3

a.

i. BSPD is a parallelogram (0.5 pt)

ii. C is the midpoint of [DS] and [BP] thus diagonals bisect each other. (1 pt)

b.

i. ACSB is a parallelogram (0.5 pt)

ii. CS equal and parallel to AB thus 1 side parallel and equal to its opposite (1 pt)

c.

i. (AC) // (DP) (0.5 pt)

ii. (AC) // (BS) since ACSB parallelogram (1 pt)

(DP) // (BS) since BSPD parallelogram

Thus 2 sides parallel to the same side are parallel to each other

d.

i. E midpoint of [PS] (0.5 pt)

ii. In triangle DSP: (1 pt)

C midpoint of [DS] and (CE) // (DP) (proved)

Then E midpoint of [PS] by converse of midpoint theorem

e. CE=DP/2 (1 pt)

f.

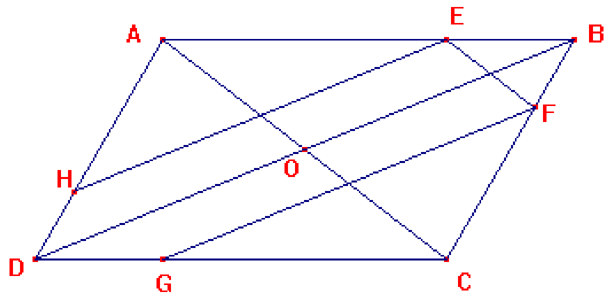
i. XY=CE/2 (midpoint theorem) (1 pt)

ii. XY=CE/2=DP/4 (by substitution) (1 pt)

**Answer-Key of Unit Test (/20)**

Question 1

a. (1 pt)




b. (HE)//(DB) (given), so AH/AD=AE/AB (Thales Property) (1 pt)

c. (FG)//(DB) (given), so CG/CD=CF/CB (Thales Property) (1 pt)

d. (EF)//(AC) (given), so AE/AB=CF/CB (Thales Property) (1 pt)

e. since AE/AB=CF/CB (proved) (1.5 pt)

So by comparing parts a and b we get AH/AD= CG/CD (by substitution)

f.

i. (GH)//(AC) (0.5 pt)

ii. Since AH/AD= CG/CD (proved) so by converse of Thales Property (2 pt)

Question 2

a. MN/AB=CM/CA=CN/CB (Thales Property) (1.5 pt)

b. MP/CD=AM/CA=AP/AD (Thales Property) (1.5 pt)

c. MN/AB+MP/CD= CM/CA+AM/CA (from parts a and b) (2 pt)

so, (CM+AM)/CA= AC/CA=1

Question 3

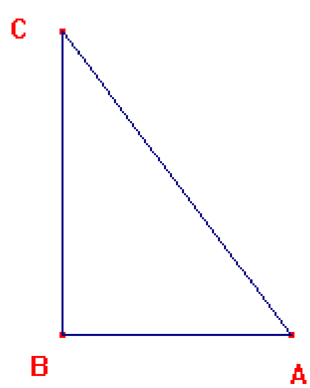
a.

i. Triangle ABC is right at B (0.5 pt)

ii. $AC^2=AB^2+BC^2$ since 25=9+16 (0.5 pt)

then by converse (Pythagoras Property) triangle is right at B

b. (0.5 pt)




c. Since (CJ)//(AB) (given), so CI/CB=CJ/CA=IJ/AB (Thales Property) (2 pt)

then, CJ=1.25 and IJ=0.75

d. (MK)//(AB) (given), so CM/CB=MK/AB (Thales Property) (1 pt) then, x/4=MK/3 then, MK=3x/4

e.

i. (LI) intersects (AC) (0.5 pt)

ii. since BL/BA(=1/4)≠BI/BC(=3/4) so by converse of Thales Property (1 pt)

f.

i. AJIL is a trapezoid (0.5 pt)

ii. (CJ)//(AL) and (IL) is not parallel (AJ) (proved) (0.5 pt)