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EFFECT OF TECHNOLOGY INTEGRATION IN TEACHING QUADRATIC FUNCTIONS ON LEBANESE STUDENTS' LEARNING, PROBLEM- SOLVING ABILITIES, AND ATTITUDES

By

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Dedication Page

To Bassem, Lara, Rana and Hani

EFFECT OF TECHNOLOGY INTEGRATION IN TEACHING QUADRATIC FUNCTIONS ON LEBANESE STUDENTS' LEARNING, PROBLEM-SOLVING ABILITIES, AND ATTITUDES

Hala Adib El-Jiryes

Abstract

Technology is being integrated in education at large and in mathematics teaching and learning in particular. This study is an action research conducted in math classes in Lebanon. The purpose of this thesis is to study whether integration of technology in teaching quadratic functions affects students' understanding, critical thinking and problem solving abilities, and whether it changes their attitudes towards mathematics. The sample is composed of two grade 10 classes of a French school in Lebanon. The total number of participants is 47 out of which 28 are girls. In the experimental group, the programmable calculator Casio Class pad 330 and the DGS Geogebra were integrated in teaching, while in the control group, the non-programmable scientific calculator was the only device used in teaching. The pre-test, post-test method is applied to compare students' achievement and problem solving abilities, in each of the groups before and after instruction, as well as to compare the achievement of the control and experimental groups after instruction. In addition, a questionnaire is filled after instruction by all participants to compare control and experimental groups' attitudes, and interviews are conducted with students from the experimental group in order to get a more analytical insight in their attitudes toward the use of technology in math learning. Although the pre-test results yielded better achievement by the control group, the quantitative analysis of the post-test showed that the experimental group's achievement has improved over time. The post-test results were not significantly different after instruction. However, the questionnaires showed that the attitudes of students exposed to technology are significantly more positive than those of the control group. The interviews with the experimental group students revealed a clear increase in their interest in, and willingness to learn mathematics.

Keywords: Technology, Quadratic Functions, Attitudes, Critical Thinking, Authentic Problem-Solving Situations.

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CHAPTER ONE

Introduction

1.1- Overview

Educational change has always been a major concern for researchers, educators, psychologists and politicians. This is partly because current students are future leaders, governors, lawyers, economists, financial managers and the like. Continuous research is undertaken to meet the main goal of education which is to provide students all the assets they need to become active, productive, and efficient citizens. Continuous research is conducted on education and ways of improving learning for several reasons, among which are two important ones. The first is that education deals with human beings who are different in mental capacities and who live in different environments which affect them differently. The second is that the mechanisms which make the brain function and acquire information and learning are still a debatable subject.

For many decades, the most widely applied educational theory has been *behaviorism* whereby students' minds are considered to be empty containers that teachers fill with information. Drill and practice is the main approach adopted under this theory. Accordingly, the teaching method applied is teacher-centered lecturing, reinforced by a long list of similar practice exercises. This theory reminds us of Pavlov's theory of reflex.

In opposition to *behaviorism*, *constructivism* advocates other approaches that place the student at the center of the educational operation. All other components of the educational system revolve around students who are supposed to construct their own knowledge with teacher's guidance: teachers provide scaffolding (Vygotsky, 1950 as cited in Van Der Stuyf, 2002) and students develop their newly acquired knowledge as the learning process takes place. Under this theory, students are active, they are exposed to activities that require them to make assumptions and conjectures, and construct knowledge on their own.

With the invasion of technology, educators' views are changing. Slowly, but surely, technology is paving its way into the educational field. Many schools over the world are using technology in teaching at various levels and to different extents. However, education is slow in responding to growth in disciplines and to evolutions in the tools for learning. Integration of technology into education started three decades ago. Maple was first developed in 1980 by the Symbolic Computation Group at the University of Waterloo in Waterloo, Ontario, Canada.(Wikipedia Encyclopedia). It was used when teaching calculus especially to engineering students along with Minitab (Joiner & Ryan, 2000) in statistics. In the 1990s, technology was integrated in courses for math majors in universities. Among many others, an innovative computer-based university math program was introduced in 2001 namely, Math Integrated with Computers and Applications (MICA) originated at Brock University (Muller, Buteau, Ralph & Mgombelo, 2009). Such programs provide pre-service teachers with knowledge about technology and skills needed for teaching. On the other hand, practicing in-service teachers are encouraged by their institutions to attend seminars and training workshops to acquire knowledge and skills in educational technology. The main goal behind the training programs is to improve students' learning.

Many research studies regarding the integration of technology in teaching and learning mathematics have been undertaken in the last decades (Applebaum, 2004; Basturk, 2005; Bauer & Kenton, 2005). Some suggest that technology made a big difference as far as achievement and understanding are concerned, while others show that no substantial difference was achieved. In general, educators who adopt an approach based on constructivism for learning are supporters of the integration of technology in teaching to enhance exploration and active roles of students in the learning process. *Behaviorists* underestimate the effect of technology on learning. In both cases, educators are becoming more and more concerned about finding, testing and applying various techniques that would help students learn more efficiently with the introduction of technology in teaching. The vital problem to be solved is to find effective strategies for integration that would challenge students' thinking. Critical thinking is the aim while students become editors of the information they are exposed to. They internalize information as they understand it. The information stored becomes then assimilated knowledge to be retrieved and used when needed, in a rational, beneficial and efficient sense.

In many schools and universities, technology has actually been integrated in mathematics teaching to various extents. Many research studies show that this integration improved students' mathematics learning for various reasons to be discussed later (Christou, Pittalis, Mousoulides & Jones, 2006; Burrill, 2005, Laborde & Sträber, 2010). An established fact is that technology is only a tool and a means to reach more important goals of learning. Bearing in mind the positive effect of integration of technology on math learning, the most efficient way to ensure better learning is still debatable. This is due to the fact that many aspects of students' mathematical thinking are still unclear.

1.2- Purpose of the Study

The present thesis aim is to investigate the effectiveness of integrating technology in mathematics teaching and learning. In Lebanese schools, traditional teacher-centered patterns of teaching have prevailed for a long time. Slowly, constructivist approaches are taking over after the 1997 reform of goals and methods of teaching mathematics. The 1997 curriculum (ECRD, 1997) advocates understanding and critical thinking as a basis for learning, as opposed to the older curriculum mainly based on memorization of facts then drill and practice. The textbooks issued after the reform propose activities that foster exploration to be conducted by the students themselves.

Many schools in Lebanon teach foreign curricula in addition to the Lebanese curriculum. These schools generally adopt more student-centered approaches. Technology-based activities are undertaken in various schools, in several mathematical topics at different levels and to various extents. However, few studies were conducted in Lebanon to test, investigate or explore whether integration of technology actually helps students learn mathematics more efficiently. The primary concern of the present study is to investigate whether the use of technology in math teaching and learning supports critical thinking that enables students to apply what they learn in authentic problem solving situations. The topic for quadratic functions is considered as a context for learning.

1.3- Research Questions

In more specific terms, the study attempts to answer the following research questions:

- Does the use of technology (programmable calculator, spreadsheet and CAS software) improve students' understanding of mathematical concepts (particularly quadratic functions) and relationships?

- Does the use of technology (programmable calculator, spreadsheet and CAS software) improve students' ability to apply those concepts (particularly quadratic functions) and relationships in authentic problem solving situations?

- How does technology integration restore connections between different mathematical representations, otherwise learned as isolated notions?

- What are the features of the used technological devices that help achieve the above goals?

- What are the attitudes of students towards the integration of technology in mathematics teaching especially: quadratic functions?

1.4 - Rationale

Technology is being integrated in teaching, and more specifically in mathematics teaching. The potential use of technology is affecting the mere contents of curricula. For example, an innovation in the French curriculum is the introduction of probabilities starting grade nine rather than in grade 12, as it used to be in older curricula. Simulations were completely disregarded before, because it was impossible to manually conduct such activities, as it would take many hours to make a simulation of tossing a dice 1000 times, for example. This is why the subject was not treated at early grade levels. But nowadays, with a spreadsheet computer program, it is possible to conduct such an experiment within fractions of a second. This is why new subjects have been introduced in the curriculum.

Another innovation is the teaching of algorithms in the French curriculum. This subject was not taught in high schools and it was reserved to math and computer majors at the university. Now, algorithms are taught starting from grade ten level, for two purposes: initiation to programming, of course, but more importantly for thinking flow and learning logistics. This subject is very important because it helps students acquire deeper knowledge of how the mechanism of technological devices functions. Innovations in technology are so quick that students should be taught in a way that allows them to adapt to any new technological device. By the time they graduate, the technological devices would have become obsolete; this is why students should understand the logical flow of steps because it is the same for all new devices with minor changes from one model to another. This is in compliance with the constructivist approach, students learn to think and to analyze in order to adapt to any situation.

For all the purposes mentioned above it is important at this point to test whether integration of technology is serving the purposes desired. It is important to test whether it is helping students acquire deeper knowledge.

Few, if not no studies were conducted in Lebanon to investigate the effect of this integration on the development of students' critical thinking. The present research study attempts to bridge this gap. The mathematical subject chosen as learning context in this research is quadratic functions because functions exist around us in real life and many students fail to make the connection between their studies and real life situations.

1.5 - Definition of Terms

The terms defined in this section namely, technology, critical thinking and authentic problem-solving situations.

1.5.1- Technology

An interesting note to be made here is the Webster's dictionary referral to the origin of the word technology. It refers to "Greek techno logia" which is a "systematic treatment of an art". It is important because it is actually an art to integrate technology in education in a way to which students' thinking would react. Another definition in Webster's dictionary is that technology is a "manner of accomplishing a task especially using technical processes, methods or knowledge."

In this study, the technological tools integrated are: graphical programmable calculators, spread sheets, and CAS software such as GeoGebra and Geoplan- Geospace.

1.5.2- Critical thinking

Thinking implies formation of an idea that will become knowledge. Thinking could be defined as “engaging the mind in reflection”. Therefore, the powers of judgment, conception and inferences are challenged.

Critical thinking means correct thinking in pursuing relevant and reliable knowledge about the world (Schafersman, 1991). Critical thinking is what enables students to become efficient, active and responsible members of the society. Critical thinking skills are problem solving skills that produce reliable knowledge (Schafersman, 1991). Critical thinking skills include many aspects such as comparing, conjecturing, generalizing, deducing, analyzing, proving, evaluating and patterning (O’Daffer & Thomquist, 1993).

1.5.3- Authentic Problem Solving

Authentic real life problem solving is an expression used in this study to indicate real life situations which call on students’ *savoir faire*. It is observed that students are usually knowledgeable when they are asked to solve problems within an academic framework. However, they face difficulties when they want to apply what they have studied to real life problems. This means that they usually lack the ability to make connections. This is why it is important to test whether integration of technology helps them make the link between a real-life problem situation that they may face, and the mathematics that they learn in class.

CHAPTER TWO

LITERATURE REVIEW

2.1- Introduction

The effect of integrating technology in mathematics teaching on students' learning has been extensively treated in the mathematics education literature. An extensive review of the literature is conducted in this chapter to provide support and theoretical background to the present study. The major concepts for the literature review are: integration of technology in mathematics teaching and understanding, as well as teachers' and students' attitudes towards integration of technology.

Some topics are more difficult to understand by students than others because they are too abstract. There is a need to mediate their learning by using concrete models or symbolic systems. Visualization may alleviate the problem as it allows students to identify interrelations between variables and makes it easier for them to make the link between several representations of the mathematical objects. Restoring this previously lost link gives meaning to the subject under study. It also allows students to make the link between algebra and geometry; for instance, students can relate the interpretation of remarkable identities to geometric figures such as rectangles, whose areas represent products of two values, and squares whose areas represent squares of numbers.

Laborde (2001, as cited in Laborde, Gutierrez, Noss & Rakov, 2009) made the distinction between three embedded levels that pertain to the integration of technology tools in mathematics learning: interaction between tool and knowledge, interaction between knowledge, tool and the learner, and interactions between the learner and the teacher, that is, the level of integration of a tool in a math curriculum and in the classroom (Laborde, Gutierrez, Noss & Rakov, 2009).

According to NCTM (2000), technology is a fundamental principle in "high quality mathematics education" (p. 11) for "it influences the mathematics that is taught and enhances students' learning" (p. 24). An important way to facilitate students' understanding is multiple representations of mathematical ideas. Technology facilitates dynamic representations to allow students to discover, make predictions, observe, manipulate and interpret results.

2.2 Presentation and critique of software programs

A large number of journal articles, book chapters and conferences report studies on the use of computer software as a useful element in the teaching/learning environment. The software programs include, among others, Logo, Boxer, Geometric Supposer, DGS Software such as Cabri and Sketchpad, and applications to calculators.

Noss and Hoyles (1996) show that computer environment "can shape and remould mathematical knowledge".

In DGS, attention is brought to three different kinds of points: totally free points, points, free on an object, and fully dependent points such as points on an intersection. The drag option allows students to work on these three kinds of points and observe the differences of the levels of freedom.

In DGS, a dynamic diagram is constructed by a sequence of operations where elements are linked by dependency relations. A DGS highlights the functional aspects of geometrical figures. Laborde (1999) introduces a new kind of geometry where understanding is affected by the tools.

2.3- Mathematical Representation and the Learner

Learning mathematics is being able to move from diagram to discourse and from recognizing spatial shapes and configurations to reasoning deductively using theoretical knowledge. Geometry and graphical representations constitute together an additional entry to learning. (Hollebrands, K., Laborde, C. and Strasser, R. (2008)).

DGS enables students to draw diagrams in response to a series of geometrical instructions. When an element is dragged, the diagram is modified while all geometric properties are preserved. Diagrams react to manipulations respecting geometric laws exactly like material objects react respecting physics laws. This is better than pencil and paper where students can modify the diagram to fit their expectations.

Computer diagrams show external objects behavior and give feedback to be interpreted by students through geometry. This is where the link between spatial-graphical and geometric aspects comes into the picture. Leung (2003) believes that simultaneity narrows down the gap between experimental and theoretical mathematics: this is the theory of variation.

Research shows that feedback is primordial in students' development of solutions to problems. With computer software, feedback is immediate and provides students with valuable information. Some researchers think, however, that the choice of tasks is critical in students' understanding. Pratt and Davidson (2003) believe that the interactive white board (IWB) together with DGS are insufficient to encourage the fusion of visual and conceptual aspects of concepts when the task is only focused on visual transformation of geometric figures.

Sinclair (2003) shows that the design of accompanying material supports the development of exploration of strategies and geometric thinking.

Tools or artifacts are created and used by humans to accomplish actions more rapidly and easily such as geometric tools namely, compass to draw circles and ruler to draw lines. Mariotti (2001) introduces the "dialectic" relationship between practice and theory.

Vygotsky (1978) views a double function of mediating tools: external orientation based on technical tools, versus internal orientation based on psychological tools.

Hollebrands (2003) considers that students' understanding of basic concepts enables them to understand transformation as a concept. Thus, drag, macro-constructions and modules (Kadunz, 2002) locus and trace (Falcade, Mariotti & Laborde 2004) are signs that contribute to math learning. When students interact, they must differentiate between what is attributable to math and what is a bias introduced by the tool.

Goldenberg and Cuoco (1998) note the difference between Euclidean geometry and DGS geometry in the behavior of a point on a circle, for example. Scher (2001) found that students failed to differentiate between behavior attributable to math and the one biased by the tool used. This is where the role of the teacher is primordial, according to Mariotti (2001), to make the student learn. The teacher should use scaffolding, questioning, and discrete interventions to attain this goal. Hoyos (2002) notes the importance of collective discussions under teachers' guidance. This is also the role of the teacher as presented by Hoyles, Noss and Kent (2004).

2.4 - Instrumental Genesis

Noss and Hoyles (1996) say that learning does not come spontaneously from interaction with the microworld. Verillon and Rabardel (1995) introduce "instrumentation schemes", i.e. when students take the tool and integrate it in their learning. Arzarello, Micheletti, Olivero, Robutti, Paola, and Gallino (1998) discuss the three ways in which students use the drag mode. Dragging for wandering is the first, students explore to draw conjectures. The other two are not spontaneously done by students. These are the "lieu muet" dragging that they make to be convinced of the conjecture. The third is the dragging to test hypothesis, to collect evidence, and to see if the hypothesis is true or false. Holzl (1996) studies "drag and link" approach in dragging under Cabri. Talmon and Yerushalmy (2004) concentrate on prediction tasks.

2.5 - Proof with New Technology

It is still an open question discussed by Furringhetti and Paola (2003), Harada, Gallow-Dumiel and Nohda (2000); Holland (2001); Keskessa (1994)…Arguments for DGS note that construction implies the need for proof, while arguments against DGS say that the facility of measuring eliminates the need for proof.

Balacheff (1987) sees measuring as a crucial experiment that plays an important role in proof. In the same line, Vadcard (1999) believes that measuring is used by students to discover higher levels of proof. Flanagan (2001) and Hollebrands (2002) consider that measurements give empirical solutions and deductive arguments. Chazan (1993) gives importance to proof in the sense it is an explanation of empirical evidence. De Villiers (1991) reinforces this view of proof as he says that mathematicians prove something only if they are convinced that it is true. Hanna (1998) explains the purpose of proof and highlights its function as "an explanation of geometry". Edwards (1998) encourages activities before proof. This coincides with "Preuves, démonstrations et écritures décimales" (Bailleul, Comiti, Dorier, Lagrange, Parzysz and Salin, 1999) which came as an interpretation of the "didactiques des mathématiques" of Lagrange. Leung and Lopez-real (2002) created "Cabri proof by contradiction". Actually DGS allows activities over long-term to foster proof as said by Sanchez and Sacristan (2003).

Four studies made by researchers in 2000 discuss the subject of proof. The first is by Jones (2000) who discusses students' proof when they draw a quadrilateral and use the drag mode and the figure remains unchanged. The second is by Mariotti (2000) who defines proof as a means to show and choose the command which will give the intended outcome. Proof is needed for two purposes namely, establishing the validity of the construction and convincing others of it. The third study is by Marrades and Gutierrez (2000). It specifies that, to analyze a construction, proof is needed; the results are noted in notebooks to be updated by theorems. The fourth study is by Hadas, Herschkowitz and Schwarz (2000) who consider proof as a need for cognitive reasons and disequilibria. The four studies mentioned stress the primordial importance of DGS to "reconcile action and deduction", as expressed by Laborde (2000).

2.6 - Tutoring Programs for Geometry Learners

Beckman (2001) stresses the importance of the role of the teacher in this field. There are three main conclusions drawn from using DGS. The first is that computers provide a window on students' understanding. It is a departure from paper and pencil methods and the mental processes of the students are externalized. The second is that under DGS, construction tasks induce the need to integrate geometrical knowledge. The third is that DGS offers an innovative perspective in addressing the issue of teaching and learning of proof. Actually, DGS allows epistemological reflection on the nature of proof to develop. (Hollebrands, K., Laborde, C., and Straber, R. ,2008).

Many other dynamic software programs are created to help students develop conceptual understanding. These programs also help students make sense of what they learn in ways that are not actualized under paper and pencil conditions.

2.6.1- Turtle Geometry

Turtle geometry is the oldest graphic-based program that has been used for basic geometric concepts such as angles, rotations, etc. and yet it is still very much in use and intensely sold, in renovated versions. It has a lot of information to teach and it proved to be effective in the development of both teaching and learning of mathematics. The learning theory advanced by Pierre and Dina Van Hiele in 1986 is composed of five levels. Level 0 is when students do not recognize figures. Level 1 is when students see figures as a whole; it is called the visual level. Level 2 is descriptive and analytical; the students start dealing with properties. Level 3 is the abstract relational level, where students explore conjectures: this is where turtle proved to be very effective. Finally, level 4 is the level in which students are able to establish theorems, in an axiomatic system.

The teaching model is composed of five phases. The first is information level in which teachers give the information. Phase 2 is the guided orientation where exploration takes place; hence here turtle has shown the best effect. Phase 3 is explanation. Phase 4 is free orientation. Finally, phase 5 is integration.

2.6.2 – Logo

Logo geometry curriculum contains paths. Here the student learns how to go and come back to the starting point. The second is shapes: starting with squares, rectangles, equilateral triangles…. Students are encouraged to construct using properties. Finally, motion is the stage where students work with congruent figures.

Logo geometry has an effect on students' mental representation of geometry concepts. Gender differences are studied. Masculine performance is higher then females, because their expertise with computers involves elevated characteristics with males. Thus gender differences are artifacts of the task and measures of performance (Ching, Kafai& Marshall, 2002, Edwards, 1991, Yelland, 2002).

Measurement is positively affected by logo because it facilitates angle measurement (Browning 1991, Clements and Battisha 1989, du Boulay, 1986) as well as length measurement. Students transform mental and physical action into concepts of turn and angles.

Better recognition of relevance of proof and problem solving was found with the use of logo. Finally, logo affected transformations positively through puzzles and symmetry.

2.7- Computer Tools

2.7.1- Dynamic Geometry Software (DGS)

Integrating DGS in mathematics teaching and learning proved to be very effective in elementary and middle schools. Students' achievement is improved when using Cabri or Geometer's sketchpad because their understanding is better (Jackiw, 1991, 2002). The software improves visualization skills and develops interrelationships between shapes and their properties. The drag option triggers analytical thinking of students. Some research concentrated on reflections and rotations (Dixon, 1997) while others dealt with transformations (Glass, 2001).

2.7.2- Computer manipulatives

Computer manipulatives are very effective because they are concrete. They have many advantages. First, they provide an alternative building tool. Second, they provide a manageable, flexible manipulative. Finally, these games are an extensible manipulative. As for math benefits, the software brings new ideas that create awareness of the material. It is also a change in the nature of manipulatives. Finally, it allows decomposition and composition processes. There is a transition from the concrete to the pictorial and afterwards to the abstract stage. Computer Assisted Instruction increased the performance of students. It also leads to better understanding of spatial geometry (Lowrie, 1998). Games improve on coordinate geometry. Video games also provide spatial knowledge, more for boys than girls.

2.8- GeoGebra

This section of the literature review focuses on GeoGebra. It includes a brief presentation of the history of GeoGebra and a summary of the results of various researches made on GeoGebra and its integration in mathematics teaching.

The International Society for Technology in Education (ISTE) requires a change in teacher education programs. The ISTE's National Educational Technology Standards for Teachers highlight the role of technology in improving teaching and learning of mathematics. Keeping in mind that students are "digital natives" and teachers are "digital immigrants", powerful technological tools are available for teaching mathematics. These are subdivided into two groups which are: Computer algebra systems such as Derive, Mathematica, Maple or MUPAD and Dynamic geometry software such as Geometer's Sketchpad, Cabri Geometry and Geoplan-Geospace. These have been mentioned in the previous sections. The problem is that in may countries these two aspects namely, algebra and geometry are taught separately with no link between them. This is why they have been referred to as "two formal pillars" (Atiyah, 2001). This problem is highlighted when studying functions. It is not enough to show figures, , pictures or diagrams to facilitate students' learning of mathematics or there use of representations and visualization. Thus, there has been an increasing need to develop a dynamic linking of various representations in facilitating visualization of students because students can explore, solve and communicate concepts of mathematics in various ways. GeoGebra came as a solution that combines the ease-of-use of dynamic geometry software with the versatile capabilities of computer algebra systems. GeoGebra provides a closer connection between symbolic manipulation and visualization possibilities of CAS and the dynamic changeability of DGS. These are the two characteristics of the default screen of GeoGebra which provides two windows: an algebra window and a geometry window, with facility to move from one window to another. The big impact of GeoGebra is that it coordinates information across representational systems and connects the visualization of the graph of the function with its algebraic equation and with tables of inputs and outputs (Tall,2009 Bohm,2008) .

The software GeoGebra was created in the master's thesis project of Markus Hohenwarter at the university of Salzburg in 2002 (Hohenwarter, Preiner, 2007). During the past years many improved versions have been created. GeoGebra is written in Java, thus it runs on any computer, either direct from the web or by installing it. It is free of charge and easy to use. Moreover, it is now available in 45 languages and ia received many educational software awards in USA and in Europe such as EASA 2002, igita 2004, Comenius 2004, eTwinning 2006, AECT 2008, BETT 2009 finalist, Eurele A 2009 finalist (Hohenwarter, Lavicza, 2009). In 2008, active members of the GeoGebra club met in a conference in Cambridge and founded the international GeoGebra Institute (http://www.GeoGebra.org/igi). (Hohenwarter, Lavicza, 2009).

The software's nature and its availability together with its ease of use are assets of GeoGebra. One proof is that the the website: www.GeoGebra.org was visited by 300,000 in 2008 and it is estimated than over 100,000 educators use it over the world in their teaching (Hohenwarter, 2008) . Actually, this is in compliance with the words of Edwards and Jones (2006) that using GeoGebra inspires a change in the classroom problems "that need high-level thinking and things that students may find themselves wanting to follow up outside of regular school lessons" (p30).

Many researches have been made on teachers and pre-service math educators to test the effect of integration of GeoGebra in their teaching.

In 2007, a study was made on undergraduate students (prospective math teachers) who were asked to integrate GeoGebra in logarithmic functions teaching. This was part of a larger project ESCEMMAT which purpose was to improve teaching functions through technology-based mathematics lesson (Gomez-Chacon, 2010). The results were that 64% of the participants show a high interest in computers. Students were more interested and they were enjoying their work. Moreover, the use of computers gave them additional freedom for experimenting new ideas. The most important result is that all students found that using GeoGebra was very convenient especially to make connections between algebraic and geometric thinking.

A project funded by the National Center for Excellence in the Teaching of Mathematics (NCETM) for England was conducted nine experienced teachers (Jones, 2009). The project team was made of researchers and teachers. The purposes were: to nurture a professional development network around the use of GeoGebra in England, to improve integration of GeoGebra into the mathematics curriculum and to enhance the professional opportunities of teachers by giving them workshops. Work on the project shows that GeoGebra can be integrated in teaching many geometry concepts of the secondary school as it allows students to develop a good vocabulary, produce accurate drawings in no time and experiment new ideas with direct feedback. Students do not loose time setting up the task, but instead they spend time exploring the mathematics that is central to the task. Teachers play a guidance role and this approach opens the door for creative thinking and problem solving abilities of students.

CHAPTER THREE

METHOD

This study is an action research aiming to investigate whether technology moulds students' thinking and whether it has an impact on their understanding. This is an action research that is of an experimental nature, with an intervention to implement specific teaching strategies and investigate their outcomes. Two groups of students are considered: one is the experimental group and the other is the control group. Moreover, besides adopting an experimental design, the study uses also mixed methods of research: an attitude survey and interviews with selected students.

The researcher is the teacher of two grade-10 classes that would serve as context for the experimental implementation of mathematics modules with, and without technology integration. The curriculum unit chosen is "Quadratic Functions" for grade 10. The experimental group is taught the whole unit with integrated activities using various technologies namely, programmable calculator (Casio ClassPad 300), a dynamic algebraic and geometric computer application: GeoGebra, in addition to a spreadsheet program (Excel 2007). During the same period of time, the control group is taught "Quadratic Functions" of grade 10, the classical way: that is, with activities and applications that do not involve technological devices, except for a non-programmable scientific calculator.

3.1-The Mathematical Unit

The mathematical topic chosen to be the context for experimentation is that of quadratic functions for grade 10 of the French renovated curriculum of the year 2010. Many reasons underlie this choice. They are discussed in this section.

First, *Quadratic functions* are at the core of the tenth grade program and make a considerable part of it, starting the tenth grade. In the French curriculum, grade 10 is undifferentiated for all secondary tracks, that is the *Scientific*, the *Economics*, and the *Literature* tracks. This all students will be exposed to this topic, irrespective of their future track. The more general topic of *Functions* is further developed in and in grades 11 and 12 in the three tracks to different extents and under different aspects. It is to be noted that under the French Program, even the *Literature* students do sit for a mathematics exam in the Baccalaureate. As far as the *Economics* track is concerned, functions are developed and used in economical problem-situations such as optimization - to choose the best financial alternative for a manufacturing company, for example. For the scientific track, *functions* are further developed and extended to *exponential and logarithmic functions* as well as functions applied to other domains such as physics, chemistry, biology and dynamics.

Second, the topic of *functions* lends itself to classical teaching approaches as well as to technology-based approaches. Many computer-based applications are available that facilitate the exploration and representation of functions. An important aspect of functions is that they are used to solve authentic real-life situations such as choosing the best pay alternative upon joining a sports club, for example. In such situations, the choice is to be made between a lump-sum amount to be paid for the whole season against a partial membership with a variable amount according to frequency of use. Such cases and the like are easily solved with linear functions.

Third, there is nowadays an important shift in the focus of the evaluation system. Students are not asked anymore, in exams, to just plot graphs of functions, because this could be perfectly done by a graphic calculator or dynamic Computer Algebra System (CAS) software. The basis of evaluation is the analysis that students can conduct having the available data or graph. The focus is hence shifted from the abstract study of the variations of functions and plotting their graphs to more applied and meaningful analytical studies of functions and their graphical representations to solve authentic real-life situations. Functions are now seen as the base of decision-making. Furthermore, sometimes students are asked to translate real-life problems into graphs to come up with solutions which are feasible and fruitful in optimization problems with constraints.

For the purpose of this research, the mathematical unit on linear and affine functions and on quadratic functions was modified (See Appendix I) to become technology-rich, by integrating technology-based activities. This modified unit is the one that was implemented in the experimental section of grade 10, while in the control section, the unit as taught usually, with no or very little technological devices, was implemented.

3.2- Sample

The study is conducted in a private French school in Beirut. The school abides by the French program reformed in 2009 and implemented as of October 2010. The sample is composed of two grade-ten sections. One section is considered to be the control group where classical methods of teaching are used. The other section is considered to be the experimental group where technological devices are incorporated in teaching.

The two sections have comparable sizes; the first includes 25 students and the other 24. The size of the sample is then 49 participants, around 60% of which are girls. More precisely, the sample includes 28 girls and 18 boys who come from comparable socio-economical and educational backgrounds. The age interval is 15-17 years. In order to guarantee similarity of achievement levels in the control and experimental groups, the averages of students' math grades over the elapsed period of the experimental year were calculated. The proportions of high-, average- and low-achievers showed to be similar in both sections.

As far as teaching methods are concerned, students are trained to work either individually or in groups. More importantly, students are used to critical learning and analytical thinking as opposed to rote memorization and parroting information. The constructivist approach is globally the basis of education in the mathematics classes. Students are usually assigned activities from which they draw relationships, properties and conjectures to be proved later. They construct their own knowledge under teachers' guidance. No formal lecturing sessions are given; rather interactive sessions take place almost exclusively throughout the year. The teacher posts the plan of the session and then students work on pre-planned activities under the teacher's guidance (See Appendix II for experimental-group activities). As students proceed with their work they reach the objectives of the lesson plan.

3.3- Procedure

The research procedure included three phases: The experimental implementation of the mathematical unit, the attitude survey, and the interviews with five selected students.

3.3.1- Implementation of the mathematical unit

Throughout the implementation of the mathematical unit, the control group attended sessions in a regular classroom equipped with a chalkboard. The tools used for conducting the class activities are paper-and-pencil, the math textbook, graphing paper, and all geometry manual instruments (compass, ruler, set square and protractor). The experimental group, however, attended all sessions in the computer laboratory that is equipped with a smart interactive board directly connected to the teacher's computer. Each student had access to a computer with GeoGebra and Excel 2007 software installed. Moreover, each student has a Casio ClassPad 330 programmable calculator.

The implementation was undertaken over a two-month period. The pre-test / post-test method was used. First, a paper-based pre-test (Appendix III) was administered to evaluate students' prior knowledge of functions, previously acquired in grade 9, and to make sure that both sections stand almost at the same level of knowledge and achievement. Afterwards, a two-month period is devoted to the implementation of the experimental unit, including a review of *linear and affine functions* and the introduction and development of *quadratic functions*.

In the control group, the only technological device used is a non-programmable scientific calculator. Teaching sessions took place in classical classroom with a chalkboard, using paper-and-pencil activities and manual geometrical instruments. Students were assigned activities and were asked to draw conjectures to be proved later. Application exercises were alternated with class activities and discussions / questioning sessions. Students were also given homework to be corrected later in class.

In the experimental group, the same procedure was followed with the same activities and examples to ensure fairness. The only difference is that the classes tool place in the computer laboratory. A smart interactive board was available and directly connected to the teacher's computer. Some activities integrated the use of a programmable calculator, namely Casio ClassPad 330. Further, each student had a computer networked with the printer. The programs used are Excel 2007 and GeoGebra dynamic software. GeoGebra is adopted for two reasons. First, it is downloadable free of charge and hence students can use it at home, which means that they may apply and revise at home whatever they have learned in class. Second, GeoGebra allows students to see side by side the algebraic and geometric representations of the functions.

It is important to note that the teaching pace was the same during a session and the number of sessions assigned was the same for both sections. On the other hand, it is important to note that students in both sections were exposed to classical routine mathematical examples as well as authentic problems. A common formative quiz (Appendix IV) was also administered to both sections.

During the two-month implementation period, a formative-evaluation quiz was administered to check the students' level of understanding and to make sure that the two sections were ready to move forward. The quiz was administered directly after *linear and affine functions* were fully covered and just before the introduction of *quadratic functions*.

The period of time dedicated to *quadratic functions* was much longer than that dedicated to *linear and affine functions,* because quadratic functions were completely new to students. It was the first time they officially deal with parabolas and see graphs with changing sense of variations, with a minimum or a maximum value of y. The sessions started with examples having different values of coefficients. Tables of values were prepared to study the sense of variation of the functions under study. Afterwards, authentic real-life problems were assigned to students in order to make sure that they are able to use what they have studied and translate problems into functions in order to find the best solution available.

In the experimental class, technological devices were used. Students used their programmable calculators to do all direct calculations. Also Excel was available on students' computers and consequently all calculations to find values of functions for certain values of the variable were overcome. Consequently, some activities took less time to be completed. For example, much time was saved when setting tables of values and tables of variations. The saved time was used by the students for interpreting results, commenting on them, and coming up with optimal solutions to the assigned problems, upon the request of the teacher.

It is important to note that, in some activities, students in both sections were allowed to work in groups. This is in compliance with the constructivist approach because it allows brainstorming that is expected to enrich students' thoughts and enhance their mathematical communication.

At the end of the implementation period, a paper-based post-test (Appendix V) was administered at the same time to both sections: the experimental and the control groups. In the post-test, exercises and problems of different nature and levels were assigned to students to investigate students' procedural knowledge, conceptual understanding, and problem-solving abilities, as per the framework adopted by the National Assessment of Educational Progress [NAEP] (National Center for Education Statistics, 2005).

In order to insure fairness among the two sections, students of the control group were assigned extra sessions after the implementation period, to expose them to technology-based activities, allowing them to develop the technological skills developed by their fellows in the experimental group.

3.3.2- The Questionnaire

At the end of the implementation phase, a questionnaire was administered to both the experimental and the control groups. The aim was to explore students' attitudes towards mathematics and the use of technology in learning it. Students were asked to fill in the questionnaire under the supervision of a substitute teacher to prevent the effect of the presence of the researcher. Students were given pens and were asked to tick the correct answer. No writing was required so that students would not fear that the teacher would recognize their handwriting. Students were assured anonymity and confidentiality. The questionnaire (Appendix VI) is a Likert-type scale, consisting of 21 statements that students were asked to evaluate on a scale of four levels: Never, Sometimes, Often and Most often. Twenty-two students from the experimental group and 24 students from the control group filled the questionnaire.

3.3.3- The Interviews

After the survey, a semi-structured interview composed of few questions (Appendix VII) was conducted with five students of the experimental group, in order to get a closer look at their attitudes and to triangulate the data. The students were interviewed individually, the interviews were taped and notes of the answers were taken. The interviewed students were three boys and two girls, selected according to the following:

- One boy who had poor results in the pre-test and much better results in the post-test.
- One girl who had poor results in the pre-test and good results in the post-test.
- One boy who was new to school coming from a foreign country. He was chosen because the unit entitled "*functions*" was a completely new subject to him, as he was not exposed to it in the other country.
- One girl who was very rebellious at the beginning of the unit and then demonstrated a considerable change in attitude towards mathematics. She showed an increasing interest in the subject as it was treated.
- One boy who had excellent scores in both tests, and who did not like technology.

The interviews were transcribed and analyzed qualitatively in an attempt to support the results of the survey in answering the research question:

What are the attitudes of students towards the integration of technology in teaching mathematics especially *quadratic functions?*

3.4- Analysis Method

The data to be analyzed was collected using the following instruments: the pre-test, the quiz and the post-test on one hand, and the questionnaire and interviews with students, on the other hand. Data are analyzed quantitatively and qualitatively. The pre-test, quiz and post-test are analyzed at both levels: quantitatively and qualitatively, while the questionnaire is analyzed quantitatively and the interviews qualitatively.

A T-test is used to compare the results of tests in-between the experimental and the control groups, as well as between the pre-test and the post-test within each group. Comparisons of the means of each group are conducted and charts are drawn to come up with conclusions and to find answers for the research questions. On the other hand, the tests and the quiz are also analyzed qualitatively: several parts of the solutions to problems proposed by students are presented and analyzed qualitatively because these show students' flow of ideas. This is an answer to one of the research questions, namely: how does the integration of technology in mathematics teaching mold students' thinking.

CHAPTER FOUR
FINDINGS

4.1- A Priori Analysis

This section presents an a priori analysis of the activities and tests assigned during the experimental unit implementation. It focuses on the purpose intended by each activity and explains the logic behind assigning such an activity. As far as tests are concerned, these have been assigned and composed in a way to answer the research questions stated in the introduction of the thesis report. It is important to note that there is a logical and rational continuation of the themes tested in the tests, for example in the quiz and in the post-test there are exercises that shed light on the effect of technology's integration on connections between algebra and geometry, as well as connections to authentic real-life problems.

4.1.1- A Priori Analysis of the Pre-test

The first checkpoint is the pre-test (Appendix III) assigned to both groups to verify their standing and check whether the two groups are comparable. This is why the questions are more concentrated on the lower levels of knowledge, namely procedural and conceptual. The pre-test is focused on the general concept of functions because the concepts of image and antecedent were briefly tackled in grade 9. It is composed of two exercises that deal with the same ideas approached from two different aspects: the algebraic aspect and the graphical aspect. The rationale is that the key words: image and antecedent are sometimes understood by some students algebraically but without making the graphical link.

The <u>first exercise</u> is composed of four questions that are direct applications of the concept of function. They test: 1) the conceptual understanding of the concepts of function, image and antecedent, and 2) the computation skills as applied to finding the image or the antecedent of a value, or to solving for the roots of a function. Students are expected to know that to compute the image of a certain value then this is the value to attribute to *x*. On the contrary, if they are asked to find the antecedent then the unknown is the *x*, and the given is the value of f(x). Students are asked to solve an equation in this situation.

On the other hand, students should know that questions 1 and 2 of the first exercise are procedurally similar except that one is stated in conceptual language "*image of 0 by the function*", while the other is stated in mathematical symbolic language. The same applies to questions 3 and 4 of the first exercise.

The <u>second exercise</u> deals with the same levels but based on representations. Students have to answer questions based on the graph of the function without knowing its algebraic formula. Using the graph, students should know how to read the value of the image given the value of the variable *x*, and inversely the value of the antecedent given the value of the function f(x). Questions are included to evaluate conceptual understanding and representational skills.

Question 3 of exercise II deals with multiple representations, as students are asked to translate the graph into a table of variations with all the notations and special values it has: Students should extract from the graph information that indicates ob which intervals the function is increasing or decreasing.

In exercise II, questions 4, 5, 7, and 8 evaluate students' representational skills and critical thinking. The purpose is to check whether students can solve equations and inequalities graphically. Many students know how to solve equations and inequalities algebraically but usually fail to translate the results graphically. This is why these questions were assigned to investigate whether the algebraic solutions have a graphical interpretation in students' thinking. Moreover, it is important to note the difference in the solution between equality and an inequality. Students' attention should be drawn to the fact that since the graph is continuous, the solution could be written in an interval form.

4.1.2-A Priori Analysis of Activities 1, 2 and 3

4.1.2.1-Activity 1 (Appendix II)

The purposes of activity 1, concerned with linear and affine functions, are numerous. First, the activity is important because it shows students graphs being constructed point by point. Students can observe the evolution of the graph instead of seeing it as a fully finished one (items 2&3). Second, activity 1 is designed to show that when the coefficient of a linear function is positive, the line goes upwards as students look at it from left to right. This is interpreted by the fact that as students look from left to right, the variable x increases and accordingly y increases (items 3&4). Third, when a is positive, as its value increases, the line becomes steeper and closer to the y axis (item 5). Fourth, when the sign of the coefficient changes, the graph is obtained is the symmetrical of the original one. This aspect is important because it gives a broader application of the symmetry concept. Here, symmetry is seen outside a geometrical context (items 6,7,8,9&10).

4.1.2.2- Activity 2 (Appendix II)

Activity 2 is different from activity 1 because it is designed to be done on the programmable calculator. Hence, there is no slider to be used to change the value of the coefficient and see the effect directly on the graph. Certainly, the coefficient's value could be changed but there is no automatic animation that changes the value of the coefficient. This is why only two values of the coefficient were chosen, namely 2 and -2.

The programmable calculator allows students to see the graph drawn gradually and point by point. Moreover, the calculator gives directly the table of values of the function. Students can directly compute the value of y given that of x and vice versa. More importantly, the table of values allows students to observe the behavior of y as x increases, by narrowing down or increasing the step between various values of x according to the teacher's or students' needs. Students can see that when the coefficient is 2 then as x increases so does y and this incarnates in students' thinking the idea of an increasing function.

On the other hand, since the screen of the programmable calculator is divided into two windows, students can see at the same time, the table of values and the graph. More importantly, the calculator has an option of having a blinking point on the graph. This allows the student to move the blinking point and record the behavior of the function by observing the values of the coordinates of each point, shown at the bottom of the screen.

Finally, when the coefficient's sign is changed (from 2 to -2), students see directly the symmetry that is involved. In addition students can have both functions drawn at the same time with both tables of values which makes the comparison easier and more meaningful.

4.1.2.3- Activity 3 (Appendix II)

This activity is to be conducted on the computer using GeoGebra. Having in mind the activities previously conducted, this activity aims to confirm the formulated conjectures, namely: 1) *if the coefficient is positive then the function is increasing*, and 2) *as the value of the positive coefficient increases then the line is steeper*. This is performed through using a slider that is available in the GeoGebra application. The values of the coefficient are set between 0 and 5 and the software would automatically change the value of the slider given a certain increment. Students can then observe the graph moving more and more towards the y axis.

The second part of the activity (items 6&7) is interesting because when the coefficients' limits are turned into negative numbers, then the rule is reversed and students can observe that as the value of the coefficient a decreases then the line becomes steeper.

The final part of the activity (starting item 8) is important because it introduces the affine function that is derived from the linear function through a translation. Students plot the graph of the initial function $f(x) = ax$ and notice that $f(x) = ax + b$ is actually the image of the first one by a translation of vector (0, b). Then the conjecture is generalized and the activity sheet is completed.

At this point, students are exposed to another field of application of translations that is neither in the geometry domain nor in physics. Connections between the three domains, physical, geometrical and graphical, are thus reinforced through the concept of functions.

<u>4.1.3- A Priori Analysis of the Quiz</u>

The quiz (Appendix IV) is assigned at this time (after Activity 3) to close the sessions on *linear functions* and before starting *quadratic functions*. The duration of the quiz is 60 minutes. Students sit for the test with manual instruments (ruler, graph paper) and a scientific non-programmable calculator only.

The quiz is composed of six exercises that evaluate different levels of assimilation and understanding of linear functions.

The <u>first exercise</u> evaluates students' procedural knowledge and conceptual understanding. The purpose is to check basic knowledge of the concept of *linear and affine functions*. Table 4.1 shows the rationale behind each question in the first exercise.

The <u>second exercise</u> evaluates students' ability to represent the variations of a function. In this exercise, direct application of the procedure is evaluated.

The <u>third exercise</u> concentrates on applications, that is procedural abilities and applications of formulas are evaluated.

TABLE 4.1
Role of Exercise I in the Quiz

Function	Justification of each choice
$f(x)=\frac{1}{2}x$	This is a linear function written in its basic format
$f(x)=\frac{-1}{2}x+3$	This is an affine function written in its basic format
$f(x)=5\sqrt{x}+3$	Students should know that this function is not linear because its is a function of the square root of the variable
$f(x)=x\sqrt{5}+3$	Students should know that the coefficient could be a square root.
$f(x)=\frac{x}{2}+x$	Students should reduce and then state the nature of the function
$f(x)=x^2-1$	Students should notice that the variable is squared, so the function is not linear
$f(x)=\frac{1}{2}x-\frac{x}{2}+3$	Students should know that constant functions are also linear, the coefficient could be zero
$f(x)=(x+1)^2-(x-1)^2$	Students should develop then answer. Despite the squared factors, the function will turn out to be linear.

The fourth exercise investigates students' connections between algebraic expressions and graphical representations of functions. It concentrates on representations and recognition of the graph of a function among others. It is to be noted that the lines represented are more than the functions assigned, in order to reduce the effect of luck.

The fifth exercise is an authentic real-life problem; hence it evaluates students' critical thinking, reasoning and problem-solving abilities, together with connections. This exercise is more important than the others because it evaluates whether students have grasped the concept and uses of functions as a modeling tool, and whether they will be able to apply it in life. Various options are considered in the problem and a decision has to be made as to the most economical option. The problem's purpose is to evaluate students' critical thinking when faced with a real-life problem.

The last exercise is a geometrical application of functions. It further highlights the connections between geometry and algebra. It presents a geometrical situation that is to be translated into functions. Students are expected to solve the problem using functions and to produce graphical and algebraic solutions. The focus is on connections between various disciplines as well as representations.

4.1.4- A Priori Analysis of Activities 4, 5, 6, 7, 8, 9, 10 and 11 (Appendix II)

4.1.4.1- Activity 4

This activity is different from the others in the sense that its purpose is to give a real-life problem that could be solved with a quadratic function before any instruction about the concept. Students are faced with this authentic problem and they try to come up with a solution under the teachers' guidance but without teacher's interference in the solving process. Students have no notion of *quadratic functions* yet. This is why the activity is challenging. It aims to trigger students' thinking and create a need for new mathematical tools to solve the problem. This is very nourishing for students' minds because it implies special mind exercises and it involves students' private initiative.

At the end of the whole unit and after development of *quadratic functions*, it is expected that students would make a flash-back to this problem and find the solution using quadratic functions. This would give meaning to the concept of *quadratic functions* and would give students a real-life example. Thus, the *quadratic functions* concept would acquire an applicable practical aspect from the beginning and students would understand it more thoroughly because they feel the need for it.

4.1.4.2- Activity 5

This is an introductory activity to *quadratic functions* starting with the basic parabola with equation $y = x^2$. It is important because this is the first time students are faced with a graph that is not linear, namely the parabola. Many points of discrepancy are to be commented as opposed to *linear functions*. First, in the case of the parabola (the basic parabola in this activity), the sense of variations is not the same throughout the domain of definition. Second, the parabola has a vertex, which in this case is a minimum, and this is a new keyword in the study of functions. This is the main reason for which the activity is conducted on the programmable calculator as it explicitly gives the value of the minimum of a function. When solving question 9 of the activity, the calculator gives an answer, whereas in question 10, the calculator would reply that the minimum is "not found". Students would be asked to justify this by saying that since the line is infinite then it has no minimum and no maximum. Third, the parabola has elements of symmetry namely, the ordinates axis in the case of the basic quadratic function. Finally, the number of solutions of the equation $f(x) = m$ (where m is a number) does not always have a single solution as in the case of a linear functions.

4.1.4.3- Activity 6

The purpose of this activity is to extend the findings of the previous one through the study of the function $f(x) = ax^2$ and the use of a slider to change the value of the coefficient a. Students would directly observe the effect of such a change on the shape of the parabola. They can observe that, as a increases, the parabola becomes thinner and gets closer to the y-axis, although it still has the same minimum which is the origin. Afterwards, students would observe that for negative values of a, the parabola is turned upside down and it has a maximum that is still the origin. Contrarily, when "a" is negative, then as it increases in value, the parabola becomes wider and moves away from the y-axis, while having the same maximum, the origin.

The second part of the activity introduces the general formula of a quadratic function with a slider for each coefficient. Students would write their conjectures. In question 9 of the activity students would notice that the axis of symmetry in the general case is $x = - (b/2a)$ and that the value of the origin is not the extreme anymore.

4.1.4.4- Activity 7

The focus of this activity is the sense of variation of quadratic functions and the table of variations. In this activity, the slider chosen attributes only positive values for the coefficient "a". The first purpose of the activity is to show that when "a" is positive then the parabola has a minimum which is $x = - (b/2a)$. The second purpose is to show that when "*a*" is positive, then the sense of variation is decreasing on the interval $\left]-\infty;\frac{-b}{2a}\right]$

and increasing on the interval $\left[\frac{-b}{2a};+\infty\right[$.

4.1.4.5- Activity 8

This activity is similar to the previous one except for the fact that the values attributed to coefficient "*a*" are negative. The purpose is to show that when *a* is negative then the parabola has a maximum that is $x = -\ (b/2a)$. The table of variations in such a case is the reverse of the previous one that is: the function is increasing on the interval $\left]-\infty;\frac{-b}{2a}\right]$ and decreasing on the interval $\left[\frac{-b}{2a};+\infty\right[$.

4.1.4.6- Activity 9

This activity is assigned using GeoGebra. Students can observe on the screen simultaneously the geometrical figure, together with the graph of the function being drawn point by point. This problem is important because it involves connections between geometric, algebraic and representational domains. The situation to be solved is geometrical in nature and could be solved algebraically with an equation as well as graphically with the graphical representation of two functions.

Students are expected to realize that the area of a square could be translated into a quadratic function, graphed with a parabola, whereas the perimeter of a square is a linear function, graphed with a straight line. More importantly, students should note that the domain of definition of both functions is restrained to the positive values of x because it is a length.

The graphs would intersect in two points, namely $x = 0$ and $x = 4$. Students are expected to interpret the result and deduce that the only acceptable solution is 4 because the value 0 cancels the existence of the square.

4.1.4.7- Activity 10

This activity is assigned using the software GeoGebra. It stresses the connections between geometry, algebra and functions. Students are expected to state the possible values of the variable x. Second, students are expected to move point C and write a conjecture about the changes in values of the areas of triangle DCE. Having solved the problem graphically, students are expected to write a proof using, together, their geometric and algebraic knowledge.

4.1.4.8- Activity 11

This final activity is a geometrical application of functions. It involves all mathematical fields and could be solved with a quadratic equation as well as with the comparison of a quadratic function and a linear one.

As in the previous activities, students are expected to determine the interval of values that are acceptable for the variable. They are also supposed to use their geometry formulas to find the expression of the function and finally they are expected to solve the problem graphically and algebraically. This activity was chosen on purpose because many students fail to see that the positive values of the function

$f(x)= x^2 -11x+28$ are the same as those for which the points of the parabola representing $g(x) = x^2$ are above those of the line representing $h(x) = 11x – 28$.

4.1.5- A Priori Analysis of the Post-test

The objectives behind the post-test (Appendix V) are several, namely an evaluation of conceptual and critical thinking, as well as connections and representational abilities.

Exercise I evaluates students' awareness of the fact that quadratic functions do not have the same sense of variation throughout their domain of definition, thus the interval in which *y* lies cannot be found in a linear way for all values of the variable *x*. This is why, in many lines of the given table, the variable *x* ranges between a negative and a positive value. Table 4.2 presents the purpose behind each line of the exercise.

TABLE 4.2
Purpose of Items in Exercise I of Post-Test

If…….	Then……	Purpose of the Question
$0 < x < 2$	$\ldots < x^2 < \ldots$	To recognize that f is strictly increasing on this interval
$-3 < x < -1$	$\ldots < x^2 < \ldots$	To realize that f is strictly decreasing on this interval
$-2 < x < 2$	$\ldots < x^2 < \ldots$	The numbers are chosen to be opposites on purpose because they have the same image, students should know that the sense of variation changes within this interval
$3 < x < 7$	$\ldots < x^2 < \ldots$	To realize that the function is strictly increasing on this interval

$-5 \leq x < 1$	$\ldots \leq x^2 < \ldots$	The sense of variation changes so two intervals should be considered: [-5,0[and [0 ;1[
$x \leq 4$	$< x^2 \leq \ldots$	Students are expected to answer that the image is always positive. (and not less than 16)
$- < x < 3$	$\ldots < x^2 < \ldots$	Two intervals have to be considered because the sense of variation changes within the interval
$x > 8$	$\ldots < x^2 < \ldots$	To state that the function is strictly increasing on this interval

The purpose of Exercise II is a direct evaluation of the procedure to be used to determine the sense of variations of the function.

Exercise III is given to test whether students know how to find the axis of symmetry of the graph, in addition to the study of the sense of variations of the function. Afterwards, students are tested whether they can connect the algebraic solution to the graphical interpretation. Therefore, in this exercise connections and representational abilities are evaluated.

Exercise IV is assigned to evaluate whether students are able to read a graph and use it to solve equations and in inequalities.

Exercise V is an application of geometry to functions. Students are asked to solve a geometrical problem graphically and algebraically. Here the aim is to investigate whether students make the relation between geometry and algebra. Connections and representational abilities are tested in this exercise.

4.2- Quantitative Analysis of Tests

The quantitative analysis encompasses three tests: the pre-test, the quiz and the post-test. It is conducted at two levels. First, each student's pre-test, quiz and post-test were assigned scores for both sections. For each test / section, the mean and standard deviation are calculated, as well as other descriptive statistics (see Table 4.3)

The results of the tests and the statistical calculations are shown in Appendix VIII. Table 4.3 summarizes some of the statistical indicators that reveal important aspects to be analyzed in the next paragraph.

TABLE 4.3
Descriptive Statistics of the Pre-Test, Quiz and Post-Test for the Experimental and Control Groups

	Pre-test		Quiz		Post-test	
Section	experimental	control	experimental	control	experiment	control
Mean	10.86	12.25	11.83	11.65	12.64	12.54
Standard deviation	3.65	3.19	3.44	3.73	2.48	3.04
Min X	4	6	5	4	8	6.5
Q1	8.5	10	9.5	9.5	10.5	11
Median	11	12	11	11.25	12.5	12
Q3	14	14.5	15	14.5	14.75	14.5
Max X	17	20	18.5	17.5	17	20
Mode	8.5	10	15	10	10	11
N	25	24	23	24	21	25

It seems important to note that the total number of students is not the same over the three exams because of the absence of some students. No make-up exam was administered to ensure fairness and consistency.

Following is the analysis of the statistical indicators shown in Table 4.3.

4.2.1-Mean

The mean is subject to two kinds of comparisons; one which is a longitudinal comparison, to follow the evolution of each group's scores, and the other is a horizontal comparison between groups; that is to compare the experimental to the control groups' scores for each test.

For the longitudinal comparison, the trend followed by the mean of the experimental group is upward while that of the control group decreases from the pre-test to the quiz then goes upwards again. The experimental group's mean goes from10.86 to 12.64 and this is a significant increase because it is almost at the rate of 16.5%, while the control group's mean changed from 12.25 in the pre-test to 12.76 in the post-test, achieving an increase of only 2.37%. The noticeable increase in the mean of the experimental group could be attributed to the integration of technology in teaching functions. This idea is justifiable because all the other factors are kept constant, the teacher is the same, the rhythm is the same and the environment is the same. One may safely interpret the result as showing that the integration of technology has aroused students' interest and willingness to learn, an interpretation that is reinforced by their being "digital natives". Second, Casio ClassPad together with GeoGebra has freed the experimental group's students from computation mistakes; hence they could focus on the core of the problems and forget about other trivial factors.

In the horizontal comparison of the means, it is to be noted that there has the classification of the mean of each group has changed over the research period. When comparing the pre-test means, the mean of the experimental group (10.86) is much below that of the control group (12.25), the discrepancy is almost 12.8%. This situation is already reversed in the quiz results as the experimental group's mean (11.83) surpasses the control group's mean (11.65). This trend has continued in the post-test where the experimental group's mean (12.64) is higher than that of the control group (12.54). More importantly, the difference between both groups' means is reduced to almost 0.8%. This indicator shows clearly that not only the experimental group's mean has improved over the research period in absolute terms but also in relative terms; that is the mean of the experimental group has improved over time and got closer to that of the control group which started at a much higher score in the pre-test.

4.2.2- Standard Deviation

The values of the standard deviations of both groups over the three tests are almost constant and rather moderate. This shows that the population has a closely normal distribution. This is important because it increases the reliability of the test.

A closer look at the standard deviation values shows that in the experimental group the trend is downwards with a decrease from 3.65 to 2.48. This decrease of almost 32% is a healthy indicator because it shows that the distribution of grades is closer to the mean than in the pre-test. Thus all grades revolve around the mean more than before. This could be interpreted by a general positive learning atmosphere since 50% of the grades are so close to the mean. This is an achievement because it indicates that all students were involved and working hard which is in compliance with the "No Child Left Behind" recommendations.

4.2.3- Minimum and Maximum Values of Scores

In the experimental group the range of scores is decreasing over time. It goes down from (17-4) which is 13 to (17-8) which is 9. The decrease is by 30%. This is also a positive indicator, related to the decrease of the standard deviation, because the smaller the gap, the closer are the grades and the more normal is the distribution. Hence, the class is more homogeneous.

4.2.4- Quartiles and Medians

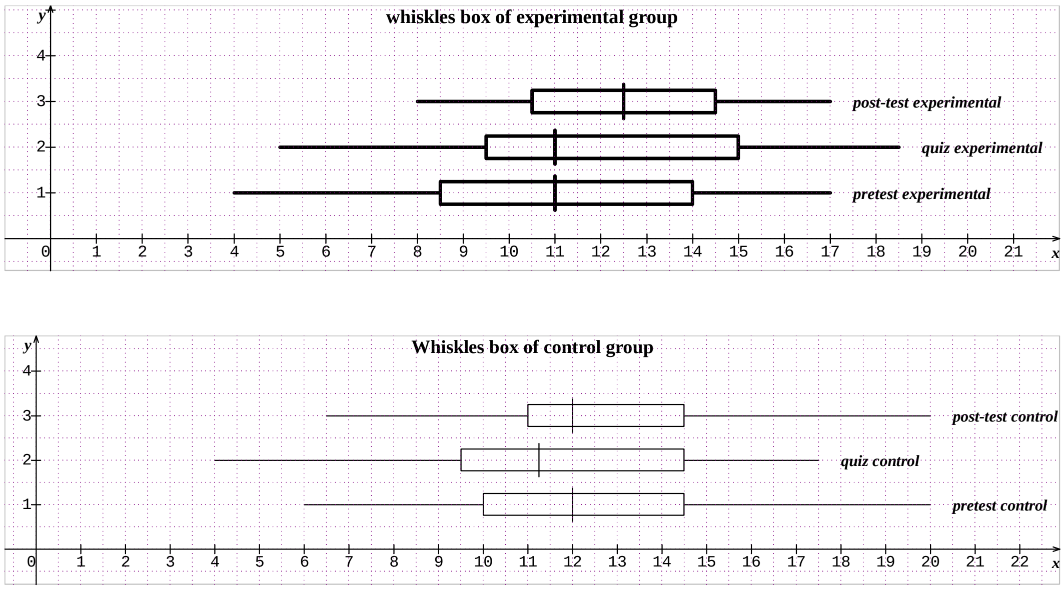


Figure 4.1: Boxplots of pre-test, quiz and post-test results for the experimental and the control groups

The boxplots in Figure 4.1 show that for the experimental group, the median has increased between the pre-test and the post-test, while it remained the same in the control group, after a decrease in the quiz. Thus, at the beginning of the research period, 50% of the experimental group's participants had a score up to 11; while at the end, 50% of the same group had a score up to 12.5. if half of the class has scored more than 12.5 on the post-test, then this means that the unit had a positive impact on students' learning.

Moreover, Figure 4.1 shows that the quartiles of the experimental group increased steadily over the implementation period, especially the first quartile (from 8.5 to 10), which indicates that low achievers in the experimental group could achieve better. On the other hand, the highest quartile increased from 14 in the pre-test to 14.5 in the post-test, passing through a score of 15 in the quiz. As to the control group, the lowest quartile increased from 10 in the pre-test to 11 in the post-test, passing through a decrease to 9.5 in the quiz, while the highest quartile remained the same across the three tests.

4.2.5- Mode

The mode of the experimental group has increased from 8.5 to 10 as opposed to that of the control group which increased only from 10 to 11. This increase reflects a positive effect of integration of technology on grades as the most frequent grade at the end of the experimental period is higher than at the beginning. The increase in the mode for the experimental group gets more importance because it shifts the most frequent score from a failing grade (8.5) to a passing grade (10).

4.3- Comparison of the Experimental and the Control Group Using the T-Test

Following is an analysis of the pre-test, the quiz and the post-test of both groups using the T-test. The T-test has been chosen because it is appropriate when one wants to compare the means of two groups. The purpose of the T-test is to assess whether the means of two groups are statistically different from each other.

Assumptions:

The assumptions of an independent T-test were examined.

Data Measured at the Interval level: Data was measured at an interval level.

Independent scores: the scores are independent because they come from different people, and there are no influences on one another

Normally Distributed Data: the Kolmogorov-Smirnov test for normality revealed that the scores are normally distributed in both sections.

TABLE 4.4
Table of T Values of the Pre-Test, Quiz and Post-Test

	Pre-Test	Quiz	Post-Test
T	-1.42	0.17	0.12
a	0.05	0.05	0.05
df	47	45	44

Table 4.4 shows that the difference in achievement between the experimental group and the control group in the post-test is not significant at the 0.05 confidence level. However, it is important to note here that, in the pre-test, the control group had a higher mean score than the experimental group.

In this section, a within-group analysis of the means of the pre-test and the post-test is undertaken for each group aside. The T-test has been chosen because it is appropriate when one wants to compare the means of two independent tests. The purpose of the T-test is to assess whether the means of the pre-test and the post-test for each group are statistically different from each other. The idea is to compare the mean of the pre-test to that of the post-test for the experimental group on one hand and for the control group on the other hand.

Assumptions: The assumptions of an independent T-test were examined.

Independent Variable: Use of Technology

Dependent variable: Scores on tests

Independence: The variables are independent.

Data Measured at the Interval level: Data was measured on an interval level.

Independent scores: the scores are independent because they come from different exams that are not related and have no influence on one another

Normally Distributed Data: the Kolmogorov-Smirnov test for normality revealed that the scores are normally distributed in both sections.

TABLE 4.5
Table of T values of the Pre-test and the Post-test for Each Group

	Experimental	Control
T	1.96	0.32
a	0.05	0.05
df	47	47

According to the rule of thumb and using a standard table of significance, since the T value is lower than that in the corresponding cell then the results are not significant at a 0.05 level. However, the T value for the experimental group is only slightly below the listed value. It is important to note that, for the experimental group, the difference between the computed T value and the value in the table of significance is minimal (2.5%) while for the control group the gap is very high (528%).

More importantly, if the alpha value considered was 0.1, then the T value of the experimental group becomes significant while that of the control group remains insignificant. Actually, the T value of the experimental group overpasses the cell value by 17% which is a high percentage difference.

On the basis of the above results, the researcher can safely claim that technology had a positive effect on students' achievement. It is to be noted that the mean of the control group in the pre-test was higher than that of the experimental group. This provides additional meaning to the significance shown per statistical calculations.

4.4-Qualitative Analysis of Tests

In order to support the results of the quantitative analysis, a qualitative analysis of the tests submitted by some participants is conducted for triangulation. The qualitative analysis focuses on the pre-test, the quiz, the post-test, and mostly on the authentic real-life problem given at the end of the post-test.

4.4.1- Qualitative Analysis of the Experimental Group in General

The review of students' responses shows that all students of the experimental group were able to translate the authentic real-life problems given in the quiz and in the post-test into functions of various natures. In the quiz (Appendix IV), the problem was to be translated into affine and linear functions and in the post-test (Appendix V), it was to be translated into a quadratic function and graphed as a parabola.

As mentioned above, the experimental group's students realized that a graphical representation of the functions would directly give the solution to the problem. They plotted the function and detected the point of indifference between both options. Afterwards, they gave the range over which the first option is the most economical and the range whereby the second option is the most economical. This shows that students are able to translate such situations into functions and, on that basis, they make a rational decision. More importantly, students were able to translate the problem into functions and solve a real-life problem graphically and algebraically.

4.4.2- Qualitative Analysis of Three Students' Solutions

The following section of the analysis deals with particular students chosen from among the experimental group. Three students were chosen from the experimental group: two girls (Tamara and Mayssa) and a boy (Toufic). Although their grades in the pre-test were acceptable, the two girls were selected for the change in their attitude and the level of their interest in the mathematical unit. As to the boy, he was selected for the tremendous improvement in his grades, from the pre-test to the post-test, passing by the quiz (Table 4.6). From a grade 0f 4 in the pre0test to a grade of 13.5 in the post-test, Toufic achieved an improvement of 237.5%. As much as Maissa's grades are concerned, the increase between the pre-test and the post-test is a moderate 16%, while Tamara's grades decreased from 14 in the pre-test to 12 in the post-test, which is a 21% decrease. It is important here to point out that Tamara's interest and understanding level had improved, as the interview with her, the analysis of her questionnaire, and her grade in the quiz showed. Her lower grade in the post-test can be explained by her being sick on that day.

Concerning the students' motivation and involvement in mathematics learning, it is worthwhile to mention that, as the teacher was proceeding with the unit, they pretended that it was too difficult for them. Maissa was literally rebellious and resistant. However, when the teacher started giving activities using computer software and programmable graphical calculator, the attitudes of these students changed and their attention was grasped by this new innovative approach. The integration of technology aroused their interest and gave them an incentive to participate and listen. In the interview, they stated that using the software made things easier because they could see the graphs being traced point by point. On the other hand, as they were relieved from the worry of tracing a graph that would be inaccurate and from the worry of making computation mistakes, they dared more than before to work on the activities proposed in class, and even more, they dared to propose solutions. They overcame their obsession of incapability and came forward with answers to the questions asked in the activities.

TABLE 4.6
Scores of Three Selected Students in Pre-Test, Quiz and Post-Test

Name of student	Score in pre-test	Score in quiz	Score in post-test
Maissa	12.5	12	14.5
Jad	4	9.5	13.5
Tamara	14	15	12

The following sections focus on each student apart.

Tamara: in the pre-test, Tamara could read graphically neither the image nor the antecedents of a value by a function, given its graph. She memorized the definitions without relating them to the graph or to their meaning. This is why she got in the table of Ex II, question 2, two lines with wrong answers (Figure 4.2). More importantly, for Ex II, question 4 and Ex II, question 5, she plugged in random answers without any explanation. Her low level of understanding is confirmed in questions 7 and 8 where she gave completely wrong answers (Figure 4.3). This proves that the graph has no meaning or role to her; it is seen just a picture that does not reveal any mathematical meaning.

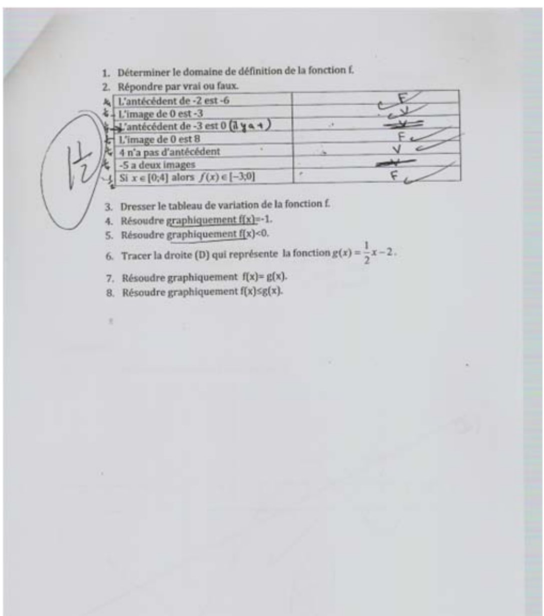


Figure 4.2: Tamara's Pre-test, Ex. II

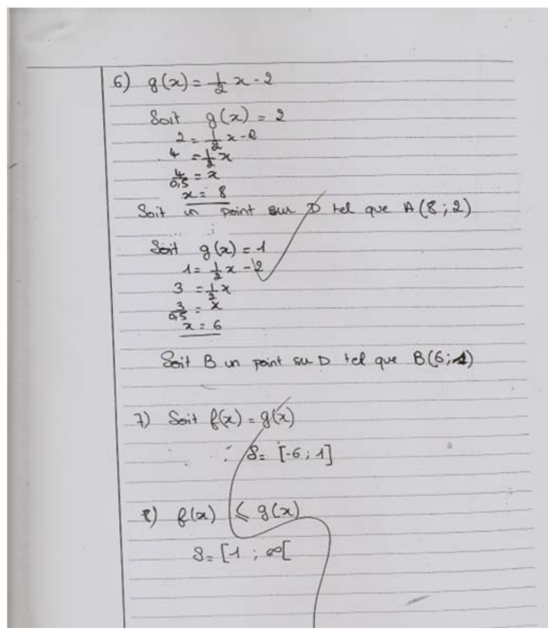
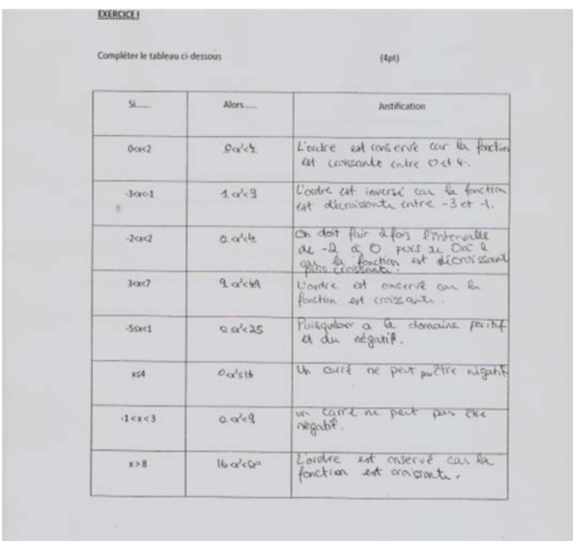


Figure 4.3: Tamara's Pre-test, Ex.II, 7 & 8

In the post-test, in Ex I, which is to be compared to Ex II of the pre-test, Tamara got 75% of the answers right. Among her correct answers were items 3 and 6 of Ex II

(Figure 4.4), which reflects a high level of understanding, because those two items deal with inequalities with negative values. A common mistake is to give an answer of:

$25 \leq x^2 < 1$ or $1 < x^2 \leq 25$ to the premise $-5 \leq x < 1$. Tamara could give a correct answer, probably because of the visual effect of the parabola, which showed students the relationships between the values of x and those of x^2 on an interval on which the function $f(x) = x^2$ is not monotonous. Tamara's high level of understanding was also confirmed by her solution of Ex IV where a graphical solution and an algebraic solution are required (Figure 4.5); Tamara got all the graphical solutions right but she failed to give the algebraic solutions to back up her answers.

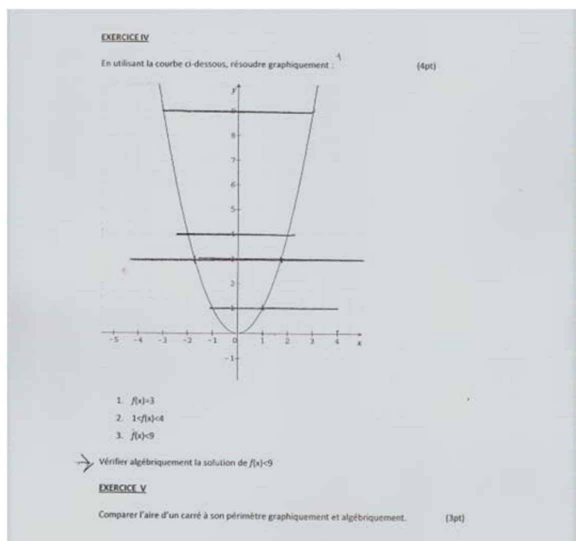


EXERCICE I

Compléter le tableau ci-dessous (4pt)

Si......	Alors......	Justification
$0<x<2$	$0<x^2<4$	L'ordre est conservé car la fonction est croissante entre 0 et 4.
$-3<x<-1$	$1<x^2<9$	L'ordre est inversé car la fonction est décroissante entre -3 et -1.
$-2<x<2$	$0<x^2<4$	On doit faire à fois l'intervalle de -2 à 0 puis de 0 à 2 car la fonction est décroissante puis croissante.
$3<x<7$	$9<x^2<49$	L'ordre est conservé car la fonction est croissante.
$-5\leq x<1$	$0\leq x^2<25$	Puisqu'on a le domaine positif et du négatif.
$x\leq 4$	$0\leq x^2\leq 16$	Un carré ne peut pas être négatif
$-1<x<3$	$0<x^2<9$	un carré ne peut pas être négatif.
$x>8$	$16<x^2<$[illegible]	L'ordre est conservé car la fonction est croissante.

Figure 4.4: Tamara's Post-test, Ex. II



EXERCICE IV

En utilisant la courbe ci-dessous, résoudre graphiquement : (4pt)

1. $f(x)=3$
2. $1<f(x)<4$
3. $f(x)<9$

Vérifier algébriquement la solution de $f(x)<9$

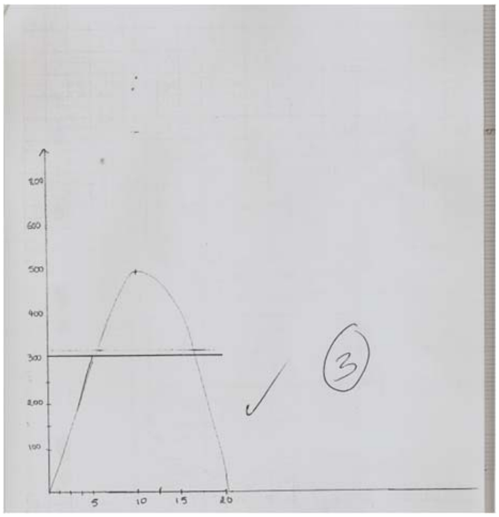
EXERCICE V

Comparer l'aire d'un carré à son périmètre graphiquement et algébriquement. (3pt)

Figure 4.5: Tamara's Post-test, Ex. IV

In the authentic real-life problem, Tamara plotted the graph of the function without any difficulty but she failed to solve the inequality algebraically. The graph is important because when the test was administered, the students had only the scientific calculator with them and not the graphical one and they had no access to the computer. This shows that

Tamara was able to connect a real-life problem to the graphs she studied in the unit and come up with a solution to the problem.



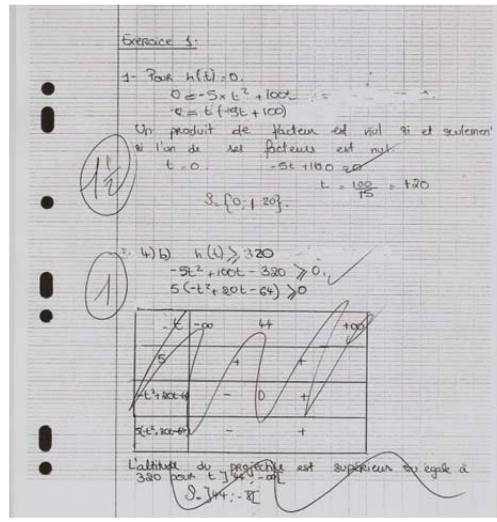
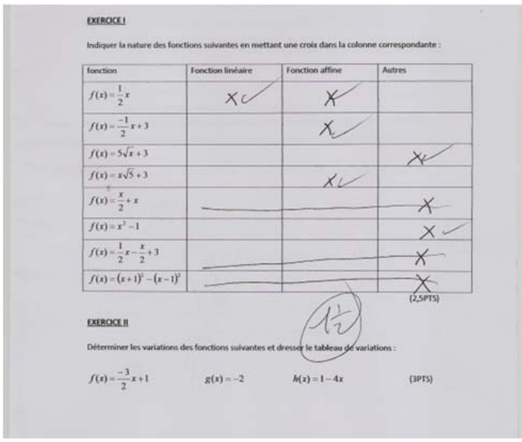


Figure 4.6: Tamara's Graphic Representation. **Figure 4.7:** Failure to Solve Algebraically

Maissa: This student was chosen because her attitude towards math in general and *functions* in particular changed drastically throughout the research period. At the beginning, she was rebellious, ready to give any offensive remark at any time of the lesson. She was even reluctant to buy the Casio ClassPad 330. When she was asked to give her answer, she deliberately responded bluntly. This rebellious attitude faded gradually as activities were progressing on the computer. She became more and more interested in doing mathematics. She even solved correctly the authentic real-life problems in both, the quiz and the post-test.

As far as grades are concerned, Maissa did well on the objective-type exercises in both, the quiz and the post-test. In the quiz, Ex I, three answers out of eight were wrong, while in the similar exercise of the post-test, Ex I, only one answer out of eight was wrong, although the level of difficulty of the questions was much higher.


EXERCICE I

Indiquer la nature des fonctions suivantes en mettant une croix dans la colonne correspondante :

fonction	Fonction linéaire	Fonction affine	Autres
$f(x)=\frac{1}{2}x$	X	X	
$f(x)=\frac{-1}{2}x+3$		X	
$f(x)=5\sqrt{x}+3$			X
$f(x)=x\sqrt{5}+3$		X	
$f(x)=\frac{x}{2}+x$			X
$f(x)=x^2-1$			X
$f(x)=\frac{1}{2}x-\frac{x}{2}+3$			X
$f(x)=(x+1)^2-(x-1)^2$			X

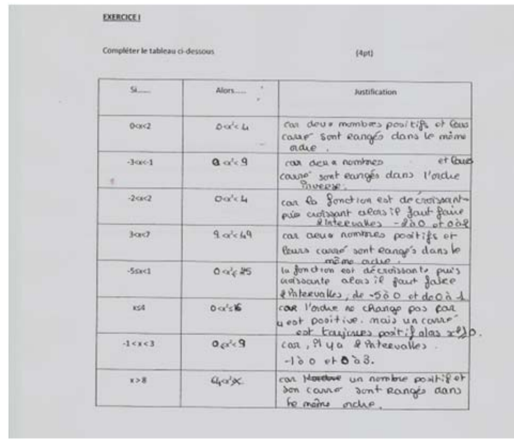
(2,5PTS)

$1\frac{1}{2}$

EXERCICE II

Déterminer les variations des fonctions suivantes et dresser le tableau de variations :

$f(x)=\frac{-3}{2}x+1$ $g(x)=-2$ $h(x)=1-4x$ (3PTS)


EXERCICE I

Compléter le tableau ci-dessous (4pt)

Si.....	Alors.....	Justification
$0<x<2$	$0<x^2<4$	[illegible]
$-3<x<-1$	$1<x^2<9$	[illegible]
$-2<x<2$	$0<x^2<4$	[illegible]
$3<x<7$	$9<x^2<49$	[illegible]
$-5\leq x<1$	$0<x^2\leq 25$	[illegible]
$x\leq 4$	$0<x^2\leq 16$	[illegible]
$-1<x<3$	$0\leq x^2<9$	[illegible]
$x>8$	$64<x^2$	[illegible]

Figure 4.8: Maissa's Quiz, Ex I **Figure 4.9:** Maissa's Post-test, Ex I

Toufic: This student was chosen for two reasons. First, his attitude and class behavior changed drastically throughout the research period. Second, his grades improved considerably throughout the research period.

Toufic started the unit with a very low profile. His grade in the pre-test was very low (4 out of 20). He was not confident about mathematics and did not have any interest in it. He was almost giving up on mathematics as a whole, because he was convinced that he would never understand it.

At the beginning of the unit, Toufic was very quiet but attentive in class. Actually, he was not interested by the lesson, probably because he had decided that *functions* were too difficult for him to understand. However, the activities on GeoGebra and on Casio ClassPad 330 aroused his interest. He felt happy being able to plot a graph and to set a table of values, overpassing his weakness in computation. Gradually, he started to ask questions in class and got involved in the class work. He even smiled in class and asked to go to the board to solve problems.

As far as grades are concerned, the pre-test was an utter failure (see Figure 4.10). Toufic could not read anything on the graph of a function. For him, the graph is nothing more than a graph line with no implications whatsoever. The numbers did not have any meaning. He could not even read the minimum or the maximum value of *x*, and gave a wrong answer when determining the domain of definition. He was unable to set the table of variations as he could not link the graph to the sense of variation of the function.

The post-test was a success for Toufic. He correctly solved all the items of Ex I (Figure 4.11), even the lines 3, 5, and 7 which require a high level of understanding, thinking, and visual reasoning.

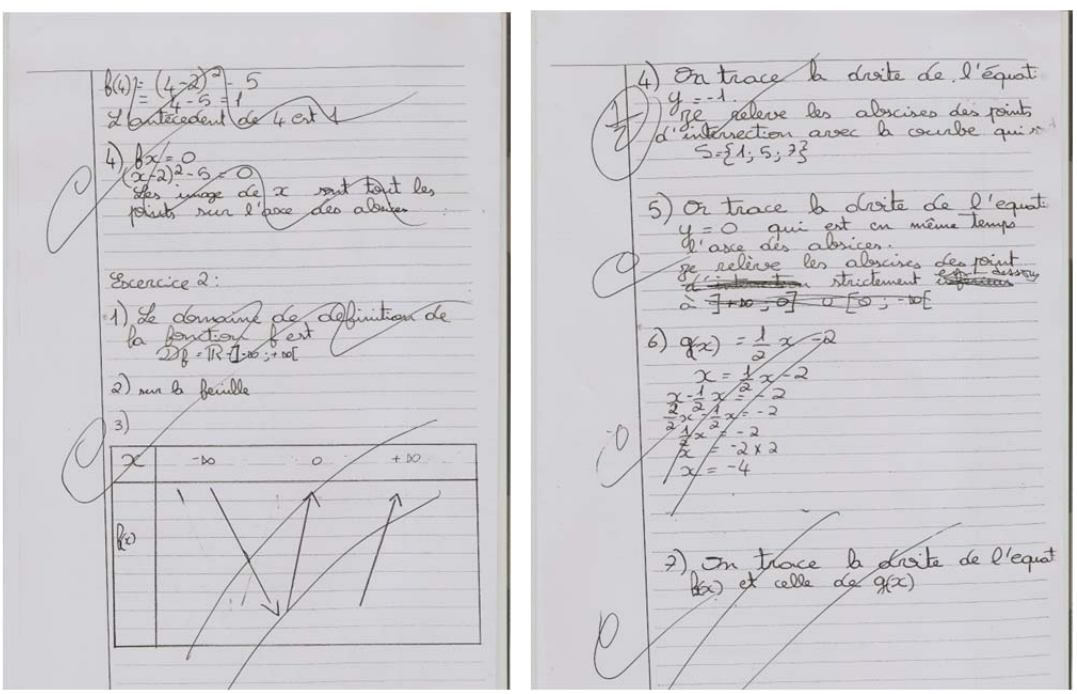
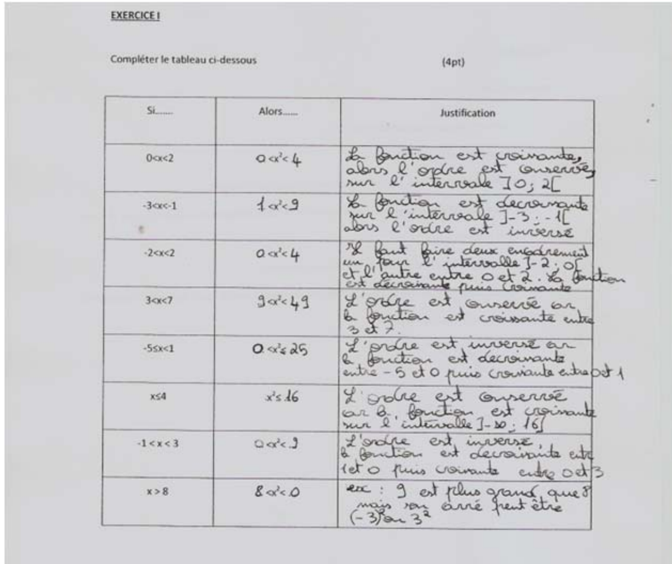


Figure 4.10: Two Pages of Toufic's Pre-test



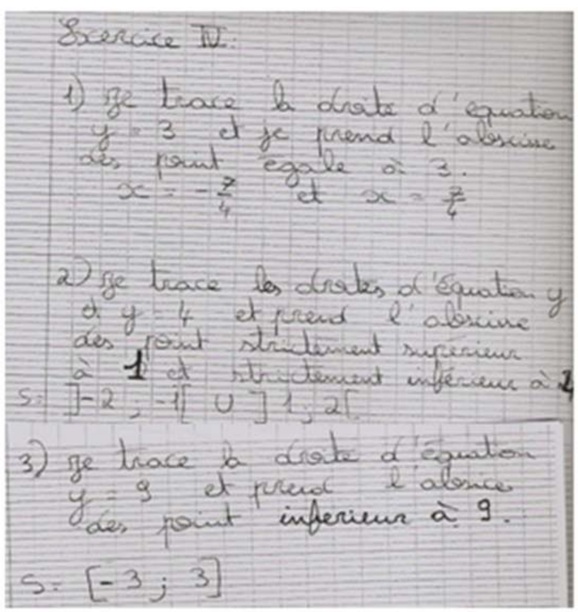
EXERCICE I

Compléter le tableau ci-dessous (4pt)

Si......	Alors......	Justification
$0<x<2$	$0<x^2<4$	La fonction est croissante, alors l'ordre est conservé, sur l'intervalle $]0;2[$
$-3<x<-1$	$1<x^2<9$	La fonction est decroissante sur l'intervalle $]-3;-1[$ alors l'ordre est inversé
$-2<x<2$	$0<x^2<4$	Il faut faire deux encadrement un pour l'intervalle $]-2;0[$ et l'autre entre 0 et 2. La fonction est decroissante puis croissante
$3<x<7$	$9<x^2<49$	L'ordre est conservé car la fonction est croissante entre 3 et 7.
$-5\leq x<1$	$0<x^2\leq 25$	L'ordre est inversé car la fonction est decroissante entre −5 et 0 puis croissante entre 0 et 1
$x\leq 4$	$x^2\leq 16$	L'ordre est conservé car la fonction est croissante sur l'intervalle $]-\infty;16[$
$-1<x<3$	$0<x^2<9$	L'ordre est inversé, la fonction est decroissante entre 1 et 0 puis croissante entre 0 et 3
$x>8$	$8<x^2<0$	ex : 9 est plus grand que 8 mais son carré peut être $(-3)^2$ ou 3^2

Figure 4.11: Toufic's Post-test, Ex. I

Even further, Toufic solved correctly Ex IV of the post test, which requires solving equations and inequalities, graphically and algebraically. This is a giant step forward for someone who started with a very low grade (4 out of 20) and ended up solving the whole problem correctly.



Exercice IV:

1) Je trace la droite d'équation $y=3$ et je prend l'abscisse des point égale à 3.
$x=-\frac{7}{4}$ et $x=\frac{7}{4}$

2) Je trace les droites d'équation y et $y=4$ et prend l'abscisse des point strictement supérieur à 1 et strictement inférieur à 4
S: $]-2;-1[\cup]1;2[$

3) Je trace la droite d'équation $y=9$ et prend l'abscice des point inferieur à 9.

S: $[-3;3]$

Figure 4.12: Toufic's Solution of Ex IV of the Post-test

When asked to plot the graph of the real-life problem, he succeeded in drawing an almost perfect parabola (Figure 4.13).

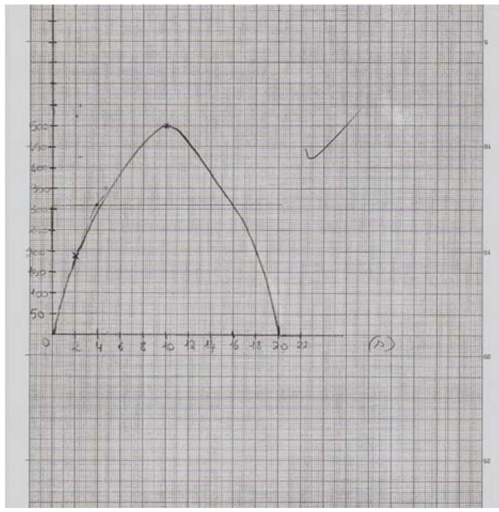


Figure 4.13: Toufic's Graphic Representation of the Real-life Problem

<u>4.5-Analysis of the Questionnaire</u>

To analyze the questionnaire, the items were classified according to four categories according to themes (Table 4.7). The four categories are: questions related to Knowledge and Understanding (K/U), questions related to Meaning (M), questions related to Liking mathematics (L), and finally questions related to Technology (T). For each item of the questionnaire, each student should select, on a Likert scale, one of the four options: n*ever, sometimes, often, and very often*. The answers were evaluated on a scale from 1 to 4 respectively. It is important to note that items were also categorized under positive and negative (Table 4.7), depending on whether the item reflects a positive or a negative attitude. In the quantitative processing of data, the coefficients 1, 2, 3 and 4 are assigned to the options *never, sometimes, often, and very often,* respectively, in the case of a positive item. However these coefficients are reversed in the case of a negative item.

TABLE 4.7
Categories of Questionnaire's items

Category	Items' numbers
Knowledge and Understanding (K/U)	Positive effect: 1 -3 -4 -19- 20 Negative effect: 13 – 14 -16 -17 -21
Meaning (M)	6 -15
Liking Mathematics (L)	2 – 5 – 7 – 11 – 12
Technology (T)	8 -9 -10 – 12 -18

The students' responses for each item were tallied (Appendix IX), then multiplied by the coefficients and the products were added to produce a score for the item. Then percentages were computed. The experimental group results were compared to the control group results.

The analysis of the questionnaire is divided into two parts. The first is an analysis of the results of each item and a comparison of the results of both groups. The second is an analysis of items grouped by categories. All results are computed in percentages and charts are drawn for each item showing both sections together (Appendix X).

The analysis per item shows similarities between both groups in some aspects and some discrepancies in other aspects. To compare the results, the mode of each group was adopted because it is the most recurrent. For example, when students are asked whether they find mathematics interesting (item 2) the mode is the same for both groups, namely for the option *very often*. When inquiring whether students find math useful in their everyday life (item 4), the mode is the same, namely for the option *sometimes*. As far as functions are concerned (item 21), the mode of both groups is *never*, that is they “never” find functions difficult. This could be explained by the fact that functions are applicable to students’ everyday life. They see functions everywhere around them, e.g. in interest rates, in discount offers, and in any decision making situation with several alternatives.

The analysis per item shows discrepancies between the two groups, especially in items directly or indirectly related to technology. For instance, in the item related to how often students use a calculator for computations (item 8), the experimental group’s mode is for the option *very often,* while that of the control group is *often*. Moreover, in asking whether students use calculators to check their results (item 9), the difference of the modes is much wider. The mode of the experimental group is for the option *very often,* while that of the control group is for *sometimes*.

This shows that the integration of technology in teaching made students realize that calculators are helpful not only for performing computations but more importantly for verification of results. Students can then concentrate on the analysis of a problem rather than drowning in the computations and worrying about computation mistakes.

On the other hand, the experimental group has a mode for *very often* to indicate the difficulty they face when constructing graphs as opposed to a mode for *sometimes* for the control group. This is a drawback of getting used to have the graphical calculator or the software plot graphs instead of trying to plot them manually. However, teachers ask students to plot graphs on technological devices only because the objectives have changed in the new curriculum. The final aim of any functions' problem is not any more to plot the graph but rather to analyze the problem and come up with solutions graphically and algebraically.

The second part of the analysis of the questionnaire is different from the first one for two reasons. First, the items are grouped under four categories. Second, the analysis is more quantitative in nature than the first one because the responses are assigned weights: that is, the response *never* is assigned a weight equal to 1. The other responses, *sometimes*, *often* and *very often* are assigned weights equal to 2, 3 and 4 respectively. The weights are reversed for items that have a negative connotation. A weighted average is computed for each category (Appendix X).

Table 4.8 summarizes the weighted average of the answers to the questionnaire according to the category involved.

Table 4.8

Table of weighted averages of questionnaires according to categories

	Knowledge and understanding	Meaning	Liking	Technology
Weighted averages of experimental group	27.318	4.77	99.6	12.23
Weighted averages of control group	27.2	4.5	116.7	10.72

A comparison of the averages shown in Table 4.8 is an indicator of the effect of integration of technology in teaching quadratic functions.

As far as knowledge and understanding are concerned, the average per student of the experimental group is above that of the control group. This could be related to the effect of visualization on understanding. Seeing the graph being constructed point by point allows students to understand better how and why the graph is formed and what is the relationship between the table of values, the table of variations and the graph itself.

As far as meaning is concerned, the average per student is higher in the experimental group than in the control group because visualization has related various aspects of functions. Observing the graph together with the figure and with the table of values (such as in Activity 10 of Appendix II) gives meaning to functions. The layout of all windows next to each other restores the link, usually lost, between the various aspects of the problem. The clear presentation of function graphs and the dynamic nature of those graphs through the use of technology and through authentic real-life problems add meaning to functions.

As far as technology is concerned, the average per student of the experimental group is obviously higher than that of the control group, simply because working with the tools raises students' interest and gets them involved in the activity. On the other hand, these students are "technology natives", which means that they are at ease with technology and they feel more secure once they have these tools within reach.

Finally, liking has a lower average per student for the experimental group than for the control group. This could be due to the fact that attitudes do not change overnight. Probably, students needed a longer period of time to get to change their taste.

4.6- Analysis of Interviews

Interviews conducted with students are important because they reveal several aspects that are related to the research and they provide answers for the research questions that cannot be obtained from the questionnaire or the tests. Five students were interviewed individually, the interviews were taped and notes of the answers were taken. The interviewed students were three boys and two girls, selected according to the following:

- One boy who had poor results in the pre-test and much better results in the post-test.
- One girl who had poor results in the pre-test and good results in the post-test.
- One boy who was new to school coming from a foreign country. He was chosen because the unit entitled "*functions*" was a completely new subject to him, as he was not exposed to it in the other country.
- One girl who was very rebellious at the beginning of the unit and then demonstrated a considerable change in attitude towards mathematics. She showed an increasing interest in the subject as it was treated.
- One boy who had excellent scores in both tests, and who did not like technology.

The interviews were transcribed and analyzed qualitatively in an attempt to support the results of the survey in answering the research question:

At this level the analysis conducted is of qualitative nature because it is related to attitudes of students towards mathematics and towards technology and its integration in teaching and learning. Students' Responses to the interviews show the effect of technology's integration on mathematics' understanding, that is it shows how this integration molds their thinking and how it helps them assimilate mathematics more quickly and more thoroughly with special emphasis on quadratic functions since the interviews took place right at the end of the research data collection process.

The responses to the interviews give answers to research questions number one, three, four and five. This is why the analysis is divided into four major parts according to the question concerned.

4.6.1- Attitudes Towards Mathematics

The first three questions are related to students' attitudes towards mathematics. As far as liking mathematics is concerned, Four of the five students said that they liked mathematics since their early school years; three of them said that they started liking mathematics since they started algebra. Only one student said that liking mathematics started in grade four because he was more into fun than into studying, but later he realized the importance of mathematics and he actually started liking math as the academic years went by.

The interviews' results also show that students face problems in mathematics mostly in grade 10, mainly because the teaching methods differed significantly. Grade 10 is the first high school year and teachers expect more autonomous work on the part of students than that provided in grade 9. This is why the whole teaching method is different. Teachers start with activities that enhance students thinking and stimulate their thoughts then direct them towards drawing conjectures. This explains why the research was done at grade 10 level rather than grade 9. Students find difficulty to adapt to the new method especially after grade 9 where the drill-and-practice method is applied due to the Brevet exam constraint. This explains the response of students as to the choice of grade 9 to be the best year as far as liking mathematics is concerned. Students feel more secure when they are taught procedures that they repeatedly use in routine exercises. It is important to note that all students are confronted with this problem when they graduate from middle school and start high school. Actually, students accommodate towards the middle of grade-10 year because they get used to the new method to be followed and they ultimately succeed, get good grades and achieve a high level of assimilation.

4.6.2- Answers to Research Questions

- *Does the use of technology (programmable calculator, spreadsheet and CAS software) improve students' understanding of mathematical concepts and relationships?*

This research question is answered in the interview through questions 1 and 8. Students said that technology (programmable calculators, spreadsheets and GeoGebra) helped them understand mathematical concepts because they make the concepts easier and clearer. For them, visualization is the key. Seeing the graphs being drawn point by point makes students understand better because the intermediate steps are essential to grasp the global idea. Students are happy because they are not asked to memorize the lesson with ready-made figures. On the contrary, they visualize the immediate effect of a change in the parameters of the function and this leads to meaningful assimilation of the concept rather than superficial and strictly procedural learning. This is exactly the purpose of activities 2, 3 and 5. The graphic effect of a change in the parameters is directly seen in GeoGebra and in the graphical calculator.

As far as the spreadsheet (Excel 2007) is concerned, students are able to automatically compute the image of any x without worrying about calculation mistakes. On the other hand, students can observe the trend of y as x increases and vice versa. They can see the exact value of the minimum or the maximum of a function and get the effect of any change in parameters on the exact values of y.

- *How does technology integration restore connections between different mathematical representations, otherwise learned as isolated notions?*

This research question is answered in questions 5 and 8 of the interviews. Students confirm that working on the computer or on graphical calculators has helped them make connections between the algebraic expressions, the table of values and the graphical representation of functions, especially with GeoGebra. Students can observe on the screen the table of values together with the graph. The fact that technological devices show the intermediate steps gives meaning the concept and makes the flow of ideas more consistent and more logical. When students follow the evolution of ideas step by step they are more convinced and they grasp the idea better.

As far as GeoGebra is concerned, activity 6 that compares the perimeter of a square to its area is very important because it turns a problem of plane geometry into a problem that can be solved with graphs representing functions. Students actually draw the square and the corresponding graphs. Afterwards, they vary the side and find the solution to the problem. Seeing the same problem from two different angles makes the connections between functions and geometry.

- *What are the features of the used technological devices that help achieve the above goals?*

As stated earlier, the main feature of the used technology that helps achieve the goals is that with those tools, students can observe, at the same time, the formula of the function, the table of values, as well as its graphical representation. Hence all problems can be solved arithmetically and geometrically.

- *What are the attitudes of students towards the integration of technology in mathematics teaching especially: quadratic functions?*

As far as attitudes are concerned, the answer comes in questions 4, 6 and 7 of the interviews. Students are very enthusiastic when technological devices are used in teaching and learning mathematics. It arouses their interest and it increases their concentration in class. The whole atmosphere is different; everyone wants to participate even the weakest students because they feel at ease with technology. They are fascinated by technological devices simply because they grew up with technology. Students state the level of concentration they have is much higher when they work on a computer than on paper.

Students feel more involved in the activity and more concerned about its solution. The mere fact that they are sitting in front of the computer increases the level of concentration.

On the other hand, students state that they feel that exercises and problems become easier once technological devices are within reach. In addition, students feel reassured when they use technology because they overcome the problem of computation mistakes that make them lose grades. This relief allows students to concentrate more on the core of the problem rather than on the peripheral calculations. For this reason, the problems assigned to students are different; students are now asked to reflect on a solution rather than to perform the classical study of a function.

It is important to note at this point that some students expressed different opinions saying that they preferred to work without technology. They see mathematics as a gymnastic of the mind and they would rather work without technology. This idea could be the reflection of parents' ideas about technology since they believe that technology would cause laziness in thinking. This is justifiable as they do not know about the new curriculum or the new trend in evaluation that is basically based on analysis and synthesis rather than repeating the same classical steps of a study of a function.

Another reason for students' reaction to technology is that a close study of these students shows that these are students who have always excelled in calculations; hence they do not really worry about computation mistakes.

Actually, the first opinion about technology is the crucial one because in education, teachers should be more concerned about getting average and weak students involved in learning rather than excellent students who can work on their own. The No Child Left Behind Act (2001-2002), is the theme of the educational system. Hence integrating technological devices is important because it gives a chance for every student to succeed at school.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1- Chapter's Outline

The first part of this chapter summarizes all the answers to the research questions obtained from the tests, the questionnaires and the interviews. In the second part, the limitations of the study and ideas for further prospective research are presented.

5.2- Discussion of Results of each Research Question

In this section, each of the research questions is dealt with and discussed according to the results obtained from the study.

5.2.1- First Question

The first question is:

- *Does the use of technology (programmable calculator, spreadsheet and CAS software) improve students' understanding of mathematical concepts and relationships?*

As shown in the results of the tests in Table 4.3, the level of knowledge and understanding improved in the experimental group throughout the research period. It is worthwhile to mention that the average of the pre-test of the experimental group was below that of the control. However, this result was reversed in the post-test. The mean of the experimental group in the post-test is higher than that of the control group.

According to the results of the questionnaire for items dealing with knowledge and understanding (K/U) shown in table 4.8, the experimental group has a higher weighted average per student than the control group. Therefore, integration of technology in quadratic functions had a positive effect on students' knowledge and understanding.

5.2.2- Second Question

- *Does the use of technology (programmable calculator, spreadsheet and CAS software) improve students' ability to apply mathematical concepts and relationships in authentic problem solving situations?*

To answer this question, it is useful to concentrate on the real-life authentic problems assigned in the quiz and in the post-test. In the quiz, the real-life situation proposed is the problem of the library where two options are suggested for students to decide which one is economically better. In the post-test, the real-life problem is that of the projectile thrown in the air and students have to calculate the maximum altitude it can reach. The results show that almost half of the students in the experimental group have solved the problem in the quiz. However, in the post-test, all students of the experimental group, except two, have solved the problem correctly. This shows that integration of technology had a positive effect on students' achievement and that they were able to solve the real-life problem.

On the other hand, as shown in table 4.8, the result of the questions related to meaning in the questionnaire is positive in the experimental group. The average per person in the experimental group is higher than in the control.

Ultimately, solving real life problems is the measure of success of students because when students know how and when to apply what they have studied it means that they have constructed meanings for the acquired knowledge.

5.2.3- Third Question

- *How does technology integration restore connections between different mathematical representations, otherwise learned as isolated notions?*

The effect of technology integration on connections is shown in the pre-test, the quiz (exercise IV) and the post-test (exercises I, IV, V). The achievement of the experimental group in the post-test is better than in the quiz exercise. This shows that technology integration does have a positive effect on connections. One may attribute this positive effect to the fact that GeoGebra has two simultaneous windows: one for algebraic expressions and one for graphics (geometrical or representations of functions). The role of visualization is important in this respect because students can observe, at the same time, the graph being constructed point by point, and the coordinates of each point in the *algebra* window together with the geometrical figure on the *graphics* window (such as in Activity 10, Appendix II).

On the other hand, the interview answers show that technology has made connections "more evident and clearer" as stated by Tamara and Toufic. These students stated that technology made things easier because the dynamic construction of the graph made them understand the process better and relate the points to the graph.

5.2.4- Fourth Question

- *What are the features of the used technological devices that help achieve the above goals: Knowledge and understanding, meaning, problem solving and connections?*

As previously mentioned, the dynamic software GeoGebra links both sides of mathematics, namely algebra and geometry because it has two windows that are shown at the same time. This was especially observed in Activities 10 and 11 of Appendix II. Moreover, in the geometry window, students can draw the figure and the corresponding graph of the function. Students can observe the point moving on a segment of a geometrical figure and, at the same time, see it plotted on the graph. Concurrently, the *algebra* window shows the coordinates of the point being plotted on the graph.

The programmable graphic calculator Casio ClassPad 330 has a screen divided into two windows: one that shows the function and its table of values and one that shows the graph of the function being traced. In parts 3 and 4 and in parts 8 and 9 of Activity 2, Appendix II, students have the table of values together with the corresponding graph. This reinforced the link between the different representations, as stated by Toufic in the interview.

As shown in table 4.8, in the questionnaire, the experimental group had a higher average than the control group for items related to technology. This could be attributed to the fact that students found that technology helps understanding; hence they became more interested in using technology than the other group.

To summarize, the most important feature of the technological devices is that they show the link between algebra and geometry that is otherwise lost in many cases. Visualization is the key, it establishes the link between algebra and geometry because students observe on a single screen two windows: one which gives algebraic results and the other that plots the graph dynamically.

5.2.5- Fifth Question

- *What are the attitudes of students towards the integration of technology in mathematics teaching especially: quadratic functions?*

This question is mainly answered through the responses to the interviews and the questionnaires. Students said it out-loud in the interview that "seeing functions being drawn on the computer and on the graphical calculator made things much easier and more understandable". They noted that the double-window feature helped a lot because it gave meaning to the coordinates that they have so far seen as only numbers.

On the other hand, working with technology made math classes "livelier and more interesting" as they stated. Students are always in favor of innovations especially because they are "digital natives".

As far as algebra is concerned, students are freed of computation mistakes because the focus of problems is not calculations anymore, but rather analysis. Hence, they are encouraged to participate because they do not have to worry about calculations anymore. This is the case of Toufic who was too shy to participate in class because he lacked confidence. However, with the integration of technology, he started to participate and to propose solutions because he felt more at ease.

Another change in attitude is that of Maissa who was very rebellious at the beginning and then, she became interested in the activities because technology was involved.

Finally, in the questionnaire, as shown in table 4.8, questions related to liking showed that the experimental group's average was below that of the control group. As mentioned before, this could be attributed to the fact that change of taste takes time to happen.

5.3- Limitations of the Study

This study has several limitations because it is an action research based on a specific sample.

First, the sample size is only 49 students chosen from one school. This sample size is too small to generalize results.

Although the sample has been chosen at random, yet there are still outside factors that play a role. For example, it happens that sometimes some generations are more technology-oriented than others. In addition, in the previous years of schooling, some students could have been more or less exposed to technological devices. This affects their reaction to technology.

Another limitation of the sample is that it is chosen from a school whose students come from a high and medium socio-economic level. This makes them more open to technology because it is part of their life. For instance, all students have a computer at home and a programmable calculator of their own. This does not apply to all families in Lebanon and not to all schools either. Hence, the results cannot be generalized to all Lebanese students because of the difference in the standards of living.

There is also another limitation for the study which is that it has been done on a specific unit which is *functions*. Hence, two important issues have to be mentioned. The first is related to the teacher. The availability of numerous computer applications related to functions makes the integration of technology easier because these are ready-made so the teacher does not have to go through an additional effort to create activities. The second issue is related to the subject itself. Some students are interested by functions because they feel that they will be useful in their prospective work field. Some others are not really into functions since they care more about statistics or space geometry because they want to become statisticians or architects etc… Here, it is important to mention that taste and future plans play a role and affect results.

Finally, duration of implementation plays a role in affecting results. Students were not given enough time to get used to studying with technology. This was their first approach to such a method. Since students were fascinated with the tools, a longer period of time would have altered the results of the study. This is shown by the fact that students became more interested in mathematics and they were more willing to participate in class and to work on activities. However, their attitudes needed more time to change, this is why in questions related to liking, the average of the experimental group remained below that of the control group.

5.4- Research Questions for Prospective Further Research

As mentioned earlier, the sample does not really represent all Lebanese students. Hence, this study could be replicated with a different sample from other parts of the country. This diversity allows more generalization of the results.

There is also a need to reinvestigate the question over a longer period of time to constitute a longitudinal study whose results could be generalized.

On the other hand, it is interesting to expand the study further to cover other mathematical subjects such as statistics, plane geometry, etc.

Finally, it is advised to expand the study of functions to grade levels 11 and 12 since functions are also part of the curriculum of these grades.

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APPENDIX I

UNIT PLAN

Quadratic functions

Unit Title: Functions, Quadratic Functions

General Objectives:

- To solve an authentic real-life problem through a graphical representation of a quadratic function
- To solve an authentic real-life problem through an equation of the second degree
- To relate graphical representations to arithmetical solutions of a problem

P.S.: Since students have very little (or practically no knowledge) of functions, the first three sessions are dedicated to linear functions

Linear Functions:

General Objectives:

1. To associate a case of proportionality to a linear function
2. To notice that all linear functions are represented by points that are on the same line, hence the name "linear"
3. To see the difference between a linear function and an affine function
4. To calculate the value of y given that of x and vice versa
5. To see that when $b=0$ then the affine function becomes linear and the line passes through the origin
6. To see that affine functions representations are obtained by a translation of vector by translation
7. To notice that when a is positive and it increases the line is closer to the y axis
8. To see that if $a \geq 0$ then the line goes "up"
9. To compare the trend of y to the trend of x given the values of a
10. To see that $y=ax$ are obtained from the line symmetry with respect to the x axis

Session One: Duration: 50 mn

Specific Objectives:

At the end of the session, students should be able to:

1. To see that linear functions are represented by points that are on the same line
2. To notice that all lines representing linear functions pass through the origin
3. To record a change in the value of *a* affects the slope of the line
4. To read the value of *a* by taking two points of the line
5. To note that different mathematical issues are correlated since the representation of $f(x)= - ax$ is obtained by a line symmetry from that of $f(x)=ax$
6. To state that, given that *a* is positive, the line gets closer to the *y* axis as the value of *a* increases

ACTIVITY ONE WAS ASSIGNED DURING THE SESSION

Session Two: Duration: 50 mn

Title: The sense of variation of a function

At the end of the session, students should be able to:

1. To create a table of values
2. To calculate the value of *y* given the value of *x* and vice versa
3. To plot on the graph, given that its coordinates are calculated algebraically
4. To relate the sign of "*a*" (the coefficient) to the sense of variation of the function
5. To recognize that if "*a*" is positive then the function is increasing
6. To recognize that if "*a*" is negative then the function is decreasing
7. To state a definition of an increasing function and a decreasing function

P.S: The same activity was given on Casio ClassPad 300

ACTIVITY TWO WAS ASSIGNED DURING THE SESSION

Session Three: Duration:50 mn

Session Title: Sense of variation

At the end of the session, students should be able to:

1. To draw a conjecture relating the sign of *a* to the sense of variation of a function
2. To realize that if $a = 0$ then the line overpasses the x axis
3. To know that if $a > 0$ then the function is increasing
4. To match an increasing function with a line that goes “upward”
5. To notice that, given that “*a*” is positive, then the line becomes straighter as the value of “*a*” increases
6. To notice that, given that “*a*” is negative, then the line becomes straighter as the value of *a* decreases
7. To find the translation of vector that turns $(y=ax)$ into $(y=ax+b)$

ACTIVITY THREE WAS ASSIGNED DURING THE SESSION

Session Four: Duration: 50 mn

The quiz (See Appendix IV)

The quiz has been given in the middle of the unit for several reasons

1. The linear functions were explained in full
2. It is a pause that shows how much of the material was understood
3. It allows the teacher to evaluate the level of students and see whether there is any improvements
4. It gives the teacher a green light to proceed with the material.

Session five: Duration: 50 mns

Title: the pisciculture problem (open problem)

Objectives: - to trigger thinking

- to create a need for quadratic functions.

Method: a real-life authentic problem is given and students are asked to come up with a solution.

Teacher's role: guidance only.

ACTIVITY FOUR WAS ASSIGNED DURING THE SESSION

Session Six: Duration:50 mn

Session Title: Quadratic functions

Note: this session is undertaken using the Casio ClassPad 300

Specific objectives:

At the end of the session, students should be able to:

1. Recognize a quadratic function (the basic type $f(x)=ax^2$)
2. Decide which definition interval to use according to the context of the problem
3. Recognize the shape of a quadratic function
4. Construct a table of values of x and the corresponding y
5. Spot the minimum value of y and its corresponding x.
6. Prove that the origin is the minimum of the function through an arithmetic proof.
7. Prove that positive values of y have two antecedents in quadratic functions as opposed to linear functions
8. Highlight the line of symmetry of the graph.
9. Compare the representations of quadratic functions versus linear functions with respect to searching for the minimum.

ACTIVITY FIVE WAS ASSIGNED DURING THE SESSION

Session Seven Duration: 100 mn

Session Title: Sense of variation of a quadratic function

(Considering “*a*” positive then “*a*” negative)

Specific objectives:

At the end of the session, students should be able to:

1. State the two possible cases that would give a parabola
2. Relate the sign of “*a*” to the shape of the parabola
3. Specify whether the function admits a minimum or a maximum and find the corresponding x
4. Study the sense of variation of the function
5. Construct the table of variations of the function
6. Note the effect of a change in “*b*” or “*c*” given that “*a*” is constant
7. Prove that the x of the extreme is $x=-b/2a$
8. Prove that the equation of the line of symmetry is $x=-b/2a$
9. Find the values of the intersection of the parabola with the x axis
10. State that these values are the roots of the function
11. Prove that the equation of the line of symmetry is the sum of the roots divided by 2.

ACTIVITY SIX WAS ASSIGNED DURING THE SESSION

Session Eight Duration: 50 mns

Session title: Sense of variation and table of variations of a quadratic function:

Note: two cases are considered:

- The coefficient “*a*” is positive.
- The coefficient “*a*” is negative.

At the end of the session, students should be able to:

1. Determine the shape of the parabola according to the sign of “*a*”.
2. State whether the function is decreasing then increasing or vice versa.
3. State whether the parabola has a minimum or a maximum.

ACTIVITIES SEVEN AND EIGHT WERE ASSIGNED DURING THE SESSION

Session Nine: Duration: 50 mns

Session Title: Quadratic functions applied to geometry: a comparison of the area and the perimeter of a square.

At the end of the session, students should be able to:

1. Solve a problem using the graphs of functions.
2. Translate the area of a square into a quadratic function.
3. Translate the perimeter of a square into a linear function.
4. Read the graph and come up with a solution to the problem.

ACTIVITY NINE WAS ASSIGNED DURING THIS SESSION

Session Ten: Duration: 90 mns

Session Title: maximal area

At the end of the session, students should be able to:

1. Connect geometry to functions
2. Produce a conjecture from the graphical solution provided by the software
3. Support the conjecture with a mathematical proof

TEACHER IS NOT SUPPOSED TO GIVE ANY HINT AT ALL.

ACTIVITY TEN WAS ASSIGNED DURING THE SESSION

Session Eleven: Duration: 50 mns

Session title: geometrical application of functions

At the end of the session students should be able to:

1. Connect a geometrical problem to functions.
2. Use representations and come up with solutions.
3. Back up the solution with calculations.
4. Propose two solutions for the problem: one with a general quadratic function and another with two functions represented by a parabola and a line

ACTIVITY 11 WAS ASSIGNED DURING THIS SESSION

Session Twelve: Duration: 100 mns

Session title: Post-test

APPENDIX II

ACTIVITIES

XYZ SCHOOL	MATH **ACTIVITY (1)**	YEAR 2010-2011
GRADE 10	FUNCTIONS	TIME: 50 mns
NAME		

1) Open GeoGebra.
2) Enter the expressions of the following functions:

    $f(x) = 0.5x$ $g(x) = x$ $h(x) = 2x$ $k(x) = 4x$

3) Plot the graphs of the functions.
4) Write a conjecture:____
5) How do the lines compare?____________________________________
6) Change the signs of the coefficients. What do you notice?
7) Which line is the closest to the *y* axis?___________________________
8) Erase all functions and enter the expression of the functions

    $f(x)=2x$ and $g(x) = -2x$

9) Plot the graphs of the functions.
10) Write a conjecture____

XYZ SCHOOL	MATH **ACTIVITY (2)**	YEAR 2010-2011
GRADE 10	FUNCTIONS- VARIATIONS	TIME: 50 mns
NAME		

1) Turn on Casio ClassPad 300.
2) Enter the function $f(x)=2x$
3) Plot the graph of the function
4) Prepare a table of values.
5) How does y change when x changes?
6) Write a conjecture:____
7) Cancel the function and enter $g(x)= - 2x$
8) Plot the graph of the function.
9) Prepare a table of values
10) Write a conjecture:____

Definitions:

1) An **increasing** function is ____
2) A **decreasing** function is____

XYZ SCHOOL	MATH **ACTIVITY(3)**	YEAR 2010-2011
GRADE 10	FUNCTIONS- VARIATIONS	TIME: 50 mns
NAME		

1) Open GeoGebra
2) Choose a slider for the number a, the limits are [0 ; 5].
3) Write the function $f(x) = ax$.
4) Plot the function.
5) Let the number a vary, what do you notice? ____________________________
6) Change the limits of the slider to [-5; 0].
7) Let the number a vary, write the conjecture_______________________________
8) Start again with the formulas

   $f(x) = ax$ and $g(x) = ax + b$. (put another slider for b)
9) Plot the functions and write the conjectures

10) How could the graph of g be derived from the graph of f ?

A translation of vector ____

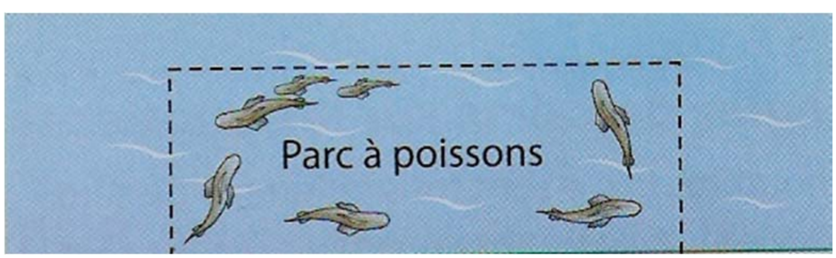
XYZ SCHOOL	MATH **ACTIVITY(4)**	YEAR 2010-2011
GRADE 10	FUNCTIONS- quadratic	TIME: 50 mns
NAME		

This is an authentic real-life problem.

The pool problem

Mr. Fish wants to create a fish park. He has a thread of 400 meters that he wants to fix to the border to determine the three sides of the rectangular pool.

Where should he fix his thread so that the area of the park is maximal? Calculate the corresponding area.



XYZ SCHOOL	MATH **ACTIVITY(5)**	YEAR 2010-2011
GRADE 10	FUNCTIONS- quadratic	TIME: 50 mns
NAME		

1) Turn on Casio ClassPad 300
2) The function studied is the area of a square.
3) Plot the function
4) Write a conjecture ____
5) Extend the values of x to include negative values and draw the function

   $f(x) = x^2$

6) What is the shape of the graph?

   This is called **a parabola.**

7) Does the graph have any element of symmetry?
8) What is the lowest value of y? what is the corresponding value of x?

   This is called the **minimum** of the function.

9) Use the calculator and search for the minimum of the function $f(x) = x^2$.
10) Repeat the activity for $g(x) = x$. Can you find a minimum?

XYZ SCHOOL	MATH **ACTIVITY (6)**	YEAR 2010-2011
GRADE 10	FUNCTIONS- Quadratic	TIME: 50 mns
NAME		

1) Open GeoGebra.
2) Choose a slider for the number a; the limits are [0 ; 5].
3) Enter the function $f(x) = ax^2$.
4) Let a vary and write the conjecture____
5) Change the limits to [-5 ; 0], write the conjecture____________________________
6) Choose two other sliders for b and c to vary between [-10; 10].
7) Enter the function $f(x) = ax^2 + bx + c$
8) Write the conjecture _____________________________________
9) Draw the line with equation $x = -b/2a$
10) Write the conjecture. ____________________________________

XYZ SCHOOL	MATH **ACTIVITY(7)**	YEAR 2010-2011
GRADE 10	FUNCTIONS	TIME: 40 mns
NAME		

1. Open GeoGebra.
2. Create a slider for a variable number between 0 and 5 with an increment of 0.1.
3. Create the function $f(x) = ax^2$
4. Change the values of *a*, using the slider.
5. Write the conjecture:

____

____

6. Set the tables of variation using $a = 1; a = 2; a = 5$.

7. Write a general conjecture that is true for all these parabolas:

____

XYZ SCHOOL	MATH ACTIVITY(8)	YEAR 2010-2011
GRADE 10	FUNCTIONS	TIME: 40 mns
NAME		

1. Open GeoGebra.
2. Create a slider for a variable number between 0 and 5 with an increment of 0.1.
3. Create the function $f(x) = ax^2$
4. Change the values of *a,* using the slider.
5. Write the conjecture:

6. Draw the tables of variation using $a = -1$; $a = -2$; $a = -5$.

7. Write a general conjecture that is true for all these parabolas:

XYZ SCHOOL	MATH **ACTIVITY(9)**	YEAR 2010-2011
GRADE 10	FUNCTIONS- inequalities	TIME: 40 mns
NAME		

Use GeoGebra to solve the following problem:

Compare the area of a square to its perimeter.

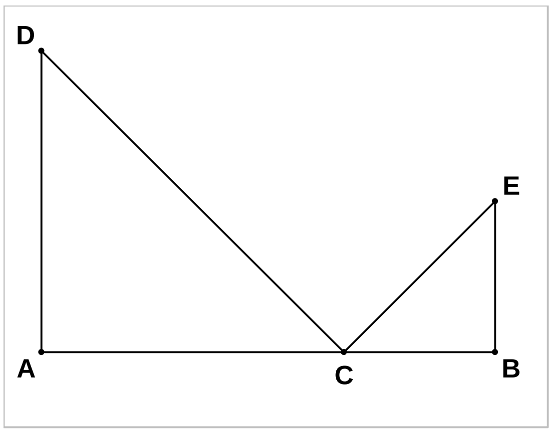
Write the conjecture which you got graphically

Solve the inequality algebraically

Note: the teacher is here to assist students in their work. Students are expected to figure out the graphical solution.

XYZ SCHOOL	MATH **ACTIVITY(10)**	YEAR 2010-2011
GRADE 10	FUNCTIONS- maximal area	TIME: 90 mns
NAME		

Use GeoGebra for the activity

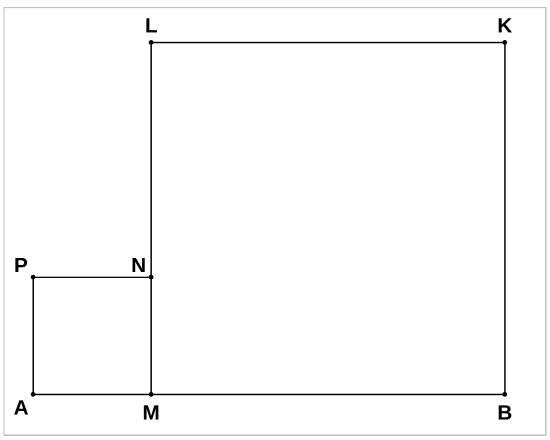


In the figure, AB = 6, C is a point on segment]AB[. Triangles ACD and CBE are isosceles triangles, right-angled in A and B respectively. Move point C and record the values of the area of triangle CDE.

1) Draw the figure using GeoGebra
2) Move *C* and observe the values of the area.
3) What is the position of C that corresponds to the maximum area?
4) Plot point *M(x,y)* such that *x* is the distance *AC* and *y* is the area of *CDE*.
5) Move *C* and plot the points.
6) Write the conjecture.____
7) Write a proof.

XYZ SCHOOL	MATH **ACTIVITY(11)**	YEAR 2010-2011
GRADE 10	FUNCTIONS- EQUATIONS- AREAS	TIME: 75 mns
NAME		

[AB] is a segment such that AB = 11 cm. M is a point of [AB] such that $AM = x$. construct two squares as shown in the diagram.



THE PURPOSE OF THE PROBLEM IS TO FIND THE POSITION OF M SUCH THAT THE SUM OF THE TWO SQAURES' AREA IS 65.

1) Draw the figure using GeoGebra (don't forget the slider).

2) Move M to find the desired position.

3) Write the conjecture:

____

4) On paper, prove that the sum of the two areas is $f(x) = 2x^2 - 22x + 121$

5) Write the equation to be solved and find its solution.

6) Show that it is: $x^2 - 11x + 28 = 0$.

7) Plot the functions $g(x) = x^2$ and $h(x) = 11x - 28$ using GeoGebra.

8) Find the intersection of the graphs.

9) Reconcile the graphical findings with the algebraic results.

APPENDIX III

XYZ SCHOOL	**(PRE)TEST** of MATHS	OCTOBER 2010
Grade 10	FONCTIONS	Time : 50 MNS
NAME		

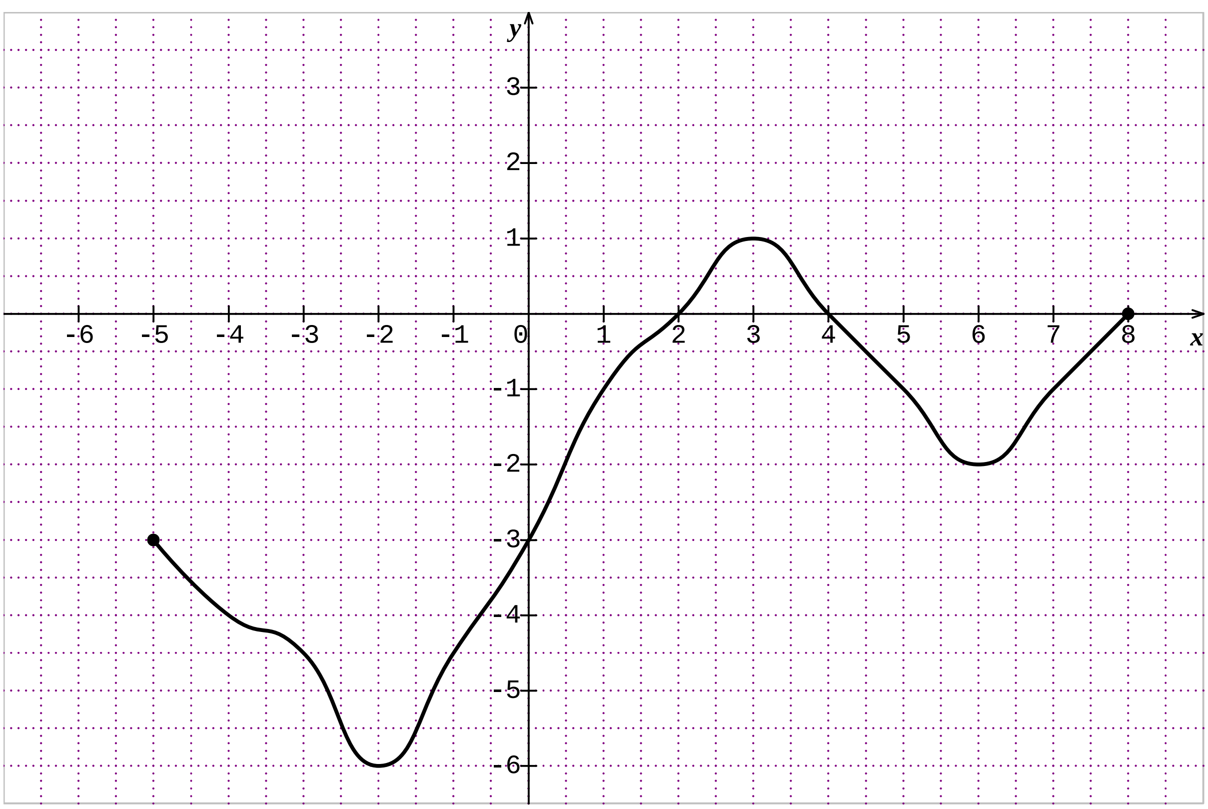
EXERCISE I

Consider the function $f(x) = (x-2)^2 - 5$

1. Calculate $f(\frac{-3}{2})$ and $f(1-2\sqrt{3})$.
2. What is the image of 0 by the function *f* ?
3. What are the antecedents of : -7 ; -5 and 4?
4. Solve algebraically the equation *f(x)* = 0. (6 pts)

EXERCISE II

Given below is the graphical representation of the function *f*.



1. What is the domain of the function f?
2. Answer by True or False.

The antecedent of -2 is -6	
The image of 0 is -3	
The antecedent of -3 is 0	
The image of 0 is 8	
4 has no antecedent	
-5 has two images	
If $x \in [0;4]$ then $f(x) \in [-3;0]$	

3. Set the table of variations of the function f.
4. Solve graphically $f(x) = -1$.
5. Solve graphically $f(x) < 0$.
6. Plot the line which represents the function $g(x) = \frac{1}{2}x - 2$.
7. Solve graphically $f(x) = g(x)$.
8. Solve graphically $f(x) \le g(x)$. (14 pts)

APPENDIX IV

XYZ SCHOOL	MATH **QUIZ**	NOVEMBER 2010
Grade 10	AFFINE FUNCTIONS	TIME: 60 MNS
NAME		

EXERCISE I

Cross the cells that represent the nature of each of the functions below:

Function	Linear function	Affine function	Other
$f(x)=\frac{1}{2}x$			
$f(x)=\frac{-1}{2}x+3$			
$f(x)=5\sqrt{x}+3$			
$f(x)=x\sqrt{5}+3$			
$f(x)=\frac{x}{2}+x$			
$f(x)=x^2-1$			
$f(x)=\frac{1}{2}x-\frac{x}{2}+3$			
$f(x)=(x+1)^2-(x-1)^2$			

(2.5 pts)

EXERCISE II

Determine the sense of variations of each of the functions below and prepare its table of variations:

$f(x)=\frac{-3}{2}x+1$ $\qquad g(x)=-2$ $\qquad h(x)=1-4x$ $\qquad$ (3 pts)

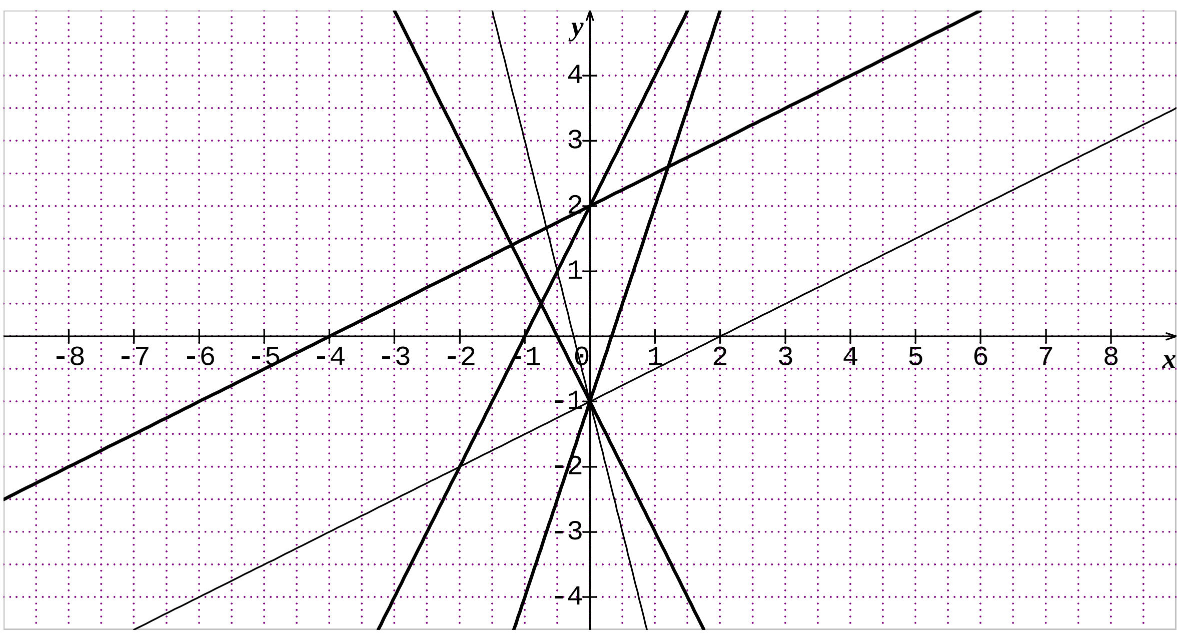
EXERCISE III

1) Determine the linear function such that $f(-3)=\frac{1}{2}$.
2) Determine the affine function such that $f(1)=2$ and $f(-2)=5$.
3) Determine the affine function such that $h(-2)=-3$ and $h(4)=-3$. (3pts)

EXERCISE IV

Relate each of the functions below to its graph (justify your answer)

$f(x)=3x-1$ $g(x)=-2x-1$ $h(x)=\frac{1}{2}x+2$ (3 pts)



EXERCISE V

A library offers two options:

Formula A: 1 euro per borrowed book.

Formula B: a down payment of 12 euros with an additional 0.2 euro per borrowed book.

Let x be the number of borrowed books, $f(x)$ and $g(x)$ the corresponding costs under options A and B respectively.

1) Determine $f(x)$ and $g(x)$ *in terms of* x.
2) Plot the graphs of both functions.
3) For which minimum number of books is option B more economical than option A?
4) Solve algebraically question 3. (4 pts)

EXERCISE VI

ABC is a right-angled triangle in C such that AB= 5cm, and AC = 4cm. D is a point of [AB] such that AD = x. The parallel to (BC) through D cuts (AC) in E. Let $f(x)$ be the distance AE.

1) Determine the domain of definition of f.
2) Express $f(x)$ in terms of x.
3) What is the nature of f ?
4) Calculate the value of x such that the area of triangle AED would be $1.5cm^2$.

(4.5 pts)

Appendix V

XYZ SCHOOL	MATH **(POST) TEST**	DECEMBER 2010
GRADE 10	QUADRATIC FUNCTIONS	TIME: 120 MIN
NAME:		

EXERCISE I

Complete the table below (3 pts)

If…….	**Then……**	**Justification**
$0 < x < 2$	$\ldots < x^2 < \ldots$	
$-3 < x < -1$	$\ldots < x^2 < \ldots$	
$-2 < x < 2$	$\ldots < x^2 < \ldots$	
$3 < x < 7$	$\ldots < x^2 < \ldots$	
$-5 \leq x < 1$	$\ldots \leq x^2 < \ldots$	
$x \leq 4$	$< x^2 \leq \ldots$	
$-1 < x < 3$	$\ldots < x^2 < \ldots$	
$x > 8$	$\ldots < x^2 < \ldots$	

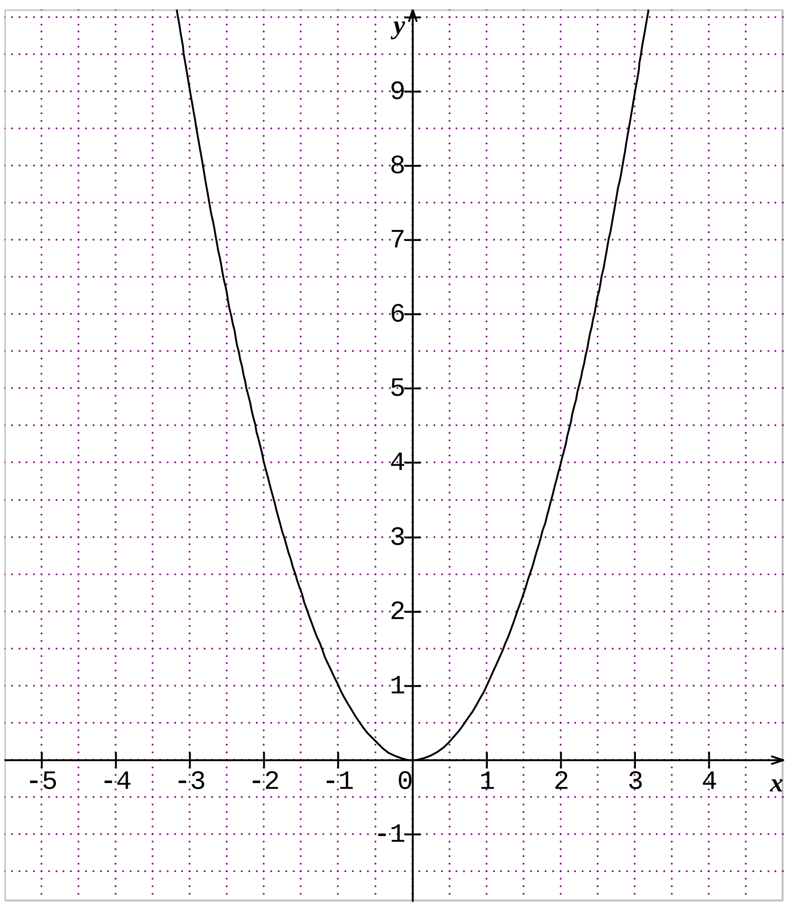
EXERCISE II

Let $f(x) = x^2$, prove that f is increasing over the interval $[0 ; +\infty[$ (2pt)

EXERCISE III

Let $f(x) = x^2 - 5x + 4$ (4pt)

1. What is the domain of definition of f?
2. Prove that $f(x) = (x - 1)(x - 4)$.
3. Knowing the sign of a, what is the sense of variation of f.
4. What is the value of the extreme and what is the corresponding value of x. Draw the table of variations of f.
5. What is the equation of the symmetry axis.
6. Solve algebraically $(x - 1)(x - 4) > 0$. What are the values of x where the graph is below the x axis.



EXERCISE IV

Using the graph above, solve graphically: (3pt)

1. $f(x) = 3$
2. $1 < f(x) < 4$
3. $f(x) < 9$

Solve algebraically $f(x) < 9$

EXERCISE V

Compare the area of a square to its perimeter graphically and algebraically.

(3pt)

EXERCISE VI

In an experiment, the altitude (in meters) of a projectile thrown from the ground at any moment t (in seconds) by the formula:

$$h(t) = -5t^2 + 100t.$$

(The point of origin is $t = 0$ s.)

1. At what moment t does the projectile fall on the ground ?
2. Plot the graph of function h on the interval [0 ; 20]
3. Determine graphically the period over which the altitude of the projectile is higher than or equal to 320 m.
4. a) Show that:

$$h(t) - 320 = -5(t - 16)(t - 4).$$

b) Solve question 3 algebraically.

(4 pts)

Appendix VI

QUESTIONNAIRE

For each question choose the most convenient answer

This questionnaire is anonymous and confidential, so please answer truthfully

		Never	Sometimes	Often	Very Often
1	When I am in the math class, I know exactly what I have to do.				
2	I think math is interesting.				
3	During math lesson, I do my best to solve problems.				
4	During math course, I feel that I learn useful new ideas.				
5	At home, I do my math homework on a daily basis.				
6	I feel that I will use in my daily life what I have learned in math..				
7	I only study math before the quiz.				
8	I use the calculator to verify my results.				
9	I use my calculator to find answers.				
10	I use the computer to do my math homework.				
11	I discuss my math homework with my friends.				
12	Sometimes I speak of math on Internet or Facebook with my friends.				
13	I have difficulties understanding math.				
14	Sometimes I solve math problems without understanding what I do.				

15	I know how to solve math excercises but I have difficulties in solving problems.				
16	Constructing geometrical figures is tough and time consuming.				
17	Constructing functions' graphs is tough and time consuming.				
18	I like to use a tool that facilitates constructions of figures and mathematical graphs.				
19	I am at ease with algebra.				
20	I am at ease with geometry.				
21	*Functions* is a tough subject.				

I prefer to use:

- Programmable calculator
- Computer
- Scientific calculator

Give two reasons why you like (or dislike) math.

Thank You

Appendix VII

INTERVIEW QUESTIONS

1. Have you always loved math?
2. Speak about a period where you most hated math and explain why.
3. Speak about a period where you most loved math and explain why.
4. Do you prefer doing math using the computer or the calculator? Why?
5. How did the computer sessions help you understand math ? Give two examples.
6. How do you feel when the teacher forbids the use of the calculator during the exam?
7. Does the calculator reassure you? If yes, say in which way.
8. Explain if and how each of the following was beneficial to you in understanding functions:
    a. Programmable Calculator
    b. Geogebra
    c. Excel

Appendix VIII

Statistics of the Pre-Test, Quiz and Post-Test

Pre-Test Results

Parameter	Experimental group	Control group	percentage difference
Mean	10.79	12.25	-11.91836735
∑X	259	294	-11.9047619
∑X^2	3079	3836	-19.73409802
XσN	3.43	3.13	9.584664537
XσN-1	3.51	3.19	10.03134796
N	24	24	0
min X	4	6	-33.33333333
Q1	9	10	-10
Median	11	12	-8.333333333
Q3	13.5	14.5	-6.896551724
max X	17	20	-15
mode	9	10	-10

Post-Test results

Parameter	Experimental group	Control group	percentage difference
Mean	12.81	12.76	0.39184953
∑X	269	319	-15.67398119
∑X^2	3571	4295	-16.85681024
XσN	2.44	2.99	-18.39464883
XσN-1	2.5	3.05	-18.03278689
N	21	25	-16
min X	8	7	14.28571429
Q1	10.5	11	-4.545454545
Median	13	12	8.333333333
Q3	14.5	15	-3.333333333
max X	17	20	-15
mode	14	12	16.66666667

Quiz Results

Parameter	Experimental group	Control group	percentage difference
Mean	12.26	12.125	1.113402062
∑X	282	291	-3.092783505
∑X^2	3680	3831	-3.941529627
XσN	3.11	3.55	-12.3943662
XσN-1	3.18	3.63	-12.39669421
N	23	24	-4.166666667
min X	8	4	100
Q1	10	10	0
Median	11	12	-8.333333333
Q3	15	15	0
max X	19	18	5.555555556
Mode	10	10	0

Appendix IX

Categories of Questionnaire's items and Frequencies of Answers by Experimental and Control Groups

Questionnaire Statements	Cat.	EXPERIMENTAL GROUP A	B	C	D	CONTROL GROUP A	B	C	D
1.When I am in the math class, I know exactly what I have to do.	K/U	*0*	*9*	*10*	*3*	0	15	5	4
2.I think math is interesting.	L	*3*	*5*	*5*	*9*	0	6	8	10
3.During math lesson, I do my best to solve problems.	K/U	*1*	*4*	*6*	*11*	1	8	7	8
4.During math course, I feel that I learn useful new ideas.	K/U	*3*	*8*	*7*	*4*	1	11	6	6
5.At home, I do my math homework on a daily basis.	L	*2*	*9*	*7*	*4*	0	6	10	8
6.I feel that I will use in my daily life what I have learned in math..	M	*6*	*5*	*7*	*4*	3	14	4	3
7.I only study math before the quiz.	L	*8*	*5*	*4*	*5*	4	12	3	5
8.I use the calculator to verify my results.	T	*1*	*3*	*5*	*13*	1	12	7	4
9.I use my calculator to find answers.	T	*0*	*4*	*7*	*11*	1	13	6	4

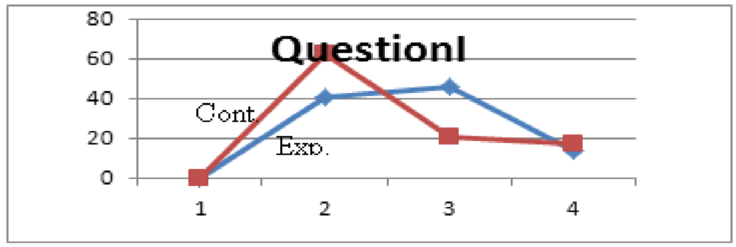
10.I use the computer to do my math homework.	T	*19*	*1*	*2*	*0*		20	3	1	0
11.I discuss my math homework with my friends.	L	*6*	*11*	*3*	*2*		4	8	9	3
12.Sometimes I speak of math on Internet or Facebook with my friends.	T & L	*6*	*4*	*8*	*4*		3	9	7	5
13. I have difficulties understanding math.	K/U	*3*	*4*	*10*	*5*		0	12	6	6
14.Sometimes I solve math problems without understanding what I do.	K/U	*7*	*11*	*3*	*1*		5	17	1	1
15.I know how to solve math excercises but I have difficulties in solving problems.	M	*2*	*12*	*6*	*2*		4	12	7	1
16.Constructing geometrical figures is tough and time consuming.	K/U	*9*	*13*	*0*	*0*		12	9	2	1
17.Constructing functions' graphs is tough and time consuming.	K/U	*2*	*3*	*8*	*9*		2	9	8	5
18.I like to use a tool that facilitates constructions of figures and mathematical graphs.	T	*5*	*15*	*2*	*0*		11	6	6	1
19.I am at ease with algebra.	K/U	*5*	*9*	*4*	*4*		13	6	2	3
20.I am at ease with geometry.	K/U	*4*	*15*	*2*	*1*		6	12	3	3
21.Functions is a tough subject.	K/U	*21*	*0*	*0*	*1*		19	2	3	0
		113	*150*	*106*	*93*		110	192	121	81

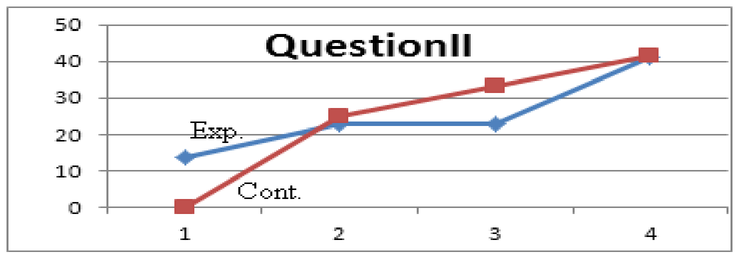
A = Never; B = Sometimes; C = Often; D = Very Often
K/U = Knowledge / Understanding; M = Meaning; L = Liking Mathematics; T = Technology

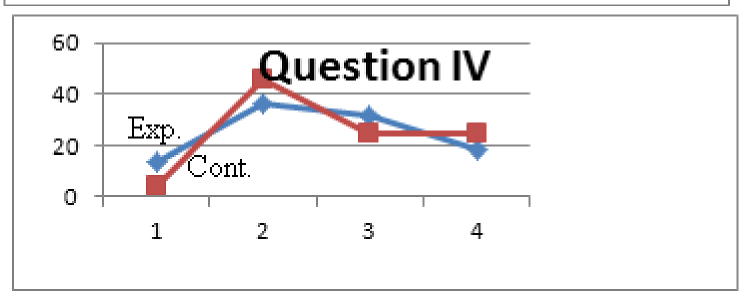
Appendix X

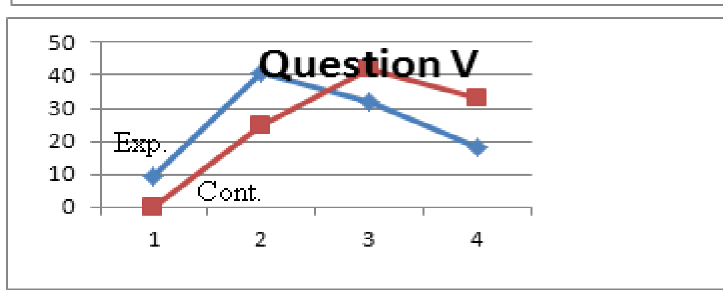
Frequencies, Percentages and Charts Representing selected Likert items by Experimental and Control Groups

	Exp. f	Cont. f	Exp.%	Cont.%
	Q	1		
A	0	0	0	0
B	9	15	40.91	62.5
C	10	5	45.45	20.83
D	3	4	13.64	16.67
			100	100
	Q	2		
A	3	0	13.64	0
B	5	6	22.73	25
C	5	8	22.73	33.33
D	9	10	40.91	41.67
			100	100
	Q	3		
A	1	1	4.545	4.167
B	4	8	18.18	33.33
C	6	7	27.27	29.17
D	11	8	50	33.33
			100	100
	Q	4		
A	3	1	13.64	4.167
B	8	11	36.36	45.83
C	7	6	31.82	25
D	4	6	18.18	25
			100	100
	Q	5		
A	2	0	9.091	0
B	9	6	40.91	25
C	7	10	31.82	41.67
D	4	8	18.18	33.33
			100	100

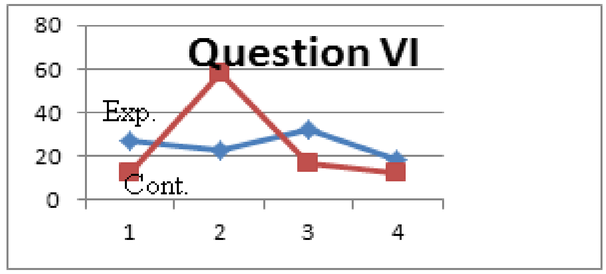


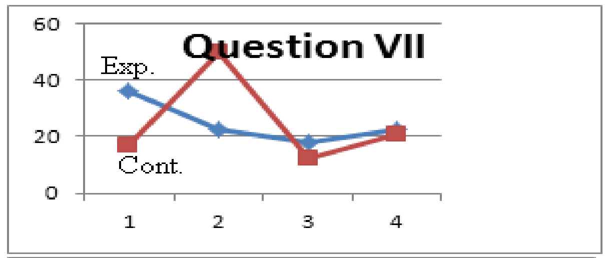


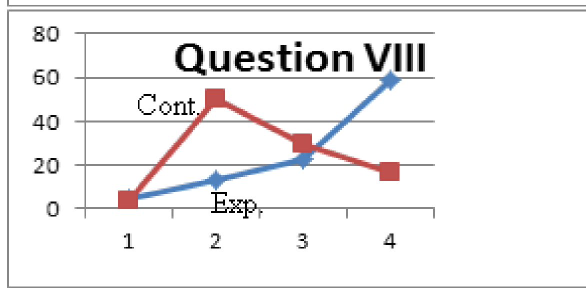


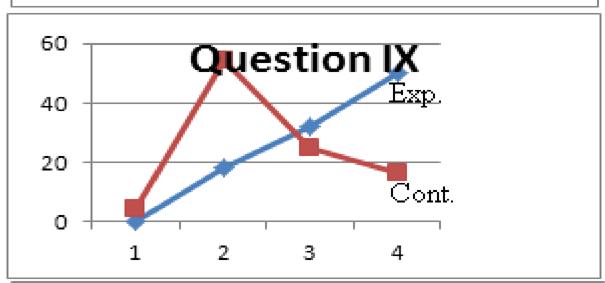


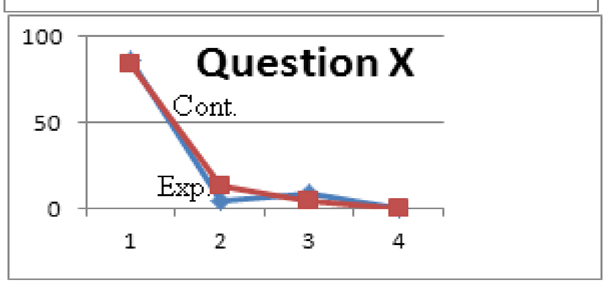
	Q	6		
A	6	3	27.27	12.5
B	5	14	22.73	58.33
C	7	4	31.82	16.67
D	4	3	18.18	12.5
	Q	7		
A	8	4	36.36	16.67
B	5	12	22.73	50
C	4	3	18.18	12.5
D	5	5	22.73	20.83
			100	100
	Q	8		
A	1	1	4.545	4.167
B	3	12	13.64	50
C	5	7	22.73	29.17
D	13	4	59.09	16.67
			100	100
	Q	9		
A	0	1	0	4.167
B	4	13	18.18	54.17
C	7	6	31.82	25
D	11	4	50	16.67
			100	100
	Q	10		
A	19	20	86.36	83.33
B	1	3	4.545	12.5
C	2	1	9.091	4.167
D	0	0	0	0
			100	100
	Q	11		
A	6	4	27.27	16.67
B	11	8	50	33.33
C	3	9	13.64	37.5
D	2	3	9.091	12.5
			100	100

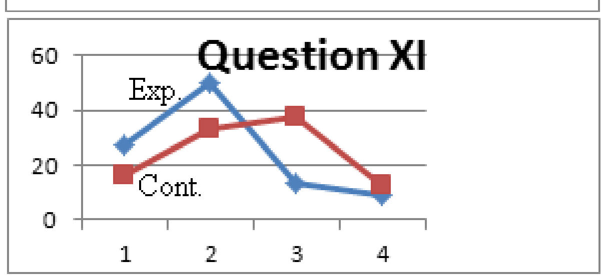




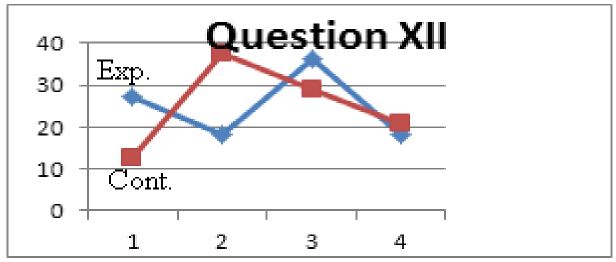


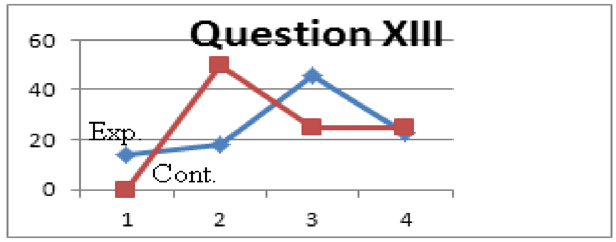


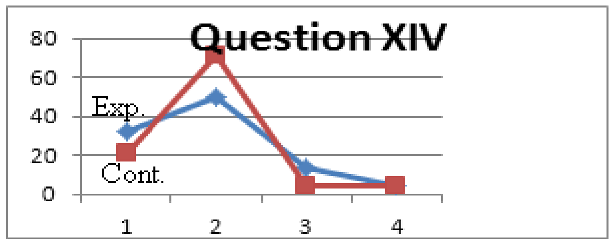


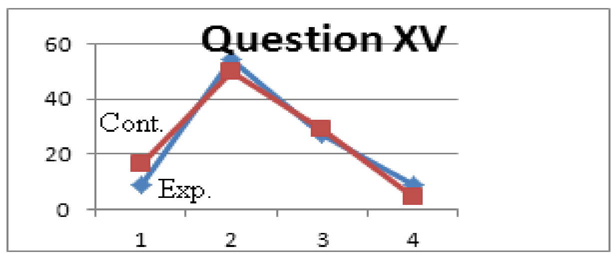


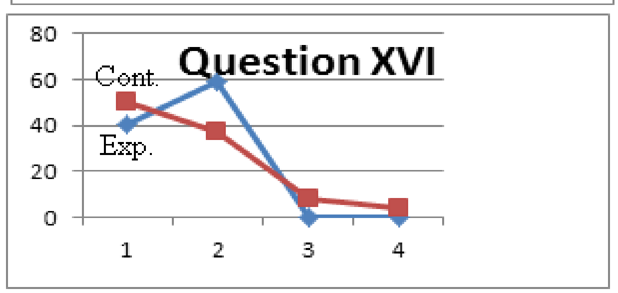
	Q	12		
A	6	3	27.27	12.5
B	4	9	18.18	37.5
C	8	7	36.36	29.17
D	4	5	18.18	20.83
			100	100
	Q	13		
A	3	0	13.64	0
B	4	12	18.18	50
C	10	6	45.45	25
D	5	6	22.73	25
			100	100
	Q	14		
A	7	5	31.82	20.83
B	11	17	50	70.83
C	3	1	13.64	4.167
D	1	1	4.545	4.167
			100	100
	Q	15		
A	2	4	9.091	16.67
B	12	12	54.55	50
C	6	7	27.27	29.17
D	2	1	9.091	4.167
			100	100
A	Q	16		
B	9	12	40.91	50
C	13	9	59.09	37.5
D	0	2	0	8.333
	0	1	0	4.167
			100	100
	Q	17		
A	2	2	9.091	8.333
B	3	9	13.64	37.5
C	8	8	36.36	33.33
D	9	5	40.91	20.83
			100	100

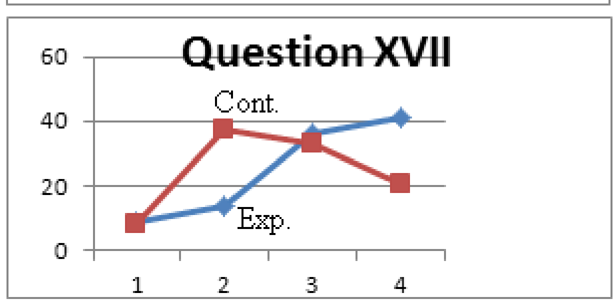




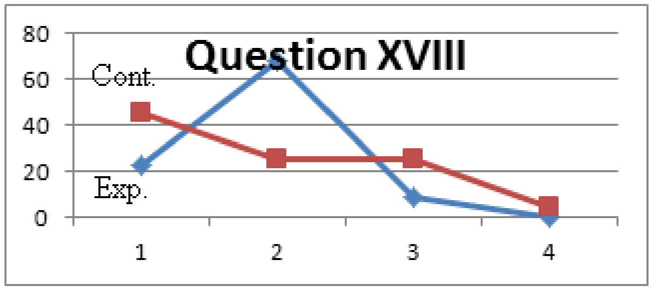


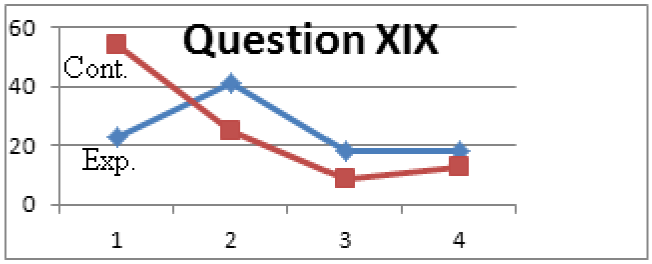


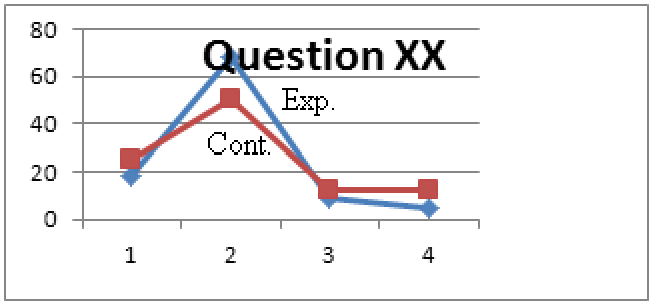


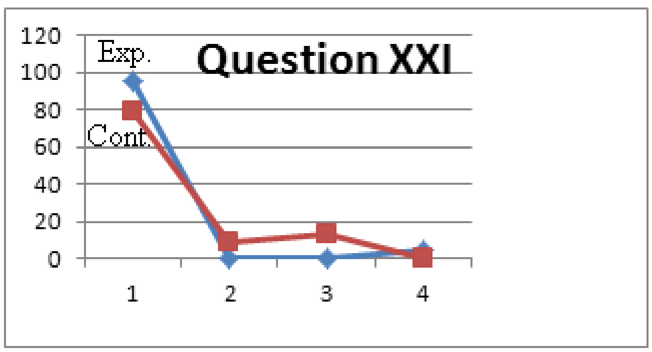


	Q	18		
A	5	11	22.73	45.83
B	15	6	68.18	25
C	2	6	9.091	25
D	0	1	0	4.167
			100	100
	Q	19		
A	5	13	22.73	54.17
B	9	6	40.91	25
C	4	2	18.18	8.333
D	4	3	18.18	12.5
			100	100
	Q	20		
A	4	6	18.18	25
B	15	12	68.18	50
C	2	3	9.091	12.5
D	1	3	4.545	12.5
			100	100
	Q	21		
A	21	19	95.45	79.17
B	0	2	0	8.333
C	0	3	0	12.5
D	1	0	4.545	0
			100	100
	tech			
	5	7	22.73	29.17
	15	13	68.18	54.17
	2	4	9.091	16.67
			100	100









A = Never; B = Sometimes; C = Often; D = Very often
Q = Question; Exp. = Experimental; Cont. = Control; f = frequency; % = percentage
Tech = Technology