An Augmentation of Fuzziness to Randomness in Project Evaluation

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An Augmentation of Fuzziness to Randomness in Project Evaluation

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Abstract: Net present value is the traditional approach to evaluate the financial viability of projects including large ones. Unfortunately, the NPV may mislead decision makers for two reasons. First, it does not take into consideration managerial flexibility which is the ability of decision-makers to react to upcoming information related to some uncertain events in a way that may change the project design and planning. Randomness based on standard deviation is usually utilized to accommodate such flexibility. Second, the NPV calculation relies on some fuzzy variables that are subject to imprecision or ambiguity such as the characteristics of the project cash flow and investment cost. The variability of these errors can also be represented in the volatility of the model. In such scenario, volatility would be randomness as well as fuzziness based. Researchers in the field propose to use project Investment and Project cost as fuzzy variables. We argue in this paper that both of these factors can be embedded in the randomness-based volatility factor. This paper investigates the use of the volatility factor as fuzzy parameter in a Real options approach to evaluate projects. Hence, the overall ambiguity would be represented in a combination of randomness as well as fuzziness to achieve acceptable evaluations.

Keywords. Fuzzy real options; Fuzzy arithmetic; Uncertainty; Project evaluation; Black and Scholes formula, Trapezoidal fuzzy numbers.

Introduction

High investment projects have a significant impact on their related decision-making process mainly due to the long period between planning and execution. This produced uncertainty about the future of the project environment implies that the project cannot be executed as it has been planned and that managers frequently react to new upcoming information. Managerial decisions that can be altered by new upcoming information might be the project launching date, its closing date, its capacity level, operating plan, etc. Unlike the tradition Net Present Value (NPV) used by executives to evaluate projects, Real Options Approach (ROA) addresses the flexibility of having the "option" to exercise the right to invest without the obligation when the project is due for execution. ROA is the mechanism adopted by researchers to address the uncertainty related to some investment decisions. Hence, an investor who is evaluating a project has no early-commitments when the decision has to be made. Obviously, at the end of conception period of the project, corresponding to the maturity period for the financial option, the project manager would have a better picture of the actual investment profitability (or possible loss) and can have the choice of actually executing or canceling off the project. Such flexibility decision is not accounted for in the traditional

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net present value (NPV). A project to build a hotel, for example, might be deemed to be unprofitable when approaching the end of execution of the project which can be converted to another option of utilizing it for other uses. Such flexibility is called a real option because, conceptually, it is similar to a financial option by which its holder has the right but not the obligation to, for instance, buy some stocks at a pre-established cost at some date in the future. Black and Sholes [1] and Merton [2] were the pioneers to propose ROA modulations, and which was adopted by many other types of research in the field (e.g. [3]). When high investment considered by corporates, the period between the investment decision and actual start of the investment projects can be huge where multiple environmental factors can vary leading to increasing cumulated uncertainty. Using the ROA, the project investor would compute a more acceptable value of the project based on the current and observable input variables whereas their uncertainty is wheeled through its variance. With respect to the option of launching the project based on its value later, note that, as long as the option is alive, its value is increasing with the level of the uncertainty. Thus, ROA is best suited to evaluate the value of a project with some sources of flexibility. This evaluation is based on the present value of its future cash flows, an uncertain variable that can be represented using stochastic processes. Nevertheless, such processes depend on critical statistical variables such as their variance or level of uncertainty that would determine how the stochastic variable would evolve in the future and determine closely the level of the project flexibility. Fortunately, even when experts diverge in estimating such variables [26], they are able to come up with acceptable approximation using fuzzy numbers that allow accounting for the ambiguity that might be related to their estimation.

Traditional fuzzy logic theory, proposed by Zadeh [5], [6] (also termed as type-1 fuzzy sets), can be used to measure the ambiguity of ROA inputs. A Generalization of type-1 fuzzy sets was also proposed by Zadeh where the objective was to rectify the initial proposal where the fuzzy sets were not actually "fuzzy" and hence a proposal of type-2 fuzzy sets emerged [7] where different researchers proposed different techniques of approaching fuzziness solutions to fuzzy sets (e.g. [9] [10]). Implementation of type-2 fuzzy sets to ROA was also very limited as can be seen from table 1.

The pace of implementing an integration of fuzzy logic (type-1 or type-2) with ROA is relatively slow in recent years. For example, table 1 shows the number of articles produced in the past 5 years (2013-2017). The table compares the total number of ROA-articles produced as opposed to Fuzzy-ROA which shows extremely low percentage (about 0.5%) of the total ROA articles. Generating this table was done via searching the "Google Scholar" and filtering the number of articles that are related to the topic (e.g. "Type-2") excluding the number of articles that just use citation of the search terms. The table also presents an example of published articles per year, per type of article (ROA, F-ROA type1, and F-ROA type-2). Although ROA implementations seem to be relatively gaining popularity among researchers in the different field of applications, however, Fuzzy integration with ROA seems to be still low. Add to this, type-2 fuzzy sets integration to ROA is relatively negligible. This indicates a high potential of exploring such research area with opportunities to integrate different techniques of type-2 fuzziness into the ROA in different fields of industry.

Carlsson et al. [4] were the pioneers to introduce fuzzy logic integration with ROA where two input variables are considered as fuzzy ones: the investment cost and the present project value. They applied Black & Sholes equation to evaluate the project value as a European option: The project is evaluated now (time zero) to be launched at time T, the project expiry date if it is deemed profitable at the end expiry date. However, another variable seems even more critical is the variance of the project present value that is assumed to follow a Geometric Brownian Model (GBM). The reason is that the project value is too sensitive to its variance that increases linearly with time [24]. In addition, the implementation of ROA is usually obstructed [25] by the difficulty of estimating the variance of the project value.

Table 1: Number of ROA Articles in the past 5 Years

# / Year	ROA (thousands)	F-ROA Type-1	F-ROA Type-2	Total F-ROA	Examples		
					ROA	F-ROA Type1	F-ROA Type-2
2013	3.88	23	---	23	[11]	[16]	---
2014	4.18	13	---	13	[12]	[17]	---
2015	4.25	17	3	20	[13]	[18]	[21]
2016	4.30	23	1	24	[14]	[19]	[22]
2017	3.61	19	2	21	[15]	[20]	[23]
Total	**20.22**	**95**	**6**	**101**			

After reviewing the different mechanisms and techniques adopted by different researchers in the field of ROA and Fuzzy-ROA, this paper considers the standard deviation or volatility associated with the stochastic process of the project present value as a fuzzy variable. The projected volatility is represented by a trapezoidal fuzzy number as proposed by Carlsson et al. [4] to represent the project value and it's investment cost as it is an easy and intuitive way for experts to express explicitly their approximation of such ambiguous variables. A Numerical example of a project described by Carlsson et al. [4] has been reproduced in this article to show the considerable sensitivity of the project value with respect to its volatility. These results are compared to those of Carlsson et al. to show that the projected volatility is too critical to the value of a project.

1. Review on the theory of Real Options Approach

Managers and executives used to apply the traditional financial analysis based on discounted cash flow technique such as net present value (NPV) and internal rate of return (IRR) to assess project financial viability. According to a survey by Graham and Harvey [26], 392 small and large American firms, that 75% of them use NPV and IRR whereas Gitman and Forrester [27] noted that less than 10% relied on NPV. Later, Bennouna, Meredith, and Marchant [28] found that 83% of firms are still relying on capital budgeting techniques for decisions related to capital budget where most of them favor IRR and NPV parts of Discounted Cash Flow (DCF). Although such studies indicate that NPV and IRR are increasingly being used by corporates, however, DCF technique is suffering from serious drawbacks. The main problem would be the difficulty of revenues and cost prediction as well as the exact project organization and re-organization during the execution. The second reason is that DCF technique does not allow the flexibility of capturing emerging valuable opportunities/resources if future conditions are favorable. Similarly, DCF does not allow the flexibility to avoid threats if future conditions are unfavorable. At the project level, sources of flexibility may include for instance delaying the project launch, suspending its implementation temporarily, or definitely abandoning the project if conditions are unfavorable. Black and Scholes [3] and Merton [4] are the pioneers of developing ROA which provides an “option to buy shares” providing the user with the opportunity without obligation to acquire these shares within a specific period of time.

Most straightforward approach to applying ROA is the Black and Scholes formula [3], [4], [30], [31]. Definition of “European Call option” is dependent on the project launch after period “t” from now, with the present value of the project (S) subject to some volatility σ , and investment cost X. In this case, corporates have the right but not the obligation to incur the investment cost *X* at the end of the maturity period *t*, if the project is deemed profitable. In these conditions, the current value *W* of the project is given by the following equation of Black and Scholes, as modified by Merton :

$$\boldsymbol{W = Se^{-\delta T}N(d_1) - Xe^{-rT}N(d_2)}$$

$$d_1 = \frac{\log(S/X) + (r - \delta + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T} = \frac{\log(S/X) + \left(r - \delta - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

Where:

S: The project’s present value; σ : Standard deviation of the present value;

X: Investment cost; *T*: Maturity period; *r*: The Risk-free rate of return;

δ: The cost to maintain the option alive, expressed as a percentage of the project value S;

$N(x) = (1/\sqrt{2\pi})\int_{-\infty}^{x} e^{-x^2}dx$: The cumulative normal function value at x, or the probability that a random draw from a standard normal distribution will be less than x.

It should be noted that if t=0 or if σ = 0, then *W* reduces to the project NPV, that is S-X. Obviously, the option value is directly proportional to the increase in σ and also in T. It should also be noted that the option value, increase with T, the possible launching date, and with S, the project present value.

2. Integration of Fuzzy-ROA enhancing decision making on project capital budget

Integration of Fuzziness and randomness is would be embedded within the integration of fuzzy logic and ROA. Fuzzy logic usually deals with the uncertainty of project variables and randomness represented by the volatility (standard deviation) in the B&S ROA equation. The inevitable ambiguity related to the project present value volatility is addressed in this paper by using fuzzy number representation. Hence, integrating fuzzy logic with ROA technique seems to be the ideal solution for efficient capital budget decision-making.

The Black and Scholes formula allows valuing the project while taking into account the flexibility of avoiding losses if the project should not be launched at the end of the expiry period. Carlsson and Fuller [4] introduced fuzziness to the mechanism of Real Options Approach where volatility assumed to be constant. Although volatility σ represents the standard deviation of the actual present value, however, such variation is not constant with respect to time. Hence instead of using "crisp" value of σ, this paper proposes to consider volatility as a fuzzy number as well. In this case, to ease their fuzzification, d_1 and d_2 can be reduced to:

$d_1 = \frac{A}{\sigma} + B\sigma$ and $d_2 = \frac{A}{\sigma} - B\sigma$.

Where A= $\frac{\log(\frac{S}{X}) + (r-\delta)T}{\sqrt{T}}$ and $B = 0.5\sqrt{T}$.

Carlsson and al. [4] argue the suitability of expressing explicitly estimated variables by experts using a trapezoidal fuzzy number. Assuming that the standard deviation is a trapezoidal fuzzy number denoted by $\tilde{\sigma} = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$. Its membership function A (t) and its α-cut interval are the following:

$$A(t) = \begin{cases} 1 - \frac{\sigma_2 - t}{\sigma_2 - \sigma_1} & if\ \sigma_1 \leq t \leq \sigma_2 \\ 1 & if\ \sigma_2 \leq t \leq \sigma_3 \\ 1 - \frac{t - \sigma_3}{\sigma_4 - \sigma_3} & if\ \sigma_3 \leq t \leq \sigma_4 \\ 0 & if\ t \notin [\sigma_1, \sigma_4] \end{cases}$$

$[\tilde{\sigma}]^\alpha = [a_1(\alpha), a_2(\alpha)], \alpha \in [0,1]$,

$a_1(\alpha) = \sigma_2 - (1-\alpha)(\sigma_2 - \sigma_1)$,

$a_2(\alpha) = \sigma_3 + (1-\alpha)(\sigma_4 - \sigma_3)$.

Recalling that the results of additions and subtractions of trapezoidal fuzzy numbers are trapezoidal fuzzy numbers that can be computed as follows:

(x_1, x_2, x_3, x_4) (+) $(y_1, y_2, y_3, y_4) = (x_1 + y_1, x_2 + y_2, x_3 + y_3, x_4 + y_4)$

(x_1, x_2, x_3, and x_4)(-) $(y_1, y_2, y_3, y_4) = (x_1 - y_4, x_2 - y_3, x_3 - y_2, x_4 - y_1)$

For exact calculation of the multiplication, division, and inverse of trapezoidal fuzzy numbers, the membership functions shall be used and the result need not be trapezoidal fuzzy numbers. However, it is often acceptable to approximate the results by using α –cut intervals for $\alpha = 0$ and 1.

$B\tilde{\sigma} = (B\sigma_1, B\sigma_2, B\sigma_3, B\sigma_4)$

$-B\tilde{\sigma} = (-B\sigma_4, -B\sigma_3, -B\sigma_2, -B\sigma_1)$

$A/\tilde{\sigma} = (A{\sigma_4}^{-1}, A{\sigma_3}^{-1}, A{\sigma_2}^{-1}, A{\sigma_1}^{-1})$

Hence achieving:

$$\tilde{d}_1 = (A\sigma_4{}^{-1} + B\sigma_1, A\sigma_3{}^{-1} + B\sigma_2, A\sigma_2{}^{-1} + B\sigma_3, A\sigma_1{}^{-1} + B\sigma_4)$$

$$\tilde{d}_2 = (A\sigma_4{}^{-1} - B\sigma_4, A\sigma_3{}^{-1} - B\sigma_3, A\sigma_2{}^{-1} - B\sigma_2, A\sigma_1{}^{-1} + B\sigma_1)$$

Similarly, we can approximate the integrals N ($\tilde{d}_1$) and N ($\tilde{d}_2$) using the α –cut intervals. Thus, the fuzzy value of the option becomes:

$$\widetilde{\boldsymbol{W}} = \boldsymbol{S}\boldsymbol{e}^{-\boldsymbol{\delta T}}\boldsymbol{N}(\tilde{d}_1) - \boldsymbol{X}\boldsymbol{e}^{-\boldsymbol{rT}}\boldsymbol{N}(\tilde{d}_2)$$

$$\widetilde{W} = (Se^{-\delta T}N(A\sigma_1{}^{-1} + B\sigma_1) - Xe^{-rT}N(A\sigma_1{}^{-1} - B\sigma_1),$$

$$Se^{-\delta T}N(A\sigma_2{}^{-1} + B\sigma_2) - Xe^{-rT}N(A\sigma_2{}^{-1} - B\sigma_2),$$

$$Se^{-\delta T}N(A\sigma_3{}^{-1} + B\sigma_3) - Xe^{-rT}N(A\sigma_3{}^{-1} - B\sigma_3),$$

$$Se^{-\delta T}N(A\sigma_4{}^{-1} + B\sigma_4) - Xe^{-rT}N(A\sigma_4{}^{-1} - B\sigma_4)).$$

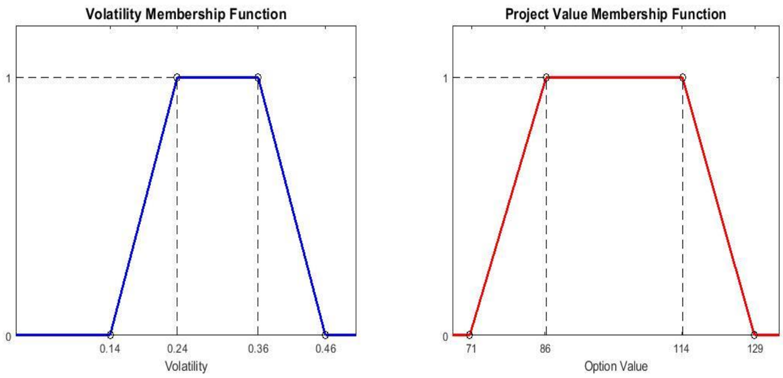
3. Example of oil extraction project

In order to provide practical insight to the proposed volatility fuzziness, this paper will follow the same example as defined by Carlsson et al. [4], describing an oil company acquiring a five-year license in a land with an expectation of yielding 50 million barrels of oil. Today's price of the oil barrel is assumed to be $10 and the current development cost is $600 million. Hence we can start by regenerating the output of ROA without any fuzziness as can be seen in table2. The same assumptions of Carlsson et al. also used in here of $15 million to keep the option alive, thus:

*S=500 m$; X*=600 m$; t=5 y; *r=5%*; δ=3%.

We assume that: $\tilde{\sigma} = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$= (0.1;0.2;0.3;0.4).

This is presented graphically in Figure 1(a). Figure 1(b) shows the execution of B&S equations resulting in the fuzzy output of project value. As discussed in the literature of real options, the uncertainty of the project adds value to the project option the project. To measure the sensitivity of σ, the same central values (centroid of Trapezoidal function) of S and X will be adopted as defined by Carlsson et al. and changed to vary by 20% to form its core interval 0.24-0.36. Note that in the defined example by Carlsson et al., the core interval of S is +/-20% of its central value S0=500 m$ and the core interval of X is +/- 8% of its central value 600 m$. We find that the core interval of the project value is 86-114 m$ in the same order as the one generated by Carlsson et al. 40-166 m$ when the project value and its cost are both fuzzy.

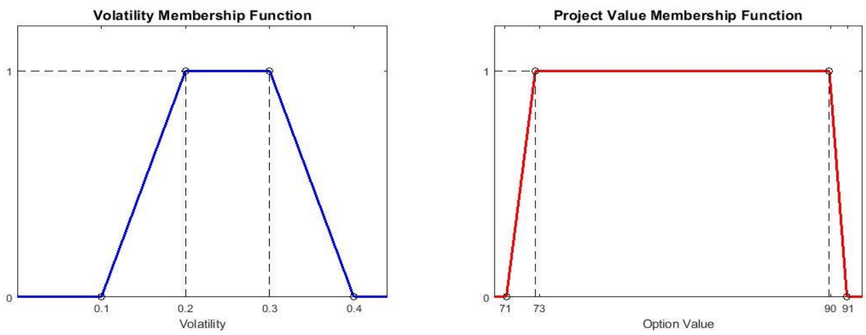


(a) (b)

Figure 1 (a) Fuzzifying Volatility with central value σ= 30%;
(b) Fuzzy project Value (F-ROV) result from B&S equation

Moreover, to test the sensitivity of fuzzy volatility, we assume that the volatility is reduced to 25% instead of being 30% as in case of Carlsson et al. This would produce a shorter interval still between {73, 90} as can be seen from figure 2. Table 2 summarizes these results showing that the interval was reduced when only volatility was assumed to be fuzzy in nature instead of S & X being fuzzy and the volatility is constant.The fact that the interval of the options value generated when σ is fuzzy would indicate the practicality and acceptability of using the proposed volatility as an indicator of flexibility in fuzziness. In other words, Ambiguity of project value has been reduced to the lower level of uncertainty using fuzzyσ.

Ambiguity, randomness, and uncertainty have been used interchangeably by many researchers. Obviously, there is a distinct difference between randomness and fuzziness [32, 33] where some researchers proposed to combine both concepts for better optimization (e.g. [34] where randomness is probability space which fuzziness measure the strength level of the specific variable represented by fuzzy membership value, hence a measure of ambiguity.



(a) (b)

Figure 2: (a) Proposed Volatility Trapezodial Function with central value σ=25%
(b) Fuzzy Project Value Trapezoidal function

Table 2: Comparison between projects Value of Carlsson et al. as opposed to the proposed output of project value based on fuzzy-volatility

Inputs	**W (FROV)**
S and X are fuzzy, σ =0.3	{40, 166} [4]
S and X Crisp, σ= [0.14,0.24,0.36,0.46]	{86,114} Figure 1
S and X Crisp, σ = [0.1,0.2,0.3,0.4]	{73, 90} Figure 2

The reason for this discussion is to show that volatility defined as a fuzzy variable can reduce the ambiguity of the final required project value uncertainty interval measured using fuzzy membership value. This is apparent from table 2 where the proposed fuzzy-volatility produces less uncertainty than fuzzy-S&X as proposed by Carlsson et al. Sensitivity of volatility is also proportional to the final output of project value. A reduction of sigma by 5% produced a reduction in the interval by [13, 24] where both solutions of fuzzy sigma produced intervals as a subset of the solution provided by Carlsson et al. (Table 2).

It should also be noted that, based on the previous discussion of Randomness and fuzziness, the proposed membership function is "fixed" symmetrical fuzzy set shape (function). Although such fuzzy set interval can be fixed for short period of time (less than 6 months for example), however, for long complex projects this trapezoidal shape cannot be fixed (type-1 fuzzy set) but rather "fuzzy" in nature (fuzziness of fuzzy set: type-2 fuzzy set). To be able to accommodate type-2 fuzzy sets to the volatility modulation, and without changing the fuzzy interval, the trapezoidal shape of the fuzzy set cannot be fixed but rather skewed with a viability of “skewness” with respect to time. As the value of sigma is inversely proportional with respect to time, then this skewness would be biased towards the higher end of the interval at the beginning of “long period before execution” and become closer to the lower end of the interval when the period approaching execution time. The modulation of time-dependent σ and with the use of type-2 fuzzy sets are beyond the scope of this article. However, this argument raises the potential for further research of tackling the same problem using type-2 fuzzy sets by comparing different techniques of achieving type-2 fuzzy sets. As mentioned in the review section of this article Mendel et al. [10] approach is well adopted by other researchers and seems to be easy enough to follow. However, Kouatli [35] also proposed one kind of type-2 where fuzziness can be skewed left or right depending on a proposed parameter termed as “Degree of Fuzziness” (DOF) which can be used to skew the fuzzy set vertically and horizontally. The full analysis of such technique with comparisons of other techniques in this specific volatility problem is far beyond the objective of this article but will be discussed in a future research on this topic.

4. Conclusion

After reviewing the Fuzzy implementations to Real Options Approach (F-ROA), this paper proposes to use the volatility σ as being a fuzzy number to introduce a new model based on this fuzzy volatility. Although volatility usually represents the variations of possible factors such as the price and the quantity, however, this variation may change with time as new emerging factors may necessitate the recalculation of the volatility factor. For this reason, this paper proposes the consideration of introducing fuzziness to σ where most variability of other factors can be embedded in this indicator. Hence introducing the augmentation of fuzziness to the randomness of nature ofσ. An example introduced by Carlsson et al about oil investment was regenerated and compared with the project value based on the proposed fuzzy volatility which shows that the proposed fuzzy-volatility can be a subset of the solution provided by Carlsson et al.

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