Simple-DF versus Selective-DF Relaying over Rayleigh Turbulence-Induced FSO Fading Channels

Chadi Abou-Rjeily, *Member IEEE*
Department of Electrical and Computer Engineering
Lebanese American University (LAU), Byblos, Lebanon
chadi.abourjeily@lau.edu.lb

***Abstract*—In this paper, we consider the problem of relay-assisted free-space optical (FSO) transmissions. We evaluate the diversity orders that can be achieved by simple-DF and selective-DF relaying protocols over quantum-limited FSO systems that are subject to Rayleigh fading. We prove that for a N_r-relay system, selective-DF captures the full spatial diversity order of N_r+1 while simple-DF achieves a reduced order of $\lceil \frac{N_r}{2} \rceil + 1$ making this scheme highly suboptimal for FSO communications.**

I. Introduction

There has been a growing interest in applying the co-operative techniques in the context of Free-Space Optical (FSO) communications [1]–[9]. User cooperation emerged as a candidate fading mitigation technique for high speed FSO communications that suffer from pronounce levels of fading (or scintillation) that results from the variations of the index of refraction due to inhomogeneities in temperature and pressure changes [10]. Cooperative FSO solutions are disadvantaged by the non-broadcast nature of FSO transmissions where the message transmitted from one node can be overheard only by the corresponding destination node but not by the neighboring nodes as in cooperative Radio-Frequency (RF) systems. Despite the fact that this simplifies the cooperation strategies since the absence of interference results in simpler transceiver structures where joint encoding/decoding is avoided, the main disadvantage resides in dedicating a fraction of the total power for delivering the information messages to the relays. However, despite this power penalty, significant performance gains have been reported in the literature [1]–[9].

Several Amplify-and-Forward (AF) protocols have been studied in the context of FSO [1], [2]. Decode-and-Forward (DF) relaying was considered in [1], [3]–[5] where various simple-DF and selective-DF protocols have been analyzed. In simple-DF, all symbols received by a certain relay are forwarded to the destination node [3], [5] while in selective-DF, a quality-guaranteing criterion is imposed on the forwarded symbols in order not to confuse the destination with inaccurate estimates of the information messages [1], [3], [4].

In this paper, we consider the simple-DF strategy with any number of relays (denoted by N_r) and we prove that this strategy is not suitable for FSO systems with intensity-modulation and direct-detection (IM/DD) since it results in a reduced diversity order of $\lceil \frac{N_r}{2} \rceil + 1$ even in the absence of background radiation (the function $\lceil x \rceil$ rounds the real number x to the smallest integer that is larger than x). This finding shows that, unlike RF systems where simple-DF is capable of achieving the full diversity order of N_r+1, this strategy achieves only a fraction of this diversity order in FSO systems. In this context, the systems that were considered in [3] (with $N_r = 1$ relay) correspond only to a special case where the simple-DF and selective-DF strategies achieve the full diversity order since in this case $\lceil \frac{N_r}{2} \rceil + 1 = N_r + 1 = 2$. The findings in this paper are based on an asymptotic analysis that ignores background noise compared to fading and quantum noise for high signal energies [11].

II. System Model and Cooperation Strategies

Consider a relay-assisted FSO system with N_r relays. The relays will be denoted by $\text{R}_1, \dots, \text{R}_{N_r}$ and they will assist the communications between a source node S and a destination node D. We denote by a_0, $a_{\text{s},1}, \dots, a_{\text{s},N_r}$ and $a_{1,\text{d}}, \dots, a_{N_r,\text{d}}$ the random path gains between S-D, S-$\text{R}_1, \dots,$ S-R_{N_r} and R_1-D, ..., R_{N_r}-D, respectively. In this work, we adopt the Rayleigh turbulence-induced fading channel model [11] where the probability density function (pdf) of the path gain ($a > 0$) is given by: $f_A(a) = 2ae^{-a^2}$. This channel model captures the statistical behavior of long FSO links that are subject to severe fading conditions [11].

Consider Q-ary pulse position modulation (PPM) with IM/DD. The average number of photoelectrons generated by the incident light signal in a PPM slot is given by [11]:

$$\lambda_s = \eta \frac{P_r T_s / Q}{hc/\lambda} = \eta \frac{E_s}{hc/\lambda} \quad (1)$$

where T_s is the symbol duration, h is Planck's constant, c is the speed of light, $\lambda = 1550$ nm is the wavelength and $\eta = 0.5$ is the detector's quantum efficiency. P_r stands for the optical signal power that is incident on the receiver and $E_s = P_r T_s / Q$ corresponds to the received optical energy per symbol along the direct link S-D.

As a first step in the cooperation strategies, a PPM symbol $s \in \{1, \dots, Q\}$ is transmitted from S to D and the relays. Denote by $\mathbf{Y^{(n)}} = [Y_1^{(n)}, \dots, Y_Q^{(n)}]$ the Q-dimensional decision vector observed at D for $n = 0$ and at R_n for $n = 1, \dots, N_r$ where $Y_q^{(n)}$ corresponds to the number of photoelectrons detected in the q-th slot along the link S-D for $n = 0$ and along the link S-R_n for $n \neq 0$. In the absence of background radiation, the only source of photoelectrons is the information-carrying light signal resulting in $Y_q^{(n)} = 0$ for $q \neq s$. In this case, $Y_s^{(n)}$ can be modeled as a Poisson random variable (r.v.) with parameter:

$$\mathrm{E}[Y_s^{(n)}] = \begin{cases} \frac{1}{2N_r+1} a_0^2 \lambda_s, & n = 0; \\ \frac{1}{2N_r+1} \beta_1^{(n)} a_{\mathrm{s},n}^2 \lambda_s, & n = 1, \dots, N_r. \end{cases} \quad (2)$$

where $\beta_1^{(n)} = \left(\frac{d_{SD}}{d_{SR_n}}\right)^2$ is a gain factor associated with the link S-R$_n$ where d_{SD} and d_{SR_n} stand for the distances from S to D and from S to R$_n$, respectively. The total power is distributed evenly among the $2N_r+1$ S-D, S-R and R-D links.

For simple-DF, R$_n$ decides in favor of the slot of $\mathbf{Y^{(n)}}$ having the maximum number of photoelectrons. In the case of ties (all slots are empty since the system is operating under the quantum limit), R$_n$ breaks the tie randomly and forwards the corresponding symbol to D. On the other hand, in selective-DF, a symbol is retransmitted to D only in the case where a nonzero count was observed at R$_n$ or, otherwise, R$_n$ backs off. Denote by $\hat{s}^{(n)}$ the symbol transmitted by R$_n$. The corresponding decision vector at D can be written as $\mathbf{Z^{(n)}} = [Z_1^{(n)}, \dots, Z_Q^{(n)}]$ where $Z_q^{(n)} = 0$ for $q \neq \hat{s}^{(n)}$ and $Z_{\hat{s}^{(n)}}^{(n)}$ is a Poisson r.v.:

$$\mathrm{E}[Z_q^{(n)}] = \frac{1}{2N_r+1} \beta_2^{(n)} a_{n,\mathrm{d}}^2 \lambda_s \quad (3)$$

where $\beta_2^{(n)} = \left(\frac{d_{SD}}{d_{R_n D}}\right)^2$ with $d_{R_n D}$ corresponding to the distance between R$_n$ and D.

For simple-DF, D decides in favor of the nonempty slot of $\mathbf{Y^{(0)}}$. In case where $\mathbf{Y^{(0)}} = \mathbf{0}_Q$ where $\mathbf{0}_Q$ corresponds to the Q-dimensional all-zero vector, D inspects the decision vectors $\mathbf{Z^{(1)}}, \dots, \mathbf{Z^{(N_r)}}$ and performs a majority decision among the positions of the nonzero counts of these vectors. In other words, D decides in favor of the position that is repeated the largest number of times. On the other hand, the decoding strategy implemented at the relays in the case of selective-DF ensures that these relays are either backing-off or forwarding the correct symbol. In fact, in the absence of background radiation, a nonzero count in $\mathbf{Y^{(n)}}$ implies that the PPM symbol was detected correctly at R$_n$. Consequently, in selective-DF, D decides in favor of any non-empty slot of $\mathbf{Y^{(0)}}, \mathbf{Z^{(1)}}, \dots, \mathbf{Z^{(N_r)}}$. Note that both cooperation strategies can be implemented in the absence of channel state information (CSI) at the transmitter and receiver sides.

III. Performance Analysis

The channel state is defined by the vector $A \triangleq [a_0, a_{\mathrm{s},1}, \dots, a_{\mathrm{s},N_r}, a_{1,\mathrm{d}}, \dots, a_{N_r,\mathrm{d}}]$. For the sake of notational simplicity, we define $k_0 \triangleq P a_0^2 \lambda_s$, $k_1^{(n)} \triangleq P \beta_1^{(n)} a_{\mathrm{s},n}^2 \lambda_s$ and $k_2^{(n)} \triangleq P \beta_2^{(n)} a_{n,\mathrm{d}}^2 \lambda_s$ where $P \triangleq \frac{1}{2N_r+1}$. In what follows, $P_{e|A}^{(N_r)}$ and $P_e^{(N_r)}$ stand for the conditional symbol-error probability (SEP) and average SEP with N_r relays, respectively.

1) Selective-DF: For this scheme, an error occurs with probability $\frac{Q-1}{Q}$ (tie breaking) only when $\mathbf{Y^{(0)}} = \mathbf{Z^{(1)}} = \dots = \mathbf{Z^{(N_r)}} = \mathbf{0}_Q$. On the other hand, $\mathbf{Z^{(n)}} \neq \mathbf{0}_Q$ if and only if R$_n$ is not backing off and a nonzero photoelectron count was observed along the link R$_n$−D. In other words, $\mathbf{Z^{(n)}} \neq \mathbf{0}_Q$ if and only if $Y_s^{(n)} > 0$ with probability $1 - e^{-k_1^{(n)}}$ and $Z_{\hat{s}^{(n)}}^{(n)} > 0$ with probability $1 - e^{-k_2^{(n)}}$ resulting in:

$$P_{e|A}^{(N_r)} = \frac{Q-1}{Q} \Pr(\mathbf{Y^{(0)}} = \mathbf{0}_Q) \prod_{n=1}^{N_r} \left(1 - \Pr(\mathbf{Z^{(n)}} \neq \mathbf{0}_Q)\right) \quad (4)$$

$$= \frac{Q-1}{Q} e^{-k_0} \prod_{n=1}^{N_r} \left(e^{-k_1^{(n)}} + e^{-k_2^{(n)}} - e^{-k_1^{(n)}} e^{-k_2^{(n)}}\right) \quad (5)$$

Averaging the above probability results in:

$$P_e^{(N_r)} = \frac{Q-1}{Q} \frac{1}{1+P\lambda_s} \prod_{n=1}^{N_r} \left(\frac{1}{1+P\beta_1^{(n)}\lambda_s} + \frac{1}{1+P\beta_2^{(n)}\lambda_s} - \frac{1}{1+P\beta_1^{(n)}\lambda_s} \frac{1}{1+P\beta_2^{(n)}\lambda_s} \right) \quad (6)$$

showing that $P_e^{(N_r)}$ scales asymptotically as $\lambda_s^{-(N_r+1)}$ implying that selective-DF achieves a diversity order of $N_r + 1$ which is the best that can be achieved with N_r relays.

2) Simple-DF: For $N_r = 1$ relay, the conditional SEP of simple-DF was derived in [5] and it was shown that the diversity order is equal to 2.

For $N_r = 2$, assume that the symbol $s \in \{1, \dots, Q\}$ was transmitted. Denote by $p_e^{(n)}$ the conditional probability of error at R$_n$. An error occurs at R$_n$ only when $Y_s^{(n)} = 0$; in this case, R$_n$ makes a random decision among the Q slots resulting in:

$$p_e^{(n)} = \frac{Q-1}{Q} \Pr(Y_s^{(n)} = 0) = \frac{Q-1}{Q} e^{-k_1^{(n)}} \quad (7)$$

On the other hand, a correct decision will be made at D when $Y_s^{(0)} > 0$. Consequently:

$$P_{e|A}^{(2)} = \Pr(Y_s^{(0)} = 0) \Big[\Pr(Z_{\hat{s}^{(1)}}^{(1)} = 0) \Pr(Z_{\hat{s}^{(2)}}^{(2)} = 0) p_{0,1}^{(2)} + \Pr(Z_{\hat{s}^{(1)}}^{(1)} > 0) \Pr(Z_{\hat{s}^{(2)}}^{(2)} = 0) p_{1,1}^{(2)} + \Pr(Z_{\hat{s}^{(1)}}^{(1)} = 0) \Pr(Z_{\hat{s}^{(2)}}^{(2)} > 0) p_{1,2}^{(2)} + \Pr(Z_{\hat{s}^{(1)}}^{(1)} > 0) \Pr(Z_{\hat{s}^{(2)}}^{(2)} > 0) p_{2,1}^{(2)} \Big] \quad (8)$$

where $p_{i,j}^{(n)}$ is defined as the probability of error with n relays when nonzero photoelectron counts (at D) are observed from i relays. The integer j is introduced for indexing the possible choices of these i relays out of the n available relays ($j = 1, \dots, \binom{n}{i}$). In (8), $p_{0,1}^{(2)} = \frac{Q-1}{Q}$ since the case $Y_s^{(0)} = Z_{\hat{s}^{(1)}}^{(1)} = Z_{\hat{s}^{(2)}}^{(2)} = 0$ implies that $\mathbf{Y^{(0)}} = \mathbf{Z^{(1)}} = \mathbf{Z^{(2)}} = \mathbf{0}_Q$ resulting in a random decision taken at D. On the other hand, $p_{1,1}^{(2)} = p_e^{(1)}$. In fact when $Y_s^{(0)} = 0$, $Z_{\hat{s}^{(2)}}^{(2)} = 0$ and $Z_{\hat{s}^{(1)}}^{(1)} > 0$, D will decide in favor of $\tilde{s} = \hat{s}^{(1)}$ resulting in an erroneous decision with probability $p_e^{(1)}$. In the same way, $p_{1,2}^{(2)} = p_e^{(2)}$. When $Y_s^{(0)} = 0$, $Z_{\hat{s}^{(1)}}^{(1)} > 0$ and $Z_{\hat{s}^{(2)}}^{(2)} > 0$, D will decide in favor of either $\tilde{s} = \hat{s}^{(1)}$ or $\tilde{s} = \hat{s}^{(2)}$. In fact, when $\hat{s}^{(1)} = \hat{s}^{(2)}$, the decision will be $\tilde{s} = \hat{s}^{(1)} = \hat{s}^{(2)}$ and when $\hat{s}^{(1)} \neq \hat{s}^{(2)}$, D will decide randomly in favor of either $\tilde{s}=\hat{s}^{(1)}$ or $\tilde{s}=\hat{s}^{(2)}$. Consequently:

$$p_{2,1}^{(2)} = p_e^{(1)}p_e^{(2)} + \frac{1}{2}(1-p_e^{(1)})p_e^{(2)} + \frac{1}{2}p_e^{(1)}(1-p_e^{(2)}) \quad (9)$$

$$= \frac{Q-1}{2Q}\left[e^{-k_1^{(1)}} + e^{-k_1^{(2)}}\right] \quad (10)$$

where an erroneous decision will be made at D when both relays are making errors. In the case where one relay is in error and the other not, the random tie breaking between the correct and wrong symbols will result in an erroneous decision with probability $1/2$.

Replacing $\{p_{0,1}^{(2)}, p_{1,1}^{(2)}, p_{1,2}^{(2)}, p_{2,1}^{(2)}\}$ in (8) results in:

$$P_{e|A}^{(2)} = e^{-k_0}\Bigg[e^{-k_2^{(1)}}e^{-k_2^{(2)}}\frac{Q-1}{Q} + (1-e^{-k_2^{(1)}})e^{-k_2^{(2)}}\frac{Q-1}{Q}e^{-k_1^{(1)}} + e^{-k_2^{(1)}}(1-e^{-k_2^{(2)}})\frac{Q-1}{Q}e^{-k_1^{(2)}} + (1-e^{-k_2^{(1)}})(1-e^{-k_2^{(2)}})\frac{Q-1}{2Q}\left(e^{-k_1^{(1)}} + e^{-k_1^{(2)}}\right)\Bigg] \quad (11)$$

Equation (11) scales asymptotically as: $P_{e|A}^{(2)} \approx \frac{Q-1}{2Q}e^{-k_0}\left[e^{-k_1^{(1)}} + e^{-k_1^{(2)}}\right]$ which results in: $P_e^{(2)} \approx \frac{Q-1}{2Q}\frac{1}{1+P\lambda_s}\left[\frac{1}{1+P\beta_1^{(1)}\lambda_s} + \frac{1}{1+P\beta_1^{(2)}\lambda_s}\right]$. This constitutes a rather surprising finding associated with simple-DF where increasing the number of relays from $N_r = 1$ to $N_r = 2$ does not result in any increase in the diversity order that remains equal to 2.

Before tackling the general case of a N_r-relay network, we consider the special case $N_r = 3$ that will shed more light on the behavior of the system. The conditional SEP of simple-DF with 3 relays can be written as:

$$\begin{aligned} P_{e|A}^{(3)} = e^{-k_0}\Bigg[& e^{-k_2^{(1)}}e^{-k_2^{(2)}}e^{-k_2^{(3)}}p_{0,1}^{(3)} \\ &+ \sum_{i=1}^{3}(1-e^{-k_2^{(i)}})e^{-k_2^{(\pi(i))}}e^{-k_2^{(\pi^2(i))}}p_{1,i}^{(3)} \\ &+ \sum_{i=1}^{3}(1-e^{-k_2^{(i)}})(1-e^{-k_2^{(\pi(i))}})e^{-k_2^{(\pi^2(i))}}p_{2,i}^{(3)} \\ &+ (1-e^{-k_2^{(1)}})(1-e^{-k_2^{(2)}})(1-e^{-k_2^{(3)}})p_{3,1}^{(3)}\Bigg] \end{aligned} \quad (12)$$

where the function $\pi^k(.)$ performs a cyclic permutation of order k over the elements of $\{1,2,3\}$:

$$\pi^k(i) = (i+k-1) \bmod 3 + 1 \quad (13)$$

such that $\{i, \pi(i), \pi^2(i)\} = \{1,2,3\}$ for all values of $i \in \{1,2,3\}$.

In (12), the probability $p_{0,1}^{(3)}$ corresponds to the event where zero photoelectron counts are observed from all relays as well as the source. In this case, D decides randomly in favor of any one of the slots resulting in $p_{0,1}^{(3)} = \frac{Q-1}{Q}$. The probability $p_{1,i}^{(3)}$ corresponds to the event where a nonzero photoelectron count is observed only from the relay R_i. In this case, D decides in favor of $\tilde{s}=\hat{s}^{(i)}$ resulting in $p_{1,i}^{(3)} = p_e^{(i)} = \frac{Q-1}{Q}e^{-k_1^{(i)}}$ which is the error probability at R_i. The probability $p_{2,i}^{(3)}$ corresponds to the event where nonzero photoelectron counts are observed only from the two relay R_i and $R_{\pi(i)}$. An analysis similar to the one performed for calculating $p_{2,1}^{(2)}$ in (9) and (10) shows that $p_{2,i}^{(3)} = \frac{Q-1}{2Q}\left[e^{-k_1^{(i)}} + e^{-k_1^{(\pi(i))}}\right]$. Finally, $p_{3,1}^{(3)}$ stands for the error probability when nonzero counts are observed from the 3 relays. In this case, D follows the decision taken by the majority of these relays and $p_{3,1}^{(3)}$ can be written as:

$$p_{3,1}^{(3)} = p_e^{(1)}p_e^{(2)}p_e^{(3)}p_{3,1,0}^{(3)} + p_{3,1,2}^{(3)}\sum_{i=1}^{3}(1-p_e^{(i)})(1-p_e^{(\pi(i))})p_e^{(\pi^2(i))} + (1-p_e^{(1)})(1-p_e^{(2)})(1-p_e^{(3)})p_{3,1,3}^{(3)} + p_{3,1,1}^{(3)}\sum_{i=1}^{3}(1-p_e^{(i)})p_e^{(\pi(i))}p_e^{(\pi^2(i))} \quad (14)$$

where $p_{3,1,0}^{(3)} = 1$ (resp. $p_{3,1,3}^{(3)} = 0$) since in this case all the relays are making erroneous (resp. correct) decisions. In the same way, $p_{3,1,2}^{(3)} = 0$ since in this case 2 relays (out of 3) are making correct decisions and D will follow the decision made by these relays that form the majority. $p_{3,1,1}^{(3)}$ corresponds to the case where one relay R_i is making a correct decision. In this case, two scenarios are possible. (i): $\hat{s}^{(i)} = s$ and $\hat{s}^{(\pi(i))} = \hat{s}^{(\pi^2(i))} \neq s$ implying that the 2 relays that are making errors decide by chance in favor of the same symbol. In this case, D decides in favor of the majority resulting in a error with probability 1. (ii): $\hat{s}^{(i)} = s$ and $\hat{s}^{(\pi(i))} \neq \hat{s}^{(\pi^2(i))} \neq s$ and D makes a random choice among 3 possible values resulting in an error with probability $2/3$. Consequently, $p_{3,1,1}^{(3)} = 1\frac{1}{Q-1} + \frac{2}{3}\frac{Q-2}{Q-1} = \frac{2Q-1}{3(Q-1)}$. Therefore, (14) simplifies to:

$$p_{3,1}^{(3)} = \frac{Q-1}{Q}\left[\frac{2Q-1}{3Q}\left[e^{-(k_1^{(1)}+k_1^{(2)})} + e^{-(k_1^{(1)}+k_1^{(3)})} + e^{-(k_1^{(2)}+k_1^{(3)})}\right] - \frac{Q-1}{Q}e^{-(k_1^{(1)}+k_1^{(2)}+k_1^{(3)})}\right] \quad (15)$$

Replacing $p_{0,1}^{(3)}$, $\{p_{1,i}^{(3)}, p_{2,i}^{(3)}\}_{i=1}^{3}$ and $p_{3,1}^{(3)}$ by their values in (12) and performing an asymptotic analysis results in:

$$\begin{aligned} P_{e|A}^{(3)} \approx \frac{Q-1}{Q}e^{-k_0}\Bigg[&\frac{1}{2}\Big[e^{-k_2^{(3)}}(e^{-k_1^{(1)}} + e^{-k_1^{(2)}}) \\ &+ e^{-k_2^{(1)}}(e^{-k_1^{(2)}} + e^{-k_1^{(3)}}) + e^{-k_2^{(2)}}(e^{-k_1^{(3)}} + e^{-k_1^{(1)}})\Big] \\ &+ \frac{2Q-1}{3Q}\left[e^{-(k_1^{(1)}+k_1^{(2)})} + e^{-(k_1^{(1)}+k_1^{(3)})} + e^{-(k_1^{(2)}+k_1^{(3)})}\right]\Bigg] \end{aligned} \quad (16)$$

Since $P_{e|A}^{(3)}$ is approximated by the weighted sum of different terms corresponding to the product of three decreasing exponential functions, then $P_e^{(3)}$ scales asymptotically as λ_s^{-3} showing that the diversity order with 3 relays is equal to 3.

For $N_r > 3$, the expressions of the conditional SEP become cumbersome. Moreover, since cooperation results in the highest performance gains for large values of λ_s, we further proceed with an asymptotic analysis that allows us to reach the following main result.

Proposition 1: For cooperative FSO systems with N_r relays, simple-DF achieves a diversity order of $\lceil \frac{N_r}{2} \rceil + 1$.

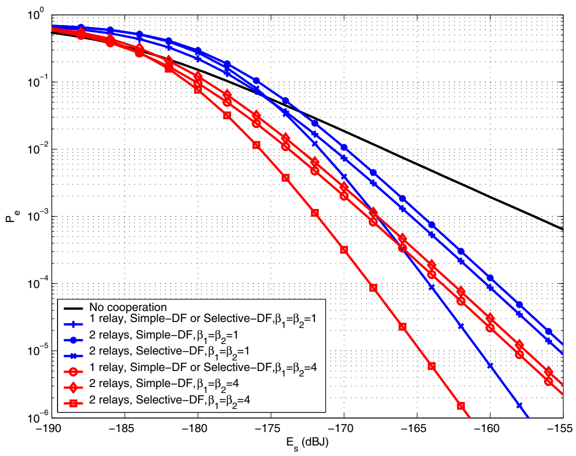

Fig. 1. Performance of 4-PPM with 1 relay and 2 relays.

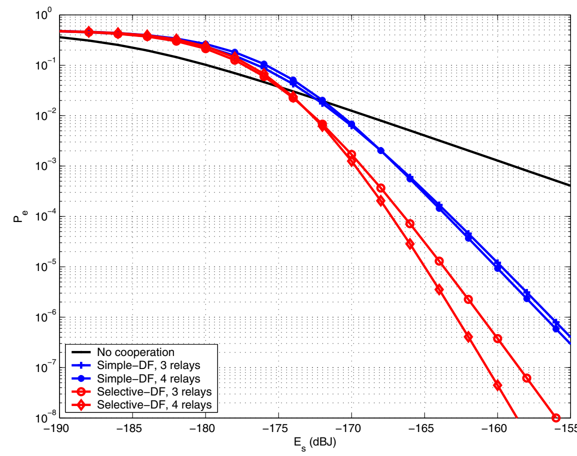

Fig. 2. Performance of 2-PPM with 3 and 4 relays for $\beta_1 = \beta_2 = 1$.

Proof: The proof is provided in the appendix.

Comparing simple-DF with selective-DF shows that the latter is capable of achieving higher diversity orders since $N_r + 1 \geq \lceil \frac{N_r}{2} \rceil + 1$. On the other hand, it is always more advantageous to deploy an odd number of relays with simple-DF. For example, $2k-1$ relays result in the same diversity order (of $k+1$) as $2k$ relays with a reduced system complexity.

IV. Numerical Results

For simulation purposes, we assume that all relays are at the same distances from the source and the destination resulting in $\beta_1^{(1)} = \cdots = \beta_1^{(N_r)} \triangleq \beta_1$ and $\beta_2^{(1)} = \cdots = \beta_2^{(N_r)} \triangleq \beta_2$.

Fig. 1 shows the performance of 4-PPM with one relay and two relays. For $N_r = 1$, simple-DF and selective-DF result in exactly the same performance. The slopes of the SEP curves indicate that both strategies result in the same diversity order of two. For $N_r = 2$, the results support the finding of section III where simple-DF achieves a diversity order of 2 while selective-DF achieves the full diversity order of 3. This figure also shows that deploying simple-DF with 1 relay is better than deploying it with 2 relays.

Fig. 2 shows the performance of 2-PPM with three and four relays. This figure shows that increasing the number of relays with simple-DF from 3 to 4 does not result in any increase in the diversity order and the only advantage resides in a negligible performance gain in the order of 0.3 dB observed at large values of E_s. As a conclusion, in order to take advantage from the presence of more relays in the neighborhood of the source and the destination, a selective-DF protocol needs to be implemented at the relays.

V. Conclusion

We investigated simple-DF and selective-DF as candidate solutions for relay-assisted FSO communication systems with any number of relays. The theoretical asymptotic analysis and the numerical results showed that the simple-DF protocol is highly suboptimal for FSO systems since it is not capable of exploiting the entire underlying spatial diversity.

Appendix

We start with the following definition: we say that a function $f(\lambda_s)$ is exponentially equal to b, denoted by $f(\lambda_s) \doteq b$, when this function is equal to a weighted sum of the product of different decreasing exponential functions in λ_s and in one path gain such that the minimum number of terms in these products is equal to b. For example, from (11), $P_{e|A}^{(2)} \doteq 2$. In an equivalent manner, the average of $f(\lambda_s)$ over the pdf $p(A)$ of the channel state Rayleigh distributed vector A will scale asymptotically as λ_s^{-b}: $\lim_{\lambda_s \to +\infty} \frac{\log \int f(\lambda_s) p(A) \mathrm{d}A}{\log \lambda_s} = -b$. For example, from (11), $P_{e|A}^{(2)} \doteq 2$ meaning that $P_e^{(2)}$ behaves asymptotically as λ_s^{-2} for large values of λ_s.

Extending (8) and (12), the conditional SEP of simple-DF with N_r relays can be written as:

$$P_{e|A}^{(N_r)} = e^{-k_0} \sum_{i=0}^{N_r} \sum_{j=1}^{\binom{N_r}{i}} p_{i,j}^{(N_r)} \Pi_1 \Pi_2$$

$$\triangleq e^{-k_0} \sum_{i=0}^{N_r} \sum_{j=1}^{\binom{N_r}{i}} p_{i,j}^{(N_r)} \prod_{m_1 \in \mathcal{I}_j^{(N_r,i)}} \left(1 - e^{-k_2^{(m_1)}}\right) \prod_{m_2 \in \{1,\ldots,N_r\} \setminus \mathcal{I}_j^{(N_r,i)}} e^{-k_2^{(m_2)}} \quad (17)$$

where the probability $p_{i,j}^{(N_r)}$ is defined in the same way as in (12). The sets $\mathcal{I}_1^{(N_r,i)}, \ldots, \mathcal{I}_{\binom{N_r}{i}}^{(N_r,i)}$ are all possible subsets of $\{1, \ldots, N_r\}$ having i elements each. For example, for $N_r = 3$, $\mathcal{I}_1^{(3,1)} = \{1\}$, $\mathcal{I}_2^{(3,1)} = \{2\}$, $\mathcal{I}_3^{(3,1)} = \{3\}$ and $\mathcal{I}_1^{(3,2)} = \{1,2\}$, $\mathcal{I}_2^{(3,2)} = \{1,3\}$, $\mathcal{I}_3^{(3,2)} = \{2,3\}$. Evidently, $\mathcal{I}_1^{(N_r,0)}$ is empty and $\mathcal{I}_1^{(N_r,N_r)} = \{1, \ldots, N_r\}$. Since $\Pi_1 \doteq 0$ and $\Pi_2 \doteq N_r - i$, then the diversity order d_{N_r} with N_r relays is given by:

$$P_{e|A}^{(N_r)} \doteq \min_{i=0,\ldots,N_r} \{1 + \min_{j=1,\ldots,\binom{N_r}{i}} f(N_r,i,j) + N_r - i\} \triangleq d_{N_r} \quad (18)$$

where $p_{i,j}^{(N_r)} \doteq f(N_r, i, j)$.

We will prove proposition 1 by induction. From (11) and (16), the induction holds for $N_r = 2$ and $N_r = 3$, respectively. Assume that $d_{N_r - 1} = \lceil \frac{N_r - 1}{2} \rceil + 1$ and prove that $d_{N_r} = \lceil \frac{N_r}{2} \rceil + 1$.

The main building block in our proof is that $f(N_r, i, j)$ depends only on i. Evidently, $f(N_r, i, j)$ does not depend on j which is nothing but an index used for numbering the different events. For simple-DF, the majority choice is made exclusively among the i relays that result in nonzero photoelectron counts at D while the remaining $N_r - i$ relays will be ignored in the decision process. In other words, D receives nothing from these $N_r - i$ relays and it proceeds very simply as if they do not exist. For example, $p_{0,1}^{(N_r)} = \frac{Q-1}{Q}$ while $p_{1,j}^{(N_r)} = p_e^{(j)} = \frac{Q-1}{Q} e^{-k_1^{(j)}}$ from (7) implying that $f(N_r, 0, 1) = 0$ and $f(N_r, 1, j) = 1$ for all values of N_r. Now writing $f(N_r, i, j)$ as $f(i)$ in (18) results in:

$$\begin{aligned} d_{N_r} &= \min_{i=0,\dots,N_r} \{1 + f(i) + N_r - i\} \\ &= \min\left\{ \min_{i=0,\dots,N_r-1} \{1+f(i)+N_r-i\}, 1+f(N_r)+N_r-N_r \right\} \\ &= \min\left\{ 1+ \min_{i=0,\dots,N_r-1} \{1+f(i)+(N_r-1)-i\}, 1+f(N_r) \right\} \end{aligned} \tag{19}$$

$$\begin{aligned} &= \min\{1 + d_{N_r-1}, 1 + f(N_r)\} \\ &= \min\left\{ \left\lceil \frac{N_r - 1}{2} \right\rceil + 2, 1 + f(N_r) \right\} \end{aligned} \tag{20}$$

where $p_{N_r,1}^{(N_r)} \doteq f(N_r)$. This probability can be written as:

$$p_{N_r,1}^{(N_r)} = \sum_{i=0}^{N_r} p_{N_r,1,i}^{(N_r)} \sum_{j=1}^{\binom{N_r}{i}} \prod_{m_1 \in \mathcal{I}_j^{(N_r,i)}} \left(1 - p_e^{(m_1)}\right) \prod_{m_2 \in \{1,\dots,N_r\} \setminus \mathcal{I}_j^{(N_r,i)}} p_e^{(m_2)} \tag{21}$$

where the sets $\mathcal{I}_j^{(N_r,i)}$ are defined in the same way as in (17) and $p_e^{(m)}$ is the error probability at the m-th relay given in (7). The probability $p_{N_r,1,i}^{(N_r)}$ corresponds to the probability of error when N_r nonzero photoelectron counts (at D) are received via the N_r indirect links where i of these counts are observed in the correct slot while the other $N_r - i$ counts are distributed among the remaining $Q - 1$ slots. Consequently, $p_{N_r,1,i}^{(N_r)}$ depends on the manner in which the majority among the N_r relays is selected. Therefore, $p_{N_r,1,i}^{(N_r)}$ is a function of N_r, Q and i and it does not depend on any channel gain in A. As a conclusion, $p_{N_r,1,i}^{(N_r)} \doteq 0$. For example, $p_{2,1,1}^{(2)} = \frac{1}{2}$ and $p_{3,1,1}^{(3)} = \frac{2Q-1}{3(Q-1)}$ from section III. In general, a closed-form general expression of $p_{N_r,1,i}^{(N_r)}$ can not be reached and the evaluation of this probability becomes tedious for large values of N_r. On the other hand, our analysis will be based on $p_{N_r,1,i}^{(N_r)} \doteq 0$ independently from the specific value of $p_{N_r,1,i}^{(N_r)}$.

On the other hand, $p_{N_r,1,i}^{(N_r)} = 0$ when $i > N_r - i$ because even in the extreme case where the $N_r - i$ nonzero counts happen to be in the same erroneous slot, the majority will remain for the i nonzero counts in the correct slot implying that a correct decision will be made in this case. The above inequality implies that $i > \frac{N_r}{2}$ and this inequality is satisfied (and hence $p_{N_r,1,i}^{(N_r)}$ will be zero) when $i \geq \lceil \frac{N_r}{2} \rceil$ if N_r is odd and when $i \geq \lceil \frac{N_r}{2} \rceil + 1$ if N_r is even. Finally, since $p_{N_r,1,i}^{(N_r)} \doteq 0$ and $p_e^{(n)} \doteq 1 \ \forall\ n$ (from (7)), then (21) implies that:

$$\begin{aligned} p_{N_r,1}^{(N_r)} &\doteq f(N_r) \\ &= \begin{cases} \min_{i=0,\dots,\lceil \frac{N_r}{2} \rceil - 1} [N_r - i] = N_r - \lceil \frac{N_r}{2} \rceil + 1, & N_r \text{ odd;} \\ \min_{i=0,\dots,(\lceil \frac{N_r}{2} \rceil + 1) - 1} [N_r - i] = N_r - \lceil \frac{N_r}{2} \rceil, & N_r \text{ even.} \end{cases} \\ &= \begin{cases} k, & N_r = 2k-1; \\ k, & N_r = 2k. \end{cases} \end{aligned} \tag{22}$$

When $N_r = 2k - 1$ is odd, (20) and (22) imply that:

$$\begin{aligned} d_{N_r} = d_{2k-1} &= \min\left\{ \left\lceil \frac{(2k-1)-1}{2} \right\rceil + 2, 1 + k \right\} \\ &= \min\{k+1, k+1\} = k+1 \end{aligned} \tag{23}$$

When $N_r = 2k$ is even, (20) and (22) imply that:

$$\begin{aligned} d_{N_r} = d_{2k} &= \min\left\{ \left\lceil k - \frac{1}{2} \right\rceil + 2, 1 + k \right\} \\ &= \min\{k+2, k+1\} = k+1 \end{aligned} \tag{24}$$

Equations (23) and (24) can be written as $d_{N_r} = \lceil \frac{N_r}{2} \rceil + 1$ thus completing the proof of proposition 1.

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