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A Scalable Family of Unitary and Differential Space-Time Codes for Ultra Wideband Communications

By

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Karl Fayad

Abstract

Higher data rates have always been the main demand ever since the emergence of telecommunications. This quest for faster and more reliable communications has led to the evolution of numerous generations of wired and wireless communication. One of the most recent and most promising wireless technologies is the Ultra Wideband (UWB) communication. This technology, associated with the usage of Multiple Input Multiple Output (MIMO) techniques and Space Time (ST) codes, can be considered as the future for short-range wireless communications. Following from the excessive delay spreads of the UWB channels, channel estimation is rather challenging and, consequently, coherent solutions are being slowly disregarded shifting the focus to differential techniques. While the literature related to coherent ST codes for UWB is quite rich, limited contributions targeted the more challenging problem of differential ST codes.

In this work, we propose a novel unipolar ST code for UWB. The proposed technique can be completely implemented in an analog manner while still ensuring full spatial diversity. Being differential and unitary, the presented system further guarantees the optimal detectability of the received signals without any need for channel estimation. The above features keep the system complexity at a minimum while ensuring full

reliability. In addition, the proposed codebook can be partitioned into a number of sub-codebooks with predefined structure resulting in additional scalability capabilities that provide a compromise between data rate and decoding complexity.

Keywords: Ultra-Wideband, Space-Time, Codebook, Codewords, Differential, Diversity, Unitary.

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Chapter 1

Introduction

Ever since the first data transmissions using wired and wireless systems, the demand for increased data rates has always been there. This was made possible due to the brisk improvements in semiconductors, reaching a phase where the wireless personal area networks (WPAN) and the local area networks (WLAN) which are being highly deployed, can barely undergo the rates and demands of the available wireless services. This was a major motivation to improve the performance of this specific short range wireless connectivity area, which led to the Ultra Wideband (UWB) technology. Section 1.1 presents a short historical review, section 1.2 provides a quick overview of the UWB technology. After that, different signaling formats adopted for UWB will be explained in section 1.3. In addition, section 1.4 describes the different multiple access techniques. Finally, the various applications and challenges of the UWB technology will be discussed in section 1.5 and 1.6 respectively.

1.1 History

UWB, is considered as one of the oldest forms used for radio communications. Its main idea was first used in the beginning of the 19th century. Its roots can be linked back to the primary Marconi spark gap radio. Guglielmo Marconi broadcasted Morse code sequences using a radio transmitter and relying on spark gap [8]. At that time, UWB's main advantages were could not shoot up and shine. It was only until the late 1960's

that UWB's promising future was discovered and heavy research was conducted at the Sperry Research Center out of which emerged most of the basic concepts and patents for UWB. Until 1989, different terms including "impulse", "baseband", "short-pulse", "monopulse", and "carrier-free" were employed to refer to what is now known as UWB [8]. Kept as a secret, UWB was exclusively used by the Defense Departments in military applications until the early 2000's when the permission of the technology's commercial usage was granted by the Federal Communication Commission (FCC). An unlicensed UWB operation at a frequency range of 3.1 GHz to 10.6 GHz with a low EIRP level of -41.3 dBm/MHz, was authorized in 2002 by FCC. The spectral mask set by the FCC is shown in the figure below. The power spectral density at its highest values is yet limited to -41.3 dBm/MHz as set by the Part 15 emission boundaries for radio interference control [7]. Before its wide-spreading, UWB was specifically used for

Figure 1.1: FCC spectral mask for UWB communication

certain applications which mainly include but are not restricted to radars. Nevertheless, subsequent to that phase, UWB communication systems faced immense improvements and became widely used as a replacement to narrow band systems.

1.2 Overview

As previously mentioned, UWB systems are quite different as opposed to their counterpart conventional wireless systems. They actually transmit signals over a very wide frequency spectrum compared to narrowband and spread spectrum. Furthermore, short pulses with ultra low power are used in UWB against continuous waves used in the other communication systems. The figure below shows the power spectral density of the different technologies against their frequencies [20].

Figure 1.2: Power spectral density comparison of different wireless systems

The initial definition of a UWB signal was provided in 1990 by DARPA. A fractional bandwidth which is computed through:

$$f_{BW} = \frac{f_H - f_L}{f_c} \quad (1.1)$$

where f_{BW} is the fractional bandwidth, f_L and f_H are the lowest and highest cutoff frequency at -3 dB spectrum and $f_c = 0.5(f_H + f_L)$ is the center frequency. Based on the previous definitions, a signal would be classified as a UWB signal when the fractional bandwidth f_{BW} has a value greater than 0.25. Along with the restrictions set by the FCC in 2002 as described earlier, came a small change in UWB's fractional bandwidth acceptance threshold which has been modified to 0.2 instead of 0.25. Furthermore, the cutoff frequencies are set to be measured at -10 dB Besides the stated definitions, a

signal is also considered as UWB if its bandwidth has a value greater or equal to 500 MHz [14, 20].

In comparison to the other conventional wireless communication systems, UWB systems have several advantages. The main advantage would be its high data rate which exceeds the 500 Mb/s for short and medium ranges due to its ultra wide bandwidth of several GHz. In addition, it provides speed at low cost and low complexity. Besides that, its low power spectral density which is even below the noise level, allows it to coexist with the remaining services. Furthermore, this specific trait makes it quite undetectable and hard to interfere with. On top of that, UWB systems are considered immune to multi path fading making it quite robust given that it can operate in non line of sight (NLOS) environments with low loss. Not to forget that the UWB pulses used have durations of a fraction of a nano second making it useful and accurate for tracking applications [20].

1.3 Modulation Formats

In this section, some of the mostly employed signaling schemes for the Ultra Wideband technology will be explained. The modulation process bases itself on the variation of different signaling features in order to convey the required information. Some of the signal traits that can be altered to do the job are the frequency, the amplitude and the phase.

1.3.1 Pulse Amplitude Modulation

Pulse Amplitude Modulation (PAM) is based on the variation of the amplitude of the different pulses. For instance, a 2-ary PAM would transmit pulses of two different amplitudes, where the higher amplitude pulse would represent a "1" and the lower amplitude pulse would represent a "0" [17]. In the case of a M-ary PAM M different amplitudes will be adopted and the k -th signal would be expressed as follows:

$$s_k(t) = a(k)w(t - kT_f) \quad (1.2)$$

where $a(k)$ denotes the amplitude of the k -th pulse, T_f denotes the frame's period and w is the pulse waveform [17]. The figure below, describes a 4-ary PAM scheme.

Figure 1.3: Example of 4-PAM

1.3.2 On-Off Keying

On-Off Keying (OOK) is another signaling scheme that is non scalable that only provides two different signal shapes and thus is used on a binary level. The absence of a pulse would signify the transmission of the "0" symbol, whereas the presence of a pulse designates that the "1" symbol has been conveyed [11]. The k -th OOK signal would be expressed as follows:

$$s_k(t) = a(k)w(t - kT_f) \quad (1.3)$$

where a can only hold two values, namely "0"s and "1"s. The figure below, describes the defined OOK scheme.

Figure 1.4: Example of OOK

1.3.3 Binary Phase Shift Keying

Binary Phase Shift Keying (BPSK) is a modulation scheme that is based on the polarity of the pulses. A "0" is expressed by a pulse with a negative polarity, whereas a "1"

would be represented by a pulse with a positive polarity [17]. The k -th BPSK signal would be expressed as follows:

$$s_k(t) = a(k)w(t - kT_f) \quad (1.4)$$

where a can only hold two values, namely "-1"s and "+1"s. The figure below, describes the defined BPSK scheme.

Figure 1.5: Example of BPSK

1.3.4 Pulse Position Modulation

Pulse Position Modulation (PPM) is a modulation scheme that is based on the position of the conveyed pulses. An M-ary PPM would have its symbol time frame decomposed into M different slots of duration δ each. The location of the transmitted pulse is directly related to the symbol being sent [2]. The k -th signal of a M-ary PPM would be expressed as follows:

$$s_k(t) = w(t - \delta m(k) - kT_f) \quad (1.5)$$

where $m(.) \in \{0, ..., M - 1\}$ is the slot position of the symbol being transmitted. The figure below, describes the defined PPM scheme.

1.4 Multiple Access Schemes

Up until now, single pulse usage is considered for symbol transmission and thus, multiple access capability has not been considered. For this purpose, and in order to further reduce the potential interference of the uninterrupted pulse transmissions,

Figure 1.6: Example of 4-PPM

different techniques have been developed. Direct Sequence (DS) and Time Hopping (TH) are two of the mostly considered multiple access schemes when it comes to UWB.

1.4.1 Direct Sequence (DS)

DS-UWB, similarly to spread spectrum techniques, employs user distinguishable sequences of UWB pulses to convey the information, where the pulses can be analogously associated to the spread spectrum's chips. The sequences are user specific and are generated pseudo randomly. They will be used to form a continuous chain of UWB pulses which can be inverted by the corresponding user. DS-UWB transmission is not suitable for PPM which has time hopping like features [4, 18]. In the case of PAM and OOK modulation, the k -th signal of the DS-UWB can be mathematically expressed as follows:

$$s^{(j)}(t) = \sum_{n=0}^{N_s-1} c_n^{(j)} a_k^{(j)} w(t - kT_s - nT_p) \quad (1.6)$$

where $a_k^{(j)}$ and $c_n^{(j)}$ -th are the j -th user's k -th data bit and n -th pseudo random code respectively. $w(t)$ is the pulse waveform, T_p is the pulse duration which equates the chip length T_c , N_s is the number of frames per data bit, and T_s is the length of a single data bit equal to $N_s T_c$. A time domain representation of the DS-UWB spreading technique is shown in the figure below.

1.4.2 Time Hopping (TH)

TH-UWB is yet another widely used transmission technique which allows multiple access. Diverse users will be associated with unique TH codes which specify different

Figure 1.7: DS-UWB time domain representation

time shifts within every time frame. In other words, a same data symbol will be transmitted several times with specific delays indicated by a TH code. TH-UWB can be used with PAM and PPM modulation schemes [18]. The k -th signal of the TH-UWB's j -th user can be represented as follows for the case of PAM:

$$s^{(j)}(t) = \sum_{n=0}^{N_s-1} a_k^{(j)} w(t - kT_s - nT_f - c_n^{(j)}T_c) \quad (1.7)$$

and as follows for the case of PPM:

$$s^{(j)}(t) = \sum_{n=0}^{N_s-1} w(t - kT_s - nT_f - c_n^{(j)}T_c - a_k^{(j)}\delta) \quad (1.8)$$

where $a_k^{(j)}$ and $c_n^{(j)}$ -th are the j -th user's k -th data bit and n -th pseudo random code respectively. $w(t)$ is the pulse waveform, T_c is the frame's slot duration, N_s is the number of frames per data bit, and T_s is the length of a single data bit equal to N_sT_f . A time domain representation of the TH-UWB transmission technique is shown in the figure below.

Figure 1.8: TH-UWB time domain representation

1.5 Applications

At the outset, the heavy usage of the UWB technology was focused around ground penetrating sounding and radar sectors by the Department of Defense for military purposes. Later on, along with the release of its secrecy, UWB got exposed to diverse kinds of research which turned out to be convenient for different commercial applications. Different major features of UWB led to its widespread, out of which its ability to provide high data rate at low cost and complexity stood out. Two different application areas are distinguished and assigned to two task groups created for exploration purposes.

1.5.1 High Data Rate Applications

This specific kind is basically for the short range indoor application. One of the major high data rate applications would be the Wireless Personal Area Network (WPAN) connecting the different personal computers and wearable devices. In addition, the next generations of Bluetooth, FireWire and Wireless Universal Serial Bus (USB), will also be based on a UWB physical layer. Similarly, the fourth generation's (4G) wireless system would also have the UWB as its physical basis. Furthermore, the different portable media players and personal computer peripherals, including the TVs, the DVDs, Printers, and Camcorders, could also be sharing high data rate connectivity through UWB without the need for personal computers. Besides that, other high data rate applications include, medical and police programs. These incorporate but are not limited to, physiologic softwares, radar monitoring for organs, and imaging applications for police rescue services enabled by UWB's wall see through capabilities. On top of that, the high data rate of UWB may possibly enable wireless monitors [5, 6].

1.5.2 Low Data Rate Applications

The number of low data rate applications has also been increasing rapidly due to UWB's extremely precise tracking capabilities. One of the major low data rate applications is the tagging sector, namely the radio frequency identification (RFID) tags. The required data

rate for RFID tagging could be much lower than 100 b/s making the UWB technology perfect for such tasks. Radar based control for the different means of transportation is also one of these applications whose job is to be aware of a vehicles' surrounding and avoid crashes. Another area would be the positioning systems, which are used for real time accurate tracking. Furthermore, there's the location aware communication systems, which incorporate both data and radar communications making full use of the UWB pulses for tracking and conveying data about different items [9].

1.6 Challenges

Many difficulties are faced with the UWB technology. When it comes to designing and implementing a UWB device, several things have to be taken into consideration, including the practicality and the cost of the formed devices. The ultra large bandwidth of the UWB technology and the unique features held by its signals, do indeed pose a bunch of challenges would it be hardware or signal processing issues. Some of these challenges will be discussed below in two different subsections. The first subsection would target some of the signal processing issues whereas the second one would target part of the hardware problems faced.

1.6.1 Signal Processing Challenges

First of all, one of the difficulties that is faced with UWB is the channel estimation. Given the nature of the technology's ultra short pulses, several multi path components would be produced which would yield a large number of different delays and various amplitudes that need to be estimated. This would require very tough estimations at incredibly high sampling rates to be able to resolve the issue [12].

A second difficulty would be pulse shaping. Given that the UWB's signal is of carrier less nature, a spectral mask has been set in order to limit any probable interference with other wireless services and provide a safe operation mode. This poses another limitation to UWB signals, leading to the necessity of applying preprocessing to the UWB pulses.

In addition, preprocessing is also needed in order to enhance the signal to noise ratio [19].

In addition, there's the synchronization issue. This timing problem has been around for a few decades. However, UWB brought this issue to a whole new level due to its carrier-less signal nature. Due to the ultra short timeframe of the UWB pulses, a small time misalignment would result in an insufficient energy extraction leading to erroneous symbol detection. For this, several data and non data aided timing synchronization techniques have been proposed [13].

1.6.2 Hardware Challenges

The second group of challenges in addition to the signal processing group would be the hardware difficulties. A first hardware challenge to the UWB technology would be the analog to digital conversion. In accordance with the Nyquist criterion, the sampling rate required for the analog to digital conversion is in the order of giga samples per second (Gsp/s). This speed is considered as a limitation making the analog to digital conversion the congestion of the digital implementations of the UWB receivers. This increased the interest of novel analog UWB schemes [10].

Another challenge would be dealing with antenna distortion. This is quite a difficult task taking into consideration the ultra large bandwidth of multiple gigahertz and the consistency needed in that huge range of frequencies. Moreover, the need of integrating the antenna structures into relatively compact devices and using quite inexpensive material, would further increase this task's difficulty [21].

Chapter 2

Technical Background

In this chapter, the main technologies related to the scalable scheme proposed in this thesis will be described. Section 2.1 will provide an overview of the Multiple Input Multiple Output technique. The Space-Time Code (ST) will be described in the subsequent section 2.2. Finally, section 2.3 will provide a description of the Space-Time Code for Ultra Wideband communications.

2.1 Multiple Input Multiple Output

Higher data rates, greater performance and better quality of service has always been a major request in the field of communications. In order to meet these requirement, different techniques were developed and explored. One of the fastest technologies that came into view is the usage of multiple antennas at the transmitter and the receiver side known as Multiple Input Multiple Output (MIMO). The latter makes use of the multiple links through smart antenna functions in order to provide better communication performance, as well as enhance link reliability and spectral efficiency. Besides that, if used properly, MIMO could offer multiplicative gains of different types in network performance [15].

Figure 2.1 shows the possible antenna arrangements. Single input single output (SISO) is the basic configuration which cannot involve smart antenna functions. Single input multiple output (SIMO) is formed of a single transmitter and multiple receivers,

Figure 2.1: Transmit and receive antenna configurations

multiple input single output (MISO) is constructed out of multiple transmitters and a single receivers, MIMO has multiple transmit and receive antennas, while MIMO multiuser (MIMO-MU) is the case where multiple transmit or receive antennas communicate with multiple users with multiple transmit or receive antennas each.

Let's consider a MIMO system with P transmit and Q receive antennas. Each of the different receive antennas would discern a different version of a transmitted signal due to the different channel responses between each of the transmit antennas and the receive antennas. The channel response of the i -th receive antenna from the j -th transmit

antenna is denoted by $h_{i,j}$. The MIMO channel can now be denoted as:

$$H = \begin{bmatrix} h_{1,1} & h_{1,2} & \dots & h_{1,P} \\ h_{2,1} & h_{2,2} & \dots & h_{2,P} \\ \vdots & \ddots & \ddots & \vdots \\ h_{Q,1} & h_{Q,2} & \dots & h_{Q,P} \end{bmatrix} \quad (2.1)$$

known as the channel transfer function, the MIMO channel matrix H defines the relation between the output and the input of the system in hand. The conveyance of the different transmitter signals can be seen as a signal vector $x = [x_1 \ x_2 \ \dots \ x_P]^T$ conveyed from the transmitter array, and thus, the signal seen at the receiver array $y = [y_1 \ y_2 \ \dots \ y_Q]^T$ can be described as follows:

$$y = Hx + n \quad (2.2)$$

where n is the $Q \times 1$ noise matrix. Multiple users can be easily integrated into this model by dedicating a different MIMO channel matrix to the receive antenna array from each of the multiple users:

$$y = \begin{bmatrix} H_1 & \dots & H_U \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_U \end{bmatrix} + n \quad (2.3)$$

where H_u is the MIMO channel of the u -th user to the receiver and x_u is the signal vector conveyed by the u -th user.

2.1.1 Array Gain

The increase in signal to noise ratio (SNR) that occurs when the signals of multiple transmit or receive antennas are combined coherently, is known as the array gain. In the case where multiple receivers are used, array gain occurs since the signals received by the multiple antennas are seen differently. They have different phases and amplitudes and thus, their coherent combination would yield an enhanced resultant signal, increasing the signal quality. To be more specific, the SNRs of the different links would be summed up to yield the global SNR. Hence, the number of receivers

would proportionally affect the improvement in signal power at the receiver. In the case of multiple transmitters, array gain can be exploited similarly. However, channel knowledge at the transmitter is needed to make proper use of it [15].

2.1.2 Diversity Gain

Similarly to array gain, diversity gain happens when several signals are combined to counter up fading issues. Fading occurs when a wireless channel's signal power fluctuates or faces rapid power variations. For this, diversity is used to diminish the losses and that is through the usage of different independent diversity branches or fading links. Consequently, when the diverse signals are combined, the probability that the global signal is in fade would decrease since each and every signal should happen to be in fade at a same instant of time. Diversity gain can occur with the usage of multiple receive antennas, known as receive diversity, or with the usage of multiple transmit antennas, known as transmit diversity. Nevertheless, the latter requires suitable design of the signals to be transmitted. One of the most popular and powerful transmit diversity techniques is the Space-Time coding [15].

2.1.3 Multiplexing Gain

Space Multiplexing (SM) is a MIMO specific feature which allows throughput improvement, an enhancement in transmission rate without the expenditure of supplementary power. The increase in transmission rate is proportional to the minimum number of transmit or receive antennas. Through demultiplexing, multiple sub streams of a fractional rate are composed out of an initial bit stream to be transmitted. The sub streams can be separately seen at the receiver under satisfactory channel conditions and having knowledge of the channel. Hence, the original sub streams can be generated after demodulation. These can be combined to yield the bit stream initially transmitted [15].

2.1.4 Interference Reduction

Since the wireless transmission is unguided, interference of the co-channels occurs frequently and thus, should be exploited. With the required knowledge of the spatial channels, interference reduction can be exploited by differentiating between the different spatial signatures and improve the signal to interference ratio. Similarly, interference reduction could also be utilized at the transmitter's side. The aim would be to adjust the signal power in a way that is optimized for its corresponding receiver and minimized for the remaining co-channel users [15].

2.2 Space-Time Code

The usage of multiple transmit or receive antennas in wireless communication systems introduced a new dimension referred to as space, which led to its naming as we now know it - Space-Time communications, also known as smart antennas or multi antenna communications. It basically relies on the redundant conveyance of a data stream, hoping that some of the transmitted copies would get through the transmitter-receiver path in an acceptable state allowing correct decoding. One of the major features that ST offers is the ability to exploit transmit diversity for the diverse antenna fading channels. Before ST codes (STC), other diversity were used to deal with fading, such as receive antenna diversity, temporal diversity, as well as frequency diversity. Nevertheless, transmit diversity gained popularity due to the desirability of keeping the complexity and burden at the base station instead of the remote units.

The space time coding scheme, first introduced by Tarokh et al. in 1998, presents a combinational design of coding, diversity and modulation. Two basic types of STC are available, the space time trellis codes (STTC) and the space time block codes (STBC). STTCs are very hard to design and have complex encoder and decoder structures. However, in addition to supplying full diversity, STTC, unlike STBC provide coding gain. Nevertheless, STBC is much easier to design and has a very simple decoding structure. In this thesis, the main focus is on the space-time block code [15, 16].

2.2.1 Space-Time Block Code Model

The STBC model is formed out of P transmit and Q receive antennas, forming a channel which is made up of $P \times Q$ sub channels. All transmit antennas convey their signals simultaneously, and thus, P signals are transmitted at any time interval. The transmitted signals can be picked up from a specific codebook which holds L different codewords [15, 16].

$$\mathcal{C} = \{C_0, C_1, \dots, C_{L-1}\} \quad (2.4)$$

where each of the various codewords is a $P \times 1$ matrix of the following form:

$$C_l = [C_l^0 \quad C_l^1 \quad \dots \quad C_l^{P-1}]^T \quad (2.5)$$

On the other hand, the overall channel matrix H would hold $P \times Q$ sub channels.

$$H = \begin{bmatrix} h_{1,1} & h_{1,2} & \dots & h_{1,P} \\ h_{2,1} & h_{2,2} & \dots & h_{2,P} \\ \vdots & \ddots & \ddots & \vdots \\ h_{Q,1} & h_{Q,2} & \dots & h_{Q,P} \end{bmatrix} \quad (2.6)$$

The received signal can thus be described as:

$$y = Hx + n \quad (2.7)$$

where y is the received signal, x is the transmitted signal which can be any one of the codebook's codewords, and n is a $Q \times 1$ noise matrix. We define the $P \times L$ error matrix B as:

$$B = \begin{bmatrix} c_0^0 - \hat{c}_0^0 & c_0^1 - \hat{c}_0^1 & \dots & c_0^{L-1} - \hat{c}_0^{L-1} \\ c_1^0 - \hat{c}_1^0 & c_1^1 - \hat{c}_1^1 & \dots & c_1^{L-1} - \hat{c}_1^{L-1} \\ \vdots & \ddots & \ddots & \vdots \\ c_{P-1}^0 - \hat{c}_{P-1}^0 & c_{P-1}^1 - \hat{c}_{P-1}^1 & \dots & c_{P-1}^{L-1} - \hat{c}_{P-1}^{L-1} \end{bmatrix} \quad (2.8)$$

where $\hat{c}$ is the decoded codeword.

Based on what has been previously stated, the maximum or full diversity is provided if and only if a full rank of the matrix B is achieved for any two different codewords of the codebook in hand.

Space-Time Codes can be further classified by the receiver's knowledge on the channel acquired through training and estimation. In the case where the receiver knows the channel impairments, the code is classified as coherent STC. If the receiver does not know the channel impairments but has information about the statistics of the channel, then the code is categorized as non-coherent STC. Finally, in the case where the receiver has no clue neither about the channel impairments nor about its statistics, then the code is considered as differential STC.

2.2.2 Coherent Space-Time Code

As mentioned previously, coherent STCs have knowledge about the channel impairments. In other words, the overall channel matrix H is known to the receiver. Hence, in the case of a coherent STC, determining the transmitted codewords can be based on the a Maximum Likelihood (ML) criterion which consists of trying all the possible codewords and picking up the one which yields the closest value to the received signal y as described below:

$$\hat{x} = \arg \min_{\hat{x} \in \{C_0, \dots, C_{L-1}\}} \|y - Hx\|^2 \quad (2.9)$$

A sample coherent STC is the Alamouti code, a two branch transmit diversity scheme with any number of receivers. The two branch transmit diversity scheme with a single receiver will be described below [3].

Two different signals are transmitted from the two transmit antennas at any given period. Two combined periods will be used to submit two different symbols. During the first period, antenna zero transmits the signal s_0 and antenna one transmits the signal s_1 . Whereas, during the complementary symbol period, $-s_1^*$ and s_0^* are transmitted from antenna zero and antenna one respectively. Given that the two channels for transmit antenna zero and transmit antenna one are denoted by $h_0 = \alpha_0 e^{j\theta_0}$ and $h_1 = \alpha_1 e^{j\theta_1}$ respectively, the received signals during the two consecutive symbol

periods are expressed as follows:

$$\begin{aligned} r_0 &= h_0 s_0 + h_1 s_1 + n_0 \\ r_1 &= -h_0 s_1^* + h_1 s_0^* + n_1 \end{aligned} \quad (2.10)$$

where r_0 is the received signal at time t , r_1 is the received signal at time $t + T$, n_0 is the received noise during the first period, and n_1 is the received noise during the second period. At the receiver's stage, the received signals would be combined as:

$$\begin{aligned} \hat{s}_0 &= h_0^* r_0 + h_1 r_1^* = (\alpha_0^2 + \alpha_1^2) s_0 + h_0^* n_0 + h_1 n_1^* \\ \hat{s}_1 &= h_1^* r_0 - h_0 r_1^* = (\alpha_0^2 + \alpha_1^2) s_1 - h_0 n_1^* + h_1^* n_0 \end{aligned} \quad (2.11)$$

Finally, signal s_i would be chosen based on the following maximum likelihood decision rule:

$$d^2(\hat{s}_0, s_i) \leq d^2(\hat{s}_0, s_k), \quad \forall i \neq k \quad (2.12)$$

where $d^2(x, y)$ is computed as follows:

$$d^2(x, y) = (x - y)(x^* - y^*) \quad (2.13)$$

2.2.3 Differential Space-Time Code

In the case of a differential STC, neither the transmitter nor the receiver have any knowledge of the channel. For this, given that the channel matrix H is not known, the differential code bases itself on a direct relation among the successive signals such that the transmitted signal at time t , x^t is related to the transmitted signal at time $t - 1$, x^{t-1} as follows:

$$x^t = C_l x^{t-1} \quad (2.14)$$

where C_l is a codeword picked from a codebook of L different codewords. These are built based on a set of specific constraints. First, each and every codeword has to be unitary. Second, they have to verify the full rank constraint, meaning that the resulting difference of any two codewords should have a full rank which is equal to the number of transmit antennas in use. Based on the inter signal mathematical connection, the transmitted codeword can be estimated based on a maximum likelihood relation

among the consecutive received signal by iterating through all the possible codewords as follows:

$$\hat{C}^t = \arg \min_{\hat{C}^t \in \{C_0, \dots, C_{L-1}\}} \|y^t - \hat{C}^t y^{t-1}\| \quad (2.15)$$

Differential Space-Time Codes can be viewed as an extension to Differential Binary Phase Shift Keying (DBPSK). The DBPSK is a phase modulation form that conveys the bit x^t at time t that can take one of the two possible binary values, namely a '0' and a '1'. Assuming that z^t is the bit intended to be transmitted, then the actual bit to be transmitted x^t would hold a value related to both the bit intended to be transmitted and the previously transmitted bit as follows:

$$x^t = x^{t-1} \oplus z^t \quad (2.16)$$

where $\oplus$ is the binary addition. If the intended bit to be transmitted is a '0', then the transmitted signal would remain the same. However, if the intended bit is a '1', then if the transmitted signal bit would change its state from a '0' to a '1' or the inverse. Hence, the intended bit conveyed can be reconstructed differentially as such:

$$z^t = x^{t-1} \oplus x^t \quad (2.17)$$

The differentially decoded bit will be correct regardlessly of whether the actual bits were inverted or not, since it depends on the difference between two successively transmitted bits and not their actual values. This phase modulation form can be further generalized by using any function $f(x, a)$ instead of the binary addition operator $\oplus$ such that $y_0 = f(x_0, a)$ has a single solution for any combination of y_0 and x_0 .

2.2.4 Coherent vs Differential Space-Time Code

As a comparison between coherent and differential STC, we state the advantages and disadvantages of the two types of code. Coherent STC provides lower bit error rates (BER) at the cost of complexity, whereas the differential STC requires a much simpler design providing high BER. The simplicity of the differential code and the complexity of the coherent code is mainly due to the need of estimating the channel matrix H .

2.3 Space-Time Code for UWB

In this section, the difficulties of the coherent STCs for UWB are presented in the subsection 2.3.1 which would lead to the importance of the differential STCs for UWB. For this, some of the previous differential STCs for UWB will be described in the subsequent subsection 2.3.2.

2.3.1 Coherent Space-Time Code for UWB

As previously stated, the UWB's conveyance of symbols is achieved through the transmission of a train of ultra short pulses. The information in the case of coherent coding can be encoded in the amplitude through the previously described modulation format PAM or in the position by using the PPM modulation. However, the usage of the extremely short pulses would lead to the birth of several multi path components. This would give birth to a large number of channel parameters that would need to be considered when estimating the channel matrix H which is indeed needed in order to use coherent STCs. This procedure is quite complicated and would lead to a high cost and complexity in STCs for UWB. Not to forget that an integration time T_i is required to collect the energy of the different transmitted pulses. Yet, the larger the integration time the better the results the greater the complexity. The complexity growth can be explained since the number of multi path components that has to be estimated would augment if the integration time is increased.

Based on the previously stated points, differential STCs for UWB increased in popularity. A key feature in these types of schemes is the diversity gain. Several diversity schemes were proposed out of which we present a few in the following subsection.

2.3.2 Differential Space-Time Code for UWB

This subsection will provide a description of two differential STCs for TH-UWB found in [1]. The two schemes are able to achieve full spatial diversity and can be used with

any number of receivers.

The first scheme can be used for any number of transmit antennas and can be associated with the M-ary PPM, BPSK and PPM-2-PAM constellations. This system is a single user TH-UWB that is formed of P and Q transmit and receive antennas respectively. N_f pulses of different polarities are sent by each transmit antenna with a period T_f in order to convey the information symbol. Every symbol is represented by the pulse position $d \in \{0, \dots, M-1\}$ and its amplitude $a \in \{-1, +1\}$. The modulation delay δ and the time frame T_f are chosen in a way to satisfy orthogonality among the diverse positions and eliminate ISI. The transmitted pulses are normalized to have a unit energy, and the transmitted signals are factored in a way that assures a same total transmitted power for the case of any number of antennas. The conveyed pulses of each of the transmit antennas are encoded through a set of unique polarities. For this, a specific symbol is conveyed by the transmission of a same pulse at a same position for N_f consecutive periods but with different polarities.

At the receiver stage, partial Rake receivers of order L are used. The different fingers will have delays equal to the multiple of the pulse width. This would ensure that no particular channel state information is needed. The output of each of the receiver antenna is correlated with the reference signal of every transmit antenna for each modulation slot m and for every finger delay l . This would form $QPLM$ different outputs for every transmitted symbol k . These values are stored in an $M \times PQL$ decision matrix Y_k .

The differential encoding system is based on the transmission of an encoding vector A_k related to its predecessor encoding vector A_{k-1} through the amplitude s_k and the position Δ_k of the symbol intended to be sent:

$$A_k = s_k \Omega^{\Delta_k} A_{k-1} \quad (2.18)$$

with A_0 being the identity matrix's first column $I_{M,0}$ and Ω being the permutation matrix expressed as follows:

$$\Omega = \begin{bmatrix} \Theta^T & 1 \\ I_{M-1} & \Theta \end{bmatrix} \quad (2.19)$$

where Θ is the null matrix of size $M - 1 \times 1$.

Finally, the differential decoder would be using two successive codewords based on the following equation:

$$(\hat{s}_k, \hat{\Delta}_k) = \arg \max_{\substack{s \in \{-1, +1\} \\ p \in \{1, \dots, M-1\}}} (maxdiag(sY_k Y_{k-1}^T \Omega^{M-p})) \quad (2.20)$$

where the resultant of $maxdiag(.)$ is the maximum of the inputted matrix's diagonal.

The second scheme can be used for exactly two transmit antennas and can only be associated with the constellations that incorporate amplitude variations. This diversity scheme encodes the pulses of two successive symbols instead of encoding the pulses of the same symbol similarly to the scheme described earlier. Similarly to the previous scheme, the k -th and the $k - 1$ -th symbols defined by a position value and an amplitude value each. The first transmit antenna would transmit the first symbol with its exact polarity in the first symbol period and the second symbol with an inverted polarity in the second symbol period. The second transmit antenna would convey the second symbol with its exact polarity in the first symbol period and the first symbol with its exact polarity in the second symbol period.

At the receiver stage, partial Rake receivers of order L would be used in a way that assures that no particular channel state information is needed. The different fingers will have delays equal to the multiple of the pulse width. The output of all of the receive antennas are correlated with a unique reference signal for each modulation slot m and for every finger delay l where the first receiver's output would be integrated over the first symbol period and the second receiver's output would be integrated over the second symbol period. This would form $4MQL$ different outputs for every transmitted symbol k . These values are stored in an $2M \times 2QL$ decision matrix Y_k .

The differential encoding system is based on the transmission of an encoding vector A_k through the amplitude s_k and the position Δ_k of the symbol intended to be sent:

$$A_k = \begin{bmatrix} a_{k,1} I_{M,d_{k,1}} & a_{k,2} I_{M,d_{k,1}} \\ a_{k,2} I_{M,d_{k,2}} & a_{k,1} I_{M,d_{k,2}} \end{bmatrix} \quad (2.21)$$

where a_k is related to a_{k-1} through:

$$a_k = s_k a_{k-1} = \begin{bmatrix} s_{k,1} & s_{k,2} \\ -s_{k,2} & s_{k,1} \end{bmatrix} \begin{bmatrix} a_{k-1,1} & s_{k-1,2} \\ -a_{k-1,2} & 1_{k-1,1} \end{bmatrix} \quad (2.22)$$

with $d_{k,p} = \Delta_{k,p} + d_{k-1,p} \pmod{M}$ for $p \in \{1, 2\}$.

The maximum likelihood receiver would first start by computing:

$$G_{p_1,p_2} = Y_k Y_{k-1}^T \begin{bmatrix} \Omega^{M-p_1} & \Theta \\ \Theta & \Omega^{M-p_2} \end{bmatrix} \quad (2.23)$$

where Θ is a null matrix of dimensions $M \times M$.

Afterwards, a 2×2 matrix g_{p_1,p_2} with its element at (m, n) being the (f_m, f_n) -th element of the previously defined matrix G_{p_1,p_2} , where f_i corresponds to the index of the maximum magnitude of the i -th M diagonal elements of G_{p_1,p_2} .

The last step for detection would be the following receiver's decision equation:

$$(\hat{s}_k, \hat{\Delta}_k) = \arg \max_{\substack{s_1, s_2 \in \{-1, +1\} \\ p_1, p_2 \in \{0, \dots, M-1\}}} \text{trace}(s^T g_{p_1,p_2}) \quad (2.24)$$

Chapter 3

Proposed Scalable Scheme

As previously stated, differential space-time codes for ultra wideband communications is gaining importance, due to the extreme challenges of the channel estimations enforced by the UWB pulse nature. Furthermore, analog to digital conversion is yet another important issue that needs to be dealt with. For these reasons, we focused our work on developing a unitary and differential STC for TH-UWB that can be implemented in a fully analog manner. The scheme proposed is scalable providing the users with different data rate. The higher the data rate chosen the higher the complexity and the higher the error rates. The first subsection will describe the encoder of the proposed scheme, whereas the decoder will be explained in the second subsection.

3.1 Encoder

The single user structure proposed is a unitary and differential STC for TH-UWB that follows an extension of the Pulse Position Modulation (PPM) constellation. The system is composed of P transmit and Q receive antennas where the number of transmit antennas is limited to two at this stage ($P = 2$).

3.1.1 Modified Pulse Position Modulation

The modulation format used for the proposed scheme can be considered as an extension to the PPM modulation format described in chapter 1. We define this modulation

scheme to be based on the position of the conveyed pulses similarly to the PPM format, yet, multiple consecutive pulses could be sent which would incur amplitude variations in order to keep the same power values among the different symbols, and ensure a unitary scheme. The time frame, comparably to the PPM format, is decomposed into M different slots of duration δ each. The symbol being sent is not only related to the location of the transmitted pulse, but also to the number of consecutive pulses $\theta \in \{0, \dots, \Theta\}$ that are sent. The maximum number of pulses can vary between 1 and $M - 1$ pulses, $\Theta \in \{0, \dots, M - 1\}$. The k -th signal of the modified M-ary PPM would thus be expressed as follows:

$$s_k(t) = \sum_{i=0}^{\theta(k)-1} \frac{1}{\sqrt{\theta(k)}} w(t - \delta m(k) - kT_f) \quad (3.1)$$

where $m(.) \in \{0, \dots, M - 1\}$ is the slot position of the symbol being transmitted, $\theta(.) \in \{1, \dots, \Theta - 1\}$ is the number of pulses being transmitted. The figures below, describes the defined M-PPM extension with an order of $\Theta = 3$.

Figure 3.1: Example of 4-PPM extension with $\theta = 1$

Figure 3.2: Example of 4-PPM extension with $\theta = 2$

The position of the leading pulse indicates the symbol code in the figures above.

Figure 3.3: Example of 4-PPM extension with $\theta = 3$

3.1.2 Differential Encoding Scheme

The modified M-PPM constellation described above is used in the scalable scheme proposed and thus each signal period is decomposed into M slots of duration δ each. Two transmit antennas will be conveying their data streams iteratively, meaning that one antenna transmits its data during the first time frame remaining idle during the second time frame, while the other antenna remains idle in the first time frame and transmits its data during the second time frame. For this, the symbol period T_s is decomposed into two sub-periods T_{sp} consisting of M slots of duration δ each. Thus, every symbol would be encoded into PM slots, where every other data stream can hold from 1 to $\Theta - 1$ pulses, $\Theta \in \{1, \dots, M - 1\}$ being the order of the scheme. Thus, every transmitted symbol can be encoded into one of ΘL different codewords. Note that the base scheme, or lowest order scheme (scheme of order 1) follows the M-PPM constellation and holds a codebook of size $2L$.

For a scheme of order Θ , the k -th symbol conveyed $\Delta_k \in \{0, \dots, \Theta L - 1\}$ can be encoded into one of ΘL different codewords. The differential encoding scheme corresponds to transmitting the codeword picked by a differential "codeword index" c_k which, similarly to the symbol conveyed Δ_k , can hold any value between 0 and $\Theta L - 1$, $c_k \in \{0, \dots, \Theta L - 1\}$. However, the codeword index of the k -th symbol, c_k will be differentially related to the $(k - 1)$ -th codeword index c_{k-1} and the symbol index Δ_k through the following equation:

$$c_k = c_{k-1} + \Delta_k \pmod{\Theta L} \quad (3.2)$$

where the computed c_k index is used to pick a specific codeword to be conveyed among a set of multiple codewords, known as the codebook $\mathcal{C}$.

3.1.3 Codeword Format

As mentioned earlier, the codewords of the proposed system are based on space and time and are generated in a way that ensures the full spacial diversity of the system for all orders as explained in the Diversity Appendix. For this, each codeword is represented by a matrix where each column would refer to a specific transmit antenna being used providing space diversification each row would refer to a specific time slot providing time diversification. The matrix is composed of P columns and L rows, where each cell would hold a zero or a one, representing the absence or the presence of a pulse respectively. For instance, a one in the (l, p) cell indicates the presence of a pulse in the l -th slot conveyed by the p -th transmitter. In this thesis we only consider the case of two transmitters, and thus, the codeword would be represented by a $L \times 2$ matrix.

For further illustration, a sample codeword of a scheme using the 4-ary PPM constellation is provided below:

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (3.3)$$

$\underbrace{\hspace{1.5cm}}_{v_0} \quad \underbrace{\hspace{1.5cm}}_{v_1}$

where v_0 is the first transmit antenna's code vector and v_1 is the second antenna's code vector. Assuming that the above codeword is used as the actual base of the signal transmitted, the first antenna would be transmitting a pulse in the first slot of the first

sub period T_{ss} remaining idle in all other slots, whereas the second antenna would be transmitting a pulse in the first slot of the second sub period T_{ss} remaining idle in all other slots, as displayed in the figure below.

Figure 3.4: Codeword based transmitted signals

The codewords are listed in the codebook in a specific order for indexing purposes as shown below:

$$\mathcal{C} = \{C_{0,0}, C_{1,0}, \dots, C_{\Theta-1,0}, C_{0,1}, \dots, C_{\theta,l}, \dots, C_{\Theta-1,L-1}\} \quad (3.4)$$

where $\theta \in \{0, \dots, \Theta - 1\}$ and $l \in \{0, \dots, L - 1\}$. The codewords can be further divided into Θ families, where each family is identified by the number of consecutive pulses in its codewords. The codewords of the f -th family contain f different pulses per transmitter. A family is denoted by $\mathcal{C}_\theta$ and holds the following codewords:

$$\mathcal{C}_\theta = \{C_{\theta,0}, C_{\theta,1}, \dots, C_{\theta,l}, \dots, C_{\theta,L-1}\} \quad (3.5)$$

Each codeword $C_{\theta,l}$ is defined as follows:

$$C_{\theta,l} = \psi^l \times C_{\theta,0} \quad (3.6)$$

where ψ is a $L \times L$ permutation matrix defined as:

$$\psi = \begin{bmatrix} \Theta_M & \Omega \\ I_M & \Theta_M \end{bmatrix} \quad (3.7)$$

with Θ_M being a $M \times M$ dimensional null matrix and Ω is the $M \times M$ dimensional permutation matrix expressed as follows:

$$\Omega = \begin{bmatrix} \Theta_{1,M} & 1 \\ I_{M-1} & \Theta_{M,1} \end{bmatrix} \quad (3.8)$$

where $\Theta_{M,N}$ is the $M \times N$ dimensional null matrix.

The codewords of a specific family are similarly generated by raising the same permutation matrix Ω to a certain power $l \in \{0, \dots, L-1\}$ and multiplying it by the reference codeword matrix of the θ -th family $C_{\theta,0}$ expressed by the following equation:

$$C_{\theta,0} = \frac{1}{\sqrt{\theta+1}} \times \sum_{i=0}^{\theta} C_i \quad (3.9)$$

where C_i is a $L \times P$ matrix of zeros with ones exclusively in the $(pM + i, p)$ cells with $p \in \{0, \dots, P-1\}$. The reference codewords of the different families are also distinguished by the number of their consecutive ones and their amplitudes. For instance, the reference codeword matrix of the first family ($\theta = 0$) and the reference matrix of the third family ($\theta = 2$) of a same scheme with $M = 4$ and $\Theta = 3$ are respectively expresses as:

$$C_{0,0} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad C_{2,0} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad (3.10)$$

The codewords, based on the generation equations described above, can be seen to hold two different forms. The first format would have the first transmit antenna convey its information in the first sub period (or the first M slots) while remaining idle in the

second sub period, whereas the second transmit antenna would transmit the exact same information in the second sub period while remaining idle in the first one. On the other hand, the second format would have the first transmit antenna convey its information in the second sub period while remaining idle in the first one, whereas the second transmit antenna would transmit a one slot shifted version of the information transmitted by the first antenna in the first sub period, while remaining idle in the second one. The described process of a family specific codebook can be expressed in a mathematical equation as follows:

$$C_\theta = \frac{1}{\sqrt{\theta+1}} \times \left\{ \begin{bmatrix} \sum_{i=0}^{\theta} \Omega^{m+i} e_1 & \Theta_M \\ \Theta_M & \sum_{i=0}^{\theta} \Omega^{m+i} e_1 \end{bmatrix} \right. \quad (3.11)$$

$$\left. \begin{bmatrix} \Theta_M & \sum_{i=0}^{\theta} \Omega^{m+i+1} e_1 \\ \sum_{i=0}^{\theta} \Omega^{m+i} e_1 & \Theta_M \end{bmatrix} \right\}$$

where $e_1 = C_{0,0}$, $\theta \in \{0, \dots, \Theta - 1\}$ and $m \in \{0, \dots, M - 1\}$. This

For further illustration, we provide a concrete example below. The codewords of the first family ($\theta = 0$) of a scheme with $M = 3$ and $\Theta = 2$ are expressed as:

$$C_{0,0} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad C_{0,2} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad C_{0,4} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad (3.12)$$

$$C_{0,1} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad C_{0,3} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \quad C_{0,5} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

whereas the reference matrix of the second family ($\theta = 1$) of the same scheme are expressed as follows:

$$\begin{aligned}
C_{1,0} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} & C_{1,2} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} & C_{1,4} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \\
C_{1,1} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} & C_{1,3} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} & C_{1,5} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}
\end{aligned} \tag{3.13}$$

All codewords $C_{\theta,l}$ with an even index l are considered of type 1, whereas the codewords $C_{\theta,l}$ with an odd index l are considered of type 2.

3.1.4 Scheme Data Rate

From the previously described subsections, it can be stated that the codebook can be decomposed into Θ different families, where the codewords of the f -th family contain f different pulses per transmitter, and that every family is formed of L different codewords generated by the permutation matrix ψ , taking one of two different forms as shown in the equation above. Hence, given that the number of codewords held by the described codebook is ΘL , the data rate of the proposed system would be:

$$R = \frac{\log_2(\Theta L)}{PT_s} \tag{3.14}$$

where R is in bits/s, P is the number of transmit antennas, and $L = PM$ as stated earlier in this paper. For $M = 4$, a scheme of the third order ($\Theta = 3$) would have a codebook of $\Theta M = 24$ codewords providing a data rate of $R = \log_2(24)/2T_s$ bits/s compared to a scheme of the first order ($\Theta = 1$) which would have a codebook of 8 different codewords and provide a lower data rate of $R = \log_2(8)/2T_s$ bits/s. Nevertheless, the system complexity would increase for the higher order schemes which provides indeed a higher data rate. Furthermore, the higher the order, the higher the number of codewords, the more difficult the received codeword estimation would be, and the higher the probability error rate.

3.1.5 Transmitted and Received Signal

As per the description of the scheme's codebook, the codewords are defined as space-time matrices of zeros and ones representing the presence and the absence of a pulse transmission by the corresponding transmit antenna at the corresponding time slot. Each vector of a given codeword represents the transmission action of a specific antenna over a period of time T_s consisting of $L = PM$ time slots. Based on this, the signal conveyed by the p -th transmitter during the k -th symbol duration is expressed as follows:

$$s_{k,p}(t) = \sum_{l=0}^{L-1} d_{k,p,l} w(t - l\delta) \quad (3.15)$$

where $d_{k,p}$ is the p -th vector of the k -th codeword noted as the position vector, $d_{k,p,l}$ is the l -th element within the position vector, δ is the modulation delay and $w(t)$ is the emitted pulse waveform of duration T_w normalized to have unit energy. Note that $d_{k,p,l}$ can either be a zero, meaning that no pulse should be conveyed by the corresponding transmit antenna at the given time slot l , or a certain nonzero value representing the amplitude of the pulse that should be transmitted. The amplitude values are taken into consideration by the codeword generator and are generated in a way that assures unit energy for all transmitted symbols. Unit energy is ensure by satisfying the following equality:

$$C_{\theta,l}^T \times C_{\theta,l} = I \quad (3.16)$$

In case of a system without inter symbol interference (ISI), the received signal at the q -th antenna during the k -th symbol duration can be expressed as follows:

$$r_{k,q}(t) = \sum_{p=0}^{P-1} s_{k,p}(t) * g_{q,p}(t) + n_{k,q}(t) \quad (3.17)$$

where $*$ denotes the convolution product and $n_{k,q}(t)$ is the Gaussian noise generated during the k -th symbol at the q -th antenna. $g_{q,p}(t)$ symbolizes the impulse response of the frequency selective channel between the p -th transmitter and the q -th receiver. Every one of the sub-channels is formed out of $U_{q,p}$ multi-path components each having a different amplitude $\alpha_{q,p,u}$ and arriving at a different instant $\tau_{q,p,u}$ within the pulse duration T_p :

$$g_{q,p}(t) = \sum_{u=0}^{U_{q,p}} \alpha_{q,p,u} \delta(t - \tau_{q,p,u}) \quad (3.18)$$

where the multi path delays fulfill the following equation:

$$\tau_{q,p,0} < \tau_{q,p,1} < \dots < \tau_{q,p,L_{q,p}-1} \quad (3.19)$$

Furthermore, the maximum sub channel specific delay spread $T_{q,p} = \tau_{q,p,L_{q,p}-1}$ is ensured to be extremely larger than the pulse duration T_w .

The maximum of all the delay spreads satisfies the following relations in order to satisfy the orthogonality between the positions:

$$\max(\tau_{q,p,u}) \leq \delta - T_w \quad (3.20)$$

Furthermore, in order to eliminate ISI, the time frame T_s is ensured to satisfy:

$$T_s \geq (L-1)\delta + \max(\tau_{q,p,u}) + T_w \quad (3.21)$$

By setting $h_{q,p}(t) = w(t) * g_{q,p}(t)$ and replacing the equation (3.14) in (3.16) the received signal at the q -th receive antenna would be expressed by the following equation:

$$r_{k,q}(t) = \sum_{p=0}^{P-1} \sum_{l=0}^{L-1} d_{k,p,l} h_{q,p}(t - \delta l) + n_{k,q}(t) \quad (3.22)$$

3.2 Decoder

The proposed encoder's transmitted signals can be decoded at any of the Q receive antennas available. As previously mentioned, the decoder would use a differential manner in order to estimate the received codewords. Thus, every two consecutive received symbols would be used as an input to the decoder. Furthermore, an advantage of this presented scheme, would be the ability of easily implementing its decoder in an analog way, for any of the possible orders. This would be quite beneficial since one of the main hardware challenges related to the UWB technology is actually the analog to digital conversion. The reason is that the sampling rate needed to achieve proper conversion for the UWB technology is in the order of giga samples per second (Gsps), making the analog to digital conversion a main data rate limitation. Later on in this section, the correlator's function is described and a sample analog implementation is provided.

3.2.1 Correlation Receiver

At the receiver's side, once two consecutive signals are obtained, a correlation operation would be applied in a differential manner for symbol estimation purposes. This step consists of delaying the newly received signal before multiplying the two signals and integrating the result. A threshold detector would be then used in order to estimate the new symbol. This course of action is applied in order to estimate the matching level of the two successive signals. For this, the correlation receiver would apply this procedure to all possible permutations of the $(k - 1)$ -th symbol received. In other words, the $(k - 1)$ -th symbol would be permuted in all of the ΘL possible ways, and each of the resulting signals would be multiplied with the k -th symbol prior to integrating the result. The different results would then be fed to a maximum likelihood (ML) detector allowing the decoder to determine the actual differential code value Δ of the k -th symbol. All of the correlations combined would form a sufficient statistic allowing the receiver to properly detect the received symbol.

In order to mathematically express the above description of the correlation receiver, it would be beneficial to define an integration matrix X holding the integrations of all the possible combinations of the slot by slot multiplications of the two received signals. Meaning that each of the L slots of the $(k - 1)$ -th symbol is multiplied with each of the L slots of the k -th symbol before being integrated and stored in the matrix X . For the purpose of expressing the matrix, we define a vector holding delayed versions of the k -th received signal which, under a duration δ would yield all of the possible slot based sub signals required. This vector is defined as:

$$R_k = \begin{bmatrix} r_k(t) \\ r_k(t - \delta) \\ \vdots \\ r_k(t - (L - 1)\delta) \end{bmatrix} \quad (3.23)$$

where R_k is of size L . Every cell holds a delayed version of the received signal that starts at the beginning of each of the L slots for $t = 0$. The integration matrix X can now be defined as:

$$X = \int_0^{T_i} R_{k-1} R_k^T dt \quad (3.24)$$

The value of any cell $X(i, j)$ in the i -th row and the j -th column can be exclusively computed though:

$$X(i, j) = \int_0^{T_i} r_{k-1}(t - i\delta) r_k(t - j\delta) dt \quad (3.25)$$

where T_i is the integration time whose value is less than or equal to the modulation delay $T_i \leq \delta$. The larger the value of T_i the lower the resulting bit error rate (BER) would be.

The matrix X of a scheme using the 4-ary PPM constellation would have the

following form:

$$X = \begin{bmatrix} X(0,0) & X(0,1) & X(0,2) & X(0,3) & X(0,4) & X(0,5) & X(0,6) & X(0,7) \\ X(1,0) & X(1,1) & X(1,2) & X(1,3) & X(1,4) & X(1,5) & X(1,6) & X(1,7) \\ X(2,0) & X(2,1) & X(2,2) & X(2,3) & X(2,4) & X(2,5) & X(2,6) & X(2,7) \\ X(3,0) & X(3,1) & X(3,2) & X(3,3) & X(3,4) & X(3,5) & X(3,6) & X(3,7) \\ \hline X(4,0) & X(4,1) & X(4,2) & X(4,3) & X(4,4) & X(4,5) & X(4,6) & X(4,7) \\ X(5,0) & X(5,1) & X(5,2) & X(5,3) & X(5,4) & X(5,5) & X(5,6) & X(5,7) \\ X(6,0) & X(6,1) & X(6,2) & X(6,3) & X(6,4) & X(6,5) & X(6,6) & X(6,7) \\ X(7,0) & X(7,1) & X(7,2) & X(7,3) & X(7,4) & X(7,5) & X(7,6) & X(7,7) \end{bmatrix} \quad (3.26)$$

where the upper left quadrant is denoted by $X_{0,0}$, the upper right quadrant is denoted by $X_{0,1}$, the lower left quadrant is denoted by $X_{1,0}$, and the lower right quadrant is denoted by $X_{1,1}$. Any of the 4 quadrants $X_{i,j}$ would have the following form:

$$X_{i,j} = \begin{bmatrix} X(0,0) & X(0,1) & X(0,2) & X(0,3) \\ X(1,0) & X(1,1) & X(1,2) & X(1,3) \\ X(2,0) & X(2,1) & X(2,2) & X(2,3) \\ X(3,0) & X(3,1) & X(3,2) & X(3,3) \end{bmatrix} \quad (3.27)$$

where $i, j \in \{0, 1\}$.

In order to translate the explained code into a mathematical equation, it would be beneficial to go through a few direct examples of the previously defined scheme ($M = 4$). For this purpose, we decompose the given problem into three different parts. The first part would tackle the case where the two consecutive symbols belong to a same family. The second one would handle the case where the $(k - 1)$ -th symbol is of the base family ($\theta = 0$) and the k -th symbol is of the second family ($\theta = 1$). Finally, the third instance would deal with a general permutation from the $(a + 1)$ -th family ($\theta = a$) to the $(b + 1)$ -th family ($\theta = b$), with $a \leq b$. Note that the case where $a > b$ is not considered since a simple symbol inversion brings us back to one of the cases considered. For instance, the case where the $(k - 1)$ -th symbol belongs to the

$(a + 1)$ -th family and the k -th symbol belongs to the $(b + 1)$ -th family with $a > b$ can be interpreted as a shift from the $(b + 1)$ -th family to the $(a + 1)$ -th family by simply inverting the two symbols and thus, getting back to the third case considered.

Permutations within a same family

Considering that the first and the second actual codewords are $C_{0,0}$. The maximal correlation in this case would occur with no modification at all. Thus, the i -th slot of the first signal should be correlated with the i -th slot of the second signal. This can be translated by summing up the values of $diag(X)$.

Figure 3.5: $C_{0,0}$ received signal

An even shift of l within the same family would keep the codeword of the same type and can be translated by circularly shifting the two sub periods of the first signal to the right by a same number of slots equal to $\lfloor \frac{l}{2} \rfloor$. Thus, in order to compute the maximal correlation, the values of the shifted version of $diag(X_{0,0})$ to the right by a number of cells equal to $\lfloor \frac{l}{2} \rfloor$ should be summed up with the values of the shifted version of $diag(X_{1,1})$ to the right by the same number of cells $\lfloor \frac{l}{2} \rfloor$.

An odd shift of l within the same family would modify the codeword's type. This can be translated by inverting the two sub periods of the first signal, circularly shifting the second sub period to the right by a single slot and finally circularly shifting each of its two sub periods to the right by a same number of slots equal to $\lfloor \frac{l}{2} \rfloor$. Thus, in order to compute the maximal correlation, the values of the shifted version of $diag(X_{1,0})$ to

Figure 3.6: $C_{0,2}$ received signal

the right by a number of cells equal to $\lfloor \frac{l}{2} \rfloor$ should be summed up with the values of the shifted version of $diag(X_{0,1})$ to the right by a number of cells equal to $\lfloor \frac{l}{2} \rfloor + 1$.

Figure 3.7: $C_{0,3}$ received signal

Permutations from first to second family

Considering that the first and second actual codewords being used have the same l component with $l = 0$. The two consecutive codewords would thus be, $C_{0,0}$ and $C_{1,0}$ respectively. The leading pulses of the two received signals are both in the first slot. However, the second signal has two consecutive pulses compared to the first one which only has a single pulse. For this, the modifications that needs to be done to the old received signal would include copying the signal within the first slot of each of the two sub periods into the second slot of the two sub periods. This would ensure the exact same number of pulse as well as the exact same pulse positioning for maximal

Figure 3.8: $C_{1,0}$ received signal

correlation. Nevertheless, since the system proposed is unitary, amplitude modifications needs to be adjusted to establish proper detection. The amplitudes of the pulses of the codewords belonging to the first family are equal to 1 compared to the amplitudes of the pulses of the codewords belonging to the second family which are equal to $\frac{1}{\sqrt{2}}$. Thus, the first signal's amplitude should be multiplied by a factor of $\frac{1}{\sqrt{2}}$. For further illustration, the correlation function of this specific case computed through the cell values of the matrix X is stated below:

$$Z = \frac{1}{\sqrt{2}}(X_{0,0}(0,0) + X_{0,0}(0,1) + X_{0,0}(2,2) + X_{0,0}(3,3) + X_{1,1}(0,0) + X_{1,1}(0,1) + X_{1,1}(2,2) + X_{1,1}(3,3)) \quad (3.28)$$

Similarly to the previous case of permutations within a same family, an even shift of l within the family would keep the codeword of the same type and can be translated by circularly shifting the two sub periods of the first signal to the right by a same number of slots equal to $\lfloor \frac{l}{2} \rfloor$. Thus, the same structure of the previously stated function would be used with a shift of $\lfloor \frac{l}{2} \rfloor$ to the right in each of the two quadrants. For instance, considering that the first and second codewords are $C_{0,0}$ and $C_{1,2}$ respectively ($l = 2$), the new correlation function would become:

$$Z = \frac{1}{\sqrt{2}}(X_{0,0}(0,1) + X_{0,0}(0,2) + X_{0,0}(2,3) + X_{0,0}(3,0) + X_{1,1}(0,1) + X_{1,1}(0,2) + X_{1,1}(2,3) + X_{1,1}(3,0)) \quad (3.29)$$

Again, likewise to the previous case of permutations within a same family, an odd shift of l within the family would change the codeword's type and can be translated by

Figure 3.9: $C_{1,2}$ received signal

inverting the two sub periods of the first signal, circularly shifting the second sub period to the right by a single slot and finally circularly shifting each of its two sub periods to the right by a same number of slots equal to $\lfloor \frac{l}{2} \rfloor$. Thus, the same structure of the

Figure 3.10: $C_{1,3}$ received signal

previously stated function would be used with quadrant modifications, and a shift of $\lfloor \frac{l}{2} \rfloor$ and $\lfloor \frac{l}{2} \rfloor + 1$ to the right in each of the $X_{1,0}$ and the $X_{0,1}$ quadrants respectively. For instance, considering that the first and second codewords are $C_{0,0}$ and $C_{1,3}$ respectively ($l = 2$), the new correlation function would become:

$$\begin{aligned}
 Z = \frac{1}{\sqrt{2}} & (X_{1,0}(0, 1) + X_{1,0}(0, 2) + X_{1,0}(2, 3) + X_{1,0}(3, 0) \\
 & + X_{0,1}(0, 2) + X_{0,1}(0, 3) + X_{0,1}(3, 0) + X_{0,1}(0, 1))
 \end{aligned} \tag{3.30}$$

Permutations from the (a+1)-th to the (b+1)-th family

In this part, it is assumed that $a \geq 0$ and $b \geq a$, and that the two codewords have the same l component with $l = 0$. In this specific case, one signal has $a + 1$ consecutive pulses whereas the other has $b + 1$ consecutive pulses. For this, the $b - a$ consecutive pulses should be added to the first signal for matching. This is done by averaging all the $a - 1$ pulses of each of the two sub periods and copying the result in the remaining slots. Once that step is done, the two signals would have the same number of pulses at the exact same slot positions. However, the amplitude issue remains. The latter is solved by multiplying the first signal by a factor of $\sqrt{a+1}/\sqrt{b+1}$. To apply the state operation through matrix usage, the first $(a + 1)$ cell values and the last $M - b$ of $\text{diag}(X_{0,0})$ and $\text{diag}(X_{1,1})$ should be summed up. Furthermore, the intersection of the first $a + 1$ rows and the columns between the $(a + 1)$ -th column and the b -th column inclusive of the $\text{diag}(X_{0,0})$ and $\text{diag}(X_{1,1})$ should be summed up and divided by $(a + 1)$ before being added to the previous result. Finally, the total should be multiplied by $\sqrt{a+1}/\sqrt{b+1}$. This would be the maximal correlation value.

Similarly to the two previous parts, if there's a shift of l within the family, the same procedure would be applied by keeping the structure of the last part described above.

Mathematical Rendering

In this section, the previously delineated examples are set into actual mathematical expressions. For this, and in order to apply the ML criterion, we define a decision function that would compute the likelihood of all the possible transformations of the $(k - 1)$ -th symbol:

$$Z_{a,b,l,m,u}(X) = \sqrt{\frac{a+1}{b+1}} \left(\sum_{i=0}^a (\alpha_{i,i} + \beta_{i,i}) + \frac{1}{a+1} \sum_{i=a+1}^b \sum_{j=0}^a (\alpha_{i,j} + \beta_{i,j}) + \sum_{i=b+1}^{M-1} (\alpha_{i,i} + \beta_{i,i}) \right) \quad (3.31)$$

where $\alpha_{i,j}$ and $\beta_{i,j}$ are defined as:

$$\alpha_{i,j} = X_{l(mod\ 2),0} \left(j + m - \lceil \frac{l}{2} \rceil + u \right. \\ \left. (mod\ M), i + m + u (mod\ M) \right) \quad (3.32)$$

$$\beta_{i,j} = X_{1,1+l(mod\ 2)} \left(j + m - \lfloor \frac{l}{2} \rfloor \right. \\ \left. (mod\ M), i + m (mod\ M) \right) \quad (3.33)$$

where $X_{0,0}$ is the upper left quadrant of the matrix X , $X_{0,1}$ the upper right quadrant, $X_{1,0}$ the lower left quadrant and $X_{1,1}$ the lower right quadrant. a is the assumed family of the $(k-1)$ -th symbol, and b the assumed family of the k -th symbol. l is the assumed intra family distance and m the assumed position of the pulse. u is for synchronization among the two different quadrants indicating the two sub periods. In the case of a scheme of order 1 ($\Theta = 1$), the above equation would reduce to the following equation:

$$Z_{l,u}(X) = \sum_{i=0}^{M-1} (\alpha_{i,i} + \beta_{i,i}) \quad (3.34)$$

where the $m = 0$ in the function of both parameters α and β .

Maximum likelihood criterion

The previously delineated set of equations are used to fill a likelihood decision matrix D of size $\Theta \times L$ where the (γ, l) -th cell would hold the likelihood value $D_{\gamma,l}$ of an inter family jump of γ and an intra family distance of l :

$$D_{\gamma,l} = \max \left(Z_{\theta,\theta+\gamma,l,m,u}(X), \right. \\ \left. Z_{\theta',\theta'+(\Theta-\gamma),L-l-1,m,u}(X^T) \right) \quad (3.35)$$

where $\gamma \in \{0, \dots, \Theta-1\}$, $l \in \{0, \dots, L-1\}$, $\theta \in \lceil \frac{\gamma}{\Theta} \rceil \times \{0, \dots, \Theta-1\}$, $\theta' \in \{0, \dots, \gamma-1\}$, $m \in \lceil \frac{\gamma}{\Theta} \rceil \times \{0, \dots, M-1\}$, and $u \in \{0, 1\}$. In the case of a scheme of order 1, the likelihood decision matrix would be reduced to a $1 \times L$ matrix where the l -th cell would hold the likelihood value D_l of an intra family distance of l :

$$D_l = \max (Z_{l,u}(X)) \quad (3.36)$$

In order to determine the received symbol, the maximum likelihood criterion would be applied to the previously defined decision matrix:

$$(\hat{\gamma}, \hat{l}) = \arg \max_{\substack{\hat{\gamma} \in \{0, \dots, \Theta-1\} \\ \hat{l} \in \{1, \dots, L-1\}}} D_{\hat{\gamma}, \hat{l}} \quad (3.37)$$

where $\hat{\gamma}$ specifies the estimated inter family jump and $\hat{l}$ specifies the determined intra family distance. Finally, the information transmitted can be computed based on the previously defined values through:

$$\hat{\Delta} = \hat{l}\Theta + \hat{\gamma} \quad (3.38)$$

for the case of an order 1 scheme ($\Theta = 1$), the inter family jump is always equal to zero ($\gamma = 0$) and the information transmitted is simply computed through:

$$\hat{\Delta} = \hat{l} \quad (3.39)$$

3.2.2 Analog Block Diagram

In this subsection, the analog implementation of the base scheme of the first order ($\Theta = 1$) will be considered. The system is assumed to use the M-ary PPM constellation with $M = 4$ and $\Theta = 1$.

First, we start by naming the k -th and the $(k - 1)$ -th signal over the whole symbol period T_s as S_k and S_{k-1} respectively. The k -th and the $(k - 1)$ -th signal over the first sub symbol period T_{ss} are denoted by s_k^0 and s_{k-1}^0 respectively, whereas the k -th and the $(k - 1)$ -th signal over the second sub symbol period T_{ss} are denoted by s_k^1 and s_{k-1}^1 respectively.

Moreover, we define the signal's circular permutation operator $(.)^{P_i}$ as being a time slot (δ) shift to the right. Having a signal $(s_k)^{P_0}$ that starts at the first time slot and ends at the n -th time slot, then the signal $(s_k)^{P_x}$ would be a permuted version of x slots of the initial signal. This signifies that $(s_k)^{P_x}$ starts at the x -th time slot and ends at the $(x - 1)$ -th time slot.

Using the above parameters of the base scheme along with the defined notations and operators, the codeword based maximum likelihood decision values can now be set and

are expressed as follows:

$$\begin{aligned}
(l = 0) : & \quad (s_{k-1}^0)^{P_0}(s_k^0)^{P_0} + (s_{k-1}^1)^{P_0}(s_k^1)^{P_0} \\
(l = 1) : & \quad (s_{k-1}^1)^{P_1}(s_k^0)^{P_0} + (s_{k-1}^0)^{P_0}(s_k^1)^{P_0} \\
(l = 2) : & \quad (s_{k-1}^0)^{P_1}(s_k^0)^{P_0} + (s_{k-1}^1)^{P_1}(s_k^1)^{P_0} \\
(l = 3) : & \quad (s_{k-1}^1)^{P_2}(s_k^0)^{P_0} + (s_{k-1}^0)^{P_1}(s_k^1)^{P_0} \\
(l = 4) : & \quad (s_{k-1}^0)^{P_2}(s_k^0)^{P_0} + (s_{k-1}^1)^{P_2}(s_k^1)^{P_0} \\
(l = 5) : & \quad (s_{k-1}^1)^{P_3}(s_k^0)^{P_0} + (s_{k-1}^0)^{P_2}(s_k^1)^{P_0} \\
(l = 6) : & \quad (s_{k-1}^0)^{P_3}(s_k^0)^{P_0} + (s_{k-1}^1)^{P_3}(s_k^1)^{P_0} \\
(l = 7) : & \quad (s_{k-1}^1)^{P_0}(s_k^0)^{P_0} + (s_{k-1}^0)^{P_3}(s_k^1)^{P_0}
\end{aligned} \tag{3.40}$$

where $l \in \{0, \dots, L - 1\}$ is the codeword index with $L = 2 \times M = 8$.

The fully analog implementation of the scheme can thus be done based on the above decision functions. In order to illustrate the implementation, a decision variable generate (DVGEN) block diagram is designed. The $(k - 1)$ -th signal needs to be modified in all possible ways handled by the codebook before correlating each with the k -th signal. For this purpose, a permutation generator (PGEN) is built. However, the latter would need different versions of a periodic rect function in order to be able to apply the circular time slot shifts. Hence, the needed periodic rect generator (RGEN) is designed first.

The figure below is the RGEN's block diagram:

Figure 3.11: Periodic rect functions generator

where PRECT is a periodic rect function that is based on the following equation:

$$prect(t) = \begin{cases} 1 & \text{if } rem(t, 4\delta) \leq \frac{1}{4} \\ 0 & \text{elsewhere} \end{cases} \quad (3.41)$$

where $rem(a, b)$ is the remainder of the division of a by b . The above generator would generate six different periodic rect functions using delay blocks of δ and summation blocks. The period of all the functions is equal to 4δ . The first function would yield a one in the first slot of every period. The second function would yield a one in the first two slots. The third would yield a one in the first three slots. The fourth yields a one in the last three slots. The fifth in the last two slots and the sixth in the last slot.

The RGEN is then used in the design of the permutation generator as illustrated in the block diagram shown below. The input of the PGEN component is a certain signal,

Figure 3.12: Permutation generator

which would be managed in order to output four different signals with the usage of delay, multiplication and summation blocks. Within a duration of four time slots, all the possible circular permutations of the four slot delayed version of the signal would be returned. For instance, if $(s_k^0)^{P_0}$ is inputted into PGEN, the outputted signals would be: $(s_{k-1}^1)^{P_0}$, $(s_{k-1}^1)^{P_1}$, $(s_{k-1}^1)^{P_2}$, and $(s_{k-1}^1)^{P_3}$.

Finally, the block diagram that generates the decision variable previously delineated is shown below. Note that the integration should start at the beginning of each of the second sub periods to provide the decision variables at the end of that same second sub period, meaning that the DVGEN would stay idle during the first sub period and all the work would be done during the second sub period, outputting the results at its very end. The corresponding codeword of the decision variable with the maximal value would be the estimated codeword.

Figure 3.13: Decision Variables generator

Chapter 4

Results

4.1 Simulation Settings

The second derivative of the Gaussian Pulse was chosen to be used as the shape of the pulse waveform $w(t)$ with a duration of 0.5 ns. The PQ sub-channels are generated independently using the standard IEEE 802.15.3a channel model CM2 corresponding to the non line of sight (NLOS) conditions, and thus, the transmit and the receive arrays should be adequately spaced. The extended M-PPM modulations used are orthogonal with $\delta = 100$ ns and with a frame time of $T_s = L\delta$.

The first set of results compares the proposed scheme with the Repetition Coding (RC) scheme, whereas the second set compares the different orders of the scheme in different constellation sizes M as well as different integration times T_i .

The RC scheme used is based on the M-PPM constellation and uses P transmit antennas with $P = 2$. Within the same symbol period T_s , the same data stream will be emitted twice, once during every sub period T_{ss} . The first transmit antenna would emit the symbol data during the first sub period remaining idle during the second sub period, while the second transmit antenna would convey the same data during the second sub period remaining idle during the first sub period. This scheme introduces full spacial diversity to the system, dropping down the error rates.

4.2 Results Evaluation

The experiments done can be decomposed into two sections. The first part would compare the first order of the proposed scalable scheme with the repetition coding scheme, whereas the second part would highlight the performance of the different orders of the proposed scalable scheme.

4.2.1 Scalable Scheme vs Repetition Coding

In this section, the performance gain of the first order scheme over the RC scheme will be shown. A number of simulations are done for different constellation sizes and different integration times.

The integration time is set to have one of the two values $T_i = 1$ ns and $T_i = 5$ ns, whereas the constellation sizes picked are 3-PPM, 4-PPM and 8-PPM.

After testing the two codes under the proposed constraints, it is shown that there is a 3-dB difference in SNR as an advantage for the proposed scheme of the first order over the RC scheme.

4.2.2 Performance of the Scalable Scheme's Different Orders

In this section, the different orders of the same scalable scheme will be tested. We show the error rate variations for a set of different orders. Again, different integration times and constellation sizes will be used.

The integration time is set to have one of the two values $T_i = 1$ ns and $T_i = 5$ ns, whereas the constellation sizes picked are 3-PPM, 4-PPM and 8-PPM.

As highlighted previously in this paper, the rate of the proposed scalable scheme is $\log_2(\Theta L)/N_p T_s$ bits/s, where Θ is the order of the scheme. The gain that can be achieved from the lowest order to the highest order scheme for any constellation size is

Figure 4.1: Performance of RC vs Order 1 Scheme for 3-MPPM with $T_i = 1$ ns

Figure 4.2: Performance of RC vs Order 1 Scheme for 4-MPPM with $T_i = 1$ ns

thus limited to:

$$\frac{\log_2 2^{n-1} - 1 + \log_2 2^n}{\log_2 2^n} < \frac{2n-1}{n} < 2 \quad (4.1)$$

Figure 4.3: Performance of RC vs Order 1 Scheme for 8-MPPM with $T_i = 1$ ns

Figure 4.4: Performance of RC vs Order 1 Scheme for 3-MPPM with $T_i = 5$ ns

For instance, in the case of the 4-PPM constellation, there's a gain of 1.52 in data rate from the lowest order scheme to the highest order scheme. Whereas, in the case of the

Figure 4.5: Performance of RC vs Order 1 Scheme for 4-MPPM with $T_i = 5$ ns

Figure 4.6: Performance of RC vs Order 1 Scheme for 8-MPPM with $T_i = 5$ ns

8-PPM constellation, there's a gain of 1.70 from the lowest to the highest order scheme. Note that this gain is provided at the cost of a loss of in SNR as an advantage to the

lower order scheme as shown in the displayed results.

Figure 4.7: Performance of the different orders for 3-MPPM with $T_i = 1$ ns

Figure 4.8: Performance of the different orders for 4-MPPM with $T_i = 1$ ns

Figure 4.9: Performance of the different orders for 8-MPPM with $T_i = 1$ ns

Figure 4.10: Performance of the different orders for 3-MPPM with $T_i = 5$ ns

Figure 4.11: Performance of the different orders for 4-MPPM with $T_i = 5$ ns

Figure 4.12: Performance of the different orders for 8-MPPM with $T_i = 5$ ns

Chapter 5

Conclusion

This thesis presented a novel unipolar space time code for ultra wideband communications. The scheme described benefits from a fully analog implementation structure, while still securing full spatial diversity. Its differential and unitary nature allows it to handle the received signals without dealing with the challenges and difficulties of channel estimation. Yet, it ensures optimal detection through the use of numerous correlation variables that form sufficient statistics. Besides that, the proposed system further provides scalability options, allowing for increases in data rates at the cost of decoding complexity.

Compared to the repetition coding system, the proposed scalable scheme, with the different advantage it provides, made it possible to achieve higher performance levels for the usage of two transmitters. The multiple experiments executed, highlighted that higher order schemes would provide a higher data rate at the cost of probability error complexity and decoding complexity.

Since the proposed scheme only handles the usage of two transmitters, a possible direction for future work would be extending the proposed scalable scheme to handle the usage of any number of transmit antennas. Another direction would be further increasing the data rate. This could be done by increasing the size of the proposed codebooks while keeping a fully diverse system. Other permutation matrices could be proposed that would allow for larger set of families or a larger number of codewords within each family. Finally, an alternative improvement would be reducing the complexity of the

higher order schemes' analog implementation.

Appendix A

Diversity

The purpose of this appendix is to prove that the proposed scalable scheme is fully diverse for any order Θ . For this, what has to be proven is that the resulting difference of any two of the codebook's codewords has a full rank, or in this scheme's case, a rank of 2. In other words, the following equation has to be verified:

$$\begin{aligned} \text{rank}(C_{\theta,m} - C_{\theta',m'}) &= 2 \\ \forall \theta, \theta' &\in \{0, \dots, \Theta - 1\}, \\ m, m' &\in \{0, \dots, M - 1\}, \\ \text{with } \theta &\neq \theta' \text{ or } m \neq m' \end{aligned} \tag{A.1}$$

This equation covers all of the possible orders of the proposed scheme, where θ which indicates the order, can take any of the possible order values. $C_{\theta,m}$ can take one of the following two different types:

Type 1 which covers the codeword formats where the first transmit antenna conveys its data stream in the first sub period and the second transmit antenna conveys its data stream in the second sub period with no difference between the two data streams:

$$\mathcal{C} = \frac{1}{\sqrt{\theta + 1}} \times \begin{bmatrix} U & \Theta_M \\ \Theta_M & U \end{bmatrix} \tag{A.2}$$

Type 2 which covers the codeword formats where the first transmit antenna conveys its data stream in the second sub period and the second transmit antenna conveys its

data stream in the first sub period with a single time slot shift between the two data streams:

$$\mathcal{C} = \frac{1}{\sqrt{\theta+1}} \times \begin{bmatrix} \Theta_M & V \\ U & \Theta_M \end{bmatrix} \quad (\text{A.3})$$

where the time slot shift between the two transmit antenna's data stream is covered in the two matrix variables U and V , where $U = \sum_{i=0}^{\theta} \Omega^{m+i} e_1$, and $V = \sum_{i=0}^{\theta} \Omega^{m+i+1} e_1$.

Let's start first with the case where $C_x, C_y \in \text{Type 1}$:

$$C_x - C_y = \frac{1}{\sqrt{\theta+1}} \times \begin{bmatrix} \underbrace{U - U'}_{v_1} & \underbrace{\Theta_M}_{v_2} \\ \Theta_M & U - U' \end{bmatrix} \quad (\text{A.4})$$

where $U' = \sum_{i=0}^{\theta'} \Omega^{m'+i} e_1$, and $V' = \sum_{i=0}^{\theta'} \Omega^{m'+i+1} e_1$.

To prove that the above matrix has a rank of 2, we need to prove that there is no relation whatsoever between the two vectors of the matrix v_1 and v_2 .

This comes down to verifying the following equation:

$$c_1 v_1 + c_2 v_2 = 0 \Rightarrow c_1 = c_2 = 0 \quad (\text{A.5})$$

which would lead to:

$$\begin{aligned} c_1(U - U') &= \Theta_M \\ c_2(U - U') &= \Theta_M \end{aligned} \quad (\text{A.6})$$

The above equation is satisfied in one of two different cases. Either $c_1 = 0$ and $c_2 = 0$ or $U = U'$. However, U is a vector of θ consecutive ones and U' is a vector of θ' consecutive ones, with $\theta, \theta' \geq 1$. For this, U and U' can only be equal if and only if they have the same number of consecutive ones and at the exact same positions. This translates to,

$$U = U' \Leftrightarrow \begin{cases} m = m' \\ \theta = \theta' \end{cases} \quad (\text{A.7})$$

implying that $c_1 = c_2 = 0$ and consequently, the first case's proof is covered since it leads to a rank of 2.

Let's now consider the second case where $C_x, C_y \in \text{Type 2}$:

$$C_x - C_y = \frac{1}{\sqrt{\theta+1}} \times \begin{bmatrix} \underbrace{\Theta_M}_{v_1} & \underbrace{V - V'}_{v_2} \\ U - U' & \Theta_M \end{bmatrix} \quad (\text{A.8})$$

To prove that the above matrix has a rank of 2, we need to prove that there is no relation whatsoever between the two vectors of the matrix v_1 and v_2 .

This comes down to verifying the following equation:

$$\begin{aligned} c_1(U - U') &= \Theta_M \\ c_2(V - V') &= \Theta_M \end{aligned} \quad (\text{A.9})$$

The above equation is satisfied in one of two different cases. Either $c_1 = 0$ and $c_2 = 0$ or $U = U'$ and $V = V'$. However, U and V are vectors of θ and $\theta + 1$ consecutive ones respectively, and U' and V' are vectors of θ' and $\theta' + 1$ consecutive ones respectively, with $\theta, \theta' \geq 1$. For this, U, U' and V, V' can only be equal if and only if they have the same number of consecutive ones and at the exact same positions. This translates to,

$$\begin{aligned} U &= U' \\ V &= V' \end{aligned} \Leftrightarrow \begin{cases} m = m' \\ \theta = \theta' \end{cases} \quad (\text{A.10})$$

implying that $c_1 = c_2 = 0$ and consequently, the second case's proof is covered since it also leads to a rank of 2.

Finally, we consider the last case where $C_x \in \text{Type 1}$ and $C_y \in \text{Type 2}$:

$$C_x - C_y = \frac{1}{\sqrt{\theta+1}} \times \begin{bmatrix} \underbrace{U}_{v_1} & \underbrace{-V'}_{v_2} \\ -U' & U \end{bmatrix} \quad (\text{A.11})$$

Analogously to the two previous cases, to prove that the above matrix has a rank of 2, we need to prove that there is no relation whatsoever between the two vectors of the matrix v_1 and v_2 .

This comes down to verifying the following equation:

$$\begin{aligned} c_1 U &= -c_2 V' \\ c_2 U &= -c_1 U' \end{aligned} \tag{A.12}$$

The above equation is satisfied in one of two different cases. Either $c_1 = 0$ and $c_2 = 0$ or $U = \alpha V'$ and $U = \alpha U'$. However, U is a vector of θ consecutive ones, and U' and V' are vectors of θ' and $\theta' + 1$ consecutive ones respectively, with $\theta, \theta' \geq 1$. For this, U, V' and U, U' can only be proportional if and only if they have the same number of consecutive ones and at the exact same positions. This translates to,

$$U = V' \Leftrightarrow \begin{cases} m = m' + 1 \\ \theta = \theta' \end{cases} \tag{A.13}$$

and,

$$U = U' \Leftrightarrow \begin{cases} m = m' \\ \theta = \theta' \end{cases} \tag{A.14}$$

implying that $c_1 = c_2 = 0$ and consequently, the third case's proof is covered since it also leads to a rank of 2.

This covers all the different combination of codewords that can be picked from the scheme's codebook of any given order. Thus, it is proven that the proposed scheme is fully diverse.

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