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**Achieving Full Transmit Diversity for PPM
Constellations with any Number of Antennas via
Double Position and Symbol Permutations**

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Abstract

Encoding and decoding techniques present in the literature, mainly designed for radio frequency (RF) applications, face many problems when applied in optical wireless (OW) communications. These schemes seem to have lacks in some places like diversity, rate, or delay criteria. Therefore, we present a general technique for constructing minimal-delay, fully-diverse and rate-1 Space-Time (ST) block codes for Pulse Position Modulations (PPM) applied with an arbitrary number of transmit antennas and modulation positions. Introducing the novel idea of time-domain constellation extension as well as joint position and symbol permutations permitted to maintain unipolar transmissions the fact that renders the proposed code suitable for Free Space Optical (FSO) communications with direct detection at the receiver. To construct this code, it has been shown that for conveying a M -ary PPM symbol, each symbol duration must contain $M + 1$ time slots (or positions) rather than M . This increases the system dimensionality by 1 but guarantees a code with full diversity for an arbitrary number of transmit antennas and modulation positions in the PPM constellation. In parallel, an M -PPM decoder was developed to be applied with the same systems without any obstacles.

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List of Abbreviations

ASK: Amplitude Shift Keying

BEP: Bit Error Probability

BER: Bit Error Rate

BFSK: Binary Frequency Shift Keying

BPSK: Binary Phase Shift Keying

BW: Bandwidth

CSI: Channel State Information

dB: decibels

FO: Fiber Optics

FSK: Frequency Shift Keying

FSO: Free-Space Optics

QAM: Quadrature Amplitude Modulation

MAN: Metropolitan Area Network

MIMO: Multiple-Input Multiple-Output

MISO: Multiple-Input Single-Output

ML: Maximum Likelihood

MRRC: Maximal Ratio Receive Combining

OFDM: Orthogonal Frequency-Division Multiplexing

OW: Optical Wireless

PAM: Pulse Amplitude Modulation

PCU: Per Channel Use

PDF: Probability Density Function

PPM: Pulse Position Modulation

PSK: Phase Shift Keying

R_c : Channel Rate

RF: Radio Frequency

SER: Symbol Error Rate

SIMO: Single -Input Multiple-Output

SISO: Single -Input Single –Output

SNR: Signal-to-Noise Ratio

ST: Space-Time

STBCs: Space-Time Block Codes

STC: Space Time Code

STTCs: Space-Time Trellis Codes

TH-UWB: Time-Hopping Ultra-Wideband

ZF-DFE: Zero-Forcing Decision Feedback Equalization

1 Introduction

Figure 1: Communication is a Challenge

Wireless is the transfer of information between two or more sides over short or long distances (up to million of kilometers). This transmission is mainly based on radio frequency (RF) spectrum to convey the information. With the development of communications, we are currently in need for an unaffordable amount of RF spectrum to meet the user requirements. Many techniques are done to achieve this goal. The use of Multiple-Input Multiple-Output (MIMO) made an improvement because it uses a multi-antenna array instead of a single antenna one, where more spatial multiplexing is added to the transmission leading to a higher capacity. The use of Space-Time codes as well made an improvement in terms of code diversity leading to reliable codes and better throughput. But this was not enough, because more people are using the internet applications as well as the mobile systems and therefore we need higher and better bandwidth for wireless transmission. The need for a highly connected system is also a major challenge in communication, where RF spectra do not have the high capability to achieve this mission.

Figure 2: a highly connected world

The possibility of using unregulated and unlicensed bandwidth of optical wireless (OW) channels is an interesting characteristic of OW transmission and a major point in the world of communications. Therefore, we tried to apply all RF techniques in OW communications called Free Space Optics (FSO) in order to get the required throughput where light is used as the signal and the air or space as the transmission medium. So MIMO systems, as well as spatial and Space-Time (ST) multiplexing are applied in FSO, leading to an increase in capacity, better performance and an enhanced transmission range.

Fading exists in the nature and it is a problem caused by many factors that affects the channel reliability, so many encoding techniques exists to combat such distortions. Such techniques are initially considered with QAM or PAM modulation schemes that consist on modifying the amplitude, phase or polarity of the information symbols. But in optical communications, it is almost impossible to perform such modifications on the light signal, and if done they are undetected by the receiver. Therefore, unipolar modulation schemes such as On-Off Keying (OOK) and Pulse Position Modulation (PPM) became popular in FSO field where

the information can be conveyed only by the presence or absence of the light pulses, as well as the time delays of the light pulses.

We present the Alamouti ST code, paper [8], an example representing hundreds of other codes used for RF transmission. These codes introduce phase rotations, amplitude scaling, or polarity modification to achieve a full transmit diversity order and consequently they are not suitable for FSO transmissions using unipolar modulation techniques such as PPM. Applying Alamouti code with the unipolar PPM symbols, necessitates the transmission of pulses having positive and negative polarities. In fact, all ST codes suffer from the problem of sending the negative of pulses which is impossible to negate the positive PPM constellation symbols.

Some solutions exist to solve this problem in [9]-[12], which are based on sending unipolar pulses, modulated via PPM techniques, achieving the full diversity. But these solutions cannot be applied in any system having arbitrary number of transmitters n and modulation positions M . For example, in [12], the set of values of M for which a code exists and is fully diverse diminishes as n increases, and there are no unipolar ST codes in the literature that can be associated with any (M, n) combination, especially for large transmitters n and small modulation positions M . So our purpose is to solve this lack and develop a new solution code applicable for any system.

Therefore, a minimal-delay, fully-diverse and rate-1 Space-Time (ST) block code is proposed to be applied with an arbitrary number of transmit antennas and modulation positions. This code uses unipolar PPM symbols, having the same amplitude and maintaining a positive polarity. It is based on the novel idea of time-domain constellation extension as well as joint position and symbol permutations

that permit to achieve full diversity, a important criterion explained in detail throughout the report and it is radical for wireless transmissions. These characteristics rendered the proposed code suitable for Free Space Optical (FSO) communications with direct detection.

To achieve this code, it has been shown that for conveying an M -ary PPM symbols, each symbol duration must contain $M + 1$ time slots (or positions) rather than M . This augments the system dimensionality by 1 but guarantees a code with full spatial diversity for an arbitrary number of transmit antennas and modulation positions. This extension could be done either in the amplitude domain or time domain, but since amplitude modifications are prohibited in FSO, the only remaining solution resides in the time domain.

In parallel of the coding schemes, we take a look at the decoding schemes present in the nature. The main decoders are based on QAM or PAM detection [13], which is basically used with RF symbols. These schemes cannot be applied in optical communications, since the decoding is based on polarity or amplitude modifications, both undetected by the photodetector receivers. In addition, such decoders are inefficient in other cases, where PPM symbols are used as inputs, which are based on the time delays of the transmitted pulses. A development of such decoders was done in a more recent paper [14], that builds the 2-PPM decoder, which is suitable for optical transmissions because the decoding is based on PPM symbols. But the main limitation of this code is that it is based on 2 modulation positions only and still suffers the problem of decoding multi-dimensional symbols having arbitrary number of time slots (not only 2). Therefore, we developed a new decoder applied to any system transmitting multi-dimensional symbols.

The new decoding scheme proposed is based on M-PPM where M is an arbitrary number that can have any value. This scheme is a development of the previous 2-PPM scheme and is based on the maximum likelihood criterion to find the input symbols by minimizing the noise, i.e. knowing some received point, the algorithm find the closest point to this one by spanning in all directions. Therefore, two multi-dimensional spans are applied. The first one is based on spanning upward and downward between layers or levels, where each level represents an antenna in the MIMO systems, and we move to the upper level to generate valid intervals or to the downward level when all the positions at a given level are spanned. The second span is based on moving in all directions in a specific level, to span all components or sub-layers representing the modulation positions, in order to find the right pulse position.

The report begins with an overview on optical communication, its advantages and why it is preferred on wireless communication, in chapter 2. The channel used to transmit the information is subject to different types of fading which are presented in chapter 3, as well as the modals used to represent all types of channels. In chapter 4, we present all modulation schemes present in the literature, and the introduction of the new PPM modulation scheme designed for optical cases. In chapter 5, the use of multi-antenna array instead of the single antenna system is represented by MIMO systems, with MIMO advantages and the improvement it adds in terms of capacity, transmission, range, throughput and performance. Space-Time (ST) coding is a basic technique of MIMO systems that achieves diversity coding, for a more reliable transmission and is presented in chapter 6. In addition, we present the design criterion for ST codes in terms of rate and coding diversity as well as the trade-off between rate and diversity. In chapter 7, we present the encoding schemes found in

the literature, and in chapter 8, the decoding schemes are explored in parallel. The new solution code with a detailed proof are presented in chapter 9. The new decoding scheme is presented in chapter 10, with the corresponding simulations done over three million symbols, different types of channels, and all types of codes. Chapter 11 concludes. Finally, we present an appendix for proving the reliability of the new proposed code, and list the references used to complete this work.

2 Optical Communication

2.1 Definition

Optical communication is a transmission that uses light as the transmission medium. Free Space Optics (FSO) stands for the wireless part of optical transmissions. It is a line of sight technology that uses invisible infrared (IR) beams of light to provide an unaffordable bandwidth for transmitting and receiving voice, video and data information. Like fiber, FSO uses lasers to transmit data, but instead of using a glass fiber as the transmission medium, it is transmitted through the air or space. It works on the same principle as infrared television remote controls, wireless keyboards or wireless Palm devices. The light is sent from one telescope to another using lasers, where at the receiver side, photon detector receivers i.e. telescopic lenses collect the photon stream and transmit digital data containing internet messages, video images, radio signals or computer files. Practically, available systems offer capacities in the range of 100 Mbps to 2.5 Gbps, and data rates can reach 160 Gbps. FSO can function over distances of several kilometers as long as there is a clear line of sight between the source and the destination, and enough transmitter power.

This technology was adapted for two main reasons: First, the need for high bandwidth connections which cannot be provided in the case of the regulated and licensed RF wireless channels. Second is the problem of installing fiber optic (FO) cables in locations where the transmit and receive sides are difficult to be connected physically, or the laying of cables is expensive. In fact, light travels through the air faster than it does through the glass, which is used in the case of FO connections, so we classify FSO technology as optical communications at the speed of light. For example, to communicate between satellites, FSO is used because it doesn't require

expensive FO cables or securing spectrum licenses for RF channels, while FSO requires simply light to build connections.

In figure 3, wireless transmission is presented. In the optical case, the transmitters are replaced by lasers, the receivers by photodetectors, and the RF signal by light.

Figure 3: Wireless Communication

2.2 History

FSO is developed by the military and NASA, and it was used for more than three decades in many forms to provide fast communication links in remote locations. While FO communications gained worldwide acceptance, FSO is considered the newest and best technology nowadays. It enables bandwidth transmission capabilities used in fiber optics, using similar optical transmitters and receivers with the advantages of wireless transmission. In fact, this technology is not new in the literature.

Old Roman records indicate that metal plates were used as mirrors to reflect sunlight for long range signaling. The US military used similar sunlight powered devices to send telegraphs from mountain top to another in the early 1800's. For centuries, the

navies of the world used blinking lights to send messages between ships. In 1880, Alexander Graham Bell used the photophone that works on sunlight reflections, vibrating mirrors and a selenium photo cell to send telephone signals over 600 feet. During world wars, light signals were experimented, but radio and radar had more success. In the 1960's, the invention of laser led to the development of new semiconductor devices and optical fibers, and optical communication became applied in most of the practical cases.

2.3 Advantages

FSO has many advantages:

1. By using a modulated light instead of the RF carrier, it offers a limitless and unregulated bandwidth. Unlike radio and microwave systems, it is an optical technology where no spectrum licensing or frequency coordination with other users is required.
2. It is immune to radio frequency interference or saturation which is present in our atmosphere and affect RF signals but is not a concern in FSO. Since the optical radiation has no interference to electrical equipments, OW can be used in sensitive environments where radio transmission is not allowed.
3. It is easy deployed with a low cost, so it emits the necessity of wiring and cabling and it only needs a line of sight between the two ends of the link.
4. It is impossible to detect the narrow and invisible laser signal, making it hard to intercept and therefore extremely secure.

5. FSO finds solution for the last mile problems of the Metropolitan Area Networks (MAN) where new customers are added and are allowed to get the high BW and speed of MAN's, with lower data rates.

Figure 4: FSO Constellation

2.4 Comparison between Optical and Wireless Communications

Unlike RF and microwave systems, the optical channel uses intensity or light modulation and direct detection. The modulation schemes used for OW transmission are different from the conventional RF schemes, and they are based on the presence and absence of the light pulses as we are going to see throughout the report, due to the fact that light beam must maintain a positive polarity and a constant average amplitude. So the RF scheme can be used but perform differently when they are applied in FSO.

In the case of RF spectrum, licenses and conditions must be taken into consideration, and a limited amount of BW can be explored. In the optical case, an unaffordable

amount of spectrum is presented for a highly connected world with the possibility of using unregulated and unlicensed BW.

Unlike radio signals, the nature of the optical beam gave it the possibility of hitting many physical obstructions during its establishment. So we are getting high directivity using beam shaping elements the fact that leads to more security and wavelength reuse. Since FSO has no interference to electrical equipments, it can replace radio transmission in many sensitive environments.

But optical radiation can affect human eye and cause damage, therefore, the maximum power that can be transmitted is limited by eye safety regulation, and extra equipments are required when high power transmissions are working to ensure safety.

In optical systems, light is detected at the photodetector receiver with direct detection that is incoherent such that the detector responds linearly with the power rather than amplitude as is the case with radio detection. So the receiver sensitivity in OW is lower than for radio channels, and more path loss can affect the laser beam.

3 Fading

3.1 Wireless Fading

Each channel is subject to distortion called fading. The reflectors in the environment divides the RF signal and creates multiple paths that each version can take, which creates a superposition of signals detected at the receiver. Fading affects the amplitude and phase of the transmitted signal and results in a weak resultant signal. Each path experiences different attenuation, delay, and phase shift. This causes modification of the power and constructive or destructive interference that affect the reception where the last one refers to deep fade and can cause failure of communication. This is combated by spatial, time and frequency multiplexing.

- Slow fading or shadowing arises when the coherence time of the channel is larger than its delay. It is caused by the physical obstructions (mountains, hills, buildings...) that obscure the main path between the transmitter and receiver. This result in a signal amplitude modification and is modeled by a log-normal distribution which is overviewed in following sections.
- Fast fading is the rapid fluctuations of the RF signal over small areas. It occurs when the coherence time of the channel is smaller than its delay, i.e. the channel impulse response changes rapidly with respect to the symbol duration. It is caused by reflections and movements of objects in the environment.
- Flat fading occurs when all the frequency components of the signal experience the same magnitude of fading. So the channel has constant gain and linear phase response over a BW which is greater than that of the

transmitted signal. Since the signal is narrow with respect to the channel BW, the flat fading channel is an amplitude varying narrowband channel.

- Frequency-selective fading occurs when the coherence BW of the channel is smaller than that of the signal such that different frequency components of a signal experience decorrelated fading.

Figure 5: Amplitude Fading

3.2 Optical Fading

As RF systems, OW systems are subject to fading caused by many factors present in the atmosphere. Fading must be taken into consideration when designing any system because it affects its transmission, capacity, reliability and throughput. We present the main issues that should be considered when deploying systems based on FSO technology:

- Fog is a vapor composed of small water drops, which are different from rain and snow and have microscopic dimensions. It can modify light characteristics, or totally cut the passage of the signal during its transmission.

Fog can be combated by making short FSO link distances and adding redundancies (see chapter 6).

- Absorption occurs when water coming from rain or other source in the atmosphere extinguishes photons. This causes the attenuation of the light signal and can be combated by using appropriate power and spatial multiplexing (see chapter 5).
- Scattering is a type of fading when light is forced to deviate from a straight trajectory by one or more non-uniformities in the medium. In this case light collides with one of the factors. It causes no loss of energy but a redistribution of it, that may reduce the signal intensity for long distances. Therefore, we have multipath propagation due to scattering on many obstacles.
- Physical obstructions are caused by buildings, mountains or hills... and can cause small interruption intervals during the transmission. Because FSO is a line-of-sight technology, the interconnecting points must be free from physical obstruction and able to see each other. Multiplexing techniques can be used to address this problem and maintain signal availability.
- Scintillation happens when heated air coming from natural sources or from electrical devices create temperature variations and affect the signal, so we have fluctuations in signal amplitude.

The fading coefficients affect the channel capacity, transmission and performance in any transmission system whether in RF wireless or OW communication. Therefore, we need efficient coding schemes to combat such distortions. Coding is based on modulating the information symbols in different ways using any of the modulation schemes that we are going to present throughout the chapter. Modulated codes then

travels in a channel that is subject to the different types of distortion. Fading channels are modeled by one of the models in the mathematical world. Basically, we present the Guassian, lognormal, Rayleigh, and exponential models. Each one of them has its own characteristics, to meet any of all the real life situations. These models can be divided to two sections: Complex models designating RF wireless channels because their mathematical equation can have any sign, amplitude or polarity modifications, and positive models applied for OW channels because their equation have always positive results meeting the light beam characteristics.

Figure 6: Fading caused by Physical Obstructions

3.4 Wireless models

3.4.1 Complex Guassian Channels

Basically, we have the complex Guassian model characterized by its variable $Z = X + iY$, where X and Y are real and independent Guassian variables with equal variances σ_r^2 . The PDF of the joint variables is:

$$f(x, y) = \frac{1}{2\pi\sigma_r^2} e^{-\frac{(x^2+y^2)}{2\sigma_r^2}}$$

And knowing that $\sigma_z = \sqrt{2} \sigma_r$, the resulting PDF for the complex Gaussian variable Z is

$$f(z) = \frac{1}{\pi\sigma_z^2} e^{-\frac{|z|^2}{\sigma_z^2}}$$

3.5 Optical models

3.5.1 Lognormal Channels

These models are adapted because their PDFs are strictly positive (due to the fact that optical communication is based on the transmission of light which cannot be presented by a negative signal).

In the lognormal case, the amplitude of the path gain A is: $A = e^X$, where X is normal with mean μ_X and variance σ_X^2 . Thus, the logarithm of the amplitude scale is normally distributed. The intensity $I = A^2$ is log-normally distributed. The PDF for A is

$$f_A(a) = \frac{1}{a(2\pi\sigma_X^2)^{\frac{1}{2}}} \exp\left(-\frac{(\log_e a - \mu_X)^2}{2\sigma_X^2}\right), \quad a > 0.$$

So that the mean path intensity $E[A^2] = 1$, and $\mu_X = -\sigma_X^2$. The scintillation index

$S.I. = \frac{E[A^4]}{E^2[A^2]} - 1$ is a measure of the degree of fading and is related to the parameter

σ_X^2 by $S.I. = e^{4\sigma_X^2} - 1$.

Figure 7: Lognormal PDF for variances 1, 3 and 6

The figure represents the PDF of a lognormal function where the variance varies from 1, to 3 to 6.

3.5.2 Rayleigh Channels

In the Rayleigh case, the fading comes when non-dominating scatterers affect the optical field. Each one of non-uniformities contributes random optical phase when arriving at the photodetector. This is modeled by:

$$f_A(a) = 2ae^{-a^2}, \quad a > 0$$

where $E[A^2] = 1$. The random intensity $I = A^2$ is a one-sided exponential random variable. The scintillation index for the Rayleigh situation is 1, but the distribution differs from the log-normal case.

Figure 8: Rayleigh PDF

In the figure, the PDF of a Rayleigh channel is presented, for many variance values like 1,2, and 3.

3.5.3 Exponential Channels

A new model called exponential can be used based on the following PDF function:

$$f(a) = \frac{1}{\mu} e^{-\frac{a}{\mu}}, \quad a > 0$$

The exponential distribution occurs when considering the lengths of the inter-arrival times in a communication system, $\mu > 0$ is the scale parameter, $E[a] = \mu$, and a is the duration of time that the system manages.

Figure 9: Exponential PDF

The figure represents the PDF of an exponential function for variance 1, 2, and 3.

3.5.4 Positive Guassian Channels

Finally, we have also the real Guassian model (previously seen as a complex model in RF wireless communications) characterized by the following PDF

$$f_A(a) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(a-\mu)^2}{2\sigma^2}}, \quad a > 0$$

Where a the a real positive signal, $\sigma > 0$ is the standard deviation, the real parameter μ is the expected value, and the mean path intensity is normalized such that $E[A^2] =$

1. The probability density function has important properties including: symmetry about its mean μ , the mode and median both equal the mean μ , the inflection points of the curve occur one standard deviation away from the mean i.e. at $\mu - \sigma$ and $\mu + \sigma$.

4 Modulation Schemes

After the brief look on the channel models, we should have a look at the different types of modulation schemes to modulate signals prior to the transmission on the channels. Modulation is the process of varying a periodic waveform in order to use that signal to convey a message. We distinguish here again between radio wireless and optical wireless schemes, due to the fact that wireless is based on RF signals, and optical is based on light, which are two different types of signals. We differentiate between digital and analog schemes. Digital modulation is used to transfer a digital bit stream over an analog bandpass channel, for example over the public switched telephone network, or a limited radio frequency band, and consists mainly of: FSK, ASK, PSK, QAM, PAM, OFDM, OOK and PPM. While analog modulation is used to transfer an analog lowpass signal, as an audio or TV signal, over an analog bandpass channel, for example a limited RF band or a cable TV network channel, and consists mainly of: AM, FM, PM, SSB, PAM and SM. We will consider the digital schemes and insist on the optical ones. The exact requirements of a particular scenario under consideration determine which scheme is used. From these schemes, only OOK and PPM are used for optical modulation, while all others are used for RF wireless modulation and are inefficient in for the optical field for many reasons.

4.1 Radio Modulation

4.1.1 Amplitude-Shift Keying (ASK)

ASK is a modulation scheme that represents digital data as variations in the amplitude of a carrier wave where RF signals are used for transmission, while maintaining the frequency and the phase constant. Amplitude levels represent binary

0s and 1s where four levels represent 2 bits, 8 levels represent 3 bits... The carrier represents an on-off switch where the “on” case is translated by 1 and represent the presence of the carrier and the “off” case is translated by 0 and represent the absence of the carrier for the signal. As AM, ASK detects noise and interference present in the different routes and maintain linearity while modulating. ASK can be used to modulate optical signal, but modifying light amplitude is rarely detected by the photo-detector receiver due to the nature of the light beam.

4.1.3 Frequency-Shift Keying (FSK)

FSK is a modulation scheme that conveys data by modifying or modulating the frequency of a carrier wave while maintaining the same amplitude and phase. The simplest FSK form is binary FSK (BFSK) that uses two discrete frequencies binary 0s and 1s, and can be extended to more than 2 levels.

4.1.4 Phase-Shift Keying (PSK)

PSK is a scheme that modifies the carrier phase, while maintaining a constant amplitude and frequency. PSK uses a number of special phases that are used to represent the information bits we have. At the receiver, the demodulator compares the received phase with a reference one to recover the input data.

4.1.5 Quadrature Amplitude Modulation (QAM)

QAM modulate the amplitude of the two carrier waves, which are in general sinusoids, out of phase by 90 degrees, therefore they are called quadrature carriers. PSK just presented is a special case of QAM where the phase is changed and the amplitude of the carrier remains constant. ASK is a special case as well, where the amplitude varies maintaining a constant phase.

We present a constellation diagram where points are arranged in a square grid equally spaced. Since data in digital communication is binary, and the QAM is square, therefore, the constellation will be represented as 16, 64, 128 or 256 QAM. Higher order QAM can be achieved but leading to more error rates because points become closer to each other for the same average constellation energy. A rectangular 16-QAM constellation diagram is shown here:

Figure 10: 16-QAM constellation

A circular 8-QAM constellation diagram is another form of the QAM constellation which are optimal in terms of the number of points that we get in the constellation, but they are not often used since rectangular QAMs are much easier to modulate and demodulate.

Figure 11: 8-QAM constellation

4.1.6 Pulse-Amplitude Modulation (PAM)

PAM is a pulse modulation scheme that convey data by varying the voltage or power amplitude of individual pulses in a timed sequence of electromagnetic pulses i.e. the input data is encoded in the amplitude of a series of signal pulses. Therefore, in the analog case, the pulse amplitude can be infinite, while in the digital case a two or more pulse levels can be used to digitize the symbol. For example, a two-bit modulator i.e. PAM-4 takes 2 bits at a time and maps the signal amplitude to one of the four possible levels which are -3, -1, 1 and 3 volts. At the receiver side, the demodulator detects the amplitude level of the pulses at every symbol period. In practical applications, PAM is replaced by Pulse Position Modulation (PPM) since it is more suitable for modulating light beams. QAM and PAM modulations are strongly used in RF encoders and decoders explored throughout the report.

4.1.7 Orthogonal Frequency-Division Multiplexing (OFDM)

OFDM is a frequency-division multiplexing (FDM) scheme utilized as a digital multi-carrier method. A large number of orthogonal sub-carriers close to each others are used to carry data, therefore the data are divided into several parallel low-rate data streams one for each sub-carrier. Each one of the new carriers is modulated using one of the different schemes such as QAM or PSK at low symbol rates maintaining the total data rates similar to the original single carrier modulation schemes in the same BW.

4.2 Optical Modulation

4.2.1 On-Off Keying (OOK)

On-off keying (OOK) is a type of modulation that represents digital data as the presence or absence of a carrier wave. In optical communication, the presence of

light for some small duration denotes a binary 1, while its absence for the same duration denotes binary 0, as it is shown in the following figure.

Figure 12: OOK Constellation

This modulation scheme is used because light is a positive parameter, where a modification of its phase, amplitude or polarity is prohibited and undetected. OOK represent the two binary symbols 0 and 1, so it encodes 1 bit. But what if we need more than 2 binary constellation? If we need to represent a binary 4 constellation, i.e. 00, 01, 10, 11, OOK cannot be the right model because it represents either 0 or 1. Therefore, pulse position modulation (PPM) became the solution.

4.2.2 Pulse-Position Modulation (PPM)

Pulse-position modulation is a form of signal modulation in which M message bits (not only 1 bit) are encoded by transmitting a single pulse in one of 2^M time slots. So M time slots are transmitted every T seconds leading to a bit rate which is M/T bits per second. As the figure show:

Figure 13: PPM encoding

In figure 12, one pulse is transmitted in one of the M slots. The pulse duration is small (1 ns in this case) which is the duration of each time slot. A high peak to mean power ratio corresponds to the pulse, while the power is zero for all remaining time slots. Therefore, PPM is still a power-efficient modulation technique.

Figure 14: 2-PPM Constellation

The 2-PPM constellation is based on sending a pulse in the first time slot to represent the first position, and a pulse in the second time slot to represent the second position.

The 4-PPM constellation that we proposed in OOK section is represented by the figure, where the first line represent the "00" symbol, the second line for "01", the third line for "10", and the fourth line represents "11". In fact, each of these symbols is represented by the presence of light in one of the 4 time slots or modulation positions. Its advantage is that it can be generalized to M -PPM, where a signaling interval of length T_s , is subdivided into M slots, each of length $T = T_s/M$, and a digital message comprised of $\log_2 M$ bits is sent by pulsing the laser in one of these time slots. Therefore, to represent a symbol having position k in the M -PPM constellation, we take the k -th of the $M \times M$ identity matrix, because the position of 1 in this column represents the position of the pulse in one of the time slots.

Therefore, for $P \in \{1, 2, \dots, M\}$ the 2×2 PPM constellation are represented by:

$$\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}, \text{ the } 3 \times 3 : \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}, \text{ the } 4 \times 4 \text{ will be: } \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}, \dots,$$

$$\text{the } M \times M \text{ will be } \left\{ \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \dots, \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \right\}$$

Therefore, OOK is simpler than PPM but limited to two symbols, while PPM is more complex to implement but more efficient because it represents a wider number of cases.

Figure 15: 4-PPM Constellation

PPM is primarily useful for optical communications systems, where there tends to be little or no multipath interference. One of the difficulties of implementing this technique is that the receiver must be properly synchronized to align the local clock with the beginning of each symbol. Therefore, it is sometimes implemented differentially as Differential Pulse-position modulation, where each pulse position is encoded relative to the previous pulse, such that the receiver only measures the difference in the arrival time of successive pulses. It is possible to limit the

propagation of errors to adjacent symbols, so that an error in measuring the differential delay of one pulse will affect only two symbols, instead of affecting all successive measurements.

Both optical schemes are used, but when to use each, and what are the advantages and drawbacks of each scheme? We are going to see in the next section a sample of comparisons between the two schemes.

4.2.3 Comparison between OOK and PPM

Many comparisons were made between OOK and PPM under different circumstances. For example, in paper [1], FSO communication between satellites networked together is examined. It permits high data rates between different places on earth using satellite connections based on the use of optical radiation as a carrier leading to narrow signal divergence. Due to the narrow divergence, the large space distance and the vibration caused by tracking noise and mechanical impacts, establishing a point-to-point connection between satellites is difficult.

At this point, the comparison between OOK and PPM for system modulation is presented to find the appropriate performance making the network as simple and inexpensive as possible. Notably, even low vibration of one satellite pointing system decreases the network performance dramatically. The satellites are placed at equal orbit altitude, distance, bit rate, same optical wavelength, optical power and other parameters.

For a change in normalized vibration variance from 0.05 to 0.5, it is seen that the PPM method yields a higher SNR than OOK. In fact, in OOK, for half the bits transmitted, the power is kept low in order to keep the average transmitted power constant. While in PPM, we transmit very short duration high-power pulses, so it is

expected to have a higher SNR than OOK. The Bit Error Rate (BER) for PPM is lower than OOK which means it acquires a better performance as well. The result can be explained on the basis of power criterion, where a higher transmitted signal power is for PPM because more positions are in use. It is assumed that the problems of OOK threshold adaptation and PPM synchronization are solved and no penalty is expected. It is important to mention that the synchronization of PPM is most problematic, while the adoption of the OOK threshold is necessary. Therefore, it is seen that PPM is the preferred modulation method, but if synchronization is considered, OOK would be better.

In Jinlong paper [2], Intensity Modulated Direct Detection system is used with OOK modulation, which is proposed by its simplicity, and Pulse Position Modulation (PPM) proposed by its power efficiency. In terms of power, PPM has a better power efficiency than OOK, using a high SNR in order to have a precise result. In terms of BW, OOK requires less BW than PPM, probably because less modulation positions are present. Finally, in terms of transmission capacity, both modulation schemes have the same performance.

In Costas paper [3], we explore the signaling over high-speed optical inter-satellite links. As the demand for increased throughput increases, this paper shows that PPM became inefficient and alternative modulation schemes should be investigated. In this paper, the throughput and error-probability performances of various modulation schemes, including PPM, overlapping PPM (OPPM), multi-pulse PPM (MPPM) and on-off keying (OOK) are investigated. Overlapping PPM (OPPM) is a generalization of PPM that allows more than one pulse-positions per pulse-width and preserves some of the desirable properties of PPM, such as equal energy signals and low duty-

cycle. Another modulation scheme that has been considered is MPPM which is another generalization of PPM, that allows more than one pulse per symbol interval.

The capacity is a fundamental limit on the rate, while it was shown that the cutoff-rate of a system, which is upper-bounded by capacity, is a practical limit. So in terms of those two variables, all modulation schemes can do better than PPM, especially at the higher rates. So the best performance for OOK, then OPPM, then MPPM and finally PPM. Then we look at the peak power requirements of PPM, MPPM, OPPM and OOK as the data rate r in nats per second increases and uncoded error-probability is kept fixed. OOK has the best performance, then PPM, OPPM, and finally MPPM. The symbol-error probability for the four modulation schemes is compared as a function of the energy per bit for the same duty cycle as PPM, i.e. MPPM is best, followed by OOK and PPM, and lastly by OPPM.

After seeing the modulation schemes under a different number of criterion, we can say as a conclusion that no modulation scheme considered was seen to be uniformly superior to all other under all constraints and all parameter values. OOK was better in some cases in terms of simplicity but its drawback is that it's not an equal energy signaling scheme (which means that an estimate of the received power is needed by the receiver before decisions are made). PPM was better in other cases where synchronization, complexity and bandwidth are not taken into account.

5 MIMO Systems

Free space laser communication systems based on a single input and single output suffer from the fading problems listed previously, as well as capacity constraints and issues related to quality and coverage. Therefore, we have limits on the reliability of many high speed data transmission applications. We present the multiple-input multiple-output (MIMO) architecture, which has the potential of resolving or significantly reducing the effect of these problems and exploiting the space dimension to improve wireless systems capacity, range and reliability. Notably, the power requirements for an MIMO system are lower than those of single input single output (SISO) system where higher data rates can be supported using same transmit powers.

In addition, time multiplexing could be achieved using coded symbols transmitted over time from the different transmit antennas of the MIMO system. Therefore, the space and time are both explored using Space-Time (ST) codes getting a high rate and reliable transmission. ST codes generated from orthogonal designs can be used as well to achieve a simple decoding at the receiver with linear operations.

MIMO is used in the famous wireless systems WIFI as a short range, and WIMAX on a wider range. The third generation 3G mobile system is based on MIMO systems as well as the future fourth generation 4G systems will also employ MIMO technology.

5.1 Definition & Characteristics

MIMO is the use of multi-antenna array at the transmitter and receiver instead of the SISO systems to send multiple parallel signals in order to improve communication

performance. It is one of the several forms of smart antennas and has attracted attention in wireless communications, since it offers significant increases in data capacity and link range without additional BW or power. It achieves this by spatial multiplexing, Space-Time (ST) multiplexing and diversity coding. MIMO can have many special types depending on the practical case:

- Multiple-Input Single-Output (MISO) is a degenerate case when the receiver has a single antenna.
- Single-Input Multiple-Output (SIMO) is a degenerate case when the transmitter has a single antenna.
- Single-Input Single-Output (SISO) is the standard system where neither the transmitter nor receiver has multiple antennas.

Figure 16: MIMO types

Figure 17 shows the improvement of MIMO over SIMO and MISO, in terms of channel capacity, with respect to the number of antennas elements.

In an urban environment, the signals sent from any transmitter will hit mountains, trees, buildings, etc. and continue their way to the receiver but in different directions. Therefore, the signal travels through multiple paths before reaching its destination. So multipath occurs and the different signals arrive at the receiver at various times. With MIMO, the receiver process and sort the multiple signals to produce one signal that has the original transmitted data.

Figure 17: high capacity for MIMO over SIMO and MISO

5.2 Basic Functions

MIMO can be sub-divided into three main categories, precoding, spatial multiplexing, and diversity coding:

Precoding is a signal processing technique applied to the signals prior to transmission. The processing is based on phase modification or any other

modulation techniques to reduce multipath fading. This step requires channel state information (CSI) at the transmitter.

Spatial multiplexing delivers parallel data signals from the different antennas at the transmitter side. So a high rate signal is split into multiple lower rate data streams and each stream is sent from a different transmit antenna in the same frequency channel. This increases channel capacity at higher SNR because the space dimension is reused and multiplexed more than one time leading to an increase of the number of bits per second and per Hz. The maximum spatial order or number of streams consists of the minimum between the number of transmit antennas and t

hat of the receive antennas if a linear receiver is used. Spatial multiplexing can be used with or without transmit channel knowledge. Figures 18 and 19 represent spatial multiplexing.

Figure 18: Spatial Multiplexing in the case of a single user system

Figure 19: Spatial Multiplexing in the case of multi-user system

We see that the space antennas provide multiple uncorrelated copies of the same signal.

Diversity Coding technique consists of sending a single redundant stream coded using Space-Time coding techniques. Each coded version of the signal is emitted from each of the space antennas. Orthogonality could be introduced to facilitate the decoding function at the receiver.

5.3 Case Study

Many research were done in this domain. We explore in Wilson paper, see [4], the Bit Error Probability (BEP) in MIMO systems, where M laser transmitters combined with N photodetectors are studied for terrestrial, line-of-sight optical communication. The resulting channel has the potential for combating fading effects on turbulent optical channels. The modulation type used is Q -ary PPM across lasers, with intensity modulation. Ideal photodetectors are assumed with and without background radiation where both Rayleigh and log-normal fading models are treated. The sources and detectors are physically situated that fading between source-detector pairs is independent and thus, diversity benefits can be achieved using the MIMO channel. Because it's an FSO case, the noise and input signals are assumed positive.

By analyzing the BEP, we distinguish between 4 cases:

- With no fading, and no radiation, BEP is independent of M , the number of lasers, for a fixed energy because sources are non coherent. However, the effective received power increases with N , the number of photo-detectors, which is interpreted as increasing the receiving aperture size, or increasing the optical gain. Moreover, BEP is independent of Q , the number of modulation positions, for a fixed energy per symbol, but as Q increases for a fixed bit rate, the peak power increase as $Q / \log_2 Q$ to maintain fixed energy

per symbol. The required energy per bit E_b decreases as $\log_2 Q$, and small E_b can achieve small BER as Q tends to infinity.

- When taking into consideration the impact of fading, without background radiation, BEP is computed for the case of Rayleigh fading, and a diversity of order MN . In this case we have an energy division penalty that affects the efficiency of adding more sources like fading. The diversity gains are large in a Rayleigh fading regime, for various values of M , but full transmit diversity is obtained without coding at the transmitters, which is not possible in the RF channel case. In fact, the paths in FSO are additive, never able to produce destructive interference due to the all positive nature of the signal, so diversity comes automatically by adding additional independent paths, without requiring any coherent signal combining at the receiver. Adding detectors improves efficiency due to larger total aperture, and also by aperture averaging of the fading variation.
- Exploring the case of background radiation with no fading, we see that BEP decreases with increasing Q , for fixed E_b , effectively because the slot duration decreases.
- For the case of fading and radiation, we see the cumulative degradation imposed by those factors. By increasing the numbers of transmit or receive antennas, the un-faded performance can be reached.

5.4 Advantages

5.4.1 MIMO Increases Throughput

Wireless throughput scales as more radio transmissions (in the case of radio systems) or light (in case of optical systems) are added onto the same channel. We

notice from the figure, that the highest throughput exists for the 3x3 MIMO, then 2x2 MIMO, and the lowest one for 1x1 SISO. Shannon's law is defined as the theoretical maximum rate at which reliable digits can be transmitted over a bandwidth-limited channel in the presence of noise, usually expressed in the form $C = W \log_2(1 + S/N)$, where C is the channel capacity in bits per second, W is the bandwidth in hertz, and S/N is the signal-to-noise ratio. Therefore, the more transmitters are added, the more BW we get, and therefore more capacity and throughput. This is detailed in the next section that present the capacity for different types of MIMO systems.

Figure 20: MIMO increases throughput

5.4.2 MIMO Increases Channel Capacity

The following figure represents MIMO channels, where the channel capacity is the tightest upper bound on the amount of information that can be reliably transmitted over a communication channel. By the noisy-channel coding theorem, the channel

capacity of a given channel is the limiting information rate that can be achieved with arbitrarily small error probability.

Figure 21: Channel Representation

In general capacity of a single-input single-output is denoted by:

$$C = \log_2(1 + \rho h^2)$$

Where ρ is the SNR, and h represents the channel matrix.

For MIMO channels, $Y = HX + N$ where H is the $N \times M$ matrix, S is $M \times 1$ input matrix and Y is $N \times 1$ output matrix. The rank is denoted r , and λ_i denotes positive eigenvalues of HH^+ . For deterministic channel matrix without channel knowledge at the transmitter:

$$C_{\text{SISO}} = C = \log_2(1 + \rho h^2)$$

$$C_{\text{MISO}} = \log_2(1 + \frac{\rho}{M} \sum_{i=1}^M h_i^2)$$

$$C_{\text{SIMO}} = \log_2(1 + \rho \sum_{i=1}^N h_i^2)$$

$$C_{\text{MIMO}} = \sum_{i=1}^r \log_2 \left(1 + \frac{\rho}{M} \lambda_i \right)$$

SIMO capacity increases more rapidly with number of receiving antennas than MISO capacity with number of transmitting antennas because MISO has no spatial

array gain at the receiver. MIMO capacity increases linearly with $\min(M,N)$ or number of antennas and depend on the channel matrix H . The larger rank and eigenvalues HH^+ the channel coefficients have, the more MIMO capacity we have. The following figure shows that when we add new channels, the capacity is increased as:

Figure 22: MIMO increases channel capacity

5.4.3 MIMO Increases Range

Each multipath route is viewed as a separate channel that creates many virtual paths over which the different signal versions are transmitted. Traditional radios are confused by this multipath, while MIMO takes advantage of these new connections to increase range and throughput, which is modeled by figure 22.

Figure 23: MIMO increases range

6 Space-Time codes

Space-Time codes presented in the previous chapter achieves diversity and reliability for systems, but first let's look on the different types of diversity, and then exploring spaces-time codes in details due to the advantage they offer in addition to the MIMO systems.

6.1 Diversity

Diversity is a method for improving the reliability of data by coding the same redundant signal with different types of modulation schemes to combat fading and interference and to avoid erroneous messages.

Diversity is based on the fact that channels experience different levels of fading and interference. Therefore, it provides two or more inputs at the receiver such that the fading among these inputs is uncorrelated. If one path undergoes deep fade at a particular point in time, another independent or highly uncorrelated path may have a strong signal at that input. Diversity techniques may exploit the multipath propagation, resulting in a diversity gain, often measured in decibels (dB).

The following classes of diversity schemes can be identified:

- **Time Diversity:** it's a temporal diversity where redundant copies of the signal are sent at different time instants. A redundant forward error correction code may be added and the message is spread in time by means of bit-interleaving before it is transmitted. Thus, error bursts are avoided, which simplifies the error correction.
- **Frequency Diversity:** the signal is sent over different frequencies and affected by frequency- selective fading.

- **Space Diversity:** the space dimension is explored where the signal is transferred over different paths each corresponding to an antenna in the multi-antenna array MIMO system.
- **Polarization Diversity:** this scheme is based on transmitting many versions of a signal received via antennas with different polarization. A diversity combining technique is applied on the receiver side.
- **Multiuser Diversity:** is a scheme that schedule users at either the transmitter or receiver side, where each side selects the best user depending on the quality required by the channel.
- **Cooperative diversity:** achieves antenna diversity gain by utilizing the cooperation of distributed antennas belonging to each node.

6.2 STC Definition & Types

Space-Time Code (STC) is used to improve the reliability of the transmitted data in MIMO wireless systems. It relies on sending multiple redundant copies of the same signal each modulated differently and sent from one of the space antennas in the system, in the hope that at least one of them is correctly detected by the receiver. Therefore, STC's make use of both the spatial and time diversity listed earlier to improve the system performance.

The fact that the transmitted signal must traverse a difficult environment with scattering, reflection, refraction etc. and then may be corrupted by thermal noise, means that some of the received copies of the data will be better than others. This redundancy gives the high possibility to the receiver of being able to use one or more of the received copies to correctly decode the signal. in other words, ST coding

combines all copies of the received signal in an optimal way to extract as much information from them as possible.

STC's can be subdivided according to whether the receiver knows the channel impairments. In the coherent case, the receiver knows the channel impairments and CSI through training or some form of estimation. These codes are simpler than their non-coherent ones. In non-coherent STC, the receiver does not know the channel impairments or the CSI, but knows the statistics of the channel. Looking from another point of view, STC can be classified in two main classes: Space-Time Trellis Codes (STTCs) and Space-Time Block Codes (STBCs) each having its own properties.

6.2.1 Space-Time Trellis Codes (STTCs)

An STTC distributes a trellis code over multiple antennas in the space and multiple time slots in the time domain to provide both coding gain and diversity gain. The multiple different copies are used by the receiver to reconstruct the input message. To use an STTC we must have multiple transmit antennas but not necessarily multiple receive antennas, however they could be used to enhance performance.

6.2.2 Space-Time Block Codes (STBCs)

An STBC acts on a block of data at once therefore providing only diversity gain, but are much less complex than STTCs in terms of implementation. This scheme transmits copies of a block of data stream across a number of antennas, at different time instants, and the receiver exploits the various versions of the data improving the input's reliability. Because of its simplicity and advantages, STBC will be adopted throughout the text as the main form of STC.

6.3 STC Characteristics

An STC is usually represented by a matrix where each row represents one antenna and each column represents a time slot.

The code rate of an STC measures how many symbols per time slot it transmits on average over one block. If a block encodes K symbols, and T represents the time slots, the code-rate is

$$R_c = \frac{K}{T}$$

Using the spatial multiplexing, we have seen that the rate is an increasing variable proportional to the addition of new antennas. So the rate can achieve higher values than the normal rate for SISO systems. Using STC's, the full rate achieved is 1, because in this case the number of antennas equals the number time slots. This is achieved in many codes such as the Alamouti code for radio communications, and its solution codes for optical communications, as well as the new proposed code.

STC's orthogonality is an important issue. It means that the code is designed such that the vectors representing any pair of columns taken from the coding matrix are orthogonal. The result of this is technique is a simple, linear and optimal decoding of the input signal can be achieved at the receiver. But to satisfy this, some codes must sacrifice some proportion of their data rate.

The design of STC's is based on the diversity derived by Tarokh in its paper "Space-Time Codes for High Data Rate Wireless Communication: Performance and Code Construction", paper [7], which is overviewed in the next section.

6.4 Case Study: Design of STCs

In Tarokh paper [7], the design of channel codes for improving the data rate and the reliability of communications is presented over fading channels using multiple transmit antennas. Data is encoded by a channel code and the encoded data is split into n streams that are simultaneously transmitted using n transmit antennas. The received signal at each receive antenna, where we have m receive antennas, is a linear superposition of the n transmitted signals perturbed by noise. The purpose is therefore to find a solution to the problem of designing a physical layer (channel coding, modulation, diversity) that operates at BW efficiencies that are twice to four times as high as those of today's systems using multiple transmit antennas.

6.4.1 System model

Data is encoded by the channel encoder to feed a serial-to-parallel converter, then it is divided into n streams of data. Each stream is used as the input to a pulse shaper where the output is then modulated. At each time slot t , the output of modulator i is a signal transmitted using transmit antenna i for $1 \leq i \leq n$. The n signals are transmitted simultaneously each from a different transmit antenna and all of them have the same transmission period T . At each receive antenna, a noisy superposition of the transmitted signals is corrupted by Rayleigh or Rician fading. The demodulator at the receiver then computes a decision statistic based on the signals arriving at each receive antenna $1 \leq j \leq m$. The signal d_t^j received by antenna j at time t is given by

$$d_t^j = \sum_{i=1}^n \alpha_{i,j} c_t^i \sqrt{E_s} + \eta_t^j$$

where the noise η_t^j at time t is modeled as independent samples of a zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension. The coefficient $\alpha_{i,j}$ is the path gain from transmit antenna i to receive antenna j . It is assumed that these path gains are constant during a frame and vary from one frame to another.

6.4.2 Math Review

This overview is necessary to prove the diversity and coding criterion in the design of STC's.

The inner product of $x = (x_1, x_2, \dots, x_k)$ and $y = (y_1, y_2, \dots, y_k)$ is $x \cdot y = \sum_{i=1}^k x_i \bar{y}_i$

Where $\bar{y}_i$ denotes the complex conjugate of y_i . For any matrix A , the Hermitian (transpose conjugate) of A is A^* such that an $n \times n$ matrix is *Hermitian* if $A = A^*$.

The matrix is *nonnegative definite* if $xAx^* \geq 0$ for any $1 \times n$ complex vector x . An $n \times n$ matrix V is *unitary* if $VV^* = I$ where I is the identity matrix. An matrix $n \times l$ matrix B is a *square root* of an $n \times n$ matrix if $B^* = A$.

An eigenvector of v an $n \times n$ matrix A corresponding to eigenvalue λ is a $1 \times n$ vector of unit length such that $va = \lambda v$ for some complex number λ . The vector space spanned by the eigenvectors of A corresponding to the eigenvalue zero has dimension $n - r$, where r is the rank of A .

Any matrix A with a square root B is nonnegative definite.

For any nonnegative-definite Hermitian matrix A , there exists a lower triangular square matrix B such that $BB^* = A$.

Given a Hermitian matrix A , the eigenvectors of $\text{span } \mathbb{C}^n$, the complex space of n dimensions and it is easy to construct an orthonormal basis of $\mathbb{C}^n$ consisting of eigenvectors A . Furthermore, there exists a unitary matrix V and a real diagonal D matrix such that $VAV^* = D$.

The rows of V are an orthonormal basis of $\mathbb{C}^n$ given by eigenvectors of A . The diagonal elements of D are the eigenvalues $\lambda_i, i = 1, 2, \dots, n$ of A counting multiplicities.

The eigenvalues of a Hermitian matrix are real.

The eigenvalues of a nonnegative-definite Hermitian matrix are nonnegative.

6.4.3 Design Criterion

In this design, we are considering the case of independent case coefficients where the signals transmitted from different antennas undergo independent fades. Each one of them is modeled by $\alpha_{i,j}$ and they are considered as independent samples of complex Gaussian random variables with non-zero complex mean $E\alpha_{i,j}$. It is assumed that each element of the signal constellation is contracted by a scale factor $\sqrt{E_s}$ chosen so that the average energy of the constellation is 1. The probability that a maximum-likelihood receiver decides erroneously in favor of a signal is denoted by:

$$e = e_1^1 e_1^2 \dots e_1^n e_2^1 e_2^2 \dots e_2^n \dots e_t^1 e_t^2 \dots e_t^n$$

Assuming

$$c = c_1^1 c_1^2 \dots c_1^n c_2^1 c_2^2 \dots c_2^n \dots c_t^1 c_t^2 \dots c_t^n$$

was transmitted.

Assuming ideal channel state information (CSI), the probability of transmitting c and deciding in favor of e at the decoder is approximated by

$$P(c \rightarrow e | \alpha_{i,j} \ i = 1, 2, \dots, n, j = 1, 2, \dots, m) \leq \exp(-d^2(c, e)E_s/4N_0)$$

Where $N_0/2$ is the noise variance per dimension and

$$d^2(c, e) = \sum_{j=1}^m \sum_{t=1}^l \left| \sum_{i=1}^n \alpha_{i,j} (c_t^i - e_t^i) \right|^2$$

This is the standard approximation to the Gaussian tail function. Setting

$\Omega_j = (\alpha_{1,j}, \alpha_{2,j}, \dots, \alpha_{n,j})$ we rewrite the previous equation as

$$d^2(c, e) = \sum_{j=1}^m \sum_{i=1}^n \sum_{i'=1}^n \alpha_{i,j} \overline{\alpha_{i',j}} \sum_{t=1}^l (c_t^i - e_t^i) \overline{(c_t^{i'} - e_t^{i'})}$$

After simple manipulations, we see that

$$d^2(c, e) = \sum_{j=1}^m \Omega_j A \Omega_j^*$$

Where $A_{pq} = x_p \cdot x_q$ and $x_p = (c_1^p - e_1^p, c_2^p - e_2^p, \dots, c_l^p - e_l^p)$

For $1 \leq p, q \leq n$. Thus

$$P(c \rightarrow e | \alpha_{i,j} \ i = 1, 2, \dots, n, j = 1, 2, \dots, m) \leq \prod_{j=1}^m \exp(-\Omega_j A(c, e) \Omega_j^* E_s / 4N_0)$$

Where $A_{pq} = \sum_{t=1}^l (c_t^p - e_t^p) \overline{(c_t^q - e_t^q)}$

Knowing that $A(c, e)$ is Hermitian, there exists a unitary matrix V and a real diagonal matrix D such that $VA(c, e)V^* = D$. By construction, the matrix

$$B(c, e) = \begin{pmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \dots & \dots & e_l^1 - c_l^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \dots & \dots & e_l^2 - c_l^2 \\ e_1^3 - c_1^3 & e_2^3 - c_2^3 & \dots & \dots & e_l^3 - c_l^3 \\ \dots & \dots & \dots & \dots & \dots \\ e_1^n - c_1^n & e_2^n - c_2^n & \dots & \dots & e_l^n - c_l^n \end{pmatrix}$$

is clearly a square root of $A(c, e)$. Thus the eigenvalues of $A(c, e)$ are nonnegative real numbers.

We express $d^2(c, e)$ in terms of the eigenvalues of the matrix $A = (c, e)$ and $(\beta_{1,j}, \dots, \beta_{n,j}) = \Omega_j V^*$, then

$$\Omega_j A(c, e) \Omega_j^* = \sum_{i=1}^n \lambda_i |\beta_{i,j}|^2$$

Next, $\alpha_{i,j}$ are samples of a complex Gaussian random variable with mean $E_{\alpha_{i,j}}$. Let $K^j = (E_{\alpha_{1,j}}, \dots, E_{\alpha_{n,j}})$

Since V is unitary, $\{v_1, v_2, \dots, v_n\}$ is an orthonormal basis of $\mathbb{C}^n$ and $\beta_{i,j}$ are independent complex Gaussian random variables with variance 0.5 per dimension and mean $K^j \cdot v_i$. Let $K_{i,j} = |E_{\alpha_{i,j}}|^2 = |K^j \cdot v_i|^2$. Thus $|\beta_{i,j}|$ are independent Rician distributions with PDF

$$p(|\beta_{i,j}|) = 2|\beta_{i,j}| \exp(-|\beta_{i,j}|^2 - K_{i,j}) I_0(2|\beta_{i,j}| \sqrt{K_{i,j}})$$

For $|\beta_{i,j}| \geq 0$, where $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind. Thus to compute an upper bound on the average probability of error, we average

$$\prod_{j=1}^m \exp\left(-\frac{E_s}{4N_0}\right) \sum_{i=1}^n \lambda_i |\beta_{i,j}|^2$$

with respect to independent Rician distributions of $|\beta_{i,j}|$ to arrive at :

$$P(c \rightarrow e) \leq \prod_{j=1}^m \left(\prod_{i=1}^n \frac{1}{1 + \frac{E_s}{4N_0} \lambda_i} \exp\left(-\frac{K_{i,j} \frac{E_s}{4N_0} \lambda_i}{1 + \frac{E_s}{4N_0} \lambda_i}\right) \right)$$

At this point we examine some special cases.

The Case of Rayleigh Fading: In this case, $E_{\alpha_{i,j}}$ and $K_{i,j} = 0$ for all i and j . Then the previous inequality can be written as

$$P(c \rightarrow e) \leq \left(\frac{1}{\prod_{i=1}^n (1 + \frac{\lambda_i E_s}{4N_0})} \right)$$

Let r denote the rank of matrix A , then the kernel of A has dimension $n - r$ and exactly $n - r$ eigenvalues of A are zero.

Say the nonzero eigenvalues of A are $\lambda_1, \lambda_2, \dots, \lambda_r$, then it follows from previous inequality that

$$P(c \rightarrow e) \leq (\prod_{i=1}^r \lambda_i)^{-m} \left(\frac{E_s}{4N_0} \right)^{-rm}$$

Thus a diversity advantage of mr and a coding advantage of $(\lambda_1, \lambda_2, \dots, \lambda_r)^{1/r}$ is achieved. Recall that $\lambda_1, \lambda_2, \dots, \lambda_r$ is the absolute value of the sum of determinants of all the principal $r \times r$ cofactors of A . Moreover, we see that the ranks of $A(c, e)$ and $B(c, e)$ are equal.

We note that the diversity advantage is the power of SNR in the denominator of the expression for the error probability derived above. The coding advantage is an approximate measure of the gain over an uncoded system operating with the same diversity advantage. Therefore, from the above analysis, we arrive at the following design .

Design Criterion for Rayleigh Space-Time Codes:

- The Rank : In order to achieve the maximum diversity mn , the matrix $B(c, e)$ has to be full rank for any codewords c and e . If $B(c, e)$ has minimum rank r over the set of two tuples of distinct codewords, then a diversity of rm is achieved.
- The Determinant : Suppose that a diversity benefit of rm is our target. The minimum of r^{th} roots of the sum of determinants of all $r \times r$ principal cofactors of $A(c, e) = B(c, e)B^*(c, e)$ taken over all pairs of distinct codewords e and

c corresponds to the coding advantage, where r is the rank of $A(c, e)$. Special attention in the design must be paid to this quantity for any codewords e and c . The design target is making this sum as large as possible. If a diversity of nm is the design target, then the minimum of the determinant of $A(c, e)$ taken over all pairs of distinct codewords e and c must be maximized.

We next study the behavior of the right-hand side of the inequality for large signal-to-noise ratios. At sufficiently high signal-to-noise ratios, one can approximate the right-hand side by

$$P(c \rightarrow e) \leq \left(\frac{E_s}{4N_0} \right)^{-rm} (\prod_{i=1}^t \lambda_i)^{-m} \left[\prod_{j=1}^m \prod_{i=1}^r \exp(-K_{i,j}) \right]$$

Thus a diversity rm of and a coding advantage of

$$(\lambda_1, \lambda_2, \dots, \lambda_r)^{1/r} \left[\prod_{j=1}^m \prod_{i=1}^r \exp(-K_{i,j}) \right]^{\frac{1}{rm}}$$

is achieved. Thus the following design criterion is valid for the Rician space-time codes for large signal-to-noise ratios.

Design Criterion for The Rician Space-Time Codes:

- The Rank : This is the same as that given for the Rayleigh channel.
- The Coding Advantage : Let $\Lambda(c, e)$ denote the sum of all the determinants of $r \times r$ principal cofactors of $A(c, e)$, where r is the rank of $A(c, e)$. The minimum of the products

$$\Lambda(c, e)^{1/r} \left[\prod_{j=1}^m \prod_{i=1}^r \exp(-K_{i,j}) \right]^{\frac{1}{rm}}$$

taken over distinct codewords c and e has to be maximized. Note that one could still use the coding advantage, since the performance will be at least as good as the right-hand side of the inequality.

6.5 Rate- Diversity Trade off

Diversity renders the transmission more efficient, but it has certain bound. For example, in spatial multiplexing, the rate can get higher but the diversity is 0. While in the space-time multiplexing, we have a higher diversity but the rate is constant for the codes (in average rate-1). In addition, the SNR can have a role in achieving diversity.

Therefore, two main tradeoffs regarding ST coding for MIMO systems have been identified:

- First is the tradeoff between multiplexing and diversity gains. It states that a given code can achieve the full multiplexing gain and the full diversity gain, but SNR can be used to either increase throughput or reduce probability of error.
- Secondly, there is a tradeoff between diversity gain and the code rate, and indicates that as the code rate goes up, less redundancy and therefore less transmit diversity is possible in code. These tradeoffs identify that for a code to achieve the multiplexing diversity tradeoff frontier, the code rate must be less than $1/n_t$, where n_t is the number of transmit antennas, and the rate R_C vary between 0 and 1. In addition, if a code has a higher rate, then a more tradeoff exists. R_C relate throughput with receiver complexity, since complexity is in general exponentially related to the number of coded bits transmitted per symbol. It has been proven that the highest rate any antenna code can achieve is

$$r_{max} = \frac{n_0 + 1}{2n_0} \quad ; \quad \text{where } n_t = 2n_0 \text{ or } 2n_0 - 1$$

Figure 19 represents rate-diversity trade-off for Gaussian (in red) and QAM (in blue) cases, where the source is transmitting $r \log(\text{SNR})$ bits per channel use.

d versus r

Figure 24: Rate-Diversity trade-off

Such trade-off 's should be maintained in the designing of any code. In this report, trade-off analysis on different types of codes is presented in the simulations section (10.2) with the corresponding Matlab figures.

7 Encoding Schemes

In this part, ST codes are explored for RF and OW communications. So we are going to have a look at many encoding techniques that uses QAM, PAM and other RF modulated symbols in the first part of the chapter. Applying these codes and modulation techniques in OW domain, is inefficient for many cases, that why we present solutions based on unipolar codes in the second part of the chapter to meet the OW transmissions requirements.

7.1 Space-Time Codes for RF Wireless Transmissions

We first begin with Alamouti paper [8], which explore a new scheme for wireless communications that uses two transmit antennas and one receive antenna, providing the same diversity as MRRC (Maximal Ratio Receive Combining), where one transmit and two receive antennas are used. It can be generalized to two transmit antennas and M receive antennas to provide a diversity order of 2M. Therefore, we have the advantage of placing two transmit antennas at the base station, instead of placing two transmit antennas at each of the remote terminals, which is less expensive. Therefore the ST Alamouti block code for this scheme is

$$C(s_1, s_2) = \begin{bmatrix} s_0 & s_1 \\ -s_1^* & s_0 \end{bmatrix}$$

It uses the encoding and transmission sequence of information symbols at both transmitters, and the combining scheme at one receiver, followed by a decision rule for maximum likelihood (ML) detection. At time t , s_0 is sent from antenna 1, s_1 from antenna 1, where s_0 and s_1 are QAM or PAM symbols. At time $(t + T)$, $-s_1^*$ is sent from antenna 0, s_0 is sent from antenna 1. The channel at time t is modeled by a complex multiplicative distortion $h_0(t)$ for transmit antenna 0 and $h_1(t)$ for

transmit antenna 1. We assume that fading is constant and T is the symbol duration, we can write

$$h_0(t) = h_0(t - T) = h_0 = \alpha_0 e^{j\theta_0}$$

$$h_1(t) = h_1(t - T) = h_1 = \alpha_1 e^{j\theta_1}$$

The received signals are then expressed as

$$r_0 = r(t) = h_0 s_0 + h_1 s_1 + n_0$$

$$r_1 = r(t - T) = -h_0 s_1^* + h_1 s_0^* + n_1$$

Where r_0 and r_1 are the received signals at time t and $(t + T)$, and n_0 and n_1 are complex random variables representing receiver noise and interference which are always present in real life cases. The combiner builds the following two combined signals that are further sent to the ML detector

$$\hat{s}_0 = h_0^* r_0 + h_1 r_1^*$$

$$\hat{s}_1 = h_1^* r_0 - h_0 r_1^*$$

Substituting we get

$$\hat{s}_0 = (\alpha_0^2 + \alpha_1^2) s_0 + h_0^* n_0 + h_1 n_1^*$$

$$\hat{s}_1 = (\alpha_0^2 + \alpha_1^2) s_1 - h_0 n_1^* + h_1^* n_0$$

These combined signals are sent to the ML decoder which, for each of the signals s_0 and s_1 , uses the decision rule for PSK signals, which is to choose s_i if and only if

$$d^2(\hat{s}_0, s_i) \leq d^2(\hat{s}_0, s_k)$$

for $s_i \neq s_k$, and same decision for $\hat{s}_1$ as well. This is the RF wireless scheme with one receiver and it can be extended to M receivers leading to more general results leading to a diversity of order $2M$. This is represented by the figure:

Figure 25: two-branch transmit diversity with one receiver

This scheme has many advantages. First, it has a diversity 2, which means it is fully diverse and if is generalized to M receive antennas, the diversity is $2M$. One of its advantages the added reliability due to multiple receive chains. If one of the receive chains fail, and the other don't, then the performance loss is on the order of the diversity gain. In other words, the signal may still be detected, but with bad quality which is called a soft failure. Fortunately, the new transmit diversity scheme provides the same soft failure. To illustrate this, it is assumed that the transmit chain for antenna 1 in the figure is disabled, i.e., $h_1 = 0$. Therefore, the received signals becomes

$$r_0 = h_0 s_0 + n_0$$

$$r_1 = -h_0 s_1^* + n_1$$

The combiner shown in the figure builds the following two combined signals:

$$\hat{s}_0 = \alpha_0^2 s_0 + h_0^* n_0$$

$$\hat{s}_1 = \alpha_0^2 s_1 - h_0 n_1^*$$

These combined signals are the same as if there was no diversity. Therefore, the diversity gain is lost but the signal may be detected. For the scheme with two transmit and two receive antennas, both the transmit and receive chains are protected by this redundancy scheme.

The scheme needs less power, less delay, and is sensitive to channel estimation errors. It also needs good antenna configurations, separated by ten wavelength at the base station, and 3 wavelength apart at the remote stations, to provide diversity at both the base and remote stations. Simulations were done to show the Bit Error Rate BER performance of un-coded coherent BPSK for MRRC and the new transmit diversity scheme in Rayleigh fading (figure 26).

Figure 26: BER performance comparison of coherent BPSK with MRRC and two-branch transmit diversity in Rayleigh fading

The performance of the new scheme with two transmitters and a one receiver is 3 dB worse than two-branch MRRC because the simulations assume that each transmit antenna radiates half the energy in order to ensure the same total radiated power as with one transmit antenna. If each transmit antenna in the new scheme was to radiate the same energy as the single transmit antenna for MRRC, however, the performance would be identical. In other words, if the BER was plotted against the average SNR per transmit antenna, then the performance curves for the new scheme shifts 3 dB to the left and overlaps with the MRRC curves. Even with the equal total radiated power assumption, the diversity gain for the new scheme with one receive antenna at a BER of 10 is about 15 dB. Similarly, assuming equal total radiated power, the diversity gain of the new scheme with two receive antennas at a BER of 10 is approximately 24 dB, which is 3 dB worse than MRRC with one transmit and four receive antennas.

Problem formulation

ST coding can be a possible solution for solving the last mile problem using FSO links since spatial diversity can combat the atmospheric turbulence that degrades the performance of such systems. The previous Alamouti code is an example of hundreds of STC's applied in wireless communications. These codes are mainly considered with QAM, PAM and PSK modulation schemes. Such techniques introduce phase, polarity or amplitude modifications, for example in Alamouti code both the signal and its negative are transmitted. In optical communication systems employing direct detection at the receiver, intensity modulations such as on-off keying (OOK) or pulse-position modulation (PPM) are commonly used to convey the information. Applying alamouti code with unipolar modulation is inefficient because it necessitates to transmit the negative of symbols.

Therefore, a modification is necessary for the RF Alamouti code, in order to be used with such unipolar modulations, preserving the same desirable properties as the conventional code, but does not rely on the necessity of transmitting the negative of a signal. Such modifications are needed in order to apply the coding function in FSO field before transmitting any signal.

So now there is a growing interest in applying the ST techniques with unipolar OOK and PPM. But since OOK is limited and encodes 1 bit only, PPM is adopted in general because it encodes up to M message bits for arbitrary modulation positions M while it remains power-efficient for most cases. This modulation scheme found applications in Impulse-Radio Ultra-wideband (IR-UWB) communications where the control the amplitude and the phase of the very low duty-cycle sub-nanosecond pulses is getting complicated. PPM is also popular in FSO communication with direct detection where information can be only conveyed by the presence or absence of light pulses.

Therefore, Alamouti code, considered mainly with QAM and PSK constellations, is tailored to binary PPM, and then extended to M -ary PPM. But are these space-time codes orthogonal or they breaks down the orthogonality between the transmitted data streams? In fact, orthogonal ST codes are interesting because of their simple decoding strategy, and should be taken into consideration.

However, some ST block codes may result in an additional constellation extension when associated with PPM. For example, the constellations in the Alamouti paper we presented at the beginning, the code does not introduce a constellation extension since s^* (the complex conjugate of s) and $-s$ are both QAM (resp. PAM) symbols whenever s is a QAM (resp. PAM) symbol. Therefore, Alamouti does not introduce

any constellation extension, and the code is shape preserving in this case. But when it is the case of optical communications, do the codes still shape preserving for any value of M knowing that phase rotations or amplitude modifications are not admissible with PPM constellations?

And will the codes succeed in achieving full transmit diversity if these additional constellations are not introduced?

how should the M -ary PPM constellation be modeled to ensures the full diversity of the codes as well as the shape-preserving property?

Can we achieve the full diversity for any number of transmitters n and modulation positions M ?

Many papers presenting different codes and schemes were done to solve the previous problems and to answer some or all of these interrogations. Most importantly, in paper 1, Simon and Vilnrotter modified the original Alamouti code proposed for RF wireless applications, in order to allow it to be applied in scenarios such as FSO communication with direct detection where unipolar modulations like pulse-position modulation and ON-OFF keying are used to convey the information. The code was modified such as to maintain all of the desirable properties of the original scheme.

We take a look also at orthogonal space time (ST) codes for binary PPM in paper 2 for Abou-Rjeily, see [10], where he proposed schemes that verify the additional constraint of achieving a full transmit diversity order, without introducing any phase rotations, but preserving the code's orthogonality. This renders the proposed codes suitable for FSO communications with direct detection and for UWB

communications. At the receiver side, optimal detection can be achieved with linear operations because the orthogonality of the transmitted input.

Another paper for Abou-rjeily and Belfiore [11], proves the full diversity 2×2 ST code with M-ary PPM constellations for all values of M (not only for M= 2). The M-dimensional constellation is represented by

$$C_{PPM} = \{e_m ; m = 1, \dots, M\}$$

Each information symbol is represented by a M-dimensional vector belonging to this constellation, where e_m is the m -th column of the $M \times M$ identity matrix I_M .

As a continuation to this work, paper [12] for Abou-rjeily and Fawaz, present a new codeword that achieve full transmit diversity for many values of transmitters n and modulation positions M . for a given set of values (n,M) , this family of codes achieves a rate of 1 symbol per channel use (PCU) which is the highest possible achievable rate when no constellation extensions are introduced.

7.2 Space-Time Codes for Optical Transmissions

A first solution is a modification for Alamouti code, see [9], and after modification, the signals are described by the following block code:

$$\begin{bmatrix} s_0 & s_1 \\ \bar{s}_1 & s_0 \end{bmatrix}$$

Where $s_i \in \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$ for $i = 0,1$

And the bar notation denotes the complement of the signal, i.e. for $s_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$,

$\bar{s}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and the reverse is true. In the case of OOK

$$s_1 = 0, \quad 0 \leq t \leq T$$

$$s_2 = A, \quad 0 \leq t \leq T$$

Where A is a positive constant related to the intensity of the light and t is the signal duration. In the case of binary PPM, the signals are expressed as

$$s_1 = \begin{cases} 0, & 0 \leq t \leq T/2 \\ A, & T/2 \leq t \leq T \end{cases}$$

$$s_2 = \begin{cases} A, & 0 \leq t \leq T/2 \\ 0, & T/2 \leq t \leq T \end{cases}$$

For either modulation, we can express one of these waveforms in terms of the other by property (1):

$$s_i = -s_j + A \text{ for } j \neq i$$

Mathematically, the complement of a signal will be:

$$\text{If } x_i = s_1, \text{ then } \overline{x_i} \triangleq s_2 = -s_1 + A = -x_i + A$$

Likewise

$$\text{If } x_i = s_2, \text{ then } \overline{x_i} \triangleq s_1 = -s_2 + A = -x_i + A$$

Therefore, using the 2-PPM constellation, the signals look like:

Figure 27: the modified Alamouti Code

In this figure, a pulse is sent in the first bit interval from antenna 1 at time 1 to represent $s_0 = 0$, a pulse in the first bit interval at time 2 to represent $s_1 = 0$. From

antenna 2, a pulse is sent in the second bit interval, at time 1, to represent $\overline{s_1} = 1$, and a pulse at the first bit interval, at time 2, to represent $s_0 = 0$.

By similarity with the conventional Alamouti code, the received signal in the first bit and second bit intervals are given by:

$$y_1 = h_{11}x_1 + h_{12}x_2 + n_1$$

$$y_2 = h_{11}x_2' + h_{12}x_1 + n_2$$

Or equivalently

$$y_1 = h_{11}x_1 + h_{12}x_2 + n_1$$

$$y_2 - h_{11}A = -h_{11}x_2 + h_{12}x_1 + n_2$$

Assuming perfect knowledge of the channel gains and the transmitted signal level, the receiver uses the sequence y_1, y_2 to construct the input waveforms:

$$\widetilde{x}_1 = h_{11}y_1 + h_{12}y_2 - h_{11}h_{12}A$$

$$\widetilde{x}_2 = h_{12}y_1 - h_{11}y_2 + h_{11}^2A$$

Finally, we get:

$$\widetilde{x}_1 = (h_{11}^2 + h_{12}^2)x_1 + N_1$$

$$\widetilde{x}_2 = (h_{11}^2 + h_{12}^2)x_2 + N_2$$

Therefore, each symbol can be detected separately, with a diversity of order 2 (because 2 antennas are sending in the space) using ML decision.

As the conventional Alamouti code, this modified code is generalized to L_r receive antennas leading to similar results

$$y_l = h_{11}x_1 + h_{12}x_2 + n_l$$

$$y_{l+Lr} = h_{11}x_2' + h_{12}x_1 + n_{l+Lr}$$

Where y_l for $l = 1, \dots, L_r$ represents the signals received by the L_r received antennas in the first bit interval, and y_{l+L_r} for $l=1,2,\dots, L_r$ represents the signals received by the L_r received antennas in the second bit interval. At the receiver, we form

$$\widetilde{x}_1 = \sum_{l=1}^{L_r} h_{l1} y_l + \sum_{l=1}^{L_r} h_{l2} y_{l+L_r} - A \sum_{l=1}^{L_r} h_{l1} h_{l2}$$

$$\widetilde{x}_2 = \sum_{l=1}^{L_r} h_{l2} y_l - \sum_{l=1}^{L_r} h_{l1} y_{l+L_r} + A \sum_{l=1}^{L_r} h_{l1}^2$$

Substituting and making use of the property (1) results in

$$\widetilde{x}_1 = \sum_{l=1}^{L_r} (h_{l1}^2 + h_{l2}^2) x_1 + N_1$$

$$\widetilde{x}_2 = \sum_{l=1}^{L_r} (h_{l1}^2 + h_{l2}^2) x_2 + N_2$$

Which are the identical statistics achieved by the conventional Alamouti code and prove the achievement of a diversity of order $2L_r$ as well as the ability to detect each symbol separately.

In the second solution, see [10], the codeword is a 2-PPM 2-dimensional constellation given by

$$C = \{[1 \ 0]^T; [0 \ 1]^T\}$$

where the scalar 1 indicates the presence of a light waveform similarly to the previous code. The poisson arrivals are approximated by Gaussian noise leading to a linear dependence between the input and output MIMO channel represented by:

$$X = HC + N$$

For $n=2,4$ the codewords structure are:

$$C(s_1, s_2) = \begin{bmatrix} s_1 & s_2 \\ \Omega s_2 & s_1 \end{bmatrix}$$

$$C(s_1, s_2, s_3, s_4) = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ \Omega s_2 & s_1 & \Omega s_4 & s_3 \\ \Omega s_3 & s_4 & s_1 & \Omega s_2 \\ \Omega s_4 & \Omega s_3 & s_2 & s_1 \end{bmatrix}$$

Where $\Omega s = -s + 1_2$ and replaces the bar notation in the previous code meaning the complement of a code. For any $S \in C$ is a pulse permutation related directly to the signal compliment. And $\Omega s \in C$ whenever $S \in C$ and the code is shape preserving. Therefore C becomes

$$C(s_1, \dots, s_n) = C'(s_1, \dots, s_{n'}) + C_0$$

$$C_0 = \begin{bmatrix} 0_2 & 0_2 \\ 1_2 & 0_2 \end{bmatrix}$$

For 2 symbols, and for 4 symbols it is the following

$$C_0 = \begin{bmatrix} 0_2 & 0_2 & 0_2 & 0_2 \\ 1_2 & 0_2 & 1_2 & 0_2 \\ 1_2 & 0_2 & 0_2 & 1_2 \\ 1_2 & 1_2 & 0_2 & 0_2 \end{bmatrix}$$

These codewords achieve full transmit diversity because $C(s_1, \dots, s_n) - C'(s'_1, \dots, s'_{n'})$ has full rank for $(s_1, \dots, s_n) \neq (s'_1, \dots, s'_{n'})$ i.e. full transmit diversity is achieved if the $(2n \times n')$ matrix $C'(a_1, \dots, a_{n'})$ has full rank for $(a_1, \dots, a_{n'}) \in A^{n'} \setminus \{(0_2, \dots, 0_2)\}$ where A denotes the set of all possible differences between two information vectors:

$$A = \{s - s' ; s, s' \in C\} = \{[0 \ 0]^T, [1 \ -1]^T, [-1 \ 1]^T\}$$

The rank of a matrix does not change when permuting its rows, then $C'(a_1, \dots, a_{n'})$ has the same rank as the matrix:

$$C''(a_1, \dots, a_{n'}) = [(C'_1(a_1, \dots, a_{n'}))^T (C'_2(a_1, \dots, a_{n'}))^T]^T$$

Where $C'_i(a_1, \dots, a_{n'})$ is the $n \times n'$ matrix composed from the rows of $C'(a_1, \dots, a_{n'})$ having odd (resp. even) indices for $i = 1$ (resp. $i = 2$). From previous equations, an element $a_i \in A$ can be written as $a_i = a'_i [1 \ -1]^T$ with $a'_i \in \{0, \pm 1\}$ for $i = 1, \dots, n'$. Consequently:

$$C'_i(a_1, \dots, a_{n'}) = (-1)^{i-1} C_{\text{orth}}(a'_1, \dots, a'_{n'})$$

Where $C_{\text{orth}}(a'_1, \dots, a'_{n'})$ corresponds to the $n \times n'$ real orthogonal codeword and associated with the scalars $a'_1, \dots, a'_{n'}$. Consequently, $\text{rank}(C_{\text{orth}}(a'_1, \dots, a'_{n'})) = n$ unless when $a'_i = 0$ for $i = 1, \dots, n'$. Therefore, $C''(a_1, \dots, a_{n'})$ will have a full row rank unless when $a_i = 0_2$ for $i = 1, \dots, n'$. As a conclusion, all the non-zero 2-dimensional vectors that result in a rank-deficient matrix $C(a_1, \dots, a_{n'})$ do not belong to the set A and the proposed code achieves a full transmit diversity order. The code can be readily modified in order to be applied with the 1-dimensional OOK constellation given by $C = \{[0, 1]\}$ where $s_1, \dots, s_{n'} \in C$ are scalars, while Ωs_i must be replaced by $-s_i + 1$ for $i = 1, \dots, n'$. In fact, both families of codes can be applied with 2, 4, and 8 transmitters without introducing any extension to the 2-PPM or OOK constellations.

In the third solution, paper [11], the proposed code is constructed as follows. The $2M \times 2$ codeword has the form

$$C = \begin{bmatrix} s_1 & s_2 \\ \Omega s_2 & s_1 \end{bmatrix}$$

where Ω is a $M \times M$ cyclic permutation described by

$$\Omega = \begin{bmatrix} 0_{1 \times (M-1)} & 1 \\ I_{(M-1)} & 0_{(M-1) \times 1} \end{bmatrix} = \begin{bmatrix} 0 & \dots & 0 & 1 \\ 1 & 0 & \dots & 0 \\ & \ddots & \ddots & \\ 0 & \dots & 0 & 1 \end{bmatrix}$$

And s_1, s_2 are the M -dimensional vector representations of the information symbols. For PPM constellations, $\Omega s \in C_{PPM}$ whenever $s_1, s_2 \in C_{PPM}$ and the code is shape preserving with PPM where $C_{PPM} = \{e_m ; m = 1, \dots, M\}$ and e_m is the m -th column of the $M \times M$ identity matrix.

Exploring the 4-PPM case, we see the role of the cyclic permutation matrix Ω or π . In fact, as its name shows, each pulse after being multiplied by Ω occupies the next time slot, but the pulse in the last time slot occupies the first one, in a cyclic way as the figure shows:

Figure 28: Permutation in a 4-PPM Constellation

We want to show that the proposed code permits to achieve full transmit diversity with M-PPM-M'PPM constellations for $M \geq 3$ and M' and with M-PPM constellations for $M \geq 2$.

$$\Delta C(a_1, a_2) = \Delta C(s_1 - s'_1, s_2 - s'_2) = \begin{bmatrix} a_{1,1} & \dots & a_{1,M} & a_{1,\pi(1)} & \dots & a_{2,\pi(M)} \\ a_{2,1} & \dots & a_{2,M} & a_{1,1} & \dots & a_{1,M} \end{bmatrix}^T$$

where $\pi(\cdot)$ the permutation matrix is given by

$\pi(i) = (i - 2) \bmod (M + 1)$ $a_{i,M}$ is the M -th component of the vector a_i for $i = 1, 2$ and $m = 1, \dots, M$. a_1 and a_2 belong to the set A that denotes the set of all differences between any two information vectors:

$$A = \{ s - s'; s, s' \in C_{PPM} \}$$

The proposed code is fully diverse if the matrix ΔC has full rank for $(a_1, a_2) \in A^2 \setminus \{0_M, 0_M\}$ where 0_M stands for the M -dimensional all-zero vector i.e. the code is fully diverse for a given value of M if all the non-zero M -dimensional vectors that result in a rank-deficient matrix $C(a_1, a_2)$ do not belong to A .

as can be seen, elements of A can have a maximum number of two non-zero components. The transmit diversity order is achieved because of this particular structure of A . supposed that $C(a_1, a_2)$ is a rank deficient matrix if there exists a non-zero rational number k such that $C_1(a_1, a_2) = KC_2(a_1, a_2)$ where $C_i(a_1, a_2)$ stands for the i -th column of $C(a_1, a_2)$. Therefore, we are going to have the following equation:

$$\frac{a_{2,1}}{a_{1,1}} = \dots = \frac{a_{2,M}}{a_{1,M}} = \frac{a_{1,1}}{a_{2,\pi(1)}} = \dots = \frac{a_{1,M}}{a_{2,\pi(M)}} = k$$

after some manipulations, $a_{2,M} = k^{2i} a_{2,M-i}$; $i = 1, \dots, M-1$

Since $k \neq 0$, all the components of a_2 must be zero or different from zero simultaneously. Since the elements of A can have a maximum number of two non-zero components then for $M \geq 3$ at least one component of a_2 is zero. Consequently, $a_2 = 0_M$ when $a_2 \in A$ and $M \geq 3$. $a_2 = 0_M$ implies that $\text{rank } C(a_1, a_2) = 2$ unless when $a_{1,1} = \dots = a_{1,M} = 0$. Therefore, the only rank-deficient matrix $C(a_1, a_2)$ associated with two elements a_1, a_2 of A is the all-zero matrix. Since the above proof is independent from the value of M' , then code given permits to achieve a transmit diversity order of 2 when associated with M -PPM- M' -PAM constellations for $M \geq 3$ and for all values of M' . In particular, a full transmit diversity order can be achieved for $M \geq 3$ with M -PPM.

Considering the case of M-PPM with $M = 2$, $a_{2,2} = k^2 a_{2,1}$. For $a_{2,1} = 0$, then a similar proof for M-PPM- M' -PAM can be applied implying that all the non-zero matrices $C(a_1, a_2)$ have a full rank. On the other hand, one of the components of a_2 must be equal to +1 while the other must be -1 when $a_2 \neq 0_M$ is an element of A . Consequently, the relation $a_{2,2} = k^2 a_{2,1}$ will imply that $k^2 = -1$ which is not possible given that all the entries of the different codewords are real-valued. Consequently, the code is fully diverse with M-PPM for $M = 2$ in addition to $M \geq 3$. Therefore, we conclude that this code is a rate-1 code, shape preserving and fully diverse.

The fourth solution [12] is an extension to the third one, where for M-PPM with $n = P$ transmit antennas, the minimal delay codewords are represented by the matrices of dimensions $nM \times n$ having the following structure

$$C(s_1, \dots, s_n) = \begin{bmatrix} s_1 & s_2 & \dots & s_n \\ \Omega s_n & s_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & s_2 \\ \Omega s_2 & \dots & \Omega s_n & s_1 \end{bmatrix}$$

where the symbols and permutation matrix are the same as before.

The case of $n = 2$ was explored in the previous case. Consider now the case $n > 2$,

C is rank-deficient if there exists $n-1$ rational numbers $k_1, \dots, k_n$ such that

$$C_n = \sum_{i=1}^{n-1} k_i C_i$$

After some manipulations

$$[C(-1, k_1, \dots, k_n)]^T [a_n^T \dots a_1^T]^T = 0_M$$

Where Ω and a_i are considered as parameters rather than a $(M \times M)$ matrix. The equation is verified if $\det(R)a_i = 0_M \quad ; \quad i = 1, \dots, n$

Where $R = [C(-1, k_1, \dots, k_n)]^T$

And when calculating the determinant, R is considered as $n \times n$ matrix. In this case, $a_i \in A$ for $i = 1, \dots, n$

Since Ω is contained in the upper triangular part of R , then $\det(R)$ is a polynomial of degree $n - 1$ in Ω . Therefore, the linear dependence between the columns of the codeword implies the following relation

$$(\sum_{i=0}^{n-1} \lambda_i \Omega^i) a_i = 0_M ; i = 1, \dots, n$$

Following from the structure of R , it can be easily shown that for all values of n . in what follows we take the case $M \geq n$, the equation becomes :

$$MX = 0_M ; X = a_1, \dots, a_n$$

$$M = \sum_{i=0}^{M-1} \lambda_i \Omega^i ; \lambda_i = 0 \text{ for } i = n, \dots, M-1$$

Being a function of the rational numbers $k_1, \dots, k_n$, then λ_i is a rational number for all values of i . For $n > 2$, the proposed code is fully diverse with M -PPM if M verifies at least one of the three propositions stated in what follows:

1) $\varphi(M) \geq n$ for all values of M and $\varphi(.)$ stands for Euler's totient function

2) $n \leq M - d(M)$ if M is odd or if M is multiple of 4 and

$n \leq M - d(M) + 1$ if M is an even number that is not a multiple of 4

Where $d(M)$ stands for the biggest integer divisor of M in the set $\{1, \dots, M-1\}$

3) $n = M$ for prime values of M

Finally this code is a rate-1 code (transmitting n symbols in n symbol durations) and is fully diverse (the difference of any 2 codes is full rank), minimal delay (no more than $T = n$ time slots for n symbols) for the values given in the following table.

Table 1: normalized rates achieved by the code

	$M = 2$	$M = 3$	$M = 4$	$M = 5$	$M = 6$	$M = 7$	$M = 8$	$M = 9$	$M = 10$	$M = 11$	$M = 12$
$n = 2$	1	1	1	1	1	1	1	1	1	1	1
$n = 3$	-	1	0.8617	1	1	1	1	1	1	1	1
$n = 4$	-	-	0.8444	1	1	1	1	1	1	1	1
$n = 5$	-	-	-	1	-	1	0.9486	1	1	1	1
$n = 6$	-	-	-	-	-	1	0.9465	1	1	1	1
$n = 7$	-	-	-	-	-	1	0.9450	-	0.9608	1	0.97
$n = 8$	-	-	-	-	-	-	0.9438	-	0.96	1	0.9694

$R = 1$ implies that the code is a rate-1 code, while $R < 1$ implies that another family of code must be applied because the code is not fully diverse for these values, but this new family of code is not a rate-1 code. When no value of R is given, this means that the proposed code cannot be applied for these values of (n, M) because full diversity cannot be achieved.

Problem Formulation

We notice from the empty spaces in the previous table, as well as from the numbers which are less than 1, that the proposed code is not fully diverse for all (n, M) values, which means no code exists for these systems. But this is needed in practical life, and our purpose is to find codewords for all values of n transmitters and M modulation positions.

8 Decoding Schemes

Many decoding schemes exist in the literature that work on QAM, PAM codes in the RF wireless domain. In optical communications, these schemes cannot be applied, since the decoding is based on polarity or amplitude detection, both undetected by the photodetector receiver. Therefore, a PPM decoder is presented based on 2-PPM modulation positions. This scheme can be applied in the optical domain because it uses the PPM concept, where symbols are modulated by the time delays of the transmitted pulses, all having the same amplitude and positive polarity.

Problem Formulation

The main idea behind decoding is to find X in the system model equation

$$y = Hx + n$$

In fact, x is the information input, H represents the channel, n models the noise, and Y represents the received output. So the maximum likelihood (ML) decoder consists of minimizing the noise term to find the input:

$$\hat{x} = \operatorname{argmin}_{x \in \mathcal{C}} \|y - Hx\|^2$$

Therefore, at the receiver, we need to find X from Y , and this is done by multiplying Y by H^{-1} to get the input as the following:

$$y = Hx + n$$

$$z = H^{-1}y = x + H^{-1}n$$

So our purpose is to minimize the noise term $|H^{-1}n|$ i.e. minimizing the term $|z - x|$. This is known as a zero-forcing method. We are going to see next the

decoding techniques that are trying to achieve this goal. We begin by PAM, QAM decoding techniques followed by PPM decoding techniques.

8.1 QAM & PAM Decoders

We apply the maximum likelihood (ML) lattice sphere decoder to the received point $y = x + v$ to determine the nearest point to y . Therefore, $|y - x|$ should be minimized. The main idea of this decoder is to enumerate lattice points inside a hypersphere centered on the received point y . Then we reduce the search radius each time a new point is found. This will decrease the number of points while ensuring the closest point criterion. If no point is found, the radius is enlarged to find new points. This is presented by the following figure:

Figure 29: Sphere Decoder for Multiple Antenna Systems

Two non-trivial lattice decoding strategies are introduced to achieve our purpose:

- The sphere decoder based on Pohst strategy (called VB), to enumerate lattice points inside the sphere. The search complexity is sensitive to the choice of initial radius.
- the sphere decoder based on Schnorr-Euchner strategy (called AEVZ), to view the lattice as laminated hyperplanes. The radius limit the search region to a sphere. AEVZ complexity is insensitive to the choice of the initial radius.

Both strategies can be accelerated by a factor up to 10, if M is reduced via KZ or LLL algorithms. They have a main drawback if the lattice constellation is finite. In

fact LLL and KZ convert the constellation cube into a parallelotope when the modulation assumed is QAM with boundaries difficult to determine. In addition, practically, the complexity decrease obtained by taking into consideration the finite constellation is more profitable than applying LLL and KZ algorithms. On multiple antenna channels, VB and AEVZ complexities are similar at moderate and high SNR but AEVZ shows a speed gain over VB by a factor varying from 1 up to 4 at low SNR. We propose a modified version of the AEVZ sphere decoder that takes into consideration the boundaries of the finite QAM constellation, without including any lattice reduction. Here is the accelerated sphere decoder algorithm:

Input : a received point y , the generator matrix M ($n_s \times n_s$) of the lattice, the radius R of the sphere, and the bound $z_{\min}$ and $z_{\max}$ of the constellation. The radius R can be set to $+\infty$. a gain of speed 30% at most can be obtained if R is linked to the Gaussian noise variance N_0 or to the minimum distance $D_{\text{Emin}}(\Lambda)$.

Output : the ML point $z_{M \text{ L}}$ belonging to the constellation, and its squared Euclidean distance to y .

Step 1: (Pre-processing) compute the Gram matrix $G = M M^T$ and do a Cholesky decomposition $G = V V^T$, where V is lower triangular. Compute the inverse $V^T = V^{-1}$

Step 2: (Initialization) Set $\text{bestdist} = R^2$, $k = n_s$, $\text{dist}_k = 0$, $e_k = y V^T$, $z_{kk} = [e_{kk}]$, $z_k = \max(z_k, z_{\min})$, $z_k = \min(z_k, z_{\max})$, compute $\rho = (e_{kk} - z_k) / (V^T_{kk})$, $\text{step}_K = \text{sign}(\rho)$

Step 3: Compute $\text{newdist} = \text{dist}_k + \rho^2$. if $\text{newdist} < \text{bestdist}$ and $k \neq 1$ then go to step 4 else go to step 5 endif

Step 4: Compute for $i = 1, \dots, k-1$

$e_{k-1,i} = e_{k,i} - \rho V_{ki}^1$, decrement k , set $\text{dist}_k = \text{newdist}$, $z_k = [e_{kk}]$, $z_k = \max(z_k, z_{\min})$,

$z_k = \min(z_k, z_{\max})$, compute $\rho = (e_{kk} - z_k) / (V_{kk}^1)$, $\text{step}_k = \text{sign}(\rho)$ go to step 3

Step 5: if $\text{newdist} < \text{bestdist}$ then set $\check{z} = z$, $\text{bestdist} = \text{newdist}$,

Else if $k = n$ then return $\check{z}$ and terminate,

Else increment k , endif.

Compute $z_k = z_k + \text{step}_k$, if $z_k < z_{\min}$ or $z_k > z_{\max}$ then $\text{step}_k = -\text{step}_k - \text{sign}(\text{step}_k)$, $z_k = z_k + \text{step}_k$ endif

if $z_k < z_{\min}$ or $z_k > z_{\max}$ then go to step 5 endif

$\rho = (e_{kk} - z_k) / (V_{kk}^1)$, $\text{step}_k = -\text{step}_k - \text{sign}(\text{step}_k)$ go to step 3

8.2 PPM Decoders

For the maximum likelihood (ML) decoding algorithm, the equation of the system model is $Y = HC + N$ which can be written as

$$y = \text{vec}(Y) = I_n \times H \Phi [s_1^T \dots s_n^T]^T + \text{vec}(N) \equiv \hat{H}s + n$$

because we are dealing with the 2-PPM case and P antennas, s is composed of P layers having 2 dimensions or sub-components each. This decoder is based on the enumeration and closest point strategy found in QAM that consist of minimizing the squared Euclidean distance, with two new additions to render it suitable for 2-PPM decoding scheme. First, a multidimensional zero-forcing decision feedback equalization (ZF-DFE) is be applied to determine the order with which the two components of each layer must be tested. Secondly, a multidimensional applied to

the different layers in upward and downward directions to determine which layer the decoder must examines.

The previous equation can be written as $z=Q^T y = Rs + Q^T n$. Q is a unitary and R an upper triangular matrix, with positive diagonal elements, obtained by applying QR decomposition on the matrix H . Let $V = R^{-1}$ and $X(p, p')$ is the $M \times M$ matrix composed of the elements X_{ij} of the $PM \times PM$ matrix X for $i = (p - 1)M + 1, \dots, pM$ and $j = (p' - 1)M + 1, \dots, p'M$ for $p, p' = 1, \dots, P$. The m -th column of X will be denoted by $X_{:,m}$.

$e(k)$ the 2-dimensional vector obtained by applying ZF on the layers $1, \dots, K-1, K+1, \dots, P$ and by taking hard decisions on the layers $K+1, \dots, P$. The influence of the K -th layer on the distance between the received decision vector and the candidate output of the algorithm when a pulse is transmitted at the m -th time slot is given by:

$$d_k^2 = \| R^{(k,k)} (e^{(k)} - I_{:,m}) \|^2 = \| E^{(k)} - R^{(k,k)}_{:,m} \|^2$$

where $E^{(k)} = R^{(k,k)} e^{(k)}$ and $I_{:,m}$ corresponds to the m -th column of the 2×2 identity matrix I . Therefore at the K -th layer, the algorithm must first test the symbol corresponding to the modulation position given by:

$$\text{argmin}_{m=1,2} \{ -2 (R^{(k,k)}_{:,m})^T E^{(k)} + (R^{(k,k)}_{:,m})^T R^{(k,k)}_{:,m} \}$$

ρ is a 2-dimensional vectors, e and E are $2P \times P$ matrices and $X(p)$ corresponds to the elements $2(p - 1) + 1, \dots, 2p$ of the $2P$ -dimensional vector X for $p=1, \dots, P$. The ML decoding algorithm works as follows:

Input: $z, R, P, C, (m=2)$

Output: $\hat{s}$

Step 1: Set $k = P+1$, $\text{dist}(K) = 0$, $V = R^{-1}$, $\rho = 0_2$, $f = 0_P$, $\text{bestdist} = C$ (sphere squared radius)

Step 2: $\text{newdist} = \text{dist}(k) + \rho^T \rho$

if ($\text{newdist} < \text{bestdist}$) & ($K \neq 1$) go to Step 3

else go to Step 4 endif

Step 3: if $K = P+1$, $e_{:,k-1} = Vz$ else

For $i = 1, \dots, k-1$

$e_{:,k-1}^{(i)} = e_{:,k}^{(i)} - V^{(i,k)} \rho$ endfor endif

$K = K - 1$

$\text{dist}(K) = \text{newdist}$

$E_{:,k}^{(k)} = R^{(k,k)} e_{:,k}^{(k)}$

$\text{pos}(k) = \text{argmin}_{m=1,2} \{ -2 (R^{(k,k)}_{:,m})^T E^{(k)} + (R^{(k,k)}_{:,m})^T R^{(k,k)}_{:,m} \}$

$x_{:,k} = I_{:, \text{pos}(k)}$

$\rho = E_{:,k}^{(k)} - R^{(k,k)}_{:, \text{pos}(k)}$

go to Step 2

Step 4: if $\text{newdist} < \text{bestdist}$,

for $i=1, \dots, P$, $\hat{s} = x_{:,i}$

endfor, $\text{bestdist} = \text{newdist}$

elseif $k=P$ terminate

else $k = k+1$ endif.

If $f(k) = 1$ go to step 4 endif,

$\text{Pos}(k) = \text{pos}(k) \bmod 2 + 1$

$X_{:,k} = I_{:, \text{pos}(k)}, f(k) = 1, \rho = E^{(k)}_{:,k} - R^{(k,k)}_{:, \text{pos}(k)}$ go to Step 2

Problem Formulation

The main limitation of this scheme is that it works on 2-PPM modulated symbols only. So we need a new decoder that works in any system in optical communications, where multi-dimensional codes, having arbitrary number of time slots or modulation positions M are used. In chapter 10, we present the new decoding scheme that meets these requirements.

9 New Proposed Code

We begin our analysis by presenting spatial codes and multiplexing schemes, which are simple, then we explore ST codes which have more complex coding but ensure reliable transmissions. Finally, we present our new ST code, which is suitable for systems having any numbers of antennas and modulation positions.

9.1 System Model

The system is represented by $Y = HC + N$, where we have n_t transmitters, n_r receivers, equal n in case they have the same dimension, M is the number of modulation positions, and T is the number of columns in the matrix which represent the time multiplexing and diversity of the ST code, in other words, it is the number of symbol durations of the information symbols. In the case of spatial multiplexing only, T is 1 meaning the absence of time multiplexing. H is the $(n_r.M \times n_t.M)$ matrix, representing the channel coefficients. It can be any of the exponential, Rayleigh or log-normal channels seen in previous section, with variance 1 and it models the real-life transmission channels. It does not have a relation with spatial or space-time multiplexing because it transmits the code whatever was the encoding at the transmitter. C represents the information code to be transmitted and it is an $(n_t.M \times T)$ matrix. The information symbols C transmits $(s_1, \dots, s_n) \in C_{PPM}$. In spacial codes, the special case of ST codes, T is 1 meaning that no other versions of the symbols is sent. N models the random noise and interference in any transmission system, which are present in the real-life cases and cannot be ignored. It is an $(n_r.M \times T)$ matrix verifying the equation dimensions.

9.2 Spatial Codes

In this part, $Y = HC + N$ is presented in details in terms of spatial multiplexing. In this equation H is obtained from the initial matrix h by $H = h \otimes I_M$

And h is the $(n_r \times n_t)$ matrix that is multiplied by the $M \times M$ identity matrix I_M (by means of the chronical product), to give the channel coefficient matrix H :

$$\begin{bmatrix} h_{11} & & 0 & h_{1n} & & 0 \\ & \ddots & & \dots & \ddots & \\ 0 & & h_{11} & 0 & & h_{1n} \\ h_{n1} & & 0 & h_{nn} & & 0 \\ & \ddots & & \dots & \ddots & \\ 0 & & h_{n1} & 0 & & h_{nn} \end{bmatrix} = \begin{bmatrix} h_{11} & \dots & h_{1n} \\ \vdots & \ddots & \vdots \\ h_{n1} & \dots & h_{nn} \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & 1 \end{bmatrix}$$

The M - dimensional symbols $(s_1, \dots, s_n)$ are represented by $\begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}$, where no time

multiplexing is used for simplicity of the codeword (i.e. $T=1$). For some cases, time multiplexing could be needed for more efficient transmission, but leads to a more complex decoding, unless code orthogonality is preserved.

The noise terms are obtained randomly by random functions and then multiplied by the noise variance term:

$$\text{Noise-variance} = n_t \times 10^{\frac{SNR}{10}} / n_t \times \log_2(M)$$

Where SNR is the signal-to-noise ratio that we vary during our simulation from 0 to 35 during our next simulations. Therefore, we finally get:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} h_{11} & 0 & h_{1n} & 0 \\ & \ddots & \dots & \ddots \\ 0 & h_{11} & 0 & h_{1n} \\ h_{n1} & 0 & h_{nn} & 0 \\ & \ddots & \dots & \ddots \\ 0 & h_{n1} & 0 & h_{nn} \end{bmatrix} \times \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_n \end{bmatrix}$$

This codeword is simple to encode and decode, while the next ST code is more complex due to time coding, but provide more reliable transmissions. We note here the rate-diversity trade-off, where we don't have diversity but the rate is greater than 1, and keep on increasing proportionally with the number of antennas in the transmission space.

9.3 Space-time (ST) Codes

The same formula is explored in details in this case but with time multiplexing

$$\text{added to the encoding scheme. The codeword } C(s_1, \dots, s_n) = \begin{bmatrix} s_1 & s_2 & \dots & s_n \\ \Omega s_n & s_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & s_2 \\ \Omega s_2 & \dots & \Omega s_n & s_1 \end{bmatrix}$$

is used where each of the M -dimensional symbols $(s_1, \dots, s_n)$ belongs to C_{PPM} .

$Y = HC + N$ gives the following equation:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = [h_1 \quad h_2 \quad \dots \quad h_n] \times \begin{bmatrix} s_1 & s_2 & \dots & s_n \\ \Omega s_n & s_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & s_2 \\ \Omega s_2 & \dots & \Omega s_n & s_1 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_n \end{bmatrix}$$

$$\Rightarrow Y = h \otimes I_M \cdot C' + N$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = [h_1 \quad h_2 \quad \dots \quad h_n] \otimes \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 \end{bmatrix} \times \begin{bmatrix} s_1 \\ \Omega s_n \\ \vdots \\ \Omega s_2 \\ s_2 \\ s_1 \\ \vdots \\ \Omega s_3 \\ \vdots \\ s_n \\ \vdots \\ s_2 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_n \end{bmatrix}$$

Where C' is the vertical concatenation of the codeword columns and it's an $(n_t^2.M \times 1)$ matrix, and $h.I_M$ is a $(n_t \times n_t^2.M)$, h is $(1 \times n_t.M)$ represents the horizontal concatenation of each h_i is related to one of the transmitters.

$\Rightarrow Y = h \otimes I_M . \Phi . C'' + N$ where Φ is the $(n_t^2.M \times n_t.M)$ and C'' is the $(n_t.M \times 1)$

$$\text{matrices: } C'' = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix} \text{ and } \Phi = \begin{bmatrix} I_M & 0_M & \dots & 0_M \\ 0_M & \dots & 0_M & \Omega_M \\ 0_M & \dots & \Omega_M & 0_M \\ \vdots & \ddots & \ddots & \vdots \\ 0_M & \Omega_M & \dots & 0_M \\ 0_M & I_M & \dots & 0_M \\ I_M & 0_M & \dots & 0_M \\ 0_M & \dots & 0_M & \Omega_M \\ \vdots & \ddots & \ddots & \vdots \\ 0_M & \Omega_M & \dots & 0_M \\ \vdots & \vdots & \vdots & \vdots \\ 0_M & \dots & 0_M & I_M \\ 0_M & \dots & I_M & 0_M \\ \vdots & \ddots & \ddots & \vdots \\ I_M & 0_M & \dots & 0_M \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = [h_1 \quad h_2 \quad \dots \quad h_n] \otimes \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 \end{bmatrix} \times \begin{bmatrix} I_M & 0_M & \dots & 0_M \\ 0_M & \dots & 0_M & \Omega_M \\ 0_M & \dots & \Omega_M & 0_M \\ \vdots & \vdots & \vdots & \vdots \\ 0_M & \Omega_M & \dots & 0_M \\ \vdots & \vdots & \vdots & \vdots \\ 0_M & \dots & 0_M & I_M \\ 0_M & \dots & I_M & 0_M \\ \vdots & \vdots & \vdots & \vdots \\ I_M & 0_M & \dots & 0_M \end{bmatrix} \times \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_n \end{bmatrix}$$

Where the predefined Y and N represents the received output and the noise term respectively.

Regarding the rate-diversity trade-off, these codes achieve full diversity, at the expense of a constant rate which is equal to 1 in this case because we are sending n symbols in n symbol durations. In the contrary of the spatial multiplexing, where the rate is in increase at the expense of zero diversity or coding.

This code is fully diverse for many systems having many number of antennas and modulation positions but not all of them. We developed a new codeword that ensures the diversity for all (n, M) combinations without any exceptions. As we are going to see in the following section, this new code is based on the STC we have just presented but with some modifications.

9.4 New Solution STC

9.4.1 Introduction

We propose a new solution for the design of unipolar ST block codes that can be applied with M-PPM and n transmit antennas for all values of M and n . With respect to the existing unipolar codes that we presented earlier in the report, the introduction of an additional constellation extension rendered such a construction possible. But unlike other codes, the proposed scheme is the first known code that is based exclusively on a time domain constellation extension i.e. a new modulation position is added, while maintaining full diversity of the code. Therefore, for conveying the M-PPM symbol, **each symbol duration must contain $M + 1$ time slots or modulation positions rather than M , under the condition that the added position must be empty.** This time-domain constellation extension corresponds to an increase by 1 of the dimensionality of the transmitted signal.

Note that the constellation extension could be introduced in the amplitude domain or time domain, but since amplitude extensions are prohibited with unipolar transmissions, the only possible remaining solution resides in time-domain. This is equivalent to an increase constellation dimensionality, and consequently, an increased receiver complexity since the basis on which the received signal must be projected will have additional components to detect the added empty position.

The proposed code is the first code that guarantee full spacial diversity for an arbitrary number of transmit antennas and modulation positions. For example, in paper [12], we have codes associated with $M = 2$ for $n = 3, 5, 6, 7$ antennas; for these values of (M, n) , the solution we propose necessitates $M + 1 = 3$ positions

instead of 2, which constitutes a main contribution since there are no codes that are fully diverse with 3 positions for $N > 3$. It can be seen also that the code introduces no amplitude scaling, phase rotations or multi-pulse transmissions per symbol duration. In other words, only one unipolar pulse is transmitted per symbol duration which constitutes the challenge of the code.

We are going to present a detailed proof for the correctness of new code and prove its full diversity and reliability.

9.4.2 Code Construction

Proposition:

For any (n, M) combination, if the codeword $C(n, M)$ is fully diverse, therefore it is fully diverse for $C(n, M + 1)$ i.e. for the same number of transmitters n but with $M+1$ modulation positions, given that the $(M + 1)$ -th position added is empty (mathematically zero).

Proof I:

For $n = 2$, we had previously demonstrated that the codeword C is fully diverse for all M values, see papers [11], [12].

For $n > 2$, the minimal delay codeword has the same structure listed in [12]:

$$C(s_1, \dots, s_n) = \begin{bmatrix} s_1 & s_2 & \dots & s_n \\ \Omega s_n & s_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & s_2 \\ \Omega s_2 & \dots & \Omega s_n & s_1 \end{bmatrix}$$

but now each **symbol duration is divided into $M + 1$ positions** rather than M positions for M-PPM. Therefore, C is now $n(M + 1) \times n$ matrix, Where

$s_1, \dots, s_n \in C_{PPM} = \{e_m, m = 1, \dots, M + 1\}$ are the $(M + 1)$ -dimensional vector representation, where the $(M + 1)$ -th position is zero.

Ω is $(M + 1) \times (M + 1)$ having the same structure as before with an added position.

$$\Omega = \begin{bmatrix} \mathbf{0}_{1 \times M} & \mathbf{1} \\ \mathbf{I}_M & \mathbf{0}_{M \times 1} \end{bmatrix}$$

And it works in a cyclic manner, but now the pulse in the last position, occupy the new added time slot instead of the first because the constellation is augmented by one position. It is because of this form that the new code achieves its full diversity. We examine again the same 4-PPM case again in order to notice the development of Ω , that is working now on a 5-PPM constellation not only 4. We see that each pulse still occupy the next time slot, and the last pulse occupy the new added empty slot rather than going back to the first position.

Figure 30: from 4 to 5-PPM Constellation, the new permutation matrix role

Where $\Omega s \in C$ whenever $s \in C$ and the code is still shape preserving with $(M + 1)$ positions.

Denote $A = \{s - s'; s, s' \in C_{PPM}\}$ the set of all possible differences between two information vectors. The code is fully diverse if the matrix $C(a_1, \dots, a_n)$ is full rank for

$$(a_1, \dots, a_n) \in A_n \setminus \{\mathbf{0}_{M+1}, \dots, \mathbf{0}_{M+1}\}.$$

In other words, the code $C(s_1, \dots, s_n)$ is fully diverse for given values of n and M , if all the non-zero M -dimensional vectors that result in a rank deficient matrix C do not belong to the previously defined set A .

Elements of A can have a maximum number of two non-zero components, where one of them must be $+1$, and the other must be -1 , even after adding the new position (because this position is zero). The transmit diversity is achieved because of this particular structure of A . Given that the code is fully diverse for n transmitters and M modulation positions, we need to prove the diversity for $M + 1$ positions.

This is proved by manipulating the codeword matrix equations.

For notational simplicity, C_i will refer to the i -th column of the codeword $C(a_1, \dots, a_n)$ which is denoted by C for simplicity, and which is explained in the detailed matrix C_d . The codeword C , or its detailed matrix C_d , are rank-deficient if there exist $n-1$ rational numbers $k_1, \dots, k_n$ such that we get equation 1:

$$C_n = \sum_{i=1}^{n-1} k_i C_i \quad \text{eq. (1)}$$

where C becomes the difference of 2 arbitrary codes from the set of M^n codewords:

M^n codewords

Figure 31: the huge number of codewords for n transmitters and M modulation positions

So we take any 2 codes:

$$C' = \begin{bmatrix} s'_1 & s'_2 & \cdots & s'_n \\ \Omega s'_n & s'_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & s'_2 \\ \Omega s'_2 & \cdots & \Omega s'_n & s'_1 \end{bmatrix}$$

and

$$C'' = \begin{bmatrix} s''_1 & s''_2 & \cdots & s''_n \\ \Omega s''_n & s''_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & s''_2 \\ \Omega s''_2 & \cdots & \Omega s''_n & s''_1 \end{bmatrix}$$

their difference leads to

$$C = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \\ \Omega a_n & a_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & a_2 \\ \Omega a_2 & \cdots & \Omega a_n & a_1 \end{bmatrix}$$

Where $a_i = [a_{i,1}, a_{i,2}, \dots, a_{i,M}, a_{i,M+1}]^T$

Therefore, $\Omega a_i = [a_{i,\pi(1)}, a_{i,\pi(2)}, \dots, a_{i,\pi(M)}, a_{i,\pi(M+1)}]^T$

$$\text{i.e. } \Omega \mathbf{a}_i = [\mathbf{a}_{i,M+1}, \mathbf{a}_{i,1}, \dots, \mathbf{a}_{i,M-1}, \mathbf{a}_{i,M}]^T$$

we notice the permutation matrix role applied to each vector of the codeword where elements representing positions and modified in a cyclic manner.

This will lead to the detailed matrix $\mathbf{C}_d$ which represents the matrix $\mathbf{C}$, but each element is presented in terms of a vector of positions rather than a simple element. The positions could be listed in order or permuted after being multiplied by the permutation matrix. Therefore, $\mathbf{C}_d$ will be:

$$\left[\begin{array}{ccccc} \mathbf{a}_{1,1} & \mathbf{a}_{2,1} & \cdots & \mathbf{a}_{n,1} & (\text{Slot } 1) \\ \vdots & \vdots & & \vdots & \vdots \\ \mathbf{a}_{1,M+1} & \mathbf{a}_{2,M+1} & \cdots & \mathbf{a}_{n,M+1} & (\text{Slot } M + 1) \\ \mathbf{a}_{n,M+1} & \mathbf{a}_{1,1} & \cdots & \mathbf{a}_{n-1,1} & (\text{Slot } 1) \\ \vdots & \vdots & & \vdots & \vdots \\ \mathbf{a}_{n,M} & \mathbf{a}_{1,M+1} & \cdots & \mathbf{a}_{n-1,M+1} & (\text{Slot } M + 1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{a}_{3,M+1} & \cdots & \mathbf{a}_{1,1} & \mathbf{a}_{2,1} & (\text{Slot } 1) \\ \vdots & & \vdots & \vdots & \vdots \\ \mathbf{a}_{3,M} & \cdots & \mathbf{a}_{1,M+1} & \mathbf{a}_{2,M+1} & (\text{Slot } M + 1) \\ \mathbf{a}_{2,M+1} & \mathbf{a}_{3,M+1} & \cdots & \mathbf{a}_{1,1} & (\text{Slot } 1) \\ \vdots & \vdots & & \vdots & \vdots \\ \mathbf{a}_{2,M} & \mathbf{a}_{3,M} & \cdots & \mathbf{a}_{1,M+1} & (\text{Slot } M + 1) \end{array} \right] \begin{array}{l} \text{row } 1 \\ \\ \text{row } 2 \\ \vdots \\ \text{row } n - 1 \\ \\ \text{row } n \end{array}$$

As we notice, multiplying by Ω , permutes vector $\mathbf{a}_i$ such that $\mathbf{a}_{i,M+1}$ occupies the first position or time slot $\mathbf{a}_{i,1}$, and $\mathbf{a}_{i,1}$ occupies the second slot $\mathbf{a}_{i,2}$, etc. till each pulse occupies its next time slot.

Suppose that the matrix $\mathbf{C}$ is rank-deficient:

Therefore, equation (1) applies. Looking at (row n position 1), in the detailed matrix $\mathbf{C}_d$, all elements, except $\mathbf{a}_{1,1}$ are **zeros**. Therefore, due to equation (1), $\mathbf{a}_{1,1}$ will be **0**, because it represents the sum of all other columns in the assumption.

Next, looking at (row $n-1$ position 1) in $\mathbf{C}_d$, all elements, except $\mathbf{a}_{2,1}$, are zero therefore applying equation (1) yield $\mathbf{a}_{2,1} = 0$.

Next looking at (row $n-2$ position 1), then (row $n-3$ position 1) till (row 1 position 1), we get consequently $\mathbf{a}_{1,1}$ till $\mathbf{a}_{n,1}$ all equal to **zeros**.

We repeat the same process again with rows n then $n-1$ then $n-2$, consequently till row 1, but now with position 2, and we get elements from $\mathbf{a}_{1,2}$ consequently till $\mathbf{a}_{n,2}$ all equal to **zeros**.

Repeating the process again with all rows, but now with positions 3 then 4 till position $M + 1$, we get all elements of the matrix equal **zero**, leading to the **all-zero matrix** $\mathbf{C}$. Therefore, the codeword $\mathbf{C}$ will still be full rank for $(a_1, \dots, a_n) \in A_n \setminus \{0_{M+1}, \dots, 0_{M+1}\}$ because it's the $M + 1$ zero matrix.

The proposition was proved because it leads to the all-zero matrix. And due to the fact that it is full rank for M positions, then it is going to be for $M + 1$ as well, for arbitrary values of M and n , leading to the reliability and full diversity of all (n, M) combinations. A second mathematical proof is provided in the appendix at the end of the report.

9.4.3 Examples

To better understand the code's functionality, we take an example on a 3×3 system, which is then extended to 4×4 system. They both work on 4-PPM

constellation or symbols, which are extended to 5-PPM following the new proposed code. The 3×3 code ensures space and time diversity such that:

$$C(s_1, s_2, s_3) = \begin{bmatrix} s_1 & s_2 & s_3 \\ \Omega s_3 & s_1 & s_2 \\ \Omega s_2 & \Omega s_3 & s_1 \end{bmatrix}$$

Where $s_1 = 1$, $s_2 = 4$ and $s_3 = 2$. From antenna 1, s_1 is transmitted at time 1, s_2 at time 2 and s_3 at time 3. From antenna 2, $\Omega s_3 = 3$ is transmitted at time 1, s_1 at time 2 and s_2 at time 3. Here, Ωs_3 is the permuted version of s_3 and its value is 3 instead of 2. From antenna 3, $\Omega s_2 = 5$ is transmitted at time 1, $\Omega s_3 = 3$ at time 2 and s_1 at time 3. Here, Ωs_2 is the permuted version of s_2 and its value is 5 instead of 4. We see that 5 is the new added time slot, and because it is empty the pulse could occupy it rather than going back to position 1. The figure shows the matrix code with the corresponding elements.

Figure 32: Example on 3 x 3 System

This system can be extended to a 4×4 case where the ST codeword is an extension of the 3×3 one with s_4 added in the time and space domain.

$$C(s_1, s_2, s_3, s_4) = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ \Omega s_4 & s_1 & s_2 & s_3 \\ \Omega s_3 & \Omega s_4 & s_1 & s_2 \\ \Omega s_2 & \Omega s_3 & \Omega s_4 & s_1 \end{bmatrix}$$

Here all values are kept the same, $s_4 = 3$ is transmitted from antenna 1 at time 4, $\Omega s_4 = 4$ is transmitted from antenna 2 at time duration 1 with a pulse at time slot or position 4, from antenna 3 at time duration 2 with a pulse at position 4, and from

antenna 4 at time duration 3 with a pulse at position 4. All the details are translated by the figure:

Figure 33: Example on 4 x 4 System

Both figures show the different symbol permutations in the code matrix, and the position permutations inside each symbol in the lower triangular part of the code, thus, justifying the title of the report: Achieving Full Transmit Diversity for PPM Constellations with any Number of Antennas via Double Position and Symbol Permutations.

9.4.4 Advantages

This code has many advantages which make it suitable for all current FSO MIMO systems.

- First, it is based on transmitting unipolar symbols where polarities are always kept positive and the modulation is based on the time delays of the transmitted symbols. Therefore, no amplitude or phase modifications are introduced, making the code optimal for FSO applications.
- It is fully diverse which means strongly modulated and therefore provide secure transmission

- Applied to arbitrary number of modulation positions and number of antennas, therefore, filling table 1 in [12] presented in chapter 7. So this code can be applied to any $n \times n$ systems to convey the information.
- To transmit n symbols, n antennas are transmitting at n time intervals, therefore it's a rate-1 or full rate code.
- It's minimal-delay because we cannot achieve less than n symbol durations to transmit an n -symbols code with such diversity.

10 New Solution Decoder

In parallel of improving the M-PPM code, an M-PPM decoder was developed in parallel. In fact, during our research, pseudocode and implementation of 2-PPM decoding was done in [14], on any number of antennas. From this concept, we developed the maximum likelihood (ML) M-PPM decoder presented in section 1, and we present many simulations that prove the correctness of the different schemes in section 2. So this scheme is based on the one in [14], where we had only 2-PPM positions, but deal with the general case where it can be applied to any number of antennas n and modulation positions M .

10.1 Pseudocode

The maximum likelihood (ML) decoding algorithm works as follows with $m = M$ -PPM:

Input: z, R, P, C, m

Output: $\hat{c}$

Step 1: Set $k = P+1$, $\text{dist}(K) = 0$, $V = R^{-1}$, $\rho = 0_m$, $f = 0_P$, $\text{bestdist} = C$ (sphere squared radius), $\text{counter} = 1_P$

Step 2: $\text{newdist} = \text{dist}(k) + \rho^T \rho$

if ($\text{newdist} < \text{bestdist}$) & ($K \neq 1$) go to Step 3

else go to Step 4 endif

Step 3: if $K = P+1$, $c_{:,k-1} = Vz$ else

For $i = 1, \dots, k-1$

$$\mathbf{e}_{:,k-1}^{(i)} = \mathbf{e}_{:,k}^{(i)} - \mathbf{V}^{(i,k)} \rho \text{ endfor endif}$$

$$K = K - 1$$

$$\text{dist}(K) = \text{newdist}$$

$$\mathbf{E}_{:,k}^{(k)} = \mathbf{R}^{(k,k)} \mathbf{e}_{:,k}^{(k)}$$

$$\text{pos}(k) = \underset{\text{for all } m}{\text{argmin}} \{ -2 (\mathbf{R}^{(k,k)}_{:,m})^T \mathbf{E}^{(k)} + (\mathbf{R}^{(k,k)}_{:,m})^T \mathbf{R}^{(k,k)}_{:,m} \}$$

$$\mathbf{x}_{:,k} = \mathbf{I}_{:, \text{pos}(k)}$$

$$\rho = \mathbf{E}_{:,k}^{(k)} - \mathbf{R}^{(k,k)}_{:, \text{pos}(k)}$$

go to Step 2

Step 4: if newdist < bestdist,

$$\text{for } i=1, \dots, P, \hat{\mathbf{c}} = \mathbf{x}_{:,i}$$

$$\text{endfor, bestdist} = \text{newdist}$$

else if k=P terminate

else k = k+1 endif.

If f(k) = 1 go to step 4

Else go to Step 5 endif

Step 5: counter(k) = counter(k) + 1

$$\text{pos}(k) = \text{next minimum}(\mathbf{A})$$

$$\mathbf{X}_{:,k} = \mathbf{I}_{:, \text{pos}(k)},$$

$$\rho = \mathbf{E}_{:,k}^{(k)} - \mathbf{R}^{(k,k)}_{:, \text{pos}(k)}$$

if counter(k) == m

f(k) = 1, go to Step 2

else go to step 5

The algorithm is based on the 2-PPM, with some modifications. First, $e(k)$ the M -dimensional vector (instead of 2) obtained by applying ZF on the layers $1, \dots, K-1, K+1, \dots, P$ and by taking hard decisions on the layers $K+1, \dots, P$. Therefore, we move to the upper layer to generate valid intervals, and to the downward layer when all the intervals at the given levels are spanned. The influence of the K -th layer on the squared Euclidean distance between the received decision vector (considered in the transmitted signal subspace) and the candidate output of the algorithm when a pulse is transmitted at the m -th position is given by:

$$d_k^2 = \| R^{(k,k)} (e^{(k)} - I_{:,m}) \|^2 = \| E^{(k)} - R^{(k,k)} I_{:,m} \|^2$$

where $E^{(k)} = R^{(k,k)} e^{(k)}$ and $I_{:,m}$ corresponds to the m -th column of the $(M \times M)$ identity matrix .

ρ is an m -dimensional vectors .e and E are $mP \times P$ matrices . The multi-dimensional span is applied to determine the order with which the m candidate sub-vectors of each layer must be tested. All the m results are sorted in a matrix A to facilitate which position to take each time. The use of counter is radical in this case to ensure that all elements of A are tested i.e. in order to insure that all positions, of each antenna (or layer), are examined; after examining all positions, we can move to the next layer, and this is presented in the next figure:

Figure 34: spanning between layers

In this figure, each level represents an antenna, and the lines or components inside levels represent the modulation positions or time slots. It shows the spanning between layers, which is to move upward to generate valid intervals, and downward when all positions at a given level or layer are completely spanned. This is repeated until all layers or levels are spanned. Spanning of positions inside each layer is represented in the next figure. The points are the closest points deduced from the received point, from which the decoder is trying to find the input symbol. For example, the received point x is (1), and the decoder is moving in all directions between components to find the correct input point.

Figure 35: spanning between positions

Figure 36: Flowchart of the new decoding algorithm

In addition to this, zero forcing decoding, a simpler algorithm presented in the beginning of the chapter, that works on finding the input by multiplying the received point by H^{-1} , instead of applying a QR decomposition of H and thus complicate things. Although this method is simpler but it leads to a higher number of errors or BER, as we are going to see in the following simulations. So both algorithms are

implemented, and simulations were done to compare both decoding schemes, in terms of on spatial, ST, and the new solution code.

10.2 Simulations

A number of simulations were done via the Matlab software on three million symbols and different types of channels in order to show the high performance level and the enhanced diversity order achieved by the proposed scheme. In all simulations, the symbol error rate (SER) is plotted with respect to the signal-to-noise ratios (SNR) per bit in decibels (dB). We notice that the SER (or bit error rate (BER)) is high for low SNR, but it gets lower for higher SNR, till it reaches 0 error level, the optimum level of transmission achieved with high SNR. Therefore, the two variables SER and SNR perform inversely proportional to each others. In addition, each time we are adding more antennas to the system, the degree of multiplexing is getting higher, and therefore less errors we have, but this result differs from one type of multiplexing and coding to another. Three types of codes are explored: spatially multiplexed (uncoded) codes, Space-Time (ST) codes, and the new solution code. In addition, the new solution code performs well on 3 different types of channels: exponential, Rayleigh and lognormal; as well as on any number of antennas and modulation positions.

Figure 37: Performance of spatial multiplexed code on exponential channel with 2-PPM and ML decoding

The first figure shows the maximum likelihood (ML) decoding on 2-PPM with different number of antennas. In this case, the spatial multiplexing degree is higher but the diversity coding is zero. So we are transmitting at a higher rate, proportional to the number of antennas, at the expense of zero diversity. And the more antennas added to the system, the less SER with respect to SNR we have. This is proved for 1x1, 2x2, 4x4 and 8x8 systems.

Figure 38: Comparison on 2x2 systems between ML decoding and zero forcing decoding, with 2-PPM and 8-PPM, on spatial codes

We notice that the ML decoding we present performs better than the simple zero forcing decoder, on any PPM constellation and any system, and leads to less SER. This is shown for 2x2 systems with 2 and 8-PPM constellations.

Figure 39: comparison between un-coded, ST-coded and zero forcing decoding schemes

The performance of ST- coded symbols is better than the uncoded symbols. This is because of the time multiplexing that gives a higher diversity coding, at the expense of a constant rate which is 1 and maybe a decreasing rate with respect to a higher coding gain. So the trade-off between rate and diversity, or coded and uncoded symbols, should be taken into consideration depending on the practical needs.

Figure 40: Performance of the proposed code on Exponential channel with P transmit antennas and 4-PPM for $P=1, 2, 3, 4$ and 5

The simulations just presented performs on the new coding scheme, with 4-PPM on exponential channel. The highest SER occurs mostly in SISO systems, which is translated by the highest 1x1 curve. This is followed by the other MIMO systems curves in order from 2x1 till 5x1. The lowest errors for 5x1 happens because more transmitters and therefore more diversity are to the block code.

Figure 41: Performance of the proposed code on Lognormal channel with P transmit antennas and 4-PPM for $P=1, 2, 3, 4$ and 5

Similar results occurs for the same systems in the case of lognormal channel with 4-PPM.

Figure 42: performance of the proposed code on Rayleigh channel with P transmit antennas and 4-PPM for $P=1, 2, 3, 4$ and 5

Similar results also occurs for the same systems in the case of Rayleigh channel with 4-PPM.

The three figures above shows the performance of the new proposed scheme with 4-PPM, on the three different channels: exponential, lognormal and Rayleigh, for systems having P transmitters where P varies from 1 to 5. All channels give similar results and comparisons. For the three channels, the SER is less for high SNR, and became zero for some high SNR. Furthermore, when antennas are added to the system, more capacity, multiplexing and diversity are added by each new extra-transmitter.

Figure 43: performance of the proposed code on Lognormal channel with P transmit antennas and 3-PPM for $P=1, 3, 5, 6$ and 10

The new proposed code made an improvement where it is used with 3-PPM modulation schemes, for $P=1, 3, 5, 6, 10$ and even more transmitters could be added all leading to the same full diversity and SER in terms of SNR, and all MIMO performs better than SISO for a specific SNR. The solutions for such values of PPM, as 3-PPM, didn't exist before.

11 Conclusion

This project is a result of researches, designs and practical work that led to the new proposed code which is efficient for any optical systems having any number of antennas and signal-set dimensionality. This code is achieved by adding one empty position to the PPM constellation such that each symbol is divided into $M + 1$ time slots, and it is because of this time extension that the code achieves its full diversity. The introduction of the special permutation matrices as well as the new modulation schemes like PPM helped in proving the result. For example, the permutation matrix was behind changing each pulse position to occupy the next position and the time duration of the pulse was a main idea where each symbol transmitted from different antenna occupy different time duration as was shown in the codeword matrix C and which justify the title of the topic. A detailed code explanation based on mathematical proofs is provided at the end of the report to prove the correctness of our proposition.

Therefore, we have shown that it is a rate-1, minimal delay, fully diverse, unipolar ST code, that works in any optical system having arbitrary number of modulation positions and number of antennas at the transmitter and/or receiver sides. The idea of PPM replaces other modulation schemes because it is based on the time delays of the transmitted pulses, and therefore suitable for FSO communications. Similarly, the novel idea of time-domain constellation extension renders such construction possible and opens the door to more sophisticated and more powerful codes.

We developed as well the M-PPM decoder capable of decoding uncoded or spatially multiplexed codes, as well as Space-Time ST-coded symbols. The new decoder also performs well on the new coding scheme, where symbols have arbitrary number of

modulation positions. Many simulations are done to prove the reliability of the codes and schemes presented. The results show that we are getting the lowest symbol error rate (SER) for high signal-to-noise ratios (SNR). They show as well the trade-off between rate and diversity where at high spatial multiplexing degree (translated by high number of transmit antennas), and zero diversity, we can have an increasing transmission rate proportional with the number of antennas added; while at a constant space multiplexing and higher diversity coding, the rate is constant and maybe decreasing at the expense of more diversity i.e. more reliable throughput at the receiver. So depending on the practical needs, we choose the code, channel, and decoding to meet the situation requirements.

The result is submitted for publications in the *IEEE* transactions on communications. So the code is available and people can benefit from it in the future, develop it and discover new schemes to achieve the higher throughputs or better reliability. This can be done by adding new constellation extensions, introducing new permutation matrices to achieve diversity, finding a new modulation scheme other than that based on the time delays of the transmitted pulses, or developing new ideas that were not highlighted during our work.

Appendix

Proof 2:

The mathematical proof of the new proposed code works as follows and is presented in paper [6] which is the submitted paper for this thesis. Consider the case of M -ary PPM with $P = n$ transmit antennas. In what follows, $T = n$ symbol durations, and the n information symbols $p_1, \dots, p_n \in \{1, \dots, M\}$, where each symbols is divided into $M+1$ positions (rather than M positions for M -PPM). Based on this new partitioning, the position of the pulse transmitted from the i -th transmit antenna during the j -th symbol duration will be determined from (i, j) -th element of the $n \times n$ matrix P given by:

$$P_{i,j} = \begin{cases} p\sigma^{i-1}(j), & j \geq i \\ \pi^{-1}(p\sigma^{i-1}(j)), & j < i \end{cases}$$

where $\sigma^k(.)$ and $\pi^k(.)$ stand for the cyclic permutations of order k over the $(M + 1)$ extended positions and n symbol durations respectively:

$$\sigma^k(i) = (i - k - 1) \bmod n + 1$$

$$\pi^k(i) = (i - k - 1) \bmod (M + 1) + 1$$

Equation (1) shows that the proposed diversity scheme corresponds to introducing convenient permutations among the positions and symbol durations justifying the title of the report. Even though $p_i \in \{1, \dots, M\}$ for $i = 1, \dots, n$, eq. (2) shows that $\pi^{-1}(p_i) \in \{1, \dots, M + 1\}$. This corresponds to the time-domain constellation extension introduced by the proposed scheme.

For a system equipped with Q receive antennas and a L -th order Rake, the linear dependence between the baseband inputs and outputs of the channel can be expressed as :

$$Y = HC + N$$

where Y and N are $QLM' \times T$ matrices corresponding to the decision variables and the noise terms respectively with $M' \equiv M + 1$. Note that M' (rather than M) correlators are needed per receive antenna and Rake finger since the encoded pulses can occupy M' positions. H is the $QLM' \times PM'$ channel matrix whose $((q - 1)LM' + (l - 1)M' + m, (p - 1)M' + m')$ -th element corresponds to the impact of the signal transmitted during the m' -th position of the p -th antenna on the m -th correlator (corresponding to the m -th position) placed after the l -th Rake finger of the q -th receive antenna.

C is the $PM' \times T$ codeword with $P = T = n$. From equations (1)-(3), C can be written as:

$$C(S_1, \dots, S_n) = [(C^{(0)}(S_1, \dots, S_n))^T, \dots, (C^{(n-1)}(S_1, \dots, S_n))^T]^T$$

$$C^{(i)}(S_1, \dots, S_n) = \begin{bmatrix} S_{n-i+1,\pi(1)} & \cdots & S_{n,\pi(1)} & S_{1,1} & \cdots & S_{n-i,1} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ S_{n-i+1,\pi(M')} & \cdots & S_{n,\pi(M')} & S_{1,M'} & \cdots & S_{n-i,M'} \end{bmatrix}; \quad i = 0, n-1$$

where $C^{(i)}$ is a $M' \times n$ matrix. $S_1, \dots, S_n$ are M' -dimensional vectors that belong to the set:

$$C = \{e_m; m = 1, \dots, M\}$$

where e_m is the m -th column of the $M' \times M'$ identity matrix. In other words, $S_i = e_{p_i}$ for $i = 1, \dots, n$. The m -th component of S_i is denoted by $S_{i,m}$ for $m = 1, \dots, M'$.

Proposition 1: [main result] The proposed code permits to achieve a full transmit diversity order with M -PPM and n transmit antennas for all values of M and n .

Proof: Based on [2] and because of the linearity of the code, the ST code is fully diverse if the matrix $C(X_1, \dots, X_n)$ has full rank for $(X_1, \dots, X_n) \in X^n \setminus \{(0_{M'}, \dots, 0_{M'})\}$ where $0_{M'}$ is the M' -dimensional zero vector and X corresponds to the set of possible differences between the elements of C given in eq. (8): $X = \{s - s'; s, s' \in C\}$. The diversity order is achieved because of this particular structure of X .

$C(X_1, \dots, X_n)$ will be denoted by C for simplicity while $C_j^{(i)}$ stands for the j -th row of $C^{(i)}$. We first observe that $\det(C^T C)$ verifies the following relation:

$$\begin{aligned} \det(C^T C) &= \sum_{i_1=1}^{nM'} \sum_{i_2=i_1+1}^{nM'} \dots \sum_{i_n=i_{n-1}+1}^{nM'} (\det([C_{i_1}^T \dots C_{i_n}^T]^T))^2 \geq \\ &\sum_{i_1=1}^{M'} \sum_{i_2=M'+1}^{2M'} \dots \sum_{i_n=(n-1)M'+1}^{nM'} (\det([C_{i_1}^T \dots C_{i_n}^T]^T))^2 \\ &= \sum_{i_1=1}^{M'} \sum_{i_2=1}^{M'} \dots \sum_{i_n=1}^{M'} \left(\det([C_{i_1}^{(0)T} \dots C_{i_n}^{(n-1)T}]^T) \right)^2 \end{aligned}$$

The proof is based on the following propositions. In what follows, $m \in \{1, \dots, M'\}$.

Proposition 2: $X_{1,m} = \dots = X_{n-1,m} = 0$ implies that $\det(C^T C) \geq 1$ unless $X_{n,m} = 0$.

Proof: Construct the $n \times n$ matrix C' as:

$$\begin{aligned}
C' &= \left[\left(C_{\pi^{-1}(m)}^{(1)} \right)^T \cdots \left(C_{\pi^{-1}(m)}^{(n-1)} \right)^T \left(C_{(m)}^{(0)} \right)^T \right]^T \\
&= \begin{bmatrix} X_{n,m} & X_{1,\pi^{-1}(m)} & \cdots & \cdots & X_{n-1,\pi^{-1}(m)} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ X_{2,m} & \cdots & \cdots & X_{n,m} & X_{1,\pi^{-1}(m)} \\ X_{1,m} & \cdots & \cdots & X_{n-1,m} & X_{n,m} \end{bmatrix}
\end{aligned}$$

Since $X_{1,m} = \dots = X_{n-1,m} = 0$, C' is upper triangular. From eq. (9), this implies that: $\det(C^T C) \geq (\det(C'))^2 = (X_{n,m})^{2n}$. Moreover, based on the structure of the set X , $X_{n,m} \in \{0, \pm 1\}$ completing the proof of proposition 2.

Proposition 3: $X_{1,m} = \dots = X_{n'-1,m} = X_{n'-1,\pi(m)} = \dots = X_{n,\pi(m)} = 0$ implies that $\det(C^T C) \geq 1$ unless $X_{n',\pi(m)} = 0$.

Proof: Let $k \equiv \sigma^{-1}(1 - n')$, we observe that following matrix is a lower triangular matrix:

$$\begin{aligned}
C' &= \begin{bmatrix} C_m^{(k)} \\ \vdots \\ C_m^{(n-1)} \\ C_{\pi(m)}^{(0)} \\ \vdots \\ C_{\pi(m)}^{(k-1)} \end{bmatrix} \\
&= \begin{bmatrix} X_{n',\pi(m)} & \cdots & \cdots & X_{n,\pi(m)} & X_{1,m} & \cdots & X_{n'-1,m} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ X_{2,\pi(m)} & \cdots & X_{n',\pi(m)} & X_{n'+1,\pi(m)} & \cdots & X_{n,\pi(m)} & X_{1,m} \\ X_{1,\pi(m)} & \cdots & \cdots & X_{n',\pi(m)} & X_{n'+1,\pi(m)} & \cdots & X_{n,\pi(m)} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ X_{n'+1,\pi^2(m)} & \cdots & X_{n,\pi^2(m)} & X_{1,\pi(m)} & \cdots & \cdots & X_{n',\pi(m)} \end{bmatrix}
\end{aligned}$$

Consequently, from eq. (9): $\det(C^T C) \geq (\det(C'))^2 = (X_{n', \pi(m)})^{2n}$ completing the proof. Now the proof of the main proposition will follow from proposition 2 and proposition 3. Since none of the n information symbols initially occupies the position $M' = M + 1$ (before applying the pulse permutation), then $X_{1, M'} = \dots = X_{n, M'} = 0$.

Since $X_{1, M'} = \dots = X_{n-1, M'} = 0$, applying proposition 3 with $n' = n$ and $m = M'$ results in $\det(C^T C) \geq 1$ unless when $X_{n, \pi(m)} = 0$. Now, $X_{1, M'} = \dots = X_{n-2, M'} = 0$ and $X_{n, \pi(M')} = 0$ imply that $X_{n-1, \pi(M')} = \dots = X_{1, \pi(M')} = 0$ from the recursive application of proposition 3 for $n' = n - 1, \dots, 1$. In other words, $\det(C^T C) \geq 1$ unless when the M' -th and $\pi(M')$ -th components of $X_1, \dots, X_n$ are all equal to zero. Repeating the same procedure with $\pi(M')$ rather than M' we conclude that $\det(C^T C) \geq 1$ unless when the $\pi(M')$ -th and $\pi^2(M')$ -th components of $X_1, \dots, X_n$ are all equal to zero. By recursion, we conclude that $\det(C^T C) \geq 1$ unless when the components $\{M', \pi^1(M'), \dots, \pi^M(M')\}$ of $X_1, \dots, X_n$ are equal to zero implying that $X_1 = \dots = X_n = 0_M$ since this last set is equal to $\{1, \dots, M\}$.

References

- [1] Arnon, Shlomi, Stanley R. Rotman and Norman S. Kopeika. Performance limitations of free-space optical communication satellite networks due to vibrations: direct detection digital mode. *Optical Engineering* 36.11 (1997).
- [2] ZHANG, Jinlong. *Modulation analysis for outdoors applications of optical wireless communications*. International conference on communications technology proceedings. Beiginj, China, 2000. 3 May 2008 <http://ieexplore.iee.org/xpls/abs_all.jsp?arnumber=890940>.
- [3] Georghiades, Costas N. *Signaling Over High-speed Optical Intersatellite Links*. Global telecommunicationas conference, 1993, including a communication theory mini-conference, technical program conference record, Houston, TX: GLOBECOM, 1993, 23 Apr 2008 <http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=318215>.
- [4] Wilson, Stephen G., et al. Free-space optical MIMO transmission with Q-ary PPM. *IEEE Transactions on Communications* 53.8 (2005).
- [5] O'Brien, Dominic C. and Marcos Katz. *Short-range optical wireless communications*. Wireless World Research Forum (WWRF). 18 May 2008 <<http://cictr.ee.psu.edu/research/wc/Short%20range...>>
- [6] Abou-Rjeily, Chadi and Zeina Baba. *Achieving full transmit diversity for PPM constellations with any number of antennas via double position and symbol permutations*. School of Engineering and Architecture, Lebanese American University. 22 May 2008. <<http://services.dea.lau.edu.lb/faculty/carjeily/Publications.html>>.

- [7] Tarokh, V., N. Seshadri and A. Calderbank. Space-time codes for high data rate wireless communication: performance and code construction. *IEEE Trans. Inform. Theory*. 44 (1998).
- [8] Alamouti, Siavash M. A simple transmit diversity technique for wireless communications. *IEEE Journal on Select Areas in Communications* 16.8 (1998).
- [9] Simon, Marvin K. and Victor A. Vilnrotter. Alamouti-type space-time coding for free-space optical communication with direct detection. *IEEE Transactions on Wireless Communications*. 4.1 (2005).
- [10] Abou-Rjeily, Chadi. *Orthogonal space-time block codes for binary pulse position modulation* (n.d.). School of Engineering and Architecture, Lebanese American University. 20 Feb 2008. <<http://services.dea.lau.edu.lb/faculty/carjeily/Publications.html>>.
- [11] Abou-Rjeily, Chadi and Jean-Claude Belfiore. *A rate-1 2 x 2 space-time code without any constellation extension for TH-UWB communication systems with PPM*. School of Engineering and Architecture, Lebanese American University. 17 Mar 2008. <<http://services.dea.lau.edu.lb/faculty/carjeily/Publications.html>>.
- [12] Abou-Rjeily, Chadi and Wissam Fawaz. *Space-Time codes for MIMO ultra-wideband communications and MIMO free-Space optical communications with PPM*. School of Engineering and Architecture, Lebanese American University. 10 Mar 2008. <<http://services.dea.lau.edu.lb/faculty/carjeily/Publications.html>>.
- [13] Boutros, Joseph, et. al. *Soft-input soft-output lattice sphere decoder for linear channels*. GLOBECOM, 2003. 8 Feb 2008 <<http://ieeexplore.ieee.org/iel5/8900/28134/01258504.pdf?arnumber=1258504>>.

[14] Abou-Rjeily, Chadi and Jean-Claude Belfiore. A Space-Time Coded MIMO TH-UWB Transceiver with Binary Pulse Position Modulation *IEEE Communications Letters*. 11.6 (2007)

[15] Abou-Rjeily, Chadi. *A maximum-likelihood decoder for joint pulse position and amplitude modulations*. School of Engineering and Architecture, Lebanese American University. 2 Mar 2008.
<<http://services.dea.lau.edu.lb/faculty/carjeily/Publications.html>>.

Readings

Free space optics (FSO): an introduction. fSONA Optical Wireless. 29 April 2008.
<<http://www.free-space-optics.org/>>

Free space optical communication: *Wikipedia*. 09 May 2008. 10 May 2008
<http://en.wikipedia.org/wiki/Free_space_optics>

Fading: *Wikipedia*. 26 May 2008. 1 June 2008
<<http://en.wikipedia.org/wiki/Fading>>

Fabio Belloni. *Fading Models*. Signal Processing Laboratory, Helsinki University of Technology. 23 November 2004. 16 February 2008.
<http://www.comlab.hut.fi/opetus/333/2004_2005_slides/Fading_models.PDF>

What is Free Space Optics (FSO)?. LightPointe Communications. 2005. 5 April 2008. <<http://www.freespaceoptics.org/freespaceoptics/default.cfm>>

Amplitude-shift keying: *Wikipedia*. 4 June 2008. 7 June 2008.
<http://en.wikipedia.org/wiki/Amplitude-shift_keying>

Phase-shift keying: *Wikipedia*. 22 June 2008. 23 June 2008.
<http://en.wikipedia.org/wiki/Phase-shift_keying>

Frequency-shift keying: *Wikipedia*. 13 April 2008. 20 April 2008.
<http://en.wikipedia.org/wiki/Frequency-shift_keying>

Quadrature amplitude modulation: *Wikipedia*. 9 June 2008. 9 June 2008.
<http://en.wikipedia.org/wiki/Quadrature_amplitude_modulation>

Orthogonal frequency-division multiplexing: *Wikipedia*. 10 June 2008. 11 June 2008. <http://en.wikipedia.org/wiki/Orthogonal_frequency-division_multiplexing>

What is Pulse Amplitude Modulation?. Tech-FAQ. 2008. 7 May 2008.
[<http://www.tech-faq.com/pulse-amplitude-modulation.shtml>](http://www.tech-faq.com/pulse-amplitude-modulation.shtml)

Pulse-amplitude modulation: Wikipedia. 19 June 2008. 19 June 2008.
[<http://en.wikipedia.org/wiki/Pulse-amplitude_modulation>](http://en.wikipedia.org/wiki/Pulse-amplitude_modulation)

On-off keying: Wikipedia. 13 May 2008. 15 May 2008.
[<http://en.wikipedia.org/wiki/On-off_keying>](http://en.wikipedia.org/wiki/On-off_keying)

Pulse-position modulation: Wikipedia. 19 June 2008. 24 June 2008.
[<http://en.wikipedia.org/wiki/Pulse-position_modulation>](http://en.wikipedia.org/wiki/Pulse-position_modulation)

MIMO: Wikipedia. 09 June 2008. 12 June 2008.
[<http://en.wikipedia.org/wiki/MIMO>](http://en.wikipedia.org/wiki/MIMO)

Spatial multiplexing: Wikipedia. 2 March 2008. 18 March 2008.
[<http://en.wikipedia.org/wiki/Spatial_multiplexing>](http://en.wikipedia.org/wiki/Spatial_multiplexing)

Shannon's law. FS-1037C. 23 August 1996. 20 April 2008.
[<http://www.its.bldrdoc.gov/fs-1037/dir-033/4816.htm>](http://www.its.bldrdoc.gov/fs-1037/dir-033/4816.htm)

Lindroos, Petri. Channel Capacity MIMO capacity: S-72.333 Postgraduate course in radio communications. 25 January 2005. 19 March 2008.
[<http://www.comlab.hut.fi/opetus/333/2004_2005_slides/Capacity.PDF>](http://www.comlab.hut.fi/opetus/333/2004_2005_slides/Capacity.PDF)

Hourani, Hafeth. Overview of diversity techniques in wireless Ccommunication Ssystems. Helsinki University of Technology. 2004/2005. 17 Feb 2008.
[<http://www.comlab.hut.fi/opetus/333/2004_2005_slides/Diversity.PDF>](http://www.comlab.hut.fi/opetus/333/2004_2005_slides/Diversity.PDF)

Space-time code: Wikipedia. 23 May 2008. 30 May 2008.
[<http://en.wikipedia.org/wiki/Space-time_coding>](http://en.wikipedia.org/wiki/Space-time_coding)

Space-time trellis code: Wikipedia. 27 May 2008. 30 May 2008.
[<http://en.wikipedia.org/wiki/Space%E2%80%93time_trellis_code>](http://en.wikipedia.org/wiki/Space%E2%80%93time_trellis_code)

Space-time block code: Wikipedia. 27 May 2008. 30 May 2008.
[<http://en.wikipedia.org/wiki/Space%E2%80%93time_block_code>](http://en.wikipedia.org/wiki/Space%E2%80%93time_block_code)

Diversity scheme: Wikipedia. 22 May 2008. 30 May 2008.
[<http://en.wikipedia.org/wiki/Diversity_reception>](http://en.wikipedia.org/wiki/Diversity_reception)